

THE
DIELECTRIC CONSTANT OF WATER VAPOR
AT A
FREQUENCY OF 42 MEGACYCLES

THESIS BY
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ABSTRACT

A heterodyne beat apparatus for the determination of the dielectric constant of water vapor at 42 megacycles was constructed. With it the Clausius-Mosotti equation was proved to be valid for water vapor at constant temperature at this frequency, and the value of the dielectric constant of water vapor at 100 degrees centigrade and 760 mm of mercury was determined. The value obtained was 1.0052 .

PURPOSE OF THE INVESTIGATION

This investigation was undertaken because of certain irregularities in the transmission of radio signals between Pasadena and Palomar. The stations operate at a frequency of 41 megacycles, and over a distance of 100 miles. At this distance only the ground wave need be considered, as the sky wave, even when reflected from the Heaviside layer, comes back to earth only at distances much greater than this. This ground wave, being electromagnetic in character, should follow the ordinary optical laws, which would cause it to be diffracted around obstacles and bent in passing through media in which a gradient of the refractive index exists.

The optical path between Pasadena and Palomar is obstructed, the line of sight cutting through a mountain at a vertical distance of 1800 feet below the crest. Therefore in this case both diffraction and refraction may take place, and it was to give an idea of the amount of bending due to different concentrations of water vapor in the atmosphere that the dielectric constant of water vapor in this frequency region was needed. This is to be used in a research problem on the variation of field strength between these two stations.

No previous investigation of the dielectric constant of water vapor has been undertaken at a frequency greater than 1 megacycle; although no anomalies in the dielectric constant at this higher frequency were really expected, yet it was thought well to actually measure the dielectric constant at 42 megacycles.

HISTORICAL REVIEW

The first measurement of the dielectric constant of a gas was undertaken in 1874 by Boltzmann¹ who varied the dielectric between two condenser plates and found the resulting changes in potential with an electrometer. Seven gases, namely air, hydrogen, carbon dioxide and monoxide, nitric oxide, methane, and ethylene were measured; and the Clausius-Mosotti relation, that $(\epsilon-1)$ varies linearly with the density, was shown to be valid. He found for these gases that the dielectric constant was approximately equal to the square of the optical refractive index. Maxwell's relation $n^2 = \mu\epsilon$ where n is the refractive index of the medium, ϵ its dielectric constant and μ its magnetic permeability, at the frequency of the wave, is practically impossible to check directly, but this served as an indication of its validity.

Klemencic² checked Boltzmann's work on these gases and also investigated ethyl chloride and bromide, ethyl ether and sulphur dioxide. His method was not a static one, but involved the charging of a condenser at 60 cycles and its discharge through a galvanometer at the same frequency. Although he agreed with Boltzmann in his values on the seven gases measured by the latter; for the vapors in question he found that the relation $n_o^2 = \epsilon$, where n_o is the optical refractive index, was not satisfied, the differences lying well outside his limits of error.

Lebedew³ next worked on this subject, measuring nine gases, namely toluene, benzene, methyl and ethyl alcohol, and several esters, at 100 to 126 degrees centigrade. Of

these only benzene had a dielectric constant which was the square of its optical refractive index. His method was to take two condensers, one of which contained the gas to be measured, and, charging them with the same amount of electricity, measure the potential difference.

Other early workers in this field were Linde⁴, who studied carbon and sulphur dioxides, nitrous oxide, and chlorine under their own vapor pressures, Goodwin⁵, who investigated ammonia, and Hasenoechri⁶, who investigated nitric oxide and oxygen.

The next important research in this field was done by Bädeker⁷, who followed the method given by Nernst⁸ and improved by Philip⁹, which consists of a simple bridge arrangement compensated for conductivity of the dielectric. He found that liquid films became adsorbed on his condenser plates and insulators, so that his values would be too high immediately after admitting new gas, but would decrease to a lower value on standing, when equilibrium would be attained. The vapors he studied were carbon disulphide, ammonia, sulphur dioxide, ethyl and methyl alcohol, nitric oxide, hydrochloric acid, and water vapor. The result he obtained for water vapor was $\epsilon = 1.00705(1 - .02(t - 145))$, where t is the temperature in degrees centigrade, which has been reduced to 76 cm pressure by the Clausius-Mosotti equation. To attain this value, Bädeker took the mean of the determinations of the dielectric constant of air by Klemencic and Boltzmann ($\epsilon = 1.000588$ at NTP i.e. 0 degrees centigrade and 760 mm pressure) for what he actually measured was the difference in dielectric constants of water vapor and air.

Tangl¹⁰, in 1907, used the simple bridge method to investigate the variation of the dielectric constant of air with pressure. He used pressures up to 100 atmospheres and found that the ratio of $(\epsilon - 1)$ to density remained constant. The value he found for this quantity was $\frac{\epsilon - 1}{\text{density}} = 540 \times 10^{-6} \pm 5 \times 10^{-6}$ at 19 degrees centigrade. (The density of air at NTP is taken as 1 in this result.)

Pohrt¹¹ investigated the dielectric constant of a great number of substances, using a simple bridge method, with 120 volt 50 cycle alternating current as his source and a special Dolezalek electrometer as a balance indicator. He claims an average error in $\epsilon - 1$ of .2% and gives the value $\epsilon = 1.000586$

Another method which might be mentioned at this point is that used by Rohmann¹². This method involves the use of a sensitive dynamometer invented by Mandelstam and Papalex¹³ in 1910, which measures the time integral $\int_0^{\infty} i_1 i_2 dt$, where i_1 and i_2 are the currents in the two fixed coils of the instrument. These coils are set with their planes at right angles, and a metal ring is hung so as to rotate about their common diameter. Two tuned circuits (with inductances L_1, L_2 , and capacitances C_1, C_2) are connected to the two coils of the instrument, and loosely coupled together. The circuit attached to the coil perpendicular to which the ring lies when no currents flow is closely coupled to a high-frequency circuit which may be energised by momentarily closing a telegraph key. If the inductances and capacitances satisfy the equation $L_1 C_1 = L_2 C_2$ then

there is no deflection of the ring on closing the key, but since this zero is one of unstable equilibrium, the instrument is very sensitive.

Rohmann dried the air he worked with by passing it through tubes containing sodium hydroxide, sulphuric acid, and phosphorus pentoxide, and obtained the value $\epsilon = 1.000580$ for the dielectric constant at 0 degrees centigrade and 760 mm. of mercury. Riegger¹⁴ used this same method in 1919 to compare hydrogen, methane, and carbon dioxide and monoxide with air.

The next worker in the field was M. Jona¹⁵. He used an a.c. bridge null method similar to that of Bädeker. However, he did not interchange gases in the gas condenser arms of the bridge, as did Bädeker, but used only one gas condenser. Jona assumed the validity of the Clausius - Mosotti relation at any one temperature and reduced the result to 76 cm. of mercury. The results of his temperature run on air gave values ranging from 1.000525 to 1.000589 when reduced to NTP, the average value being $\epsilon = 1.000556$. For saturated water vapor in the range from 390 to 450 degrees he obtained the following data, which have been reduced to 76 cm:

Temperature °K	390.4	397.5	402.0	415.1	423.8	436.2	451.4
$\epsilon - 1$ observed	$612 \cdot 10^{-5}$	584	575	530	511	489	462
$\epsilon - 1$ Bädeker	$754 \cdot 10^{-5}$	740	732	709	695	675	652

He did not claim any particular absolute accuracy, merely stating that if the value of the dielectric constant were

subsequently to be determined at any one temperature for any of the gases he investigated, then his results would give the correct temperature dependence. In this same manner he studied carbon dioxide, ammonia, sulphur dioxide, and methyl alcohol.

In 1919 Pungs and Preuner¹⁹ made a noteworthy contribution to the study of substances at this high frequency by inventing the heterodyne beat method which has been used by nearly all investigators since that time. Their circuit consisted of two triode oscillators which were loosely link - coupled together. In the link circuit is a rectifier and a condenser, across which is a pair of earphones in series with an inductance. This inductance is linked magnetically with an audio oscillator, causing a beat note between the audio signal and the heterodyne whistle of the two high frequency oscillators. They used the method for measuring small capacities by comparison with a standard, working at a frequency of one megacycle.

