RADIO FIELD STRENGTH MEASUREMENTS AT 41 MEGACYCLES

Thesis by

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SUMMARY

A radio receiver was modified, and the necessary calibration instruments constructed, to make reliable field strength measurements at 41 megacycles. The accuracy of the equipment was sufficient to measure field intensities to within 1/2 decibel above an arbitrary reference.

Measurements were made of the intensity of the radio signals as received from Mt. Palomar. A comprehensive series of readings was obtained for two months during the daylight hours. Although tests were made under a variety of weather conditions, it was found that atmospheric disturbances had no appreciable effect on signal intensity. As far as could be determined, the field strength remained essentially constant during this period. It was found that the average field intensity was in the neighborhood of 140 microvolts per meter.

Tests were conducted at Mt. Wilson, with the conclusion that the effect of the mountains in the Pasadena-Mt. Palomar path was to introduce an attenuation of approximately 15 decibels.

Some antenna calculations were made, and the complete mathematical solution of the self impedance of a linear antenna was obtained.

I. INTRODUCTION

An ultra short wave, single link, radiotelephone circuit was projected early in 1936 as a means for establishing two-way direct telephone communication between the site of the observatory at Palomar Mountain, and the California Institute of Technology in Pasadena. Since the Bell Telephone Laboratories were interested in making a field experiment under topographical and atmospheric conditions such as were expected to be encountered over the 90-mile Pasadena-Palomar path, a large portion of the required radio, and telephone, apparatus was loaned by them. It was understood that this equipment would be available for such experimental work as the Institute wished to perform.

A program of research was initiated by the Electrical Engineering Department of the Institute, under the supervision of Dr. S. S. Mackeown. The aims were twofold: (1) to develop a reliable and accurate method of measuring field strength at ultra high frequencies, and (2) to investigate the effect of weather conditions upon the transmission of ultra short waves, and perhaps to arrive at some conclusion as to the mechanism of their propagation in the absence of an optical path. This thesis describes the work done to date.

II. DESCRIPTION OF PERMANENT EQUIPMENT

Modified ultra short wave radiotelephone transmitters and receivers of the 5-watt mobile type, together with some standard and special telephone equipment, were supplied by the Bell Telephone Laboratories. Only the transmitters and receivers will be described, since the remainder of the equipment was of little importance in experimental work.

Transmitters

Two type 21A Western Electric radio transmitters, modified for the particular application, were supplied for service at Pasadena and Palomar Mountain. The 21A transmitter is designed for operation in the frequency band from 30 to 42 megacycles. A Western Electric quartz plate is used to maintain the frequency well within 0.025 per cent in the temperature range from 0 to 60 degrees Centigrade. The high frequency stability of these transmitters enables them to be used as frequency standards in all test work.

The 21A transmitter is rated at only 5 watts, and it was found that more power was needed to guarantee reliable communication. The Astrophysics Department of the Institute constructed power amplifiers which increased the power output of each station to 40 watts. After the installation of the amplifiers no difficulty was encountered in establishing reliable communication.

Receivers

Two type 19A Western Electric radio receivers (modified) were supplied for permanent installation at Pasadena and Palomar Mountain. These receivers were used for regular communication, and although the Pasadena receiver was further modified for field strength measurement, it was used very little for testing. A Western Electric type 18B radio receiver was supplied for test purposes. This set was equipped for mobile operation, and was suitable for measurements in the field. The receiver was considerably modified, as shown later, and was used in making practically all tests to date.

The permanent radio equipment at both Mount Palomar and Pasadena was installed in individual radio rooms. For communication purposes the equipment needed no attendants, all operations being carried out by remote control. Of the several frequencies on which the stations were licensed to operate, 41.0 megacycles was chosen for permanent operation since interference from other stations was least at this frequency.

III. DESCRIPTION OF FIELD STRENGTH MEASUREMENT EQUIPMENT

A. Field Strength Meter

Numerous methods are suggested in the literature for the measurement of field strength. The most common method makes use of a superheterodyne receiver with an adjustable attenuator in the intermediate frequency amplifier for adjusting the gain of the receiver in accurately known amounts. For high frequencies such a method appeared inadvisable because of the difficulty of building a suitable intermediate frequency attenuator. Since the Western Electric model 18B receiver was available, it was decided to modify it to serve as a field strength meter. This necessitated some means of calibration, since the receiver at best could serve only as a comparison device. The standard signal generator, and the standard field generator, constructed for calibration purposes, will be discussed later.

The most obvious method of adapting a superheterodyne receiver to the measurement of signal strength is to measure the plate current of one of the tubes controlled by the automatic volume control circuit. The automatic volume control circuit, after modification, is shown in Fig. 1. When a modulated intermediate frequency voltage is applied to the plate of the detecting diode, T_1, a

