THE SCATTERING OF SLOW ELECTRONS IN GASES

Thesis

by

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The earliest attempts to investigate the nature of the atom using an electron stream directed into a gas as the investigating tool were the experiments of Lenard (1), who determined the total cross section offered to an electron by the gas through which it was directed. Using fast cathode rays, and photo-electrons, he covered the high velocity electron range. The results obtained, indicated that the molecular cross section decreased monotonically with increasing electron velocity. Later work of Lenard and others (2) showed that the molecular cross section for fast electrons was proportional to the sum of the atomic numbers of the atoms in the molecule. During the same period, the experiments of Bragg and Moseley were laying the foundation for the modern view of the constitution of the atom. The conception developed slowly, culminating in the model advanced by Bohr (3), a very useful and reasonably accurate picture, which has been merely slightly modified conceptually by the newer quantum mechanics.

Using the Bohr picture of the atom, Bragg's and Lenard's experiments show that fast particles penetrate the atom, and suffer deflection only when their path happens to approach the nucleus closely. This view had been developed mathematically by Rutherford in 1911 (4) and an expression obtained for the fraction of the incident beam scattered at any definite angle from its original path. For molecules, evidently, the scattering of fast particles could roughly be considered as the sum of that due to the separate atoms constituting the molecule.

In between the time of Lenard and the advent of the Bohr theory, a number of investigators worked on the variation of molecular cross

section with electron velocity, and with the nature of the gas. The next notable advance was made when Ramsauer (5) discovered that, for the noble gases, (and later for others) the molecular cross section with decreasing incident electron velocity went through a maximum and decreased again. Later investigations extending the results showed a rise again at lower velocities, which seemed monotonic. The principal experimenters on this phase of the subject were Beuthe, Brode, Bröse, Brücke, Mayer, Normand, Palmer, and Ramsauer. A paper by Bröse and Saayman (6) gives a good bibliography of the work done on the subject up to that time. Later work has been in the direction of extension to lower electron velocities, and a greater number of gases. Theoretically the subject was investigated by Zwicky (7) in a series of papers discussing the effects which might be expected when an electron beam is sent into a gas, from the classical standpoint and on the older quantum theory, obtaining a good qualitative explanation of the Ramsauer effect. Later theoretical investigations have used the newer quantum theory and will be mentioned later.

The experiments of Lenard and later workers, and of Ramsauer and his contemporaries determine the total molecular cross section. This is an index of the proportion of the total beam of electrons which leaves it due to any cause while it traverses the length of the apparatus. When a beam of electrons of a definite mean velocity is directed into a gas, measurements may be made of the effects produced on the gas molecules, or on the electrons of the beam. The latter may suffer change of energy or change of momentum, or both. In Ramsauer's type of apparatus either of these effects cause the electron to leave the beam, whereas in Mayer's, the latter alone does so.

Change of the electron's energy can be determined by magnetic or electrostatic analysis of velocities, or by retarding potential curves. However, no complete analysis of the phenomena occurring upon impact is possible without considering both energy and momentum changes of the parties to the impact. Since the mass of the electron is so much less than that of the atom, it is sufficient to consider the deviation of the electron from its original path.

The first effort to determine the change of momentum of the incident electron in gases was that of Dymond (8) in 1926. The apparatus determined the intensity of scattered current at angles up to 90° from the original path. His curves showed maxima which were later found to be spurious (9). A number of other investigators entered the field, and studied various gases for various electron velocities (10). In general the following results were obtained: monotonic decrease of scattered current with increase of scattering angle, which did not follow Rutherford's law for inverse square scattering but fell off usually more slowly; scattering would tend to concentrate at small angles with increase in the initial electron velocity; an electron scattered with loss of energy would be less apt to be scattered through a large angle than one scattered without loss of energy. However, sufficient technique was not developed to enable them to carry their measurements out to the larger angles, a deviation of 90° being the greatest measured, and the results usually being of the order of experimental error when that angle was reached.

Both McMillen's and Arnot's work gave indications of non-monotonic relationships between intensity of scattering and angle of scattering.

Accordingly both workers set up investigations to determine this more accurately and at larger angles. Also Pearson at this institution attacked the problem. The first publications of these more accurate results was by Arnot (11) and Bullard and Massey (12), both at Cambridge. They have been followed by a number of other papers by various experimenters (13). All of these investigators find definite maxima and minima in their results, somewhat similar to those shown in the diffraction of light by a medium consisting of a random distribution of spheres. The analogy is qualitatively explainable in terms of the new quantum theory, for to a slow electron of a definite velocity may be ascribed a wavelength $\lambda = \frac{h}{mv}$, and a wave treatment applied, whence one might expect diffraction effects. The phenomenon is not quite so simple though. A large amount of work has been done on the theoretical approach to the problem.

The classical treatment is due to Rutherford (4). Treating the scattering as due to an inverse square field of force, he obtains the classic formula

$$\text{fract. scattered} = \frac{r}{Q} = \left(\frac{2Ne^2}{mv^2}\right)^2 \frac{nt}{16r^2} \csc^4\left(\frac{\theta}{2}\right)$$

which is easily reduced to fit scattering in gases.
Rutherford's expression is deduced for a repulsive force, but the same result is obtained if the case is similarly treated for an attractive force.

Since it is evident that an inverse square force is merely an approximation to the field of the atom, further work has been done. On the basis of quantum mechanics, Rutherford's expression has been obtained by Wentzel (14) and others (15) both with approximations, and rigorously,

for a coulomb field. A treatment due to Mott (16) taking account of the shielding of the nucleus by the electron atmosphere applies a correction, giving the expression

$$\left[f(\theta)\right]^{1/2} = \frac{e^2}{2mv^2}\left[N - F(\theta)\right]\csc^2\frac{\theta}{2}$$

$F(\theta)$ = structure factor

Another correction for shielding is given by Sommerfeld (17), who considers all the shielding electrons to be in the K shell, and obtains

$$f(\theta) = \left(\frac{e^2 N}{mv^2\tau}\right)^2 \frac{1}{(\sin^2\frac{\theta}{2} + \alpha^2)^2} \qquad (\alpha = \frac{\lambda}{a}\frac{N}{2\pi}) \quad (= \frac{2\pi e^2}{hv}N)$$

The most important treatment using simplified assumptions has been that of Born (18), considering the incident electron beam as a plane wave, and the scattered current as small spherically symmetrical wavelets superimposed on this by the presence and effect of the scattering center (which, as shown by Mott, is equivalent to using Huygens' principle to determine the scattered intensity of DeBroglie waves due to a static force field). His result may be obtained by transforming the Schrodinger differential equation, $\nabla^2\Psi + k^2\Psi = k^2\frac{V}{E}\Psi$, by the use of Green's function into the integral equation

$$\Psi = Ae^{ik\bar{n}_0\cdot\bar{r}} + \frac{2\pi m}{h^2}\int V(r')\frac{e^{ik|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|}\Psi(r')\,dv'$$

giving the asymptotic expression for Ψ

$$\Psi \cong Ae^{ik\bar{n}_0\cdot\bar{r}} + \frac{2\pi m}{h^2}\frac{e^{ikr}}{r}\int V(r')e^{ik\bar{n}\cdot\bar{r}'}\Psi(r')\,dv'$$

in which the second term represents the scattered wave. Born's approximation of undistorted incident wave gives us

$$\Psi(r') = A e^{ik\bar{n}_0 \cdot \bar{r}'}$$

and therefore

$$f(\theta) = \left| \frac{2\pi m}{h^2} \int V(r) e^{ik(\bar{n}_0 - \bar{n}_1)\cdot \bar{r}} dv \right|^2$$

for the ratio of the scattered current to the incident main beam. The limits of validity of Born's formula have been considered by Mott (19) and by Möller (20). The principal faulty assumptions in Born's treatment are neglect of the effect of electron exchange, and of the effect of the distortion of the atomic field by the advancing electron, as well as the distortion of the electron wave by the atom. The close check of the Born expression with experiment over a wide range of values has recently been ascribed by Massey and Mohr (21) ~~as being due~~ to the fact that the two effects work in opposite directions except at very slow velocities. (See also reference 57).

Oppenheimer (22) first pointed out the necessity of considering electron exchange, and gave a general treatment of its effects. Later the theory developed by him was applied to particular cases by Massey and Mohr (23) with some qualitative success. The effect has also been treated by Feenberg (24).

The distortion of the atomic field by the electron has been treated by Faxen and Holtsmark (25), and by Holtsmark (26). Distortion

corresponds to an excitation of all the eigenfunctions of the atom during the scattering. If a particular eigenfunction is excited permanently the collision is inelastic, with loss of energy by the scattered electron. Therefore only those eigenstates may be excited permanently which correspond to excitation energy less than that of the incident electron. The process becomes more complex and will not be considered. Faxen and Holtsmark considered the distortion of a spherically symmetrical field. Holtsmark obtained results for argon (27) and krypton (28) from which with quite a bit of labor in computation may be obtained the angular scattering.

Holtsmark's treatment built out of eigenfunctions, a solution that reduced to a plane wave at infinity. Recently Allis and Morse (29) and Sommerfeld(30) have used the same method of treatment and extended it, obtaining good agreement with experiment. Lately Massey and Mohr (31) and McDougall (31) have published the first of some articles treating the subject taking into account both distortion and electron exchange. The check with experiment is more satisfactory than previous treatments. The application of the Thomas-Fermi statistics has been done by Mitchell (32) and Bullard and Massey (33). The results differ little from the Rutherford formula. Other treatments of various factors concerned and minor problems connected with the theory are given in references under (34). The subject calls for further theoretical treatment.

The treatments of the problem, of necessity start from an assumed atomic model, and calculate the scattered intensity and its dependence on angle. The inverse problem, (of starting from the observed scattering, and determining the force field of the atom, which would cause such scattering) is amenable to treatment in the case where the scattering

shows itself to be a function of $v \sin \frac{\theta}{2}$, where v stands for the velocity of the incident electron. This functional dependence of the scattering on velocity and angle is approximately fulfilled in many cases, and is indicated theoretically in the Born treatment. The obtaining of the force field of the scatterer is carried out in Appendix II.

Langmuir (10) has given the empirical formula which agrees well with the scattering measurements at low angles ($\theta = 60°$) of scattering

$$I = K e^{-(\theta/\theta_0)^2}$$

At angles of more than sixty degrees this drops off too fast.

Definition of Terms

TOTAL CROSS SECTION is the value of the cross section of a sphere which would classically deflect (or cause to leave the beam) the same number of electrons as in the beam are observed to leave it through atomic scattering. It is defined by

$$dI = -\sigma N I dx$$

whence $$I = I_0 e^{-\sigma N x}$$

Here N is the number of molecules per unit volume, x the distance traveled, and I the electron current at any point. Sometimes in place of $N\sigma$ there is used the coefficient q or q_p, the definitions of the two q's being obvious.

MEAN FREE PATH is the mean distance that an electron goes without interception. A simple calculation shows this is equal to $\frac{1}{N\sigma}$.

Ramsauer expresses results in terms of $\frac{1}{N_1\sigma}$. $N_1\sigma$ (since N_1 is defined as the number of molecules in 1 cc. of the gas at 0°C. and 1 mm pressure) thus measures the area which one cc of the gas at 1 mm pressure and 0°C. presents to the electron.

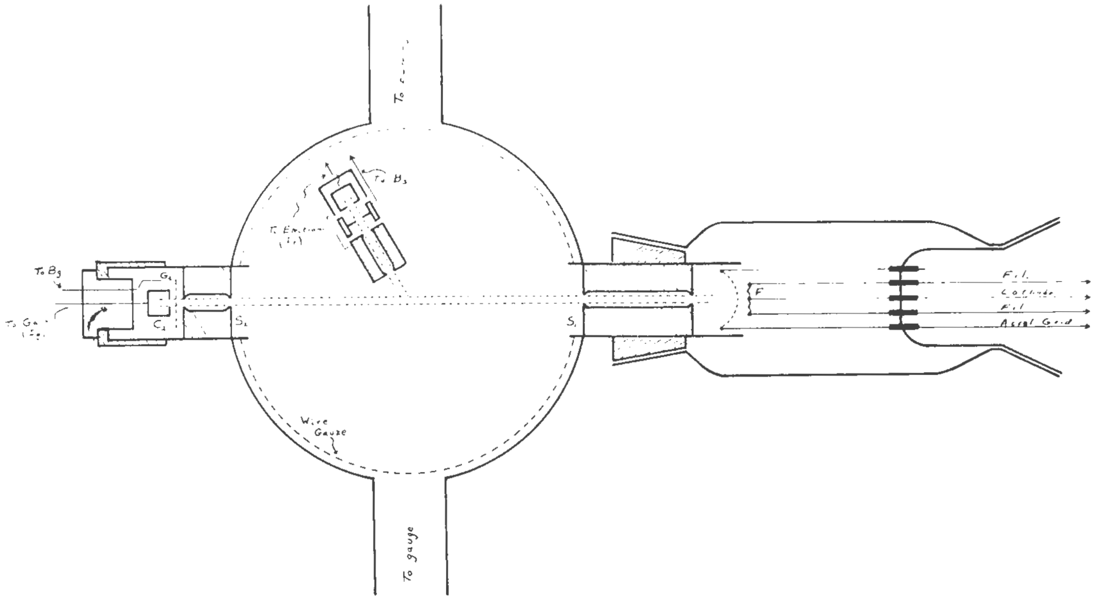
SCATTERING FUNCTION is the ratio of the current scattered at a definite angle θ with the main beam to the total scattered current. It may be expressed in two ways: the scattering per unit angle, or the scattering per unit solid angle. The former may be defined as the fraction of the total scattered current passing through the surface of a sphere (surrounding the scattering center) of a definite angular width, at an angle θ to the main beam. The latter may be defined as the fraction of the total scattered current which passes through a definite fraction of the surface of the above sphere, the area considered being that part of the surface of the sphere which is intersected by a radius at an angle θ to the main beam. Evidently the second is obtained from the first by multiplication by a constant times $\sin\theta$.

ELASTIC IMPACT is an impact in which the impacting electron loses a negligible amount of energy (i.e. small compared to the lowest resonance potential of the atom impacted). An inelastic impact will mean an impact in which the loss of energy is of the order of or greater than the lowest resonance potential. Due to the low resolution of electron velocity which results from use of a retarding potential, short Faraday cages, and a 3 to 6 volt drop across the filament, only elastic scattering was measured in the present work.

Apparatus

The general apparatus used in this field consists of a source, collimating slits, a collector with collimating slits for the scattered current, and a means of determining the main beam value. According to the experimenter, the source of electrons or the collector is rotated; also the means of measuring the main beam may be a collector or the sides of the scattering chamber. Ramsauer and Kollath collect the scattered current on circular segments symmetrical about the main beam. Jones and Whiddington (35) in investigating energy losses of electrons in He have used photographic film to record beam intensity, instead of a Faraday cage. The results were not successful, as a number of losses known to occur did not show on the film. Whiddington and Taylor (36) and Weidner (37) discuss the action of slow electrons on photographic films. However, it may be concluded that this method of detecting the main beam is not satisfactory. The velocity analysis has been accomplished by retarding potentials usually. Dymond (10) and Jones and Whiddington used electromagnetic analysis. McMillen used electrostatic analysis, with a cylindrical first power field, obtaining refocusing of the electrons of one velocity at the second slit. With the arrangement most used, the scattering volume varies with $\frac{1}{\sin \theta}$. Therefore results are multiplied by $\sin \theta$ to get the scattering per unit solid angle, and again by $\sin \theta$ to get the scattering per unit angle.

In order to interpret the experimental results with any degree of certainty, it is necessary to have the distance traveled by the electron and the gas pressure chosen such that only single scattering occurs. In


