The Measurement of Dielectric Loss in Solid Dielectrics at Very High Frequencies.

A Thesis by

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1. Object of This Investigation

The object of the research described in this thesis has been to develop a method for the measurement of the losses in solid dielectrics at very high frequencies, particularly in the vicinity of 100 megacycles. No references to accurate work at such high frequencies could be found in the literature at the time this work was started, although some rather qualitative comparisons of different kinds of solid insulation had been reported.[1] These qualitative tests indicated roughly the relative losses in various kinds of insulation material under rather special conditions of field distribution in the insulation. However, they gave no accurate information from which it would be possible to calculate the actual amount of loss under given conditions of field strength and frequency. Such information would be highly desirable in the design of certain types of high frequency equipment in which it is impossible to avoid the use of insulation in regions where the electric field strength is quite high.

A further reason for attempting to make rather precise measurements on insulation losses at high frequencies

1. A partial list of the papers published on this subject since 1933 will be found in the bibliography.

is that it is only in this way that the variation of the loss with frequency may be determined. In other words, it is desirable that it be possible to compare the results of high frequency tests with those which have been made by other observers at lower frequencies. From such comparisons, it might be possible to justify an extrapolation of the results to still higher frequencies, thus enhancing the value of the tests. Moreover, the theory of dielectric losses in solids is not yet fully developed and additional information, obtained at high frequencies in such a way as to be directly comparable with the present knowledge, might be of aid in the further development of this theory.

Because of the well known difficulties encountered in making accurate dielectric loss measurements at very high frequencies, it was felt that it would be desirable to make a rather thorough experimental and analytical investigation of the most promising method. Actual results were not considered to be of primary importance in this work, but rather the emphasis was placed on an endeavor to obtain data on which to base the design of future equipment. Consequently, considerable time was spent on an investigation of various preliminary arrangements, obtaining experience with the difficulties to be encountered in the final arrangement. An account of this experience, together with the necessary mathematical analyses, is to be included in this thesis. The most important results to be presented here comprise those suggestions regarding the design of future equipment, and estimates of the accuracy to be realized, rather than the results of actual measurements with the present equipment.

2. Choice of the Method to be Investigated.

2a. Desirable Characteristics of the Method.

At the outset, it was decided that the method adopted should, if possible, possess the following features:

1. The high frequency equipment should be as simple as possible.

2. It should be arranged so that it might be completely shielded.

3. The loss in the sample of insulation should be large enough in comparison with the unavoidable losses in the measuring equipment so that it might be measured conveniently and reasonably accurately.

4. The losses inherent in the measuring equipment should be capable of independent measurement, the conditions under which such a measurement would be made being as nearly

as possible the same as those existing with a sample in place.

5. The distribution of the electric field in the sample of insulation should be calculable and, if possible, there should be negligible "edge effects".

6. It should be possible to check experimentally the magnitude of any unavoidable edge effects which would disturb the field in the sample.

7. The actual magnitude of the electric field within the sample should be known, at least roughly.

8. The sample should be inserted into the system in such a way that it would be unnecessary to consider the residual inductance or capacitance of the conductors through which the current to the sample must flow.

9. It should be possible to make each determination depend on a great number of readings, each set of readings obeying some simple law. This would make it possible to check all the readings in each set for consistency and thus to eliminate some of the observational errors.

10. The results should be easily convertible into some standard form, such as a statement of the power factor or "loss factor" of the material.

2b. Review of Methods for the Measurement of Dielectric Losses

A review was made of the methods of measurement which have been used successfully at lower frequencies, and careful consideration was given to a method proposed by Dr. A. V. Haeff. It was found that the various methods might be grouped under the following general headings:

1. Actual or indirect electrical measurement of the power input to the test circuit.

2. Calorimeter measurements of power loss.

3. Resistance substitution method.

4. Distuning method.

1. The method suggested at the start of this work by Dr. Haeff (see Appendix A) would come under the first heading, since it involved the indirect measurement of the increased radiation necessary to supply the power to the sample. The main advantage of the method would result from its use of a super-regenerative receiver as the voltage-indicator in the measuring equipment. By this means it was hoped that the losses in the voltage-indicator could be kept at a low value; sufficiently low so as not to over-shadow the losses in the sample. This precaution might possibly be necessary at somewhat higher frequencies where the unavoidable losses in the

usual kinds of voltage indicator become relatively large, but for measurements at 100 megacycles this difficulty does not arise. On the other hand, the method has certain inherent disadvantages which made it undesirable, at least for frequencies at which simpler and more direct methods may be used.

Perhaps the most important of these disadvantages is that the entire measurement would have to be based on either calculated values or rather indirectly determined values of the losses in a "standard" tuned transmission line. Because of the scarcity of data indicating the validity of transmission line calculations at high frequencies, it was considered highly desirable to apply a rather direct experimental check to such calculations. Consequently, the method finally adopted is such that calculations of the losses in tuned transmission lines do not form the basis of the results, but are used only for the calculation of small correction factors.

The proposed method would also involve the use of a great deal of rather complicated high-frequency equipment, thus increasing the complexity of the necessary adjustments and the consequent possibility of error. Another difficulty which would have arisen is that of body-capacity effects; it would have been practically impossible to provide complete shielding.

It was accordingly decided that other methods should be investigated, although for future work at higher frequencies Dr. Haeff's method certainly merits further investigation.

2. The calorimeter method might be quite suitable for qualitative comparisons between different kinds of dielectric material, but it would be extremely difficult to determine the power factor or loss factor of high-frequency insulation by such a method. This is true because the power loss in the sample of material is not measured directly as an electrical quantity. Instead, it is measured by determining the amount of heat which is developed when the sample is subjected to a high-frequency field. Before the result of such a test can be expressed in electrical terms an absolute measurement of one of the electrical quantities, either the current flowing to the sample or the electromotive force between the electrodes, must be made. A determination of either one of these

quantities would be difficult, not only because of the difficulty in calibrating suitable instruments for very high frequencies, but also because of the effect of distributed capacitance which may cause a large variation in total current along the length of a high-frequency conductor. Thus the current which is actually measured might be quite different from that which it was desired to measure. Because of this sort of difficulty it was decided that the calorimeter method would be unsuitable for ultra high frequency work.

3. The methods falling under the last two classifications depend directly on the properties of resonant circuits, these resonant circuits usually being made up of a coil and variable condenser. The sample is usually inserted in a small condenser arranged so that it may be connected in parallel with the main tuning condenser in the resonant circuit. Some form of indicator, often a thermocouple instrument, is used to indicate the magnitude of the current in the resonant circuit and the resonant circuit is coupled very loosely to a radio frequency oscillator of rather large output. In the resistance substitution method a reading is first taken of the resonance current with the sample in place. Then, holding the oscillator

output constant and with the sample removed, the resonance current is adjusted by means of a series resistor in the resonant circuit until it has the same value as it had with the sample in place. Thus the effective resistance of the sample of insulation is measured in terms of a series resistance in the resonant circuit which produces the same effect on the resonance current. A simple calculation is then required to convert the result to its final form.

This method possesses the advantage of simplicity but is quite unsuitable for very high frequency work because of the difficulty in the construction and calibration of suitable resistors for use at ultra-high frequencies. Another difficulty which may become quite serious at high frequencies results from the use of a variable condenser whose losses vary as its capacitance is changed. This effect is not important at low frequencies, because of the small relative change which must be made in the setting of the variable condenser when the sample is inserted. However, at high frequencies the maximum capacitance of the condenser is, of necessity, rather small, and the change required on the insertion of the sample becomes important. In fact, it has been shown that, unless elaborate precautions are taken, the reduction of the loss in the condenser when the sample is inserted and the circuit retuned may actually be greater than the increase in loss due to the sample. From these considerations it was concluded that any method involving the use of known resistances or of coil-and-condenser resonant circuits would not be sufficiently accurate for use in this investigation.

4. The distuning method, as used at low frequencies, employs the same sort of resonant circuit, but the equivalent resistance of the resonant circuit, with and without the sample, is measured in terms of a known change in reactance rather than in terms of a known change in resistance. This change in the reactance of the resonant circuit is usually effected by changing the capacitance of the tuning condenser a known amount, although the same result may be obtained by changing the inductance of the coil by a known amount or by slightly altering the frequency of the supply oscillator. In either case, the introduction of reactance into the resonant circuit causes a reduction in the resonant circuit current and the equivalent resistance of the resonant circuit may be calculated in terms of the known change in reactance and the resultant decrease in the resonant circuit current. Thus

the effect of the sample of dielectric in increasing the equivalent resistance of the resonant circuit may be determined by the difference in two sets of readings, with and without the sample in place. Although this method does not involve the use of calibrated non-inductive resistors, a coil-and-condenser resonant circuit is usually used and in this form it would not be suitable for ultra-high frequency use. However, the variation of the current in a resonant circuit as the reactance is changed is known to follow a relatively simple law. This is a distinct advantage, in that it allows the readings to be checked with the possibility that spurious effects may be detected and eliminated. Consequently it was decided that the method to be investigated should be some modification of the usual distuning method, provided that a resonant circuit could be constructed whose losses and reactance could be calculated to a reasonable degree of accuracy.

2. Preliminary Analysis of the Possibilities of a Coaxial Line.

Fortunately, at frequencies of the order of 100 megacycles, tuned transmission lines form very desirable resonant circuits and, in addition, their behavior can be calculated quite accurately.[2,3,4,5,6] This is particularly true of coaxial tuned lines, short-circuited at each end and approximately a half-wave length long when in resonance. For such lines, the only departure from a completely-shielded simple geometrical configuration occurs at the points where the energy is introduced and the measuring current taken out. By making these necessary openings very small, their effect can be made negligible, in which case there results a resonant circuit which is practically completely shielded, has very low losses, and in addition is susceptible to reasonably accurate calculation.

However, before these advantages could be realized, a suitable method had to be devised for introducing the sample

2. Schmidt, O. "Das Paralleldrahtsystem als Messinstrument in der Kurzwellentechnick". Hochfrequenztechnik und Electroakustik, 41, p.2, January 1933.

3. King, R. "Eine zussammenfassende Untersuchung uber stehende elektrische Drahtwellen". Ann. der Phys. Folge 5, 7 p. 806 '30.

4. Sterba, J. and Feldman, C. B. "Transmission Lines for Short-Wave Radio Systems." Proc. I.R.E. 20 p. 1163 July '32

5. Shelkunoff, S.A. "The Electromagnetic Theory of Lines and Shields". Bell Sys. Tech.Jour. p. 532, Oct. '34.

6. Terman, F. E. "Resonant Lines in Radio Circuits." Elect. Eng. 53 , p. 1046, July '34

of dielectric into the resonant circuit. Before the experimental work was started, it was decided that it might be possible to use a sample in the form of a ring about one or two centimeters in length and fitting tightly in the space between the line conductors. Such a sample would produce the greatest effect on the losses if placed at the voltage maximum in the line, one quarter wave length from the "sending end". The relative magnitude of the resulting increase in the resonant circuit losses, according to a preliminary calculation, would, it was believed, be adequate for measurements on most of the available materials. In addition, there would be no additional leads required to carry the current to the sample, and the field in the sample should be almost entirely free from edge effects. In fact, the field in the sample should be exactly radial except for the effect of second-order propogation effects and these, it was believed, should be negligible if the space between the line conductor surfaces were kept small.

2d. Description of Original Line

In order to check some of the foregoing conclusions, it was decided that a coaxial tuned line should be built, to be operated from the output of a constant-frequency oscillator. The equivalent reactance of the line was to be altered by changing its length a measured amount, rather than by changing the oscillator frequency. The latter method would have required two high-frequency sources, one being of variable frequency and the other a fixed reference frequency of a highly stable type. In addition, the beat-note frequency would have had to be measured by means of a frequency bridge or some equivalent scheme, since the frequency variation required would have been greater than the audible range. On the other hand, the fixed-frequency method would require the use of only one oscillator of fixed frequency. In addition the measurement of the change in line length to an adequate degree of accuracy would be a relatively simple task, and the mathematical analysis would be simplified since the electrical length of the line between sending end and sample would be constant.

However, the decision to make the line length mechanically variable introduced an important source of possible difficulty. This difficulty was that which might be encountered in reducing, to a negligible quantity, the effects of contact resistance variations. In order to do this, the movable shorting piece was designed in a form intended to maintain the best possible contact with both the inner and outer conductors. The contacts were arranged, as shown in fig. 1, so as to have rather long contact portions, slotted in order that they might press against the line conductors.

The dimensions of the two conductors were chosen so that the radial distance between their current-carrying surfaces would be relatively small. In addition, the outer conductor was made slightly less than an inch in diameter. These dimensions (shown in fig.1) were chosen so as to reduce the importance of second and higher-order modes of propogation in the line.[5] In this way, the fundamental errors involved in the assumptions underlying the usual transmission line theory were reduced to a practical minimum.

The radio frequency energy was introduced into the line by means of a "feeding circuit" mounted at the fixed or "sending end" of the line. This feeding cirćuit consisted of a short section of two-conductor line, shorted at one end and tuned at the other by a small variable condenser. It was coupled to the oscillator by means of a twisted-pair transmission line. The shorted end of the feeding circuit was made of relatively small-diameter wire, projecting through

5. Shelkunoff, S. A. "The Electromagnetic Theory of Lines and Shields".
Bell Sys. Tech. Jour. p. 532, Oct. '34.

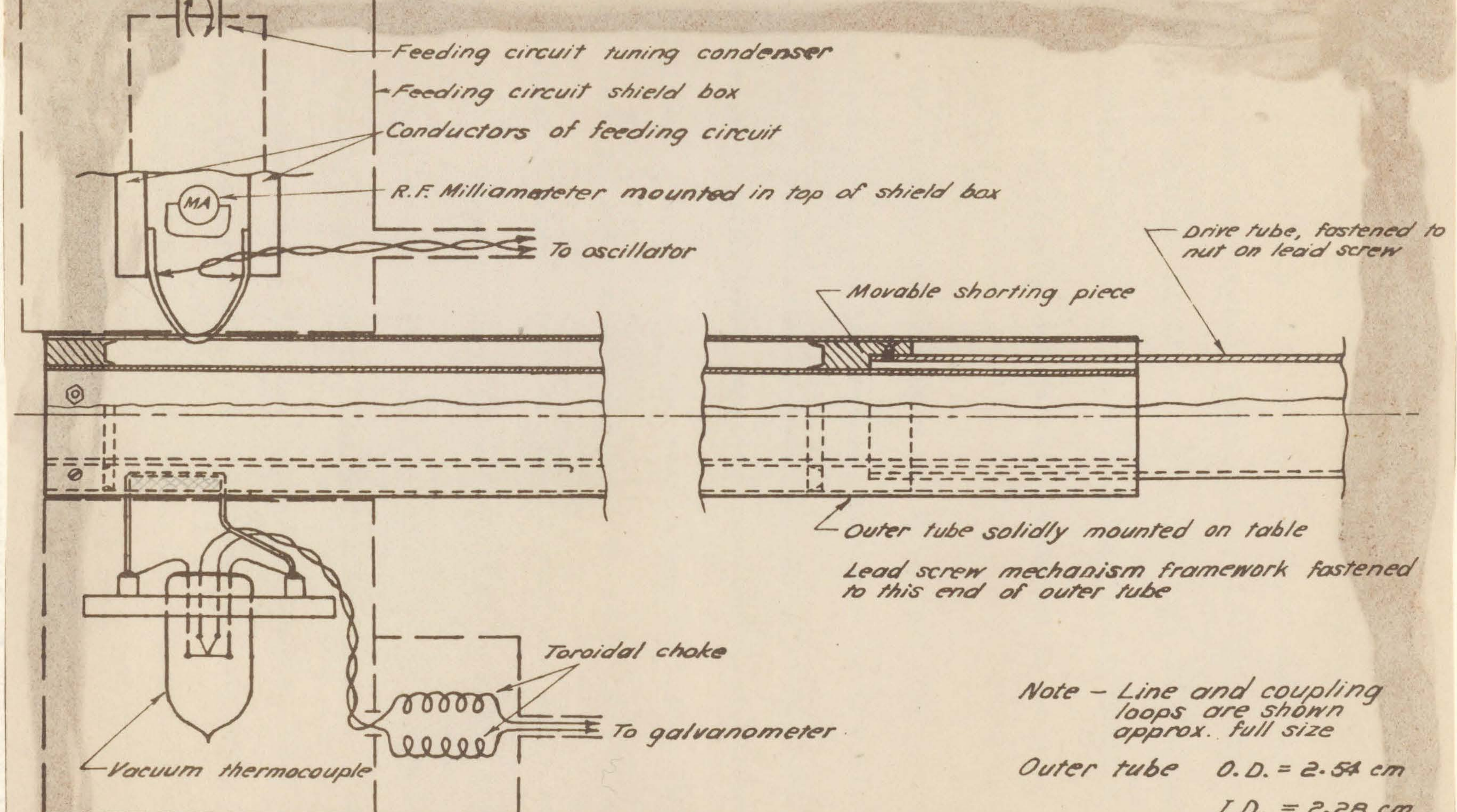

FIG. 1.

SCHEMATIC DIAGRAM OF
THE FIRST ARRANGEMENT OF
LINE AND ASSOCIATED CIRCUITS

a slot into the interior of the outer conductor of the coaxial line. This slot was parallel to the axis of the line and was immediately adjacent to the shorting piece at the fixed end of the line. Consequently, if a constant radio frequency current were flowing in the feeding circuit a voltage of constant magnitude would be introduced at this end of the line.

The current at the same point in the line would then be dependent on the equivalent impedance of the line, as measured from the sending end. In Appendix B, part I, the relations between this impedance and the line losses, line length and constants of the sample are derived. The results show that a loss measurement could be made if the relative magnitude of the current at the sending end could be measured. Consequently, it was decided to mount a thermocouple near the sending end of the line and to employ inductive coupling between line and thermocouple. Then, as the line length was varied through resonance, it would be possible to take readings of thermocouple current, plotting them in the form of a resonance curve. From this curve the line losses could be obtained. Then a similar run would be taken with a sample in place and the difference in loss, attributable to the sample, determined by subtraction.

3. Description of Oscillator

a. Requirements.

Before the proposed line could be tested, it became necessary to obtain a suitable oscillator to act as the high-frequency source. This oscillator had to meet the following conditions:

1. It should be relatively simple to build, since the time available for developing a suitable high-frequency source was limited.

2. Its frequency should be highly stable.

3. Its power output should be relatively large, so that it might be coupled very loosely to the tuned line.

4. It should operate at a frequency of about 100 megacycles.

Some consideration was given to the possibility of picking up a broadcast signal, removing its modulation, and multiplying its frequency in a number of steps, then amplifying the output to a suitable level. However, such a scheme would have required a considerable complication of equipment. The same considerations applied to the alternative of a crystal oscillator, controlling the output of high-frequency amplifier by means of frequency-multiplying stages. Instead, it was decided to construct a simple tuned-grid, tuned-plate push-pull oscillator, controlled by a high Q tuned line in the grid circuit. It was believed that such an arrangement, using Western Electric 304-A tubes, would provide the best compromise between absolute frequency stability and simplicity.

3b. Description of Final Oscillator

Several preliminary models were built using tuned lines in both plate and grid circuits. These were found to be rather critical in adjustment, and undesirable magnetic coupling was believed to exist between the two tuned circuits. In addition, the necessary space required for shielding was rather excessive. Consequently, a single-turn coil and a condenser were substituted in the plate circuit, the tuned line being retained in the grid circuit for frequency control.