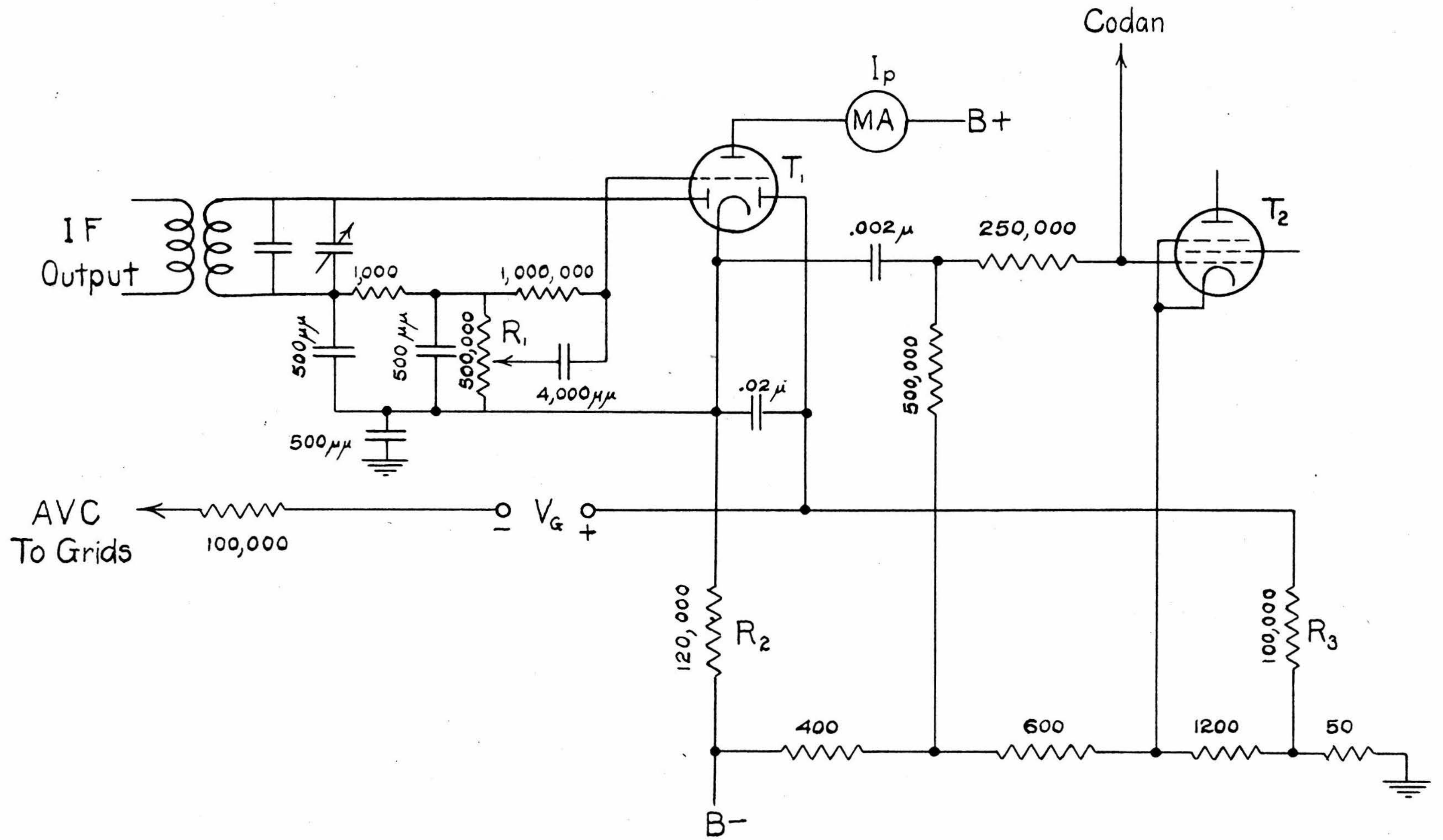


FIG. 1

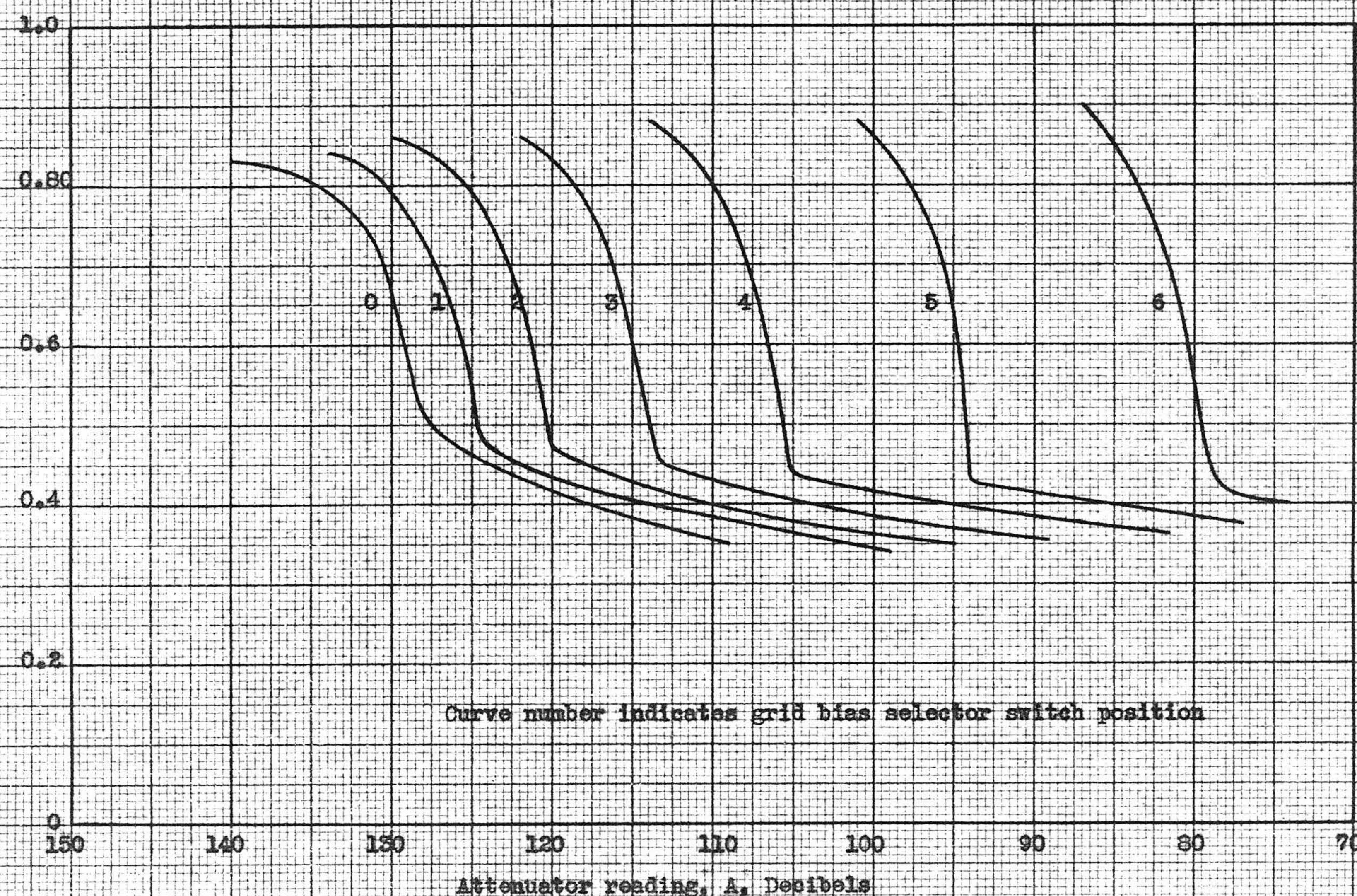
Modified AVC Circuit

direct current voltage, as well as an audio frequency voltage, appears across its load resistor, R_1. These voltages are applied to the triode section of T_1, and the amplified voltages appear across the load resistor, R_2, of this tube. When no signal is being received the diode automatic volume control plate of T_1 is maintained at a negative potential with respect to the cathode, by biasing resistors in other parts of the receiver circuit. However, when a signal is received the grid becomes more negative, causing the current through R_2 to decrease. This lowers the voltage across R_2, and allows the cathode to become negative with respect to the a.v.c. diode plate. The a.v.c. diode plate then draws current, causing a voltage drop to occur across R_3. This voltage is used to decrease the gain of the set by increasing the grid bias of the radio frequency amplifier, the modulator, and the intermediate frequency amplifiers. In this way the plate current of T_1 is seen to be dependent upon the input signal intensity.

It was found that the plate current of T_1 was very sensitive to change in signal input over a limited region. Since it was desired to measure a relatively large range of field strength, some means had to be found to increase the sensitive range of the set. The a.v.c. circuit described is of the delayed type, and thus comes into action only after the signal has reached a given value. By introducing additional fixed bias into the a.v.c. circuit the set obviously demands a stronger signal to cause a.v.c. action to take place. This additional grid bias was introduced as shown at V_G. In this way the set could be made sensitive at any desired signal strength. A set of curves for different values of V_G, showing the plate current of T_1 as a function of the input signal, is given in Fig. 2. Due to the lack of a suitable vacuum tube voltmeter to indicate the one volt level, the curves are plotted against the attenuation, A, in decibels below an arbitrary reference, as determined by the standard signal generator.

If continuous recording of field strength over a limited range is desired, a recording milliammeter need only be inserted into the plate circuit of T_1. The value of V_G may be chosen so that quite reliable measurements may be made over a range of about fifteen decibels. In such an application the stability of the set is of course very important. It was found that the set could be depended upon to maintain its calibration over a period of 24 hours within about 1/2 db. For continuous measurements this accuracy appeared to be sufficient.

For manual operation a more convenient method was chosen. It is to be noticed that the curves in Fig. 2



CALIBRATION CURVES (PLATE CURRENT vs. SIGNAL INPUT)

FIG. 2

are very sensitive in the neighborhood of 0.6 ma. This suggested the possibility of adjusting the additional grid bias voltage to bring the plate current to 0.6 ma. In this way a reliable and sensitive indication of input signal intensity was obtained. Using a signal generator in the substitution method, measurements could be relied upon to about 1/4 db.

It was found that the set was quite sensitive to small variations in plate supply voltage. For portable use the entire power for the set is supplied from a storage battery in conjunction with a dynamotor. Since the state of charge of the battery had a very pronounced effect upon the plate supply voltage, it was necessary to find some means of controlling the voltage. It was decided that this could best be done by inserting small dry batteries in series with the plate supply. For operation from a.c. lines the set was provided with a rectifier unit, in which the output voltage was easily regulated by means of a potentiometer.

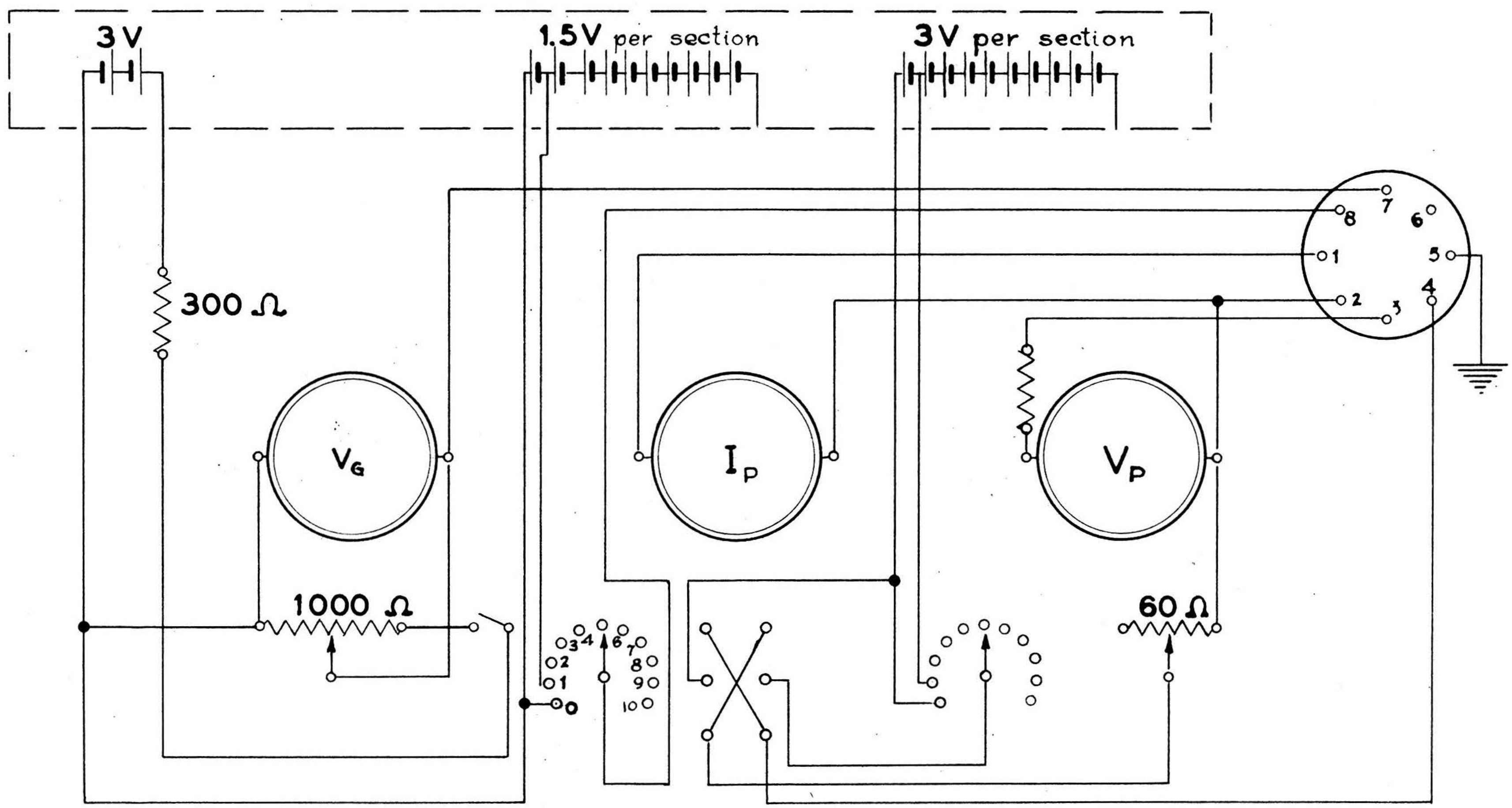
The set was modified as outlined above for field strength measurements so that all connections could be made through a single eight-conductor cable. All of the auxiliary equipment, including dry batteries, was assembled in a cabinet which was attached as an integral part of the receiver. The complete wiring diagram of the auxiliary cabinet is shown in Fig. 3. The cabinet includes:

1. A 1 ma. d.c. meter to measure the plate current, I_p, of V_1.

2. A selector switch to change the grid bias voltage in steps of 1½ volts. This circuit is supplemented with a potentiometer connected across a 3-volt battery, and connected to a 2-volt meter to interpolate between the fixed 1½-volt stages. The voltage of the 1½-volt steps remains constant since no current is drawn by the grid circuit. The reading of the selector switch gives the curve number, as indicated on the curves in Fig. 4 and Fig. 5, while the reading of the voltmeter, V_g, indicates the additional voltage necessary to bring the plate current to 0.6 ma. when operating upon a given curve.

