V-PARTICLE PRODUCTION

Thesis by

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Dr. Robert Leighton supervised and actively participated in obtaining all the data contained in this paper. Each section of this paper reflects the ideas and modes of approach offered by Dr. Leighton in our numerous discussions. It was Dr. Leighton's insight of the various V-particle phenomena that suggested some useful information could be obtained by a study of the penetrating shower particles, their interactions in the lead plate, and a correlation of the various momenta and angular distribution of the charged particles contained in stars.

Nearly every phase of the paper was discussed at various times with Dr. Carl Anderson. His numerous suggestions and questions are reflected throughout the entire paper.

Numerous helpful suggestions were offered by Dr. R. F. Christy. This was especially true in our discussions of the possibility of a high angular momentum in the V^0-particle.

The results obtained in the cloud chamber operated by the author were not impressive nor of the same quality as was obtained in the cloud chamber operated by Dean Wanlass and William Alford. The data contained in the present paper is their data.

ABSTRACT

In a set of about 23,000 cloud chamber photographs, 37 V^0-particles and 9 charged V-particles were produced in nuclear interactions in a lead plate between two cloud chambers in a magnetic field. An analysis of the circumstances of occurrence of V-particles strongly indicates that the V-particles were produced principally by mesons. Production of V^0-particles by nucleons is also indicated. By a study of the multiplicity of the stars containing V^0-particles, the average momentum of the particles producing V^0-particles is found by several methods to be about 10 Bev/c. At present it appears that all V^0-particles were produced by particles having momenta in excess of about 2 Bev/c. It is found that the V^0-particle is produced in the center-of-mass system with about 400 Mev/c momentum.

In order that the cross-section for V^0-particle production be checked, an interaction length for penetrating shower particles is found to be 340 ± 40 gr/cm^2, and is a value that is consistent with the results obtained in other cloud chamber experiments. About one in twenty interactions by penetrating-shower particles whose momenta are in excess of 1 Bev/c results in a V^0-particle being produced. About 2 percent of all of the shower particles produced in penetrating-shower particle interactions are V^0-particles.

A measurement which might indicate a high angular momentum in the V^{o}-particle is obtained for each V^{o}-particle produced in the lead plate. The best examples are recalculated. There is a weak indication that the V^{o}-particle has a high angular momentum.

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I. INTRODUCTION

Recent cosmic-ray observations have established the existence of a number of new unstable particles. Correlating the Greek alphabet with the masses of sub-nuclear particles has been nearly rampant. The V-particles, first reported in 1947[1], differ from the other new particles in at least two respects. The name V-particles describes the characteristic angular appearance of the decays. High energy interactions of cosmic-ray particles with nuclei produce both charged and neutral V-particles. Cloud chambers have been the principal instruments for V-particle detection. Since only charged particles produce visible tracks in a cloud chamber, the neutral V-particles are not detected until they decay into charged particles. The observed neutral V-particle decays have resulted in two oppositely charged particles, which in many instances have been identified as protons (positive) and π-mesons (negative). Neutral V-particles having such a decay scheme are designated V_1^0-particles. Some neutral V-particle decays have been observed in which the positive particles are not protons, but not much information on these particles is available. In the subsequent material in this paper, the subdivision of the neutral V-particles will not be employed but they will be collectively called

V^o-particles. Examples of V^o-particle decays are given in Figures 2, 3, and 4.

The V^o-particles have been the subject of extensive investigations, while very little is definitely known regarding the charged V-particles. Unstable charged particles, positive and negative, do exist and are heavier than the unstable π-mesons (276 m_e). However, the existence of charged V-particles comparable to the neutral V-particles in mass and lifetime has not been established with certainty. Indeed, the articles regarding charged V-particles should be viewed with some caution since much of the information was obtained before the K-mesons were reported. Unstable particles, positive and negative, with masses about 1250 m_e and having relatively long lifetimes, have been observed(2). The examples of charged "V-particles" referred to in the subsequent pages are tacitly assumed to be V-particles. Although the charged V-particle decays were not completely analyzed, in no particular case could the decay be readily identified as that of a K -meson. The large number of charged particles decaying in the neighborhood of their origin, indicates the existence of a shorter lived particle than the K-meson. Examples of charged V-particle decays are given in Figures 5 and 6.

In 23,000 successive cloud chamber photographs, 134 examples of the V^o-particle decay and 18 examples of the charged

V-particle decay were obtained. In these examples, the Q-values for the V^o-particle decays and some of the other properties of the V^o-particles have been measured(3). Confining the analysis to these same examples, the aim of the present paper is to further these investigations by presenting the information obtained in regard to the nature and circumstances of V-particle production.

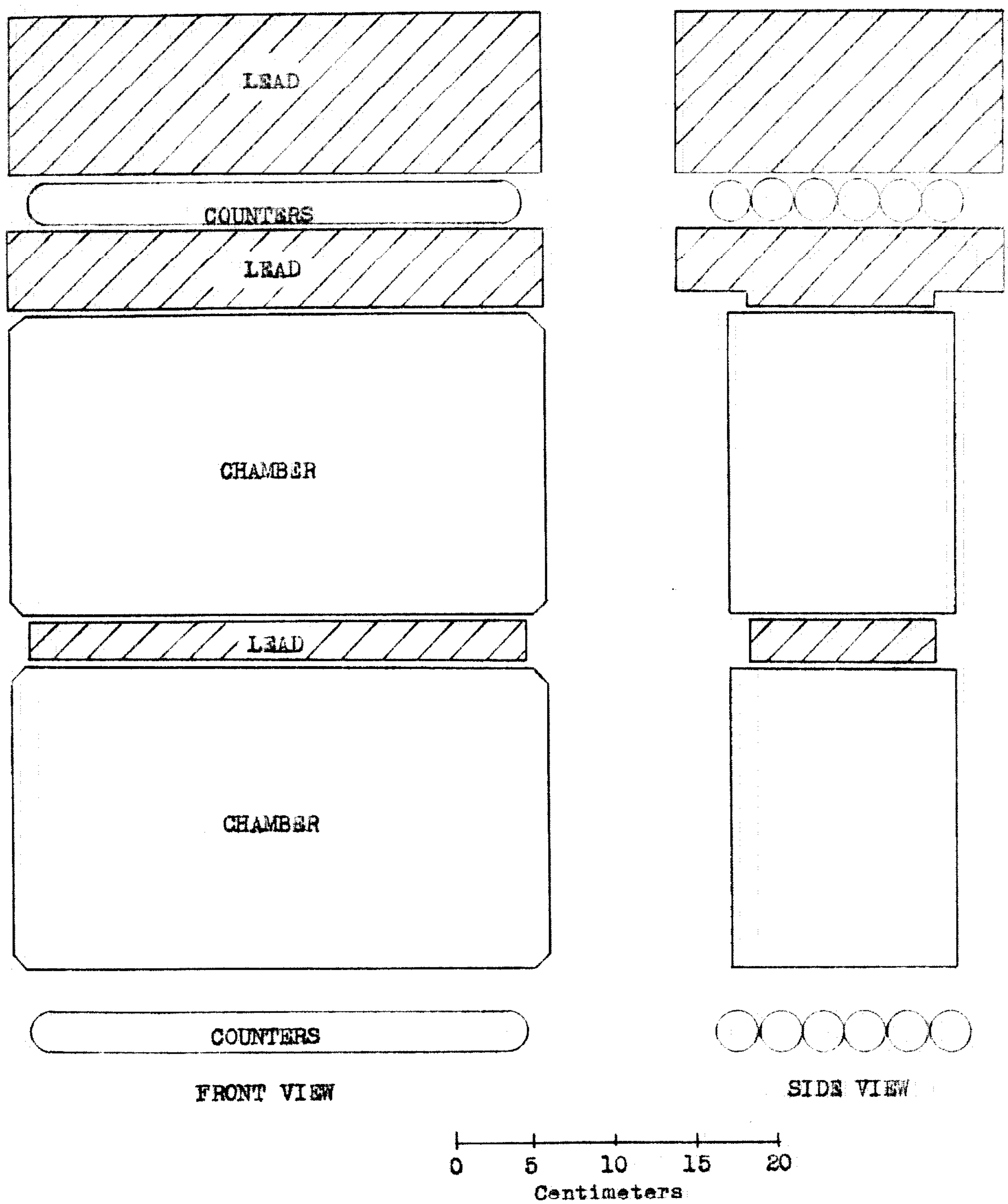
II. APPARATUS

The data for this paper was obtained in a double cloud chamber. The two cloud chambers were arranged, one above the other, as shown in Figure 1. A 2.5 cm lead plate occupied the space between the two cloud chambers. The lead plate was the site of many of the analyzable interactions. Lead absorbers above the cloud chambers shielded them from low energy particles and provided material for high energy interactions. The pole structure, yoke and magnet windings provided additional absorber, but are omitted from Figure 1. The cloud chambers were between the poles of a magnet normally operated at an induction of 5000 gauss. The cloud chambers were enclosed in a copper lined, insulated box, thermostatted to approximately $\pm 0.1^{\circ}$ C, to reduce cloud chamber distortions. Stereoscopic photographs were taken from the front of the two cloud chambers using side illumination. About two-thirds of the data was taken at an elevation of 220 m and the remainder at 1750 m.

Two trays consisting of six counters each were located above and below the cloud chambers (figure 1). A counter is pulsed by the passage of one or more charged particles through its surface. The cloud chambers were actuated and a photograph was taken when a proper coincidence of pulses occurred in the two trays of counters.

FIGURE 1

Arrangment of Lead and Counters



All of the data at 220 m, and three-fourths of the data at 1750 m, was taken with a coincidence requirement of one or more counters in the top tray and three or more counters in the bottom tray. With this requirement, about 20 percent of the photographs at 1750 m contained penetrating showers. The remainder of the data at 1750 m was taken with the coincidence requirements for the top tray increased to two or more counters, whereas the bottom tray requirements were unchanged. The photographs containing penetrating showers were increased to 40 percent. There was a corresponding increase in the small fraction of photographs containing V-particles.

III. CIRCUMSTANCES OF OCCURRENCE OF V-PARTICLES

Before investigating in detail the production of V-particles, several conclusions can be drawn from a classification according to the circumstances of their occurrence.

A. Multiple Occurrences

Among the photographs containing 134 V^o-particles and 18 charged V-particles, ten photographs contained more than one V-particle. The photographs with multiple occurrences were: 1 with three V^o-decays; 3 with two V^o-decays; 4 with one V^o-decay and one charged V-decay; and 2 with two charged V-decays. Figure 2 is an example of a photograph with two V^o-particle decays.

The orientation of the V-particles in the multiple occurrences was such that none indicated the origin of a V-particle as the decay product of a previous V-particle. However, in three cases the same nuclear event could have been the origin of two V-particles. Assuming comparable lifetimes of about 3×10^{-10} seconds, it has been shown that if V-particles were always produced in pairs, the number of such examples should have been much larger [3]. The conclusion that V-particles are not always produced in pairs was further supported by the fact that the number of multiple decays from different origins

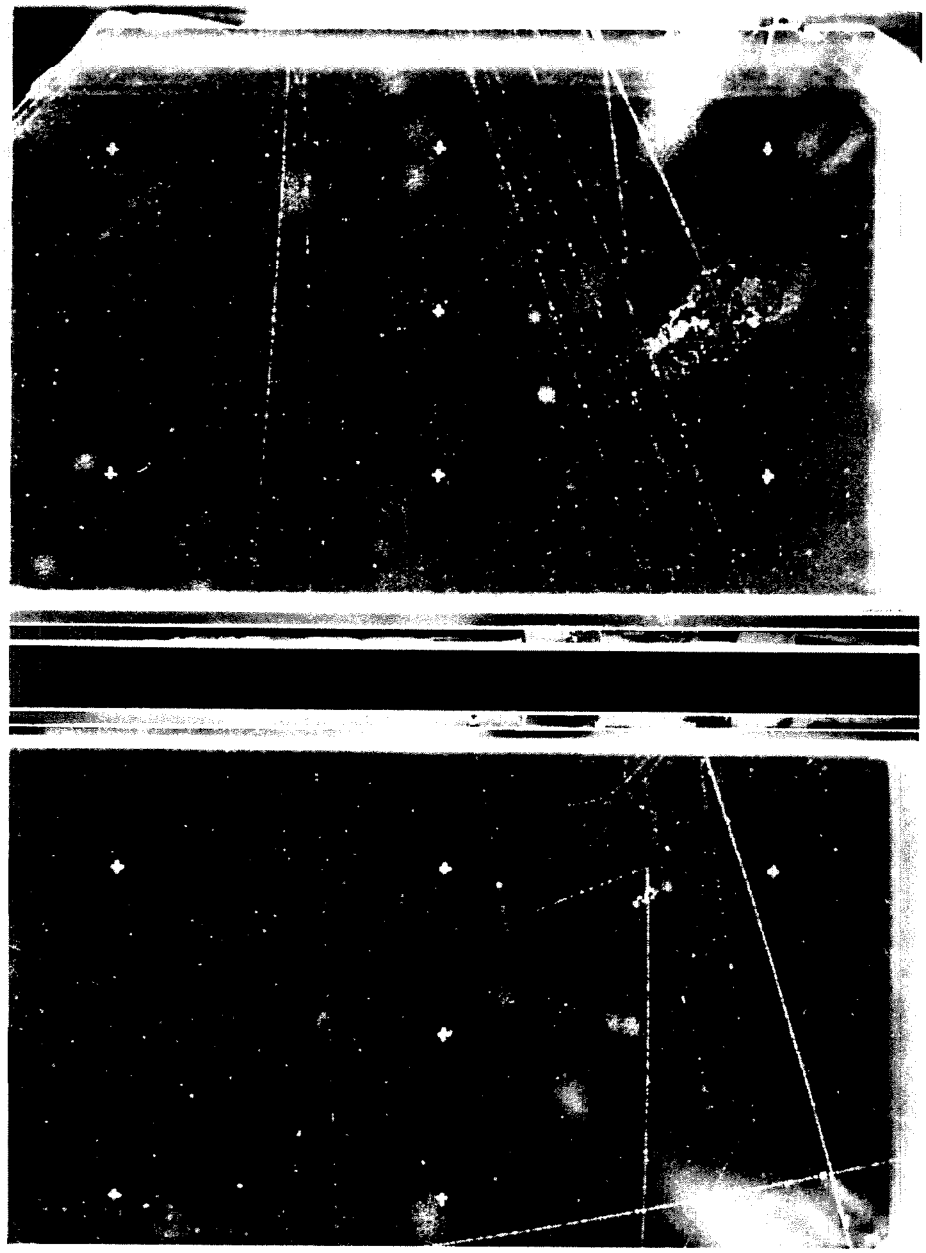


FIGURE 2

V^o-Particle Production By A Penetrating-Shower Particle

Frame 23942. The top chamber shows a typical penetrating-shower. One of the penetrative shower particles initiates a star in the lead plate. One of the particles in the star is a heavily ionizing proton. The interaction is also the origin of the V^o-particle which decays into a proton and a negative π-meson. There is a second V^o-particle decay in the upper left portion of the top cloud chamber. This V^o-particle could have originated with the penetrating shower.

exceeded the number from the same origin.

Ten photographs with multiple occurrences suggest a rather copious production of V-particles. The multiple occurrences can be interpreted as a large production probability when the proper circumstances prevail. The simultaneous presence of a large number of high energy penetrating-shower particles was the circumstance in nearly every case of multiple occurrence. Figure 2 is illustrative of these conditions.

B. V-Particle Production in the Lead Above the Cloud Chambers with an Accompanying Penetrating Shower

A penetrating shower is defined as two or more parallel or clearly associated particles, at least one of which penetrates the lead plate between the two cloud chambers. A penetrating-shower particle usually penetrates the plate with very small scattering and remains at minimum ionization. Such a particle has high momentum, showing little or no visible curvature in the 5000 gauss magnetic induction. Measurements show that the penetrating-shower particles often have momenta exceeding 2 Bev/c. The penetrating-shower particles are secondaries of high energy nuclear interactions in the iron or lead, and sometimes the air, above the top cloud chamber. About one-half of the penetrating showers are in excess of four particles. The penetrating shower particles are very nearly parallel and are oriented in a generally vertical direction. There were 72 examples of V^o-particles and 9 charged V-particles decaying in the cloud chambers, with origins in the lead above the cloud chambers, and at the same time were accompanied by penetrating showers. By observing that a large fraction of the V^o-particles originating in the lead plate was produced by penetrating-shower particles, it is inferred that the interactions above the cloud chambers were also largely by penetrating-shower particles. Figure 2 has a V^o-particle produced in the lead above the cloud chambers.

C. V-Particle Production in the Lead Plate between the Cloud Chambers

Thirty-seven V^o-particles and nine charged V-particles had origins in the lead plate between the two cloud chambers. The origins of the V-particles were established by obvious interactions in the lead plate. An average of about four high-momentum particles accompanying the V-particles were projected back into the lead plate to locate the origins. By observing the examples with definite origins, it was found that an extension of the line of flight of the V^o-particle, before decaying, passes between the paths of the decay products. When more than one interaction was present in the lead plate, the V^o-particle was ascribed to the proper origin by requiring a fulfillment of this condition. Little difficulty was found in ascribing a charged V-particle to its proper origin, since its track can be projected back into the lead plate along with the other charged particles to the interaction.

Each example of V-particle production by a charged particle was very carefully analyzed to ascertain whether the charged particle producing the V-particle passed, within the accuracy of measurement, through the star origin. It was further required that it should have been visible in the lower chamber, had it penetrated the lead plate, and that it was not observed in the lower chamber unless it was scattered at the star origin through an angle greater than five degrees. This procedure eliminated the possibility of production by an unseen neutral particle. Four V^o-particles were produced by neutral particles. Figure 4 is an example of V^o-particle production by a neutral particle.

The nine charged V-particles having origins in the lead plate were all produced by charged particles. The statistics are too poor to say this is significant, since only one in nine V^o-particles was produced by a neutral particle. Examples of charged V-particles produced in the lead plate are illustrated in Figures 5 and 6.

D. V-Particles with Unidentified Origins

Twenty-five V^o-particles occurred alone in the cloud chambers, or were not clearly associated with any nearby events.

