AN INVESTIGATION OF THE DEFLECTION CURVE FOR A PILE

DUE TO A HORIZONTAL LOAD

By

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PRESENTATION

I, Charles L. Cobb submit this thesis to the faculty of the California Institute of Technology, Department of Civil Engineering, in partial fulfillment of the requirements for a Degree of Master of Science in Civil Engineering.

ACKNOWLEDGEENT

I wish to extend my thanks to Professor R. R. Martel for the help and suggestions he has given me in the preparation of this Thesis.

PROCEDURE

I have chosen to investigate the deflection curve for a horizontally loaded sheet pile, and to draw from this investigation information that may lead to a better understanding of the behavior of sheet piling due to horizontal loading.

I proceed from the basis that as the pile is first loaded, it is infinitely stiff and simply deflects in a straight line. The forces acting upon this infinitely stiff beam or pile are in equilibrium and hence any additional forces that might be applied to it due to the release of this infinitely stiff pile must also be in equilibrium. These forces may be found from the differences in the deflections between the assumed and computed pile.

The first assumption that must be made is in what manner does the soil pressure vary with depth? For the sake of simplicity I have chosen that it varies as a straight line. This assumption is in close accordance with the manner in which sandy soil will act, but varies from that of clay considerably.

In the first calculations I will prove that the infinitely stiff pile, when loaded horizontally at the top, will rotate about a point three-quarters the length from the top. From this deflected position I will find the deflection curves for the first approximation, provided the pile was released from its infinitely stiff condition and allowed to bend. This added deflection will cause a new set of forces to be set up and thus these forces will cause an added deflection. This process will be continued until the amount of added deflection will become negligible.

The force applied to a pile is proportional to the soil pressure "e" and to the deflection "u". The soil pressure "e" is proportional to the depth, varying as a straight line, thus making the force applied to the pile proportional to the depth and the deflection:

> Equ. 1. f = e u yy being the depth

If the total forces acting on the pile are in equilibrium, then the summation of these forces must be equal to zero.

Equ. 2.
$$H = 0$$

 $H = \int_{0}^{0} e u Y dy = 0$

The pile being a free ended beam, the summation of the moments about the end must also be equal to zero.

Equ. 3.
$$M = 0$$

 $M = \int_{e}^{l} u Y^{2} dy = 0$

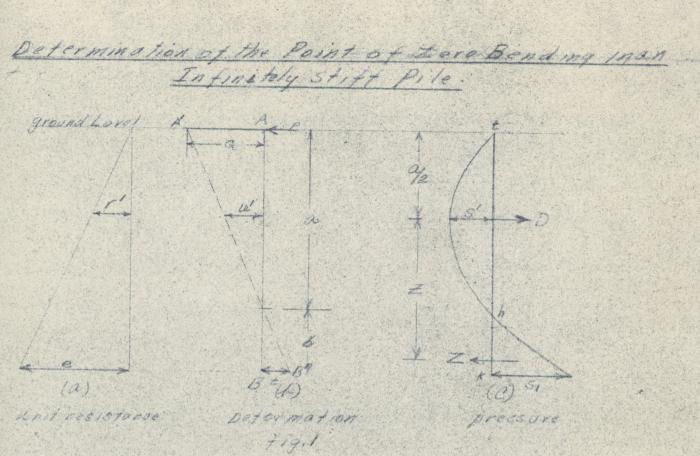
The equations 2 and 3 make it possible then to find the forces applied and the amount of deflection.

If these two unknowns are found for each subsequent deflection, and these dampen to a zero value, the total deflection and forces may be found.

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For my first deflection I will calculate both analytically and graphically, and the subsequent deflections I will do only graphically. The graphical method may introduce a small human error, but I will put this as small as possible.

I will now proceed to determine the point of zero deflection for the infinitely stiff pile. After this I will determine the first deflection analytically, and finally give the results of the subsequent deflections from the graphical method in tables 2 and 3.