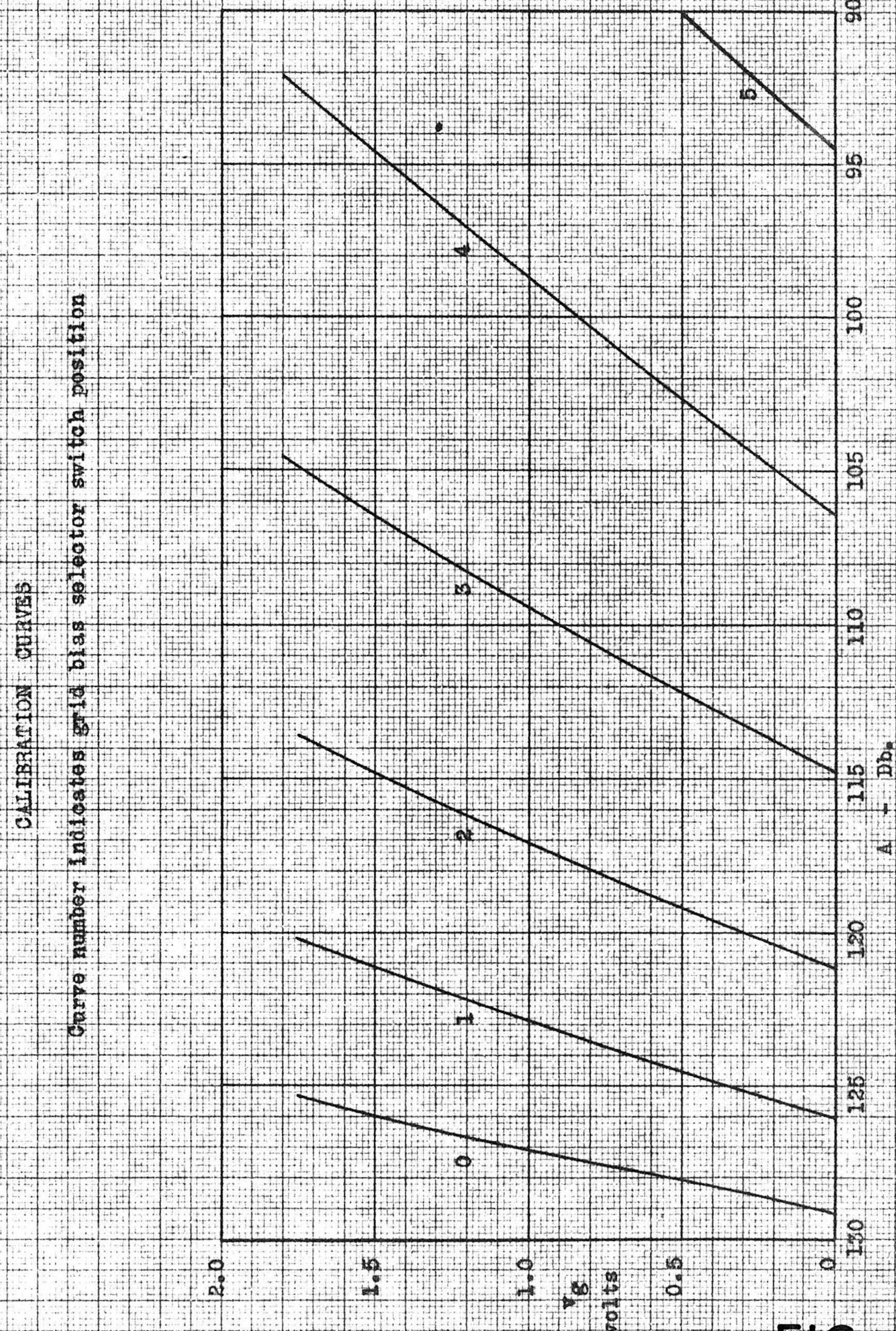
3. A selector switch to introduce dry batteries in steps of three volts, to increase or decrease the plate supply voltage of the dynamotor. A series rheostat serves to adjust the voltage between steps.

Figure 6 is a picture of the complete field strength meter. It is to be noted that the entire set is portable, and quite compact. When using a simple dipole antenna on a short mast, two men can easily operate the equipment in the field.



AUXILIARY CABINET

FIG. 3



CALIBRATION CURVES
Curve number indicates grid bias selector switch position
2.0
1.5
1.0
0.5
0
V_g
volts
130
125
120
115
110
105
100
95
90
A - Db.
0
1
2
3
4
5

FIG. 4

CALIBRATION CURVES

Curve number indicates grid bias selector switch position

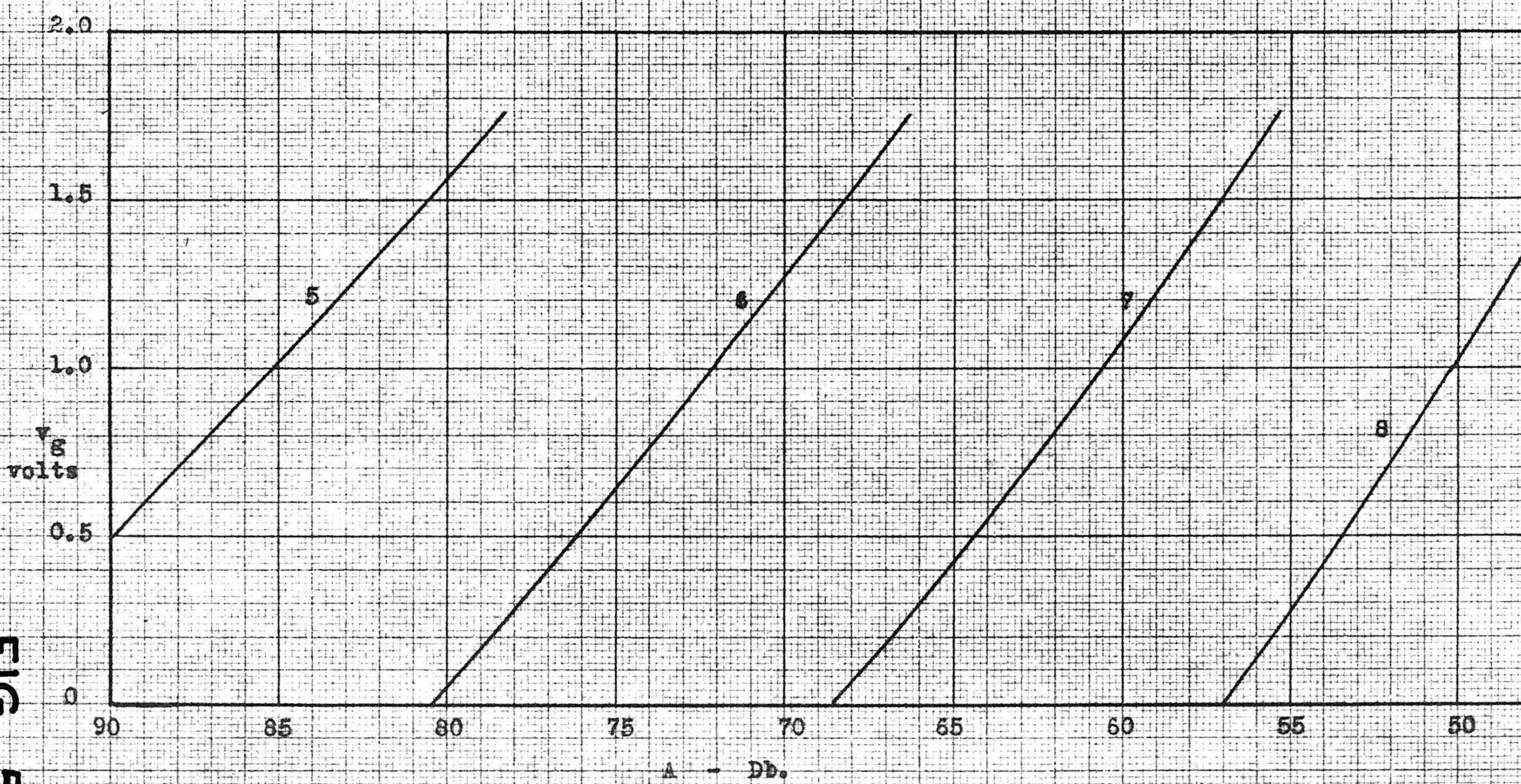


FIG. 5

Field Strength Meter

Fig. 6

In order to have all readings consistent, a standard routine of measurement was adopted. The method is outlined below.

1. The plate supply voltage was adjusted to give $I_p = 0.85$ ma. with the antenna disconnected, and no external grid bias applied. This corresponds to a plate supply voltage of 195 volts. The plate current was so sensitive to fluctuations in voltage that it was used for indication, rather than the plate voltmeter.

2. The antenna was connected to the set, and the set tuned by adjusting I_p to a minimum. The grid bias voltage selector switch was then set to the point just below the position which allowed I_p to become greater than 0.6 ma. The grid bias potentiometer was then adjusted to bring I_p to 0.6 ma. In general, the tuning was again checked at this point because of the greater sensitivity of the meter reading in this region. The position of the selector switch was recorded as #0, #1, #2, #3, etc., and was called the "curve number." The reading of the 2-volt voltmeter was recorded as V_g. This completed the adjustment of the receiver.

From a set of calibration curves of the type shown in Figs. 4 and 5, the signal could then be referred to some reading, A, of the attenuator on the standard signal generator. This calibration could be relied upon to within 1/2 db. if the set was allowed to warm up for about 1/2 hour before starting operation. Because a calibration seemed advisable about every 24 hours, a more direct method was the introduction of an equal signal from the standard signal generator, after the set had been adjusted as outlined in step 2. The attenuator reading then gave a direct measure of the incoming signal. To avoid the use of a calibration curve, this method was adopted whenever the signal generator was available.

