SENIOR

THESIS

A NEW DESIGN AC-DC MEASURING INSTRUMENT

Electrical Engineering Department

Presented by

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INDEX

A New Design AC-DC Measuring Instrument

This paper will cover rather completely all work done in the development, both practical and theoretical, of this instrument. The consideration will be divided into the following classifications: basic principle, development, operating characteristics, theory, sources of error in the different types, directions in which development must be continued, and conclusion.

Basic Principle

The original idea was acquired from the study of the Kelvin current balance and De La Rive's battery. The Kelvin current balance is amply explained by illustration 1. These balances show rather low sensitivity in actual operation. They are rather likely to have high losses in the form of eddy currents when they are used on alternating currents. This principle was combined with the experimental stunt known as De La Rive's battery, which is shown in figure 2. It consists of a floating battery which has a coil of

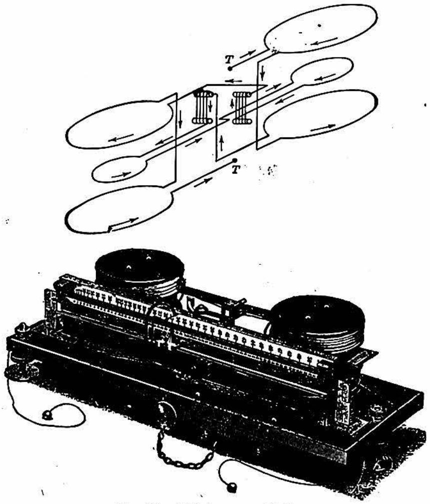


Fig. 40.—Kelvin current balance.

Figure 1.

wire connected to its plates. Bringing the magnet up to the coil will cause the coil either to encompass the magnet, or to turn around and encompass it. If one takes the stationary coils on the Kelvin balance, and replaces them with a ring of iron, to which magnetism is applied either externally or internally, one can cause the moveable coils to experience a turning couple if they are properly connected. This connection is such that the one coil experiences a force tending to carry it to the center of the piece of iron, and the other one a force trying to push itself off the iron. This is the basic principle underlying the instrument.

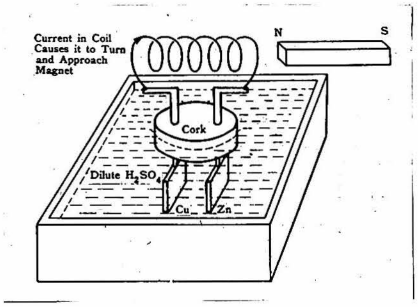


Figure 2

Development

All of the original development work was done with direct current models. The first method tried was the use of an external magnet to excite the iron ring on which the coils moved. This of course was inspired by the present D°Arsenval

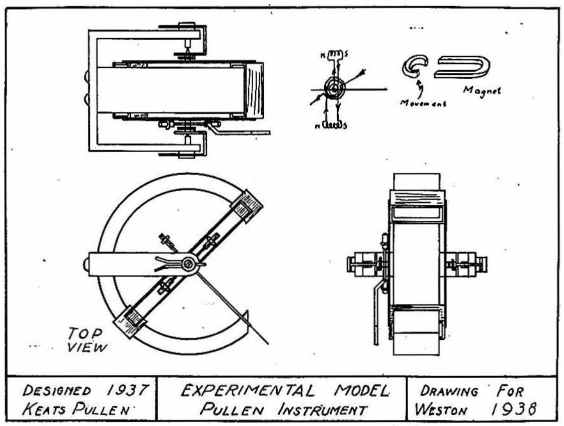


Figure 3

instruments. The general layout is shown in figure 3. The two coils are arranged as shown in the print, with a frame of brass wire carrying the pointer, pivots, springs, etc. The ring was of soft iron, being excited by an external magnet. It was found that the distance of the magnet from the faces of the ring was rather important. When the magnet was placed near to the ring, the sensitivity was rather low. If, on the other hand, the spacing were about three quarters of an inch on each pole, the sensitivity turned out to be rather high. The first set of coils fit closely to the ring. When a pair of coils having a considerably larger crossection was made and installed, the sensitivity was found to be very low compared to the value shown under the previous test. This of course meant that the flux was leaving the ring and turning back along it, thereby permitting the flux to escape passing through the coils in such a manner that they are acted on. Various types of flux concentrators were tested with the magnet to see if applying the flux over a very small area of the end of the ring segment would change sensitivity. It was found that this did not work. In all of these experiments, gravity return was resorted to. With three hundred ohms in the moving assembly, and this gravity return, sensitivity of about a hundred microamperes full scale was achieved. That was with the ~~air~~gap to magnet distance of three quarters of an inch. The magnet used in the experiments was rather weak, being of ordinary tungston steel, and being pretty thoroughly demagnetized at the time it was used. The flux density being used was probably not more than three or four gauss. This would lead one to suspect that fairly good sensitivity should be obtainable. When this form of the instrument was developed as far as was practical, the possibility of an internal magnet for the excitation of the instrument was considered. A schematic diagram of this particular model is shown in figure 4. A special magnet was designed by the author and made by Indiana Steel Products Company. A rather heavy moving assembly was built and installed. Springs used were strong enough to hold the period to about a second and a half. The moving assembly could have been built with about a quarter the weight it had, so the period could have been halved. After attempts made to use iron balancing screws resulted in

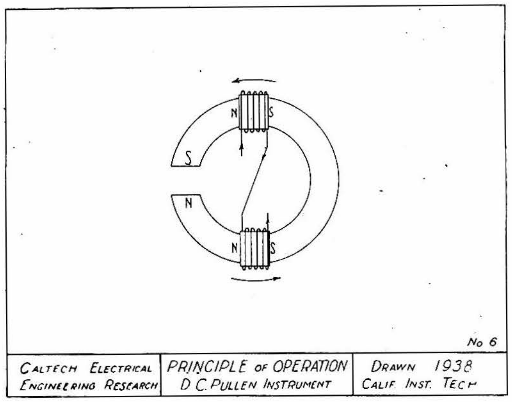


Figure 4.

large quantities of attraction being in evidence, brass 2-56x1/2 balancing screws over the moving coils were used. The actual structural assembly can be seen from the diagramatic and photographic views shown here. The method of assembly is very similar to that of the previous model. The addition of the magnet in place of the externally excited ring resulted in a very great increase of sensitivity. This is because the magnetic flux is being used much more efficiently. An even more efficient design has been evolved which makes use of two magnets, meeting in the center of the area on which the coil moves. They are mounted so that they have their magnetic fields opposing each other, with the result that here the flux crosses the center of the ring instead of going through the iron. The magnetic flux output along the magnet is approximately constant for

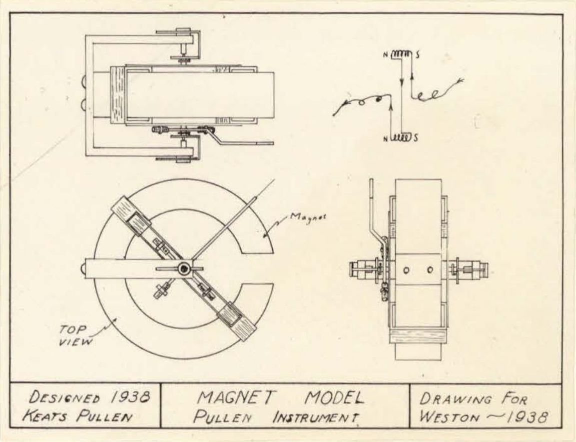


Figure 5.

