A STUDY OF THE SHORT WAVE-LENGTH LIMIT OF THE CONTINUOUS X-RAY SPECTRUM

Thesis by

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ABSTRACT

Because of the troublesome discrepancy among the natural atomic constants $\underline{e}$, $\underline{h}$ and $\underline{m}_o$ which has existed since 1935, a precision measurement of $\underline{h}/\underline{e}$ by the short wave-length limit of the continuous x-ray spectrum is undertaken. The method used is the customary one of observing an isochromat, but several changes over previous practice are made in the apparatus and the experimental technique, and these are described in detail. Chief among these are: (a) an improved filament supply circuit and an electronic robot for stabilizing the voltage, (b) an elaborately calibrated precision resistance for measuring the voltage, (c) improved sensitivity in the ion chamber measurements of intensity, (d) the use of freshly cleaved crystals in the two-crystal monochromator to improve its resolving power and thereby reduce the fillet at the foot of the isochromat (made possible with the high intensity available from the Watters Memorial power supply and x-ray tube), (e) the use of Ross balanced filters to suppress the effect of non-coherently scattered radiation and to eliminate the integrated effect of the wings of the spectral transmission "window" of the monochromator, (f) an indirect cathode to preclude the existence of excess energy secondary electrons, and (g) a very clean target of high atomic number to insure a good intercept on the isochromat. The calibration of the voltmeter and of the spectrometer are not yet in a satisfactory state, but preliminary results give $\underline{h}/\underline{e} = (1.3805_4 \pm .0011_5) \times 10^{-17}$ erg sec/esu. This is to be compared with the indirect value (obtained from combining the Rydberg constant, the x-ray value of $\underline{e}$ and the best value of $\underline{e}/\underline{m}_o$) of $(1.37929 \pm .00023) \times 10^{-17}$ erg sec/esu. The overlapping of these probable errors demonstrates the reso-

lution of the discrepancy. The experiment is quite capable of five times the stated precision after certain checks are made, and better results are expected soon.

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I INTRODUCTION

The ratio of Planck's constant of action to the electrical charge on the electron h/e has been determined by a great many investigators using several different methods. The most accurate method is by a measurement of the short wave-length limit of the continuous x-ray spectrum, which yields h/e directly from the energy relation. This was first done by Duane and Hunt[1], and the work was continued for several years in the same laboratory[2]. More recent and accurate work has been done by Kirkpatrick and Ross[3], Schaitberger[4] and DuMond and Bollman[5]. The inverse phenomenon of the photoelectric effect similarly gives a direct value of h/e, though the results are less accurate and the theory necessary to interpret the results is far more complicated and in a less satisfactory state. The most accurate work has been done by Millikan[6], Lukirsky and Prilezaev[7] and Olpin[8]. Another direct determination of h/e comes from the measurement of excitation and ionization potentials. The best work has been done by Lawrence[9], Van Atta[10], Löhner[11] and Whiddington and his co-workers[12]. The constant c_2 in Wien's displacement law, when combined with the velocity of light, the Faraday and the gas constant, also yields a value of h/e. The most precise determinations of c_2 are the optical pyrometrical ones by Wensel[13]. The results of five other experiments (electron diffraction with measured voltage, electron diffraction with measured velocity, the Compton effect, x-ray photoelectrons and any measurement of e/m_o) can be combined in pairs to

determine h/e; and the results of three more experiments (Stefan-Boltzmann constant, fine structure constant, the Rydberg constant equated to Bohr's formula) can be used if the direct value of e is also employed in the calculations.

Unfortunately the results of all these experiments do not agree, and the probable errors fail to overlap by large amounts in some cases. In the worst case two results differ by ten times the mean probable error, as follows. The Bohr formula gives, with e always in abs esu,

$$R_\infty = \frac{2\pi^2 m e^4}{ch^3} = \frac{2\pi^2 e^2}{c(h/e)^3(e/m)}.$$

Using Birge's latest values[14] for the four physical quantities involved, and solving for h/e, one obtains

$$h/e = (1.37929 \pm .00023) \times 10^{-17} \text{ erg sec/esu.}$$

The best experimental value[15] is, on the other hand,

$$h/e = (1.37646 \pm .00029) \times 10^{-17} \text{ erg sec/esu.}$$

These differ by 0.00283×10^{-17} erg sec/esu. Stating the problem more generally, without this "unsymmetrical" reducing of all possible experimental results to a value of h/e, the theoretical relationships which are believed to exist between these various experimental results fail to obtain within the limits computed from the respective probable errors of the individual experiments.

This fundamental and very annoying discrepancy among the values of the natural constants e, h and m_o was first pointed out by Birge[16], and has since been studied in great detail with various methods of approach and various means of presenting the situation proposed. Birge[17] has continued to publish in this field, using chiefly as a means of representation the Birge-Bond diagram[18] but recently developing general formulas for the various types of consistency charts. He has suggested

possible causes for the discrepancy, but has advocated from the start further study of h/e as the field most likely to clear up the situation. DuMond has frequently discussed[19] the situation and has invented[20] an isometric consistency chart which is believed by many to be the most satisfactory way of (a) exhibiting the experimental results on a single diagram, (b) showing clearly the degree of interconsistency of these results, (c) indicating the effect of changes in single experiments or in single points of theory, and (d) maintaining symmetry among e, h and m_o. (The Birge-Bond diagram fails in the last two of these objectives, though the last is not necessarily of importance.) The isometric chart is reproduced in Fig. 1, where all the significant experimental results preceding the present work are shown, after DuMond, with the addition of the excellent value* of h/e obtained by Ohlin[21]. On this diagram any set of experiments is consistent if the lines representing these experiments intersect in a common point. Thus the consistency situation (or lack of it) can be seen at a glance. Similar charts have been devised independently by Beth[22] and by Darwin[23]. Dunnington[24] has given an excellent review of the situation and has analyzed it using the Birge-Bond diagram and also an elaborate least squares solution. Other analyses have been made by Ladenburg[25], von Friesen[26] and Kirchner[27].

All of these authors agree that, from considerations of the situation as a whole, the most suspicious experiment (that is, the single experiment whose omission leaves that set of experiments which has a greater interconsistency than any other such set) is the direct measurement of h/e, and that if, by the discovery of some hidden systematic

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*--The value plotted is his first result, namely $h/e = 1.3777 \times 10^{-17}$. His latest value is still better, namely $1.379_1 \times 10^{-17}$.

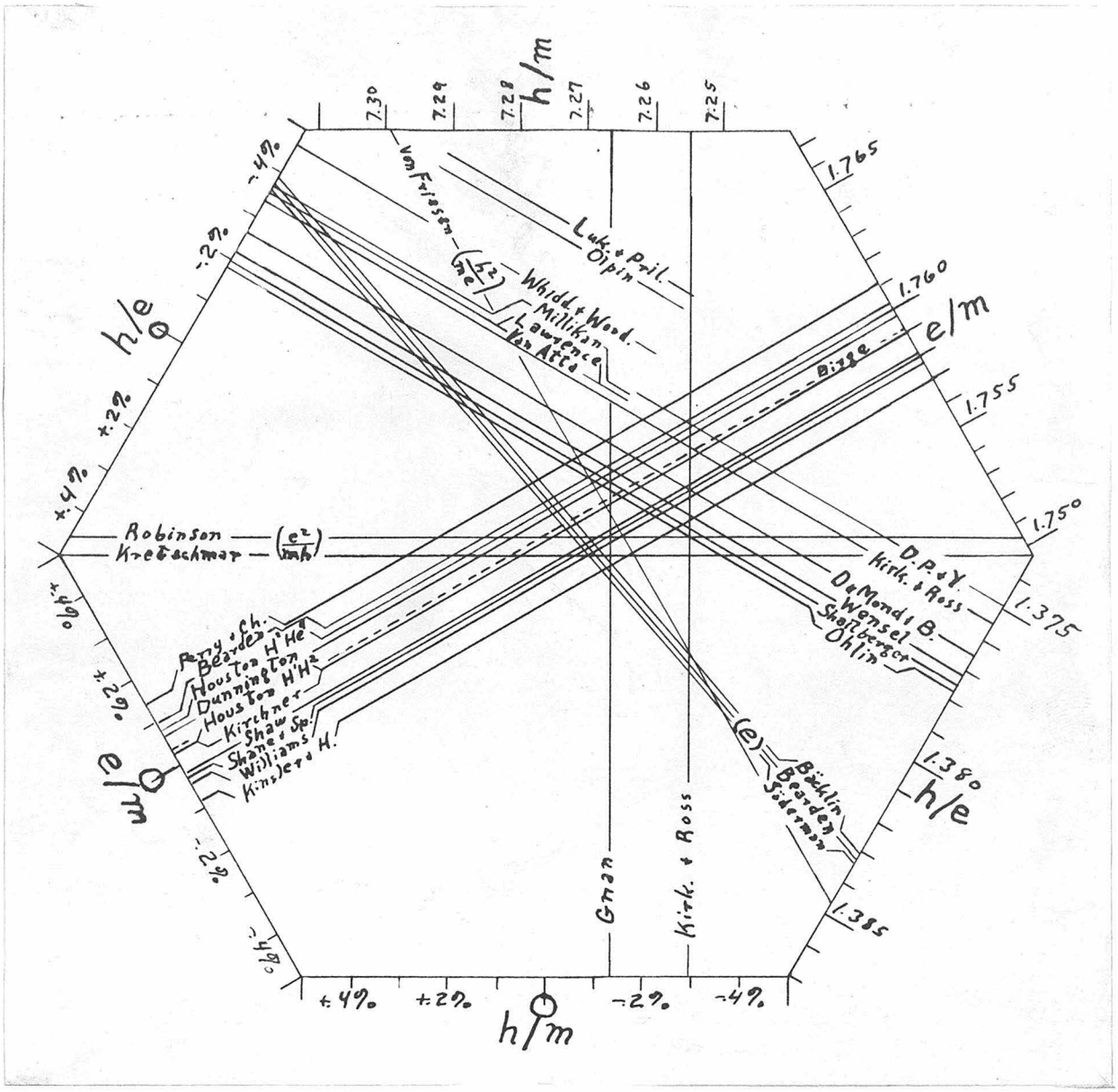

Fig. 1. A complete isometric consistency chart after DuMond, showing all significant experimental results preceeding the present work. The various functions are indicated though for simplicity the scales for some of these have been omitted.

errors or in any other way, the experimental value of h/e could be raised to a value consistent with the three most accurately known combinations of constants (ruled grating - x-ray value of e, specific charge e/m_o, and the Rydberg constant R_∞), then the discrepancy would be essentially resolved and the resulting values of e, h, and m_o could be assumed with full confidence. The only remaining problem in this field would be gradually to bring the less accurate experiments into consistency with these four. Actually now only the radiation constants c_2 and $\mathfrak{C}$ are inconsistent with these "good" experiments as can be seen on Fig. 1. Also, according to a statement by Birge at the Pasadena meetings of the American Physical Society, December 21, 1940, a least squares solution of all the recognized experiments except the three good ones and the two radiation constants yields exactly the same values of e, h and m_o as the solution for the three good experiments alone. Therefore, another precision determination of h/e is undertaken.

II EXPERIMENTAL PROCEDURE

The present experiment, like most precision determinations of h/e, consists of observing the short wave-length limit of the continuous x-ray spectrum by the method of isochromats. A very constant D.C. high-voltage which can be varied at will is applied to an x-ray tube. The radiation produced passes through a 2-crystal spectrometer into an ionization chamber, and the resulting current (suitably amplified) is observed as a function of the increasing voltage applied to the tube. With the spectrometer fixed in an anti-parallel position, thus defining the wave-length within very narrow limits, the voltage at which radiation just begins to get through the "monochromator" is V_{min} in the equation*

$$V_{min}\, e = h\nu = h\, c\, /\lambda \ .$$

The spectrometer is calibrated by means of the known wave-lengths of certain x-ray emission or absorption lines. Knowledge of the angular readings of the two crystal tables is then all that is required to determine the wave-length λ of the center of the band passed by the spectrometer. The value of c is of course known with great precision from other experiments. The voltage is measured by means of a specially constructed "volt-box" which gives a known very small fraction (about 1/200,000) of the voltage across the tube to an ordinary potentiometer, which in turn compares this voltage with a set of standard cells. Thus, with suitable corrections and in the proper units, the desired ratio is found to be

$$h/e = \lambda\, V_{min} \,/\, c \ .$$

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*--This equation is more often written $V\, e = h\, \nu_{max} = h\, c/\lambda_{min}$ where one thinks of the spectral distribution of the radiation coming from a tube with a fixed voltage across it, but the form given here is obviously more suitable for this experiment.

The many refinements of technique and the apparatus itself will now be described in detail.

The X-ray Tube and Pumping System

The x-ray tube and power supply used in the present experiment constitute the 30 kw continuous input high intensity x-ray equipment of the Watters Memorial Research Laboratory at the California Institute of Technology and have been previously described[28]. However, several changes and additions have been made since that article was written, and these will now be described with a brief statement of their purpose and their performance.

There were some large changes of great practical usefulness, though fundamentally inessential. For example, a partition was removed and the whole set was rotated through 90° and expanded so as to fill up the original room, with some of the accessories (spectrometer, voltmeter and voltage regulator) extending half way into the second room. The general arrangement is shown in Fig. 2. Bigger and better catch basins were installed for the two outdoor shower bath sprays which insulate the target cooling water from ground, the one for the inlet situated on the roof now being enclosed in a small pent-house which serves to keep both wind and passers-by from the shower. Three specially designed valves were installed in the vacuum system, one directly underneath the center of the x-ray tube between it and the liquid-air trap leading to the diffusion pump (see below), and one each in the lines leading to the second diffusion pump from the two rectifying tubes of the power supply. These valves are manually operated by means of a lever pivoted in a ball-and-socket joint which is surrounded by a vacuum-tight sylphon. The lever thus turns a stud inside the vacuum that forces a plate against a ring

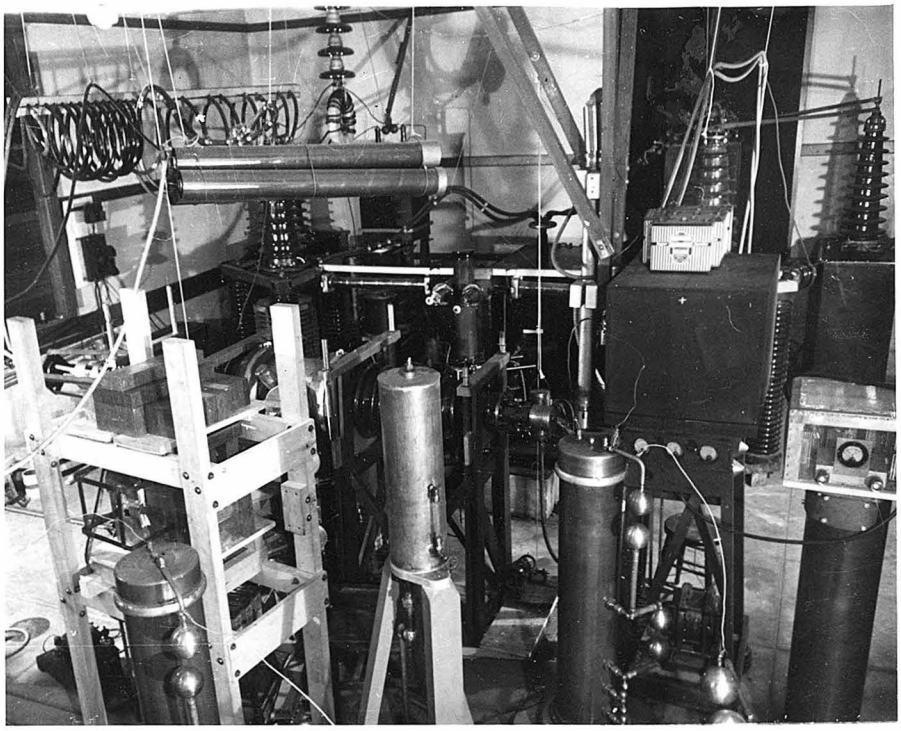

Fig. 2. General view of the Watters Memorial Research Laboratory, showing some of the accessory equipment used in the present experiment.

gasket. The valves can be opened wide enough to offer negligible resistance to the pumping, and proved very useful during periods of searching for leaks.

Other changes in the pumping system were more important. The 2-stage diffusion pump for the x-ray tube (3" and 6" diameters) was rebuilt, the design being changed in three respects. This pump now stands some 2 feet behind its former position immediately under the tube, being connected thereto through a liquid-air trap and one of the valves just described. It has the self-fractionating feature, first suggested by Hickman[29], by which the lighter fractions of the oil are automatically delivered to and retained in the higher pressure stage. This is accomplished (in both pumps, as a matter of fact) by means of: first, a groove cut into the exterior wall to catch most of the descending condensed oil in the second stage, whence it is led into the boiler of the first stage by a small pipe that terminates close to the free surface of the boiling oil; and second, another pipe connecting the bottoms of the two boilers through which the oil levels can be equalized. The pumping oil used now is "Octoil-s", also developed by Hickman and obtainable from Distillation Products, Incorporated, Rochester, New York, a subsidiary of Eastman Kodak Company.

The third change in the x-ray pump was part of a concerted effort to discourage hydrocarbon contamination of the target of the x-ray tube. The prevention of contamination of the target, especially by substances of low atomic number, is extremely important in the present experiment inasmuch as any such thin film would slow down the impinging electrons but would give rise to so low an intensity of x-rays (intensity proportional to Z) as probably to be unobservable. The x-rays emitted

by this thin film alone would have the limiting frequency sought at the short wave-length limit, and would give rise on an isochromat to a slight increase in slope at voltages below the main increase, which slight increase could hardly be distinguished from the natural fillet to be found at the "foot" of such curves due to other causes.

