THE MECHANICAL AND NOMOGRAPHIC SOLUTION OF RAOB AND APOB DATA

a thesis by

Robert E. Fleming

In Partial Fulfillment of the Requirements for the Degree of Master of Science in Meteorology

California Institute of Technology

Pasadena, California

1941

TABLE OF CONTENTS

Part I Separate Nomographical Solutions

Ch. I An Outline of General Nomography --------1

Ch. II The Calculation of w ------------------9

Ch. III The Calculation of θ -----------------17

Ch. IV The Calculation of θ_d -----------------19

Ch. V The Calculation of θ_E -----------------25

Ch. VI The Calculation of Altitude -----------34

Ch. VII A Criterion of Stability --------------41

Part II Master Nomograms 44

Part III Slide Rule Solutions

Ch. VIII The Calculation of w and θ -----------51

Ch. IX The Calculation of θ_E ----------------54

Conclusions ------------------------------------59

INTRODUCTION

Anyone having frequent recourse to the Stuve, Neuhoff or Rossby diagrams must soon realize the tediousness of the operations involved in calculating repeatedly such quantities as w θ_E and altitude. There are two distinct objections to these types of diagrams: The first is the complexity of nets or grids (systems of lines for arbitrary constant values of variables). Not only is time wasted in following a given line to its coordinate or value, but the process is physically detrimental to the eyes and is a frequent source of error. The second objection is the small number of soundings for which one sheet may be used, if w or altitude must be found. The following treatise is an attempt to replace these diagrams within a limited scope by other means: one nomographical, and one mechanical. The scope covered by these means is confined to the calculation of thermodynamical quantities, and cannot present a graphic picture of a RAOB or APOB sounding, nor is it applicable at present where the concept of work is involved, as in fog formation and dissipation, or in the probability and characteristic levels of a thunderstorm.

Since at the present time RAOB and APOB teletype reports include altitude, pressure, temperature, relative humidity and mixing ratio, the only essential thermodynamic quantity each office must calculate itself is θ_E , the equivalent potential temperature. The computation of this quantity is easily accomplished either by a nomogram or a slide rule. However, both these instruments may be employed further, particularly the nomogram. With the latter will be developed methods for obtaining w, θ, θ_d , z and necessary associated quantites, of use both at the sounding station and at the transmitting center, where these quantities must at present be computed.

An examination of the following charts will indicate their simplicity of construction and of operation as compared to the diagrams now in use. The slide rule has one added advantage, namely its compact size, but has not been considered as thoroughly as the nomogram, because of the possibility of adding numerous other diagrams upon the latter without being cramped for space.

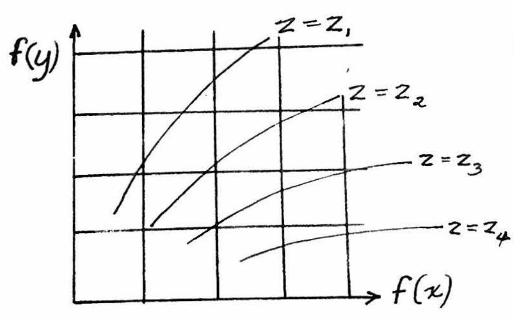
Chapter I

An Outline of General Nomography.

Before any charts are to be developed it would be well to consider briefly the construction of nomograms in general, and the properties of the types herein employed. Unfortunately the nomogram is known too little in proportion to its usefulness in the repeated solution of a formula or equation containing three or more variables. In any Cartesian graph this involves a set of lines in addition to, and intersecting, the lines parallel to the values of ordinates and abscissae. Thus the general equation

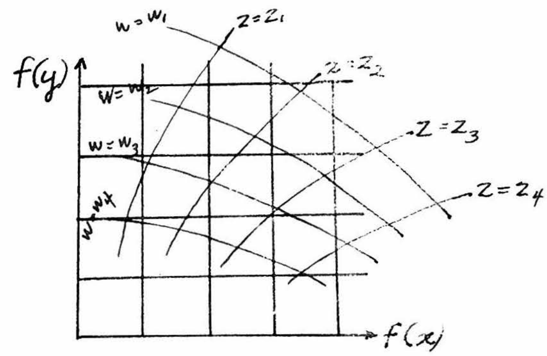
$$f(x) + f(y) + f(z) = 0$$

when plotted upon Cartesian principles would appear as in Fig I. If now a fourth variable appears in our equation, still another net, forming a grid with the former net, is added, as shown in Fig II:



f(x) f(y) f(z) = 0

Fig I



f(x) f(y) f(z) f(w) = 0

Fig II

There is but a single nomogram--termed a third order nomogram from the number of its variables--corresponding to Fig I, but there are a number of alignments or nomograms corresponding to Fig II, as we shall see later. The general third order nomogram will consist merely of three lines, as shown in Fig III. Each of these lines is graduated according to definite functions of the type

$$x_o = g(x) \qquad y_o = h(x)$$

$$x_o = g(y) \qquad y_o = h(y) \qquad \text{etc.,}$$

where x_o and y_o are the corresponding Cartesian grid supports, and are never shown upon the nomogram in practice, but only considered in choosing suitable scale factors and graduating the necessary lines. These supports may be of the simple type indicated in Fig IV, or of the more general type shown in Fig V, involving oblique Cartesian coordinates, so that the range of values to be considered will be included upon a reasonably sized sheet.

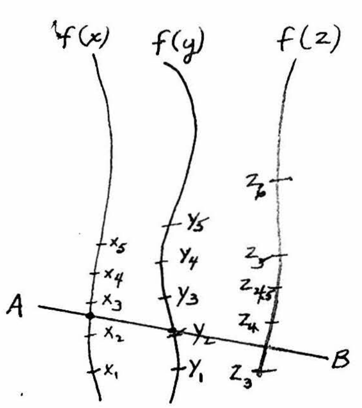


Fig III

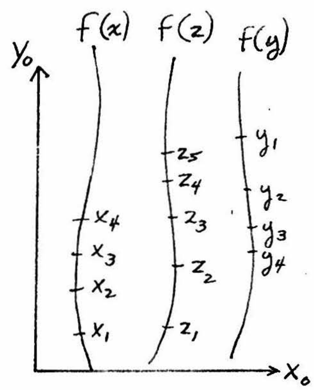


Fig IV

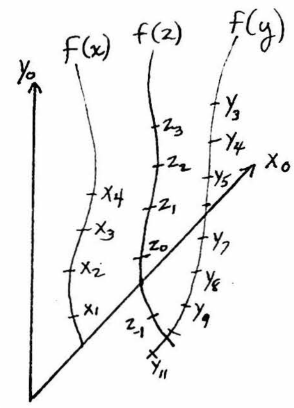


Fig V

The procedure of solving for a dependent third variable can most readily be seen from Fig III. A straight line or edge--sometimes referred to as the index--will be laid from A to B in such a manner as to intersect the given variables at $y_{2.2}$ and $z_{3.8}$ for instance. Then the value of x uniquely corresponding to the index would be $x_{2.7}$.

For a rigorous proof of the validity of these operations one should refer to a standard treatise on Nomography, several of which are listed in the Bibliography. Here but a general idea of the development of this chart will be sketched. From Cartesian geometry it will be recalled that the general condition that three points of P_1 $(x_1\ y_1)$, P_2 $(x_2\ y_2)$, P_3 $(x_3\ y_3)$ lie upon a straight line is given by

$$(1) \qquad \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

A relation must then be found between P_1, P_2, and P_3, and the position of three corresponding variable points so that for every given set of values of P_1, P_2, and P_3 there will correspond three collinear points. This is accomplished by considering the parametric form of the equations for each P. The parametric forms corresponding to equation (1) above may be written

$$x_0 = x_1 = f(x) \qquad y_0 = y_1 = g(x)$$
$$x_0 = x_2 = f(y) \qquad y_0 = y_2 = g(y)$$
$$x_0 = x_3 = f(z) \qquad y_0 = y_3 = g(z),$$

and if maintained in this order x_0 and y_0 may be considered the Cartesian grid support upon which the loci of the curves for x, y, and z may be constructed.

Quite frequently the set of values interested in, result in scales of widely differing size. In some cases it is possible to introduce arbitrary constants, termed scale factors, so distorting the scales as to even up their extreme values. Thus if we have the equation

$$f(t) + f(w) + f(z) = 0$$

and arbitrarily let

$$x = \mu_1 f(t)$$

$$\text{and} \quad y = \mu_3 f(w)$$

we will have three equations in x and y:

$$1 \cdot x + 0 \cdot y - \mu_1 f(t) = 0$$

$$0 \cdot x + 1 \cdot y - \mu_3 f(w) = 0$$

$$\frac{1}{\mu_1} x + \frac{1}{\mu_3} y + \quad f(z) = 0$$

The condition that any three points as loci correspond to given values of t, w, and z gives us the determinant

$$(2) \qquad \begin{vmatrix} 1 & 0 & - \quad f(t) \\ 0 & 1 & - \quad f(w) \\ \frac{1}{\mu_1} & \frac{1}{\mu_3} & \quad f(z) \end{vmatrix} = 0.$$

The latter will be termed the primary constructional form, as contrasted to the primary basic form

$$(3) \qquad \begin{vmatrix} f(t) & 0 & 1 \\ f(w) & 1 & 0 \\ -f(z) & 1 & 1 \end{vmatrix} = 0.$$

There are no set rules for the method of filling in the various rows and columns of the latter; and experience and trial and error are of prime necessity. Nevertheless, in the forms to be developed hereafter, as close to an inductive reasoning as possible will be presented, rather than merely the final or reduced form.

To obtain the reduced form, from which the nomographical functions may be constructed, one must realize first of all the form towards which he is striving. For a third order determinant an examination of equation I will show that no row may contain more than one variable. Hence even in the primary form the terms must be so arranged that no two variables appear in a single row. Further, one column must consist of unit values only. Hence we shall manipulate the determinant by columns, never by rows, so as to obtain unit values in one column.

It would be appropriate to mention here the properties of determinants which we shall use:

1) If all the terms in a row or column are multiplied or divided by the same number, the value of the determinant will be multiplied or divided by that number.

   It follows a zero-valued determinant may have a column multiplied or divided by a number without change in value.

2) If the terms of two columns are identical or any single column elements all consist of zeros, the value of the determinant is zero.

3) The value of any determinant remains unchanged if to the elements of one column are added a constant times the corresponding elements of any other column.

4) The sum of two determinants, two of whose columns are identical, is a determinant consisting of these columns and a column whose elements

are the respective sums of
corresponding elements of the
dissimilar columns.

In the light of the above we would add the second and third columns of equation 3 and divide the third row by 2 obtaining

$$(4)\qquad \begin{vmatrix} f(t) & 0 & 1 \\ f(w) & 1 & 0 \\ -f(z) & 1 & 1 \end{vmatrix} = \begin{vmatrix} f(t) & 0 & 1 \\ f(w) & 1 & 1 \\ -f(z) & 1 & 2 \end{vmatrix} = \begin{vmatrix} f(t) & 0 & 1 \\ f(w) & 1 & 1 \\ -\frac{1}{2}f(z) & \frac{1}{2} & 1 \end{vmatrix} = 0$$

The nomogram corresponding to the reduced form is thus seen to consist of three straight lines, with f(z) equidistant between f(t) and f(w).

A similar manipulation of the constructional determinant gives us:

$$(5)\qquad 0 = \begin{vmatrix} 1 & 0 & -f(t) \\ 0 & 1 & -f(w) \\ \frac{1}{\mu_1} & \frac{1}{\mu_3} & f(z) \end{vmatrix} = \begin{vmatrix} 1 & 0 & f(t) \\ 1 & 1 & f(w) \\ \frac{1}{\mu_1}+\frac{1}{\mu_3} & \frac{1}{\mu_3} & -f(z) \end{vmatrix} = \begin{vmatrix} 1 & 0 & f(t) \\ 1 & 1 & f(w) \\ 1 & \frac{\mu_1}{\mu_1+\mu_3} & \frac{\mu_1\mu_3}{\mu_1+\mu_3}f(z) \end{vmatrix}$$

Comparison of (5) with (4) shows that a certain latitude of scale scale factors has been introduced into the constructional determinant, which may permit of a better distribution of curves.

There are a number of methods of forming nomograms of the fourth order and higher. One is to resort to grids. Since this is what we are seeking to avoid, we will merely mention it in passing by indicating the general solution of

$$(6)\qquad f(w)+f(z)+f(t)+f(v) = 0$$

is possible with a net as shown in Fig VI:

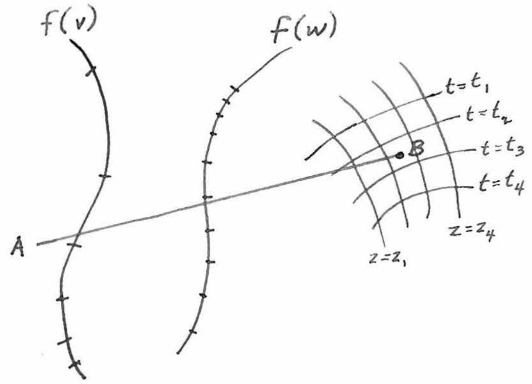


Fig VI

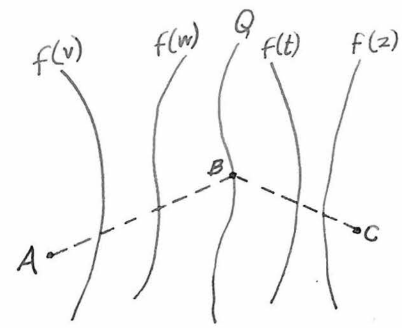


Fig VII

Another method is often possible when the functions are capable of being separated into two separate parts, of two variables each, neither of which appears in the other. Thus if in (6) we let

$$f(u)\ f(w) = Q = f(t)\ f(z),$$

we may form two separate nomograms, as shown in Fig VII, with the Q function identical for both alignments, and hence unnecessary to scale or graduate. This type will be used frequently, and will be termed a double-set nomogram.

A third method, termed the set-square, will also be frequently employed. Essentially it consists of two nomograms, as does the double-set, but with the supporting Cartesian grid of one rotated through 90^o, so that the two indices are at right angles. By this device a formula involving four variables may be solved by a single setting. Without going through the projective transformations and rotations necessary to develop this type, it will merely be stated here that if a determinant of the form

$$\begin{vmatrix} f(u) & g(u) & 1 & 1 \\ f(v) & g(v) & 1 & 1 \\ f(w) & g(w) & 1 & 0 \\ f(t) & g(t) & 1 & 0 \end{vmatrix} = 0$$

may be found, the four necessary functions will be constructed as shown in Fig VIII for ordinary rectilinear Cartesian supporting grids or axes, and as in Fig IX for oblique axes.

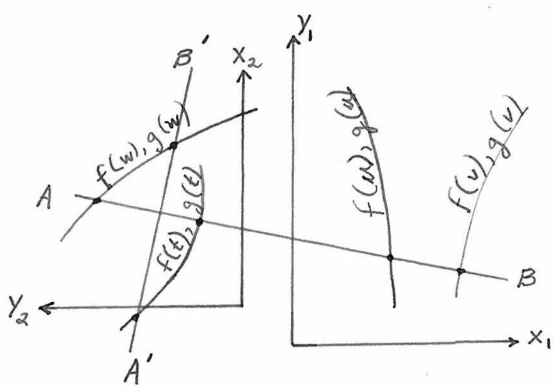


Fig VIII

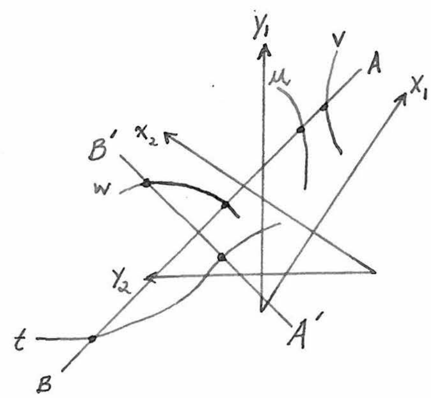


Fig IX

The most complex type we shall wish to solve will be of the form

$$f(u)+g(u)f(v)+f(w)+f(t) = 0,$$

or of such a nature as to be reducible to the above form. The solution of such forms as

$$f(u)+g(u,w)f(v)+f(w)= 0$$

will not be attempted because of the excessive number of settings required.

Chapter II

THE CALCULATION OF w

The mixing ratio, w, for air is given by the expression

(1) $w = .622\frac{er}{p-er}$,*

where e is the saturation vapor pressure

r is the relative humidity, in %

and p is the total pressure of the air.

To conform to general practice, we shall express p and e in millibars. The factor .622 is merely the ratio of the gas constants for dry air and water vapor, and is used in the definition of w to simplify other thermodynamical expressions. For the purposes of nomography it is best that each variable be in a separate term; hence we shall take the logarithm of both sides of (1), obtaining:

(2) $\log w - \log .622 - \log r + \log (p-er) = 0.$

If now we assume the product er small in comparison to p, we obtain the approximation

(3) $\log w - \log .622e - \log r + \log p = 0.$

With this relationship between our variables, we shall construct a double-set and a set-square nomogram.

---

* Brunt, pp. 33, Equation 7

A: DOUBLE-SET NOMOGRAM

Equation (3) may be rearranged so that

$$\text{(4)} \qquad \log w - \log r = \log .622e - \log p.$$

The reason for this particular grouping will become apparent later in our choice of scale factors. To arrive at a constructional determinant, let

$$x = \mu_1 \log w$$
$$y = \mu_2 \log r$$

in the left hand part of equation (4). We may then form the three equations:

$$\text{(5)} \qquad \begin{aligned} 1\cdot x + 0\cdot y - \mu_1 \log w &= 0 \\ 0\cdot x + 1\cdot y - \mu_2 \log r &= 0 \\ \frac{1}{\mu_1} x - \frac{1}{\mu_2} y - Q &= 0. \end{aligned}$$

The condition for consistency of this set in x and y is

$$\text{(6)} \qquad \begin{vmatrix} 1 & 0 & -\log w \\ 0 & 1 & -\log r \\ \frac{1}{\mu_1} & -\frac{1}{\mu_2} & -Q \end{vmatrix} = 0.$$

Upon changing the signs of the third column, adding the first and the second columns, dividing the third row by $\frac{\mu_2 - \mu_1}{\mu_1 \mu_2}$, and rearranging the order of the columns, we obtain

$$\text{(7)} \qquad \begin{vmatrix} 1 & 1 & \log w \\ 0 & 1 & \log r \\ \frac{1}{\mu_1} & \frac{\mu_2 - \mu_1}{\mu_1 \mu_2} & Q \end{vmatrix} = \begin{vmatrix} \log w & 1 & 1 \\ \log r & 0 & 1 \\ \frac{\mu_1 \mu_2 Q}{\mu_2 - \mu_1} & \frac{\mu_2}{\mu_2 - \mu_1} & 1 \end{vmatrix} = 0.$$

In the same manner we may construct from the right hand part of (4), the following determinants:

$$(8)\qquad \begin{vmatrix} 1 & 0 & -\mu_3 \log .622e \\ 0 & 1 & -\mu_4 \log p \\ \frac{1}{\mu_3} & -\frac{1}{\mu_4} & -Q \end{vmatrix} = \begin{vmatrix} \mu_3 \log .622e & 1 & 1 \\ \mu_4 \log p & 0 & 1 \\ \frac{\mu_3 \mu_4 Q}{\mu_4 - \mu_3} & \frac{\mu_4}{\mu_4 - \mu_3} & 1 \end{vmatrix} = 0.$$

If r is to vary from .10 to 1.00, w from 0.5 x 10 to 20 x 10, p from 100 to 1000 mb., and e as a function of T, the absolute temperature of the air, is to vary from 2 to approximately 150 mb., the apparent spreads in the various functions will be given roughly by:

log .10	to	log 1.00,	or	-1.00	to	0.00	(for r)
log .0005	to	log .02,	or	-3.3	to	-1.7	(for w)
log 100	to	log 1000,	or	2.0	to	3.0	(for p)
log .622 x 2	to	log .622x150,	or	.15	to	2.0	(for e)

Appropriate scale factors would then be:

$$\mu_1 = \mu_3 = 0.50$$

$$\mu_2 = \mu_4 = 1.00.$$

As in many cases to follow, the absolute limit of one or mone variables will not be considered, because of the tendency for the function to go rapidly towards infinity, as would the functions log p and log r above. Nevertheless, we may choose as end values such limits as are not usually exceeded in practice. As for the lower limit on e, this will depend upon whether the vapor pressure is taken over super-cooled water or over ice, and the dilemma is avoided here because of the infrequent use of w at low temperatures.