4. The above routine is sufficient to measure the <u>relative</u> values of field strengths when a fixed antenna system is used. As will be shown later, the <u>actual</u> field at the antenna may be measured with the aid of the standard field generator. In this way, the field strength can always be measured in db. above some arbitrary reference, such as 1 mv./meter, for any antenna system. A sample data sheet, as actually recorded, is shown in Appendix C. Also in Appendix C, it is shown how such data were converted to give the field strength in db. above 1 millivolt per meter, in order to facilitate comparison with other data.

B. Standard Signal Generator

When measuring signal strength it is always necessary to have some means of introducing a known comparison voltage into the indicating device, or field strength meter. At first, it was intended to use a method due to R. C. Shaw, as described by Schelleng, Burrows, and Ferrell[1]. In this method the calibrations were made by introducing into the antenna itself a known field from a local source, to which the name "standard field generator" has been given. The standard field generator is a small portable oscillator, which is shielded, except for a radiating loop. The current in the loop, from which the field may be calculated as is shown in Appendix C, is measured by means of a thermogalvanometer.

Such a system has several disadvantages. Due to the long non-optical transmission path between Pasadena and Mt. Palomar, and to the low power of the transmitter, it was expected that the fields to be measured would be quite weak. This meant that if the local field were to be of the same order of magnitude, an extremely small current would have to flow in the radiating loop. Although small radio-frequency currents may be measured in the laboratory by means of vacuum thermocouples and sensitive galvanometers, any simple metering method demands currents of nearly 100 ma. Since the loop must be placed close to the antenna (not farther away than one wave length) in order to avoid ground effects and other disturbing reflections, such large currents would give excessive fields. These excessive fields could not, then, be compared directly with the field strength to be measured. In order to calibrate the receiver it would therefore be necessary to have an accurate means of measuring the relative magnitude of the signals to be measured and the signal introduced into the antenna by the field generator. Ordinarily this difficulty could be overcome if the intermediate frequency attenuator type of instrument were being used. Without such an instrument it becomes necessary to develop a means for calibrating the set to measure an extremely wide range of signals. The field strength generator gives only a relatively narrow range of signal intensity because its attenuation range depends on the meter range, which for a square law meter cannot exceed 4:1 with accuracy. Consequently, it was decided to construct a "standard signal generator" which could be used to introduce directly into the set a wide range of signals with known attenuation below an arbitrary reference. If an actual measure of the field in volts per meter were desired, it was concluded that it would be relatively simple to introduce a strong known signal into the antenna, and thus calibrate the signal generator input in volts per meter at the antenna. It was also felt, at first, that the receiver calibration would not remain constant, and consequently, that a substitution method of measurement would have to be used. This of course would necessitate turning off the signal to be measured every time a substitution was to be made with the field generator. A standard signal generator, on the other hand, would eliminate this difficulty since it would introduce a known signal directly into the receiver, with the antenna disconnected.

A standard signal generator should possess four very desirable characteristics.

1. Frequency stability.
2. Constancy of oscillator output.
3. Nearly perfect shielding.
4. An attenuation device which permits the constant oscillator output to be attenuated accurately in small steps, or continuously, over a wide range.

The requirements of frequency stability and constant output were met quite easily. Since measurements were to be made at but one frequency, and since a standard source was always to be present in the form of the crystal-controlled transmitters, there was no need to know the frequency of the oscillator with any great accuracy. Practically any of the simple oscillator circuits maintains a given frequency if the power supply is kept constant. This was done by using dry batteries entirely, chiefly as an aid to portability and to simplify shielding.

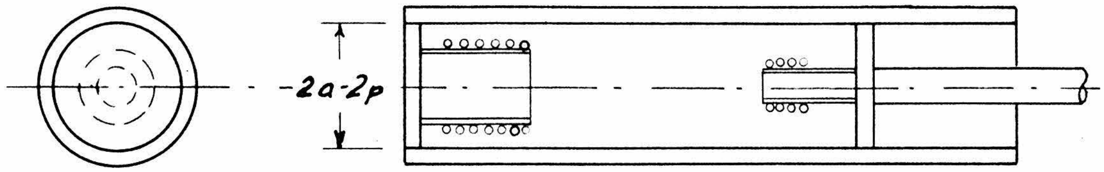
The problem of adequate shielding in signal generators is very difficult. Most of the principles ultimately used in shielding were arrived at by the cut and try process. When the final signal generator is discussed the method of shielding will be described.

Commercially available standard signal generators almost invariably use resistance type attenuators. In general they are quite cheap to build, and may be relied upon for the ordinary range of radio frequencies. However, for high frequencies and high attenuations, stray capacities, and inductive effects become very important. It was felt that since the attenuator would be the ultimate standard of comparison, it would be desirable to seek a superior method.

It was finally decided to use a mutual inductance "piston type" attenuator, as described by Harnett and Case[2]. In this type of attenuator the output and input coils are enclosed in a metal tube. The signal voltage delivered to the output terminals is proportional to the field at the pickup coil, which is movable along the axis of the tube. Mr. Harold A. Wheeler, of the Hazeltine Corporation, has developed a mathematical treatment of the piston attenuator by using Byerly's[3] solutions of temperature problems involving circular cylinders and square cylinders. A series of exponential terms results for the solution, in which the first term is the only one of consequence. Thus the field varies exponentially with spacing, and the scale on the piston may be calibrated linearly in decibels. (Mr. Wheeler has not published his treatment of the problem, but supplied the above information upon request.)

Figures 7 and 8 show two possible types of piston attenuators. The attenuation formulas as calculated by Mr. Wheeler were taken directly from the paper by Harnett and Case. It is to be noted that the depth of penetration is to be added to the actual cylinder diameter. At 41 megacycles this depth of penetration is roughly only 5×10^{-4} inches, and is negligible for a cylinder of ordinary dimensions.

In the coaxial type of attenuator there are two major sources of error. One is incidental coupling, and the other is coupling due to transverse fields arising from departures from axial symmetry. Both of these fields

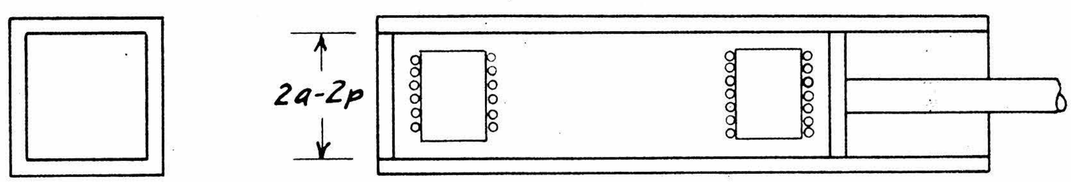


Depth of penetration $p = \frac{2.6}{\sqrt{f}}$ mils (f in mc)

Attenuation = 33.3 decibels per radius
(27.3 db. in square cylinder)

Piston attenuator, coaxial coils in circular cylinder

Fig. 7



Attenuation = 16.0 decibels per radius
(13.64 decibels in square cylinder)

Piston attenuator, coplanar coils in circular or
square cylinder

Fig. 8

are attenuated much less rapidly than the coaxial field, and hence the errors introduced by them increase with increased attenuation, and in fact, at sufficiently high attentuation they will ultimately become the dominating fields. The coplanar type of attenuator utilizes the transverse field, and since the transverse field is attenuated less rapidly than either the capacitive coupling or the coaxial coupling, the accuracy in this type of attenuator increases with attenuation. Since a large attenuation was desired, this factor could not be neglected. It is to be noted from Figs. 7 and 8 that the attenuation in the coplanar type of attenuator is roughly only half as much per radius as for the coaxial type. This suggests immediately that the accuracy of reading the scale in decibels would be considerably greater in the case of a coplanar attenuator, than in a coaxial one of the same diameter. For these reasons, it was decided to build a signal generator using a mutual inductance attenuator with coplanar coils in a square cylinder.

The square attenuator was constructed to give an attenuation of five db. per centimeter of axial displacement. The input coil of the attenuator formed the tank circuit of the oscillator. Since a measure of the input current was necessary, a radio-frequency thermomilliammeter was coupled to the input coil by a single turn. The meter could not be put in series with the oscillator coil, since this prevented ample oscillator output. The entire oscillator, the batteries, and the input end of the attenuator were mounted inside of an aluminum shield.

The first signal generator, as described in the above paragraph, proved inadequate for several reasons. In the first place the rate of attenuation was checked by the following method. Since the output of the attenuator is the product of the input and the attenuation, it follows that if the input current be increased by a certain ratio, say 3 to 1 (9.54 db.), an increase of attenuation of 9.54, as given by the piston reading, should restore the output to its original value, as indicated by the receiver plate current. It was found that the actual rate of attenuation differed from the predicted value by as much as 25%, and, further, that the rate of attenuation was by no means constant for different piston positions. This was explained in two ways:

1. The input coil necessarily carried a very large current in order to give an indication on the coupled meter. This would naturally cause large currents to flow in the attenuator case. Ordinarily it would be expected that this would be of little consequence since

just such currents would be expected to flow in order to give the desired attenuation. The attenuator tube was made from copper plates soldered together, and because of the difficulties in construction, it appeared entirely possible that these soldered joints did not provide uniformly good electrical contact. If this were the case, an unsymmetrical distribution of currents would be set up, and could easily cause errors in attenuation. The remedy appeared to be the use of an attenuator made from seamless tubing.

2. The inner half of the attenuator was exposed to all of the fields put out by the unshielded oscillator. There was no reason to assume that the currents thus induced in the attenuator tube would be symmetrically distributed, and so it appeared that this could be a source of trouble.

3. The shielding was so inadequate that large currents flowed in the outer shield, causing strong external fields and pronounced body capacity effects. Such fields naturally preclude any possibility of making accurate measurements.