To gauge
Wire Gauze
To B_3
S_1
S_2
C_2
F
Fil.
Cathode
Fil.
Axial Grid

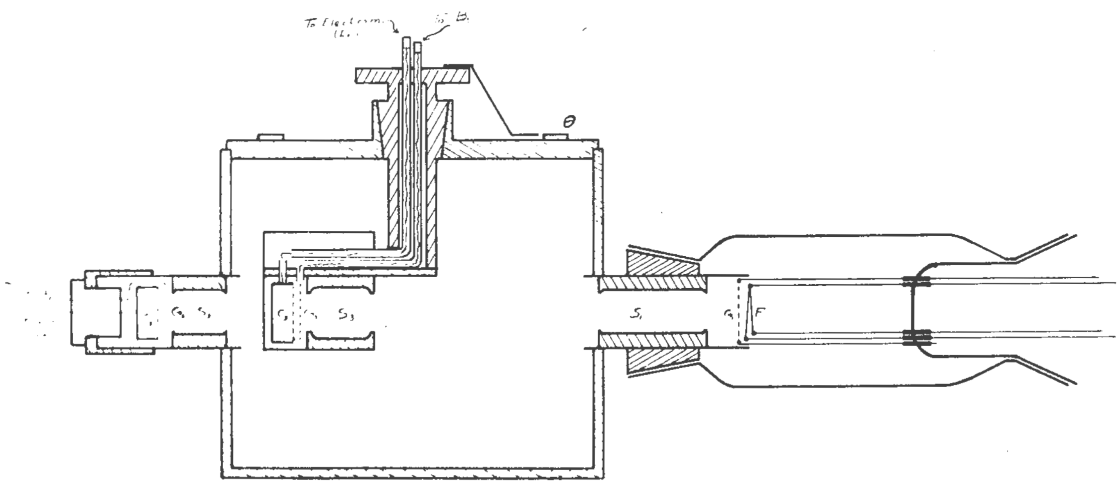

θ
S_1
S_3
C_3
F

Fig. 7

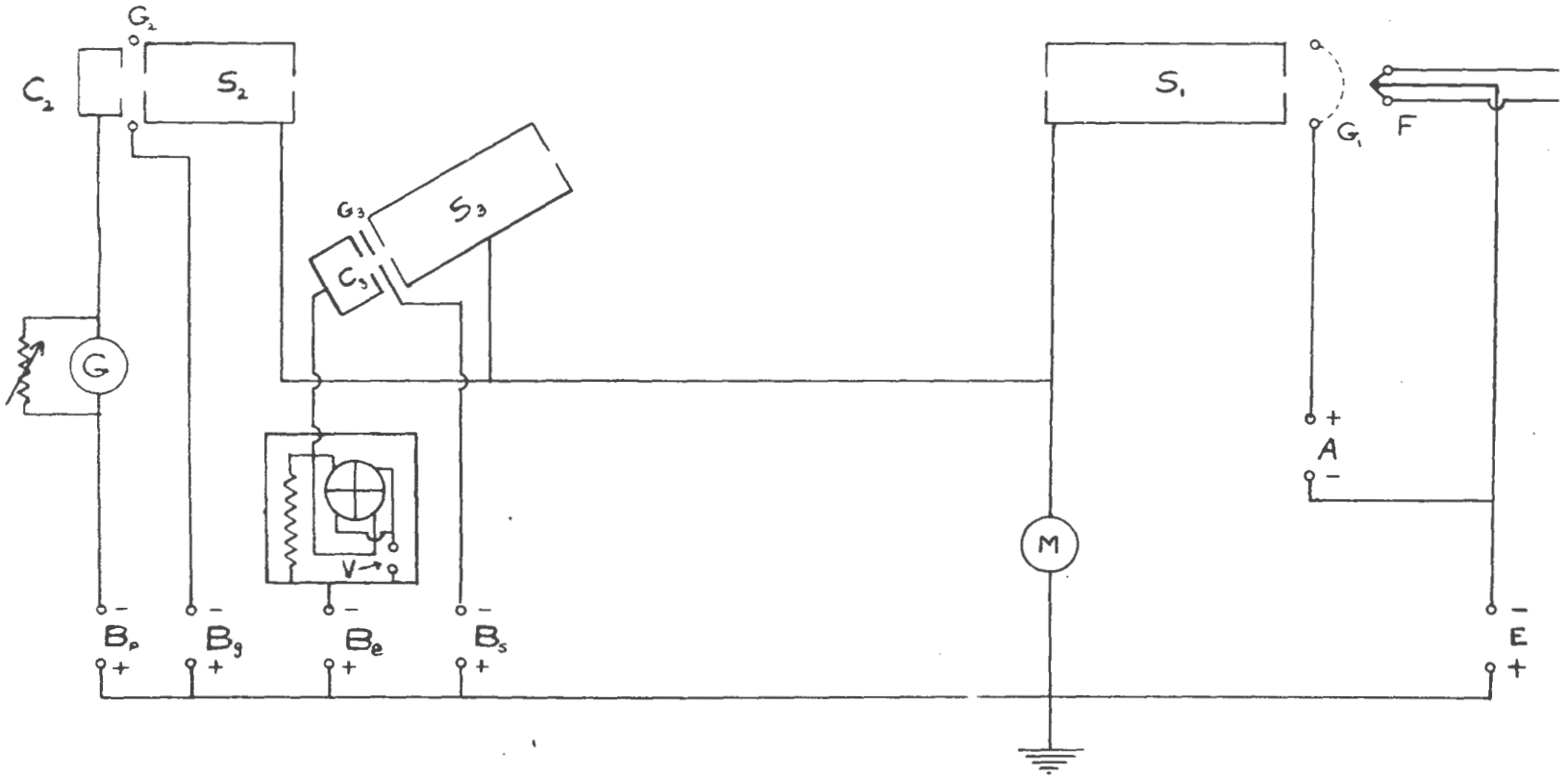


Fig. II

all the present work, this has been done. A criterion for single scattering is given by Wentzel (38) which can be adapted to the case of scattering in gases, but here single scattering was obtained by choosing the gas pressure such that the electron mean free path was one and a half or more times the dimensions of the apparatus.

The apparatus used in the present investigation has been described by Pearson (thesis, C.I.T. 1930) and Arnquist and Pearson (15). Therefore it will suffice to give a brief resume, and an account of the changes which have been incorporated into it.

The source of electrons was an eight mil tungsten filament, heated with AC stepped down from the 110 volt line. At first the filament was twisted tightly about its supports. However, a tendency to bend out of line with the slits was found and so in the major part of the work the filament was spot welded to its supports, which were leads (nickel to tungsten) through a pyrex stopper with a ground joint set in a glass tube slightly off center. By rotating the stopper and the tube, the filament could be centered opposite the slits. The alignment was done visually, looking through the two sets of sollimating slits from the side of the main beam collector. The electrons were accelerated to an accelerating grid of nickel gauze bent so as to focus the beam on the slits, after its passage through the grid.

The scattering chamber itself was five inches in diameter, and of brass to eliminate electrostatic fields in its interior. Magnetic fields were taken care of by horizontal and vertical Helmholz coils, supplied current from storage batteries. The shape of the main beam could be determined by varying the vertical field and thus swinging the beam across the slits.

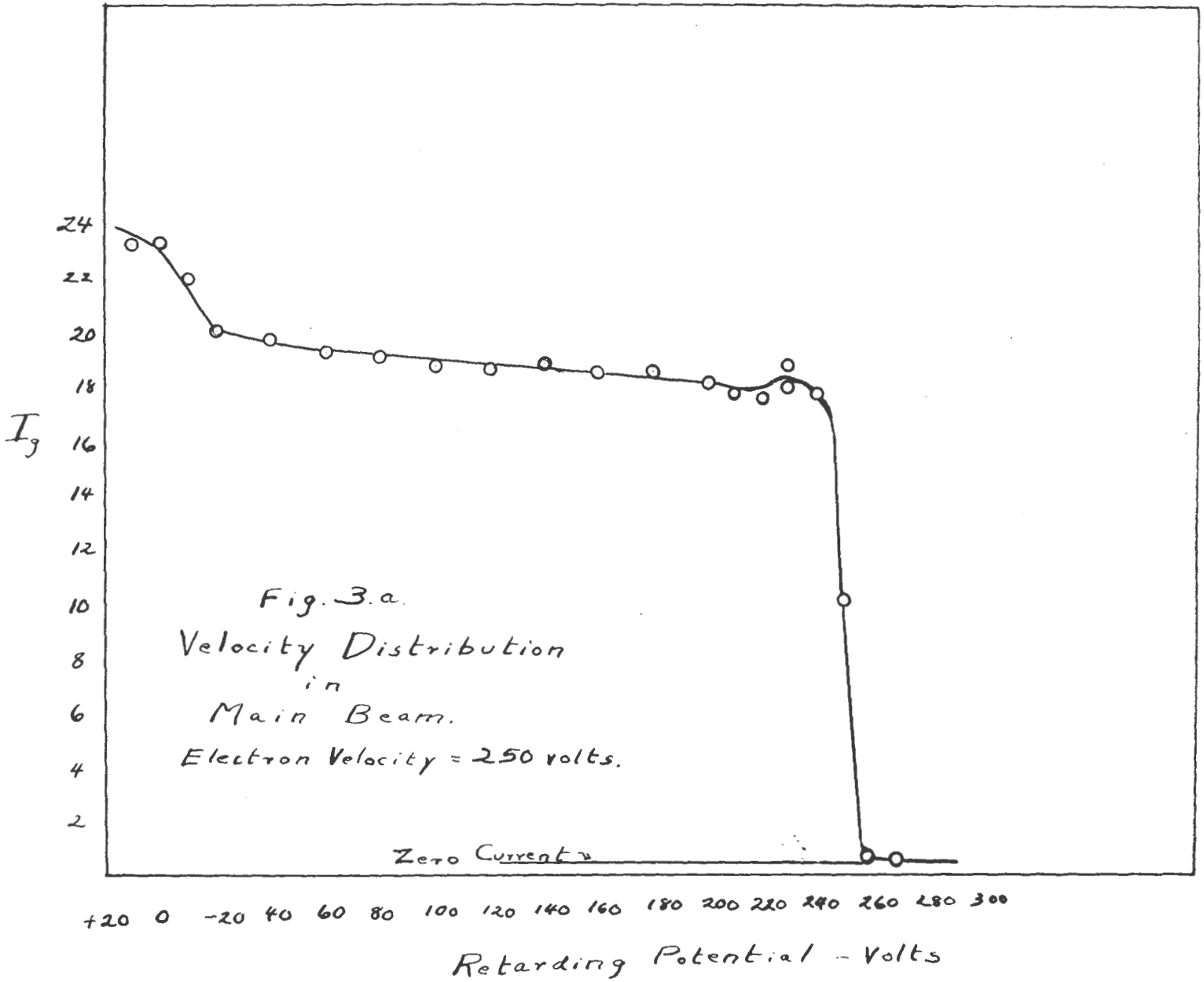


Fig. 3.a.

Velocity Distribution

in

Main Beam.

Electron Velocity = 250 volts.

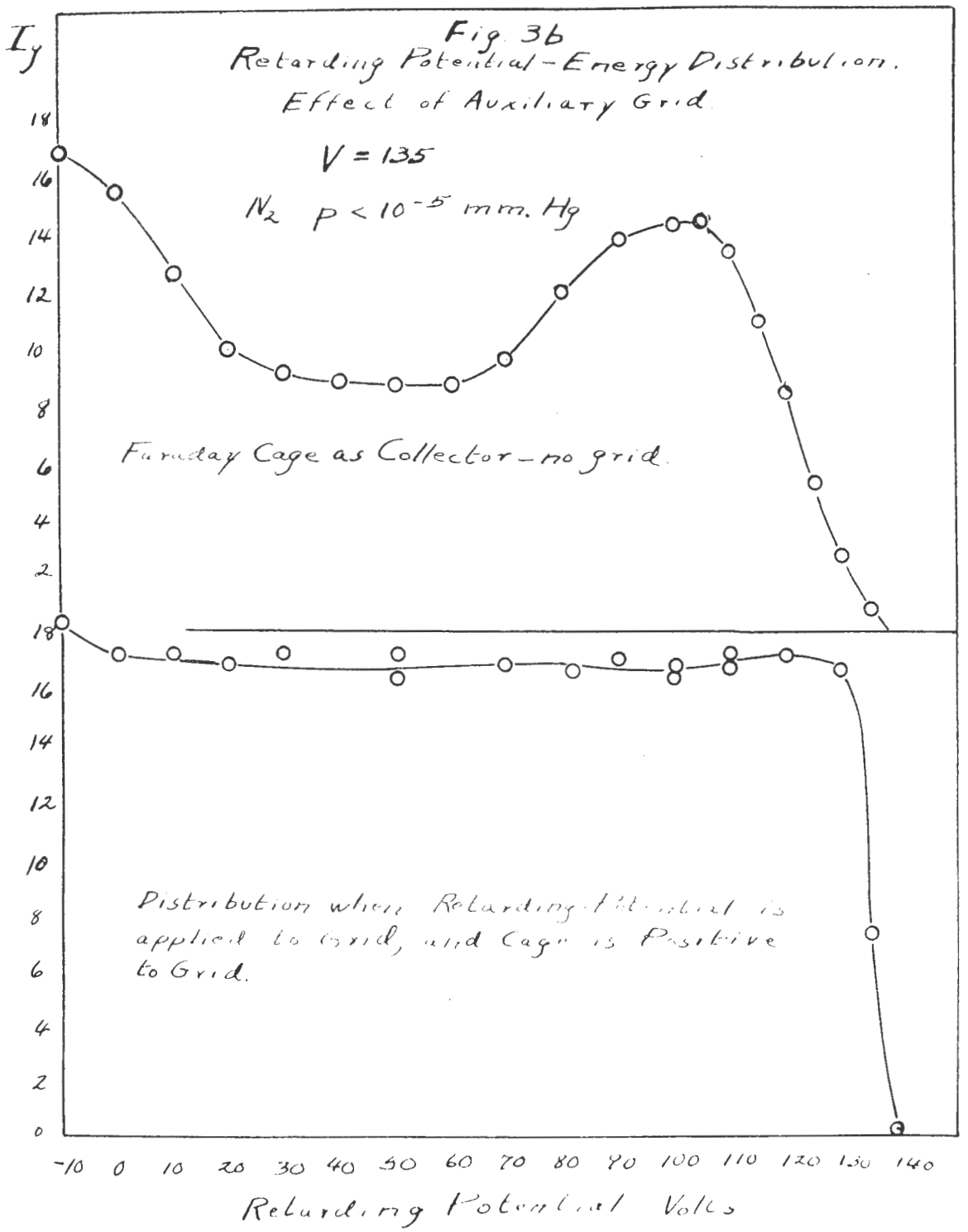


Fig. 3b
Retarding Potential – Energy Distribution.
Effect of Auxiliary Grid.
V = 135
N2 p < 10-5 mm. Hg
Iy
Faraday Cage as Collector – no grid.
Distribution when Retarding Potential is applied to Grid, and Cage is Positive to Grid.
18
16
14
12
10
8
6
4
2
0
-10 0 10 20 30 40 50 60 70 80 90 100 110 120 130 140
Retarding Potential Volts

The current in the main beam was measured by a Faraday cage behind collimating slits, and directly opposite the source. This was insulated from the chamber with redmanol. In order to collect only elastically scattered electrons, a retarding potential was applied either to this cage, or to a mesh grid placed in front of the cage, the cage being at the same potential as the scattering chamber. Energy distribution curves for the two cases are shown in Fig. 36. The difference is due to the reflection of electrons from the Faraday cage, due to surface films of impurities, and to the small size of the cage, which reflection did not take place when the cage was at a high positive potential with respect to the retarding grid.

The scattered current was measured by a rotating collector shown as C_3 in Fig. 1. The collector was insulated by quartz from its support, and the lead to it was brought out through the axis of rotation. A continual haphazard current to the collector was traced to the portion of quartz tube in air, which was hygroscopic and absorbed enough moisture to allow variable currents large enough to be measured by the electrometer. When coated with ceresin, this disappeared.

In the work with mercury vapor, the rotating collector was merely held at the appropriate retarding potential. With nitrogen, the number of positive ions was sufficient to make it necessary to include as extra slit (G_3 in Fig. 1) which could be maintained at an independent potential to sweep them away. This also could be used to prevent reflection of electrons from the collector. With this addition, duplicable results were obtainable.

The main beam was measured by a galvanometer of sensitivity 8×10^{-9} amp/cm deflection, and was of the order of a microamp or less.

The scattered current was measured by a Dolezalek electrometer, modified to increase its sensitivity (39). The simplest and most accurate method of measuring the scattered current was to use the electrometer in a null circuit. Inasmuch as it was several feet from the apparatus with a shielded lead between the two, its capacity was large. Also, it was desired to be able to read currents of different orders of magnitude, whence the voltage deflection method was inconvenient. Accordingly one pair of quadrants were shunted by a large resistance (shown in Fig. 1) the other pair being brought to the desired potential by an Ayerton shunt across a source of potential. Several different types of resistance were used. India ink lines on paper boiled in paraffin were found to develop contact potentials, and took several months to come to a steady value of resistance. Further it would be just a matter of chance, in making up a group of such resistors, that one would be of the right order of magnitude. Alcohol and xylol mixture resistors were tried, but were not steady, varying in resistance continually with time. Graphite on redmanol, between two contacts was also variable. The most successful material was a compressed rod of carbon obtained from R.C. Burt's laboratories (used there as the base for the K O perfume burning stick, but here used as rod only). One was obtained, constant to less than one per cent with respect to voltage across its terminals in the range from thirty volts to one one hundredth volt, and to one tenth of a per-cent with respect to time, with a resistance of 1×10^{11} ohm. This was used in the work on nitrogen. In addition, another resistance (an india ink line on paper boiled in paraffin under a vacuum, was used to measure the larger currents when necessary.

(Since, there has been put on the market by the S. S. White Dental Supply Co. resistors of a vulcanized, non-hygroscopic, homogeneous material, which seem from reports, to be even more satisfactory, and are supplied up to 5 x 10^{12} ohm.)

As the sensitivity of the electrometer could be raised to 1000 mm/volt, it was possible to measure scattered currents down to the order of 10^{-14} amps. The insulation resistance of the lead to the collector was sufficient to make corrections unnecessary. The electrical connections are shown in Fig. 2. The accelerating and retarding potentials were obtained from a DC motor generator set, capable of delivering two independent potentials.

Evacuation of the apparatus was accomplished by two stages of mercury pumps, backed by an oil pump. Metal to metal joints, and glass to metal joints, were sealed with picein wax. It was a metal apparatus, and under good conditions, it could be operated at a pressure of 10^{-6} cm. with the pumps going, and would hold a vacuum of the order of 10^{-5} cm. for a day or two. It could not be baked out, as the chamber was of brass, but after prolonged pumping, and electron bombardment, the vacuum conditions were good. In the work with mercury, the pumps were continually going, and the pressure was determined by keeping liquid air trap (which contained some mercury) at 0°C. by a mixture of ice and water, thoroughly agitated. This determined the pressure of the mercury vapor in the apparatus as 1.8 x 10^{-4} mm. (Int. Crit. Tables). The other side of the chamber was connected to the McLeod gauge through a liquid air trap, by a long narrow glass tube, and so had little effect on the pressure of the mercury. For nitrogen the narrow tube was removed, the liquid air

trap placed close to the chamber, and a large tube connected the chamber to the McLeod gauge. Thus the gauge would read more accurately the pressure in the chamber when the constant flow method was used. Since the pressures used were such that the scattering is a linear function of the pressure, and only relative results can be obtained, the accurate knowledge of the absolute pressure was not necessary, as long as it could be retained constant throughout a run.

Experimental Procedure

The filament was aligned opposite the slits visually, and was found to require realignment several times before the main beam would be symmetrical in its shape, as determined by moving it across the main beam collector magnetically. The alignment, of course, was done at atmospheric pressure and then the apparatus pumped out till vacuum conditions were good again, before determining the shape of the beam. The life of a filament would be sufficient to obtain several runs if conditions were good. No difference was noted in curves obtained with different filaments when lined up properly.

Having obtained a centrally aligned filament, and good vacuum conditions, the receiving slits and Faraday cages were bombarded with electrons, the beam being moved back and forth magnetically, with the pumps going. This aided in the outgassing of the apparatus, and with several hours more pumping, the scattering in vacuum would be negligible at angles of more than ten degrees to the beam.