Our constructional determinants may now be written as

$$(9) \qquad \begin{vmatrix} 0.5 \log w & 1 & 1 \\ \log r & 0 & 1 \\ Q & 2 & 1 \end{vmatrix} = \begin{vmatrix} 0.5 \log .628e & 1 & 1 \\ \log p & 0 & 1 \\ Q & 2 & 1 \end{vmatrix} = 0.$$

It must be borne in mind that Q will necessarily have the same value (be coincident) in both of the third order nomograms, whose scales, grid supports, and values are sketched below in Fig I and Fig II:

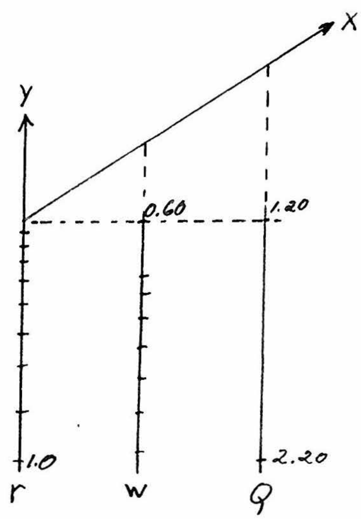


Fig I

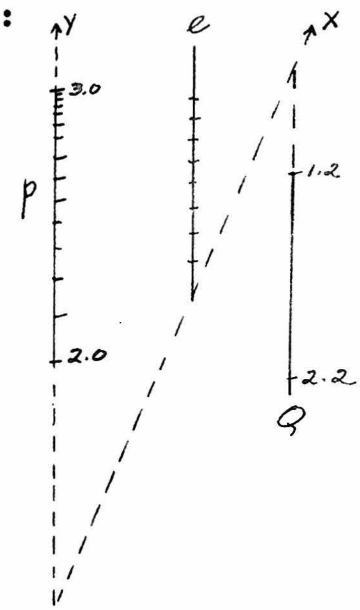


Fig II

The nomogram, in Chart I, is then constructed from the values tabulated in Table I. A 10 scale (i.e. 12 units per foot) was chosen, with each unit considered one tenth its assigned size in the y direction; and because the second column may be multiplied by any satisfactory scale factor, the scales are merely separated horizontally by equal distances of 3.5 on the ten scale.

It will be noted that the scales of p and r are constructed to the same logarithmic scale factor, and hence may serve as the A and B or C and D scales of a slide rule if necessary.

Since these scales coincide, one of them must be reproduced upon a different line, preferably that of Q. Values are then multiplyable provided that the index line remains parallel for the two settings. If values of e are graduated upon the T scale, then values of e_r, the actual vapor pressure may readily be computed if desired.

B: SET-SQUARE NOMOGRAM

Equation (3) may again be regrouped as:

$$(10) \quad \left[\tfrac{1}{2}(\log e - \log \tfrac{P}{.622}) - \log w\right] + \left[\tfrac{1}{2}(\log e - \log \tfrac{P}{.622}) + \log r\right] = 0$$

It will be noted that two of the variables are split and hence appear identically in the two groups. The reason for this will become apparent below when it will be desired to combine two third-order determinants into a single fourth-order determinant. We may now write each bracketed group, expressed as a determinant, so that

$$(11) \quad \begin{vmatrix} \tfrac{1}{2}\log e & 0 & 1 \\ \log w & 1 & 0 \\ \tfrac{1}{2}\log \tfrac{P}{.622} & -1 & 1 \end{vmatrix} + \begin{vmatrix} \tfrac{1}{2}\log e & 0 & 1 \\ -\log r & 1 & 0 \\ \tfrac{1}{2}\log \tfrac{P}{.622} & -1 & 1 \end{vmatrix} = 0.$$

Equation (11) may now be expressed as a single determinant, because we have two rows identical. We obtain:

$$(12) \quad \begin{vmatrix} -\log r & 1 & 0 & 1 \\ \log w & 1 & 0 & -1 \\ \tfrac{1}{2}\log e & 0 & 1 & 0 \\ \tfrac{1}{2}\log \tfrac{P}{.622} & -1 & 1 & 0 \end{vmatrix} = 0$$

Upon shifting the minus sign of the fourth column into the first row and changing the sign of the latter, and then adding the fourth column to the third column, we obtain:

$$(13) \quad \begin{vmatrix} \log r & -1 & 0 & 1 \\ \log w & 1 & 0 & 1 \\ \tfrac{1}{2}\log e & 0 & 1 & 0 \\ \tfrac{1}{2}\log \tfrac{P}{.622} & -1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} \log r & -1 & 1 & 1 \\ \log w & 1 & 1 & 1 \\ \tfrac{1}{2}\log e & 0 & 1 & 0 \\ \tfrac{1}{2}\log \tfrac{P}{.622} & -1 & 1 & 0 \end{vmatrix} = 0.$$

It will be seen that the above form is that of a fourth-order nomogram employing a set-square index, as mentioned in Chapter I, with all four variables being straight line functions, due to the absence of functions involving variables in the second column. Although a constructional determinant could be developed, the consideration of the end values of the various functions precludes the necessity of proceeding beyond the basic, for the apparent spreads will be given roughly by:

-1.00 to 0.00	(for log r)
-3.3 to -1.8	(for log w)
0.20 to 1.1	(for $\frac{1}{2}$ log e)
1.10 to 1.60	(for $\frac{1}{2}$ log $\frac{p}{.622}$)

The scales, values, and grid supports may then be diagrammatically sketched as in Fig III:

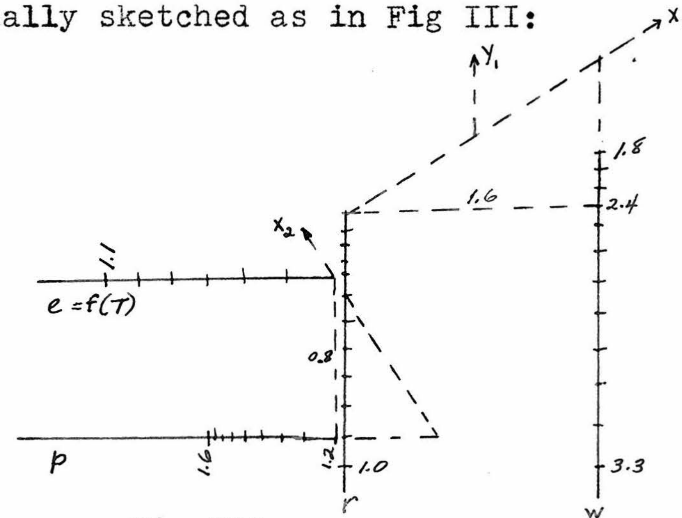


Fig III

The values of major points are calculated in Table II, and plotted upon a 20 scale, whose unit values may be considered to be divided by 10. The ratio of obliquity of the axes, y/x, was taken as $\frac{3}{2}$ or $\frac{2.4}{1.6}$, and it will be noted that this ratio must be applied to both grid supports.

As in the case of Chart I, two of the scales constitute an ordinary slide rule; these scales in this case are the r and w scales. Hence the quantity e_r may be calculated by a single operation upon these scales.

If greater accuracy is desired, a constructional determinant may be used, but the basic is deemed satisfactory, the maximum scale-spread ratio being only 2.

Chapter III

THE CALCULATION OF θ

The potential temperature, θ, is defined as

(1) $\theta = T \left(\frac{p_0}{p}\right)^{.288}$, *

where T is the absolute temperature

p is the pressure of the parcel of air

and p_o is the standard-level pressure of the air

Since p_o is usually taken as 1000 mb, p must also be expressed in mb. The quantity .288 is the numerical value of $\frac{\gamma-1}{\gamma}$ or $\frac{AR}{c_p}$ for air. If instead of p we use p_d, the partial pressure of dry air, we obtain θ_d, the partial potential temperature. If now we arrange (1) in logarithmic form, we obtain

(2) $$-\log\theta + \log T + .288 \log\frac{1000}{p} = 0$$

or $$\begin{vmatrix} \log T & 0 & 1 \\ .288 \log\frac{1000}{p} & 1 & 0 \\ \log\theta & 1 & 1 \end{vmatrix} = 0.$$

Upon adding the second and third columns of the latter, and dividing the third row by 2, this becomes

$$\begin{vmatrix} \log T & 0 & 1 \\ .288 \log\frac{1000}{p} & 1 & 1 \\ \frac{1}{2}\log\theta & \frac{1}{2} & 1 \end{vmatrix} = 0.$$

If we allow T and θ to vary from 240 to 340 C, and p from 1000 to 200, the ratio of spreads does not

---

* Brunt, pp. 38, Equation (34)

exceed 2, so that a basic determinant will suffice for illustrative purposes. An oblique system of grid supports must be introduced, such that the ratio $y/x = \frac{118}{6.0} = 19.67$. The diagrammatic sketch of the chart thus obtained, reproduced as Chart II, is shown below in Fig I.

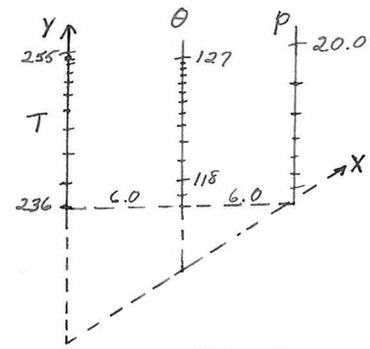


Fig I

All values of y have been arbitrarily multiplied by a scale factor of 100, and the chart is laid off on a 20 scale.

In practice the quantity θ is not usually desired; instead θ_d is sought. Rossby* has prepared a table of corrections to be applied to θ to obtain θ_d. However the computer in practice is more bothered than helped by the necessity of using another table, so we should develop either a chart giving us θ_d directly, or else obtain it as a special case from a θ_E chart, considering w=0. Both these methods will now be developed.

Chapter IV

THE CALCULATION OF θ_d

A: DOUBLE-SET NOMOGRAM

While the quantity θ_d is not employed sufficiently in practice in an accurate fashion to justify its inclusion as a separate series of scales upon a master nomogram, when θ or θ_d may be obtained as a special case of θ_E, it will nevertheless be instructive to construct such a chart.

The exact definition of θ_d occurs in:

$$(1) \qquad \theta_d = T\left(\frac{1000}{P-er}\right)^{.288}$$

While in the form above any nomogram is difficult to construct, if we substitute the variable w for r, where

$$(2) \qquad w = .622\frac{er}{P-er} = .622 \quad ,$$

we have approximately:

$$\theta_d = T\left[\frac{1000}{P(1-\frac{w}{.622})}\right]^{.288} = T\left(\frac{1000}{Pw'}\right)^{.288},$$

$$\text{where} \qquad w' = 1-\frac{w}{.622},$$

The latter is separable into two distinct equations, such that

$$(4) \qquad \left(\frac{\theta}{T}\right)^{3.47} = Q = \frac{1000}{Pw'},$$

where (3) has been raised to the exponent $\frac{1}{.288}$. Upon taking logs of (4), we obtain

$$(5) \qquad \log Q \log + \log w' = \log Q + 3.47\log T - 3.47\log \theta_d = 0$$

If we let

$$x = \mu_1 \log Q$$
$$y = \mu_3 \log w',$$

the three equations obtained are:

$$(6)\qquad \begin{aligned} 1\cdot x + 0\cdot y - \mu_1 \log Q &= 0 \\ 0\cdot x + 1\cdot y - \mu_3 \log w' &= 0 \\ \frac{1}{\mu_1} x + \ y + \ \log \frac{P}{1000} &= 0. \end{aligned}$$

The consistency of (6) requires that

$$(7)\qquad \begin{vmatrix} 1 & 0 & \mu_1 \log Q \\ 0 & 1 & \mu_3 \log w' \\ \frac{1}{\mu_1} & \frac{1}{\mu_3} & -\log \frac{P}{1000} \end{vmatrix} = 0.$$

Upon adding columns one and two, and dividing the third row by $\frac{\mu_1+\mu_3}{\mu_1\mu_3}$, we obtain

$$(8)\qquad \begin{vmatrix} 1 & 0 & \mu_1 \log Q \\ 1 & 1 & \mu_3 \log w' \\ 1 & \frac{\mu_1}{\mu_1+\mu_3} & \frac{\mu_1\mu_3}{\mu_1+\mu_3} \log \frac{1000}{P} \end{vmatrix} = 0.$$

Similarly, for the second part of (5), if we let

$$x = \mu_2 \log Q$$

$$y = 3.47\mu_4 \log T,$$

we find that

$$(9)\qquad \begin{vmatrix} 1 & 0 & \log Q \\ 1 & 1 & 3.47 \log T \\ 1 & \frac{\mu_2}{\mu_2+\mu_4} & \frac{3.47\mu_2\mu_4}{\mu_2+\mu_4} \log \theta_d \end{vmatrix} = 0.$$

A consideration of the spreads desired in our variables leads to the choices:

$$\mu_1 = .667 \qquad \mu_2 = .667$$

$$\mu_3 = 20.0 \qquad \mu_4 = 2.0,$$

so that the scale factor for Q is the same in both nomograms, and there is a great magnification in the w scale. A glance at the diagrammatic sketch Fig I and Fig II will indicate that this magnification is accomplished at the expense of a tendency for coincidence in the Q and p scales; since the p scale is used in conjunction with the w scale, no loss in accuracy results, as long as w is not the variable sought.

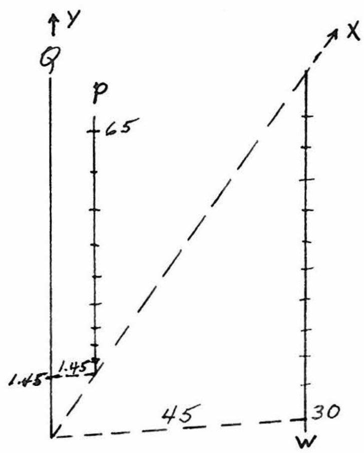


Fig I

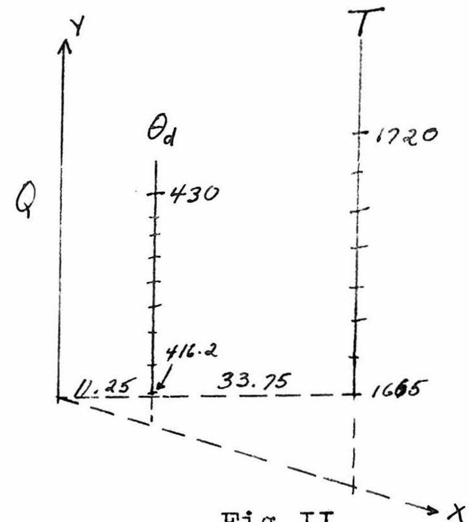


Fig II

Chart IV has been constructed upon a 60 scale, from values tabulated in Table

Chapter IV

B: SET-SQUARE NOMOGRAM

While a satisfactory double-set chart may be constructed, it has not been possible to construct a set-square diagram with scales more evenly distributed than that developed by Millar. The reason for this will become apparent when we find that the constructional determinant employs the same scale factor for the two variables with ranges most dissimilar.

Equation (3) may be written:

$$(10)\quad \left[\tfrac{1}{2}(3-\log p-\log w')+3.47\log T\right]+\left[\tfrac{1}{2}(3-\log p-\log W')-3.47\log\theta_d\right]=0.$$

If we let

$$(11)\quad Q=\tfrac{1}{2}(3-\log p-\log w')+3.47\log T=-\tfrac{1}{2}(3-\log p-\log w')+3.47\log\theta_d,$$

two separate determinants may be formed. To obtain the constructional forms, let

$$x=\mu_2(3-\log p)$$

$$y=\mu_4(\log w');$$

if we include the Q term in the T and 0 functions, we have:

$$(12)\quad \begin{vmatrix} 1 & 1 & -\mu_2(3-\log p) \\ 0 & 1 & -\mu_4\log w' \\ \frac{1}{\mu_2} & \frac{1}{\mu_4} & 6.94\log T-2Q \end{vmatrix} = \begin{vmatrix} 1 & 0 & -\mu_2(3-\log p) \\ 0 & 1 & -\mu_4\log w' \\ \frac{1}{\mu_2} & \frac{1}{\mu_4} & -6.94\log\theta_d+2Q \end{vmatrix} = 0$$

Each of (12) may be considered as composed of two separate determinants.

$$(13)\quad \begin{vmatrix} 1 & 0 & -\mu_2(3-\log p) \\ 0 & 1 & -\mu_4\log w' \\ \frac{1}{\mu_2} & \frac{1}{\mu_4} & 6.94\log T \end{vmatrix} - \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{\mu_2} & \frac{1}{\mu_4} & 2Q \end{vmatrix} = 0,$$

and

$$(14)\quad \begin{vmatrix} 1 & 0 & -\mu_2(3-\log p) \\ 0 & 1 & -\mu_4\log w' \\ \frac{1}{\mu_2} & \frac{1}{\mu_4} & -6.94\log \theta_d \end{vmatrix} - \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{\mu_2} & \frac{1}{\mu_4} & -2Q \end{vmatrix} = 0.$$

Expansion of the right-hand members of (13) and (14) shows us that these determinants have the values 2Q and -2Q respectively. We may then add the left-hand members:

$$(15)\quad \begin{vmatrix} 1 & 0 & -(3-\log p) \\ 0 & 1 & -\log w' \\ \frac{1}{\mu_2} & \frac{1}{\mu_4} & 6.94\log T \end{vmatrix} + \begin{vmatrix} 1 & 0 & -\mu_2(3-\log p) \\ 0 & 1 & -\mu_4\log w' \\ \frac{1}{\mu_2} & \frac{1}{\mu_4} & -6.94\log \theta_d \end{vmatrix} = 0,$$

or, upon combining into a fourth-order determinant:

$$(16)\quad \begin{vmatrix} 1 & 0 & -\mu_2(3-\log p) & 0 \\ 0 & 1 & -\mu_4\log w' & 0 \\ \frac{1}{\mu_2} & \frac{1}{\mu_4} & 3.47\log T & -1 \\ \frac{1}{\mu_2} & \frac{1}{\mu_4} & -3.47\log \theta_d & 1 \end{vmatrix} = 0.$$

We shall clear the first two columns of fractions and subtract one from the other, change the sign of the third row, add timesthe final column to the first, and divide the rows so that the first column contains only 1's:

$$\begin{vmatrix} \mu_2 & 0 & -(3-\log p & 0 \\ -\mu_4 & \mu_4 & -\log w' & 0 \\ 0 & -1 & 6.94\log T & 1 \\ 0 & 1 & -6.94\log \theta_d & 1 \end{vmatrix} = \begin{vmatrix} \mu_2 & 0 & (3-\log p) & 0 \\ -\mu_4 & & -\log w' & 0 \\ \mu_2 & -1 & 6.94\log T & 1 \\ \mu_2 & 1 & 6.94\log \theta_d & 1 \end{vmatrix} = 0,$$

or,

$$
(17) \qquad \begin{vmatrix} 1 & 0 & (3-\log p) & 0 \\ 1 & -\mu_2 & \log w' & 0 \\ 1 & -1 & 6.94 \log T & 1 \\ 1 & 1 & 6.94 \log \theta_d & 1 \end{vmatrix} = 0.
$$

We see that the same factor multiplies both the p function, and w. Since the spread in log w' is very much less than that in log p, no satisfactory chart has been found. It is possible that other solutions exist, but, there being no inductive method of obtaining such, we must be satisfied with (17) or its equivalent, developed by Millar.

