The Fine Structure of the Line λ 4686

of Ionized Helium

Thesis by

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CONTENTS

Summary

The fine structure of the line $\lambda 4686$ of ionized helium emitted from a discharge tube with hollow cathode cooled by liquid air was measured with a Fabry-Perot interferometer. The formula for the Fabry-Perot interferometer pattern expressed in Fourier series was applied in the analysis. Of the eight components predicted by theory four were definitely observed, while the remaining weak ones were inferred to be present. Relative intensities and positions of the eight components were found to be in general agreement with theory. The separation between the two strongest components was $.0995 \pm .0018$ I.A. or $.4529 \pm .0082$ cm^{-1} differing from the theoretical value by less than one percent. The Rydberg constant of helium was calculated with two different values of indices of fraction. e/m was also calculated with both the old and the latest values of atomic masses.

INTRODUCTION.

The study of the fine structure of hydrogen lines and that of ionized helium lines has been considered so important ever since the advent of the Bohr-Sommerfeld theory on atomic structure, that the general theory of atomic phenomena has been built on it as a cornerstone. Although many more investigations have been devoted to hydrogen than to ionized helium the spectrum of the latter is theoretically more favorable than that of the former in several respects : first, He^+ is heavier than H and its lines are therefore less broadened by the Doppler effect; second, the He^+ nucleus has twice the charge of the H nucleus and the spectrum therefore is less influenced by the Stark effect; third, helium shows no nuclear spin and no complication will arise from that cause; and fourth, the separation of the fine structure components is much greater than in H due to the larger nuclear charge. Sommerfeld's theory of fine structure was first confirmed by Paschen[1] in his measurement of the fine structure of helium lines. Of all the measured lines of the Fowler series and Pickering series, the line 4686 was the brightest and its structure, especially that excited by continuous current, accorded with the theory most satisfactorily. Later, Leo[2] worked on the fine structure of the line 4686 and obtained almost the same result. In the present work it is attempted to make a fresh measurement on the same line with a Fabry-Perot interferometer. This instrument can give a greater

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1. Paschen F., Ann. d. Physik 50 901 (1916)
2. Leo, W., Ann. d. Physik 81 757 (1926)

precision in the determination of wave-length and of the separations between the components than the concave gratings such as that used by Paschen and Leo. It can also enable one to study the relative intensities quantitatively by means of a recently developed method of analysis[3] even if there is overlapping of different orders in a pattern of such complex structure as the line 4686 . The source of illumination is a hollow cathode glow tube, essentially like those of Paschen and Leo, but so constructed that the cathode is immersed in liquid air in order to diminish the Doppler effect.

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3. Houston, W. V., Phys. Rev., 51, 446 (1937)

THEORY.

I. Positions of the components.

Sommerfeld[4], applying the special theory of relativity to the electron mass, obtained the term values for a hydrogen-like atom as follows :

$$T = R \cdot \frac{z^2}{n^2} + R\alpha^2 \cdot \frac{z^4}{n^4}\left(\frac{n}{k} - \frac{3}{4}\right) + \text{higher orders of } \alpha^2 \qquad (1)$$

where R is the Ryderberg constant varying with the mass of the nucleus , z the atomic number of the nucleus, n the total quantum number, k the azimuthal quantum number, and α ($= 2\pi e^2/ hc$) the fine structure constant. The first term corresponds to Bohr's[5] formula derived from non-relativistic mechanics. In the case of small z , the relativity correction for the term value becomes :

$$T = R\alpha^2 \cdot \frac{z^4}{n^4}\left(\frac{n}{k} - \frac{3}{4}\right) \qquad (2)$$

when the terms after the second of equation (1) are neglected.

With the introduction of electron spin another quantum number j is used to specify the state of an atom in addition to n and k (or ℓ), where

$$j = \ell \pm \frac{1}{2} \quad \text{and} \quad \ell = k - 1$$

From a quantum mechanical treatment, Heisenberg and Jordan [6] have calculated both the relativity correction and the correction due to spin-orbit interaction. The combined result is

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4. Sommerfeld, A., Ann. d. Physik 51 1, (1916) or Atomic Structure and Spectral Lines p. 467 (1923).

$$T = R\alpha^2 \cdot \frac{z^4}{n^4} \left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4} \right) \qquad (3)$$

This is exactly the same as Sommerfeld's formula (2), except that j is to be used instead of k. Later, Gordon[7] obtained the same result from Dirac's theory of the hydrogen atom which is now regarded as the accepted method of derivation. Since there are in general two values of k (or l) corresponding to each j, each energy level for a given j is really a coincidence of two levels for $l = j + \frac{1}{2}$ and $l = j - \frac{1}{2}$. Thus more possible transitions are permitted by the selection rule $\Delta k = \pm 1$, than when each level corresponds to one value of k only. As a matter of fact, the component III_d found by Paschen in the line 4686 is not permitted by such a rule, $\Delta k = \pm 1$, used in connection with formula (2) but is permitted by the selection rules $\Delta l = \pm 1$ and $\Delta j = 0, \pm 1$ used in connection with the formula (3). This seems to be a triumph of the quantum mechanical theory over the older quantum theory.

The structure of the line 4686 consists of the transitions between the levels for $n = 3$ and those for $n = 4$ permitted by the selection rules $\Delta l = \pm 1$, $\Delta j = 0, \pm 1$, as shown in Fig. 1. The coincident levels (of same j but different l) are drawn separately in order to be distinctly visualized. The values ΔT are calculated by taking $R_{He} = 109722.4$[8] and $\alpha^2 = 5328 \cdot 10^{-5}$[9]

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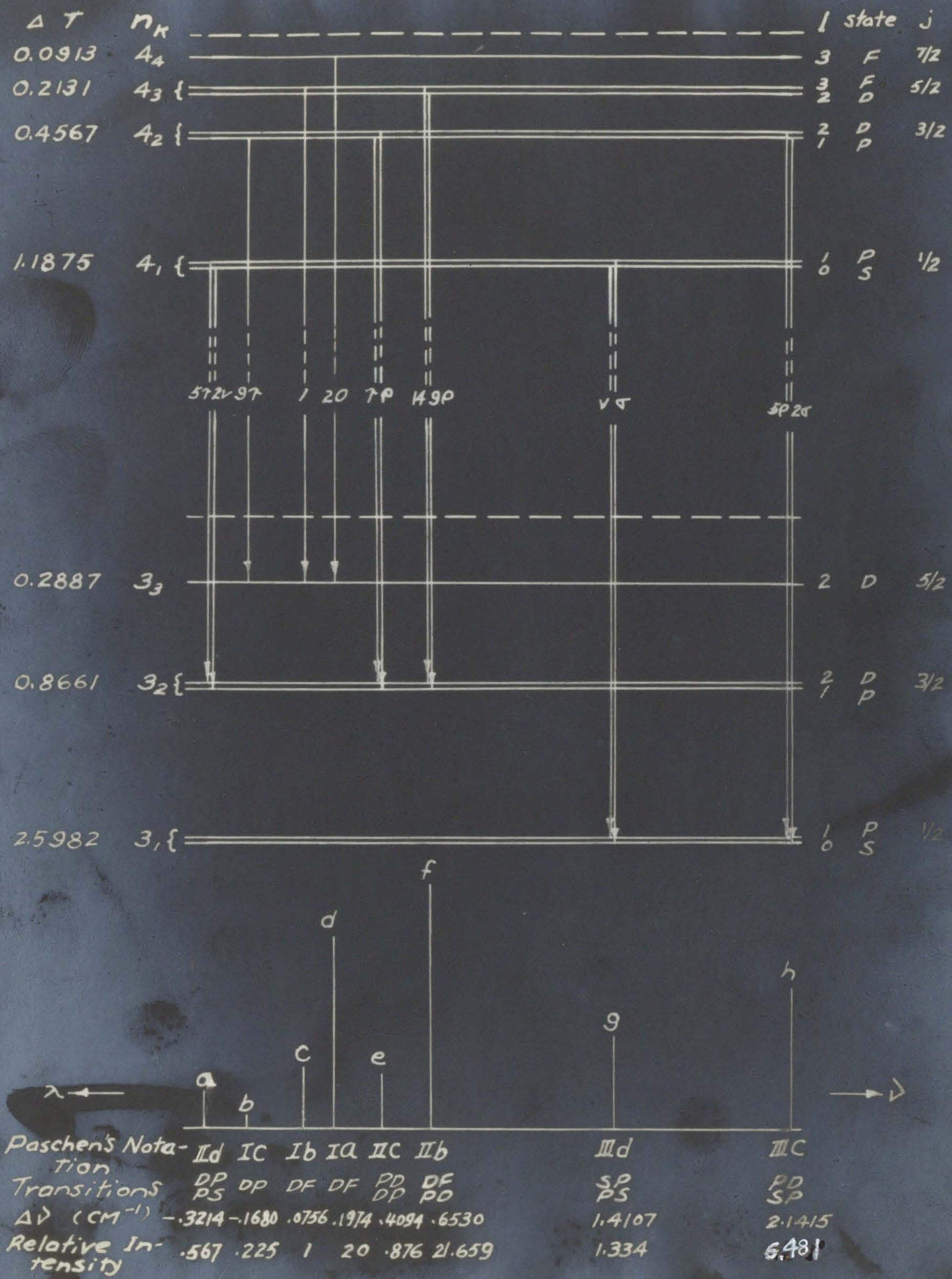
5. Bohr, N., Phil. Mag. 26 1, 476 (1913)