From the experiences with the first signal generator it was decided to build a second signal generator, incorporating the following principles:

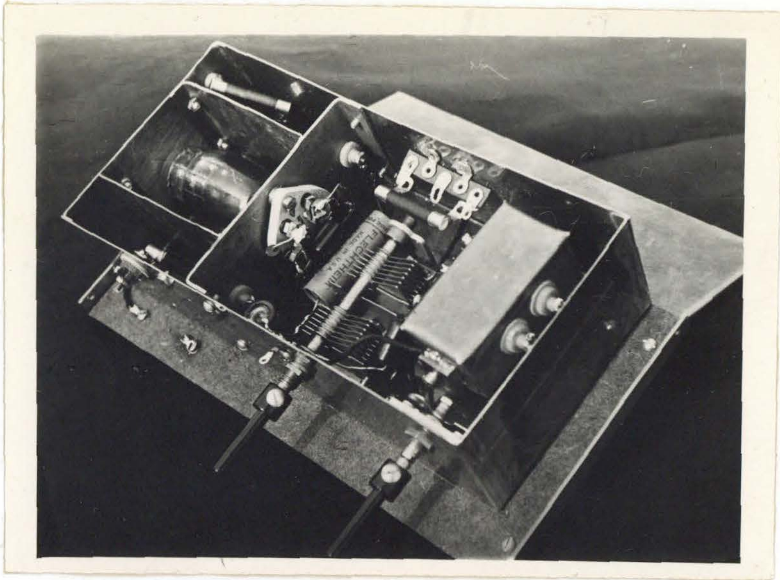
1. An attenuator made from seamless copper tubing. This called for a circular tube, and, for the reasons already given, it was decided to use coplanar coils. The tubing chosen gave an attenuation of 12.8 db. per inch, as calculated by the equation in Fig. 8. The actual useful attenuation which could be obtained without loss of accuracy due to too close coupling was at least 80 db. (very conservative).

2. A method of measurement of the current in the attenuator input coil without such large currents as in the first signal generator. This was accomplished by using a more powerful oscillator with a separate tank circuit, to which the attenuator input coil was coupled by induction. The current in the latter circuit was measured by placing a thermocouple directly in series with it. The leads from the thermocouple to the d.c. meter were thoroughly filtered.

3. Entirely different shielding. To obtain adequate shielding was an extremely tedious process. Little reason can be offered for some of the principles involved, except to say that they ultimately gave the best results. Since the experiences with the first attenuator indicated that stray fields from the oscillator, external to the attenuator tube, were a source of error, it was decided to place the entire oscillator inside a second copper

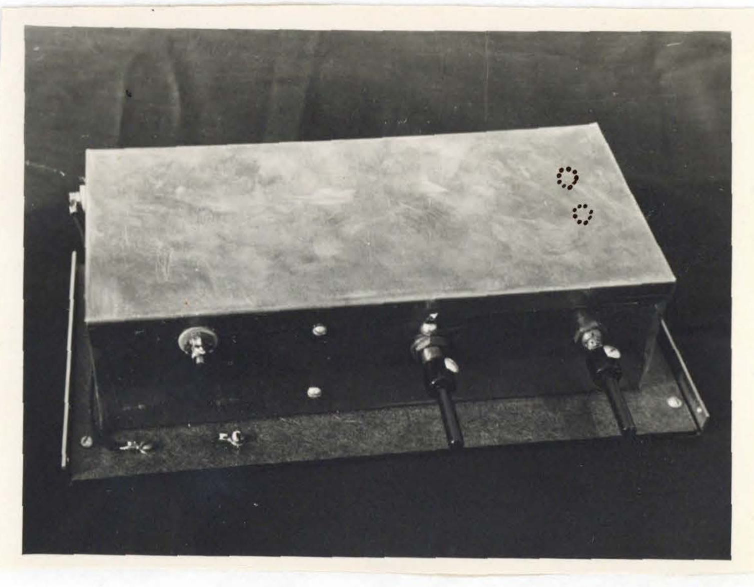
shield. It was further decided to filter all power leads, both before and after leaving the inner shield. This filtering prevented large fields from being produced by the battery leads. The filter chokes were enclosed in separate shields to remove them as much as possible from the main field of the oscillator. In addition, the tank circuit coils, and the attenuator pickup coil, were enclosed in a third shield inside of the inner shield. All of the ground connections within the inner shield were made to one point on the shield. This eliminated the principal paths for circulating currents in the shield, and thus considerably increased the shielding effect. The oscillator and the inner shield are shown in Figs. 9 and 10, both with the cover off and on. The separate tank coil shield and separate filter shields are clearly visible. The dotted circles on the cover indicate the openings through which the only external radio-frequency leads were passed.

The attenuator and shielded oscillator were mounted within an outer aluminum shield. A copper chimney was constructed from the inner shield to the input end of the attenuator to give complete shielding for the radio-frequency leads. The thermocouple element was mounted inside this chimney, and the d.c. leads were filtered before they were led out to the panel. In the first



Oscillator in Inner Shield, Cover Removed

Fig. 9

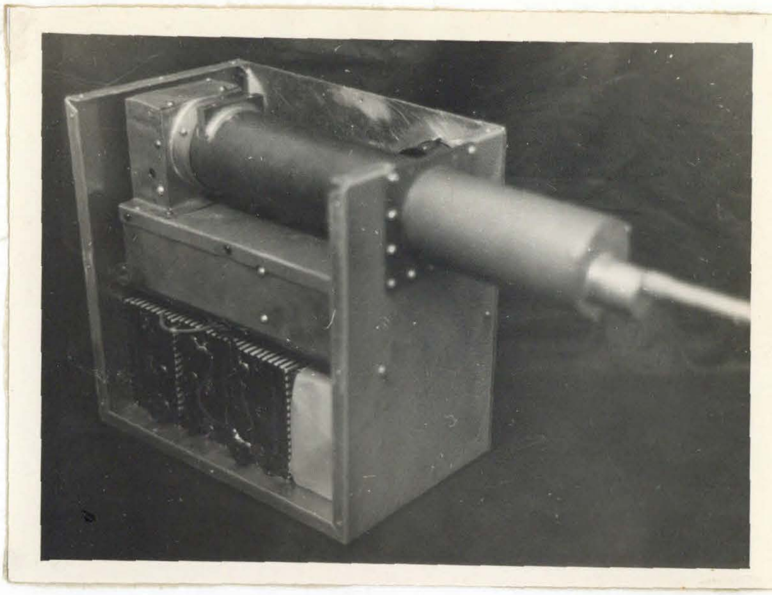


Inner Shield, and Oscillator

Fig. 10

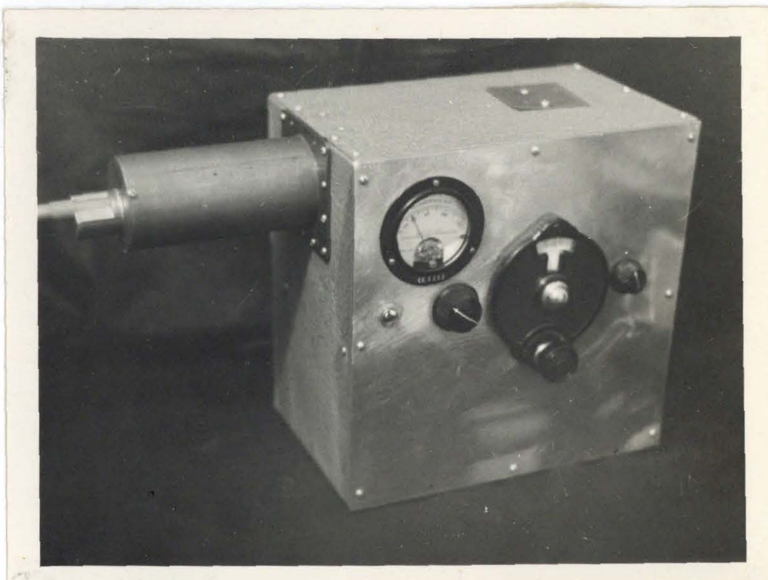
signal generator the radio-frequency meter on the panel was the cause of much of the stray fields external to the shield. Definite body capacity effects were noticed with the hand within six inches of the meter. This effect was entirely eliminated in the second signal generator by placing the thermocouple inside the chimney. In order to reduce current paths between the inner and outer shields to a minimum, only one connection was desirable between the two shields. The most successful method of making this connection was found to be by the attenuator tube itself, in conjunction with the connecting chimney. The two condenser shafts from the oscillator were passed through the outer shield by bakelite shafts. A back view of the interior of the signal generator is shown in Fig. 11, in which the method of mounting the inner shield and the attenuator are clearly shown.

Figure 12 shows a front view of the entire oscillator. Connection to the attenuator output is made by a simple bayonet type shielded connector. The front panel contains a filament switch, a rheostat for adjusting the oscillator output, the main tuning condenser control, and a meter for reading the radio-frequency input current to the attenuator. The attenuator scale is calibrated directly in db., and the numbers increase as the output voltage decreases.