This method was first used for the measurement of dielectric constant by Hyslop and Carman²⁰, who measured the dielectric constant of kerosene, toluene, ethyl ether, petroleum ether, and olive oil, using a wavelength of 5000 meters. Hans Weigt²¹ next reported a measurement of carbon monoxide and dioxide by this same method.

Fritts²² determined the dielectric constants of air, carbon dioxide, nitrogen, hydrogen, and oxygen by this means, using simple triode oscillator tubes. He compared the beat note photographically with an electrically driven tuning fork, using two shutters in series, which gave him a slit of variable length.

Projections on one side corresponded to the vibrations of the fork and so gave him a time axis, while on the other side of the line they gave him the beats of the heterodyne signal with the fork. He claimed that he could measure the dielectric constant absolutely to within $\frac{1}{2}\%$.^{*} Since the whole record was taken within $\frac{1}{2}$ minute, he felt that drift need not be considered. Working at 393 meters he checked Boltzmann on hydrogen and carbon dioxide, but got a slightly lower value for air than the latter, differing by 8% in his value for $\epsilon - 1$.

Zahn²³ reported in 1924 a study of the halogen hydrides and air, using triode oscillators and the usual heterodyne circuit. The best note was amplified and fed into a telephone, where it was compared by ear with the output of a tuning fork. He used a frequency of one megacycle in his main circuits and adjusted his measuring condenser to return the beat note to 1000 cycles after admitting the gas. For air he took 39 readings and found a maximum deviation from the mean of $\frac{1}{2}\%$,^{*} giving a probable error of 1/10%.^{*} His final value for air is $\epsilon = 1.000572$ at NTP.

In 1928 Stuart²⁴ used this same method to study the temperature dependence of the dielectric constant, working at a frequency of 300 kilocycles. He claimed that the drift was linear and less than 1 part in 10^6 per minute, with no sudden fluctuations. As a detector he used a vibration galvanometer. He employed a mercury switch by means of which he could

*

error in $\epsilon - 1$

substitute a standard condenser for his gas condenser and so check the drift from time to time. Readings were taken by alternating from vacuum to air repeatedly. He studied air and carbon dioxide, getting the value for air $\epsilon = 1.000582 \pm 0.000007$.atNTP.

Forro²⁵ reported a determination of the dielectric constant of air by the beat frequency method in 1928, giving the value $\epsilon = 1.000539$. In a paper published a short time later²⁶ the frequency dependence of ϵ is studied and the value $\epsilon = 1.000586$ at .6 megacycles and $\epsilon = 1.000593$ at 4.3 megacycles given, with an accuracy in $\epsilon - 1$ of 1.25% claimed. No reason was given for the difference between this value and the earlier one.

Sanger and Steiger²⁷ report a determination of the dielectric constant of water vapor by the beat frequency method, using simple triode oscillators and zero beats with a tuning fork as obtained by judging the point of zero beat aurally. Nitrogen was the comparison gas and Zahn's values were taken as correct. Their results follow: the density was .4181mgm/cc throughout.

Temperature, °K	393	423	453	483
$(\epsilon - 1) \times 10^5$	400.2	371.67	348.81	328.73

H. E. Watson, Rao, and Ramaswamy²⁸ made a precision determination of the dielectric constants of 17 gases at 800 kilocycles by the heterodyne beat method. In the course of this investigation they calibrated their General Radio

condenser by a step-by-step method, and discovered that the lead-in hole of the condenser caused an error of 2% in the readings at the frequency in question. (This of course would cause an error in Zahn's work, for he also used a General Radio condenser.) To measure their capacity changes they made a special condenser, as designed by Wolf.³⁶ This is a cylindrical condenser with the centre member made of two cylinders of different diameters joined together. This central barrel was then moved to and fro by means of a very accurate screw, giving a precise knowledge of the capacity change. The screw was calibrated optically. To reduce the drift of their oscillators they used crystal control on one of them. Their value for the dielectric constant of air was 1.000528 at 76 cm and 25 degrees centigrade, the results showing a $\frac{1}{2}$ % deviation.*

Jordan, Broxon, and Walz²⁹ studied the dependence of the dielectric constant of air on frequency. Working at values up to 70 kilocycles, they found no dependence greater than their error of measurement. An improved double bridge method was used in this work. Claiming only 1% accuracy,* they give the value of the dielectric constant of air at NTP as 1.000591.

Finally, there is the determination of Hector and Schultz²⁵ who used the beat frequency method, obtaining their standard frequency by removing the modulation from the carrier wave of Radio Station WBEN, after which they generated harmonics of this frequency. They made a photographic comparison of the beat note with commercial 60 cycle current, by means of an

* in $\epsilon-1$

oscillograph, and obtained the value of the dielectric constant of air as $1.00058986 \pm .00000050$

Summary of Previous Results:

The results obtained up to the present for the dielectric constant of air are tabulated below:

Investigator	Reference	$(\epsilon-1) \times 10^6$	Accuracy Claimed
Boltzmann	1	590	
Klemenčič	2	586	
Tangl	10	576	1%
Pohrt	11	586	.2%
Riegger	14	550	2%
Jona	15	556	
Fritts	22	540	.5%
Zahn	23	572	.1%
Stuart	24	582	1.2%
Forro	25	{ 586 } { 593 }	1.25%
Watson	28		576
Jordan	29	591	1%
Hector	30	589.86	.1%
Ghosh	31	579.7	
Carman	32	594	2%
Bedesu	33	623	1%
Bryan	34	589.3	.3%
McAlpine	35	581	.4%

The arithmetical mean of the above values is 581.

The researches mentioned above on water vapor give indication that its dielectric constant satisfies the Debye equation:

$$(\epsilon - 1) = 4\pi N \left(\alpha_0 + \frac{\mu^2}{3kT} \right)$$

where:

N = number of molecules per cubic centimeter

k = Boltzmann's constant

μ = dipole moment

ϵ = dielectric constant

For our purposes we may put this equation in the form:

$$(\epsilon - 1) = \left(A + \frac{B}{T} \right) \times \text{density}$$

where:

$$A = 4\pi N_A \frac{\alpha_0}{m}, \quad B = 4\pi N_A \frac{\mu^2}{3km}$$

Now the original work of B ddiker gave a value of μ which was much too high, being 2.5×10^{-18} . Jona had trouble with adsorption on the plates of his condenser and so when he plotted $(\epsilon - 1)T/\text{density}$ he did not get a straight line. His value for μ however is fairly close to the accepted value, being 1.87×10^{-18} which gives the value of $B = 3590$. Sanger actually found both constants A and B above, getting the values $A = .600$ and $B = 3507$. The accepted value of μ , from these and other different kinds of experiment is 1.85×10^{-18} , leading to a value for B of 3520, which is very close to Sanger's value.

THE PRESENT RESEARCH

The Standard Oscillator:

In this research the heterodyne beat method was also used. The standard oscillator is seen in figure 1. The first stage is a standard crystal oscillator circuit using for a generator the type 77 tube T_1 . In the grid circuit of this tube is located the crystal X and paralleled with the radio frequency choke L_3 and the 8,000 ohm grid leak resistance R_1 . The crystal used is a Western Electric AT cut quartz crystal rated at 10.58330 megacycles, which is one of the spares for the Pasadena-Palomar transmission system. This crystal has a frequency which is practically independent of temperature variations. It was necessary to make a mounting for it, so this was constructed out of Victron insulating material in order that losses should be kept as small as possible. The screen of the tube is at the same d.c. potential as the plate, but is brought to r.f. ground potential by use of the .01 microfarad condenser C_4 . The tuned circuit L_1C_1 is connected to the plate through the .005 microfarad condenser C_3 . C_1 is a small variable condenser of 30 micromicrofarad capacity. L_1 is an open coil consisting of 21 turns of number 12 bare copper wire, 1 1/8 inches in diameter and 3 inches long, supported on celluloid spacers to which it is cemented with Duco cement. This was necessary to prevent vibration and consequent modulation of the generated wave. The plate of this tube was connected through the

r.f. choke L_4 and the jack J_1 to the 90 volt battery used.

