AN INVESTIGATION OF THE DEFLECTION CURVE FOR A PILE DUE TO A HORIZONTAL LOAD

By

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TABLE OF CONTENTS

PRESENTATION

I, Charles L. Cobb submit this thesis to the faculty of the California Institute of Technology, Department of Civil Engineering, in partial fulfillment of the requirements for a Degree of Master of Science in Civil Engineering.

ACKNOWLEDGMENT

I wish to extend my thanks to Professor R. R. Martel for the help and suggestions he has given me in the preparation of this Thesis.

PROCEDURE

I have chosen to investigate the deflection curve for a horizontally loaded sheet pile, and to draw from this investigation information that may lead to a better understanding of the behavior of sheet piling due to horizontal loading.

I proceed from the basis that as the pile is first loaded, it is infinitely stiff and simply deflects in a straight line. The forces acting upon this infinitely stiff beam or pile are in equilibrium and hence any additional forces that might be applied to it due to the release of this infinitely stiff pile must also be in equilibrium. These forces may be found from the differences in the deflections between the assumed and computed pile.

The first assumption that must be made is in what manner does the soil pressure vary with depth? For the sake of simplicity I have chosen that it varies as a straight line. This assumption is in close accordance with the manner in which sandy soil will act, but varies from that of clay considerably.

In the first calculations I will prove that the infinitely stiff pile, when loaded horizontally at the top, will rotate about a point three-quarters the length from the top. From this deflected position I will find the deflection curves for the first approximation, provided the pile was released from its infinitely stiff condition and allowed to bend. This added deflection will cause a new set of forces to be set up and thus these forces will cause an added deflection. This process will be continued until the amount of added deflection will become negligible.

The force applied to a pile is proportional to the soil pressure "e" and to the deflection "u". The soil pressure "e" is proportional to the depth, varying as a straight line, thus making the force applied to the pile proportional to the depth and the deflection:

Equ. 1. $f = e\,u\,y$

y being the depth

If the total forces acting on the pile are in equilibrium, then the summation of these forces must be equal to zero.

Equ. 2. $H = o$

$$H = \int_0^{\ell} e\,u\,Y\,dy = 0$$

The pile being a free ended beam, the summation of the moments about the end must also be equal to zero.

Equ. 3. $M = 0$

$$M = \int_0^{\ell} e\,u\,Y^2\,dy = 0$$

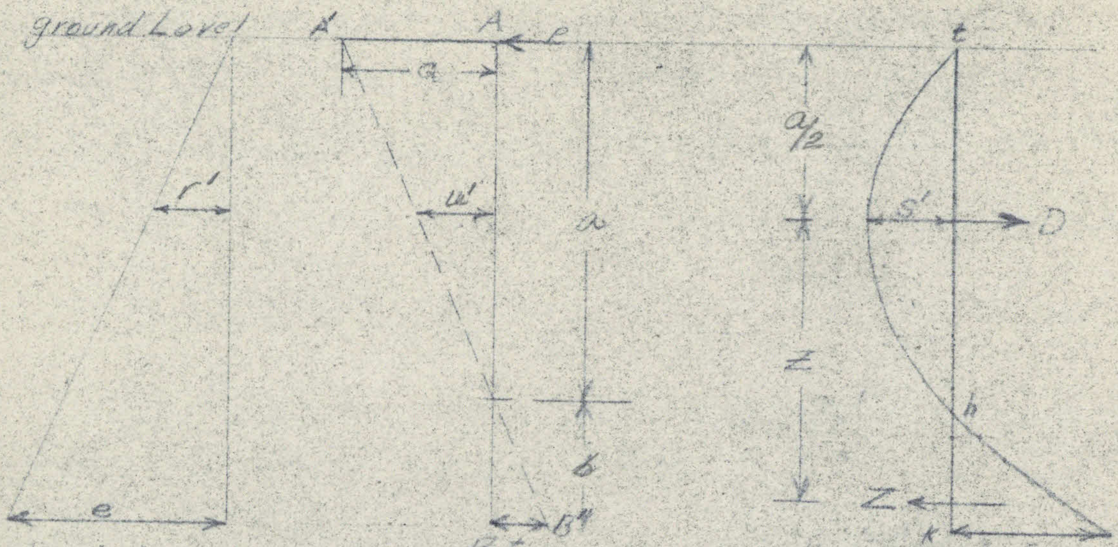
The equations 2 and 3 make it possible then to find the forces applied and the amount of deflection.

If these two unknowns are found for each subsequent deflection, and these dampen to a zero value, the total deflection and forces may be found.

For my first deflection I will calculate both analytically and graphically, and the subsequent deflections I will do only graphically. The graphical method may introduce a small human error, but I will put this as small as possible.

I will now proceed to determine the point of zero deflection for the infinitely stiff pile. After this I will determine the first deflection analytically, and finally give the results of the subsequent deflections from the graphical method in tables 2 and 3.

Determination of the Point of Zero Bending in an Infinitely Stiff Pile.



(a) Unit resistance (b) Deformation (c) pressure

Fig. 1

Let AB, fig. 1 (b) be the original position of a pile embedded in the earth and A'B' the position after a force P is applied at A. The resistance of the earth varies with the depth as shown in Fig. 1 (a), which is a diagram of the passive resistance per unit deformation.

The maximum pressure $S' = r'u'$

$$r' = \frac{ea}{2(a+b)}$$

$$u' = \frac{a}{2} \cdot \frac{Q}{a} = \frac{Q}{2}$$

Equ. 1 $\therefore \quad S' = \dfrac{eaQ}{4(a+b)}$

The Maximum pressure $S_1 = et$

$$t = \frac{Qb}{a}$$

$$\therefore \quad S_1 = \frac{eQb}{a}$$

Equ. 2 $\quad eQ = \dfrac{aS_1}{b}$

Substituting Equ. 2 into Equ. 1

$$S' = \frac{a^2 S_1}{4b(a+b)}$$

Equ 3, $\frac{a^2}{4} = \frac{S'b(a+b)}{S_1}$

Equ 3 can be shown to be a parabola; we have $2p = \frac{b(a+b)}{S_1}$ as the parameter of a parabola of the form $y^2 = 2px$; $S' = \frac{a^2}{8p}$

Move the origin to the right a distance S'

$$y^2 = 2p(x - S') = 2px + \frac{a^2}{4}$$

$$x = \frac{4y^2 - a^2}{8p}$$

D = force represented by the area of the curve in Fig. 1(c) from t to h

then

$$D = 2\int_0^{a/2} x\,dy = 2\int_0^{a/2} \left(\frac{4y^2 - a^2}{8p}\right) dy$$

$$= 2\left|\frac{4y^3}{24p} - \frac{a^2 y}{8p}\right|_0^{a/2}$$

$$D = \frac{a^3}{24p} - \frac{a^3}{8p}$$

$$D = \frac{a^3}{12p}$$

from $2p = \frac{b(a+b)}{S_1}$

we have

$$D = \frac{a^3 S_1}{6b(a+b)}$$

Z = force represented by the area of the curve in Fig. 1(c) from h to k.

Then:

$$Z = \int_{a/2}^{a/2+b} x\,dy$$

$$= \frac{1}{8p}\int_{a/2}^{a/2+b} \frac{4y^2 - a^2}{8p}\,dy$$

$$Z = \frac{bS_1(3a+2b)}{6(a+b)}$$

The moment of Z about D

$$M = \int_{a/2}^{a/2-b} xy\,dy = \frac{bS_1(a+b)}{4}$$

$$z = \frac{M}{Z} = \frac{6(a+b)}{S_1 b(3a+2b)} \times \frac{S_1 b(a+b)}{4}$$

$$z = \frac{3(a+b)^2}{2(3a+b)}$$

Equating moments about D

$$\frac{Pa}{2} = \frac{S_1 b(3a+2b)}{6(a+b} \times \frac{3(a+b)^2}{2(3a+2b)}$$

$$\frac{Pa}{2} = \frac{S_1 b(a+b)}{4}$$

$$A = (a+b)$$

$$b = A - a$$

$$\frac{Pa}{2} = \frac{S_1(A-a)A}{4}$$

$$2Pa = S_1 A^2 - S_1 a A$$

hence

<u>Equ. 4</u> $\quad a = \dfrac{S_1 A^2}{2P + S_1 A}$

The sum of the forces on the pile have to equal zero.

or

$$P + Z = D$$

<u>Equ. 5.</u> $\quad P + \dfrac{S_1 b(3a+2b)}{6(a+b)} - \dfrac{a^3 S_1}{6b(a+b)} = 0$

from Equs. 4 and 5

$$b = \frac{2PA}{2P + S_1 A}$$

and

<u>Equ 6.</u> $\quad S_1 = \dfrac{6P}{A}$

Substituting Equ 6 into Equ 4 we have

$$a = \frac{6PA^2}{A(2P+6P)}$$

$$a = \frac{3A}{4}$$

$$A = (a+b) = l$$

$$a = \frac{3l}{4}$$

"a" being the point of zero deflection. We concluded that for an infinitely stiff

Pile the deflection varies as a straight line, and becomes zero at a point 3/4 l from the top, and then becomes negative the remainder of the length.

Due to the fact that it is impossable to have an infinitely stiff pile we must consider what take place as the pile bend away from this straight line condition and apply these added deflections to the deflections of the infinitely stiff one. I will try to show these added deflections in the following work. As I mention before I will carry through the first deflection in analytical form and the remaining ones will be done grapically.

<u>First added Deflection to that of the Infinitely stiff pile by analytical method.</u>

The force acting on the infinitly stiff pile is in relation to the amount of movement of the pile and the earth pressure.

so,

force $\propto$ e, Q, l

where e = the intensity of the earth pressure at the lower end of the pile

Q = the movement of the upper end of the pile.

from fig. 1, (a, b)

$$\text{force} = \frac{ey}{l} \times \frac{Q(3l-4y)}{3l}$$

$$\text{force} = \frac{eQy}{l} - \frac{4eQy^2}{3l^2}$$

The shear at any point along the pile is equal to the acting force P less the sum of the force due to the earth pressures.

or

$$\text{Shear} = P - \int_0^y (\text{force})\, dy$$

$$= P - \int_0^y \left(\frac{eQy}{l} - \frac{4eQy^2}{3l^2}\right) dy$$

$$\text{Shear} = P - \frac{eQy^2}{2l} + \frac{4eQy^3}{9l^2}$$

but when

$y = l$, the shear = 0

so.

$$0 = P - \frac{eQl}{2} + \frac{4eQl}{9}$$

$$P = \frac{eQl}{18}$$

<u>Equ.7</u> $$Q = \frac{18P}{el}$$

therefore:

Equ.8. $$\text{Shear} = P - \frac{9Py^2}{l^2} + \frac{8Py^3}{l^3}$$

The moment at any point in the pile is equal to the area of the shear curve Equ. 8 is the equation of the shear so the intergal of this is the moment if intergrated between zero and the point in question

so

$$\text{Moment} = \int_0^y (\text{shear})\,dy.$$

$$= P\int_0^y \left(1 - \frac{9y^2}{l^2} + \frac{8y^3}{l^3}\right) dy$$

Equ. 9. $\text{Moment} = P\left(y - \frac{3y^3}{l^2} + 2\frac{y^4}{l^3}\right)$

The slope at any point is the area of the moment curve Equ. 9 is the equation of the moment.

so.

$$\text{slope} = \int_0^y (\text{moment})\,dy$$

$$= P\int_0^y \left(y - \frac{3y^3}{l^2} + 2\frac{y^4}{l^3}\right) dy.$$

Equ. 10. $\text{slope} = P\left(\frac{y^2}{2} - \frac{3y^4}{4l^2} + \frac{2y^5}{5l^3}\right) + K.$

The deflection is the area of the slope curve.

so

$$\text{deflection} = \int_0^y (\text{slope})\,dy$$

$$= \int_0^y \left[P\left(\frac{y^2}{2} - \frac{3y^4}{4l^2} + \frac{2y^5}{5l^3}\right) + K\right) dy.$$

$$\text{deflection} = \frac{P}{60EI}\left(10y^3 - \frac{9y^5}{l^2} + \frac{4y^6}{l^3}\right) + Ky$$

When $y = 0$ the deflection $= 0$

$$0 = \frac{P}{EI \times 60}\left(10l^3 - 9l^3 + 4l^3\right) + Kl$$

$$K = -5l^2$$

so the deflection $= y$

Equ. 11. $y = \frac{P}{60EI}\left(5l^2y - 10y^3 + \frac{9y^5}{l^2} - 4\frac{y^6}{l^3}\right.$

This deflection y must be applied to the straight line deflection in such a manor that the summation of force caused by the added deflections are equal to zero and the moments caused by these forces are also zero.