Chapter V

THE CALCULATION OF θ_E

A: DOUBLE-SET NOMOGRAM

The defining equation for θ_E, the equivalent potential temperature, will be taken as

$$(1) \qquad \theta_E = \theta_d e^{\frac{Lw}{c_p T}},*$$

or upon taking logs:

$$\ln\theta_E = \ln\theta_d + \frac{Lw}{c_p T}.$$

Substituting the value previously given for θ_d, we obtain

$$(2) \qquad c_p \ln T - AR\ln p/p_o + \frac{Lw}{c_p T} - c_p \ln\theta_E = 0$$

We shall regroup (2) so that

$$(3) \qquad \begin{aligned} &AR \ln p + (c_p \ln\theta_E - AR\ln p_o) - Q = 0 \\ &c_p \ln T + \frac{Lw}{c_p T} - Q = 0, \end{aligned}$$

where the arbitrary function Q has been introduced so that each equation contain only three variables. The second of (3) suggests the following constructional substitutions:

$$x = \mu_2 AR\ln p$$

$$y = \mu_4 Q \qquad ,$$

or

$$x + 0\cdot y - \mu_2 AR\ln p = 0$$

$$0\cdot x + y - \mu_4 Q + 0 = 0$$

$$\frac{1}{\mu_2}x - \frac{1}{\mu_4}y + C_p \ln\theta_E - p_1 = 0,$$

where

$$P_1 = AR\ln p_o.$$

The consistency of this set of equations requires that:

---

* Brunt, pp. 97, equation 55

$$(4) \qquad \begin{vmatrix} 1 & 0 & \mu_2 AR\ln p \\ 0 & 1 & \mu_4 Q \\ \frac{1}{\mu_2} & \frac{-1}{\mu_4} & -C_p \ln\theta_E + P_1 \end{vmatrix} = 0.$$

Upon adding the first and second columns, and dividing the third row by , and rearranging, we obtain

$$(5) \qquad \begin{vmatrix} \mu_2 AR\ln p & 0 & 1 \\ \mu_4 Q & 1 & 1 \\ \frac{\mu_2 \mu_4}{\mu_4 - \mu_2}(P - C_p \ln\theta_E) & \frac{-\mu_2}{\mu_4 - \mu_2} & 1 \end{vmatrix} = 0.$$

The first of equations (3) may most easily be put into a basic determinant:

$$(6) \qquad \begin{vmatrix} Lw & -1 & 0 \\ C_p \ln T & \frac{1}{T} & 1 \\ Q & 0 & 1 \end{vmatrix} = 0.$$

In the above form the variation of L with T has been neglected; in some forms this would not be advisable, but we shall see that the scale of T chosen will be too small to be affected by this simplification. If desired, the variation may be easily included. Upon adding the second and third columns of (6), dividing the second row by $\frac{1+T}{T}$, and changing the sign of the first row, there results:

$$(7) \qquad \begin{vmatrix} -\frac{Lw}{2} & 1 & 1 \\ \frac{C_p T \ln T}{2(1+T)} & \frac{1}{1+T} & 1 \\ Q/2 & 0 & 1 \end{vmatrix} = 0.$$

As we shall see, the scale of w and T are so dissimilar that a constructional determinant is required. This may be derived if the variables of (3) are rechosen as separable terms involving non-separable functions of several variables. Without developing this form in detail, we may immediately form the constructional determinant corresponding to (7) by analogy to one of the basic types listed in Allcock and Jones*, as

$$(8)\quad \begin{vmatrix} -\frac{\mu_3}{\mu_1}\frac{LW}{2} & 1 & 1 \\ \frac{\mu_3 C_p T \ln T}{2\left[(1+T)\Delta\mu\left(\frac{1}{1+T}\right)+\mu_3\right]} & \frac{\mu_1\left(\frac{1}{1+T}\right)}{\Delta\mu\left(\frac{1}{1+T}\right)+\mu_3} & 1 \\ Q/2 & 0 & 1 \end{vmatrix} = 0,$$

where $\Delta\mu = \mu_1 - \mu_3$.

If we clear the denominator of the second row, we have:

$$(9)\quad \begin{vmatrix} -\frac{\mu_3}{\mu_1}\frac{LW}{2} & 1 & 1 \\ \frac{\mu_3 C_p T \ln T}{2T'} & \frac{\mu_1}{T'} & 1 \\ Q/2 & 0 & 1 \end{vmatrix} = 0,$$

where $T' = \Delta\mu + \mu_3(1+T) = \mu_1 + \mu_3 T$

Equation (9) will be reversed with respect to its grid supports by subtracting the third column from the second:

$$(10)\quad \begin{vmatrix} -\frac{LW}{2}\frac{\mu_3}{\mu_1} & 0 & 1 \\ \frac{\mu_3 C_p T \ln T}{2T'} & 1-\frac{\mu_1}{T'} & 1 \\ Q/2 & 1 & 1 \end{vmatrix} = 0.$$

Close comparison of (10) and (5) leads us to the conclusion that the most practical position of scales is such that the Q scale is approximately in the center, with the T and w scales slightly smaller than those of p or θ_E. The value of Q will then be centered somewhere near the center of the diagram, and no solution of θ_E will then occur beyond the chart, even with maximum magnification of the scales. This may be accomplished

---

*Allcock and Jones:

if we impose the conditions that μ_1 and μ_2 are negative, while μ_3 and μ_4 are positive.

Consideration must now be given the spreads of the basic functions:

θ_E from 250 to 360 , or $c_p \ln\theta_E$ -P from .85 to .94

p from 100 to 1000, or ARlnp from .32 to .48

T from 250 to 300 , or $\frac{c_p T \ln T}{T'}$ and $1-\frac{1}{T'}$ variable

w from 0 to 20 or $\frac{Lw}{2}$ from 0.00 to 5.9

Whatever oblique system we adopt, changing Q and the spread of T with reference to the grid supports, but not in mathematical value, we must still impose the condition $\mu_1 \gg \mu_3$, in order that the spread upon the w scale be reduced to comparable size with other scales.

We may now construct diagrammatic sketches of the two nomograms sought:

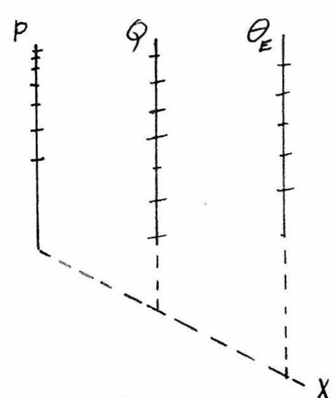


Fig I

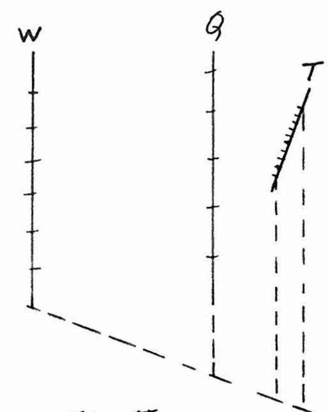
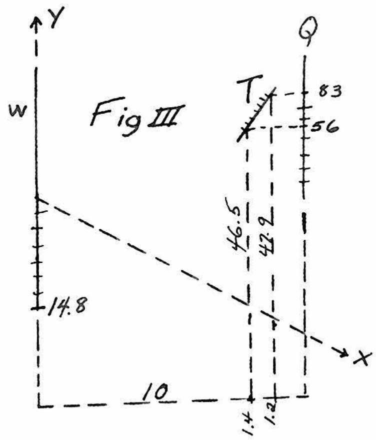
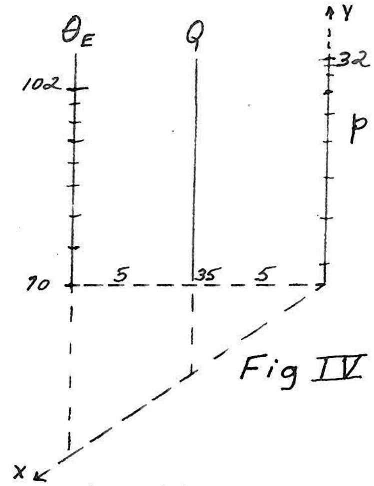


Fig II

The nomograms of Fig I and Fig II will be employed later in the master nomogram. For the purposes of simplified induction we shall here employ a set of positive scale factors, whose values may be determined more easily. In this case the Q scale will be an end scale in the w-T nomogram, leading to a smaller spread upon the T scale. The sketches for this case are shown below in Figs III and IV:





The values chosen were $\mu_1 = 200$, $\mu_3 = 5$, $\mu_2 = 1$, $\mu_4 = \frac{1}{2}$. It will be noted that the latter values of μ_2 and μ_4 are those reducing (5) to a basic form. The values have been calculated and listed in Table V , and plotted upon Chart V , using a 20 scale. It will be noted that the extreme obliquity of coordinates in Fig III further contracts that T scale spread, decreasing accuracy somewhat.

B: SET-SQUARE NOMOGRAM

From equation (2) we saw that

$$(2) \qquad c_p \ln T - AR \ln P/p_0 \quad - c_p \log \theta_E = 0.$$

This may be separated in such a way that two of the variables are repeated:

$$(11) \qquad (\tfrac{c_p}{2} \ln T - AR \ln p + \tfrac{Lw}{2T}) - (-\tfrac{c_p}{2} \ln T + [c_p \ln \theta_E - AR \log p_0 - \tfrac{Lw}{2T}) = 0.$$

For the sake of brevity let

$$(12) \qquad p_2 \; - \tfrac{AR}{c_p} \ln p_0$$

The constant p_2 has been included with the term involving θ_E merely to maintain positive logarithmic values, and could be employed with any of the other functions. If, now, we assign the value Q to each of the quantities in parentheses, we find:

$$(13) \begin{vmatrix} w & -1 & 0 \\ \tfrac{c_p}{2} \ln T & \tfrac{L}{2T} & 1 \\ AR \ln p & 0 & 1 \end{vmatrix} = Q = \begin{vmatrix} -w & 1 & 0 \\ (\ln T)(-\tfrac{c_p}{2}) & \tfrac{L}{2T} & 1 \\ c_p(\ln \theta_E - p_2) & 0 & 1 \end{vmatrix}$$

Since these determinants are not zero-valued, we must carry along upon both sides any operation we perform; however, it will be found that these operational factors finally cancel out. Upon changing the sign of the first row of the first of (13), adding its latter columns, and dividing its middle row to obtain unity there results:

$$(14) \qquad \begin{vmatrix} -w & 1 & 1 \\ \frac{c_p T \ln T}{L + 2T} & \frac{L}{L+2T} & 1 \\ AR \ln p & 0 & 1 \end{vmatrix} \left(-\frac{L + 2T}{2T}\right) = Q.$$

Once again the derivation of a constructional form corresponding to (14) involves the formation of a new set of variables; rather than develop this, we shall take the general analogous form listed in <u>Allcock and Jones</u>, obtaining:

$$(15)\qquad \begin{vmatrix} -\frac{\mu_3}{\mu_1}W & 1 & 1 \\ \frac{\mu_3 c_p T \ln T}{T'} & \frac{\mu_1 L}{T'} & 1 \\ AR\ln p & 0 & 1 \end{vmatrix} \left(-\frac{L+2T}{2T}\right) = Q.$$

Similar operations upon the second of (13) gives us

$$(16)\qquad \begin{vmatrix} -\frac{\mu_3}{\mu_1}W & 1 & 1 \\ \frac{\mu_3 c_p T \ln T}{T'} & \frac{\mu_1 L}{T'} & 1 \\ c_p(\ln\theta_E - P_2) & 0 & 1 \end{vmatrix} \left(\frac{L+2T}{2T}\right) = Q,$$

where $T' = L(\mu_1-\mu_3)+\mu_3(L+2T)$

$= L\mu_1+2\mu_3 T.$

Subtracting the last two determinants, we have

$$(17)\qquad \begin{vmatrix} -W & 1 & 1 \\ \frac{\mu_3 c_p T \ln T}{T'} & \frac{\mu_1 L}{T'} & 1 \\ AR\ln p & 0 & 1 \end{vmatrix} + \begin{vmatrix} -\frac{\mu_3}{\mu_1}W & 1 & 1 \\ \frac{\mu_3 c_p T \ln T}{T'} & \frac{\mu_1 L}{T'} & 1 \\ c_p(\ln\theta_E - P_2) & 0 & 1 \end{vmatrix} = 0,$$

or upon combining into a single determinant:

$$(18)\qquad \begin{vmatrix} c_p(\ln\theta_E - P_2) & 0 & 1 & 1 \\ AR\ln p & 0 & 1 & -1 \\ \frac{\mu_3 c_p T \ln T}{T'} & \frac{\mu_1 L}{T'} & 1 & 0 \\ -\frac{\mu_3}{\mu_1}W & 1 & 1 & 0 \end{vmatrix} = 0.$$

If we change the sign of the second row, add the fourth column to the second, and divide the third column by $\frac{\mu_1 L}{T'}$, we find

$$(19)\qquad \begin{vmatrix} c_p(\ln\theta_E - P_2) & 1 & 1 & 1 \\ -AR\ln p & 1 & -1 & 1 \\ \dfrac{\mu_3 c_p T\ln T}{\mu_1 L} & 1 & \dfrac{T'}{\mu_1 L} & 0 \\ -\dfrac{\mu_3}{\mu_1} w & 1 & 1 & 0 \end{vmatrix} = 0.$$

A consideration of the desired spread of variables leads to the choice

$$\mu_3 = 2.5, \mu_1 = 1;$$

actually these scale factors are not separate; rather the ratio $\frac{\mu_3}{\mu_1}$ is the single arbitrary quantity, for

$$\frac{T'}{\mu_1 L} = \frac{\mu_1 L + 2\mu_3 T}{\mu_1 L} = 1 + \frac{2}{L}\frac{\mu_3}{\mu_1} T.$$

The diagrammatic sketch resulting from (19) is shown below in Fig V and Fig VI, with the values taken from Table VI .

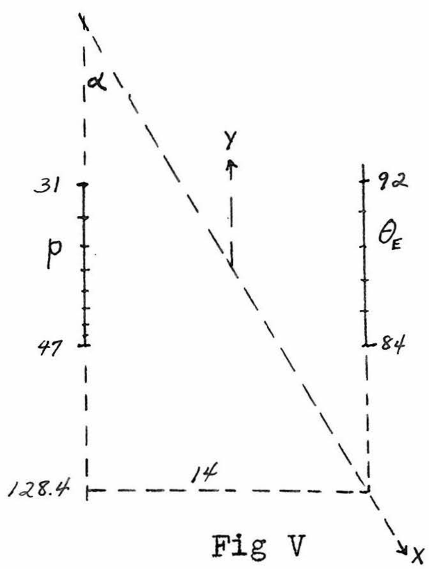


Fig V

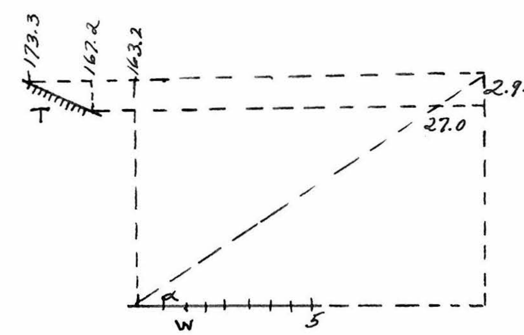


Fig VI

The angle α has the assigned value

$$\frac{14}{128.4} = \frac{17.8}{163.2} = .109.$$

From these figures Chart VI has been constructed, using a 20 scale. Approximately the greatest magnification possible has been utilized, but the w scale is still smaller than desirable; while this scale may be increased very simply by increasing the ratio $\frac{\mu_3}{\mu_1}$, it will be found that the distance between the T and w scales increases rapidly. The possibility of a negative ratio has also been investigated; it is found that the spread in the T scale decreases too rapidly because of the extremely small α necessary.

Chapter VI

THE CALCULATION OF ALTITUDE

A: DOUBLE-SET NOMOGRAM

From a consideration of the hydrastatic pressure of a column of air, Equation (1) may be deduced:

$$(1) \qquad P = P_o \, e^{-\frac{gz}{RT}},$$

where P is the pressure at the altitude z,

P_o is the pressure at altitude z = 0

T is the temperature of the column of air between z = 0 and z = z.

g is the value of gravity

and R is the gas constant for air

Equation (1) is strictly valid only for an isothermal column, but little error is introduced if the mean temperature of the column is employed, as is the case with the Stuve diagram. Also variations in g with height and locality will be neglected, and the standard value of the acceleration of gravity employed. Extracting the natural logs of (1) we have:

$$(2) \qquad \ln P/P_o + \frac{g \Delta z}{RT} = 0*,$$

where Δz is the difference in elevation between the pressure levels p and p_o,

and T the mean temperature between p and p_o

If now we let

$$(3) \qquad x = \mu_1 \ln \frac{P}{P_o}$$
$$y = \mu_2 g \, \Delta z,$$

---

*Brunt, pp. 34, Equation 14

the following set of equations is obtained:

$$(4)\qquad \frac{1}{\mu_1}x + \frac{1}{RT}\frac{1}{\mu_2}y + 0 = 0$$
$$x + 0\cdot y - \mu_1 \ln P/P_0 = 0$$
$$0\cdot x + y - \mu_2 g \Delta z = 0.$$

The condition that these be consistent in x and y is:

$$(5)\qquad \begin{vmatrix} \frac{1}{\mu_1} & \frac{1}{\mu_2 RT} & 0 \\ 1 & 0 & -\mu_1 \ln P/P_0 \\ 0 & 1 & -\mu_2 g \Delta z \end{vmatrix} = 0.$$

Upon changing the sign of the third column, adding the first to the second, and dividing the first row by we have:

$$(6)\qquad \begin{vmatrix} \frac{\mu_2 RT + \mu_1}{\mu_1 \mu_2 RT} & \frac{1}{\mu_2 RT} & 0 \\ 1 & 0 & \mu_1 \ln \frac{P}{P_0} \\ 1 & 1 & \mu_2 g \Delta z \end{vmatrix} = \begin{vmatrix} 1 & \frac{\mu_1}{\mu_1 + \mu_2 RT} & 0 \\ 1 & 0 & \mu_1 \ln \frac{P}{P_0} \\ 1 & 1 & \mu_2 g \Delta z \end{vmatrix} = 0.$$

The conditions we shall impose upon end values are:

$g\Delta z$, 0 to 5×10^8, $cm^2 \cdot sec^{-2}$

RT , $3 \times 10^6 \times 2 \times 10^2$ to $3 \times 3 \times 10^8$

P/P_0 , 0.10 to 1.00.

