A STUDY OF HEAT TRANSFER BY TURBULENCE

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Abstract

This paper consists of four main divisions. The first of these is a discussion of turbulence in a flowing fluid and of how heat and momentum are transferred thereby. The second is a presentation of the problems met in constructing the apparatus with which to measure the pertinent quantities connected with turbulent transfer. The third is a description of the method of calibration of the measuring instruments. The fourth is a presentation of the experimental results obtained; these results, however, are so limited in quantity that they serve merely as an indication of those to be expected in the future.

The Theoretical Aspect

W. F. Durand (1)* states, "The present position of the theory of heat transfer in eddying flow is very unsatisfactory. It appears quite certain that the exchange of quantities having scalar properties, such as heat energy, does not always obey the same law as the exchange of a quantity having vector properties, such as momentum............" This indicates a lack of knowledge as to how heat and momentum are transferred by a turbulently flowing fluid. To gain this knowledge one must first have an understanding of what turbulence is.

The hypotheses as to what constitutes turbulence, and the functional relationships that have been used in connection therewith , have undergone two stages of development and are now beginning upon a third. The first of these stages was one of strictly limited knowledge. Recognition was given to the differences between laminar and turbulent flow, but functional relationships were restricted to the use of empirical constants, valid only for limited ranges.

The second stage, however, marked a decided step forward. Osborne Reynolds' work upon the subject, leading to the formulation of the ratio that bears his name, showed that turbulence in a flowing fluid depends upon the balance between inertial and viscous forces. When the former predominate, the flow is turbulent; when the latter predominate, the flow is laminar. Thus, by the formulation of the dimensionless ratio known as the Reynolds number, correlation between different systems under different flow conditions

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* See page 55 for bibliography

was possible. Still, however, the exact mechanism of turbulence remained unknown; within a limited range of values of the Reynolds number, the flow could be either laminar or turbulent, depending upon the absence or presence of disturbances.

Now the third stage of development of knowledge has been entered upon with the hope that the laws which govern turbulent motion can be ascertained. Concommitant therewith it is hoped that relationships can be found that will permit the calculation of velocity and pressure distributions in turbulent flow and of the energy transfers across a turbulent flow without resort to empirical elements.

The essence of turbulence is now known to reside in the fact that the velocity and the pressure at a given point in a turbulent flow are continuously fluctuating in an irregular fashion about a mean value. Both the variation and the irregularity are of prime importance. It is essential that the variations have no definite frequency. However, there is a definite correlation between the components of the velocity fluctuations at a given point; for, without that condition, there could be no transmission of shearing stress (or transfer of the momentum which generates shearing stress) across a plane parallel to the direction of mean flow, except for that contributed by laminar friction. Consequently, the fundamental equation of shearing stress along a plane parallel to the mean flow in a region where the viscous forces are inconsequential is:*

$$\tau = -\rho \overline{u'v'} \tag{1}$$

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* For a tabulation of the nomenclature see Appendix 1, page 45

or in terms of heat transferred across this plane:

$$Q = c\rho\,\overline{\theta' v'} \qquad (2)$$

The evaluation of these fluctuations in terms of other properties of the flowing system requires further consideration. The mechanism of the transfer of a quantity may be thought of in the following manner. Existing in the turbulent flow are regions of equal potential, where potential may refer either to velocity or to temperture. Also in the turbulent flow are eddies or small gusts that move across the turbulent stream with random motion and exist only momentarily. When one of these eddies forms and leaves a given region, it leaves with the potential of that region associated with it. When it has completed its motion, it finds itself, in general, in a region of different potential from that of the region from which it came. As a consequence, its potential must be equalized with that of its surroundings, either by its giving up momentum (or heat) if it has an excess, or by absorbing momentum (or heat) if it is deficient in this quantity. This continuous formation, movement, and dissolution of eddies is the mechanism by which turbulence effects a transfer of a quantity. The component, perpendicular to the direction of mean flow, of the average distance an eddy travels before it loses its identity by transferring its excess potential to its surroundings has been given the name "mixing length", here denoted by L. This general idea is subscribed to by most theorists upon the mechanism of transfer by turbulent flow. However, the evaluation of this mixing length in terms of other properties of the flow has received varied treatment.

L. Prandtl (2) has proposed that the essence of the transfer is that of momentum in such a manner that momentum is conserved in the direction of mean flow. Using this concept it seems justifiable to consider u' as being proportional to the product of the mixing length and the rate of change of velocity with respect to distance perpendicular to the mean flow. Also, as there must be a correlation between the velocity fluctuations at a given point, v' may be set equal to a constant times u'. Thus, absorbing all constants of proportionality into L, Prandtl obtained from equation (1):

$$\tau = \rho L^2 (du/dy)^2 \qquad (3)$$

and from equation (2):

$$Q = -c\rho L^2 (d\theta/dy)(du/dy) \qquad (4)$$

G. I. Taylor (3), however, started upon the premise that the essential feature of transfer was that of vorticity under conservation of vorticity. Correspondingly, his equation derived from (1) is:

$$\tau = \rho |v'| L \, d^2u/dy^2 \qquad (5)$$

where L has a different numerical value from that given by the Prandtl equations.

Th. von Karman (4) has started from a different base than either of his predecessors. He desired not to be limited by the assumptions of conservation of momentum or of vorticity and, instead, postulated the following two conditions:

(A) The mechanism of turbulent interchange is independent of viscosity except near walls.

(B) The local flow pattern is statistically similar in the neighborhood of every point, and only the time and length scale vary.

This second condition is equivalent to saying that the different components of the velocity fluctuations are governed by a constant correlation. Using these assumptions, von Karman was led to the same equations as was Prandtl and, in addition, to the following equations, all provided that L is small in comparison with the linear dimensions of the flow:

$$L = K\left|(du/dy)/(d^2u/dy^2)\right| \quad (6)$$

$$L = K\left|(d\theta/dy)/(d^2\theta/dy^2)\right| \quad (6)$$

where K is a universal constant equal to 0.4. Whether these two values of L are equivalent is not explainable by theory alone, and experimental data must be obtained to clarify this point.

The available experimental data are too meager to verify any of the foregoing formulations as to the essence of turbulent transfer. However, as a starting point for the present investigation, let the assumptions of von Karman be taken as the criteria for turbulent transfer.

When consideration is given to transfer from a solid wall to a moving fluid, the simplifications made with regard to derivations in the above theories are no longer applicable; for now the transfer through the boundary layer must be considered, and viscous forces are no longer inconsequential. In this region, the transfer of momentum is equal to the gradient of the velocity in the direction normal to the surface upon which the boundary layer exists multiplied by a constant of proportionality, the coefficient of viscosity. Likewise, the transfer of heat is equal to the corresponding temperature gradient multiplied by a constant of proportionality, the thermal conductivity. Thus, for the complete transfer from wall to flowing fluid, equations (1) and (2) become:

$$\tau = \mu\, du/dy - \rho\, \overline{u'v'} \quad (7)$$

$$Q = -k\, d\theta/dy + c\rho\, \overline{v'\theta'} \quad (8)$$

Reynolds has suggested that turbulent transfer may be expressed in a form analogous to that of laminar transfer by the introduction of the concepts of eddy viscosity and eddy conductivity. Assuming that these two quantities are numerically equal, the above equations may be written:

$$\tau = (\mu + e\rho)\ du/dy \tag{9}$$

$$Q = -(k + ec\rho)\ d\theta/dy \tag{10}$$

or rewritten as :

$$\tau/\rho = (\nu + e)\ du/dy \tag{11}$$

$$Q/c\rho = -(k/c\rho + e)\ d\theta/dy \tag{12}$$

Thus, it may be seen that the transfer of heat is directly proportional to the transfer of momentum if either of the following conditions is satisfied:

(A) ν and $k/c\rho$ are negligible compared to e

(B) ν and $k/c\rho$ are numerically equal; that is, the Prandtl number $= \sigma = \mu c/k = 1$

The first condition is satisfied quite generally in the main body of any turbulent flow, but not near the walls. The second condition is satisfied completely only in the case of an ideal gas, but it is closely approximated by the common gases. For liquids, however, it is not even approximately true.

The ratio τ/ρ has the dimensions of velocity squared; thus, the quantity $2\tau_0/\rho U^2$, where U is some reference velocity, is dimensionless. It is usually designated as the coefficient of friction, C_f. Similarly, $Q/c\rho$ has the dimensions of velocity times temperature. Using a reference temperature Θ, the quantity $Q/c\rho U\Theta$ is dimensionless and may be called the heat transfer number, C_h. Thus:

$$\tau_0 = C_f \rho U^2/2 \tag{13}$$

$$Q = C_h \rho c U \Theta \tag{14}$$

For the case where ν equals $k/c\rho$, as stated in condition (B) above, von Karman (5) has shown that:

$$C_h = C_f/2 \tag{15}$$

However, for the cases where ν is not equal to $k/c\rho$ and where ν and

k/c_p are not negligible compared to e throughout the range of transfer, the approximations allowable in deriving equation (15) no longer are applicable. In this case, von Karman has arrived at the relation:

$$1/C_h = 2/C_f + 5(2/C_f)^{1/2}\left\{\sigma - 1 + \ln\left[1 + 5/6(\sigma - 1)\right]\right\} \qquad (16)$$

If C_f and C_h can be determined from equations (13) and (14), the validity of equations (15) and (16) can be substantiated or refuted. To do this, however, τ_0 and Q must be determined experimentally for a series of values of U and θ. In addition , by the determination of point values of the temperature and velocity at suitably located points in a turbulently flowing stream, the mixing lengths of equations (6) and (6') can be computed and compared. Thus, to proceed further with the study of heat transfer by a turbulently flowing fluid, experimental data are necessary. Consequently, a discussion of the factors which were pertinent to the construction of an apparatus with which the quantities needed for this furtherance could be measured is appropriate.

Construction of the Apparatus

The prime consideration before construction was the decision as to the physical set-up in which the turbulent motion was to be produced and studied. It was decided that the production of a turbulent flow between two parallel plates, being held at different temperatures, in such a manner that the flow would be two-dimensional would yield a maximum amount of information with a minimum number of complicating factors. This goal created the following three main problems: (1) the obtaining of two-dimensional turbulent flow between the parallel plates, (2) the maintenance of the parallel plates at specified temperatures, (3) the arrangement of devices to measure the essential quantities connected with the flow - namely, the temperature and the velocity of the turbulently flowing stream at many known points between the plates, the total quantity of heat being transferred between the plates, and the shearing stress existing in the fluid at the side-wall.

Dimensions of experimental channel. The first problem resolved itself into two simpler ones. The first was to provide the correct width and the correct length of plates to correspond to a given distance between them; that is, the determination of the correct dimensions of a channel in which to make experimental measurements. After study of the data bearing upon this question, it was found that the ratio of length to width to depth should be about 60:18:1. This would give a sufficient length so that the turbulence would be fully adapted to the channel by the time the flow had reached its end; and the high ratio of width to depth would create two-dimensional flow in at least the middle third of a cross-section. The smallest depth with which it would be convenient to work was thought to be 3/4 of an inch. A suitable width of one foot and a length of five feet were then chosen, thus giving the ratios 80:16:1.

Production of the Turbulent Flow. The second part of the first problem was the creation of a proper stream of fluid through the channel. Proper in this case reduced to (1) known quantity, (2) correct bulk temperature, and (3) sufficient propulsion. Since air at bulk temperature of 130°F and a pressure of approximately atmospheric was arbitrarily selected as the initial fluid to be used in these measurements, the question as to quantity reduced to a consideration of the values of the Reynolds number with which it was desired to work. It was felt that if it could be conveniently done, it would be extremely informative if the Reynolds number could be varied from 2000 to 100,000. Under the specified conditions, this meant that the quantity of air should vary from 0.205 cubic feet per second to 10.25 cubic feet per second. This latter figure then gave the maximum capacity of the blower needed to circulate the air - namely, 615 cubic feet per minute. This was the first quantity needed in the complete specification of a blower; in addition, the deliverable static head had to be specified. For a proper determination of this latter quantity, the detailed elements of the path to be followed by the flowing air had to be known, because the frictional losses entailed by this path represented the minimum value of the static head that the blower had to deliver. Any lower value would have provided insufficient propulsion to drive the air throughout the complete circuit, which, of necessity, had to consist of more than the blower and the experimental channel. Consequently, attention had to be turned to those details of path.

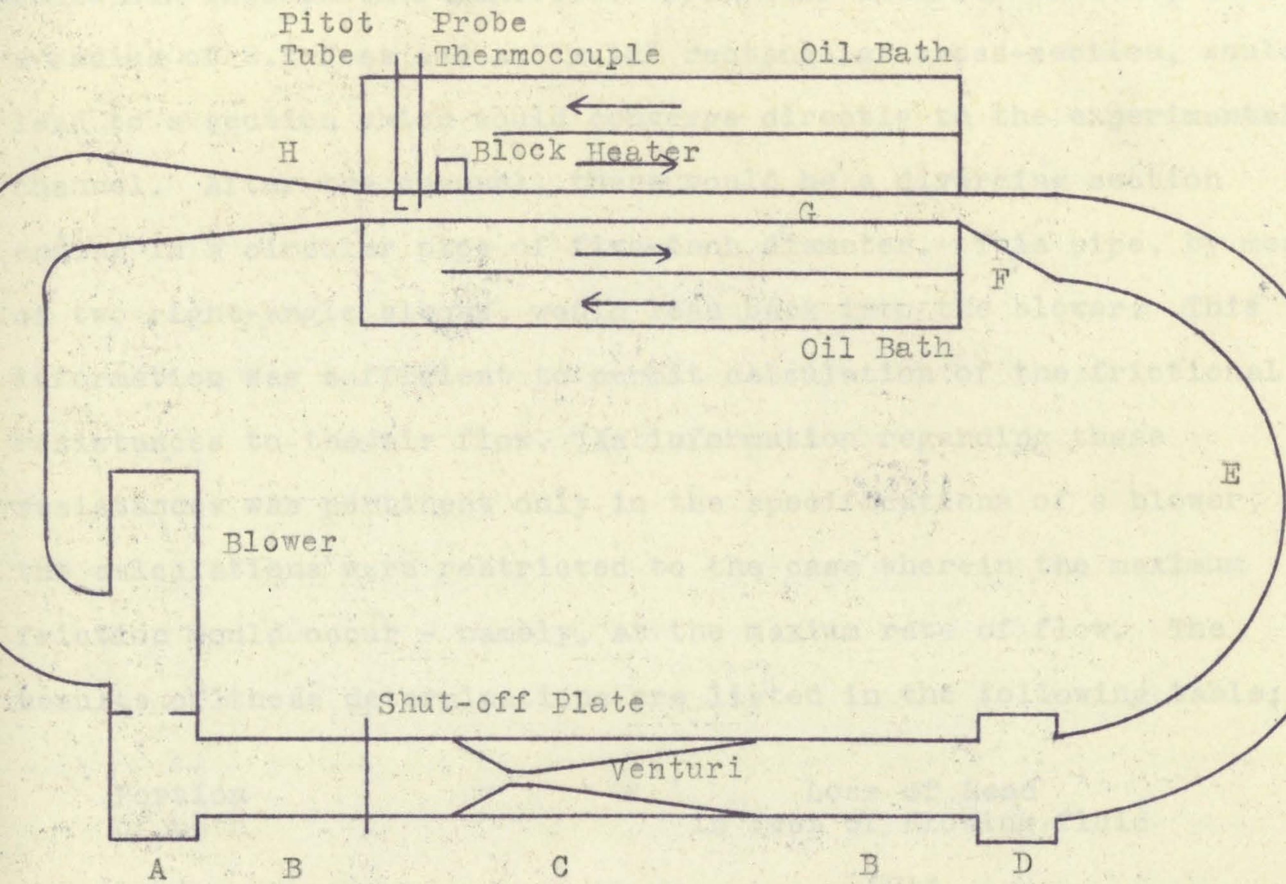
Measurement of Rate of Air Flow. Somewhere in the flow path there had to be a metering device to give a measurement of the total quantity of air that would pass through the experimental channel. It was first thought that an orifice meter would be the proper choice, but, when it was realized how great energy losses due to other factors were going to be, the Venturi meter was selected as having smaller flow resistances. Calculations (details of which appear in Appendix II) showed that three Venturi meters of the sizes tabulated below were necessary in order to measure the desired range of flows.

Nominal pipe-size of Venturi housing (inches)	Actual Pipe Diameter (inches)	Throat Diameter (inches)	Air Flow Rate Minimum (cubic feet/sec)	Air Flow Rate Maximum (cubic feet/sec)
2	2.05	0.66	0.205	0.658
3	3.09	1.21	0.658	2.22
4	3.88	2.39	2.22	9.25

Construction of the Venturi Meters. In the actual construction of the Venturi meters, economy and efficiency dictated the use of a cement form in an iron pipe. This was accomplished by constructing wood forms, consisting of two interlocking pieces, in the shape of the air-cavity in the finished meter. These forms were then greased to prevent the adherence of concrete and inserted in the appropriate pipes. The concrete was then poured in, allowed to set, and the wood forms withdrawn.