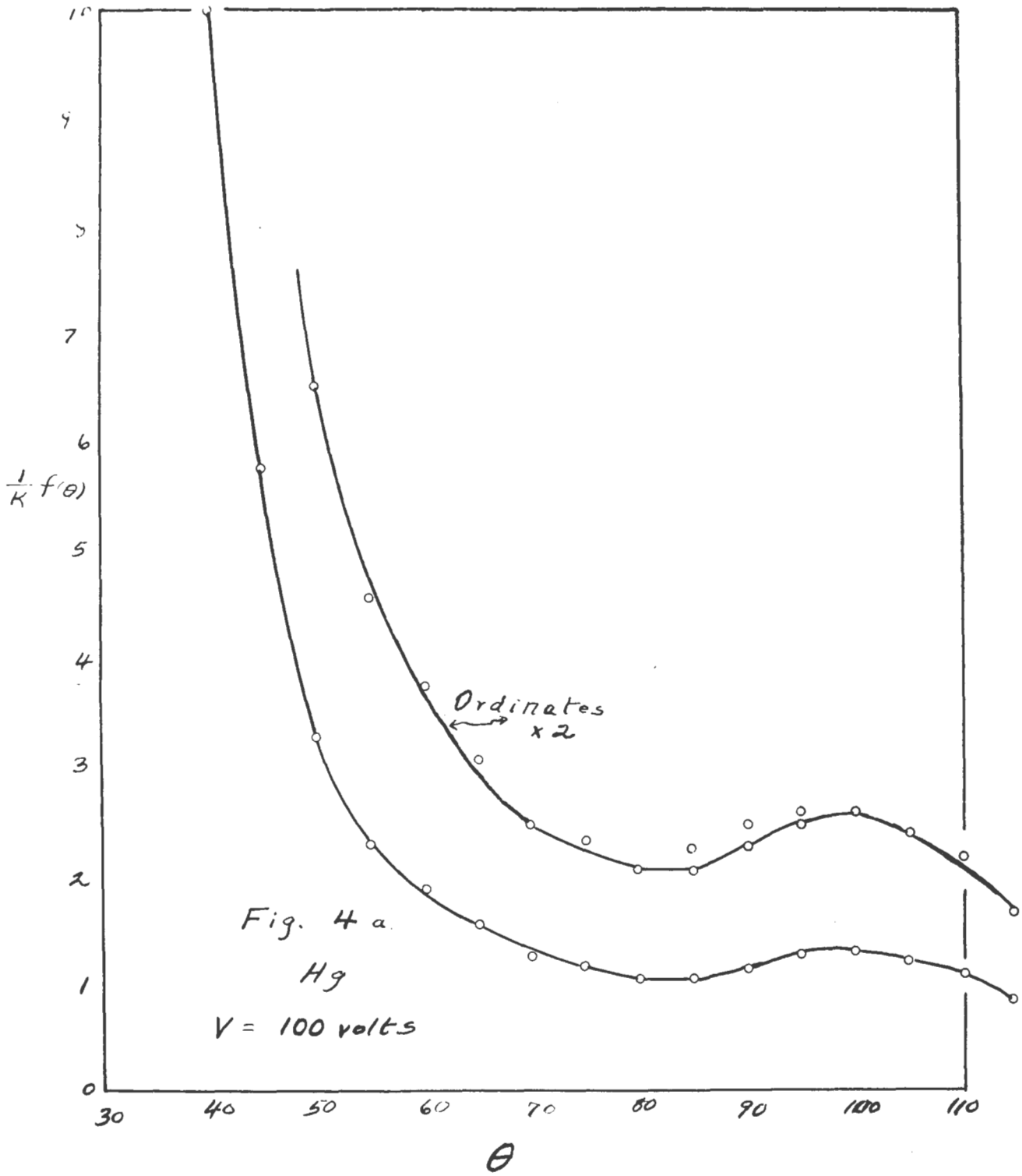
Before taking a run, several hours were allowed for the gas to come into, and stay at, a pressure equilibrium. During the latter part of this, the filament would be burning, as otherwise changes would occur

in the pressure on lighting the filament, presumably due to driving out of absorbed gases, and a "getter" effect.

With the gas at the proper pressure, the retarding potential to use was determined by a velocity analysis of the main beam. A typical curve is shown in Fig. 3. The retarding potential used was always the highest voltage just before the drop off. (In the figure the retarding potential used would be 240 volts).

The amount of current collected increases quite steadily and gradually with decrease of retarding potential, down to 10 volts, at which voltage slow speed secondaries begin to be collected. One important feature of the diagram is the absence of a depression in the curve at retarding potentials intermediate between zero and the electron velocity. Such a depression would be due to reflection of electrons from the collector, and has been found in the present apparatus when the collector alone is used; it has been overcome by the use of the auxiliary grid.

The scattering measurements were done as follows. The center of the main beam would be adjusted on the main beam collimating slits (maximum current then went to the galvanometer). The rotating collector would be set at the angle desired and readings of both currents taken. Simultaneous readings were taken when possible. Otherwise the readings were taken twice and accepted if they agreed to a few percent. Then the retarding potential was increased to an amount such that only positives were being received, and the readings taken. The differences of the two readings of the current from the scattered current collector would give the scattered current. In mercury, the positives were small in number compared to the electrons. In nitrogen, the positive current was quite large.


$\frac{1}{K} f(\theta)$
Ordinates
x 2
Fig. 4 a
Hg
V = 100 volts
θ
0
1
2
3
4
5
6
7
30
40
50
60
70
80
90
100
110

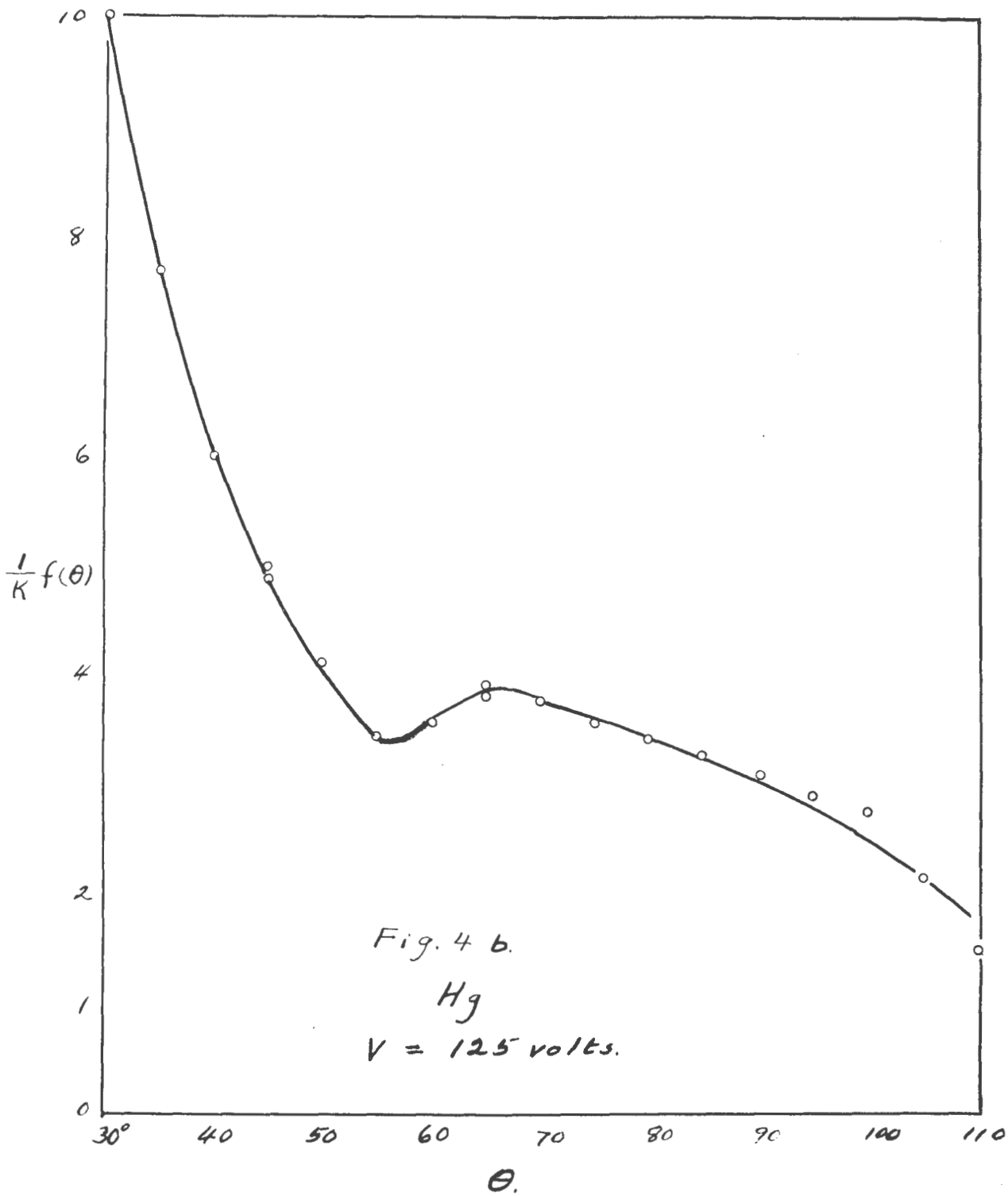


Fig. 4 b.

Hg

V = 125 volts.

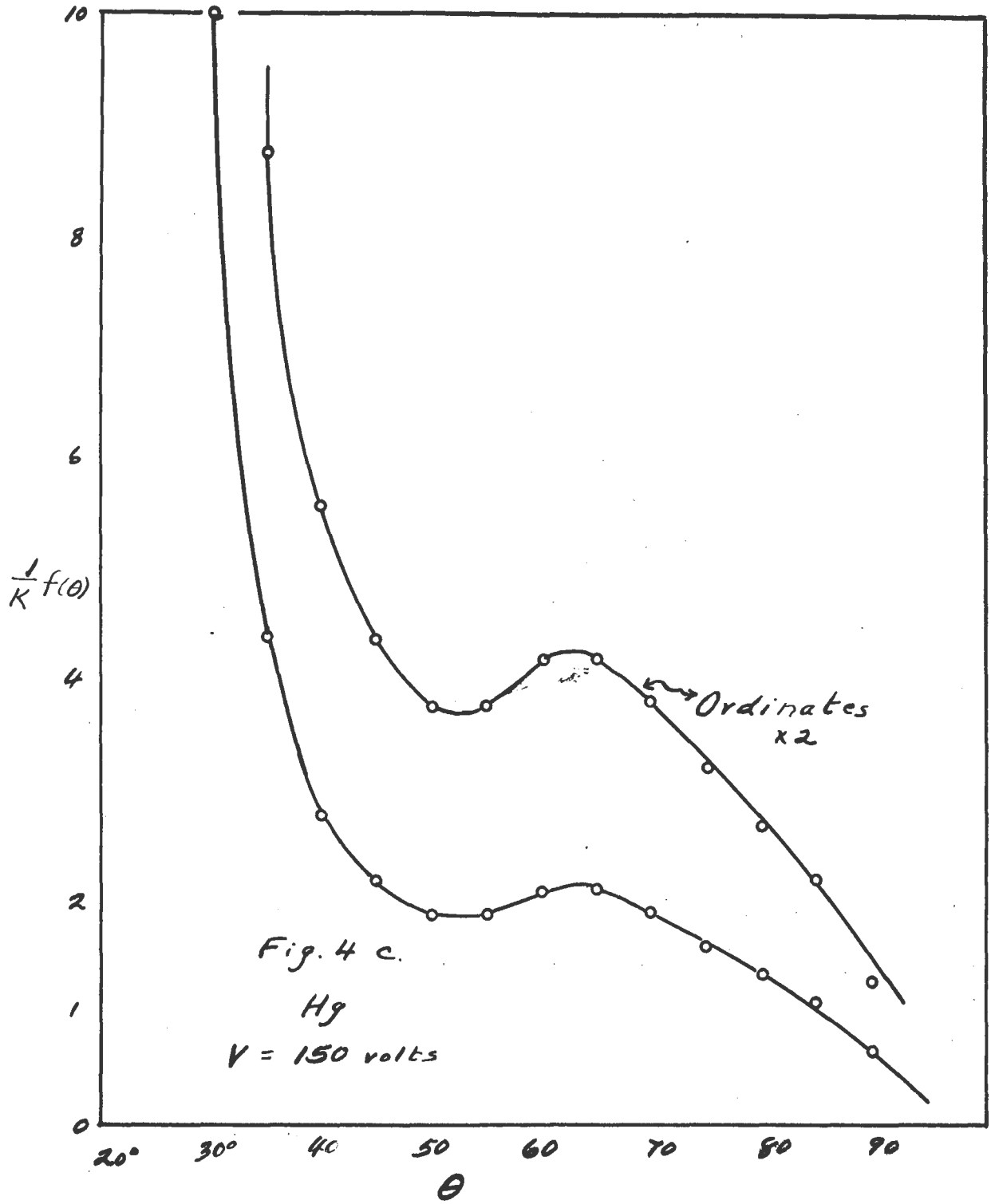


Fig. 4 c.

Hg

V = 150 volts

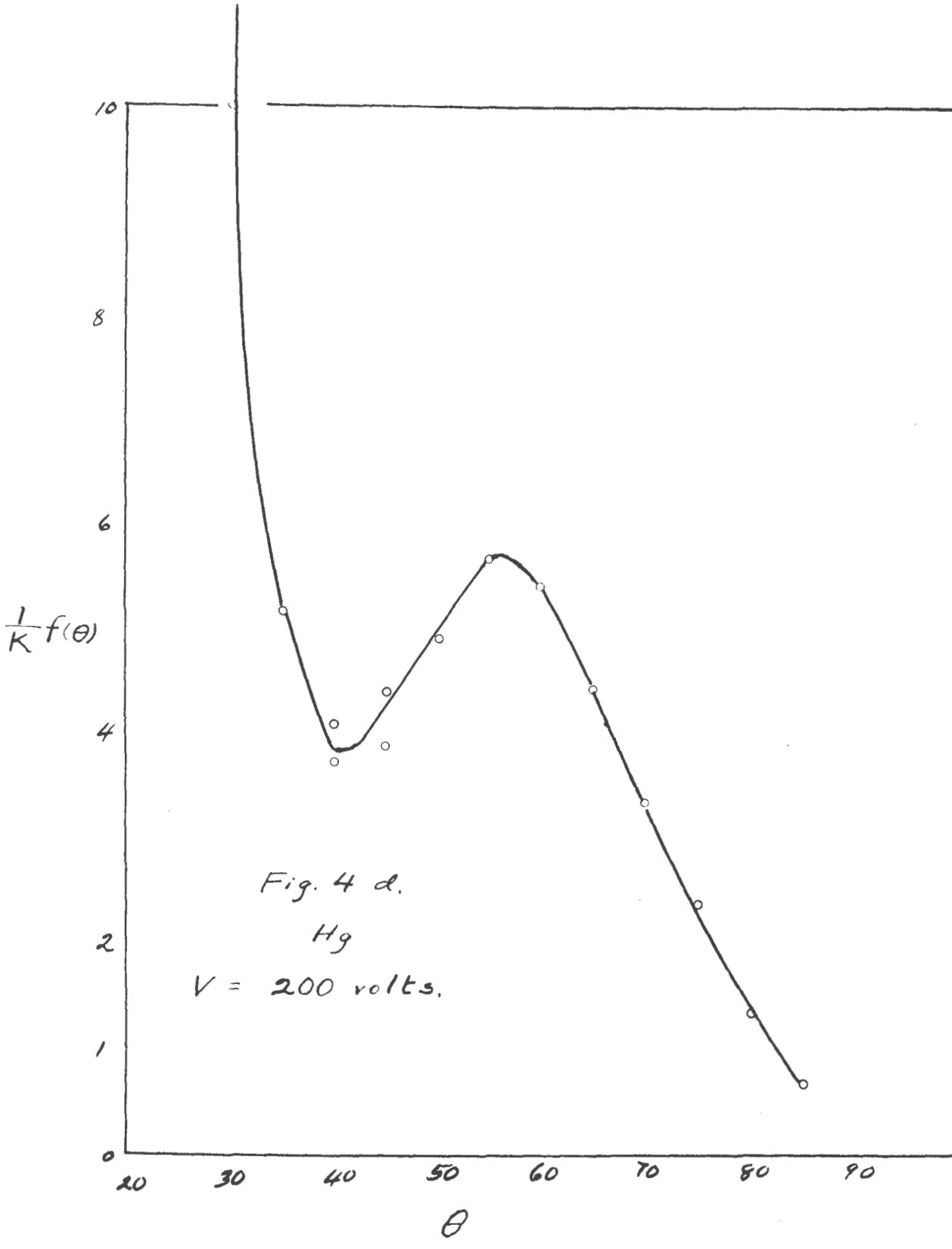


Fig. 4 d.

Hg

V = 200 volts.

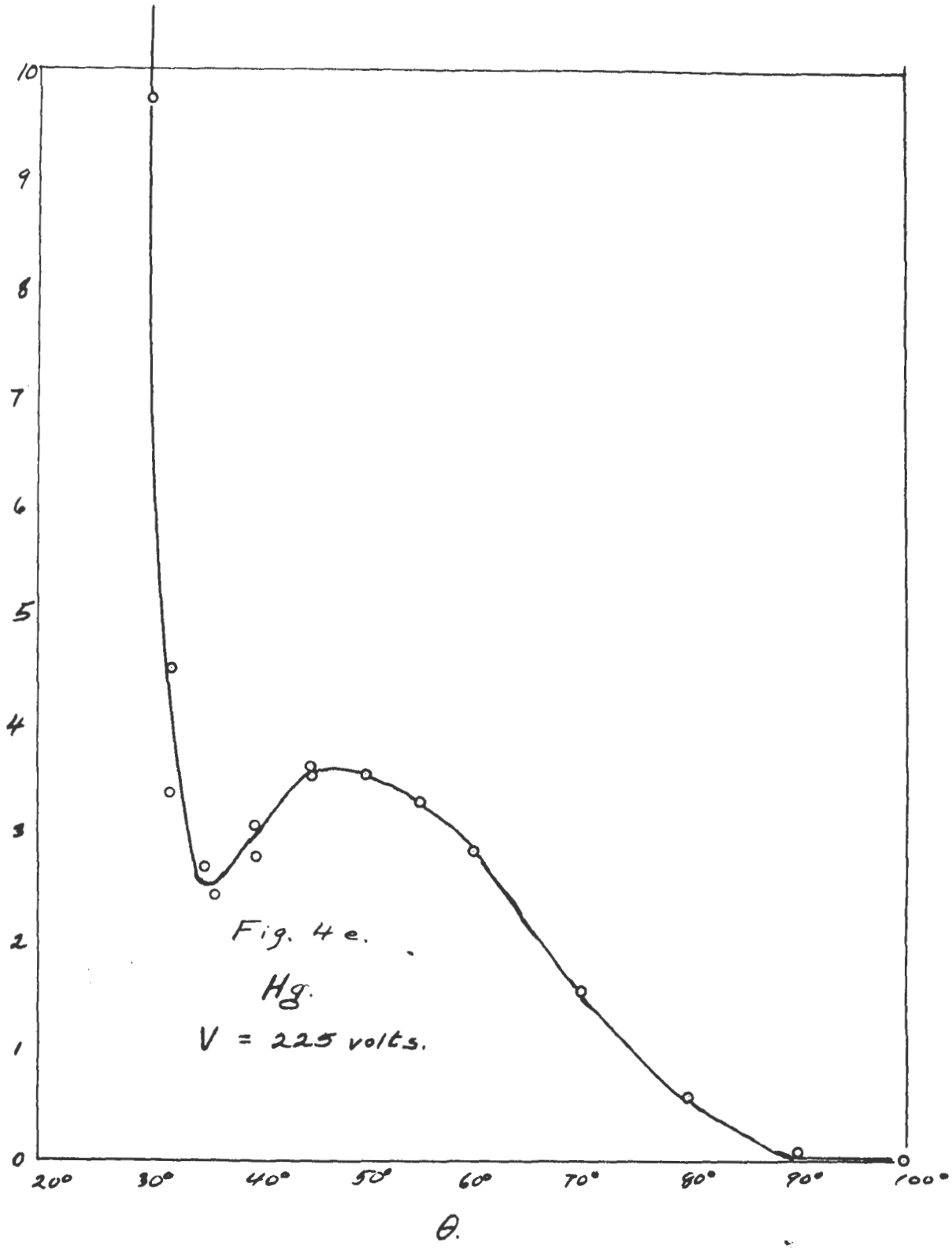


Fig. 4e.