A schematic of the oscillator circuit is shown in fig.2, with pictures of the actual arrangement in fig.3. The grid line was made of two 5/8" O.D. copper tubes, spaced 1 1/8" between axes, and mounted vertically in a heavy brass shorting block fastened below the wooden baseboard. The length of the tubing, above the shorting block, was 55.1 cm., the grid connections being approximately 16 cm. from the shorted end of the line. The plate tank circuit originally consisted of

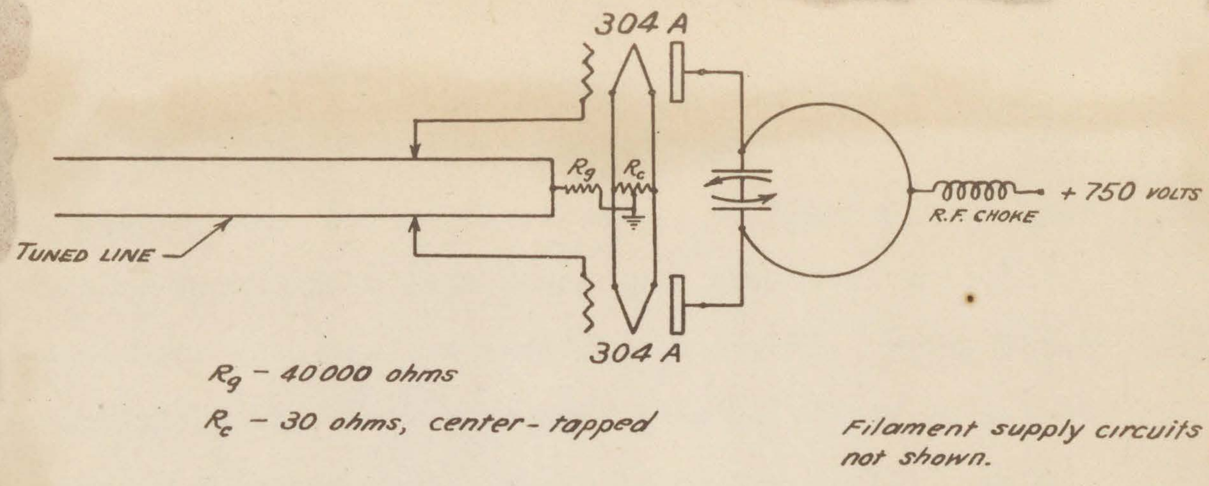

FIG. 2
OSCILLATOR CIRCUIT SCHEMATIC

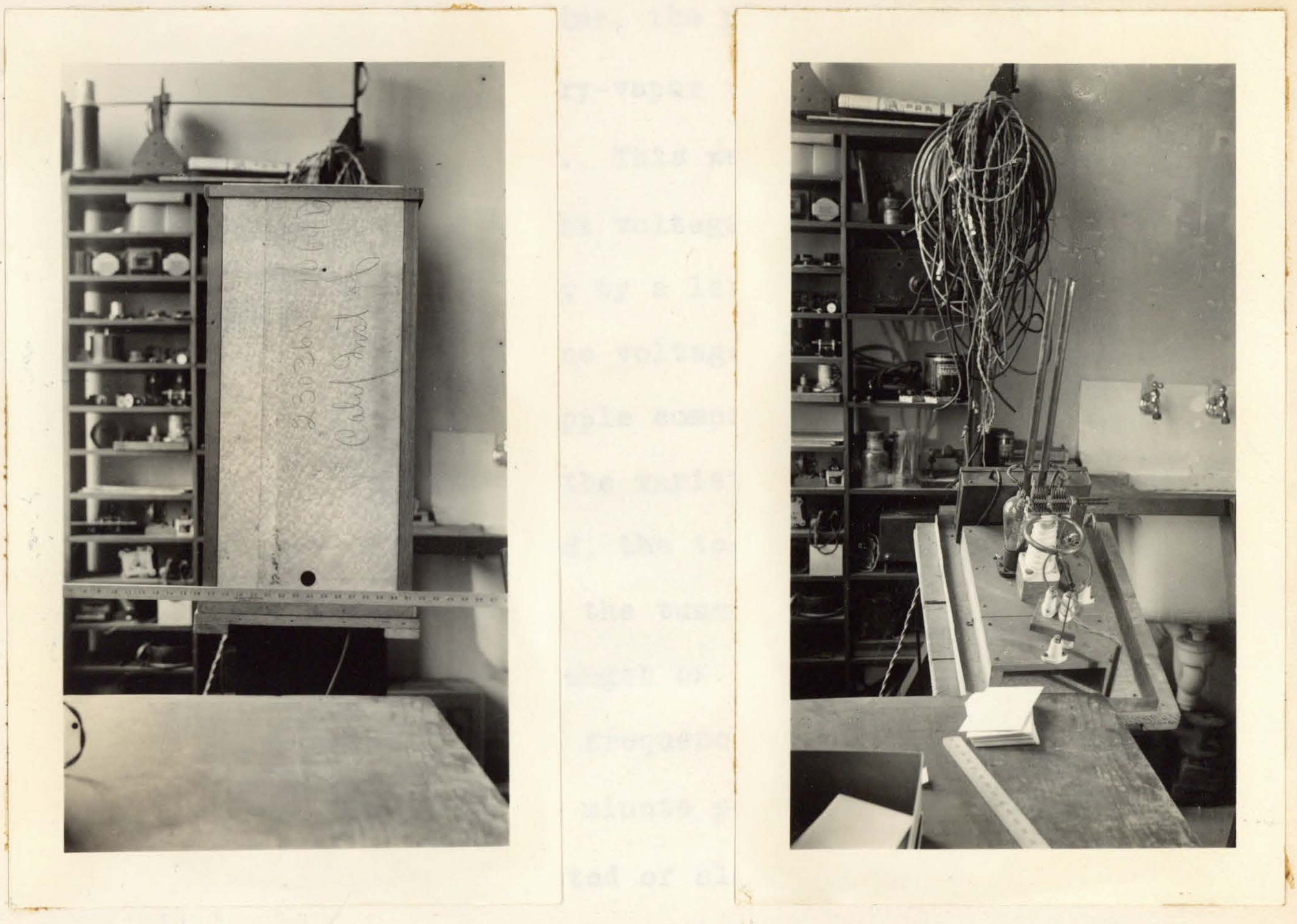

Fig. 3. Oscillator, with and without shield.

a single turn, 9.7 cm in diameter, of 3/16" O.D. copper tubing, tuned by a Hammarlund type MCD-35-x variable condenser with its two sections in series. Under these conditions the oscillator gave a satisfactory output at about 2.9 meters, with 700 volts on the plates. Later the plate coil was replaced by one of about the same mean diameter but made of 3/8" O.D. copper tubing.

As operated for some time, the plate voltage supply consisted of a simple mercury-vapor tube full-wave rectifier and two-section filter. This was later improved by the addition of a vacuum tube voltage regulator, (described in Appendix K) thus reducing by a large factor the possible errors due to changes in line voltage, or modulation of the oscillator output by the ripple component of the plate voltage.

In order to determine the variation in output frequency during the warming-up period, the test recorded in fig.4 was made. In making this test, the tuned line was used to determine the apparent wave-length of the oscillator output. It will be evident that the frequency became practically constant at the end of a 20 minute period. Nevertheless, a standard procedure was adopted of allowing at least one hour for the oscillator to warm up before taking any readings.

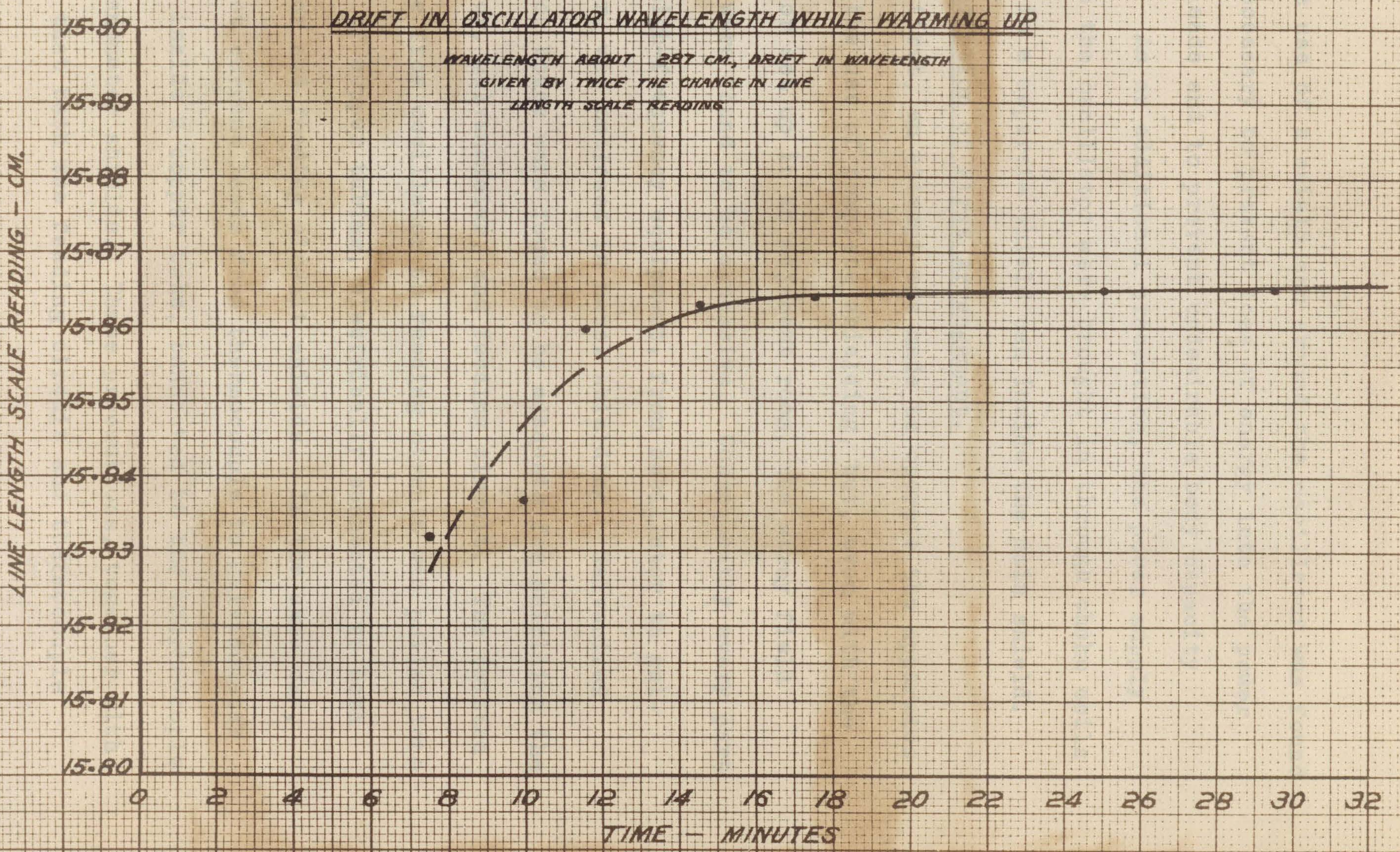

FIG. 4

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The use of a shield can around the oscillator was found to be necessary to prevent loss of energy into the power supply leads and to prevent undesirable body capacity effects. As an additional precaution, the power supply leads to the oscillator were brought in through choke coils mounted in a separate shield can inside the main shield.

4. Description of First Arrangement.

At first, an attempt was made to operate the apparatus without the use of complete shielding, but, even with the oscillator set up in an adjacent room, the stray field effects were found to be excessive. Consequently every part of the high-frequency equipment, from oscillator to galvanometer, was enclosed in metal shielding. Galvanized sheet iron boxes were found to be adequate for shielding the oscillator, galvanometer, and feeding circuit. The twisted pair line from oscillator to feeding circuit was pulled into a length of copper tubing, electrically connected to both oscillator and feeding circuit shields. In the same way, the direct current leads from thermocouple to galvanometer were run inside a copper tube, connected to the thermocouple and galvanometer shields. These precautions, together with the fact that both the feeding circuit and thermocouple

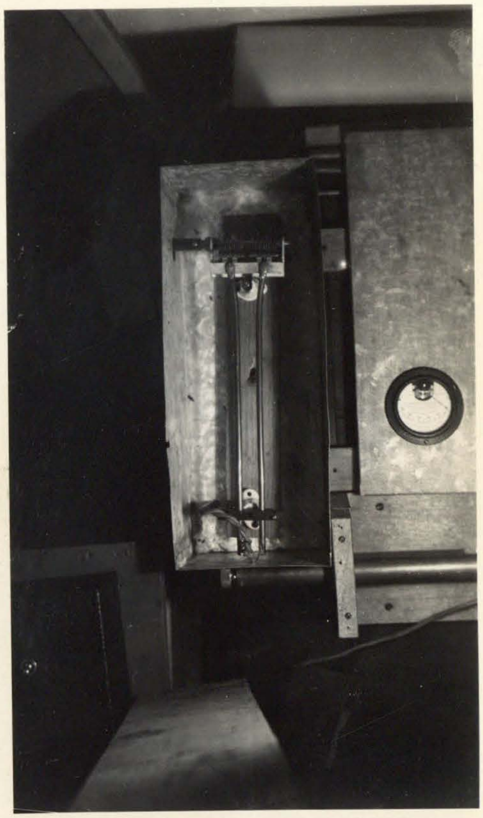


Fig. 5. Feeding Circuit, cover off.

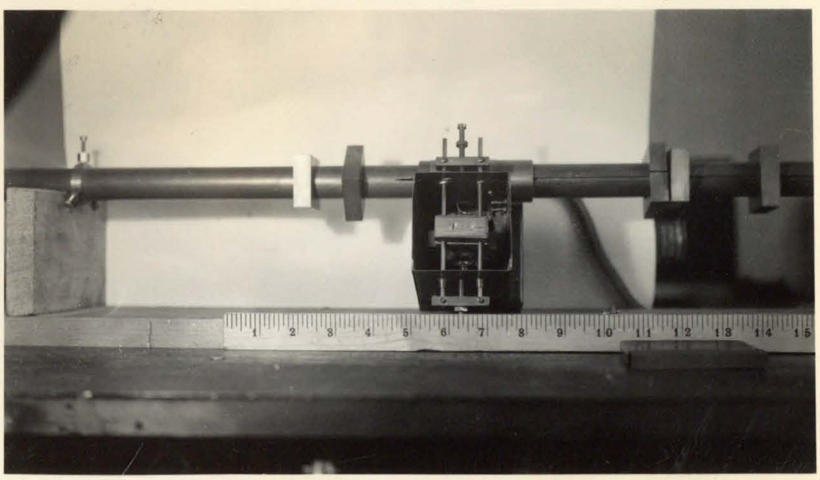


Fig.6. Top View of Thermocoulpe Box, cover off.
(at movable end of line, as in third arrangement)

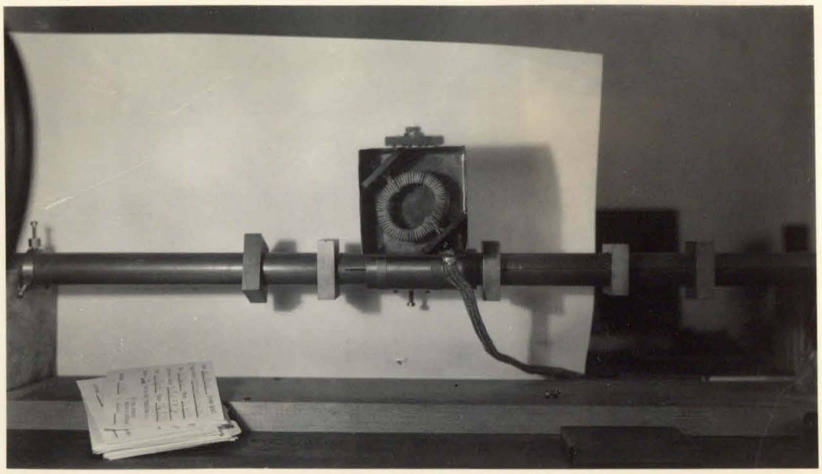


Fig. 6a. Bottom View of Thermocouple Box, showing toroidal choke and flexible shielded lead to galvanometer.

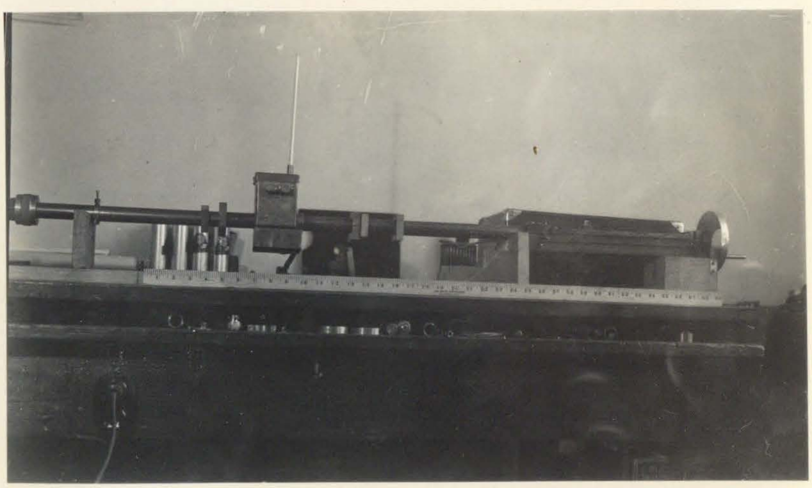


Fig. 7. Movable End of Line, showing metric lead screw mechanism and thermocouple box as used in third and fourth arrangements.

boxes were clamped to the outer conductor of the tuned line, reduced the stray fields to a quite negligible magnitude.

No attempt was made to match the impedance of the twisted-pair feeding line to the oscillator plate impedance. Instead, it was simply terminated in a rigid loop of wire mounted about six inches from the plate tank coil. However, undesirable power loss in the twisted pair was avoided by matching its impedance to that of the feeding circuit. This was done by adjusting the position on the feeding circuit at which the twisted-pair line was connected. In this way, the oscillator was made to produce, with a relatively low input, considerable circulating current in the feeding circuit, when that circuit was adjusted to resonance.

To indicate the relative magnitude of the feeding circuit current, a thermocouple galvanometer having a full-scale reading of 115 m.a. was mounted in the top of the feeding circuit box. It was coupled to the feeding circuit by the mutual inductance between a short loop of wire connected between its terminals and the conductors of the feeding circuit.

The feeding circuit was mounted on a separate baseboard so that its coupling with the coaxial line could be varied by moving the whole feeding circuit horizontally in a direction perpendicular to the axis of the line. As usually used, the coupling loop on the end of the feeding circuit projected only a very small distance into the coaxial line.

The vacuum thermocouple was clamped to a vertical insulating panel to which was fastened the terminals carrying the coupling loop. This panel was fastened at its upper edge to a parallel pair of brass rods, arranged to slide in a direction perpendicular to the axis of the coaxial line. Thus the coupling to the thermocouple could be varied by sliding the thermocouple and its mounting back and forth. Screws were arranged to act as stops, limiting the motion of the thermocouple and its loop, and providing a means for locking it in place when properly adjusted. The thermocouple pick-up loop was self-supporting, being of rectangular shape and made of about 24 gauge copper wire. Its length parallel to the axis of the coaxial line was about one and a half centimeters.

The leads from the couple to the galvanometer were completely shielded by copper tubing and a choke of toroidal form was inserted near the thermocouple. This choke consisted of two self-supporting windings, one connected in each lead and arranged to set up a magnetomotive force in the same direction around the toroid. A small shielding can was provided for this choke.

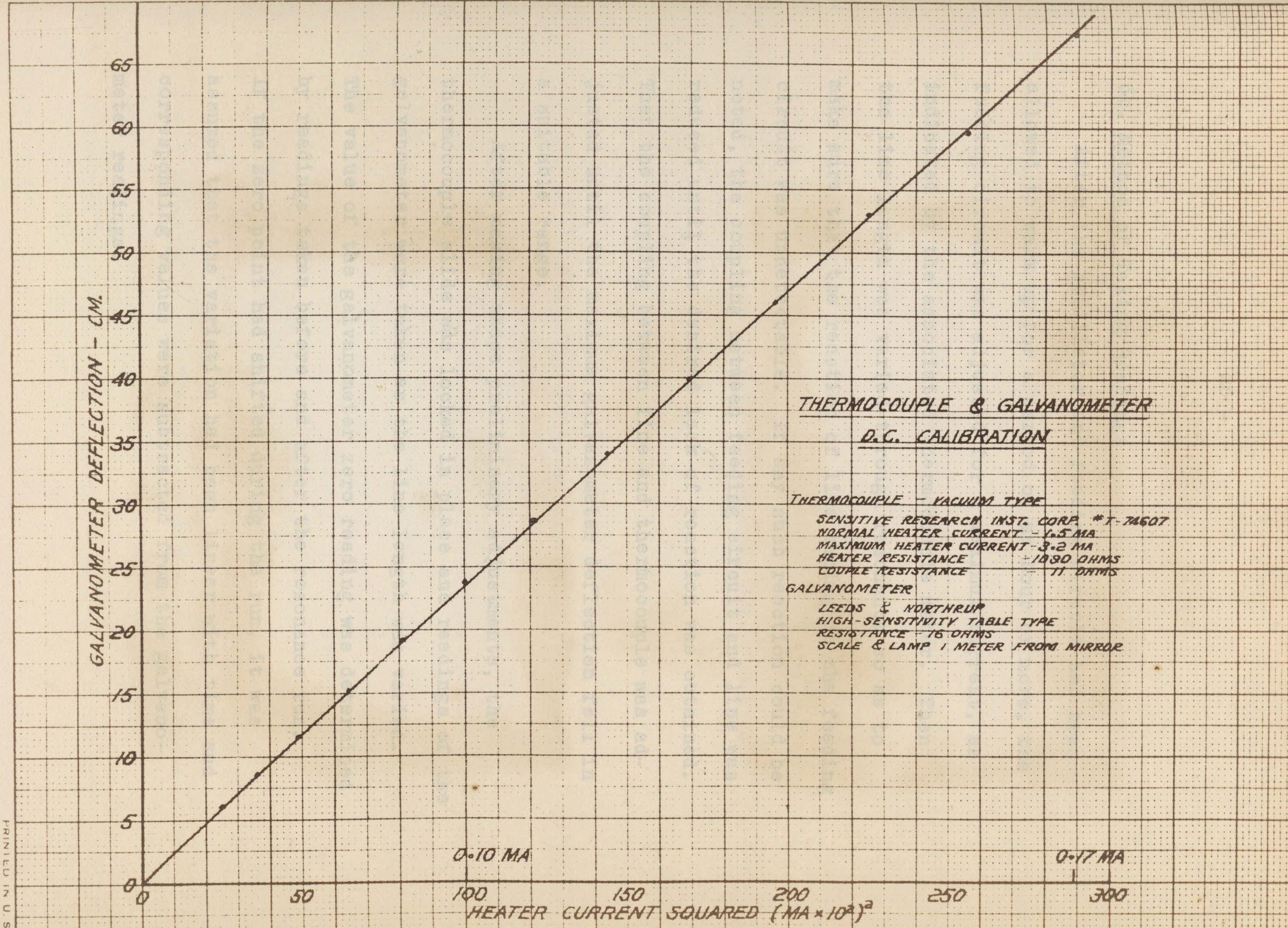
Several mechanical arrangements were used in turn to adjust and measure the length of the line. At first, a simple lead screw of about 16 threads per inch was used in conjunction with a short length of metric scale. Sufficient precision could not be obtained with this simple arrangement, and a cathetometer arrangement was next employed. However, the difficulties encountered in measuring relatively large changes in line length with this apparatus were found to be too great for conveniently rapid work. Consequently, a new lead screw was constructed, having a pitch of two millimeters. It was fitted with a graduated dial, having twenty equally-spaced divisions, thus making it possible to change the line length by as little as one tenth of a millimeter at a time.