Let AR stights, be the original position of a pile embedded in the earth and A B' the pasetion after aforee P is applied at A. The resistance of the earth Varies with the depth as shown in Fig. 1 (as the bisk is a diagram of the passive resistance per unit defermation

The source and pressure
$$s' = r'u'$$

 $r' = \frac{2u}{2iars}$
 $u' = \frac{2}{2} \cdot \frac{2}{a} = \frac{2}{3}$
Equil: $s' = \frac{2aa}{4tars}$
The maximum pressure sizet
 $z = \frac{2a}{3}$
Equil: $s_1 = \frac{2a}{3}$
 $z = \frac{2a}{3}$
Equil: $a = \frac{2}{3}$

Substituting Equ. 2 into Equ.1 2 = A & 3, A & (a+b) Equ3, 2 = 5 (atb) Equis dan be shown to be a paralola, we have 2p= blat bl as the paremeter of a parabola of the form g2: 2pd; 5'= = Move the exigin to the right a distance s' $g^2 = gP(q-s) = gPq + \frac{q}{4}$ D: for as represented by the area of the Corve in Fig. 1(c) from ttoh 1401 D= a S 4. dy = 2 S. (442-2) dy 50/ 34 - 27 / 12 $D = \frac{4^3}{34p} - \frac{a^2}{3p}$ $D = \frac{1}{12} \frac{1}{p} \frac{b(a+b)}{b}$ From $3p = \frac{b(a+b)}{b}$ we have $\frac{a^3}{b} \frac{5}{b}$ $D = \frac{a^3}{ab} \frac{5}{a+b}$ Z = force represented by the area of the Corte 11 + 19.1 (c) + rom h to k. Then: Sahtb Z = Say 4dy = 3 p 1a 442-02 xy $Z = \frac{bs_1(barsb)}{b(a+b)}$ moment of Z about D M= 1/2-bydy = bis (atb) The

 $z = \frac{44}{5} - \frac{6(a+b)}{5(3a+2b)} \times \frac{5(b(a+b))}{4}$ Z= 3/016/2 2(32+6) Equating moments about D Pe = 516 (33+26) × 3(2+6)2 $\frac{2}{\frac{Pa}{2}} = \frac{5/6(a+b)}{4}$ (33+2b) A= (a+b) 6 = A - a $\frac{Pa}{2} = \frac{S_1(A-a)A}{4}$ 2 Pa= SIA2-SiaA Equ. 4 as SIAZ The sum of the forces on the prichave P+Z=D $\frac{E945}{670} p + \frac{51613a+26}{67a+6} - \frac{3351}{667a+6} = 0$ from E gus. 4 and 5 and. SP+S,A Equ6. 51= 6 F ubstituting Equ 6 into Equ 4 are have EPAZ $a = \frac{e}{A(ap+bp)}$ a: JA A = (a+b)= 2 a= 31 We concluded that for an infinitely stiff

-9-Pile the deflection varies 250 straight fine, and becomes zero at a point 3/4 & from the top, and then becomes negative the remainder st the length. Due to the fact that it is impossible to have an infinitely Stiff Pile we must Consider what take place Ispile bend away from this straight line condition and apply these ad dea deflections to the deflections of the infinitely stiff one. I will try to show these added deflections In The following Work. As I meation before I will carry through the first deflection is analytical formand the remaining ones will be done grop our aly.

10-Eirst added Deprest we to that of the Enfinitely stuff pile by analyteal method. The force sating on the infinitly stiff pile is in relation to the smoot of move ment of the pile in a the earth pressure. force & e, R, l Where e= the intensity of the earth pressure at the lowerend of the pile a = the more ment of the upper end of the pile. from fig. 1, (2, b) force = ey x Q (3l-au) ill. force = eay - 4e ay2 3.12 The shear at any point along the pile is equal to the acting force Pless the sum of the force due to the earth pressures. Shear - P - So (force) dy = P - Jo" (= gy - 4 e gy 2) dy Shear = $p - \frac{ea_{42}}{2l} + \frac{4ea_{43}}{9l^2}$ but when g=1, the shear=0 so: $0=p=\frac{e_{0}1}{2}+\frac{4e_{0}1}{2}$ p= egl $Equit: Q = \frac{18P}{eT}$ Equis. Shear = P = 9 py + 8 pys