So if the deflections "y" are ploted the curve will be, c d e f, in fig. 2. Then to this we apply the straight line, g h, which is the stright line deflection of the infinitely stiff pile, the added deflections "u" must satify the requirement given above of $\Sigma H = 0$ and $\Sigma M = 0$.

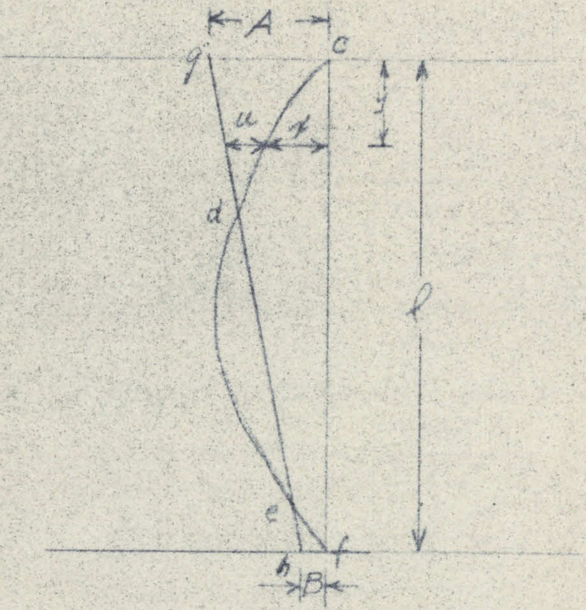


fig 2

A and B are the deflections of the end points of the straight line curve away from the axis of the superimposed curve.

by similar triangles we have

$$\frac{u+y}{\frac{Al}{A-B} - y} = \frac{A}{\frac{Al}{A-B}}$$

or u the added deflection =

$$u = \frac{By}{l} + \frac{A(l-y)}{l} - y$$

Where y is given by equ. 11

$$\Sigma H = 0 = \int_0^l \left(\frac{By}{l} + \frac{A(l-y)}{l} - y \right) y \, dy$$

$$0 = \int_0^l \left[\frac{By^2}{2} + Ay - \frac{Ay^2}{2} - \frac{P}{60EI}\left(5l^2y^2 - 10y^4 + \frac{9y^6}{12} - \frac{4y^7}{10}\right)\right] dy$$

$$0 = \frac{Bl^2}{3} + \frac{Al^2}{2} - \frac{Al^2}{3} - \frac{Pl^5}{60EI}\left(\frac{5}{3} - 2 + \frac{9}{7} - \frac{1}{2}\right)$$

$$0 = \frac{Bl^2}{3} + \frac{Al^2}{6} - \frac{Pl^5}{60EI}\left(\frac{19}{42}\right)$$

<u>Equ. 12</u> $\quad 2B + A = \dfrac{19Pl^3}{420EI}$

$$\Sigma M = 0 = \int_0^l \left[\frac{By^3}{2} + Ay^2 - \frac{Ay^3}{1} - \frac{P}{60EI}\left(5l^2y^3 - 10y^5 + \frac{9y^7}{12} - \frac{4y^8}{10}\right)\right] dy$$

$$0 = \frac{Bl^3}{4} + \frac{Al^3}{3} - \frac{Al^3}{4} - \frac{Pl^6}{60EI}\left(\frac{5}{4} - \frac{10}{6} + \frac{9}{8} - \frac{4}{9}\right)$$

$$0 = \frac{Bl^3}{4} + \frac{Al^3}{12} - \frac{Pl^6}{60EI}\left(\frac{19}{72}\right)$$

<u>Equ. 13</u> $\quad 3B + A = \dfrac{19Pl^3}{360EI}$

solve Equs. 12 + 13 for A + B.

$$3B + A = \frac{19Pl^3}{360EI}$$

$$2B + A = \frac{19Pl^3}{420EI}$$

$$B = \frac{19Pl^3}{2520EI}$$

$$A = \frac{26Pl^3}{2520EI}$$

Now by having A + B the slope of the line, gh, is known, and the difference between the gh, line and the curve, cdef, is the added de-flection to be added to the deflection of the infinitely stiff Pile.

These added deflections are given in table 1.

Table 1

* Deflections by the Analytical Method

point	Values of Equ. 11	Values of the 9 f. unc. fig. 2	Value of "U"
.0l	0	1.85366	+1.854
.1l	0.49009	1.71493	+1.225
.2l	0.92262	1.57591	+0.653
.3l	1.24895	1.43688	+0.188
.4l	1.43578	1.29786	-0.138
.5l	1.50375	1.15884	-0.345
.6l	1.35222	1.01951	-0.333
.7l	1.11203	0.88049	-0.232
.8l	0.78054	0.74146	-0.040
.9l	0.39865	0.60344	-0.203
1.0l	0	0.46342	-0.463

* all values given in terms of $\frac{Pl^3}{10EI}$

The analytical method could be continued on, but due to the fact that the graphical method is much faster and gives results within 5 to 7 per cent of the analytical I have resorted to it to carry the investigation on.

The graphical Method of finding Deflections

In the graphical method of finding deflections the operations are exactly the same as those for the analytical method. First a deflection times a pressure gives a force; then these forces are sumated to give the shear, and the process is continued exactly as in the analytical method only the intergration is done graphical. It is found that by careful work the results of the two method are within seven percent of each other.

The caculations for the results after the curves, cdef, of Fig. 2, is gotten graphical are done in the same manor as for the analytical method only in the graphical method a summation is used instead of the intergration. The two equations for A and B, Equ. 12 and 13, may be stated more generaly so that they be used for the summation

Equ. 12 becomes,

$$2B + A = \Sigma H \times 6$$

Equ. 13 becomes

$$3B + A = \Sigma M \times 12$$

The results of the graphical methods are given in table 2 and the graphical work is found in the appendices.

Calculations for the "gh" Line of Fig. 2

$2B + A = 6 \times \Sigma H$
$3B + A = 12 \times \Sigma M$

1st trial.

$2B + A = 6 \times 0.4324 \frac{Pl^3}{60EI}$
$3B + A = 12 \times 0.2562 \frac{Pl^3}{60EI}$
$B = 0.4800 \frac{Pl^3}{60EI}$
$A = 1.6344 \frac{Pl^3}{60EI}$

2nd trial

$2B + A = 6 \times 0.13204 \frac{Pl^3}{60EI}$
$3B + A = 12 \times 0.07885 \frac{Pl^3}{60EI}$
$B = 0.15396 \frac{Pl^3}{60EI}$
$A = 0.48432 \frac{Pl^3}{60EI}$

3rd trial

$2B + A = 6 \times 0.03059 \frac{Pl^3}{60EI}$
$3B + A = 12 \times 0.01866 \frac{Pl^3}{60EI}$
$B = 0.04038 \frac{Pl^3}{60EI}$
$A = 0.10278 \frac{Pl^3}{60EI}$

4th trial

$2B + A = 6 \times 0.01156 \frac{Pl^3}{60EI}$
$3B + A = 12 \times 0.00703 \frac{Pl^3}{60EI}$
$B = 0.01502 \frac{Pl^3}{60EI}$
$A = 0.03932 \frac{Pl^3}{60EI}$

5th trial.

$2B + A = 6 \times 0.00384 \frac{Pl^3}{60EI}$
$3B + A = 12 \times 0.00231 \frac{Pl^3}{60EI}$
$B = 0.00468 \frac{Pl^3}{60EI}$
$A = 0.01368 \frac{Pl^3}{60EI}$

6th trial.

$2B + A = 6 \times 0.00070 \frac{Pl^3}{60EI}$
$3B + A = 12 \times 0.00043 \frac{Pl^3}{60EI}$
$B = 0.00096 \frac{Pl^3}{60EI}$
$A = 0.00228 \frac{Pl^3}{60EI}$

7th trial

$2B + A = 6 \times 0.00023 \frac{Pl^3}{60EI}$
$3B + A = 12 \times 0.000133 \frac{Pl^3}{60EI}$
$B = 0.00023 \frac{Pl^3}{60EI}$
$A = 0.00094 \frac{Pl^3}{60EI}$

8th trial.

$$2B + A = 6 \times 0.000081 \frac{Pl^3}{60EI}$$

$$3B + A = 12 \times 0.000049 \frac{Pl^3}{60EI}$$

$$B = 0.00010 \frac{Pl^3}{60EI}$$

$$A = 0.00018 \frac{Pl^3}{60EI}$$

Table 2

Results of Graphical Intergration to find Deflection
(all terms given in $\frac{PL^3}{60EI}$)

	Point	Def	Def X Load	H Area of Load	Mom. arm	M Statical Mom	Def. of Line AB	U
1st	0	0	0				1.6344	+1.6344
				.002445	.067	.0002		
	.1l	.489	.0489				1.5190	+1.0300
				.011895	.180	.0021		
	.2l	.895	.1790				1.4035	+0.5085
				.026745	.278	.0075		
	.3l	1.193	.3579				1.2881	+0.0951
				.045375	.377	.0171		
	.4l	1.374	.5496				1.1726	-0.2014
				.062505	.500	.0313		
	.5l	1.401	.7005				1.0572	-0.3438
				.073515	.549	.0400		
	.6l	1.293	.7758				0.9418	-0.3512
				.077460	.648	.0496		
	.7l	[illegible]	.7632				0.8263	-0.2497
				[illegible]	.747	.0506		
	.8l	.750	.6000				0.7109	-0.0391
				.047910	.846	.0406		
	.9l	.398	.3582				0.5954	+0.7974
				.01790	.923	.0167		
	1.0l	0	0				0.4800	+0.4800
	Σ			.4324		.2562		
2nd	0l	0	0				.48432	+.48432
				0.00061	.067	0.00004		
	.1l	0.1215	0.01215				.45128	+.32978
				0.00295	.155	0.00046		
	.2l	0.2345	0.04690				.41805	+.28375
				0.00716	.254	0.00181		
	.3l	0.3175	0.09525				.38601	+.06791
				0.01246	.353	0.00440		
	.4l	0.3848	0.15392				.35315	-.03262
				0.01800	.452	0.00813		
	.5l	0.4123	0.20615				.31914	-.09316
				0.02246	.551	0.01240		
	.6l	0.4050	0.24300				.28616	-.11890
				0.02420	.650	0.01573		
	.7l	0.3443	0.24101				.25307	-.09123
				0.02206	.750	0.01654		
	.8l	0.2503	0.20021				.22003	-.03027
				0.01610	.848	0.01365		
	.9l	0.1353	0.12177				.18700	+.05170
				0.00809	.933	0.00667		
	1.0l	0	0				.15396	+.15396
	Σ			0.13204		0.07885		

Table 2, Cont.