5. <u>Results with First Arrangement</u>

a. <u>Calibration of Thermocouple and Galvanometer</u>

The thermocouple and galvanometer arrangement was calibrated without disturbing the leads from couple to galvanometer. In fact, the calibrating arrangement was identical with that used in actual operation, except that the couping loop was removed and the heater was connected in a direct current circuit. Because of the lack of suitable instruments, an alternating current calibration was not made. However, it was found that reversal of the heater current caused no detectable change in the galvanometer reading. It was therefore assumed that the direct current calibration would also hold for alternating currents of low frequency. At very high frequencies, it was realized that the absolute calibration would no longer be valid. However, there seemed to be no reason to doubt that the form of the calibration curve would be unchanged at even the highest frequency. Of course this assumption implied that all the impedances in the thermocouple and galvanometer circuit would be constant at any particular frequency.

As will be evident from the curves of fig. 8, the relation between the square of the heater current and the galvanometer deflection was found to be quite linear over the desired range. In fact, the errors in observations due to drift of the zero point seemed to be much greater than any actual departure from the linear calibration.


THERMOCOUPLE & GALVANOMETER
D.C. CALIBRATION
THERMOCOUPLE - VACUUM TYPE
SENSITIVE RESEARCH INST. CORP. #T-74607
NORMAL HEATER CURRENT - 1.5 MA
MAXIMUM HEATER CURRENT - 3.2 MA
HEATER RESISTANCE - 1080 OHMS
COUPLE RESISTANCE - 11 OHMS
GALVANOMETER
LEEDS & NORTHRUP
HIGH-SENSITIVITY TABLE TYPE
RESISTANCE - 16 OHMS
SCALE & LAMP 1 METER FROM MIRROR
GALVANOMETER DEFLECTION - CM.
0
5
10
15
20
25
30
35
40
45
50
55
60
65
0
50
100
150
200
250
300
0.10 MA
0.17 MA
HEATER CURRENT SQUARED (MA × 10^2)^2
FIG. 8

5b. Method of Taking Readings

After the oscillator and feeding circuit had been allowed to warm up for a period of an hour or more, the feeding circuit was adjusted for a maximum current, as indicated by the associated thermocouple meter. Then the line length was varied through resonance so as to make sure that the reaction of line tuning on the feeding circuit was undetectable. If any such reaction could be noted, the coupling between feeding circuit and line was reduced until the desired lack of reaction was obtained. Then the coupling between line and thermocouple was adjusted until the maximum galvanometer deflection fell in a suitable range.

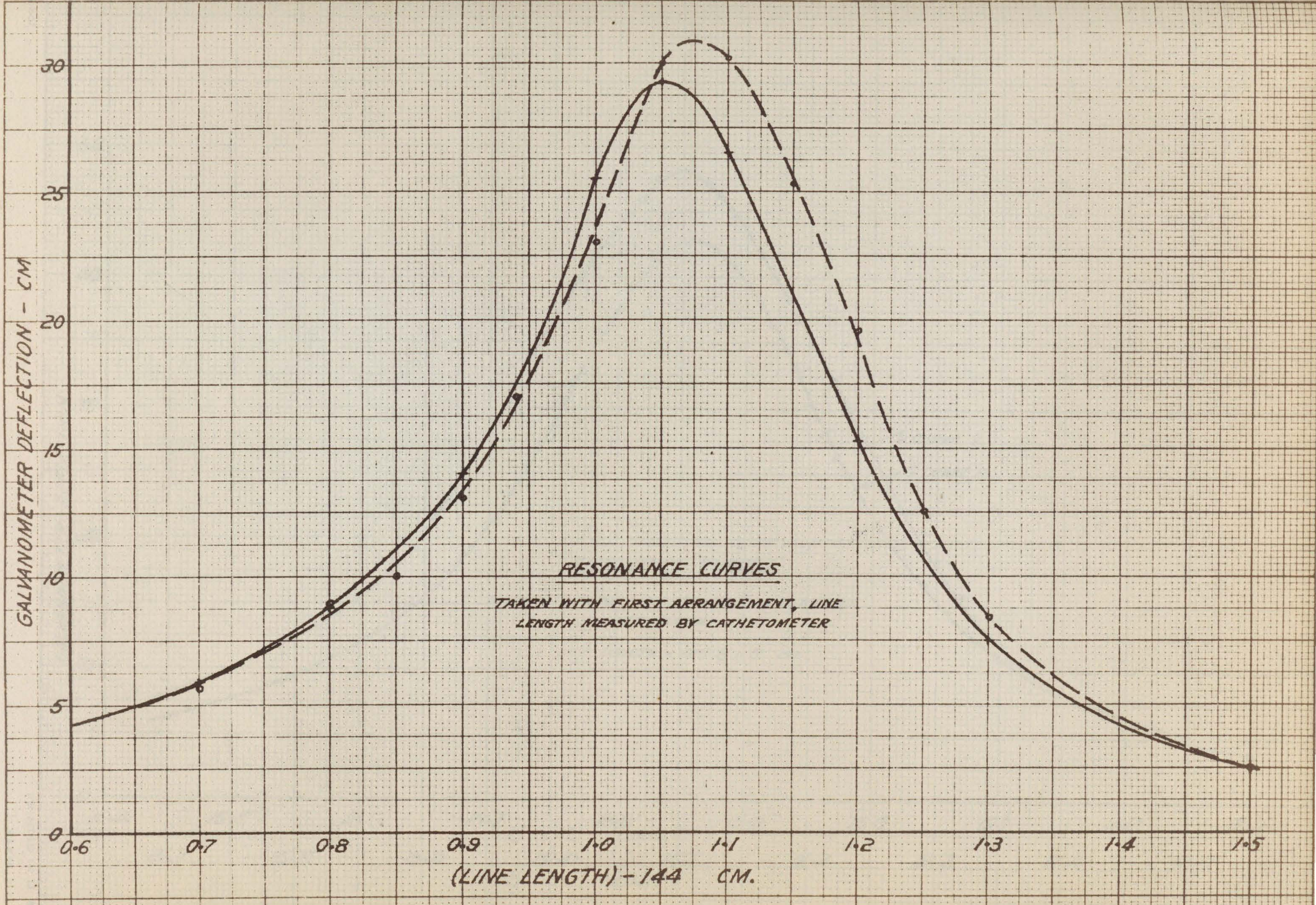
After making these preliminary adjustments, the thermocouple slide was locked in place and readings of the galvanometer were taken as the line length was varied. The value of the galvanometer zero reading was determined by readings taken before and after the resonance run. If the zero point had shifted during the run, it was assumed that the variation had been linear with time and corresponding values were subtracted from the galvanometer readings.

5c. Sample Curves

The runs made with the first lead screw and simple scale did not include sufficient points to form useful curves. However, on replacing this preliminary arrangement by the cathetometer set-up, much better results were obtained, which are illustrated by the sample curves shown in fig.9. When the cathetometer set-up was replaced by the metric pitch lead screw and graduated dial, an even greater improvement was evident, as indicated by the curves of fig. 10. An inspection of these curves will show that the experimental points, taken at intervals of one-tenth millimeter of line length, fell on the smooth curve remarkably well. One or two exceptions will be noted on curve 1 of this figure, but these merely served to indicate the presence of some mechanical backlash in the lead screw system. On locating and correcting this trouble, curves like that of curve 2 were obtained.

5d. Discussion of Results.

Perhaps the most important conclusion drawn from these curves was that the contact resistance at the junctions of the movable shorting piece and the line conductors was relatively constant. At least it did not



GALVANOMETER DEFLECTION - CM
0
5
10
15
20
25
30
0·6
0·7
0·8
0·9
1·0
1·1
1·2
1·3
1·4
1·5
(LINE LENGTH) - 144 CM.
RESONANCE CURVES
TAKEN WITH FIRST ARRANGEMENT, LINE
LENGTH MEASURED BY CATHETOMETER

FIG. 9

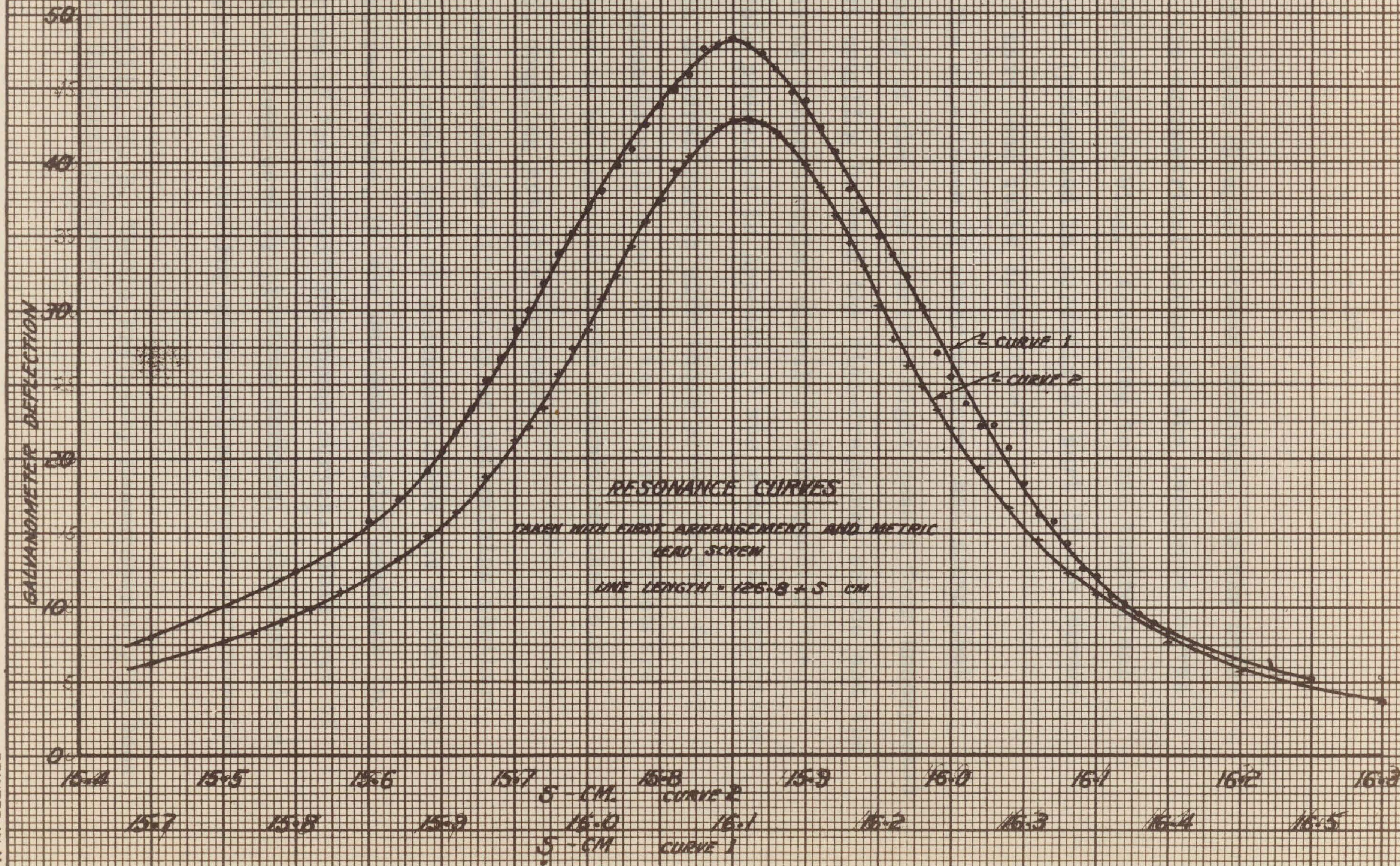

RESONANCE CURVES
TAKEN WITH FIRST ARRANGEMENT AND METRIC
LEAD SCREW
LINE LENGTH = 126.8 + S CM
CURVE 1
CURVE 2
GALVANOMETER DEFLECTION
0
10
20
30
40
50
15.4
15.5
15.6
15.7
15.8
15.9
16.0
16.1
16.2
16.3
S - CM. CURVE 2
15.7
15.8
15.9
16.0
16.1
16.2
16.3
16.4
16.5
S - CM CURVE 1

FIG. 10

suffer sudden changes between readings. On the other hand, the asymmetrical nature of the resonance curves was rather disturbing. According to the theoretical analysis (presented in Appendix B, part I) the curves should have been quite symmetrical.

The experience of other workers[7] had shown that asymmetry of the kind observed might result from unsuspected direct coupling between the high-frequency source and the measuring (thermocouple) circuit. Consequently the analysis presented in Appendix G was made. A comparison of the observed curves with those shown in fig. G.1 of that appendix will show that the observed effects are of the type to be expected on the assumption that such direct coupling actually existed.

Because of the difficulty of determining whether or not the usual type of resonance curve was of exactly the correct shape, it was decided to plot readings in the form of "rectified" resonance curves. This type of curve had been used by previous workers,[7] although these workers had been dealing with coil-and-condenser resonant

7. Moullin, E. B. "Radio Frequency Measurements", (a book) Oxford University Press.

circuits. However, the analysis presented in Appendix C showed that the same method could be used to advantage in connection with the readings taken with the tuned line. In addition, it became evident that the slope of an ideal rectified resonance curve would yield the most accurate value for the losses in the line. If this method were used, the value for the losses would depend, not only on one or several pairs of readings, but on all the readings taken during any one run. In addition, those points obviously in error could be easily located and eliminated from the calculation of the desired results.

The existence of direct coupling from feeding circuit to measuring circuit would alter the shape of the rectified resonance curve as shown in fig. G.2, Appendix G., and in fig.11. Such a curve obviously would not yield the required results as rapidly nor with such probable accuracy as would a curve obtained with negligible direct coupling. In the latter case, the curve should be a straight line.

6. <u>Description of Second Arrangement</u>.

a. <u>Thermocouple Mounting</u>.

In order to avoid the effects of direct coupling,

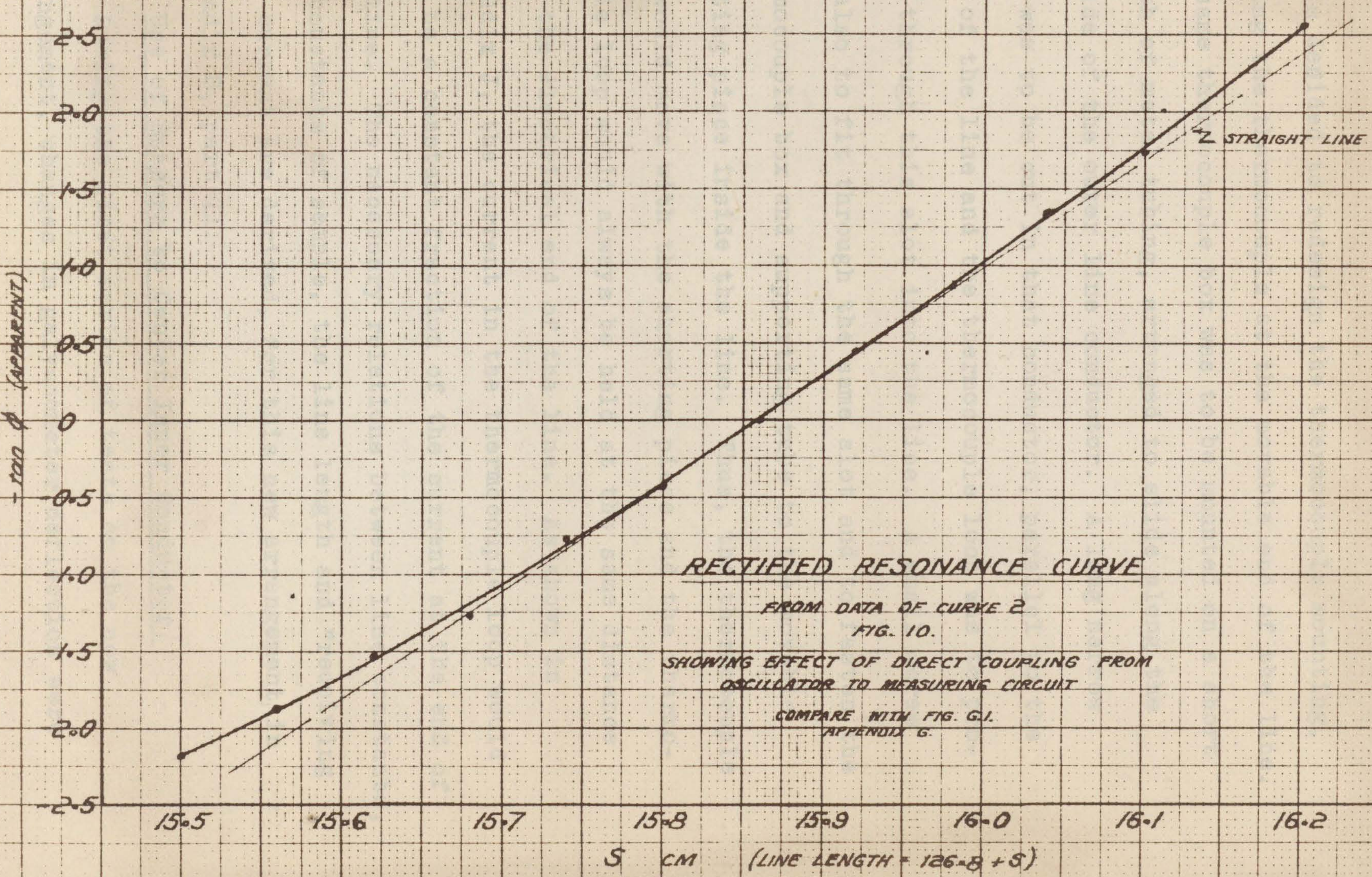
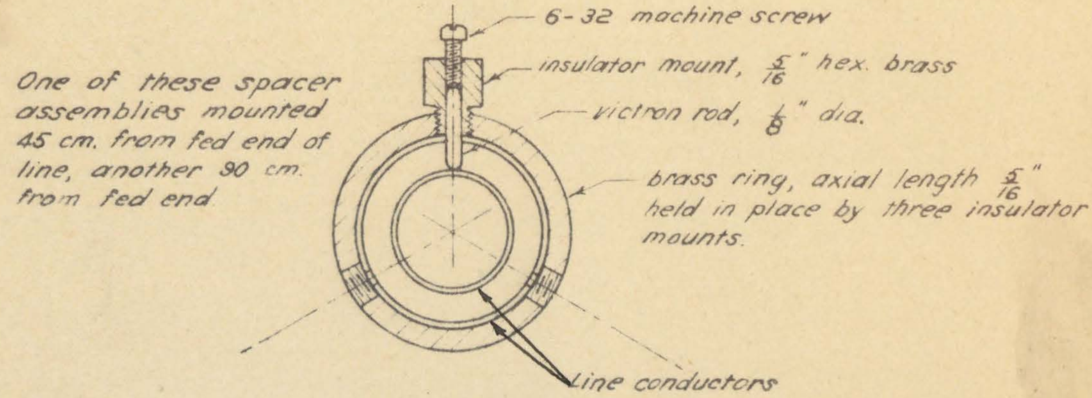

RECTIFIED RESONANCE CURVE
FROM DATA OF CURVE 2
FIG. 10.
SHOWING EFFECT OF DIRECT COUPLING FROM
OSCILLATOR TO MEASURING CIRCUIT
COMPARE WITH FIG. G.I.
APPENDIX G.
STRAIGHT LINE
- tan φ (APPARENT)
2.5
2.0
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
-2.0
-2.5
15.5
15.6
15.7
15.8
15.9
16.0
16.1
16.2
S CM (LINE LENGTH = 126.8 + S)

FIG. 11.

it was decided to redesign the thermocouple mounting, placing the thermocouple at the movable end of the line. The same thermocouple box was to be mounted on a short length of metal tubing, arranged to slide along the outside of the outer line conductor. A long narrow slot was to be cut in that conductor, parallel to the axis of the line and the thermocouple loop was to project through this slot into the line. A small screw was also to fit through the same slot and to fasten the thermocouple box and supporting tube to the movable shorting piece inside the line. Thus, the thermocouple box would move with the shorting piece and the thermocouple loop would always be held at the same distance from the electrical end of the line. As shown in Appendix J, the current in the thermocouple loop would then be a constant fraction of the current at the end of the line. The necessary relations between line constants, the constants of sample, the line length and "receiving end" current are derived, for this new arrangement, in Appendix B, part II.

6b. Use of Spacers to Center Inner Conductor.