Since the diagrammatic sketch of Fig I shows us that and z are graduated in opposite directions, necessitating an oblique system of grid supports, appropriate values for the scale factors would be:

$$(7)\qquad \mu_1 = 5.0$$
$$\mu_2 = 5.0 \times 10$$

The constructional determinant then becomes:

$$\begin{vmatrix} 11.51\log P/P_0 & 0 & 1 \\ 4.9\times10\Delta z & 1 & 1 \\ 0 & \frac{5}{5+.0143T} & 1 \end{vmatrix} = 0.$$

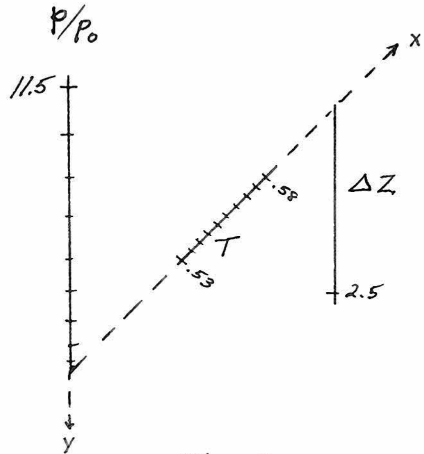


Fig I

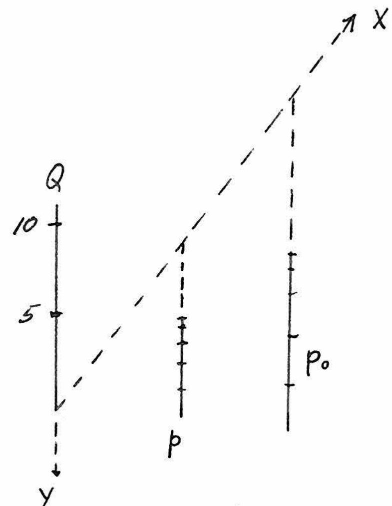


Fig II

The values according to which the scales of Chart VII have been graduated are listed in Table VII, which also contains the scalings for Fig II. A larger scale for T and Δz could be used, but was not deemed justified by experimental data. Since in practice the ratio P/P_0 is not known, while p and p_0 are, a nomogram solving this ratio must be included, with the same P/P_0 scale. Since we want

$$Q = P/P_0 \qquad (8)$$

or

$$\log Q - \log p + \log p_0 = 0,$$

where the Q scale is the same as the P/P_0 scale in the altitude nomogram, the determinant resulting will be:

$$(9) \qquad \begin{vmatrix} \log Q & 0 & 1 \\ \log p_0 & 1 & 0 \\ \log p & 1 & 1 \end{vmatrix} = \begin{vmatrix} \log Q & 0 & 1 \\ \log p_0 & 1 & 1 \\ \frac{1}{2}\log p & \frac{1}{2} & 1 \end{vmatrix} = 0.$$

The second determinant results from adding the second and third columns of the first, and then dividing the third row by 2. A diagrammatic sketch of the resulting nomogram is shown in Fig II. It will be noted that the direction of the y grid support has been chosen positive downwards; this leads to a pressure scale with decreasing pressures upwards, to which the computer is accustomed upon adiabatic charts, and which will enable this scale to be used with but a change in scale factor upon the composite master nomogram later to be developed.

There is one undesirable feature of this choice of variables: Δz is calculated rather than z, and any mistake in a previous calculation will enter into all succeeding calculations of that particular sounding, and is not readily detected. Personal errors in setting, of course, will tend to cancel out, however.

B: SET-SQUARE NOMOGRAM

The differential altitude may also be found in a single setting by a set-square nomogram. We may split (2) into two parts such that

$$(10) \quad \ln p + \frac{g\Delta z}{2RT} = Q = \ln p_o - \frac{g\Delta z}{2RT}.$$

If now we let

$$x = \mu_1(\ln p - Q), \text{ and } x = \mu_2(\ln p_o - Q)$$
$$y = \mu_3 g\,\Delta z \qquad\qquad y = \mu_4 g\,\Delta z$$

we obtain the sets of equations:

$$(11) \quad \begin{aligned} x + 0\cdot y - \mu_1(\ln p - Q) &= 0 \\ 0\cdot x + y - \mu_3 g\Delta z &= 0 \\ \frac{1}{\mu_1} x + \frac{1}{\mu_3} y \frac{1}{2RT} + 0 &= 0 \end{aligned}$$

and

$$(12) \quad \begin{aligned} x + 0\cdot y - \mu_2(\ln p_o - Q) &= 0 \\ 0x + y - \mu_4 g\Delta z &= 0 \\ \frac{1}{\mu_2} x - \frac{1}{\mu_4} y \frac{1}{2RT} + 0 &= 0. \end{aligned}$$

The conditions that these be consistent in x and y is that

$$(13) \quad \begin{vmatrix} 1 & 0 & -\mu_1(\ln p - Q) \\ 0 & 1 & -\mu_3 g\Delta z \\ \frac{1}{\mu_1} & \frac{1}{2RT\mu_3} & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 & -\mu_2(\ln p_o - Q) \\ 0 & 1 & -\mu_4 g\Delta z \\ \frac{1}{\mu_2} & \frac{-1}{2RT\mu_4} & 0 \end{vmatrix} = 0.$$

Equations (13) may be expanded in the third columns so that

$$(15)\quad \begin{vmatrix} 1 & 0 & \mu_2 \ln p_o \\ 0 & 1 & \mu_4 g\Delta z \\ \frac{1}{\mu_2} & \frac{-1}{2RT\mu_3} & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & -\mu_2 Q \\ 0 & 1 & 0 \\ \frac{1}{\mu_2} & \frac{-1}{2RT\mu_4} & 0 \end{vmatrix} = 0.$$

An expansion of the right hand members of (14) and (15) will show that these determinants have the value -Q. We can then subtract the two left hand components, obtaining

$$(16)\quad \begin{vmatrix} 1 & 0 & \mu_1 \ln p \\ 0 & 1 & \mu_3 g\Delta z \\ \frac{1}{\mu_1} & \frac{1}{2RT\mu_3} & 0 \end{vmatrix} - \begin{vmatrix} -1 & 0 & \mu_2 \ln p_o \\ 0 & 1 & \mu_4 g\Delta z \\ \frac{1}{\mu_2} & \frac{-1}{2RT\mu_4} & 0 \end{vmatrix} = 0$$

If we now impose the condition

$$(17)\quad \mu_1 = \mu_2,\ \mu_3 = -\mu_4$$

and change the signs of the third column and first row of the right hand determinant, we may combine the two determinants into the single fourth-order determinant

$$(18)\quad \begin{vmatrix} -1 & 0 & \mu_1 \ln p_o & 1 \\ 1 & 0 & \mu_1 \ln p & 1 \\ 0 & 1 & \mu_3 g\Delta z & 0 \\ \frac{1}{\mu_1} & \frac{1}{2RT\mu_3} & 0 & 0 \end{vmatrix} = 0.$$

Upon multiplying the fourth row by $2RT\mu_3$, and adding the second and fourth columns, we obtain

$$\begin{vmatrix} -1 & 0 & \mu_1 \ln p_o & 1 \\ 1 & 0 & \mu_1 \ln p & 1 \\ 0 & 1 & \mu_3 g\Delta z & 0 \\ \frac{2RT\mu_3}{\mu_1} & 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 1 & \mu_1 \ln p_o & 1 \\ 1 & 1 & \mu_1 \ln p & 1 \\ 0 & 1 & \mu_3 g\Delta z & 0 \\ \frac{2RT\mu_3}{\mu_1} & 1 & 0 & 0 \end{vmatrix} = 0.$$

A suitable scale factor for both z and T is not immediately apparent, but if we multiply the second column by λ and add it to k times the first column, we will have a suitable form:

$$(20)\quad \begin{vmatrix} \mu_1 \log p_0 & \lambda + k & 1 & 1 \\ \mu_1 \log p & \lambda - k & 1 & 1 \\ \mu_3 g \Delta z & \lambda & 1 & 0 \\ 0 & \lambda - 2RT\frac{\mu_3}{\mu_1}k & 1 & 0 \end{vmatrix} = 0.$$

Assuming the same end values of our functions as for the double-set nomogram, appropriate values of our arbitrary constants will be:

$$k = 4.50\ ,\ \lambda = 10.00$$

$$\mu_1 = 8.00\ ,\ \mu_3 = 2 \times 10^{-8}$$

The diagrammatic sketch according to which Chart VIII will then be constructed is shown in Fig III:

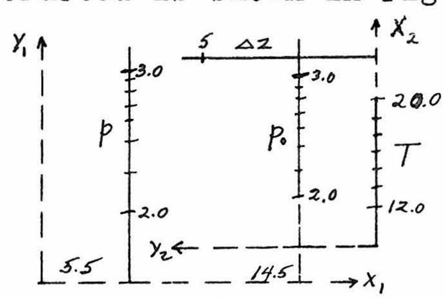


Fig III

The second grid support is so placed that the T scale is placed upon the same line as p_o, eliminating the construction of an extra line. It will be found advisable upon the master nomogram to reverse the coordinate systems, so that p and p_o decrease upwards; Chart VIII is reversed because of the easy transition from the grids of the θ_E chart to those of this one. The values according to which it has been graduated are shown in Table VIII.

While a setting is eliminated in the set-square diagram, as compared to the double-set, the same difficulty remains, in that any initial error in z carries over into succeeding calculations.

Chapter VII

A CRITERION OF STABILITY

One of the main disadvantages of a nomographic substitute for such charts as the Stuve is the inability to determine the degree of stability of a given sounding directly. Of course, we could proceed to calculate $\frac{\partial \theta_E}{\partial z}$, leading to a simple three scale nomogram scaled logarithmically; however, it is sometimes desirable to have an idea of stability without first calculating θ_E. In practice, once θ_E is known, the computer can estimate for the purposes he requires without computation. A method of determining stability, without knowing θ_E, will now be developed.

$$(1) \quad \frac{\partial p}{\partial z} = -\frac{\rho g}{RT},$$

where ρ is the density of the air;

while the adiabatic decrease of temperature with height is given by:

$$(2) \quad \frac{\partial T}{\partial z} = -\frac{Ag}{c_p}.$$

We may divide these equations, obtaining

$$(3) \quad \frac{\partial T}{\partial p} = \frac{ART}{pc_p} \equiv \delta_d.$$

The quantity δ_d is, then, the adiabatic increase of temperature with respect to hydrostatic pressure, and hence is analogous in use to the quantity γ_d, the dry adiabatic lapse rate. Any δ we should obtain by estimation from the given increments of temperature and pressure, if less than the δ_d for the mean temperature and pressure of the incremental column, will indicate a stable layer, while a δ longer than δ_d will indicate an unstable layer.

Taking the logs of (3), we have

$$(4) \qquad \log p + \log \frac{c_p \delta_d}{AR} - \log T = 0.$$

A constructional determinant may be formed by letting

$$x = \mu_1 \log p$$

$$y = \mu_3 \log T$$

or

$$1 \cdot x + 0 \cdot y - \mu_1 \log p = 0$$

$$(5) \qquad 0 \cdot x + 1 \cdot y - \mu_3 \log T = 0$$

$$\frac{1}{\mu_1} x - \frac{1}{\mu_3} y + \log \frac{c_p \delta_d}{AR} = 0$$

Equations (5) require that

$$(6) \qquad \begin{vmatrix} 1 & 0 & \log p \\ 0 & 1 & \log T \\ \frac{1}{\mu_1} & \frac{-1}{\mu_3} & -\log \frac{c_p \delta_d}{AR} \end{vmatrix} = 0,$$

or, upon adding the first and second columns, and dividing the third row by $\frac{\mu_3 - \mu_1}{\mu_1 \mu_3}$:

$$(7) \qquad \begin{vmatrix} 1 & 0 & \log p \\ 1 & 1 & \log T \\ 1 & \frac{-\mu_1}{\mu_3 - \mu_1} & \frac{-\mu_1 \mu_3}{\mu_3 - \mu_1} \log \frac{c_p \delta_d}{AR} \end{vmatrix} = 0.$$

From previous knowledge of the spread in the p and T functions, we will choose

$$\mu_1 = 1$$

$$\mu_3 = -5,$$

obtaining the constructional determinant

$$(8) \qquad \begin{vmatrix} -\log p & 0 & 1 \\ 5 \log T & 1 & 1 \\ \frac{5}{6} \log \frac{c_p \delta_d}{AR} & \frac{1}{6} & 1 \end{vmatrix} = 0.$$

The scale factors were chosen of opposite sign in order that p would be upon an outer scale, to fit into the master nomogram. The diagrammatic sketch resulting from (8) is shown below in Fig I. A 10 scale has been used to graduate the values, upon Chart IX , listed in Table IX .

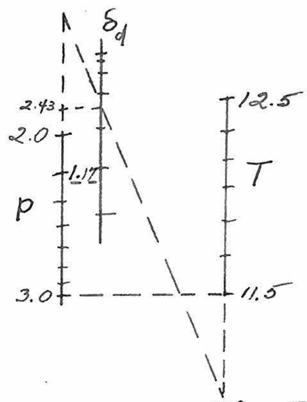


Fig I.

MASTER NOMOGRAMS

A: DOUBLE-SET NOMOGRAM

The double-set charts previously developed will now be combined into a single chart, with as much duplication of lines as practical, so that the fewest possible number of lines will remain. A comparison of the various determinants shows us that all the p functions may be made to coincide, but that the T scales can not. If the chart is to be approximately 11" x 16", the θ_E chart, previously plotted upon a 20 scale, may readily be plotted upon a 10 scale. The values used in plotting this are indicated in Fig I and Fig II.

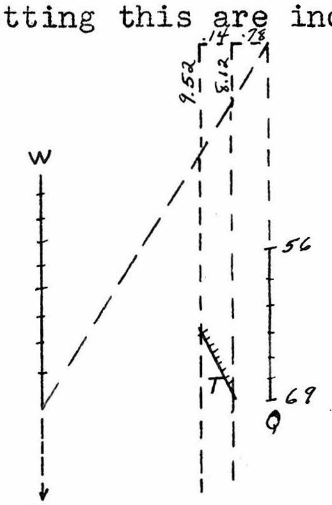


Fig I

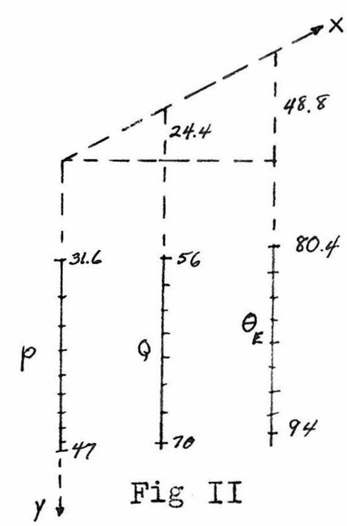


Fig II

It will be noted that the y axes have been inverted, and the x axes then reversed, in order that the p function be graduated with lowest pressures at the top, corresponding to the direction of pressures upon other charts. We will not be able to graduate all T functions in the same direction, however, and the computer will have become accustomed to each scale individually. It is recommended, of course, that the T scales be graduated in degrees Centigrade, while θ_E be in degrees absolute.

The pressure-altitude chart is now constructed, with its p scale superimposed upon that already graduated. This necessitates multiplying the y coordinates of all functions by 1.373, or $\frac{1.58}{1.15}$, effectively multiplying the p scale by 1.58, the spread on the former p scale, and dividing by 1.15, the scale factor formerly employed upon the p function. As shown in Fig III and IV, the x axes have been reversed, allowing the maximum horizontal distance between scales.

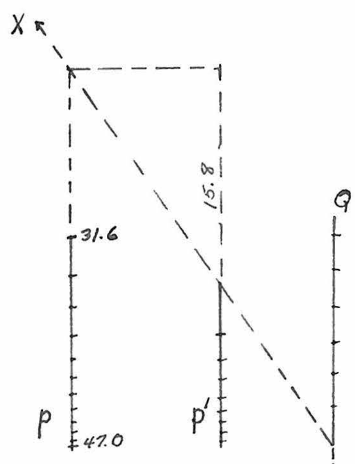


Fig III

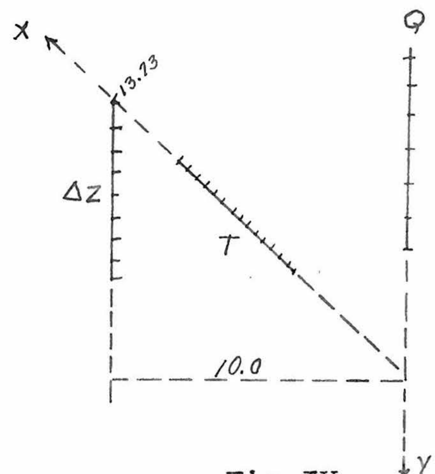


Fig IV

The p' or p scale has been graduated upon the Q scale of Fig I and Fig II, and the z function upon the w line, thus eliminating the addition of two lines.

The w chart must also have its y coordinates multiplied before the p scale coincides with that of θ_E and z. In this case the factor is 1.58. A further simplification has been effected by bringing the r and w scales into coincidence. The values with which this has been accomplished are indicated in Figs V and VI.