The momentat any point in the pile is equal to the area of the shear curve Equ. 8 is the equation of the shear So the intergal of this is the man ent if intergrated between zero any the point in greation Moment = Sod (spear)dy. = p 10 (1- 4= + 84") dy Equip. Moment = P (y-343+2 43) The slope at any point is the area of the moment curve Equ.9.1s the equation of the moment. so. slope= 6 (moment) dy $\frac{= P \int_0^y (y - \frac{3}{4} + 2\frac{4}{4}) dy}{= P (\frac{y^2}{2} - \frac{3}{4} + 2\frac{4}{4}) dy} f K.$ The deflection is the area of the slope curve: so deflection = 6 (slope) dy = [P(z - zzz) + 1c) dy. deflection: Lot (10 yo - 42 + 445) + 16y When you the deflection : 0 $O = \frac{1}{16\pi m} \left(\frac{10}{10} - \frac{9}{9} + \frac{9}{10} + \frac{9}{10} \right) + \frac{1}{16} \left(\frac{10}{10} - \frac{9}{9} + \frac{9}{10} + \frac{9}{10} \right) + \frac{1}{16} \left(\frac{10}{10} - \frac{9}{10} + \frac{9$ This deflection of must be applied to the straight line deflection in such a manor that

the sammation of force coused by the added deflections are equal to zero and the moments caused by these forces are also zero. So if the deflections "y" are ploted the curve will be odet, in fig. 2. Then to this we apply the straight line, gh, which is the stright line deflection of the infinitely stiff pite; the added deflections "" must satify the requirement given a bore of EH=0 and EM=0. K-A-C a a a

719 L Aand B are the deplections of the endpoints of the straight line curre away from the axis of the Superimposed Curve. by similar triangles we have

 $\frac{\mu + \psi}{AE} = \frac{A}{AU}$ $\frac{AE}{A-B} - \psi = \frac{A}{A-B}$

or in the added deflection = H = By + A(1-4) - 4

Where & is given by equ. 11 2H=0= Jo (By + A(2-4) - 4) y dy

0= Jo [# + Ay - # - Tost (5 12 4 - 10 y + # - #] by 0: 3 + 42 - 412 P15 15-2+3-12) $0 = \frac{84^2}{3} + \frac{44^2}{6} - \frac{Pt^2}{20ET} \left(\frac{42}{42}\right)$ Equ. 12 28 + A = 430ET ZM=0=6 2 + Ay 2 - Ay - 60 EI (5123 - 10y + $O = \frac{B + 3}{7} + \frac{A + 3}{3} + \frac{B + 3}{4} + \frac{B + 3}{60 \epsilon r} \left(\frac{5}{7} + \frac{10}{6} + \frac{9}{8} + \frac{4}{7}\right)$ $O = \frac{B + 3}{4} + \frac{A + 3}{72} - \frac{B + 6}{60 \epsilon r} \left(\frac{19}{72}\right)$ $O = \frac{B + 3}{4} + \frac{A + 3}{72} - \frac{B + 6}{60 \epsilon r} \left(\frac{19}{72}\right)$ 19 P.1 Equ.13 3 B + A = 360 EI Solve Egus 12+13 for A+B 3.8+ A = 19P4 3 360EI 2B+A= 19 213 420ES B= 19P13 9520EI A = 26 P/3 550 EI Now by having At B the slope of the hne, gh, is Known, and the difference between added deflection to be added to the defect.

ion of the infinitely stift Pile. These added deflections are given in

Toble 1.