Our solution of this problem was as follows. The old charcoal trap was removed, though the lead ring-and-disk baffle below it was retained as an x-ray shield. Then first, the pump was moved further from the tube, as already indicated. Second, refrigerated baffles were installed in the top of the pump. These baffles consist of two sets of plates, an annular ring and a disk in each set, all of which are in good thermal contact with a pipe in the line of an old household G.E. methyl chloride refrigerator unit. The plates are kept between -3° and -7°C continuously, and have proved to be very effective in condensing practically all of the oil that diffused backward through the pumping nozzle, and this in a place whence the oil could return to the pump. Third, a liquid-air trap was installed between the baffles and the valve into the x-ray tube. After several designs were attempted, an inside trap consisting of a copper cylinder roughly 3" in diameter and 3" high supported by 4 inches of .005"-walled German-silver tubing was installed in a cylindrical cavity lined with bright Aluminum foil and allowing at least 1" clearance on all sides. The pumping speed of this whole system is quite ample, and its effectiveness was ascertained by actual examination after several weeks of operation. The baffle plates were found to be quite wet with oil. A slight dark oily deposit was found on the pump side of the liquid-air trap, but none on the tube side or on the walls leading to the tube. Scrapings were taken from the target itself and

analyzed spectroscopically by Hassler[30] who found only tantalum (evaporated from the filament) and estimated that this accounted for at least 70 per cent of the matter present.

Returning now to the x-ray tube itself, the target was replaced (due to a leak in the wall of the Shelby tubing stem!) by another of nearly identical design. More attention was paid to minimizing flow resistance next to the target (the back of the target is now conical) and to increasing the heat insulation (now a double wall) between the two concentric water lines in the stem. Also the soft-soldered vacuum joint between the stem proper and the steel sleeve carried by the journal in the eccentric carriage was changed to a more replaceable gasket joint clamped by a double nut. But the target proper is again a thin layer of gold formed on a copper base as before, and the whole has practically the same dimensions as its predecessor.

The Indirect Cathode

The cathode has been altered rather fundamentally. Primarily because of Ohlin's article[21], it was thought advisable to construct a cathode in which the filament would be shielded from direct positive ion bombardment and could be maintained at any desired potential with respect to the surrounding shield.[31] The latter was already possible since both filament leads were insulated from the shield system. The indirect cathode design is as follows, illustrated by Figs. 3 and 4. The leads and the shield system from the end of the tube as far forward as the rear of the cathode box are unchanged. Two long bolts, parallel to the axis and placed diametrally, hold the "anti-target" - a solid disk of $\frac{1}{4}$" steel at shield potential upon which most of the positive ions fall. Two special pieces of steel clamp with set screws onto the two electrical leads

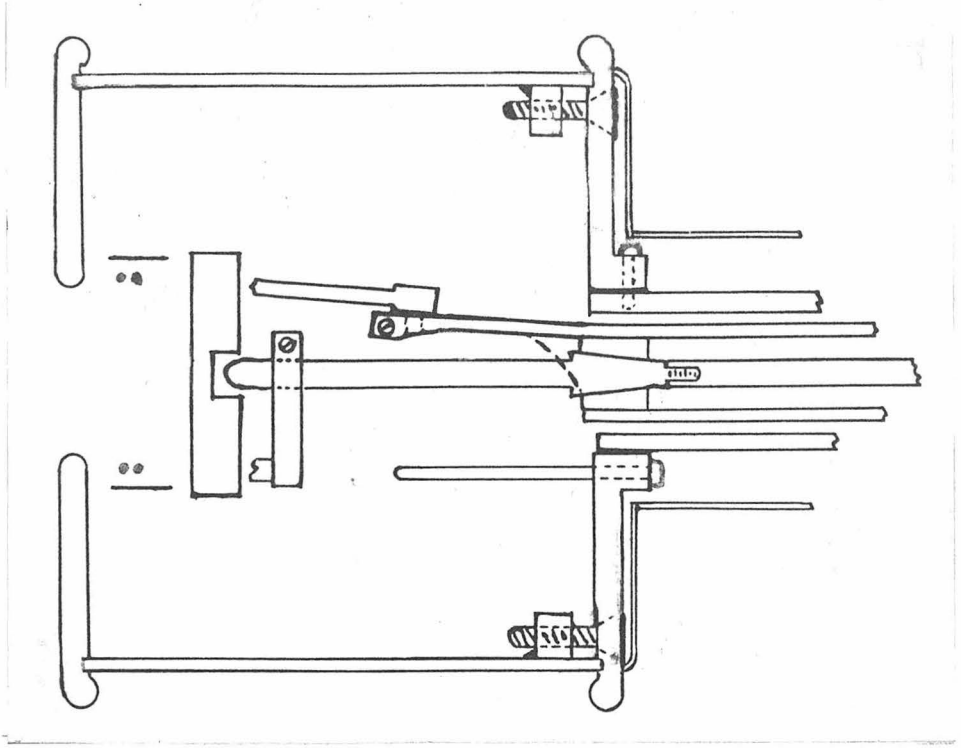

Fig. 3. Full-scale cross-sectional view of the indirect cathode. The section is a diametral plane, but azimuthal orientations are altered at will in the interest of clarity.

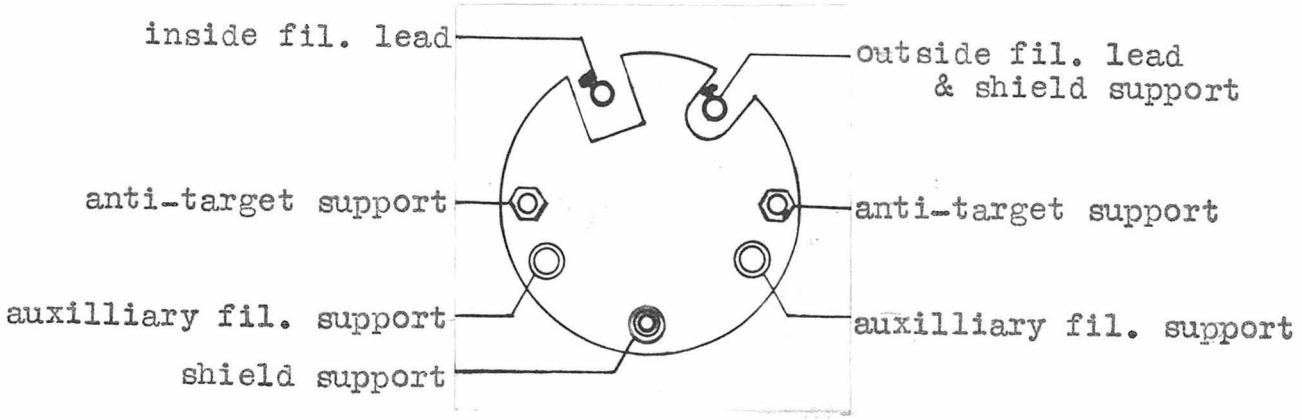

Fig. 4. The "anti-target", showing true orientation of the various parts.

and project forward through notches in the anti-target. Into these is mounted with set screws the filament itself, about 7" (6" effective for emission) of 40-mil tantalum wire as before, but now formed into nearly two complete turns with 1" diameter. The filament is also held in place by two small loops of steel wire mounted on insulated plugs in the anti-target. In addition there is a light cylindrical shield enclosing the filament, spot-welded together out of .006" nickel and held on by two ears, one of which is fastened to the negative terminal of the filament, and the other to a third insulated plug in the anti-target. These plugs are standard tapered plugs pressed into reamed holes with mica insulation, and fitted with tiny caps on the larger end (toward the anode) to shield the mica from collecting condensed electrically conductive films of metal. This whole outfit is surrounded by a welded steel box which has a 7/8" rounded hole in the front plate. The relative size of this hole and the filament circle, and the various spacings involved, were determined solely by intuition. As a matter of fact, they are far from critical since the electric field immediately outside the hole is strong enough (of the order of 10,000 volts/cm) to "reach" inside around the corner quite easily and to produce essentially straight line trajectories from the hole over to the anode. (The focal spot was observed flouroscopically to be roughly 2 cm in diameter.) The assembly and disassembly of this cathode is a fairly complicated procedure, but complete directions are available.

With the original normal cathode, the tube had the nearly "flat" V-I characteristic of a saturated diode; but with the indirect cathode the current is space-charge limited and the characteristic is linear as with most triodes. This, of course, means that the stability of the set

as a whole is less dependent on filament current fluctuations than formerly but more sensitive to fluctuations in applied emf. The net result has been a marked increase in stability and ease of operation. This cathode, by the very prevention of positive ion bombardment of the filament for which it was designed, should also increase the life of the latter and conversely it should serve to hinder the evaporation of tantalum onto the target. As yet no tests have been made of the effect, if any, of the biasing voltage on the foot of the isochromats, all curves having been run with the shield at -45v with respect to the filament. However, Ohlin's criticisms[32] of the usual conduct of this experiment are met in two ways. In the first place, the pressure in our tube during operation is observed continuously with a Knudsen gauge (new model identical with the previous one[33] except that the damping is supplied with a permanent magnet) and was 10^{-5} mm of mercury. In the second place, all but an exceedingly minute fraction of the positive ions striking the cathode must fall on surfaces whose potential is over 40v more positive than the center of the filament. Now it has been shown[34] that, while a very few secondary electrons emitted by positive ion bombardment of metal surfaces may have energies up to 1000 electron-volts, less than 10% of all the electrons have energies over 10 ev and "the great majority" have an energy less than 30 ev. It therefore seems exceedingly unlikely that enough secondary electrons can exist in our tube with energies greater than that possessed by the electrons coming direct from the filament to produce an observable shift in the isochromat.

The Filament Supply Circuit

The 30 amp filament supply circuit has been completely rebuilt with three aims in view: the elimination of contact fluctuations, the

elimination of drift due to heating of the resistances and the attainment of maximum stability against the fluctuations of the generator output. The conditions for the latter are easily obtained from consideration of the simplified circuit with constant load illustrated in Fig. 5. Taking circulating currents in the most significant manner, as indicated, Kirchoff's law gives

$$\begin{cases} i_1(L + R) + i_2R = V \\ i_1R + i_2(R + r) = V-E . \end{cases}$$

The solution of these equations is

$$i_1 = (ER + Vr)/(rL + rR + LR).$$

From this it immediately follows that

$$di_1/dV = 1/(L + R + LR/r)$$

so that $\underline{R}$ must be large and $\underline{r}$ as small as possible. The importance of the stability of the filament current cannot be overestimated, even with the indirect cathode, inasmuch as the heat dissipated in the filament varies as i^2 and the emission goes up as T^4. With the circuit as finally constructed (see Fig. 6) and about to be described, a potentiometer across the filament leads to the x-ray tube indicated random fluctuations of 1 part in 100,000 and no drift in steady operation when the charging current was properly adjusted.

The generators for the filament supply are from the U. S. Army Air Service and are rated at 13v and 30 amp each. They are mounted on the same redwood stand as the previous automobile generators and they are driven by 72" V belts from the same 1 H.P. induction motor, but all else is new save a couple of meters. The field circuits of the generators are brought out to two small panel rheostats which have been calibrated to provide a slight charging current for any given filament cur-

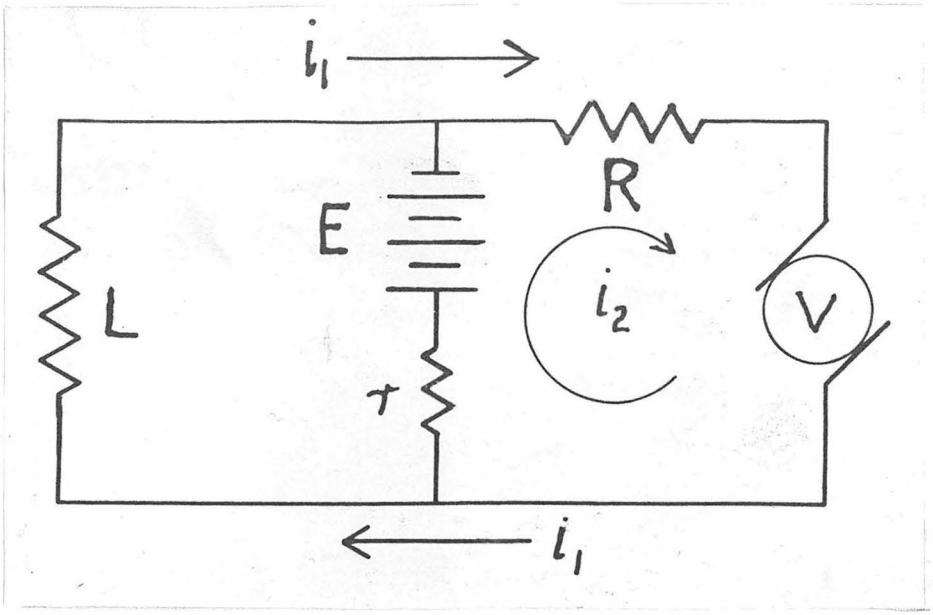

Fig. 5. Simplified circuit for deriving stability conditions for the filament supply.

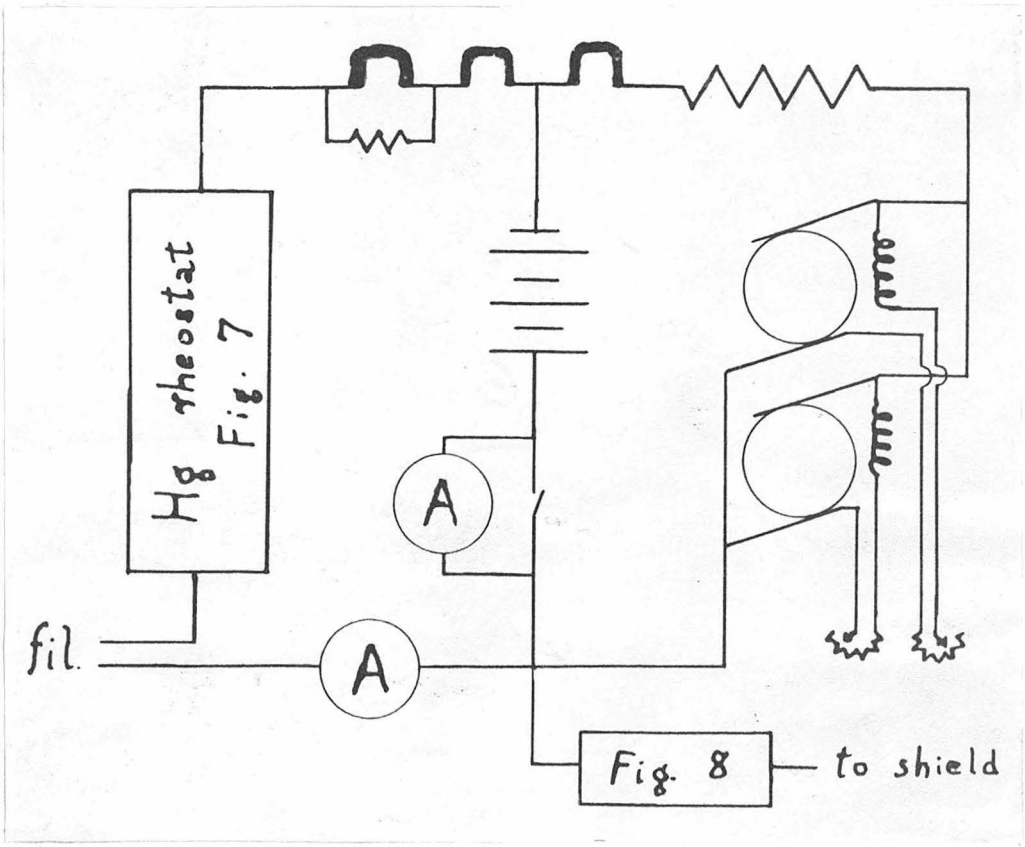

Fig. 6. Schematic diagram of the filament supply circuit.

rent. The overload and constant-voltage relays were removed. A special 100-watt resistor was constructed and arbitrarily added to the generator circuit (corresponding to R of Fig. 5). This resistor consists of 4 spirals of #16 Nichrome in parallel, making approximately 1/8 ohm, immersed in transformer oil to provide electrical insulation and at the same time transfer of heat to the outside, and surrounded by a water jacket. The electrical leads emerge through a bakelite bushing in the top, the remainder of the construction being of brass soft-soldered together. The water jacket is in series with that of the mercury rheostat (to be described presently) and the line from the adjacent rectifying valve which supplies the surge resistance in the last segment of the electrical circuit, so that no high voltage insulation is required.

The batteries are three 21KL Globe-Union glass jar cells (this manufacturer's largest size, rated 1000 amps) in series. They are connected together with two parallel strips of lead $\frac{1}{2}$" x 1/8" and to the rest of the circuit by means of 3/8" copper cable with appropriate lugs. This cable can be broken at one point for purposes of reading the charging current, but this is very seldom necessary after calibration of the generator field rheostats. The overall resistance of the whole battery shunt on the main circuit (corresponding to r of Fig. 5) is certainly less than .007 ohm. The main filament ammeter (permanently connected) is a 50 millivolt meter with a built-in 50 amp shunt.