Hg.

V = 225 volts.

The angular setting of the collector could be done with an accuracy of approximately a degree. The pointer setting, of course, would be accurate, and the backlash was small. The possible error lay in the centering of the 0° - 180° line of the scale with the direction of the main beam. This was done visually, and checked by scattering measurements at positive and negative angles.

Experimental Results

The results obtained for the scattering in mercury are shown in Figs. 4a - 4e. The ordinate is $\frac{1}{K} f(\theta)$ which is the ratio of the scattered current scattered by a fixed volume of the gas, to the main beam current. The work on mercury was undertaken as a check on the previous work done here on mercury. It will be seen that the intensity of the scattered current decreases montonically with the increase of the angle of scattering, except for a single maximum, and its attendant minimum, in each case. The angular positions of the maxima and minima vary with the velocity of the electrons. Only the position of the minima was consistent, and duplicable. Fig. 5 gives the relation between the position of the minima and the electron velocity. The results of Pearson and Arnquist are also shown on the figure for comparison. The general tendency is a decrease of the angle at which the minimum occurs, with increase of electron velocity. Arnquist's results closely approximate a cot θ relationship. This is approximated by the present results, but not so closely. There is no theoretical reason for such a relationship at present known. An increase in the prominence of the maximum with increase in velocity of impacting electron, up to two hundred volts is

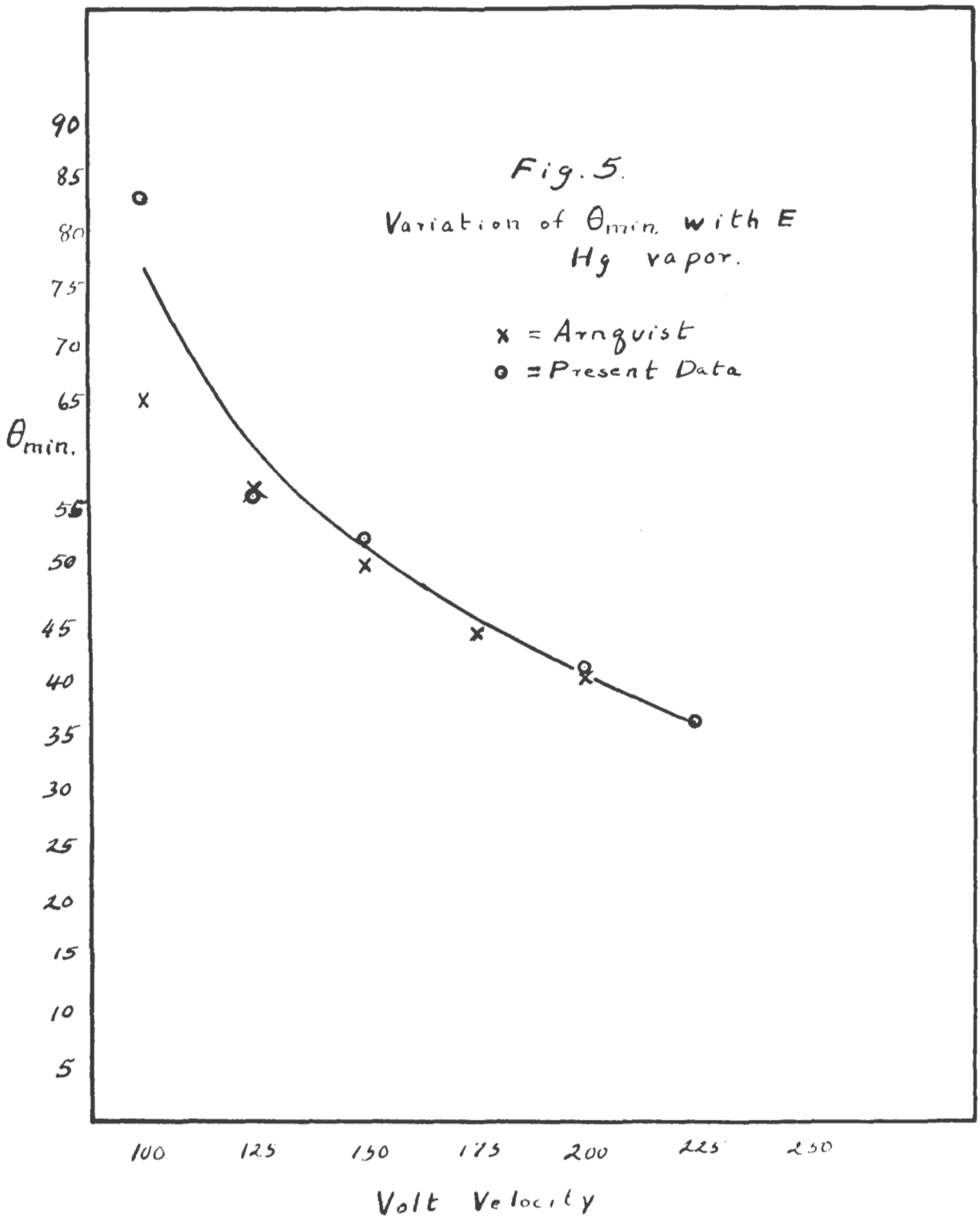


Fig. 5.

Variation of $\theta_{min.}$ with E
Hg vapor.

x = Arnquist
o = Present Data

apparent. Tate and Palmer (13) noticed just lately the same behavior. The fact may be explained as partly due to the decrease in the relative scattering at large angles as compared to that at smaller angles, which occurs with increase of velocity.

Pearson and Arnquist have shown that with this apparatus, the fraction of the total scattered current per unit solid angle in the direction θ is given by

$$f(\theta) = K \frac{I_e}{I_g} \frac{\sin\theta}{P}$$

where I = scattered current (measured by electrometer)

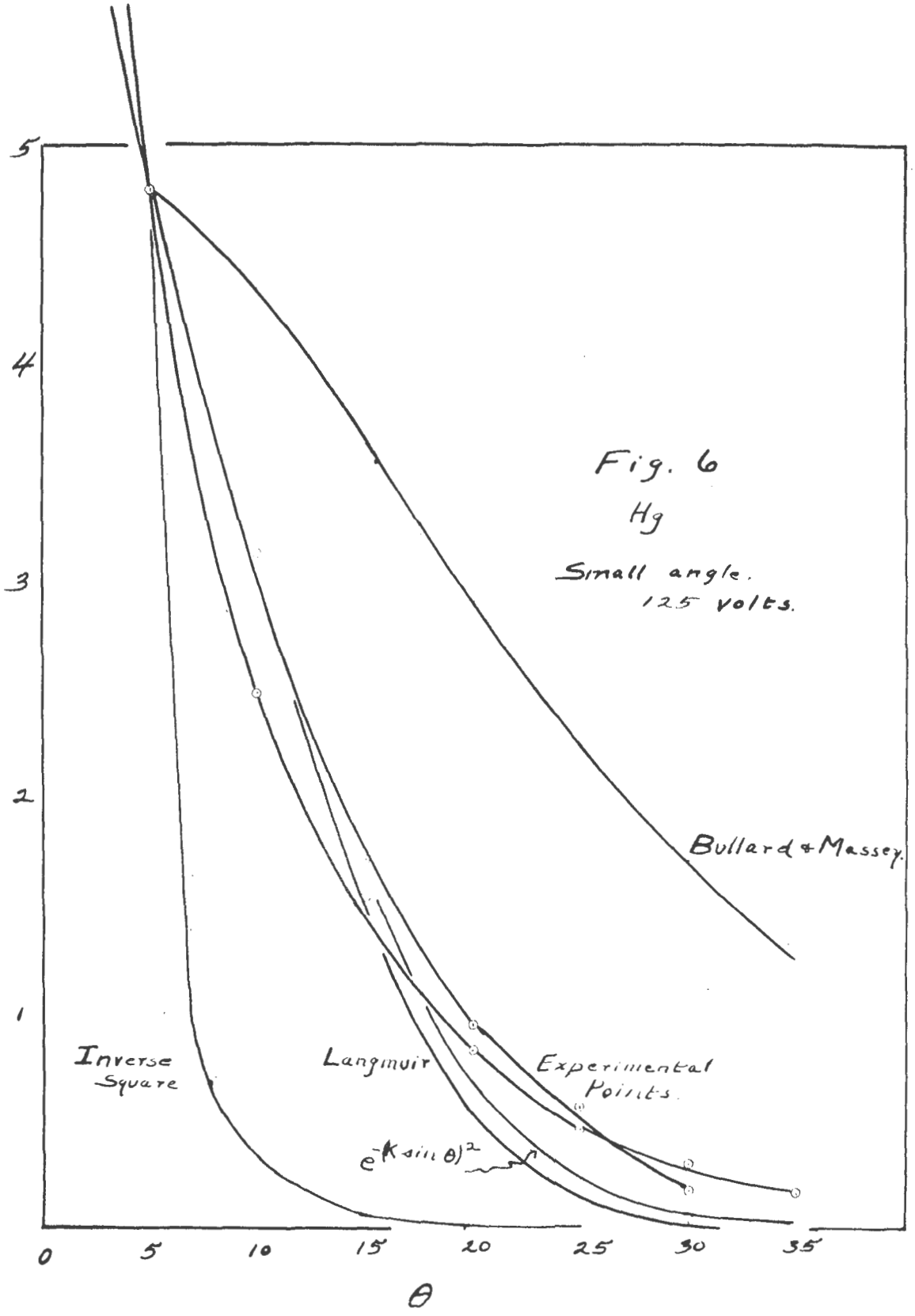
I = main beam current (measured by galvanometer)

and K is a correction factor including constants of the apparatus, absorption coefficient, slit scattering, etc.

Accordingly for the scattering per unit solid angle, there is plotted in arbitrary units

$$\frac{1}{K} f(\theta) = \frac{I_e}{I_g} \frac{\sin\theta}{P}$$

and to obtain the scattering per unit angle, the results are to be multiplied by sin θ. If K is known, the total absorption coefficient of the gas for electrons of the definite velocity could be determined by integration of the latter curves. However, the scattering at the smaller angles is so much more than that at the angles this apparatus was built to measure, and which were desired, that the extrapolation to zero degree upto 180 degree scattering is not possible. Besides, relative results are all that can be obtained with the apparatus, and the value of K


5
4
3
2
1
0
5
10
15
20
25
30
35
θ
Fig. 6
Hg
Small angle.
125 volts.
Bullard & Massey.
Inverse Square
Langmuir
Experimental Points.
$e^{-K \sin\theta)^2}$

could not be determined. It was found that though the value of K_Λ would as determined by values of $\frac{1}{K}f(\theta)$ on different days and different runs change from day to day, the form of the curves would remain quite reasonably constant. This was also Arnquist's experience in his work.

Below 100 volts, electron velocity, the current received by the collectors became too small for satisfactory measurements. This is due to several causes, particularly to slit scattering which increases considerably for small velocities, to the greater effect of slight irregularities in adjustment, etc.

Inasmuch as the agreement of theoretical treatments with experiments is better at small angles, a couple of runs were taken for small angle scattering for 125 volt electrons. The results are shown in Fig. 6 . For purposes of comparison there are plotted also the relationships which would be expected on the basis of various theoretical treatments. The curves are fitted at five degrees. It will be seen that the Rutherford treatment gives too small a scattering at the larger angles. Also it has the theoretical disadvantage of giving infinite scattering at zero degrees, and of not allowing for screening of the nucleus by the planetary electrons. The Bullard and Massey treatment, using the Thomas-Fermi force field and Born's approximate scattering formula, gives too great a scattering at large angles. The Sommerfeld formula, (not pictures) gives even less of a decrease of scattering as angle of scattering increases. This is to be expected, for in the Sommerfeld model all the outher electrons are supposed to be in the K shell, giving an exceptionally well shielded nucleus. The Thomas-Fermi field lies in between the Sommerfeld and Rutherford pictures.

The Langmuir empirical expression gives a good agreement for angles of twenty five degrees or less. It will be seen that the expression developed by the author in Appendix II gives an even closer approximation. This would indicate the possibility of the atomic force field given by the expression developed there as an approximation to the actual force field seen by the approaching electron.

Nitrogen

The results obtained in the scattering in nitrogen are shown in Figs. 7a - 7g. As before $\frac{1}{K} f(\theta)$ is plotted giving the scattering per unit solid angle. To obtain the scattering per unit angle at any angle, the results given have merely to be multiplied by sin θ. In all cases there is a monotonic decrease of scattered intensity with increase of scattering angle up to approximately ninety degrees. At the lower velocities, there occurs a minimum, and an increase of scattering at the angles of scattering larger than the angle of the minimum. As the velocity of the impacting electron increases, this minimum moves to greater angles, and the subsequent rise is not so great. At the high velocities (more than 350 volts) the minimum is no longer present in the range of angles observable. This is in accordance with the general fact that increase of electron velocity causes the scattering to become more nearly like nuclear scattering. The variation of the angle at which the minimum occurs as affected by the velocity of the electron is shown in Fig. 8. It is seen that the trend is in the opposite direction from that shown in the results on mercury.

The electron velocity range studied has been that between one hundred and four hundred volts in steps to cover the range. At

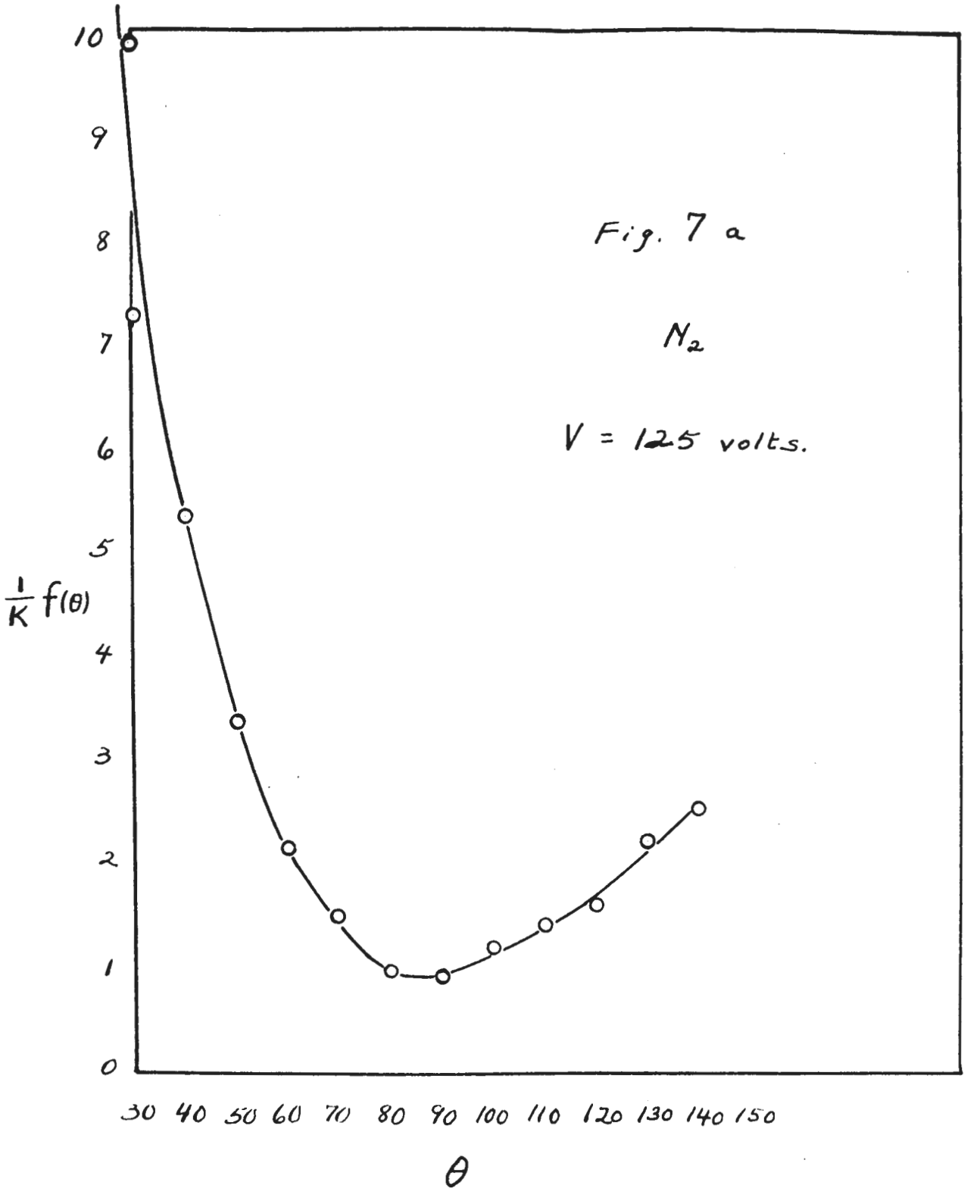


Fig. 7 a

N_2

$V = 125$ volts.