6. Heisenberg, W., and Jordan, P., Zeits. f. Phys. 37 263 (1926)

7. Gordon, W., Zeits. f. Phys. 48 11 (1928)

8. Houston, W.V., Phys. Rev., 30 60 (1927)

9. Birge, R.T., Phys. Rev. 48 918 (1935)



ΔT
n_K
l state j
0.0913 4_4 3 F 7/2
0.2131 4_3 3 F 2 D 5/2
0.4567 4_2 2 D 1 P 3/2
1.1875 4_1 1 P 0 S 1/2
5↑2ν 9↑
1 20
↑ρ
14 9ρ
ν σ
5ρ 2σ
0.2887 3_3 2 D 5/2
0.8661 3_2 2 D 1 P 3/2
2.5982 3_1 1 P 0 S 1/2
a b c d e f g h
λ ←
→ ν
Paschen's Notation IId IC Ib IA IIC IIb IIId IIIC
Transitions DP/PS DP DF DF PD/DP DF/PD SP/PS PD/SP
Δν (cm⁻¹) -.3214 -.1680 .0756 .1974 .4094 .6530 1.4107 2.1415
Relative Intensity .567 .225 1 20 .876 21.659 1.334 6.481

Fig. I.

It is easily seen that from the 13 possible transitions one can get 8 components which are denoted by a, b, c, d, e, f, g, and h. Below the energy diagram, these components are drawn according to their theoretical positions calculated from ΔT, and their relative intensities are roughly represented by the heights (see the following section). The states and the transitions between them, resulting in 5 different multiplets (D-F, P-D, S-P, D-P, and P-S) are also shown.

II. Relative Intensities.

Theoretical relative intensities of the fine structure of 4686 were first calculated by Sommerfeld and Unsold [10] from Shroedinger's form of wave mechanics, and the same result was obtained by Saha and Banerji [11] from Dirac's theory in the form developed by Darwin and Weyl. According to them, the relative intensities of the components of a multiplet follow the sum rule, namely :

(DF)	(PD)	(SP)	(DP)	(PS)
1 : 20 : 14	1 : 9 : 5	1 : 2	5 : 9 : 1	2 : 1

and the relative intensities of the 5 multiplets are :

(DF)	(PD)	(SP)	(DP)	(PS)
35	15ρ	3σ	15τ	3ν

where $\rho = 0.851$, $\sigma = 1.113$, $\tau = 0.025$, $\nu = 0.221$. In Fig. 1, the intensities of each transition are indicated in the middle of the above diagram, and the numerical values of the intensities of the components are given at the bottom.

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10. Sommerfeld, A., and Unsold, A., Zeits. f. Phys. <u>36</u> 259 (1926)
<u>38</u> 237 (1926)

11. Saha, M. and Banerji A.C., Zeits. f. Phys. <u>68</u> 704 (1931)

PREVIOUS EXPERIMENTS.

The line 4686 was first discovered in the spectrum of ζ-Puppis and was thought to be a hydrogen line. Fowler in 1912, obtained a series of lines, among which 4686 was the strongest, by passing strong condensed discharges through helium tubes containing hydrogen as an impurity, but they were not considered as helium lines until Bohr[5] founded his theory of the hydrogen atom. Evans[12] first obtained the line 4686 from the spectrum of a helium tube free from hydrogen, thus giving strong support to Bohr's theory. However, Fowler[13] pointed out that the lines observed by him were not accurately represented by Bohr's formula. Then Bohr[14] showed that the deviations could be accounted for by taking into account the mass of the nucleus which had been assumed to be infinitely large in comparison with the mass of an electron in his original paper; and Fowler[14] admitted the close agreement between the theoretical and the observed values. Afterwards, Fowler undertook a further investigation[15] and concluded from the analogy with spark spectra of other elements such as Mg. etc. that the series beginning with the line 4686 and the associated Pickering series are enhanced lines of helium. Additional evidence that the line 4686 is an ionized helium line in conformity with Bohr's theory was also given by Stark, Rau, and Evans[16].

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12 Evans, E.J., Nature 92 5 (1914)

13 Fowler, A., Nature 92 95

14 Nature 92 pp. 231-232 (1914)

15 Fowler A., Roy. Soc. Phil. Trans. A 214 225 (1914); Roy. Soc. Proc. Ser. A 90 426 (1914).

16 Evans, E.J., Phil. Mag. 29 284 (1915) and the references there.

Before Sommerfeld's theory of fine structure was published, Evans and Croxson[17], hoping that a knowledge of the structure of the line 4686 would serve as a guide in testing different hypotheses for explaining the doubling of the hydrogen lines, investigated this structure with an echelon spectroscope. They found the line emitted by a condenser discharge broad and diffuse, but it bec me very sharp when excited by direct current. All the photographs showed that this line was a close doublet with components of nearly equal intensity. The separation measured from the best photographs was 0.094 Å. After Sommerfeld's paper[18] on fine structure was published, in which he quoted certain unpublished measurements by Paschen on the fine structure of the 4686 series and some others, Evans and Croxson re-examined all their photographs and found a third component.

A few months later, Paschen[1] published his work and gave strong support to Sommerfeld's theory. Instead of an echelon which has the disadvantage of overlapping of different orders, he used a large concave grating having a dispersion 2.3 Å / mm. The wave lengths were measured with reference to other helium lines calibrated with iron standard of Buisson and Fabry, and the observed values were correct within a few thousandths of an Angstrom. The best resolved line of all those observed was the line 4686 and the results were as follows .

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17. Evans, E.J., and C. Croxson Nature <u>97</u> 56 March (1916)

18. Sommerfeld, A., Bay. Akad. d. Wiss. Munich (1916).

Spark Picture.

Theor.	Int.	λ_{air}	Probable Error
I_d	0	4686.050*	0.01
II_d	0.5	5.926	0.004
I_a, II_c	7	5.803	0.003
I_a	7	5.808	0.003
II_c	6	5.774	0.006
$II_{a,b}$	6	5.700	0.005
II_b	5	5.717	0.01
II_a	6	5.687	0.006
III_d	0	5.549*	0.005
III_c	0.5	5.397	0.005
$III_{a(b)}$	4	5.307	0.002

* hardly observable.

Continuous Current Picture.

Theor.	Int.	λ_{air}
I_d	0.5	4686.028*
I_a, II_d	2	5.905
$I_{a,b}$ II_c	7	5.809
$II_{(a)b}$	7.5	5.703
III_d	1	5.549
III_c	3	5.388

* faint but still visible.

Probable error of λ smaller than 0.002 A and that of $\Delta\lambda$ smaller than 0.001 A.