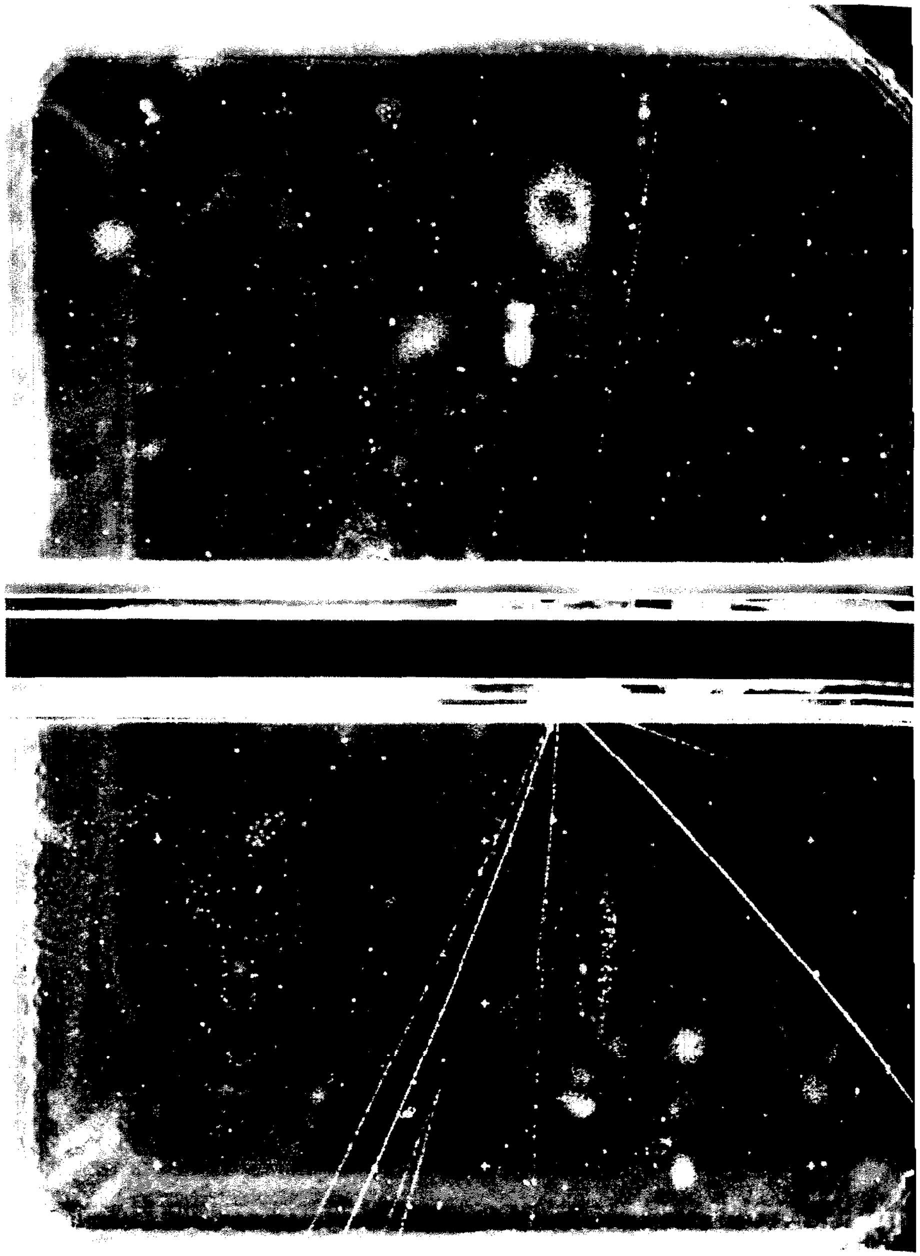


FIGURE 3

V^{o} Production by a Lone Charged Particle

Frame 17685. A single particle traverses the top chamber and initiates a star in the lead plate. The star has 6 charged particles. At least two of the star secondaries are protons. The V^{o}-particle decays near the bottom of the lower chamber.

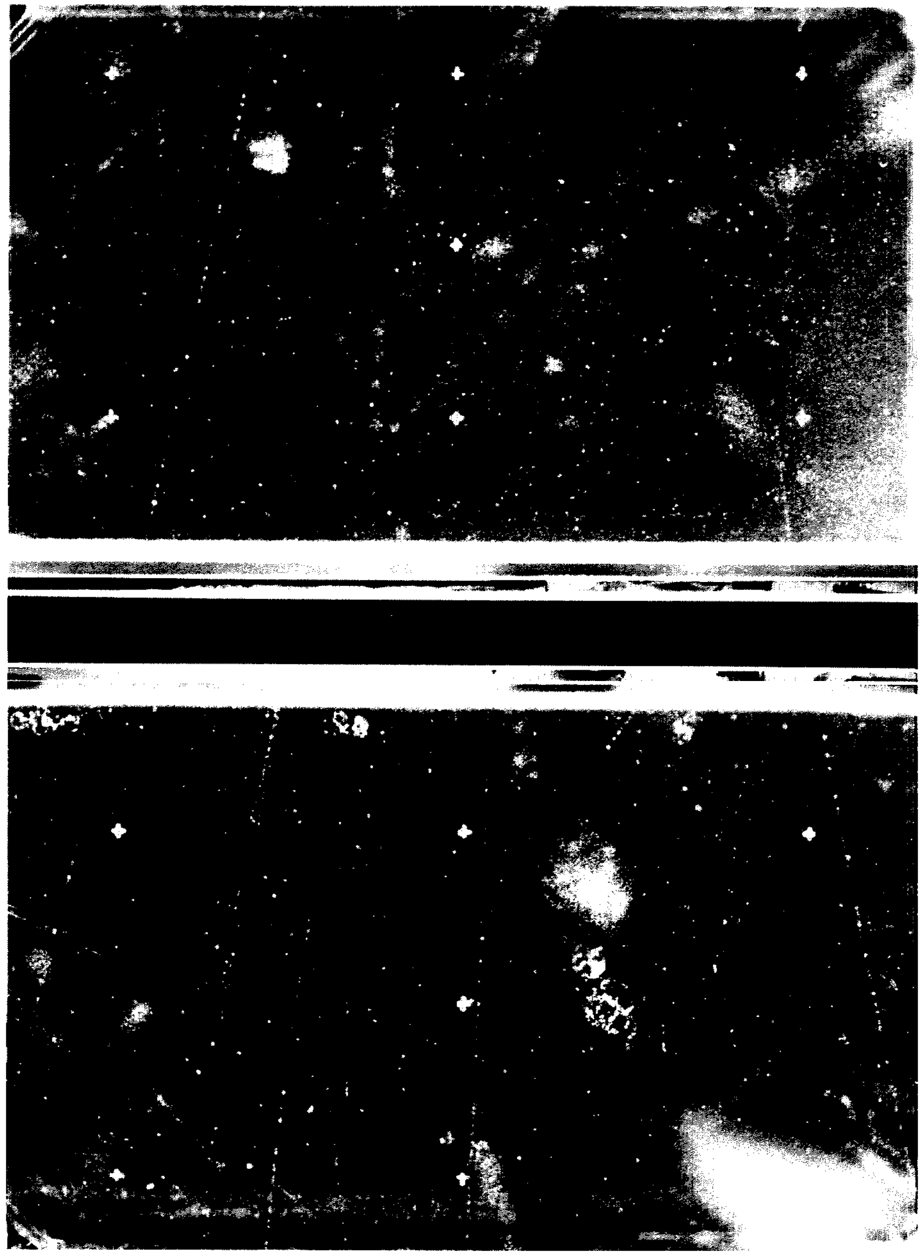


FIGURE 4

V^o Production by a Neutral Particle

Frame 24390. In the lower chamber are 4 charged particles and a V^o-particle from an interaction in the lead plate. The interaction occurs almost midway between the front and back of the chambers. Projecting the general axis of the star back into the top chamber indicates that the producer would almost certainly have been seen, if it was a charged particle.

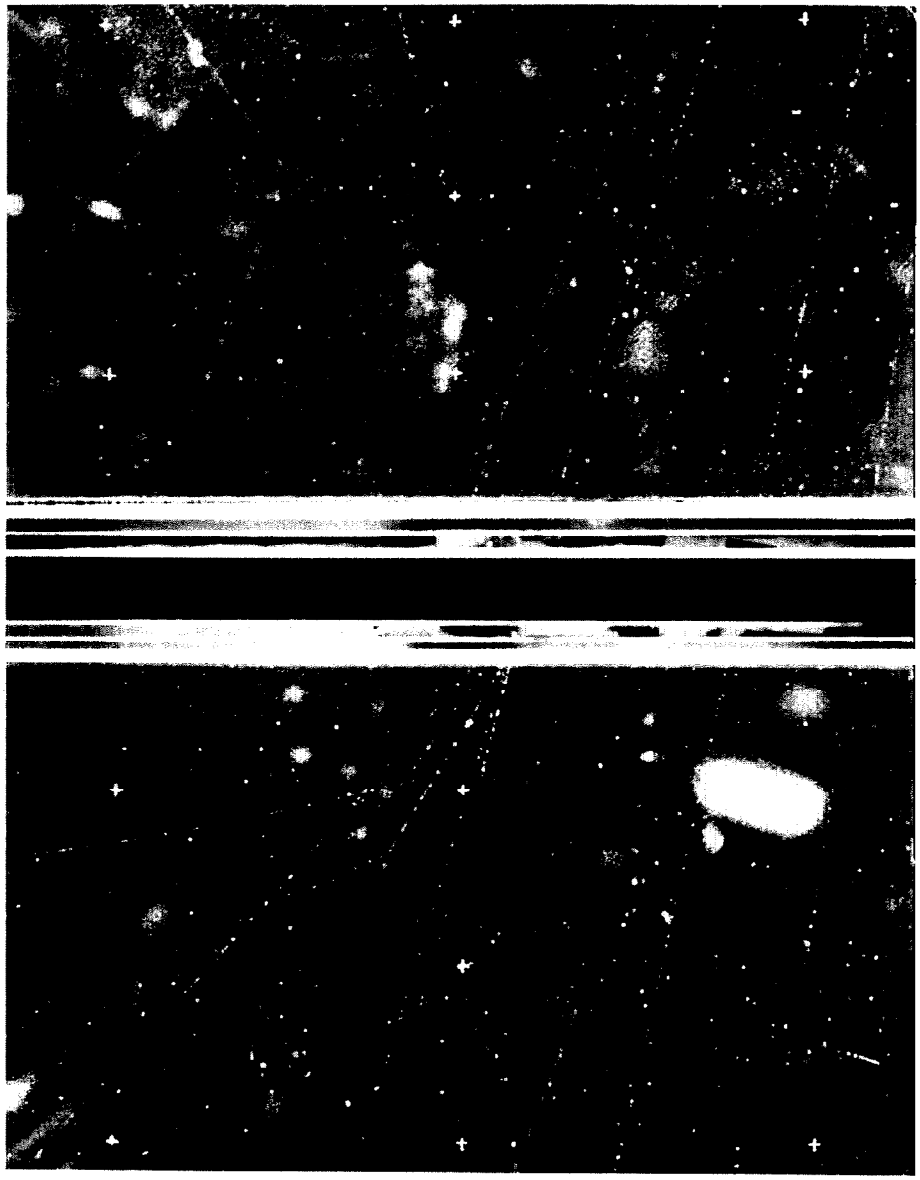


FIGURE 5

Charged V-Particle Production by a Penetrating-Shower Particle.

Frame 26622. A penetrating-shower particle in a typical shower interacts in the lead plate producing a star. One of the particles in the star is a charged V-particle.

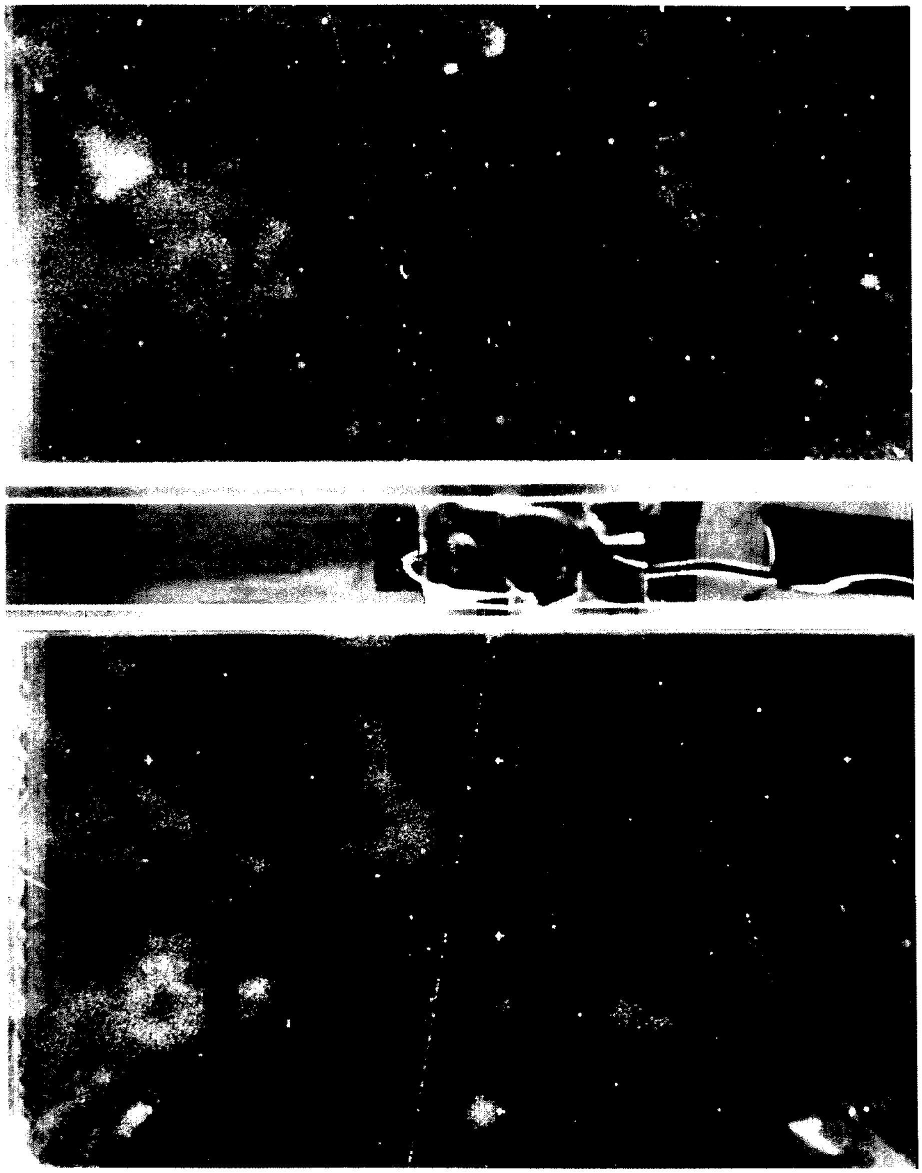


FIGURE 6

Charged V-Particle Production by a Penetrating-Shower Particle.

Frame 13484. Three penetrating-shower particles traverse the top cloud chamber. One of the shower particles produces a star having a charged V-particle and another particle. One of the shower particles decays below the plate.

A summary of the above data is given in Table 1.

E. What the Circumstances Seem to Imply

1. The Initiating Particles

Both charged and neutral particles impinging upon a nucleus can produce V^{o}-particles. There are two theories as to how a nuclear collision results in a number of particles being emitted. In the multiple theory the mesons result from an interaction of the incident particle with a single nucleon in the nucleus. However, in the plural theory, the mesons are produced in the nucleus by collisions involving nucleons recoiling from the incident particle. If V^{o}-particles are produced according to the multiple theory, then their production process may not be charge dependent since they were produced by both charged and neutral particles. Since all the observed charged V-particles were produced by charged particles, their production process may be charge dependent.

A neutron is the simplest interpretation of the neutral particle producing a V^{o}-particle. By the assumption of a charge independent interaction between nucleons, it is possible to account for the qualitative features of currently known n-p and p-p scattering data[4]. From the equivalence of nucleons, protons should produce V^{o}-particles in about the same number as neutrons, since it is estimated that the number of

TABLE 1

Circumstances Of Occurrence Of V-Particles.

$V^{\pm}$	V^{0}	
9	72	Occurred in penetrating showers whose origins were in the lead above the chambers.
9	37	Occurred in nuclear interactions (stars) in the lead plate between the two chambers.
0	25	Occurred alone in the chambers, or were not clearly associated with any nearby event.
18	134	

Circumstances In The Lead Plate.

$V^{\pm}$	V^{0}	
8	27	Were produced by charged penetrating shower particles.
1	6	Were produced by lone charged particles unaccompanied by penetrating showers or other nuclear particles.
0	4	Were produced by neutral particles. 1 case oriented parallel with a penetrating shower. 3 cases unaccompanied by penetrating showers.
9	37	

neutrons is only 1.25 times the number of protons in penetrating showers[5]. The majority of the V^0-particles were produced by penetrating-shower particles. The meson percentage in penetrating showers will be shown (Part V, Section F) to be of the order of 80 per-cent. Mesons could be the main producers of V^0-particles. The mesons are probably not all π-mesons. Because the energies involved in producing the stars containing V-particles are large, the number of heavy mesons may be significant[2]. No single type of initiating particle would be consistent with what was observed and production by nucleons and mesons is strongly indicated. It should be recalled that the counter coincidence requirements were biased to take pictures of penetrating-shower components. The counters are further biased for charged particles since they are not affected by neutral particles. Nucleons and mesons could be equally effective when possessing the necessary energy.

2. Why the Difference in the Numbers of Charged and Neutral V-Particles?

There is a large difference in the observed number of charged and neutral V-particles observed in the cloud chambers. The difference does not seem to be due to a large difference in the necessary energy for production. The multiplicity (number of particles) of the stars, which is a measure of the energies of the initiating particles, is shown in Part IV to be about the same for stars containing charged and neutral V-particles. A case was found in which both a charged V-particle and a V^0-particle were produced in the same interaction. This example also indicates that the necessary energy for the production of either V-particle is about the same.

In part, the difference in the numbers of charged V-particles and V^0-particles that are observed can be accounted for by the relative ease of identifying a V^0-decay. The V^0-decay is characterized by two charged particles from a point in the gas of the cloud chamber. A high momentum V^0-particle decay is likely to be seen. However, a high momentum charged V-particle decay is apt to be missed since there is only a small angular deviation somewhere along an otherwise single, charged particle track. Further, a charged V-particle decay near the top of either cloud chamber is very difficult to identify.

A significant difference in the lifetimes would account for a difference in the numbers of charged and neutral V-particles that are recorded. Returning to Table 1, it is seen that the highest ratio of charged V-particles to V^0-particles occurs for origins in the lead plate. In these cases, the region in the vicinity of production is being observed. A smaller ratio of charged V-particles to V^0-particles is found for those originating above the cloud chambers and no charged V-particles are without origins. These circumstances would be consistent with the charged V-particles having the shorter lifetime, providing they are of comparable momenta, and hence have about the same time dilations in their lifetimes. The charged V-particles appear to have momenta about equal to the average for the V^0-particles. The observed number of charged V-particles and V^0-particles should be about the same if produced in equal numbers. The conclusion that the cross-section for V^0-particle production is larger than that for charged V-particle production by a ratio of about three to one would be consistent with the number seen.

IV. AN ANALYSIS OF THE STARS CONTAINING V-PARTICLES

The interaction of a high momentum particle with a nucleus usually results in a number of secondary particles being emitted from the nucleus. Such an event is termed a star. Occasionally a V-particle is also among the emitted particles. In a star in which a V-particle is produced in the lead plate between the two cloud chambers, the incident particle that undergoes the nuclear collision can usually be seen traversing the top cloud chamber. By a study of the incident particles it has been hoped that the signs of the charges could be detected. The incident particles carrying a negative charge would have substantiated the previous indications that in the main V-particles are meson produced. It has been assumed that electrons do not produce V-particles. Due to their very high momenta, a study of the individual incident particles was fruitless except for a lower limit of 2 to 5 Bev/c being placed on their momenta. A lower limit on the momenta of straight tracks is one that is consistent with the small distortions observed in photographs obtained with no magnetic field. Although several of the incident particles possessed curvature, the curvature was not uniform along the length of the track and was therefore attributed to distortions.

Since the momentum of the particles producing the V-particles could not be measured directly, a detailed study of the stars containing

V-particles was undertaken to obtain an order of magnitude for their momenta. There is considerable information concerning stars observed in photographic emulsions and in cloud chambers. The energy of the incident particle has been correlated with average star size. The angular distribution of the star secondaries has been plotted. There is some evidence that meson-produced stars have characteristics which differentiate them from stars produced by nucleons. A comparison of stars containing V^o-particles and charged V-particles with this data seemed most profitable.