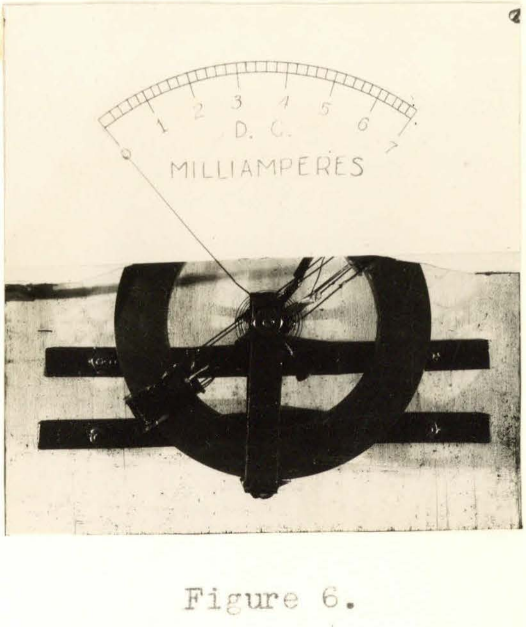


Figure 6.

different air gaps and paths, so that all the flux from the magnets, assuming they are balanced, will be useful flux. There is of course a demagnetizing effect on the magnets where they have long airgaps, but experiments indicate that with the choice of proper magnet materials, this can be held to a sufficiently small value. The model shown in the photograph went on a journey to Newark, New Jersey and back without indicating any appreciable change in sensitivity. Differences could not be checked exactly, as it was impossible to compare against the same meter as was used for calibrating. After this original model was built, the structure of the instrument was given careful consideration to see how it could be redesigned for more ruggedness. In the model shown the support for the meter itself was by way of the assembly carrying the pivots and springs. Obviously this would put the support assembly under strain with very good possibilities that it would get sprung in the course of use. As the magnet is the heaviest part of the thing, the logical thing to do is to place the supports on the magnet, and have it hold the rest. A much improved design is shown in figure 7. Actually, the

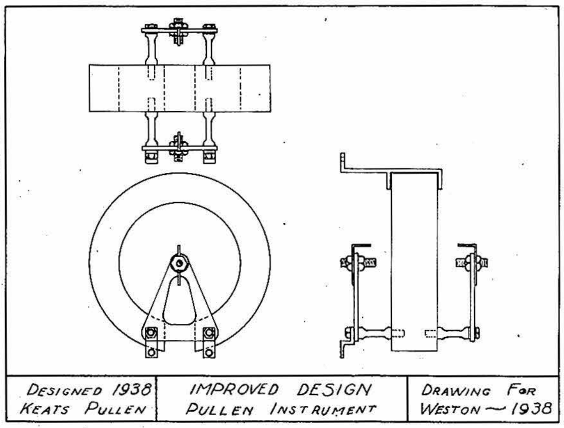


Figure 7.

support pins should go completely through the magnet for one to get the maximum of rigidity. Also the mounting brackets should be placed next the magnet. As further work on the magnet model would have necessitated large expenditures for experimental magnets, since the funds were not available, work with the DC magnet model was temporarily discontinued.

Naturally, the next application to come to mind was possible use on alternating currents. Since measurements are rather easily made on experimental designs for AC structures, it was decided that rather careful investigation of the effect of the many variables present in the instrument on the actual sensitivity should be carried out. A large quantity of ring stampings of a convenient size were acquired for this purpose. After the edges of the rings had been smoothed off to help cut down the eddy current losses, a number of stacks of these rings were given different lengths of airgaps, so the effect of this length on the sensitivity and scale distribution could be determined. The actual method of making these determinations was as follows: A clamping devide was built which would hold the various rings in a secure position, while an alternating current of about a half an ampere was passed through a fifteen or thirty turn coil wound opposite the airgap. In the center of the ring was a round ~~wooden~~ block of wood, so mounted that it could be turned. On this block was mounted a coil of fine wire which faced the ring surface. As a result, the magnetic flux leaking from the ring would generate a voltage in this coil. At the airgap itself, of course, the coil showed no voltage, as the voltage generated in each side of the coil opposed that in the other side. A diagram of this device will be included in the appendix. The voltage generated across the coil was led out to a calibrated vacuum tube voltmeter. Information on this also will be in the appendix. It consists of a two stage amplifier driving a detector stage with a one milliampere meter for the actual measurements. The magnitude of voltages measured was from about three to ten or twenty thousandths of a volt. An ammeter was used in series with the excitor coil so that the conditions from one ring to another could be duplicated. The airgaps tried in the samples ran from a quarter of an inch to an inch and a quarter. Not only was the flux on these samples measured, but also the voltage drop across the excitor coil. Naturally, the best conditions for operation are those which show a maximum flux as measured by the coil with a minimum voltage drop in the excitor. Therefore this was the condition which was sought. It took considerable time to get the equipment operating properly, but when this was done, the following results were obtained. The most uniform distribution of flux occurred, as might be expected, with the largest airgap. Also the smallest voltage drop occurred under this condition. Still, however, considerable flux was known to be passing across the airgap itself. As this flux was worthless as fas as the operation of the instrument was concerned, either the complete removal of the flux passing across there or minimizing of it was necessary. There was but one way to do this, namely, place a coil at that location so connected that it would be opposed to the flux trying to cross, thereby, if the coil is of the right size, preventing that flux from crossing. This caused all flux to pass across the ring. When this change was introduced, the ring was again closed for the purposes of symmetry. Since the inductance of the coil is dependent on the flux flowing through the coil, the inductance of the combination was greatly reduced by the double coil assembly compared to the single for the same flux. A schematic diagram of both the single and double coil combinations can be seen in figures 8 and 9 respectively. Flux distribution curves for three typical rings will also be found in the appendix. They are for the same total ampere turns excitation. Approximate inductances will be found stated on the graphs. The design of the actual instruments themselves is very similar to the improved DC construction with the addition of excitor coils.

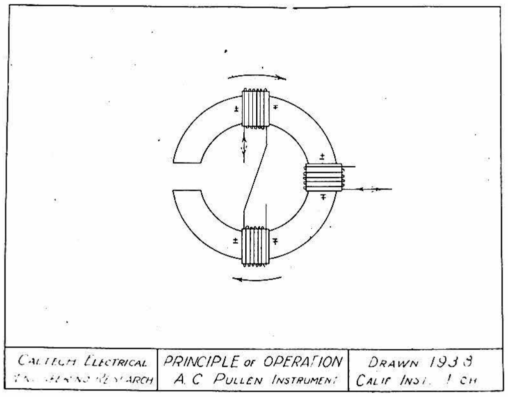


Figure 8.

The double coil instrument was the first instuemtn built on the revised pattern, with the result that some difficulties were encountered in working out satisfactory assembly methods. A drawing of the assembly for the single coil model is shown in figure 10. The only difference with the models built is the fact that the second coil is added and the ring closed. All the rings are cut into two segments, most of them half way between the two stationary coils. A few are cut under the stationary coils so as to hold the whole assembly together.

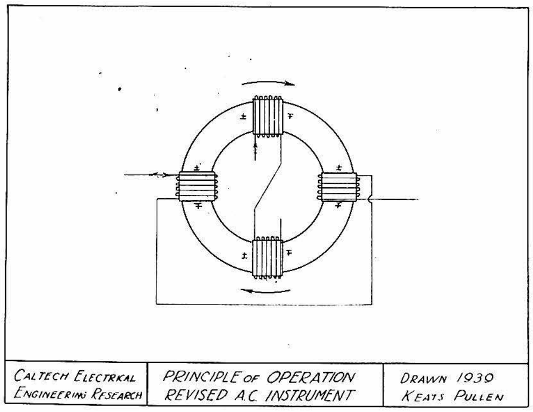


Figure 9.

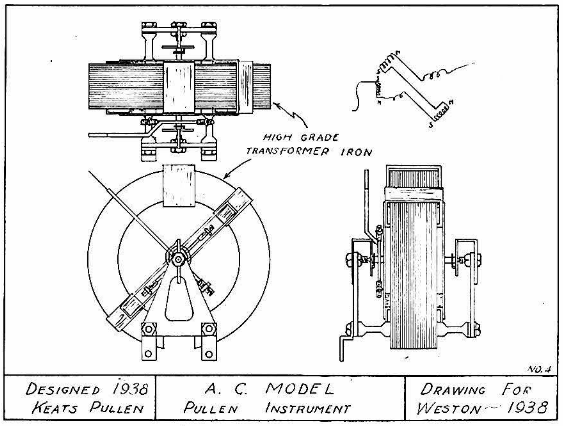


Figure 10.