He also calculated the theoretical wave lengths of the 12 components (no selection rule used) :

Scheme 4686.

	m, m'	4, 0	3, 1	2, 2	1, 3
n, n'		a	b	c	d
3, 0	I	<u>4685.810</u>	5.837	5.890	<u>6.050</u>
2, 1	II	<u>5.684</u>	<u>5.710</u>	<u>5.764</u>	<u>5.924</u>
1, 2	III	<u>5.304</u>	5.331	<u>5.384</u>	<u>5.544</u>

where n, n' denote the azimuthal and radial quantum numbers respectively, (same with m, m'). The underlined components were observed in the spark picture, while in the continuous current picture, fewer components were observed. Comparing these two groups of components, one may notice that (1) the component 5.307 did not appear in the continuous current picture (2) the component 5.703 was the strongest in this picture (3) the components 6.050 in the first picture and 6.028 in the second were very weak, and (4) the relative intensities of the components were different in the two pictures. From the $\Delta\lambda$ of these components, Paschen calculated the separation of the hydrogen doublet :

$$\Delta\nu_H = 0.3645 \pm 0.0045 \ \mathrm{cm}^{-1}$$

which is exactly the value required by Sommerfeld's theory. However, this value deviates from those obtained from the direct measurements of the hydrogen doublet by other investigators. In view of this deviation, Kunze[19] measured the fine structure of 4686 with four different arrangements of apparatus and four different light sources and found it to be a doublet or a triplet. His results of five measurements out of the sixteen possible arrangements of sources and apparatus may be summarized as follows (in Å) :

	Hollow cathode tube with condensed discharge		Hollow cathode tube with continuous current.	
	$I_{ab} - II_{ab}$	$I_{ab} - III_{ab}$	$I_a - II_b$	$I_a - III_c$
Hilger echelon	.0995	.4991	----	-----
Goerz echelon	.117	.4955	.1025	.4221
Hilger Lummer plate	.108	----	.1021	-----

He also concluded that the component III_{ab} was not properly a part of this group of lines.

Leo[2] , in view of both the above mentioned deviation and the occurrence of a helium band at 4686, investigated the fine structure of the first

19 Kunze, P., Ann. d. Phys. 79 610 (1926)

two members of the Fowler series (4686 and 3203) by means of a high dispersion grating. Of the line 4686 the structure was found as follows :

Spark Picture		Continuous Current Picture		
Int.	λ_{air}	Int.	λ_{air}	
0	4685.931	0	4685.923	poorly localized
7	.812	6	.813	
7	.710	6	.709	
1	.559	1	.557	
3	.412	3	.395	
5	.309 sharp			

The component 4685.309 (III_a), which was supposed as due to the transition $4_4 - 3_1$ by Paschen, was also found only in the spark spectrum. Leo ascribed it to a helium band, in agreement with other investigations. He calculated $\Delta \nu_H$ from the mean separations in the continuous current picture and in the spark picture, and got a smaller value :

$$\Delta \nu_H = .3459 \pm .005 \text{ cm}^{-1}$$

He also pointed out that the continuous current picture and the spark picture differ from each other only in line breadth and intensity graduation and not in the number of components.

After the papers by Sommerfeld and Unsold [10] were published, Paschen[20] re-examined his previous work by taking photometer curve of the continuous current picture, and recalculated $\Delta \nu_H$ (the theoretically prohibited component III_a which had been taken as a most important factor in the earlier calculation was rejected in these later calculations.

20. Paschen, F., Ann. d. Phys. 82 689 (1927)

	From $\Delta\lambda$ in ocular measurement	From $\Delta\lambda$ in photometer measurement	From $\Delta\lambda$ in Leo's measurement
$\Delta\nu_H$ in cm^{-1}	.3603	.3633	.3585

The last result was obtained by taking Leo's measurement in the continuous current case only. As to relative intensities, he pointed out the general agreement of his measurement with the theoretical calculation of Sommerfeld and Unsold. Finally, he concluded that his measurement of the absolute wave length in the continuous current picture is in accord with theory and that the separations lead to the theoretical value of $\Delta\nu_H = 0.3645\,cm^{-1}$

Houston[8] first used a Fabry-Perot interferometer for measurement of the line 4686. As he was interested in the determination of R_{He}, only the two strongest components I_a, II_b were measured (4685.803 and 4685.703) He concluded that these values are not in essential disagreement with the general scheme of Paschen's measurement. His value of R_{He} calculated from such value is considered to be the most accurate one.[21]

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21. Birge, R. T., Phys. Rev., Supplement 1, 1 (1929).

EXPERIMENTAL METHOD.

I. APPARATUS.

In Fig. 2, is shown the arrangement of apparatus which consists of a discharge tube T, a 2-meter spectrograph with prism P, a Fabry - Perot interferometer I, the photometric arrangement C_2 - Hg (on the right side of

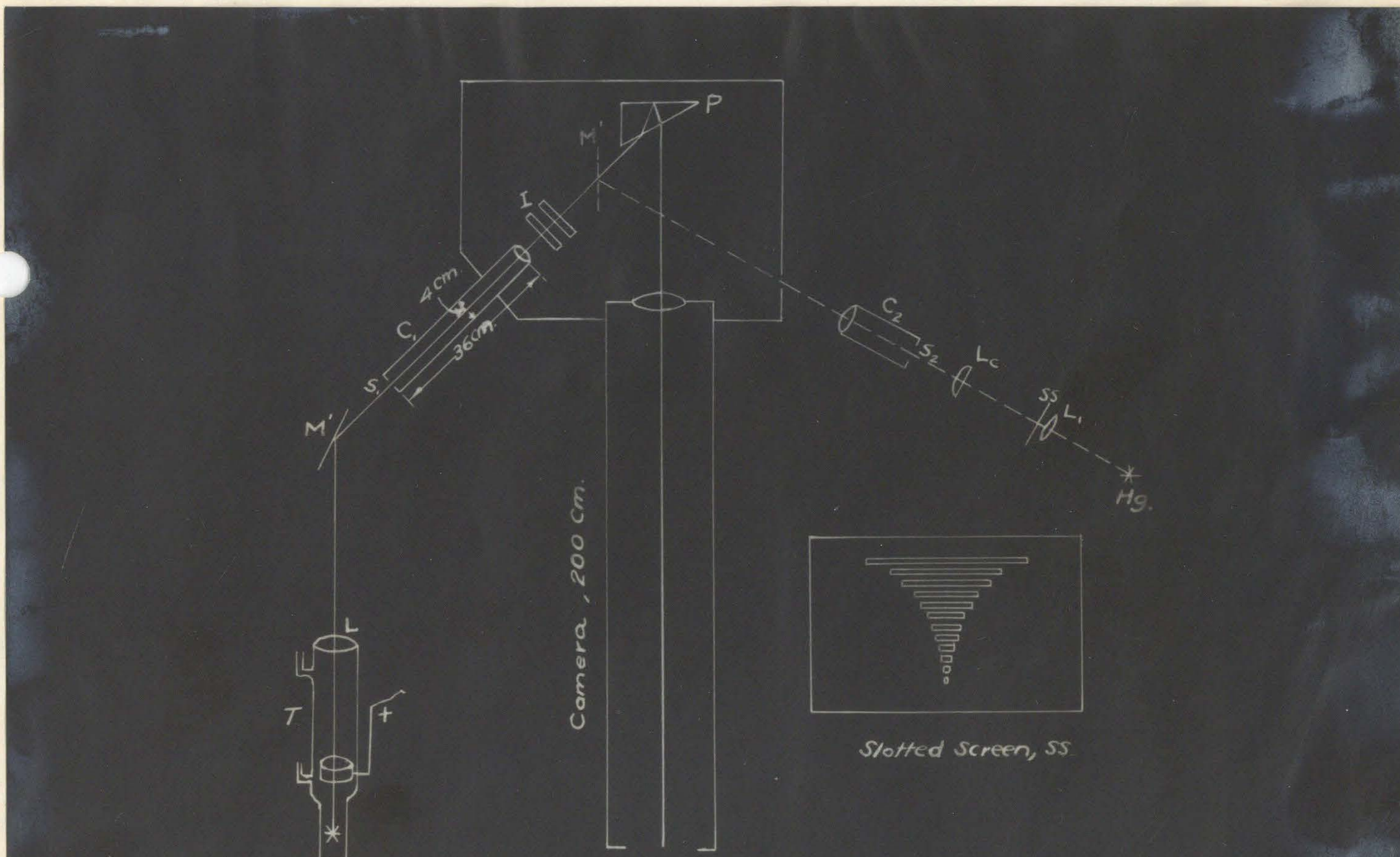


Fig. 2

the diagram) and the circulation system (see Fig. 3). The discharge tube stands vertically and the light from its cathode is reflected by a mirror M to the collimator slit. M' is another mirrow, to be put in place only when intensity marks are taken. Let us describe the different parts in the following sections.

