SKEWED RIGID FRAME BRIDGE

Thesis by

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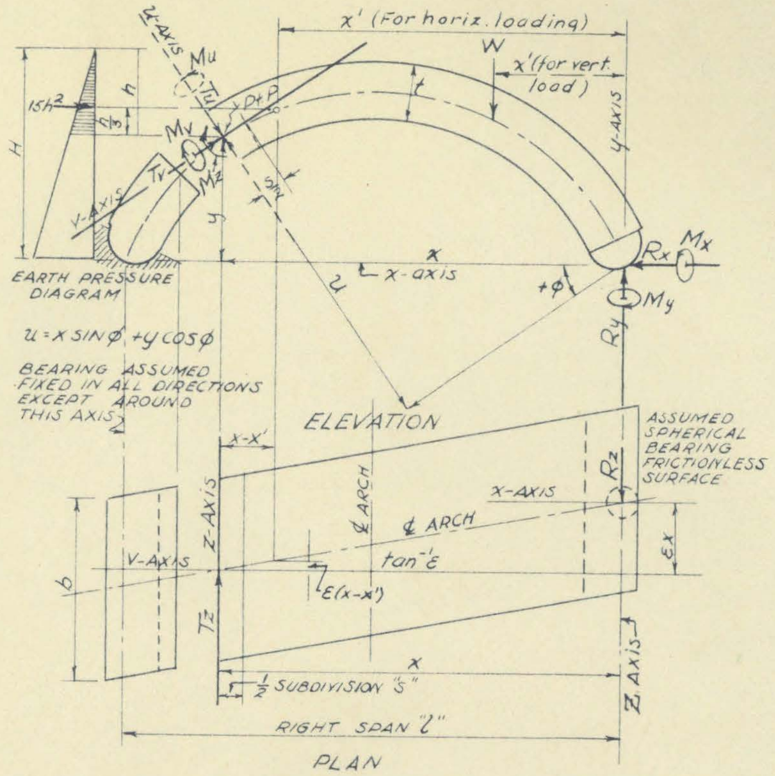
AN INVESTIGATION OF THE VARIATION OF MOMENTS
AND THRUSTS IN A SKEWED RIGID FRAME BRIDGE
WITH A CHANGE IN HEIGHT OF THE SPRINGING LINE

The purpose of this investigation is to attempt to establish a relationship between the moments and thrusts of a skewed, rigid frame bridge and the height of its springing line as the latter is varied, the angle of skew being held constant.

In changing the height of the springing line special treatment had to be made of the legs of the structure in order to maintain a constant thickness of the elements at the springing line and at the abutments. Reference to Fig. 2 shows the manner in which this was accomplished, the thicknesses of the intermediate elements being varied to maintain constant values for the thickness at the top and bottom of the leg. The same dimensions for the barrel were used in all cases.

In the investigation a structure with a right span of 55.9 ft. and a skew angle of 38 degrees was analyzed. The span and angle of skew were held constant as the height was varied, the latter being taken as 12.9 ft., 17.2 ft., and 21.5 ft. The analysis considered elements 4.3 ft. in length. Thus the changes in the height of the springing line which were investigated allowed the addition of whole elements to the legs.

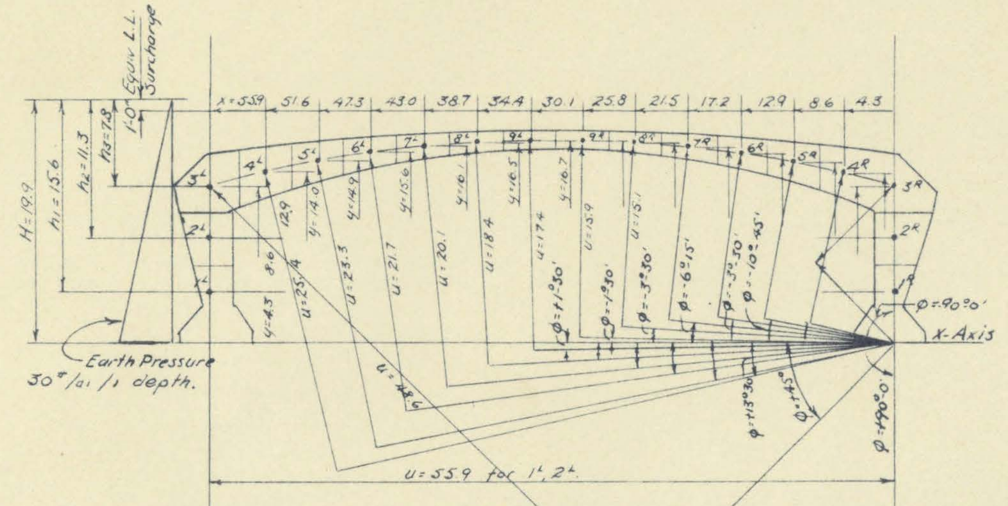
The curves plotted from the analyses are shown on the following pages and indicate the variation in R_x, R_z, M_x, and M_y for a unit load placed at points A and B shown in the diagram, for earth pressure of 30 # /sq. ft./ft. depth acting on the left end of the bridge only, and for a temperature change of plus or minus 50 degrees F.