* Deflections by the Analytical Method 12 hes of values of value of Point Egu. H the, 9. F. U. Une, fig. 2 ol. 1. O. . . 1.85366 +1.854 .14 +1.225 0.49.009 1.71493 24 1.57591 10.653. 0.73362 13.P 334895 1.43688 +0.188 44 1.43578 1.29786 -0.138 .51 1.50375 1.158.84 -0.345 .61 1.3.5.022 1.01951 - 0.333 .74 111203 0.88049 -0.232 Sil 2.78054 0.74146 -0.040 .9.8 0.39865 060244 -0.203 1.01 0 046342 -0.463 all values given in terms of LOFI

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Table 1

The analytical method could be continued on but due to the fact that the graphical method is much fastor and gives resolts with Mr Stotper ten of the analytical Expresented to it to Carry the investigation on.

The graphical Method of finding Deflections

In the graphical method of finding set lections the operation are exactly the same as those for the analytical method. First a deflection times a pressare y ires a force; then these tores are sumated togive the shear, and the process is continued exactly as in the analytical method cally the intergration is done graphical. It is found that by care ful work the results of the two method are within seven percent of each other ...

The cale lations for the results after The curve, caef, of 19.2, is gotten grapping are donien the same manor as for the analytical method anly in the graphical method a summation is used instead of the intergration. The Two equations for A and B, Equ. 12 and 13, may be stated more generaly so that they be used for the SUmmation

2 94.12 61000 es,

Equila becomes 3Bt A = EMX12

Theresults of the graphical methods are given in table 2 and the graphical Work is tond In the appendies.

-18-Cocalistions for the "gh" Line of Fig.2. 2B+A=6XZH 38+A= 12 X 2 M 1st frial. 2 B + A = 6 X 0. 4324 50F 3 B + A = 12 X 0. 2.562 6024 B= 0.4800 6053 A= 1.6344 6053 and trial P.13 2 B + A = 6 × 0. 132 04 60 EI 3 B + A = 12 X 0.07 885 60 FI B = 0.15396 405 A = 0.48432 605 3th trial 3.B+A = 6X 0.03059 3B+A= 12X0.01866 60 B= 0.04038 600 A = 0.10278 P33 4th trial. 2.B + A = 6X0.01156 60EI 3 B + A = 10x 0.00703 B= 0.01500 A A= 0.03932 213 5th trial 2B+A= 6×0.00384 TOFI 3 B + A = 12x0.00231 EL B = 0.00 468 E A= 0.01368 1051 6th trial. 2 B + A = 6 X00070402 3B+A= 12×0.00043 60 B = 0.00096 4 A= 0.00228 20022 7th trial · 2 B+A= 6 × 0.00023 20 F2 3B+A=12X0.000133 6061 B= 0.00023 to A= 0.00094 -011

-17-8th trial. 2 B + A = 6 × 0.00008/ 2023 $B + A = A \times 0.000049 BB$ B = 0.00010 BEA = 0.00018 BBCOEL

Results of Graphical Intergration to find Deflection (all terms given in the part ion to find Deflection MA