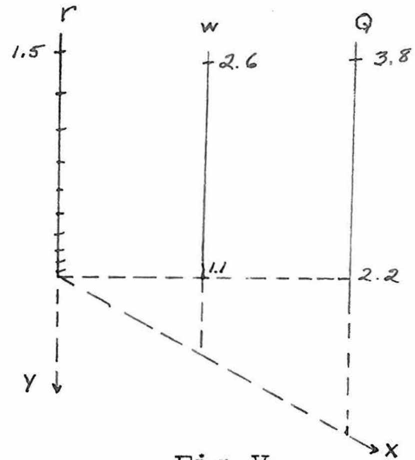


Fig V

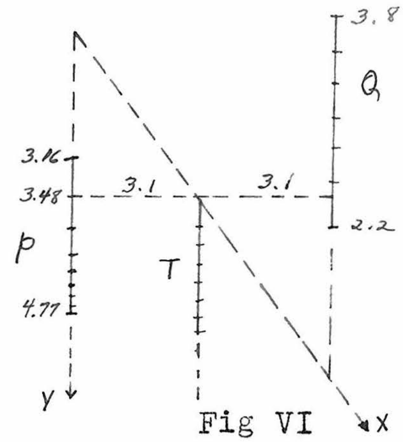


Fig VI

The use of another Q scale has been eliminated by using the temperature scale of the θ_E chart.

Finally, a stability chart has been added. The y coordinates of this chart must also be multiplied by 1.58 to achieve coincidence upon the p scales. Otherwise no alteration has been found necessary from the type of Chart IX. It was found useful to graduate the T scale upon the dummy line of the w chart.

Because a number of lines are found upon the master chart, some of them for the same quantity, the cases where these may be confused have been distinguished by adding a subscript to the variable, indicating the calculation with which the scale is to be used. A brief summary is also printed upon the chart, indicating the combinations one uses in solving the various problems. A subscript n placed before the variable indicates that this variable is to be used as a dummy index only, about which the alignment index rotates, and the value graduated is not the true value.

B: SET-SQUARE NOMOGRAM

The various separate set-square charts are combined in the most advantageous manner feasible in Chart XI. Unlike Chart X, several scales have been used to graduate its functions.

This chart is best begun by considering the best possible axes and scale factors for the θ_E diagram. In Chart VI the spreads in variables require a nearly square chart; hence a new set of oblique axes have been developed for a 16" x 11" sheet. The new angle between axes has a tangent equal to 0.0763, considerably less than that of Chart VI. This increased obliquity does not decrease the spread of the temperature scale noticeably, as a check thru Figs I and II will indicate:

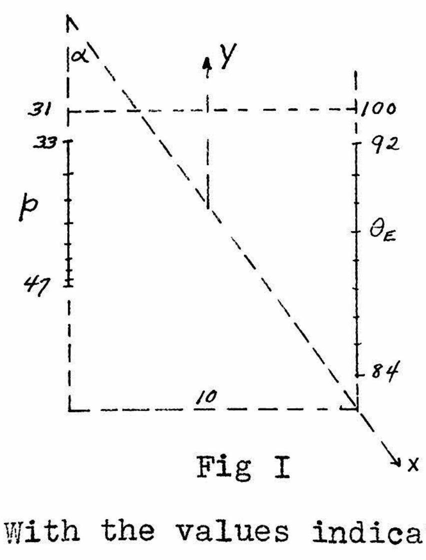


Fig I

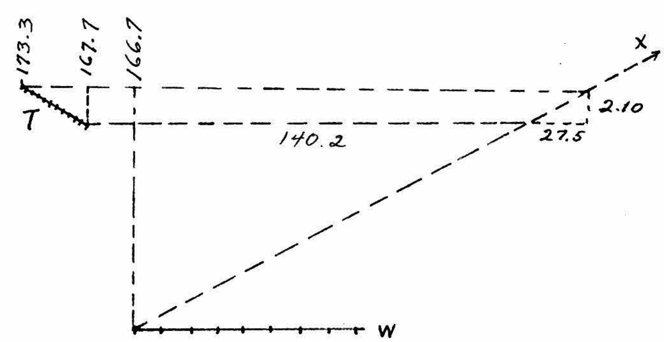


Fig II

With the values indicated a 10 scale fills the chart very well.

In superimposing the altitude diagram a reversal of axes is first required, in order that p may decrease upwards. Because the cycle of the p scale differs from that in the θ_E diagram, an adjustment must be made. Two possibilities exist; one is merely to multiply the column of the constructional determinant by the suitable quantity, and the other is to change the scale factor in Equation (17) of Chapter VI. Closer scrutiny reveals that any decrease in μ_1,--as would be necessary--increases the spread of the T scale. Since the latter is already calibrated for points beyond the spread in p, a decrease in the spread of the former is more desirable. This is effected by a decrease in k; it is still possible to decrease μ_1, but a larger decrease in the spread in k is then necessitated, with consequent rapid convergence in the p and p scales. Hence the former alternative is elected, with a multiplying factor of $\frac{.158}{.184} = 0.859$, and a new k = 3.60 is also selected. The spreads in the variables thus resulting is indicated in Figs III and IV:

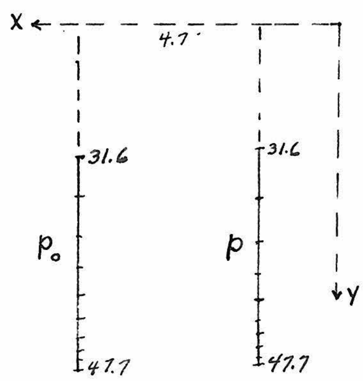


Fig III

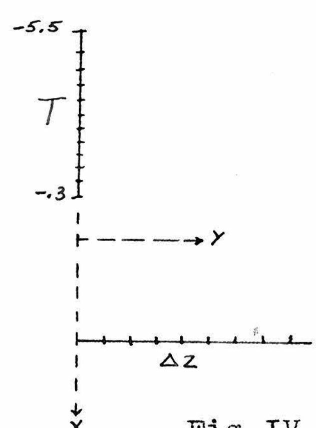


Fig IV

A consideration of the variables occuring in the w diagram might lead one to believe that its pressure scale could also be made to coincide with the other pressure scales; however, the remaining scales have larger spreads than that of p, and we have made the pressure scale the largest one in the other diagrams. It is possible to develop a constructional determinant and satisfactorily warp the scales; however, this has not been done in Chart XI because it would either lead to confusing double graduations upon a single side of a line or to additional lines. The lines already resulting from the θ_E and Δz diagrams may be employed in limited ranges that coincide with the spreads of Chart VI, so that no alteration from the latter other than a small compression of the x axes has been made. The actual spreads are shown below in Figs V and VI, which, unlike the other diagrams, is laid off still upon a 20 scale.

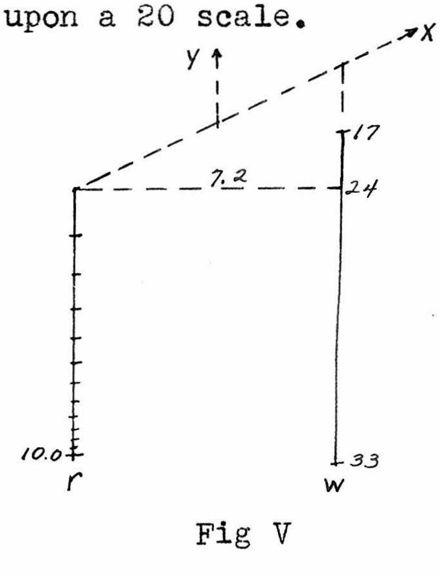


Fig V

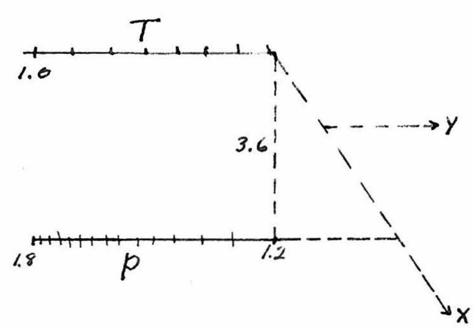


Fig VI

While accuracy is lessened by this arrangement, it's justified not only by the elimination of extra lines, but because of the less frequent use of this diagram, compared to that of θ_E.

Finally a stability chart has been added, with its p scale superimposed upon that of the θ_E and Δz diagrams, and its temperatures graduated upon the same line as w (labeled w') upon the w diagram. The same scale factors are employed in this as in the double-set chart, but the slope of the axes has been altered and the horizontal distances between lines changed to allow T upon w'. The diagrammatic sketch of its construction is indicated below in Fig VII:

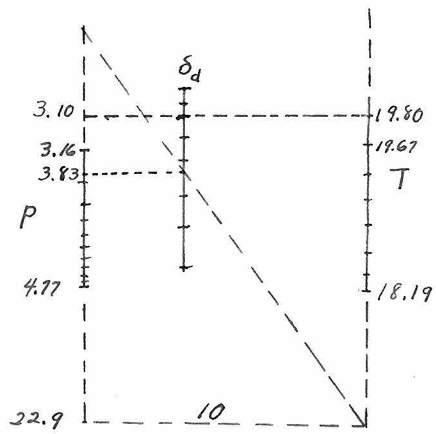


Fig VII

A short note has been added to the chart to indicate which scales are used in conjunction.

Chapter VIII

THE CALCULATION OF w AND θ

Millar, in the Bulletin of the American Meteorological Society*, outlines a satisfactory method of obtaining w and θ upon an ordinary slide rule. A new scale must be graduated upon the slide rule for each quantity.

We have seen that w is defined from

$$(1) \qquad w = .622\frac{e_r}{p_d}$$

Hence if e_r, the existing vapor pressure, and p_d, the partial pressure of the dry air, are known, and set upon the A and B or C and D scales, w is readily obtained. Ordinarily, however, e_r is not known, but T and r, the relative humidity are. The relationship between these quantities is

$$(2) \qquad e_r = re = rf(T),$$

where e is a function of T only, and may be obtained from the Smithsonian Tables. We may use the B scale for r, and the A scale for e_r and e; since, however, e is not given, we must graduate corresponding values of T above the values graduated upon the A scale.

The procedure in calculating w is then to set the B scale index line (1 or 100) opposite the given value of T, and read the value of e upon A opposite r upon B. Scale B is then reset with the value of p_d under the value of e_r, and w obtained upon

---

*Millar, F. G. Bull A. M. S. Oct 35 Page 229

A opposite 62.2 upon B.

Since p_d is obtained by mentally deducting e from p, which is given, a slight mental effort is required. However, p may be used in place of p_d if accuracy is not paramount; since, however, no additional calculations are necessary to obtain e_r, it will not be found difficult to employ p_d.

Another scale must be constructed to obtain θ or θ_d. We have seen that

$$(3) \qquad \theta = T\left(\frac{1000}{p}\right)^{.288},$$

or upon taking logs

$$(4) \qquad \log \theta = \log T + .288 \log \frac{1000}{p}.$$

Inspection of (4) will show two alternatives are possible; one is to construct a scale for T and θ, and the other, to construct one for p. Since the exponent .288 compresses the scale, accuracy will be greater if the entire scale is raised exponentially:

$$(5) \qquad 3.47 \log \theta = 3.47 \log T + \log \frac{1000}{p},$$

and the p function is graduated along the D scale. Now upon most slide rules the lenght of the cycle for the C and D scales is 29.50 upon a 30 engineering scale. Hence we must multiply (5) by this quantity in order that p may be used upon the D scale:

$$(6) \qquad 102.4 \log \theta = 102.4 \log T + 29.5 \log \frac{1000}{p}.$$

The same scale will be used for T and θ, then, and graduated according to the equation

$$(7) \qquad y = 102.4 \log T = 102.4 \log \theta.$$

The values to be used in this graduation are tabulated in Table . The position of the point $y = 0$ is quite arbitrary, but should be sufficiently far off the left end of the slide rule that the usual range of T and θ be included within the cycle of C and D.

The quantity θ_d may be obtained if instead of p we use p_d; in general, however, this results in extra computation and is not done. An empirical method of correction will later be given, or else Rossby's correction table may be employed.

It will be noted that the above method of obtaining θ requires but a single setting: the index point of the C scale is set opposite T upon the new scale, and θ found opposite p upon the C scale. All other slide rule calculations to be outlined will require at least two settings. This fact is intimatety correlated with the necessity of using fourth-order nomograms in all cases except for θ, since a slide rule is but a special type of second order nomogram: one in which the aligning index shrinks to a point, along with the third scale.

Chapter IX

THE CALCULATION OF θ_E

A: FROM θ_d

From the previous chapter we saw that an ordinary slide rule could be used to calculate θ_d directly, providing a special scale was constructed for θ_d. We shall now show that this same scale may be used as a scale for θ_E; however, a linear scale will have to be added.

We have defined θ_E as

$$(1) \qquad \theta_E = \theta_d e^{\frac{Lw}{c_pT}} = \theta_d e^{w'}$$

$$\text{where} \quad w' = \frac{Lw}{c_pT}$$

The above may be written as

$$(2) \qquad \ln\theta_E = \ln\theta_d + w'.$$

Since the C and D scales are constructed upon a 10 log base, we shall rewrite (2) as

$$(3) \qquad \log \theta_E = \log \theta_d + \frac{w'}{2.30}.$$

The constructive scale of θ_d was taken previously as

$$y = 102.4 \log \theta_d;$$

hence we will multiply (3) by a factor of 102.4:

$$(4) \quad 102.4 \log \theta_E = 102.4 \log \theta_d + w''$$

$$\text{where} \quad w'' = \frac{102.4}{2.30} w' = 109.8 \frac{w}{T}$$

Equation (4) shows that the θ_d scale will also be the θ_E scale. However, we must construct a new scale, linear in $\frac{w}{T}$, according to the scale

$$(5) \qquad y = 109.8 \frac{w}{T}.$$

These values have been tabulated in Table . The zero value of y may be placed at any convenient point along the scale. Since the zero value of must be placed opposite the given value of θ_d , it is recommended that the point y = 0 be chosen to coincide with the left end of the C scale, and that the scale be graduated above the C scale, or above the CI scale, if the slide rule has the latter. It will be noted that the cheaper slide rules, with but A, B, C, and D scales on the face are ideal for this purpose, but that space is lacking on better rules.

The procedure, then, is to obtain the ratio $\frac{w}{T}$ first. The computer will probably find it best to run through a whole sounding, jotting down these ratios before finding θ_E. With the scale factor chosen above w will be expressed as grams per kilogram, or 10 times its assigned value. The computer will also find that less sliding need be done if T is set upon the D scale and w upon the C, and the ratio read upon the C scale opposite either end of the D scale. Having obtained the ratio $\frac{w}{T}$, the y = 0 or index point for this scale is set opposite the given value of θ_d, and the value of θ_E found opposite the calculated value of $\frac{w}{T}$.

Since the station is not particularly interested in θ_d, and the latter must first be known, in order to obtain θ_E in this manner a direct method of obtaining θ_E will now be developed.

B: FROM p, T, AND w DIRECTLY

As before let

$$\theta_E = \theta_d e^{w'}.$$

But we saw from Equation (1), Chapter , that

$$\theta_d = T\left(\frac{1000}{p_d}\right)^{.288}.$$

Hence we may write

$$(6) \qquad \theta_E = T\left(\frac{1000}{p_d}\right)^{.288} e^{w'}.$$

Upon taking logs to the base 10 equation (6) becomes

$$(7) \qquad \log \theta_E = \log T - .288 \log \frac{p_d}{1000} + \frac{w'}{2.30}.$$

It will be noted that the pressure ratio has been inverted, in order to obtain a minus sign in front of the term. We are thus able to consider the pressure scale as an inverted scale and add the magnitude of the function to the value of log T by merely placing the value of p opposite the value of T, thus eliminating an ordinary setting.

There are now two main possibilities in the choice of scale for T and θ_E. One is to construct another scale, similar to the C or D scale, but ranging in value only from 200 to 360--the usual range of T and θ_E--, thus obtaining an accurate setting, or else it is possible to employ the D scale (or the C scale, for that matter) for T and θ_E, and merely construct one composite scale of p and $\frac{w}{T}$. For the sake of simplicity we shall choose the latter alternative. We have seen that

(8) $$y = 29.5 \log T$$

would be the equation of graduating T upon the C scale, using the 30 engineering scale. Therefore we shall rewrite (7) as

(9) $$29.5 \log \theta_E = 29.5 \log T - 8.50 \log \frac{p_d}{1000} + 31.6 \frac{w}{T}.$$

Here again w is expressed in grams per kilogram. The equations of graduation for p and w will then be:

$$y = 8.50 \log \frac{p_d}{1000}$$

$$y = 31.6 \frac{w}{T} \quad .$$

The point y = 0 may be arbitrarily placed for graduating p_d, but the point y = 0 must coincide with p = 1000 for the $\frac{w}{T}$ scale. Since p does not greatly exceed 1000, the few graduations of p_d above this value might conveniently be placed above or below the line of graduation, which could then be used for $\frac{w}{T}$.

Once again the procedure requires the calculation of the ratio $\frac{w}{T}$, though we shall indicate a method of obtaining an approximate solution below, that dispenses with this calculation. Having obtained $\frac{w}{T}$, the value of p_d is set opposite the value of T upon the C scale, and θ_E obtained upon the C scale opposite the value of $\frac{w}{T}$.

Where the calculation is wanted rapidly, a rough value of θ_E may be obtained by considering T constant in the expression $\frac{Lw}{c_pT}$. This assumption leads only to a second order error, because of the small fluctuation in T compared to that in w, and because the total term is small compared to other terms. Such an assumption has been used to plot the Rossby diagram. Here, however, it would be best to assume two mean values. One could be T = 250, and the other T = 295. Two scales could then be constructed for w, one in black and one in red, say, and the proper graduation for $\frac{w}{T}$ estimated according to the temperature prevailing.

Another simplification may be used, in that p may be substituted for p_d. The error arising from this approximation may be compensated for by adding increments according to the Rossby table.* For the sake of rapidity however, the author has formulated the empirical rule that there should be no addition until w $\eqsim$ 4, that 1 should be added until w = 10, that 2 should be added from w = 11 to w = 17, and that 3 should be added if w = 18 or more. This rule will give θ_E upon the border of experimental and calculating error for the case of the approximate w graduations.

---

*Rossby's correction table : see bibliography

Conclusions

Two similar charts have been developed, both of which will compute RAOB data with greater rapidity than the old type charts. Moreover, one of these, the set-square, actually affords greater accuracy than the old charts of similar size. For the solution of thermodynamical problems such as fog dissipation, however, the old type charts still must be used, a nomogram being unable to express a relation between empirical data and a fixed relationship.

For the restricted purpose of merely computing θ_E , an even faster and more compact method of computation has been devised; since a slide rule is an instrument of almost universal usage, the beginner will not find it difficult of use, and since every office has need of this quantity before all others, the slide rule solution is of prime importance.

It is hoped that enough latitude for revision has been indicated, that each individual office may alter the general types herein developed, so that maximum use may be obtained.