DIAGRAM

SHOWING FORCES ACTING AND QUANTITIES USED IN DERIVATION

FORCES AND MOMENTS SHOWN POSITIVE



DIAGRAM

SHOWING RIGHT ANGLE PROJECTION OF FRAME, LOADING, AND SCALED QUANTITIES USED IN ANALYSIS

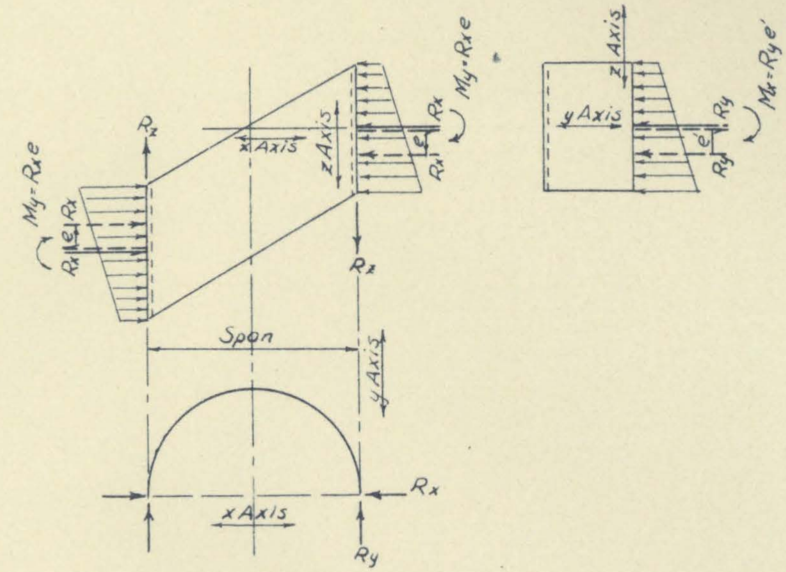
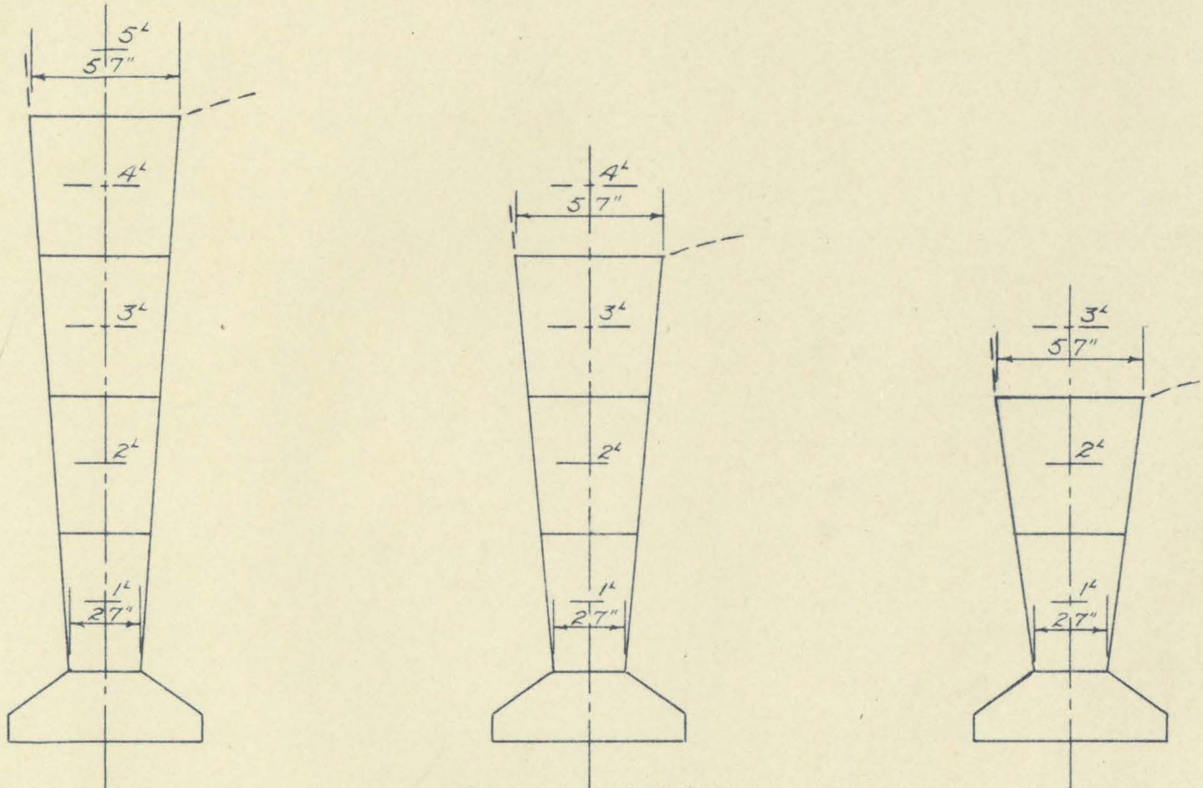


FIG. 1



Elements 4.3' tall.

FIG 2

THEORY OF ANALYSIS

The skew arch is a structure containing forces which cannot generally be made parallel to a single plane. They must therefore be analyzed in three dimensions instead of two as is done in the case of a right arch. As the structure is on a skew, forces and moments are introduced which either do not exist at all in the right arch or are present only in a small degree.

The problem is a three-dimensional one, and therefore a system of reference axes for working in three planes will be adopted. Reference is made to Fig. 1. The horizontal axis perpendicular to the abutments is called the X axis, and the vertical axis the Y axis. The axis parallel to the abutment is called the Z axis. Each axis is perpendicular to the other two and each of the three planes formed by the axes is perpendicular to the other two.