A. Classification of the Star Particles

By an analysis of only the V-particles produced in the lead plate between the cloud chambers and neglecting those produced above the cloud chambers, fewer particles in the stars were lost due to chamber geometry, and the observed number of particles in each star is more indicative of the true star size. The angular distribution of the star particles is more meaningful if it is with respect to the incident particles undergoing the nuclear collisions. This can be done only for interactions in the lead plate between the cloud chambers. In these cases the incident particles can be seen traversing the top cloud chamber. Although the number of examples of V^o-particle production is greatly reduced by an analysis of only those produced in the lead plate, the thirty-seven occurrences in the lead plate are thought to be statistically sufficient in number.

The cloud chamber tracks, produced by the charged particles emerging into the gas, in stars originating in the lead plate between the two cloud chambers were divided into three classifications.

1. Tracks with Ionization Less Than 1.5 (n_s)

The total number of shower particles in a star is termed n_s. The particles with ionization less than 1.5 consist of π-mesons with momenta greater than 170 Mev/c, and protons with momenta greater than 1 Bev/c. In those particles termed n_s, no attempt was made to identify the particles as actually mesons or protons. Since many of the particles are very straight in the magnetic field, and are at minimum ionization, the division would have been very incomplete. The average star containing a V^o-particle will be found to have n_s equal to four. Because of their high momenta, almost no shower particle produced in the lead plate fails to emerge into the gas of the lower cloud chamber, and the n_s recorded in the cloud chamber is essentially that for photographic emulsions. The assumption is made that a star and its constituents produced by a high momentum meson or proton interaction with a silver or bromine nucleus in a photographic emulsion is essentially the same as an interaction with a lead nucleus. The designation n_s in photographic emulsions is for tracks with ionization less than or equal to 1.4[5]. Visual ionization estimates in the cloud chamber can be approximately given as less than about 1.5, or minimum.

2. Tracks with Ionization Greater Than 1.5 (N_h)

The total number of heavily ionizing particles in a star is designated as N_h. The mesons and protons with ionization greater than about 1.5 have momenta less than those recorded above as n_s. Few evaporation protons exceed energies of 25 Mev and have ranges too small to emerge from the lead plate, except when produced very near the surface. Many protons with larger energies fail to enter the gas. This classification is smaller than that for the actual star. However, few of the heavily ionizing particles that enter the gas can be mistaken. This classification corresponds to the photographic emulsion criterion for N_h, except in that case the lower limit of ionization is 1.4.

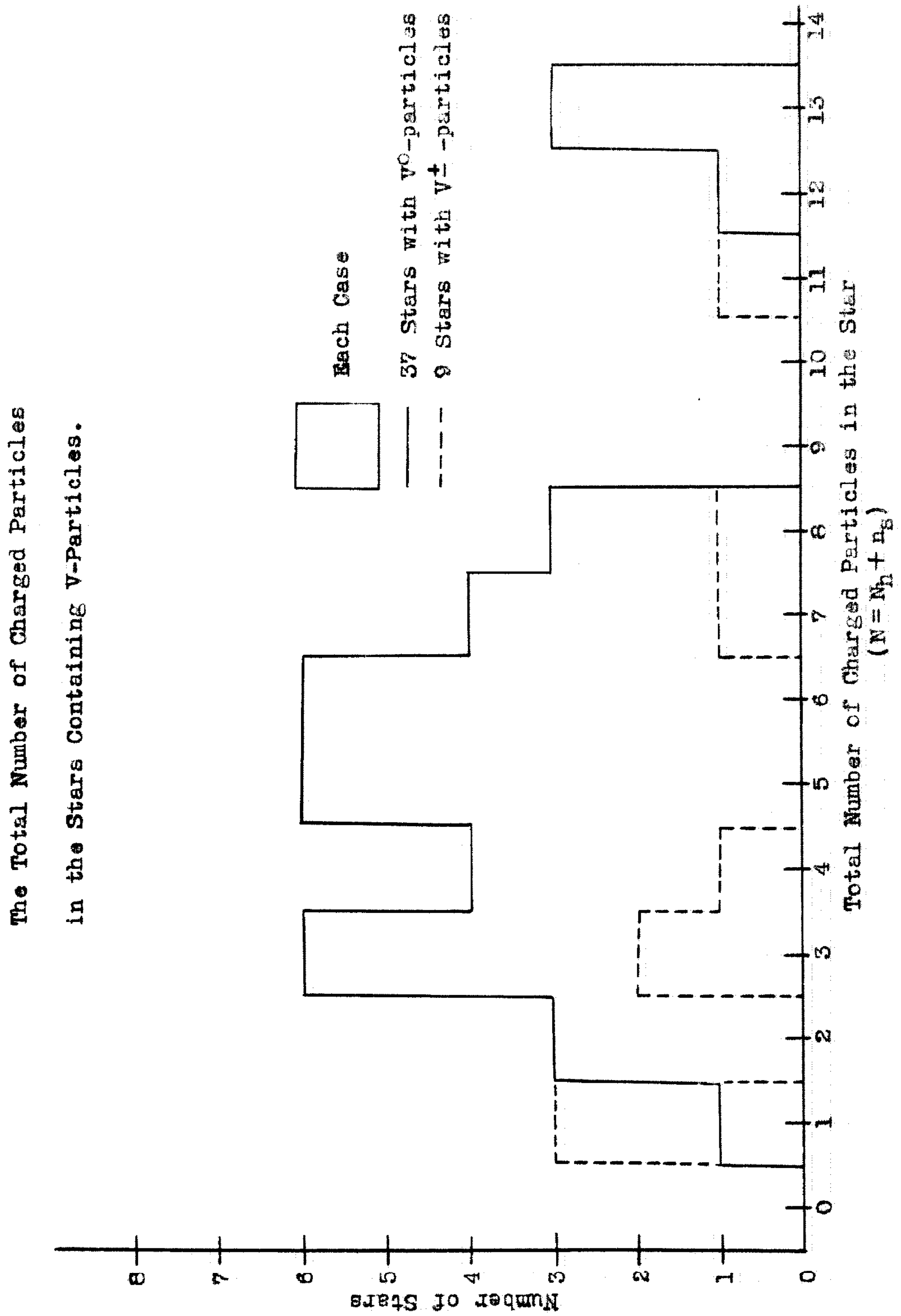
3. The Electrons (n_e)

The electrons issuing from the general star area were neglected since some are knock-ons and others pair-produced beyond the region of the star origin. The number of π^0-mesons is about one-third the sum of the charged mesons and must contribute to the electron flux in stars, since the π^0-mesons decay in the vicinity of the nucleus into photons which produce electron-pairs. Nearly all of the electrons are easily identified by large curvatures in their paths through the magnetic field while possessing minimum ionization. However, the electrons with very high momenta are difficult to distinguish from the n_s component consisting mainly of π-mesons and protons.

B. The Total Number (N) of Charged Particles in the Stars

Figure 7 is a histogram of the number of cases and the total number of charged particles ($N = n_s + N_h$) in the stars in which V^o-particles and charged V-particles were associated. The number of particles recorded in each star is the number actually observed and whose projection back into the lead plate intersected the star origin. The low momentum particles (N_h) were recorded if they could possibly have the star as an origin, since scattering is important at their low momenta. The actual number of charged particles in the stars exceed the number seen due to passage out of the chamber region when produced in the lead plate near the walls of the cloud chambers. The histogram does not include the V-particles. The average number of charged particles in the stars containing V^o-particles is five. The number of examples of stars containing charged V-particles is small. The average star containing a charged V-particle appears to have fewer charged particles than the average V^o-particle star.

FIGURE 7


The Total Number of Charged Particles
in the Stars Containing V-Particles.
Each Case
37 Stars with V^0-particles
9 Stars with $V^{\pm}$ -particles
Number of Stars
Total Number of Charged Particles in the Star
$(N = N_h + n_s)$
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

C. The Number of Shower Particles (n_s) in the Stars

Figure 8 is a histogram for the high momentum particles (n_s) in the stars with V-particles. The average star produced in the lead plate and containing a V^o-particle has about four lightly ionizing charged particles. The stars containing charged V-particles appear to contain fewer shower particles (n_s) than the stars with V^o-particles. If the charged V-particles were also included in the number of particles in the stars, the star sizes would be increased by one since nearly all of the charged V-particles are near minimum ionization. However, using momentum as a criterion rather than ionization, the number of shower particles (n_s) in the stars with neutral V^o-particles would also be increased by one since the momenta of the V^o-particles average nearly 1 Bev/c (the momenta of the V^o-particles are given in Section E). Figure 4 has a star in which all the particles are shower particles.

D. The Number of Heavily Ionizing Particles (N_h) in the Stars

Figures 7 and 8 each have an average of about four or five particles in the stars and their similarity emphasizes the relatively small number of heavily ionizing particles (N_h) found in the stars containing V-particles. Figure 9 is a histogram of the number of stars and the number of particles with an ionization greater than 1.5. The average number of heavily ionizing particles in the stars containing V-particles

FIGURE 8

The Number of Shower Particles in the Stars

Containing V-Particles.

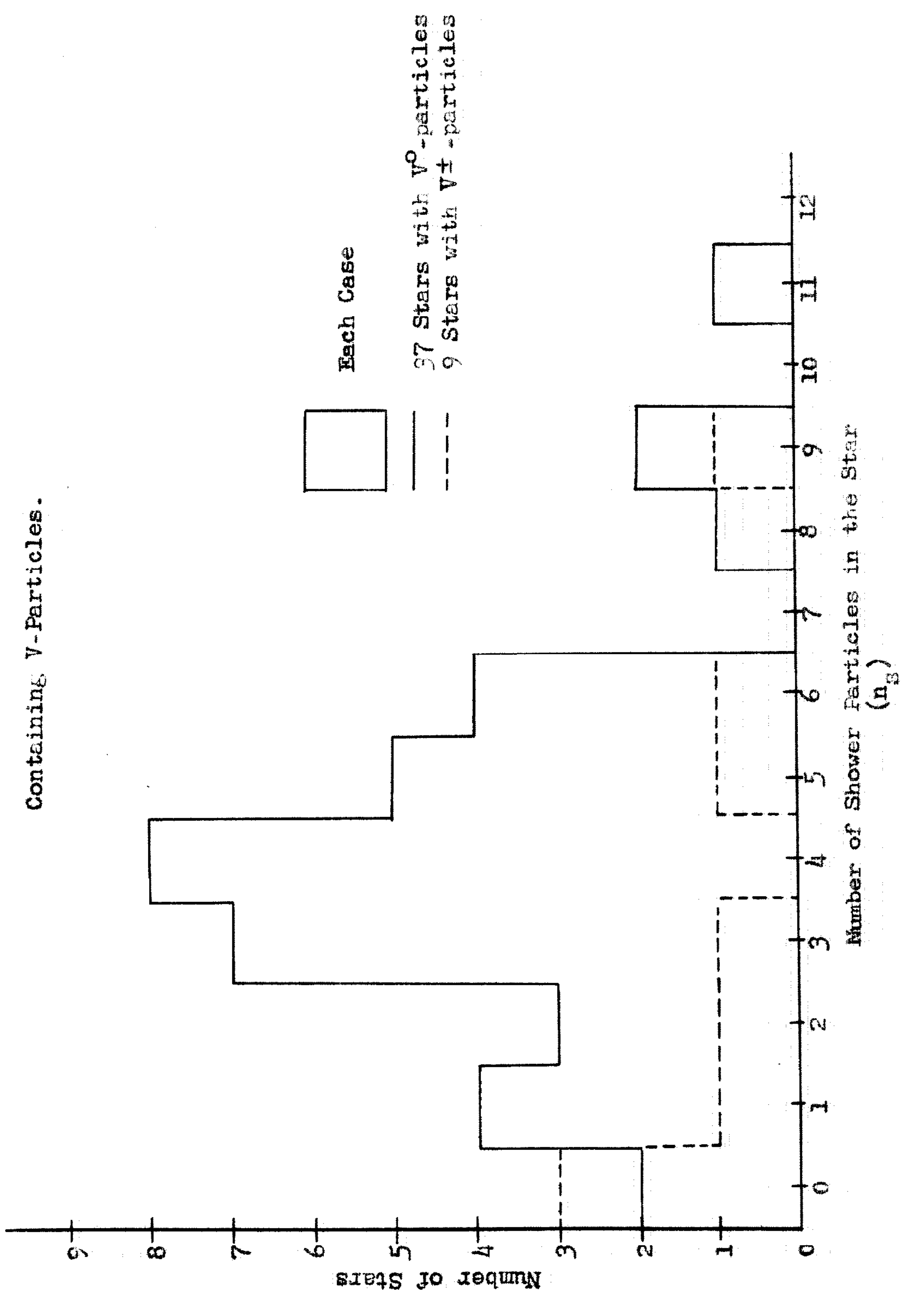
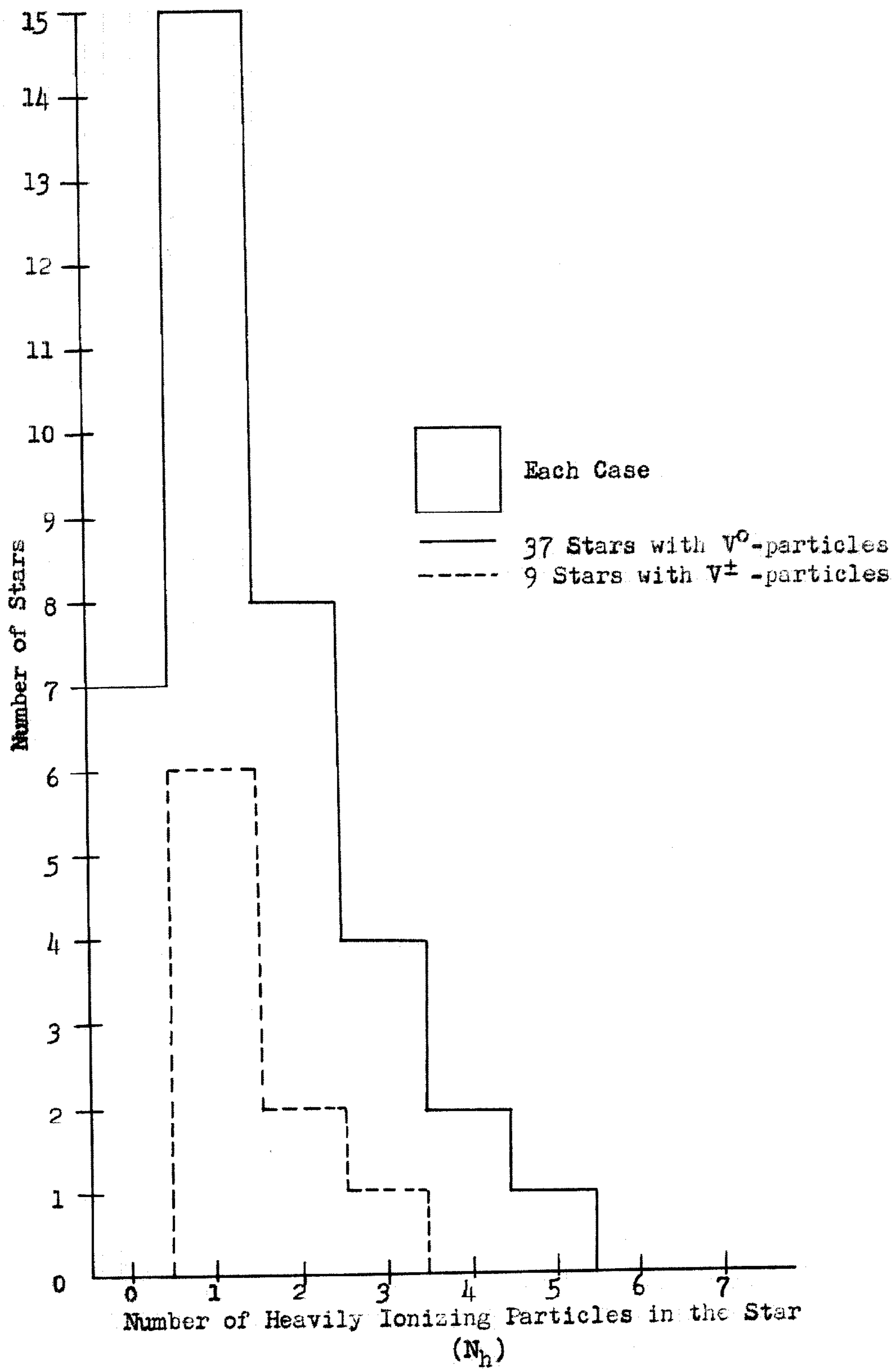


FIGURE 9

The Number of Heavily Ionizing Particles in the Stars,

Containing V-Particles.



is about one. Figure 3 has a typical star containing heavily ionizing particles. Although the number of heavily ionizing particles observed in the stars is small, it will be shown in Section I that about the correct number emerged from the lead plate.

E. The Momenta of the Incident Particles Producing V^o-Particles

The average number of shower particles (n_s) in the stars containing V^o-particles is four (fig. 8). These high momentum particles will be found in the next section (Section F) to be distributed in small angles with respect to the forward direction of the incident particles creating the stars. Since the incident particles are in a general vertical direction, few of the high momentum star particles (n_s) that are produced in the lead plate between the two cloud chambers are lost due to chamber geometry. The number of shower particles recorded is nearly correct for each of the actual stars in the lead plate. In stars in which there was an average of about four shower particles, the average momentum of the shower particle was found by Camerini et al[(5)*] to be about 1 Bev/c. The numerical value of n_s is about equal to the total forward momentum of the charged particles in the star in units of

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*Table IV in the reference.

Bev/c. In addition, the momenta of the V^o-particles average 1 Bev/c (Section H). Since other neutral particles in the stars are neglected, a minimum estimate for the average momentum of the initiating particles is 5 Bev/c.

The momentum of sufficient incident particles has been measured to allow the characteristics of the stars to be studied as a function of the incident momentum, and the variation of the average number of shower particles (n_s) with the incident momentum is given by Camerini et al[5]. Figure 10 is a partial reproduction of these results. It is reported that a particle with a momentum near 10 Bev/c produces on the average a star with 4 shower particles. This is, however, not quite the circumstance for the distribution in Figure 8, in which particles with various momenta produced a distribution in which there is an average of 4 shower particles. Figure 11 shows the distribution of multiplicity for the shower particles (n_s) in stars produced by particles in three ranges of energies, as reported by Camerini et al[5]. The distribution of n_s for the stars containing V^o-particles would be consistent with having been produced by particles with energies in excess of 4300-9400 Mev. It is estimated that the V^o-particles were produced by particles with energies in a range about the value 10 Bev. At these high energies the momenta of the particles, whether π-mesons or protons, would be in a range of momenta about the value 10 Bev/c, and

FIGURE 10

The Variation of the Average Number of Shower Particles in Stars (n_s) With the Energy of the Primary Particle.