Flow Path of the Air. With the knowledge of the number and the kind of metering devices to be employed, the total flow path could be more fully described. (A schematic diagram of this flow path is shown in Figure 1, page 12). The blower would discharge directly into a manifold from which would extend the three pipes containing the Venturi meters. The discharge ends of these pipes, each of which would be equipped with a mechanism to cut it out of the flow circuit,

Figure I


Pitot Tube
Probe Thermocouple
Oil Bath
H
Block Heater
G
F
Oil Bath
I
E
Blower
Shut-off Plate
Venturi
A
B
C
B
D

would run into another manifold. From this a curved conduit, with a radius of 2.5 feet and a 6"x 12" rectangular cross-section, would lead to a section which would converge directly to the experimental channel. After the channel, there would be a diverging section ending in a circular pipe of five-inch diameter. This pipe, by means of two right-angle elbows, would lead back into the blower. This information was sufficient to permit calculation of the frictional resistances to the air flow. As information regarding these resistances was pertinent only in the specifications of a blower, the calculations were restricted to the case wherein the maximum friction would occur - namely, at the maxium rate of flow. The results of these determinations are listed in the following table:

Portion of path	Loss of Head in feet of flowing fluid
A	214
B	123.5
C	138.5
D	214
E	0.64
F	16.7
G	327
H	30.8
I	161
Total	1226.14

This total drop in head corresponded to 15.9 inches of water (the static head at which the blower had to discharge), and this, in turn, required a blower horsepower of 1.54. Thus, the complete blower specifications were: a maximum capacity of 615 cubic feet per minute, delivered at a static head of 16 inches of water, and requiring at least one and one-half horsepower to drive.

Bulk Air Temperature. In connection with the first main problem, there remained at this point only the question of maintaining the air at the desired bulk temperature of 130°F while passing through the experimental channel. It seemed likely that, owing to the friction developed and to the recirculation of the air between the heated plates, an equilibrium would be established so that the air would enter the experimental channel at a temperature sufficiently high to obviate the necessity of a pre-heater. But only experimental testing could show the validty of this assumption, since the factors which would affect the heat losses from such a system would develop into equations whose complexity would out-weigh their utility.

Control of the Temperature of the Parallel Plates. The second main problem was the maintenance of the parallel plates, which form the top and bottom of the channel, at specified temperatures. This could best be done by using a good heat conductor for the plates, and an insulating material for the sides. The materials selected were, respectively, copper and wood. It was desired to maintain the temperature of each plate constant to within 0.05°F uniformly along the length; and, to do this, circulating oil baths in direct contact with the plates were necessary.

Circulation of the Oil. These considerations again brought up their more specific problems, which were: (1) proper circulation of the oil, (2) proper heating of the oil, (3) measurement of the temperature of the plates. The first of these was solved by the installation of two axial-flow pumps, which were designed and constructed (in accordance with the calculations given in Appendix III) to give a volumetric rate of flow of 0.175 cubic feet of oil per second when running at 1000 RPM.

Heating of the Oil. In order to provide for the proper heating of the oil, each bath was equipped with three heaters of 500 watt capacity (as specified in Appendix IV), two being externally mounted and one immersed. One of the external heaters was designed solely to help bring the bath to temperature, whereas the other external unit and the immersion heater were to keep the bath at the desired temperature under the equilibrium condition. For this reason, one external heater on each bath was connected across a Variac so that the potential applied could be varied from 0 to 130 volts. Thus, by experimental test, the proper voltage could be determined so that this heater could be left on continuously to supply the heat that was withdrawn during the flow. The immersion heater was connected through a relay system to a mercury regulator which would throw this heater off when the oil became too hot, and on when it became too cold. Thus, at the equilibrium condition, one external heater would be off, one external heater would be on at some voltage lower than 130, and the immersion heater would be going on and off at 40 second intervals (the delay period of the relays).

Measurement of Point Velocities. There remained then only the problem of the provision of the measuring devices. The first of these to be considered was the one which was to measure the velocity of the turbulent flow at various points between the parallel plates. It was decided to accomplish this measurement by the location of a Pitot, or impact, tube of extremely small diameter (0.0176 inches) in the flowing stream. If this tube were connected to one arm of a manometer, the pressure exerted upon the surface of the liquid therein would equal the sum of the impact pressure and of the static pressure at the point where the Pitot tube was located. If now it were assumed that the static pressure was constant across the width of the channel, the connection of a static pressure tap in the side-wall of the channel to the other arm of the manometer would produce upon the surface of the liquid therein only the static pressure. The difference in the level of the liquid in the two arms of the manometer multiplied by the density of the liquid would, therefore, be a direct measure of the impact pressure. Dividing the impact pressure by the density of the flowing air would give the velocity head, from which the point velocity of the air stream could be determined by the use of the fundamental equation of a Pitot tube:

$$u = c' \sqrt{2gH'} \qquad (17)$$

where c' could be assumed as unity for the flow under consideration.

Measurement of the Point Temperature. It was decided to determine the point temperature from the electromotive force developed between the two junctions of a copper-constantan thermocouple (hereafter referred to as the probe thermocouple), the one junction being placed at the point in question and the other being immersed in an ice bath in order to obtain a reproducible reference temperature. This electromotive force could then be directly measured by a potentionmeter; and, from a knowledge of the relationship between the electromotive force developed and temperature difference existing between the junctions, the latter could be calculated. The smallest wires with the required tensile strength were desired for the construction of the probe thermocouple. This necessitated the use of copper wire having a diameter of 0.0037 inch and of constantan wire of 0.0026 inch. To measure the temperature of each copper plate, one junction of a copper-constantan thermocouple was imbedded therein, the other junction being placed in an ice bath.

Determination of Location of the Pitot tube and of the Probe Thermocouple. For the adjustment of the position of the Pitot tube and that of the probe thermocouple, each of which was designed to move only vertically in the center of the channel, each was equipped with a micrometer screw. Thus, the exact position of either with respect to the plates could be determined from the micrometer reading. In this connection, the sag of the thermocouple, which is 0.003 of an inch at the junction, as well as the diameter of the wires and of the Pitot tube, would have to be taken into account.

Measurement of Heat Transferred. The next part of this problem was how to measure the total quantity of heat that would be transferred from the top plate to the bottom plate. To determine this value, a heater in the form of a cylindrical block was provided. This fitted into a hole in the top copper plate in such a way that the surface exposed to the flowing air was perfectly smooth. This block was insulated from the rest of the plate by a bakelite ring and from the oil by a metal cylinder and a Dewar flask filled with wax. Then, by a measurement of the amount of heat needed to keep this block at the same temperature as the rest of the top plate (temperature equality being indicated by a differential thermocouple) and from a knowledge of the area of the surface of the block, the heat transferred per unit area of the plate could be calculated.

Measurement of Shearing Stress. How to measure the shearing stress in the flowing fluid at the side-wall presented itself as the final problem in the construction of the apparatus. This problem became greatly simplified when cognizance was taken of the fact that:

$$\tau_0 = -m \, dP/dx \tag{18}$$

Thus, to determine τ_0 only the rate of change of the static pressure in the direction of flow would be needed. This could be measured by placing a pressure tap at each end of the channel and measuring the pressure drop from end to end with a manometer.

Calibration of Apparatus

Following the completion of the construction of the apparatus, the calibration of all measuring devices was undertaken. In this regard, only the thermocouples and the Venturi meters needed consideration.

Calibration of the Thermocouples. Because of the inaccessibility of the lower plate in the finished apparatus, only the probe thermocouple and the thermocouple in the top plate could be calibrated directly against a standard. This was accomplished by immersing the standard thermometer in the oil of the top bath until it was in direct contact with the copper plate (actually in a depression in the plate). Meanwhile the probe thermocouple was raised until its junction was in practically direct contact with the upper plate. Simultaneous readings of the standard and the two thermocouples were then made. The probe thermocouple was then lowered until it touched the bottom plate, and simultaneous readings were taken of it and the lower-plate thermocouple. This procedure was repeated at several different temperatures, the upper and lower plates being maintained at constant and substantially equal temperatures during each set of readings. Thus, there was a chance for only a negligibly small error in the calibration of the probe and lower-plate thermocouples, since the air between the copper plates could not vary appreciably from their temperature as no air was allowed to flow during this calibration.

Calibration Curves for the Thermocouples. Because of the small range of temperature involved, the plot of electromotive force in microvolts versus temperature difference between the junctions in $^{\circ}$F. is a straight line for each of the thermocouples. The calibration curves for the three thermocouples are shown as Figures II to IV.

Calibration of the Venturi Meters. If the throats of the Venturi meters could be accurately measured, it would be unnecessary to calibrate them; for, as is seen from the fundamental equation of a Venturi meter:

$$q = c' M \sqrt{2gH} \qquad (19)$$

$$M = A_1 / \sqrt{(D_1/D_2)^4 - 1} \qquad (20)$$

every quantity, with the possible exception of c', would be known, and sufficient data are available that values of c' could be predicted without serious error. However, it was impossible to measure accurately the throats of the completed Venturi meters, and it was inadvisable to employ the diameter of the wood forms used in their construction as correct. Consequently, the required calibration was accomplished while the meters were in position in the apparatus, and before the divergent section was connected to the end of the experimental channel, by taking Pitot tube readings at various points across the cross-section of the channel. For several different distances from the side-wall, velocities were taken at a sufficient number of points in a vertical plane so that a plot of velocity versus distance from the top plate would give a smooth curve. The areas under these curves were then integrated, and the integrated values plotted against distance from the side-wall. The area under this curve, which was volumetric rate of flow, was then determined. Then,by means of the Venturi equation (19), and by assuming a discharge coefficient of 0.98, the value of M was calculated, and then the diameter of the throat of the meter. The calibrations, sample curves of which will be found as Figures V to XII, yielded these results:

Venturi Meter	Actual Throat Diameter (inches)	Throat Diameter of wood form (inches)
3"	1.18	1.21
4"	2.41	2.39

The two-inch Venturi meter was not calibrated because the multiplying manometer needed for the reading of Pitot tube differentials at these low volumetric rates of flow was not yet available.

Figure II

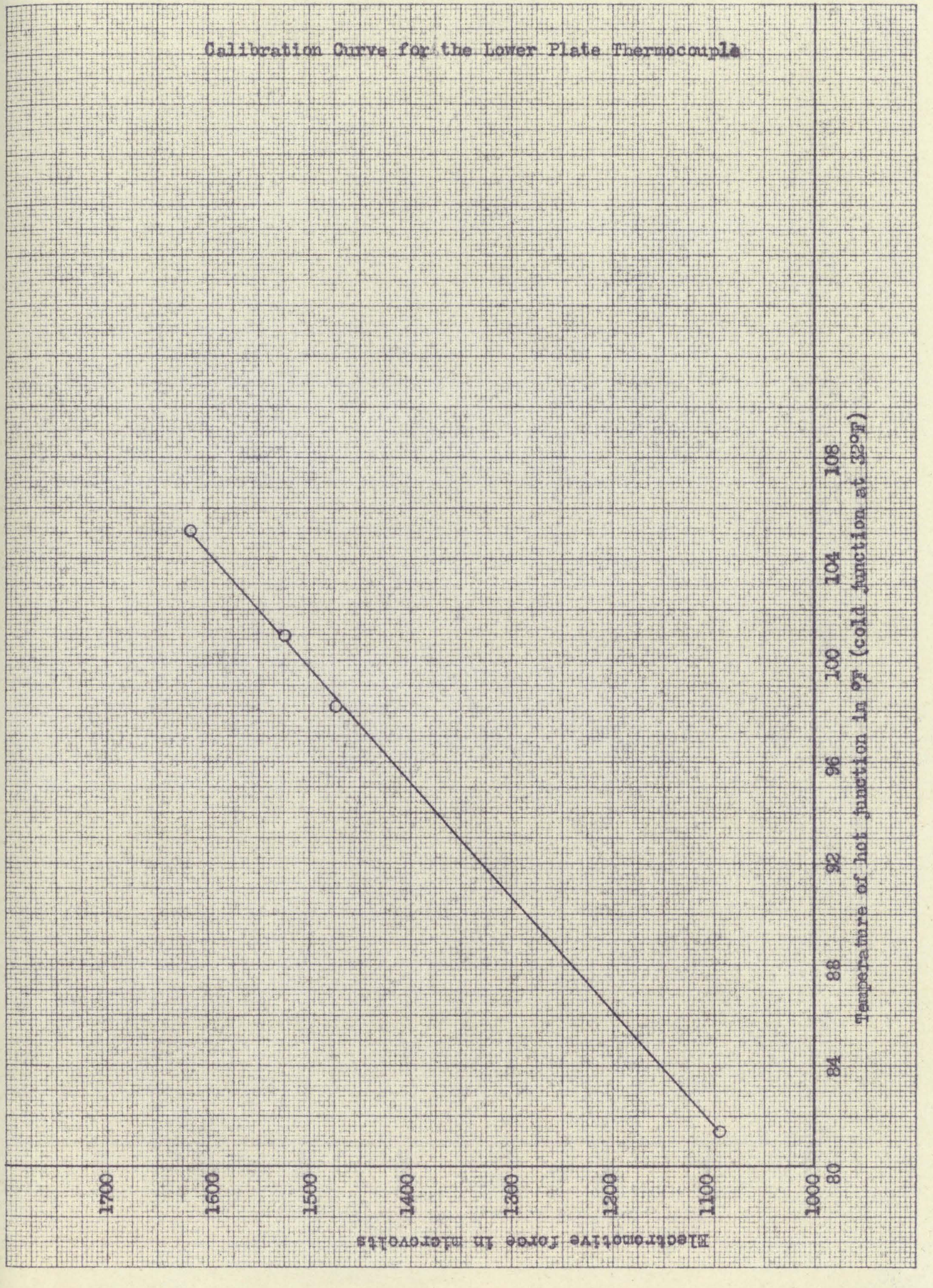
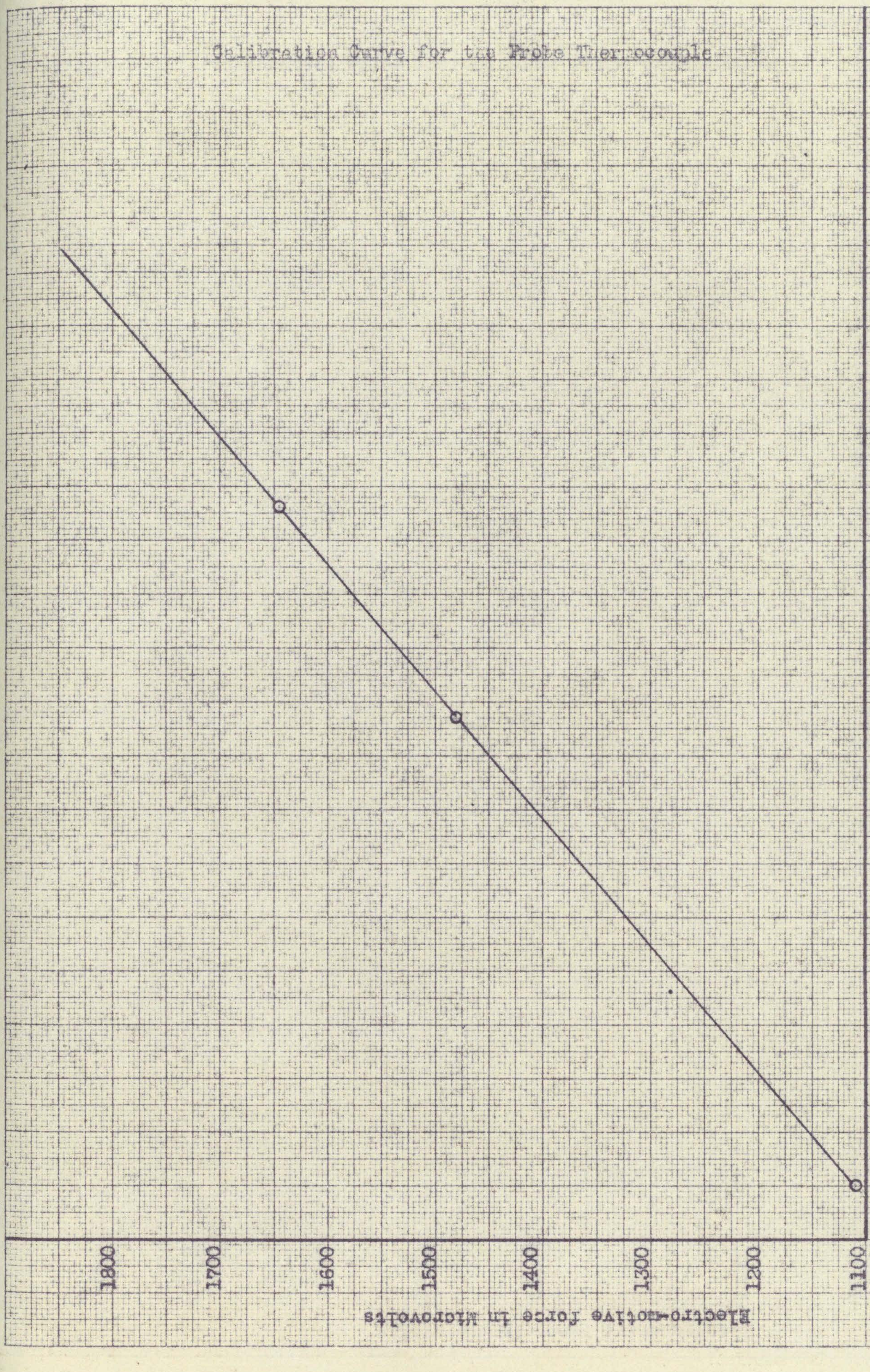