The total reactions R_y are the same as for a simple beam of the same span. The total horizontal reactions R_x are equal to each other and depend upon the elastic and geometric properties of the arch. In order to make the analysis general, the assumption is made that reactions R_y and R_x are not uniform along the abutments. The reactions R_x may be represented by placing a force R_x at the center of the abutment and introducing a moment or couple M_y equal to $R_x \cdot e$ about the Y axis, where e is the distance from the center of the abutment to the center of gravity of the horizontal pressure diagram. Likewise R_y may be represented by placing a force R_y at the middle of the abutment and introducing a moment or couple $R_y \cdot e$ about the X axis. The two horizontal reactions R_x form a couple which tends to rotate the structure as a whole. To resist this, the two cross shears R_z form a couple which balances in part the moment produced by the two reactions R_x.

Reactions R_y and R_x produce moments, shears, and thrusts in the vertical planes perpendicular to the Z axis. Reactions R_z produce shear parallel to the Z axis. Reaction moments M_y and M_x produce torsion in the vertical and horizontal portions of the arch, respectively. Reactions R_y with moment arms along the Z axis and reactions R_z with moment arms along the Y axis produce torsion in the horizontal sections. Reactions R_x with moment arms along the Z axis produce torsion in the vertical sections. Torsion in the inclined sections of the arch is produced by combinations of all of the torsion moments. The torsion moments in turn produce vertical shears in the YZ plane, which are small and may be neglected. They also produce horizontal shears parallel to the Z axis which must be combined with the shears due to reaction R_z. A moment about the Y axis is also developed by all the reactions and reaction moments parallel to the XZ plane. This moment is resisted by the entire structure acting as a horizontal beam between reaction supports and therefore the stresses produced are very small.

In the derivation of equations to be used in solving for the reactions, the supports are assumed to be so altered by the removal of the redundant reactions that the structure becomes a simple span beam. By using the laws of static equilibrium the stresses in all parts of the structure due to any given loading may be found. Likewise, the linear and angular deflections at the right support may be calculated. Use is made of the fact that the total resultant deflection in any support at the reaction must equal zero. The directions of the forces for positive signs are as shown in the diagram.

The internal stresses at any section such as P may be represented by a system of external forces and moments exerted by the portion of the structure to the left of the cut section on the portion of the structure to the right of the cut section. The forces to the right of the section must be in equilibrium with the forces to the left of the section.

REACTIONS FOR TWO-HINGED SKEWED ARCH

By inspection of the diagram, general expressions may be written for M_v, M_u, M_z, T_v, T_u, T_z, respectively, in terms of the loading and the unknown reaction components. Since M_u and T_u are very small they may be neglected.

FOR VERTICAL LOADING AS INDICATED ON DIAGRAM

$$M_z = R_y x - W(x - x') - R_x y = M_0 - R_x y$$

$$M_v = (R_x \sin\phi)ex - R_z u + M_x \cos\phi - M_y \sin\phi$$
$$+ (R_y \cos\phi)ex - W\cos\phi(x - x')e$$
$$= (R_x \sin\phi)ex - R_z u + M_x \cos\phi - M_y \sin\phi + (M_0 \cos\phi)e.$$

$$T_v = -R_y \sin\phi + W\sin\phi + R_x \cos\phi = V_0 \sin\phi + R_x \cos\phi.$$

$$T_z = R_z.$$

In which M_0 = moment for simple beam of right span 1;

In which V_0 = shear for simple beam of right span 1.

The above equations for vertical loading can be made to apply to the horizontal earth pressure loading shown on diagram (30 #/sq. ft./ft. depth) without rewriting by making the following substitutions:

For simple span moment M_0 in expression for M_z:

substitute $R_y x - 5h^3$

For $M_0 \cos\phi$ in expression for M_v:

substitute $R_y x \cos\phi - 15h^2 \sin\phi\ (x - x')$

For $V_0 \sin\phi$ in expression for T_v:

substitute $-R_y \sin\phi - 15h^2 \cos\phi$

DERIVATION OF EQUATIONS FOR REDUNDANT REACTIONS

Consider the deflections at right support due to moments M_Z and M_V at any section such as P in the diagram. (Deflections due to M_u and to the thrusts T_Z, T_u, and T_V are comparatively small and may be neglected.)

Let

M_z = moment at P about Z axis due to all forces acting;

M_v = moment at P about v axis due to all forces acting;

$m_{v(x)}$ = moment at P about v axis due to unit thrust at right reaction along R_X;

$m_{v(z)}$ = moment at P about v axis due to unit thrust at right rea ction along R_Z;

$m_{v(ox)}$ = moment at P about v axis due to unit couple at right reaction along M_X;

$m_{v(oy)}$ = moment at P about v axis due to unit couple at right reaction along M_y;

$m_{z(x)}$ = moment at P about Z axis due to unit thrust at right reaction along R_X;

$m_{z(z)}$ = moment at P about Z axis due to unit thrust at right reaction along R_Z;

$m_{z(ox)}$ = moment at P about Z axis due to unit couple at right reaction along M_X;

$m_{z(oy)}$ = moment at P about Z axis due to unit couple at right reaction along M_y;

G = shearing modulus of elasticity;

e = tangent of the skew angle;

s = a division of the arch axis;

k = ratio of modulus of elasticity for direct stress to modulus for shear = E/G.