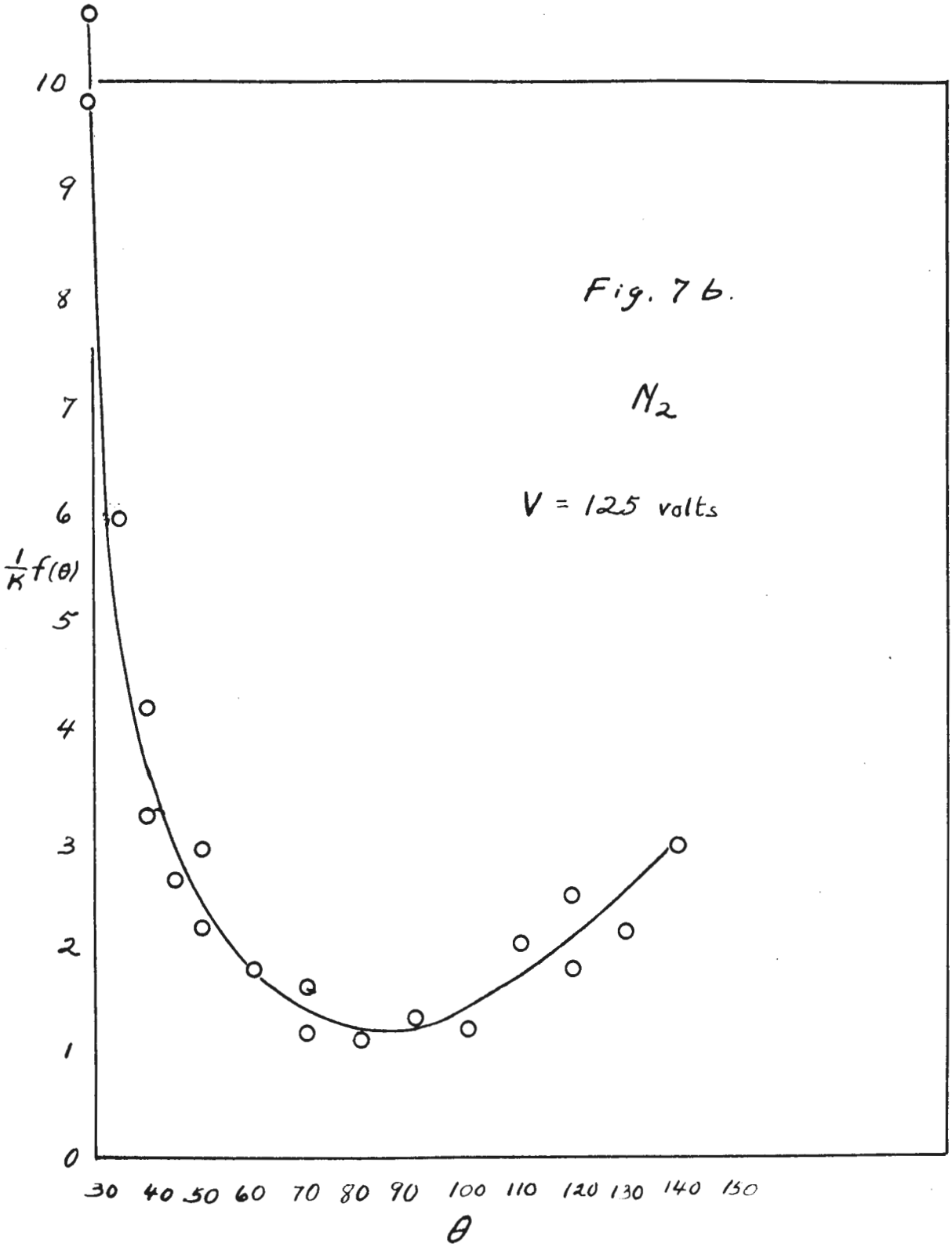


Fig. 7 b.

N_2

V = 125 volts

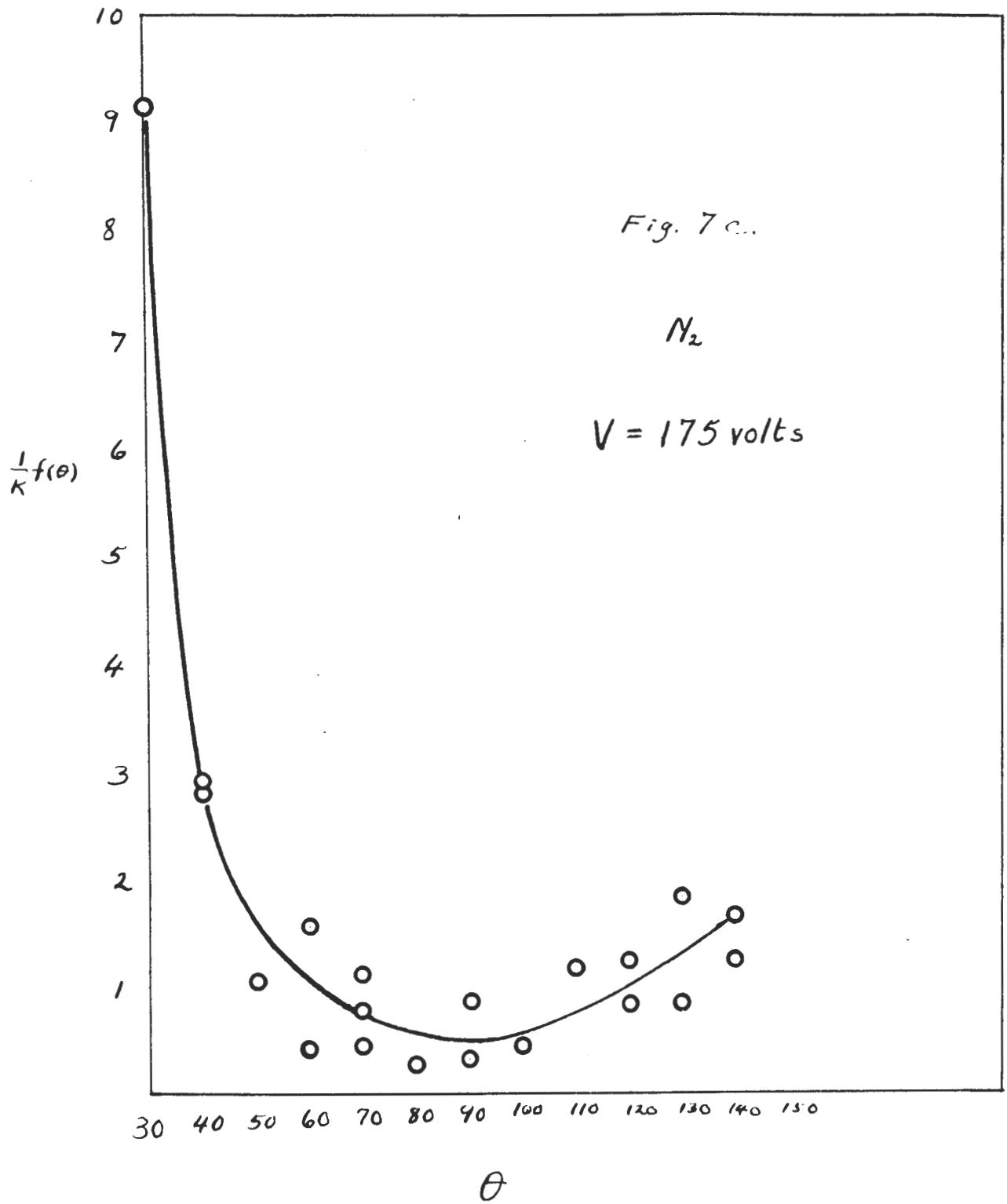


Fig. 7 c..

N_2

V = 175 volts

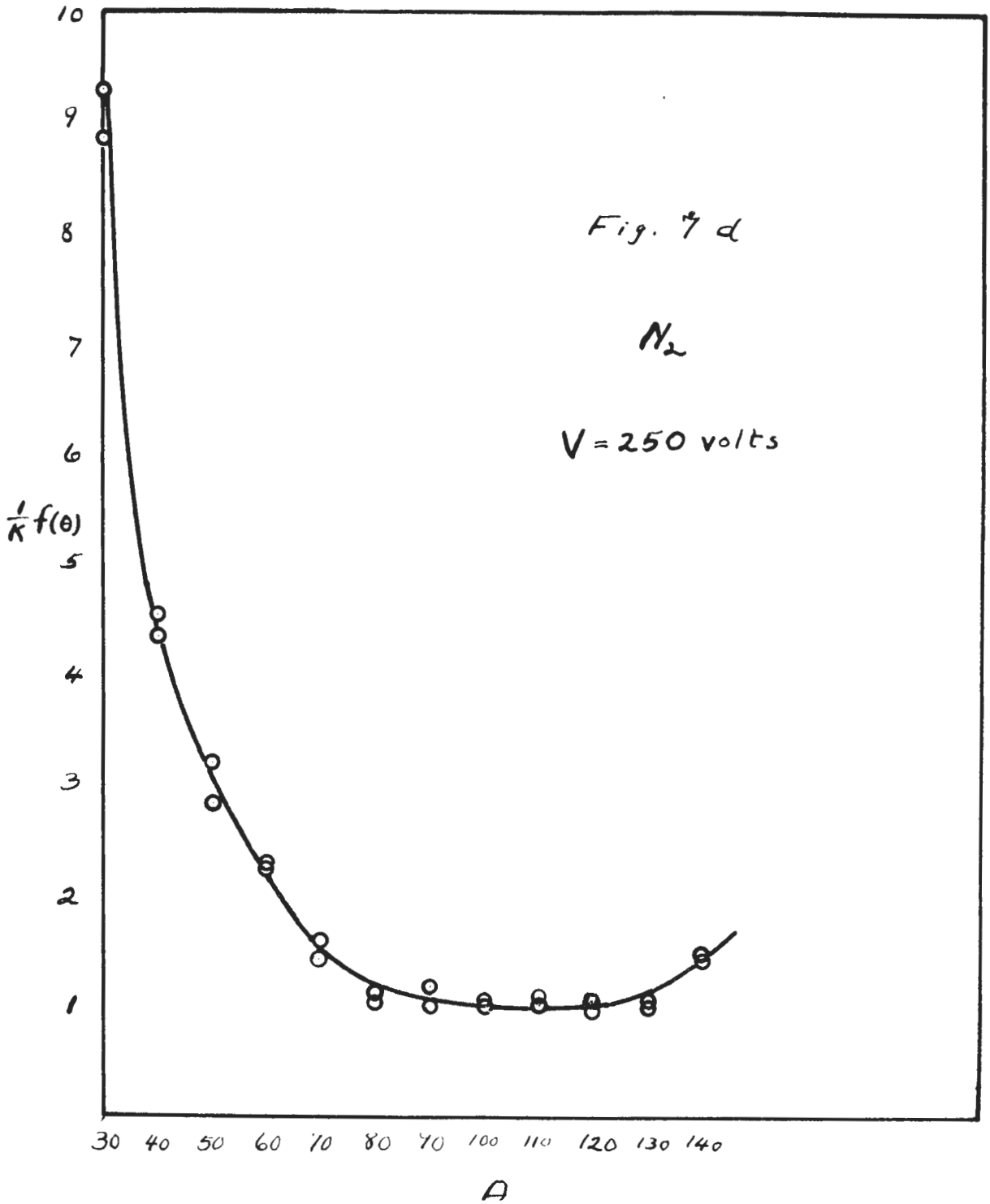
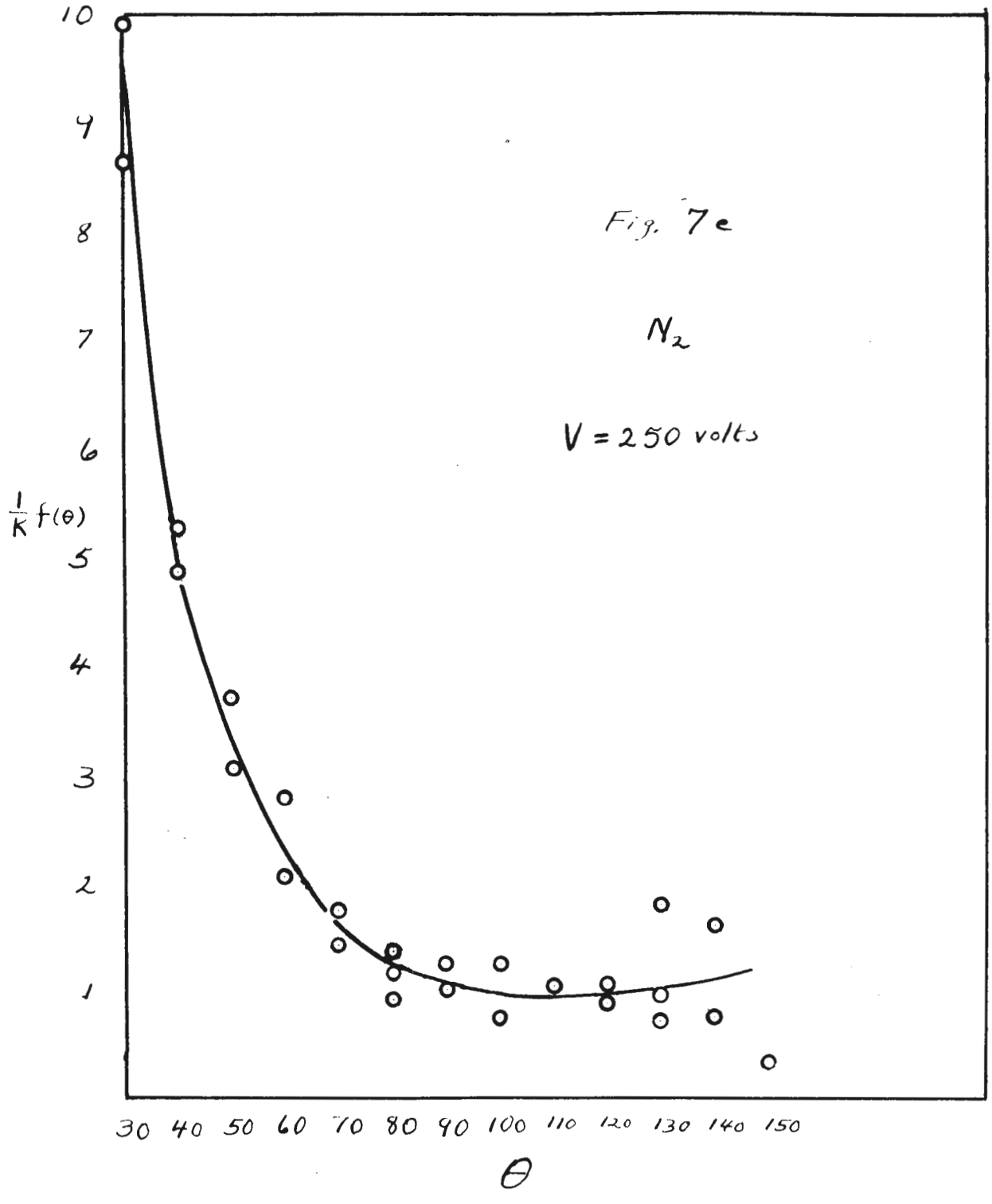


Fig. 7 d

N_2

V = 250 volts


10
9
8
7
6
1/K f(θ)
5
4
3
2
1
30 40 50 60 70 80 90 100 110 120 130 140 150
θ
Fig. 7e
N2
V = 250 volts

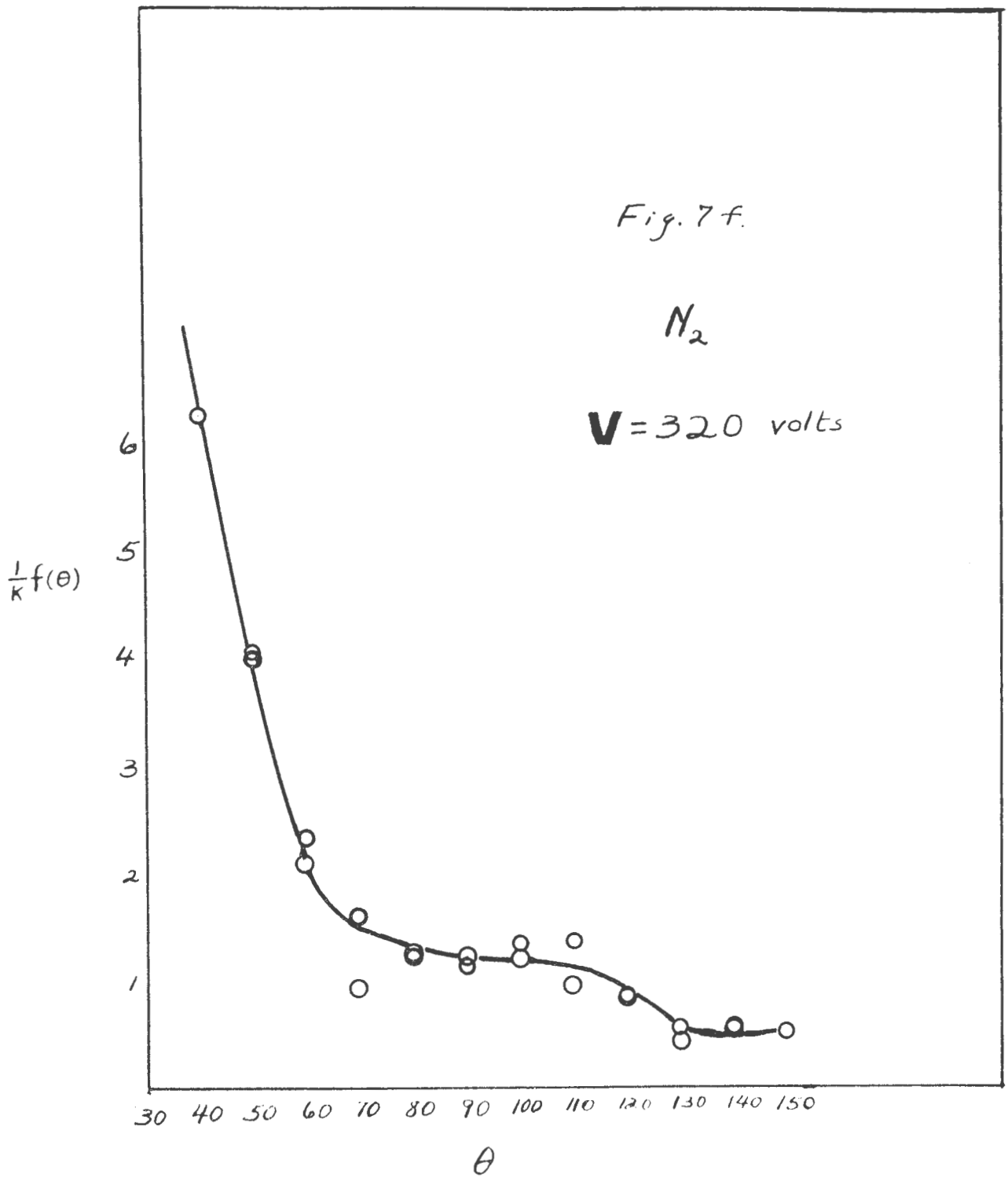


Fig. 7 f.
N_2
V = 320 volts
$\frac{1}{K} f(\theta)$
6
5
4
3
2
1
30 40 50 60 70 80 90 100 110 120 130 140 150
θ