Calibration Curve for the Lower Plate Thermocouple
Temperature of hot junction in °F (cold junction at 32°F)
80
84
88
92
96
100
104
108
Electromotive force in microvolts
1000
1100
1200
1300
1400
1500
1600
1700

Figure III


Calibration Curve for the Probe Thermocouple
Temperature of hot junction in °F (cold junction at 32°F)
80
85
90
95
100
105
110
115
120
Electro-motive force in Microvolts
1100
1200
1300
1400
1500
1600
1700
1800

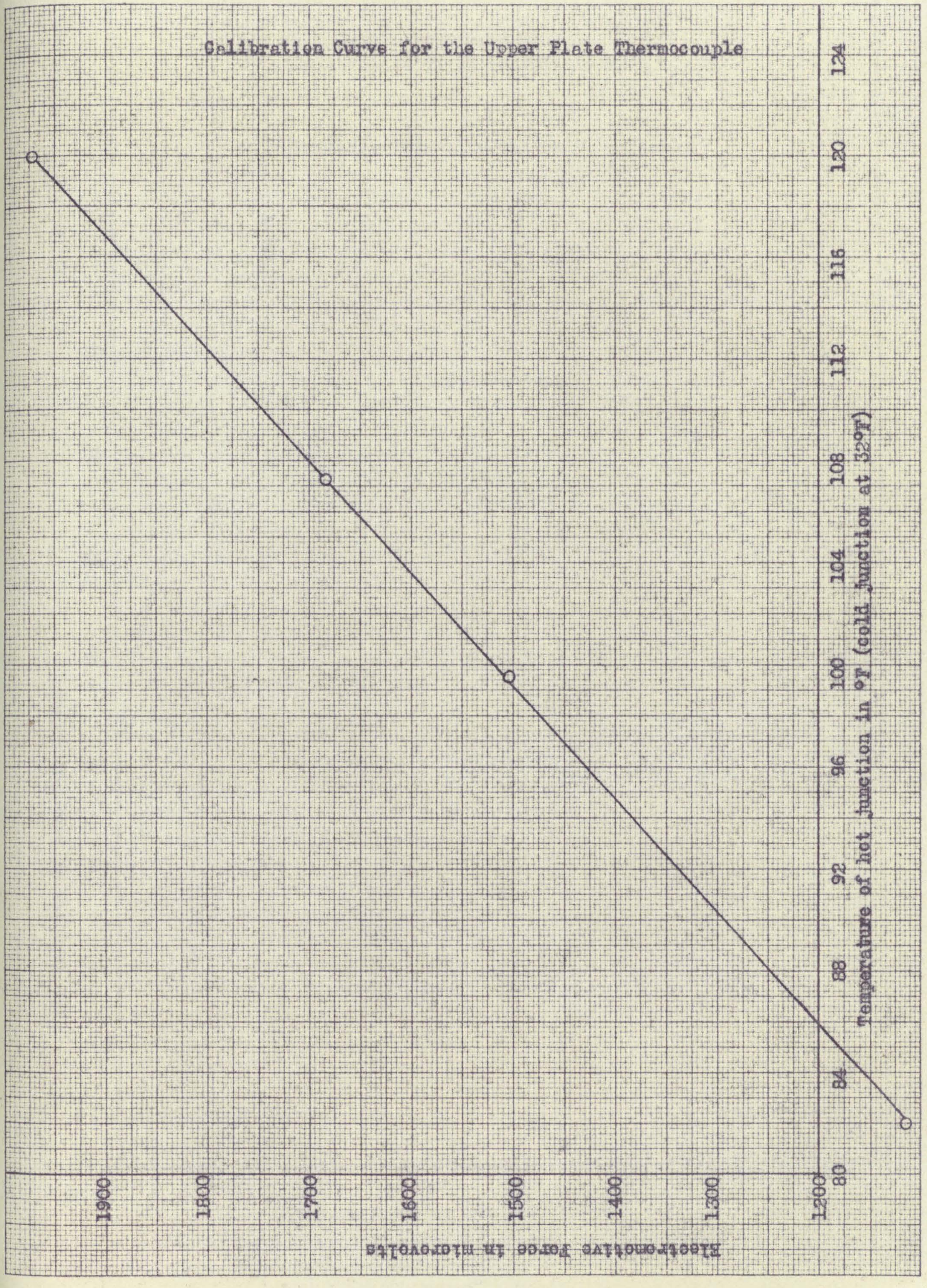
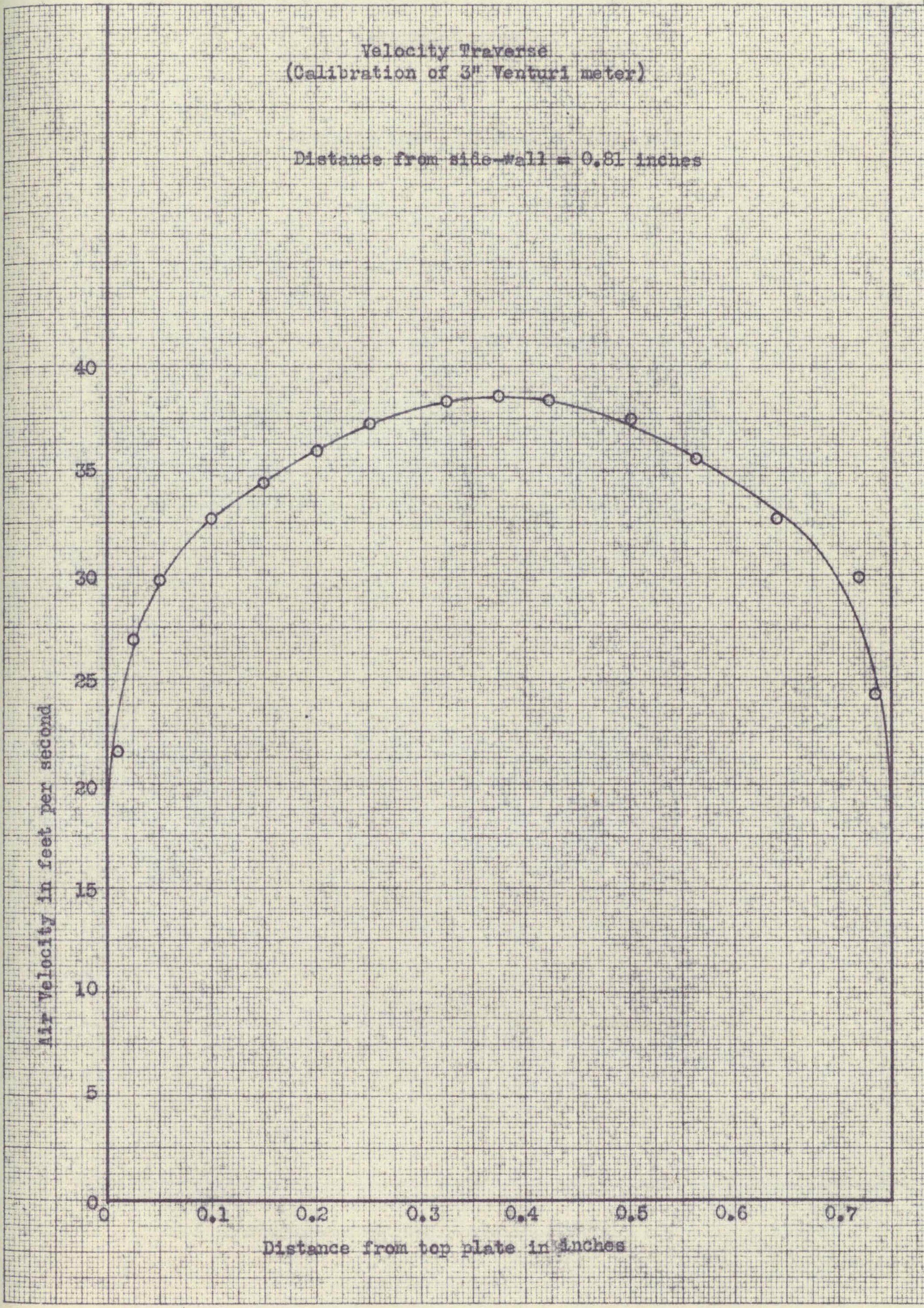

Calibration Curve for the Upper Plate Thermocouple
Temperature of hot junction in °F (cold junction at 32°F)
80
84
88
92
96
100
104
108
112
116
120
124
Electromotive Force in microvolts
1200
1300
1400
1500
1600
1700
1800
1900

Figure V


Velocity Traverse
(Calibration of 3" Venturi meter)
Distance from side-wall = 0.81 inches
Air Velocity in feet per second
40
35
30
25
20
15
10
5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches

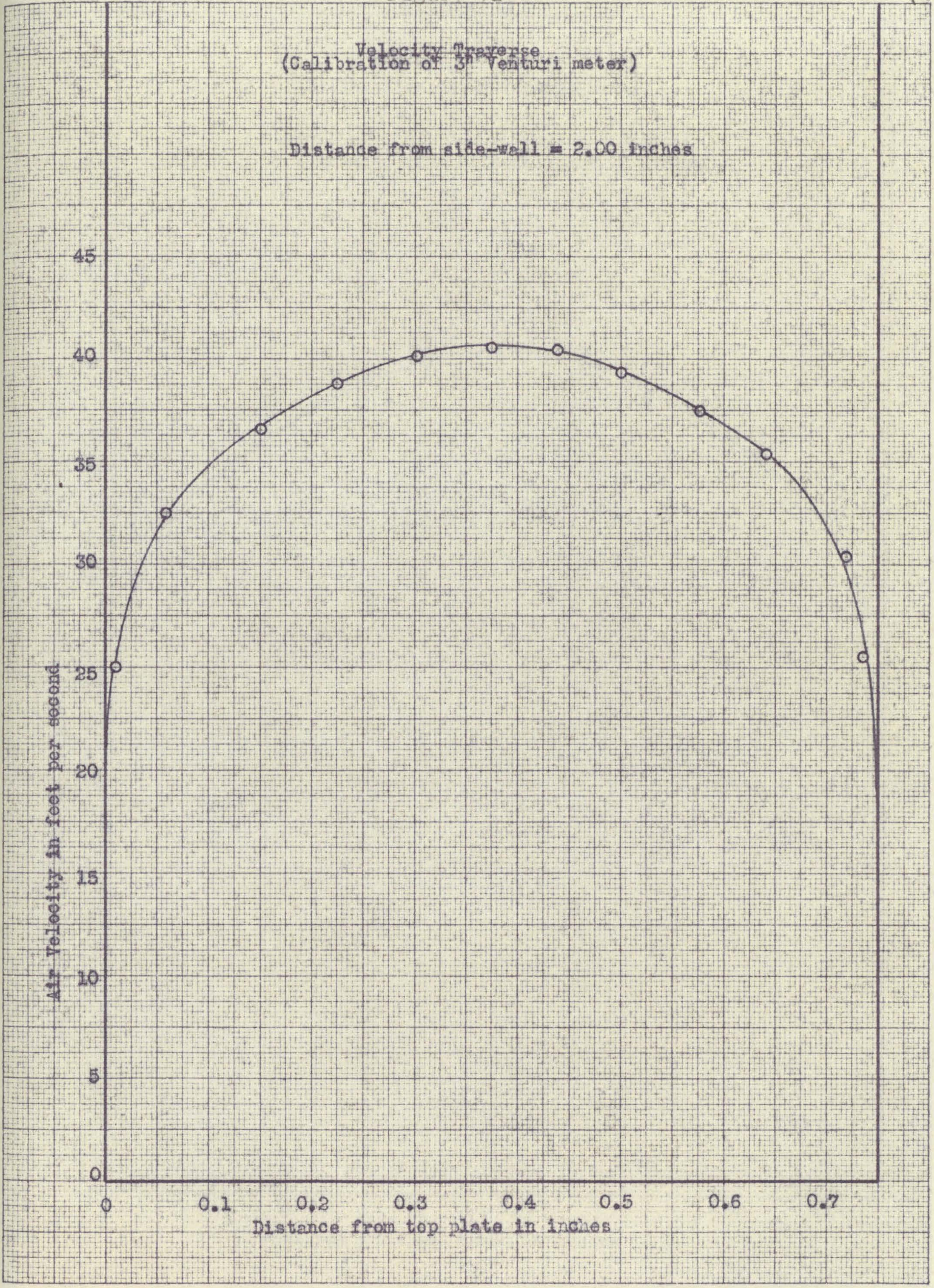

Velocity Traverse
(Calibration of 3" Venturi meter)
Distance from side-wall = 2.00 inches
Air Velocity in feet per second
0
5
10
15
20
25
30
35
40
45
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches

Figure VII

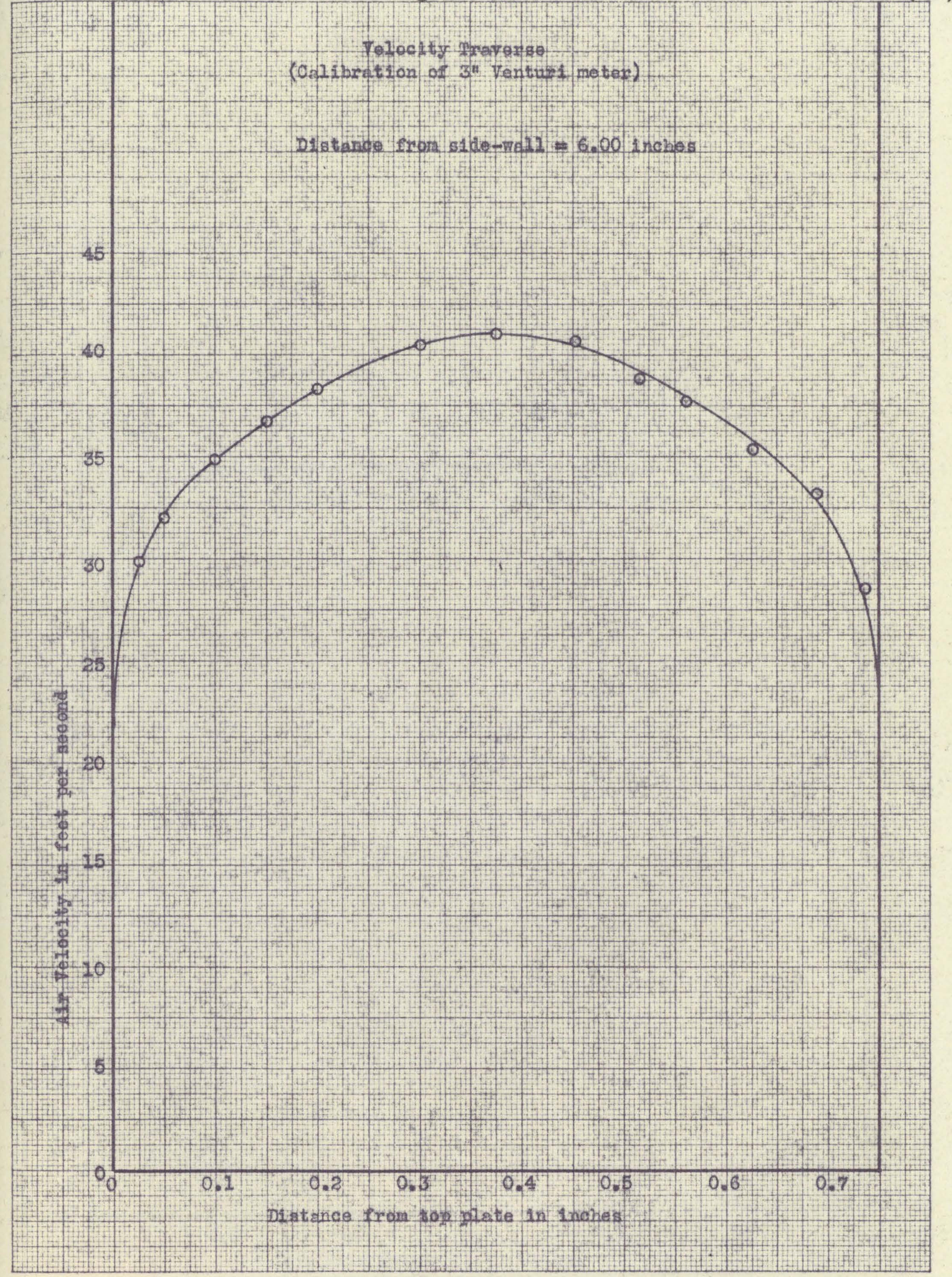
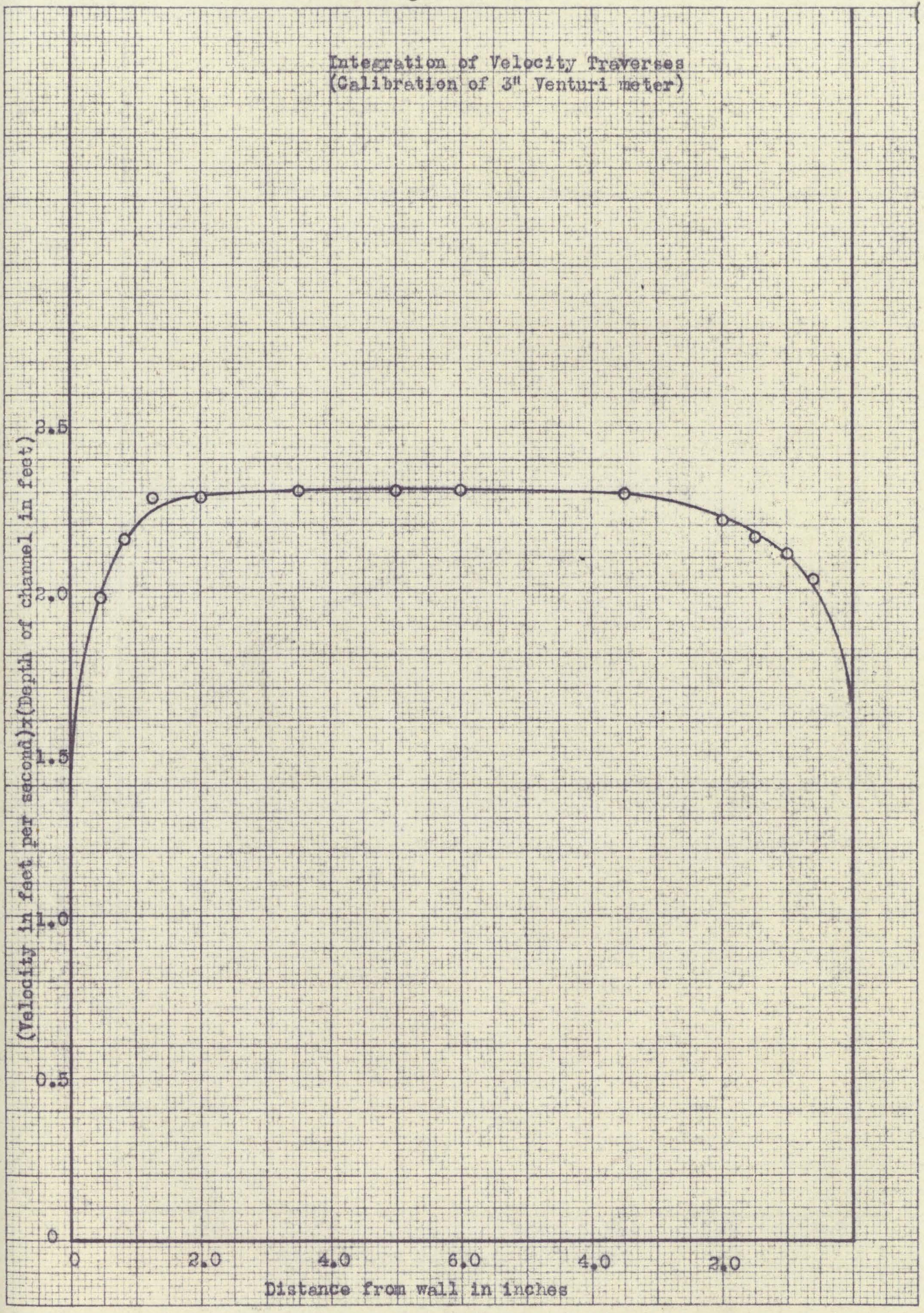