A. Discharge Tube.

This is a hollow cathode discharge tube cooled by liquid air. The cathode made of copper forms the bottom of the tube (Fig. 3), its tapering top being sealed into the glass tube. It is immersed in liquid air almost to its top. The anode is an aluminum ring just above the cathode and is of such a size that it will not intercept the light from the cathode to the window of the tube. It is also cooled considerably by the liquid air vapor. The distance between the anode and cathode is made as small as possible because it was found that in an old tube so constructed that the distance between the electrodes was very great, the space between them was strongly luminous, thus probably affecting the relative intensity of the line 4686 with respect to the other helium lines. As a matter of fact, this new tube gives the line 4686 much brighter than the old one both in absolute intensity and in relative intensity with respect to the other lines. A lens L is sealed on the top, condensing the light from the cathode upon the collimator slit and serving as a window at the same time. The length of the tube and the size of its top are so designed that when the cathode and slit are in conjugate positions, the solid angle subtended by the top of the tube at the slit is nearly equal to that subtended by the collimator lens. The diameter of the cathode is such that the image on the slit is sufficiently large to secure fairly uniform il-

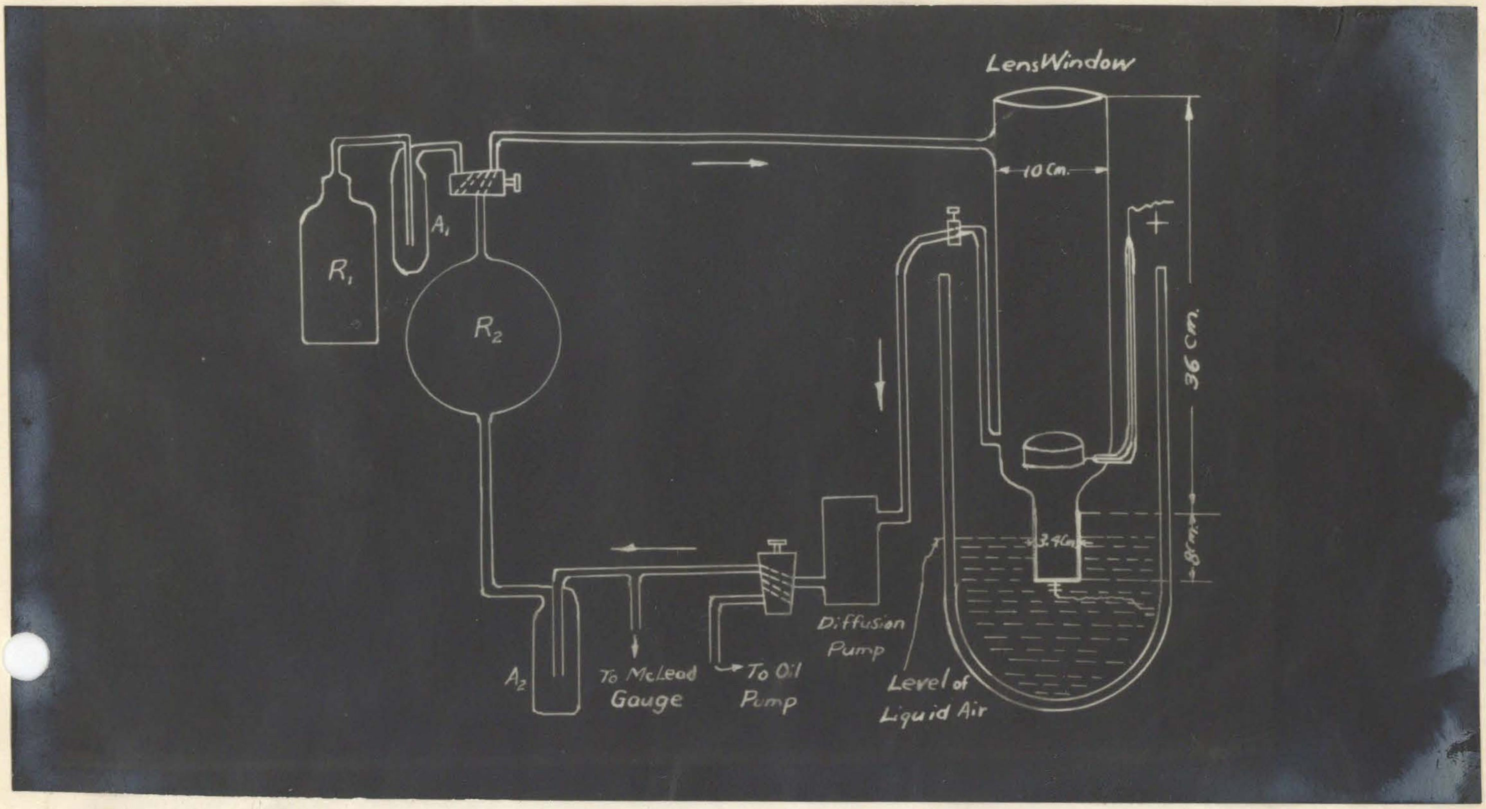


Fig. 3

lumination. The tube is run with a d.c. generator of high voltage in series with a lamp bank resistance.

B. Circulation System.

In Fig. 3, R_1 is a bottle containing helium at a pressure of several cm. to be supplied to the discharge tube. Before being admitted to the circulation system which contains a bulb R_2 as reservoir, the helium is first partially purified with charcoal A_1 cooled by liquid air. It is then purified by the charcoal absorber A_2 during circulation which is produced by the diffusion pump. The spectrum is quite pure as the circulation is going on or if it has been going on for quite a while before discharge. It is found that as the pressure is reduced the relative intensity of the line 4686 becomes greater as compared to the other lines, but there is a lower limit of pressure (about .2 - .3 mm.) below which the discharge ceases to start. During discharge the helium is absorbed so that finally the discharge stops when the pressure drops to about .1 mm. or below. Hence before each exposure, helium is usually supplied from R_1 to such a pressure that the discharge can be started.

C. Fabry-Perot Interferometer.

The interferometer plates are 11 cm. in diameter and are half silvered on the inside surfaces by evaporation. They are held in a hollow cylinder with a separator which consists of 3 invar posts held in a brass ring. The parallelism between the silvered surfaces is secured by adjusting the pressure against each other by means of three springs compressed with screws.

D. Spectrograph.

This is a prism spectrograph with a half prism and a 2-meter camera. The ratio of the focal lengths of the collimator lens and of the camera lens is about 5.5 so that the slit image on the plate is wide enough to lessen the effect of photographic grain. The opening of the camera is 8.2 cm. high so that the maximum inclination of the beam is only 4.1 / 200, thus justifying the substitution of cos θ by $1 - \theta^2/2$ in our calculation.

E. Photometric Arrangement.

Intensity marks were put on each plate by means of the arrangement shown on the right of Fig. 2. SS was a screen with twelve slots of lengths from 0.5 to 48 mm. and was illuminated by a mercury discharge tube Hg and a condensing lens L_1. A cylindrical lens L_c situtated with axis horizontal at the conjugate position of Hg, formed an image of SS at the collimator slit S_2 The light was reflected by a mirror M' to the prism and 12 marks, whose intensities are proportional to the length of the corresponding slot, were taken on the plate. The marks from the line 4916 which is the closest Hg line to the line 4686 were used in taking the microphotometer deflections which were then plotted against their relative intensities to give a curve for reducing the microphotometric deflections x of the line 4686 to intensities.

II. Experimental Difficulties and the methods of overcoming them.

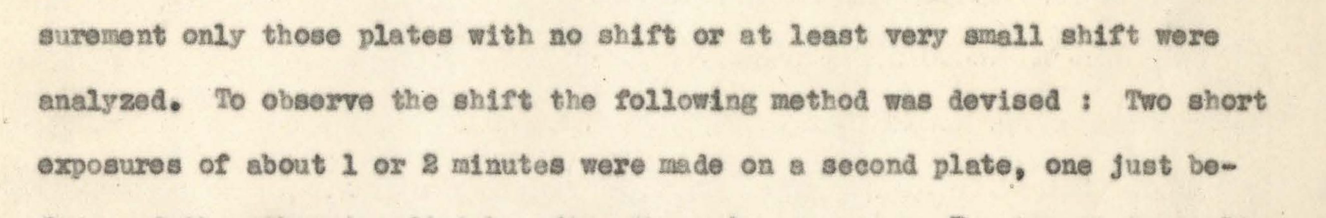
A. Copper Lines from Cathode.

It was found that the spectrum of this type of discharge tube contained a number of lines due to ionized copper. One of the copper lines had a wave-length 4682 so close to the line 4686 that the prism could not separate them. To avoid this confusion of the interferometer pattern, it was found satisfactory to aluminize the inside of the cathode by evaporation. Afterwards not copper but aluminum lines appeared, which however did not interfere with our experiment but had some use (see sect-ion d p.25).