Table I

$w \times 10^3$	.5 log w	r	log r	p	log p
.5	-1.65050	.00	-	100	2.0000
.6	-1.61090	.10	-1.0000	150	2.1761
.7	-1.57745	.15	-0.8239	200	2.3010
.8	-1.54845	.20	-0.6990	250	2.3979
.9	-1.52290	.25	-0.6021	300	2.4771
1.0	-1.50000	.30	-0.5229	350	2.5441
1.25	-1.45155	.35	-0.4559	400	2.6021
1.50	-1.41195	.40	-0.3979	450	2.6532
1.75	-1.37850	.45	-0.3468	500	2.6990
2.00	-1.34950	.50	-0.3010	550	2.7404
2.5	-1.30105	.55	-0.2596	600	2.7782
3.0	-1.26145	.60	-0.2218	650	2.8129
4.0	-1.19895	.65	-0.1871	700	2.8451
5.0	-1.15050	.70	-0.1549	750	2.8751
6.0	-1.11090	.75	-0.1249	800	2.9031
7.0	-1.07745	.80	-0.0969	850	2.9294
8.0	-1.04845	.85	-0.0706	900	2.9542
9.0	-1.02290	.90	-0.0458	950	2.9777
10.0	-1.00000	.95	-0.0223	1000	3.0000
11.0	-0.97930	1.00	-0.0000	1030	3.0128
12.0	-0.96040				
13.0	-0.94305				
14.0	-0.92695				
15.0	-0.91195				
16.0	-0.89795				
17.0	-0.88480				
18.0	-0.87235				
19.0	-0.86060				
20.0	-0.84950				

T	e	.5log.622e
260	2.25	.0730
265	3.35	.1594
270	4.89	.2416
275	7.06	.3213
280	10.02	.3973
285	14.02	.4702
290	19.38	.5405
295	26.44	.6080
300	35.65	.6229
305	47.55	.7354
310	62.76	.7957
315	82.00	.8538
320	106.10	.9097

Table II

$w \times 10^3$	log w	r	log r	P	$\frac{1}{2}\log\frac{P}{.622}$
.5	-3.3010	.10	-1.0000	100	1.10314
.6	-3.2218	.15	- .8239	150	1.19119
.7	-3.1549	.20	- .6990	200	1.25359
.8	-3.0969	.25	- .6021	250	1.30206
.9	-3.0458	.30	- .5229	300	1.34166
1.0	-3.0000	.35	- .4559	350	1.37514
1.25	-2.9031	.40	- .3979	400	1.40410
1.50	-2.8239	.45	- .3468	450	1.42969
1.75	-2.7570	.50	- .3010	500	1.45257
2.00	-2.6990	.55	- .2596	550	1.47327
2.5	-2.6021	.60	- .2218	600	1.49217
3.0	-2.5229	.65	- .1871	650	1.50956
4.0	-2.3979	.70	- .1549	700	1.52557
5.0	-2.3010	.75	- .1249	750	1.54067
6.0	-2.2218	.80	- .0969	800	1.55462
7.0	-2.1549	.85	- .0706	850	1.56772
8.0	-2.0969	.90	- .0458	900	1.58023
9.0	-2.0458	.95	- .0223	950	1.59196
10.0	-2.0000	1.00	- .0000	1000	1.60310
11.0	-1.9586				
12.0	-1.9208				
13.0	-1.8861				
14.0	-1.8539				
15.0	-1.8239				
16.0	-1.7959				
17.0	-1.7696				
18.0	-1.7447				
19.0	-1.7212				
20.0	-1.6990				

T	e	$\frac{1}{2}$ log e
260	2.25	.1761
265	3.35	.2625
270	4.89	.3447
275	7.06	.4244
280	10.02	.5002
285	14.02	.5734
290	19.38	.6437
295	26.44	.7111
300	35.65	.7760
305	47.55	.8386
310	62.76	.8988
315	82.00	.9569
320	106.10	1.01286

Table III

T	Log T	P	Log p	.864- .288 logp	θ	½Log θ
240	2.3802	1030	3.0128	-0.003	240	1.1901
245	2.3892	1000	3.0000	0.000	250	1.1989
250	.3979	950	2.9777	0.006	260	1.2075
255	.4065	900	2.9542	0.013	270	1.2157
260	.4150	850	2.9294	.020	280	1.2236
265	.4233	800	2.9031	.028	290	1.2312
270	.4314	750	2.8751	.036	300	1.2385
275	.4393	700	2.8451	.045	310	1.2457
280	.4472	650	2.8129	.054	320	1.2526
285	.4548	600	2.7782	.064	330	1.2592
290	.4624	550	2.7404	.075	340	1.2657
295	.4698	500	2.6990	.087	350	1.2720
300	.4771	450	2.6532	.100	360	1.2781
305	.4843	400	2.6021	.114		
310	.4914	375	2.5740	.122		
315	.4983	350	2.5441	.131		
320	.5052	325	2.5119	.141		
325	.5119	300	2.4771	.151		
330	.5185	275	2.4393	.162		
335	.5250	250	2.3979	.173		
340	.5315	225	2.3522	.187		
345	.5378	200	2.3010	.202		
350	.5441					
355	.5502					
360	.5563					
365	.5623					

Table IV

$20 \log(1-\frac{w}{.622})$	$wx10^3$	T	6.94log T	$1.735\log\theta_d$	P	$.645\log\frac{1000}{P}$
0.0000	0	250	16.642	4.1604	100	.6450
- .0140	1	255	16.701	4.1753	150	.5314
- .0278	2	260	16.760	4.1900	200	.4508
- .0418	3	265	16.817	4.2043	250	.3884
- .0560	4	270	16.874	4.2184	300	.3372
- .0700	5	275	16.929	4.2322	350	.2940
- .0842	6	280	16.983	4.2458	400	.2566
- .0984	7	285	17.036	4.2591	450	.2236
- .1124	8	290	17.089	4.2722	500	.1941
- .1266	9	295	17.140	4.2851	550	.1674
- .1408	10	300	17.191	4.2978	600	.1431
- .1550	11	305	17.241	4.3103	650	.1206
- .1692	12	310	17.290	4.3225	700	.0998
- .1834	13	315	17.338	4.3346	750	.0805
- .1976	14	320	17.386	4.3464	800	.0625
- .2118	15	325	17.432	4.3581	850	.0454
- .2264	16	330	17.478	4.3696	900	.0295
- .2552	17	335	17.524	4.3809	950	.0142
- .2696	18	340	17.568	4.3921	1000	.0000
- .2838	19	345	17.612	4.4031	1050	-.0137
- .2982	20	350	17.656	4.4140		
- .3128	21					
- .3272	22					
- .3418	23					
- .3562	24					

Table V

θ_E	$.552\log\theta_E - .474$	P	.158 log P	W	.295w
250	8496	100	.3160	0.0	0.00000
255	8544	125	.3313	0.5	0.00369
260	8591	150	.3438	1.0	0.00737
265	8636	175	.3544	1.5	0.01106
270	8681	200	.3636	2.0	0.01474
275	8725	250	.3789	2.5	0.01843
280	8768	300	.3914	3.0	0.02211
285	8811	350	.4020	4.0	0.02948
290	8852	400	.4111	5.0	0.03685
295	8893	450	.4192	6.0	0.04422
300	8934	500	.4264	7.0	0.05159
305	8973	550	.4330	8.0	0.05896
310	9012	600	.4390	9.0	0.06633
315	9015	650	.4444	10.0	0.07370
320	9088	700	.4495	11.0	0.08107
325	9126	750	.4543	12.0	0.08844
330	9162	800	.4587	13.0	0.09581
335	9198	850	.4628	14.0	0.10318
340	9234	900	.4668	15.0	0.11055
345	9269	950	.4705	16.0	0.11792
350	9303	1000	.4740	17.0	0.12529
355	9337	1050	.4773	18.0	0.13266
360	9371				

T	T'	$5\left(\frac{.276 T \log T}{T'}\right)$	$\frac{200}{T'}$
250	1450	.56970	.1380
260	1500	.57685	.1333
270	1550	.58365	.1290
280	1600	.59015	.1250
290	1650	.59640	.1230
300	1700	.60234	.1176

Table VI

θ_E	$.522\log\theta_E - .474$	P	.158 log P	$w \times 10^3$	2.5 w
250	.8496	100	.3160	1	.0025
255	.8544	125	.3313	2	.0050
260	.8591	150	.3438	3	.0075
265	.8636	175	.3544	4	.0100
270	.8681	200	.3636	5	.0125
275	.8725	250	.3789	6	.0150
280	.8768	300	.3914	7	.0175
285	.8811	350	.4020	8	.0200
290	.8852	400	.4111	9	.0225
295	.8893	450	.4192	10	.0250
300	.8934	500	.4264	11	.0275
305	.8973	550	.4330	12	.0300
310	.9012	600	.4390	13	.0325
315	.9051	650	.4444	14	.0350
320	.9088	700	.4495	15	.0375
325	.9126	750	.4543	16	.0400
330	.9162	800	.4587	17	.0425
335	.9198	850	.4628	18	.0450
340	.9234	900	.4668	19	.0475
345	.9269	950	.4705		
350	.9303	1000	.4740		
355	.9337	1050	.4773		
360	.9371				

T	.06234 T log T	T	T'	$\frac{T'}{u_1 L_1}$
250	1.402	250	1840	3.118
255	1.435	255	1865	3.161
260	1.469	260	1890	3.203
265	1.502	265	1915	3.245
270	1.535	270	1940	3.288
275	1.569	275	1965	3.330
280	1.603	280	1990	3.372
285	1.636	285	2015	3.415
290	1.670	290	2040	3.457
295	1.704	295	2065	3.500
300	1.738	300	2090	3.542

Table VII

$P/P_0 = Q$	11.5 log P/P_0	P	11.5x$\frac{1}{2}$log P
1.000	0.0000	100	11.5128
.900	0.5273	200	13.2455
.800	1.1156	300	14.2592
.700	1.7833	400	14.9787
.600	2.5536	500	15.5365
.500	3.4654	600	15.9924
.400	4.5810	700	16.3775
.300	6.0201	800	16.7114
.200	8.0475	900	17.0056
.100	11.5129	1000	17.2692

Δz	4.905x10	T	$\frac{5}{5+.01435T}$
.5	2.4525	250	.5822
1.0	4.9050	260	.5727
1.5	7.3575	270	.5634
2.0	9.8100	280	.5544
3.0	14.7150	290	.5458
4.0	19.6200	300	.5373
5.0	24.5250	310	.5292

Table VIII

$p=p_o$	18.40 log p	$\Delta z(x10^{-5})$	$1.96\Delta z$	T	$6.4575\ x10^{-2}T$
1050	55.5900	.5	.980	200	12.9150
1000	55.2000	1.0	1.960	210	13.5608
950	54.7897	1.5	2.940	220	14.2065
900	54.3573	2.0	3.920	230	14.8523
850	53.9010	3.0	5.880	240	15.4980
800	53.4170	4.0	7.840	250	16.1438
750	52.9018	5.0	9.800	260	16.7895
700	52.3498			270	17.4353
650	51.7574			280	18.0810
600	51.1189			290	18.7268
550	50.4234			300	19.3725
500	49.6616			310	20.0183
450	48.8189				
400	47.8786				
350	46.8114				
300	45.5786				
250	44.1214				
200	42.3384				
175	41.2722				
150	40.0040				
125	38.5830				
100	36.8000				

Table IX

P	Log P	T	log T	δ_d	$\frac{5}{6}\log 3.4934\,\delta_d$
100	2.0000	200	11.505	.00	–
125	2.0969	205	11.559	.05	-.63149
150	2.1761	210	11.611	.06	-.56551
175	2.2430	215	11.662	.07	-.50971
200	2.3010	220	11.712	.08	-.46139
225	2.3522	225	11.761	.09	-.41876
250	2.3979	230	11.809	.10	-.38062
275	2.4393	235	11.856	.125	-.29970
300	2.4771	240	11.901	.150	-.23388
350	2.5441	245	11.946	.175	-.17809
400	2.6021	250	11.990	.20	-.12977
450	2.6532	255	12.033	.25	-.04902
500	2.6990	260	12.075	.30	.01697
550	2.7404	265	12.117	.35	.07277
600	2.7782	270	12.157	.40	.12111
650	2.8129	275	12.197	.45	.16371
700	2.8451	280	12.236	.50	.20185
750	2.8751	285	12.274	.55	.23616
800	2.9031	290	12.312	.60	.26782
850	2.9294	295	12.349	.65	.29680
900	2.9542	300	12.386	.70	.32362
950	2.9777	305	12.422		
1000	3.0000	310	12.457		
1050	3.0212				

Table XII

P	8.50 log p/1000	θ_E	102.4 log θ_E	W/T	$\frac{W}{T}$x31.6
1050	.18012	260	247.293	0	0
1000	.00000	265	248.141	.005	.158
950	-.19448	270	248.971	.010	.316
900	-.38896	275	249.787	.015	.474
850	-.59993	280	250.589	.020	.632
800	-.82374	285	251.376	.025	.790
750	-1.06199	290	252.150	.030	.948
700	-1.31665	295	252.910	.035	1.106
650	-1.59027	300	253.657	.040	1.264
600	-1.88573	305	254.392	.045	1.422
550	-2.20694	310	255.115	.050	1.580
500	-2.55876	315	255.827	.055	1.738
450	-2.94772	320	256.527	.060	1.896
400	-3.38249	325	257.217		
350	-3.87541	330	257.895		
300	-4.44448	335	258.564		
275	-4.76570	340	259.224		
250	-5.11751	345	259.873		
225	-5.50647	350	260.513		
200	-5.94125	355	261.144		
175	-6.43416	360	261.765		
150	-7.00324				
125	-7.67627				
100	-8.50000				

BIBLIOGRAPHY

Brunt : Physical and Dynamical Meteorology, Cambridge University Press 1939

Rossby's Correction Table : Elements of Weather Prediction : Krick " mimeographed lab notes", Ch. VI Page 19

Allcock and Jones : The Nomogram : the theory and practical construction of computation charts, London Pitman, 1932

Hewes and Seward : Design of Diagrams for Engineering Formulas, McGraw Hill, 1923

Millar, F. G. : Bull. A. M. S., Oct. 35, Page 229

Smithsonian Meteorological Tables : Fifth Revision Wash., 1939

The author wishes to express his sincere appreciation for the invaluable assistance of Jack Guerin in preparing the tables and drafting the charts and Charles Hight, and also to acknowledge the valuable suggestions of Dr. H. J. Stewart, particularly in the development of the inverted θ_E slide rule.

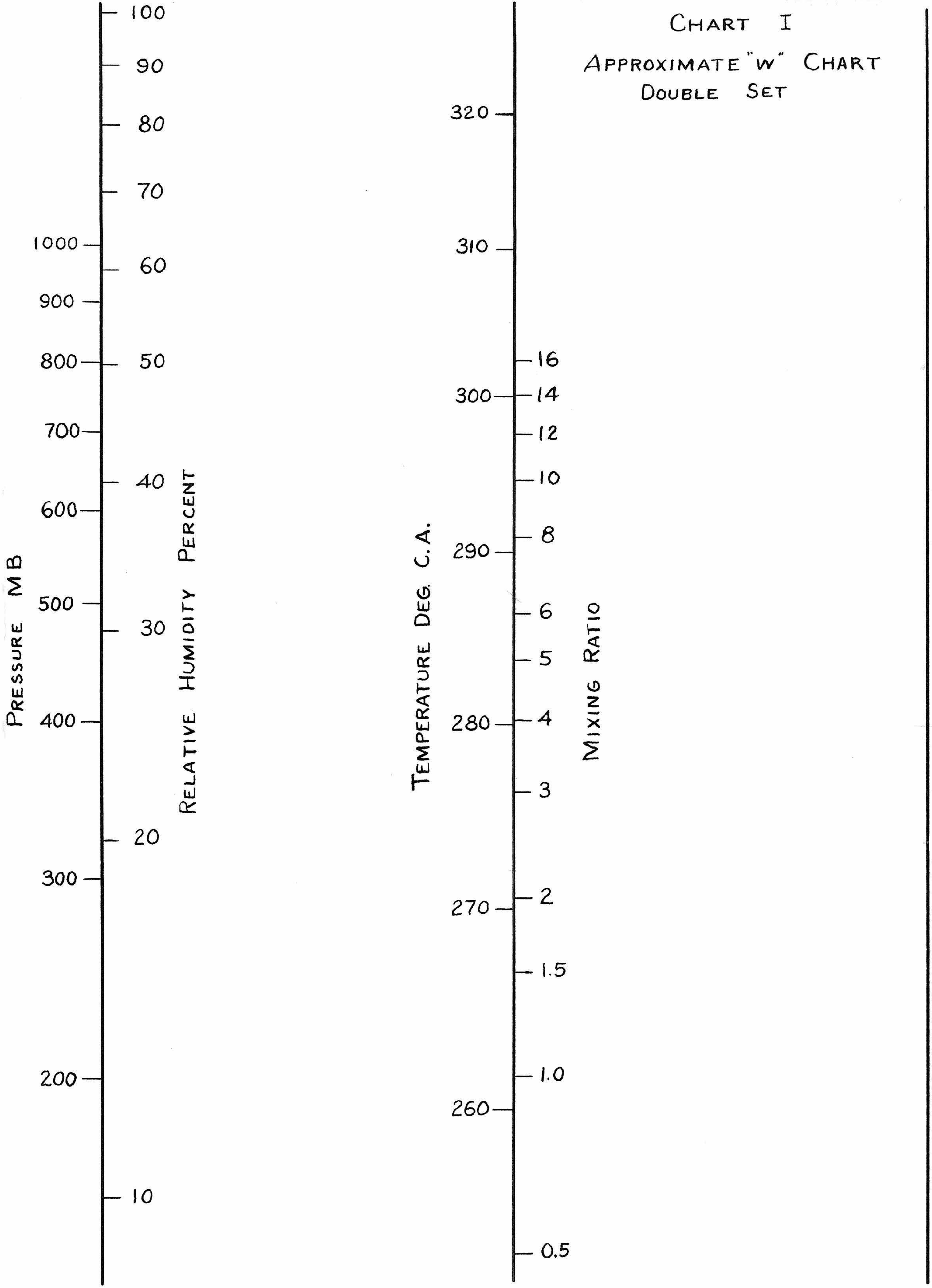


Chart I
Approximate "w" Chart
Double Set
Pressure MB
1000
900
800
700
600
500
400
300
200
Relative Humidity Percent
100
90
80
70
60
50
40
30
20
10
Temperature Deg. C.A.
320
310
300
290
280
270
260
Mixing Ratio
16
14
12
10
8
6
5
4
3
2
1.5
1.0
0.5