Velocity Traverse
(Calibration of 3" Venturi meter)
Distance from side-wall = 6.00 inches
Air Velocity in feet per second
45
40
35
30
25
20
15
10
5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches

Figure VIII


Integration of Velocity Traverses
(Calibration of 3" Venturi meter)
(Velocity in feet per second)x(Depth of channel in feet)
2.5
2.0
1.5
1.0
0.5
0
0
2.0
4.0
6.0
4.0
2.0
Distance from wall in inches

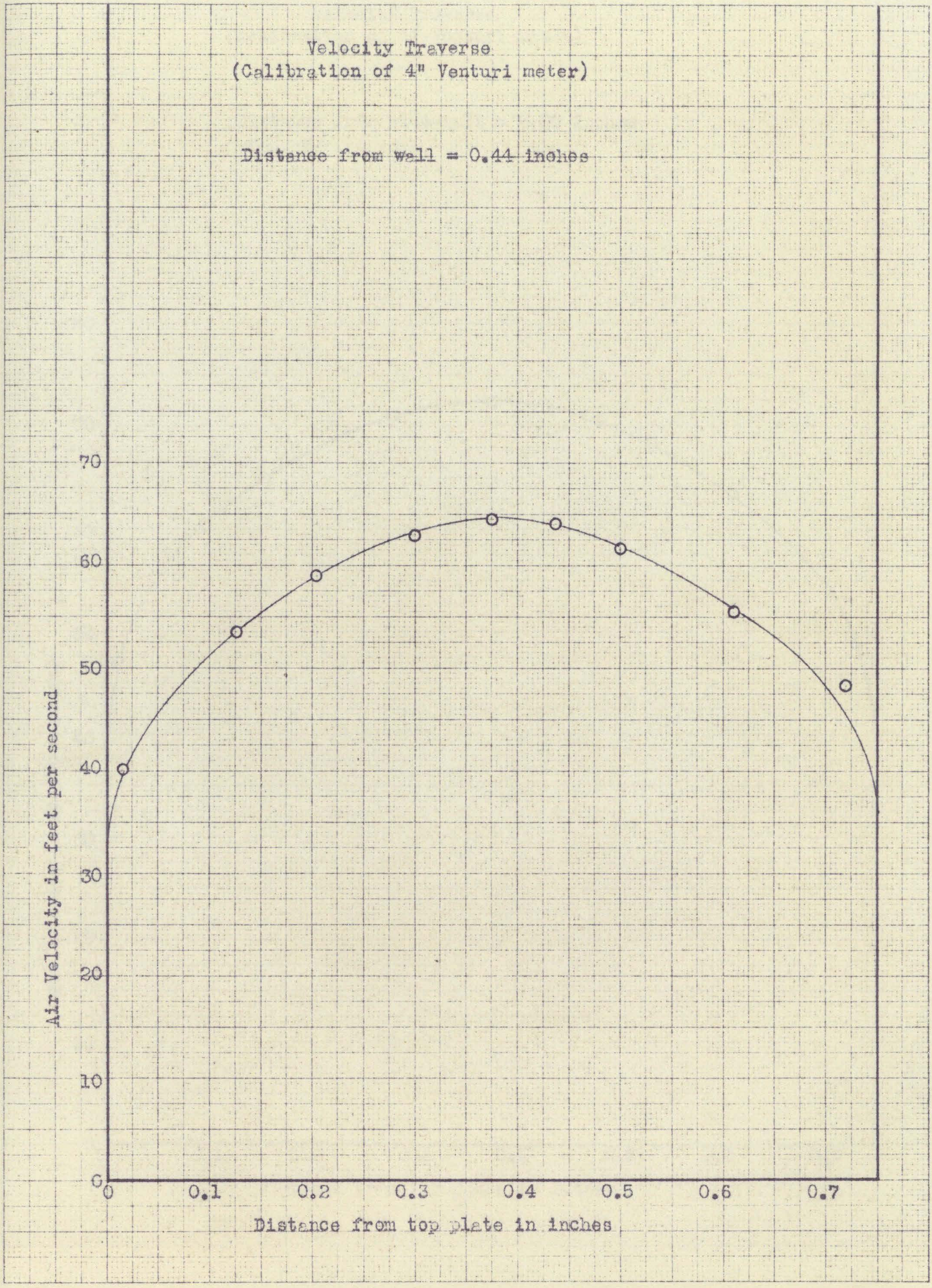

Velocity Traverse
(Calibration of 4" Venturi meter)
Distance from wall = 0.44 inches
Air Velocity in feet per second
0
10
20
30
40
50
60
70
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches

Figure X

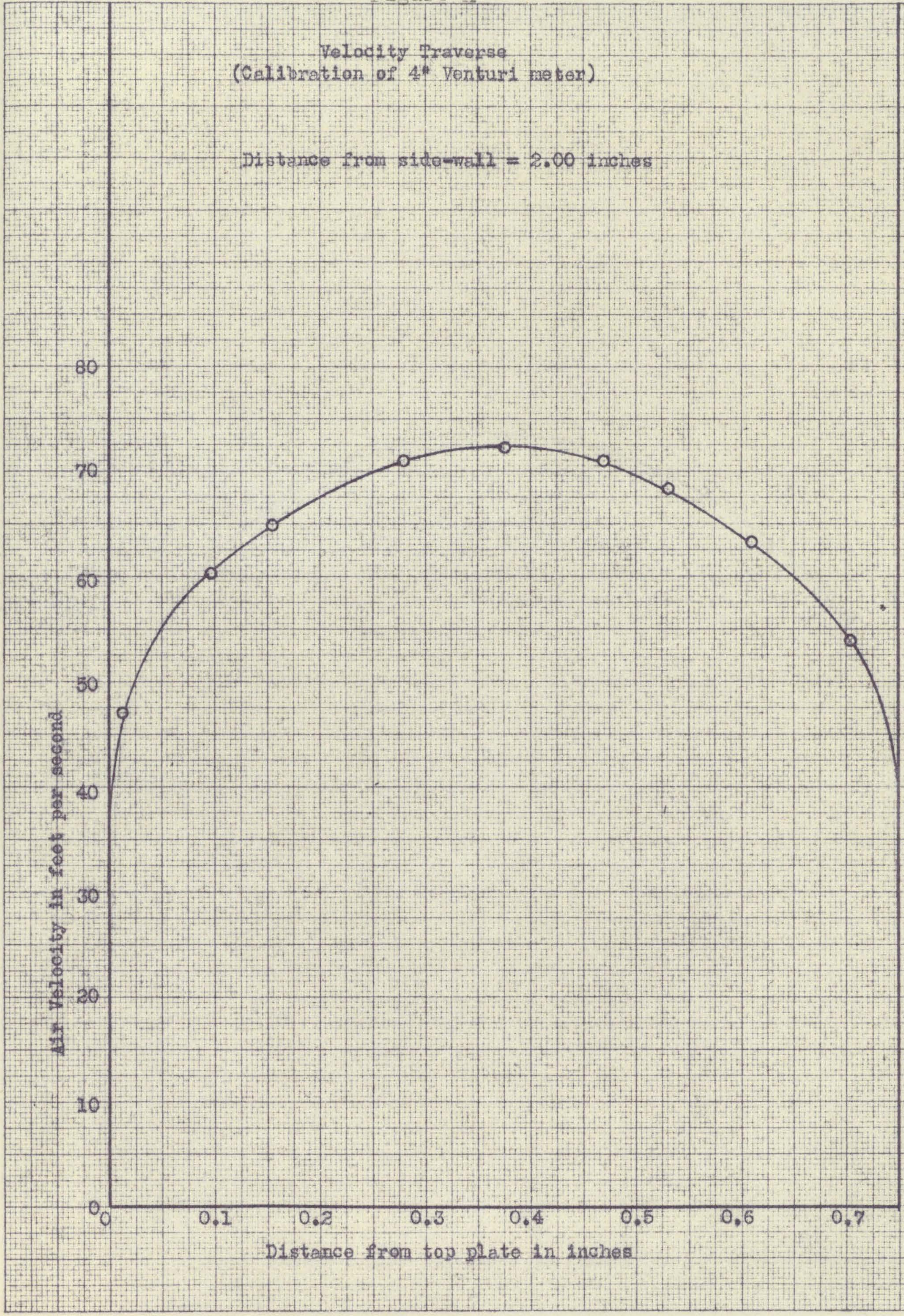

Velocity Traverse
(Calibration of 4" Venturi meter)
Distance from side-wall = 2.00 inches
Air Velocity in feet per second
0
10
20
30
40
50
60
70
80
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches

Figure XI

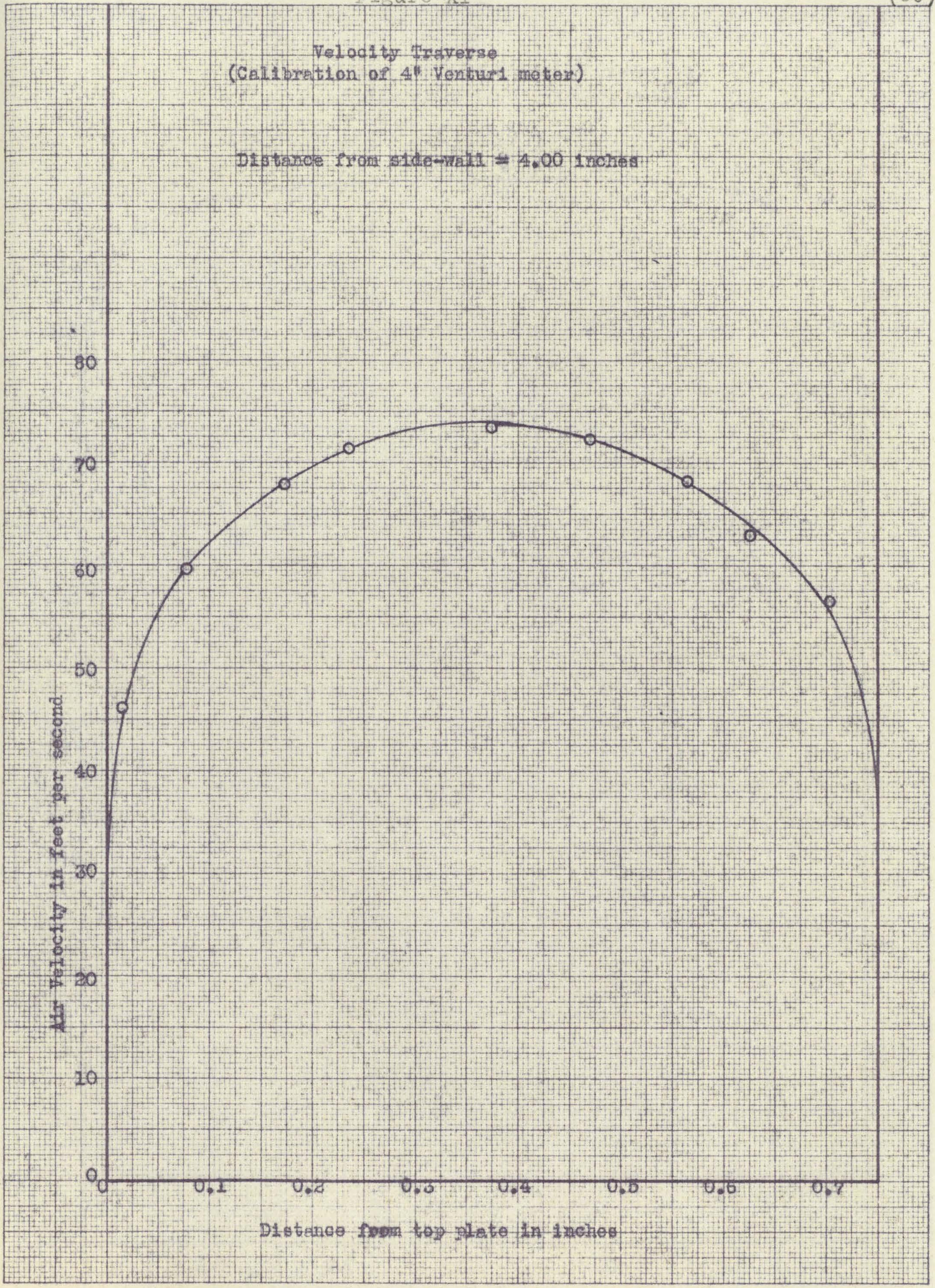
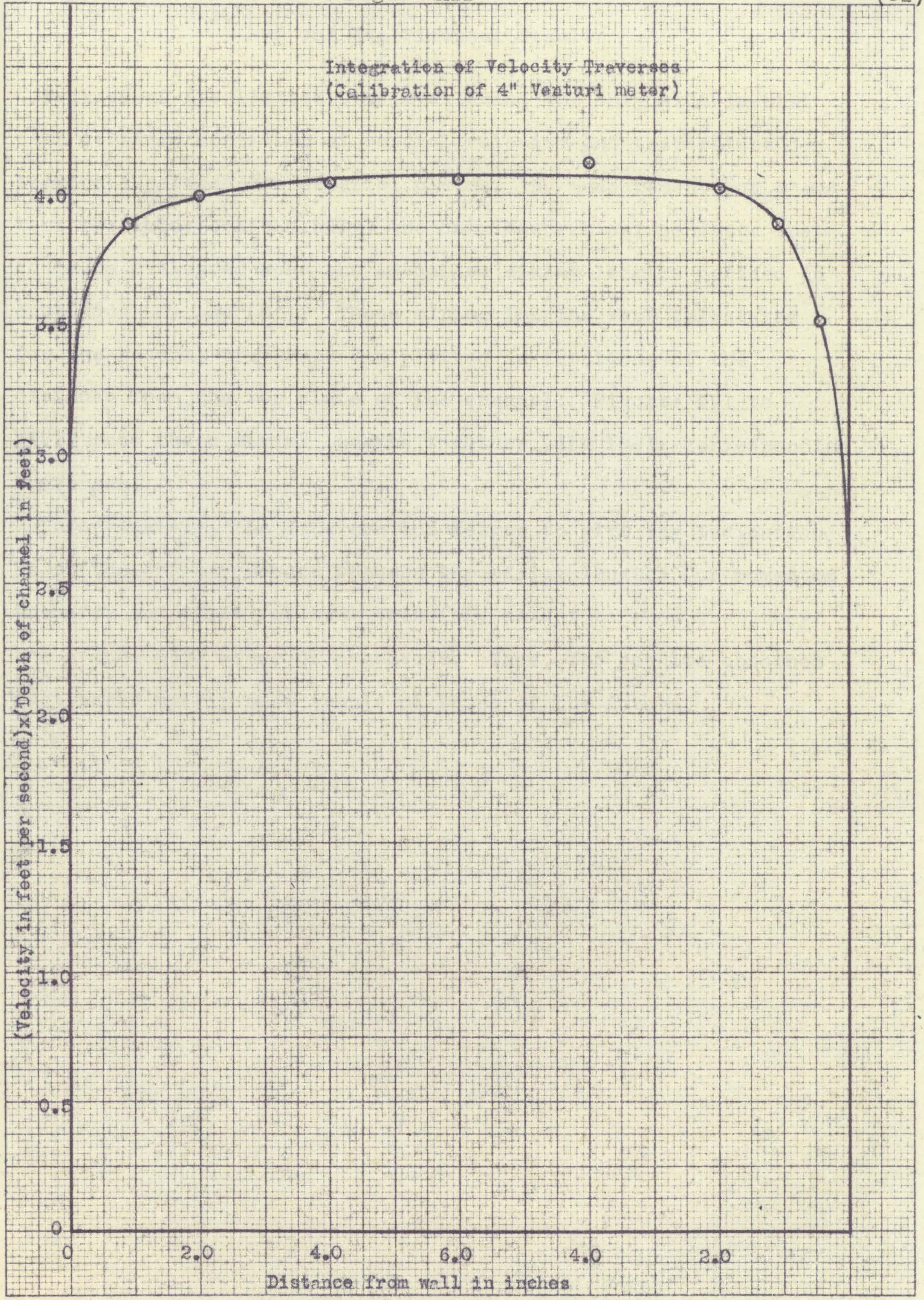