Rear View of Signal Generator, Showing Shielding

Fig. 11



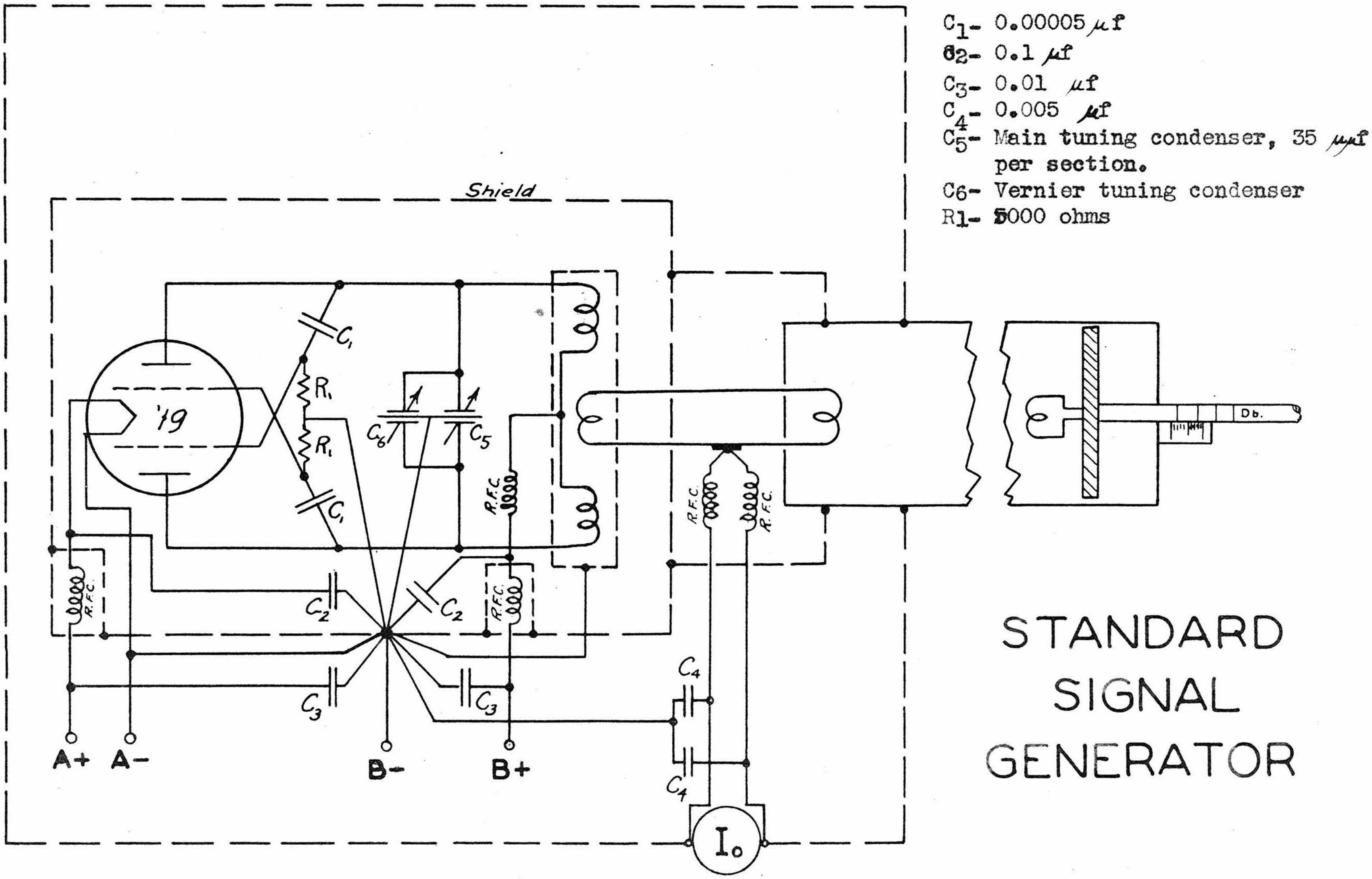
Completed Standard Signal Generator

Fig. 12

The attenuator piston is prevented from turning by a steel ball which rides under spring compression in a "vee" groove in the piston. The piston may easily be adjusted to within 1/10 db.

The complete wiring diagram of the signal generator is shown in Fig. 13. The diagram also shows the single ground point connections, and shows the location of shields. Power is supplied by three 45-volt "B" batteries, and one 3-volt filament battery.

The frequency of the signal generator was first tested by a Lecher wire system, to be certain that the fundamental frequency was in the vicinity of 41 megacycles. At first, many spurious responses were obtained when tuning the signal generator to the receiver. These responses were due to parasitic oscillations and intermittent oscillations due to improper choice of grid leak and condenser combinations. These responses were eliminated by proper choice of grid coupling condensers. The actual oscillator circuit is conventional, and presents no points for discussion. Although it was of little importance, it was found that a change in oscillator output of ten to one produced extremely little frequency change. The frequency stability under continuous operation was found to be entirely satisfactory.



STANDARD SIGNAL GENERATOR

FIG. 13

The stray fields from the signal generator were tested by connecting a 4-turn coil, by means of a coaxial line, to the receiver. No appreciable pickup was obtained, except at a few cracks in the outer shield, and in the vicinity of the meter. These stray fields were only noticeable within a half inch of the case. It is to be recognized that this was an extremely severe test of shielding, since the receiver was very sensitive. No body capacity effects whatsoever were found.

Using the thermocouple as a standard the rate of attenuation was checked by changing the attenuator input current over a ratio of three to one. The test data obtained are recorded below.

Curve No.	A $I_0 = 40$	A $I_0 = 120$	Exp. Db.	Theor. Db.
0	126.3	135.8	9.5	9.54
1	132.2	132.7	9.5	9.54
2	118.4	127.9	9.5	9.54
3	112.1	121.6	9.5	9.54
4	103.6	113.2	9.6	9.54
5	91.9	101.4	9.5	9.54
6	77.8	87.3	9.5	9.54
7	65.8	75.3	9.5	9.54
8	54.3	63.8	9.5	9.54
9	44.6	54.1	9.5	9.54

The test was performed by finding the piston position (indicated by A) to give a fixed response for attenuator currents of 40 and 120 ma. This was done at 10 different points, as given by different grid bias values. The difference between the two values of A corresponded to the measured db. change.

The above data show that the rate of attenuation remained constant over a range of 80 db. The discrepancy in the readings is well within the accuracy of the meter, and cannot necessarily be ascribed to the attenuator. The discrepancy of four parts in a thousand is so small that it can be neglected, and the rate of attenuation has been assumed exact as calculated.

It may be concluded that the standard signal generator provides an entirely reliable means of producing a signal which is known over a range of at least 80 db.

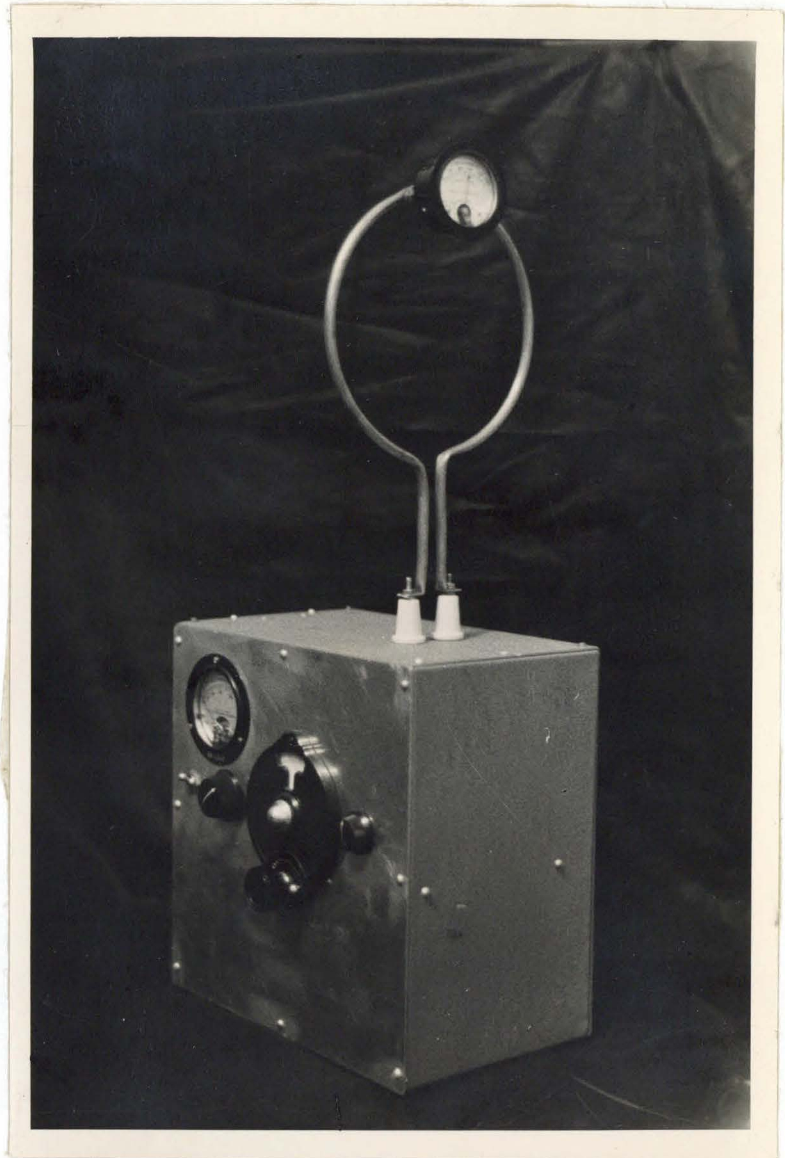
C. Standard Field Generator

It has already been pointed out that the direct measurement of field intensity with a standard field generator was impractical, since the output of such a generator is quite high. However, having a method of accurately measuring a wide range of signals immediately suggested a means of overcoming this difficulty. The method consisted simply of using the calibrated field strength meter to measure the db. difference between the weak unknown signal, and the strong known local signal from a field generator. The absolute value of the unknown signal could then be easily calculated.

Since permanent antenna structures were to be used most of the time, it was felt that it would be inadvisable to construct a separate field generator. Instead, the standard signal generator was constructed so that it could be modified into a field generator by simply removing the attenuator and installing a radiating loop outside of the case. Such an operation could be effected in less than half an hour.

The loop installed was 18 cm. in diameter, and was used to calibrate the antenna system as shown in Appendix C. Burrows[1] has discussed the use of such field generators, and the generator was used in the manner recommended by him. The spacing between the loop and the antenna was chosen at one half wave length to minimize ground effects.

Figure 14 shows the completed standard field generator.



Standard Field Generator

Fig. 14

IV. DESCRIPTION OF TESTS

A. Palomar Antenna Current

In order that the measurements made at Pasadena could be of any significance, it was necessary to know the reliability of the signal sent out by the transmitter at Mt. Palomar. Since the antenna structure was a permanent installation it was assumed that if the input to the antenna stayed constant, the output stayed constant. With this assumption, the antenna current as determined at the transmitter appeared to be an indication of the output signal. In order to test this, the Palomar antenna current was changed, and the change in field strength measured at Pasadena. The predicted and measured changes as referred to an antenna current of 1.0 ampere are given in the following table.

Ant. I	Db.above I=1.0 Meas.	Theor.
.20	-12.2	-14
.40	- 7.1	- 8
.60	- 4.1	- 4.4
.80	- 1.9	- 1.9
1.00	0	0
1.22	1.5	1.7

At first glance it may appear that there is a rather large discrepancy between the predicted and measured values.