(as given in Fig. 9 Camerini et al.(5))

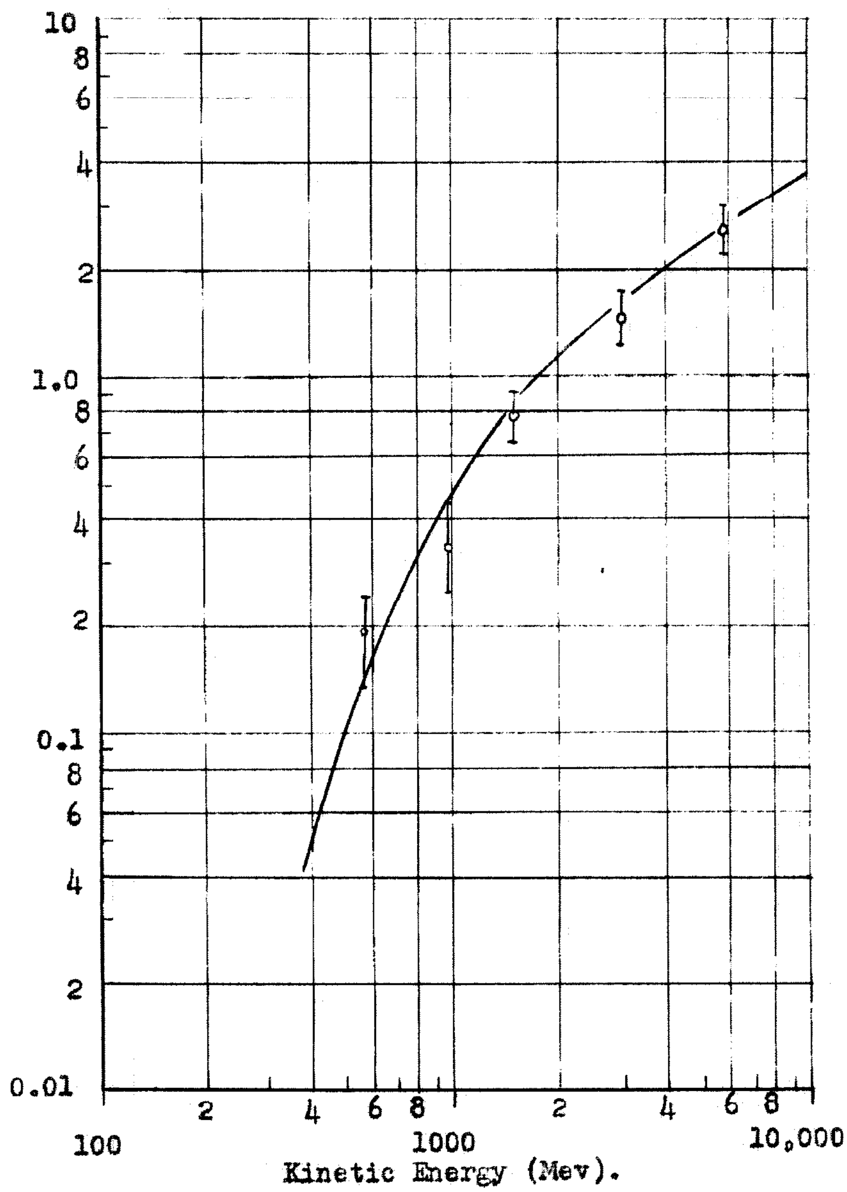
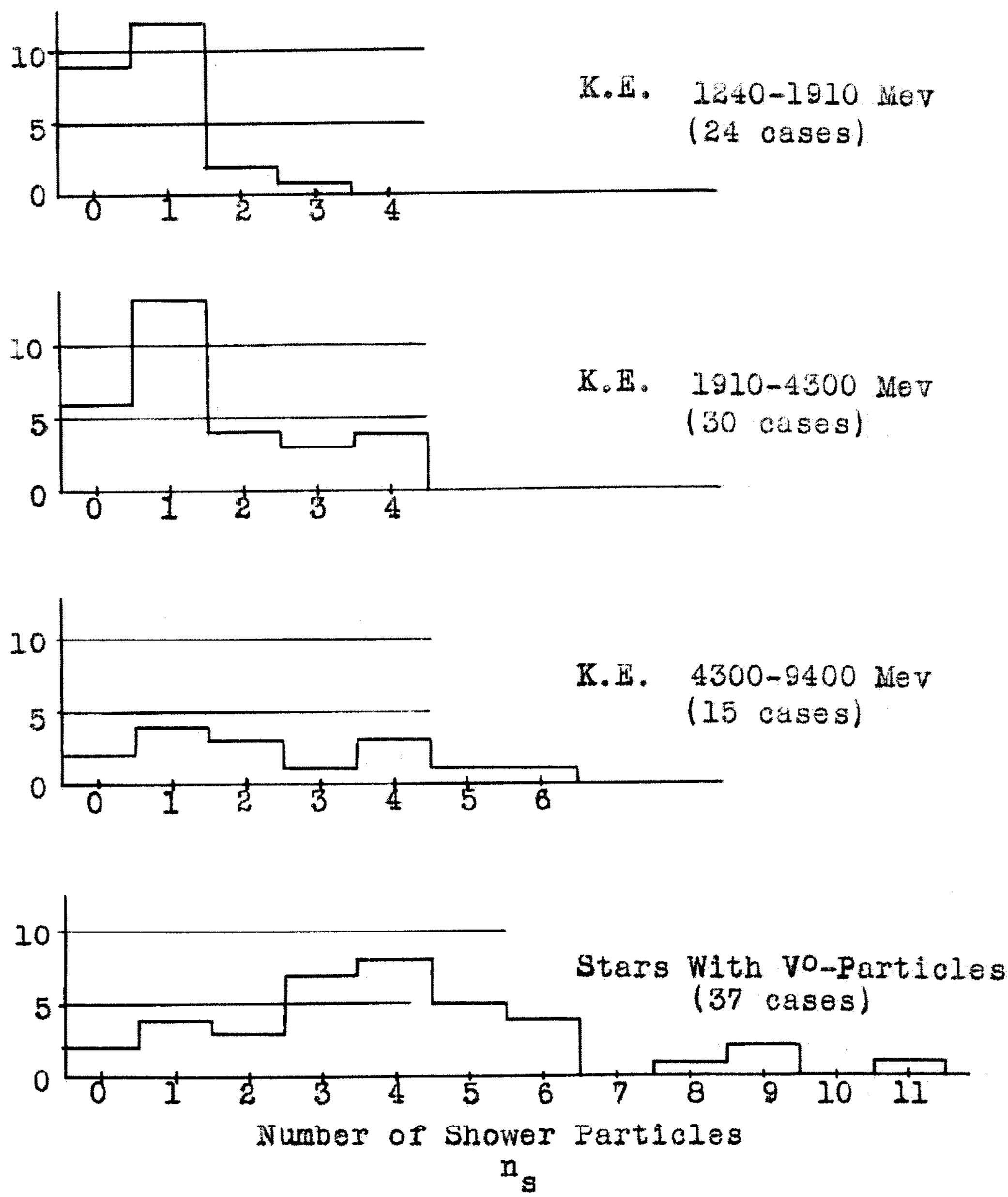


FIGURE 11

Distribution of Multiplicity, n_s, as a Function of Energy

(Camerini et al. [5] Fig. 11)



5 - 15 Bev/c seems realistic. The value 10 Bev/c is also consistent with the fact that a particle with this momentum produces a star with an average of 4 shower particles.

It will be shown (Part V, Section E) that only about one in twenty star producing interactions seem (among other variables) to be energetic enough to produce stars containing V^o-particles. By integrating a differential momentum spectrum for penetrating-shower particles, to include only those with momenta in the highest 1/20, it is found that the V^o-particles were produced by particles with momenta in excess of 7.4 Bev/c.

The Fermi[6] thermodynamic equation is $N_c = 1.2 \times \gamma^{1/4}$, in which N_c is the number of charged shower particles (n_s) in the star, and γ is $(1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$ for the initiating particle. A π-meson producing a star with 4 shower particles would have a momentum of 17 Bev/c according to this equation. E. P. George reported in a recent lecture that experimentally a better numerical constant in the Fermi equation is 1.3. Using this latter value, the momentum of a π-meson producing a star with 4 charged shower particles is 12 Bev/c.

The momentum of the incident particle producing an average star containing a V^o-particle, and consistent with each estimate, would be about 10 Bev/c. Because the momenta of the incident particles could not be measured in the magnetic field, the smallest stars containing

V^{o}-particles were produced by particles in excess of at least 2 Bev/c.

F. The Angular Distribution of the Shower Particles (n_s) in the Stars

The angular distribution of the shower particles in the stars containing V^{o}-particles is given in Figure 12. The angles are the true angles, in the laboratory system, of the star particles with respect to the particles producing the stars. The angular distribution of the shower particles in the stars containing V^{o}-particles is essentially the same as ordinary stars when compared with the cloud chamber results of Brown and McKay(7). Of even more interest is the fact that it is also the same as obtained in photographic emulsions(5). This similarity of distribution seems to indicate that the efficiency for seeing these high momentum particles is good irrespective of the lead plate and the chamber geometry. The angular distribution for the V^{o}-particles will be found in the next section to differ markedly from the distribution found in this section for the shower particles.

G. The Angular Distribution of the V-Particles

The angular distribution of the V^{o}-particles with respect to the incident particles is shown in Figure 13. The angle is the true angle of the V^{o}-particle in the laboratory system. The bulk of the V^{o}-particles

FIGURE 12

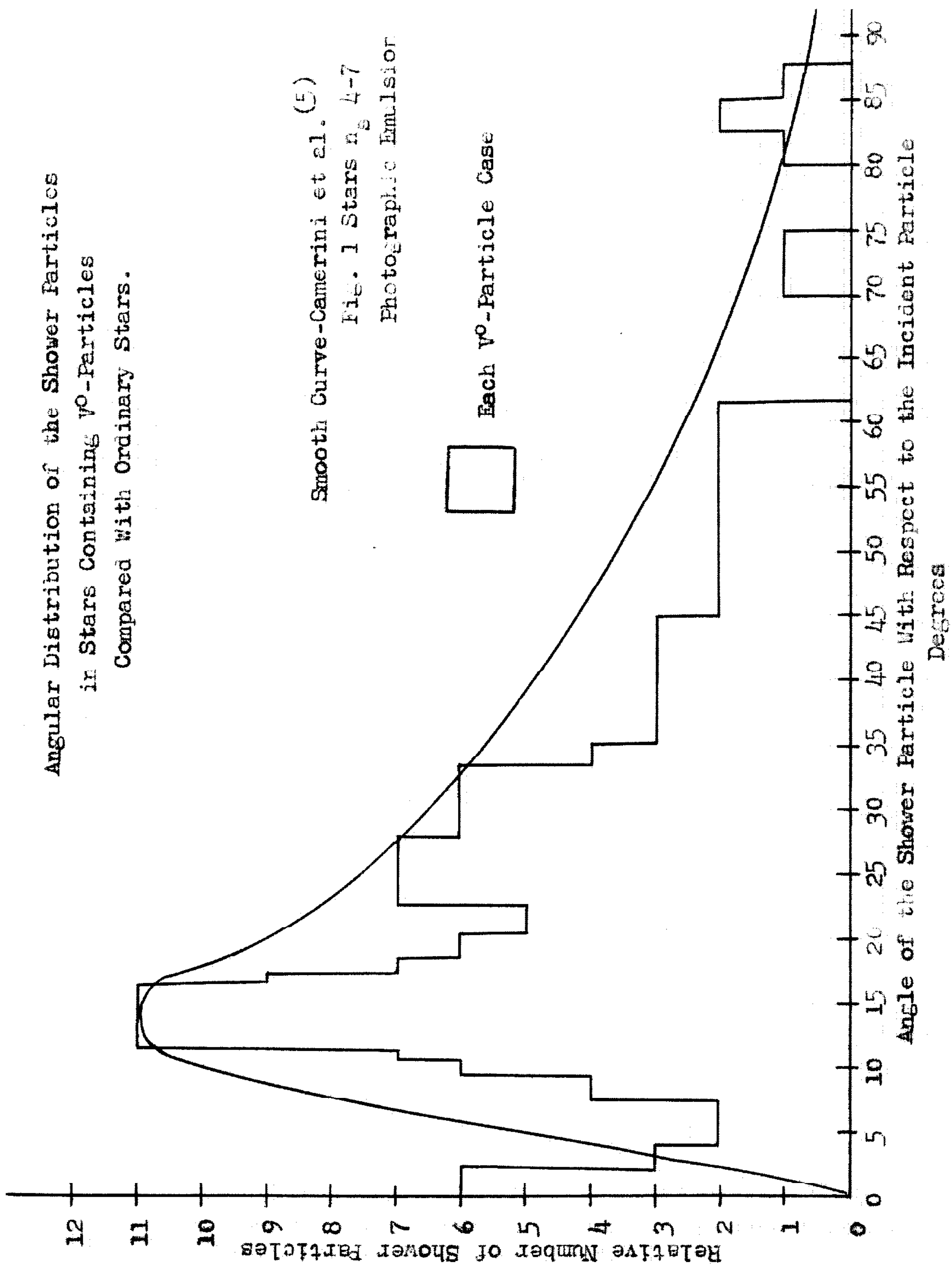
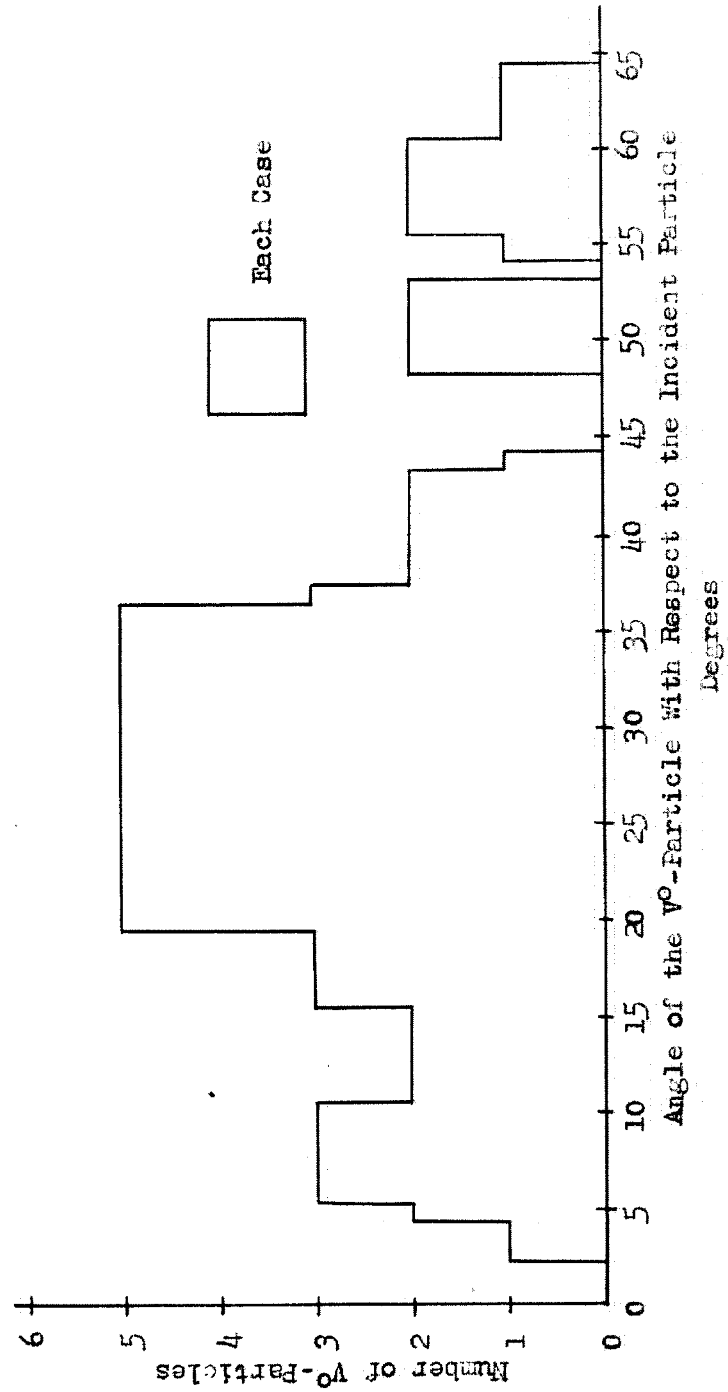

Angular Distribution of the Shower Particles
in Stars Containing V°-Particles
Compared With Ordinary Stars.
Smooth Curve-Camerini et al. (5)
Fig. 1 Stars n_s 4-7
Photographic Emulsion
Each V°-Particle Case
Relative Number of Shower Particles
0
1
2
3
4
5
6
7
8
9
10
11
12
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
Angle of the Shower Particle With Respect to the Incident Particle
Degrees

FIGURE 19

Angular Distribution of the V^{o}-Particles.



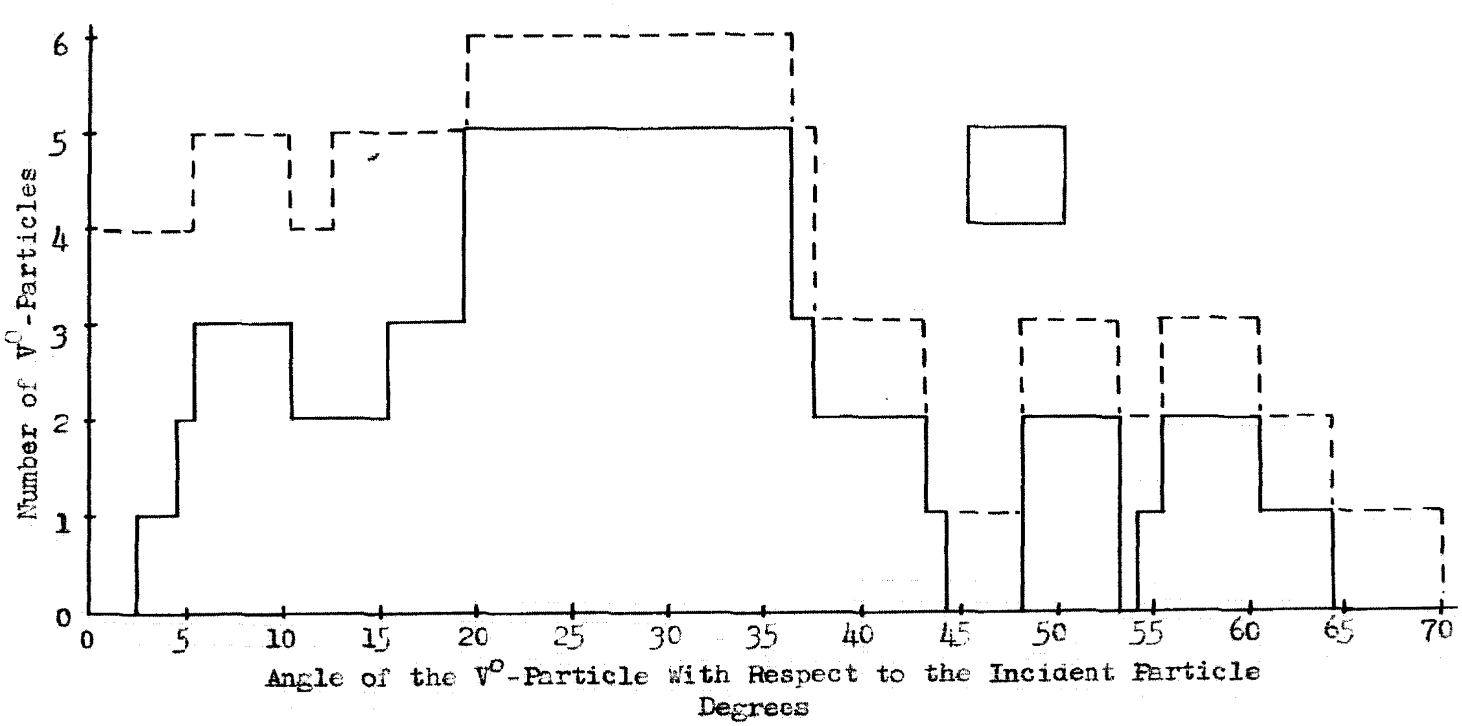
are emitted at angles between 20° and 35° with respect to the particles producing the stars. The angle of each V^{o}-particle (θ_{Vo}) was calculated from the vector components of the incident particle $\overline{I}$ and the line of flight of the V^{o}-particle $\overline{V^{o}}$ using $\overline{I} \cdot \overline{V^{o}} = IV^{o} \cos \theta_{Vo}$, except when the angle θ_{Vo} was small it was obtained from $\overline{I} \times \overline{V^{o}} = IV^{o} \sin \theta_{Vo}$. Only examples of V^{o}-particle production in the lead plate were used in order that the incident particles be seen.