The next stage is the doubler stage, which is coupled to the preceding one by means of the .0001 microfarad condenser C_5 . The tube T_2 is a type 76 triode with its grid biased to -22 volts. The oscillating circuit of this tube is very much like that of stage 1. L_2 is a coil of 7 turns of the same wire, wound on the same diameter and $2\frac{1}{2}$ inches long. This in conjunction with the 30 micromicrofarad air condenser C_2 resonates at the double frequency or 21.16 megacycles.

Originally a third stage was used to double this frequency again to 42.38 megacycles, but this was later found to be unnecessary, sufficient of the second harmonic of the second stage being present to give the required signal strength.

Mounted on this same chassis are the detector T_3 and audio amplifier T_4 . The signal from stage 2 and the one from the test oscillator described below are brought together on the grid of T_3 which is a type 76 triode. R_2 is a 150,000 ohm resistor and C_8 and C_9 are very small capacities formed by merely twisting an inch or two of insulated wires together. The load resistance R_3 is 40,000 ohms, C_{10} is a .1 microfarad coupling condenser, and C_{11} a .001 microfarad by-pass condenser. The coupling resistance R_4 is 15,000 ohms. The type 76 tube T_3 is cathode-biased by the 350 ohm resistance R_6 and the .005 microfarad bypass condenser C_{13} . The plate circuit of this tube is completed through the plate resistance

R_5 , which has a value of 50,000 ohms. The .1 microfarad condenser C_{12} and jack J_4 serve as the output circuit. J_4 consists of two jacks in parallel, into which are plugged an R.C.A. oscilloscope and a loud speaker.

As may be seen from the diagram, all these stages are in separate shielding containers to prevent mutual interaction. The shields and chassis are made from 1/16 inch aluminium sheet, and the choke coils L_4 , L_5 , and L_6 are for the purpose of keeping the radio frequency within bounds. The jacks $J_{1,2,3}$ are present so that a d.c. milliammeter may be inserted in the respective plate circuits during the tuning of the circuit. While the oscillator was in use the milliammeter was usually left connected to jack 2 to serve as a visual indicator of the conditions in the circuit.

The Test Oscillator:

The test oscillator is shown in figure 2. It consists of a symmetrical Hartley circuit, employing a type 30 tube T_1 . The tuned circuit consists of the inductance L_1 and the three condensers C_g , C_m , and C_t , in parallel. C_g is the gas condenser and C_m is the measuring condenser. These are described more fully below. The tuning condenser C_t is a dual air condenser with a grounded rotor, each side of which has a capacity of 50 micromicrofarads, and is driven with a vernier drive. It is used merely to adjust zero beat to the proper position on C_m before a run is made. The coil L_1 is wound with 12 gauge bare copper wire, with a diameter of 1 inch, and

has five turns in a length of 2 inches. This coil is supported on celluloid spacers to reduce vibration.

All leads from the oscillating circuit to the tube are of number 14 wire, which is necessary to give the required rigidity, as also to reduce losses. These leads are also spaced very closely together, in order to reduce their inductance at the expense of their capacitance. This was done so that the change in C_m should equal that of C_g more accurately.

The tuned circuit is coupled to the plate of T_1 through the .002 microfarad condenser C_1 . The operating voltage from the 90 volt dry battery B_2 is controlled by the resistor R_2 and measured by the meter M_2 , which has a resistance of 1,000 ohms per volt. The radio frequency choke L_2 and the .002 microfarad bypass condenser C_2 serve to keep the radio frequency out of the battery.

The filament is lighted by the voltage from the 3 volt dry battery B_1 , reduced through the resistor R_1 . Meter M_1 is a direct current voltmeter of 3 volts full-scale reading. The oscillation frequency varies rapidly with filament voltage, so that this meter was needed to indicate any changes occurring in this voltage. It was necessary to use dry batteries in the oscillator because any ungrounded wires coming out of the cabinet caused large body-capacity effects.

The cabinet is made from 1/16 inch aluminium and is held together with screws, it being impractical to solder it. Copper was not used because it tends to flow under stresses, and iron does not give adequate shielding. There are no pipes.

shafts, or wires coming out of the cabinet which are not grounded to it. Shafts made of insulating material were never used, because this leaves a virtual hole in the wall at this point. However, it is not well to ground the oscillating circuit at more than one point, so that shafts were especially constructed with an insulating section in them to avoid this.

For adequate shielding it was necessary to screen the meters, for when these were merely mounted on the panel in the customary fashion, large quantities of radiation came through them. It was also necessary to be able to read the meters, so pieces of bronze screen were annealed and placed over them, the edges of the screen being inside the cabinet and grounded to it. This proved successful in removing the difficulty. By means such as this, virtually all body capacity effects were eliminated, it being possible to even touch the cabinet on the operating side without causing a change in the beat note. The only holes in the cabinet not filled with metal are the lead-in hole for the connection to the other oscillator, which is on the far side of the cabinet, and the thermometer hole, which is no larger than absolutely necessary. In order to reduce effects due to external mechanical vibrations, the whole cabinet was mounted on rubber supports.

The Measuring Condenser:

The measuring condenser had to be especially constructed since the capacity change is extremely minute, being of the

order of .01 micromicrofarad. To overcome this difficulty earlier investigators have used a multiplying circuit which consists of a large condenser in series with the gas condenser. A small change in the capacity of the gas condenser will have to be compensated by a large change in the capacity of the measuring condenser, which is what is desired.

The objection to this method is the difficulty of calibration, for it is \longleftrightarrow necessary that the capacity of both condensers and their respective capacity to ground be known \longleftrightarrow accurately. These earlier workers were using a frequency of less than a megacycle, that is to say a wavelength greater than 300 meters, at which frequency the grounding problem is not so serious; it is possible to ground a wire at one point and know that a point an inch away on the same wire is also at ground potential. This, unfortunately, is not the case at 42 megacycles, and hence distributed capacitance is well - nigh impossible to measure. This fact also precluded the making of an absolute determination to better than an order of magnitude.

Since the measuring condenser had to be at one time small and symmetrical, the split-stator type seemed to be the logical one to construct. Accordingly a small Cardwell air condenser of this type was dismantled and parts of it used to build the measuring condenser. The original shaft and bushing, although new, had too much play to allow their use in this connection, so a new shaft and bushing were made from brass, in which the minimum tolerance that would allow

turning without jerking was used.

The two stator plates were used exactly as they were, but since these were connected to opposite sides of the circuit it was necessary to insulate them from each other. Accordingly a glass sleeve was drawn which would fit snugly over the mounting studs, and separators were made from lucite to keep the plates apart and hold them in place. The spacing is about $\frac{1}{4}$ inch. Since the original rotor plate was much too large, it was cut down to $\frac{1}{3}$ inch in radius, and the edges polished before it was put in place. Although the rotor plate is theoretically at ground potential, the rotor was insulated from ground to prevent anomalous effects.

This condenser is mounted in a box made from $\frac{1}{16}$ inch brass, which is cubical, 3 inches on a side, in a rigid fashion, the box being large enough so that the condenser is fairly symmetrical with respect to it. The box is screwed to the National vernier drive used, and both to a brass plate; which again is fastened to a block of wood which is securely anchored to the chassis.

The condenser so constructed has a capacity of about 2 micromicrofarads, with a total variation from maximum to minimum of only .1 micromicrofarads. Since the dial is readable to one part in 5000 this gives an adequate measuring device.

The chassis is made from $\frac{1}{8}$ inch copper, and because of the flowing property mentioned above, is braced by being

filled with a slab of transite, $\frac{1}{8}$ inch thick. This allows no warping to take place, a thing which would be very probable in view of the great weight of the 90 volt batteries resting on it. This chassis is supported directly on the floor of the cabinet. It was necessary to block up the various condensers and the tube to get them on a level with the top of the gas condenser, for leads had to be as short as possible. These blocks, however, were all of large section and were bolted in place, so that no vibration could occur.

The Gas Condenser:

This in its final form was the product of long experimentation. In the earlier models it was not thought necessary to use symmetry, and so a condenser using the bottom plate of the container as one electrode, with another plate resting on three equidistant quartz spacers as the other electrode, was employed. The troubles involved in this arrangement were mainly due to distortion on evacuation, for the bottom plate, though heavy, would buckle up sufficiently to cause a reverse effect fully ten times that being measured. This seems incredible, when it is known that the maximum motion which could occur is less than $1/1000$ inch, and that the top plate is supported directly on the bottom one by spacers, therefore moving upward with it; but at any rate this shows the type of difficulties constantly encountered in the work.