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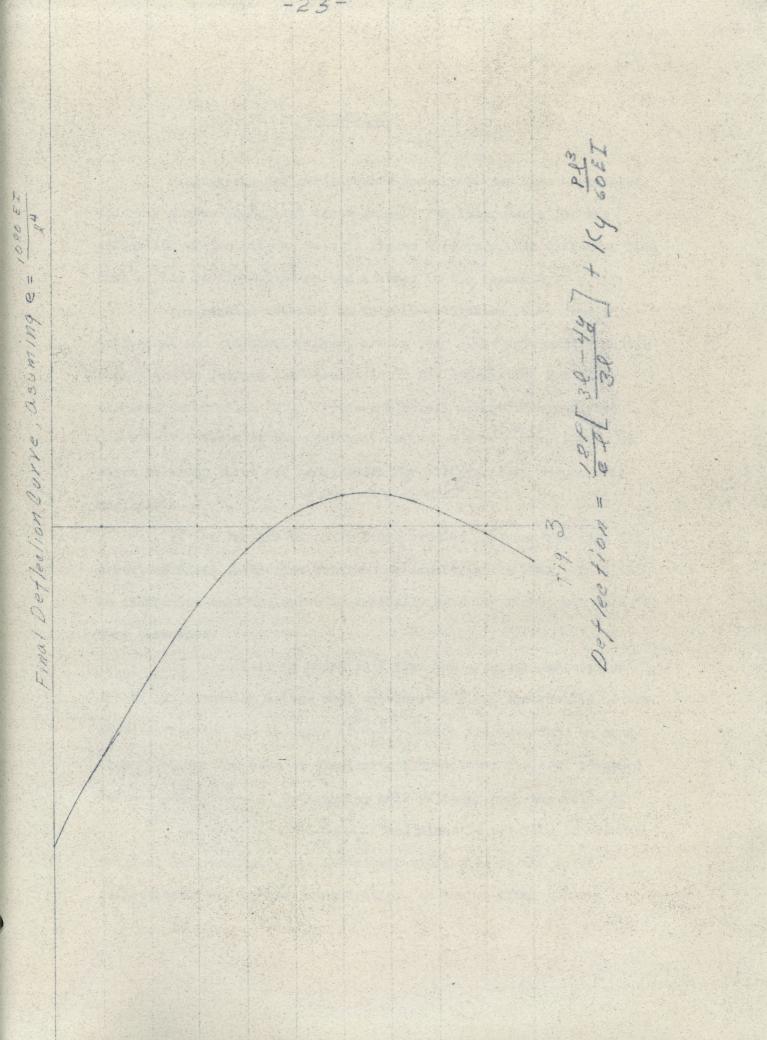
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CONCLUSIONS

Finding the deflection curve for a pile due to a horizontal load has proven to be most interesting. The final curve for the deflection of the pile as seen in figure 3 is very much different from that of the straight line as was assumed in the beginning.

The results obtained in this investigation, both from the analytical and graphical method, are in very close agreement, and the best place to compare the results is in the calculation for the original deformation "Q". By the analytical method from equation 7, $Q = \frac{18 \text{ F}}{\text{e l}}$, where by the graphical method $Q = \frac{18.5 \text{ P}}{\text{e l}}$, giving an error of about three per cent, which for all practical purposes is negligible.

If the results of table 2 are studied, one can see that the amount of added deflection obtained by each trial is less. This is an indication that the curve is gradually becoming fixed, but at a very slow rate.

It is noticed in equation 7 that the original deformation "Q" is in proportion to the soil pressure "e", but the results of the added deflection are in terms of $\frac{1^3}{E I}$ which are not found in equation 7, so for the sake of simplicity I have taken the soil pressure "e" = $\frac{1080 E I}{1^4}$. By assuming this value of "e", the value of $Q = \frac{P I^2}{60 E I}$ and makes it possible to add the original deformation of the infinitely stiff pile to the added deflections found by the investigation, so that a final curve can be obtained. The final results may be found in table 3, and the deflection curve from the results is shown in figure 3.

The shape of the final deflection curve of figure 3, is quite different from the deflection curve of the infinitely stiff pile. Upon examination of this curve we find that we have two points of zero bend, and the lower end of the pile is deflecting in the same direction as the top.

I was able to carry the investigation through only eight trials, and the final deflection curve would only be obtained after an infinite number of these trials. If this had been done the curve in the lower portion would have looked like a damped sine curve instead of being as it is shown in figure 3.

The points of zero deflections in the general case an almost infinite in number, because of the curve being a damped sine curve, which will cross the zero line several times, but I believe for practical purposes this curve shows the general shape.

The distance the first point of zero bend moves up from the three-quarter point of the infinitely stiff pile, is in relation to the moment of inertia of the pile, and must be considered in design.