Point	Def. $\frac{Pl^3}{60EI}$	Def. X Load	Area of Load "A" $\frac{Pl^3}{60EI}$	Mom. Arm	Statical Moment "M" $\frac{Pl^3}{60EI}$	Def. of the A-B Line in $\frac{Pl^3}{60EI}$	U " $\frac{Pl^3}{60EI}$
3rd -							
0	0	0				.10278	+.10278
			.00012	.067	.00001		
.1l	.0240	.0024				.09654	+.07254
			.00058	.160	.00009		
.2l	.0464	.00928				.09030	+.05390
			.00146	.257	.00038		
.3l	.0670	.02010				.08406	+.01706
			.00241	.35[illegible]	.00093		
.4l	.0900	.03200				.07782	-.00218
			.00391	.452	.00177		
.5l	.0935	.04625				.07158	-.02092
			.00514	.552	.00286		
.6l	.0940	.05760				.06534	-.02866
			.00580	.651	.00378		
.7l	.0835	.05845				.05910	-.03440
			.00548	.750	.00411		
.8l	.0642	.05120				.05286	-.01114
			.00400	[illegible]	.00339		
.9l	.0320	.02880				.04662	+.01462
			.00144	.93[illegible]	.00134		
1.0l	0	0				.04038	+.04038
Σ			.03059		.01866		
0	0	0				.03732	+.03732
			.00004	.067	.00000		
.1l	.00[illegible]	.00284				.03689	+.02849
			.00021	.155	.00003		
.2l	[illegible]	.00658				.03446	+.01756
			.00054	.250	.00014		
.3l	[illegible]	[illegible]				.03203	+.00763
			.00097	.352	.00035		
.4l	.0211	[illegible]				.02960	.00150
			.0151	.452	.00068		
.5l	[illegible]	.01780				.02717	-.00843
			.00198	.551	.00109		
.6l	[illegible]	[illegible]				.02474	-.01166
			[illegible]	.650	.00145		
.7l	[illegible]	[illegible]				.02231	-.01009
			.00206	.750	.00155		
.8l	.0231	.01820				.01988	-.00322
			.00146	.847	.0012[illegible]		
.9l	.0120	[illegible]				.01745	+.00545
			.00054	.93[illegible]	[illegible]		
1.0l	0	0				.01502	+.01502
Σ			.01180		.00763		

Table 2, Cont.

Point	Def.	Def. x Load	H Area of Load	Mom. Arm	M Statical Mom.	Def. of Line AB	u
5th 0	0	0				.01368	+.01368
			.00001	.067	.00000		
.1l	.0028	.00028				.01278	+.00998
			.00007	.155	.00001		
.2l	.0056	.00112				.01188	+.00628
			.00028	.254	.00007		
.3l	.0082	.00246				.01098	+.00278
			.00033	.353	.00012		
.4l	.0104	.00416				.01008	-.00032
			.00050	.451	.00023		
.5l	.0116	.00580				.00918	-.00242
			.00064	.551	.00035		
.6l	.0117	.00702				.00828	-.00342
			.00070	.650	.00046		
.7l	.0101	.00707				.00738	-.00272
			.00065	.749	.00049		
.8l	.0075	.00600				.00643	-.00102
			.00048	.848	.00041		
.9l	.0039	.00351				.00558	+.00168
			.00018	.933	.00017		
1.0l	0	0				.00468	+.00468
Σ			.00384		.00231		
6th							
0	0	0				.00228	+.00228
			0	.067	0		
.1l	.0005	.00005				.00215	+.00165
			.00001	.160	0		
.2l	.0009	.00018				.00201	+.00111
			.00003	.259	.00001		
.3l	.0014	.00042				.00188	+.00048
			.00006	.357	.00002		
.4l	.0018	.00072				.00175	-.00005
			.00009	.455	.00004		
.5l	.0021	.00105				.00162	-.00048
			.00011	.553	.00006		
.6l	.0021	.00126				.00149	-.00061
			.00013	.651	.00008		
.7l	.0018	.00126				.00135	-.00045
			.00012	.748	.00009		
.8l	.0013	.00104				.00122	-.00008
			.00010	.846	.00008		
.9l	.0007	.00063				.00109	+.00039
			.00005	.933	.00005		
1.0l	0	0				.00096	+.00096
Σ			.00070		.00043		

Table 2. Cont.

	Point	Def. [illegible]	Def. of load	Area of load	Mom. Arm	Static Moment	Def. of the R.B. [illegible]	U in [illegible]
[illegible]	0	0	0				.00094	+.00094
				0	.064	0		
	1 l	.0002	.00008				.00087	+.00067
				.00001	.150	.000001		
	2 l	.0004	[illegible]				.00080	+.00040
				.00001	[illegible]	.000002		
	3 l	.0006	.00018				.00072	+.00012
				.00002	[illegible]	.000007		
	[illegible]	[illegible]	.00028				.00065	-.00005
				.00003	.455	.000015		
	5 l	.0008	.00040				.00058	-.00022
				.00004	.553	[illegible]		
	6 l	.0008	.00048				.00051	-.00029
				.00005	[illegible]	[illegible]		
	7 l	.0006	.00042				.00044	-.00016
				.00004	.750	.000060		
	8 l	.0004	[illegible]				.00036	-.00004
				.00002	.845	.000017		
	9 l	.0002	.00018				.00029	+.00009
				.00001	.933	[illegible]		
	10 l	0	0				.00022	+.00022
				.00023		.000137		
8th	0	0	0				.00018	+.00018
				0	.047	0		
	1 l	.00002	.000006				.00017	+.00011
				.000002	.160	0		
	2 l	.00012	.000021				.00016	+.00004
				.000004	.258	.000001		
	3 l	.00019	.00051				.00016	-.00001
				.000007	.357	.000002		
	4 l	[illegible]	.00088				.00015	-.00007
				.000011	[illegible]	[illegible]		
	5 l	[illegible]	[illegible]				.00014	-.00011
				.000014	[illegible]	.000008		
	6 l	[illegible]	[illegible]				.00013	-.00012
				.000015	[illegible]	.000010		
	7 l	[illegible]	[illegible]				.00012	-.00010
				.000024	[illegible]	.000011		
	8 l	[illegible]	.00125				.00012	-.00004
				.000010	[illegible]	[illegible]		
	9 l	.00005	.00012				.00011	+.00003
				.000004	[illegible]	.000004		
	10 l	0	0				.00010	+.00010
				.00021		.000049		

Table-3

*Total Deflection of Pile by Graphical Method, assuming $e = \frac{1080EI}{l^4}$

point	Total added Def.	Def. of infinitely stiff Pile	Total Def. of Pile
0	+2.2777	+1.000	+3.2777
.1l	+1.4732	+0.867	+2.3402
.2l	+0.9176	+0.733	+1.6506
.3l	+0.1909	+0.600	+0.7909
.4l	-0.2382	+0.467	+0.2288
.5l	-0.4699	+0.333	-0.1369
.6l	-0.5149	+0.200	0.3149
.7l	-0.3889	+0.067	-0.3219
.8l	-0.0849	-0.067	-0.1519
.9l	+0.4953	-0.200	+0.6715
1.0l		-0.333	+0.3625

* all values given in terms of $\frac{Pl^3}{60EI}$

Final Deflection Curve, asuming $e = \frac{1080\,EI}{l^4}$

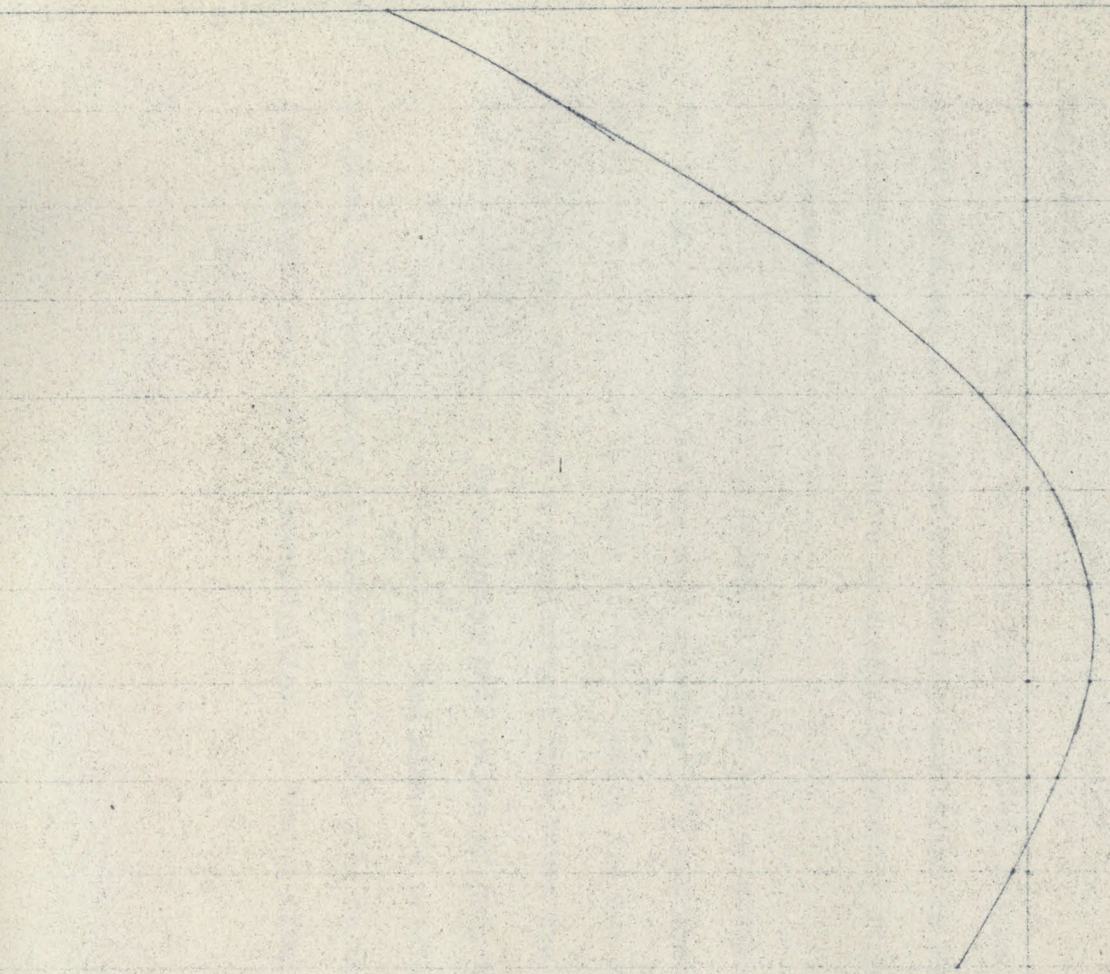


Fig. 3

$$\text{Deflection} = \frac{18P}{el}\left[\frac{3l-4y}{3l}\right] + K_y\,\frac{Pl^3}{60EI}$$

CONCLUSIONS

Finding the deflection curve for a pile due to a horizontal load has proven to be most interesting. The final curve for the deflection of the pile as seen in figure 3 is very much different from that of the straight line as was assumed in the beginning.