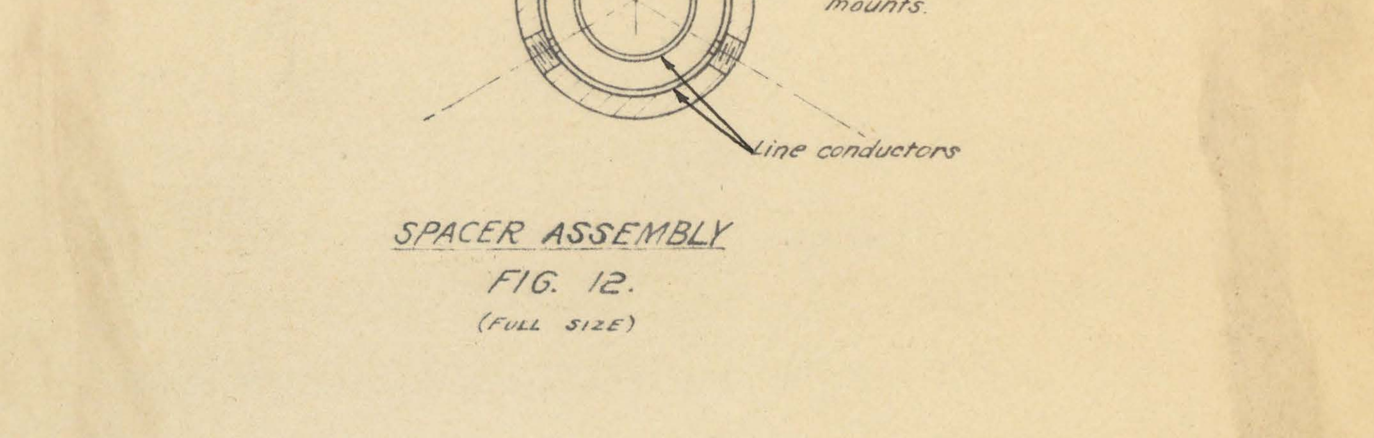
During the first series of tests on the new arrangement, changes in galvanometer deflection were noticed when a slight transverse mechanical force was applied to the outer conductor of the line. This was found to be due to a slight change in the capacitance of the center portion of the line as the axes of the conductors were displaced slightly. To overcome the possibility of erroneous results from this source, two sets of insulating spacers were installed. These were arranged as shown in fig. 12, adjustment of the centering of the inner conductor being accomplished by means of the three radial screws in each spacer assembly. Some difficulty was anticipated in determining the position of the inner conductor, since the presence of the outer conductor would make an actual measurement rather difficult. Actually, however, the anticipated difficulties did not materialize. Instead of attempting a mechanical measurement, the adjustment was made by finding the position of the adjusting screws for which the length of the line at resonance was a maximum. Then the capacitance of the center section of the line was a minimum, showing that the conductors were properly centered. This method proved to be quite sensitive and was used throughout the work on this and the third arrangement of the line.



SPACER ASSEMBLY

FIG. 12.

(FULL SIZE)



SPLIT CENTER SECTION

FIG. 13.

(FULL SIZE)

6c. Split Center Section for Insertion of Samples.

In order to avoid the necessity of taking the line apart each time a sample was inserted or removed, a short section of the outer conductor, one-quarter wave-length from the fixed end of the line was made removable. For this purpose, a section about three inches in length was removed from the outer conductor. Then a new center section of copper was made in the form shown in fig. 13, forming two half-cylinders with inner surfaces to replace the inner surface of the removed section of the outer conductor. Threads were provided at each end on the outer surface so that two nuts fitting over rings fastened on the outside of the line could be used to clamp the new center section in place. This same center section was also used in the third and fourth arrangements, being slightly altered in the latter case so as to provide a mounting for the parallel-plate sample condenser.

In the arrangement under discussion, samples were made in the form of split rings, to the dimensions shown in fig. 14. They were inserted by removing the upper half of the center section and sliding each half of the ring into place between the inner conductor and

TABLE I

Results With Second Arrangment

Sample Material	Length	l_2	m	$R_T \times 10^2$	$\alpha \times 10^5$
Simple line		72.52	6.69	7.16	2.28
" "		72.48	6.57	7.29	2.32
" "		72.47	6.55	7.31	2.32
" "		72.47	6.69	7.16	2.28
			k	$\tan\delta$	$k\tan\delta$ (Loss factor)
Victron	3/8"	71.17	2.36	2.22×10^{-3}	5.24×10^{-3}
"	3/4"	69.46	2.57	1.81×10^{-3}	4.65×10^{-3}
"	3/4"	69.51	2.55	2.58×10^{-3}	6.58×10^{-3}
"	3/4"	69.51	2.55	2.34×10^{-3}	5.96×10^{-3}
"	1"	68.79	2.45	2.01×10^{-3}	4.93×10^{-3}
"	1-1/4"	67.83	2.45	2.04×10^{-3}	5.00×10^{-3}
Redmanol	1.5 cm	67.99	4.0	7.4×10^{-2}	2.9×10^{-1}
Hard rubber	3/4"	69.06	2.8	1.1×10^{-2}	3.1×10^{-2}

Note Calculated value of α for this simple line was 2.08×10^{-5}.

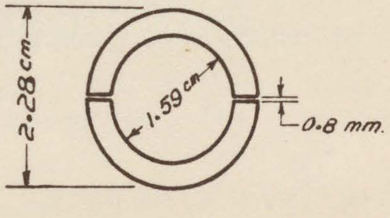


FIG. 14

DIMENSIONS OF SPLIT SAMPLE RINGS

the lower half of the center section. Then the upper half was replaced and the center section clamped by means of the clamping nuts.

Later, the inner conductor was also re-constructed, being made in two lengths arranged to be joined at the center. Then a sample in the form of a solid ring could be used, being inserted when both conductors were separated at the center. After insertion of the sample, the inner conductor was drawn together again and the split center section of the outer conductor replaced.

7. Results with Second Arrangement.

a. Sample Curves.

The results obtained with this arrangement seemed to indicate the complete absence of the effects caused by direct coupling from feeding circuit to thermocouple. The rectified resonance curves departed only slightly from the ideal straight line, as indicated by the sample curves of fig. 15. Furthermore, the accidental deviations from the correct curve were also very small; so small that the imperfections of the lead screw mechanism were clearly reproduced on the curves. This effect is demonstrated in fig. 16, where curve I is a

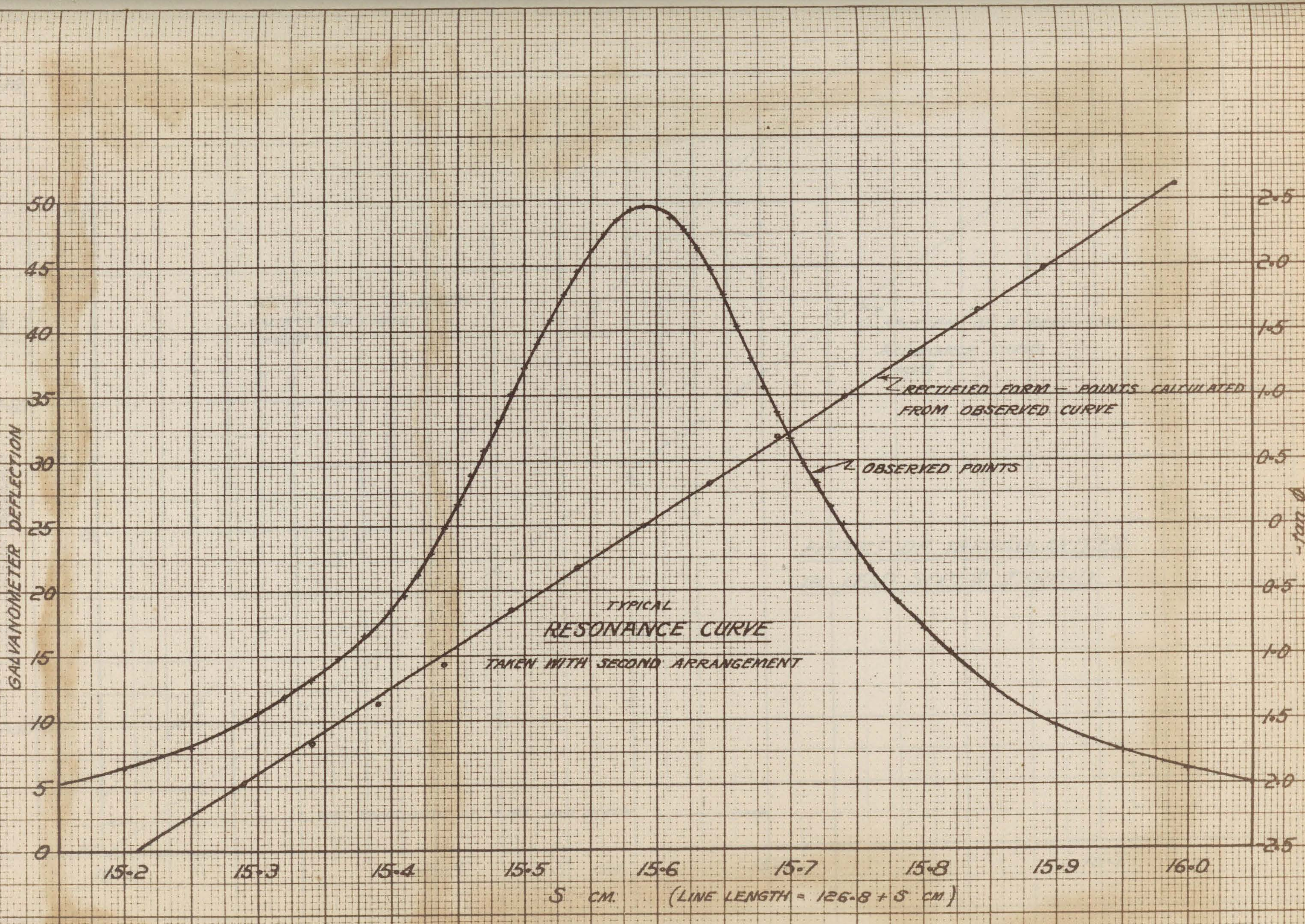


FIG. 15

E-342 (4-30)
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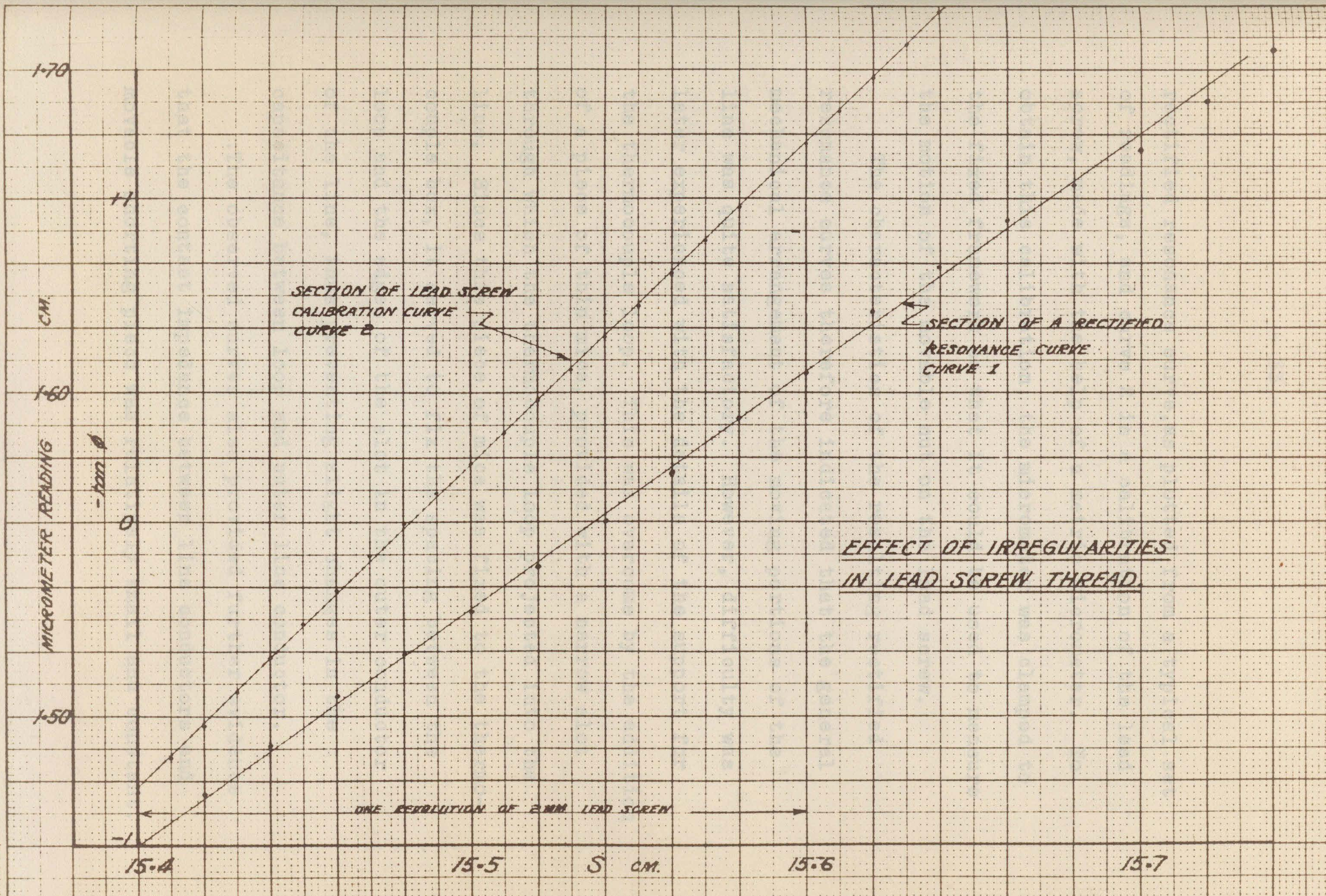


EFFECT OF IRREGULARITIES
IN LEAD SCREW THREAD.
SECTION OF LEAD SCREW
CALIBRATION CURVE
CURVE 2
SECTION OF A RECTIFIED
RESONANCE CURVE
CURVE 1
ONE REVOLUTION OF 2MM LEAD SCREW
MICROMETER READING CM.
-tan ϕ
1·70
1·60
1·50
+1
0
-1
15·4
15·5
15·6
15·7
S CM.

FIG. 16.

rectified resonance curve, as plotted from a typical set of readings, and curve 2 is a calibration of the lead screw, made with the help of a metric micrometer. To obtain this calibration, the micrometer was clamped to the fixed framework so that it could be used to measure the motion of the movable nut on the lead screw.

The characteristics of the resulting rectified resonance curves therefore indicated that the general mechanical arrangement of the moving portions of the line was quite satisfactory. However, difficulty was later experienced with the details of the support for the thermocouple loop. This was overcome by the addition of a piece of thin mica, provided with a narrow slot through which the thermocouple loop projected into the line. Since this piece of mica was fixed to the thermocouple box, it served to fix the spacing between the loop and the edges of the slot in the outer conductor of the line, thus preventing slight changes in the capacitance between loop and outer line conductor.

.The observed curves also provided further evidence that the contact impedance between line conductors and movable shorting piece was relatively small and constant.

The fact that the curves were quite straight showed, at least, that this impedance was very constant during each individual run. Consequently, the assumption became reasonable that the contact impedance could be assumed constant for all positions of the shorting piece.

7b. Measured Values of Line Attenuation Constant.

The object of the first series of tests made with this arrangement was to determine the value of the equivalent transfer resistance R_T of the coaxial line. Then the apparent magnitude of the attenuation factor α could be calculated and compared with the theoretical value.

The equations which formed the basis of these determinations are derived in Appendix B, part II. For the case under consideration here, equations (27) and (28) of the derivation become:

$$X_T = Z_o \frac{2\pi}{\lambda} \Delta l_2 \tag{1}$$

and

$$R_T = Z_o (l_1 + l_2)\alpha \tag{2}$$

But, in this case, $(l_1 + l_2)$ is equal to $\frac{\lambda}{2}$, where λ is the wave-length. Furthermore,

$$\tan\phi = \frac{X_T}{R_T}$$

and, as is demonstrated in Appendix C, the slope of the rectified resonance curve is proportional to the constant ratio, $\tan\phi/X_T$. However, the actual rectified resonance curves were plotted in terms of Δl_2, and it therefore became necessary to substitute from equation (1) above. When this substitution was made, the slope m was given by,

$$m = \frac{\tan\phi}{\Delta l_2} = \frac{Z_o 2\pi}{\lambda} \cdot \frac{1}{R_T}$$

or,

$$R_T = \frac{Z_o 2\pi}{\lambda} \frac{1}{m} \qquad (3)$$

Also,

$$\alpha = \frac{4\pi}{\lambda^2} \frac{1}{m} \qquad (4)$$

Consequently, in order to determine R_T and α it was only necessary to measure the slope m of the curve of $\tan\phi$ vs. Δl_2. The value of the characteristic impedance Z_o could be calculated quite accurately from the dimensions of the line and λ was taken as twice the actual length of the line at resonance. This length was compared to that of a standard Lecher wire system and was found to agree with the distance between nodes on the Lecher wires to within a millimeter.

The first step in determining the points on the rectified resonance curve was to determine the galvanometer deflection θ_r corresponding to the maximum point on the resonance curve. This was found, during the early part of the work, by plotting the top portion of the actual resonance curve, but later an analytical method, suggested by L. W. Baldwin, was adopted. Then, for each reading in the set, the value of $\tan\phi$ was determined from the relation

$$\tan\phi = \sqrt{\frac{\theta_r}{\theta} - 1} \qquad (5)$$

(see appendix C).

The slope m of the resulting curve was measured and substituted in equations (3) and (4). In a number of cases, the most probable slope was found by the method of least squares but this refinement was found to be unnecessary since the slope could be determined to within the observational error by simply fitting a straight line to the calculated points by eye.

Some of the values obtained for R_γ and α in this way have been tabulated in Table I. The values of α should be compared with the calculated value of α for a line of the same dimensions, on the assumption that the shorting pieces were of zero impedance. This calculated value was 2.08×10^{-5}.

7.c. Tests on Samples of Dielectric.

The next series of tests was made to determine the dielectric constant and power factor angle for various samples of insulation. These samples included one of redmanol, a second of hard rubber, and several of victron. The actual procedure in taking readings was essentially the same as that for the line alone. However, in this group of tests, each determination was based on two sets of readings, one with the line alone and the other with a sample in place. From the first run, the value of α was determined, in order that the losses due to the line itself could be calculated for the conditions of the second run.

With the sample in place, the line equations assumed the form,

$$X_T = Z_o \frac{2\pi}{\lambda \sin \frac{2\pi l_2}{\lambda}} \qquad (5)$$

$$R_T = Z_o \sin \frac{2\pi l_2}{\lambda} \left(1 + \frac{2\pi \Delta l_2}{\lambda} \cot \frac{2\pi l_2}{\lambda}\right) \left\{ \alpha \left[l_1 + l_2 \left(\cot^2 \frac{2\pi l_2}{\lambda} + 1\right) \right] + \frac{Z_o}{R} \right\} \qquad (6)$$

as given in equations (27) and (28), Appendix B. Since the total line-length for resonance was only slightly less than one-half wave-length, the term in cot $\frac{2\pi l_2}{\lambda}$, appearing in the expression for R_T, could be neglected. Thus, in terms of the slope m of the rectified resonance curve,

$$R_T = Z_o \frac{2\pi}{\lambda \sin \frac{2\pi l_2}{\lambda}} \cdot \frac{1}{m}$$

or $$\alpha\left[l_1 + l_2\left(\cot \frac{2\pi l_2}{\lambda} + 1\right)\right] + \frac{Z_o}{R} = \frac{2\pi}{\lambda \sin^2 \frac{2\pi l_2}{\lambda}} \cdot \frac{1}{m}$$

or $$\frac{Z_o}{R} = \frac{2\pi}{\lambda \sin^2 \frac{2\pi l_2}{\lambda}} \cdot \frac{1}{m} - \alpha\left[l_1 + l_2\left(\cot \frac{2\pi l_2}{\lambda} + 1\right)\right] \tag{7}$$

Knowing α, $l_1 \left(= \frac{\lambda}{4}\right)$, l_2, λ and m it was therefore possible to calculate Z_o/R and thus to determine the resistance R of the sample.

The tangent of the power factor angle of the dielectric in a simple condenser is given by the well-known expression

$$\tan \delta = \frac{1}{RC\omega}$$

where C is the capacitance and R the effective parallel resistance of the condenser. The magnitude of R, for the particular samples employed, could be determined as outlined above. It remained to evaluate the quantity C. This was not the same as the capacitance C appearing in the analysis of Appendix B. In that case, C represented the increase of capacitance at the voltage loop on the line, as produced by the presence of the sample. Consequently it was only (k-1)/k as large as the capacitance required in the above expression. When using the method under discussion, therefore, the above expression should be written:

$$\tan\delta = \frac{1}{RC\omega} \cdot \frac{k-1}{k} \qquad (8)$$

where C has now the same meaning as in the equations of Appendix B. The actual expression used in calculations of $\tan\delta$ was:

$$\tan\delta = \frac{Z_o}{R} \cdot \frac{1}{Z_o C\omega} \cdot \frac{k-1}{k} \qquad (9)$$

In this form, the first two factors were obtained respectively from (7) above and from the condition of resonance, when

$$Z_o C\omega = \cot\frac{2\pi l_2}{\lambda} \qquad (10)$$

The value of k was also determined from equation (10) under the assumption that edge effects were negligible and consequently that the following expression for the increase of capacitance due to the sample would be valid:

$$C = \frac{(0.241)(k-1)\,l_s \times 10^{-12}}{\log_{10}\frac{b}{a}} \text{ farads}$$

Thus, in terms of the measured quantities,

$$(k-1) = \frac{\cot \frac{2\pi l_2}{\lambda} \log_{10} \frac{b}{a} \times 10^{12}}{(0.241)\, Z_0\, \omega\, l_s} \qquad (11)$$

where b was the outer radius and a the inner radius of the sample, and l_s was the axial length of the sample in centimeters.