The present mercury rheostat is the result of several attempts. No wire-wound affair could be made with sufficiently constant contacts nor could the heating be prevented with only air blasts (from fans on the generators). Vibrations made the former mercury-in-glass fine-control rheostat unsatisfactory. A design was tried involving a vertical

thin-walled German-silver tubing, filled with mercury to a variable height to determine the amount of the tubing in the circuit. But the mercury was found to slowly attack the German-silver, and no successful mercury vs. oil seal could be made. The present design is a modification of this last, and is depicted in Fig. 7. It consists of 40" of .008"-walled Monel tubing inside of a concentric 1/16"-walled copper return lead, all immersed in transformer oil and surrounded by a water jacket. The top of the oil chamber is merely plugged with a large cork. The copper tubing return lead is provided with several small holes to allow circulation of oil to and from the actual resistance. It is insulated from the Monel by three bakelite washers, and soldered thereto (through a copper ring) at the base. The bottom plate of the oil chamber is insulated by means of a bakelite bushing, press-fitted with bakelite cement and baked mildly in an oven, and is very easily made oil-tight against the side walls with a Koroseal gasket as shown. The mercury seal is also accomplished with a Koroseal gasket in the device illustrated, a steel sleeve being provided inside the Monel tubing to withstand the compressive force of the seal. The height of the mercury in the tube is controlled by means of a small reversing motor and pulley system which raises or lowers a pot of mercury connected with Koroseal tubing to the rheostat proper. The pot is simply a 200 cc glass flask with a short piece of tube in the bottom, resting in a felt-lined steel cylinder. The impossibility of open-circuiting this rheostat is obvious and essential.

All the wiring in this circuit is unusually heavy and all connections are either soldered or made with $\frac{1}{4}$" lugs. The three necessary switches consist of $\frac{1}{4}$" copper links dipping at least 3/4" into mercury

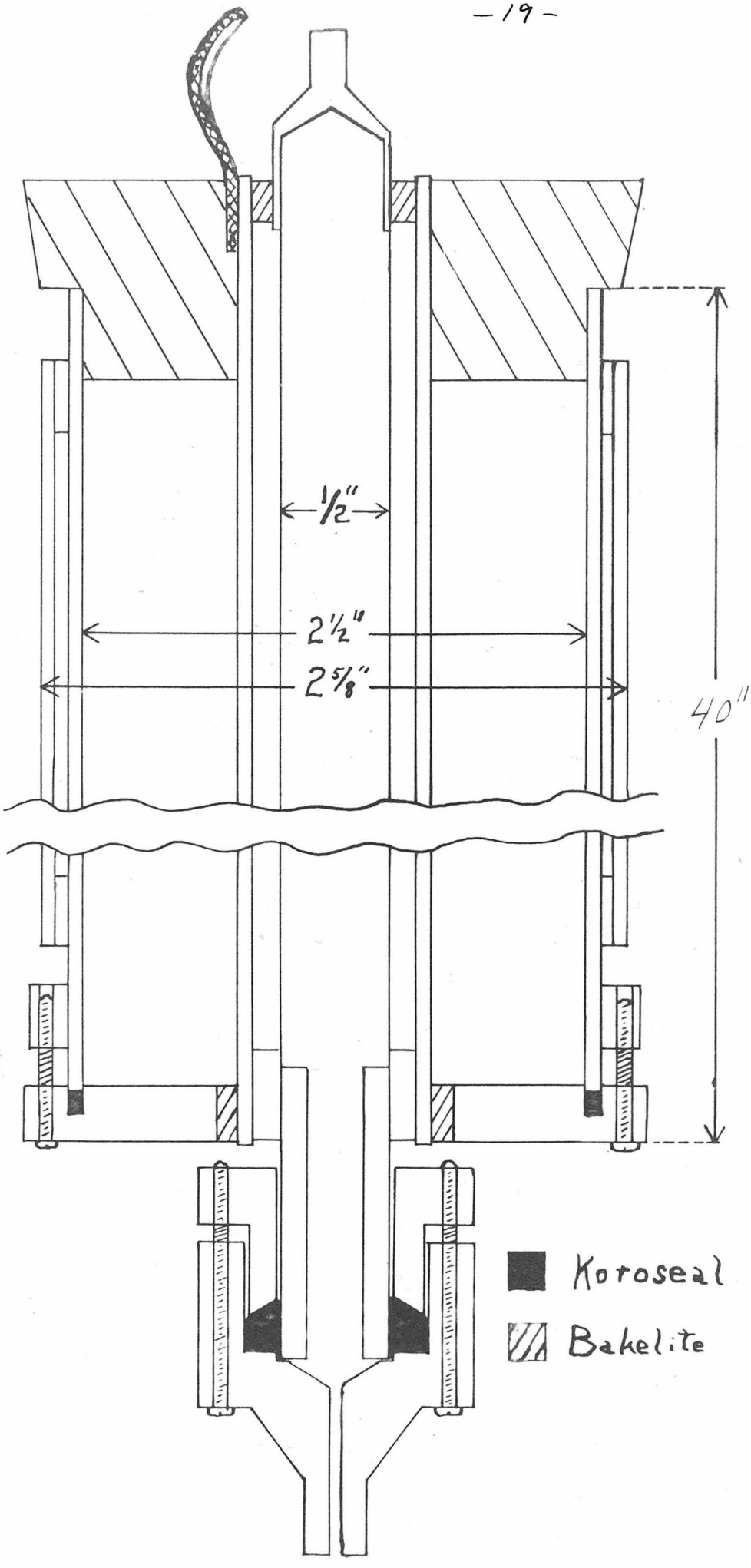

Fig. 7. Diagram of the mercury rheostat in the filament supply circuit. Scale is 3:2.

cups. Furthermore, the entire circuit is electrically shielded. Making use of the insulated frames on the generators, providing ordinary flexible shielding cable for all leads and the Koroseal tubing full of mercury and building a couple of galvanized-iron boxes for instruments was all easy, but the design of the generator resistance and the mercury rheostat was considerably complicated by this requirement of providing an electrical shield. The necessity for it comes from the desire (and necessity in this experiment) of measuring the actual current across the x-ray tube independent of any corona losses or leakage currents to ground.

The high-voltage lead of the main power supply comes to the shield, so that a meter, itself inside the shield, connected between the shield and the filament circuit will measure the desired current. An ordinary milliammeter, protected against possible surges, and a vacuum tube "current vernier" are provided as shown in Fig. 8. If the 1/8 amp fuse "blows", the cigarette paper gap will puncture and carry the current. In the "current vernier" the potential drop between A and B is balanced against a B battery, the difference being applied to the grid of an 1852 pentode. The plate current is read directly on a microammeter, and the overall current gain is approximately unity so that changes in the main current are read directly (but in the reverse sense). No protection is necessary as a sudden increase in current will merely drive the grid to cut-off.

The Main Power Supply

Aside from the voltage regulator, the changes to the main power supply have been very slight. The circuit diagram is reproduced in Fig. 9. The rectifying valves have proven quite satisfactory with one

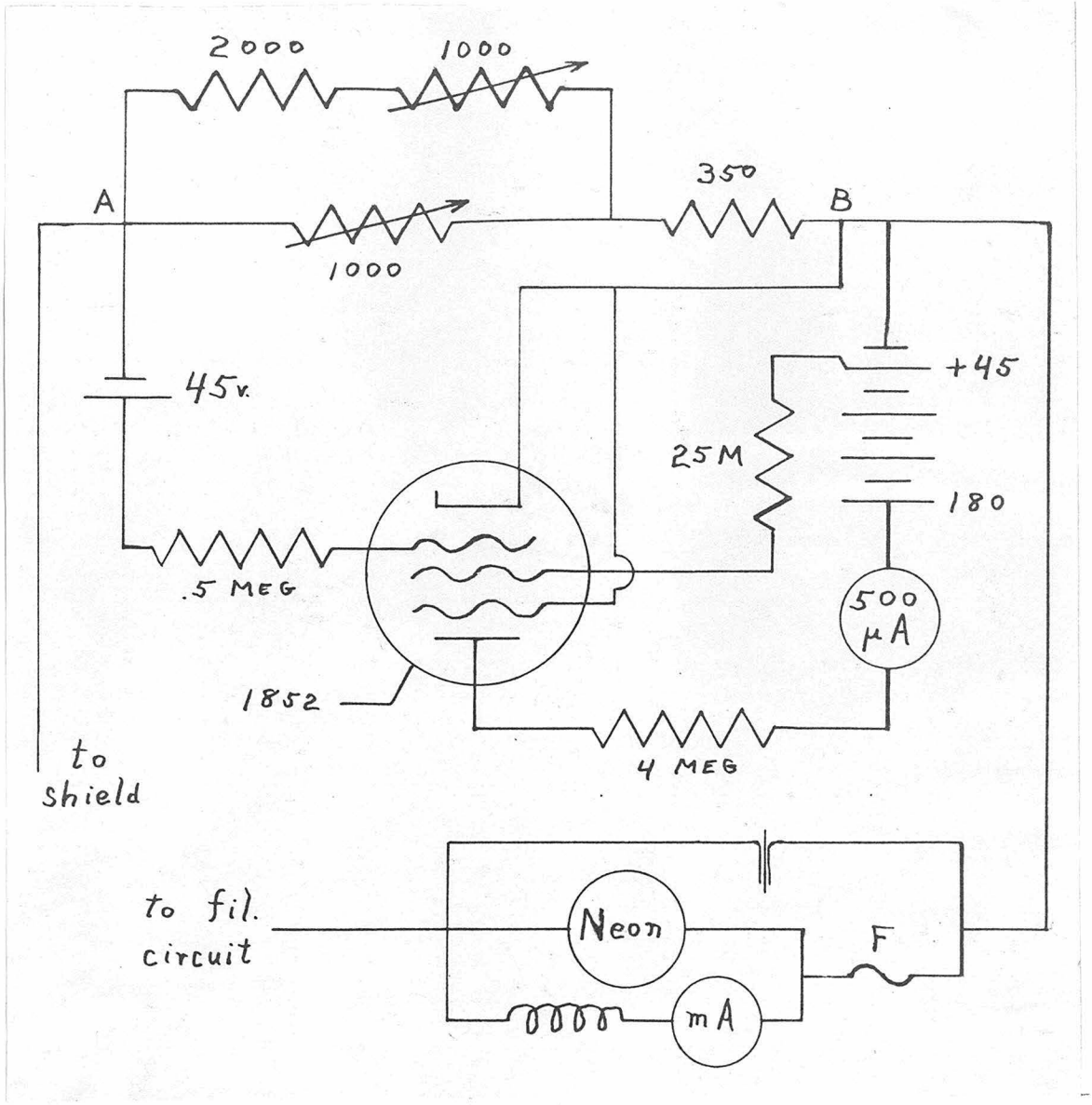

Fig. 8. Current measuring and safety devices between the filament circuit and its shield.

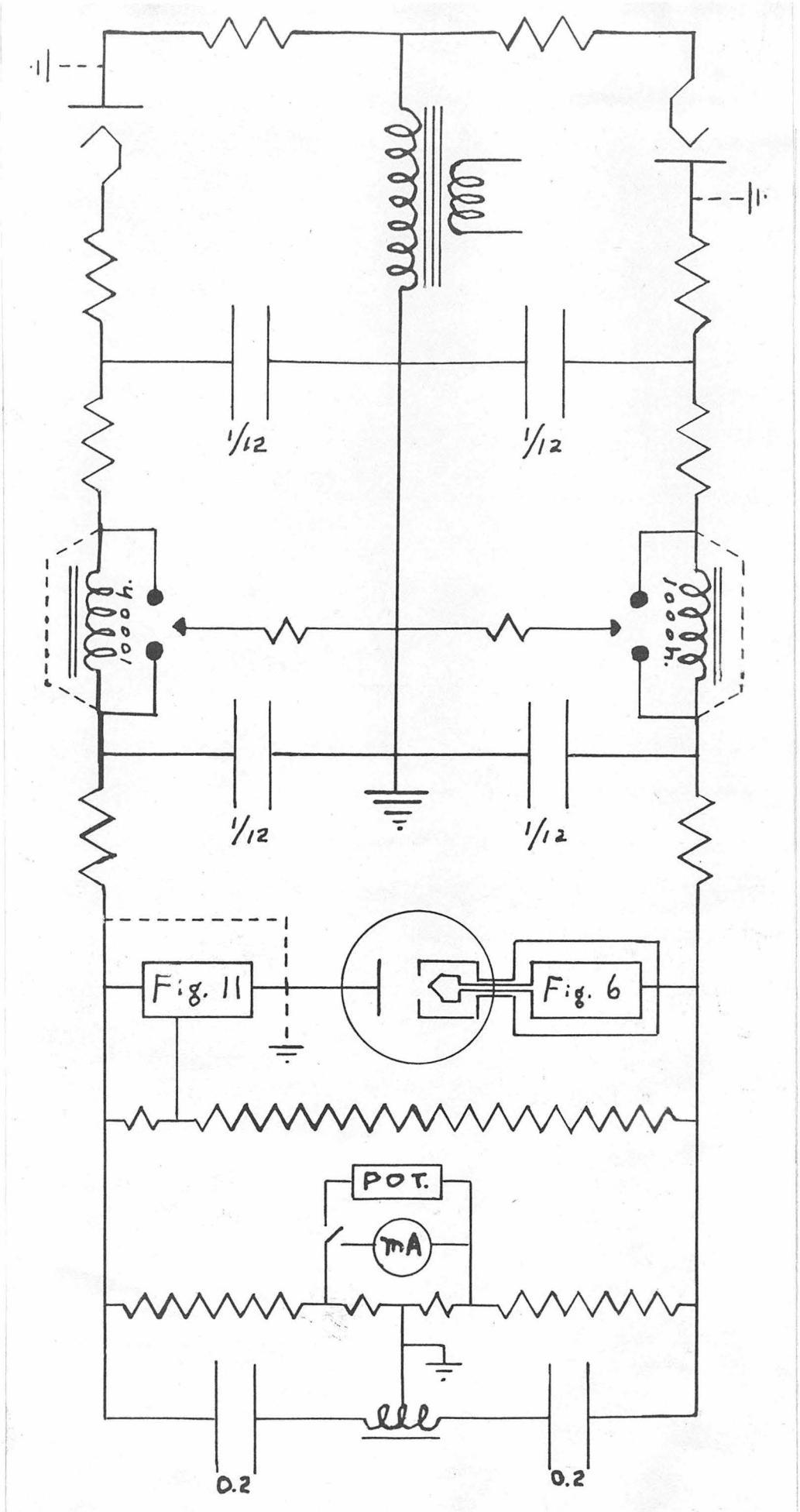

Fig. 9.
Circuit diagram of the high-voltage rectifying system. Dotted lines indicate high resistance water hose connections.

minor exception. An interesting source of high-frequency currents was traced to an arc to the plate of one of these valves, made possible by excessive stretching of the filament when hot. The weights holding these filaments in tension were easily reduced in both tubes. Another stage of condensers was rather accidentally added to the filter system. Two new G. E. pyranol condensers rated 0.2 μfd and 50 kv each (purchased for other purposes) were placed as blocking condensers in series with a grounded center-tapped transformer primarily for purposes of observing on an oscilloscope the ripple in the high voltage, and they have since been retained as part of the permanent circuit (for work under 100 kv). More adequate grounds were established throughout the high-voltage circuit and tied to the water system and to a specially sunk pipe in the ground.

The primary source of power, a 60 H.P. synchronous motor, is untouched. One pair of brushes was removed on the alternator to provide greater current density for the two remaining pairs. These and the contact rings were carefully cleaned with fine sandpaper in the orthodox manner, contributing considerably to the constancy of the voltage delivered to the high-tension transformer. Due to the increase in the primary power frequency from 50 to 60 cycles, thus increasing the voltage output of the alternator, an auto-transformer was installed to reduce the voltage going to the main transformer and restore the current to its former value.

A new set of power rheostats and resistances was installed to control the field current of the alternator, and hence the high voltage. Fixed resistances were designed to adapt available rheostats to give a coarse and two successive overlapping fine adjustments, as shown in

Fig. 10. They were made by winding various coils on "polar cub" units, and were mounted in air. With this combination the voltage has been kept constant within 300v (easily handled by the voltage regulator) for an hour at a time.

Several safety devices and relays have been incorporated into the set. Three of the mercury contact switches now commercially available for "silent" light switches were installed in series in a 110v A.C. circuit that operates a breaker in the line supplying the two insulation transformers for the rectifying valves. These require that the fore-pump (by means of the compressed-air operated valve[35] immediately in front of this pump), the outlet water from the two rectifying valves (by means of a can with a perforated bottom hung on a phosphor-bronze strip) and the target cooling water (by means of a similar can in the discharge line from the second catch basin, which is grounded) all must be running before the valve filaments can be turned on. There is also a 220v D.C. line that operates a large circuit breaker capable of interrupting the output of the alternator. In addition, there is another 110v A.C. relay, normally closed but opened by a single push-button near the controls, which opens the 220v line just described and also releases two over-head cords. This allows to fall (speed being controlled at the end by a simple spring device) two large copper strips on 3' arms which ground through G.E. surge resistors the sphere gaps protecting the two chokes in the filter system, and hence the condensers as well. Thus in case of any emergency, the power can be turned off and the whole set grounded quickly and harmlessly. Finally, there are across the 220v line a red light inside the room and a lighted "High Voltage" sign just outside the door which must be on whenever the power is on.