The results obtained in this investigation, both from the analytical and graphical method, are in very close agreement, and the best place to compare the results is in the calculation for the original deformation "Q". By the analytical method from equation 7, $Q = \frac{18\ P}{e\ l}$, where by the graphical method $Q = \frac{18.5\ P}{e\ l}$, giving an error of about three per cent, which for all practical purposes is negligible.

If the results of table 2 are studied, one can see that the amount of added deflection obtained by each trial is less. This is an indication that the curve is gradually becoming fixed, but at a very slow rate.

It is noticed in equation 7 that the original deformation "Q" is in proportion to the soil pressure "e", but the results of the added deflection are in terms of $\frac{l^3}{E\ I}$ which are not found in equation 7, so for the sake of simplicity I have taken the soil pressure "e" $= \frac{1080\ E\ I}{l^4}$. By assuming this value of "e", the value of $Q = \frac{P\ l^3}{60\ E\ I}$ and makes it possible to add the original deformation of the infinitely stiff pile to the added deflections found by the investigation, so that a final curve can be obtained. The final results may be found in table 3, and the deflection curve from the results is shown in figure 3.

The shape of the final deflection curve of figure 3, is quite different from the deflection curve of the infinitely stiff pile. Upon examination of this curve we find that we have two points of zezo bend, and the lower end of the pile is deflecting in the same direction as the top.

I was able to carry the investigation through only eight trials, and the final deflection curve would only be obtained after an infinite number of these trials. If this had been done the curve in the lower portion would have looked like a damped sine curve instead of being as it is shown in figure 3.

The points of zero deflections in the general case an almost infinite in number, because of the curve being a damped sine curve, which will cross the zero line several times, but I believe for practical purposes this curve shows the general shape.

The distance the first point of zero bend moves up from the three-quarter point of the infinitely stiff pile, is in relation to the moment of inertia of the pile, and must be considered in design.

In general it can be said that the deflection curve for a pile due to a horizontal applied load is:

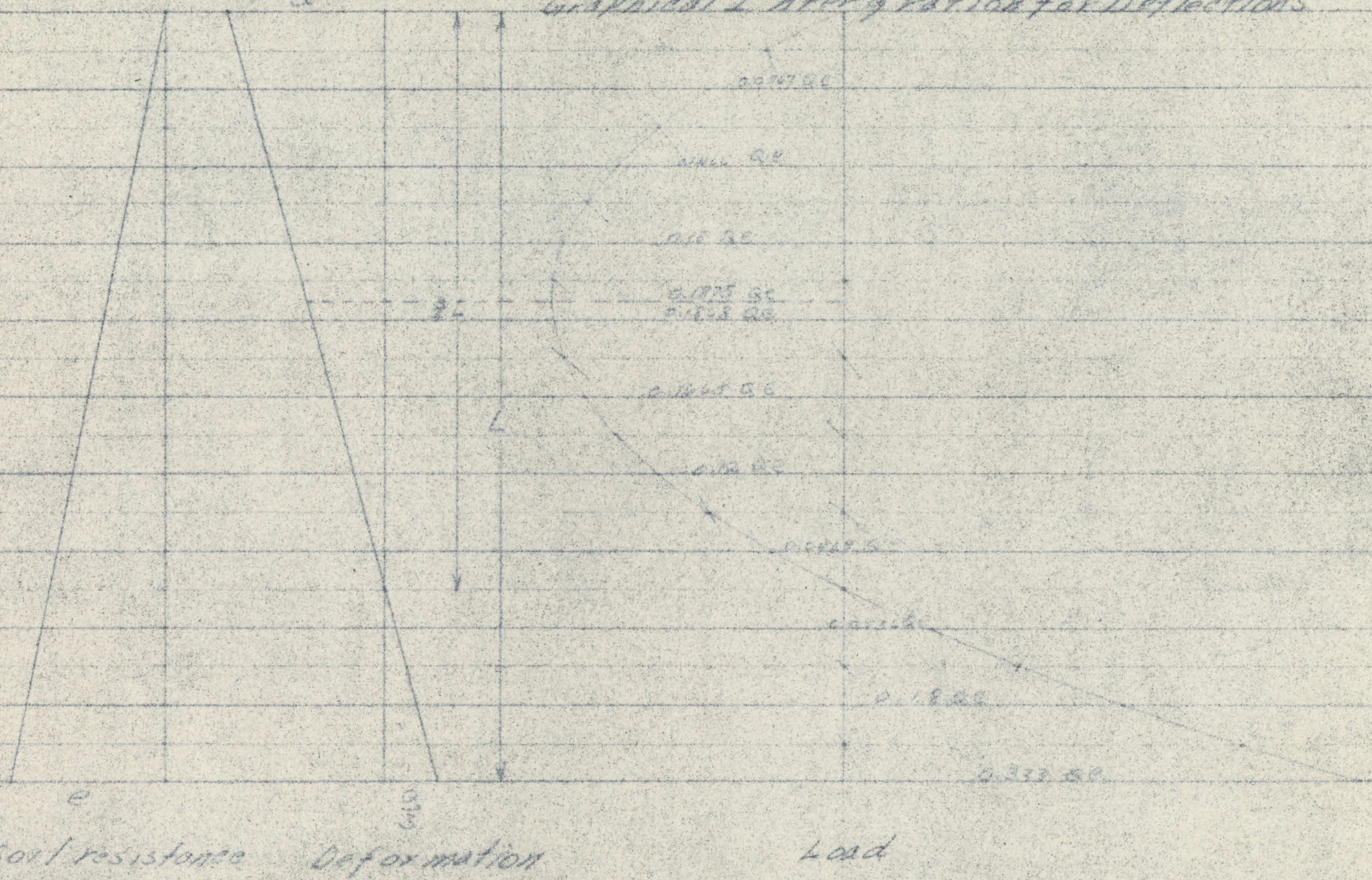
$$\Delta = \frac{18\,P}{e\,l}\left[\frac{3\,l - 4\,y}{3\,l}\right] + K_y \frac{P\,l^3}{60\,E\,I}$$

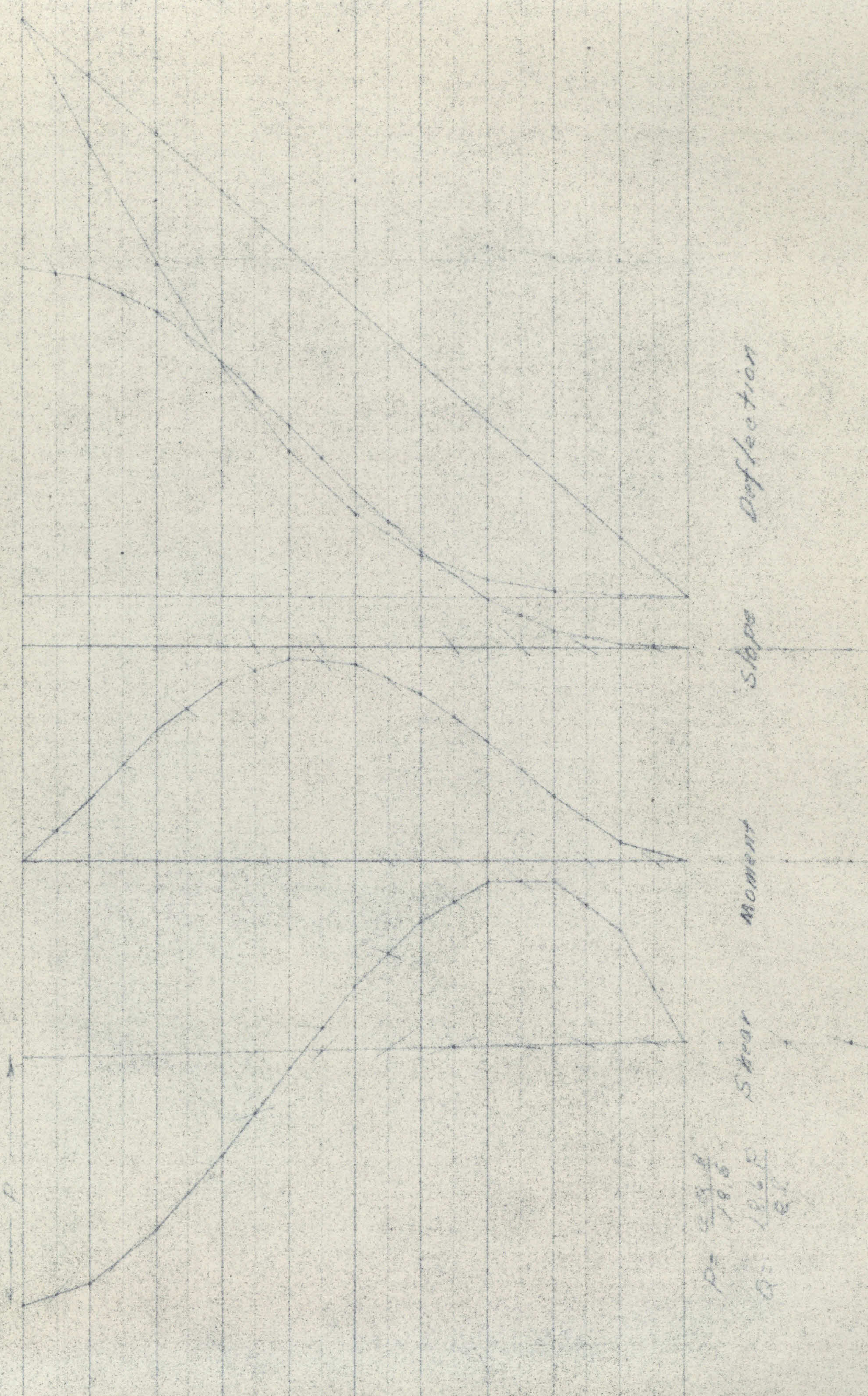
K_y being a variable with the depth, and may be found by determining the added deflection.