Figure 13 is the angular distribution of the V^{o}-particles that decayed in the gas of the lower cloud chamber. The angular distribution of the V^{o}-particles is about constant from about 20 to 35 degrees and falls rapidly at smaller and larger angles. However, in the same stars in which the V^{o}-particles occurred, the shower particles (fig. 12) have a distinct maximum in their distribution in the vicinity of 15 degrees. The expected corrections to the angular distribution are: (1) The low momentum V^{o}-particles have a high probability of decaying in the lead plate. (2) The V^{o}-particles emitted at large angles can pass through the walls of the cloud chamber before decaying. (3) The V^{o}-particles emitted at small angles with respect to the incident particle tend to have high momenta and can pass through the bottom of the cloud chamber before decaying. Corrections to the angular distribution of the V^{o}-particles will be postponed until their momenta have been studied. However, the results of these corrections are given in Figure 14.

FIGURE 14

Angular Distribution of the V^0-Particles

Corrected For Those That Failed to Decay.



The angular distribution of the charged V-particles, in which there were only nine cases, is composed of isolated cases from 0 to 70 degrees. The charged V-particles did not tend to have small angles with respect to the producing particles, which would have been the tendency if they were produced with relatively high momenta.

H. The Momenta of the V^o-Particles

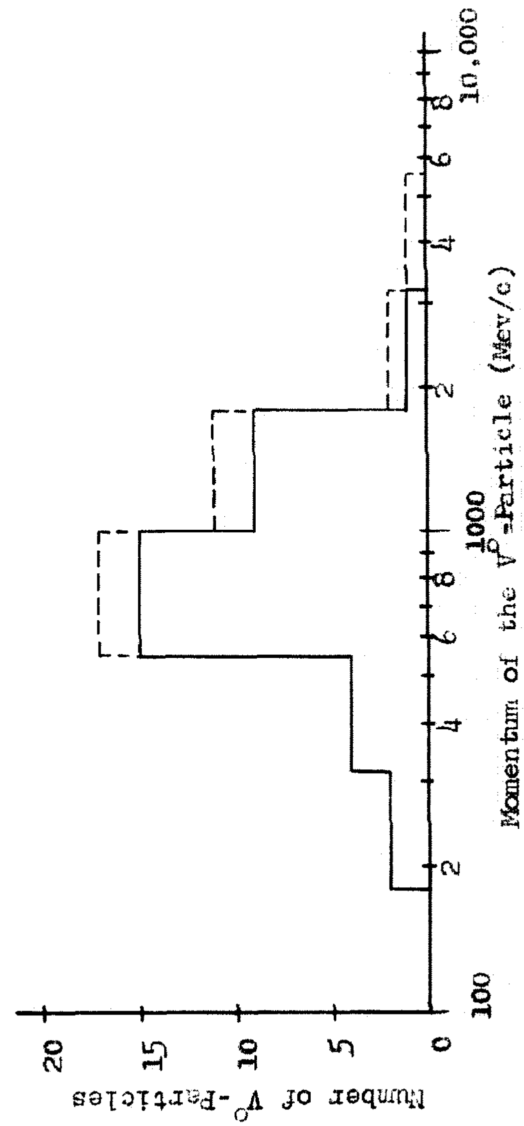
In about five-sixths of the cases of V^o-particle production in the lead plate between the two cloud chambers, the momenta of the positive and negative decay particles can be measured. In each of these cases the momentum of the V^o-particle (P_o) was obtained by vector addition of the momenta of the two decay particles[3]. Figure 15 is a histogram of the momenta of the V^o-particles that could be measured. The dashed lines of Figure 15 represent the contribution from the unmeasurable cases. The manner in which their momenta was estimated will be given below. The distribution given in Figure 15 is for the examples of V^o-particles that decayed in the lower cloud chamber and does not include the cases which were produced in the lead plate and failed to decay in the gas of the cloud chamber.

Using the angle θ_{Vo} (calculated in the previous section) which the V^o-particle makes with respect to the incident particle initiating the star, and using the momentum of the V^o-particle (P_o), the component of the momentum of the V^o-particle that is normal to the line of

FIGURE 15

Momentum Distribution of the V^o-Particles

Produced in the Lead Plate.



flight of the incident particle is $P_o \sin \theta_{V^o}$. A histogram of the normal components of the momenta ($P_\perp$) is given in Figure 16. This perpendicular component of the momentum is invariant in a transformation from the laboratory system to the center of mass system. The normal component of momentum is an indication of the momentum with which the V^o-particle is "created" in the center of mass system. This momentum is about 400 Mev/c (fig. 16). A plot of the normal components of momenta ($P_\perp$) vs the momenta of the V^o-particles (P_o) is given in Figure 17. The normal component of momentum ($P_\perp$) is nearly constant. A similar correlation has been found for π-mesons in stars[8], and it was reported that the normal components of the momentum were of the order of $m_\pi c$. For the V^o-particles the normal components are of the order of $M_{V^o} c/2$.

For the V^o-particles produced in the lead plate and decaying in the gas, the angles (θ_{V^o}) were measured in every case. When the actual momentum of a V^o-particle cannot be obtained from the decay particles, the momentum can be estimated from $P_\perp = P_o \sin \theta_{V^o}$, using $P_\perp$ equal to 400 Mev/c (the dashed lines in Figure 15).

The probability that a V^o-particle with a momentum P and a mean lifetime τ decays within a path length d is

$$P_{decay} = 1 - e^{-t/\tau} = 1 - e^{-\frac{dM_o c^2}{\tau P c^2}}$$

FIGURE 16

Distribution of the Normal Components of Momentum ($P_\perp$)

of the V^0-Particles.

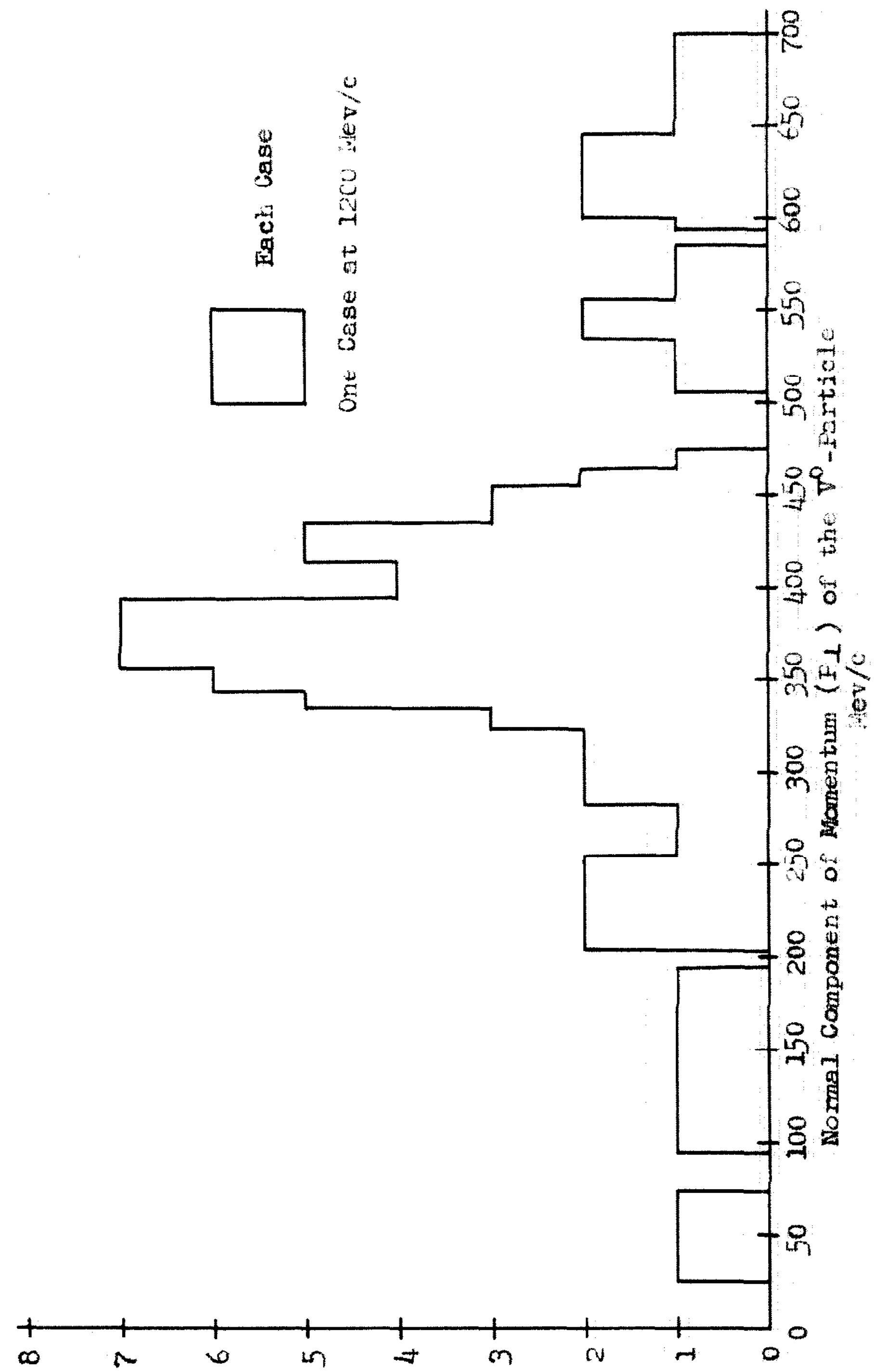
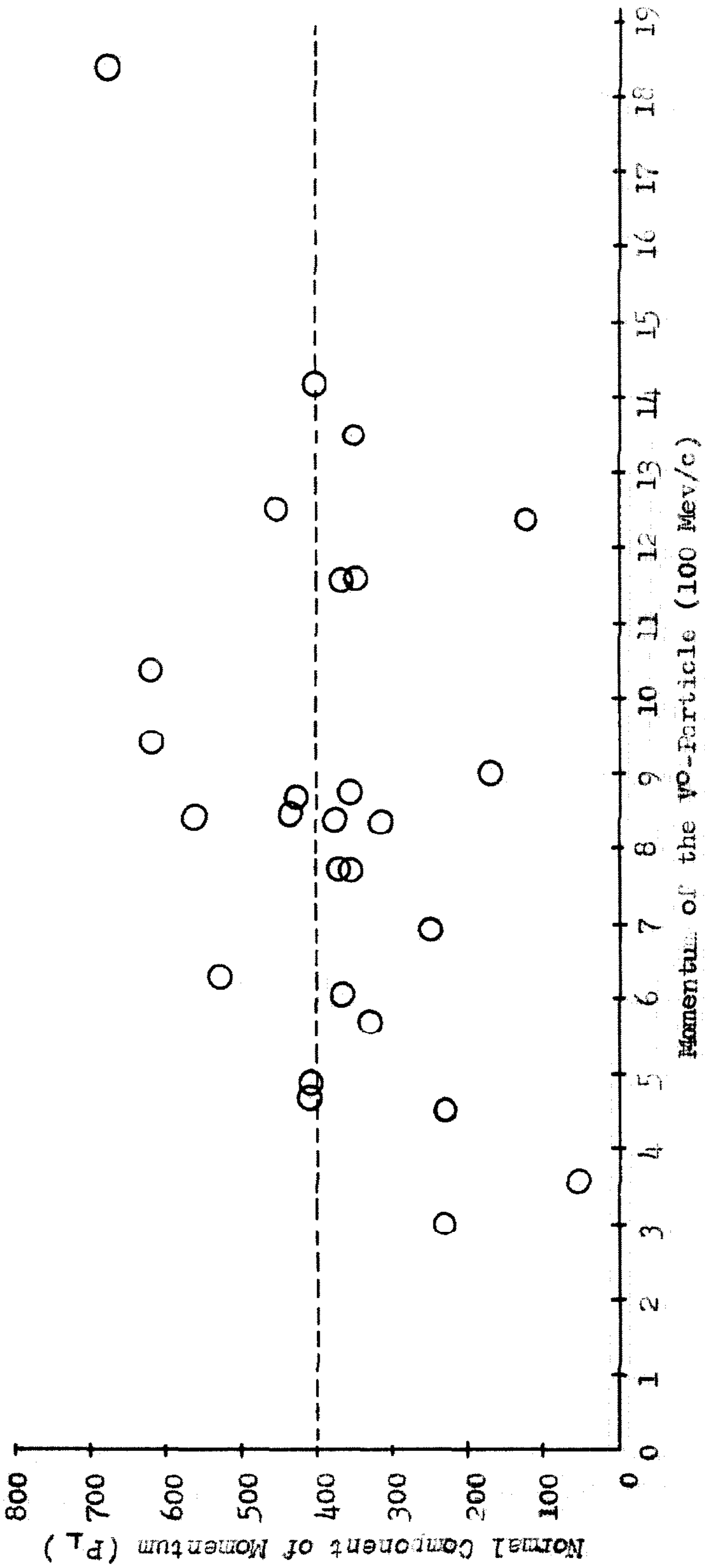


FIGURE 17

The Normal Component of Momentum ($P_\perp$) vs the Momentum (P_o)

For V^o-Particles.



in which t is the time in the center of mass system to traverse the distance d, M_oc^2 is the rest mass of the V^o-particle. If N V^o-particles with a momentum P are observed to decay while traversing the height of the lower cloud chamber, there were probably N_o particles actually passing this region where

$$N_o = \frac{N}{1-e^{-t/\tau}}$$

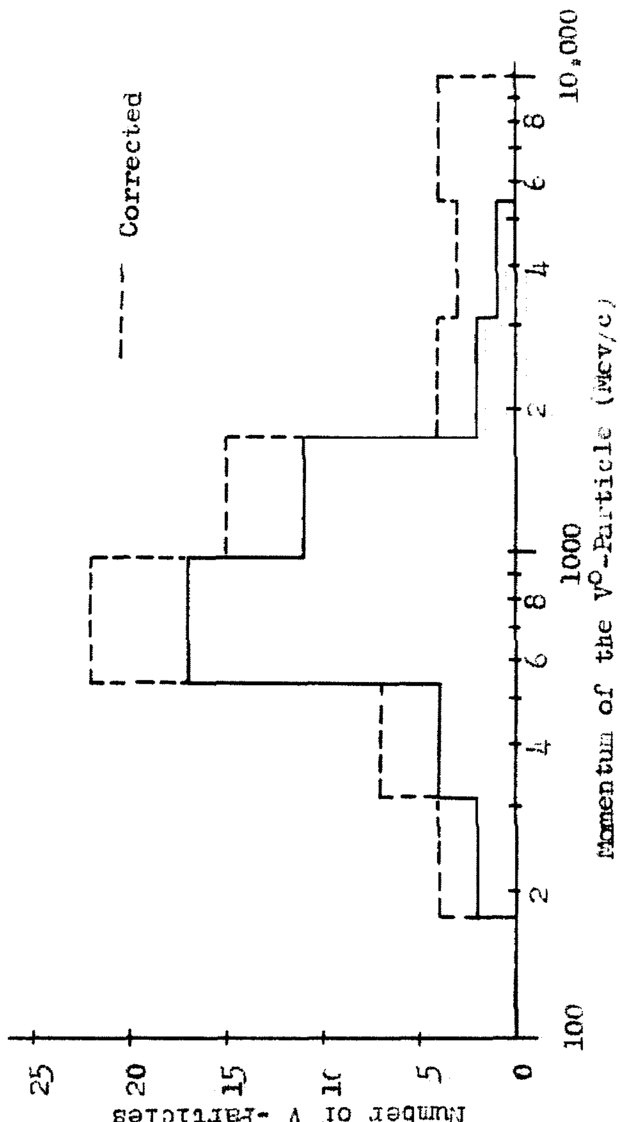
Taking the values N from Figure 15 for each range of momenta, and using the average momentum in each range, the momentum distribution given in Figure 15 was thus corrected for the number of V^o-particles that passed beyond the cloud chamber before decaying. It was assumed that each V^o-particle's path was from the bottom of the lead plate to the bottom of the lower cloud chamber. This is, of course, an over estimate of the path lengths in the cloud chamber of the V^o-particles that are emitted at large angles. It is estimated that this will, however, correct for the number that pass out of the cloud chamber walls before decaying.

In a similar manner the number that failed to emerge from the lead plate was estimated. The length (d) was taken to be one-half the thickness of the lead plate. This correction was important for correcting the low momentum portion of the momentum distribution.

The results of these corrections to the momentum distribution are given in Figure 18. The 37 V^o-particles observed to decay would be

FIGURE 18

Corrected Momentum Distribution of the V^o-Particles.



increased by about 22 for an estimate of the probable number that were produced.

The number of particles that failed to decay in the gas of the lower cloud chamber were calculated at various momentum ranges in the corrections to the momentum histogram. Again using $P_{\perp} = P_o \sin \theta_{V^o}$, with $P_{\perp} = 400$ Mev/c, the particles that failed to decay in the gas can be ascribed to the proper angles θ_{V^o}. Those V^o-particles with momenta less than 400 Mev/c were distributed uniformly at the large angles. The results are given in the corrected histogram for the angular distribution of the V^o-particles in Figure 14.

1. A Comparison of Stars Containing V^o-Particles with Other Stars Known to Have Been Meson and Nucleon Produced

Nucleon produced stars have been studied in considerable detail by Camerini et al[5]. Some of their stars were produced at an elevation of about 68,000 feet in photographic emulsions and represented mainly primary protons, with an estimated 6 percent π-mesons locally produced. The most important characteristic of a nucleon produced star is the large number of heavily ionizing protons emitted. This means that a nucleon-nucleus interaction highly excites and disrupts the nucleus in the process of meson formation. A typical star, produced by a nucleon, which has 4 lightly ionizing particles (n_s) also has about

11 heavily ionizing particles (N_h) according to Camerini et al[5]. The heavily ionizing particles consisted of about 7 low energy evaporation protons and 4 with higher kinetic energy but with an ionization greater than 1.4, and sum of the two is N_h.

The stars observed by Lal et al[9] were also in photographic emulsions, but were the interactions of mesons which had been produced in a very large star resulting from a single primary interaction. These meson interactions correspond with the manner in which V-particles are produced in the lead plate by penetrating-shower particles originating above the cloud chambers. In both cases the interactions are by secondary particles in which the π-meson percentage is high. The authors, Lal et al[9], reported the meson produced stars to have very few heavily ionizing particles (N_h) and a proportionately larger number of high momentum particles (n_s).

The stars containing V^o-particles cannot be directly compared with the results obtained in the photographic emulsions. However, assuming the stars containing V^o-particles were nucleon produced and were accompanied by the large number of relatively low momentum particles (N_h), it is possible to calculate the average number that would enter the gas region of the lower cloud chamber. This calculation is possible since the Camerini et al[5] paper gives the variation with n_s, nearly all of which are seen in the cloud chamber, of the average number of protons of different energies in their stars. The distance from the gas in the bottom chamber was recorded for each star in the lead plate containing a V^o-particle. For various depths in the lead plate, Figure 19 gives the minimum momentum of a star secondary in order that it emerge into the gas with 100 Mev/c. An extra 100 Mev/c momentum allows the paths to be oblique through the lead plate and still emerge into the gas.