The next development was to free the bottom plate from the container by supporting it too on quartz insulators. A flexible connection to the outside wiring was taken from the upper plate, and the lower plate was similarly connected to the can. It would now seem virtually impossible to experience any pressure distortion effects, but, none the less, sufficient distortion occurred to give a reverse effect about twice the value under consideration. In view of this, the final form was designed to eliminate not only all possible pressure effects, but also several features which were less likely to cause trouble.

The full - scale drawings figures 3 and 4 show mutually perpendicular diametral sections of the condenser and oil bath. The parallel plate condenser which forms the nucleus of the arrangement is composed of two brass discs a, $1\frac{1}{2}$ inches in diameter, which are made conical in form to give mechanical strength while keeping the edge effects small. These plates are separated by three equidistant quartz pins projecting from the quartz ring b. As may be seen, the pins are $1/8$ inch in diameter at the ring, but have had a step ground in them so that the portion between the plates is only .04 inches in thickness. This was done after the ring was made, by means of a special grinding jig of copper.

The ring was fastened to the lower part of the jig with sealing wax so that the body of the ring was parallel to the plane of the grinding surface, and the grinding thoroughly started. Then the ring was melted off, and both it and the jig

freed from wax. The ground portion was next held in place by hand on the lower half of the jig, resting by its own weight on the three ground surfaces, and the top half ground down nearly to size. Then the ring was turned over once again and the grinding operation completed. In this manner the plane of the condenser plates was made parallel to that of the ring, and the spacing made uniform. This particular procedure was necessitated by the fact that the pins, as welded on, did not lie quite in the plane of the ring. In order to hold the plates on the ring, three lugs c were put on each side of the latter, so that bronze leaf springs attached to the plates could be hooked behind them.

The container in which this assembly is mounted is of extremely heavy section. It is made from a solid piece of brass and is cylindrical in form, having side walls $\frac{1}{2}$ inch thick and a bottom $\frac{7}{16}$ inch thick. The lid d is made from $\frac{3}{8}$ inch brass and is soldered in place. The ring hangs from this lid by the single cemented joint e, which is made with litharge and glycerine cement. The leads to the condenser are in two parts, the lower parts f being of very flexible "pigtail" wire which are soldered at g to the upper threaded portion. This upper portion is threaded so that nuts h could be used to give firm mechanical support. These nuts were screwed down hard against the unthreaded portion so that no solder was needed to hold them in place. The vacuum seal at this point is made by the glass bushings i, which are ground

plane on their lower surface and fit snugly in the holes machined to receive them. They were coated with glyptal on the ground surface and then baked in situ. The brass was cut away around the leads so that the working capacity should be in the plates themselves as far as possible. The leads were cemented in place in the bushings with litharge - glycerine cement and the whole outer surface of the container and bushings coated generously with glyptal and allowed to become thoroughly dry. This container was supported from the lid of the oil bath j by the 5/16 inch brass pipe k, which was threaded and soldered into both lids. Thus the condenser is doubly shielded electrically in the best manner by this arrangement and also freed more completely from external mechanical disturbances. The brass pipe is the connection to the water vapor system.

The Oil Bath:

Since some form of stirring is necessary to maintain a uniform temperature in a bath of fluid, and in view of the restrictions regarding wires coming out of the cabinet, it was deemed advisable to make the oil bath in two parts, with the part containing the controlling and stirring mechanism located entirely outside the cabinet. The connection between the two baths is by means of the two $\frac{1}{2}$ inch pipes m. As may be seen from figures 3 and 4, the lower of these pipes enters radially, whereas the upper one is offset. The oil enters the inner chamber through this pipe and the tangential

entry causes a whirling motion of the oil and hence very rapid changing of the contents of this part of the bath.

The outer portion of the bath contains a mercury thermostat, two heating elements and an impeller. This latter works in a short wide vertical tube in such a manner that only fluid from the inner bath is drawn through it, and is run at a speed adequate to maintain a difference in level in the two baths of 2 inches. Two heaters were necessary, one to keep the temperature at roughly 100 degrees centigrade, and the other to set it accurately at this point; only the second heater is controlled by the thermostat. This was done because much smoother control is got under such conditions than when the thermostat controls the full heater current.

Originally, trouble was experienced from vibration due to the motor and to jarring as the knot in the cord passed over the pulleys. This was rectified by placing the motor on the floor and splicing the twine used as a driving belt, instead of knotting it.

The inner bath is covered with the cap in which is the mica window, for reasons which will be discussed later. This bath is bolted to the chassis (the bolts are not shown) and covered over with felt for thermal insulation. An ordinary 0 - 150 degree thermometer is used to tell the temperature of the oil in the inner bath; this method is sufficiently precise, for the overall accuracy obtainable would not be sensibly affected by even a 1 degree error in

this measurement. (This represents an error of only .2% in the absolute temperature.)

The Water Vapor System:

It was decided to determine the pressure of the water vapor used in this experiment from the ^{relation between} pressure and temperature of saturated water vapor. Accordingly a bulb was constructed which could be evacuated and then have a little liquid water put in it, the bulb then being placed in a thermostated water bath. The pressure in the system finally reaches the equilibrium value corresponding to that of saturated water vapor at the temperature of the bath, and this renders the use of a pressure gauge unnecessary.

Figure 5 shows a schematic diagram of the connections to the bulb. The pipe lines E and F are connected to the vacuum pump and the gas condenser respectively. All the stopcocks had to be immersed in the bath so that condensation would not occur in them, and for the same reason the pipeline F had to be heated. This is a copper pipe with a glyptal joint to the pipe k of figure 4, and a rubber tube - glyptal joint to the stopcock C, which, like the rest of the stopcocks and the bulb itself, is made of glass. This pipe was wrapped with asbestos, and a chromel heating coil of about 100 watts' capacity wound around it. The rubber hose and about 2 inches of the copper pipe are under water, so that the bulb is quite definitely the coolest part of the whole system.

The two - way stopcocks B and C are so arranged that the pump may be connected either to the bulb or through a bypass line to the stopcock C, while stopcock C will connect the condenser either to the bulb or to the bypass line. The pipeline in which stopcock D occurs is used for admitting water to the bulb, and for evacuating large quantities of water in case the bulb has to be dried out. This is done through a sulphuric acid bubbler, and so prevents the drying tubes in the regular pumping line from becoming saturated with moisture. A closed tube manometer is attached to the Cenco Hyvac pump, used for evacuation, to show that the system is vacuum tight.

The water bath used to heat the bulb is of about 2½ gallons capacity and is supported on a separate stool so that vibrations of the stirrer motor may not be transmitted to the oscillator cabinet. Here again two heaters and a mercury thermostat are used to control the temperature. A 1/10 degree thermometer is used to read the temperature of the bath, and a second thermometer is used to give the correction for the exposed stem of the first.

Calibration of the Measuring Condenser:

Since it was not feasible to make an absolute determination of the dielectric constant, for the reasons mentioned above regarding stray capacitances, it was planned to use the known value of the dielectric constant of air and find the dielectric constant of water vapor in terms of this quantity.

Hence an absolute calibration of the variable condenser was not necessary. It was, however, found necessary to make a relative calibration of capacity against dial setting, for the condenser was by no means linear, that is to say, equal increments of capacity did not correspond to equal increments of dial reading. This calibration was accomplished in the following manner: The oscillators having run for a very long time so that they were relatively stable, the condenser C_m was turned to its middle reading - about 250 scale divisions - and the condenser C_t turned until zero beat conditions occurred. This was made apparent by an absence of sound from the loud speaker and from the fact that the oscilloscope pattern was a motionless straight line. The oscilloscope was equipped with a linear sweep, whose frequency could be varied over a wide range. This was now set at 1500 cycles, which was a convenient value to use. As the dial of C_m was turned, the frequency of the beat note would change, the wave form visible on the screen undergoing corresponding changes simultaneously. However, every time the beat frequency was an integral multiple of 1500 cycles, this picture would assume the shape of a sine curve, stationary on the screen, with a number of complete waves equal to the number of times the beat frequency was divisible by 1500. The frequency of a tuned circuit follows the equation:

$$f = \frac{1}{2\pi} (LC)^{-\frac{1}{2}} \text{ at resonance very closely, and therefore:}$$

$$df/f = - dc/2C.$$

This differential form of the equation may be legitimately used, for the total change in frequency over the whole half - throw of the dial was only 22,500 cycles, or .0005f. Hence we may say that increments of the dial setting which correspond to the above increments of frequency are also those corresponding to equal capacity changes. So that to calibrate the dial in terms of capacity it is only necessary to know the dial reading for each setting for which the beat frequency is an integral multiple of 1500 cycles.