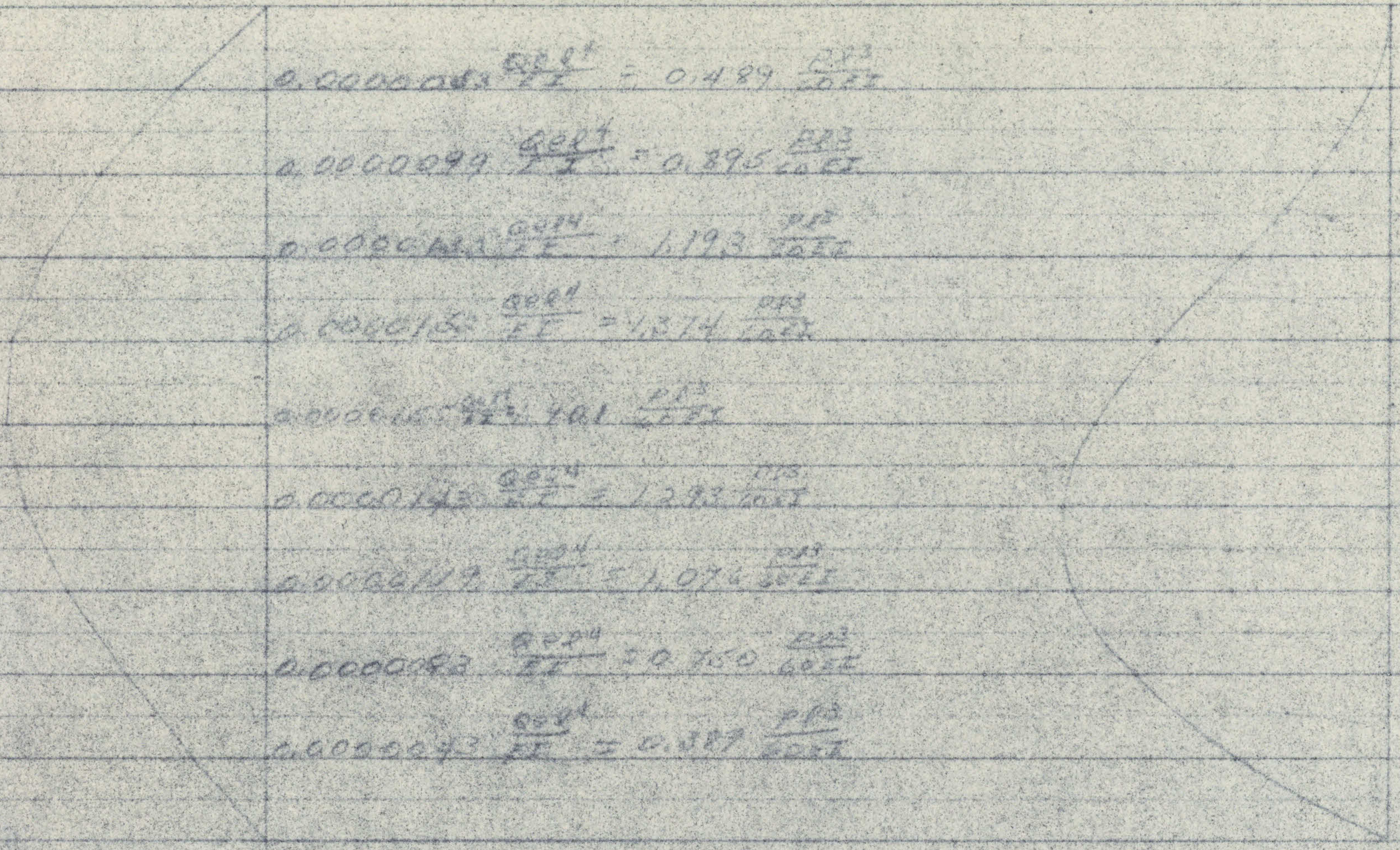
APPENDIX

1st Trial

Graphical Integration for Deflections




Deflection
Slope
Moment
Shear



$0.0000043 \frac{QL^4}{EI} = 0.489 \frac{PL^3}{60EI}$

$0.0000099 \frac{QL^4}{EI} = 0.895 \frac{PL^3}{60EI}$

$0.0000133 \frac{QL^4}{EI} = 1.193 \frac{PL^3}{60EI}$

$0.0000153 \frac{QL^4}{EI} = 1.374 \frac{PL^3}{60EI}$

$0.0000155 \frac{QL^4}{EI} = 1.401 \frac{PL^3}{60EI}$

$0.0000143 \frac{QL^4}{EI} = 1.293 \frac{PL^3}{60EI}$

$0.0000119 \frac{QL^4}{EI} = 1.074 \frac{PL^3}{60EI}$

$0.0000083 \frac{QL^4}{EI} = 0.750 \frac{PL^3}{60EI}$

$0.0000043 \frac{QL^4}{EI} = 0.387 \frac{PL^3}{60EI}$

Def By Graphics

Load Def

Example

$0.0000043 \frac{QL^4}{EI}$ Measured Def

From Shear Curve $S = \frac{15.6\,P}{L^3}$

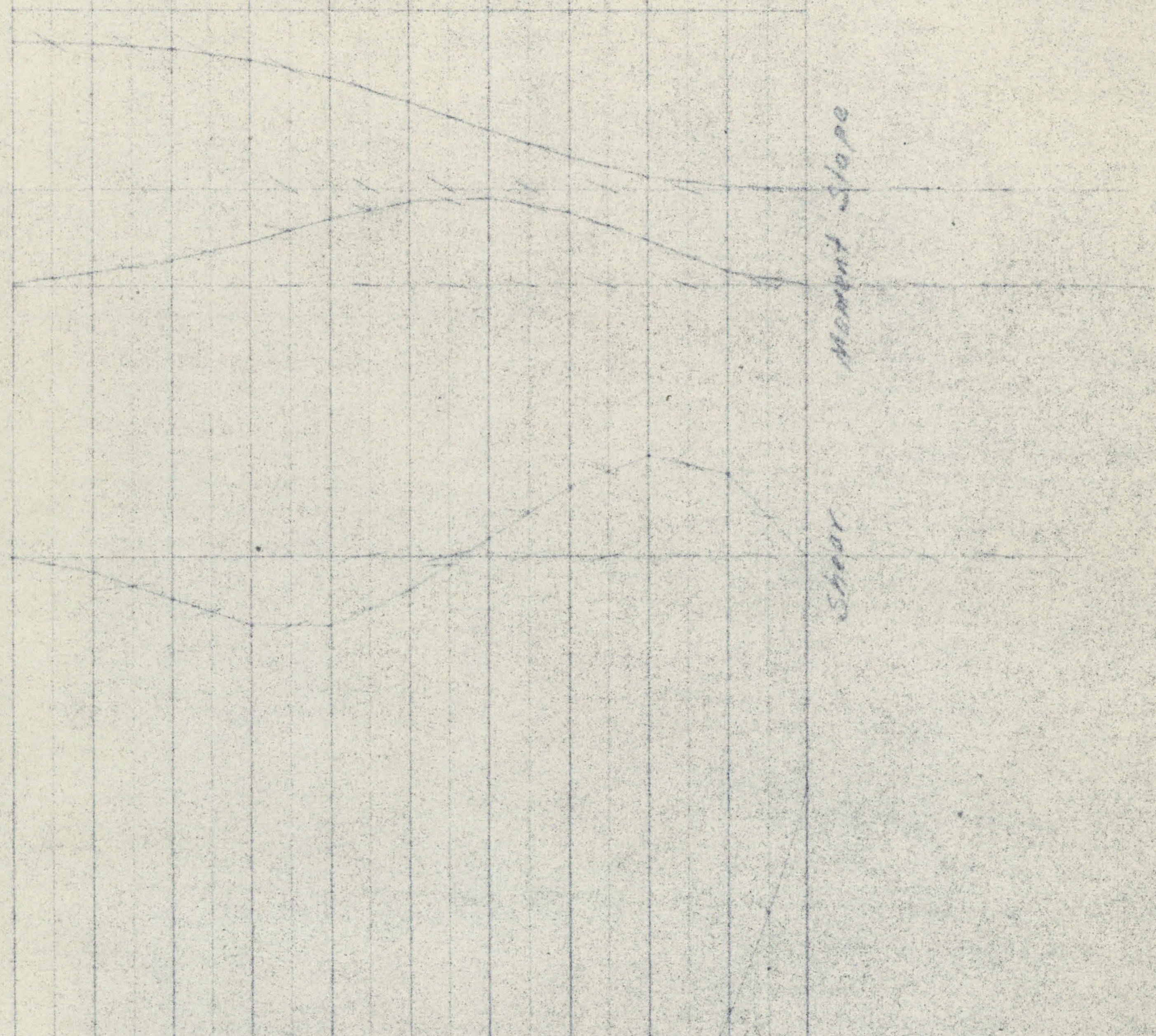
These curves have been Integrated 4 times by 3

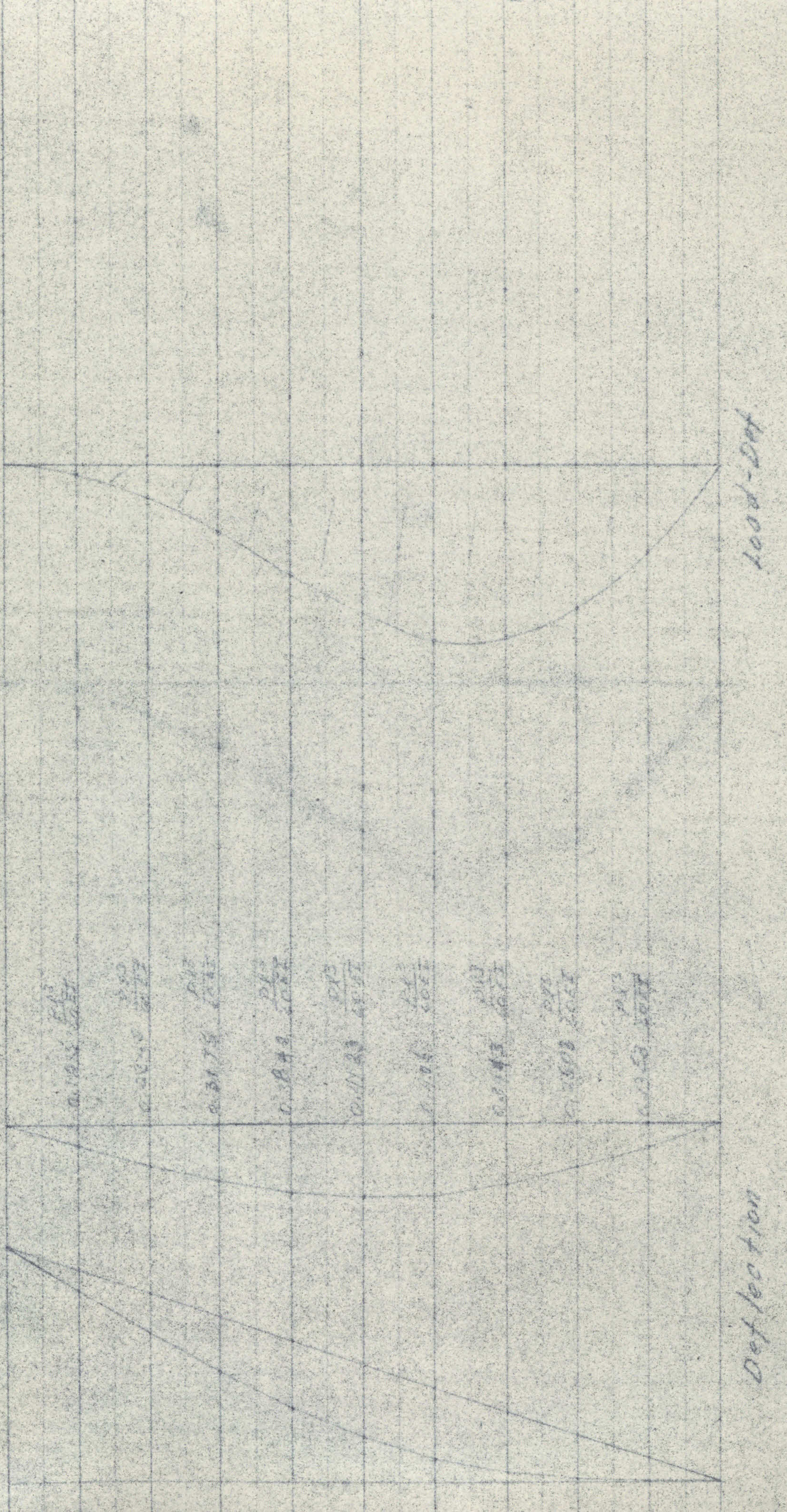
So $0.0000043 \times \frac{15.6\,P}{L^3} \times \frac{QL^4}{EI} \times \frac{3 \times 3 \times 3 \times 3}{1} \times \frac{60}{60}$