Velocity Traverse
(Calibration of 4" Venturi meter)
Distance from side-wall = 4.00 inches
Air Velocity in feet per second
0
10
20
30
40
50
60
70
80
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches

Figure XII


Integration of Velocity Traverses
(Calibration of 4" Venturi meter)
(Velocity in feet per second)x(Depth of channel in feet)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
0
2.0
4.0
6.0
4.0
2.0
Distance from wall in inches

The Experimental Results

Because of the limited time available, the only experimental data obtained consisted of several velocity and temperature traverses between the parallel plates. No measurements of Q and were possible; consequently, the only theoretical concepts that could be evaluated were the so-called mixing lengths defined in equations (6) and (6'). These mixing lengths were calculated for the several traverses and plotted as functions of the distance from the top plate of the channel. Figures XVI to XXVII show these mixing lengths together with their corresponding traverses.

Unfortunately, the data are too meager to give any indication as to the correspondence between the mixing lengths for momentum transfer and those for heat transfer, except to show that they are of the same order of magnitude. Thus, the results obtained must serve merely as a sample of those of the future.

Figure XVI

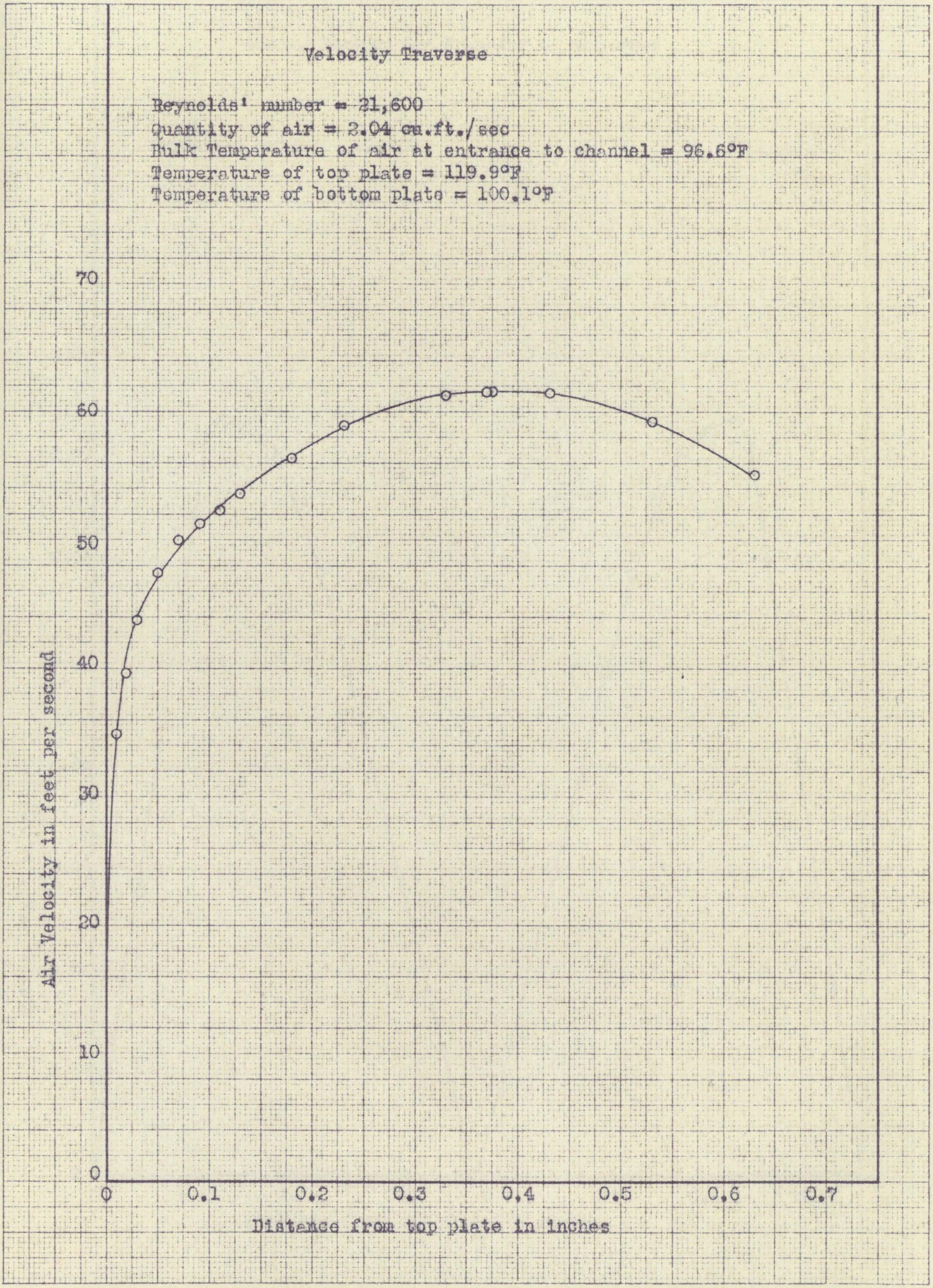
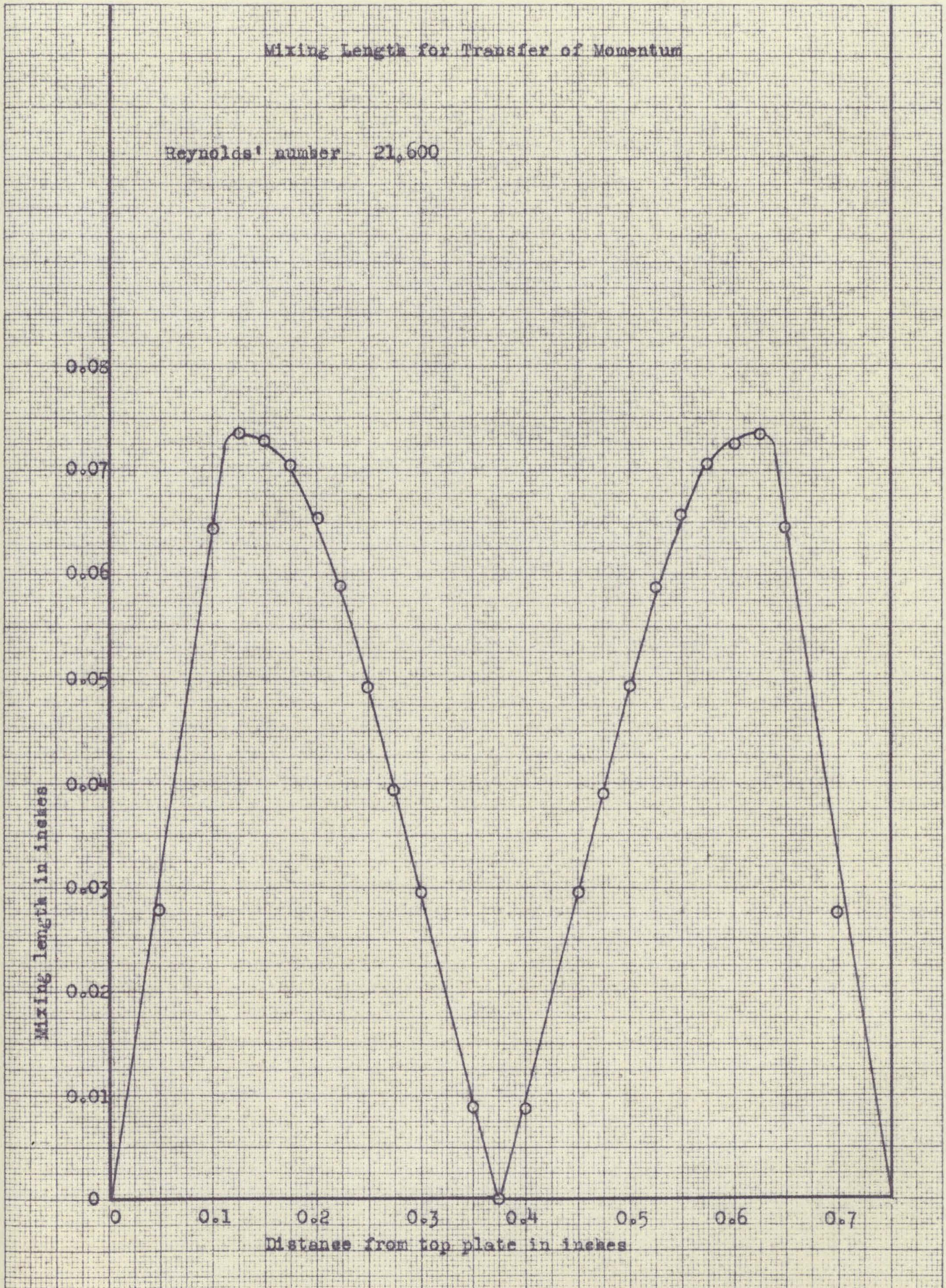
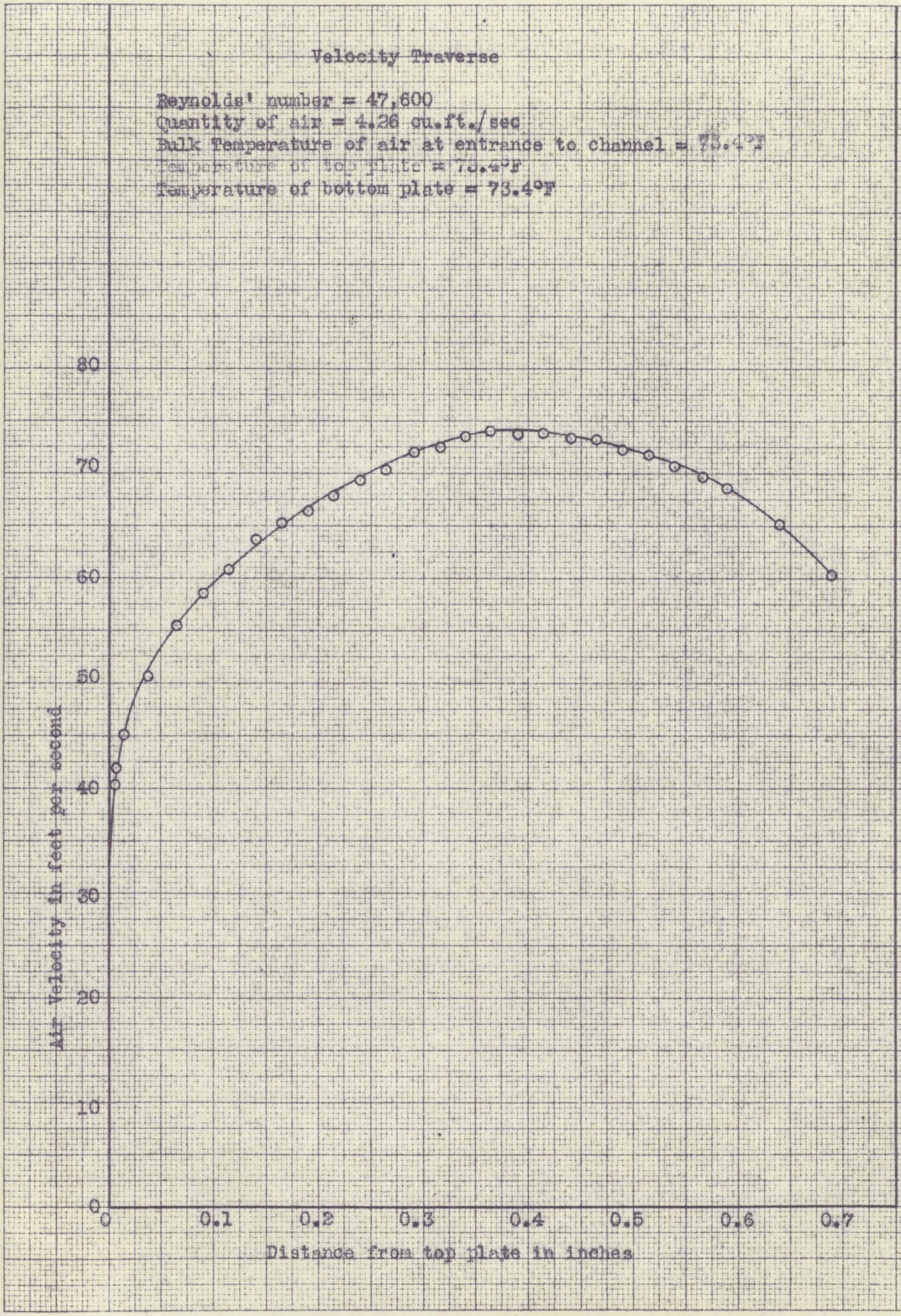
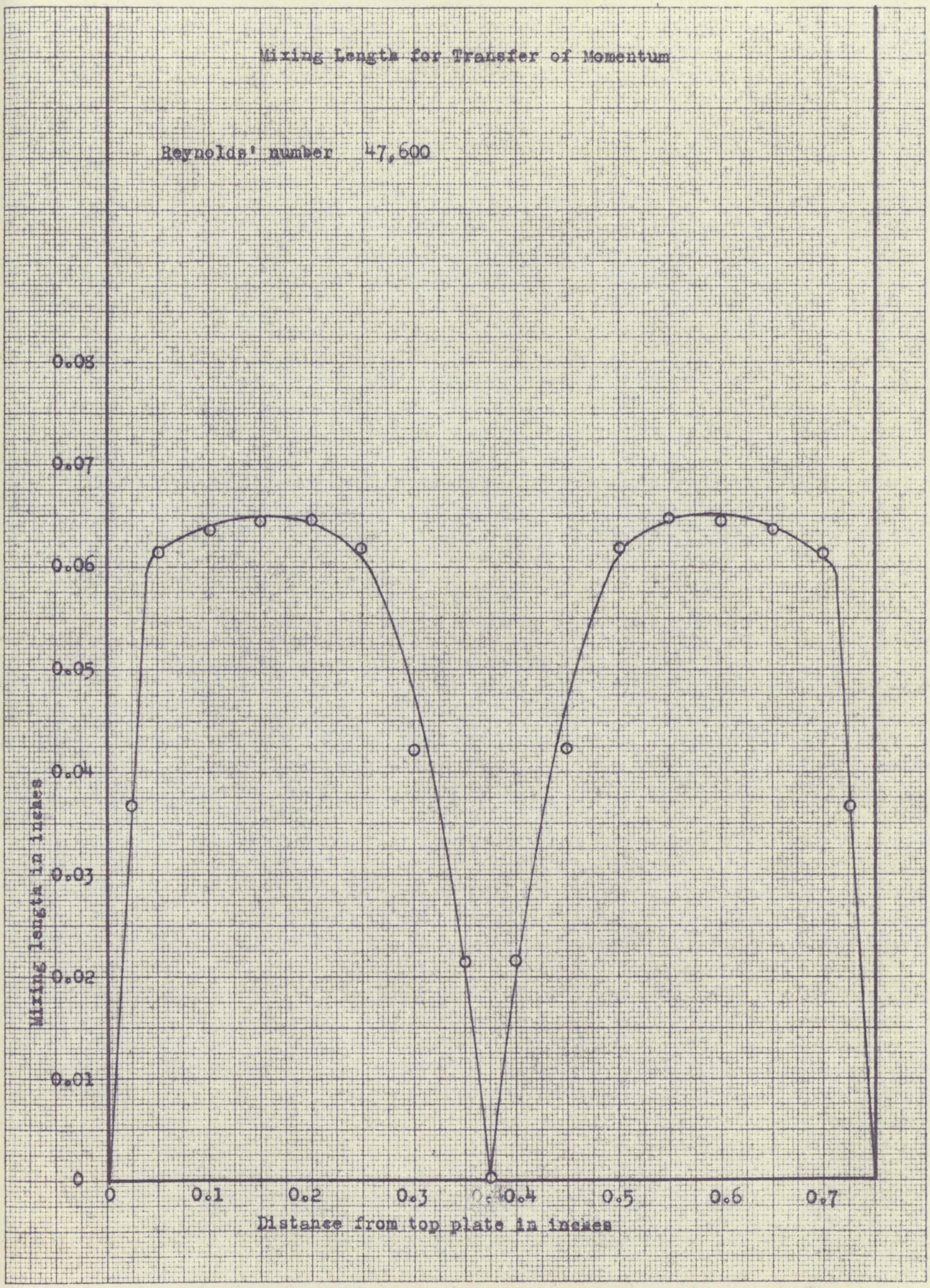
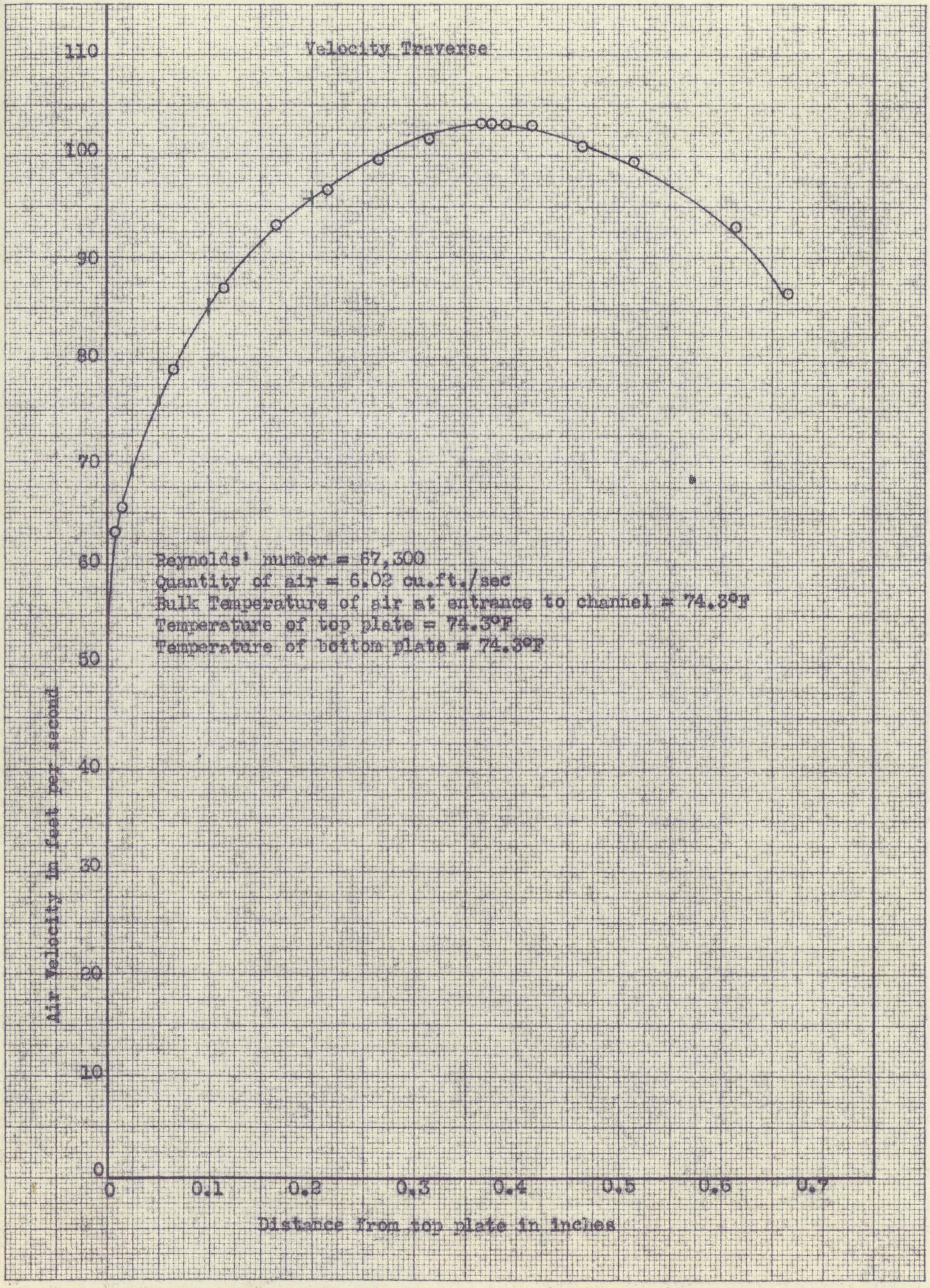