The comparisons of stars containing V^o-particles and those produced by protons and π-mesons are given in Table 2. Assuming the stars containing V^o-particles were nucleon initiated, the calculated numbers of N_h particles that should have emerged from the lead plate are generally larger than the actual numbers seen accompanying the stars. The difference is not large enough to be conclusive, but is indicative of V^o-particle production by particles other than nucleons. The stars containing V^o-particles and having a high multiplicity compare favorably with nucleon production. The stars produced by neutral particles and containing V^o-particles have almost exactly the correct number of heavily ionizing particles to have been nucleon produced. A one-inch lead plate is of such a thickness that a large fraction of the heavily ionizing protons are lost. These protons are the essential characteristics that differentiate π-meson and nucleon produced stars.

FIGURE 19

Momentum of the Particle at Various Depths in the Lead Plate to Emerge Into the Gas With a Momentum of 100 Mev/c.

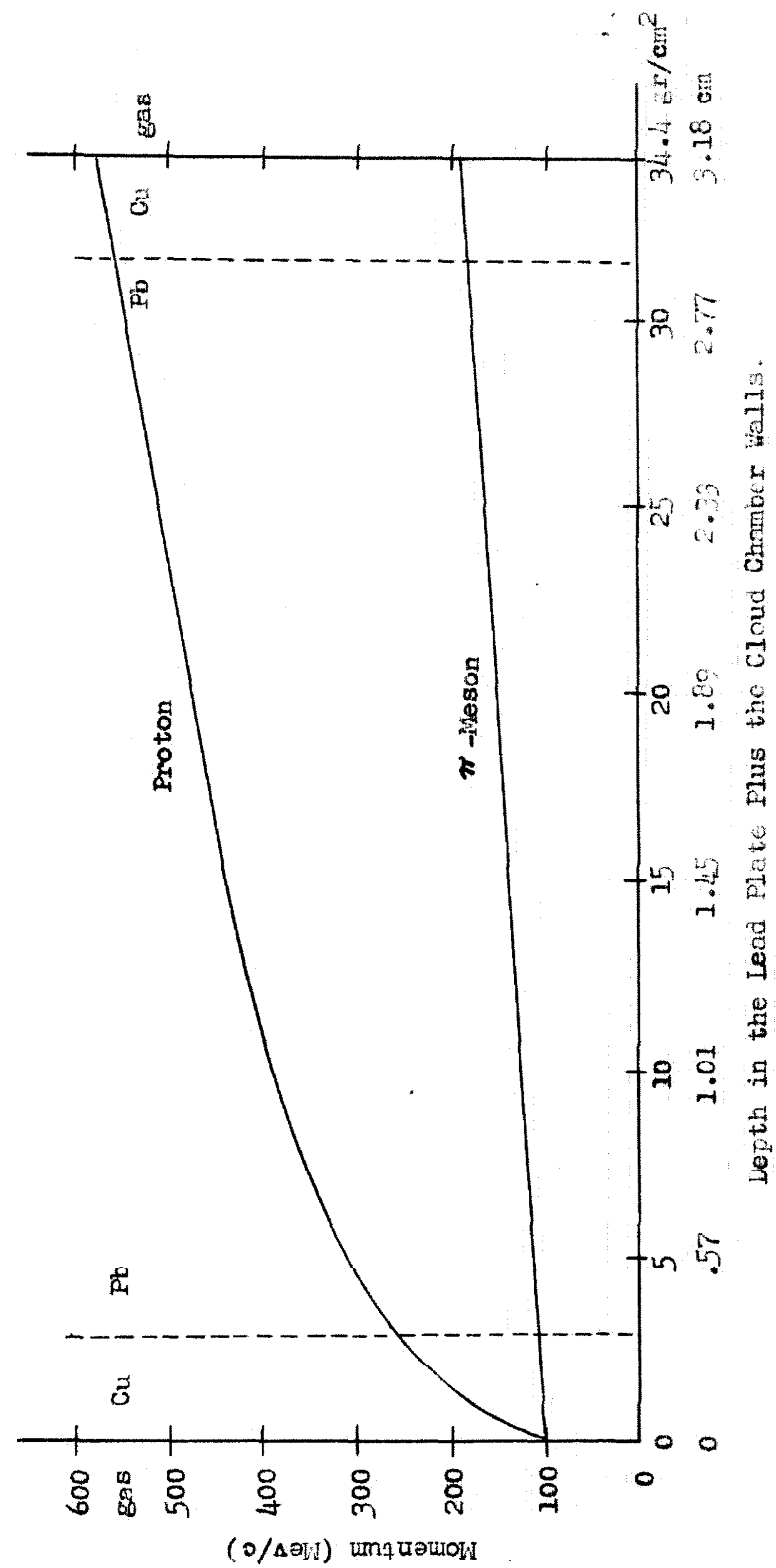


TABLE 2

A Comparison of Stars Containing V^o-Particles with Other Stars Reported to be Produced by Protons and π-Mesons.

	No. of Cases			Average Number of N_h Corresponding to n_s			
				V^o-Particles			
n_s	V^o-Particle	Lal et al.(9)	Camerini et al.(5)	Actually Seen	If Proton Produced*	π-Meson Produced Lal et al. Emulsion	Proton Produced Camerini et al. Emulsion
0	2	0	2117	1.5	1.0	-	6
1	4	0	1063	0.75	1.5	-	6
2	3	0	424	1.0	2.0	-	9
3	6	0	229	1.3	2.4	-	9
4	8	2	148	1.1	2.7	1	11
5	5	3	97	1.6	3.0	4	12
6	4	1	68	1.2	2.8	1	12
7	0	1	39	-	-	5	13
8	1	1	34	5.0	4.0	9	13
9	2	0	14	3.5	4.0	-	13
10	0	0	13	-	-	-	14
11	1	1	6	2.0	2.0	2	14
V^o-Particles Produced by Neutral Particles							
3	1			2	2.2		
4	3			2	2.5		

* If the star in the lead plate was proton produced, these are the N_h with momenta sufficient to be seen in the cloud chamber. (Calculated from Camerini et al.)

V. A CROSS-SECTION FOR V^0-PARTICLE PRODUCTION

There were 37 V^0-particles produced in the lead plate between the two cloud chambers up to a certain date of cloud chamber operation. Three modes of V^0-particle production were apparent: 27 were produced by charged penetrating-shower particles, 6 were produced by lone unassociated charged particles, and 4 were produced by neutral particles. A detailed survey was undertaken to ascertain if these were the ratios for such interactions in the lead plate irrespective of V^0-particle production, and to determine what fraction of the interactions had V^0-particles. Any essential differences in the appearances of the interactions with and without V^0-particles were noted. The number of charged V-particles produced in the lead plate is so small that a similar detailed survey for their production is not feasible. However, from the V^0-particle results, the charged V-particle production cross-section will be estimated.

The one-inch lead plate gives 18.4 percent nuclear geometrical cross-section. The two one-eighth-inch brass walls of the cloud chambers, which are in contact with the lead plate, were approximated as being copper and provided an additional 5.3 percent nuclear coverage. Thus an average of 23.7 percent of the particles traversing the materials between the cloud chambers would pass within the geometrical area of a nucleus. The total equivalent of the materials between the two gases of the cloud chambers is 3.3 cm of lead.

The survey consisted of 9 consecutive rolls of film in which there were 1800 photographs. These cloud chamber photographs had been taken at 1750 m elevation, with the counter requirements being two or more in the top tray of counters and three or more in the bottom tray (see Apparatus). Such counter requirements are referred to as a 2-3 coincidence. The counter requirements in previous photographs had been a 1-3 coincidence. This change in counter requirements resulted in a 50 percent increase in the percentage of photographs with penetrating showers at the same elevation. The counters were in adjustment over a major portion of the survey and in each roll of film the statistics were nearly identical.

In these 9 consecutive rolls of film there were 7 V^o-particles and one charged V-particle. Compared to the overall statistics, 8 V-particles being produced in the lead plate in 9 rolls of film is a large ratio, since only 47 V-particles (37 V^o-particles and 9 charged V-particles) were produced in the lead plate in the first 100 rolls of film. However, the counter requirements in the present survey doubled the percentage of photographs with penetrating-shower particles at the same elevation. The 9 rolls of film are probably a representative sampling of the interactions of penetrating-shower particles and they contained very nearly the same ratio of V-particles to penetrating-shower particles as was found in the total of 100 rolls. A total of 1630 penetrating-shower particles entered the lead plate in the present survey, of which 190 had interactions in the lead plate and 8 V-particles were produced. The remaining 1440 particles traversed the lead plate.

A. Classification of the Nuclear Events in the Lead Plate

Since V^0-particles are produced by particles with momenta too high to be measured in the 5000 gauss magnetic induction, the particles observed for a cross-section calculation were restricted to those with high momenta. The charged particles traversing the gas of the top cloud chamber and entering the lead plate were recorded if their momenta were in excess of the arbitrary value 1 Bev/c. A charged particle with 1 Bev/c momentum has a curvature in the magnetic field that is just perceptible on a photograph. Many of the charged particles that penetrate the lead plate are not traversing the cloud chambers in a vertical direction. To record with certainty that a particle penetrated the lead plate without deviation or had a nuclear interaction, observations were restricted to the central portion of the lead plate area. The outside vertical lines of fiducial marks on the backs of the cloud chambers (see fig. 2) were taken as boundaries and viewing the photographs stereoscopically, the particles and interactions were restricted from being near the front or back of the lead plate. The 8 V-particles recorded were produced in this area of the lead plate, and 2 additional V^0-particles were produced outside of this area. Except for passage through the lead plate with no detectible deviation, the interactions of the incident particles fall into the following classifications.

1. Star Production

Each star was classified as to the number of shower particles (n_s) observed in the star. Interactions which produce stars in the lead plate will be detected with a high probability if two or more high momentum particles are present, since these particles will emerge from the lead plate with a probability near one.

An interaction in the lead plate, in which only low momentum particles are present in the star, will be classified as a star if any of the particles emerge from the lead plate. Otherwise the incident particle is recorded as being stopped. In either case it is regarded as a nuclear interaction. Having restricted the interacting particles to those with momenta in excess of 1 Bev/c, this occurrence has been minimized. Barker and Butler[10] observed stars produced in a lead plate 3.4 cm in thickness, and reported almost unit probability for detecting stars when initiated by particles with momenta greater than 1-2 Bev/c.

A star containing a single high momentum particle will be recorded as a nuclear interaction if this particle is at a sizable angle (greater than 5 degrees) with respect to the incident particle. Such an interaction will be recorded as a star if any of the low momentum particles emerge into the gas, otherwise it will be recorded as a nuclear scattering.

The type of interaction that will be missed completely is one in which the low momentum particles in the star fail to emerge into the gas of the cloud chamber and a single high momentum particle in the star is emitted at a small angle with respect to the direction of the incident particle. Observing such an event in the double cloud chamber, it can only be recorded as a penetration of the lead plate without interaction. Even though the secondary particle is observed to be at a small angle with respect to the incident particle, it must not be recorded as a nuclear interaction, since this is exactly the appearance of a particle scattered by the Coulomb field. If the cross-section for nuclear interactions, obtained from cloud chamber observations, are found to be smaller than the cross-section obtained in nuclear emulsions or possibly in counter coincidence experiments, then the occurrence of the above described interaction is not rare.

2. Nuclear Scatterings

No single case can be guaranteed to be a nuclear scattering since what appears to be a scattering in the lead plate may have been a star producing collision in which a number of particles were emitted and only a single fast particle emerged from the lead. In either case it should be recorded as an interaction for calculations of a cross-section. What should not be included are the ordinary multiple and large angle single scatterings by the Coulomb fields.

The criterion used by Barker and Butler[10], in calculating an interaction length for penetrating-shower particles in lead, was to record all scatterings with angles five times the probable multiple-Coulomb scattering, calculated using Williams' formula[11]. Fretter[12] recorded scatterings greater than 15 degrees for an interaction length calculation. The arithmetic mean value of the multiple scattering angles $\overline{\alpha_1}$ for 1 Bev/c π-mesons in the present absorber is 1.55°. Five times the mean value is 7.75°. The maximum single Coulomb scattering angle ϕ_{max} is 1.35°.

From an interaction length for penetrating-shower particles, it is possible to calculate the angle α_2 at which nuclear scatterings become predominant. The mean free path for 1 Bev/c π-mesons is later found to be 340 gr/cm^2 or 30 cm of lead. The probability is about 1/10 for an interaction by a single shower particle traversing the lead plate. Integrating the Gaussian distribution for multiple scattering from α_2 to ∞, and equating to 1/10, one obtains from a table of Gaussian functions $\frac{\alpha_2}{\sqrt{\pi}\,\alpha_1} = 1.17$ or $\alpha_2 = 3.2^\circ$. Nuclear scattering becomes important for angles beyond 3.2° for 1 Bev/c π-mesons, and the angle is less for higher momentum particles. The requirement by Fretter[12] that the angles be greater than 15° in a one-half-inch lead plate seems an excessively stringent condition. For the present work, an angular deviation greater than 5° was recorded as a nuclear scattering.

3. Stopping in the Lead Plate

A number of particles with momenta in excess of 1 Bev/c were seen to enter the lead plate from above and no particles were seen below the plate. Such events would occur for low energy stars in the lead, since their secondaries are rapidly absorbed. In each such case the decision had to be made as to whether the particle entering the plate could be seen emerging from the bottom, rather than passing out of the cloud chamber walls. The efficiency is quite good in stereoscopic observations.

4. A Summary of the Results

Table 3 presents the penetrating-shower particle results obtained in the survey of 9 rolls of film. A total of 1633 penetrating-shower particles entered the lead plate and produced 156 stars in which 7 V^0-particles and 1 charged V-particle were found. In addition to the stars, 21 nuclear scatterings occurred and 20 penetrating-shower

TABLE 3

Interactions of Penetrating Shower Particles

Roll No.	Total No. of Particles	Stars	No. of V-particles and Frame No.	Scatter-ings	Stopped	Neutral stars
92	228	22	2 V^o (30045) (30095)	1	3	4
93	224	17	0	3	2	2
94	213	27	1 V^o (30514)	2	3	3
95	216	8	1 V^o (30803)	3	1	2
96	166	15	1 V^o (31044)	2	1	3
97	157	18	1 V^o (31341)	1	2	2
98*	127	12	1 V^o (31476)	2	3	1
99*	84	9	0	3	3	1
100	218	18	1 $V^{\pm}$ (31886)	4	2	3
	1633	156	7+1	21	20	19

* A portion of each of these rolls of film was taken with no magnetic field and these portions of the rolls were omitted.

particles stopped in the lead plate. Nineteen stars were observed in which the incident particles were not visible and the locations of these stars were such that the incident particles probably should have been seen. These stars are recorded as being produced by neutral particles. The charged particles, by which these stars were identified, were usually in such a direction as to make the axis of each star parallel to the general direction of an accompanying penetrating shower. These stars, produced by neutral particles, were similar to the star in Figure 4, without the V^o-particle.

Table 4 records the results for lone unassociated charged particles, all of whose momenta was in excess of 1 Bev/c, and the stars produced by neutral particles which could not be associated with penetrating showers.

A comparison of the penetrating-shower particle results obtained in the present cloud chamber with other cloud chamber results is given in Table 5. Because the height of the two cloud chambers is about 18 inches, the present cloud chamber has often been referred to as the 18-inch cloud chamber. The Barker and Butler[10] data was obtained in a cloud chamber with a magnetic field, whereas the cloud chamber operated by Fretter had no magnetic field. The Barker and Butler data has a higher ratio of nuclear scatterings, and particles stopping in the lead to the number of stars produced, than was observed

TABLE 4

Roll No.	Interactions of Lone Charged Particles: Total No. of particles	Interactions of Lone Charged Particles: No. of Stars	Interactions of Lone Neutral Particles: No. of stars
92	88	5	2
93	52	6	3
94	60	2	6
95	55	2	2
96	46	4	2
97	61	12	3
98*	32	5	4
99*	18	5	0
100	57	6	5
	419	47	28

No V^0-Particles Were Produced

No V^0-Particles Were Produced

TABLE 5

A Comparison of Nuclear Interactions in Lead Plates

	Total No. of Penetrating Particles	1 Stars by Penetrating Particles	2 Nuclear Scatterings	3 Stopped in the Lead	Stars Produced by Neutral Particle
18 inch cloud chamber 3.3 cm lead plate	1633	156	21	20	19
Barker and Butler(10) 3.4 cm lead plate	268	18	8	7	5
Fretter(12) 1.27 cm lead plates	4675	78	12	0	(0)?

in the present cloud chamber. The results obtained by Fretter are not directly comparable with the 18-inch cloud chamber results, since he used lead plates 1/2 inch thick. If the number of penetrating particles is divided by about three, the Fretter results should be the 18-inch cloud chamber results. It is seen that Fretter observed relatively fewer stars and no particles stopping in the lead.

B. The Interaction Length of Penetrating-Shower Particles

The interaction length λ_I is defined by the equation

$$N = N_o \left(1 - e^{-\frac{L}{\lambda_I}}\right)$$

which may be written in the approximate form

$$\lambda_I = \frac{N_o}{N} L \qquad N << N_o,$$

where N_o is the total number of particles entering the plate, N is the number of interactions and L is the thickness of the plate. The 18-inch cloud chamber data gives $\lambda_I = 27$ cm. The corresponding interaction length reported by Barker and Butler(10) is $\lambda_I = 26$ cm. These values correspond to what is termed "uncorrected" interaction lengths. The value of L is the thickness of the lead plate and does not take into account the angular distribution of the penetrating-shower particles. Since few of the particles traverse the plate in the direction of the normal, the particles have actually traversed a length greater than L to produce the observed number of interactions. The corrected interaction length is greater than the uncorrected. Brown and McKay[7] provide a means of correcting the data. They found the median angle of the lightly ionizing secondaries of interactions above their chamber, with respect to the zenith, and reported that the appropriate factor is 1.09. Brown and McKay also found other corrections for the low momentum interactions. This was necessary since their cloud chamber had no magnetic field. These other corrections do not seem appropriate for the data obtained in the present work, since the interactions were by particles with momenta in excess of 1 Bev/c. Barker and Butler did apply these other corrections to their data since the momenta of the shower particles had not been restricted.

The corrected[1] interaction length obtained in the 18-inch cloud chamber is 30 cm of lead or 340 gr/cm^2. In Table 6 are values for λ_I (Pb) reported by other observers using cloud chambers. The error in the 18-inch clould chamber data was computed on the assumption that all the interactions recorded were real and the error is in the total number of particles traversing the lead plate. Estimating that perhaps as many as 10 percent of the shower particles were missed, or should not have been recorded as shower particles, the error is $\pm$ 40 gr/cm^2.

It seems worthy of note that the interaction length for penetrating-shower particles is not the geometrical area of the lead

TABLE 6

Values of λ_I(Pb) from Observations in Cloud Chambers

Authors	Altitude (m.)	No. of Interactions	I (gr/cm^2)
Fretter[12] (1949)	3027	90	750
Lovati et al.[13] (1950)	3450	38	300 ± 100
Brown and McKay[7] (1950)	3260	39	316 ± 70
Barker and Butler[10] (1951)	Sea Level	33	200 ± 80
18 inch Cloud Chamber	1750	197	340 ± 40

λ_I corresponding to the geometrical cross-section of the lead nucleus is 13.9 cm or 158 gr/cm^2.

nucleus but is nearly one-half this value. This seems to indicate that the probability is only about one-half for the interaction of a high momentum particle in passing through the geometrical area of the lead nucleus.

C. A Cross-Section for V^{o}-Particle Production by Penetrating-Shower Particles

Accompanying the 197 nuclear interactions by penetrating-shower particles, 7 V^{o}-particles were observed with origins in the lead plate. These V^{o}-particles represent those produced with momenta such that there is a significant chance for them to decay in the gas of the cloud chamber. V^{o}-particles with very high momenta have large time dilations in their lifetime and pass beyond the region of the cloud chamber before decaying and hence are not recorded. In a previous section (Part IV, Section H) it was estimated that about two-thirds of the V^{o}-particles decay in the lower cloud chamber when produced in the lead plate. There were most probably 10 V^{o}-particles produced in the present analysis.