APPROXIMATE "W" CHART
SET SQUARE
CHART II

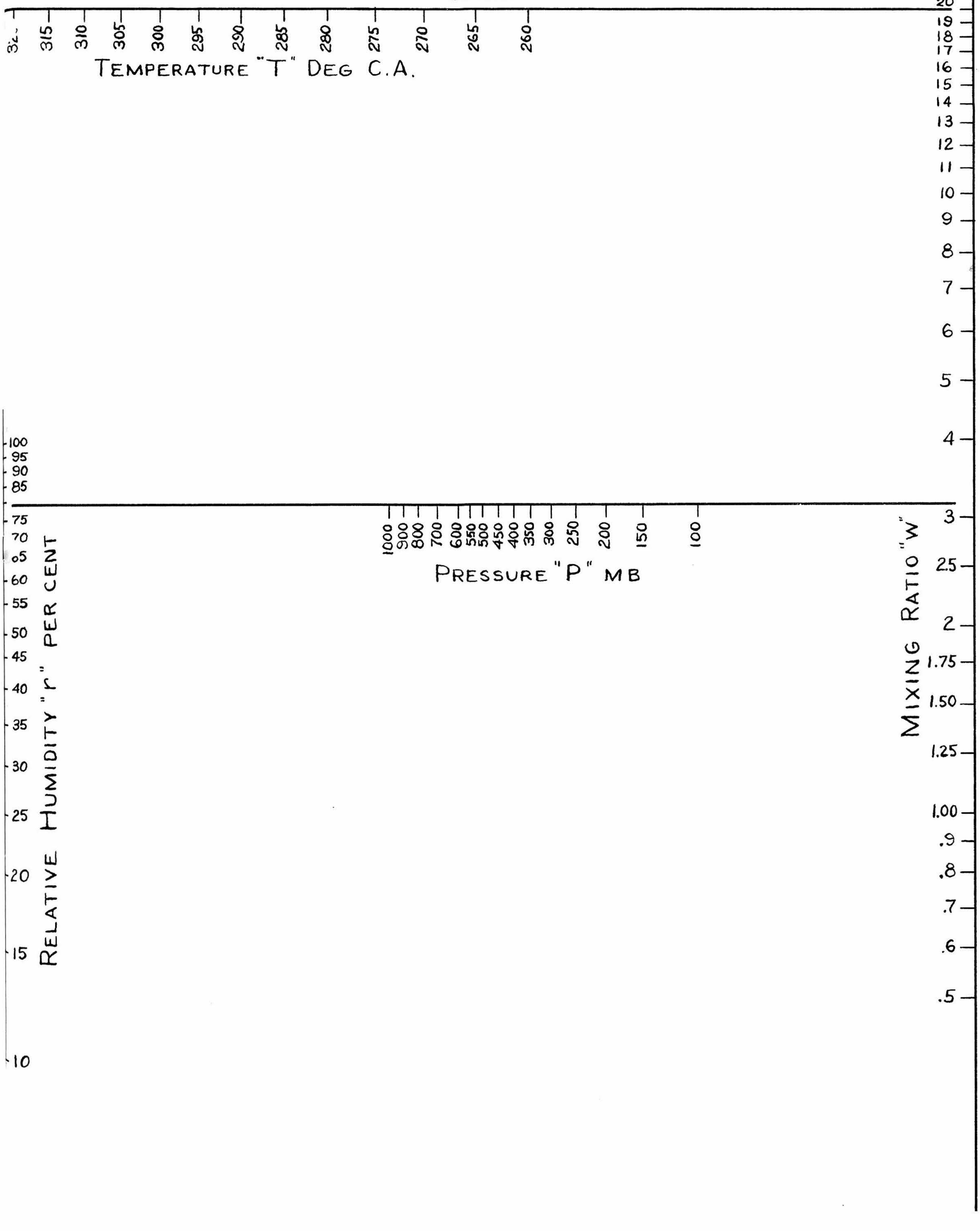
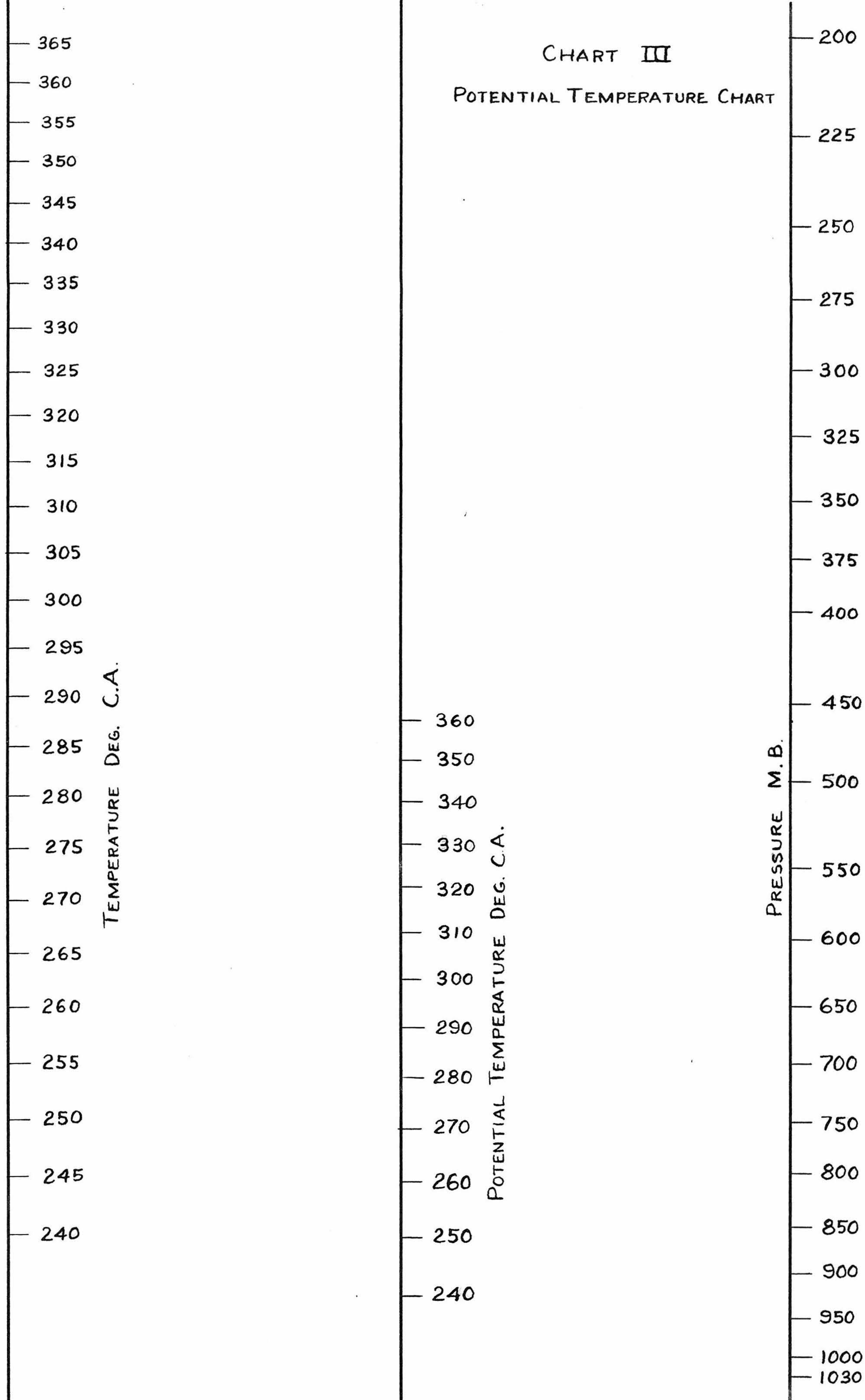


Chart III

Potential Temperature Chart



Θ_d Chart

Double Set

Chart IV

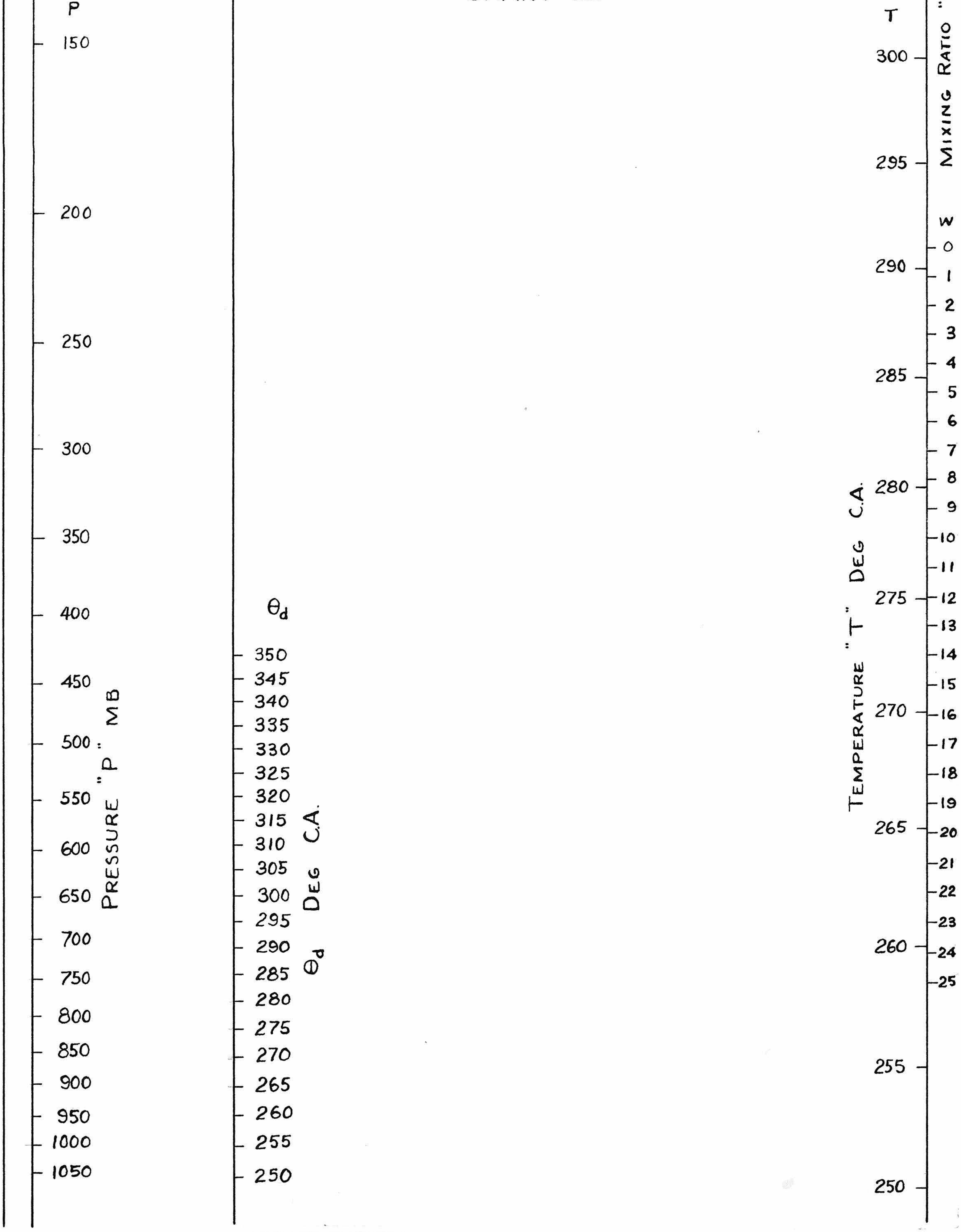
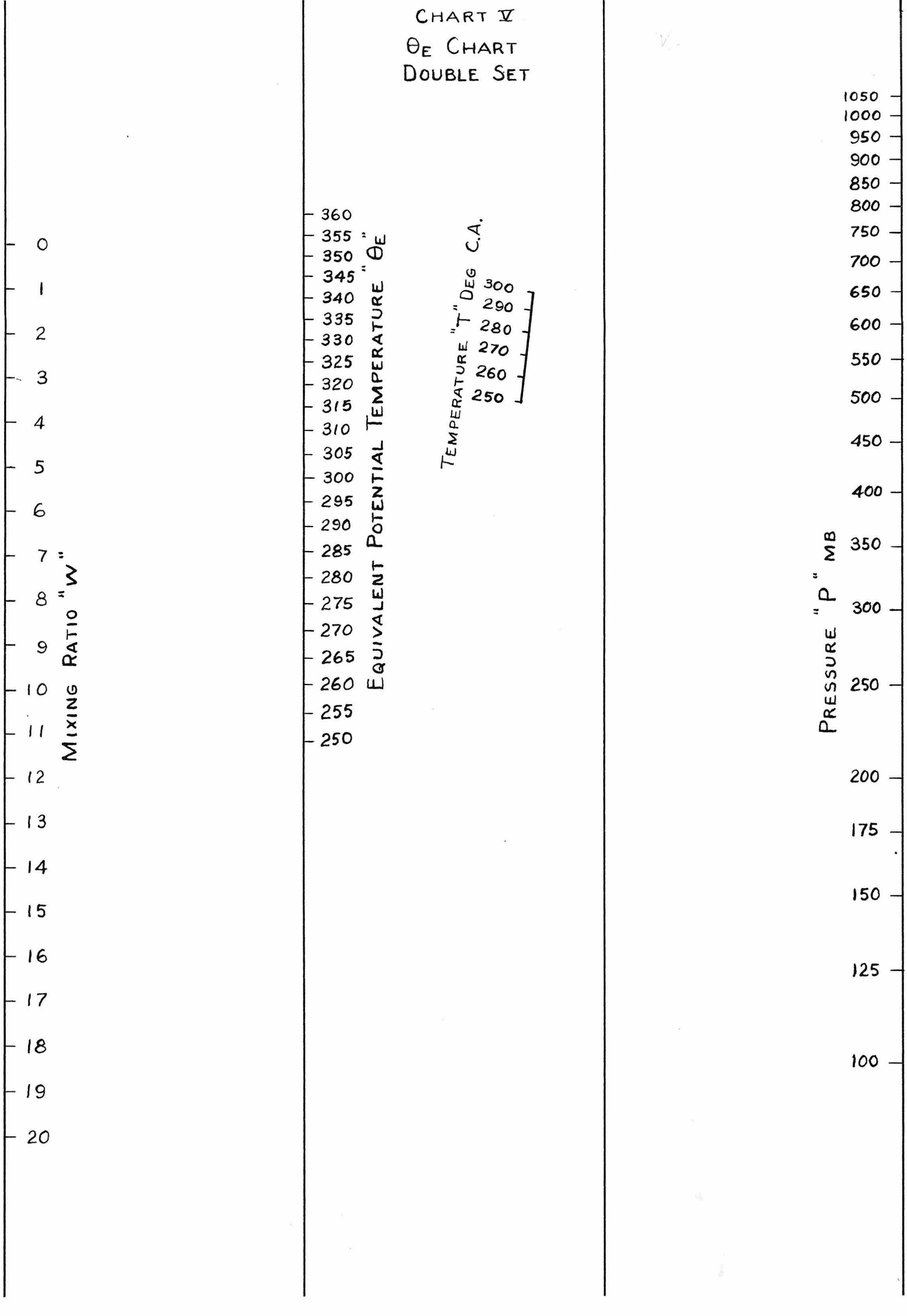
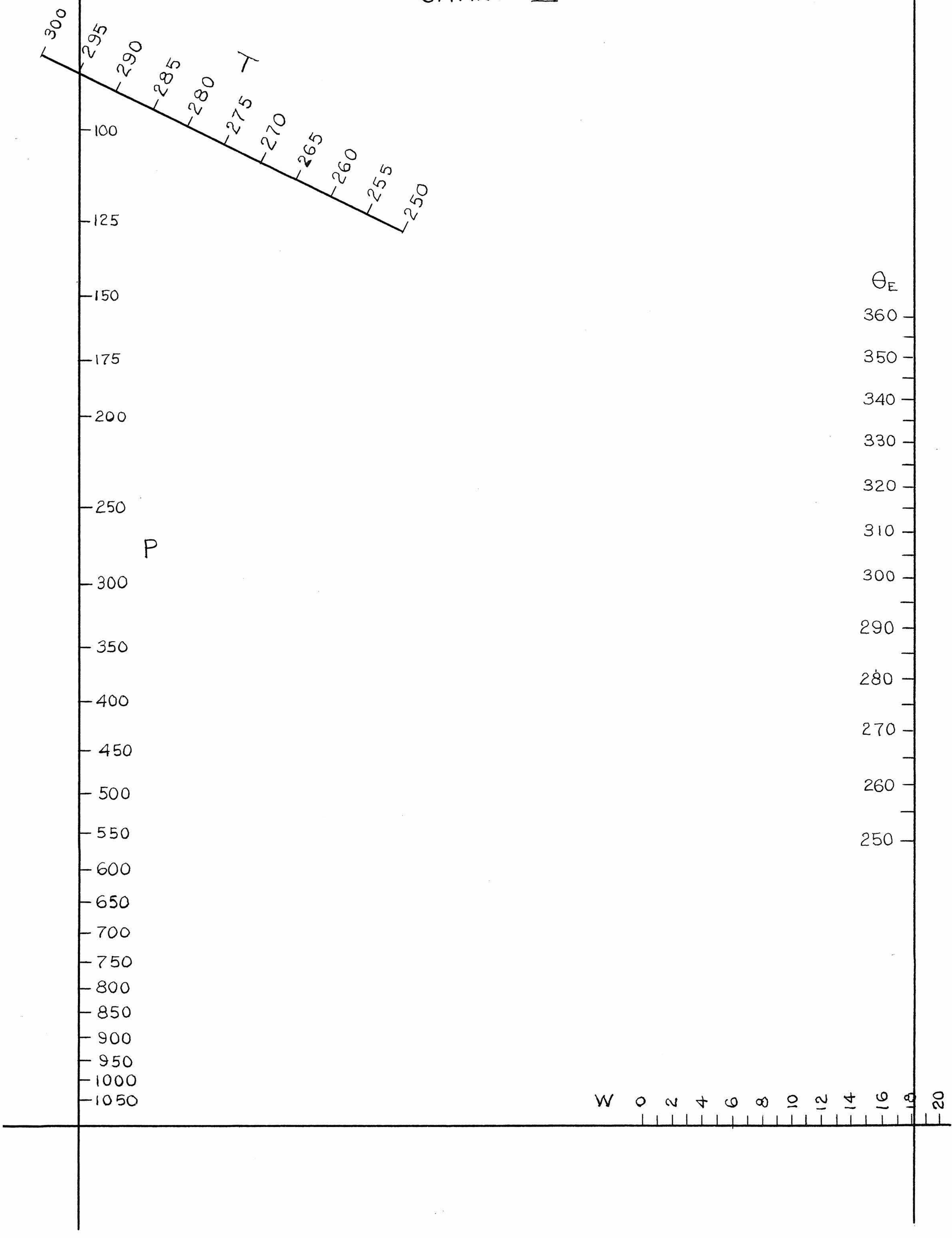