Velocity Traverse
Reynolds' number = 21,600
Quantity of air = 2.04 cu.ft./sec
Bulk Temperature of air at entrance to channel = 96.6°F
Temperature of top plate = 119.9°F
Temperature of bottom plate = 100.1°F
70
60
50
40
30
20
10
0
Air Velocity in feet per second
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches

Figure XVII


Mixing Length for Transfer of Momentum
Reynolds' number 21,600
Mixing length in inches
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches


Velocity Traverse
Reynolds' number = 47,600
Quantity of air = 4.26 cu.ft./sec
Bulk Temperature of air at entrance to channel = 73.4°F
Temperature of top plate = 73.4°F
Temperature of bottom plate = 73.4°F
Air Velocity in feet per second
0
10
20
30
40
50
60
70
80
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches


Mixing Length for Transfer of Momentum
Reynolds' number 47,600
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Mixing length in inches
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches


Velocity Traverse
Reynolds' number = 67,300
Quantity of air = 6.02 cu.ft./sec
Bulk Temperature of air at entrance to channel = 74.3°F
Temperature of top plate = 74.3°F
Temperature of bottom plate = 74.3°F
Air Velocity in feet per second
0
10
20
30
40
50
60
70
80
90
100
110
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches

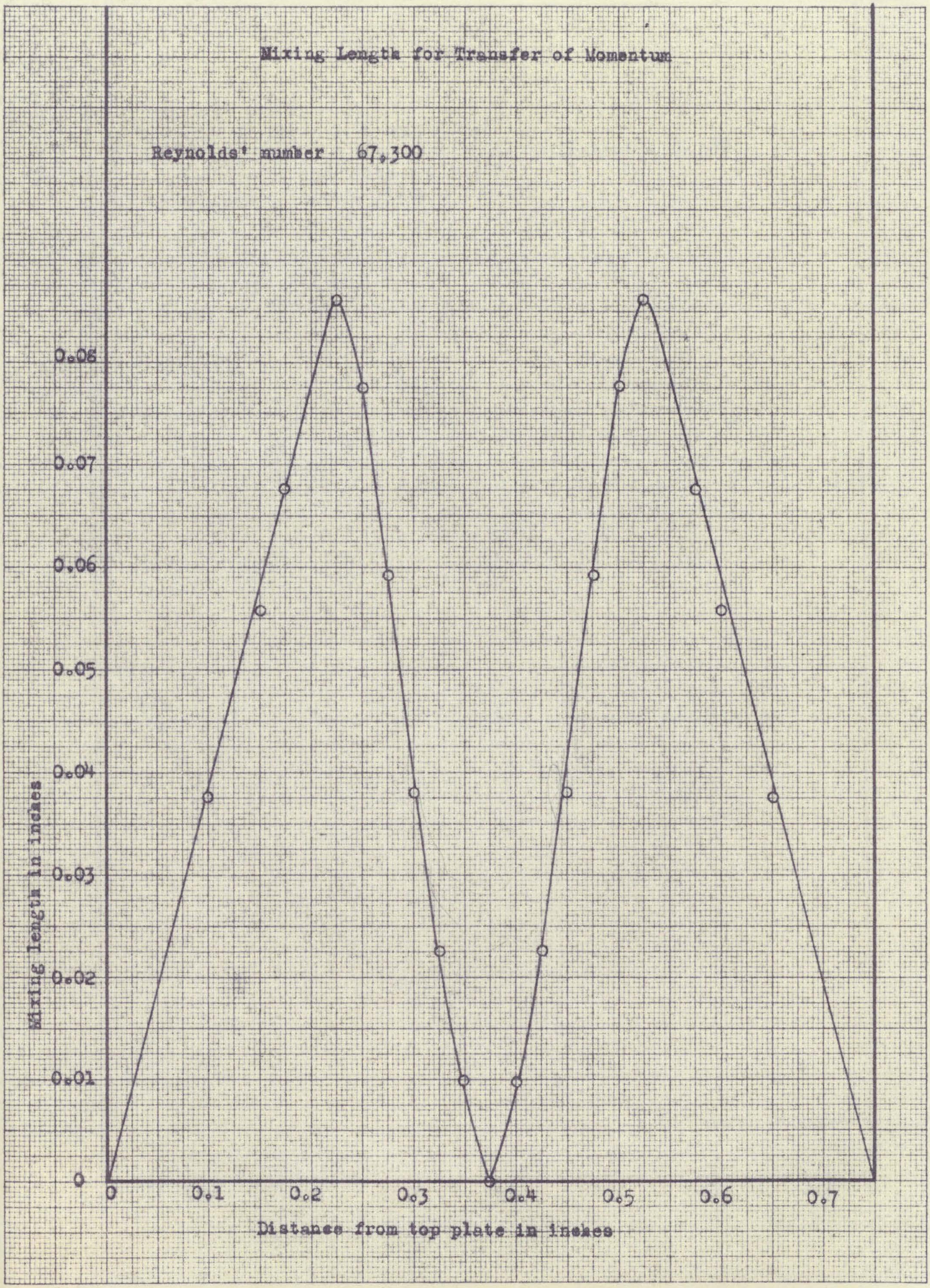
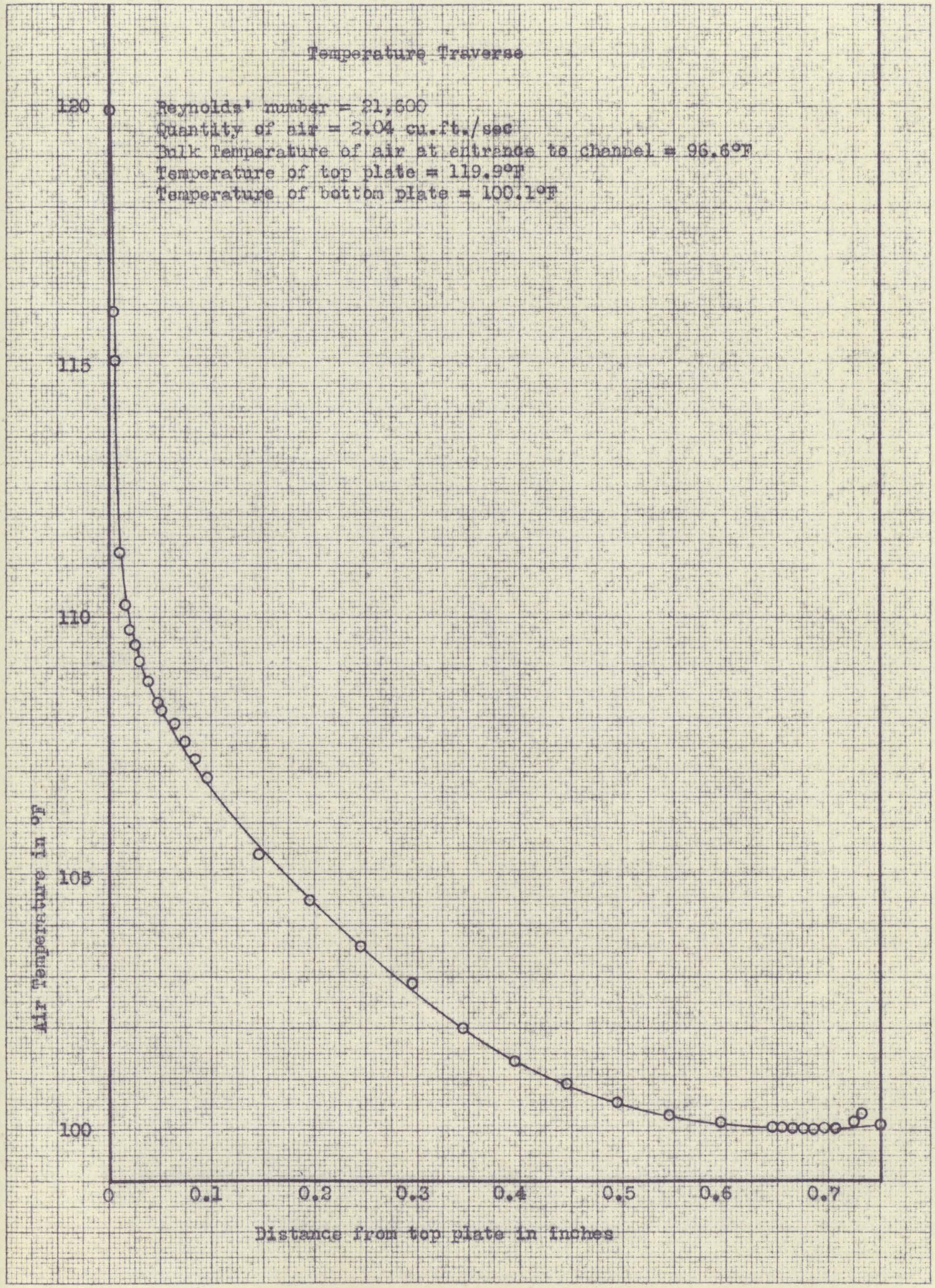

Mixing Length for Transfer of Momentum
Reynolds' number 67,300
Mixing length in inches
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches

Figure XXII



Temperature Traverse
Reynolds' number = 21,600
Quantity of air = 2.04 cu.ft./sec
Bulk Temperature of air at entrance to channel = 96.6°F
Temperature of top plate = 119.9°F
Temperature of bottom plate = 100.1°F
120
115
110
105
100
Air Temperature in °F
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches

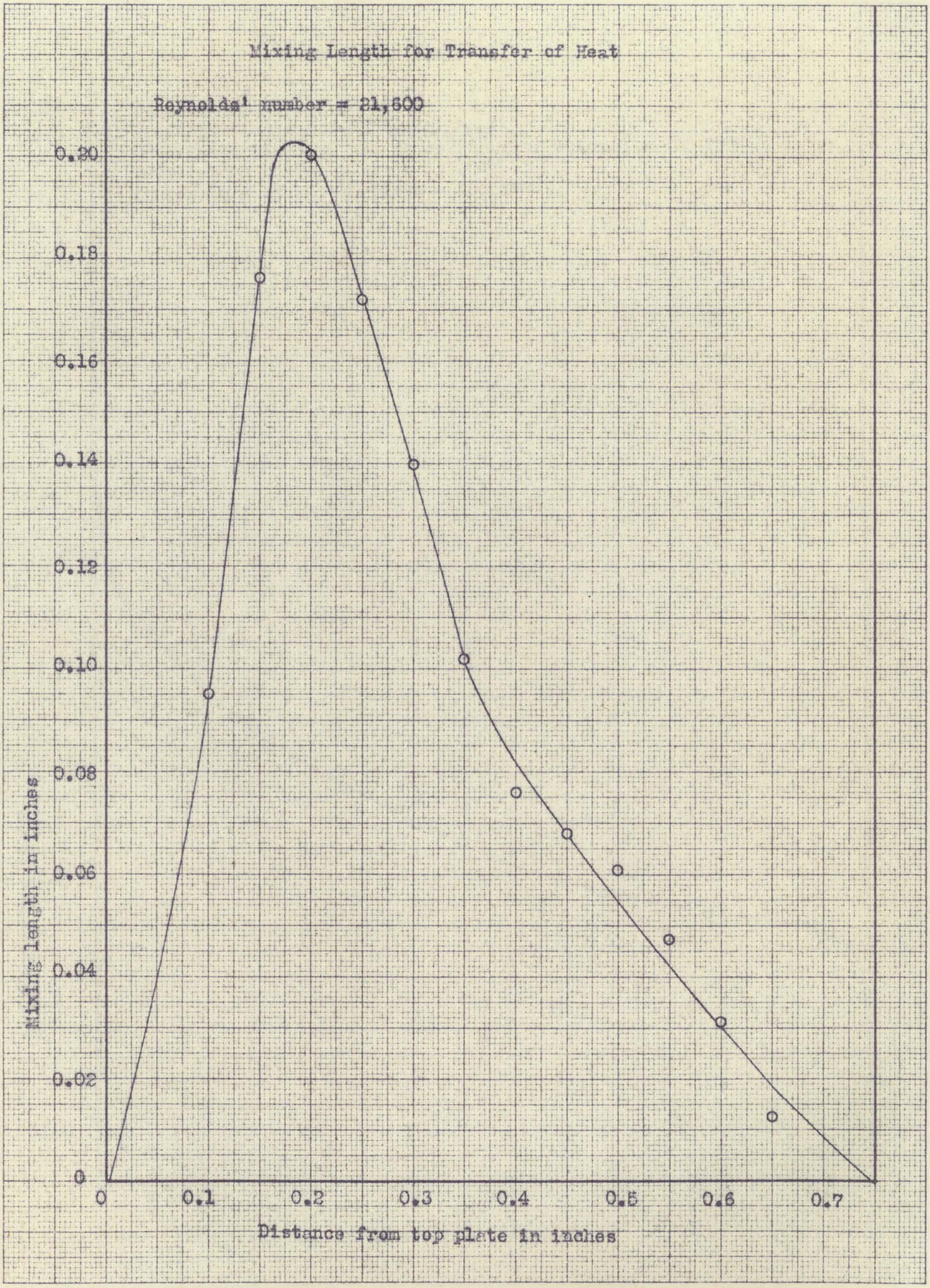

Mixing Length for Transfer of Heat
Reynolds' number = 21,600
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
Mixing length in inches
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches

Figure XXIV

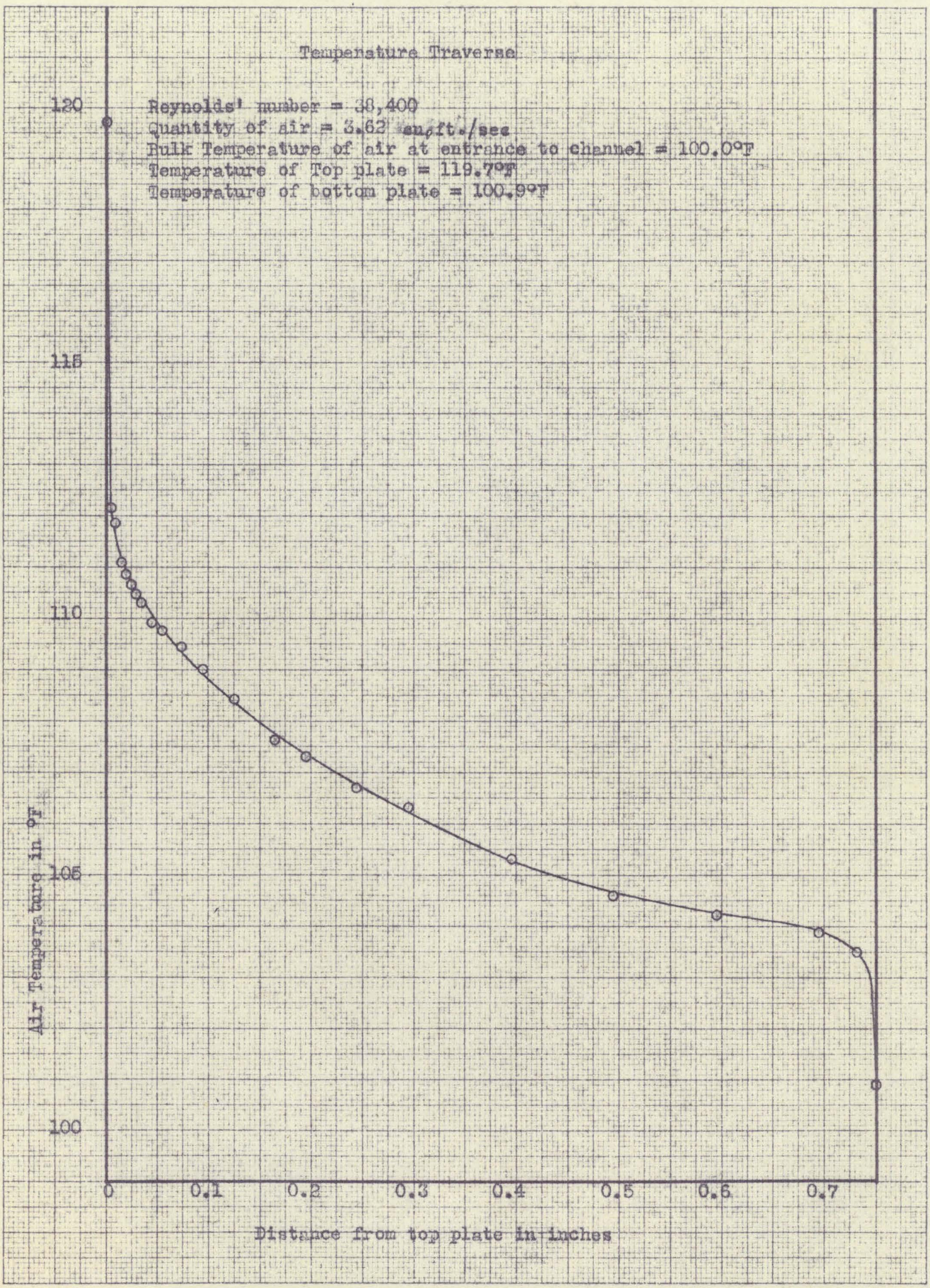
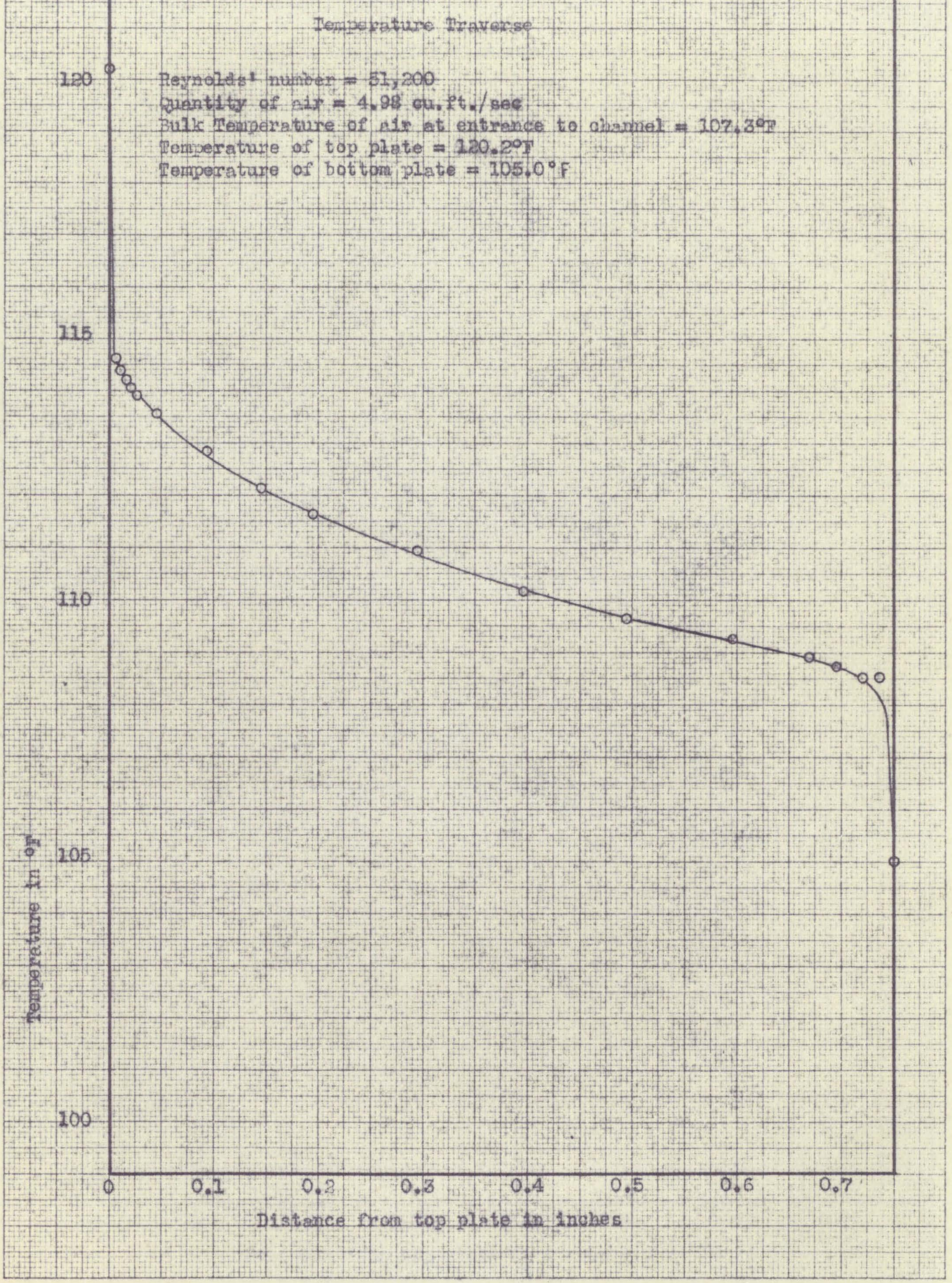
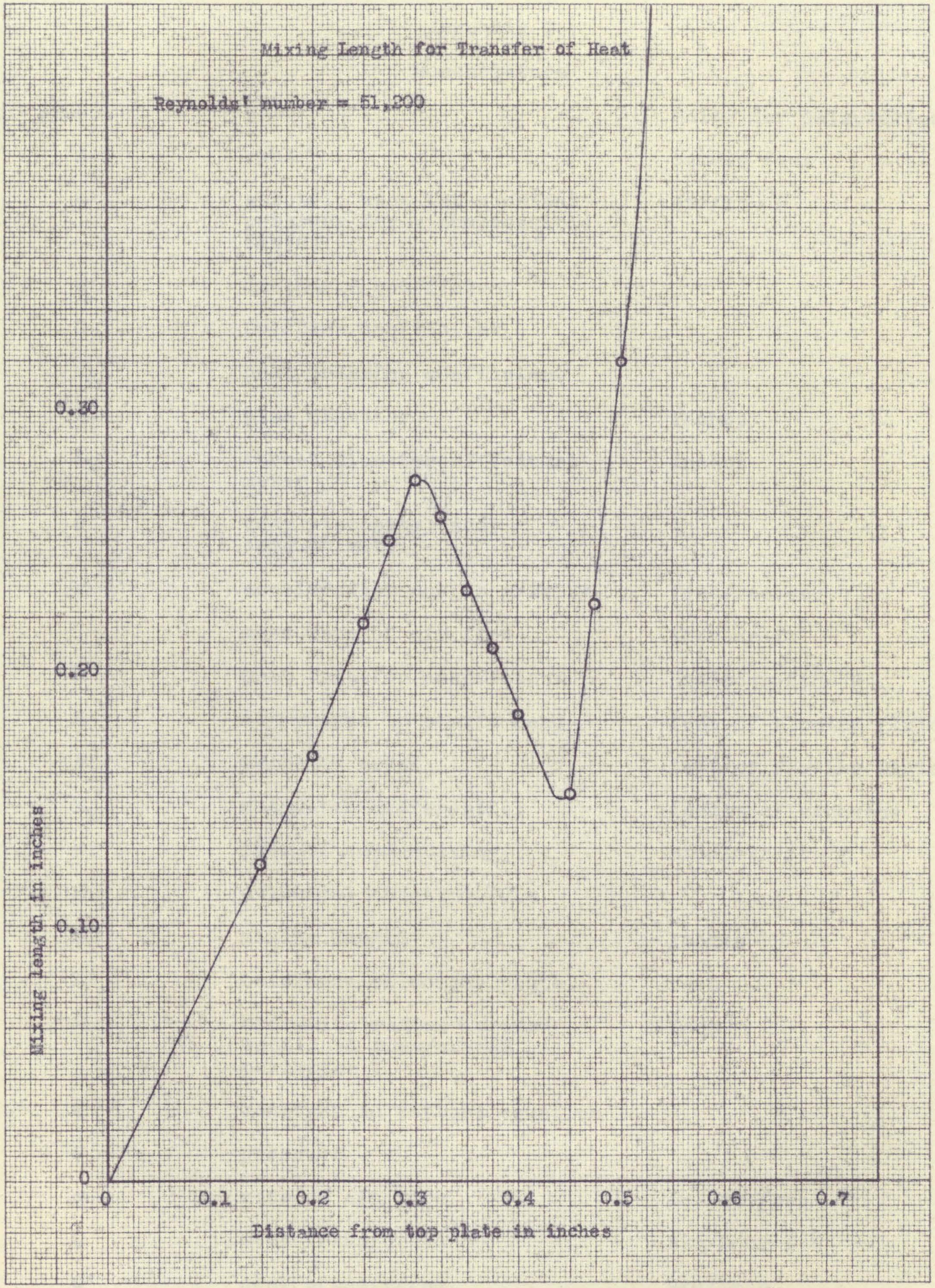

Temperature Traverse
Reynolds' number = 38,400
Quantity of air = 3.62 cu.ft./sec
Bulk Temperature of air at entrance to channel = 100.0°F
Temperature of Top plate = 119.7°F
Temperature of bottom plate = 100.9°F
120
115
110
105
100
Air Temperature in °F
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches

Figure XXV


Mixing Length for Transfer of Heat
Reynolds' number = 38,400
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
Mixing length in inches
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top in inches

Figure XXVI


Temperature Traverse
Reynolds' number = 51,200
Quantity of air = 4.98 cu.ft./sec
Bulk Temperature of air at entrance to channel = 107.3°F
Temperature of top plate = 120.2°F
Temperature of bottom plate = 105.0°F
Temperature in °F
120
115
110
105
100
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches


Mixing Length for Transfer of Heat
Reynolds' number = 51,200
0.30
0.20
0.10
0
Mixing Length in inches
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Distance from top plate in inches

Appendix I

Nomenclature

Symbol	Name	Units for Calculation
A_1	area	square feet
c	specific heat (at constant pressure for a compressible fluid)	Btu/lb $\cdot^{o}$F
c'	discharge coefficient	
C_f	friction coefficient	
C_h	heat transfer coefficient	
D_1	pipe diameter	inches
D_2	throat diameter of Venturi meter	inches
e	eddy viscosity; eddy conductivity	
g	acceleration due to gravity	feet/sec.sec
G	mass rate of air flow	lb/hour$\cdot$ft^2
h	heat transfer coefficient	Btu/hour$\cdot$ft$^2\cdot$ ^{o}F
H	difference in static head	feet of flowing fluid
H'	velocity head	feet of flowing fluid
H_k	manometer differential	inches of kerosene
k	thermal conductivity	Btu/hour$\cdot$ft $\cdot^{o}$F
K	universal constant, 0.4	
L	mixing length	inches
L'	thickness of copper plate	feet
m	hydraulic radius = cross-sectional area/wetted perimeter	feet
M	constant in Venturi equation	
P	pressure	
q	rate of air flow	cu. ft./sec
Q	heat transferred	Btu/hour$\cdot$ft^2
Q'	heat transferred	Btu/hour

Symbol	Name	Units for Calculations
Δt	temperature difference	$^{\circ}F$
U, u	velocity	ft/sec
u'	velocity fluctuation in direction of mean flow	
v'	velocity fluctuation in direction perpendicular to mean flow	
$\overline{u'v'}$	mean value of this product with respect to time	
V	overall heat transfer coefficient	$Btu/hour \cdot ft^2 \cdot {}^{\circ}F$
W	mass rate of oil flow	lb/hour
x	distance parallel to direction of mean flow	
y	distance perpendicular to direction of mean flow	
θ'	temperature fluctuation	
θ	temperature	
μ	coefficient of viscosity	lb/hour·ft
μ_f	coefficient of viscosity for the air film	lb/hour·ft
ν	kinematic viscosity	
σ	Prandtl number	
ρ	density	lb/cu ft
τ	shearing stress	
τ_0	shearing stress at the wall	
$\lvert A \rvert$	absolute value of A, where A is any quantity	

Appendix II

Venturi Meter Design

Because of its relatively low density and its clear meniscus, kerosene was selected as the fluid for the manometer with which the Venturi differential was to be read. It was desired to keep this differential between the limits of 1 inch and 20 inches of kerosene; the lower limit being dictated by the desired precision of reading and the upper by the deliverable static head of the small blower. Elementary calculations showed that for the desired range of flows one Venturi meter was insufficient. Further calculations based upon the fundamental equation:

$$q = c' M \sqrt{2gH} \qquad (19)$$

$$M = A_1 / \sqrt{(D_1/D_2)^4 - 1} \qquad (20)$$

and using the mean density of kerosene to be 49.5 pounds per cubic foot and that of air to be 0.0673, yielded the following table of values:

D_1	D_2	M	$c'M(2g)^{1/2}$	min.H_k	max.H_k	min.q	max.q
2.05	0.66	0.00239	0.0188	1.94	20.0	0.205	0.658
3.09	1.21	0.00806	0.0635	1.75	20.0	0.658	2.22
3.88	2.39	0.0335	0.264	1.15	20.0	2.22	9.25

The odd values of D_1 were, in the first two cases, due to the fact that the most easily obtainable pipe-on-hand was selected; and, in the third case, this was the actual diameter of ordered four-inch galvanized iron tubing. It was seen from the values of minimum H_k that lower values could be read with each of these meters; thus, the overlapping here insured that there would be no gaps in ability to measure rates of flow if a differential of 20 inches of kerosene could not be obtainedby the blower selected.