Commercial assembly methods would be somewhat different. The cuts in the rings are placed half way between the fixed coils, as theoretically the flux through the iron is zero at that point anyway, so the field distribution will not be changed. A clamping ring would hold the sections together. The number of turns of wire on the moving assembly should be as near to equal to those on the stationary as possible, as that is the condition for minimum voltage drop due to inductance. Of course, the resistance of the moving assembly will increase greatly on doing that. The biggest drawback to having coil ampere turns the same is the effect of stray fields. My present guess on the best relationship, taking into account these effects, is that the ratio should be about five stationary to one moving turn. Probably careful investigation should permit setting up of relations which should enable one to choose the best all around ratio. Neither equipment nor time have been available to carry out this investigation, however.

In none of the early models was a brace put in the center at the pivots. However, it was found advisable to do this, so that the instrument will retain its balance in all positions. It was not found necessary to stiffen up the crossframes to prevent other than twisting movement against the springs. The mass being carried by the crossframes is easily within their elastic limit. They probably could carry much more load without exceeding it. The method developed for assembly of these experimental models is as follows: the rings of transformer iron were first drilled for assembly. Then they were cut into the sections as described previously for the assembly. The support structure was made, it looking much like that shown in the improved design. The coils were wound up, and placed on the iron. The set of holding rings, which were cut under the stationary coil, were bolted on the main iron in such a way that each section of the main body had these holding rings on one end. The coils were placed out of the way, and the two sections pressed together. After a little adjustment, the support assemblies were placed in their proper holes. Then the coils were braced at the center of the ring, and the cross assemblies, carrying pointer, springs, and related equipment, were cemented on and allowed to dry. Then the assembly was adjusted to clearance, connected up, balanced, and the springs connected. Commercial production would require a number of changes over this method, including assembly of the moving structure before putting the whole thing together. No difficulties would be encountered if care is taken to permit the coils to be set up in proper order and then drop a clamping device in place which will permit easy assembly and dissembly. Satisfactory methods for doing this on large scale have been developed.

Operating Characteristics

In this section the various externally noticeable characteristics will be discussed, and compared with those of commercial types of instruments. By doing so one can get an idea about what the instrument at its present state of development will do and also something of an idea as to what can be expected from it.

The DC model which was built is a decidedly high inertia job. That is accountable when one notices the mistakes made in making the assembly as it is. The weight of the couls is excessive due to the celluloid coil form used in the instrument. The mass of the balancing assembly is too great, as is its inertia. However, despite these defects, it shows a sensitivity of seven milliamperes full scale deflection with an internal resistance of only two ohms by the Pullen precision Wheatestone bridge and a period of about a second and a half. This period can be cut to a fourth that value, according to results obtained on other instruments. The magnet used in the instrument is a specially made

36 percent cobalt steel magnet with a half inch airgap. The division of the scale for this instrument is almost perfectly uniform. However, proper arrangement will greatly alter this same distribution to any form one desires. Damping is also needed in this instrument, as it is far from critically damped. That is easily acquired for the DC model, however, by use of light damping frames of metal for the coils. The instrument has a maximum of torque production for a given inertia. In as much as only supersensitive lines of instruments can beat this construction on sensitivity, and they are in general instruments having a much longer period, this instrument compares very favorably with the commercial types of DC meters. An ordinary D°Arsenval ten milliampere milliammeter has about five ohms resistance, as compared to the seven milliamperes and two ohms of this one.

The AC milliammeter looks extremely interesting when compared to commercial AC instruments. The original model showed a full scale sensitivity of forty seven milliamperes with a voltage drop of only about 2.3 volts. Ordinary instruments with this range drop around thirteen volts. The period in this case was about a half second, which compares rather favorably with that of the instrument being calibrated against. It had a period of between a quarter and a half second. That, of course, means that the springs on the new design were rather strong. The model tested had no damping. This ould be best supplied by use of an aluminum vane in the space between the coils. Properly made, it would offer sufficient damping and would not noticeably increase the period of the instrument. As the scale distributes itself now on the instrument, there is a tendency at high readings toward increasing sensitivity with increasing deflction. As is obvious, if one wishes, he could under these conditions build a meter whose torque increased as the spring return torque increased, resulting in a meter which would read a certain current anywhere on the scale. For best results, a plot of movement torque and spring torque as a function of deflection in radians should have the two curves as nearly at right angles as possible, as this condition is the condition for best reliability and accuracy. Suppression of the high end of the scale so that the curves will be more nearly normal is therefore advisable. Reduction of sensitivity at high currents and increase of sensitivity at low currents would even out the scale distribution considerably. An improvement of characteristics would result quite naturally from such a change, as the voltmeters and ammeters follow a square scale. Proper placement of coils will also help this.

In the DC instrument, the equivalent circuit is the same as that of the D°Arsenval instrument, namely a resistance in series with an inductance and with a parallel circuit of inductance[spring] and capacitance[inertia] in series with that combination. On AC, the circuit consists of two inductance and resistance combinations in series with mutual coupling between the inductances, and two additional parallel circuits of a resistor and inductance in series with the rest of the reactances. Mutual inductance will exist between the pure inductive components, and between the magnetizing inductive components. The dynamometer meter has this circuit without the magnetizing components. Therefore the characteristics of the two instruments are very similar.

Theory

The basic equation by which the operation of this instrument can be most easily studied is $d\overline{F} = i\ d\overline{l} \times \overline{H}$. From this, as derived in the appendix, we find that $\theta = ki\ B/T$. Here k and T are constants depending upon the parts measurements. B is a variable in θ, so can be expanded in a Fourier series if this form would prove convenient. It would consist of a function of sine terms. This would give a rather convenient means of handling computations of sensitivity. The derivation given makes several assumptions, namely, that the flux leaves the magnet normal to the surface of the magnet, and that the conductors are near enough to the surface of the magnet that the flux is still normal, or nearly so, at that distance. These assumptions are justifieed, as the permeability of the iron in the magnet is so great compared to the air surrounding it that the flux is in effect leaving an almost perfect magnetic conductor. Careful construction will assure the coils being close enough to the flux conductor that the flux is still for all practical purposes normal to the conductor. Precise determinations of the allowable spacing between the iron and the wire of the coils has not yet been made. The results of the preliminary experiment are the major source of information on this source.

A development was undertaken to determine the effect of the reduction of the size of the instrument ot half the present model size on the sensitivity of the instrument. These calculations indicated that for a constant exciting mmf, and a constant period and resistance in the movement, the sensitivity of the instrument is increased as $1/r^3$. T his of course means that reduction in size of the instrument will definitely increase its sensitivity. In an actual milliammeter, one would achieve an increase of sensitivity of about three to one overall, if the instrument is of the AC type, or four to one in the DC type, for a reduction to half size. That is, reduction to a tenth present size, if it were practical, would yield a one milliampere AC milliammeter. The reduction in size to half that of present models actually lowers the inductance of the meter to about half that of the larger size. The relative amount of iron loss will be the same percentage, as the volume of the iron drops to an eighth of the larger size.