B. Shift of Fringes.

The change of interferometer separation resulting in a shift of fringes seemed to be due to mechanical causes rather than thermal. For it can be shown from the coefficient of expansion of invar that a change of 1° C. in temperature will cause a change of about 0.004 - 0.012 order of interference for the separators 2-6 mm, while the thermometer reading within the box containing the interferometer showed a variation rarely exceeding 0.1° C. during the whole period of exposure which was about one hour. Moreover, it was observed that in some cases, the shift appeared when the thermometer reading was constant, while in others, it did not appear when the reading showed a variation as large as 0.1° C.. Hence attention was only paid to the mechanical causes, to reduce which felt and sponge rubber pads were placed underneath the legs of the spectrograph and the springs against the interferometer plates were usually pressed tightly. Although these precautions were of some value, the shift still occurred in some pictures. To insure good x results of measurement only those plates with no shift or at least very small shift were analyzed. To observe the shift the following method was devised : Two short exposures of about 1 or 2 minutes were made on a second plate, one just before and the other immediately after the main exposure. The two pictures lie side by side on the same plate and are easily compared (Fig. 4). That this could be done was due to the fact that the plate was a little narrower than the plate holder so that it could be displaced sidewise after the first exposure was made. Fig. 4 shows the results of this observation in two cases.

Fig. 4



A. No shift.

B. Sufficient shift to cause the exposure to be discarded.

C. Circular fringes associated with other strong lines.

At a first sight we should expect the image formed by the interferometer of a narrow strip source such as a slit to consist of only a narrow section of the concentric circular fringes like those shown in Fig. 4., but as a matter of fact, the whole circles with a very bright narrow section appeared on the plate when the exposure was sufficiently long. These circles due to the adjacent strong lines such as 5015 and 4471 crossed the line 4686 and therefore introdused disturbance. These circles were due to scattering of light, as could be demonstrated by the fact that they became very bright on breathing over the interferometer plate and gradually faded down as the moisture evaporated away. In spite of very careful cleaning of the interferometer plates and even replacing by a new pair of plates* the circles were never eliminated. In the attempt to reduce the intensity of these fringes the Wratten light filter Aero. No. 1 has been used. This transmitted only 4% of the light shorter than 4500 A and thus should eliminate the circular fringes from the line 4471. At the same time the Eastman 40 photographic plate was selected, because its sensitivity for the long wave-length is small and thus the circular fringes from the line 5015 appeared less strongly. However, the filter also absorbed about 33% of the line 4686 so that the exposure was necessarily prolonged and the circular fringes from the line 5015 were proportionally strengthened. The disturbance of these fringes is the greatest source of error in this experiment, but it is believed not very serious as no apparent difference can be observed from the comparison of the microphotometric tracings of two pictures one taken with the filter used and the

* The author wishes to thank Mr. Julius Pearson for the polishing of the new plates.

other taken without it.

D. Halation.

Lines usually spread on both sides when exposed sufficiently long. Since the line 4713 is quite close to 4686, its spreading will interfere with the latter. To avoid this, the plate is painted with "Eastman Opaque" on the glass surface.

III. Method of Measurement.

A. Formulas of the Fabry-Perot Interferometer.

(a) The optical path difference between two successive rays which have been reflected twice is $2d \cos\phi$ where d is the separation between the plates and ϕ is the angle of incidence. For a maximum intensity, we have :

$$n\lambda = 2 d \cos\phi \qquad (4)$$

where n is an integer. Let f be the fractional oder of interference at the center of the fringe system and i its integral order, then :

$$(i + f)\lambda = 2d \qquad (5)$$

For the mth and the (m+1)th fringes, we have from (4),

$$(i - m + 1)\lambda = 2d \cos\phi_m \qquad (6)$$

and

$$(i - m)\lambda = 2d \cos\phi_{m+1} \qquad (7)$$

Taking their difference we get :

$$\lambda = 2d(\cos\phi_m - \cos\phi_{m+1})$$

or

$$\lambda = d(\phi^2_{m+1} - \phi^2_m) \qquad (8)$$

From (5) and (7) and substituting the value of λ from (8) we get :

$$f = \frac{\phi^2_{m+1}}{\phi^2_{m+1} - \phi^2_m} - m \qquad (9)$$

Let X_m be the distance of the mth fringe from the center of the pattern, we have, since X is proportional to ϕ ,

$$f = \frac{X^2_{m+1}}{X^2_{m+1} - X^2_m} - m \tag{10}$$

which enables us to calculate f from the measured radii of the fringes, or any multiple of them. λ is then calculated from (5) if d is known.

(b) Let ν be the wave number of a line, then from (5) we have :

$$\nu = \frac{1}{\lambda} = \frac{i + f}{2d}$$

from which it is easily derived that for two lines of nearly equal wave-length, their separation is given by

$$\Delta\nu = \frac{\Delta f}{2d} \tag{11}$$

(c) From equation (8) we see that $\phi^2_{m+1} - \phi^2_m$ and therefore $X^2_{m+1} - X^2_m$ is a constant for a given λ. This latter quantity is called the period of the interferometer pattern, because if the intensity be plotted with X^2 as abscissa, it will be a periodic function of the abscissa with period equal to $X^2_{m+1} - X^2_m$ (Fig. 5). This property of periodicity suggests Prof. Houston[22] to express the pattern as a Fourier series. Let

$$\theta = 2\pi \cdot \frac{2d}{\lambda} = 4\pi d\nu = 2\pi(i+f)$$

called angular order of x interference, be taken as the variable, then the pattern can be expressed as :

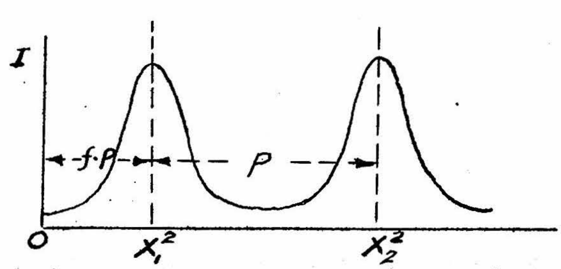


Fig. 5

$$I_1(\theta - \theta_0) = 1 + 2\sum_{n=1}^{\infty} \cos n(\theta - \theta_0) \cdot r^n \tag{12}$$

22. See Reference 3.

where θ_0 is the position of maximum intensity and r is the reflecting power of the silvered surface of the plates. This is true only for strictly monochromatic light. When we take into account the Doppler broadening and the natural broadening, the pattern for a single line may be expressed by [23] :

$$I(\theta, \theta_0) = B_0' + \sum_{n=1}^{\infty} (B_n' \cos n\theta + A_n' \sin n\theta) \tag{13}$$

where :

$B_0' = \pi^{3/2}$,

$B_n' = 2\pi^{3/2} r^n e^{-n^2/4\alpha - n\beta} \cos n\theta_0$,

$A_n' = 2\pi^{3/2} r e^{-n^2/4\alpha - n\beta} \sin n\theta_0$,

and $\alpha = \frac{M}{2RT}\left(\frac{\lambda}{4\pi d}\right)^2$, which is due to Doppler effect.

β = natural half intensity breadth, which is .56 10^{-4} Å (or $4\pi d \times \frac{5800}{\lambda^2}$ radians where λ is in Å) in the classical thoery and about the same order of magnitude in the quantum theory,

M = molecular weight of emitting gas ,

R = gas constant,

T = absolute temperature.

Since θ may be changed by any integral multiple of 2π in the trigonometrical functions we can count θ from the abscissa which corresponds to any maximum intensity. The the formula (13) is simplified to :

$$I(\theta) = I_0 \left[1 + 2\sum_{n=1}^{\infty} r^n e^{-n^2/4\alpha - n\beta} \cos n\theta \right] \tag{14}$$

If there are several components of nearly the same wave-length, the pattern must be a superposition of patterns given by (14) with I_0 proportional to their intensities and with maximum positions at intervals pro-

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23. See formulae (1) - (4) of reference 3.

portional to their separations, since

$$\Delta\theta = 2\pi\Delta f = 2\pi\cdot 2d\Delta\nu \tag{15}$$

(B) Procedure.

a. Exposure.