The total deflections at right support along the lines of action of each of the unknown reaction components, due to all forces acting, are equal, respectively, to zero. That is:

$$I.\ \frac{s}{G}\sum \frac{M_v m_{v(x)}}{F} + \frac{s}{E}\sum \frac{M_z m_{z(x)}}{I} = 0$$

$$II.\ \frac{s}{G}\sum \frac{M_v m_{v(z)}}{F} + \frac{s}{E}\sum \frac{M_z m_{z(z)}}{I} = 0$$

$$III.\ \frac{s}{G}\sum \frac{M_v m_{v(ox)}}{F} + \frac{s}{E}\sum \frac{M_z m_{z(ox)}}{I} = 0$$

$$IV.\ \frac{s}{G}\sum \frac{M_v m_{v(oy)}}{F} + \frac{s}{E}\sum \frac{M_z m_{z(oy)}}{I} = 0$$

M_v (for vertical load) $= (R_x \sin\phi)ex - R_z u + M_x \cos\phi - M_y \sin\phi + M_o e \cos\phi$

M_z (" " ") $= -R_x y + M_o$

M_v (for horizontal load) $= R_x ex \sin\phi - R_z u + M_x \cos\phi - M_y \sin\phi$
$+ [R_y x \cos\phi - 15h^2 \sin\phi (x - x')]e$

M_z (" " ") $= -R_x y + R_y x - 5h^3$

$m_{v(x)} = ex \sin\phi;\quad m_{v(z)} = -u;\quad m_{v(ox)} = \cos\phi;\quad m_{v(oy)} = -\sin\phi;$

$m_{z(x)} = -y;\quad m_{z(z)} = 0;\quad m_{z(ox)} = 0;\quad m_{z(oy)} = 0;$

Substituting the above values in equations I, II, III, and IV, simplifying, cancelling, and collecting terms, the final equations are obtained as follows:

$$I.\ R_x\left(e^2k\sum\frac{x^2\sin^2\phi}{F} + \sum\frac{y^2}{I}\right) - R_z ek\sum\frac{ux\sin\phi}{F}$$
$$+ M_x ek\sum\frac{x\sin\phi\cos\phi}{F} - M_y ek\sum\frac{x\sin^2\phi}{F}$$
$$= -e^2k\sum\frac{M_o \sin\phi\cos\phi\, x}{F} + \sum\frac{M_o y}{I}$$

$$II.\ R_x e\sum\frac{ux\sin\phi}{F} - R_z\sum\frac{u^2}{F} + M_x\sum\frac{u\cos\phi}{F}$$
$$- M_y\sum\frac{u\sin\phi}{F} = -e\sum\frac{M_o u\cos\phi}{F}$$

$$\text{III.}\quad R_x e \sum \frac{x \sin\phi \cos\phi}{F} - R_z \sum \frac{u \cos\phi}{F} + M_x \sum \frac{\cos^2\phi}{F}$$

$$- M_y \sum \frac{\sin\phi \cos\phi}{F} = - e \sum \frac{M_o \cos^2\phi}{F}$$

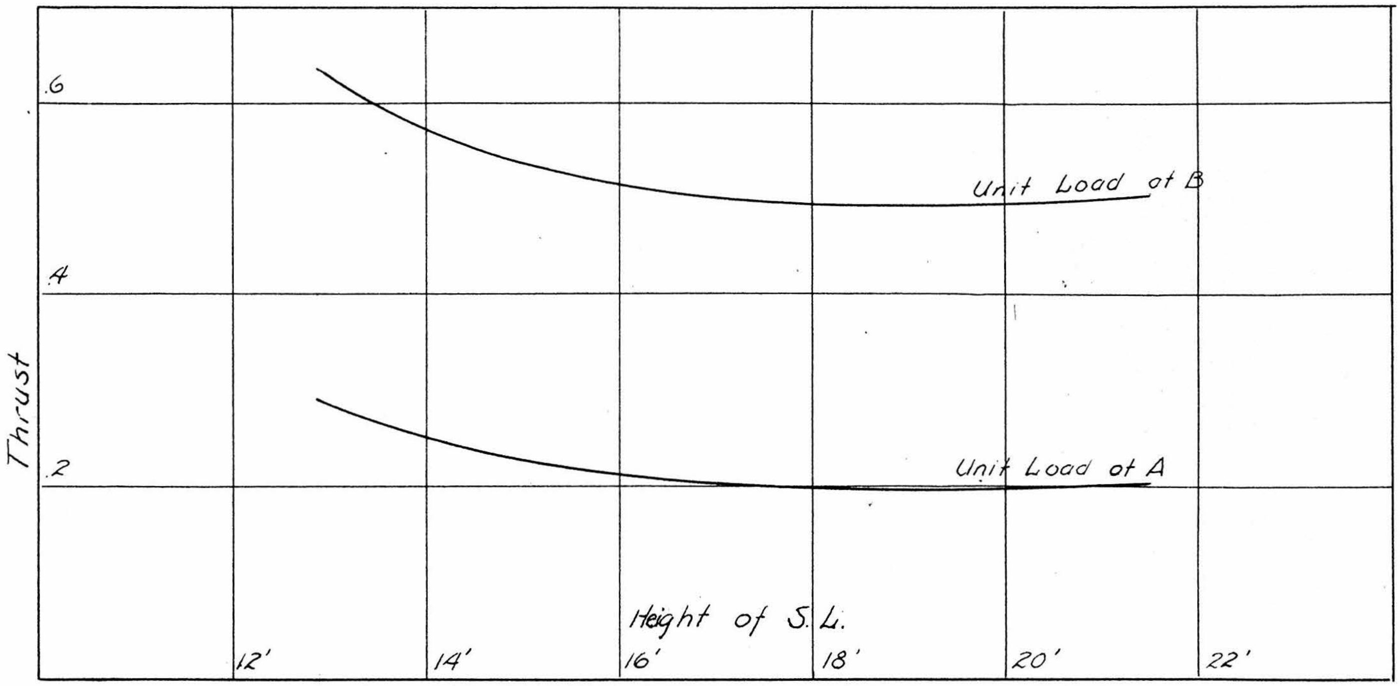
$$\text{IV.}\quad R_x e \sum \frac{x \sin^2\phi}{F} - R_z \sum \frac{u \sin\phi}{F} + M_x \sum \frac{\sin\phi \cos\phi}{F}$$