Def = $0.383 \frac{PL^3}{60EI}$

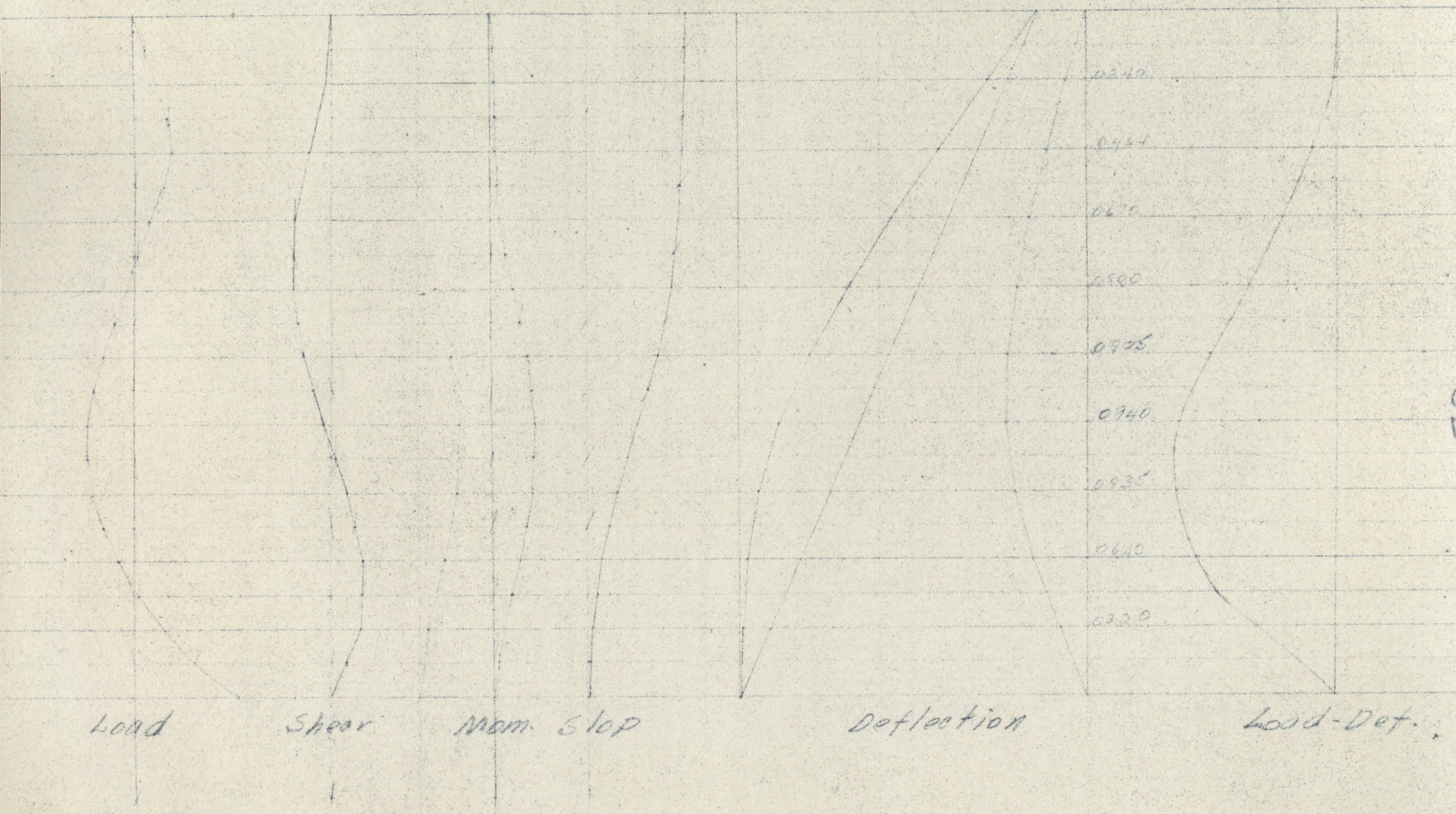
By Math Def = $0.387 \frac{PL^3}{60EI}$

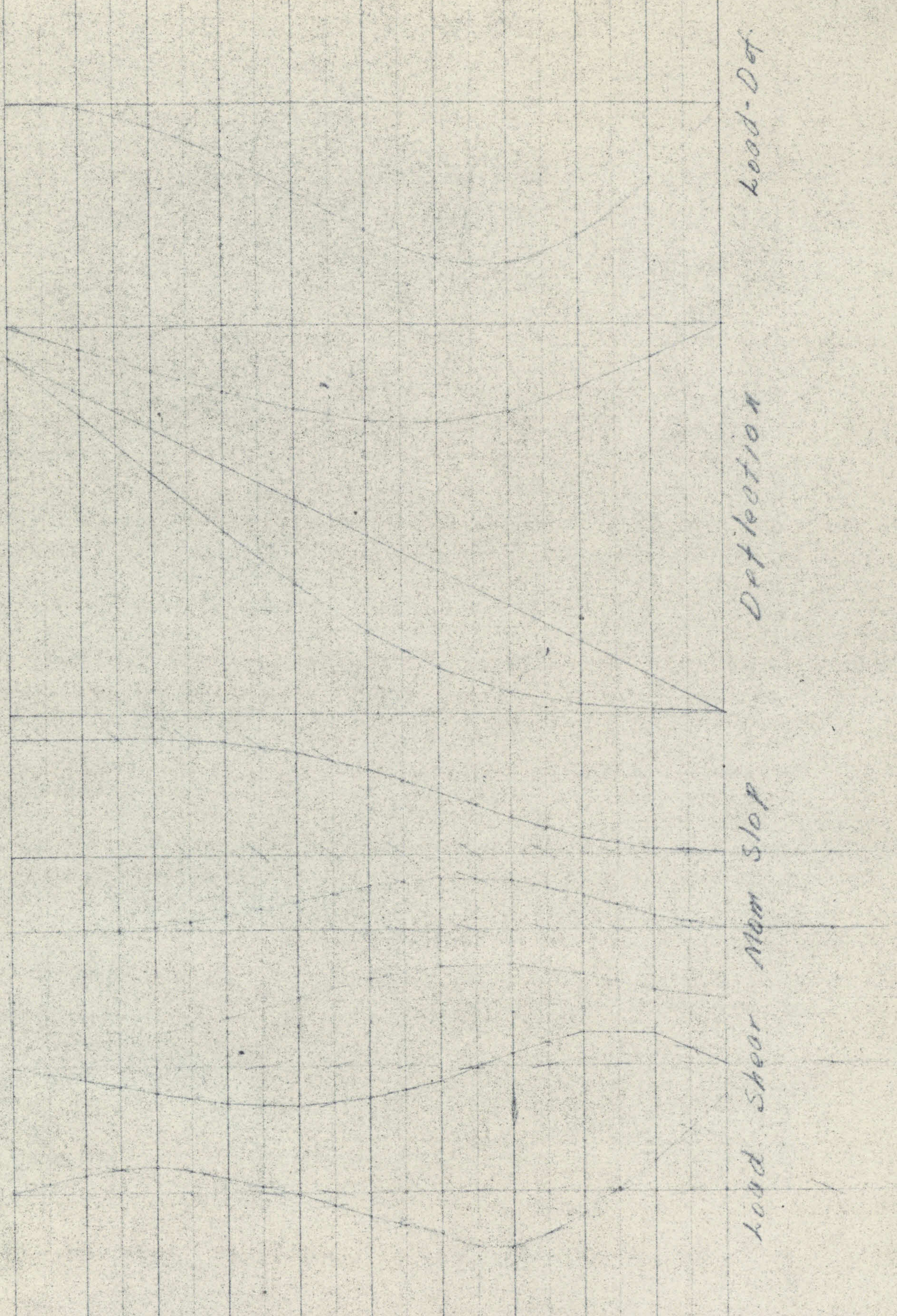
error = 1.31 %

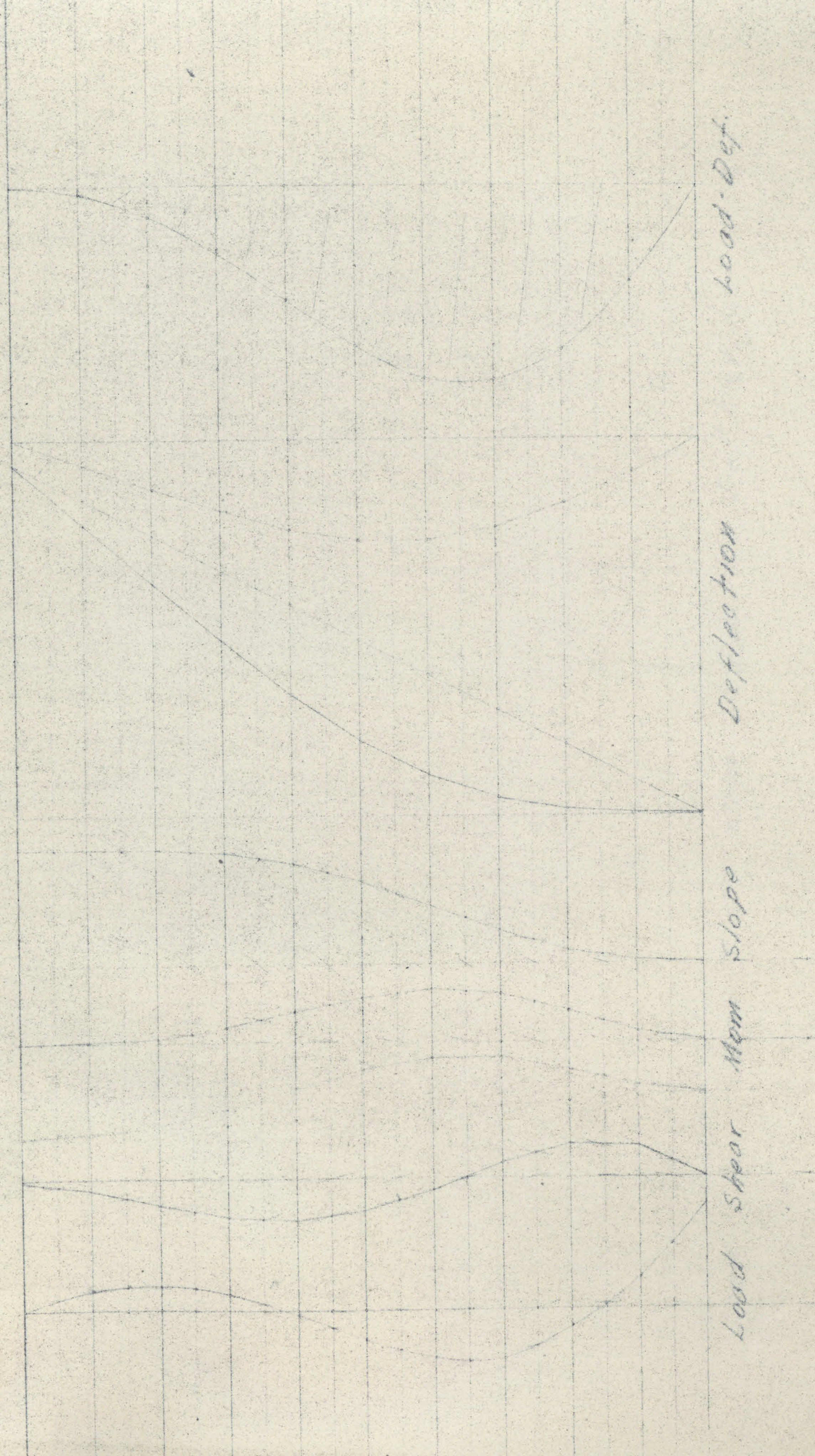

Moment Slope
Shear
Load


Load-Dia
Deflection