One V^{o}-particle is "formed" in about 20 nuclear collisions by charged penetrating particles whose momenta are in excess of 1 Bev/c. Only one in 160 penetrating-shower particles traversing the lead plate (37.5 gr/cm^2) produces a V^{o}-particle. The purpose of the above analysis, in which the interaction length for penetrating-shower particles was found, was to check the accuracy of the number of penetrating-shower particles and interactions corresponding to V^o-particle production.

The cross-section for V^o-particle production is about 1/40 the geometrical cross-section of the lead nucleus, or 5.5×10^{-26} cm^2/nucleus. The diameter of a lead nucleus is about 6 nucleon diameters ($A^{1/3} = (207)^{1/3} = 5.91$) and so 1/40 the geometrical area of the lead nucleus is slightly less than the geometrical area of a nucleon. Suppose a penetrating-shower particle is traversing a lead nucleus along a diameter and is confined to a cylinder whose cross-section is that of a nucleon. Such a particle will pass through 6 nucleons and average one collision. It thus appears that the impact parameter for V^o-particle production is of the order of 1/6 the nucleon radius.

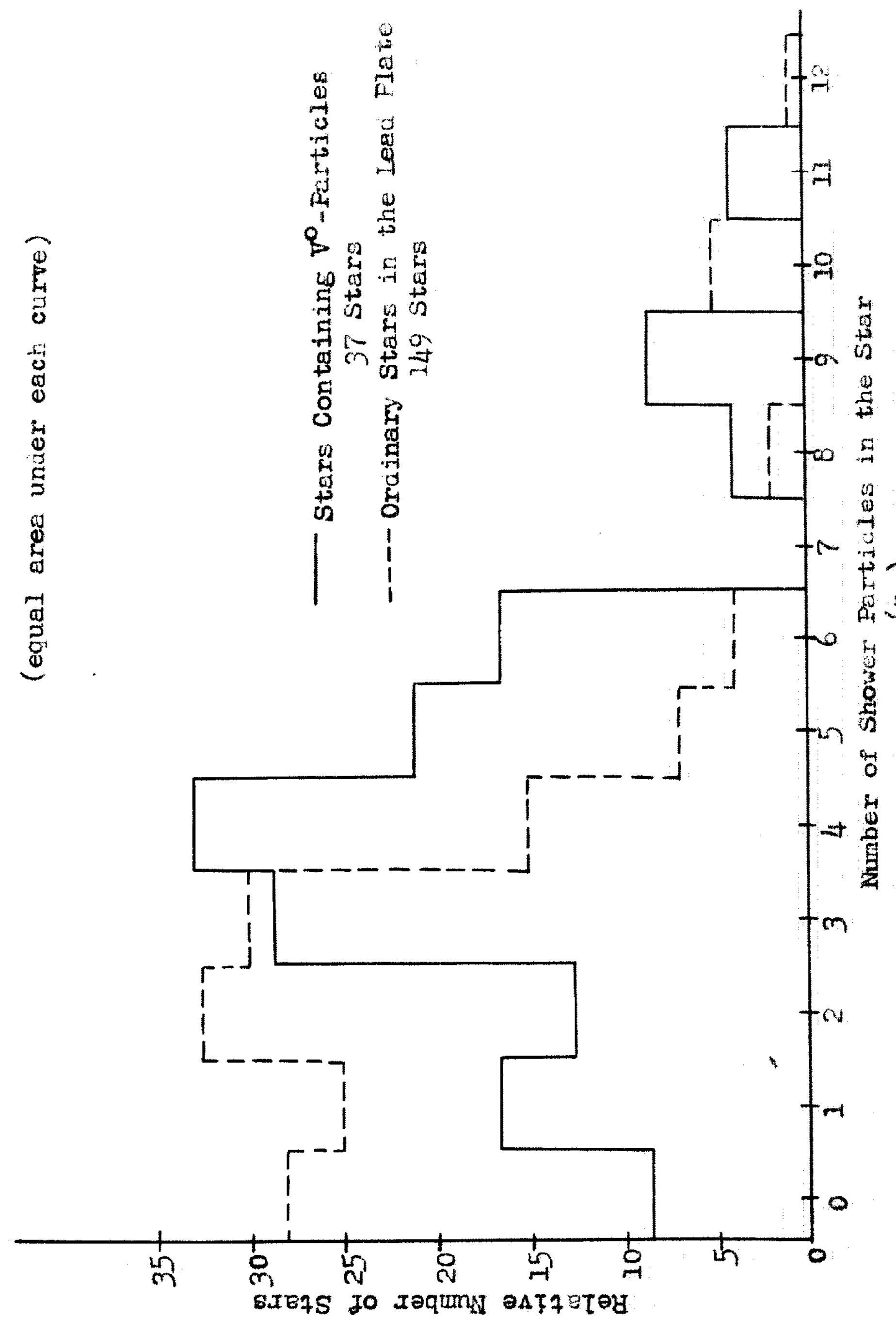
The number of shower particles (n_s) in each of the 148 ordinary stars (without V-particles) were recorded (fig. 20). The average multiplicity of these stars is two. The ratio of V^o-particles to shower particles in high momentum interactions is $7/2 \times 156$ or about two percent. An estimate of three percent was given by Seriff et al[14], and is a very reasonable estimate.

A single charged V-particle was produced in the 9 rolls of film. The overall ratio was 9 charged V-particles to 37 V^o-particles. It is estimated that the cross-section for charged V-particle production is about 1/3 that for V^o-particles.

FIGURE 20

Multiplicity of Stars Containing V^{o}-Particles
Compared With Ordinary Stars

(equal area under each curve)



D. Indications That the Cross-Section is Energy Dependent

In calculating the cross-section for V^o-particle production, it was tacitly assumed that every one of the charged penetrating-shower particles with momentum in excess of 1 Bev/c was capable of producing a V^o-particle. Experimentally, 1 Bev/c is the highest momentum that can be measured with some certainty. The multiplicity of the stars containing V^o-particles is larger than the multiplicity of the ordinary stars (fig. 20), and this is an indication that the average momentum of the penetrating-shower particles producing V^o-particles is greater than the average momentum of the penetrating-shower particles. Figure 21 is a histogram for the ordinary stars, and compares favorably with the Brown and McKay[7] stars, also observed to occur in a lead plate in a cloud chamber. Brown and McKay recorded fewer stars with no shower particles ($n_s = 0$), as would be expected, since their lead plate was 1/2 inch thick. The lead plate and cloud chamber walls in the 18-inch cloud chamber are in excess of one inch.

E. The Momenta of the Incident Shower Particles Producing V^o-Particles from the Cross-Section

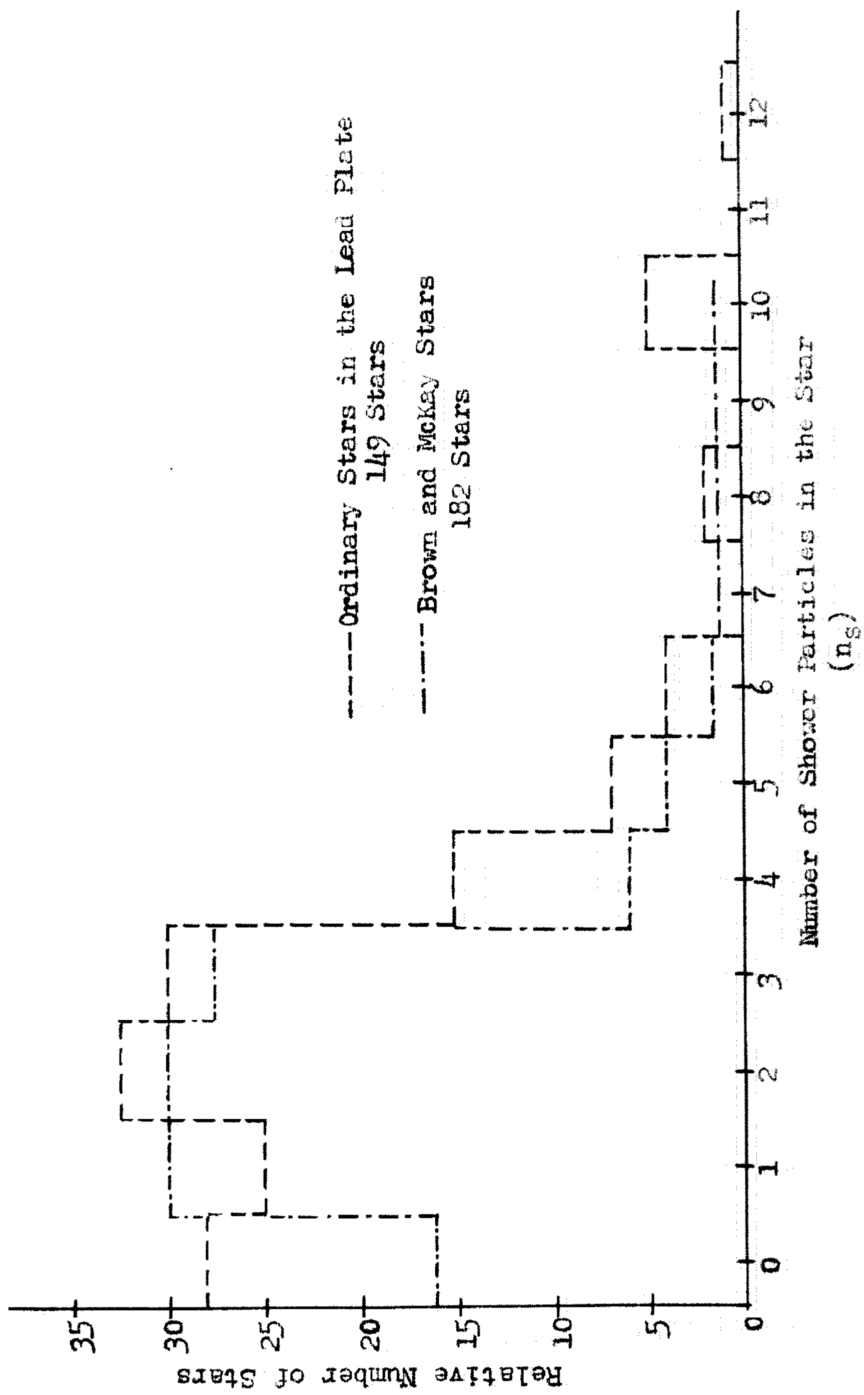
It was found that only one in 20 nuclear collisions by penetrating-shower particles produced a V^o-particle. The multiplicity of the stars containing V^o-particles indicates that this small ratio is due, in part, to

FIGURE 21

Multiplicity of the Stars Observed in the Lead Plate

Compared With the Stars Observed by Brown and McKay (7)

(equal area under each curve)



the necessity of high momentum interactions. In the region above 1 Bev/c momentum, Barker and Butler[10] observed 85 penetrating-shower particles, from which they obtained the differential spectrum $dp/p^{(2.5 \pm 0.3)}$. The penetrating-shower particles with momenta in excess of 7.4 Bev/c are in the highest 1/20 of the integral spectrum.

F. The Ratio of π-Mesons to Protons in Penetrating Showers

It has been estimated that 2/3 of the shower particles, having a momentum in excess of 1 Bev/c, are π-mesons. The method by which the estimate was obtained is quite interesting and worth relating. Camerini et al[5] explained:

> This has been done in the following way: the energy spectrum of the μ-mesons in the cosmic radiation, most of which are produced by the decay in flight of π-particles, has been determined by Sands (1950)*. From this spectrum, the energy distribution of the parent π-particles can be computed; and thence, from the low energy distribution of π-mesons observed in the present experiments, the total number to be expected among the (π, p) particles can be calculated. The result is about two-thirds of all the (π, p) particles, and the rest are therefore attributed to protons.

The (π, p) particles were defined as having momenta in excess of 1 Bev/c, and the two types of particles could not be distinguished.

The following estimate can be made for the ratio of π-mesons to protons, in the observed penetrating showers, in the present survey. In Table 3 the stars produced by neutral particles are recorded. There

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*Sands, Phys. Rev., (1950), 77, 180

was a total of 19 such stars accompanying penetrating showers. The neutral particles are assumed to be neutrons. The 19 neutron-initiated stars can only be compared with the 156 stars produced by charged penetrating showers in the classification (1), since a neutron interaction would be undetected in the other two classifications (nuclear scattering and stopping in the lead plate). The ratio is one neutron to 8 charged particles, assuming neutrons, π-mesons and protons have the same cross-section for star production. The number of neutrons should be at least equal to, or greater than, the number of protons. Using the Camerini et al[(5)] estimate that the number of neutrons is 1.25 times the number of protons, the 8 charged particles contain 7 π-mesons and one proton. The result is about 88 percent of all the shower particles, with momenta in excess of 1 Bev/c, are π-mesons. There was a total of 27 V^o-particles produced in the lead plate by charged penetrating-shower particles. Using the above result, three V^o-particles were produced by protons and 24 V^o-particles were produced by π-mesons.

VI. AN ATTEMPT TO MEASURE A LARGE ANGULAR MOMENTUM IN THE V^o-PARTICLE

A. Why a Large Angular Momentum Might Be Present

A large angular momentum would explain the long lifetime of the V^o-particles. That such would be the case was first proposed by R. Feynman (to the best knowledge of the author). The characteristic lifetime of the proton and the negative π-meson association in the nucleus is $\hbar/M_p c^2 = 6 \times 10^{-25}$ seconds. The V^o-decay(3) usually results in a proton and a negative π-meson ($V^o \rightarrow P^+ + \pi^- + Q$) and has the "long" lifetime of about 3×10^{-10} seconds. That a large angular momentum quantum number (ℓ) would give a long lifetime due to a centrifugal barrier potential was offered by W. Fowler in a discussion with E. Fermi. An angular quantum number $\ell = 5$ would probably be sufficient to account for the long lifetime.

If the incident particle is a π-meson, the least linear momentum it can possess at a maximum impact parameter equal to the range of the nuclear forces $a_n = \hbar/\mu c$, and at the same time have an angular momentum of $\ell = 5$ is 1.6 Bev/c. This momentum is readily available in typical π-mesons contained in penetrating showers. The necessary linear momentum for the π-meson is larger for smaller impact parameters, but many π-mesons possess momenta in excess of 1.6 Bev/c. Whenever an incident π-meson has sufficient linear momentum in order to have the necessary angular momentum at small impact parameters, the total energy in the center of mass system is sufficient to conserve the incident particles and produce numerous π-mesons according to the Fermi thermodynamic theory[6].

B. A Measurement That Could Indicate a High Spin in the V^o-Particle

In a typical case of V^o-particle production in the lead plate between the two cloud chambers, one can identify the track of the incident particle (I), which undergoes a nuclear collision and creates a star in which a V^o-particle is emitted. The projection of the star secondaries back into the lead plate serve to locate the origin of the star. The V^o-particle is identified after it decays into two oppositely charged particles. The line of flight of the V^o-particle before decaying is from the star origin to the decay point. If the V^o-particle is a high spin particle, it seems plausible to expect the spin vector of the V^o-particle to be normal to the plane formed by the incident particle (I) and the line of flight of the V^o-particle (V^o) before decaying. The direction of the spin vector would tend to be the direction of the vector $\overline{I} \times \overline{V^o}$. It is thought this spin direction is plausible because the nuclear field of the incident particle interacts with the nucleon at some impact parameter (p) and a sufficiently large incident momentum serves to impart the necessary angular momentum in a direction normal to the plane of the incident particle's line of flight and the impact parameter. It is an assumption that the forces involved between the incident particle and the nucleon are in the plane of the incident particle and the impact parameter. It is expected that the V^o-particle tends to move in this same plane.

If the V^o-particle possesses a high spin, the decay particles (positive proton and negative π-meson) must conserve this large angular momentum. Since the internal spin of the π-meson and proton are zero and one-half respectively, the two decay particles conserve the angular momentum during the decay process by moving with sufficient angular momentum with respect to the center of mass of the V^o-particle. The two decay particles can conserve the large angular momentum most easily by being emitted in a plane normal to the spin vector of the V^o-particle. It is tacitly assumed that there are only the two charged particles in the decay. However, if there are also neutral particles involved in the V^o-particle decay, the situation is unchanged unless they possess a large internal spin. The plane of the decay particles should be nearly parallel to the plane of the incident particle (I) and the V^o-particle (V^o).

If the vector direction of the decay particles are $\overline{A}$ and $\overline{B}$, then $\overline{A} \times \overline{B}$ should be nearly the direction of the spin vector of the V^o-particle and hence nearly parallel to $\overline{I} \times \overline{V^o}$. If the angular momentum of the V^o-particle is $\ell\hbar$, then the angular portion of the wave function for either the π-meson or the proton is

$$Y_{\ell\ell}(\theta, \phi) = P_{\ell}^{\ell}\ (\theta)e^{i\ell\phi} = \text{Sin}^{\ell}\,\theta\ e^{i\ell\phi} \tag{1}$$

The distribution of the π -mesons and the protons should be as $\text{Sin}^{2\ell}\,\theta$ where θ is measured from the direction of the spin of the V^o-particle. A distribution $\text{Sin}^{2\ell}\theta$ has a high probability near 90^o. The angle θ can be obtained from

$$(\overline{I} \times \overline{V^o}) \times (\overline{V^o} \times \overline{A}) = \overline{V^o}\ (\overline{I} \times \overline{V^o} \cdot \overline{A}) - \overline{A}(\overline{I} \times \overline{V^o} \cdot \overline{V^o}) \tag{2}$$

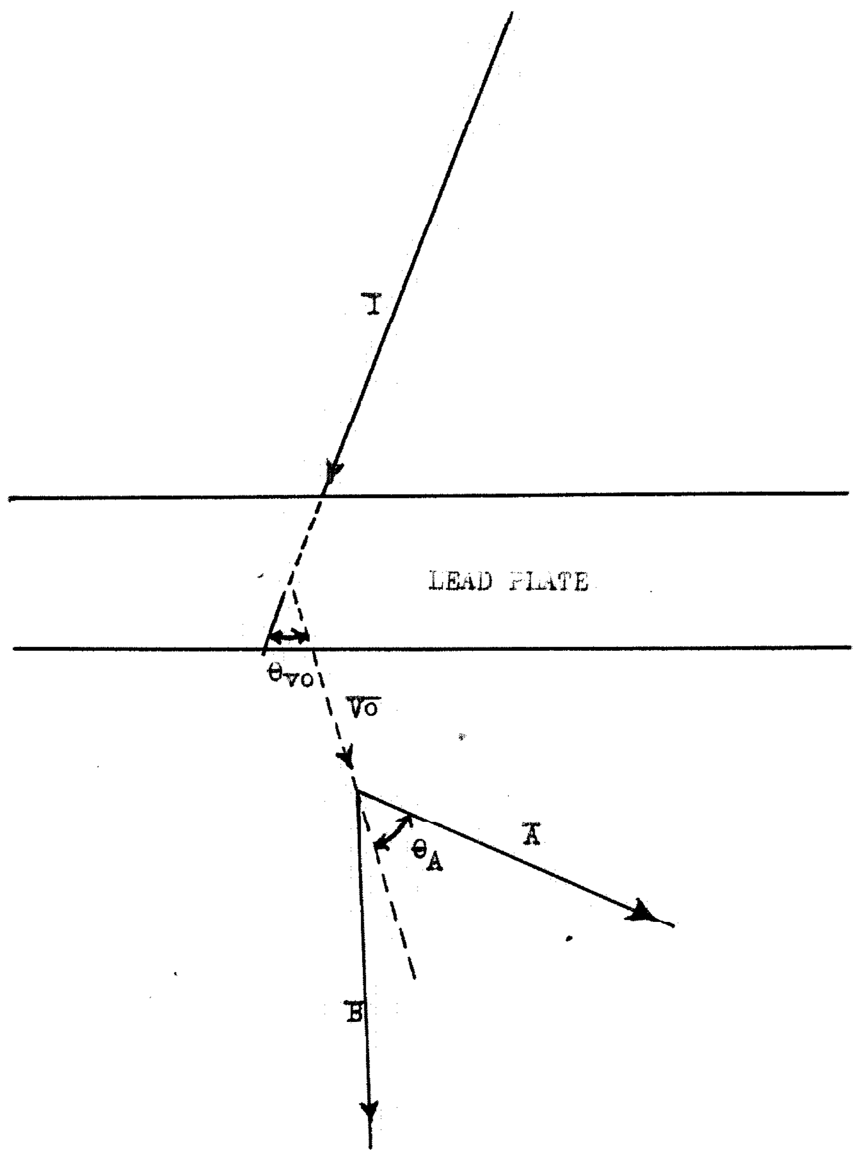
The second term on the right side of the equation is zero. Since the direction of the remaining term on the right side of equation (2) is the direction of $\overline{V^o}$, the direction of the left side of equation (2) is also $\overline{V^o}$. Expanding both sides of equation (2) it is found that

$$(I\ \text{Sin}\ \theta_{V^o}\, V^o)\ \text{Sin}\,\delta\ (V^o\ \text{Sin}\ \theta_A\ A)\overline{e}_{V^o} = \overline{V^o} \begin{vmatrix} I_x & I_y & I_z \\ V^o{}_x & V^o{}_y & V^o{}_z \\ A_x & A_y & A_z \end{vmatrix} \tag{3}$$

Where θ_{V^o} is the angle between $\overline{I}$ and $\overline{V^o}$, θ_A is the angle between $\overline{V^o}$ and the decay particle $\overline{A}$, and δ is the angle between the planes $(\overline{I}, \overline{V^o})$ and $(\overline{V^o}, \overline{A})$. These various particles and angles are in Figure 22.