As one notices from the equations of the instrument, the sensitivity depends upon the flux leaking from the ring. Terefore the distribution of the scale can be easily changed by merely changing the amount of this leakage flux. As the flux will leak most rapidly where the flux density is the greatest, all that is necessary to increase the flux at any point is the reduction of the area of the ring at that point. Precise data has not been collected on this, however, sensitivity at the low end of the scale can be made to take on the value desired, within limits, by cutting away part of the area of the ring at the point in question. In as much as the iron laminations are easily duplicable, and the flux density in the iron does not appreciably effect the accuracy, it is a simple matter to get accurate methods of changing the sensitivity of the instrument. The iron is best changed in such a manner that the outside surface remains close fitting to the coil. This can be done by taking iron out of the center of the lamination, or by trimming part of the iron along the edge on some of the laminations but leaving enough so that the flux will be held approximately as desired by its presence. Thus a very versatile instrument results, as scale modification is extremely easy to carry out and very easy to reproduce accurately.

The theory of the AC ammeter of this construction has one important consideration which is not present in the other forms of the instrument. Since the springs of an instrument will not carry heavy currents it is necessary to use means to eliminate heavy currents from the moving coils of the ammeter. The best method of doing this is by a shunt connection. In order for an instrument to be independent of frequency with this connection, it is necessary for the time constant (L/R) of the moving assembly to equal that of the stationary, assuming no reduction of torque due to increase of losses. In order to overcome a reduction in torque, the time constant of the moving assembly should be a little less, just enough to correct the error, than that of the stationary. This causes an increase in moving coil current without an appreciable change in stationary. Other than this, there is no theoretical difference from the other instruments.

It is possible to work out field distributions for the double magnet construction by methods given in Dr. Smythe°s Static and Dynamic Electricity. As time has not yet been found in which to carry out this operation, it has not yet been attacked. A simple integral, once set up, is all that has to be done in the evaluation of this problem.

Sources of Error

There are certain sources of error which are present in all instruments. These errors will be discussed here first under general errors, and then errors which effect primarily one given type of instrument.

General errors

There are several temperature errors which come under this classification. The first is the error due to temperature effects on the springs. The error introduced by the springs, according to Laws- Electrical

Measurements, is of the order of 0.03- 0.04 percent per degree Centigrade. This means that for a half percent instrument, the error over a range of ±10°C. will be less than the accuracy stated. It can be compensated for, if desired, by having a suitable resistor with a temperature coefficient which changes the resistance at such a rate as to counteract the error introduced by the springs. Another source is the expansion of the cross frames carrying the coils. The expansion coefficient (linear) for brass is 0.00002/°C or the total possible error is 0.04 percent for 20°C. This is much less than the accuracy set.

The magnetic flux errors are due to two main sources, namely changes in permeability and core loss. The permeability error would result from the change in this factor under different amounts of magnetic excitation. A calculation of the effect of this change is found in the appendix. Next we must consider core losses. At 30 kilolines per square inch, the loss in the iron is 0.16 watts per pound. Steinmetz formula if applied will give a value of loss at flux density being used which will be high, as it does not take into account eddy currents at B^2. [The exponent is taken as 1.6] Therefore we will use this equation to determine the maximum value of this loss approximately. The evaluation is given in the appendix.

The instrument will show errors due to stray fields unless some shielding is added. On DC, the error with no shielding on the AC model turned out to be a couple of percent. However, placing of a thing ring of low permeability iron surrounding a thin ring of high permeability iron to shield the moving assembly will cut this error down to a point that it is neblighble. This shielding will serve a second purpose, also. It will draw the flux out of the outside of the flux conducting ring, and can be arranged to cause additional flux to leak where it is desirable. The result is an increase in sensitivity and another means of modifying the scale distribution.

Voltmeter errors

Voltmeters are bothered by another temperature effect, namely change in resistance in windings due to change of temperature. Since the temperature coefficient of copper is 0.0038 ohms per ohm per degree Centigrade, for a temperature range of ±10°, the change in resistance is 3.8 percent. Therefore a s amping resistor having ten or more times the resistance of the copper will serve to hold this to a small value. Proper adjustment of the value of this resistor will counteract the error due to temperature effects on the springs, as the copper error is greater than the spring error, and in the opposite direction. Resistors will reduce the effect of the copper error, making possible a design which will be independent of temperature. This is general practice in instrument work at the present time. The sensitivity of the voltmeter should also be looked into. Using only a swamping resistor to hold the error due to temperature down, the maximum voltage drop allowable in the meter movement proper is ten volts for a 100 volt meter. Since a ten milliampere meter of this design drops about ten volts at full scale deflection, one can easily build a 100 ohm per volt meter in this range. [$V = W/i = 0.1/0.01 = 10$] Comparison with commercial AC voltmeters of this range is interesting. Dynamometer type meters show about 25 ohms per volt. Repulsion iron about fifteen to thirty-five, depending on acuracy of the instrument.

Wattmeter errors

We now consider the errors in the wattmeter type of construction. With a seven ampere current coil, consisting of five turns of number eight wire per coil, the current coil consumption of power is about 0.15 volt amperes. The potential coils take only 0.006 amperes at 150 volts. Z_c of the moving, or potential, coils is about fifty ohms. Therefore the potential error due to temperature is about 0.006 percent. For the phase angle error, the equations giving this are derived in the appendix. These equations give the relation between the pahse angle error, maximum error resulting in the meter, and the power factor at which this error occurs. There is also data showing that the phase angle error in the current coils will compensate for the time constant error. By taking proper values from the graph of maximum error data, one can design a wattmeter for any particular power factor range with any stated maximum error.

DC model errors.

Besides spring and wire temperature errors, the only error which has to be taken into account on the DC model is the matter of magnet stability. This is a subject on which quite a bit of research will have to be done, studying the different possible magnet materials, and putting them through very rigorous tests for stability under hard useage.

Further Development

There are many directions in which further development must be undertaken. This section will take up the different forms of the instrument and discuss the work yet to be done on them.

DC instruments

Models of the double magnet DC type of the instrument will have to be built and tested. The design which will give the best magnet permanency has to be found. Finding these proportions for the maximum sensitivity and stability is a job which will take a great deal of investigation. When this design is found, the magnet will have to be given very extensive tests to find if its stability is good enough for accurate standards, portable standards, switchboard meters, etc. It will be necessary to collect all possible data on this as by doing so only can it be found out whether the structure will have advantages sufficient to warrant making a line of commercial instruments.

AC instruments.

The major things to be done with this form are the investigation of frequency errors, ~~and~~ to look into the results of scale modification, and to investigate the smaller instrument size. Also the AC ammeter has to be experimented with.

As might be expected, the investigation of scale modification must be done in way of check practically of the ideas presented in the theory section of this subject. Tests should be carried out to check the methods, see if bending of the pointer of an instrument will introduce sizeable error, and to find out the errors incurred in present AC instruments from bending the pointer and resetting to zero. On the straight model, without any scale juggling, the error is small, due to uniformity of the scale. As soon as one changes the scale distribution from uniform, however, then errors will result when this change is made. As this is an important question, it will have to be considered carefully.

An investigation of frequency errors was started by the author, but had to be given up when the instrument being used for the tests was burned out by shorting across 110 volts. The results indicated were that the frequency error at 500 cycles was of the order of one to two percent, at 1000 cycles, about three to four percent, and at 20,000 cycles about forty percent. The exactness of these percentages can not be guaranteed, as an accurate calibration had not been placed on the instrument, estimates being made on the basis of deflection and the known fact that the instrument had a square scale, rather than uniform. These investigations will have to be continued and made completely quantitative, rather than semi-quantitative.

The characteristics of a miniature line of this type of instrument will have to be studied. Calculations, as stated, indicate a great advantage in the smaller size. Practical experiment will have to be used to either confirm or refute this. I suspect confirmation will result. However, small models suitable for ordinary use in radio and other purposes can easily be built if this does check, so it is very much worth while to carry on a careful check of this possibility.