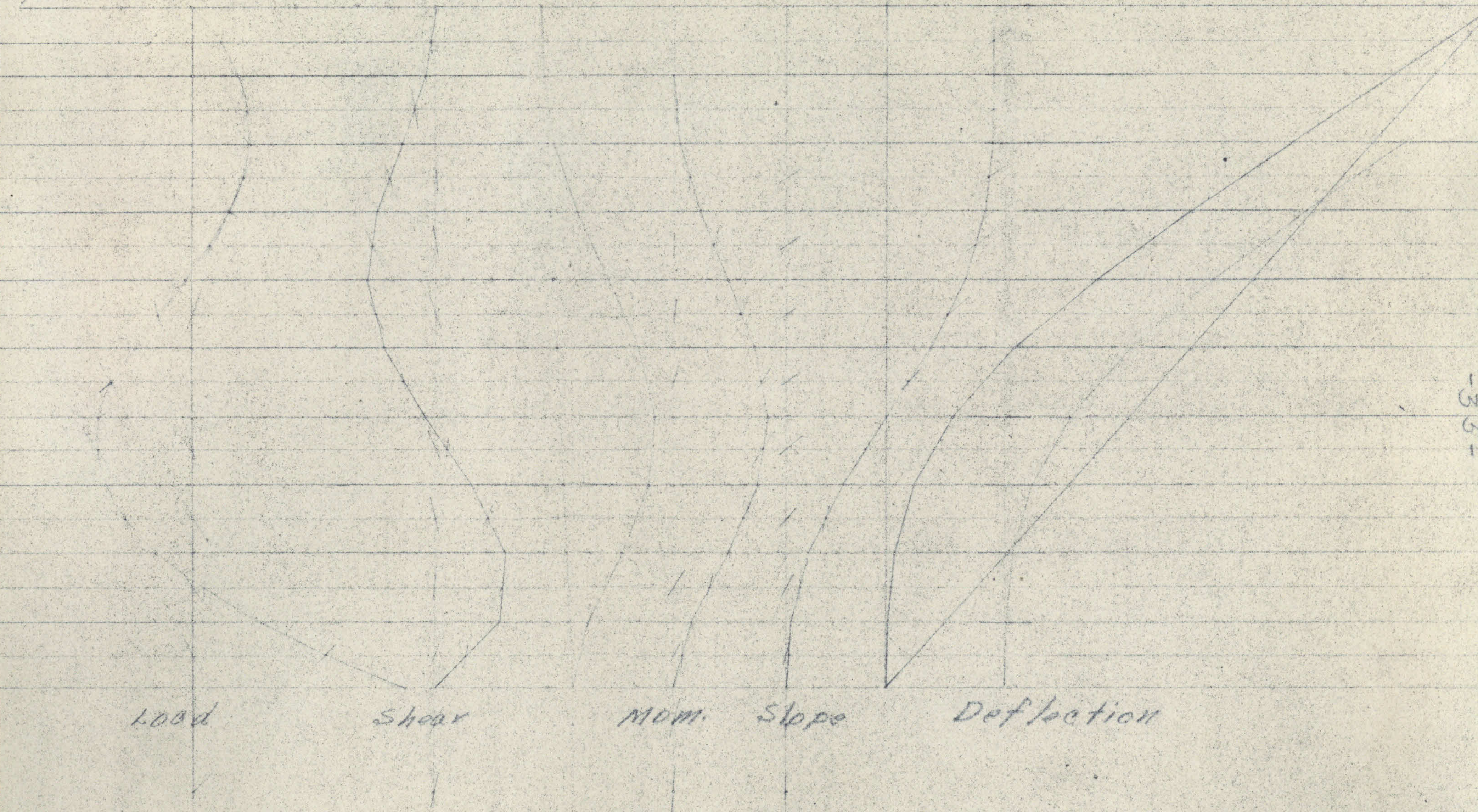
3- Trial

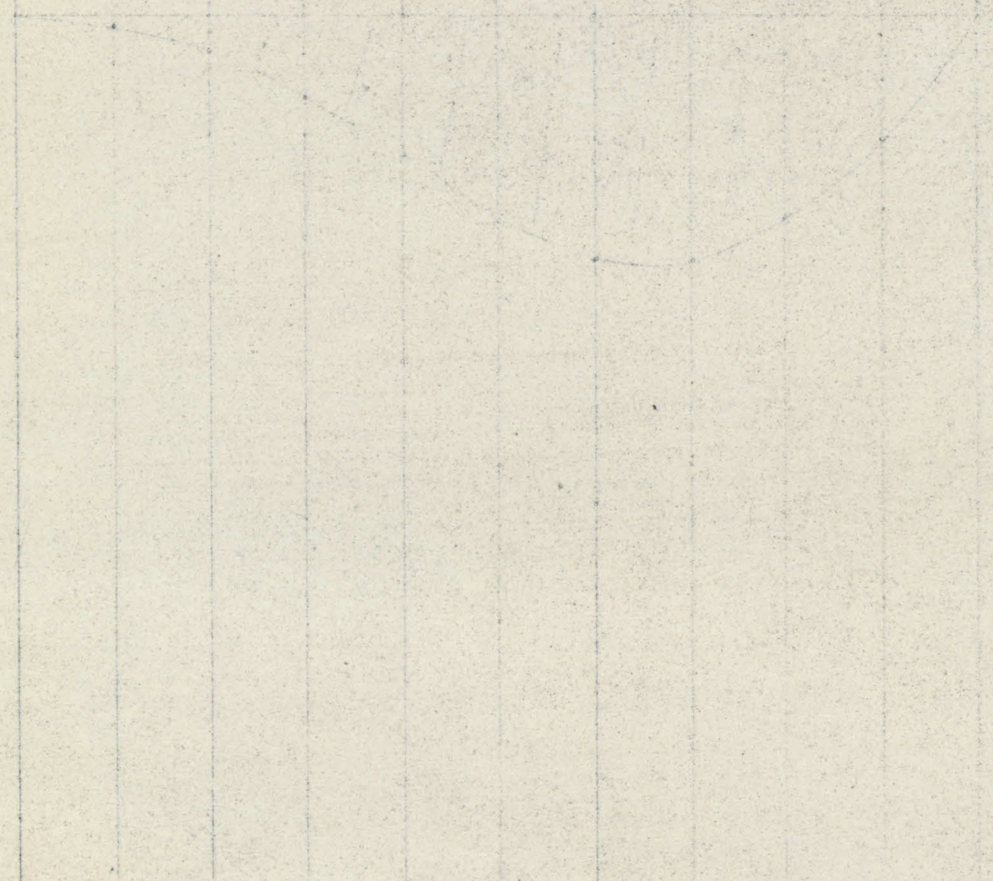

.0240
.0464
.0670
.0800
.0925
.0940
.0835
.0640
.0320
Load
Shear
Mom.
Slop
Deflection
Load-Def.


Load Shear Mom Slop Deflection Load-Def


Load
Shear
Mom
Slope
Deflection
Load-Def.