$$- M_y \sum \frac{\sin^2\phi}{F} = - e \sum \frac{M_o \sin\phi \cos\phi}{F}$$

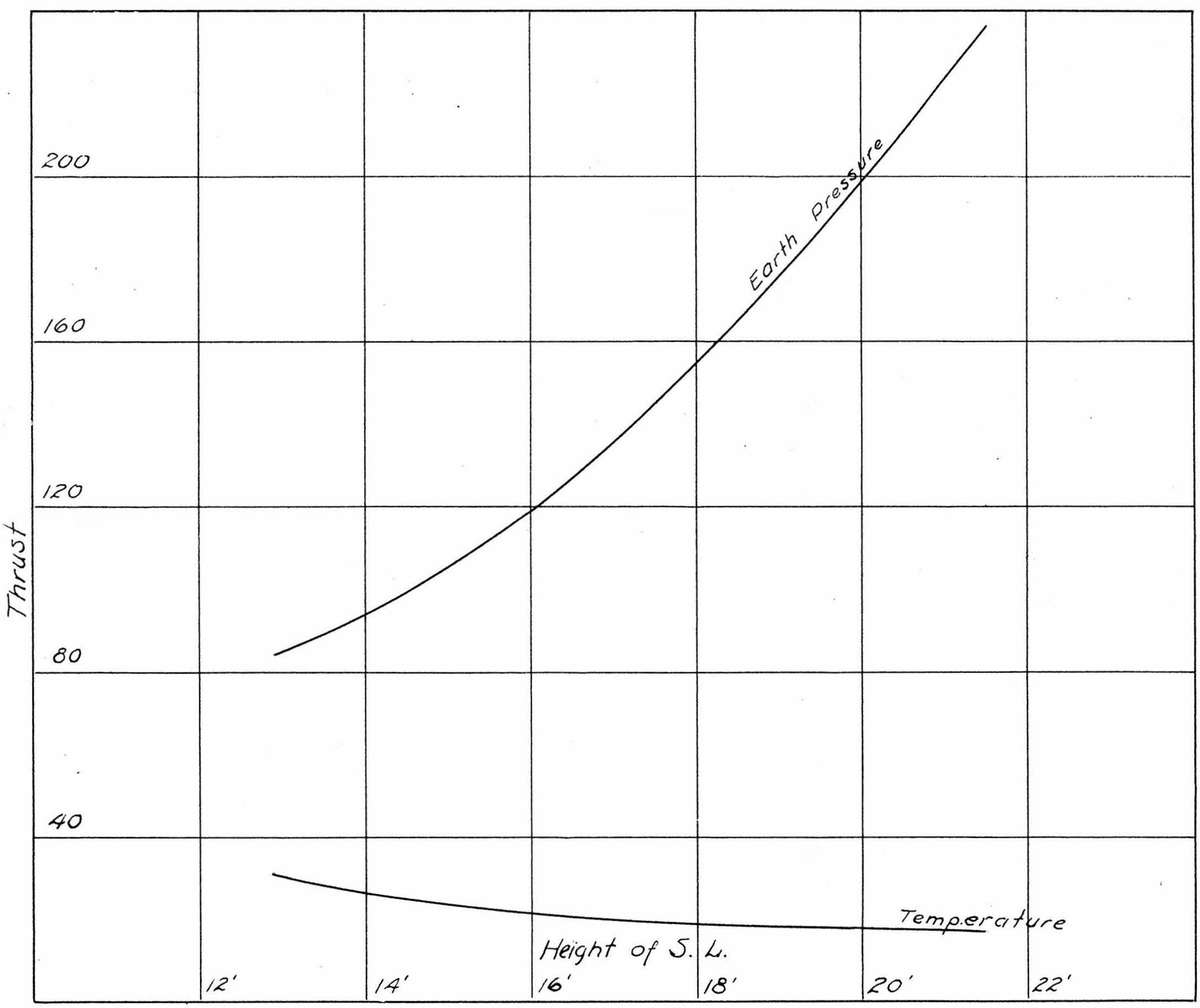
Note that left-hand members of above equations apply to any loading. Right-hand members of equations as above given apply particularly to vertical loading, for which M_o equals the simple span moment. For horizontal earth pressure use substitutions given on previous page. For temperature reactions substitute for right-hand members of equations I, II, III, and IV, respectively, the terms:

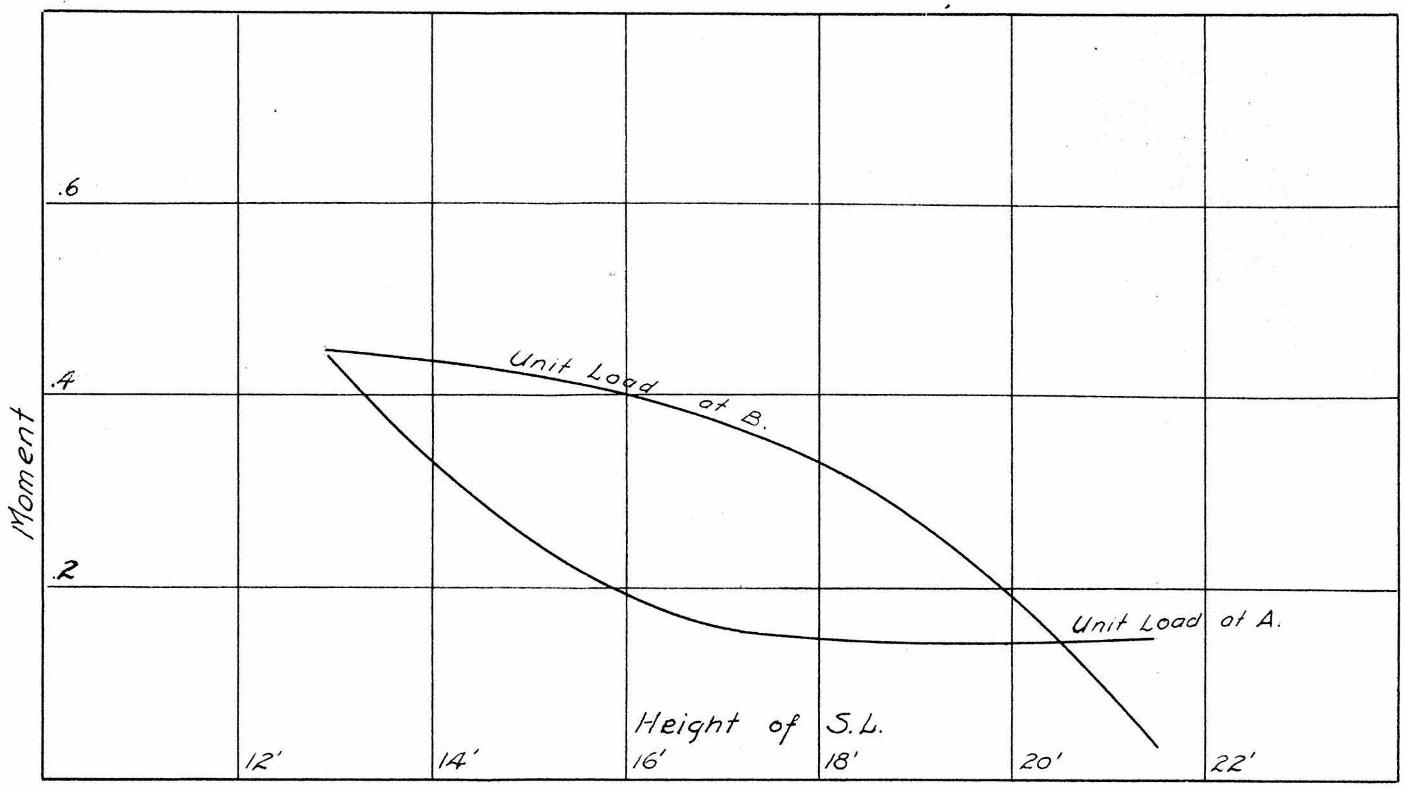
$$+\frac{Ectl}{S};\ -\frac{Eectl}{KS};\ o;\ \text{and } o.\ (t^\circ \text{ rise})$$

(c equals coefficient of expansion)