Chart V
Θ_E Chart
Double Set



Θ$_E$ Chart — Set Square

Chart VI



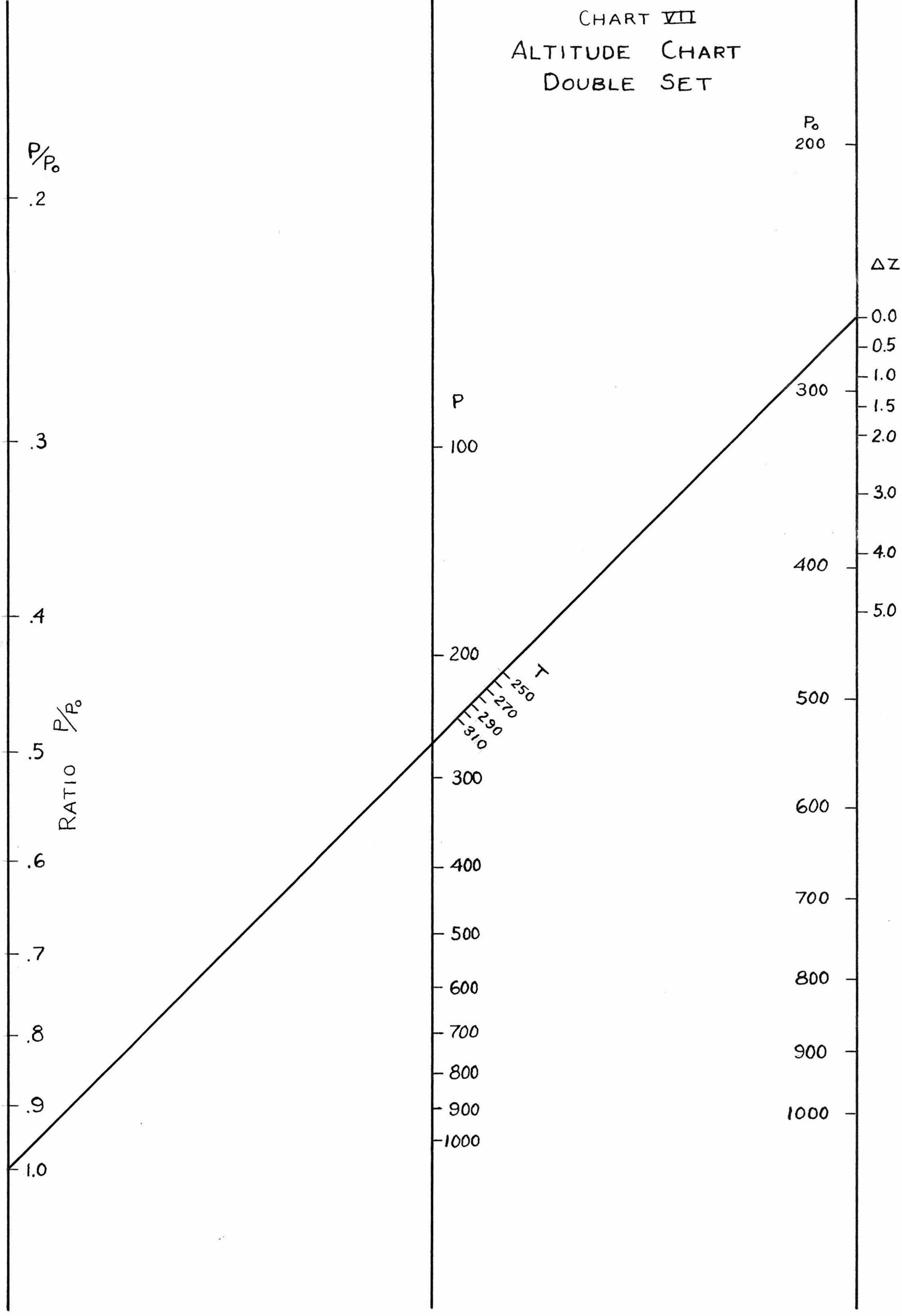


CHART VII
ALTITUDE CHART
DOUBLE SET
P/Po
.2
.3
.4
.5
.6
.7
.8
.9
1.0
RATIO P/Po
P
100
200
300
400
500
600
700
800
900
1000
T
250
270
290
310
Po
200
300
400
500
600
700
800
900
1000
ΔZ
0.0
0.5
1.0
1.5
2.0
3.0
4.0
5.0

ALTITUDE CHART
CHART VIII

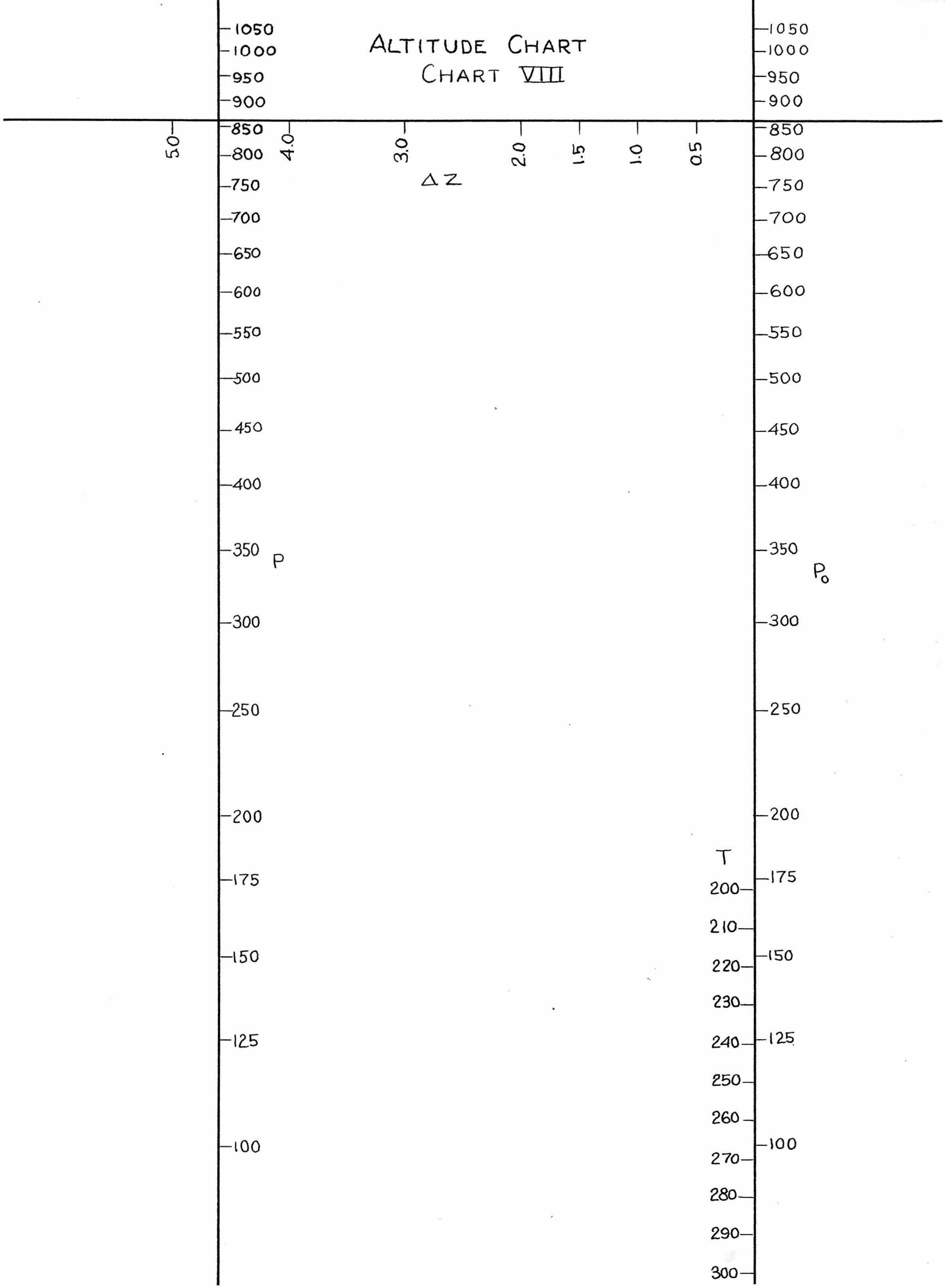
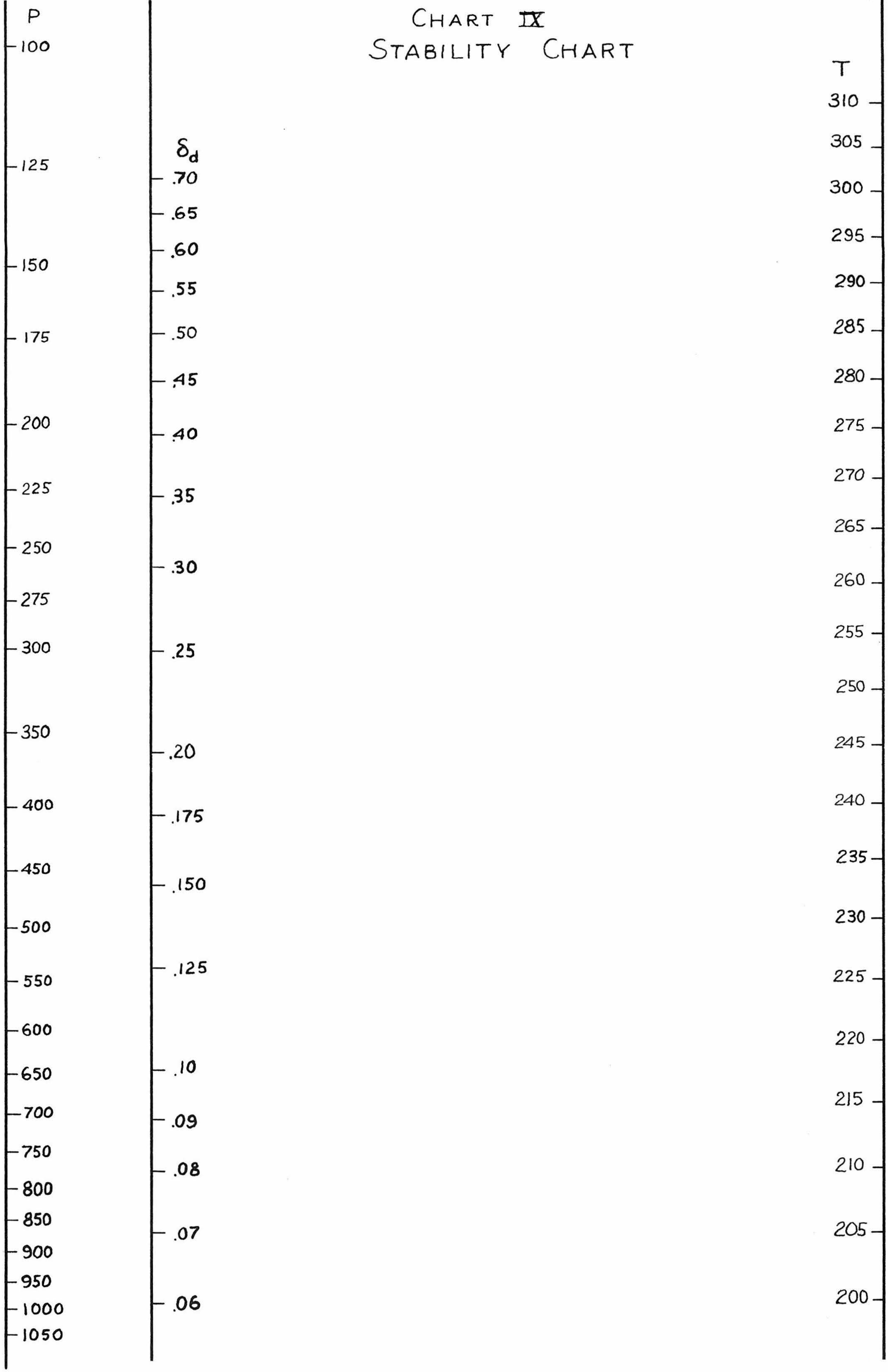


Chart IX
Stability Chart



Master Nomogram

Double-Set

CHART X

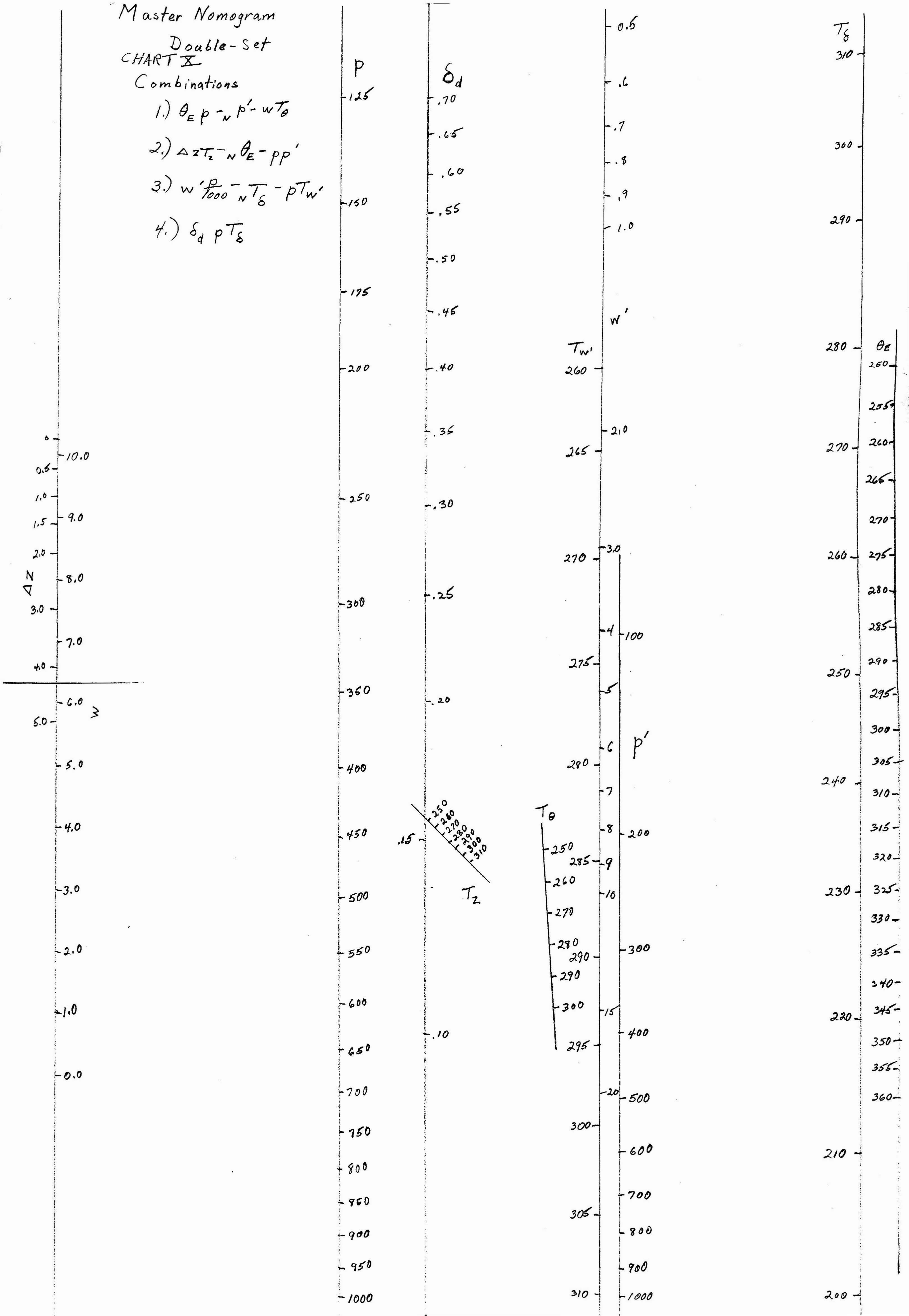
Combinations

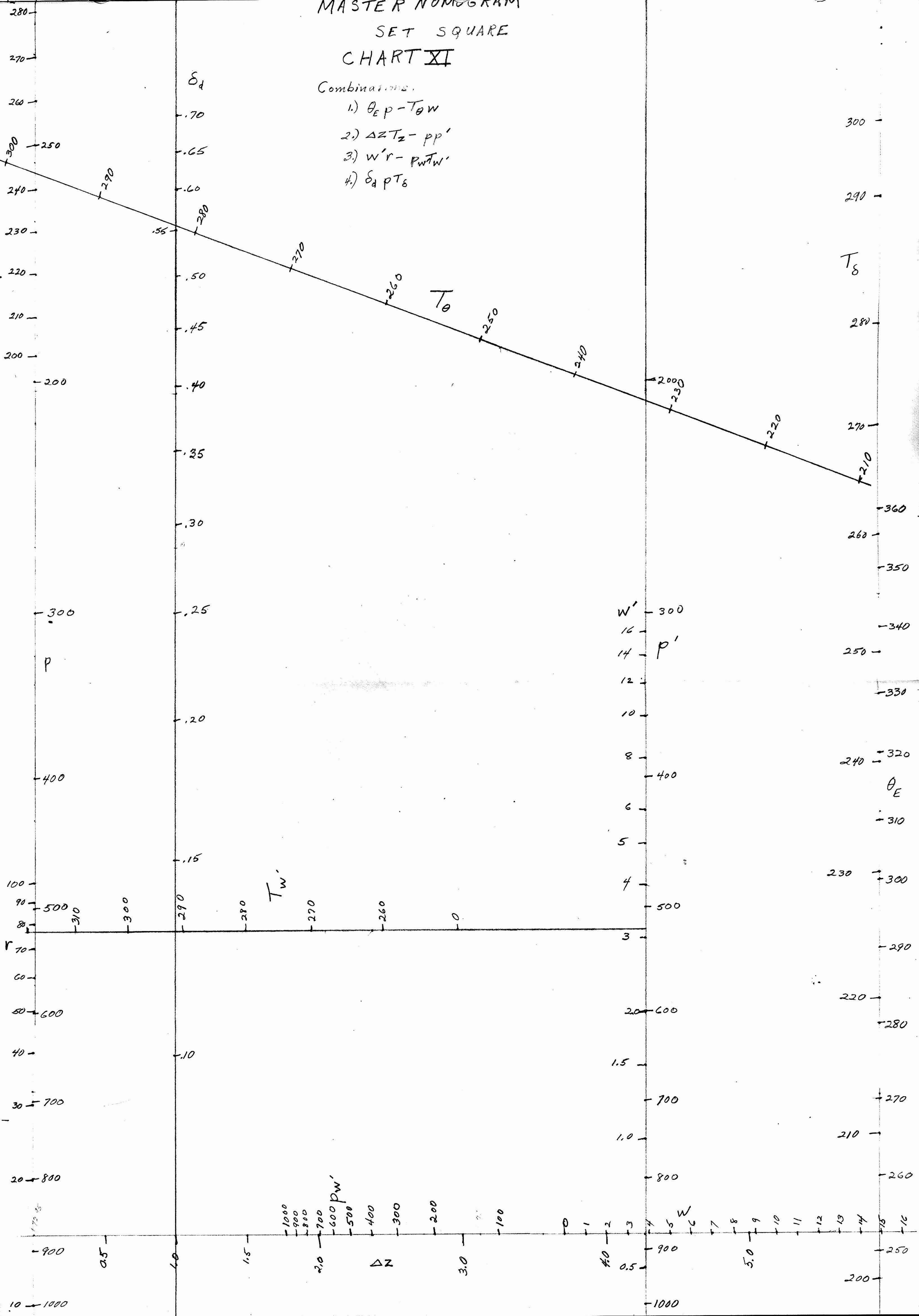
1.) $\theta_E\ p\ -_N\ p' - w T_\theta$

2.) $\Delta z T_z -_N \theta_E - p p'$

3.) $w' \frac{p}{1000} -_N T_\delta - p T_{w'}$

4.) $\delta_d\ p T_\delta$




MASTER NOMOGRAM
SET SQUARE
CHART XI
Combinations:
1.) θ_E p - T_θ w
2.) Δz T_z - pp'
3.) w'r - $p_w T_{w'}$
4.) δ_d p T_δ
δ_d
T_θ
T_δ
θ_E
p
p'
w'
r
$T_{w'}$
$p_{w'}$
W
Δz