FIGURE 22

The Particles and Angles Involved in the Calculation of a High Angular Momentum in the V^o-Particle.



Solving equation (3) for Sin δ it is found that

$$\sin \delta = \frac{\begin{vmatrix} I_x & I_y & I_z \\ V^o_x & V^o_y & V^o_z \\ A_x & A_y & A_z \end{vmatrix}}{I\, V^o\, A \sin \theta_{V^o} \sin \theta_A} \qquad (4)$$

The angle δ is (90 - θ) where θ was defined by equation (1). A correlation between the spin vector of the V^o-particle and the normal to the plane of the V^o-particle and its decay particle (A) would be given by a small angle δ.

Since the three coordinates (x, y, z) for any point on a track in the cloud chamber can be obtained, the three vectors $\overline{I}$, $\overline{V^o}$ and $\overline{A}$ can be computed. The angles θ_{V^o} and θ_A can be calculated from the three vectors. The numerical data necessary for the above calculations can be obtained from a graphical construction in the same manner as the other calculations, such as the Q values, were obtained for these same V^o-particles(3). The two sets of track images from the stereoscopic photographs are projected upon a sheet of tracing paper and careful pencil tracings are made of all the pertinent particles on the photograph.

C. The Results Indicate a High Angular Momentum in the V^o-Particles

The angle δ was calculated for each case of V^o-particle production in the lead plate in which the incident particle was a charged particle (32 cases). These results are given in Figure 23. It is seen that there is a tendency for the small angles δ and these circumstances would indicate a high angular momentum in the V^o-particle. The anticipated distribution for an indication of high angular momentum was from equation (1) as $\sin^{2\ell}\theta$, which has a very small probability for an example occurring beyond 45 degrees if ℓ is as large as 5. Since the calculated quantities are $\delta = 90 - \theta$, the distribution in δ indicating a high angular momentum would have a large concentration near zero angle. However, any tendency for the angle δ to be small is thought to be important, since the hypothesis that the direction of the spin of the V^o-particle is the direction of $\overline{I} \times \overline{V^o}$ is probably not true. The direction of the spin of the V^o-particle may be a broad distribution about this direction. The fact that a tendency for small angles δ was found seems to indicate that the V^o-particle may have a high angular momentum.

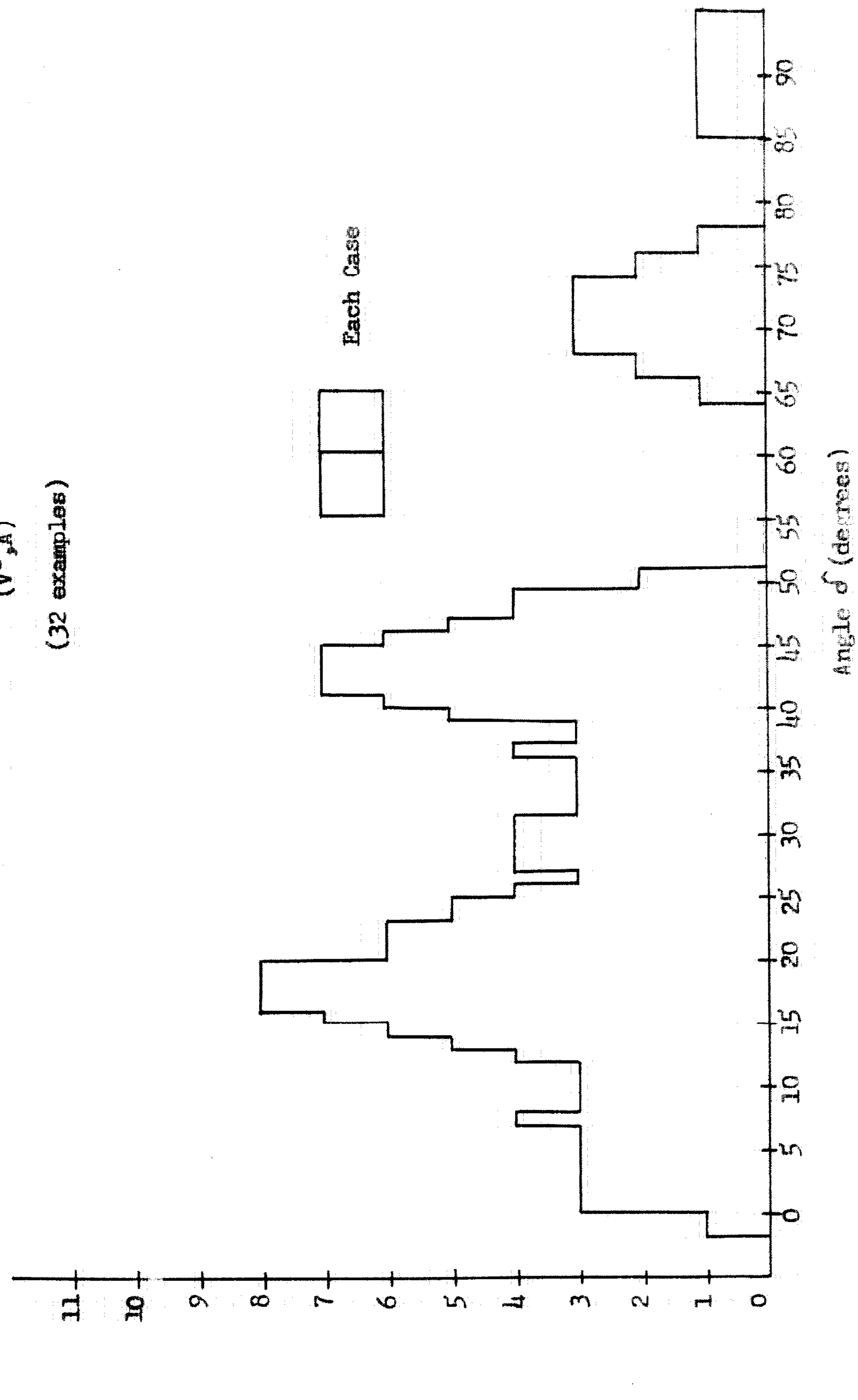
D. The Errors in the Measurements that Would Indicate a High Angular Momentum in the V^o-Particle

The angle δ will be most accurate when the two planes $(\overline{I}, \overline{V^o})$ and $(\overline{V^o}, \overline{A})$ are well defined. A plane is well defined if the two vectors are long and the angle between them is large. The lengths of the vectors $\overline{I}$, $\overline{V^o}$ and $\overline{A}$, and the angles θ_{V^o} and θ_A, vary over large ranges in the available examples of V^o-particle decays. The error for each case

Figure 23

The Distribution of the Angles δ between the Plane of the Incident Particle and the V^0-Particle (I, V^0) and the Plane of the V^0-Particle and one of its Decay Particles (V^0, A)

(32 examples)



in Figure 23 was arbitrarily assigned the value $\pm 5^{o}$, although it was known that the error in some cases was probably much larger than 5 degrees.

The z coordinate of any point on a track (depth in the cloud chamber) is obtained from a projection of the stereoscopic photograph and has an error of about 5 times the error in x or y. The particle $\overline{A}$, which has the largest angle with respect to the path of the V^{o}-particle, was usually chosen. This particle was in nearly every case the π-meson rather than the proton. The π-meson usually has the smaller momentum, and hence the larger curvature in the magnetic field. Because the angle θ_A is the angle between the V^{o}-particle and its decay particle A, it is necessary to construct the tangent at the decay point. The angle θ_A between the V^{o}-particle and one of its decay particles is usually large when the momentum of the V^{o}-particle is low. Since a low momentum V^{o}-particle will usually decay near its origin, the length of the vector $\overline{V^{o}}$ tends to be short in the cases having sizable angle θ_A. The location of the star origin in the lead plate is necessary in order that the line of flight of the V^{o}-particle be defined. The accuracy with which the origin can be located depends upon the number of particles in the star that can be projected back into the lead plate and the distance over which they must be projected. A star with a sizable number of particles is a high energy event, and the particles in the star, including the V^{o}-particle, are likely to have high energies. A high momentum (high energy)

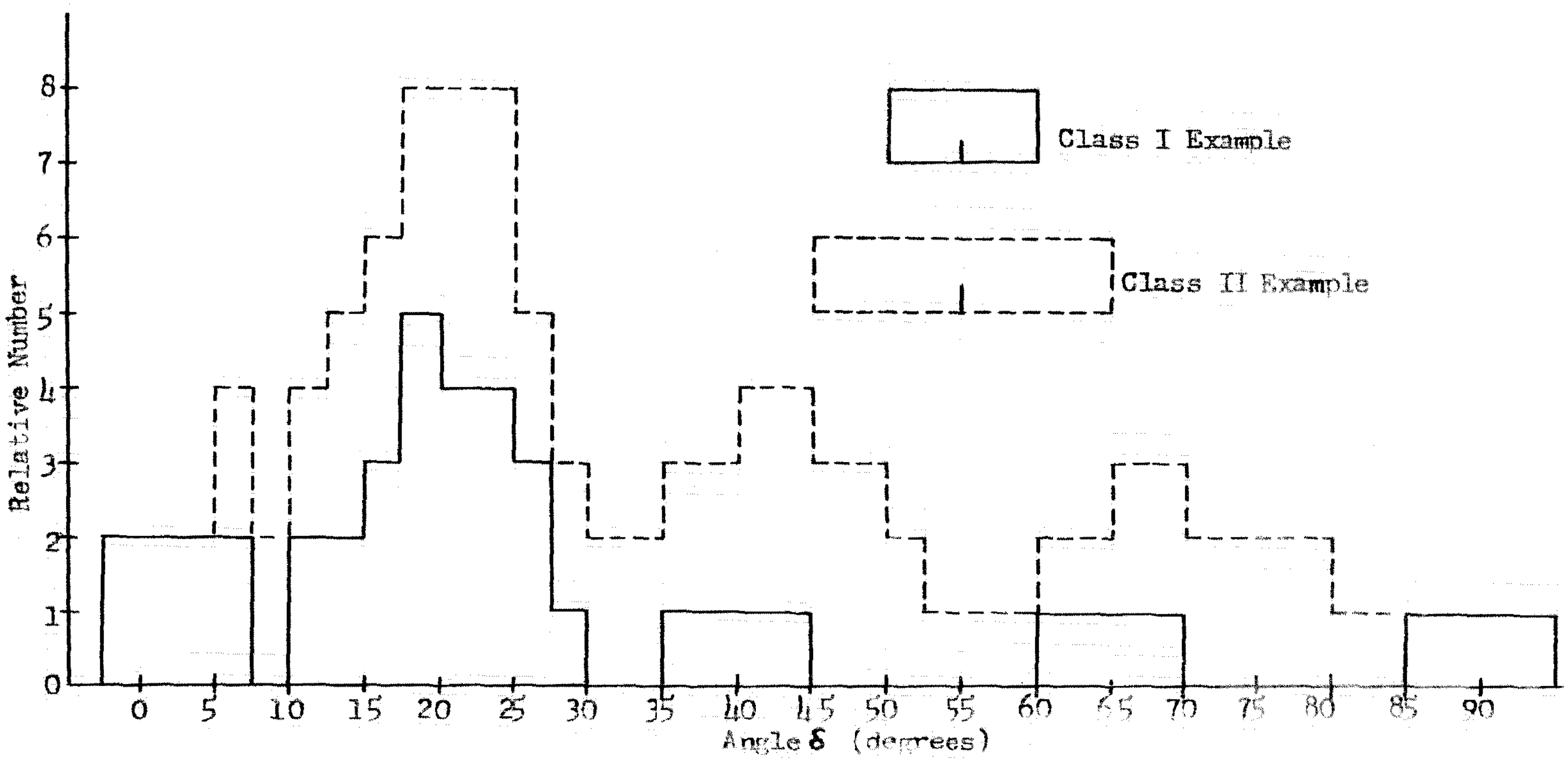
V^o-particle has a small angle between the decay particles. With so many conflicting sources of errors, no realistic error could be assigned to the angles δ that had been calculated.

As a last resort, the accuracy of the various angles δ was estimated by the tedious process of recalculating a number of the cases. What were thought to be the best cases were selected for recalculation with no influence by the original values of δ that had been obtained. A total of 20 of the best examples were selected for recalculation. These examples were subdivided into two classes (I, II), according to what accuracy it was thought the angle δ could be calculated. The basis for the selection was simply the geometrical configuration of the various particles with respect to one another and the lengths of the tracks.

The final distribution for the angle δ, which is the angle between what was thought possibly to be the spin direction of the V^o-particle and one of its decay particles, is given in Figure 24. There is a remarkably large number of cases with an angle δ less than 45 degrees. A tabulation of the various examples and their path lengths and angles is given in Table 7. The values obtained in the first calculation and the recalculation are compared in each case. The path lengths of the various particles in each specific case are essentially the same in both calculations. However, the construction procedure was refined for the recalculation and in each case this is thought to be the more

FIGURE 24

The Distribution of the Angles δ Between the Plane of the Incident Particle and the Vo-Particle (I,Vo) and the Plane of the Vo-Particle and one of its Decay Particles (V^o,A).



accurate of the two. The errors assigned are as follows: the class I examples are consistent with having about $\pm 5^{\circ}$ errors in the final values for δ, and the class II examples are generally within $\pm 10^{\circ}$. The class III examples, which were the poorest examples and were not recalculated, were sampled and found to have such large errors that it was uncertain as to whether a specific case had an angle δ greater or less than 45°.

The recalculated examples bear out the results of the first calculation and provide a check as to the accuracy with which the angle δ can be given. Finally, one must ask, "What is the probability of getting 5 cases at angles greater than 45° and 15 cases at angles less than 45° in a total of 20 cases, if they are really uniformly distributed from 0 to 90 degrees?". This is the situation in Figure 24. The answer is about 1 in 20.

TABLE 7

The V^o-particles produced in the lead plate were classified according to the apparent accuracy of the examples for a calculation of the angle δ.

Class I examples appeared to be the most accurate.

Class II examples appeared to be accurate but in each case there was some undesirable feature.

Class III examples appeared to be very inaccurate.

L_I is the usable length (cm) of the path of the incident particle in the gas of the top cloud chamber.

L_{V^o} is the length (cm) of the path of the V^o-particle in the gas of the lower cloud chamber.

L_A is the usable length (cm) of the path of the decay particle A.

θ_{V^o} is the angle (degrees) between the incident particle and the V^o-particle.

θ_A is the angle (degrees) between the V^o-particle and its decay particle A.

The first line in each case is the results of the recalculation and the second line contains the first calculation results. The same track A was used in both of the calculations of the angle δ except when indicated on the right, by the charge of the particle that was used.

TABLE 7 (continued)

Class I Examples

Frame No.	L_I	L_{Vo}	L_A	θ_{Vo}	θ_A	δ
10876	11.5	2.5	14.2	31	63	24
				33	60	26
12418	15.2	1.5	10.6	57	34	1
				60	31	5
13231	13.7	6.9	9.9	30	31	25
				29	26	20
21910	14.4	6.6	9.7	27	18	24
				30	18	21
22895	13.1	3.1	5.1	25	25	37
				29	25	44
23942	15.3	3.7	5.6	51	42	12 -
				47	30	17 +
26585	5.4	5.1	10.6	37	37	90
				40	35	90
30045	14.1	3.8	6.7	18	27	24
				24	28	5
30095	8.0	5.5	8.8	21	41	1
				22	40	4
31341	12.5	7.3	8.2	16	38	62
				20	39	69
31476	12.6	9.1	14.2	51	35	26
				50	29	15

Class II Examples

Frame No.	L_I	L_{Vo}	L_A	θ_{Vo}	θ_A	δ
9489	15.0	1.1	10.7	48	43	49
				53	41	39

TABLE 7 (continued)

Class II Examples (continued)

Frame No.	L_I	L_{V^o}	L_A	θ_{V^o}	θ_A	δ
12717	13.6	0.5	2.7	31	48	76
				28	50	70
15181	13.8	6.4	6.8	6	12	70
				8	11	69
19796	13.6	0.3	3.7	8	60	6 -
				8	50	18 +
24915	8.8	2.2	13.2	22	25	53
				33	32	44
27529	24.7	0.3	10.7	20	55	3
				8	53	22
27796	14.8	2.5	12.4	28	4	27 -
				28	22	19 +
28167	14	0.7	8.8	28	72	39
				25	76	41
30803	4.7	6.3	8.2	15	37	18 +
				27	30	40 -

Class III Examples

In nearly every Class III example the angle between the V^o-particle and its decay particle (A) was small. The value θ_A is the apparent angle (projected angle) as it appears on the photograph and was the reason for classification of most of the examples as Class III. However, the value of θ_A in the actual calculation of was the true angle in space. These Class III examples were not recalculated.

8796	13.6	0.4	16.3	37	34	46
17078	17.0	12.6	5	10	11	15
17407	15.0	1.0	10	42	15	15

TABLE 7 (continued)

Frame No.	L_I	L_{V^o}	L_A	θ_{V^o}	θ_A	δ
17685	15.0	11.5	5.7	11	6	32
18675	15.0	1.1	11.2	24	4	29
20976	10.6	7.5	6.2	6.1	11	12
21121	7.5	1.3	6.7	34	10	45
23368	16.0	1.0	11.3	23	3	71
26867	18.0	4.0	3.4	57	27	32
28738	13.0	5.5	12.0	22	3	73
31044	16.0	1.5	7.1	42	3	43

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