Some of the results of these preliminary tests have been tabulated in Table I.

8. Description of Third Arrangement.

In the course of the tests made with the second arrangement, it was realized that the introduction of a sample of victron produced rather a small increase in loss. As a result, the accuracy to be expected from measurements on victron could not be very high. To overcome this difficulty, it was decided to reduce the diameter of the inner conductor, while retaining the same dimensions for the sample. The analysis presented in Appendix D had shown that this should increase the sensitivity of the arrangement, provided that the ratio of conductor diameters was not made greater than about 9.2.

For mechanical reasons, this optimum ratio was not even approached. Instead, the inner conductor diameter was reduced to 5/16 inch, giving a ratio of 2.88, as compared with the former ratio of 1.44. According to the theory of Appendix D, this should have resulted in a sensitivity 5.32 times as great as that previously obtained. However, in order to realize this increase of sensititivity, it was necessary to provide means for the use of samples of the same dimensions as those previously used. A center section, made of copper to the dimensions of fig. 17 was therefore made to slip over the inner conductor. The sample was placed at approximately the mid-point of this center section, the position of the center section on the inner conductor being adjusted so that the sample was at the quarter-wave point.

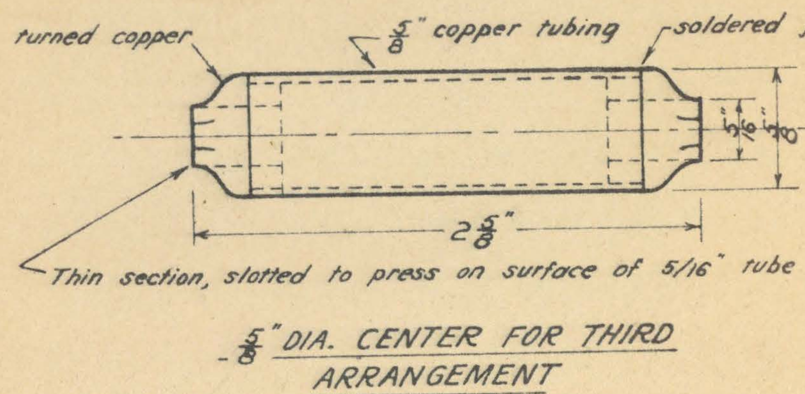
In addition, certain improvements were made in the design of both the fixed and the movable shorting pieces. These were made of copper, rather than brass, and the thin-section contact flanges were made longer. As in the previous work, vaseline was used sparingly

TABLE II

Results With Third Arrangment

Material	Sample Length	l_2	m	$R_T \times 10^2$	$\alpha \times 10^5$
Simple line		70.80	12.41	11.2	1.23
" "		70.80	12.39	11.2	1.23
5/8" cemter section only		61.27	12.58	10.9	-
		61.27	12.61	10.9	-
Victron			k	tan δ	ktan δ
"	1/4"	58.66	2.53	1.23×10^{-3}	3.08×10^{-3}
"	1/4"	58.62	2.53	1.87×10^{-3}	4.67×10^{-3}
"	1/4"	58.64	2.53	1.87×10^{-3}	4.67×10^{-3}
"	1/2"	56.39	2.45	1.66×10^{-3}	4.15×10^{-3}
"	3/4"	53.82	2.49	1.64×10^{-3}	4.10×10^{-3}

Note Calculated value of α for this simple line was 1.14×10^{-5}.



5/8" DIA. CENTER FOR THIRD ARRANGEMENT

FIG. 17.

on the movable surfaces and was found to be effective in reducing contact impedance variations.

9. Results with Third Arrangement.

Three series of tests were made with this arrangement. The first comprised a determination of the equivalent transfer resistance and attenuation factor for the simple line only. The second comprised similar tests with the 5/8" diameter center section in place, and the third included tests on various samples of victron. Some of the results are given in Table II .

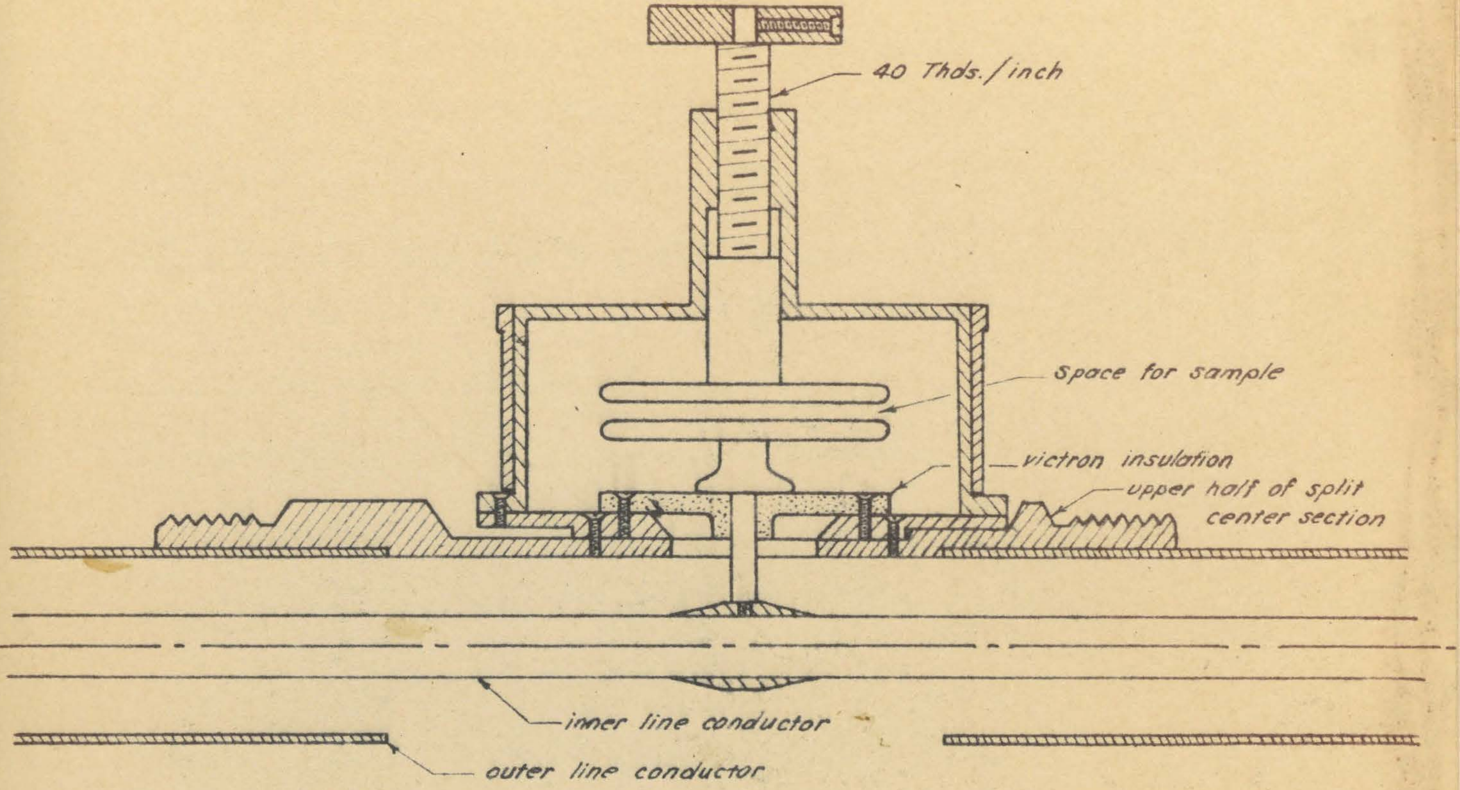
The calculations involved in obtaining these results were quite similar to those used with the second arrangement. Only one important difference should be noted. The increase of capacitance due to the sample had to be calculated from three sets of readings, rather than two. The second series of tests, on the application of equation (10) above, gave the increased capacitance due to the copper center section. Then the third series gave the increase due to both center section and sample, and, on subtraction of the result of the second series, the contribution of the sample itself was determined. This was then substituted in equations of the form of equations (9) and (11) above, thus obtaining the power factor and dielectric constant of the sample.

10. Description of Fourth Arrangement.

In order to check the results obtained with the ring samples, it was decided to construct a shielded, parallel-plate sample condenser. One terminal of this condenser was to be grounded to the outer conductor and shield, while the other was to be connected to the inner line conductor at the quarter-wave point. By the use of this arrangement, it was hoped that it would be possible to avoid at least one of the possible errors inherent in all the previous arrangements. This particular error might have arisen from the necessity of calculating the actual line losses during a test on a sample from measurements made when the line length was slightly greater than with the sample in place. In the new arrangement, it would be possible to so adjust the spacing between the condenser plates that the line length for resonance would be the same, with and without the sample. Then the current distribution and consequently the inherent conductor losses would be almost exactly the same, whether or not the sample was in place.

The actual construction of the sample condenser and its mounting is illustrated by the cross-section of fig.18 and the picture shown in fig.19. As will be evident from those figures, the cylindrical shield for the condenser was made of two snugly-fitted coaxial brass cylinders. Each of these cylinders was provided with a large opening so that the sample (a flat circular disc) could be inserted when the outer cylinder was rotated until the openings coincided. Then the outer cylinder could be rotated until the opening was completely closed, when excellent shielding was obtained.

Both electrodes of the sample condenser were made in the form of circular discs 1 1/2" in diameter with their plane faces parallel and horizontal. The upper disc was mounted on a vertical threaded spindle, which rotated in a threaded bearing mounted on the top of the shield box. On the other hand, the lower disc was mounted on a fixed spindle, connected to the inner line



SECTION THROUGH SAMPLE CONDENSER

FIG. 18.

(Full Size, some minor dimensions approx.)

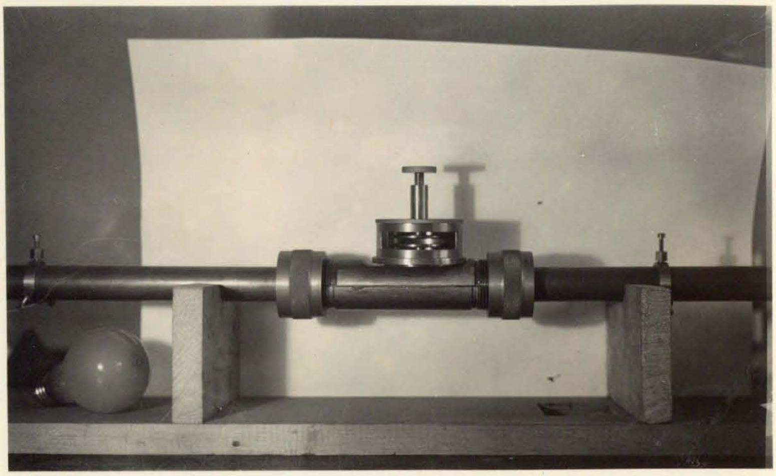


Fig. 19. Sample Condenser in Place on Line.

conductor and solidly supported by a small victron insulating piece.

11. Results with Fourth Arrangement.

The tests and calculations made with this arrangement were quite similar to those made with the third arrangement. The first curve in each set was taken with the sample in place. Then the sample was removed and the sample condenser adjusted so that resonance occurred at the same line length setting as with the sample in place. In order to check the value of the dielectric constant, some curves were taken with the sample condenser set so that the separation between the plates was the same as the thickness of the sample.

Because of the new procedure made possible by this arrangement, the calculations were somewhat simplified. Equation (7), above, could be re-written as:

$$\frac{Z_o}{R} = \frac{2\pi}{\lambda \sin^2 \frac{2\pi l_2}{\lambda}} \left(\frac{1}{m_1} - \frac{1}{m_2} \right) \qquad (12)$$

where m_1 and m_2 are the slope of the rectified resonance curves obtained respectively with and without the sample. The capacitance of the sample could be calculated by

TABLE III

Results with Fourth Arrangment

The followings results were obtained from tests made on a single disc sample of victron, 0.995" dia. and 0.066" thick.

l_2	m_1	k	$\tan\delta$	$k\tan\delta$
48.54	10.55	2.51	1.51×10^{-3}	3.78×10^{-3}
48.48	10.61	2.48	1.37×10^{-3}	3.43×10^{-3}
48.26	10.35	-	1.41×10^{-3}	3.53×10^{-3}
46.92	10.50	-	1.37×10^{-3}	3.43×10^{-3}

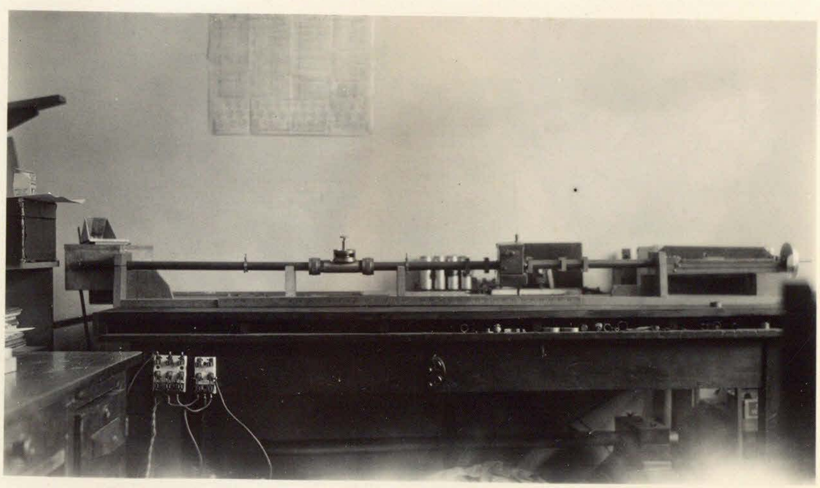


Fig. 20. Final Line Assembly.

the well-known simple formula for the capacitance of a parallel plate condenser, neglecting edge effects. This was made possible by the use of a thin sample somewhat smaller in diameter than the electrodes. In addition, the tests made to check the value of the dielectric constant were found to confirm these calculations.

12. Discussion of Results.

The tests made on the victron samples with the second arrangement indicated that the loss in the sample was only a small fraction (about 7% for three 1/4" rings) of the line conductor loss. Measurements of such small changes in total loss obviously could not be expected to yield results of sufficient precision. To overcome this disadvantage, the smaller inner conductor of the third arrangement was adopted and the loss in the sample was increased to about 27% of that in the line conductors, for three 1/4" victron rings. The fourth or final arrangement was actually not quite as satisfactory in this respect as the third arrangement, although the relative loss in the sample could have been increased from its observed value of about 20% by the use of a larger diameter or thinner sample.

However, several possible sources of error existed in the preliminary arrangements which must have been eliminated, or greatly reduced, in the final arrangement. The first of these has already been mentioned. By adjusting the sample condenser so that resonance occurred at the same line length, with or without the sample, the distribution of current in the line conductors should have been kept the same for each set of readings. Thus any change in actual line conductor losses arising from the insertion of the sample should have been eliminated. In addition, the necessity of calculating the conductor loss from transmission line theory and a measurement without the sample no longer existed.

One of the probable effects of the ring samples, as used in the first three arrangements, on the distribution of current in the line conductors is suggested by a comparison of the results obtained with the different arrangements. It will be noticed that these results gave a lower value for the apparent loss factor of the material in each successive arrangement. This may be explained, at least qualitatively, on the hypothesis that the higher apparent loss in the first arrangements was partially due to increased line conductor losses with the sample in place. Such an increase could have resulted from a re-distribution of current over the surfaces of the conductors, caused by incomplete contact between conductors and sample. It was found to be an exceedingly difficult task to machine the samples accurately enough to ensure a tight fit with both conductors. Consequently there is no doubt that some re-distribution of conductor currents must have taken place.

Some confirmation of this hypothesis is provided by the lower apparent loss factor obtained in the tests made on the third arrangement. There, the greater spacing between the line conductors reduced the effect of errors in centering the inner conductor. Also the use of a removable center section made it possible to fit the solid ring samples much more accurately. Both these changes should have reduced the effect of the sample on the distribution of current around the surface of the line conductors.

In the final arrangement, this effect must have been still further reduced. In that case, the insertion of the sample could only affect the distribution of current, and consequently the loss, in the faces of the electrodes. For this reason, and for the reasons already discussed, the values obtained with the final arrangement are considered to be the most nearly correct values so far obtained by this method.

The deviation of individual results from the mean was still rather high, however. Undoubtedly, the use of a movable thermocouple pick-up loop and the consequent necessity for a long slot in the outer conductor were partially responsible for these discrepancies. The cutting of the slot reduced the rigidity of the outer tube to such an extent that the clamping pieces, visible in the photographs of the line, were required to bring the outer conductor back to its correct diameter. These could not be placed sufficiently close to the movable shorting piece to prevent small variations in outer conductor diameter over the range of motion of the shorting piece, even for a single set of readings.

Furthermore, the thermocouple and galvanometer arrangement was somewhat unsatisfactory. Very slight changes in room temperature caused rather considerable changes in calibration factor and zero reading. It would therefore be desirable to adopt some more stable form of current or voltage indicator.

13. Suggestions for Future Apparatus.

The desirability of the following changes in the design of the equipment was suggested by the experience outlined above.

1. The radii of the line conductors should be increased by, say, a factor of two. This would reduce the line losses by the same factor, while maintaining the same characteristic impedance. Thus the sensitivity of the arrangement would be increased, while maintaining the same resonant line length. The mechanical rigidity of the line would also be improved, and an increase in the length of the contact flanges on the shorting pieces would be made practicable. The ratio of line spacing to line length would still be small enough to prevent serious effects of second and higher order modes of propogation.

2. The lower plate of the sample condenser should be supported on more rigid insulation than the present

victron. Probably some low-loss ceramic insulation could be used to advantage. In addition, the contact surfaces of the electrodes should be ground plane and parallel, and the samples should be ground to fit accurately between them.

3. The possibilities of a peak voltmeter, using a tube of the "acorn" type, to replace the thermocouple and galvanometer, should be investigated. Such a voltmeter could be arranged in a small shielded box at the same point on the line as the sample condenser, and could be coupled to the inner line conductor, or lower condenser plate, by a small capacitance. All the high-frequency portions of an arrangement of this kind could be mechanically rigid, thus avoiding the mechanical difficulties encountered with the use of a movable pick-up loop. Furthermore, the calibration and zero-reading of this type of voltmeter should be quite constant. Of course, the line equations would have to be re-written in terms of the voltage at the quarter-wave point.

14. Conclusion.

The results presented above indicate that the transmission line method under consideration could be used for measurements on materials having power factors as low as 2×10^{-4}. The final arrangement, modified according to the suggestions in the preceding section would certainly fulfill all the conditions set up in Section 2a, with the possible exception of conditions 6 and 7. However, the use of disc samples has long been established as standard practise in measurements of this kind. Consequently, the influence of edge-effects has been investigated rather thoroughly[8].

The only other condition which might not be met completely by this method is that concerned with the determination of the magnitude of the field in the sample. With the thermocouple and galvanometer as a current indicator, an approximate determination of this field strength might be made by calibrating the pick-up arrangement at broadcast frequencies. Then only a simple calculation would be required to convert the galvanometer readings into the corresponding values for the voltage at the sample. However, if a peak voltmeter were substituted for the thermocouple and galvanometer, the voltage at the sample would be

8. See, for example, Hartshorn, L. and Ward, W. H., "The Measurement of the Permittivity and Power Factor of Dielectrics at Frequencies from 10^4 to 10^8 Cycles per Second". J.I.E.E. 79, p.597, Nov. '36.

given directly by the meter readings. Experience with such voltmeters has shown that their calibration is sufficiently accurate even at frequencies of 2×10^8 cycles per second.

15. Acknowledgments.

A very grateful acknowledgment is due to all the members of the Staff in the Department of Electrical Engineering, for their encouragement and assistance during the progress of this work. In addition, much of the experimental work was done by G. Grimm, B. M. Oliver and L. W. Baldwin, who were associated successively with this research while carrying on their graduate studies. Mr. J. Millen, of the National Company, Malden, Mass., also assisted by supplying samples of victron for test.

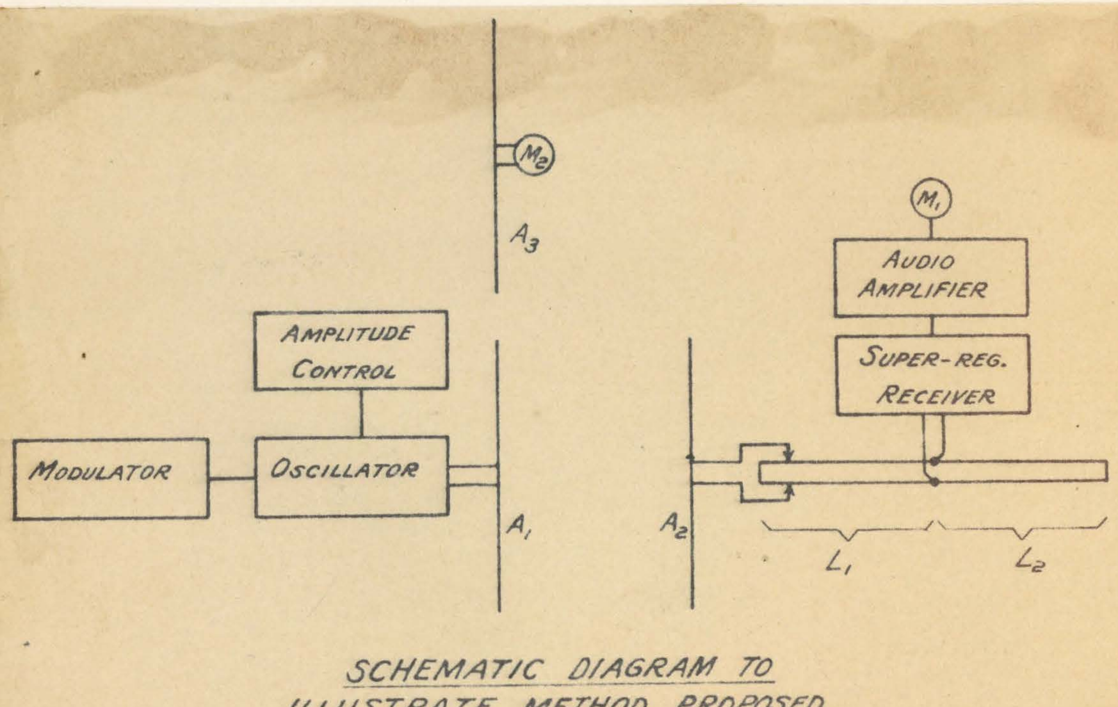
Appendix A.