It might be again emphasized here that the whole method of taking readings was designed with the fact in mind that it would be practically impossible to determine the capacity change with satisfactory accuracy, and so the necessity for doing this was eliminated. The units in which the capacity of the condenser is known may be and are entirely arbitrary, and zero units does not correspond to zero capacitance. This, however, is not an obstacle when comparisons of the dielectric constants of water vapor and air are all that are desired.

To calibrate the dial, therefore, having set the oscillator to zero beat at the middle of the dial, the dial was swiftly turned and readings taken at every integral multiple of 1500 cycles, until the top of the dial was reached, when the dial was turned in the other direction and readings taken all the way to the other end. This procedure was repeated

several times. The readings do not exactly check one another on repeating runs because of the ever - present drift.

This phenomenon is caused mainly by slow variations of battery voltage, but is also due in part to small changes in other circuit constants. These cause the frequency of the oscillating circuit to change slowly and erratically. In order to minimize this factor, the oscillators were never turned off for long, being connected to a power pack and wet battery when a run was not in progress.

The results of five such calibration runs were therefore plotted on the same sheet of coordinate paper in the following manner: The dial readings of each up and down run were averaged at every frequency setting and the values got for zero beat plotted so as to lie upon the curve first drawn, with the other values spaced at the same regular intervals apart in each case. Through the collection of 150 points thus obtained a mean line was drawn and this was used as the calibration curve. The points lay very well upon this curve, none of them being more than 1 dial division off it, and very few as far away as this. In this manner the curve was plotted and a range of values selected over which the arbitrary capacity units should vary. This was taken as 0 to 600 in order to correspond in some measure with the 0 to 500 range of the dial divisions.

Method of Taking Readings:

The method of taking readings was designed to remove as much as possible the effects of drift. This meant alternating

between vacuum and water vapor as rapidly as the physical limitations would permit. To this end the stopcock bypass system described above was designed, for with it a mere turn of the stopcock C serves to connect the condenser to the pump or to the water vapor source. When a run was to be made, the water bath was brought to the proper temperature and held there for a while before the run was started. Then the setting of C_t was made such as to give the proper setting on C_m , so that this should neither be off-scale under either condition of C_g nor rapidly reach this state. Then after the circuit had had time to settle down again, the condenser C_m was set to zero beat, and time and temperature read. Readings of zero setting, time, and temperature were taken every minute or so for a few minutes, then the stopcock C was turned and readings taken under this new condition in the same manner for a while, after which the stopcock would be turned again and so forth. It was found that satisfactory runs were of about an hour's duration. These were then plotted up and curves drawn through the points. Readings of the curves at each five minute interval were then taken from the curves and corrected with the calibration chart, the resulting difference being plotted on the same paper as the original data. The mean value of this last curve was taken as the correct value under the conditions of the run and plotted on the master curve which is shown in figure 7, and which is a plot of the ratio of ~~$\frac{p}{p_0}$~~ ~~constant~~ of water vapor at 100 degrees centigrade and the

indicated pressure to that of dry air at 100 degrees centigrade and 76 cm. of mercury. The value of the difference used to obtain this ratio was taken from the results of several calibration runs with dry air. These were taken in a manner entirely analogous to the above, the only difference being that stopcock D is opened, and to the pipeline leading from it is fastened a drying system in which atmospheric air is first passed through a sulphuric acid bubbler, then over sodium hydroxide, calcium chloride, and finally phosphorus pentoxide.

During the initial experimentation it was found that on changing over from water vapor to vacuum the reading of the condenser for zero beat would steadily decrease. This phenomenon was traced to the cooling of the plates under evacuated conditions, for they were not heated by the oil bath and had a good thermal connection to the other condensers, which formed an excellent radiating system. This was proved to be the case by allowing the oil bath to cool to room temperature, when a run was taken and the effect was seen to have disappeared. To prevent the cooling of the plates it was necessary to heat the bushings and lead wires. This was done by putting the previously mentioned cap with mica window over the oil bath to keep the leads and bushings warm, and also by the use of the following rather unique scheme*: a section about 3 inches long was removed from the wires leading to the rest of the oscillating system and pieces of number 18 gauge manganin wire substituted. This increased the direct current resistance and the

*Suggested by Dr. Bowen.

thermal resistance of this section by a factor of 80, but due to the action of skin effect, the high frequency resistance over the same path was only 6 times as great as before. Thus this method had the result of cutting down the thermal conductivity with a much smaller decrease in the electrical conductivity. It was experimentally verified that this did not cause a serious diminution of the energy of oscillation, although the putting of resistance in the oscillating circuit is not altogether conventional.

An examination of the sample run shown in figure 6 shows a similar sort of phenomenon which, however, time alone could cure. This was due to the fact that the water vapor when first admitted to the condenser is at a temperature very little higher than that of the water bath, and several minutes are required before equilibrium is reached at the higher temperature. That is the reason why the first points after changing over to water vapor are too low.

Results of the Experiment:

The results of the experiment fall into two groups, because a certain amount of remodeling of the water vapor system was done during the proceedings. This caused the air calibrations for the sets of readings taken before and after the remodeling to be different. Only one calibration run for the first set had been made with very dry air, though several had been made with air at normal room conditions. Three runs were made with dry air for the second set, and these lay very close together, as may be seen.

When it was attempted to make runs at higher water bath temperatures the absorption effects mentioned by earlier investigators occurred and also the whole apparatus acted in an erratic and generally unsatisfactory fashion, so it was decided to limit the bath temperatures to 76.5 degrees. All the runs taken at temperatures less than this are used in the plot given in figure 7, no arbitrary selection of data having occurred. Below is a list of runs made and values of the difference Δ obtained after calibration; the temperature is 100 degrees centigrade in each case:

Run	Substance	Pressure mm. Hg.	Δ	Remarks
1 to 18	various			Runs in which defects were being removed from the equipment.
19	air	740	33	Drifting too fast to be accurate
20	w.v.	26	15	
21	w.v.	98.6	57	
22	air	740	40	Air known to be actually wet.
23	w.v.	98.6	58	
24	air	740	36	Air properly dried.
25	w.v.	148	40.9	
26	w.v.	93	(57)	Too erratic to be taken seriously.
27.	w.v.	355	(164)	"
28.	w.v.			Short run.
29.	w.v.	228	135.5	
30.	w.v.	535.5	(285)	Erratic.
31.	w.v.			Test run.
32.	w.v.	486	---	Too erratic.

Run	Substance	Pressure in mm Hg	Δ	Remarks
33.	w.v.	496	(272)	Erratic
34.	w.v.	408	(205)	Erratic.
The above completes the first series.				
35.	air	740	20-25	Drifting too fast.
36.	air	740	25	
37.	air	740	25	
38.	air	740	24	
39.	w.v.	43	17	
40.	w.v.	130	50	
41.	w.v.	655	(312)	Erratic
42.	w.v.	558	(272)	Erratic.
43.	w.v.	456	(147)	Erratic.
44.	w.v.	456	(---)	Test Run.
45.	w.v.	198	78	
46.	w.v.	307	126.5	

The Δ value for air was found for each series from the above, and reduced to 76 cm. This was done with the aid of the Clausius-Mosotti relation, which is known to hold for air. Using these values it was now possible to convert the Δ readings for the various runs into the ratios of (~~dielectric constant~~ ^{$\epsilon - 1$}) of water vapor to that of air. This was done for all the low temperature runs, and the result drawn in figure 7. The linearity of figure 7 is therefore a proof that the Clausius-Mosotti equation is valid for water vapor ^{at constant temperature} at 42 megacycles. By extrapolating

this curve we may therefore find the dielectric constant of water vapor at 100 degrees centigrade and 760 mm. pressure, using the data on air given by previous investigators and summarized above.