Appendix III

Determination of Circulation Rate for the Oil

As an initial basis of calculation, the temperature of the oil on the top plate was taken as 135°F and that of the oil in contact with the bottom plate as 125°F. Thus, there would exist a five degree temperature difference between the oil and the air stream. In order to determine the rate of heat transfer between the oil and the air, the heat transfer coefficients for the three resistances to flow-namely, the oil film, the copper plate, and the air film-had to be calculated.

For the air film, an equation which has been derived from dimensional analysis (6) was used:

$$(h/cG)(c\mu_f/k)^{2/3} = 0.023\,(4mG/\mu_f)^{-0.2} \qquad (21)$$

However, for the common gases, this simplified to:

$$h = 0.0144\,c\,G^{0.8}/(4m)^{0.2} \qquad (22)$$

It was found by calculation that values of "h" obtained from these equations agreed to within 4 percent. As these particular calculations were subject to errors of greater magnitude than this, it was decided that the accuracy of equation (22) was sufficient. Thus, by its use heat transfer coefficients were calculated and plotted against the Reynolds number for the flowing air as shown in Figure XIII.

The heat transfer coefficient of each oil film was obtained as a function of oil velocity relative to the plate and this relationship is shown in Figure XIV. At a velocity of 0.7 foot per second, the values of the oil film coefficients were seen to be substantially equal to that of the air film at a Reynolds number of 100,000 for the air. Therefore, this velocity was selected for both the hot and cold oil. The corresponding values of the oil film coefficients were:

$$\text{At } 135^{\circ}F \quad h = 26.8 \ Btu/hr \cdot ft^2 \cdot {}^{\circ}F$$
$$\text{At } 125^{\circ}F \quad h = 26.2 \ Btu/hr \cdot ft^2 \cdot {}^{\circ}F$$

In calculating the overall heat transfer coefficient, there were the above-mentioned resistances to be considered. Thus:

$$1/V = (1/h)_{oil} + (L'/k)_{metal} + (1/h)_{air} \tag{23}$$

and

$$Q = V \Delta t \tag{24}$$

By the use of equation (23) and Figures XIII and XIV, V was obtained as a function of the Reynolds number of the air and was so plotted in Figure XIII. In this calculation it was found that (L'/k) for the metal was a negligible quantity and that the overall coefficient from the hot oil to the flowing air was substantially equal to that from the flowing air to the cold oil. Then, for a given temperature differential between the oil and the air, from equation (24) and from Figure XIII, Q was obtained as a function of the Reynolds number of the air, and this relationship is shown in Figure XV.

From Figure XV it was seen that the amount of heat transferred from the oil in the top bath to the air (and from the air to the oil in the bottom bath) when the Reynolds number of the air was 100,000 would be 64.5 Btu per square foot of plate per hour, thus making a total transfer of 322.5 Btu per hour for the entire surface of five square feet. The minimum advisable depth of oil flowing over the plate was thought to be three inches; therefore, as its velocity was to be 0.7 foot per second and its width of flow was to be one foot, 0.175 cubic or 10 pounds of oil would be needed per second past each plate. Then from the following equation:

$$c \Delta t = Q'/W \tag{25}$$

it was deduced that Δt, the temperature drop of the oil in passing the length of the plate, would be 0.018°F, which would be less than the 0.05°F drop set as the maximum allowable variation in temperature along the length of the plate. Thus, this rate of flow for the oil was suitable.

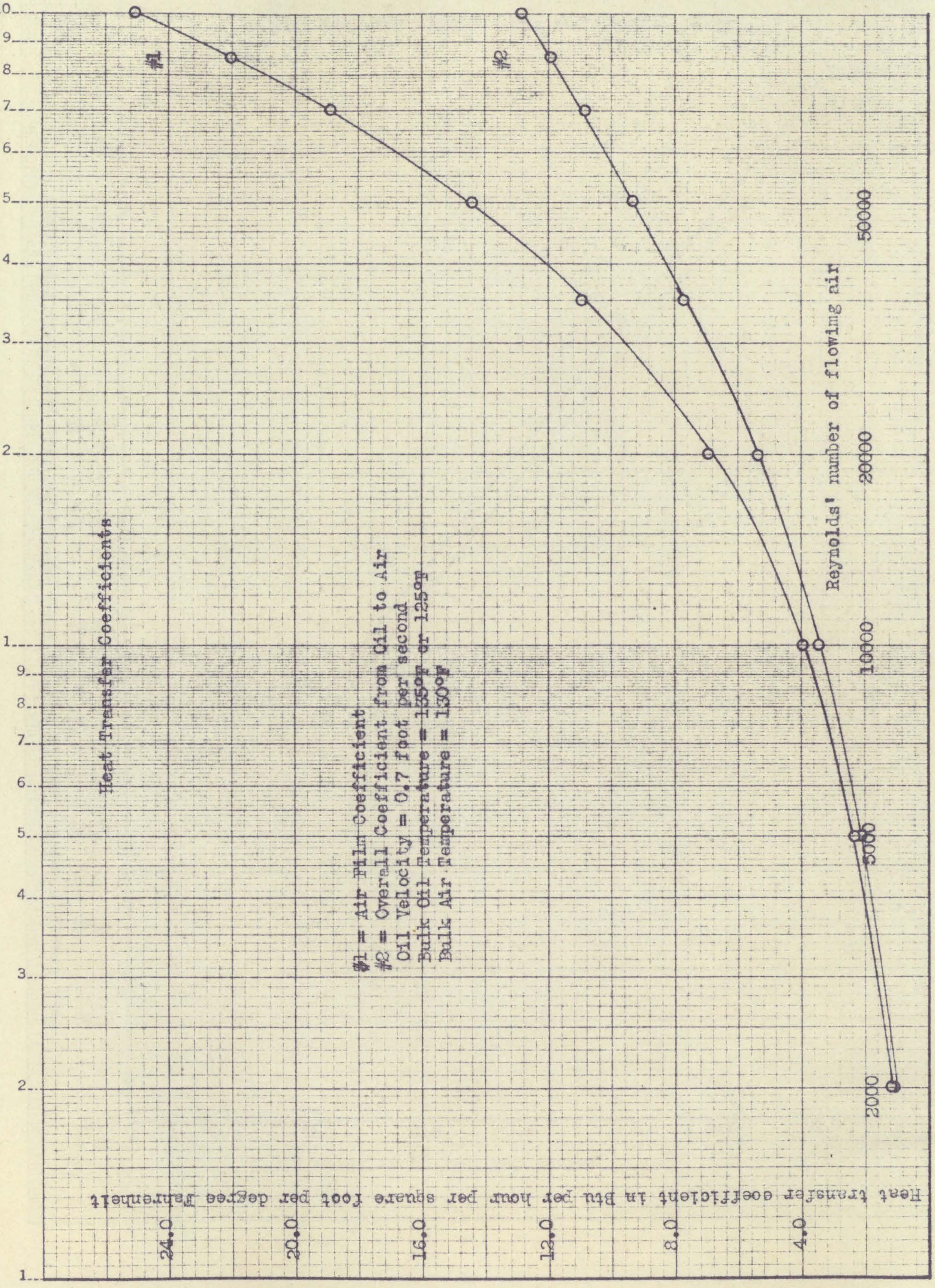

Heat Transfer Coefficients
#1 = Air Film Coefficient
#2 = Overall Coefficient from Oil to Air
Oil Velocity = 0.7 foot per second
Bulk Oil Temperature = 135°F or 125°F
Bulk Air Temperature = 130°F
#1
#2
Reynolds' number of flowing air
2000
5000
10000
20000
50000
Heat transfer coefficient in Btu per hour per square foot per degree Fahrenheit
4.0
8.0
12.0
16.0
20.0
24.0

Figure XIV

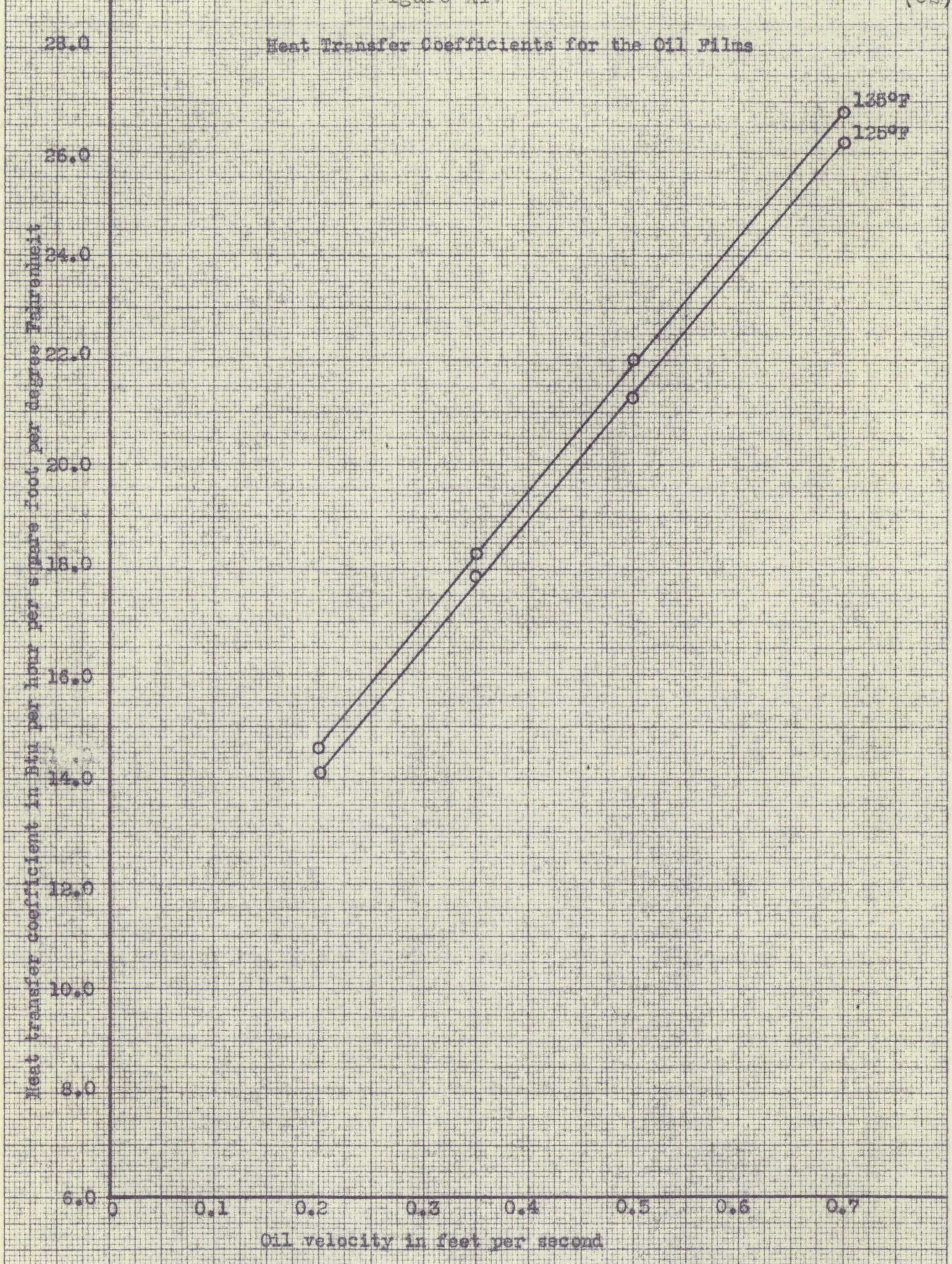
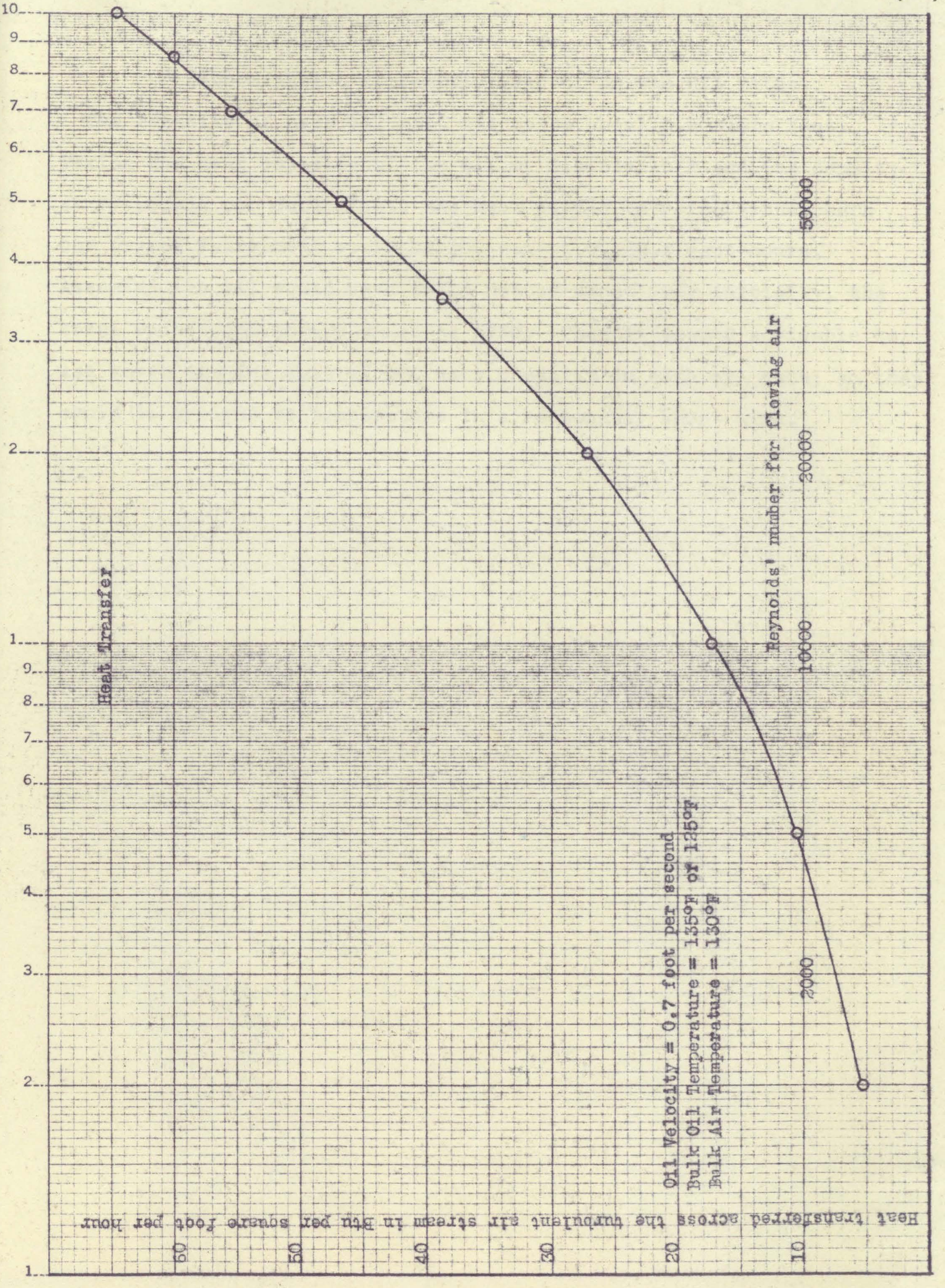

Heat Transfer Coefficients for the Oil Films
135°F
125°F
28.0
26.0
24.0
22.0
20.0
18.0
16.0
14.0
12.0
10.0
8.0
6.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Heat transfer coefficient in Btu per hour per square foot per degree Fahrenheit
Oil velocity in feet per second

Figure XV


Heat Transfer
Reynolds' number for flowing air
50000
30000
10000
2000
Oil Velocity = 0.7 foot per second
Bulk Oil Temperature = 135°F or 125°F
Bulk Air Temperature = 130°F
Heat transferred across the turbulent air stream in Btu per square foot per hour
60
50
40
30
20
10

Appendix IV.

Determination of Heaters Needed for the Oil Baths

The capacity of each oil bath was to be about 3.5 cubic feet. Taking the heat capacity of the oil at 0.5 Btu/lb.°F and its density at 57 pounds per cubic foot, the heat needed to raise either bath one degree Fahrenheit would be 100 Btu (i.e., 29.3 watt-hours.) Assuming the oil would have to be heated about 60°F initially, the total watt-hours needed would be 1750. Thus it seemed advisable to use three heaters each of 500 watt capacity on each oil bath; so they could be brought to temperature in a reasonably short time.

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