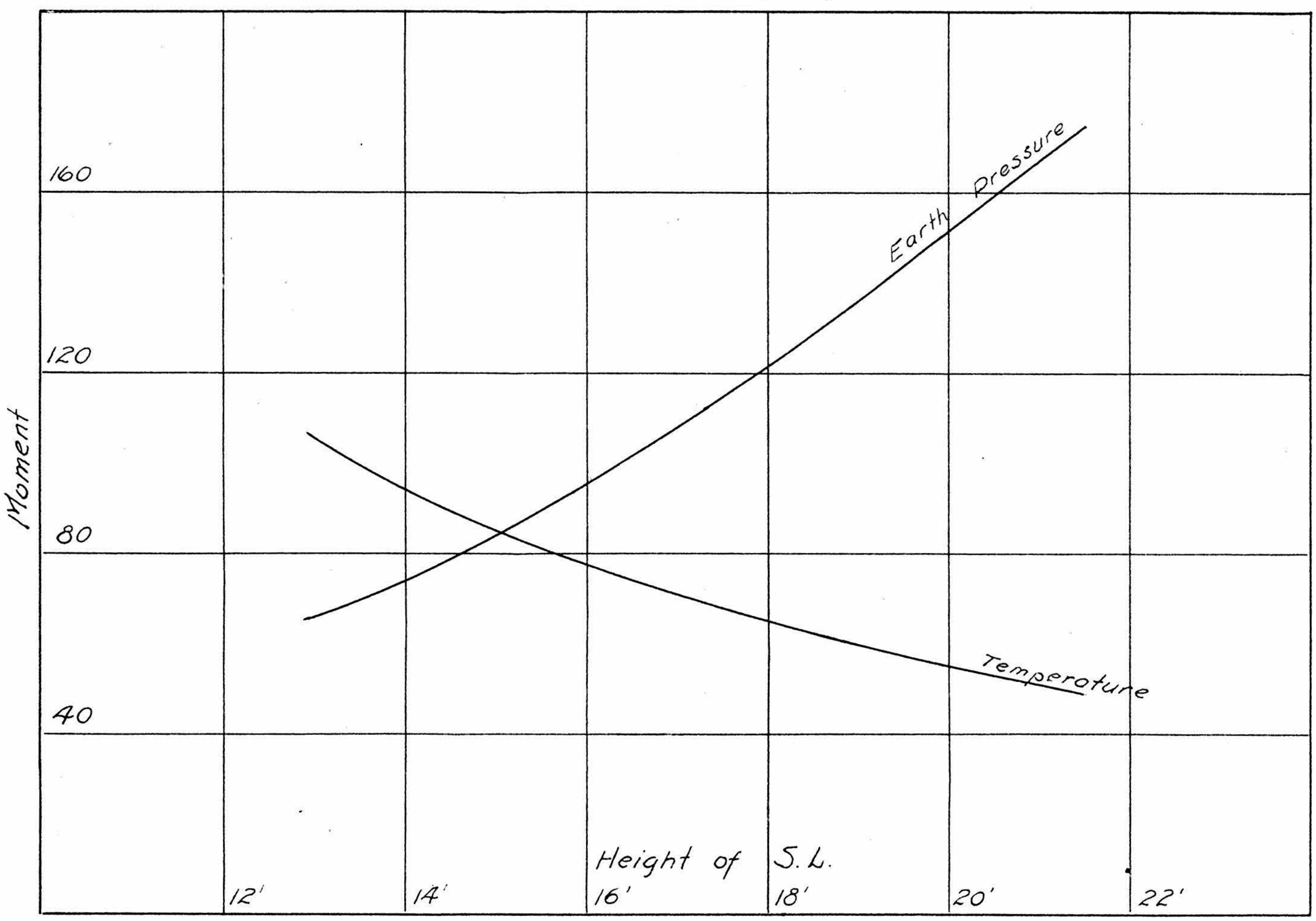


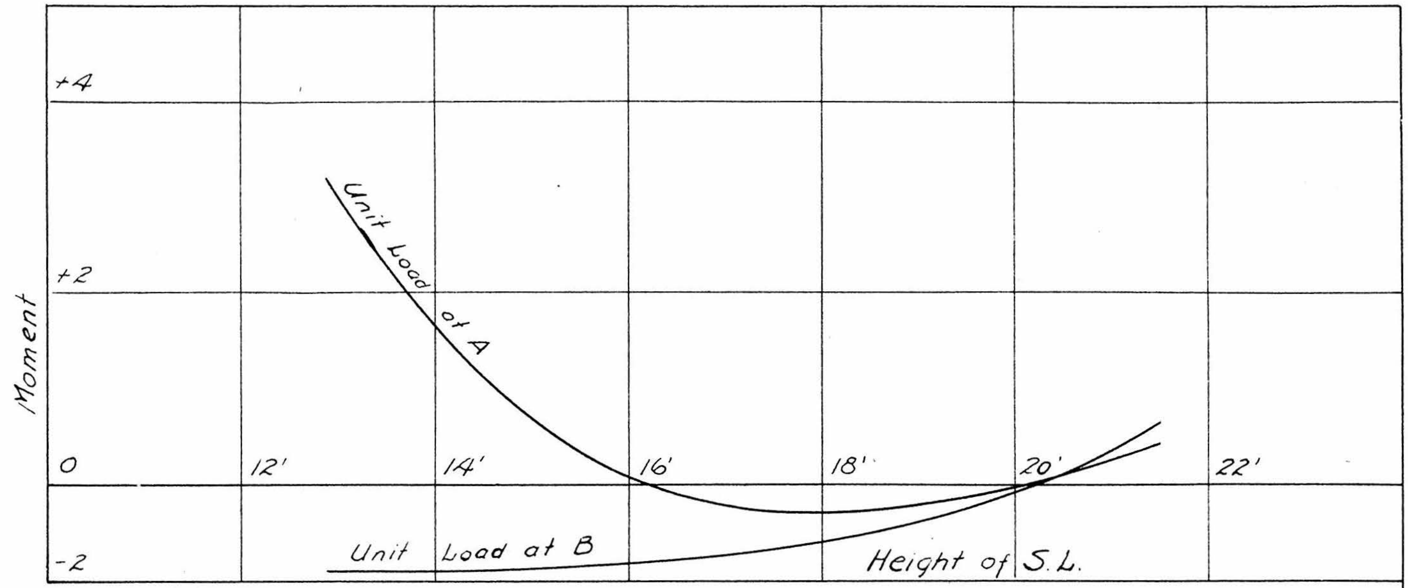
Rx Plotted Against Height of Springing Line