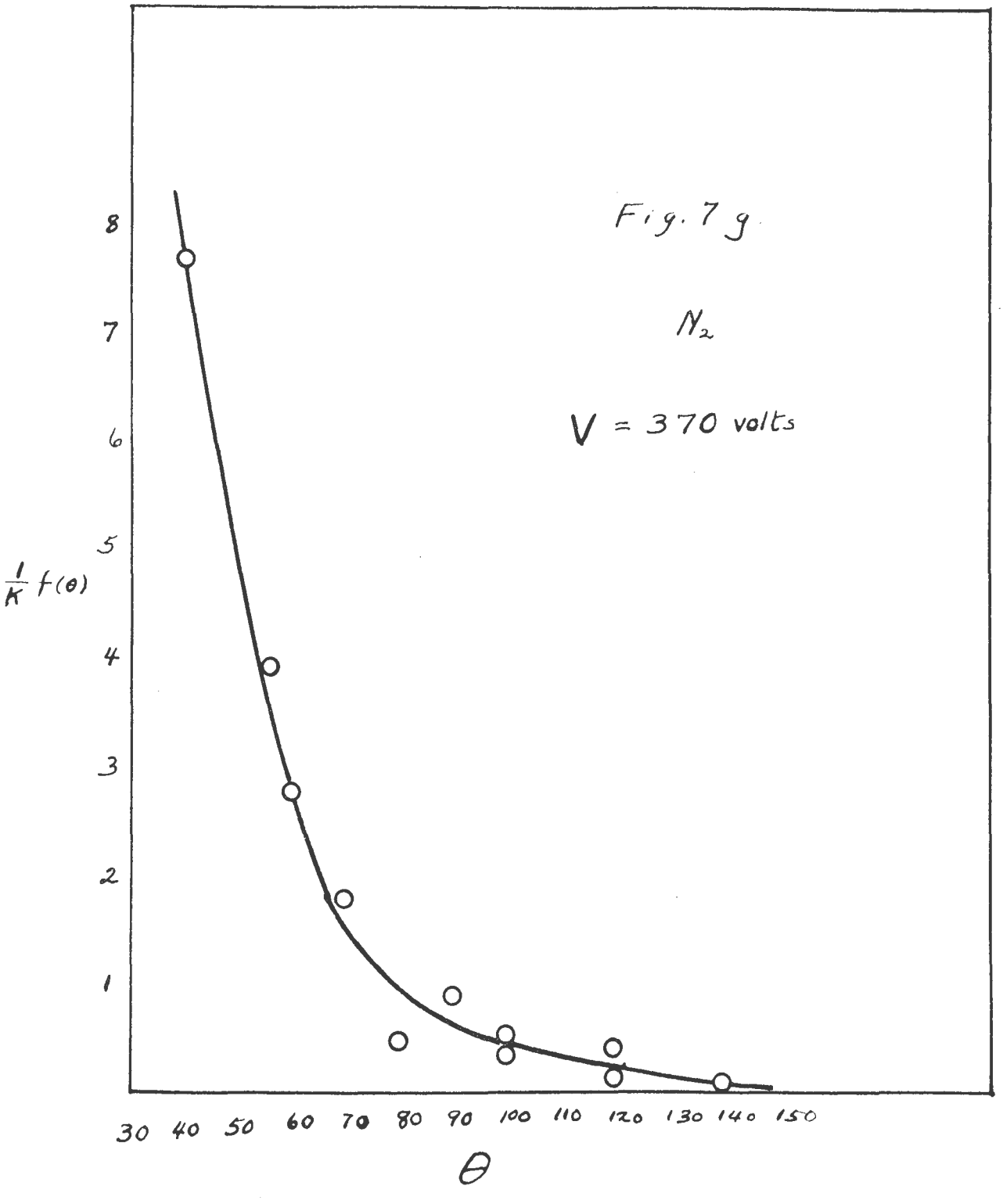


Fig. 7 g
N_2
V = 370 volts
$\frac{1}{K} f(\theta)$
8
7
6
5
4
3
2
1
30 40 50 60 70 80 90 100 110 120 130 140 150
θ

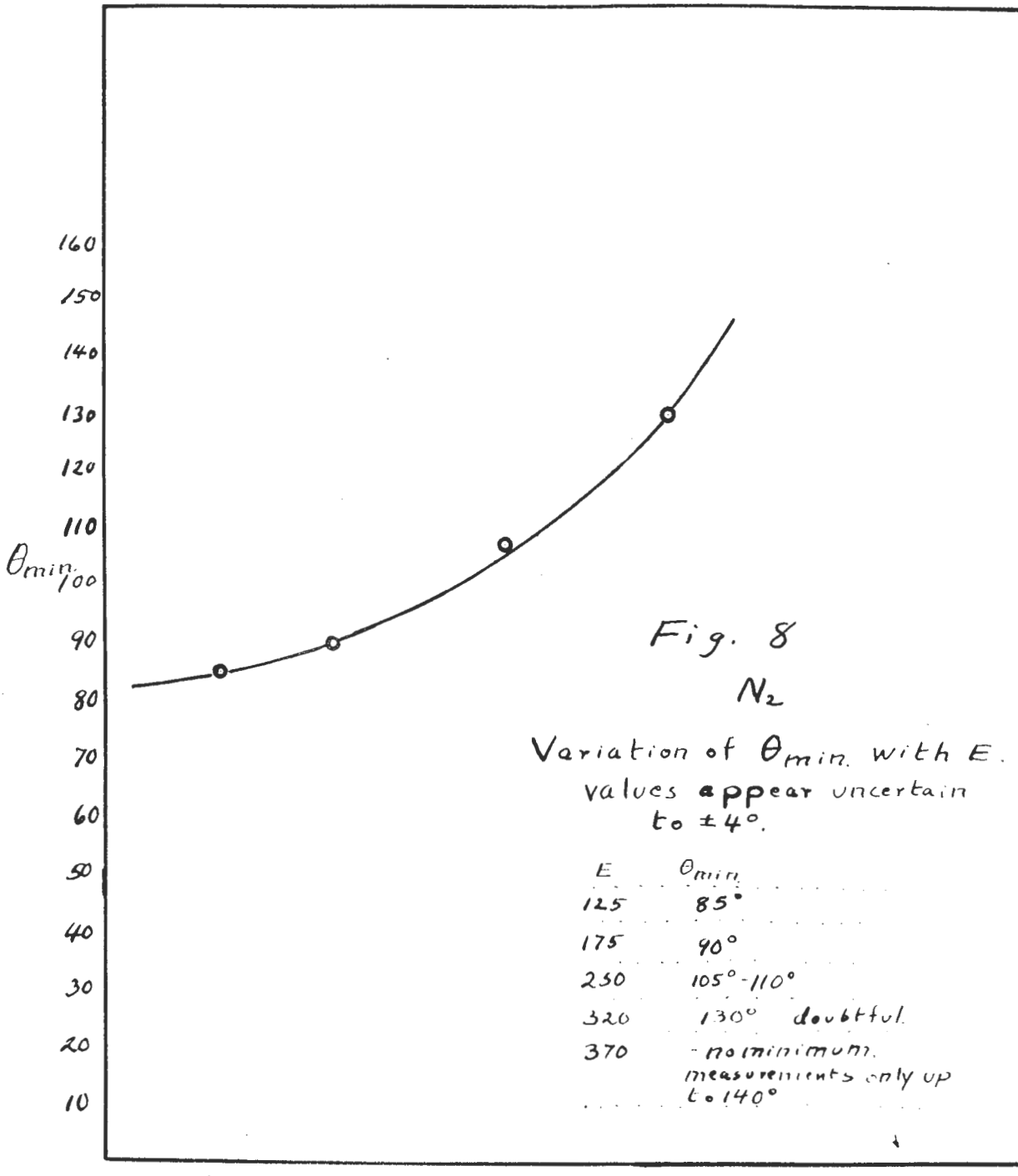


Fig. 8

N_2

Variation of $\theta_{min.}$ with E.
values appear uncertain
to ±4°.

E	$\theta_{min.}$
125	85°
175	90°
230	105°-110°
320	130° doubtful
370	- no minimum. measurements only up to 140°

these velocities, the electron is moving with sufficient velocity to be out of the velocity range in which the Ramsauer effect becomes important, and therefore the interaction terms in the theoretical treatment do not become overwhelmingly important as would be the case at the lower velocities. However, the experimental results show that the Born approximation is not a valid one below four hundred volts velocity. Since in general it is found that the Born approximation is not valid for scattering in mercury below eight hundred volts, we have a confirmation of the fact that the approximation is better for atoms of smaller atomic number, and for higher electron velocities.

Arnot (13) has found the same type of behavior in the diatomic gases which he has studied as is shown by the scattering in nitrogen obtained in the present work. The theoretical treatments are not sufficiently complete to allow an explanation rigorously. However, Arnot, in his paper, mentions that Bullard and Massey have made an unpublished calculation of the results to be expected in nitrogen, and that they agree well with his results. Arnot's results show the same type of behavior as found in the present work, and the agreement would seem to be quite good. He has only one curve (one set of readings, as apparently he did not check his work) in the range studied here, that being for two hundred and five volt electrons.

By use of the calculations which Allis and Morse (39) have made, using a simplified atomic model, (originally used by Mensing, in an unsatisfactory theoretical treatment of total absorption coefficient) it is possible to obtain certain qualitative conclusions concerning the scattering in the present case. The Allis and Morse treatment is quite

satisfactory for this. In particular it is possible to show that the observed change of the position of the minimum out to larger angles, with increase of electron velocity is what should be expected. For the case of mercury, the effect of distortion, and exchange of electrons enters to a larger degree and accordingly such a prediction can not be made.

In nitrogen the scattering is done both by nitrogen molecules and by nitrogen atoms. Also, undoubtedly, there is present, ionized nitrogen in molecular and atomic form. Friedlander, Kallman, Lasareff, and Rosen (42) have found by a somewhat doubtful means of distinguishing between N^+, and N_2^{++}, that there exist no doubly ionized molecules in nitrogen suffering impact from slow electrons. It seems probable that doubly ionized molecules, if they exist, have a very short mean life-time. However, under any circumstances, the results obtained in nitrogen must be considered as a mixture of results due to different constituents of the scattering medium.

Measurements were attempted on hydrogen, but the gas pressure would continually decrease during the course of a run. No consistent results could be obtained. This disappearance of hydrogen under such conditions has been observed before; the most recent discussion of it is by Kunsman and Nelson (43), who give other references. They state it probably occurs at the hot glass surface, and may be a process of combination with oxygen to form water.

Oxygen was tried also, but its effect on the filament (positive ion bombardment, and oxidation) was quite destructive, and considerable difficulty was encountered in obtaining a filament which would last through the preliminary processes, necessary to be gone through before taking a run. Accordingly attention was turned to nitrogen.

Discussion of Factors Introducing Possible Errors in the Scattering Measurements

1. Stray fields--electrostatic or magnetic.

The magnetic fields in the region of the apparatus are compensated for by adjusting the current through the Helmholtz coils so as to center the beam. The error involved is negligible. Electrostatic fields are eliminated because the apparatus is made of brass. The stray field which might penetrate the scattering chamber through the collimating slits is negligible.

2. Resolving power and slit width.

Pearson has shown that the error in angle of scattering due to width of slits in this apparatus is such that 90% of the scattered current has been scattered through an angle of $\theta \pm 1.6^\circ$, where θ is the angular setting of the collector.

The error due to the vertical length of the slits, he has shown for 30° causes a $d\theta = 1.9^\circ$, decreasing at $\theta = 90^\circ$ to $d\theta = 0^\circ$.

3. Spread of electron velocity and inelastic collisions.

There is approximately a 3 volt spread on other side of the mean velocity of the beam. This causes an inhomogeneity in the beam, but due to the method of choosing the retarding potential, inelastically scattered electrons are not collected. The scattering measured is the mean of the scattering curves for electrons of velocity $V \pm 3$ volts. The change in the scattering for such small velocity changes is well

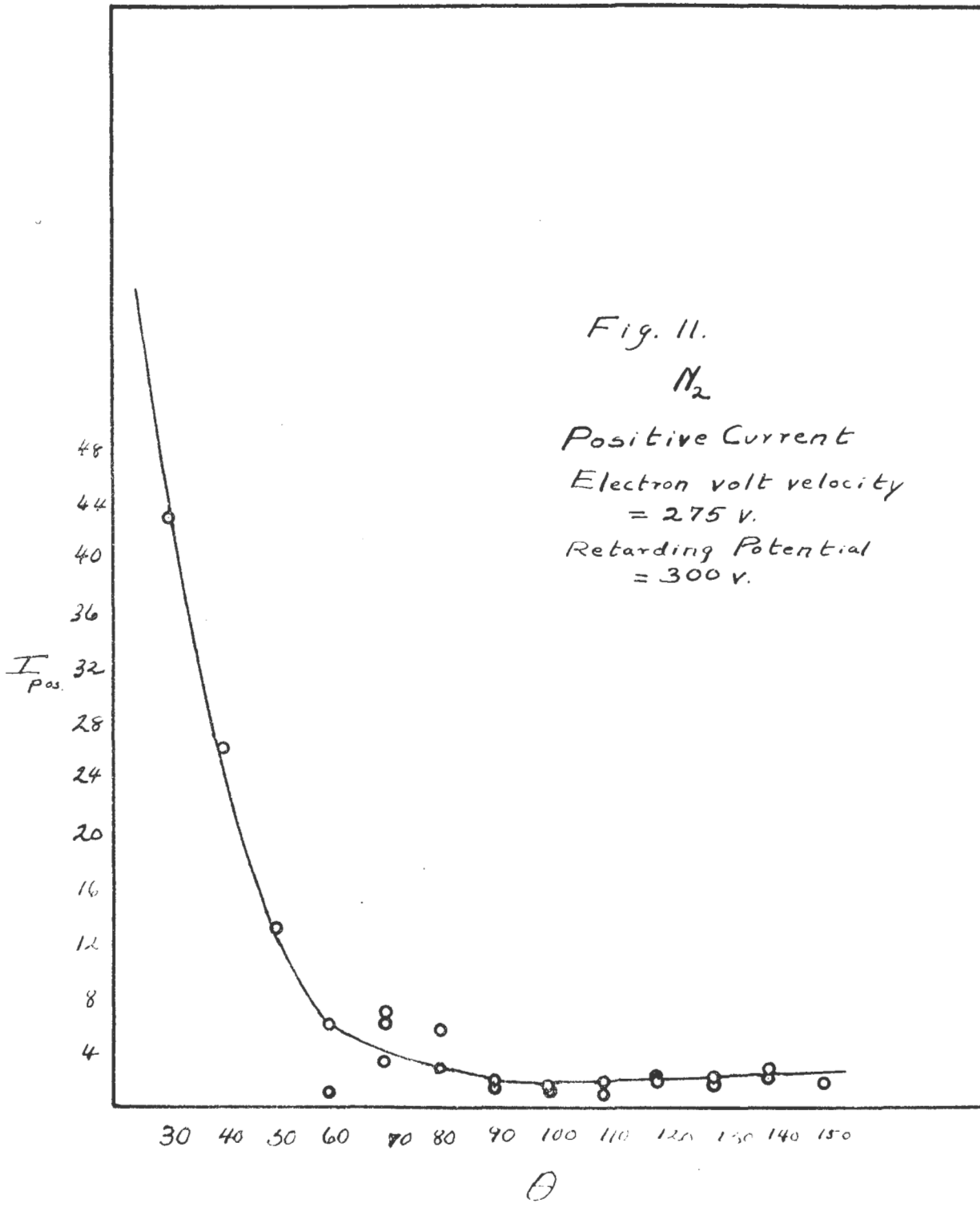


Fig. 11.
N_2
Positive Current
Electron volt velocity
= 275 v.
Retarding Potential
= 300 v.
$I_{pos.}$
48
44
40
36
32
28
24
20
16
12
8
4
30 40 50 60 70 80 90 100 110 120 130 140 150
θ

within other experimental error. The collection of inelastically scattered electrons would be a definite cause of error. The ionization potential of nitrogen is approximately 16.5 volts (40) but the average loss of energy of the impacting electrons per ionization in nitrogen is 27.9 volts (41). The difference is due to the random collisions of the electrons which have and have not made ionizing collisions (41), Renninger (19), resulting in excitation of the further impacted atoms. The scattering of inelastically scattered electrons is quite different in character from that of elastically scattered, and the above indicates that with a retarding potential of 10-15 volts less than the velocity of the beam, inelastically scattered electrons are not collected.

4. Focusing effect.

Since the value of the main beam current is read, and the scattered current collected comes only from this part of the beam, as insured by the collimating slits, the focusing effect introduces negligible error. In other apparatus, in which the measurement of the main beam current were made by collecting it on the walls of the chamber, the effect would have to be considered.

5. Positive ions.

In mercury these are negligible. In nitrogen, they may cause considerable error. However, the positive ion current was always subtracted from the electron current. The error involved will be that due to the difference in the number of positive ions collected at collector potentials of say 15 volts below the beam velocity, and 30 volts above. Since the

beam velocity was in the range of one to four hundred volts, we may assume a small error. However, this is undoubtedly the main source of error in the nitrogen scattering measurements.

6. Radial field about the beam.

Arnot (10) (both references) has shown the presence of a radial field about the beam produced by the plasma of positive ions and electrons in the space through which the beam is passing. This results in an acceleration toward the center of the beam, introducing an error in the apparent angle of scattering. To determine the error, one needs to know the potential difference between the beam and walls of the scattering chamber. However, from Arnot's data, the error in our apparatus should be no more than a degree or two.

7. Presence of other gases.

In the measurements on mercury, foreign gases are negligible, their pressure being less than one percent of that of the mercury. In the measurements on nitrogen, both constant flow and static pressure methods were tried. In either there is the possibility of a few per cent of oxygen or carbon dioxide. The error introduced is less than other errors, so it was not considered necessary to attempt strict purification of the gas. All vapors were removed by the liquid air traps.

8. Pressure determination.

In mercury the pressure was determined by the temperature of the mercury. In nitrogen the absolute pressure was not directly obtainable,

when the constant flow method was used (this method gave the purest gas in the chamber). However, the reading of the gauge was proportional to the pressure in the range used, and the large size of the connecting tube between the chamber and the gauge caused the reading to be in the proper order of magnitude. A check on this is indicated later under "focusing effect". In any case, absolute knowledge of the pressure was not important, as the factor expressing the proportionality could be absorbed in the K of the equation for the scattering.

9. Reflection.

An error entering through the reflection of the electrons from the collector and the consequent subtraction of those electrons from the measured current was compensated for by the introduction of the retarding grid for the main beam, and the retarding slit for the rotating collector. The actual receiver would then be at a positive potential with respect to the retarding field. The grid was found more successful than the slit. The chance of receiving electrons reflected from the walls of the chamber was small due to the good collimation. It was further reduced by roughening the walls through the introduction of a copper screen placed next to the walls.