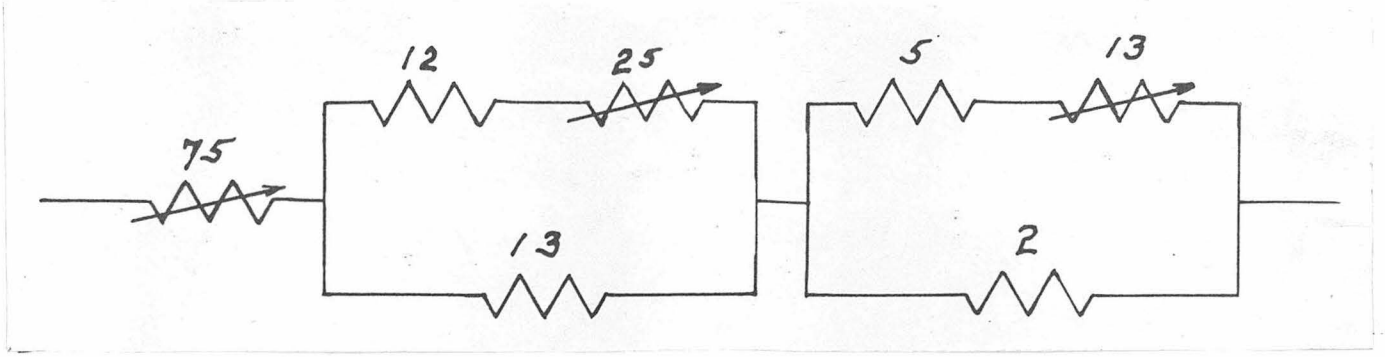

Fig. 10. Diagram of the fine control installation in the field circuit of the alternator.

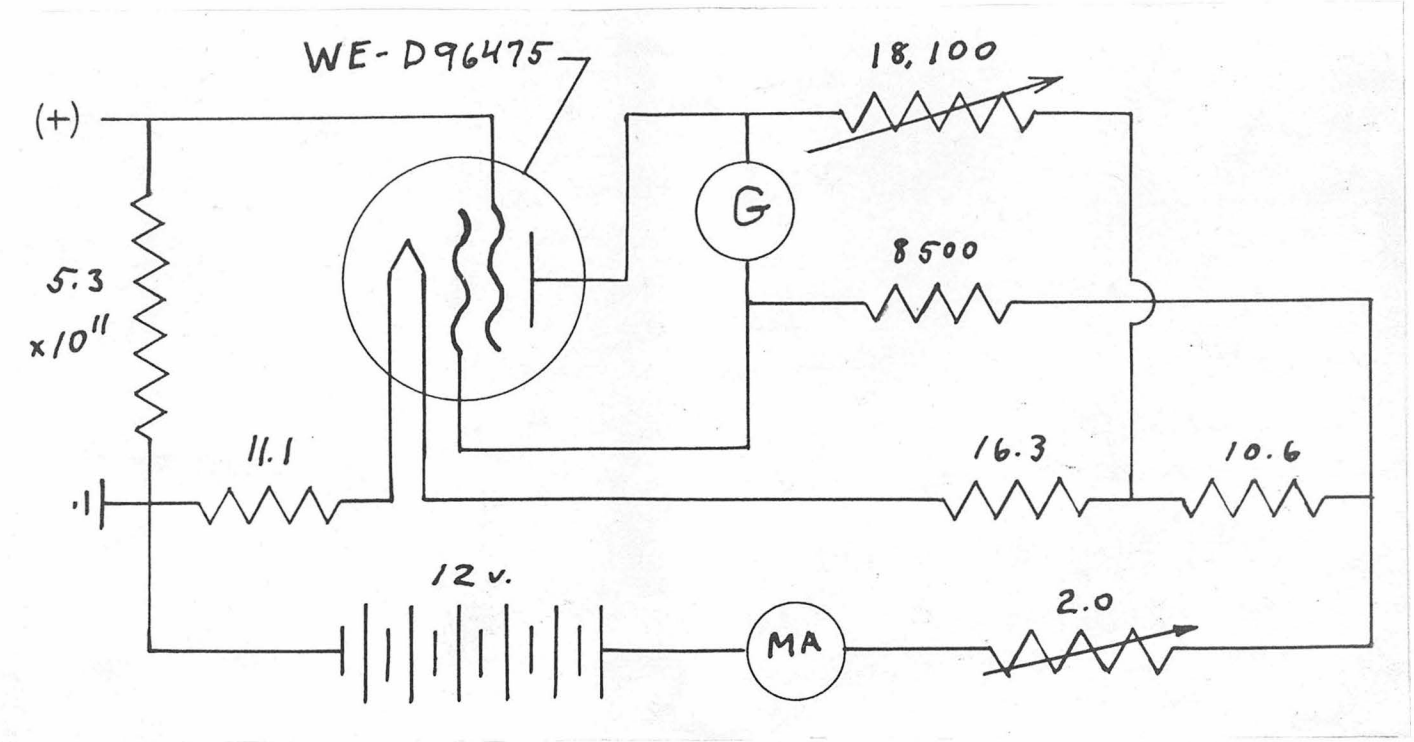

Fig. 12. Diagram of the circuit used with the electrometer tube for D.C. amplification of the ion chamber current. The resistances used with our tube are given in ohms.

The Voltage Regulator

The voltage regulator is entirely new. It is an electronic device which automatically holds the voltage across the x-ray tube exceedingly constant as long as the fluctuations in the applied voltage do not exceed a certain limit. The circuit diagram is given in Fig. 11. Its operation is as follows: A definite fraction (0.7/400) of the voltage across the tube is bucked against a constant emf (the latter being adjustable for any desired voltage by means of switches on B batteries and a small potentiometer) and the difference is applied to the first grid of a 2-stage amplifier. The output is then put on the grids of three 6L6 tubes in parallel which are inserted into the main high voltage line. Thus, suppose for example that the voltage across the x-ray tube starts to climb. This will make the grid of the first 1852 more negative, the plate of the first tube and the grid of the second tube more positive, the plate of the second tube and the grids of the 6L6's more negative, and hence the voltage absorbed in these last tubes higher, thereby reducing the drop across the x-ray tube. The coupling throughout follows a new method due to Brubaker. The gain of the whole circuit is something like 100,000 and there is no tendency to oscillations if the drop across the 6L6 tubes is over 250v.

Protection is provided by inserting across the 6L6's a 7500 ohm resistor which will carry the current should the high-voltage rise and the tubes be driven to cut-off. Should the high-voltage fall, the current is limited by the x-ray tube so no harm can be done to these tubes. A voltmeter registers the drop across these tubes at all times, and is the indicator by which the operator adjusts the high voltage (through the field of the alternator as described above) to keep within the range

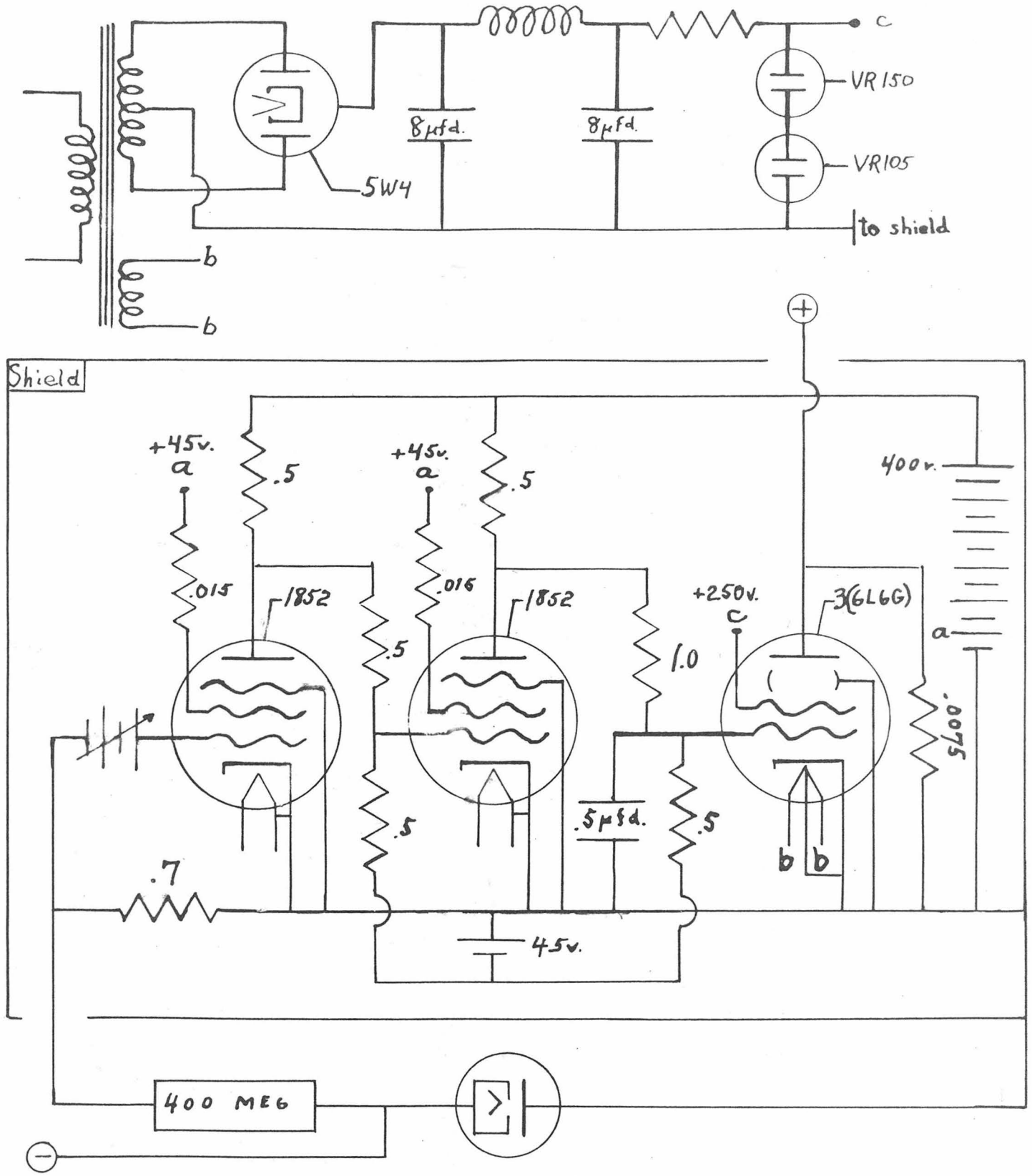

Fig. 11. Circuit diagram of the voltage regulator with its power supply. Resistances are in megohms.

that the regulator can handle. This range is theoretically 750 volts at 100 ma, but usually in practice the drop across the 6L6's is kept between 300 and 500v.

The effectiveness of the whole circuit is amply attested by the following observations. Without the regulator, the ripple observed on an oscilloscope is 0.5v rms at 20 kv and 100 ma, and the fluctuations in the high voltage as observed on the galvanometer are ± 1/500. When the voltage regulator is operating, the ripple is 1.0v rms and the random fluctuations are reduced to ± 1/10,000.

Several construction details are worthy of note. The circuit proper is made with standard parts and, together with the necessary batteries, is built into a box some 24" x 32" x 15" covered with sheet galvanized iron. Since the whole circuit is obviously at the potential of the target, namely half the tube voltage above ground, this box is placed on a 10" micarta cylinder slightly over 4' high fitted with a welded channel-iron base to provide the necessary mechanical stability. The high potential of this circuit presented some problems in connection with the power supply. Batteries were used as far as possible - a storage battery for the filaments of the 1852's and B batteries for their plate and screen potentials. For the 6L6's a special transformer was constructed on a laminated iron core in the shape of a flat annular ring 3" thick, 10" I.D. and 15" O.D. The primary consists of about 25 turns supplied from the insulation transformer which furnishes the filament current for the rectifying valve on this side of the set. Insulation must be provided only for the drop (of the order of 1000v) across the filter chokes of the main circuit (see Fig. 9) and this is easily done with empire cloth. Two secondary windings are provided, as shown in

Fig. 11, one supplying the filaments of the 6L6's directly, and one feeding a conventional full-wave rectifying circuit with voltage regulator tubes for maintaining the screen potentials of the 6L6 tubes. The 400 megohm resistance across the x-ray tube is composed of five parallel sets of two I.R.C. type MVR 1000-megohm 50-kv resistors in series. These resistors, 1" in diameter and 10" long, are centered inside of appropriate lengths of 2" bakelite tubing by means of sets of three radial supports turned out of 3/8" lucite rod (see page 31). This group of resistors, and also the special transformer just described, are clearly discernible near the top of the photograph in Fig. 2.

The Ion Chamber and Amplifying Circuit

The methyl bromide ionization chamber used to detect the x-rays is identical with the one described by DuMond and Hoyt[36]. It has, however, been painted on the inside with a suspension of lamp-black in alcohol to help cut down α-particle emission from the walls. Also it has, together with its amplifying circuit, been almost completely surrounded with 2" of lead (4" on top) to reduce the cosmic ray background. The D.C. amplifying system is completely changed. The ion collector is led by a vacuum-tight quartz bushing through the top of the chamber into a small auxiliary vacuum chamber, where it contacts by means of soldered platinum strips the grid cap of a Western Electric D-96475 electrometer tube. In this second chamber is a shaft, operated through a ground glass joint, possessing knobs by which any one of three high resistances to ground can be brought into contact with the collector-grid electrode. The drop across this resistance due to the flowing of the collector current is the potential applied to the extremely low-current grid of the electrometer tube. The tube circuit used is the one proposed indepen-

dently by Barth[37] and by Penick[38]. The resistances were adjusted according to the directions given by Penick so that the tube elements were at their rated voltages, and the galvanometer current was close to zero and passing through a rather flat maximum with respect to the filament current. This insured stability against fluctuations in the filament battery (three 12v storage batteries in parallel) but not against changes in the tube characteristics with time. The latter are allowed for only by the large rheostat (actually containing one coarse and one fine control) which adjusts the galvanometer current to zero. The circuit diagram is given in Fig. 12 (page 25) with the various values found necessary for our tube. The circuit was assembled in a shielding box close to the tube (all inside the lead), with shielded cables going to the batteries and to the galvanometer (sensitivity 6×10^{-10} amp/mm.).

With the highest grid leak resistance ordinarily used, namely 5.3×10^{11} ohms, the overall current sensitivity of the circuit is 42×10^{-17} amp/mm. With the lead in place, the cosmic ray background is of the order of 1.7×10^{-15} amp, and the fluctuations are of the order of ± 4 mm. α-particle kicks are disastrous (up to 50 mm) but occur only once every two minutes on the average.

Construction of the Voltmeter

The "voltmeter" (more accurately, a voltbox) consists of 200 wire-wound Shallcross Super-Akra-ohm resistance units of one megohm each in series with two similar units of 500 ohms each. These units are non-inductively wound in twelve separate coils on a porcelain form. The general arrangement of these units in groups of five supported in aluminum corona shield pans and the arrangement of these pans in two stacks

inside large micarta cylinders with provision for circulating oil have been described by DuMond and Youtz[28].

One important change has been made as a result of tests to determine the parallel leakage currents in the bakelite supports. These currents were computed to be as much as 1/100 the working current in many places--obviously intolerable where a precision of one part in 10,000 was being sought. After tests on samples of a dozen different insulating materials to determine their actual volume resistivity at fields up to 3,000 volts/cm, the new (quite new at that time) plastic methyl methacrylate manufactured by DuPont with the trade name lucite was adopted. Its resistivity is at least 10^{12} ohm cm at these field strengths, being about half that of amber (costing only a third as much), greater than that of any other substance tested, and 100 times the resistivity of ordinary bakelite. It is quite easily machined though, being a good insulator for heat as well as for electricity, pains must be taken to conduct the heat rapidly away from the freshly cut surface lest the plastic become sticky. The flat strips which hold the resistors in the aluminum pans, the assembly rods through the stacks with their tubular spacers between pans, and the oil lines (made in four sections, drilled through, and cemented at double-shouldered conical joints) are now all made of lucite.

The micarta cylinders and the oil itself constitute further parallel leakages. The cylinders are across the whole stacks of resistances and are consequently not as critical as the supports that shunt various sections only and could thereby produce a non-uniform current down the stacks. The cylinders were sprayed inside and out with glyptal and baked as much as possible by passing warm dry air over and through them.

(One cylinder experimentally painted with ceresin wax soon became porous to the oil.) Their leakage current (measured directly) was reduced to about 1/1000 the working current - certainly no worse than many other leakages to ground in the whole set, and in fact useful for stabilizing the potential distribution within the circulating oil.

Some conductivity tests on samples of freshly filtered transformer oil gave a value of $\rho = 10^{11}$ to 10^{12} ohm cm at 2000 v/cm in agreement with published values[39]. This corresponds to a leakage current just about at the tolerable limit; but in view of the possibility of contaminating the oil with moisture from the air during transfer to the voltmeter, the danger of the existence of particle impurities inside the stacks despite precautions, the existence of much larger fields locally (near studs, nuts, connecting wires) and the non-ohmic nature of whatever conduction does exist, it was thought advisable to search for some other fluid capable of transferring heat but possessing a higher electrical resistivity than ordinary transformer oil. After some inquiries, the Socony-Vacuum Oil Co. of New York City generously supplied us with 20 gallons of their new Gargoyle Transformer Oil A-30, for which they claimed a resistivity of 1340×10^{12} ohm cm at 2500 v/cm before shipment in sealed cans. They further claim that, after moderate exposure to water vapor, it is capable of restoration by the usual process of carefully passing through dry filter paper. Warm dry air was forced through the assembled voltmeter for 30 hours and then the oil was simply poured in.

In order to eliminate contact emf's and variable contact resistances, it was found necessary to solder <u>all</u> connections. This was done with #22 brass wire soldered directly to the studs which are cemented in

the ends of the resistance units - a fairly delicate operation inasmuch as the Nichrome (about #100) is itself soldered to these same studs under a protecting cap less than $\frac{1}{2}$" away. The least possible amount of pure rosin was used for flux. In making the connections between pans, use was made of a short round-headed bolt through the pan with a nut on the inside. The last (highest potential) megohm in a pan was connected directly to the inside of this bolt. A wire from the first megohm of the pan above came down and was soldered to the head of this bolt after assembly of the stack. These latter can be seen in the photograph of Fig. 13, every other one being in the rear. The bottom pan of each stack contains instead of megohms a 500, a 40,000 and an 80,000 ohm unit for purposes of providing low voltage taps. The four leads are soldered to studs which pass through a 3" lucite plug in the base of the cylinder, made oil-tight with glyptal. The top of the stack is soldered to a short phosphor-bronze strip which in turn is soldered to a special bolt that passes (with a cork gasket) through the brass head on the micarta cylinder and receives the high-voltage lead. These can also be seen in Fig. 13.