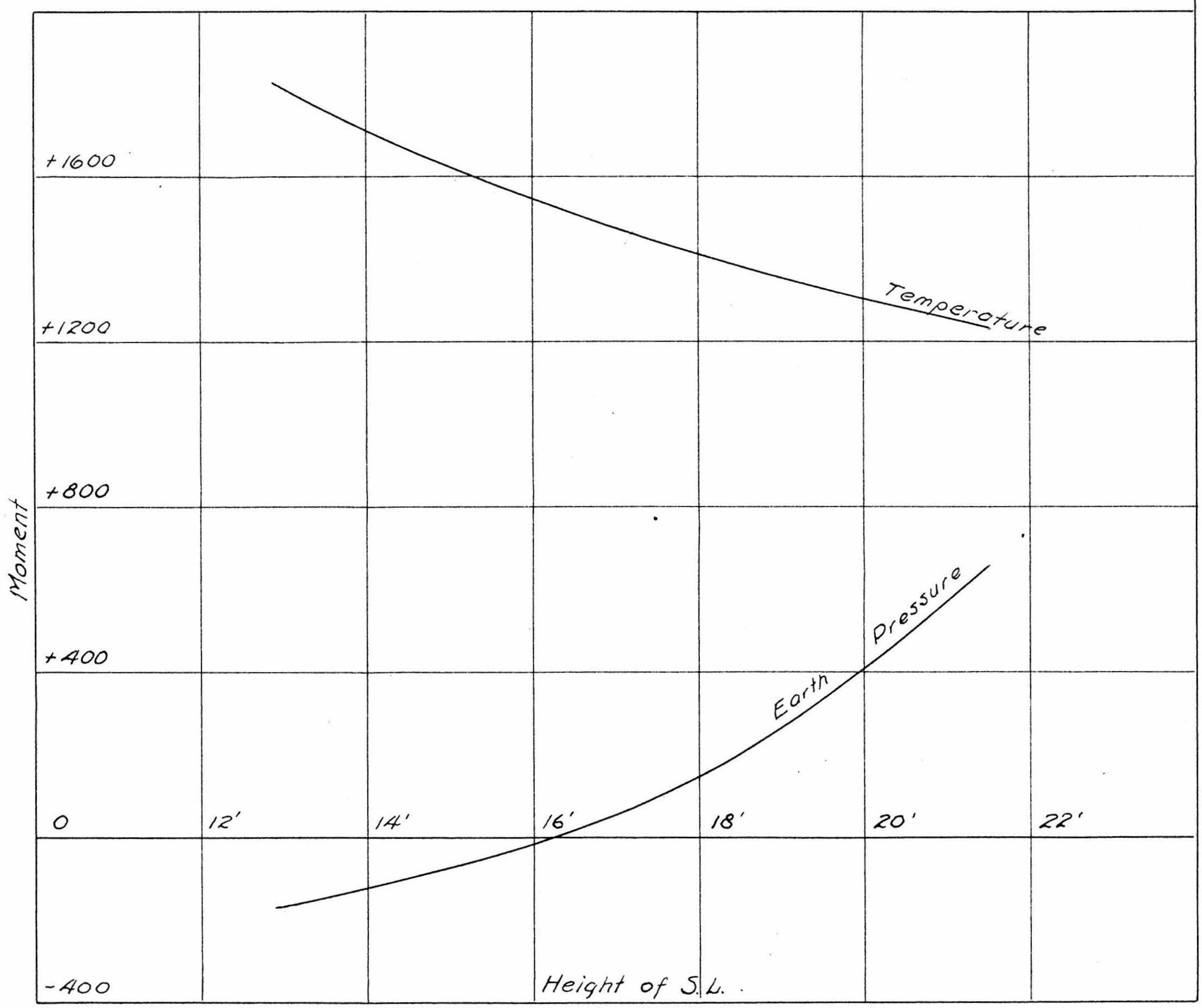


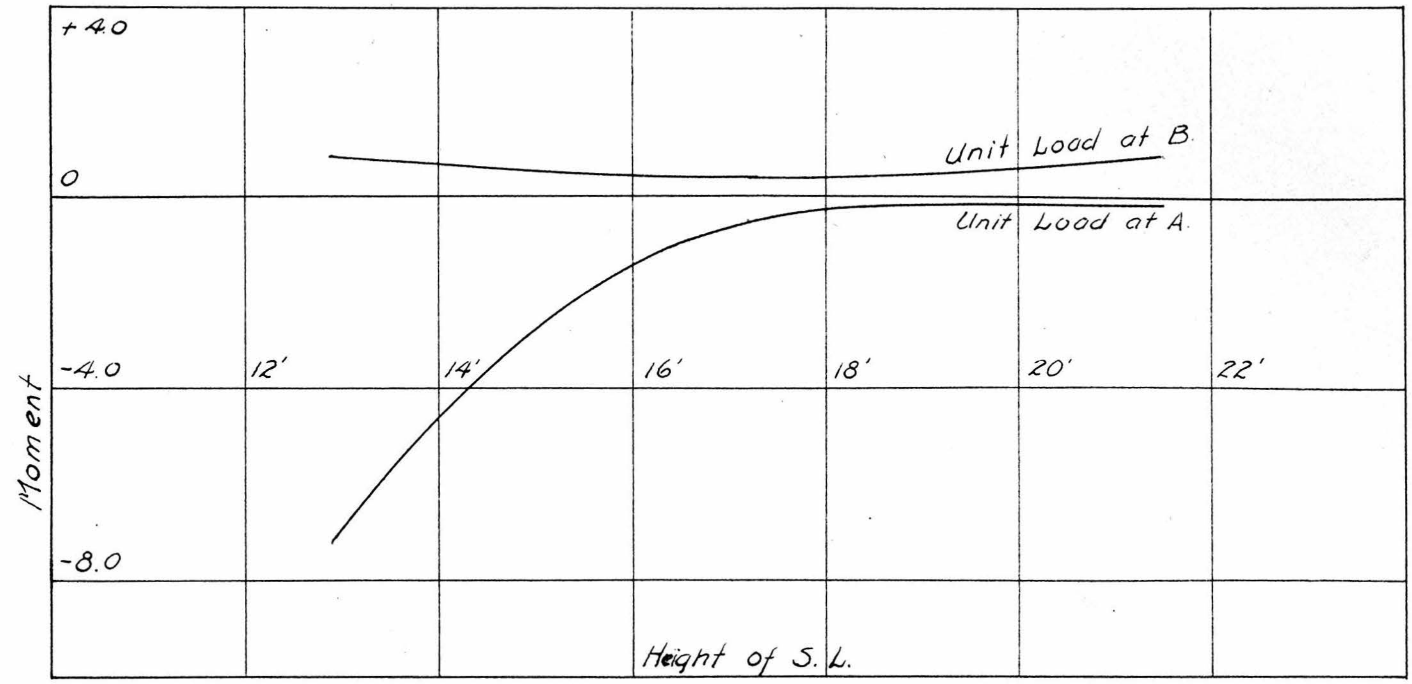
R_z Plotted Against Height of Springing Line





Mx Plotted Against Height of Springing Line





My Plotted Against Height of Springing Line