Before making an exposure, the helium was circulated through the charcoal until all traces of air and other impurities had disappeared from the spectrum. as observed with a direct vision spectroscope. The current used was around 140 m.a., as too small a current would necessitate long exposure and too large a current would increase the line breadth. The time of exposure was 40 minutes for the plate 118, 50 minutes for the plate 134 and 60 minutes for the other two plates. As mentioned in Section II (B) p. 17, two short exposures were usually made on a second plate to detect a shift of the fringes. This plate also serves for the measurement of other helium lines in order to determine the interferometer separation, for the intensity of other lines on the main plate was so strong that the fringes were not clearly defined and therefore could not be measured accurately.

b. Microphotometer Tracing and Master Curve.

For each plate to be measured, a microphotometric tracing was taken of the line 4686 with the Koch - Goos photoelectric microphotometer (A. Kruss Co., Hamburg). The magnification was usually 6. A smaple of the line and of its tracing is shown in Fig. 6 (here the tracing is of magnification 1), where four components are distinctly seen. Both coordinates of the tracing were then measured with the comparator, and the ~~f~~ deflections (ordinates) were converted into intensities by means of the deflection-intensity curve

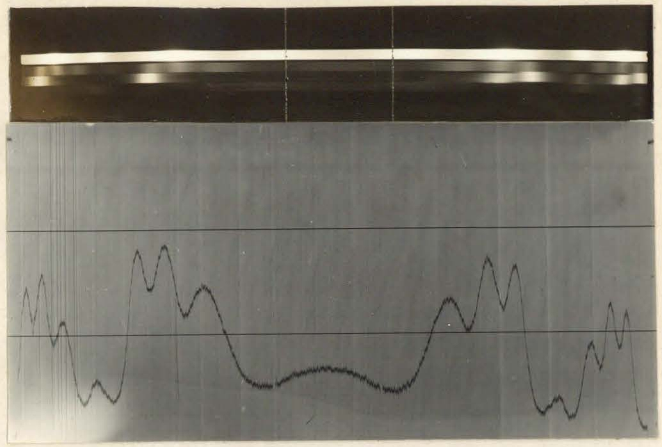

Fig. 6. Plate 134 and the microphotometer tracing.

derived from the tracing of intensity marks. These intensities were plotted against X^2 i. e. the square of the horizontal distance from the center of the tracing. Because of the non-uniformity of illumination on the collimator slit, the successive orders had not equal maxima and minima. So a curve of correction factors was derived from them, and the intensitieswere reduced to the case of uniform illumination. Then the mean was taken of intensities for succesive orders on both sides of the center and the so-called "master curve" was obtained which covers a range of one order. The number of orders taken on each side of the center depends on the separation d which determines the value of the period. In the plate 118 three orders were taken, in the plate 124 2 orders were taken, in the plate 126 and in the plate 134 one order could be taken.

c. Composition of Theoretical Pattern.

The method of analysis of this experiment was to construct a theoretical pattern by assuming relative intensities and positions of the eight components and to compare it with the master curve. The process was first x to calculate the intensity distribution of a single component, by means of the formula (14) in which r and α were found from another single line (see the following section) and β was given the classical value. It is so small that it is immaterial whether the classical or the quantum mechanical value is taken. Of the infinite series only the first few terms were used, 8 terms in the case of largest α and 12 terms in the case of smallest α, because all the following terms omitted contribut at most 0.001 of the maximum intensity which is within experimental uncertainly. The values of θ for which the function was calculated were taken at intervals of 3° or 1.5° depending upon the period p. The assumed relative intensities gave the I_0 's. The position of each component was calculated from the assumed separations by means of equation (15). For convenience of calculation, each position was usually replaced by the nearest value which is a multiple of 3°, 2° or 1.5°. The error thus introduced was still within experimental uncertainly. After getting the intensities for all components, the theoretical pattern was easily drawn and compared with the x experimental master curve. After several trials, a pattern was finally obtained which fitted the master curve best. (See Fig. 7) It is in this way that the intensities I and positions P given in Table I were obtained for each plate.

d. Determination of α and r

Since the structure is very complex, it would be extremely tedious if besides the intensities and positions we have to change also α and r in trying to get a pattern to fit the master curve. Hence it is desirable to determine α and r, from another single line. For this purpose, an aluminum

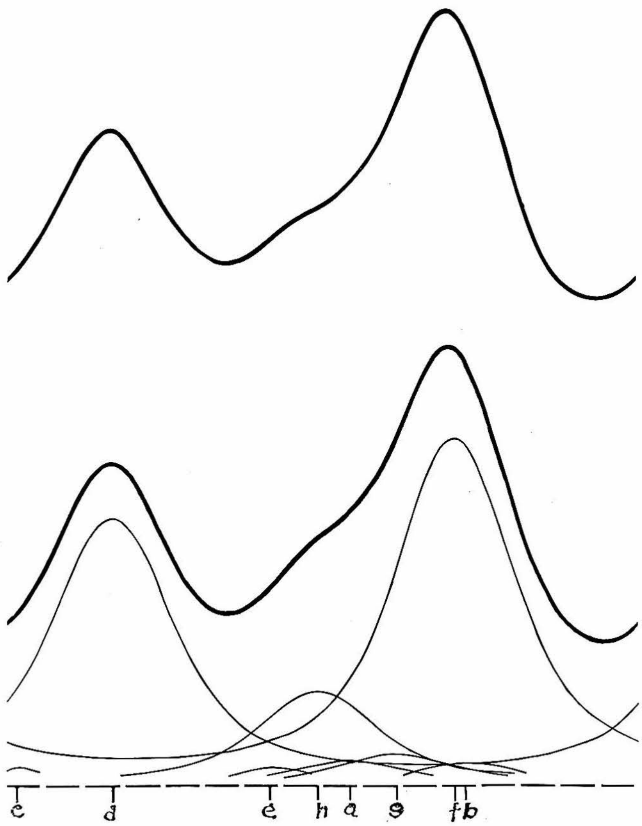

Fig. 7 A. For Plate 118

In each of the figures the upper thick curve is the experimental master curve, the lower thick curve is the theoretical one obtained by summing the thin curves representing the eight components a,b,c,d,e,f,g,h. Of the component curves only a part of the whole period is shown, except those of the two strongest components d,f.

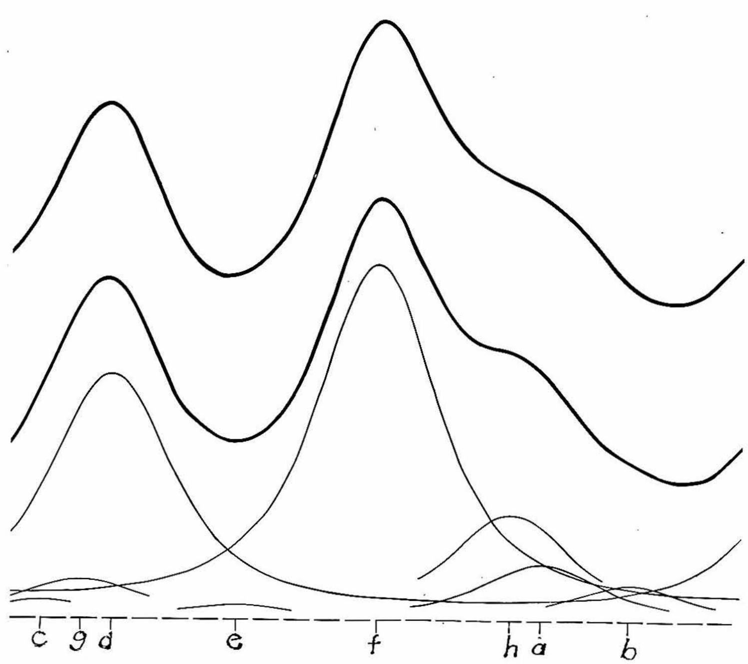

Fig. 7 B For Plate 124.