Other high-voltage terminals are provided at 5, 15, 25 and 50 megohms from ground in each stack by means of small springs (necessary for assembly) from the above-mentioned round-headed bolts on the appropriate pans and lucite bushings in the cylinder wall. A cross-section of one of these is given in Fig. 14. The most difficult stage in the assembly of the voltmeter is the lowering of the cylinders onto the stacks. For this, strings attached to the four springs and to the top bolt must be threaded through their respective holes and held under a slight tension until the cylinders are in place and the springs cleared. These

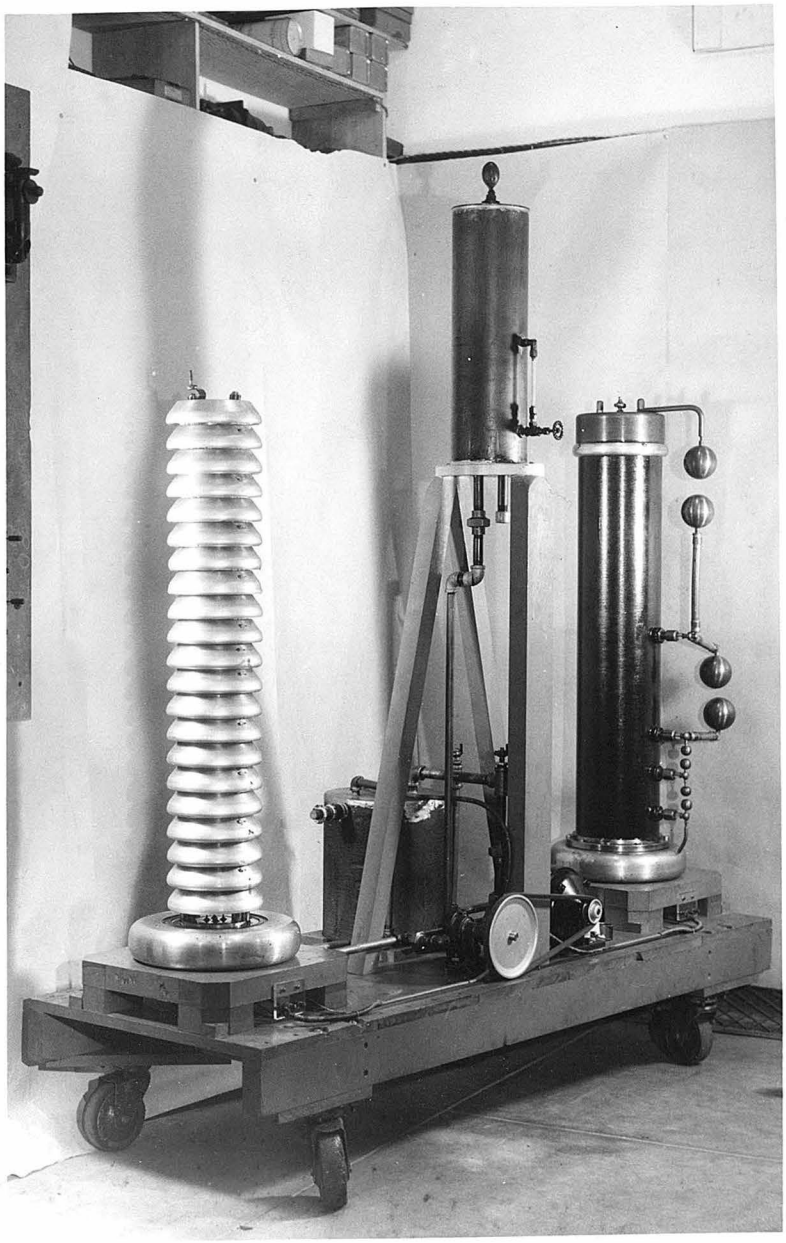

Fig. 13. Photograph of the voltmeter truck, with one of the micarta cylinders removed, showing the oil circulating system, the sphere gaps and the connections between pans.

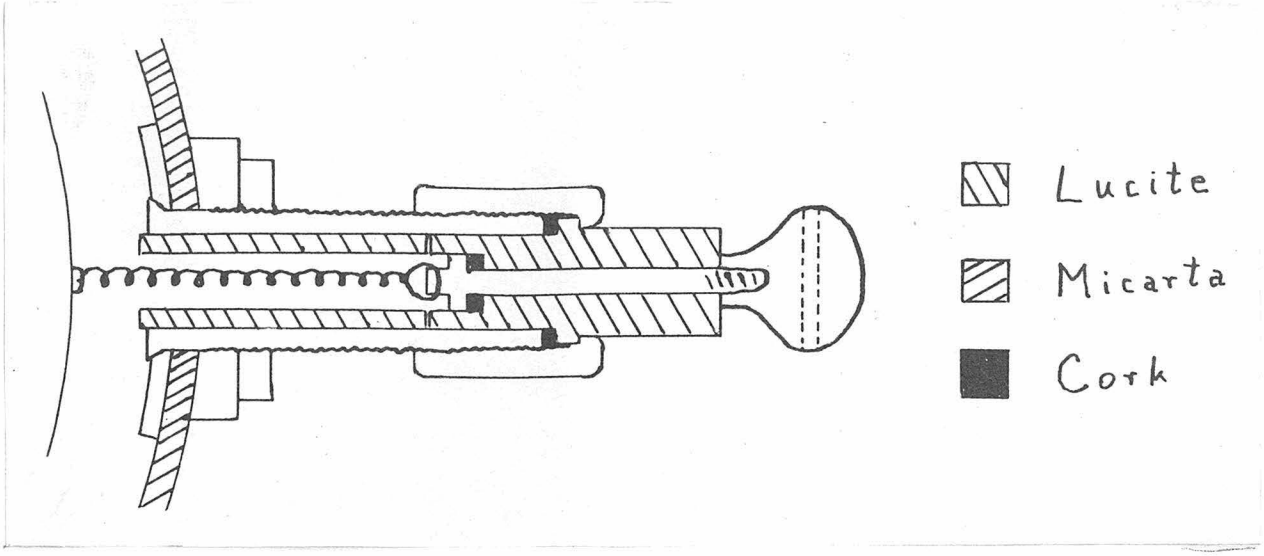

Fig. 14. Full-scale cross-sectional view of one of the high-voltage terminals through the cylinder walls of the voltmeter.

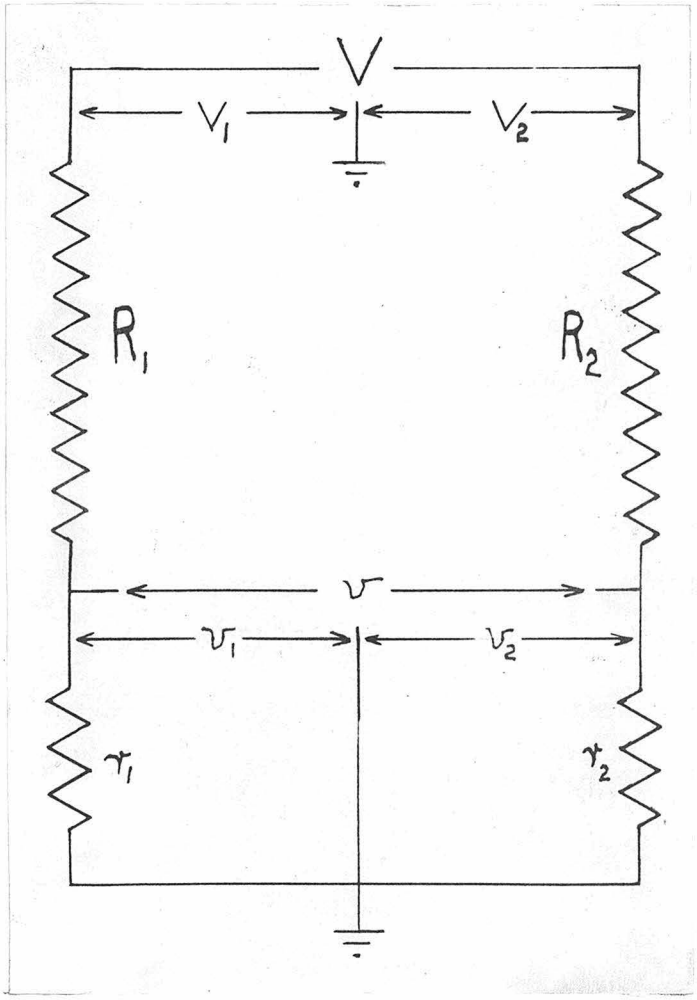

Fig. 15. Circuit illustrating the use of the voltmeter on an unbalanced supply.

are then soldered to their respective studs and the external assembly of the terminals is completed. Protective sphere gaps are provided between each adjacent pair of these terminals and to ground. The two larger gaps on each stack have 3" spheres (plumber's floats) in series with a G.E. surge resistor (~ 5,000 ohms); the smaller gaps have no resistance and the spheres are turned by hand out of brass.

The oil circulating system is provided primarily to insure uniformity of temperature throughout the stacks and hence (presumably) constancy of resistance ratios. It is also capable of maintaining a fairly constant temperature because of the inclusion of a heat exchanger - nothing more than a pair of coils of the oil line located inside a sealed tank through which tap water flows. The oil circuit is as follows: from a small motor-driven gear pump (visible just behind the pulley in the photograph of Fig. 13) to a strainer (behind the post), thence to the heat exchanger where the line divides. Each coil then supplies one stack and the return lines form a single pipe off of which lead the pump intake and a single pipe up to the reservoir in the center. All of the plumbing uses standard fixtures and the joints are standard pipe threads sealed with glyptal. The oil is introduced through the top of the reservoir and provision is made for the escape of the air as the oil level rises. At first three petcocks were employed at the level of the top of the heat exchanger and one in the top of each cylinder, but these could not be made oil-tight even after extra lapping. Now solid plugs are put in each of these places as the oil level reaches them. The oil level is maintained an inch or two above the tops of the cylinders. One petcock at the bottom of the strainer (the lowest place in the system) is fortunately tight and is used for draining the oil. The

top vent is covered with a toy balloon to allow the oil to "breathe" and yet prevent contact with the fresh air of the room.

The whole assembly is mounted on a 68" wooden truck as shown in Fig. 13. Lucite terminal boards are provided at the base of each stack, and heavy shielded cables from the 500 ohm terminals are permanently installed for the potentiometer leads. The other low voltage taps are not used at present. There is also a $\frac{1}{4}$" x 3/8" brass bar connecting the bottoms of the two stacks to which is soldered a copper strap tied to the main ground of the set. The only unsatisfactory points remaining in this construction are the mechanical supports of the large sphere gaps and the rubber balloon which is attacked by oil vapor.

Calibration of the Voltmeter

The determination of the resistance ratios in the voltmeter is, of course, one of the most essential parts of the whole experiment. However, it is first necessary that corresponding ratios in the two stacks be nearly equal if single readings of the potentiometer (across both 500 ohm resistors as indicated in Fig. 9) are to give a true value of the high voltage across the tube even when that high voltage is unbalanced with respect to ground. A somewhat unbalanced condition is in fact quite probable due to the additional leak to ground through the target cooling water on that side of the set. The stringency of this requirement is not great, and can be analyzed quite simply. Referring to Fig. 15, if we let $N_1 = R_1/r_1$ and $N_2 = R_2/r_2 = N_1(1+\rho)$,

$$\text{then } V_{true} = V_1 + V_2 = i_1(R_1 + r_1) + i_2(R_2 + r_2) = i_1 r_1(N_1+1) + i_2 r_2(N_2+1)$$
$$= v_1\left[N_1 + 1\right] + v_2\left[N_1(1+\rho) + 1\right] = v(N_1 + 1) + v_2 N_1 \rho\,;$$

but $V_{calc} = v\left[N_1(1+\rho/2)+1\right] = v(N_1+1)+vN_1\rho/2$

so the error $\Delta V = N_1\rho(v-2v_2)/2 = N_1\rho(v_1-v_2)/2.$

Furthermore, if we let $V_2 = V_1(1-\phi)$ and $r_1 = r_2$,

then $$i_1 = \frac{V_1}{(N_1+1)r_1} \quad \text{and} \quad i_2 = \frac{V_2}{(N_2+1)r_2} = \frac{V_1(1-\phi)}{r_1(N_2+1)}$$

whence $$v_1 - v_2 = (i_1 - i_2)r_1 = V_1\left[\frac{1}{N_1+1} - \frac{1}{N_2+1} + \frac{\phi}{N_2+1}\right].$$

Hence the relative error introduced is

$$\frac{\Delta V}{V} = \frac{N_1\rho}{2(2-\phi)}\left[\frac{N_2-N_1+\phi N_1+\phi}{N_1N_2+N_1+N_2+1}\right] \approx \frac{N_1\rho}{4}\left[\frac{N_1(\rho+\phi)}{N_1N_2}\right] \approx \rho\phi/4.$$

Accordingly, if ρ were reduced to 1/2000 as was actually done, and the unbalance in the high-voltage were 11:10 ($\phi = 0.1$), the error would be only 1/80,000. Even if the unbalance were 2:1 ($\phi = 1.0$), the more accurate formula above gives the error as 1/4,000.

The simplest way to accomplish equality of the resistance ratios (low value of ρ) is to make the two 500 ohm resistors as nearly equal as possible and then to distribute the megohm units between the two stacks such that their total resistances up to each of the high voltage terminals are as nearly equal as possible. Wire was unwound from the larger of the two 500 ohm units until their resistances as measured by a Leeds and Northrup open dial Wheatstone Bridge were 499.12_9 and 499.13_2 ohms respectively, and their ratio as determined with a Wolff potentiometer (comparing P.D.'s across the two resistors while carrying the same current) was 1.00000_9. The megohms, furnished by the manufacturer with a tolerance of 0.1%, had to be measured relative to each other with an accuracy of 0.01% before the distribution to and within the stacks could be made. The ratio of each megohm to a particular reference

megohm was determined, just as DuMond and Bollman[5] had done in the case of their 14 megohm stacks, by means of the Wheatstone bridge circuit given in Fig. 16.

Unfortunately, nine months after the first set of measurements had been completed, some alarming changes were discovered and a systematic repetition of the measurements on every megohm unit was made. These revealed that the changes were quite random and of both signs forming a fairly smooth Gaussian curve, that 90% had changed by less than 0.02%, but that a few had changed by more than 0.10%. Those that had changed by more than 0.07% were replaced, and a completely new allocation of the units was made in which (a) only the most stable resistors were placed in the lower pans where the necessary points of balance are closest together, (b) resistors with positive and negative changes were counter-balanced throughout as far as possible, and (c) the actual balance at the five high-voltage terminals, though perforce not as close as in the first assembly, was maintained within the above-mentioned one part in 2000.* These changes were most disturbing and their cause is not entirely known. Presumably ageing of the metal could account for a large part of them, yet the resistors had been purchased for over a year and a half at the time of the completion of the first set of tests. Also temperature variations during the tests could easily account for the observed changes in the best 90-95% (the Shallcross Co. claims a temperature coefficient of 0.00013/deg). But the large changes in both directions are still unexplained, and the fact that two or three of the resistors could possibly change by 0.2% is not only alarming, but

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*—The final ratios for the full stacks turned out to be equal within one part in 4500.

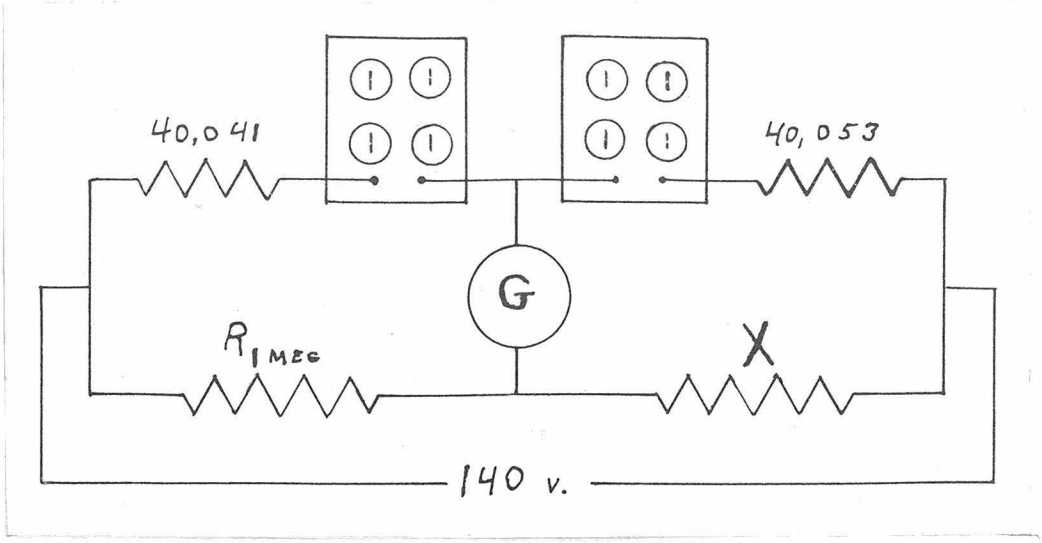

Fig. 16. Bridge circuit for comparing the megohm unit resistors.