In general it can be said that the deflection curve for a pile due to a horizontal applied load is:

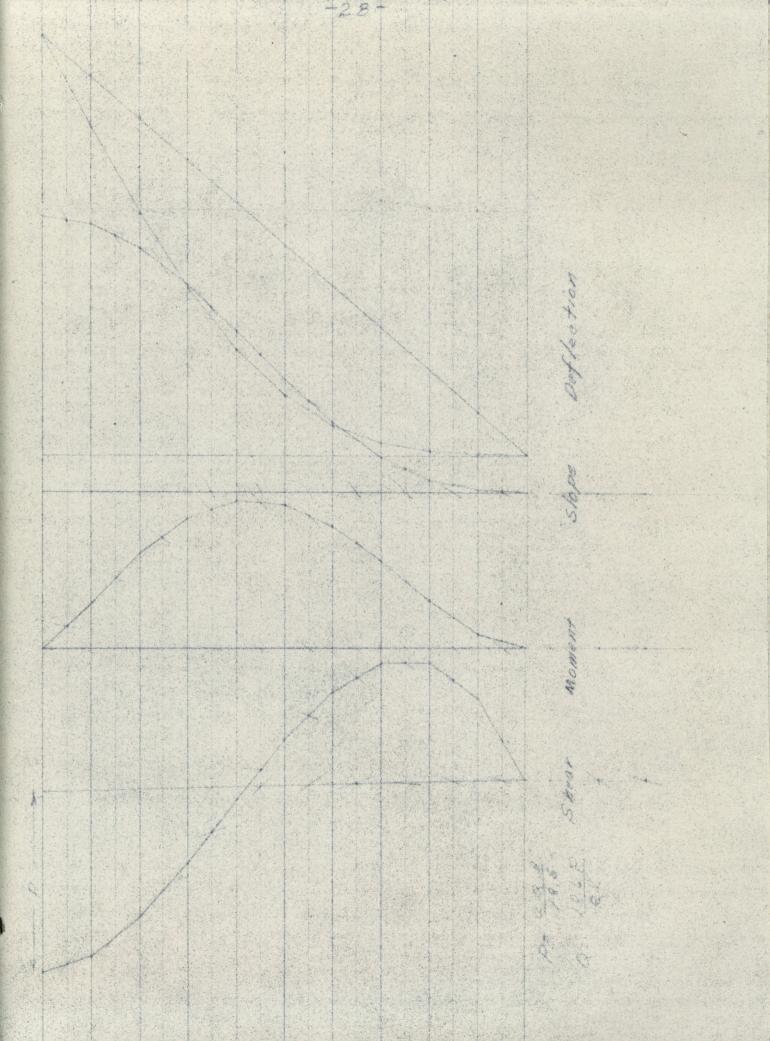
$$\Delta = \frac{18 \text{ P}}{1000 \text{ P}} \left[\frac{31 - 4 \text{ y}}{31} \right] + K_y \frac{\text{P}1^3}{60 \text{ E} \text{ I}}$$

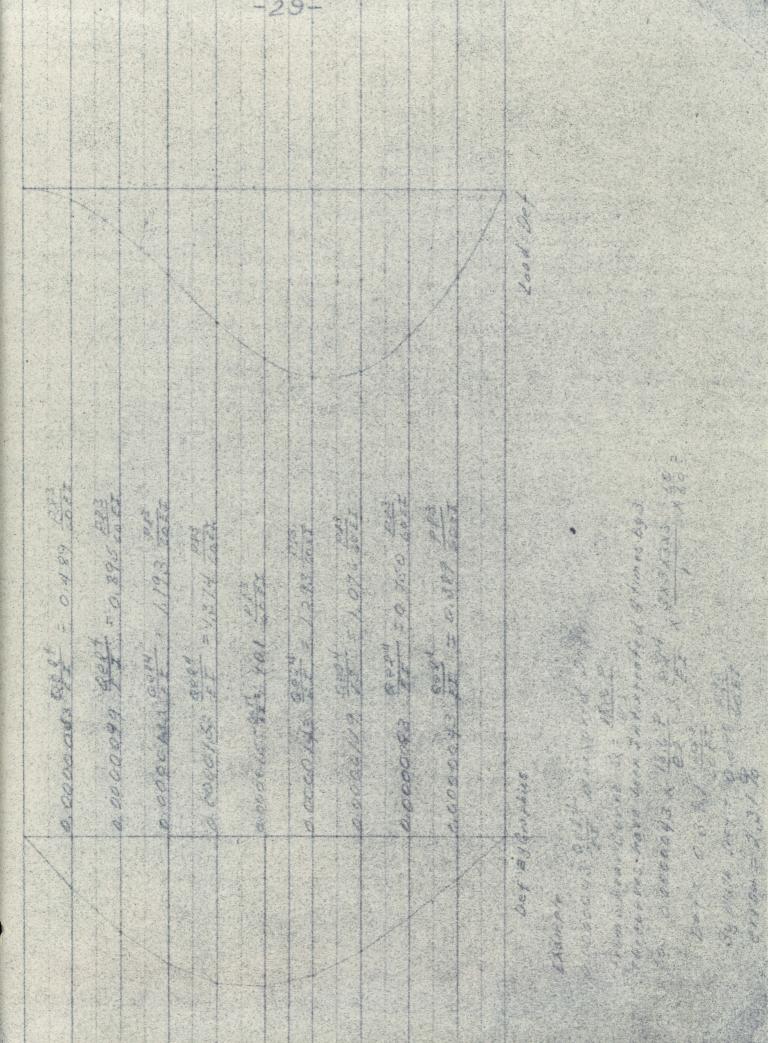
K being a variable with the depth, and may be found by determining y the added deflection.