The possibilities of construction of an AC ammeter have to be reckoned with, also. No experimental models of this type of instrument were built, for reasons to be disclosed. As has been stated, the instrument has to be one of the shunted dynamometer type of construction. Calculations of a five ampere ammeter were carried out. It was found that such an instrument could be built which would consume 0.25 volt amperes and would have the same time constant in the movina dnd stationary assemblies. One hundred milliamperes would have to pass through the moving assembly. Difficulties were encountered in design of this type of instrument in keeping the voltage drop in the moving assembly low enough to get a low power consumption and at the same time keep the period of the moving assembly low. Since the designing was difficult, it was decided not to build such an instrument for the time being.

Low power factor accuracy of the wattmeter type of instrument should be checked. As no really satisfactory method of checking this was known to be available when tests were made, only qualitative agreement has been found here. A phase shifting transformer or its equivalent should be used for this check, as one can by that means get the equivalent of zero power factor quite easily and accurately.

There are probably new types of instruments which can be built using the same design as this instrument. They must be worked out, and more checking done on the present designs to see how they can be further improved in ruggedness, sensitivity, and operating characteristics.

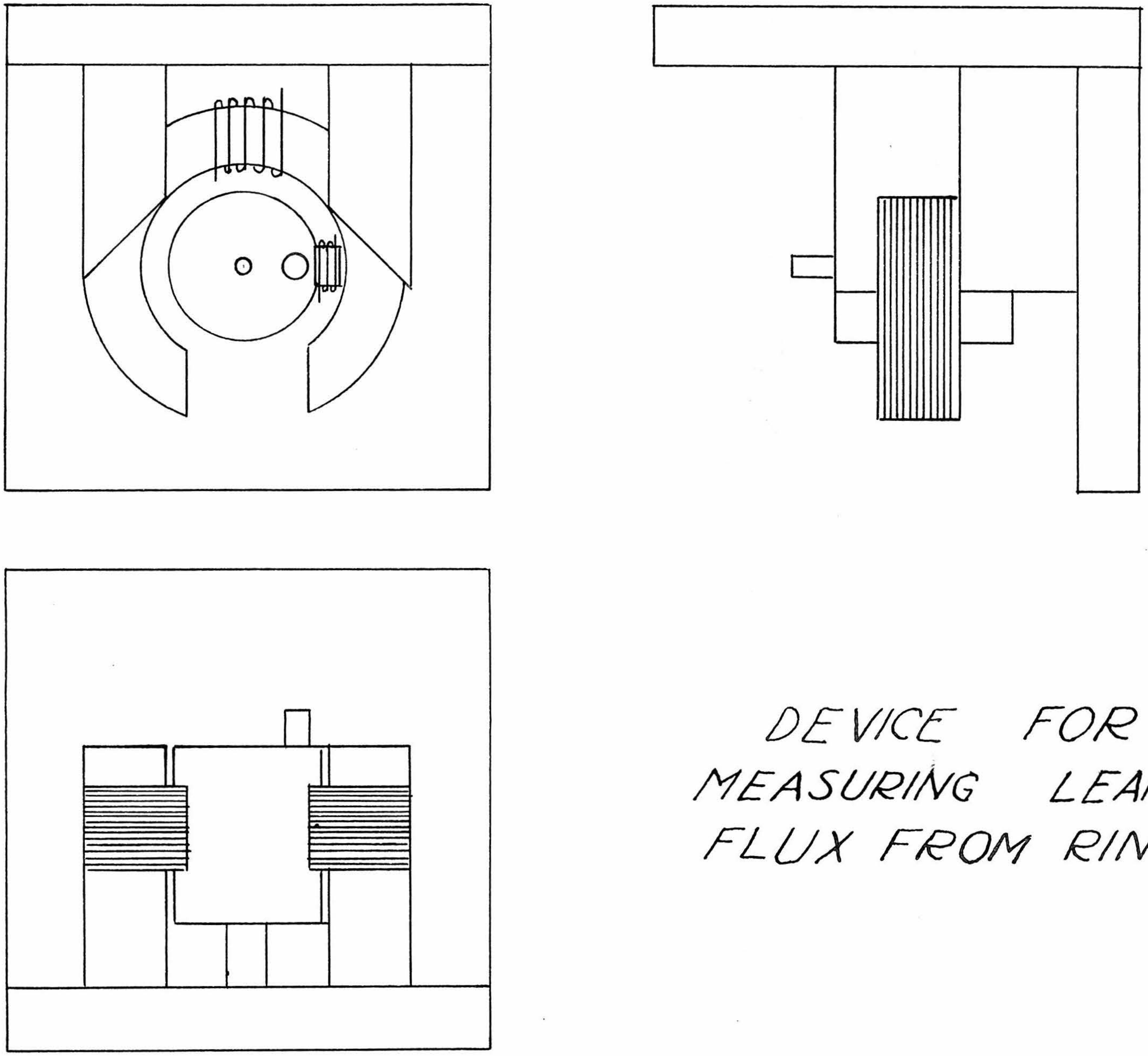
Conclusion

Here is an instrument which shows a definite improvement over the sensitivities of the instruments available in the market today. It has ruggedness, high torque, and sensitivity. When the development of the construction has been carried to a point that production is practical, the results reported here may even farther surpass the present designs. Even at the present time, however, the results represent a definite advancement in the art of measurement of AC voltages and currents, especially when these voltages and currents are small. Often measurements are difficult owing to the fact that present instruments under some conditions will radically alter the conditions in a circuit when they are inserted. This instrument shows much lower insertion error by way of the fact that it cuts power consumption to about a fifth that of existing instruments. This instrument therefore opens new fields for the measurement of those AC voltages and currents which have been rather difficult to handle with existing instruments. One finds in this instrument, therefore, a measuring devide which has versatility and flexibility, being accurate on both AC and DC, and which shows promise of great advances in the art of measuring of electrical quantities.