Measurements on the Focusing Effect

This was more pronounced in mercury than in nitrogen. In both the effects were similar. In vacuum, the beam would be broad and not sharply defined, due to the spreading caused by the mutual repulsion of the electrons. With increase of gas pressure, the beam would narrow

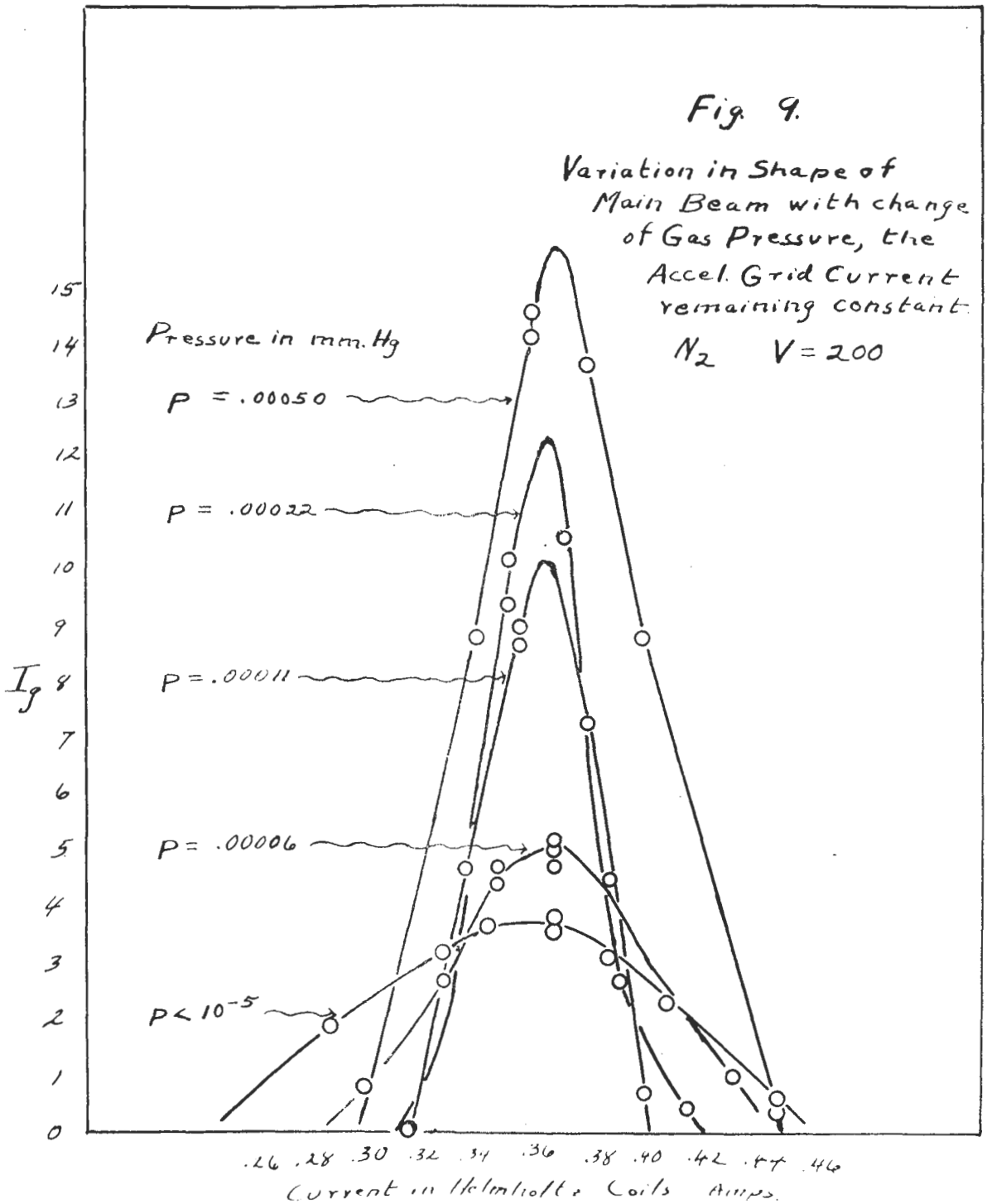


Fig. 9.
Variation in Shape of Main Beam with change of Gas Pressure, the Accel. Grid Current remaining constant.
N_2 V = 200
Pressure in mm. Hg
P = .00050
P = .00022
P = .00011
P = .00006
P < 10^{-5}
I_g
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
.26 .28 .30 .32 .34 .36 .38 .40 .42 .44 .46
Current in Helmholtz Coils Amps.

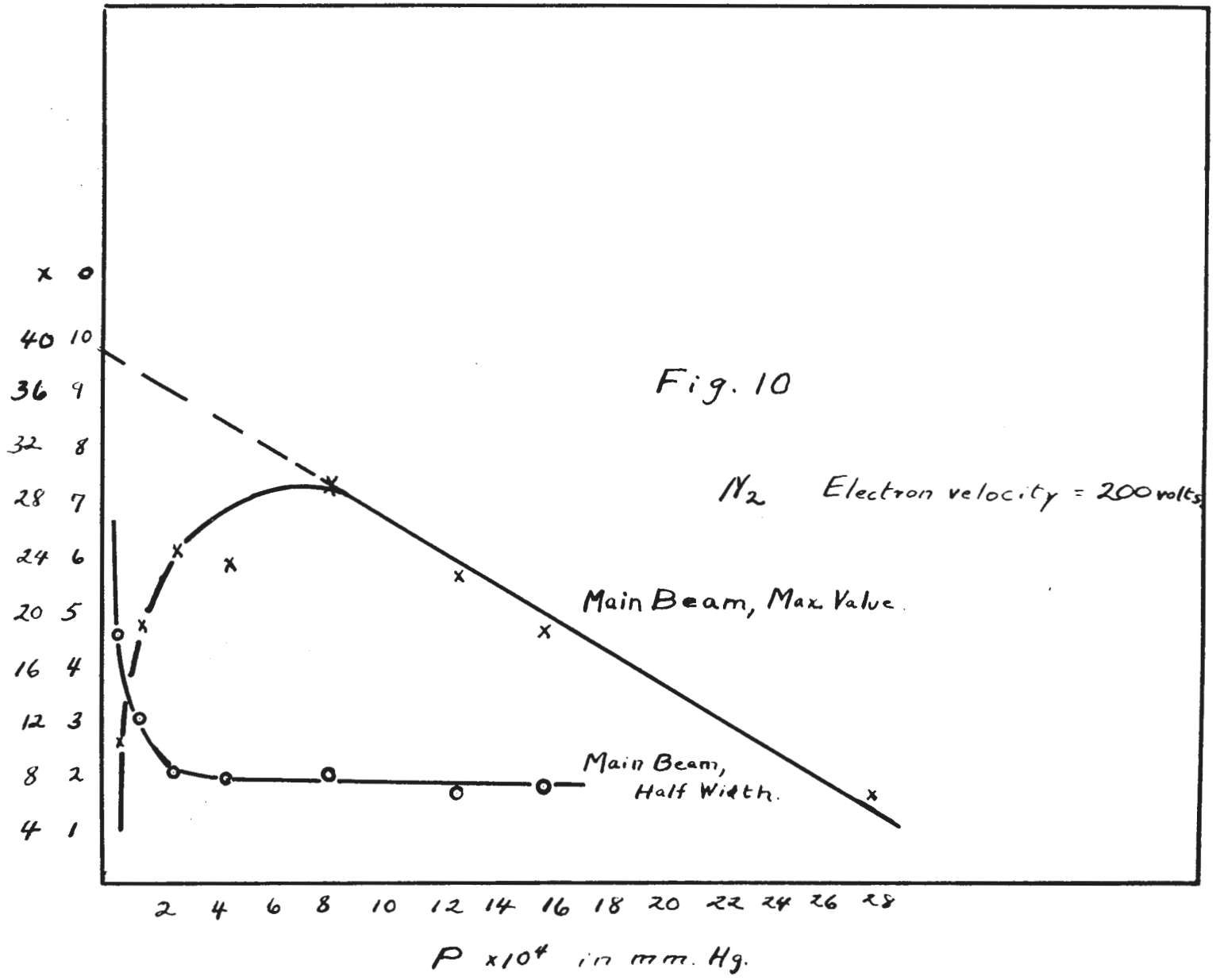


Fig. 10

N_2 Electron velocity = 200 volts

down, and become more intense. An example of such behavior is given in Fig. 9 . The change of shape with increase of pressure is seen to be a concentration within a narrower range, of the current which before was spread out. In mercury the maximum intensity in gas could be as much as five or six times that in vacuum. Fig. 10 shows the relationship of the half width of the beam to pressure in a typical case in nitrogen. It is seen that here the half width remains constant after a definite pressure is reached.

The phenomenon depends on the intensity of the beam, the velocity of the electrons, the pressure and nature of the gas. Some evidence was found that for a definite gas pressure and electron velocity, the focusing would increase very slowly up to a certain value of main beam current, and then rise rapidly, almost as if focusing had set in at a definite threshhld value of the current in the beam. In general, for one definite gas pressure, and value of main beam current, there would be a small range of electron velocity in which the focusing was more pronounced than for other velocities. Fig. 10 shows the general relationship between the value of the maximum current (intercepted out of the main beam by the slits, as the beam is moved across them) and the pressure of the gas. The straight line portion gives the decrease of maximum intensity with increase of pressure which is due to the increase of the number of scattering centers in the path of the beam. From the slope of the straight line portion, and the intercept on the axis of ordinates of this portion extended, it is possible to obtain the absorption coefficient of the gas to electrons of the given velocity, if one knows the pressure of the gas. Assuming the gas pressure to be that shown on

the McLeod gauge, we obtain a value of the absorption coefficient of 15.5, whereas Robinson (47) obtains the value 13.0 for two hundred volt electrons in nitrogen. The agreement is excellent, considering the spread of his individual results, the method of measuring pressure here, and the fact that the absorption coefficient in nitrogen changes from 3.9 to 15.1 in the range of velocities of 660 to 165 volts. Referring again to Fig. it is seen that the straight line portion does not continue to zero pressure, but suffers a drop off at low pressures. This is due to the lack of focussing effect at these low pressures, and the consequent spreading of the beam, which decreases the beam density, and therefore also the maximum value of the current which can be read.

The focussing effect has been noticed by others. In cathode ray oscillegraph tubes, it has been taken advantage of to sharpen the beam. Also it explains why an electron beam bent magnetically will remain a sharp beam. Seyfarth (48) has contributed one paragraph to a previous article by Voges (49) in which it is stated that the important factors in determining the shape of a beam are the space charge determining factors, not the defining slits. Brüche has done a little work on similar lines, and a reference is given in the article by Voges. The explanation offered, in general, is that in the beam there exists a plasma of positive ions resulting from impacts, and electrons from the beam and from impacts, which creates a radial field about the beam tending to draw the electrons toward the center of the beam. This has been discussed by Ranzi (44) (in a paper not obtained). The work by Arnot (34) has treated this positive ion plasma, and he has found a positive ion current going out in all directions from the beam, with a maximum in a

direction at right angles to the beam. No such maximum was found in the present experiments, the positive current being thoroughly random. One possible explanation is the lower gas pressures used in this work, and the fact that the apparatus is of metal, thus reducing stray fields.

The variation of current density and therefore of measured intensity across the beam would cause a noticeable effect on the variation of the measured absorption coefficient of a gas for slow electrons, as one varies the slit width of the Faraday cage receiving the electrons. (For work on variation of absorption coefficient with slit width see (45) in which Palmer obtains consistent results, and shows that Green's random results are not trustworthy).

Retarding Potential Curves.

Fig. 3α is a typical curve showing the variation of main beam current collected with the variation of the retarding potential applied to the collector. The flatness and slight rise of the curve as the retarding potential is decreased, shows that the method chosen to apply the retarding potential eliminated the effect of reflection of electrons, and of the ejection of secondaries from the collector. The rise at very low negative and all positive retarding potentials corresponds to a gathering up of the low velocity electrons in the beam, and those ejected from slits. The distribution in space of the very slow electrons ejected from atoms by the beam has been studied by McMillen (13) with interesting resulting patterns. The positive retarding potentials in general show a trend toward the horizontal, as all positives are repelled, and all available negatives are being collected. The width of the drop off of the curve at the retarding potential corresponding to the beam velocity, agrees well with the known spread of velocities in the beam due to voltage drop along the filament.

Suggestions for a new Apparatus

The following points should be considered in building a new apparatus:

1. Pressure control and measurement

The apparatus should be built of some material which can be outgassed. Wax joints should be eliminated. Either a static pressure of the gas with a McLeod gauge, or preferably an ionization gauge, with either static or constant flow method should be used. For the constant flow method, a capillary will be found quite satisfactory if one of the smallest that can be drawn is used. This will allow the desired pressure to be obtained with even a half atmosphere pressure behind the capillary.

2. Dimensions of Apparatus.

The use of low pressures and large dimensions is advantageous. Large dimensions make possible a rotating collector, instead of a rotating electron gun, thus allowing an analysis and measurement of the main beam by a fixed collector. Moreover, no trouble is experienced from the main beam striking on new parts of the chamber at each new angular setting. A rotating main beam collector and gun would be complicated. However, the apparatus should be designed so that gas pressures somewhat greater than in the present apparatus may be used, in order to have a greater scattered intensity. These two considerations involve a compromise.

3. Electron gun.

A source of electrons should be used which would give a beam a very nearly homogeneous velocity. A heater type source at the center of a cylinder with a small slit parallel to the axis would be good if not too complicated. The gun should be mounted with a sylphon mounting so as to allow it to be adjusted parallel to the slits while there is a vacuum in the apparatus.

4. Collectors.

These should be made long and deep so as to minimize reflection of electrons received. The length of the cage should be at least ten times the slit opening. Other factors are discussed by Skinner and Piper (46).

5. Retarding potentials.

These should be arranged to do away with reflection of received electrons, and ejection of secondaries. This will involve the use of two or three potentials as discussed elsewhere in this thesis. An alternative would be the means of electrostatic velocity analysis used by McMillen. However, there the focusing of the electrons of one velocity is only a first order focusing (See Voges (49) for photos of the focusing).

6. Electrometer.

Should be as close as possible to the scattered current collector. It should be preferable to use a more sensitive and dependable instrument than an shunted electrometer. (G. E. tubes?).

7. Insulation of collectors and retarding slits and grids.

This should be of quartz or amber. Redmanol or other similar materials have too low a resistance.

* * *

Summary.

There has been investigated the angular distribution of scattered intensity of electrons scattered in mercury and in nitrogen, and its dependence on velocity.

(1)

The scattering in mercury shows a definite minimum whose position is a function (approximating an inverse cotangent function) of the velocity, in the range studied.

(2)

The scattering in nitrogen is montonic except for a minimum for the lower velocity electrons, which occurs in the region around 90°. This angle of minimum scattering decreases with increase of electron velocity monotonically.

(3)

Scattering at small angles in mercury is best explained by the Langmuir empirical expression.

(4)

The focussing effect is pronounced in mercury, and exists in nitrogen. A qualitative investigation of its dependence on pressure is given.

The author takes this opportunity to express his appreciation and indebtedness to Prof. R. A. Millikan for his interest in the work, and his valuable advice and encouragement. Also thanks are rendered to Mr. Julius Pearson, and Mr. William Clancy for their technical skill and the assistance which they so kindly contributed.

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APPENDICES

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APPENDIX I

A. Derivation of the Mott Scattering Equation from the Born Formula.

According to Born's Formula, the scattered intensity per unit solid angle is

$$f(\theta) = \left| \frac{2\pi me}{h^2} \int V e^{ik(\bar{n}_0 - \bar{n}_1)\cdot \bar{r}} \, d\tau \right|^2 \qquad (1)$$

where V is the potential function of the scatterer. For radial fields, $V = V(r)$. With axes chosen as in Fig. 1, the z axis being oriented in the direction of the incident beam,

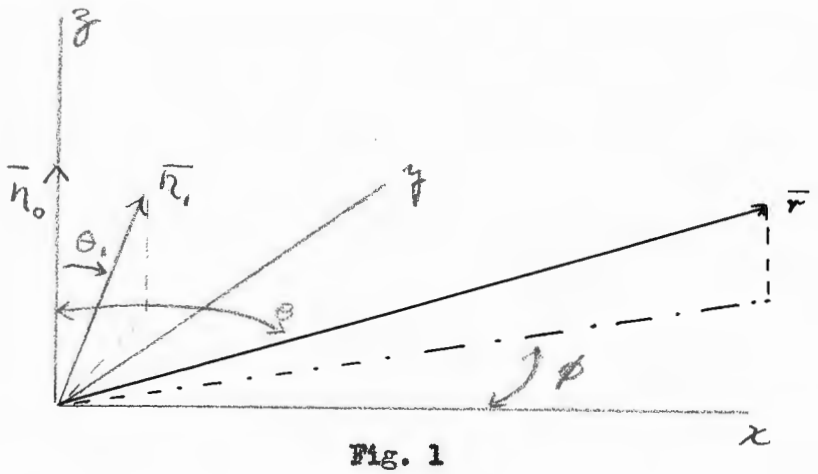

Fig. 1

(1) becomes

$$f(\theta) = \left| \frac{2\pi me}{h^2} A \right|^2 \qquad (1)a$$

with

$$A = \int_{r,\theta,\phi} V(r) \, e^{ikr[\cos\theta - \sin\theta_1 \sin\theta_2 \cos\phi - \cos\theta_1 \cos\theta]} \times r^2 \sin\theta \, dr \, d\theta \, d\phi \qquad (1)b$$

$$= \int_r V(r)\, r^2\, dr \int_\theta e^{ikr[\cos\theta(1-\cos\theta_1)]} \sin\theta\, d\theta \cdot$$

$$\int_0^{2\pi} e^{-ikr\sin\theta_1 \sin\theta\cos\phi}\, d\phi \qquad (2)$$

$$= 2\pi \int_{r=0}^{\infty} V(r)\, r^2\, dr \int_{\theta=0}^{\pi} e^{ikr(1-\cos\theta_1)\cos\theta} \cdot$$

$$J_0\left[\pm(kr\sin\theta_1)\sin\theta\right] \cdot \sin\theta\, d\theta \qquad (3)$$

In Gegenbauer's Integral (See Watson, Bessel Fns.), set $\nu = \frac{1}{2}$, $r = 0$, and apply to (3)

$$= (2\pi)^{3/2} \int_0^\infty r^2 V(r)\, dr \frac{1}{\sqrt{2kr\sin\frac{\theta_1}{2}}} J_{\frac{1}{2}}\left(2kr\sin\tfrac{\theta_1}{2}\right)$$

$$= \sqrt{\frac{8\pi^3}{u}} \int_0^\infty r^{\frac{3}{2}} V(r)\, J_{\frac{1}{2}}(ur)\, dr \qquad (4)$$

where $u = 2k\sin\frac{\theta_1}{2}$

$$\therefore\ f(\theta) = \frac{4\pi^2 m^2 e^2}{h^4} \frac{8\pi^3}{u} \left| \int_0^\infty r^{\frac{3}{2}} V(r)\, J_{\frac{1}{2}}(ur)\, dr \right|^2 \qquad (5)$$

now $J_{\frac{1}{2}}(z) = \left(\frac{2}{\pi z}\right)^{\frac{1}{2}} \sin(z)$

$$\therefore \quad f(\theta) = \frac{4\pi^2 m^2 e^2}{h^4} \frac{16\pi^4}{\pi^2 u^2} \left| \int_0^\infty r\, V(r) \sin(ur)\, dr \right|^2$$

$$k^2 = \frac{8\pi^2 m}{h^2} \left(\tfrac{1}{2} m v^2\right)$$

$$\therefore \quad u = 2k \sin\frac{\theta}{2} = \frac{4\pi m v}{h} \sin\frac{\theta}{2}$$

$$\therefore \left[f(\theta)\right]^{1/2} = \frac{2\pi e}{h v \sin\frac{\theta}{2}} \int_0^\infty V(r) \sin(ur)\, r\, dr \qquad (6)$$

which is Mott's expression. (We shall use (4) and (6) in Appendix II.)