However, it must be remembered that the type of meter used to indicate the antenna current is not very accurate over a 5 to 1 range. Assuming a meter of 2% accuracy (conservative), with a 0.2 ampere reading, the current could actually be 0.23 ampere. This would give a theoretical attenuation of 12.8 db. The error now is well within the amount by which the transmission characteristics could vary during the test.

Hence, it was concluded that if the antenna current were known, all measured values of field strength could be referred to an antenna current of 1.0 ampere as a standard, as shown in Appendix C.

B. Continuous Runs

Originally it was hoped to make continuous measurements of the field strength at Pasadena. As will be pointed out later, this was impossible, and it was necessary to be satisfied with measurements made during the hours from 9:00 A.M. to 5:00 P.M. Such measurements were conducted over a period of two months during April and May.

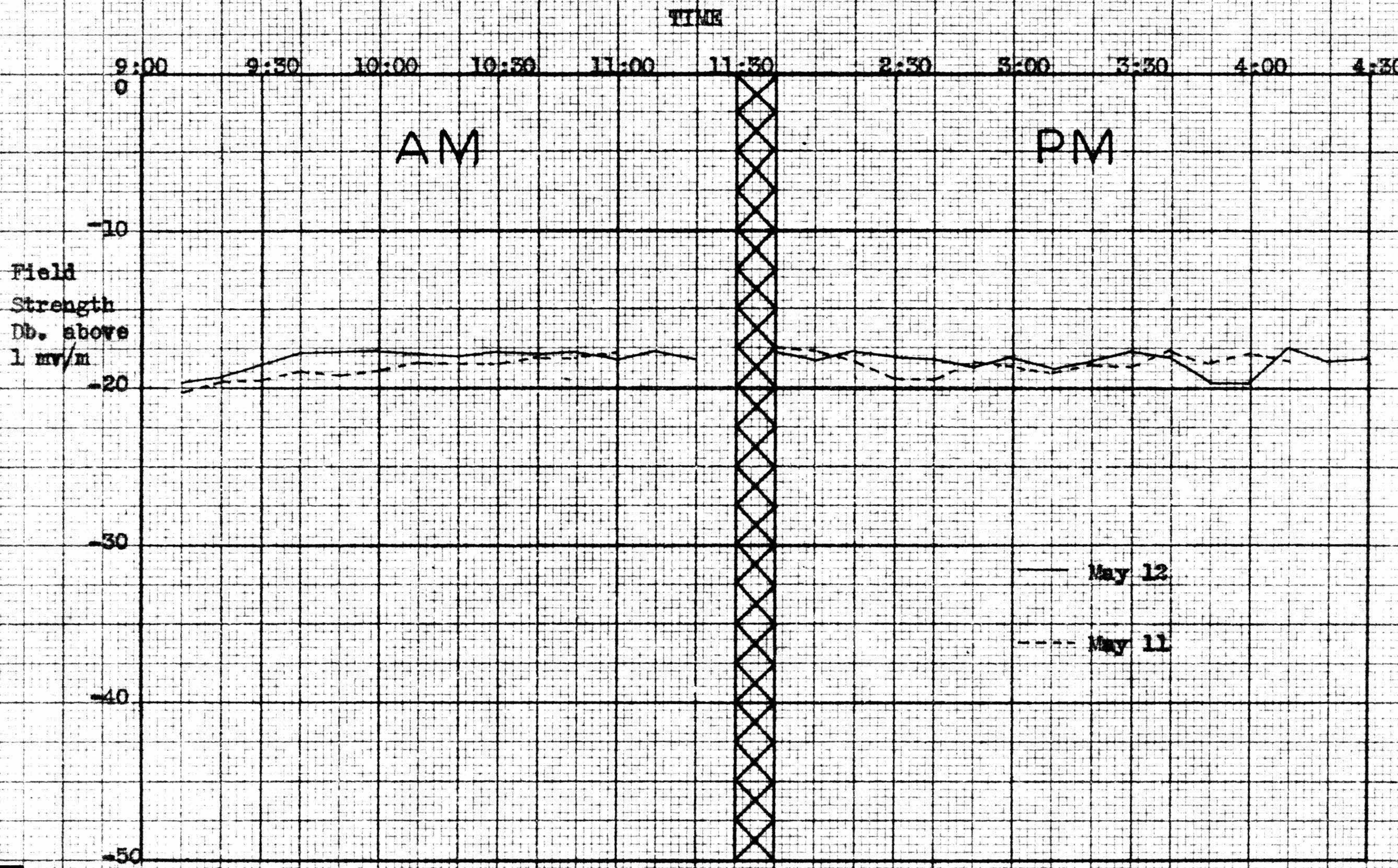
In the beginning, measurements were made every five minutes, but after a few runs it was found that the intensity remained so steady that a reading every ten minutes was ample. At first it was difficult to get reliable antenna current information, and for this reason, the records of April and May cannot be compared within about two decibels, after reduction to a standard of comparison.

Figure 15 shows a typical run of measurements made during the morning and afternoon of two successive days. It is to be noticed that the intensity remained quite constant throughout the day, and that there was no appreciable variation from day to day. The same type of data were obtained over a period of a month, without more than 4 db. variation between the maximum and minimum values.

During April it was difficult to make measurements at any time but during the afternoon. In light of later events, as indicated in the previous paragraph, this proved to be of no importance. Two typical afternoon runs, ten days apart, are shown in Fig. 16. During the month of April no appreciable variation in intensity was recorded.

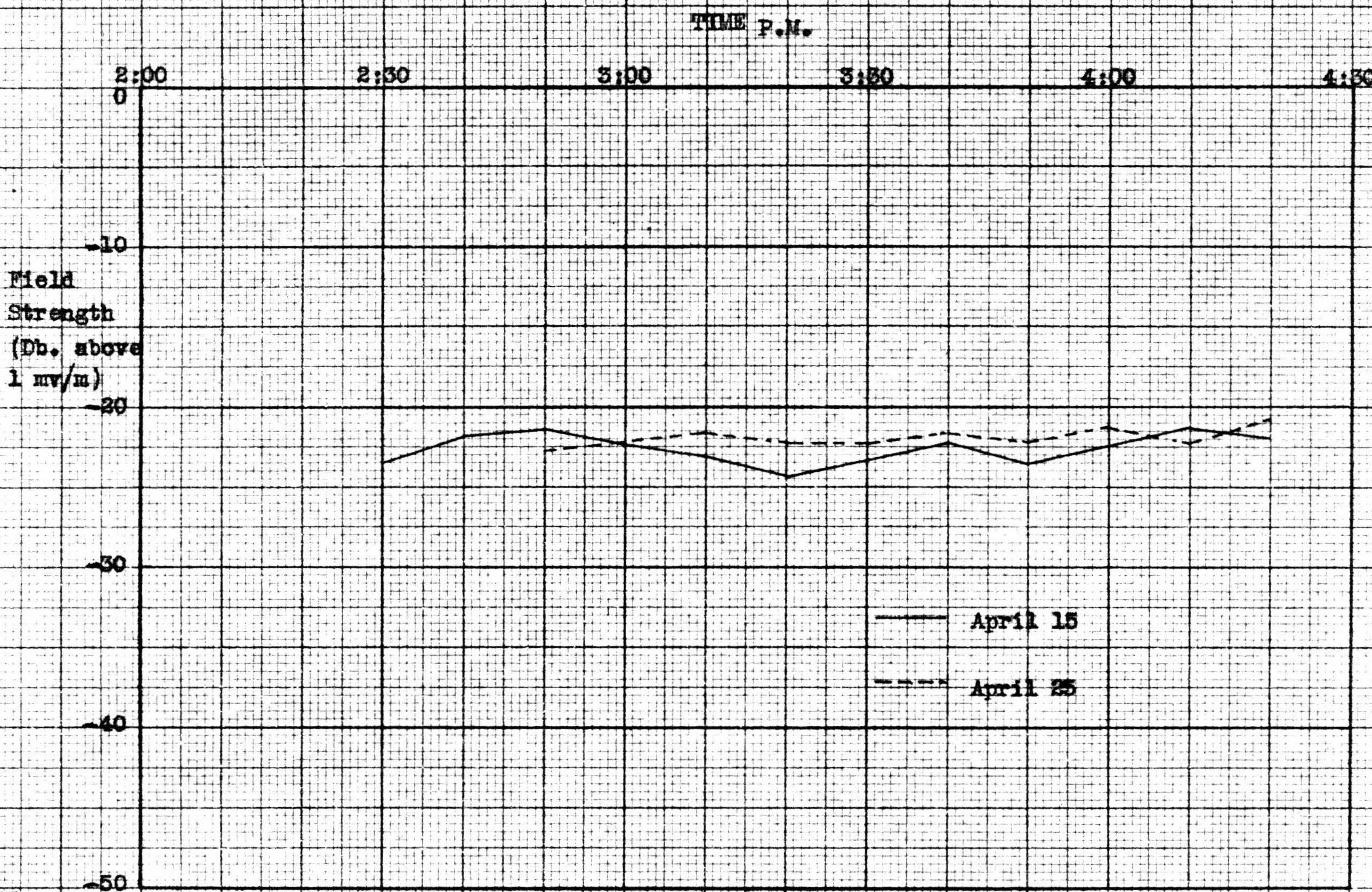
The months of April and May included about every type of weather to be experienced in southern California. The tests showed quite conclusively that even very marked changes in weather conditions had no apparent effect upon the field intensity.

It may be concluded from these tests, conducted over a two-month period, that the field intensity remained essentially constant.