Method Proposed by A. V. Haeff.

The essentials of the proposed method will be evident from the schematic diagram of fig. A.1. A modulated oscillator of constant frequency and percentage modulation, but of variable output, was to be arranged to supply energy to the antenna A_1, a portion of the resulting radiation being picked up by antenna A_2 and applied to a tuned line resonant circuit L_1. At the voltage maximum on this line, there was to be connected another tuned line L_2, together with a super-regenerative detector and audio amplifier, the latter provided with an audio frequency output meter M_1. This meter was to be used only to ensure that the maximum voltage on the line L_1 should remain constant during each set of readings. The actual determination of dielectric loss was to depend on the readings of the thermocouple meter M_2 in the auxiliary antenna A_3, together with the calculated value of the loss in a "standard" transmission line.

The standard line was first to be connected in the position indicated by L_2 and readings taken of the meters M_1 and M_2. Then the standard line was to be replaced by a line containing the sample of insulation, and the reading of M_1 returned to its previous value by increasing the transmitter output. The ratio of the loss in the line containing the sample and that in the standard line would then have been given by the two readings of the meter M_2.

Knowing the calculated value of the loss in the standard line, the total loss in the line containing the sample could then have been found by use of the observed power ratio. It would only remain to calculate the residual loss in the line containing the sample and thus to determine the loss in the sample itself.



SCHEMATIC DIAGRAM TO ILLUSTRATE METHOD PROPOSED BY A.V. HAEFF

FIG. A.1.

Appendix B.

Mathematical Analysis of Tuned Transmission Line and Effect of Sample.

Part I - Input Impedance of Half-Wave Line.

a. Without a Sample.

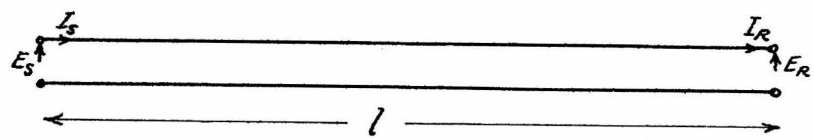


Fig. B.1.

The general equations for a four-terminal network may be written

$$E_s = A\ E_R + B\ I_R \qquad (1)$$
$$I_s = C\ E_R + D\ I_R$$

In the case of a transmission line, the constants A, B, C and D may be expressed in terms of the transmission line constants as follows:

$$A = \cosh\theta$$
$$B = Z_o \sinh\theta \qquad (2)$$
$$C = \frac{1}{Z_o}\ \sinh\theta$$
$$D = \cosh\theta$$

where $\theta = \gamma l$, the electrical angle of the line

$\gamma = \alpha + j\beta$, the propogation constant

α is the attenuation constant

β is the phase constant

l is the physical length of the line

Z_o is the characteristic impedance.

When the line is short-circuited at the receiving end, E_R becomes zero and equations (1) become

$$E_s = B\, I_R$$

$$I_s = D\, I_R$$

Consequently, the sending end impedance is

$$Z_s = \frac{E_s}{I_s} = \frac{B}{D} = Z_o \tanh \theta \qquad (3)$$

On substitution from the above definitions this becomes

$$Z_s = Z_o \tanh(\alpha + j\beta) l$$

$$= Z_o \frac{\sinh(\alpha l + j\beta l)}{\cosh(\alpha l + j\beta l)} \cdot \frac{\cosh(\alpha l - j\beta l)}{\cosh(\alpha l - j\beta l)}$$

$$= Z_o \frac{\sinh(2\alpha l) + \sinh(j2\beta l)}{\cosh(2\alpha l) + \cosh(j2\beta l)}$$

$$= Z_o \frac{\sinh 2\alpha l \quad j \sin 2\beta l}{\cosh 2\alpha l + \cos 2\beta l} \qquad (4)$$

Now, in the line under consideration here, α is very small, being of order 10^{-5}. Also, l is of order 10^{2}, hence αl is of order 10^{-3}.

Typical values are

$$\alpha = 1.2 \times 10^{-5}$$
$$l = 145 \text{ cm}$$
$$\alpha l = 1.74 \times 10^{-3}$$

Hence $\sinh 2\alpha l$ may be set equal to $2\alpha l$ with an error of less than one in 100,000. Similarly $\cosh 2\alpha l = 1$ within one in 10000. Making these approximations and simplifying;[6]

$$Z_s = Z_o\left(\frac{\alpha l}{\cos^2\beta l} + j\tan\beta l\right) \tag{5}$$

Now let

$$R_e = \frac{Z_o\alpha l}{\cos^2\beta l}$$

$$X_e = Z_o\tan\beta l \tag{6}$$

$$|Z_s| = \sqrt{R_e^2 + X_e^2}$$

Near resonance, $l = \frac{\lambda}{2} + \Delta l$, and, since at high frequencies $\beta = \frac{2\pi}{\lambda}$ with negligible error, X_e may be written

$$X_e = -Z_o\tan\frac{2\pi\Delta l}{\lambda} \tag{7}$$

where λ is the wave-length and Δl is the difference between the actual line length and one half wave-length.

If Δl is sufficiently small:

$$X_e = -Z_o\frac{2\pi}{\lambda}\Delta l \tag{8}$$

In other words, X_e is directly proportional to Δl.

As the length of the line is varied around the resonant length, there is evidently a small variation in R_e. To determine the magnitude of such variation, the expression for R_e may be differentiated with respect to l.

$$\frac{dR_e}{dl} = \frac{Z_0\alpha}{\cos^2\beta l} + \frac{2Z_0\alpha l\beta\sin\beta l}{\cos^3\beta l} \tag{9}$$

$$= \frac{Z_0\alpha l}{\cos^2\beta l}\left\{\frac{1}{l} + 2\beta\tan\beta l\right\}$$

therefore $\frac{dR_e}{R_e} = \frac{\Delta l}{l}$

at resonance, where $\tan\beta l = 0$

Consequently, if the total line length l is approximately 145 cm and if $\Delta l = \pm .3$cm., the percentage change in R_e throughout the range of variation of cannot be greater than 0.2%. Within these limits, R_e can evidently be regarded as a constant.

The input impedance of the line, near resonance, can therefore be regarded as a simple series combination of a practically constant resistance and a reactance which is directly proportional to Δl, the difference between the actual length of the line and a half wave-length. If the sending end voltage is held constant, the sending end current will therefore vary according to the familiar resonance curve as Δl is taken from its negative to its positive maximum. The interpretation of this resonance curve in terms of the line losses is discussed in Appendix C.

b. With a Sample.

It will be assumed that the sample is inserted in such a way that it may be represented as a lumped capacitance and parallel resistance connected across the line one-quarter wave-length from the sending end. Then the complete resonant system may be considered as two lines connected as shown in fig. B.2.

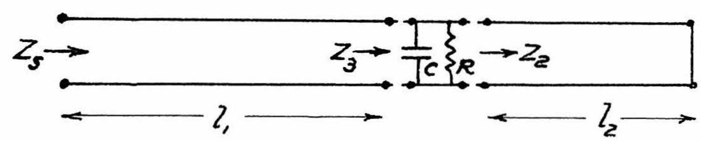


Fig. B.2.

The first line is of length $l_1 = \frac{\lambda}{4}$, while the second is of variable length l_2 and is shorted at its receiving end. The two lines are connected through the network representing the impedance of the sample.

The impedance Z_2, looking into the second line, will be

$$Z_2 = Z_0 \tanh \theta_2 \tag{10}$$

Looking into the sending-end terminals of the network representing the sample, the impedance Z_3 will be given by

$$\frac{1}{Z_3} = \frac{1}{R} + jC\omega + \frac{1}{Z_2}$$

$$= \frac{Z_2 + R + jC\omega Z_2 R}{RZ_2}$$

or $$Z_3 = \frac{RZ_2}{R + Z_2 + jC\omega Z_2 R} = \frac{R}{\frac{R}{Z_2} + 1 + jC\omega R} \tag{11}$$

To find the sending end impedance of the complete circuit, it will be necessary to consider the input impedance of a quarter-wave transmission line terminated by the impedance Z_3. This can best be found by considering the general equations for a transmission line, as follows:

$$E_{S_1} = \left(A_1 + \frac{B_1}{Z_3}\right) I_{R_1}$$

$$I_{S_1} = \left(C_1 + \frac{D_1}{Z_3}\right) I_{R_1}$$

or $$Z_S = \frac{A_1 \quad \frac{B_1}{Z_3}}{C_1 \quad \frac{D_1}{Z_3}}$$

$$= \frac{A_1 Z_3 + B_1}{C_1 Z_3 + D_1}$$

$$= \frac{Z_3 \cosh\theta_1 + Z_0 \sinh\theta_1}{\frac{Z_3}{Z_0}\sinh\theta_1 + \cosh\theta_1} \tag{12}$$

In this particular case, $\theta_1 = (\alpha + j\beta)\frac{\lambda}{4}$ or

$\theta_1 = \alpha l_1 + j\frac{\pi}{2}$. Consequently the following simplifications may be introduced:

$$\cosh\theta_1 = \cosh\left(\alpha l_1 + j\frac{\pi}{2}\right) = \cosh\alpha l_1 \cos\frac{\pi}{2} + j\sinh\alpha l_1 \sin\frac{\pi}{2}$$

$$= j\sinh\alpha l_1 \doteq j\alpha l_1$$

$$\sinh\theta_1 = \sinh\left(\alpha l_1 + j\frac{\pi}{2}\right) = \sinh\alpha l_1 \cos\frac{\pi}{2} + j\cosh\alpha l_1 \sin\frac{\pi}{2}$$

$$= j\cosh\alpha l_1 \doteq j$$

Therefore

$$Z_s = \frac{Z_3 j\alpha l_1 + jZ_o}{j\frac{Z_3}{Z_o} + j\alpha l_1} = Z_o \frac{Z_o + \alpha l_1 Z_3}{Z_o \alpha l_1 + Z_3} \qquad (13)$$

$$\doteqdot Z_o \left\{ \frac{Z_o}{Z_3} + \alpha l_1 \right\} \qquad (14)$$

In order to justify this last approximation it will be necessary to estimate the relative magnitudes of $Z_o \alpha l_1$ and Z_3. The former is approximately, for the line under consideration,

$$63.5 \times 1.2 \times 10^{-5} \times 73 = 5.6 \times 10^{-2} \text{ ohms.}$$

In the case of the same line, at resonance,

$$Z_3 = \frac{R}{\frac{R}{Z_o \tanh \theta_2} + 1 + jC\omega R}$$

$$= \frac{R}{1 + \frac{R}{Z_o} \coth \theta_2 + jC\omega R}$$

$$= \frac{R}{1 + j\frac{R}{Z_o}\left\{\omega Z_o C - \cot \beta l_2\right\} + \frac{\alpha l_2}{\sin^2 \beta l_2}}$$

$$= \frac{R}{1 + \frac{\alpha l_2}{\sin^2 \beta l_2}} \qquad (15)$$

Now $\frac{\alpha l_2}{\sin^2 \beta l_2}$ will always be much less than unity, hence at resonance Z_3 may be considered simply equal to

R. Depending on the particular sample under test, R will lie between 1000 and 20000 ohms. Consequently the error introduced into the denominator of expression (13) for Z_1 by neglect of $Z_0 \alpha l_1$ could never amount to more than say, one part in 10,000. Hence it is sufficiently accurate to write

$$Z_s = Z_0 \alpha l_1 + \frac{Z_0^2}{Z_3}$$

$$= Z_0 \alpha l_1 + \frac{Z_0^2}{R}\left\{\frac{R}{Z_2} + 1 + jC\omega R\right\}$$

$$= Z_0\left\{\alpha l_1 + \frac{\alpha l_2}{\sin^2 \beta l_2} + \frac{Z_0}{R} + j\left(Z_0 C\omega - \cot \beta l_2\right)\right\}$$

Here the line losses are represented by the terms αl_1 and $\frac{\alpha l_2}{\sin^2 \beta l_2}$, both of which will remain practically constant as l_2 is varied over the range necessary to obtain a satisfactory resonance curve.

The manner in which the reactance term in this input impedance will vary as the line length is varied near the resonance length may be found by considering the Taylor's series for $\cot \beta l_2$ about various values of βl_2. This series may be written

$$\cot \beta(l_2 + \Delta l_2) = \cot \beta l_2 - \frac{\beta \Delta l_2}{\sin^2 \beta l_2} + (\beta \Delta l_2)^2 \frac{\cot \beta l_2}{\sin^2 \beta l_2}$$

$$- \frac{(1 + 3\cot^2 \beta l_2)(\beta \Delta l_2)^3}{3 \sin^2 \beta l_2} + \ldots\ldots$$

$$= a_0 - a_1 (\beta \Delta l_2) \quad a_2 (\beta \Delta l_2)^2 - a_3 (\beta \Delta l_2)^3 \qquad (17)$$

The coefficients a_1, a_2 and a_3, together with the actual magnitude of the corresponding terms of the series for $\Delta l_2 = 0.4$ cm are given in Table B.I. In the case of the final transmission line, the values of Δl_2 necessary to obtain a satisfactory resonance curve were much less than this, usually being less than ± 0.2 cm. Thus the values given in the table are definitely pessimistic. However, it will be observed that the deviation from a linear relationship between reactance and Δl_2 is still negligibly small for a value of Δl_2 as great as 0.4cm, and even for relatively small values of βl_2. Consequently a sufficiently accurate expression for the reactance part of Z_s is

$$X_s = Z_0 C\omega + \frac{\beta \Delta l_2}{\sin^2 \beta l_2} \qquad (18)$$

Table B. I.

(Values in the last three columns are for $\Delta l_2 = 0.4$ cm.)

βl_2	a_1	a_2	a_3	$a_1(\beta\Delta l_2)$	$a_2(\beta\Delta l_2)^2$	$a_3(\beta\Delta l_3)^3$
$\frac{\pi}{2}$	1	0	.333	8.8×10^{-3}	0	2.2×10^{-7}
$\frac{\pi}{3}$	1.34	.666	.889	11.7×10^{-3}	5.1×10^{-5}	6.0×10^{-7}
$\frac{\pi}{4}$	2.0	1.414	2.66	17.5×10^{-3}	10.3×10^{-5}	17.8×10^{-7}
$\frac{\pi}{6}$	4.0	3.46	13.34	35.0×10^{-3}	26.6×10^{-5}	89×10^{-7}

Appendix B

Part II Sending End-Receiving End Transfer Impedance of a Half-Wave Line, With a Sample

Because of the difficulty of avoiding direct coupling between feeding circuit and thermocouple loop, it was found to be necessary to place the thermocouple loop at the receiving end of the line. Thus it became necessary to determine the relation between sending end voltage and receiving end current. The following analysis was therefore made for the transfer impedance

$$Z_T = \frac{E_{S_1}}{I_{R_2}} \tag{19}$$

The numerical subscripts refer to the two transmission lines of fig. B.2.

For this arrangement,

$$I_{R_1} = I_{S_2} + E_{S_2}\left(\frac{1}{R} + jC\omega\right)$$

$$E_{R_1} = E_{S_2}$$

And, since the receiving end of line 2 is short-circuited,

$$E_{R_2} = 0$$

and

$$E_{S_2} = B_2 I_{R_2}$$

$$I_{S_2} = D_2 I_{R_2} \tag{20}$$

Making use of the A, B, C, D line constants,

$$E_{S_1} = A_1 E_{R_1} + B_1 I_{R_1}$$
$$= A_1 B_2 I_{R_2} + B_1 \left\{ I_{S_2} + E_{S_2}\left(\frac{1}{R} + jC\omega\right)\right\}$$
$$= \left\{ A_1 B_2 + B_1 D_2 + B_1 B_2 \left(\frac{1}{R} + jC\omega\right)\right\} I_{R_2}$$

Hence $$Z_T = A_1 B_2 + B_1 D_2 + \frac{B_1 B_2}{R} + jB_1 B_2 C\omega \qquad (21)$$

Now since line 1 is one-quarter wave-length long,

$$A_1 = \cosh\theta_1 \doteq j\alpha l_1$$
$$B_1 = Z_0 \sinh\theta_1 \doteq jZ_0$$

Therefore

$$Z_T = j\alpha l_1 Z_0 \sinh\theta_2 + jZ_0\cosh\theta_2 + j\frac{Z_0^2}{R}\sinh\theta_2 - Z_0^2 C\omega \sinh\theta_2 \qquad (22)$$

And, since $\theta_2 = \alpha l_2 + j\beta l_2$

$$\sinh\theta_2 = \sinh\alpha l_2 \cosh j\beta l_2 + \cosh\alpha l_2 \sinh j\beta l_2$$
$$\doteq \alpha l_2 \cos\beta l_2 + j\sin\beta l_2$$
$$\cosh\theta_2 = \cosh\alpha l_2 \cosh j\beta l_2 + \sinh\alpha l_2 \sinh j\beta l_2$$
$$\doteq \cos\beta l_2 + j\alpha l_2 \sin\beta l_2$$

Substituting these approximations,

$$Z_T = -Z_0\alpha l_1 \sin\beta l_2 - Z_0^2 C\omega\alpha l_2 \cos\beta l_2 - \frac{Z_0^2}{R}\sin\beta l_2 - Z_0\alpha l_2 \sin\beta l_2$$
$$+ j\left\{\left(Z_0\alpha^2 l_1 l_2 + \frac{Z_0^2}{R}\alpha l_2 + Z_0\right)\cos\beta l_2 - Z_0^2 C\omega \sin\beta l_2\right\}$$
$$\doteq \left[-\left\{\alpha\left[l_1 + l_2(Z_0 C\omega\cot\beta l_2 + 1\right) + \frac{Z_0}{R}\right\} + j\left\{\cot\beta l_2 - Z_0 C\omega\right\}\right] Z_0 \sin\beta l_2 \qquad (23)$$

In the latter expression, the terms $Z_0\alpha^2 l_1 l_2$ and $\frac{Z_0^2}{R}\alpha l_2$ have been neglected in comparison to Z_0. This approximation may be justified by considering a typical case for the transmission line actually used. In that case,

$$\alpha = 1.2 \times 10^{-5}$$
$$l_1 = 72 \text{ cm.}$$
$$l_2 = 70 \text{ cm.}$$
$$Z_0 = 63.5 \text{ ohms.}$$
$$R = 1000 \text{ ohms}$$

Then $\alpha^2 l_1 l_2 = 7.25 \times 10^{-7}$ and $\frac{Z_0 \alpha l_2}{R} = 5.35 \times 10^{-5}$

both of which are to be compared with unity. Consequently the above approximation is exceedingly good.

In order to put this expression in a more convenient form for calculation, it will be desirable to write

$$Z_T = R_T + jX_T$$

where $R_T = \left\{\alpha\left[l_1 + l_2(Z_0 C\omega \cot\beta l_2 + 1)\right] + \frac{Z_0}{R}\right\} Z_0 \sin\beta l_2$

and $$X_T = Z_0\left\{\cos\beta l_2 - Z_0 C\omega \sin\beta l_2\right\} \qquad (24)$$

The negative sign in front of the real part of expression (23) is disregarded since it merely implies a reversal in phase, with which this analysis is not concerned.

The reactance component X_T may be expanded about its zero value as follows:

$$X_T = -Z_0\left\{(\beta\Delta l_2) + \frac{(\beta\Delta l_2)^3}{3!} + \frac{(\beta\Delta l_2)^5}{5!} + \ldots\ldots\right\}(\sin\beta l_2 + Z_0 C\omega \cos\beta l_2) \qquad (25)$$

For this value of l_2, $Z_0 C\omega$ is equal to $\cot\beta l_2$ and substituting this value in (25),

$$X_T = \frac{-Z_0}{\sin\beta l_2}\left\{(\beta\Delta l_2) + \frac{(\beta\Delta l_2)^3}{3!} + \frac{(\beta\Delta l_2)^5}{5!} + \cdots\cdots\right\} \tag{26}$$

Since $\beta\Delta l_2$ has a maximum value less than 10^{-2}, the second and succeeding terms in the series are quite negligible in comparison with the first. Consequently the value of X_T is given to a sufficiently accurate degree of approximation by

$$X_T = -\frac{Z_0\,\beta\Delta l_2}{\sin\beta l_2} \tag{27}$$

Similarly R_T may be expressed approximately by

$$R_T = \left\{\alpha\left[l_1 + l_2(\cot^2\beta l_2 + 1)\right] + \frac{Z_0}{R}\right\} Z_0\left\{\sin\beta l_2 + (\beta\Delta l_2)\ \cos\beta\Delta l_2 + \ldots\right\}$$

$$= Z_0\sin\beta l_2\left(1 + \beta\Delta l_2\cot\beta l_2\right)\left\{\alpha\left[l_1 + l_2(\cot^2\beta l_2 + 1)\right] + \frac{Z_0}{R}\right\} \tag{28}$$

From this expression, the variation in R_T as a resonance curve is taken may be estimated. Typical values of $\beta\Delta l_2\cot\beta l_2$ assuming that $\beta\Delta l_2 = 10^{-2}$ and for various values of βl_2 are given in table B.II.