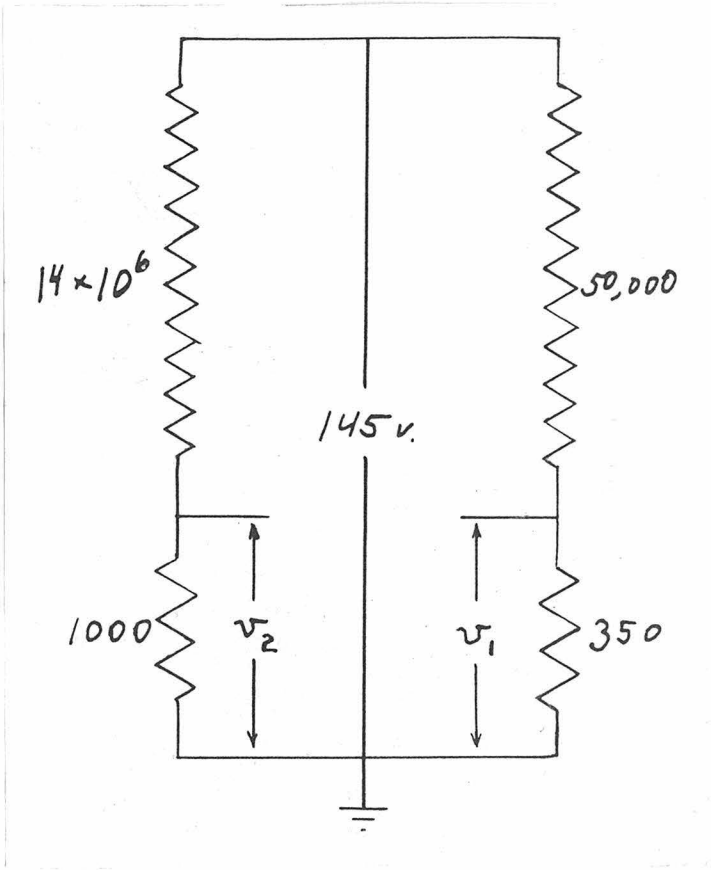

Fig. 17. Circuit for measuring the ratio of the 14 megohm stacks to their 1000 ohm resistors.

means that no final results on h/e can be given until the whole voltmeter calibration is repeated.

For determining the large resistance ratios in the stacks, it was necessary to use some intermediate ratios. The most important of these were the 14-megohm in-line stacks (hereafter referred to as the small stacks) built up by Bollman[5]. Two units that were found open-circuited were replaced, and all the units were soldered together by means of a drop of solder between each adjacent pair of end studs. The resistance ratio of these stacks was determined many times by a method very similar to one used by Bollman and illustrated in Fig. 17. First a nominal 350 ohm and a 4200 ohm resistance were placed in series with a storage battery and a further resistance, and the ratio of the first two determined with a Leeds & Northrup type K potentiometer (containing 0.1 coils). Second the ratio of the 4200 ohm to a 50,000 ohm resistance was found by an identical procedure, thus giving the ratio R of the 50,000 to the 350 ohm resistance. Finally with 145v (2 sets of 3 new B batteries in parallel) applied to the circuit of Fig. 17, the ratio of v_2 to v_1 was determined with the type K. The ratio of the 14 megohms to its 1000 ohm unit is then given by the formula

$$X = (14 \times 10^6)/1000 = (R+1) \cdot (v_1/v_2) - 1.$$

All measurements were made with both polarities of the applied emf, and a constant emf of 2.5 microvolts was found to exist in one of the 1000 ohm units and to persist through several resolderings of the junctions. Its value, computed from the change in measured emf on reversals and also observed directly with a galvanometer when no external emf was applied, is so small as to be quite negligible when the high voltage is used and the P.D. across the 1000 ohms is of the order of

1 volt. Even if it is of the nature of a thermal emf, it cannot get large enough to be significant and in any event the calibration of the large stacks in terms of these small stacks is again always made with both polarities of the applied voltage. During some of the measurements compressed air, dried by passing through $CaCl_2$ and a wad of cotton batting, was forced up the 1" glass tubes which contain the small stacks, and during some of the measurements no circulation was maintained more than the natural convection currents. The two treatments produced no real difference in the observed resistance ratios. This air current was however always maintained when high voltage was applied.

The values obtained for X + 1, the quantity by which potentiometer readings must be multiplied to give the actual high voltage applied to the stacks are*

$$\begin{cases} \text{for stack I} & 14{,}004.53 \pm .48 \\ \text{for stack II} & 14{,}007.06 \pm .43. \end{cases}$$

These values are the weighted means of at least six independent runs taken over a period of three months, each run consisting of at least ten determinations of the $\underline{v}_1/\underline{v}_2$ ratio and 4 or 5 determinations of $\underline{R}$. The internal and external (between different runs) probable errors agree satisfactorily within 20% for the poorer of the two stacks. These values have been corrected for errors in the type K potentiometer which will be discussed later, but the probable errors given do not include the uncertainty of the corrections.

There remains to examine the change in these ratios under load.

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*--These values should not agree with Bollman's values because of actual changes to the stacks. At the very beginning of this work his values were checked to better than one part in 10,000.

This change was observed relative to the large stacks, which were assumed to have no temperature effect when the oil was circulating through them and when the high voltage was applied by means of a large "key" only momentarily during actual readings of the galvanometer. A good many heating curves (relative change in resistance ratio vs. time) were obtained and various methods of correcting the calibration data were tried. The method finally adopted was to run a heating curve simultaneously with a complete set of data calibrating the large stacks in terms of the small stacks, and to interpolate on the heating curve to find the correction to be applied to each reading. One such heating curve, obtained when the positive half of 8 kv was applied to one of the small stacks and to the lowest 15 megohms of one of the large stacks, is given in Fig. 18 where the ordinates are parts in 10,000 increase. The "true zero" of these curves, that is the true value at zero time of that particular resistance ratio for which the curves were run, under conditions of no load to correspond with the condition under which the small stack ratio was determined using B batteries, was found by 10 careful observations over several days. The total corrections are of the order of 0.1%. If they are good to 2%, then the uncertainty introduced by these heating corrections should not be more than one part in 50,000.

The first attempts to calibrate the large stacks in terms of the small stacks were made by applying a constant high-voltage to both in parallel and taking simultaneous readings with two potentiometers across the respective bottom taps. However, after many devices had been tried and much effort expended, the method had to be abandoned because the voltage could not be held sufficiently constant. (The electronic voltage regulator did not exist at that time.) The solution to the diffi-

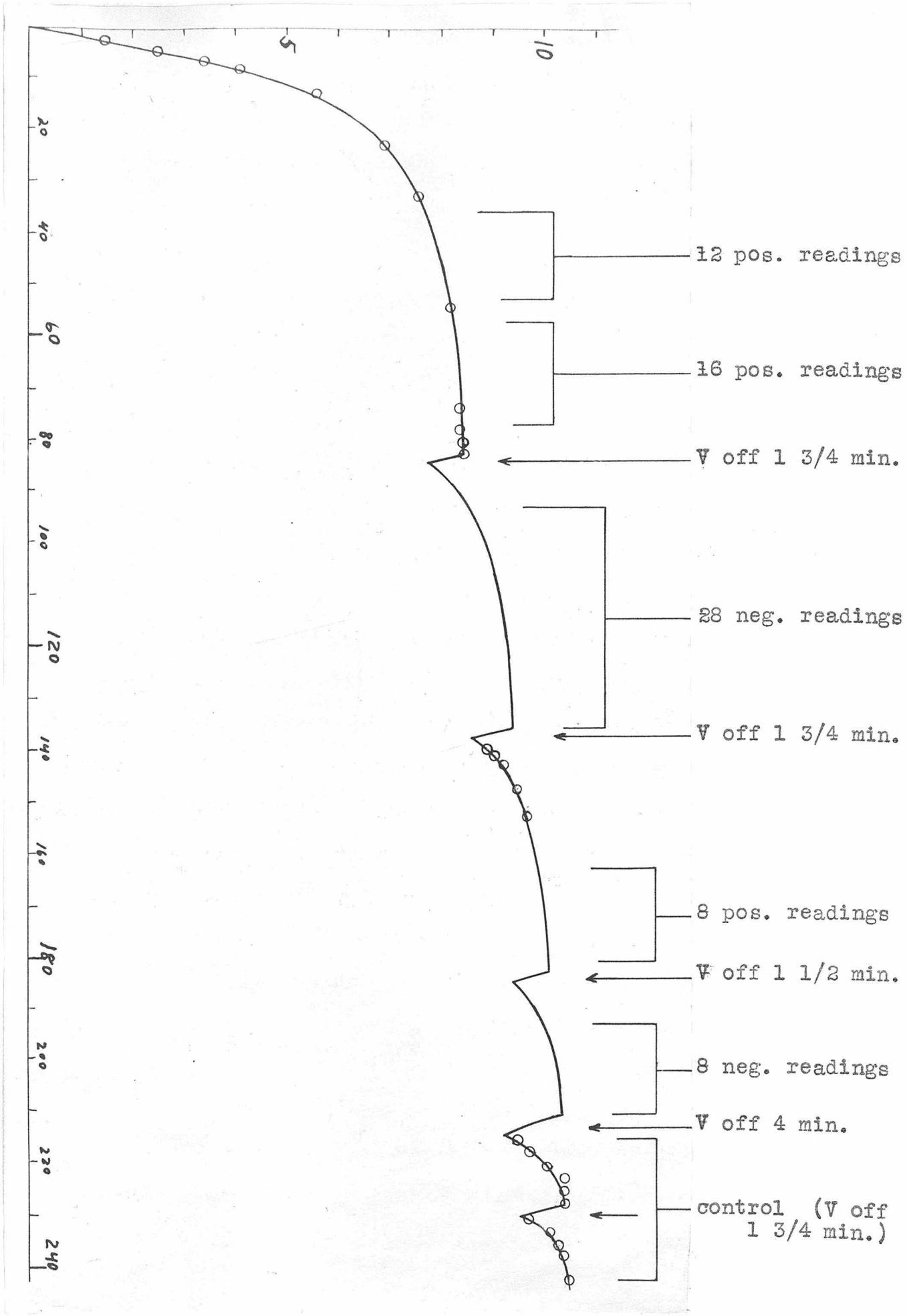

Fig. 18. One of the curves used to correct the voltmeter calibration data for heating of the small stacks.

culty was a voltage-independent method which really measures the ratio of the P.D. across the 1000 ohm unit at the foot of one of the small stacks to the P.D. across the 500 ohm unit at the foot of one of the large stacks. The circuit is given in Fig. 19. The formula for $Y+1$, the ratio of the whole stack (or that part of it being measured) to its 500 ohm unit is easily found to be

$$Y + 1 = (R_2 + r_2)/r_2 = (1/P)\left[1 + (R_1/r_1)(1 + r_1/R_p)\right]$$

where $\underline{P}$ is the ratio given by the potentiometer when the galvanometer shows no deflection. The circuit of the potentiometer used, a 20,000 ohm Wolff, is such that the total resistance $\underline{R}_p$ being used must be made to vary with $\underline{h}$, the hundreds digit, by applying the ground connection to the particular 100-ohm coil which is set on. The ratio $\underline{P}$ must therefore be computed by dividing the reading (in ohms) by the total resistance, where $\underline{R}_p = 18{,}300 + 100\,\underline{h}$. The impossibility of obtaining a continuous variation in $\underline{P}$ is obvious, and for some of the readings it was necessary to place an additional resistance in series with the potentiometer. A laboratory "secondary standard" resistance of 1000.8 ohms was used for this, and its ratio to the 18,300 ohms of the Wolff was quite easily determined to the necessary 0.1% by placing the two in series with a storage battery and a further limiting resistance, and comparing their P.D.'s with the type K potentiometer. The auxiliary ratio $\underline{r}_1/\underline{R}_p$ for each of the small stacks was determined in exactly the same way. The value of $\underline{P}$ was then observed, using each of the small stacks on each polarity, for seven combinations of the high voltage terminals on each of the large stacks. Three such complete runs were obtained, with a simultaneous heating curve for each as explained above, and the mean values computed. For the lowest terminals, 5 megohms above ground, data was

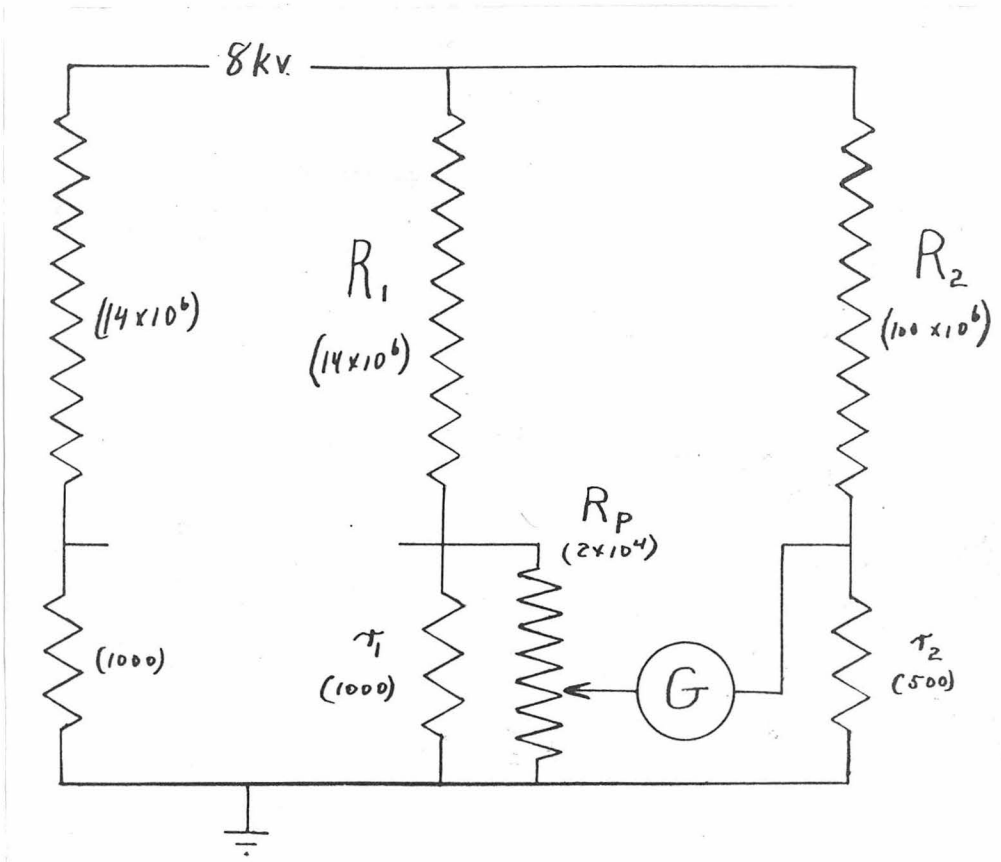

Fig. 19, Circuit used for calibrating the large stacks in terms of the small stacks.

taken in a manner similar to the above but with the roles of the two stacks with respect to the Wolff interchanged. However, slightly more reliable results were obtained by the former direct two-step B-battery method independent of the small stacks.

In order to make use of all the data taken, a system of combining the 8 observed ratios on each large stack was used. Thus the ratio of the full stack to its 500 ohm resistor is measured directly and can also be computed from the observed ratios for the two halves of the stack and from two other combinations involving the various smaller ratios. Taking the mean, the final values with all corrections for $Y+1$, the quantity by which potentiometer readings must be multiplied to give the actual high voltage applied to the stacks, when the full stacks are used, are

$$\begin{cases} \text{for stack A} & 200{,}650 \pm 3 \\ \text{for stack B} & 200{,}695 \pm 3. \end{cases}$$

The smaller ratios have not been computed, inasmuch as it is desirable to use the full stack whenever possible to avoid heating, but the complete data is available for their evaluation when necessary.

The probable error stated above is merely that computed in the usual manner from the deviations of the various determinations from their mean, and represents the internal consistency in the measurements of the P's. Now all of the measurements depend on one or both of the two potentiometers which were used. The Wolff was purchased in 1932, and was recalibrated in 1936 by F. G. Dunnington. His corrections in microvolts were applied to all readings. According to statements from the National Bureau of Standards and to generally accepted beliefs, such a potentiometer, recalibrated after a few years of ageing and known to have suffered no accidents or maltreatment, should have a precision

of at least one part in 10,000 and is probably good to 3 or 4 times that accuracy. The type K was an old but little used one borrowed from Occidental College. Its main and 0.1 coils were checked against the Wolff and found to be satisfactory within the precision of the latter, say 3 parts in 100,000. But any error which might exist would have a cumulative effect where the same coils are used in two or more successive multipliers of a product. Examination of the data shows that one set of coils is used once, another set twice, and the 0.1 coil three times. The slide wire was calibrated at every turn and the necessary corrections were always applied, but because of the low voltages involved, the uncertainty of these corrections may be as high as 10%. The geometrical sum of these four possible errors (1 x 3/100,000, 2 x 3/100,000, and 7/100,000) is 13.2/100,000, the probable error introduced into the final result by the type K. This piling up of the error is an inherent and unavoidable characteristic of the large ratios being measured, and is kept at a minimum by the use of the one-tenth coil of the type K potentiometer as much as possible.

The sources of error in the voltmeter calibration can be listed thus, giving parts in 100,000:

internal consistency in X + 1	3.1
internal consistency in P	1.5
uncertainty of the heating corrections	2.0
uncertainty of the Wolff	3.0
uncertainty of the type K	13.2
geometrical sum	14.1

Hence the overall uncertainty is of the order of 1 part in 7,000, and the final mean value of the full ratio is

$$Y + 1 = 200{,}673 \pm 28.$$

It remains to be seen, however, whether or not this will remain constant

with time. As a matter of fact, a very brief check of the two stacks in parallel indicates a relative change of one with respect to the other of $0.03_1\% \pm 0.00_8\%$.