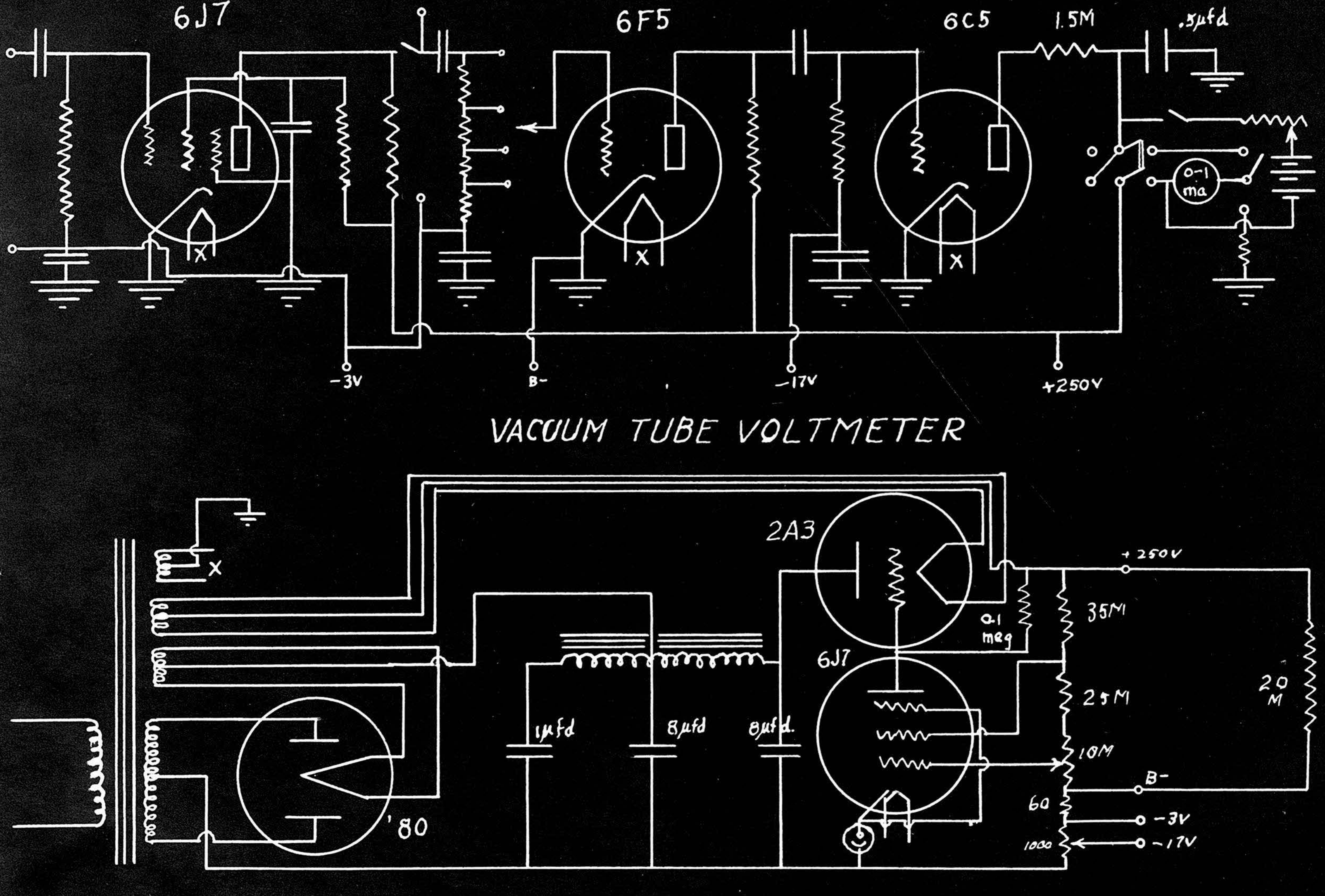
INDEX TO APPENDIX

Measurement of Flux

The first matter to be taken up is the measurement of the flux distribution of the rings under different gap conditions. A drawing of the assembly used for the picking up the voltage is shown on page 33. The coil on the ring itself consisted of thirty turns, and carried a half ampere AC in the tests. The coil on the block in the center was of five hundred turns of fine wire. The flux leaking from the ring passed through the coil, thereby setting up an alternating voltage, which was measured by the vacuum tube voltmeter. This form of voltmeter was selected as it would draw a negligible amount of current from the circuit. The instrument was working at about twenty five megohms per volt. A drain of powere would have rendered the reading inaccurate. A diagram of this voltmeter is shown on page 34. As will be noted, the voltage output is regulated by means of a neon tube regulator which holds the voltage on the tubes constant to within a percent for a voltage change in the power line of almost twenty five percent. Since it also lowers the power supply coupling impedance, coupling between the tubes due to this sonce is greatly reduced. This increases the inherent stability. It is of interest to note that the hum voltage input to the 6J7 from the power supply is less than a thousandth of a volt. The meter was used over the range of three to forty thousandths of a volt. The smallest recorded voltage is about one and a half thousandths, with a distributed coil sample on a ring with a half inch airgap. The flux can be calculated from the voltage. The formula used of course is $E = n\, d\phi/dt$. $E = 500\text{x}\ 120\pi\phi_{eff}$, in electromagnetic units. Therefore $\phi_{eff} = E_{eff}\text{x}10^8/120\pi$: This of course assumes that hysteretic errors are negligible. A sample set of data is included on page 35. The graph numbers correspond to the sample numbers given with this data. Speciman number one is a ring sample with an airgap an inch and a quarter long. Number two has an airga of a quarter of an inch. Number three shows the curves for a sample with two coils bucking each other on opposite legs of the ring. Each coil has fifteen turns, instead of the thirty present in the single coil test samples. Since the inductive reactance is of extreme importance in this case, the impedance, the resistance, and the calculated reactance are also included in the set.