B. Radially Symmetric Fields of the Type $V(r) = Cr^{-n}$

Using (5) and properties of Bessel functions, if

$$V(r) = Cr^{-n}$$

$$f(\theta) = \frac{32\pi^5 m^2 e^2}{h^4} \left[\frac{C\,\Gamma\left(\frac{3-n}{2}\right)}{2^{n-3/2}\,\Gamma(n)}\right]^2 u^{2n-6} \qquad (7)$$

$$3 > n > 1$$

$$f(\theta) = \frac{32\pi^5 m^2 e^2}{h^4} \left[\frac{C \sin\{(3-n)\pi\}\,\Gamma\left(\frac{3-n}{2}\right)}{2^{n-5/2}\,\Gamma(n)}\right]^2 u^{-n+1/2} \qquad (8)$$

$$n > 5$$

For the usual inverse square field, we have

$$V(r) = -\frac{Ze}{r} \qquad \therefore n = 1, \quad C = -Ze \tag{9a}$$

Although the equation (7) properly does not hold for n = 1, the substitution in it of the values in (9a) give the Rutherford relationship

$$f(\theta) = \frac{Z^2 e^4}{4m^2 v^4 \sin^4 \frac{\theta}{2}} \tag{9b}$$

This type of behavior of the potential $V = \frac{A}{r}$ is noticed also in Wentzel's wave mechanical treatment of scattering by this field, in which he obtains convergence of the integral by the factor e^{-ar}, later letting $a = 0$.

C. The Atomic Field Corresponding to any Observed Distribution of Intensity of Scattered Current.

By a Fourier Bessel expansion, it may be shown that with functions satisfying the usual conditions, if

$$f(x) = \int_0^\infty \lambda \phi(\lambda) J_n(x\lambda)\, d\lambda$$

then $$\phi(x) = \int_0^\infty \lambda f(\lambda) J_n(x\lambda)\, d\lambda \tag{10}$$

(see Gray, Mathews and MacRobert, "Bessel Fns." p. 243).

By (5) and since $u = 2k \sin\frac{\theta}{2}$, and $\therefore f(\theta) = I(u)$

$$\frac{h^2}{(2\pi)^{5/2} e m} \sqrt{u}\, I^{1/2}(u) = \int_0^\infty r\,(\sqrt{r}\, V(r))\, J_{1/2}(ur)\, dr \qquad (11)$$

and by (10) $$\sqrt{r}\, V(r) = \frac{h^2}{(2\pi)^{5/2} e m} \int_0^\infty u \left[\sqrt{u}\, I^{1/2}(u)\right] J_{1/2}(ur)\, du$$

$$\therefore \quad V(r) = \frac{h^2}{(2\pi)^{5/2} e m \sqrt{r}} \int_0^\infty u^{3/2} \sqrt{I(u)}\, J_{1/2}(ur)\, du \qquad (12)$$

This is an expression for the potential field of an atom which experimentally gives a scattering $f(\theta)$. The expression is valid if $f(\theta)$ turns out to be a function of u $\left(= f\left(w \sin\frac{\theta}{2}\right)\right)$, as happens approximately in many cases. The above may also be treated by the Fourier integral.

APPENDIX II.

A. The Atomic Field Corresponding to the Langmuir Empirical Expression for Electron Scattering.

Using (4) of App. I (A) with $V(r) = Ke^{-p^2r^2}$

$$f(\theta) = \frac{4\pi^2 m^2 e^2}{h^4} \cdot \frac{8\pi^3}{u} \left[\int_0^\infty r^{3/2} K e^{-p^2 r^2} J_{1/2}(ur)\, dr \right]^2 \qquad (1)$$

Using known properties of Bessel Functions, (1) becomes

$$f(\theta) = \frac{32\pi^5 m^2 e^2}{h^4} \cdot \frac{K}{u} \left[\frac{u^{1/2}}{(2p^2)^{3/2}} \right]^2 e^{-\frac{2u^2}{4p^2}}$$

$$= \frac{4\pi^5 m^2 e^2}{h^4 p^6} K e^{-\frac{u^2}{2p^2}} \qquad (2)$$

Now Langmuir's Empirical Formula, which holds good for angles up to 60°, is

$$I = I_0 e^{-\left(\frac{\theta}{\theta_0}\right)^2} \qquad (3)$$

in which I corresponds to $f(\theta)$, I_0 to $f(\theta)_{\theta=0}$

But (2) can be rewritten

$$I = I_0 e^{-K_1 \sin^2 \frac{\theta}{2}} \qquad (4)$$

$$\left(K_1 = \frac{8\pi^2 m^2 v^2}{h^2 p^2} \right)$$

And up to $\theta = 60°$, $\sin\frac{\theta}{2} \approx \frac{\theta}{2}$. (However, (4) gives slightly better agreement with experiment than Langmuir's expression (3). For example see Fig. 6 of this thesis.)

Comparing (4), (2), and (3), we find that for an identity

$$I_0 = \frac{4\pi^5 m^2 e^2}{h^4 p^4} K$$

$$\theta_0^2 = \frac{4h^2p^2}{8\pi^2 m^2 v^2} \quad \therefore \quad p^2 = \frac{2\pi^2 m^2 v^2}{h^2}\theta_0^2$$

Whence a good approximate expression for the potential of the atom as seen by the approaching electron is

$$V(r) = Ke^{-\frac{2\pi^2 m^2 v^2}{h^2}\theta_0^2 r^2} \qquad (5)$$

in which θ_0 is determined by scattering measurements, for velocities "v" in the range for which (3) or (4) hold. K may be determined as shown below.

B. Investigation of (5).

1. Since r may never be negative we see that (5) stands for a potential which increases continually with decrease of r. Therefore the applicability of this expression is limited to the range of velocities of impacting electrons which do not approach the nucleus too closely, i.e. slow electrons. Moreover the velocity range must be in the region of more than a few volts, to escape the Ramsauer effect.

2. Total Charge of Atom.

For a radial potential, $\nabla^2 V = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d}{dr} V\right)$.

Thus the expression for the total charge of the atom becomes

$$Q = \iiint_{r\,\theta\,\phi} \rho\, r^2 \sin\theta\, d\theta\, d\varphi\, dr = 4\pi \int_0^\infty \rho_{(r)}\, r^2 dr$$

$$= 4\pi \int_0^\infty \left[-\frac{1}{4\pi}\nabla^2 V_{(r)}\right] r^2 dr$$

$$= 4\pi \int_0^\infty -\frac{1}{4\pi} \cdot \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dV}{dr}\right) \cdot r^2 dr$$

$$= -r^2 \frac{dV}{dr}\Big|_0^\infty \qquad (6)$$

By (5) $V(r) = Ke^{-\rho^2 r^2}$ $\qquad \rho^2 = \frac{2\pi^2/\pi^2 v^2}{h^2}\,\theta_0^2 \qquad (7)$

∴ (6) becomes

$$Q = +K\rho^2 (2r^3 e^{-\rho^2 r^2})\Big|_0^\infty \qquad (8)$$

$$= 0.$$

Giving the correct value for total charge of the atom.

3. Since the total charge is zero, we shall investigate the distribution of charge, both positive and negative.

$$\nabla^2 V + 4\pi\rho = 0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = -4\pi\rho$$

$\therefore$ By (7)

$$\rho = \frac{Kp^4}{\pi}\left[\frac{3}{2p^2} - r^2\right] e^{-p^2r^2} \qquad (9)$$

Thus we find a positive charge density for all values of r less than $\sqrt{\frac{3}{2p^2}}$ i.e. $r < \frac{\sqrt{3}}{2}\frac{h}{\pi m v \theta_0}$,
and a negative charge density for values of

$$r > \sqrt{\frac{3}{2p^2}} \quad \text{i.e.} \quad r > \frac{\sqrt{3}}{2}\frac{h}{\pi m v \theta_0} \qquad (10)$$

4. To determine total positive charge,

$$Q_+ = 4\pi\int_{r=0}^{r=\sqrt{\frac{3}{2p^2}}} \frac{Kp^4}{\pi}\left[\frac{3}{2p^2} - r^2\right] e^{-p^2r^2}\, r^2\, dr$$

$$= 4Kp^4\left[\frac{3}{2p^2}\int_0^{\sqrt{\frac{3}{2p^2}}} x^2 e^{-p^2x^2}\, dx - \int_0^{\sqrt{\frac{3}{2p^2}}} x^4 e^{-p^2x^2}\, dx\right] \qquad (11)$$

To evaluate the integrals, let

$$J = \int_0^{\sqrt{\frac{3}{2p^2}}} e^{-p^2x^2}\, dx \qquad (12a)$$

then

$$\frac{dJ}{d(\rho^2)} = -\int_0^{\sqrt{\frac{3}{2\rho^2}}} x^2 e^{-\rho^2 x^2} dx - e^{-3/2}\sqrt{\tfrac{3}{2}}\left(\tfrac{1}{2}\cdot\tfrac{1}{\rho^3}\right)$$

$$\frac{d^2J}{d(\rho^2)^2} = \int_0^{\sqrt{\frac{3}{2\rho^2}}} x^4 e^{-\rho^2 x^2} dx + \left(\tfrac{3}{2}\right)^{3/2} e^{-3/2}\cdot\frac{1}{\rho^5}$$

$$\therefore \int_0^{\sqrt{\frac{3}{2\rho^2}}} x^2 e^{-\rho^2 x^2} dx = -\frac{dJ}{d(\rho^2)} - \sqrt{\tfrac{3}{2}}\cdot\tfrac{1}{2} e^{-3/2}\cdot\frac{1}{\rho^3}$$

and

$$\int_0^{\sqrt{\frac{3}{2\rho^2}}} x^4 e^{-\rho^2 x^2} dx = \frac{d^2J}{d(\rho^2)^2} - \left(\frac{3}{2e}\right)^{3/2}\cdot\frac{1}{\rho^5} \qquad \text{(12b)}$$

Now

$$J = \int_0^{\sqrt{\frac{3}{2\rho^2}}} e^{-\rho^2 x^2} dx$$

$$= \int_0^{\sqrt{\frac{3}{2}}} \frac{1}{\rho} e^{-y^2} dy \qquad (y = \rho x)$$

$$= \frac{A}{\rho} \qquad (A = \text{constant})$$

$$\therefore \int x^2 \ldots = +\frac{A}{2\rho^3} - \tfrac{1}{2}\sqrt{\tfrac{3}{2}}\,\frac{1}{\rho^3} \qquad \text{(13a)}$$

$$\int x^4 \ldots = +\frac{3A}{4\rho^5} - \left(\frac{3}{2e}\right)^{3/2}\frac{1}{\rho^5} \qquad \text{(13b)}$$

$\therefore$ by (11) the charge of the positive center portion (which, by (8) is equal to the negative charge outside) is

$$Q_+ = 4Kp^4\left[\frac{3}{4p^5}\left(A - \frac{\sqrt{3}}{2}e^{-3/2}\right) - \frac{1}{p^5}\left(\frac{3A}{4} - \left(\frac{3}{2}\right)^{3/2}e^{-3/2}\right)\right]$$

$$= 6\sqrt{3}\,e^{-3/2}\left(\frac{1}{\sqrt{2}} - \frac{1}{4}\right)\frac{K}{p} \qquad = 1.06\,\frac{K}{p} \tag{14a}$$

$$= 6\sqrt{3}\,e^{-3/2}\left(\frac{1}{\sqrt{2}} - \frac{1}{4}\right)K\,\frac{h}{2\pi}\cdot\frac{1}{\sqrt{e\,n_1}}\cdot\frac{1}{\theta_0\sqrt{\frac{U}{300}}} \tag{14b}$$

where e is charge of electron, and U volt velocity.

$$Z = \frac{Q_+}{e} = 61.24\,\frac{K}{\theta_0\sqrt{U}} \tag{14c}$$

where U is the number of volts accelerating potential.

K is a constant of the atom and may be determined if we know the screening, for one velocity. The fact that $(\theta_0^2 U)$ is found experimentally to be constant (approximately) for ranges of velocities considered here, shows that the screening remains approximately the same, for the velocities considered.

5. Force acting on electron (assuming that we may neglect retarded potentials, on account of our approximations, and the fact that v varies in range 0.01c to 0.03c).

$$F = -e\frac{dV}{dr} = 2Ke\,p^2 r\, e^{-p^2r^2}$$
$$= (2.18\times10^6)(\theta_0\sqrt{U})^3\left(\frac{Q_+}{e}\right) r\, e^{-p^2r^2} \qquad (15)$$

$F_{max.}$ is at $r = \frac{1}{\sqrt{2}\,p}$ (16a)

and $F_{max} = \frac{2}{1.06\sqrt{2e}}\, e^2\left(\frac{Q_+}{e}\right) p^2$

$$= (2.5\times10^{-2})\left(\frac{Q_+}{e}\right)(\theta_0\sqrt{U})^2 \qquad (16b)$$

6. Closest approach of electron to center of atom.

By conservation of energy, for the minimum "r",

$$e\,V(r)_{r=min.} = e\,U_{e.s.} \qquad (17)$$

$U_{e.s.}$ = volt velocity

$$\therefore\ Ke^{-p^2r_{min.}^2} = U_{e.s.}$$

$$r_{min.} = \frac{1}{p}\sqrt{\log\left(\frac{K}{U_{e.s.}}\right)} = \frac{1}{p}\sqrt{\log\frac{300K}{U}} \qquad (18)$$

This will be compared below with the expression for the closest approach to an inverse square center of force, namely

$$r_{min.} = \frac{Ze^2}{\frac{1}{2}mv^2} = \frac{Ze}{U_{e.s.}}$$

$$r_{min.} = \frac{Ze^2}{\frac{1}{2}mv^2} = \frac{Ze}{U_{e.s.}} \qquad (19)$$

C. Application of the above to Experiment.

1. Examination of the expressions developed shows that "p" and hence all the remaining expressions depends functionally upon $\theta_o\sqrt{U}$. Thus the rate of decay of the atomic potential and field of our representative atom, with increasing distance from the center, the positive charge included in the center kernel, the distribution of charge, the radius of the center kernel, all are determined by the value of $\theta_o\sqrt{U}$ (and of K). Now experimentally it is found that for any one gas, the product $\theta_o\sqrt{U}$ is constant.

The following is abstracted in part from K. T. Compton and I. Langmuir, Rev. of Mod. Physics, 2:233, 1930.

Gas	Volts	θ_o	$\theta_o^2 W$ (volts radians^2)
He	50	25°	9.5
	100	19°	11.0
Ne	75	21°	9.8
	100	19°	11.0
A	30	24°	5.3
	50	18°	5.2
	100	12°	4.7
	150	10°	4.5
H_2	100	15°	7.3
	250	9°	6.7
N_2	75	16°	5.9
	100	14°	5.8
Hg	30	17°	2.5
	50	11°	3.1
	100	10°	3.3
	250	6°	3.2

(In determining $\theta_o^2 W$, they evidently took into account the fractions of the degrees which were omitted in the table.)

For Arnot's results with 82 volt electrons in Hg, they found the curve fitted with $\theta_o = 11.3^\circ = 0.197$ radians. This gives $\theta_o^2 W = 3.17$

2. The Mercury Atom.

$$\text{We may take } \theta_o^2 W = 3 \text{ volts radian}^2 = 0.01 \text{ e.s. volt rad}^2 \tag{20}$$

Then

$$p = 0.62 \cdot 10^8 \tag{21}$$

Number of elementary charges in the positive kernel

$$Z = \frac{Q_{kk}}{-e} = 61.24 \frac{K}{\theta_o \sqrt{W}} = 34.2K \tag{22}$$

Now for this range of velocities, the shielding of the nucleus is such that we may take $Z = 60$

$$\therefore \quad K = 1.75 \tag{23}$$

giving

$$V(r) = 1.75\, e^{-0.4(10^8 r)^2} \text{ erg/el.} \tag{24}$$

for the approximate potential field seen by the approaching electron.

Radius of positive kernel

$$r_o = \sqrt{\frac{3}{2}}\, \frac{1}{p} = 19.8 \cdot 10^{-9} \text{ cm} \tag{25}$$

Distance of closest approach of oncoming electron (say 100 volt)

$$r_{min} = \frac{1}{p} \sqrt{\log \frac{300K}{W}} = 21.0 \cdot 10^{-9} \text{ cm} \tag{26a}$$

Distance of closest approach of same velocity electron to an inverse square center of force of same positive charge.

$$r_{\text{min.inv.sq.}} = \frac{300Ze}{v} = 86 \cdot 10^{-9} \text{ cm.} \qquad (26b)$$

Deceleration experienced by an electron just reaching r_o, at r_o, in a head on impact.

$$\frac{F_{r_o}}{m} = a \cong 3 \cdot 10^{27} \text{ cm/sec}^2 \qquad (27)$$

D. Discussion

The model obtained by the calculation from the Langmuir approximate scattering expression resembles the actual atom in consisting of a positively charged nucleus, surrounded by a negatively charged atmosphere. It may be looked upon as a smearing together of the actual charges, in such a way as to make calculations possible.

The dimensions of the model are incorrect by an order of magnitude, even when we consider the positive charge to have been smeared out over the space from the center through the first few electron shells. This is probably the most serious fault of the model.

The field is a good approximation for use in this type of work, in which the calculation of the actual field of so complicated an atom as the Hg atom would be impossible. The distance of closest approach of the electron to the atom is much less than for the electron under the influence of an inverse square field. This may be considered due to the spreading out of the charge; rather than its concentration in a point, as for example would be the case in the inverse square field.

Finally, I wish to state that the above is not offered as an actual atomic model, but has merely been investigated to determine what field the Langmuir approximate expression for the scattering at small angles (which agrees fairly well with experiment) indicates. The fact that experimentally $\theta_0\sqrt{t}$ is found to be constant makes the model useful.