Air differs from water vapor in that the former is a non-polar gas, consisting of a mixture of nitrogen and oxygen; while the latter possesses a dipole moment. The theory shows that the rotational and vibrational bands do not appear in a non-polar gas, and hence the electronic bands are all that need be considered in the case of air. The lowest of these are the $A^1\Sigma-X^1\Sigma$ band at 7620 Angströms and the $\alpha^1\Delta-X^1\Sigma$ band at 12,700 Angströms. Bender⁷⁷ has proved that these bands have a negligible effect on the refractive index of air, so that it would be expected that the dielectric constant of air at low frequency would be the square of the optical refractive index in the infra-red. Bender's value for this index is 1.000288 at NTP, which would give the expected value of the dielectric constant of air at the lower frequency as 1.000576. This is certainly quite close to the average value of 1.000581 found directly at the lower frequencies, and it seems fair to assume, in the light of both experiment and theory, that the value at 42 megacycles will also be close to the value 1.000581. In the case of water vapor, however, it would not be safe to make the same assumption, as here the oscillation and rotation bands are excited.

If we take as correct this value 1.000581 and reduce it by means of the Clausius-Mosotti equation, we obtain the value of the dielectric constant of air at 100 degrees centigrade

to be 1.000425. Extrapolating the curve of fig. 7 we find that at 100 degrees centigrade and 760 mm. of mercury the quantity $\epsilon - 1$ has a value 12.2 times as large for water vapor as for air. This gives the value 1.0052 for the dielectric constant of water vapor at 100 degrees centigrade and 760 mm. pressure.

Sanger's value for this quantity is $1 + \left(.600 + \frac{3507}{373} \right) (.0060)$
or 1.00597 .

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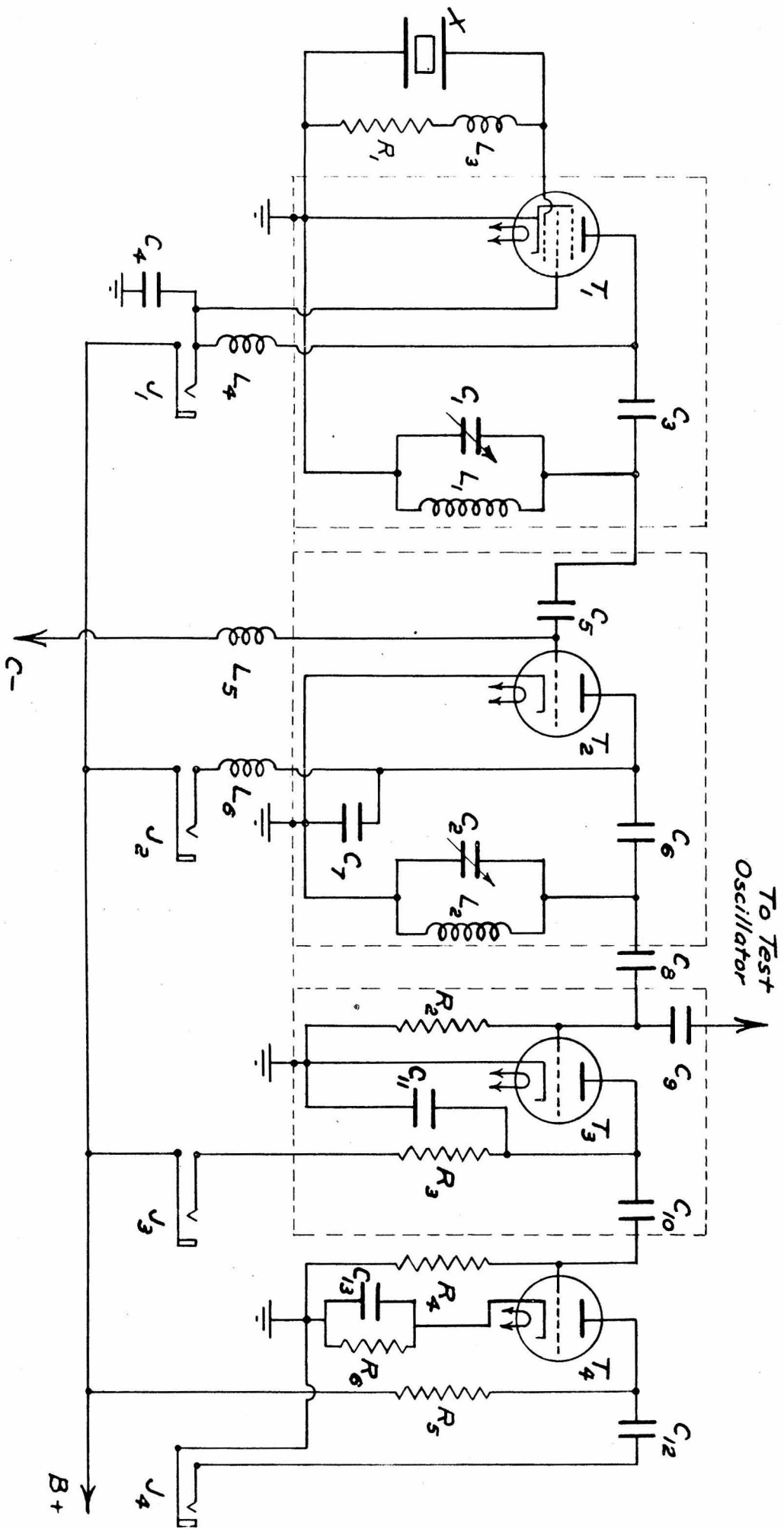