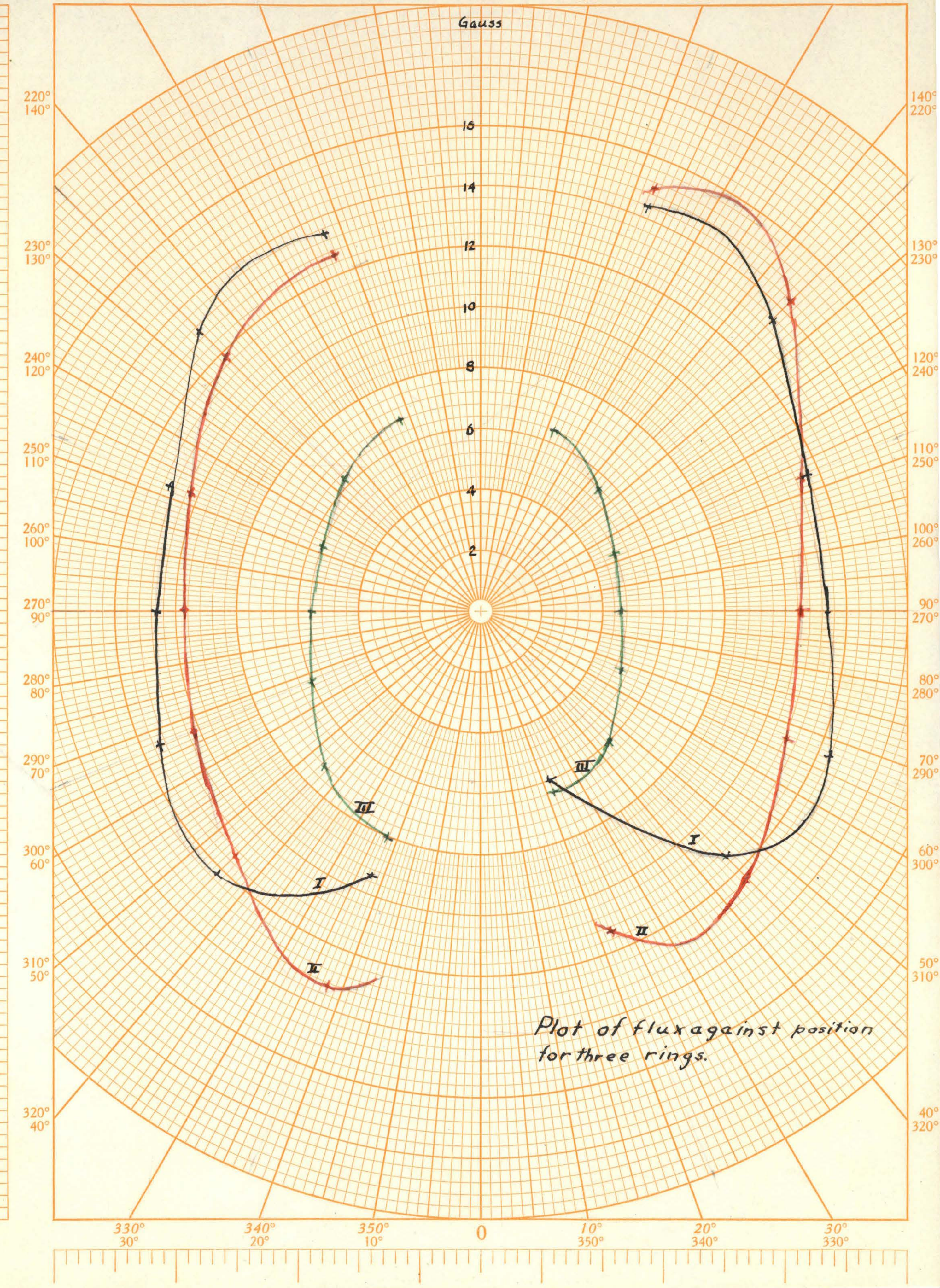
DEVICE FOR
MEASURING LEAKAGE
FLUX FROM RING



VACUUM TUBE VOLTMETER

SAMPLE OF RING DATA

Speciman I				Speciman II				Speciman III		
No.	I_m	E_ϕ	ϕ	No.	I_m	E_ϕ	ϕ	I_m	E_ϕ	ϕ
1	0.570	.0252	13.4	1	0.540	.0240	12.7	0.220	.0128	6.8
2	0.544	.0243	12.9	2	0.493	.0224	11.9	0.192	.0113	6.3
3	0.440	.0207	11.0	3	0.401	.0193	10.2	0.160	.0104	5.6
4	0.420	.0200	10.6	4	0.370	.0183	9.7	0.160	.0105	5.6
5	0.470	.0215	11.4	5	0.392	.0189	10.0	0.180	.0113	6.0
6	0.504	.0229	12.1	6	0.462	.0214	11.3	0.240	.0135	7.2
7	0.348	.0176	9.3	7	0.563	.0251	13.3	0.282	.0151	8.0
8	0.180	.0113	6.0	8	0.457	.0218	11.3	0.200	.0120	6.4
9	0.460	.0213	11.3	9	0.523	.0235	12.5	0.180	.0113	6.0
10	0.520	.0234	12.4	10	0.443	.0207	11.0	0.128	.0094	5.0
11	0.468	.0216	11.5	11	0.418	.0200	10.6	0.112	.0086	4.6
12	0.480	.0220	11.7	12	0.462	.0214	11.3	0.115	.0088	4.7
13	0.580	.0255	13.5	13	0.633	.0272	14.4	0.160	.0105	5.6
14	0.630	.0271	14.4	14	0.640	.0279	15.0	0.200	.0120	6.4
$\Sigma\phi$			161.5	$\Sigma\phi$			165.2			84.7
R		.060		R		.060			.0767	
L		.070		L		.082			.0086	
Z		.0908		Z		.103			.0772	
$\frac{\Sigma\phi}{L}$		2607		$\frac{\Sigma\phi}{L}$		2014			9849	



Plot of flux against position for three rings.

Approximate Equations Governing Instrument

From electromagnetic theory:

$$d\bar{F} = I\,d\bar{l} \times \bar{H}$$

We define:–

$r \equiv$ mean radius of ring

$F \equiv$ turning force

$l \equiv$ length of one turn

$$H \equiv (\beta_{\theta_1} - \beta_{\theta_2})\frac{A}{\varphi r l} = \delta\beta\frac{A}{\varphi r l}$$

Where $A \equiv$ crossection area of ring

$\varphi \equiv$ Angular width of coil.

$$dF = n\,I\,dl\,\delta\beta\frac{A}{\varphi r l}$$

$$\therefore T = 2rF = 2nI\,lr\,\delta\beta\frac{A}{\varphi r l} = \tau\theta$$

$$\theta = \frac{2nI\,\delta\beta A}{\tau\varphi}$$

$$= KI\delta\beta$$

This derivation is based on the following assumptions:

$\delta\beta$ is $\perp$ to magnet or ring surface near to that surface. This requires that the permeability is much greater than unity, or the ring is for practical purposes a flux conductor. The conductor carrying current must be near the surface of the flux conductor.

Relation of Size to Sensitivity

$$H_{ave} = (\beta_{\theta_1} - \beta_{\theta_2})\frac{A}{\varphi r l} = (\beta_{\theta_1} - \beta_{\theta_2})\frac{da}{\varphi r l} = \delta\beta\,\frac{ad}{\varphi r l}$$

Where $a =$ width of ring; $d =$ thickness.

$$dT = nI\,dl\cdot 2r\cdot\delta\beta\cdot\frac{ad}{\varphi r l}$$

$$\delta\beta = \text{approx}\ \frac{\text{m.m.f.}}{2r} = \frac{\eta_0}{2r}$$

$$\tau\theta = nIl\cdot 2r\cdot\frac{\eta_0 a d}{2r^2 l\varphi} = nI\,\frac{\eta_0 a d}{r\varphi}$$

$\tau = mr^2k$ for period of meter constant.

$$\therefore \theta = \frac{nI\eta_0 a d}{\tau\varphi r} = \frac{nI\eta_0 a d}{mr^2K\varphi r}$$

For resistance constant:

$$R = C_1 = \frac{l\rho}{A}$$

$$\therefore C_1 A = l\rho$$

$$\therefore m = \frac{\rho l^2}{C}$$

$$\therefore \theta = \frac{nI\eta_0 a d c}{\rho l^2\varphi K r^3}$$

$$= \frac{nI\eta c}{\rho k\varphi}\cdot\frac{ad}{r^3 l^2} = K_1\,\frac{ad}{r^3 l^2}$$

where n, I, η, C, ρ, K and φ are constants.

If $a, d, r,$ and $l = 1$,

$$\theta = K_1$$

If $a, d, r,$ and $l = 1/2$,

$$\theta = 8k_1$$

Therefore the sensitivity is inversely proportional to the cube of the radius if the same proportions are retained.

The operating equations used for the design of these instruments are quite simple. One has only to build a model, of course, and then evaluate the constants for the type of construction from measurements on this model. This was the method used in obtaining the operating equations for this instrument. Since the operation depends on the interaction of the exciting coil magnetic field, which is proportional to the ampere turns, and the current flowing through the moving assembly, the equation takes the form of a product of two terms, each consisting of the ampere turn product for its specific assembly. In other words $N_1I_1N_2I_2 = C$. This of course asssumes no change in the characteristics determined by the constants. It turned out that the models built had an ampere turn product of between 100 and 200. The full scale current squared times the product of the moving and stationary turns for one set of coils, rather than two, was the means of arriving at this figure. The setup of every coil being bucked by another coil holds the impedance much below that expected for the same total number of ampere turns.

On ammeters, another equation comes into play. That of course is $N_1^2/R_1 = N_2^2/R_2$. This relation is the one which eliminates the frequency error in the instrment. On the other forms of the instrument, it is better not to have this relation adhered to, as it will result in a maximum voltage drop from the fact that the vector voltage drops will add directly, instead of vectorially. To keep this as small as possible, in series instruments the power factors of the moving and stationary elements should differ as widely as possible. It is of interest to note that the instrument lends itself easily to this vector separation, as the wire on the moving coil has to be small in order that the period of the instrument be small, and the minimum loss in the stationary coils occurs with the largest possible wire in order to hold the resistance down. A derivation which gives the optimum value of k, the ratio between N_1 and N_2, is given on the next page. We see that the value of this constant depends on the resistance of one coil set, and the products of two constants defined divided by the products of the currents. For the value of k to be real, the only restriction is that $A^2 - \cancel{4R_2} - 2R_2^2$ must be greater than zero. ~~This is the descriminent of the quadratic in k^2.~~ ~~It happens that this also assured that the values of k^2 will be positive, since the radical gives positive roots to the equation under this condition.~~

Best Coil Relations for Ammeter

Given:- $N_1 N_2 I_1 I_2 = C;\quad N_1 = kN_2$

$\frac{N_1^2}{R_1} = \frac{N_2^2}{R_2};\quad \omega L_1 = C_1 N_1^2;\ \omega L_2 = C_1 N_2^2$

To find:- Value of k for Z to be minimum.