In actual operation, the Wolff potentiometer is used in conjunction with the voltmeter stacks as shown in Fig. 9. Its galvanometer has a sensitivity of 2.6×10^{-9} amp/mm. For rough continuous indication of the voltage, the potentiometer may be shunted by a 5,000 ohm/volt multimeter with a 0.2 ma scale which simply reads the current through the stacks, as is also indicated in Fig. 9.

The voltage measurements depend still further on the standard cells used to balance the potentiometer. Three such cells are kept in a thermostat-controlled box and their ratios are interchecked once a month. Two of these have National Bureau of Standards 0.01% certificates dated January 30, 1940. Their ratio has not varied more than 0.001% in a year and agrees with the certified value to 0.004%.

The Two-Crystal Spectrometer

The two-crystal spectrometer used in the present work to monochromatise the x-ray beam going to the ion chamber is the beautiful instrument designed and built by DuMond and Marlow respectively and previously described by them[40]. Corrections on the two worm wheels obtained from an optical calibration are given by them and were always applied. The instrument has been used in previous work[41] and found to be quite reproducible over large angles to less than 1" of arc. Ideally then, it should suffice for the calibration of the instrument to observe one known wave-length (emission line or absorption edge), thus determining the dihedral angle between the crystal faces in one position. Other angles would be known by their differences from this angle. Actually, of

course, the more fixed points and the nearer these are to the region to be used, the better.

After futile efforts to restore the good reflecting properties of two very large crystals that had been used before[41] by the usual process[42] of etching or grinding, polishing and etching, another fairly clear calcite crystal roughly $2\frac{1}{2}$" on each edge, obtained from Wm. Gaerttner, was cleaved in a milling machine. The two resulting crystals were then mounted on the spectrometer with special 3-point supports and enough beeswax to prevent them from sliding along the supporting shelf. The point supports consisted of pairs of highly polished brass buttons, one of each pair being soldered in the vertical face of the crystal holder, and the other on a phosphor-bronze clip pressing against the front face of the crystal as nearly opposite the corresponding point of support as possible. This mounting should reduce the strains in the crystal to a minimum. The focal spot of the x-ray tube was then located flouroscopically, the spectrometer placed so that a line from the focal spot to the first crystal pivot made an angle of 100° or 102° with the cathode ray beam, and the crystals were aligned optically to within about one minute of arc following the technique described by DuMond and Hoyt[36]. Finally the Au $L\alpha_1$ line was observed, thus establishing the calibration. Unfortunately a recent measurement fails to check the available values for the other gold L lines by about .03%, which corresponds to a relative error at the angle used for the best isochromat (to be described) of about .074%. Unless something serious has happened to the spectrometer, or unless the crystals are somehow moving in their supports, there should be no further appreciable error. Allowing the normally expected uncertainty of 1" of arc, the probable error in the

wave-length measurement may be taken as .075%.

To test the condition of the crystals, the observed 39" full width at half maximum of the Au $L\alpha_1$ line was compared with the accepted value[43] of 38.6", and the parallel rocking curve was run at the same wave-length ($\lambda = 1.274$Å) and found to have the satisfactory full width at half maximum of 10.5". As further checks, the Rh K absorption edge was run and a parallel rocking curve was taken at $\lambda = 0.35$ Å. The former had a width (defined as the full width at half maximum of the derivative curve) of 10", and checked the angular calibration made with the Au $L\alpha_1$ line to within the precision by which the rhodium edge is known. The latter had a width at half maximum of 7" which indicates[44] after interpolation that the width of the "spectral window curve" for a 24kv isochromat would be close to 7.5" or 10.5 v.

When this is combined (see page 54) with the two de-sensitizing or blurring effects of a 5.2v P.D. across the filament and a 0.6v ripple in the high-voltage (interpolated for 60 ma emission current), a graphical integration gives a curve almost identical in shape and 10.6v wide at half maximum. This is in contrast to the 33.6v "window" used by DuMond and Bollman[5] on their 20 kv isochromat and constitutes one of the major advantages that this experiment possesses over previous similar experiments. Moreover, the advantage is greater than appears from the simple comparison of these figures, for according to the dynamical theory of Prins et al[45] for the diffraction by a perfect crystal, the ratio $\Delta\lambda/\lambda$ or $\Delta v/v$ is constant so that much might be learned from conducting the experiment at higher voltages, whereas with ground crystals DuMond and Bollman showed that ΔV is proportional to V^2 so that experiments at high voltage are comparatively useless.

The Balanced Filters

The distribution of lead snouts and shields on the spectrometer table is shown in Fig. 20. In addition, the tube is surrounded by $\frac{1}{2}$" of lead, and the slit jaws are made of $\frac{1}{4}$" lead. This array serves well to keep x-rays scattered by the air, crystal supports, spectrometer top, etc. from reaching the ion chamber, but enough incoherent scattering from the crystals themselves was passed to give huge "tails" or fillets to the first isochromats. At this point Professor DuMond suggested the use of Ross balanced filters[46] as a coarse wave-length filter to keep out effectively the non-coherent scattered radiation except in a small wave-length band which would bracket the much narrower band passed by the bi-crystalline reflection.

The operation of such a filter is as follows. Two films are prepared of substances whose atomic numbers are adjacent and whose K absorption edges include between them the desired wave-length. The thickness of these films is adjusted so that (a) the absorption of the two films is as nearly equal as possible in the wave-length regions not included by the two absorption limits, and (b) the contrast or difference between the two absorptions within the specified region is a maximum. (Strictly speaking, condition (a) implies equal K "jump ratios" for the two substances, but in practice the condition can easily be nearly satisfied, and can be made more exactly true when necessary by the addition of a further filter to the one possessing the larger jump.) The procedure in using such a filter is to take intensity readings with first one and then the other of the balanced pair of filters in the path of the radiation. The difference between the two readings thus obtained gives the effect of the radiation within the "pass band". It is to be

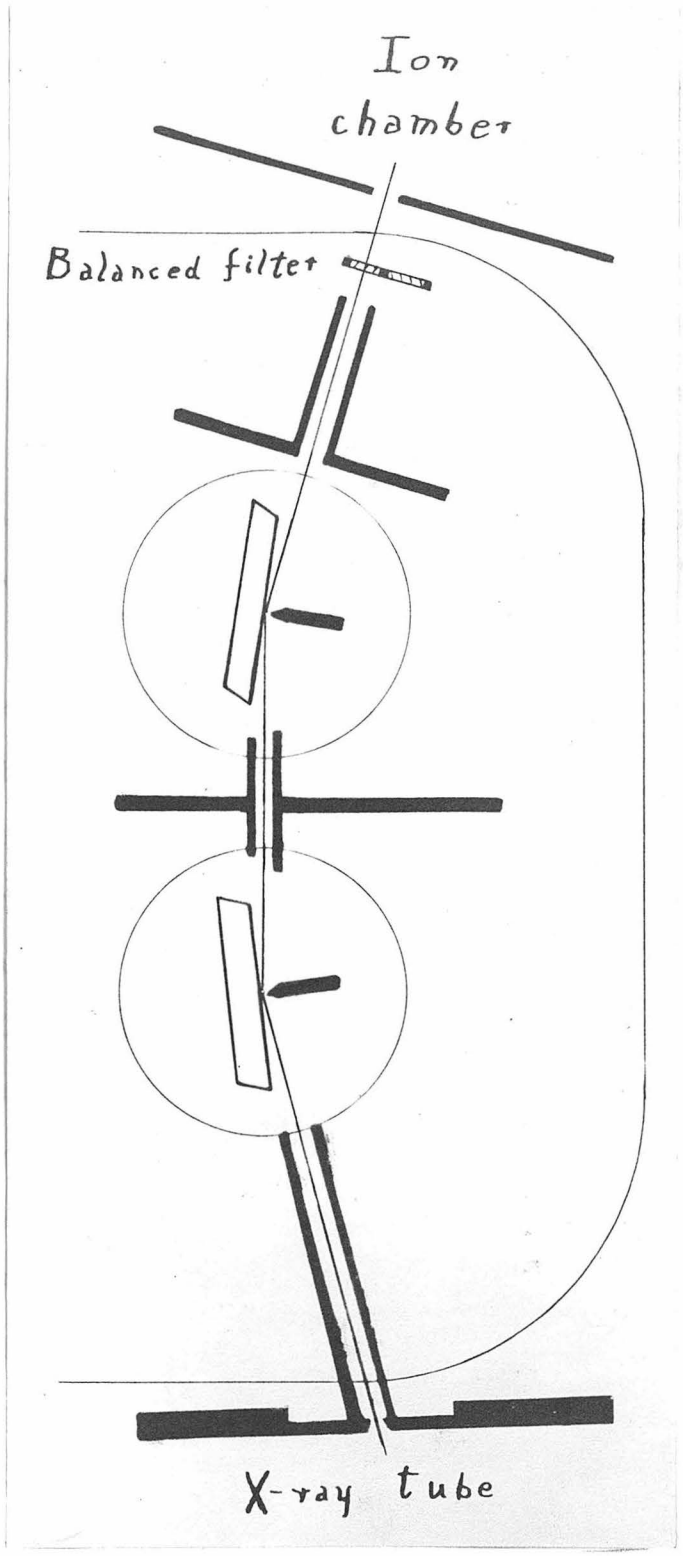

Fig. 20. Distribution of lead snouts and shields on the spectrometer top, showing also the position of the balanced filter, one quarter scale.

noted that this procedure involves no more operations than are required without any filter since the zero readings are dispensed with. Furthermore no great loss of intensity is entailed. For example, for elements of medium atomic number, the difference reading with an ideally balanced filter is of the order of 60 per cent of the reading that would be obtained with no filter. The operation of such a filter in suppressing the effect of the non-coherently scattered radiation is now apparent.

The use of such a filter also relieves a further difficulty that existed in all previous work. As has been explained before[5], any observed isochromat is really the "smear" or (mathematically speaking) the "fold" of one function which represents the spectral window curve (and other functions which represent other effects that tend to blur the foot of the isochromat, if they exist, such as high voltage ripple or filament P.D.) with another function which represents the desired isochromat or the true dependence of intensity on voltage. If $\underline{g}(\underline{x})$ represents the spectral window curve, $\underline{f}(\underline{x})$ the true isochromat, and $\underline{F}(\underline{x})$ the observed isochromat, then the situation can be represented by the statement $F(z) = \int_{-\infty}^{\infty} g(x)f(z-x)dx$. Now the $\underline{g}$ curve is very closely approximated by the "witch"

$$g(x) = (1/\pi a)\left[1/(1+x^2/a^2)\right]$$

(normalized to unit area) where $\underline{a}$ is the half-width at half maximum and $\underline{x}$ is a variable proportional to frequency but measured from the center of the $\underline{g}$ curve. The $\underline{f}$ curve may be idealized into a linear function of $\underline{x}$ for all positive $\underline{f}$. That is

$$\begin{cases} f(x) = b(z-x) & \text{for } -\infty \leq x \leq z \\ f(x) = 0 & \text{for } z \leq x \leq +\infty \end{cases}$$

where $\underline{z}$ is the value of $\underline{x}$ at the threshold and is itself a variable depending on the voltage. The limits of integration can now be taken from

$-\infty$ to z, but because the integral diverges logarithmically it will be more suitable to integrate from some large finite negative number. Thus

$$F(z) = \frac{1}{\pi a}\int_{-Na}^{z} \frac{b(z-x)\,dx}{1 + x^2/a^2} = \frac{bz}{\pi}\int_{-Na}^{z} \frac{dx}{a + x^2/a} - \frac{ab}{\pi}\int_{-Na}^{z} \frac{x\,dx}{a^2+x^2}$$

$$= \frac{bz}{\pi}\left(\tan^{-1}(z/a) + \tan^{-1}N\right) - \frac{ab}{2\pi}\log\frac{a^2+z^2}{a^2(N^2+1)}$$

$$\approx \frac{bz}{\pi}\left(\tan^{-1}(z/a) + \pi/2\right) - \frac{ab}{2\pi}\log\frac{1+z^2/a^2}{N^2+1}.$$

This dependence of $F(z)$ on the lower limit N is unavoidable with these assumptions as to the form of the spectral window curve and the true isochromat, and the possibility of the dependence being real constitutes a serious criticism of any results obtained from such a curve. As a matter of fact, it can easily be shown that the method of the projected tangent for locating the true threshold from an observed $F(z)$, if reasonable assumptions are made as to the lower limit and the point from which the tangent is drawn, will introduce an error of the order of a, a large error. (The point of maximum bending of the observed isochromat locates rigorously the true threshold, but requires a more exact knowledge of the curve in the region most difficult to observe.)

However, when balanced filters are used, the whole difficulty immediately disappears, for the integration need only be taken over the pass band of the pair. In this case $F(z)$ will have the following form, where na is the half-width of the pass band,

$$\begin{cases} F_1(z) = 0 & \text{for } -\infty \leq z \leq -na \\ F_2(z) = \dfrac{1}{\pi a}\displaystyle\int_{-na}^{z} \dfrac{b(z-x)\,dx}{1 + x^2/a^2} & \text{for } -na \leq z \leq na \\ F_3(z) = \dfrac{1}{\pi a}\displaystyle\int_{-na}^{na} \dfrac{b(z-x)\,dx}{1 + x^2/a^2} & \text{for } +na \leq z \leq +\infty. \end{cases}$$

Carrying out the integration,

$$\begin{cases} F_1(z) = 0 \\ F_2(z) = \frac{bz}{\pi}\left(\tan^{-1}(z/a) + \tan^{-1}n\right) - \frac{ab}{2\pi}\log\frac{1+z^2/a^2}{n^2+1} \\ F_3(z) = \frac{bz}{\pi}2\tan^{-1}n. \end{cases}$$

For $\underline{z \geq na}$ the slope is now constant, namely $dF/dz = (2b/\pi)\tan^{-1}n$, and furthermore the projection of this portion of the curve passes through the origin. Hence, no error is made in extrapolating the straight portion of the observed curve to the axis as a means of locating the true threshold voltage.

For these two reasons then, a balanced Rh-Pd filter was borrowed from Professor Paul Kirkpatrick of Stanford University and inserted immediately in front of the ion chamber as shown in Fig. 20. The spectrometer was then set for $\lambda = 0.521$ Å, midway between the two edges, and an isochromat was run. The success of this procedure in reducing the fillet was immediate and astounding, and the resulting curve will be given in Fig. 21.

In order to increase intensity, the vertical extent of the beam passed by the various snouts and wedges was increased to nearly 2" at the entrance to the ion chamber, which means a total angular divergence of about $3\frac{1}{2}^{o}$. In ordinary spectral work vertical divergence must be avoided as it has the effect of decreasing the dihedral angle between the crystals, therefore decreasing the Bragg angle and the wave-length of the radiations passed, and giving an erroneously high value for the wave-length of a given line. But at the short wave-length limit no shorter radiation exists, so vertical divergence cannot shift the observed threshold voltage but will in fact increase the slope of the isochromat.

Summary of Improvements

It may be well to summarize the chief improvements in equipment or in technique by virtue of which the present experiment should give more reliable results than past measurements of the short wave-length limit of the continuous x-ray spectrum have given. These will be classified according to their purpose.

The indirect cathode, the many pains taken with the filament supply circuit, and the voltage regulator can be thought of as merely contributing to the stability of the x-ray tube, making the better measurements possible but affecting in no way the values obtained from these measurements. The current vernier, the very excellent two-crystal spectrometer and the new voltmeter with its elaborate calibration provide these more accurate measurements. Perhaps more fundamental than any of these are the various factors which insure that what is observed is the true short wave-length limit. One such factor is the use of freshly cleaved crystals to substantially improve the resolving power of the monochromator. This in turn is made practicable by the greatly increased x-ray intensity available with the Watters generator, aided by the higher sensitivity of the ion chamber circuit which is possible when the cosmic ray fluctuations are reduced. Other factors in this third group of improvements are the various precautions taken to prevent contamination of the target, the indirect cathode to eliminate excess energy secondary electrons, and the balanced filters to eliminate the effect of incoherently scattered radiation and to dispel all doubts arising from the possible diverging integral which is involved. In fact, the use of freshly cleaved crystals and the balanced filter technique just mentioned, probably combined with some of the other effects to lesser degrees, produce

isochromats with sufficiently clear intercepts as to avoid entirely the necessity for complicated methods of locating the true threshold voltage, as has previously been the case.

It might be interesting to give one rather spectacular figure to indicate the complexity of the whole set-up. The number of switches, valves, rheostats, etc. which must be operated to obtain one point on an isochromat, starting with everything turned off but with all possible permanent connections made, not counting repeats or any of the calibration operations, is 105.