FIG. 1

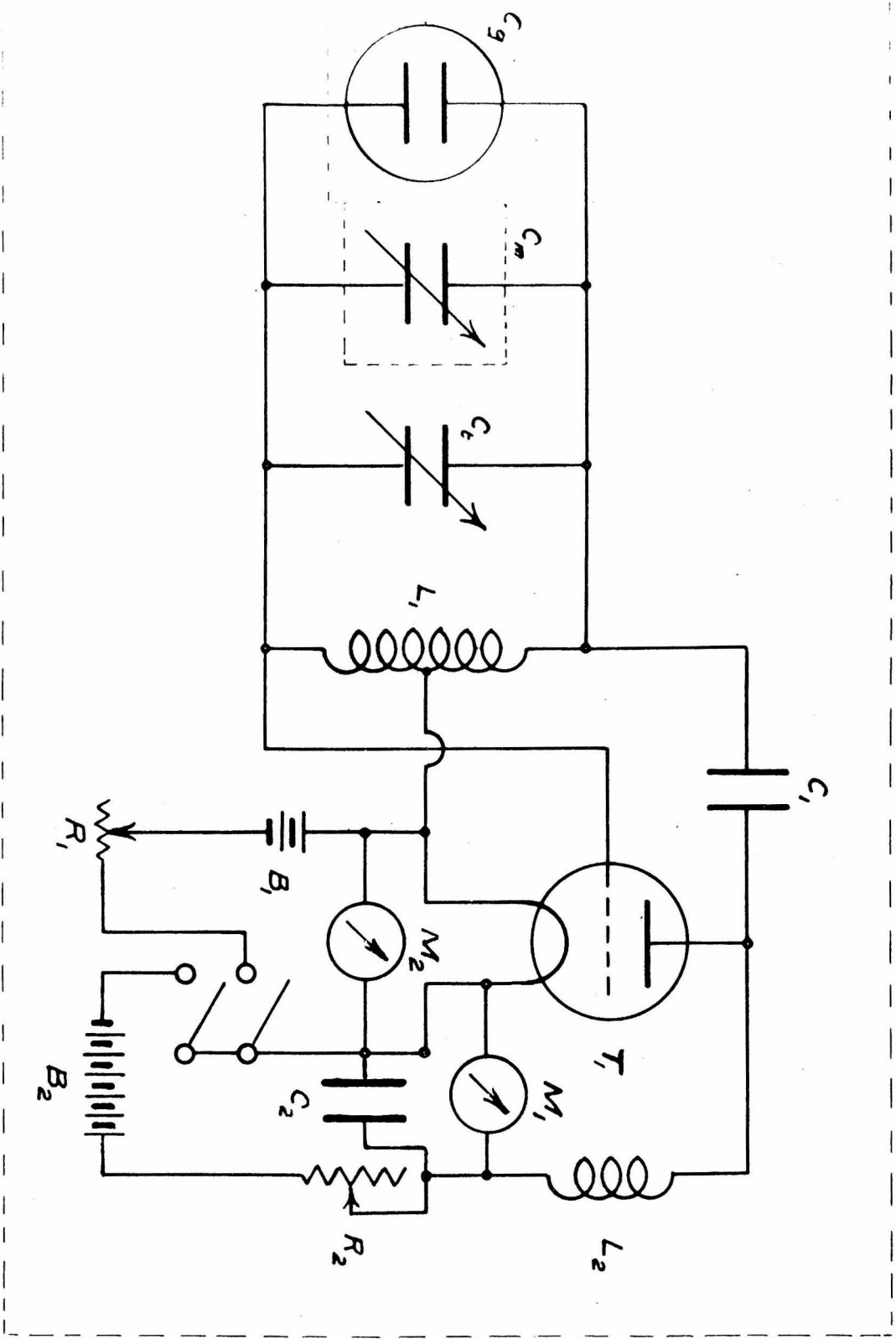


FIG. 2

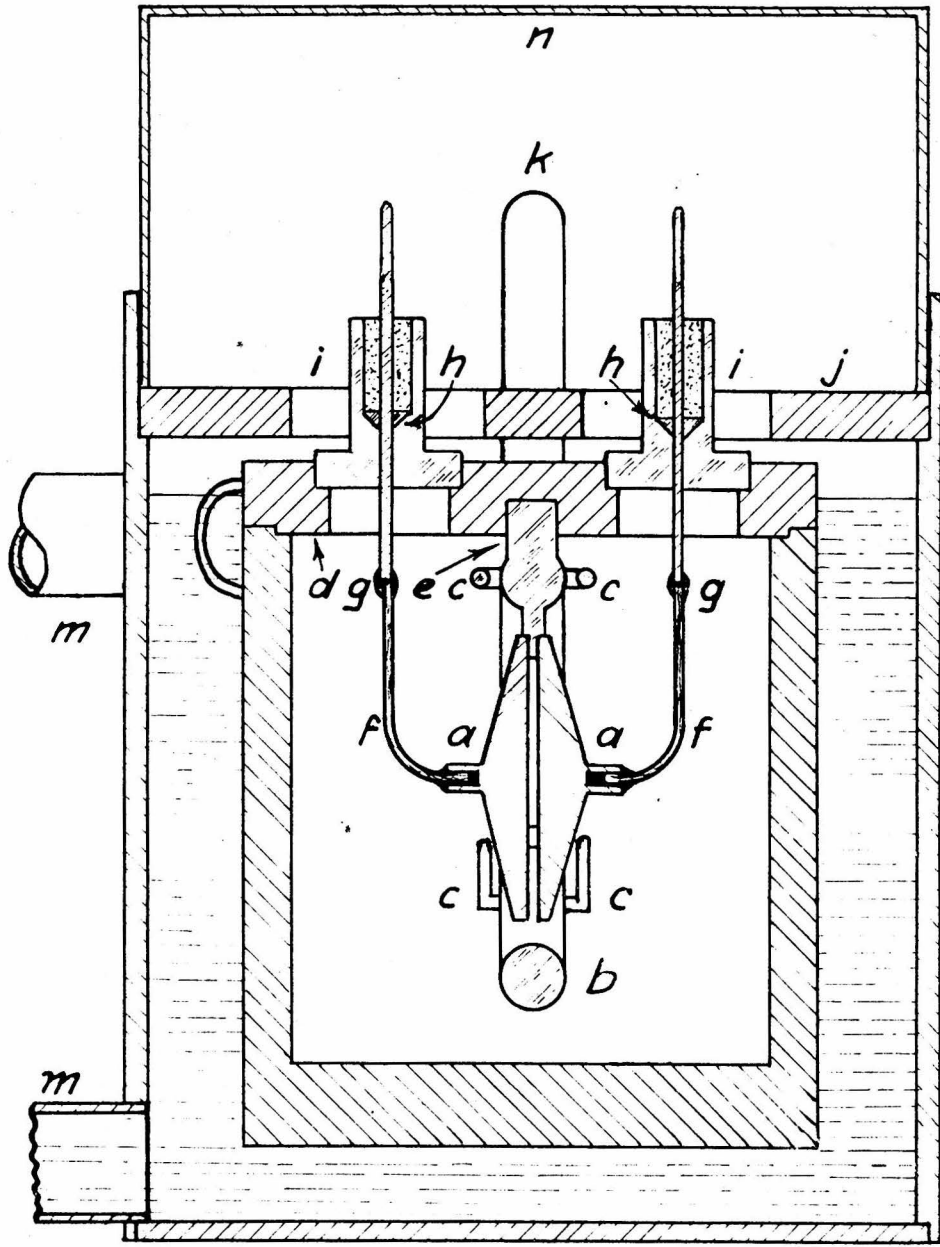


FIG. 3

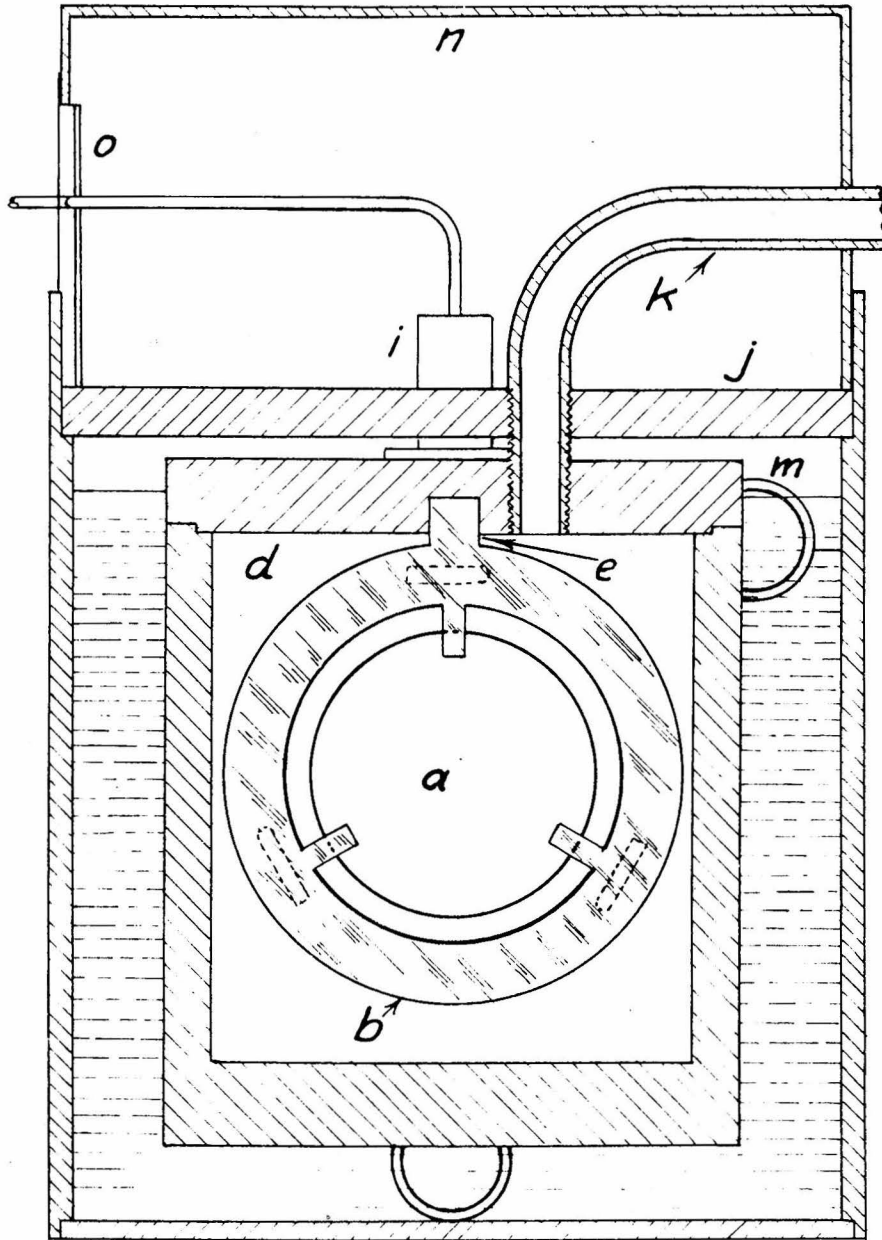