$$R_1 = k^2 R_2$$

$$k N_2^2 I_1 I_2 = C$$

$$\therefore N_2^2 = \frac{C}{kI_1I_2} \quad \text{and} \quad N_1^2 = \frac{Ck}{I_1I_2}$$

$$Z = \frac{\sqrt{R_1^2 + (C_1N_1^2)^2}\sqrt{R_2^2 + (C_1N_2^2)^2}}{\sqrt{R_1^2 + (C_1N_1^2)^2} + \sqrt{R_2^2 + (C_1N_2^2)^2}}$$

$$\therefore Z = \frac{\sqrt{k^4R_2^2 + \frac{C_1^2C^2k^2}{I_1^2I_2^2}}\sqrt{R_2^2 + \frac{C_1^2C^2}{k^2I_1^2I_2^2}}}{\sqrt{k^4R_2^2 + \frac{C_1^2C^2k^2}{I_1^2I_2^2}} + \sqrt{R_2^2 + \frac{C_1^2C^2}{k^2I_1^2I_2^2}}}$$

$$= \frac{\sqrt{k^2R_2^2 + \frac{C_1^2C^2}{I_1^2I_2^2}}\sqrt{k^2R_2^2 + \frac{C_1^2C^2}{I_1^2I_2^2}}}{k\sqrt{k^2R_2^2 + \frac{C_1^2C^2}{I_1^2I_2^2}} + \frac{1}{k}\sqrt{k^2R_2^2 + \frac{C_1^2C^3}{I_1^2I_2^2}}}$$

$$= \frac{\sqrt{k^2R_2^2 + \frac{C_1^2C^3}{I_1^2I_2^2}}}{\frac{k^2+1}{k}} \qquad \text{Take } A = \frac{C_1C}{I_1I_2}$$

$$\therefore Z = \frac{\sqrt{k^2R_2^2 + A^2}}{\frac{k^2+1}{k}}$$

$$\frac{\partial Z}{\partial k} = 0 = \frac{\frac{k^2+1}{k}\,\frac{kR_2^2}{\sqrt{\quad}} - \sqrt{\quad}\left(1 - \frac{1}{k^2}\right)}{\left(\frac{k^2+1}{k}\right)^2}$$

$$\therefore (k^2+1)k^2R_2^2 - (k^2-1)(k^2R_2^2 + A^2) = 0$$

$$k^4R_2^2 + k^2R_2^2 - k^4R_2^2 - k^2A^2 + k^2R_2^2 + A^2 = 0$$

$$2k^2R_2^2 - k^2A^2 + A^2 = 0$$

$$k^2 = \frac{A^2}{A^2 - 2R_2^2}; \qquad \therefore k = \pm\frac{A}{\sqrt{A^2 - 2R_2^2}}$$

Determination of Errors

The circuit reluctance is the sum of that of the airgap and that of the iron return path.

Assume path area in iron one twentieth that in air.

For average path

$$\mathcal{R}_1 = \frac{\pi r}{2\mu \frac{A_2}{20}} + \frac{r}{A_2} = \text{approx } \frac{3 \times \frac{9}{2} \times 20}{2 \times 2500} + 4.5$$

$$= 4.554$$

$$\mathcal{R}_2 = \frac{\pi r}{2\mu \cdot \frac{A_2}{20}} + \frac{r}{A_2} = \text{approx } \frac{3 \times 9 \times 20}{2 \times 2 \times 2800} + 4.5$$

$$= 4.548$$

$$\% \text{ change} = \frac{\mathcal{R}_1 - \mathcal{R}_2}{\mathcal{R}_1} = \frac{0.006}{4.554} = 0.12\%$$

We see that, if the effective area ratio is twenty to one, which is the case in the meters built, the error would not be noticeable.

Measurements of leakage flux on the rings indicated that the maximum flux in the ring at the coil is approximately 150 gauss = one kiloline/square inch.

$$\frac{W_1}{W_2} = \left(\frac{B_1}{B_2}\right)^{1.6}; \qquad \frac{0.16}{W} = \left(\frac{30{,}000}{1000}\right)^{1.6} = 262$$

$$\therefore W_2 < 0.000615$$

Power consumption of the meter is 0.1 volt ampere.

Therefore the iron loss is less than 0.6 percent.

Investigation of error in wattmeter due to an error $\Delta\theta$ in potential coil phase angle.

Let $P \equiv$ true power

$P_m \equiv$ power indicated by meter.

$$P - P_m = \Delta P$$

$$P = Ei\cos\theta$$

$$P_m = Ei\cos(\theta - \Delta\theta)$$

$$\therefore \Delta P = Ei\left(\cos\theta - \cos\theta\cos\Delta\theta - \sin\theta\sin\Delta\theta\right)$$

Maximum ΔP occurs at $\frac{\partial(\Delta P)}{\partial\theta} = 0$

$$\therefore -\sin\theta(1 - \cos\Delta\theta) - \cos\theta\sin\Delta\theta = 0$$

$$\tan\theta = -\frac{\sin\Delta\theta}{1-\cos\Delta\theta} = -\cot\frac{\Delta\theta}{2}$$

$$= \tan\left(90^\circ + \frac{\Delta\theta}{2}\right)$$

$$\therefore \theta = 90^0 + \frac{\Delta\theta}{2}$$

$$\frac{\Delta P}{Ei} = \left(1 - \cos\Delta\theta - \tan\left(90^\circ + \frac{\Delta\theta}{2}\right)\sin\Delta\theta\right)\cos\theta$$

$$= \left(2\sin^2\frac{\Delta\theta}{2} + 2\cos^2\frac{\Delta\theta}{2}\right)\cos\theta$$

$$= -2\sin\frac{\Delta\theta}{2}$$

Let $P_1 = Ei\cos\theta_1$ be full scale reading.

$$\therefore \frac{\Delta P}{P_1} = \frac{-2\sin\frac{\Delta\theta}{2}}{\cos\theta_1}$$

For unity power factor meter, $\Delta\theta_{max}$ is given by $0.005 = \frac{2\sin\frac{\Delta\theta_{max}}{2}}{1} = \frac{\sin\frac{\Delta\theta_{max}}{2}}{0.5}$

$$\sin\frac{\Delta\theta}{2} = 0.0025$$

$$\therefore \frac{\Delta\theta_{max}}{2} = 8'$$

$$\therefore \Delta\theta_{max} = 16'$$

For a 20% power factor wattmeter, $\cos\theta_1 = 0.20$.

$\therefore \frac{\Delta P}{P_1} = 0.005 = \frac{\sin \Delta\theta}{0.1}\,{}^{2}$

$\sin\frac{\Delta\theta}{2} = .0005$

$\frac{\Delta\theta}{2} = 2'$

$\Delta\theta = 4'$

For a 5% power factor wattmeter, $\cos\theta_1 - 0.05$

$\frac{\Delta P}{P_1} = 0.005 = \frac{2\sin\frac{\Delta\theta}{2}}{0.05}$

$\sin\frac{\Delta\theta}{2} = .00012$

$\frac{\Delta\theta}{2} = 25''$

$\Delta\theta = 50''$

To build such a meter, the constants come out as follows:

To be within error, set $\tan\Delta\theta = 0.0002$

$\therefore Z_r = 5000 Z_L$ for potential assembly.

Take $Z_L = \frac{1}{3}\omega$

$\therefore Z_r = 1500\omega$

$Z_{15\,turns} = 0.02\,\omega$

$.33 = .02\,\frac{n^2}{225}$

$\therefore n = 67$ turns

$67 \times 0.1 \times 5 \times n = \frac{200}{4} \times 20$ Spring constant $= 25$

$6.7\,n = 200$; $n = 30$ turns

$Z_L = 0.08\,\omega$

$Z_L I^2 = 2$ volt amperes for current coils.

For potential assembly, $z i^2 = \frac{(115)^2}{1500} = 8.8$ volt amps.

Drysdale and Jolley show (pages 638-9, Electrical Measuring Instruments, volume I) that current coil phase angle error due to core loss is approximately constant. Therefore it is possible, since this error tends to increase θ as potential assembly error tends to reduce θ, to cause partial cancellation of these two errors. Take $\Delta\theta = \Delta\varphi + \Delta\psi$, where $\Delta\varphi$ = potential error, and $\Delta\psi$ = current coil error. If $\Delta\varphi$ is taken so that $\Delta\varphi = -\Delta\psi$, say at sixty cycles, then the meter will have no phase angle error. $\Delta\psi$ can be held much below the 0.6 percent calculated, as the same reference states that Steinmetz constant is nearer 2 than 1.6 taken here. It happens that both of these errors are proportional to the frequency, so that if they are corrected at one frequency, except for eddy current losses, which careful preparation will keep to a very low value, they will be corrected at all frequencies. Since both errors are less than a half percent for a unity power factor wattmeter, they can easily be adjusted so that they compensate each other, thereby giving correct readings at any power factor.