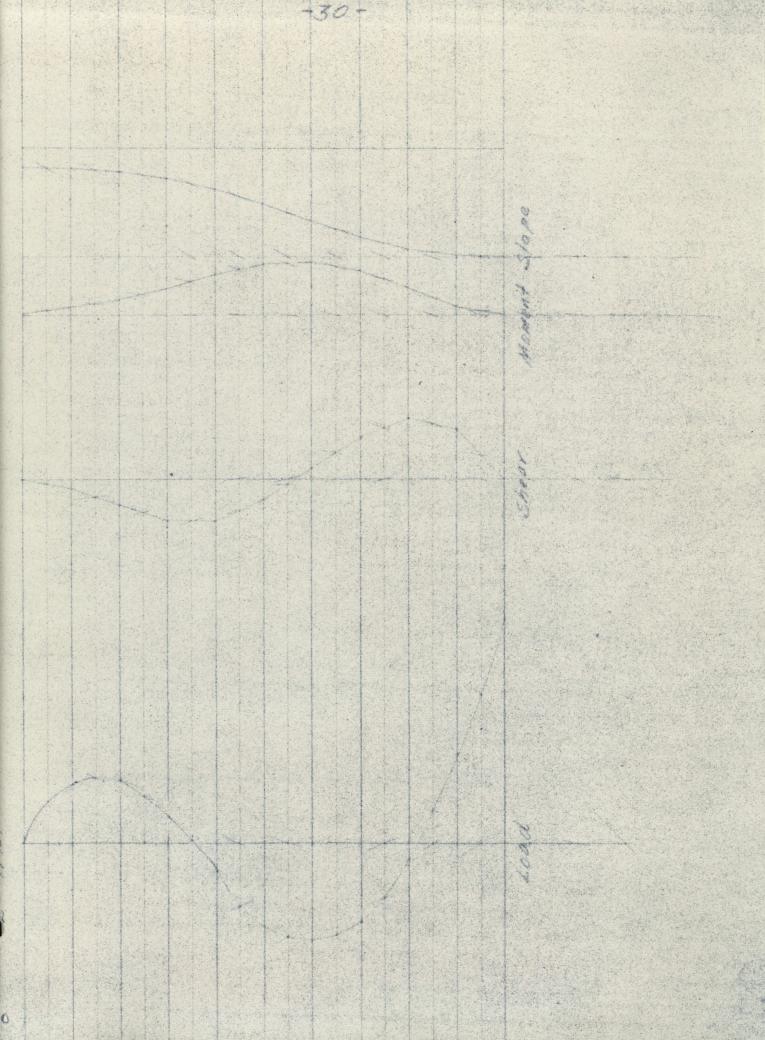
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APPENDIX

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