6th Trial

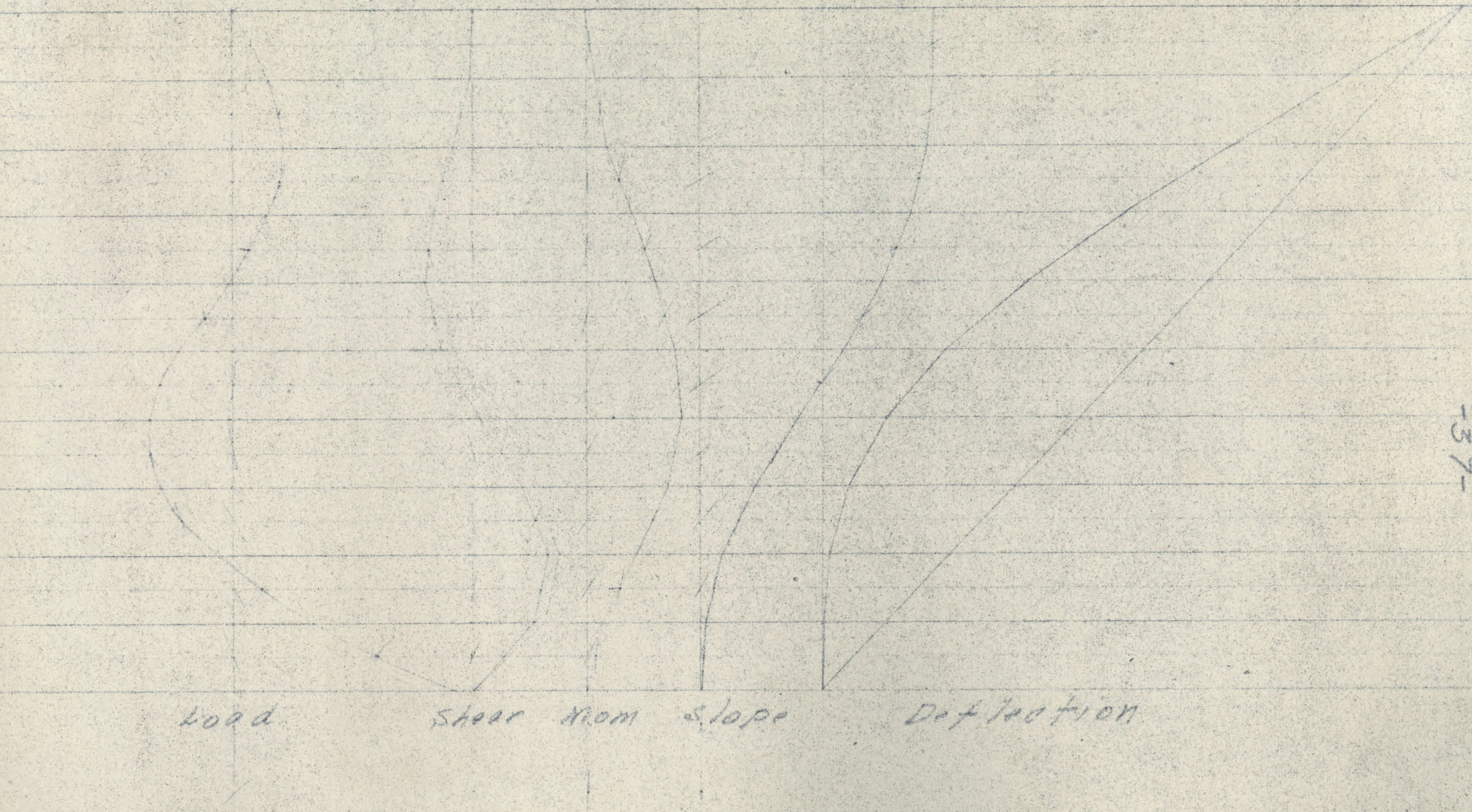


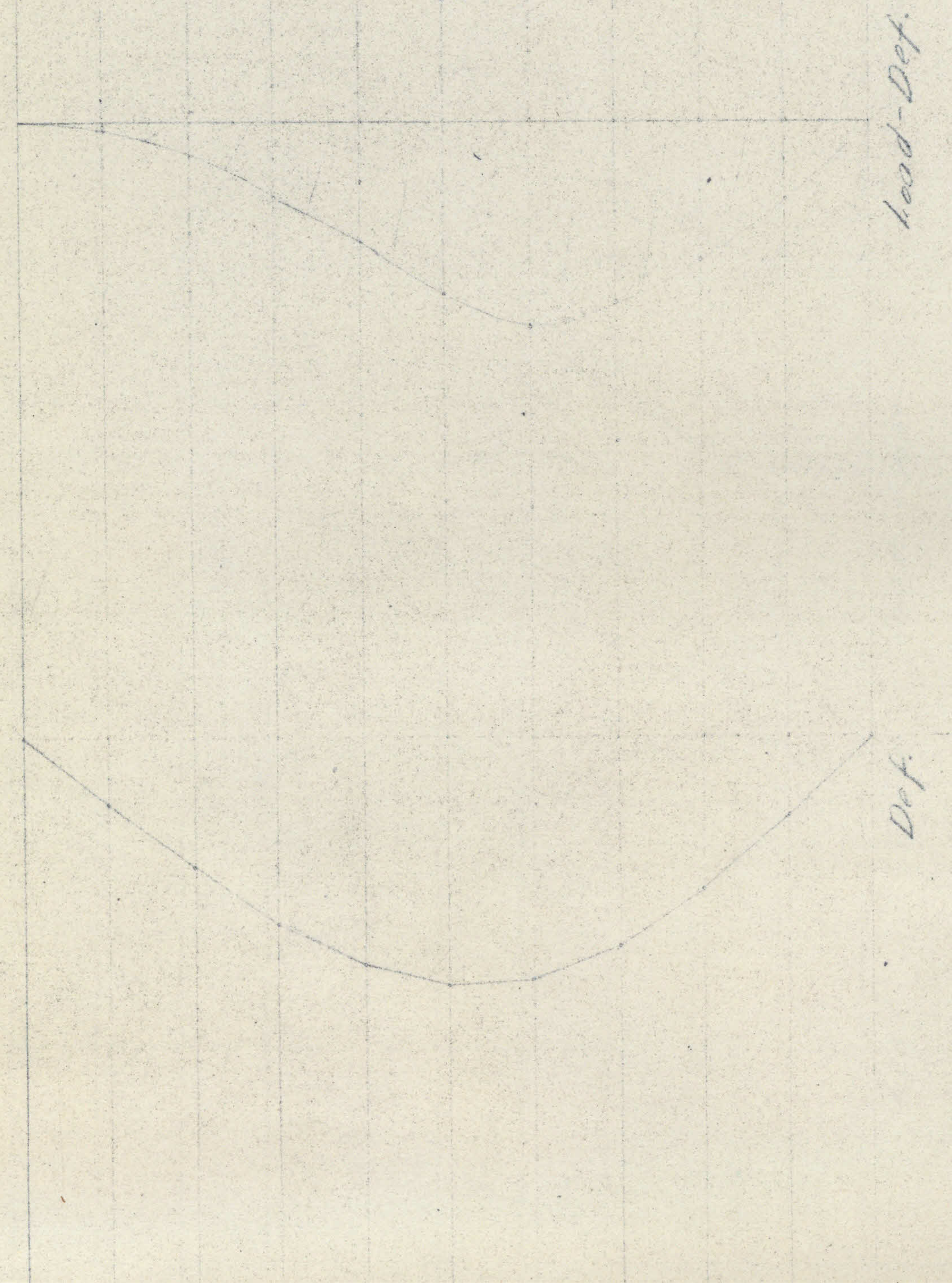


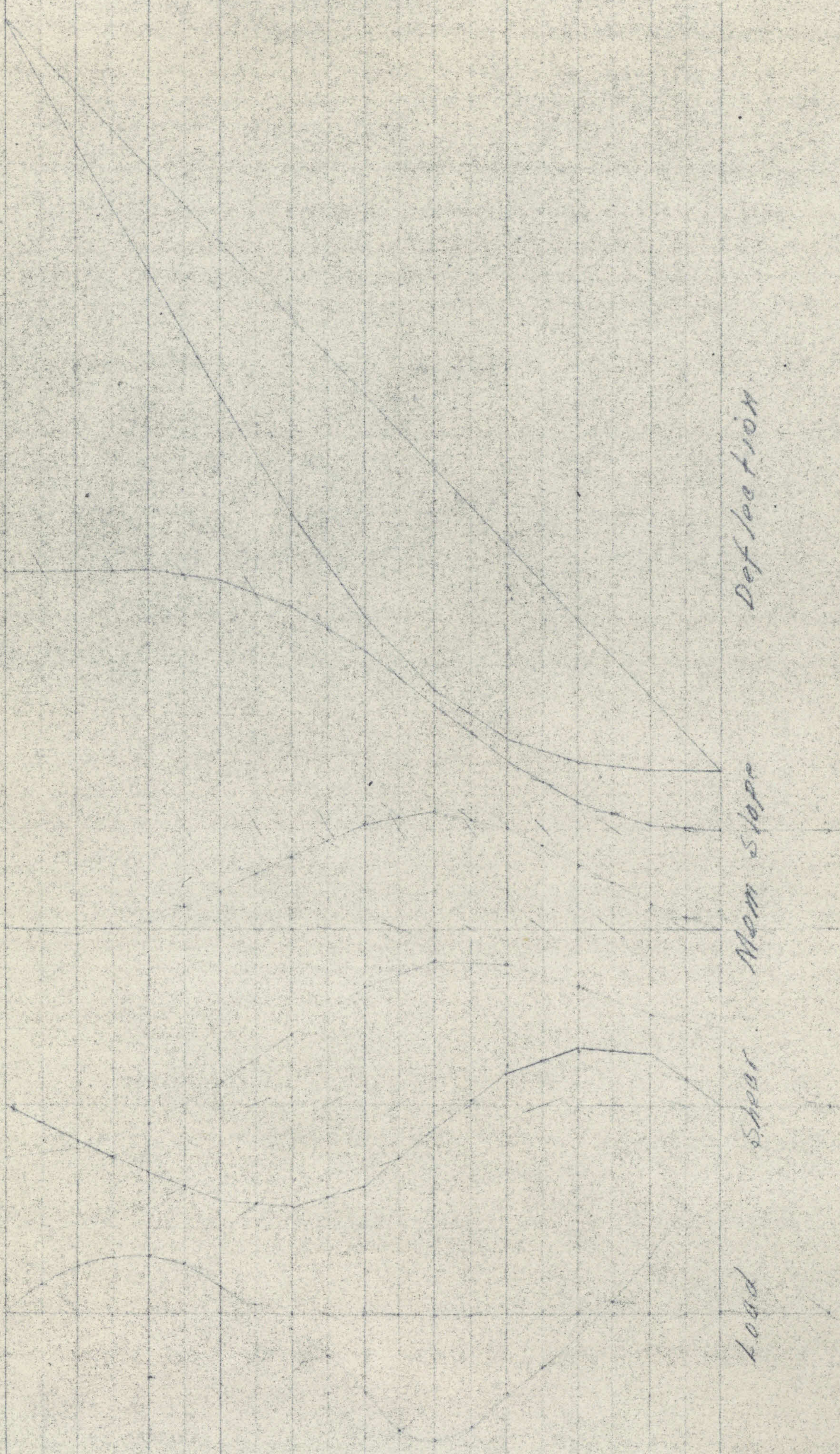
1000-Det

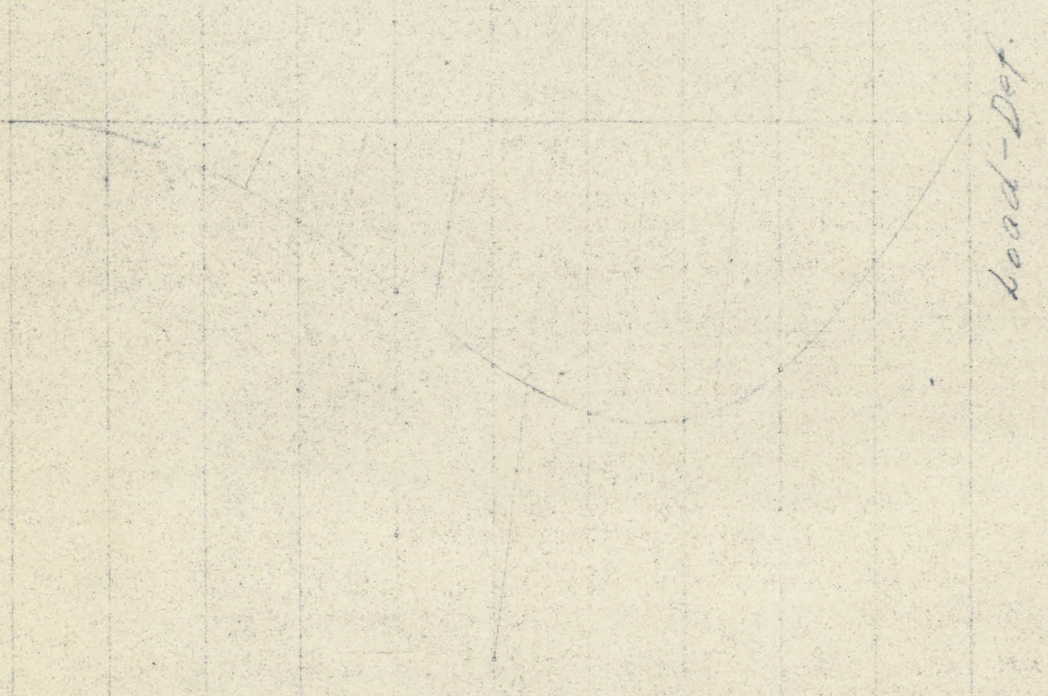
Det

Trial




Load-Def.
Def.


Deflection
Slope
Mom
Shear
Load


Load-Def.
Def.