FIG. 4

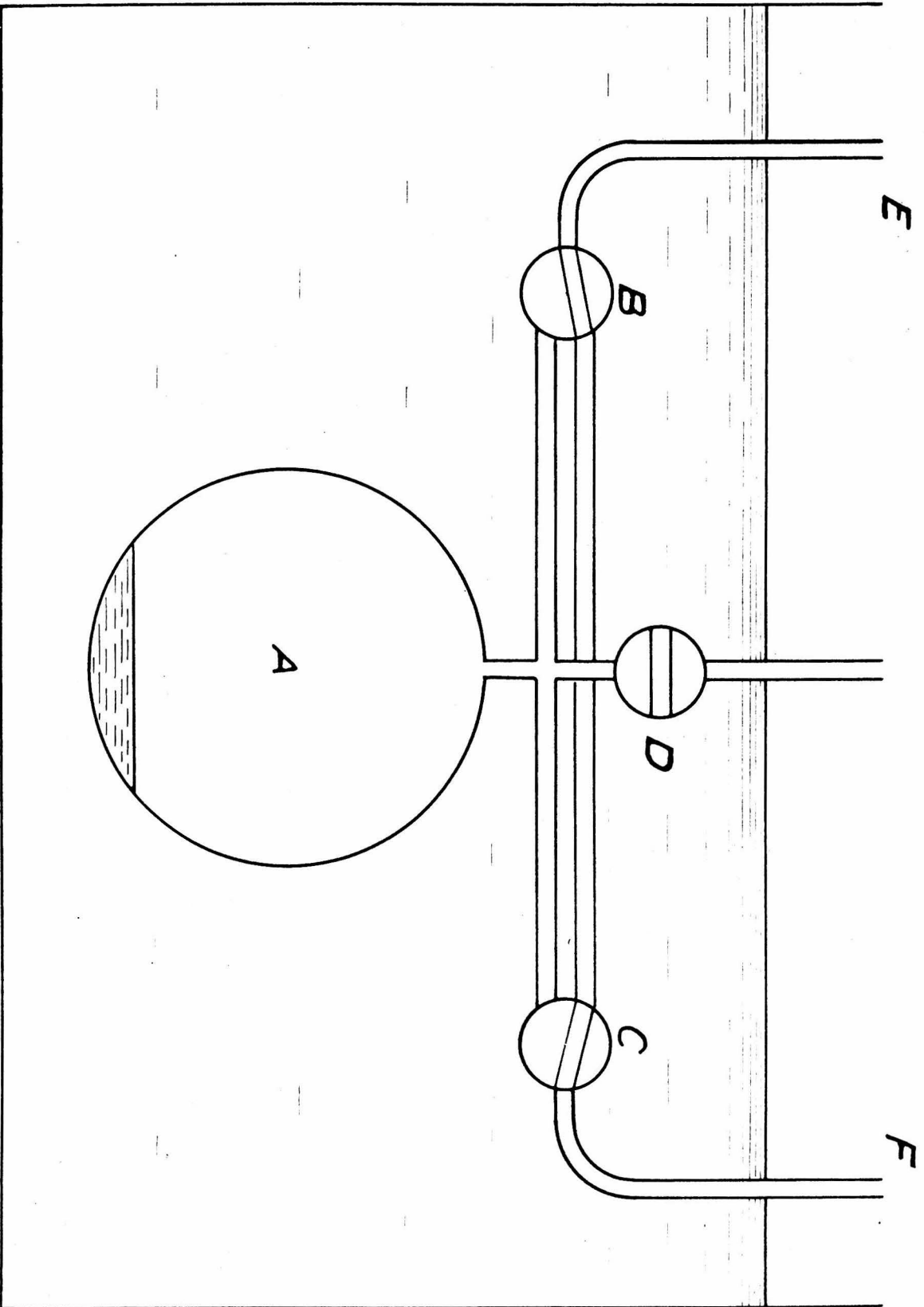


FIG.5

FIG. 6

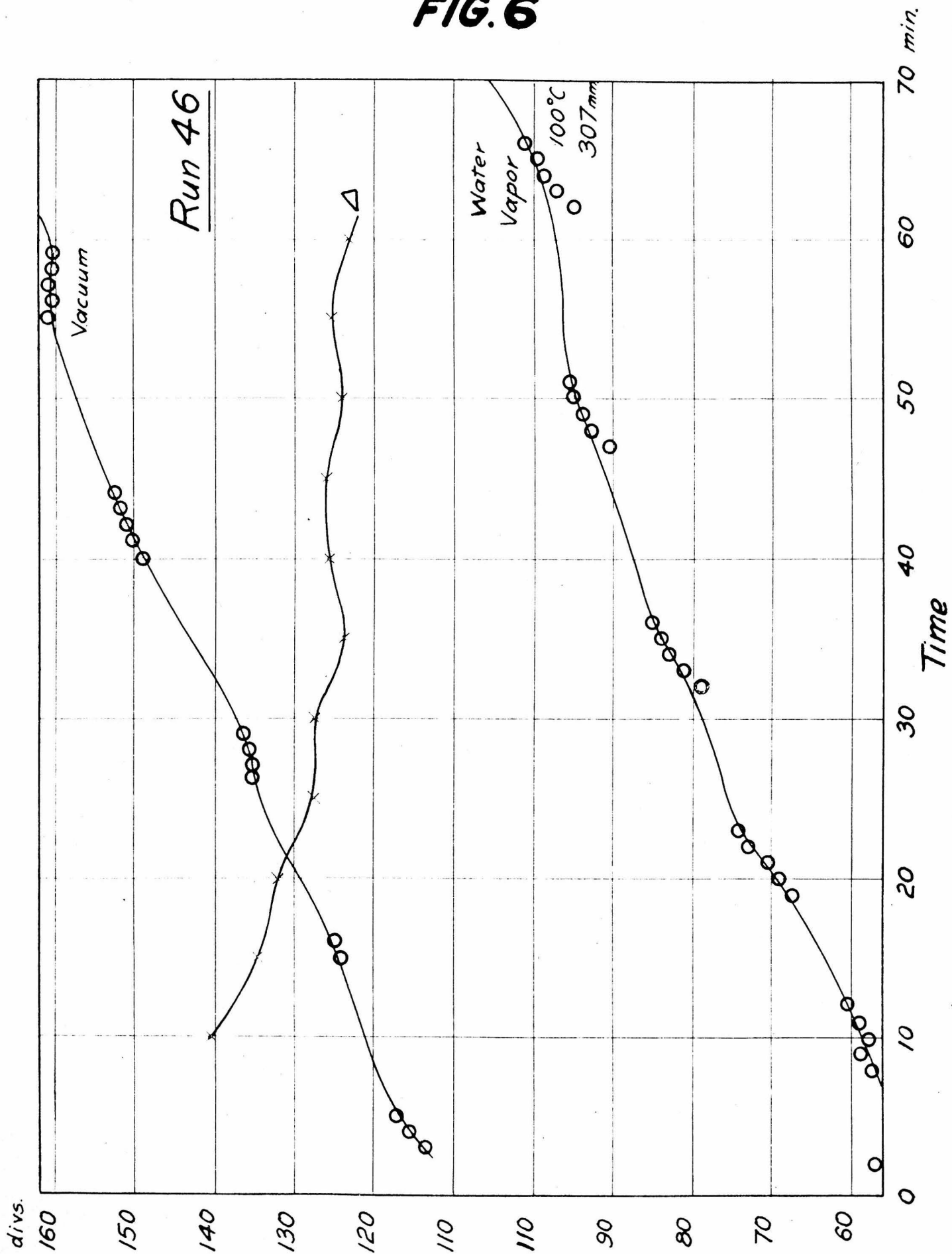


FIG. 7