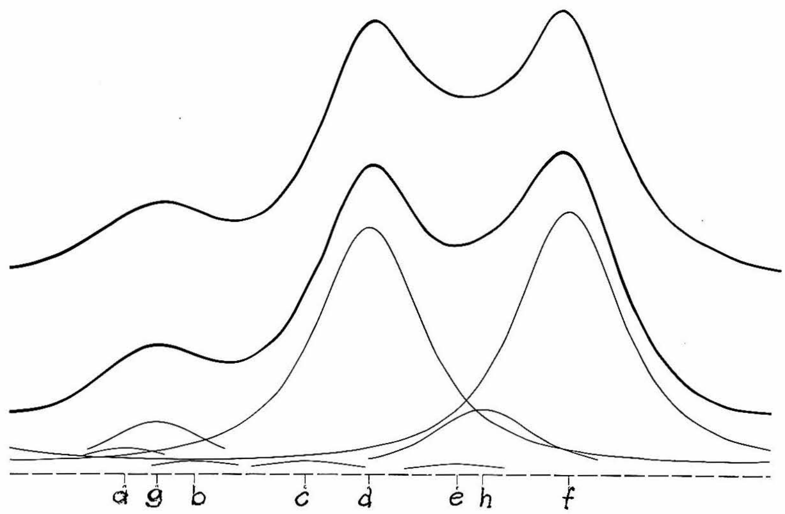

Fig. 7 C. For Plate 126

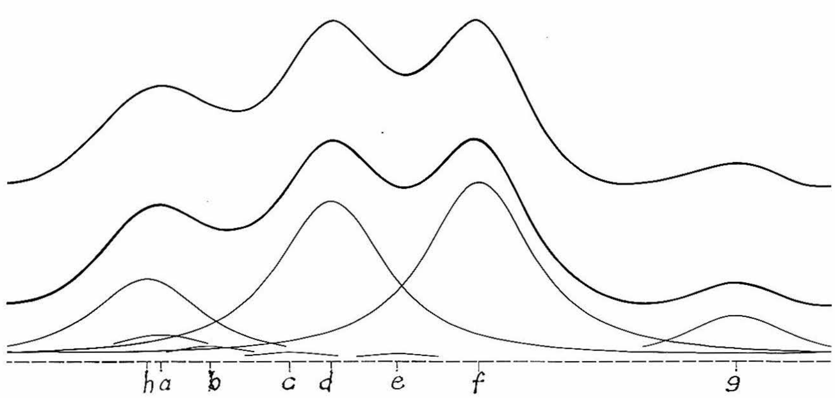

Fig. 7 D. For Plate 154.

line 4663 of about the same intensity and wave-length as 4686 was selected.

e. Determination of d.

The interferometer separation d is determined from other helium lines 5048, 5016, 4922, 4713, 4471, 4437, and 4388. Let λ_0 be the wave-length of the standard line which was taken to 5015.6750 I.A., and f_0 its fractional order of interference. Similarly, λ_1, λ_2 .... and f, f_2 .... denote those of the other lines. Then we have:

$$(i_0 + f_0)\lambda_0 = (i_1 + f_1)\lambda_1 = (i_2 + f_2)\lambda_2 = \ldots\ldots$$

from which we get:

$$i_1 + f_1 = (i_0 + f_0)\frac{\lambda_0}{\lambda_1}, \quad i_2 + f_2 = (i_0 + f_0)\frac{\lambda_0}{\lambda_2} \quad \ldots\ldots \quad (16)$$

From the approximate value of d measured with ordinary micrometer i_0 can be found approximately. Using this value and λ_m (m = 1,2, ...) the orders of interference of all other lines can be calculated by equation (16). If these calculated f_m are not equal to the observed values, other values of i_0 are tried until they are nearly equal. The last tried value of i_0 and the observed f_0 give the value of d from the equation (5) namely $(i_0 + f_0)\lambda_0 = 2d$.

f. Calculation of wave-lengths.

Wave-length is usually calculated from d, by equation (5) so that its accuracy depends upon that of d. In the last section, only a rough way of determining d was given for the change of phase upon reflection was neglected and the standard wave-lengths were given for standard conditions, (15° C. and 760 mm. of Hg) while the conditions under which the exposure was taken were usually different. To make the necessary corrections for such effects let us consider a standard line (subscript o) and a line of unknown wave-length (subscript u). If

λ_o, λ_u be wave-length at standard conditions (15° C. and 760 mm. Hg)

$\delta\lambda_o$, $\delta\lambda_u$ be their change due to variation of temperature and pressure

n_o, n_u be the observed oder of interference

δn_o, δn_u be correction to be added on account of change of phase upon reflection,

we should have the fundamental equation of the interferometer:

$$(n_o+\delta n_o)(\lambda_o+\delta\lambda_o)=(n_u+\delta n_u)(\lambda_u+\delta\lambda_u), \qquad (17)$$

or, neglecting the second order,

$$n_o\lambda_o+n_o\delta\lambda_o+\lambda_o\delta n_o = n_u\lambda_u+n_u\delta\lambda_u+\lambda_u\delta n_u. \qquad (18)$$

Since $\delta\lambda$ may be expressed in terms of δn by the relation

$$n\delta\lambda=-\lambda\delta n,$$

the last two terms on each side of (18) may be combined into one term, although their origins are different. Hence we may write :

$$n_o\lambda_o+\lambda_o\Delta n_o=n_u\lambda_u+\lambda_u\Delta n_u \qquad (19)$$

where Δ n denotes the correction for both causes. Let n_u' be a value defined by

$$n_o\lambda_o = n_u'\lambda_u \qquad (20)$$

then by subtracting from (19) we have

$$\lambda_o\Delta n_o=(n_u-n_u')\lambda_u+\lambda_u\Delta n_u,$$

or

$$n_u-n_u'=\frac{\lambda_o}{\lambda_u}\cdot\Delta n_o-\Delta n_u. \qquad (21)$$

Now from (20)

$$\lambda_u=\frac{n_o\lambda_o}{n_u'}=\frac{n_o\lambda_o}{n_u-(n_u-n_u')} \qquad (22)$$

So λ_u can be calculated from n_u and $n_u - n'_u$, since n_o and λ_o are known. The fractional part of n_u is the observed value and its integral part can be calculated from the relation $n_u\lambda_u \doteq n_o\lambda_o$ since λ_u is approximately known. To find $n_u - n'_u$, we have to use other lines of known wave-lengths $\lambda_1 \lambda_2$...... For from (21) we have :

$$n_2 - n'_2 = \frac{\lambda_o}{\lambda_2} \cdot \Delta n_o - \Delta n_2 ,$$
$$n_3 - n'_3 = \frac{\lambda_o}{\lambda_3} \cdot \Delta n_o - \Delta n_3 ,$$

....

The value on the left side of each equation is a difference between the observed values and those calculated from (16) (we must remember that i + f there corresponds to n' here) or simply the difference between the observed f and calculated f. The value on the right side is unknown, but evidently it is a function of wave-length. If these differences $n_m - n'_m$ (m = 1,2 ..) are plotted against λ_m, the difference $n_u - n'_u$ for the unknown can be found from the curve and λ_u can be calculated from (22). This value gives the wave-length directly in I.A. no matter under what conditions the exposure was made.

For the value n_o λ_o we usually calculate the product (i + f)λ for each of λ_1, λ_2..... where f is the observed value plus the correction from the curve just mentioned and is very nearly equal to the calculated value. The mean value of these products was used for the calculation of wave-lengths.

RESULTS

I. Positions and Intensities

In this experiment four plates of different interferometer separations were measured. Because of overlapping not all of the eight components could be observed. On plate 118 and 124, the two strongest components f,d were observed together with a hump which corresponded to the position of the third component h. On plate 126, a third maximum appeared which corresponded to the position of the fourth component g, while h gave neither maximum nor hump because of its position lying between the two strongest component. On plate 134, of smallest separation, the four components d,f,g,h, were manifestly observed (Fig. 7D or 6). The existence of the four weak components a,b,c,e could be inferred only by the fact that the theoretical patterns deviated too much from the master curves when they were assumed absent. Of these four, c and e could not be given too great intensities and were, therefore, always assumed to have the theoretical values throughout the analysis.