Table B.II.

βl_2	$\cot\beta l_2$	$\beta\Delta l_2\cot\beta l_2$	$\frac{R_{T\,max} - R_{T\,min}}{R_T} \times 100\%$
$\frac{\pi}{2}$	0	0	0
$\frac{\pi}{3}$	.577	6×10^{-3}	1.2
$\frac{\pi}{4}$	1.0	1×10^{-2}	2
$\frac{\pi}{6}$	1.732	1.7×10^{-2}	3.4

The values given in the table are quite pessimistic since, for the final line, $\beta\Delta l_2$ had a maximum value of less than one-half that assumed here. Consequently it was permissible to consider R_T as a constant for each resonance curve. The equivalent circuit of the arrangement therefore reduced to a simple series circuit including the constant resistance R_T and the reactance X_T .

Appendix C.

Rectification of Resonance Curves

The magnitude of the current flowing in a simple series resonant circuit may be written

$$I = \frac{E}{\sqrt{R^2 + X^2}}$$

In terms of the current I_r, the resonance current, this becomes

$$\frac{I}{I_r} = \frac{R}{\sqrt{R^2 + X^2}}$$

Thus, if a thermocouple galvanometer is used to measure the current, its readings will be given by

$$\frac{\theta}{\theta_r} = \frac{R^2}{R^2 + X^2}$$

or

$$\frac{\theta}{\theta_r} = \cos^2 \phi$$

where ϕ is the power-factor angle of the circuit corresponding to the galvanometer deflection θ. Consequently the value of ϕ may be determined directly from the ratio of the corresponding galvanometer deflection to the maximum deflection. Then it becomes possible to obtain the value of $\tan\phi$, which may be expressed as

$$\tan\phi = \frac{X}{R}$$

Evidently, if the resistance in the circuit is constant, there will be a linear relation between tan ϕ and the reactance. The slope of the curve tan ϕ vs. X will give the resistance of the circuit, while deviations from the expected linear relationship will be extremely easy to detect. Also the value of resistance so obtained will depend on a large number of readings which obey a rather simple law. This is very much more satisfactory than the usual method of plotting a resonance curve and using three points (the maximum and the two 0.707 maximum points) to determine the effective resistance of the circuit.

In the case of the tuned transmission line it has been shown (see app. B) that the equivalent reactance may be taken as directly proportional to the deviation of line length from its resonant value. Also the equivalent resistance was shown to be practically constant over the range covered by the usual resonance curve. Consequently the above method is applicable to the case of the transmission lines used in this work.

Actually, the calculations were made in the following manner: Since $\theta/\theta_r = \cos^2\phi$, $\theta_r/\theta = \sec^2\phi$ and

$\theta_r/\theta - 1 = \tan^2\phi$, the value of $\tan\phi$ was found by evaluating

$$\sqrt{\theta_r/\theta - 1}$$

Then this value of $\tan\phi$ was plotted against the line length and R_T was found by measuring the slope, m, of the resultant straight line and substitutihg in an equation of the form

$$R_T = \frac{K}{m}$$

where K is the factor relating the reactance X to the corresponding deviation Δl_2 of the line length from its resonant value. For example, for the arrangement treated in Appendix B, Part II,

$$K = \frac{Z_o \beta}{\sin \beta l_2}$$

as given by equation (27) of that appendix.

Appendix D.

Best Ratio of Inner and Outer Conductor Radii.

As given in equation (28) Appendix B, the value of the equivalent resistance of the line, with the sample in place, is

$$R_T = Z_0 \sin\beta l_2 (1 + \beta \Delta l_2 \cot\beta l_2)\left\{\alpha\left[l_1 + l_2(\cot^2\beta l_2 + 1)\right] + \frac{Z_0}{R}\right\}$$

where R is the resistance of the sample. Thus, in order that the sample should have a maximum effect on the equivalent resistance of the line, the quantity $\frac{Z_0}{\alpha}$ should be a maximum.

In terms of the constants of the line,

$$\frac{Z_0}{\alpha} \doteq \frac{2Z_0^2}{R_l} \propto \frac{\left(\log \frac{b}{a}\right)^2}{\frac{1}{a}+\frac{1}{b}} = \frac{b\left(\log \frac{b}{a}\right)^2}{\frac{b}{a}+1}$$

where R_l is the loop resistance of the line in ohms per cm., b is the outer conductor radius and a is the inner conductor radius. If b is fixed, $\frac{Z_0}{\alpha}$ may be maximized with respect to a, as follows:

let $\frac{b}{a} = x$

$$\text{then } \frac{d}{dx}\left(\frac{Z_0}{\alpha}\right) = \frac{b \log x}{x+1}\left\{\frac{2}{x} - \frac{\log x}{x+1}\right\}$$

which is zero when $x \log x - 2(x+1) = 0$

Solving this equation with the help of Newton's method, there results,

$\frac{b}{a} = 9.2$ for a maximum value of $\frac{Z_0}{\alpha}$.

This is the same result as that previously obtained by Terman[6] as the condition for the maximum sending-end impedance of a coaxial line. In addition, he has plotted the quantity $\frac{\left(\log \frac{b}{a}\right)^2}{\frac{b}{a} + 1}$ as a function of $\frac{b}{a}$. The resultant curve shows a rather broad maximum, having a value of about 0.88 at $\frac{b}{a} = 5$, as compared with a value of 1.00 at $\frac{b}{a} = 9.2$. Thus an increase, from the optimum value, in the relative size of the inner conductor is not accompanied by a very rapid decrease in the value of $\frac{Z_0}{\alpha}$. A larger inner conductor, desirable for mechanical reasons, may therefore be used without an appreciable loss in the sensitivity of the line arrangement.

6. F.E. Terman. Resonant Lines in Radio Circuits, Elect. Eng. 53, p. 1046, July '34.

Appendix E.

Effect of Frequency Drift.

The effect of a drift in frequency during the time required to obtain a complete resonance curve may be obtained by considering the effect on line reactance caused by such a drift. The effect on the equivalent resistance will be too small to have any serious effect.

The equivalent reactance of the line, with a sample in place, is given by equation (27) App. B. It is

$$X_T = \cos\beta(l_2 + \Delta l_2) - Z_o C\omega \sin\beta(l_2 + \Delta l_2)$$

Now if the frequency (or wave-length) and Δl_2 both vary in a linear fashion with time, then

$$\Delta l_2 = k_1 t$$

and $$\lambda = \lambda_o + k_2 t$$

or $$\lambda = \lambda_o + \frac{k_2}{k_1}\Delta l_2 = \lambda_o + k\,\Delta l_2$$

where t is the time measured from the instant of resonance

and λ_o is the wave-length at resonance.

Hence $$X_T = \cos\frac{2\pi(l_2 + \Delta l_2)}{\lambda_o + k\Delta l_2} - Z_o C\omega \sin\frac{2\pi(l_2 + \Delta l_2)}{\lambda_o + k\Delta l_2}$$

Approximately, this becomes

$$X_T \doteqdot \cos \frac{2\pi}{\lambda_0}(l_2 + \Delta l_2)(1-\frac{k}{\lambda_0}\Delta l_2) - Z_0 C\omega \sin \frac{2\pi}{\lambda_0}(l_2 + \Delta l_2)(1-\frac{k}{\lambda_0}\Delta l_2)$$

$$= \cos\left[\frac{2\pi}{\lambda_0}\left\{l_2 + \Delta l_2\left(1-\frac{k l_2}{\lambda_0}\right)+\ldots\ldots\right\}\right] - Z_0 C\omega \sin\left[\frac{2\pi}{\lambda_0}\left\{l_2 + \Delta l_2\left(1-\frac{k l_2}{\lambda_0}\right)+\ldots\right\}\right]$$

or $X_T = -\frac{Z_0}{\sin \beta l_2}\left\{\beta_0 (\Delta l_2)\left(1-\frac{k l_2}{\lambda_0}\right)+\ldots\ldots\right\}$

where $\beta_0 = \frac{2\pi}{\lambda_0}$

Since $R_T = \frac{X_T}{\tan \phi}$ an error in the value of X_T will produce a proportional error in R_T. Consequently the quantity $1-\frac{k l_2}{\lambda_0}$ must be equal to unity within the required limits of error. If l_2 is, say, $\frac{\lambda_0}{8}$ then in order that the error in R_T be less than 1.0%, k must have a value less than 0.08.

In a typical case, the corresponding change in wave-length during the taking of a resonance curve would be

$$2k\Delta l_2 = 2 \times .08 \times .2$$

$$= .032 \text{ cm.} = 0.32\text{mm.}$$

It is believed that deviations of frequency of this order of magnitude did not exist in the wave-length of the final oscillator (which was equipped with a regulated plate supply) at least after the expiration of the warm-up period. In addition, it should be noticed that only a linear variation of wave-length with line length would give an undetectable error of the kind treated here. For any other kind of variation, curvature would be introduced into the rectified resonance curve, which could be immediately detected and the cause eliminated.

Appendix F.

Effect of Changes in Ocsillator Frequency Due to Changes in Line Tuning.

Since this method of measuring resistance depends on calculated changes of reactance, it is essential that means be available for checking the constancy of the oscillator frequency. It has already been shown that a steady drift in oscillator frequency cannot be detected by an inspection of any one of the rectified resonance curves. On the other hand, it will now be shown that changes in frequency produced by changes in line tuning can only cause easily detectable changes in the shape of the curves. If such changes become apparent, it is then only necessary to reduce the coupling of the feeding circuit to the line until the proper rectified resonance curves are obtained.

As the line is tuned over the range necessary to obtain a resonance curve, the transferred reactance and resistance in the feeding circuit will vary through a certain range of values. This variation is given by the following equations

$$R_{1-2} = \frac{M_{12}^2 \omega^2 R_1}{R_1^2 + X_1^2}$$

$$X_{1-2} = -\frac{M_{12}^2 \omega^2 X_1}{R_1^2 + X_1^2}$$

where R_{1-2} and X_{1-2} are the resistance and reactance transferred from the tuned line to the feeding circuit.

M_{12} is the mutual inductance between the two circuits.

ω is the angular velocity corresponding to the applied frequency.

R_1 is the sending end resistance of the tuned line.

X_1 is the sending end reactance of the tuned line.

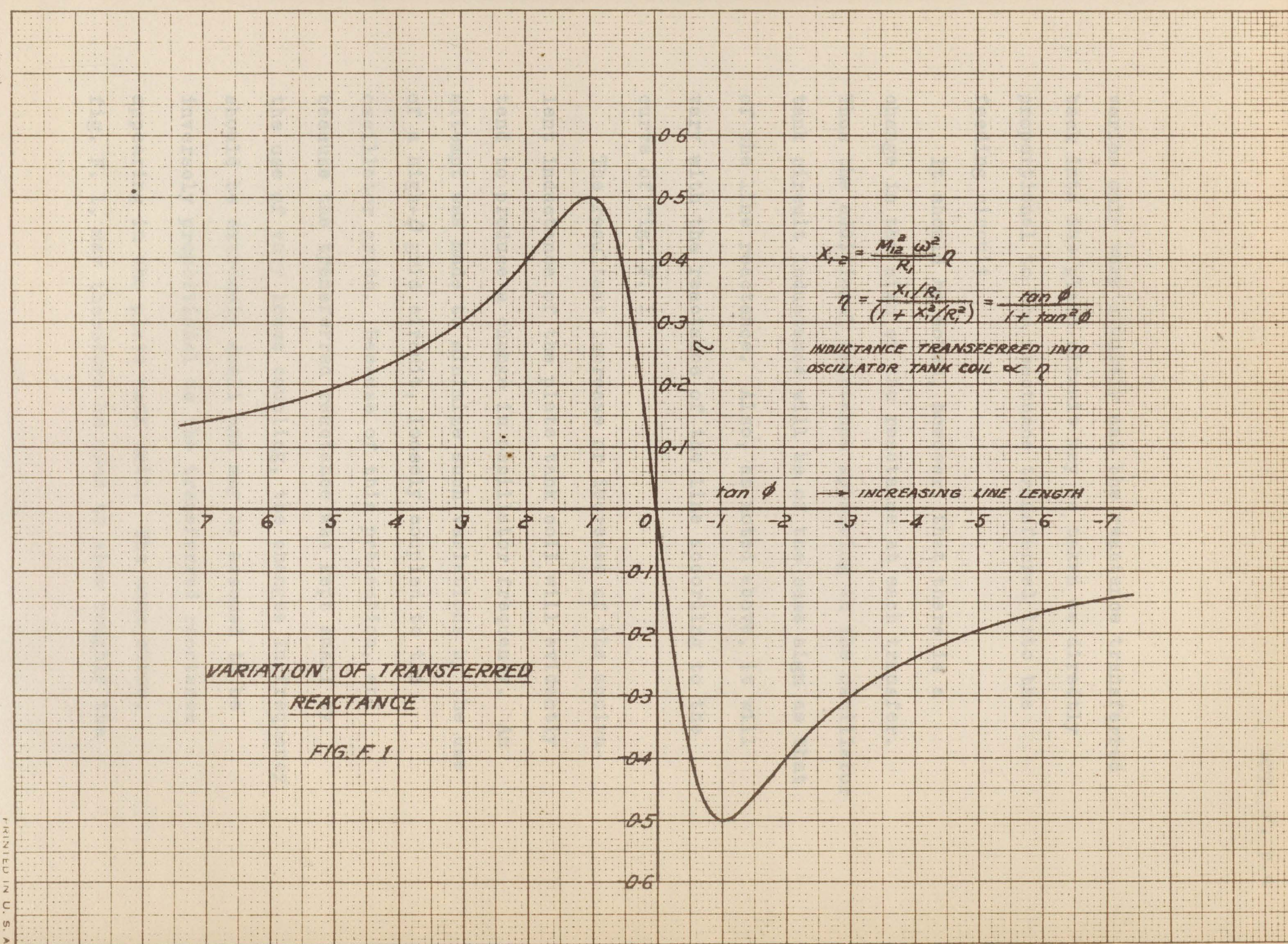
Since the Q of the feeding circuit is very much smaller than that of the tuned line, the resistance reflected back into the feeding circuit from the line must be very small compared with the actual feeding circuit resistance and will produce only a negligible effect. On the other hand, since the reactance of the feeding circuit is normally adjusted to zero, the possible effects of the transferred reactance must be investigated. To do this, it will be convenient to rewrite the above expression for the transferred reactance in the following form:

$$X_{1-2} = -\frac{M_{12}^2\omega^2}{R_1}\cdot\frac{X_1/R_1}{1+X_1^2/R_1^2}$$

$$= -\frac{M_{12}^2\omega^2}{R_1}\;\frac{\tan\phi}{1+\tan^2\phi}$$

where ϕ is the power factor angle of the sending-end impedance of the line. Since $\tan\phi$ is the only variable in this expression, the manner in which the transferred reactance varies with the line tuning can be expressed as a simple function of $\tan\phi$. This function has been plotted as fig. F.1. The significant portion of the curve, at least for this work, is that lying between $\tan\phi = +2.5$ and $\tan\phi = -2.5$, since most of the readings taken lie in this range.

Next, it becomes necessary to consider the effect of reflection of reactance from the feeding circuit into the plate tank circuit. Since the coupling between these two circuits is also a mutual inductance, the same relations will hold as for the case already considered. However, because of the small coupling between line and feeding circuit, the total variation in feeding circuit reactance will be very small. In fact, the coupling was purposely made so small that a thermocouple galvanometer, coupled to the feeding circuit, showed no detectable change in reading as the line was tuned through resonance. It follows that the power factor angle of the feeding circuit impedance



VARIATION OF TRANSFERRED REACTANCE

FIG. F 1

varies only very slightly and the reactance transferred back into the plate circuit may be taken as directly proportional to the reactance transferred into the feeding circuit.

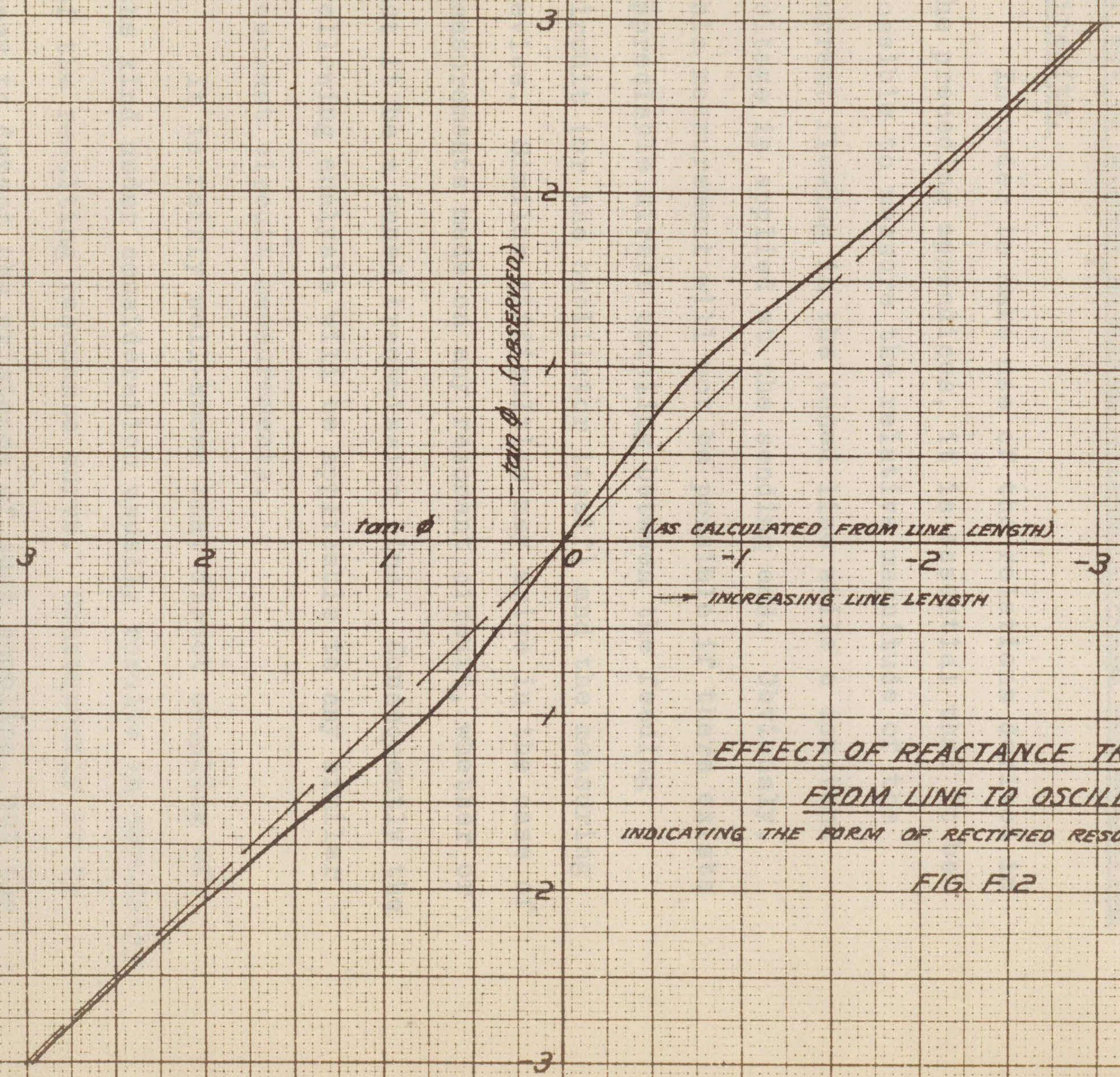
It should be noted, however, that there is a change in the sign of the reactance at each transfer. Thus the additional reactance appearing in the oscillator tank circuit inductance will be of the same sign as that of the line reactance. Also, as noted above, it will vary with the reactance of the line according to the curve of fig. F. 1.

The resultant increase or decrease of the equivalent inductance of the plate tank coil will evidently tend to produce a change in oscillator frequency. An attempt was made to minimize such variations by the use of a high-Q grid circuit, loosely coupled to the oscillator grids. Because of this precaution, and because the transferred reactance was kept small by the use of very loose coupling, the changes in frequency should be extremely small and may be assumed to be inversely proportional to the transferred reactance appearing in the plate tank coil. The same curve, fig. F. 1, may therefore be used to show roughly the drift in wave-length as the line length is altered. From the results obtained in appendix E, the effect on the shape of the rectified resonance curve may then be deduced.

At the points $\tan\phi = \pm 1$, the slope of the latter curve should be equal to the correct value, since the rate of drift in the wave-length is zero. However, as the line length is increased near resonance, the wave-length will be decreasing at a nearly constant rate. Again referring to Appendix E, this means that the indicated values of reactance will be changing more rapidly than the values calculated from equation 27, appendix B. The slope of the rectified resonance curve at the origin will therefore be greater than its correct value.