III RESULTS

One good isochromat constitutes the results to be discussed in this report. The x-ray target had just been cleaned, and the pressure in the tube during the run was 10^{-5} mm of mercury. The cathode shield was at -45v with respect to the filament. The 5.3×10^{11} ohm resistance was used in the electrometer tube circuit. The curve is given in Fig. 21, where the ordinates are the differences between ionization currents when the two parts of the balanced filter are placed in the path of the radiation, all corrected to 60 ma across the x-ray tube, and the abscissae are potentiometer readings individually corrected for the IR drop across the milliameter and choke shown in Fig. 8. The foot of the curve is read at $.11846_0 \pm .00000_6$, where the probable error is obtained purely from geometrical considerations of possible straight lines through the given distribution of points. This is the only source of error in locating the true threshold. After applying the corrections to the Wolff, we have then $.11845_8 \pm .00000_6$v. Multiplying by the B stack ratio (since during this particular run the cathode half of the high-voltage set was grounded and only the B stack was used to measure the voltage) gives 23,773.9 int volts. The correction for the P.D. across the protecting choke has already been applied, and the drop of $45+1 = 46$v in the current vernier is not applied since that instrument was shorted out during this particular run. There remain the increase of half the filament P.D. (+2.6v) and the correction for the filament work function (+4.1v). This gives the true voltage across the x-ray tube to be 23,780.6 int volts. Using $\underline{d}_1 = 3.02904$Å and $\lambda = 1.27377$Å for the Au $L\alpha_1$ line, the angle used to calibrate the spectrometer is $12^\circ\ 8'\ 15''$. From the means

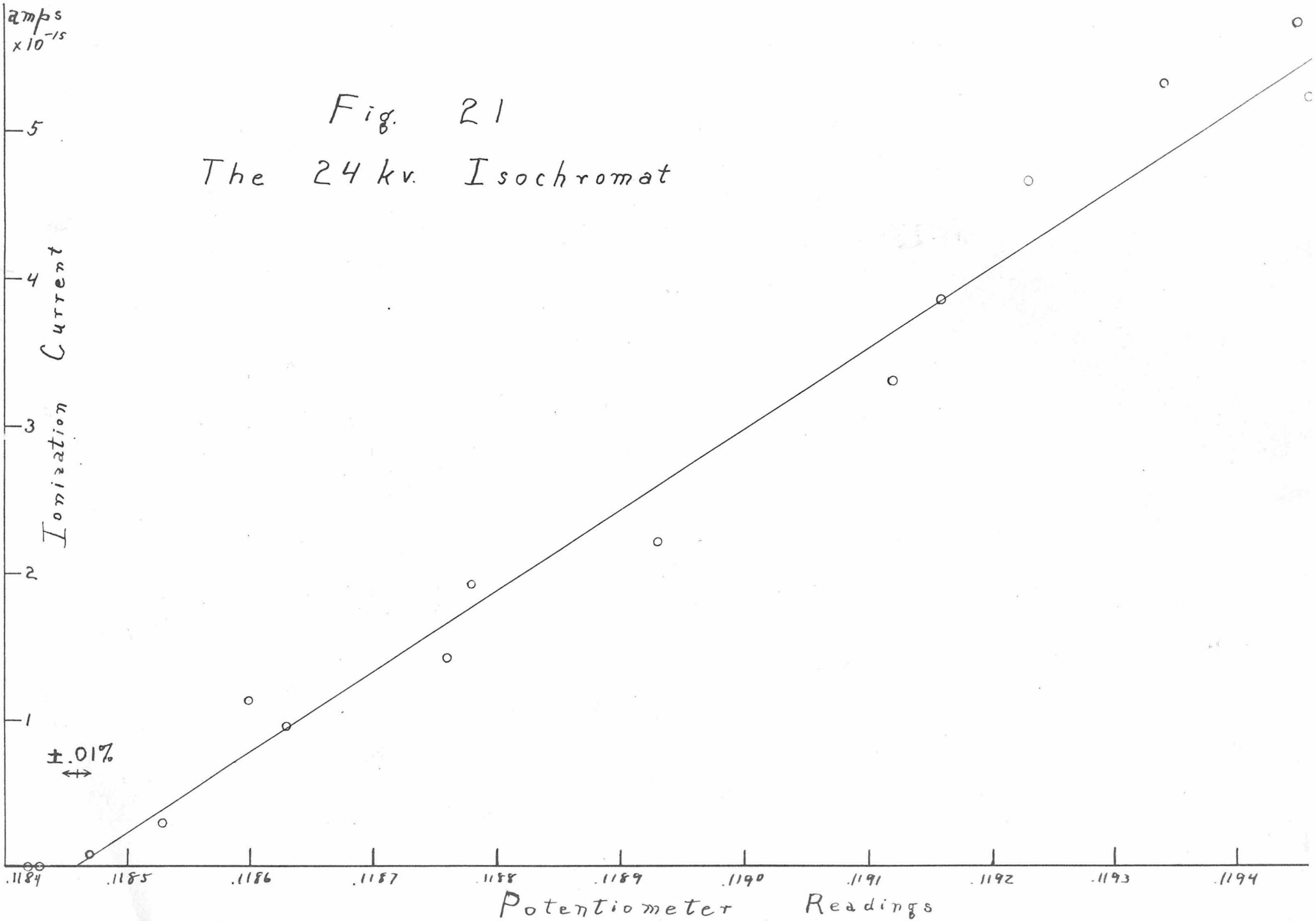

Fig. 21

The 24 kv. Isochromat

of two observations of the Au $L\alpha_1$ line and the known spectrometer setting for the isochromat run, the angle for the latter is $4^\circ\ 55'\ 42\frac{1}{2}''$. All angles were corrected for the change in grating space with temperature of the calcite crystals. The complete formula for h/e is

$$h/e = V(2d_1 \sin\theta)\ (\lambda_g/\lambda_s)\, pq/c^2$$

where V is in int volts, $d_1 = 3.02904 \times 10^{-8}$ cm, $(\lambda_g/\lambda_s) = 1.00203_4$ is the ratio of absolute to Siegbahm wave-lengths, $pq = 1.00034$ is the ratio of international to absolute volts, $c = 2.99776 \times 10^{10}$ cm/sec is the velocity of x-rays in a vacuum, and $c \times 10^{-8}$ is the ratio of absolute volts to abs esu. Substitution of these values gives

$$h/e = 1.3805_4 \times 10^{-17} \text{ erg sec/esu.}$$

It is noteworthy that this is the first time an experimental value of h/e has been obtained which was higher than the more accurate indirect value of 1.37929×10^{-17} erg sec/esu.

Discussion of Errors

The several sources of error in this experiment have already been discussed at length. It remains to summarize these and form an estimate of the probable error of the final result. We list, with the errors given as parts in 100,000.

voltmeter calibration	14
change in calibration	31 (?)
voltage measurement (Wolff)	3
standard cells	10
extrapolation on curve	5
spectrometer calibration	75
geometrical sum	83

We therefore conclude from the present data that

$$h/e = (1.3805_4 \pm .0011_5) \times 10^{-17} \text{ erg sec/esu.}$$

Obviously two points must be worked on further. First, the voltmeter calibration must be repeated to determine whether or not the change is real. The accuracy of the calibration itself will be hard to improve upon. Second, the huge uncertainty in the wave-length calibration must be cleared up. It should not be difficult to reduce this error to 6/100,000 (1" of arc), or even less by leaving the spectrometer set on some known wave-length and taking an isochromat at that wave-length. The final error should then be reducible to something like 0.02%.

Present Status of the Discrepancy

Fig. 22 shows the isometric consistency chart exactly as it was given in Fig. 1 save for the addition of our result stated above. In Fig. 23 a similar chart with scales doubled shows only three values, each with its probable error. These three values are the three most accurately known functions of the atomic constants, namely the ruled grating - x-ray value of $\underline{e}$ combined with the Rydberg constant $\underline{R}_\infty$ (mean value as given by Birge[14]), the specific charge on the electron $\underline{e}/\underline{m}_o$ (also Birge's mean value incorporating the new value of $\underline{q}$) and our value of $\underline{h}/\underline{e}$. The only justification for using the single experimental value of $\underline{h}/\underline{e}$ against the means of several determinations of the other two quantities is the belief that our value of $\underline{h}/\underline{e}$ is more reliable than any of the others, and this belief comes from the knowledge that the results of DuMond and Bollman[5], previously acknowledged to be the most accurate in the field, are now known to possess an error (of the "right" sign and order of magnitude) due to the lack of any correction for the heating under load of the resistances used by them to measure their voltage. Any weighted average (necessarily arbitrary) constructed from our value

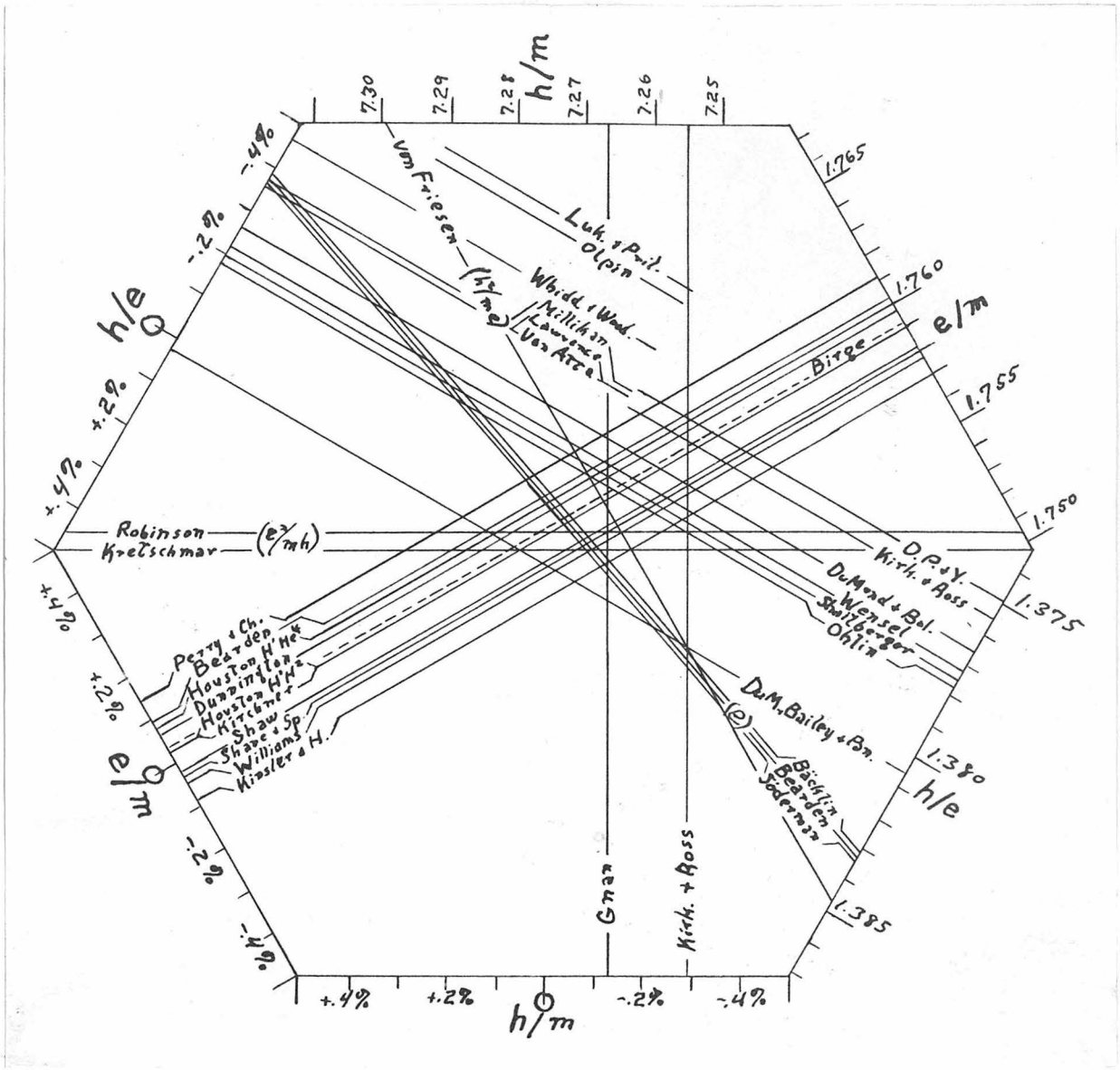

Fig. 22. An isometric consistency chart like that of Fig. 1 with the addition our present result. ~~Turn clockwise.~~

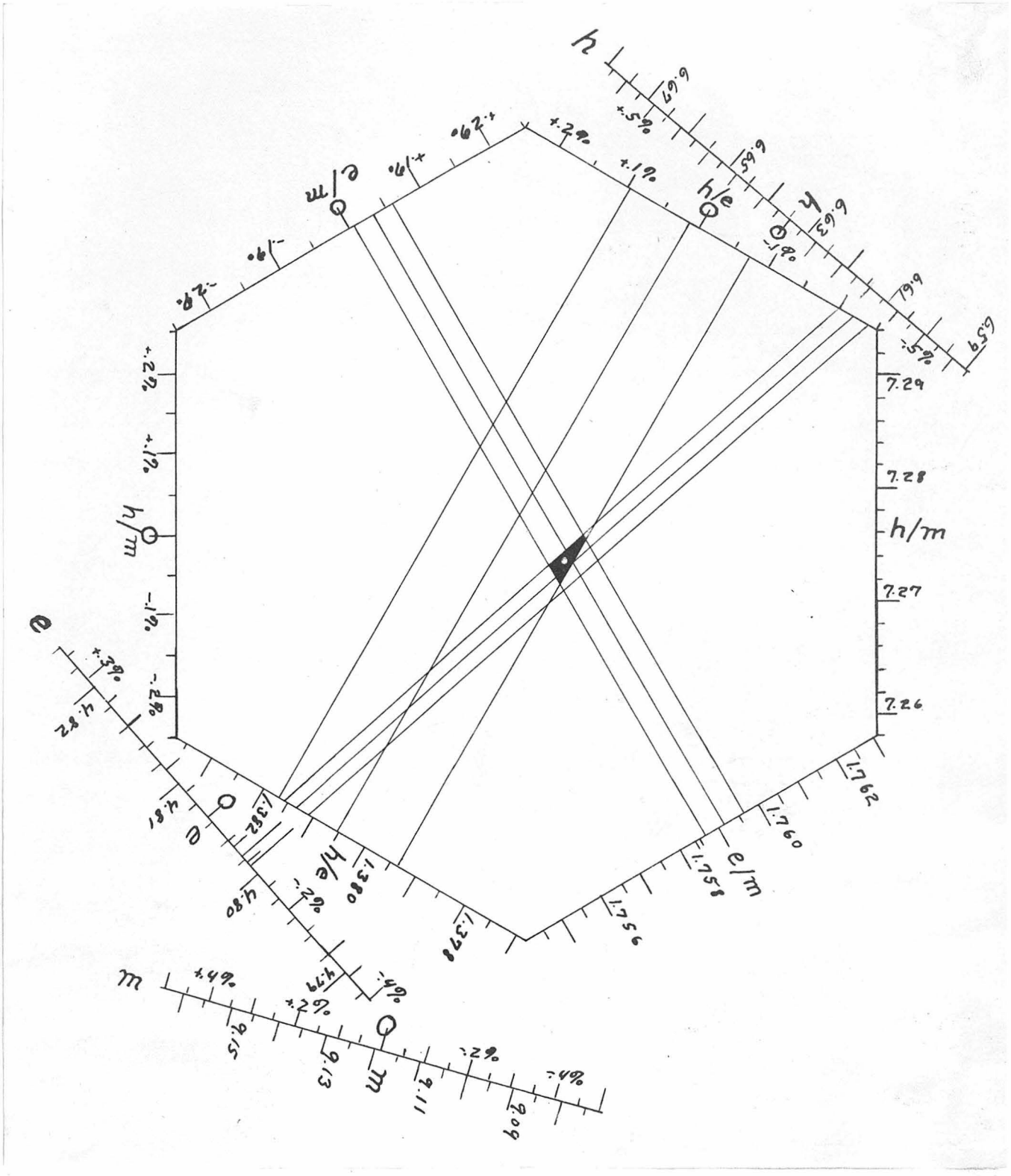

Fig. 23. An isometric consistency chart with expanded scales exhibiting the ~~r.e~~ solution of the discrepancy among the atomic constants e, h and m_0. (Turn counter clockwise.)

with some consideration for the previous values would of course only improve the agreement with the indirect value, but the temptation to do this should be resisted at least until our final value is available, if not indeed altogether.

The overlapping of the values shown in Fig. 23 demonstrates the resolution of the discrepancy among the natural atomic constants $\underline{e}$, $\underline{h}$ and $\underline{m}_o$. It therefore seems highly probable indeed that the real values of these constants lie in the overlapping region and may be tentatively given, as read from the graph, as

$$e = 4.803_0 \times 10^{-10} \text{ esu}$$

$$h = 6.625_4 \times 10^{-27} \text{ erg sec}$$

$$m_o = 9.109_6 \times 10^{-28} \text{ gm.}$$

Conclusion

It must be strongly emphasized that the results given in this report are preliminary. Before final values can be published the following things should be done: repeat the calibration of the voltmeter and of the spectrometer as mentioned above, make a definite search for any possible high frequency oscillations in the power supply, get more points on a single isochromat by maintaining reproducible conditions on the target from day to day, investigate further the "knees" first reported by DuMond and Bollman, examine the effect of the cathode shield potential on the isochromats, and conduct the experiment at voltages up to 100 kv.

After these and probably other checks have been made, if the discrepancy is still as well disposed of as at present, the resulting values of $\underline{e}$, $\underline{h}$ and $\underline{m}_o$ should be satisfactory until the time when accuracy shall be required in the next significant figure.

ACKNOWLEDGMENTS

I wish first of all to express my very deep gratitude and feeling of indebtedness to Professor J. W. M. DuMond who has worked on this problem more or less intensively for the past seven or eight years and has been a constant source of valuable suggestions, specific teachings and general inspiration to me since I joined his group. Secondly, I owe much to all those, notably Mr. J. Paul Youtz, who have worked in the past on the Watters generator and some of the auxiliary equipment. Thirdly, Mr. W. K. H. Panofsky, who will write his thesis on this same problem, has made many contributions (ranging from a safety light indicating when the high voltage is on to the all-important voltage regulator) and has been a resourceful and indefatigable co-worker for over two years. Mr. A. E. S. Green has aided materially in taking the latest data. Acknowledgment is also due Professor Paul Kirkpatrick of Stanford University for the loan of the Rh-Pd balanced filter, and the Socony-Vacuum Oil Company of New York City for the gift of 20 gallons of special transformer oil.

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