After several trials for each plate the relative intensities were found to have the values given in Table I. Of course these values are not unique ones, since the intensity of one component was so related to its neighbors that increasing the one must decrease the other in order to fit the observational master curve. For example, the component a was very close to the component h (next order) on plates 124,134, to the component g on plate 126, and was between g,h on plate 118. Thus its true intensity might be different from the values given in Table I. For the two strongest

Table I

Relative Intensities and Positions

Plate	d mm	p mm of Hg	c ma		a	b	c	d	e	f	g	h
118	5.9814	.33-.29	140	I	.07	.06	.05	.77	.05	1.00	.09	.27
				P	---	---	---	-.4526	---	0	.7593	1.4907
124	3.9794	.31-.28	145	I	.15	.10	.05	.70	.04	1.00	.11	.30
				P	---	---	---	-.4555	---	0	.7486	1.4868
126	2.9878	.33-.30	145	I	.10	.05	.05	.94	.04	1.00	.20	.25
				P	---	---	---	-.4393	---	0	.7871	1.4854
134	1.9760	.23-.18	130	I	.15	.08	.05	.89	.04	1.00	.25	.45
				P	---	---	---	-.4529	---	0	.8005	1.5167
Theoretical I					.026	.010	.046	.924	.041	1.00	.062	.299
Theoretical P, cm^{-1}					-.9744	-.8210	-.5774	-.4556	-.2436	0	.7577	1.4885

In the second column are given the interferometer separations; in the third, the pressures both at the start and at the end of exposure; in the fourth, the current. I denotes intensity, that of component f being taken as 1.00. The theoretical intensities are also given in the second row from the bottom. The theoretical positions are given in the last row, all referred to f. P denotes the position. When no value is given, it means that the position is the same as predicted by theory or assumed to be so.

components, f,d, however, the intensities could be asserted with certainty, except on the plate 124 where d was too close to g.

As to positions, those of the components a,b,c,e could hardly be determined with certainty, and they were assumed to have the theoretical values in the analysis. The components g,h were found to have separations from f greater than those predicted by theory. The two strongest components were found to have a separation in agreement with theory, except in plate 126 where d was too close to f. This was probably due to experimental error, for the maxima corresponding to this component were uncertain on the photometric tracing.

II. Wave-lengths.

Only the wave-lengths of 3 components, namely d,f,g could be calculated from the maxima on the intensity curve. Those of the two strongest components have been accurately determined and are given in Table II. They were first determined from the maxima, due corrections of fractional order of interference having been given for the change of phase upon reflection and for the deviation of temperature and pressure from standard conditions. The probable errors were estimated from the uncertainties of the positions of maxima. Then another correction due to the displacement of maxima by other components were taken into account from the theoretical patterns. The columns headed by cor. in Table II give this sort of correction. The weights were taken from the number of maxima from which the wave-lengths were calculated.

The wave-length of the component g could be calculated only from two plates. It is

$$\lambda_g = 4685.5330 \pm .0030 \text{ from plate 126,}$$

and $$\lambda_g = 4685.5266 \pm .0050 \text{ from plate 134.}$$

Table II

Wave-lengths, I. A.

Plate	λ_d				λ_f			
	from max.	cor.	λ	wt.	from max.	cor.	λ	wt.
118	4685.7030 ± .0013	−.0015	.7015 ± .0013	4	.8011 ± .0011	0	.8011 ± .0011	3
124	.7001 ± .0013	+.0012	.7013 ± .0013	3	.8031 ± .0010	−.0015	.8016 ± .0010	3
126	.7048 ± .0017	−.0029	.7019 ± .0017	2	.7962 ± .0020	.0018	.7980 ± .0020	1
134	.7052 ± .0016	−.0028	.7024 ± .0016	2	.8013 ± .0014	.0012	.8025 ± .0014	2
Mean	4685.7017 ± .0014				4685.8012 ± .0012			
$\Delta\lambda_{fd}$	0.0995 ± .0018 I.A.							
$\Delta\nu_{fd}$	.4529 ± .0082 cm^{-1}							

The probable errors were great because on plate 126 the position of the maximum corresponding to g is considerably effected by its neighboring components a,b, and on plate 134 the maximum is too broad and uncertain.

III. Computation of Rydberg Constant R_{He}.

In reducing the wave-lengths in I.A. to those in vacuum, both the formula of Meggers and Peters[24] and that of Bender[25] were used.

24) Meggers & Peters, Nat. Bur. Stand. Bull. 14, 697 (1918).
25) David Bender, Phys. Rev. 54, 179 (1938).

From the wave members the values of R_{He} were calculated by the formula

$$\nu = \frac{1}{\lambda_{vac}} = 4\, R_{He}\left(\frac{1}{3^2} - \frac{1}{4^2}\right) + \text{relativity correction.} \qquad (23)$$

Table III. Computation of R_{He}

1. $(u - 1)\cdot 10^7$ = 2789.9 from Meggers & Peters

λ_{air}	$(u - 1)\lambda$	rel. cor.	R_{He}
4685.7017	1.3072	.6530	109722.423 ± .032
4685.8012	"	..1974	109722.437 ± .028
		Mean	109722.430 ± .030

2. $(u - 1)\cdot 10^7$ = 2811.3 from Bender

λ_{air}	$(u - 1)\lambda$	rel. cor.	R_{He}
4685.7017	1.3175	.6530	109722.182 ± .032
4685.8012	1.3175	.1974	109722.196 ± .028
		Mean	109722.189 ± .030

The uncertainty of the mean is not the mean deviation but the mean of the individual uncertainties.

IV. Calculation of e/m.

From the difference between the Rydberg constant for He^+ and that for H the value of e/m can be calculated by the formula[26]

26) Birge, R. T., Phys. Rev. Supplement 1, p. 46 (1929).

$$\frac{e}{m} = \frac{FR_H(He - H - m)}{(R_{He} - R_H)(He - m)(H - m)} \qquad (24)$$

where F is the Faraday constant which is 9651.1 ± 0.8 if the atomic masses H and He are expressed in Aston scale.[27] For R_H, Professor Houston's value, namely, 109677.759 ± .016[28] is used. For R_{He}, we should use the value obtained by using the index of refraction determined by Meggers and Peters, namely 109722.430 ± .030, since R_H was obtained by using the same index of refraction. Then $R_{He} - R_H$ = 44.671 ± .034.

The latest values of atomic masses are He = 4.00391 and H = 1.00812[29]. Substituting these values in (24) we get

$$e/m = (1.7596 \pm .0013) \times 10^7 \text{ e.m.u.}$$

where the probable error is determined from the probable errors of $R_{He} - R_H$.

If we use the old values of atomic masses as used by Shane and Spedding[27] in their calculation of e/m, namely H = 1.007775

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27) Shane, C. D., and Spedding, F. H., Phys. Rev. 47, p. 36 (1935)

28) See reference 8. The uncertainty is given by the mean of the uncertainties of individual R_H obtained from the lines 6563 and 4861 instead of by the mean deviation which is .008 and, as pointed out by Prof. Houston, can hardly be taken as representing the precision of the value of R.

29) Aston, F. W., Roy. Soc. Proc. 163A, 391 (1937)

which was determined by Kenneth with reference to He = 4.00216, then we get

$$e/m = (1.7601 \pm .0013) \times 10^7 \text{ e.m.u.}$$

CONCLUSIONS

Of the eight components predicted by theory, four have been definitely observed, and the existence of the remaining weaker ones can be inferred from the master curves. The positions, in general, accord with theory, especially for the two strongest components. The relative intensities are different for different plates and not in precise agreement with the theoretically calculated values, yet the deviations are not more than can be attributed to the conditions of excitation and to photometric errors. The too high intensities of the components g,a,b suggest the hypothesis that 4p state is more populous due to the absorption of radiation 1s - 4p. On the other hand, component h does not seem to be sufficiently strong for this to be correct. The unusually high intensities of components g.h. on plate 134 are perhaps associated with the low pressure used in the same way in which corresponding variations of relative intensity with pressure are observed in hydrogen.[30] That the intensity of f is always greater than that of d is in agreement with Paschen's observation rather than with that of Leo who found them equal.

The observed separation between the two strongest components which is 0.0995 I.A. is in complete agreement both with theory within experimental error and with that observed by Prof. Houston, while the values obtained by Paschen, Kunze, and Leo are respectively 0.106, 0.1023, and 0.104, all higher than the theoretical value. However, Paschen's

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30). See reference 27).

value obtained from the microphotometer curve, .098, is a little lower than the theoretical value. In this present measurement it is believed that the value observed is more accurate than all the previous ones not only because of the accuracy which is usually attained by the interferometer but also because of the careful consideration being taken in the correction for overlapping of neighboring components.

The Rydberg constant is larger than the value obtained by Prof. Houston. This is because the wave-lengths are smaller than his values. This difference, however, is less than the difference between the values determined with the different indices of refraction. The value of e/m obtained by using the latest values of atomic masses is in close agreement with Dunnington's value who obtained 1.7597×10^7 for e/m from the deflection method.[31)]

In conclusion it is worth noting that the observed separation corresponds to a value of $\frac{1}{137.4 \pm 1.2}$ for the fine structure constant

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31) Dunnington, F. G., Phys. Rev., 52 475 (1937).

ACKNOWLEDGMENTS

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