As a check on this conclusion, the condition when $\tan\phi = +1$ may be considered. Then the reactance transferred back into the plate tank coil will be a maximum and the wave-length will be higher than when the line is at resonance. Thus the line length to give $\tan\phi = +1$ will be slightly greater than the desired value.

For very large positive or negative values of tan ϕ, the experimental curve will be asymptotic to the straight line. The actual curve, as affected by reaction on the oscillator frequency, should therefore have the shape indicated in fig. F.2. Any curvature of this kind would be easily detected on the experimental curves. However, on a simple resonance curve this cause of error would be difficult to detect and would be likely to lead to an unduly low value for the equivalent resistance of the line. This would be particularly true if the customary 0.707 I_r points were used for a calculation of the equivalent resistance.



EFFECT OF REACTANCE TRANSFERRED

FROM LINE TO OSCILLATOR

INDICATING THE FORM OF RECTIFIED RESONANCE TO BE EXPECTED

FIG. F.2

Appendix G.

Effect of Direct Coupling Between Feeding and Measuring Circuits.

In order to make use of the relations derived in the preceding appendices, it is essential that it be possible to measure the relative magnitude of the current flowing in the tuned line while a constant voltage is applied at the sending end. Obviously, this measurement will not be possible if there exists appreciable direct coupling between the feeding circuit (or the oscillator itself) and the measuring device. Similar difficulties may arise in the case of measurements made on any resonant circuit, whether or not it be a tuned transmission line. Consequently the following analysis will be applicable to any similar resonant circuit measurements.

It is fairly well-known that direct coupling of the kind under consideration usually results in asymmetry of the resulting resonance curve. Consequently it is easy to detect the existence of such coupling, but the errors which may result therefrom require some consideration. It is the purpose of the following analysis to demonstrate the reason for the asymmetry found experimentally and to indicate the nature of the resulting errors.

The arrangement to be considered consists of three circuits, each pair being coupled by a corresponding mutual inductance. The equations will first be written in the following general form, subscript 1 applying to the resonant circuit, subscript 2 to the feeding circuit and subscript 3 to the measuring circuit:

$$I_1 Z_1 = jM_{12}\omega I_2 + jM_{13}\omega I_3 \qquad (1)$$

$$I_3 Z_3 = jM_{23}\omega I_2 + jM_{13}\omega I_1 \qquad (2)$$

Eliminating I_2,

$$I_3 = I_1 \left\{ \frac{Z_1 M_{23} + jM_{12} M_{13}\omega}{Z_3 M_{12} + jM_{13} M_{23}\omega} \right\} \qquad (3)$$

This expression is rather formidable, particularly since Z_1 is a complex quantity, but it may be simplified by neglecting the second term in the denominator. The effect of this term on the resonant circuit current is discussed in appendix H. To justify this assumption, it should be noted that Z_3 is large compared to $M_{13}\omega$ (see App. H) and that M_{12} is purposely made very large compared to M_{23}. Physically, the neglect of this term implies that the voltage induced in circuit 1 by the current in circuit 3 is negligible compared with the voltage induced by the current in circuit 2. In the present case, circuit 1 is the tuned line or other resonant circuit, circuit 2 is the feeding circuit, and circuit 3 is the thermocouple pick-up circuit or measuring circuit. Thus it has been assumed that the reaction of the thermocouple current on the current in the resonant circuit is negligible. Such an assumption has been amply justified by the experience of many workers.

Written in the simplified form, equation (3) becomes,

$$\frac{I_3}{I_1} = \frac{jM_{13}\omega}{Z_3}\left\{1 - jZ_1\frac{M_{23}}{M_{12}}\right\} \qquad (4)$$

Since $\frac{M_{13}\omega}{Z_3}$ is a constant, the ratio between the resonant circuit current I_1 and the measuring current is

$$\left|\frac{I_3}{I_1}\right| = k\left|1 - j(R_1 + jX_1)\frac{M_{23}}{M_{12}}\right| \qquad (5)$$

Squaring both sides, and simplifying,

$$\frac{I_3^2}{I_1^2} = k^2\frac{R_1^2 M_{23}^2}{M_{12}^2}\left\{1 + \left(\frac{M_{12}}{R_1 M_{23}} + \frac{X_1}{R_1}\right)^2\right\} \qquad (6)$$

$$= k_2\left\{1 + (k_3 + \tan\phi)^2\right\}$$

In order to demonstrate the effect of this variable ratio between I_3 and I_1, mutually consistent values were ~~found~~ assumed for k_2 and k_3. Then the value of the ratio was determined for various values of $\tan\phi$ and the resulting ratios applied to an ordinary resonance curve. The ideal resonance curve, together with the curve which would have been observed if k_2 and k_3 were to have the assigned values, is plotted in fig. G.1.

It will be noticed that, even though the maximum value of the observed curve is only slightly greater than that of the actual curve, the shape is considerably different. A definite asymmetry has been introduced. This asymmetry arises from the fact that the current induced in the measuring circuit by direct coupling is not in phase with the current in the feeding circuit. Perhaps the reactance of the measuring circuit might be neutralized by series resonance, thus putting its current in phase with the voltage induced in it by the feeding circuit. Then the current in the resonant circuit and in the measuring circuit would be in phase when the resonant circuit was set at resonance. However, the resonance curve, while symmmetrical, would still not be of the correct shape, since the current in the measuring circuit would consist of two components, one

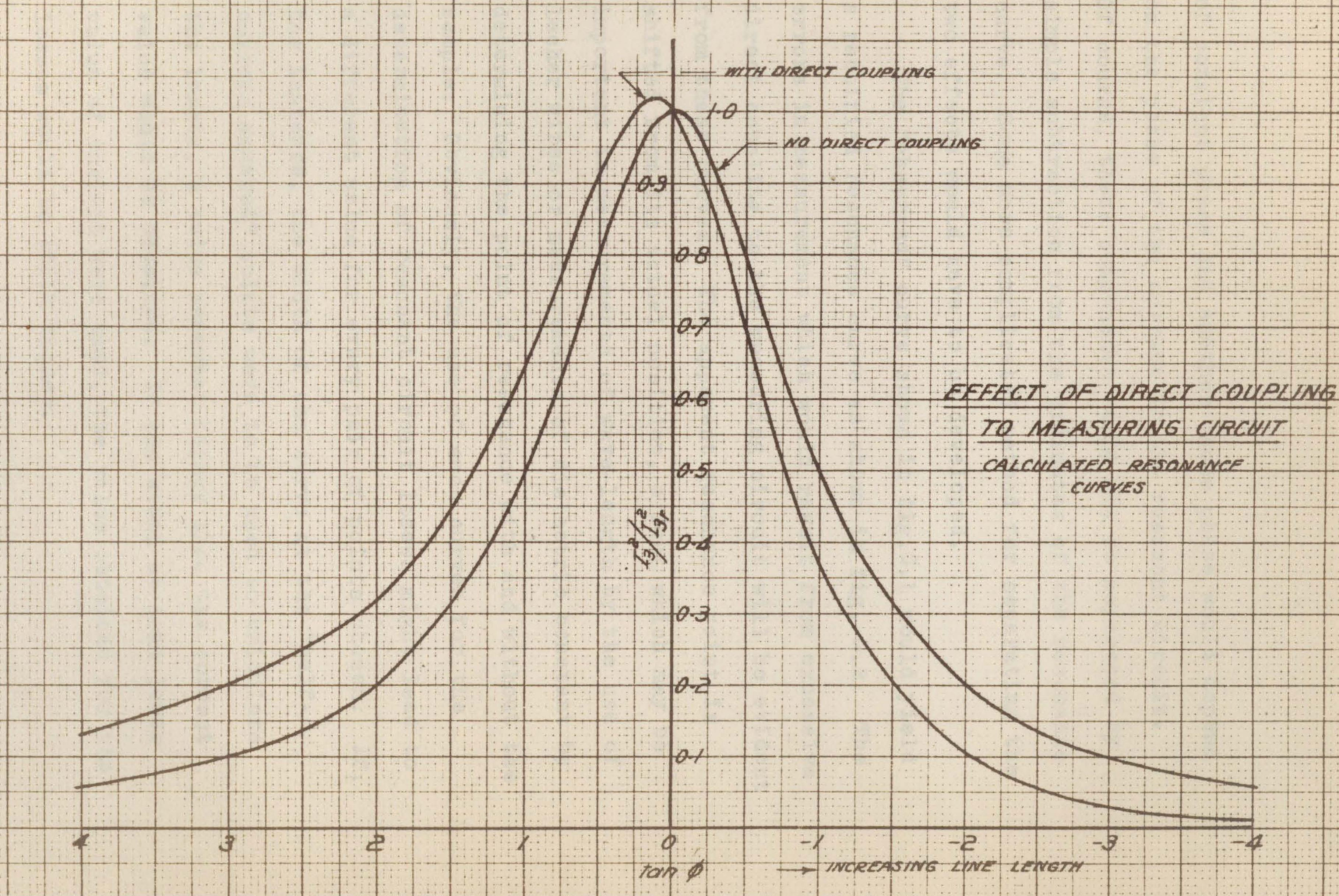
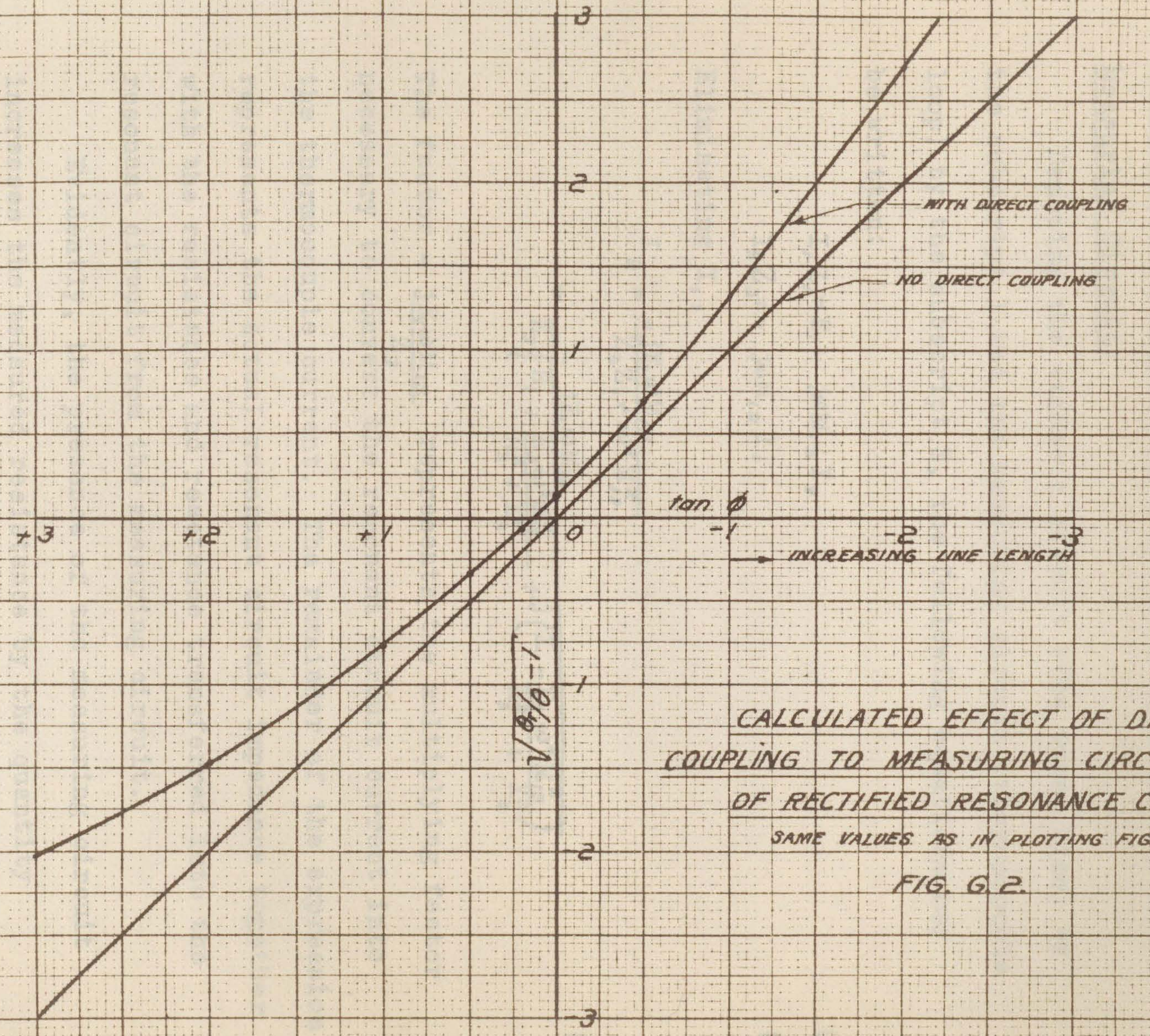

WITH DIRECT COUPLING
NO DIRECT COUPLING
EFFECT OF DIRECT COUPLING
TO MEASURING CIRCUIT
CALCULATED RESONANCE
CURVES
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
4
3
2
1
0
-1
-2
-3
-4
tan ϕ
INCREASING LINE LENGTH

FIG. G.1.

of constant phase and another whose phase would depend on the phase of the current in the resonant circuit. Of course, these components could not be separated by simple subtraction from the ordinates of the resonance curve. Some more complicated method for separating the two effects would have to be developed.

The "observed" curve shown in fig.G.1 would yield a rectified resonance curve as shown in fig. G.2. The errors in measurement which would result from excessive direct coupling to the measuring circuit will be evident from these curves. The apparent resonance point is shifted from its actual position, a fact which may be important in measurements of wave-length by the use of lecher wires or measurements of dielectric constant by determining the point of resonance with and without the sample. Obviously, the use of such curves for the determination of resonant circuit losses might lead to a different value for every pair of points chosen. If, for instance, the width of the curve at the "half-current squared" points were to be used to calculate the losses, a value somewhat higher than the correct value would be obtained. If the width at some lower value of current were used, the value obtained for the losses would be still higher.



CALCULATED EFFECT OF DIRECT COUPLING TO MEASURING CIRCUIT ON SHAPE OF RECTIFIED RESONANCE CURVES

SAME VALUES AS IN PLOTTING FIG. G.1.

FIG. G.2.

Effect of Impedance Transferred Into Tuned Line From Measuring Circuit.

Denoting the resonant circuit (the tuned line) by the subscript 1 and the measuring circuit (thermocouple loop) by the subscript 3, the following equations may be written:

$$I_1 Z_1 = E_1 - j\omega M_{13} I_3 \qquad (1)$$

$$I_3 Z_3 = - j\omega M_{13} I_1 \qquad (2)$$

Eliminating I_1,

$$I_3 = \frac{-jE_1 \omega M_{13}}{Z_3 Z_1 + \omega^2 M_{13}^2}$$

$$= \frac{-jE_1 \omega M_{13}}{Z_3 \left\{ R_1 + \frac{R_3 \omega^2 M_{13}^2}{R_3^2 + X_3^2} + j\left(X_1 - \frac{X_3 \omega^2 M_{13}^2}{R_3^2 + X_3^2} \right) \right.} \qquad (3)$$

The factor $-\frac{jE_1 \omega M_{13}}{Z_3}$ represents the multiplying factor necessary to convert the resonant circuit current into the thermocouple current. The remainder of the expression represents the actual resonant circuit impedance together with the resistance and reactance transferred into the resonant circuit from the measuring circuit.

Evidently, the presence of the measuring circuit increases the measured resistance by the quantity $\frac{R_3 \omega^2 M_{13}^2}{R_3^2 + X_3^2}$. Also the reactance is reduced by the quantity $\frac{X_3 \omega^2 M_{13}^2}{R_3^2 + X_3^2}$. Consequently it becomes necessary to estimate the approximate magnitudes of these two quantities.

Because of the simplicity of the geometrical arrangement, the approximate calculation of M_{13} is not difficult. Fig. 1. shows the actual dimensions. The coupling between line and thermocouple loop is obtained by the magnetic flux passing through the wire loop - the area shown lightly cross-hatched. This flux is given by:

$$\Phi = 1.5\int_{r=0.893}^{r=1.143} \frac{2I}{r}\,dr = 0.741 I_1$$

Hence the mutual inductance is:

$$M_{13} = 0.741 \times 10^{-9} \text{ henries}$$

and $$\omega^2 M_{13}^2 = 2.36 \times 10^{-1}$$

Now R is more than 1000 ohms, hence, even if X_3 is neglected,

$$\frac{R_3 \omega^2 M_{13}^2}{R_3^2 + X_3^2} = 2.36 \times 10^{-4}$$

For the line under consideration, the resistance R is, approximately, 0.11 ohms. Consequently, the error in the measurement of R introduced by the reaction of the measuring loop is only

$$\frac{2.36 \times 10^{-4}}{1.1 \times 10^{-1}} = 4.6 \times 10^{-3} = 0.46\%.$$

Furthermore, this additional resistance is a constant quantity and will be eliminated in the subtraction of losses to determine those present in the sample.

The reactance component is similarly very small and can only cause the resonance point to be shifted very slightly.

To sum up, both components of transferred impedance will be very small and in any case will not affect the final result.

Appendix J.

Effect of Constant Separation between Thermocouple Loop and End of Line.

Throughout the preceding analyses, it was assumed that the current affecting the measuring circuit was actually the current in the shorting piece at the end of the line. However, mechanical requirements made it necessary to mount the thermocouple loop a short distance from the actual end of the line. Furthermore, the thermocouple loop itself extended for about 1.5cm. along the length of the line. Consequently, it becomes necessary to consider the kind of relationship to be expected between the current in the thermocouple loop and that at the end of the line.

In order to determine this relationship, it will first be assumed that the flux at any point in the line, and near its end, will be directly proportional to the line current at that point. This current must obey the usual relations for a four-terminal network, which are:

$$E_x = A_x E_R + B_x I_R$$

$$I_x = C_x E_R + D_x I_R$$

where E_x and I_x are the voltage and current at a distance x from the shorted end of the line. Because of the shorted termination, E_R must be zero and thus

$$I_x = D_x I_R$$

In this expression, D_x is a constant, dependent only on the electrical distance x to the end of the line. Thus, no matter what the total length of the line, the current and flux at a given distance from the shorted end of the line will always be the same fraction of I_R. Integrating I_x from one end of the thermocouple loop to the other will therefore yield a quantity directly proportional to I_R.

The only possibility for deviation from this simple relationship lies in the possibility of relative motion between the thermocouple loop and the shorting piece. However, small accidental movements of this kind will produce only a small effect, since

$$D_x = \cos \frac{2\pi x}{\lambda}$$

and in this case x was very small compared to λ. In fact, the minimum value of D_x would occur at the far end of the thermocouple loop, about 2 cm from the end of the line. For this value of X, D_x would still be approximately 0.9990. Consequently, the assumption that the thermocouple loop current would be directly proportional to the line receiving end current is amply justified.

Appendix K

Description of Plate Supply Voltage Regulator.

The operation of this device was very similar to that of various commercial plate power supply regulators. However, the voltage reference was obtained from a 90 volt "C" battery, rather than from the more customary glow tube. Furthermore, the required voltage output was higher than that usually handled by tubes of the type used. Both of these departures from the usual design were made possible by the particular use for which the regulator was built. It was believed that practically constant supervision would prevent operation with a run-down battery, while the relatively fixed output voltage and current requirements would allow the various tubes to operate within a safe range of currents and voltages.

The method of operation will be evident from the circuit diagram, fig. k. 1. If the output voltage tends to rise, the grid of the 57 type control tube will become more positive, and permit more current to flow in the 2 megohm plate resistor. The grids of the 2A3's will consequently be driven to a more negative

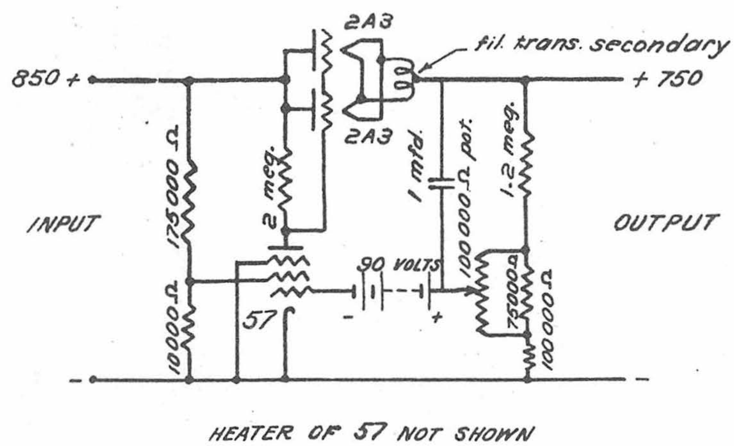


FIG. K.1.

SIMPLIFIED SCHEMATIC OF VOLTAGE REGULATOR

(PROTECTIVE CIRCUITS NOT SHOWN)

potential, with respect to the anodes, and the total voltage across these series tubes will increase. Because of the high available amplification obtained from the grid of the 57 to the grids of the 2A3's, only a small change in the output voltage will be required to produce a large change in the voltage drop through the 2A3's. Large changes in input voltage or output current will therefore produce very small changes in output voltage.

In addition, a very great reduction in the ripple component of the output voltage was obtained. This resulted from the use of the 1 mfd. condenser from the high-voltage terminal to the potentiometer arm. The entire ripple voltage was applied in this way to the grid of the control tube and thereby almost completely eliminated. A ripple voltage of about 3.5 volts peak on the 900 volt output of the rectifier and filter was reduced in this way to such an extent that it could no longer be measured with the available oscilloscope.

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