TYPICAL MORNING AND AFTERNOON FIELD STRENGTH MEASUREMENTS

FIG. 15



TYPICAL AFTERNOON FIELD STRENGTH MEASUREMENTS

FIG. 16

C. Mount Wilson

Since signals from Mt. Palomar do not have an optical path to Pasadena (Fig. 17), and since the path between Mt. Wilson and Mt. Palomar is free of any obstructions, a measure of the field intensity at Mt. Wilson appeared desirable. The measurements proved quite unsatisfactory because it was nearly impossible to get a consistent reading. The intensity was found to change by as much as 10 db. in a fifty-foot circle. Such variations made it difficult to obtain an accurate measure of the difference in field intensity between Mt. Wilson and Pasadena, and the only conclusion which could be reached was that the difference was in the neighborhood of 10-20 decibels.

Tests were conducted along the Toll Road from the top of Mt. Wilson to an elevation of 2500 feet, but the surrounding terrain, and the presence of a telephone line which hugged the road for practically the entire passable distance, had so much effect upon the measurements that no useable data were obtained.

D. Eastern Stations

During the winter months signals were heard from the east coast. The signals were quite weak and were of such a nature as to defy accurate measurement. Some general conclusions may be recorded about this long distance reception:

1. Signals only received during months of October through February, and then only periodically.

2. Signals received only during hours of from 10:00 A.M. to 3:00 P.M., with best reception shortly before noon.

3. Signals characterized by spasmodic fading. In particular, a station would be audible quite consistently over a period of one or two hours with only temporary fading interruptions, and then would suddenly fade completely, and could not be received again until the next day. Such interruptions occurred at no set time.

The above information is offered only to record the fact that signals were heard over a distance of 2500-3000 miles on 41 megacycles.

V. DISCUSSION OF RESULTS

A. Aims, Expectations, and Realizations

A good many years ago it was felt that any wave length in the ultra short range (below 10 meters) could not be received unless an optical path was present. During the last six years much experimental work has been conducted on radio signals in this region, and it has been found that the signals were quite often received when an optical path was not present. A partial list of the publications on the subject appears in the bibliography.

Burrows, Schelleng and Ferrell[1] have shown that the effect of a variable dielectric constant of the atmosphere due to pressure, temperature, and humidity gradients would be such as to bend an electromagnetic wave toward the earth. The path after this refraction would be the equivalent of the optical path on a sphere 4/3 the diameter of the earth. Figure 17 shows a profile map of the path between Mr. Palomar and Pasadena. The solid line represents the straight line path, while the dotted line represents the path predicted by the above theory. It is to be noted that in neither case is an unobstructed path presented. It was to be expected, then, that some further information as to the mechanism for the propogation of the signals between these two points could be obtained.

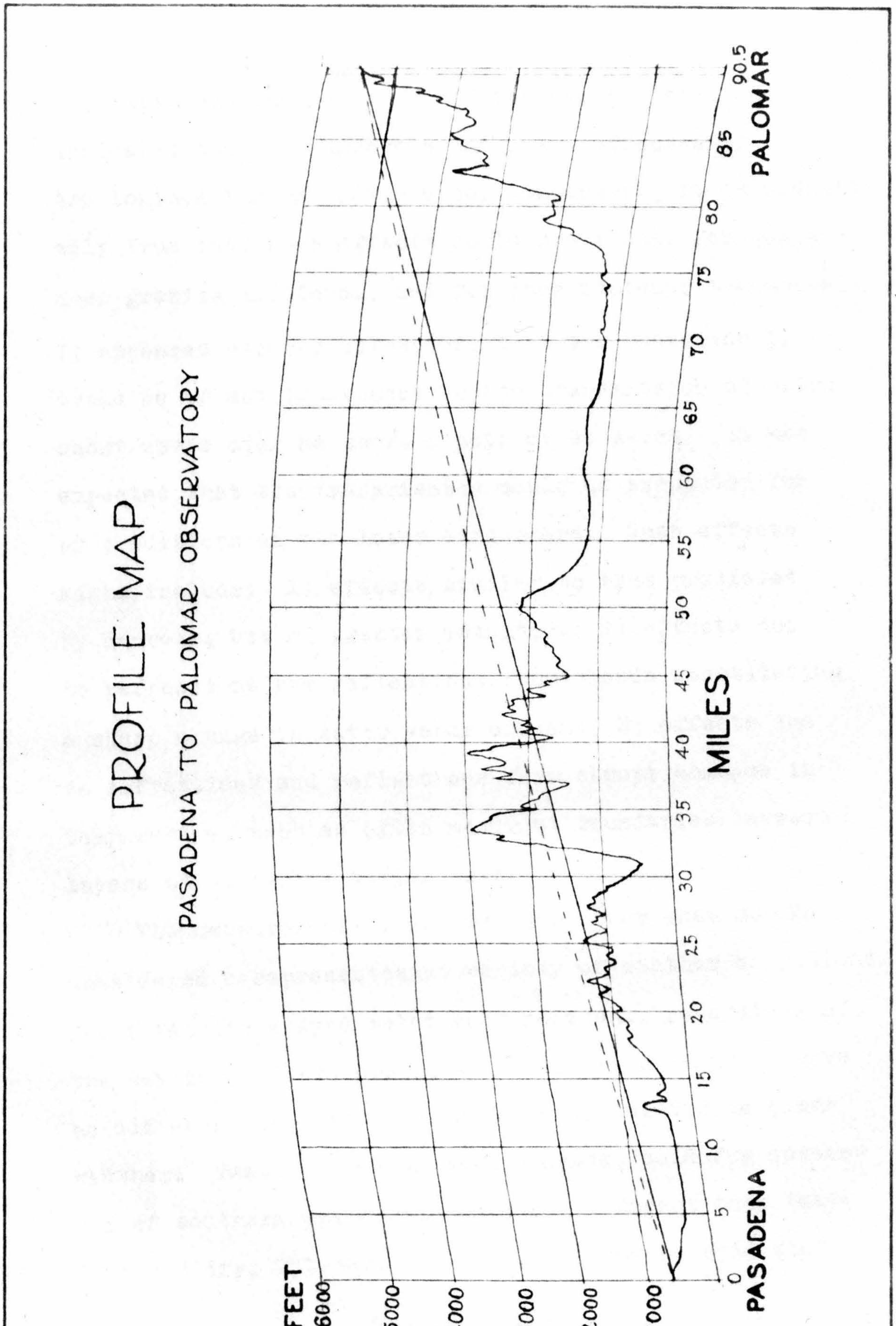

PROFILE MAP
PASADENA TO PALOMAR OBSERVATORY
FEET
6000
5000
4000
3000
2000
1000
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90.5
MILES
PASADENA
PALOMAR

FIG. 17

Although the reception of the eastern stations indicated that 41 megacycle waves are affected by the ionized layers in the upper atmosphere, it is undoubtedly true that such effects could occur only for angles near grazing incidence, and for long distance transmission. It appeared utterly unfeasible that the Heaviside layer could be of any importance to the transmission of ultra short waves over as short a path as 90 miles. It was expected that the transmission could be accounted for by conditions in the lower atmosphere. Such effects might include: 1) effects similar to that predicted by Burrows, but of greater magnitude; 2) effects due to refractions and reflections from clouds constituting a sharp change in water vapor content; 3) effects due to refractions and reflections from abrupt changes in temperature, such as often occur at boundaries between layers of different temperature.

The measurements conducted were over what may be considered a representative variety of weather conditions. The intensity stayed relatively constant, regardless of the weather. Tests conducted during rainy weather gave no different results than tests conducted during clear weather. Tests conducted when a dense, cold fog covered all of southern California were no different than tests during a dry, hot day. In short, it may be said that so far as the observations showed, no effects due to weather conditions were present.

Any efforts to measure the elevation of, or to detect the actual presence of, a marked boundary between the region of optical transmission and non-optical transmission, or a "shadow effect," were nullified by the effects of the surrounding terrain on the slopes of Mt. Wilson. It could only be concluded that a distinct difference did exist between the two types of transmission, and that the effect of the mountains in the path was to lower the signal strength by about 15 decibels.

B. Difficulties in Operation

Originally it was planned to make continuous measurements of the field intensity as measured at Pasadena. In this way some idea of the difference in transmission between night and day could be obtained. Such measurements were impossible for two reasons: 1) the Federal Communication Regulation requires that announcements be made every fifteen minutes while a station is in operation, and 2) a licensed operator must be present whenever a station is in operation.

It would have been possible to build an automatic announcing system, but the second difficulty was insurmountable. Several licensed operators were employed in various capacities by the Astrophysics Department at Mt. Palomar, but it was out of the question to expect one to be on hand 24 hours a day. It was quite difficult enough to have an operator present so that representative readings could be made during working hours. The cooperation of the Astrophysics Department was satisfactory in every respect, but it had to be remembered that any tests were a distinct inconvenience to them.

C. Inadvisability of Mobile Measurements

It was at first planned to make some measurements in the field to determine the relative field intensities at different points. Actual field tests showed that without extremely elaborate and time-consuming preparation consistent results could not be obtained because of the effect of the surrounding terrain. Such inconsistencies could probably have been overcome by the use of very high portable antenna masts. However, much work of this nature has already been done (references in bibliography) and there seemed little reason to duplicate it.

Originally it was planned to test some of the antenna calculations, such as those in Appendix B, and perhaps to carry out some intensive antenna investigations.

The inconsistency and difficulty of making measurements in the field caused this project to be abandoned.

VI. ANTENNA CALCULATIONS

In order to correlate data expected to be taken in the field it was felt that the directional characteristics of the antenna systems used by the Palomar and Pasadena transmitters should be determined. These calculations also seemed of value in that they would allow a direct calculation of the expected fields due to propagation in free space.

For these calculations, a method developed by G. H. Brown[4] was used, treating antenna elements as ordinary circuit elements with mutual impedances due to radiation coupling. The values of the mutual impedances are obtained by moving the surfaces over which the Poynting vector is ordinarily integrated directly up to the surface of the conductor. This method yields the same radiation resistances as the classical method of the sphere at infinity. In addition to the power component, a reactive power component is obtained upon integration. The latter component may be considered as being due to a radiation reactance which is analogous to the radiation resistance. Brown has obtained the values of these mutual reactances and resistances for various antenna spacings. The curves in Fig. 18 were obtained from data published in the paper referred to above.

The resistive and reactive components of the mutual impedance between two one-half wave antennas.

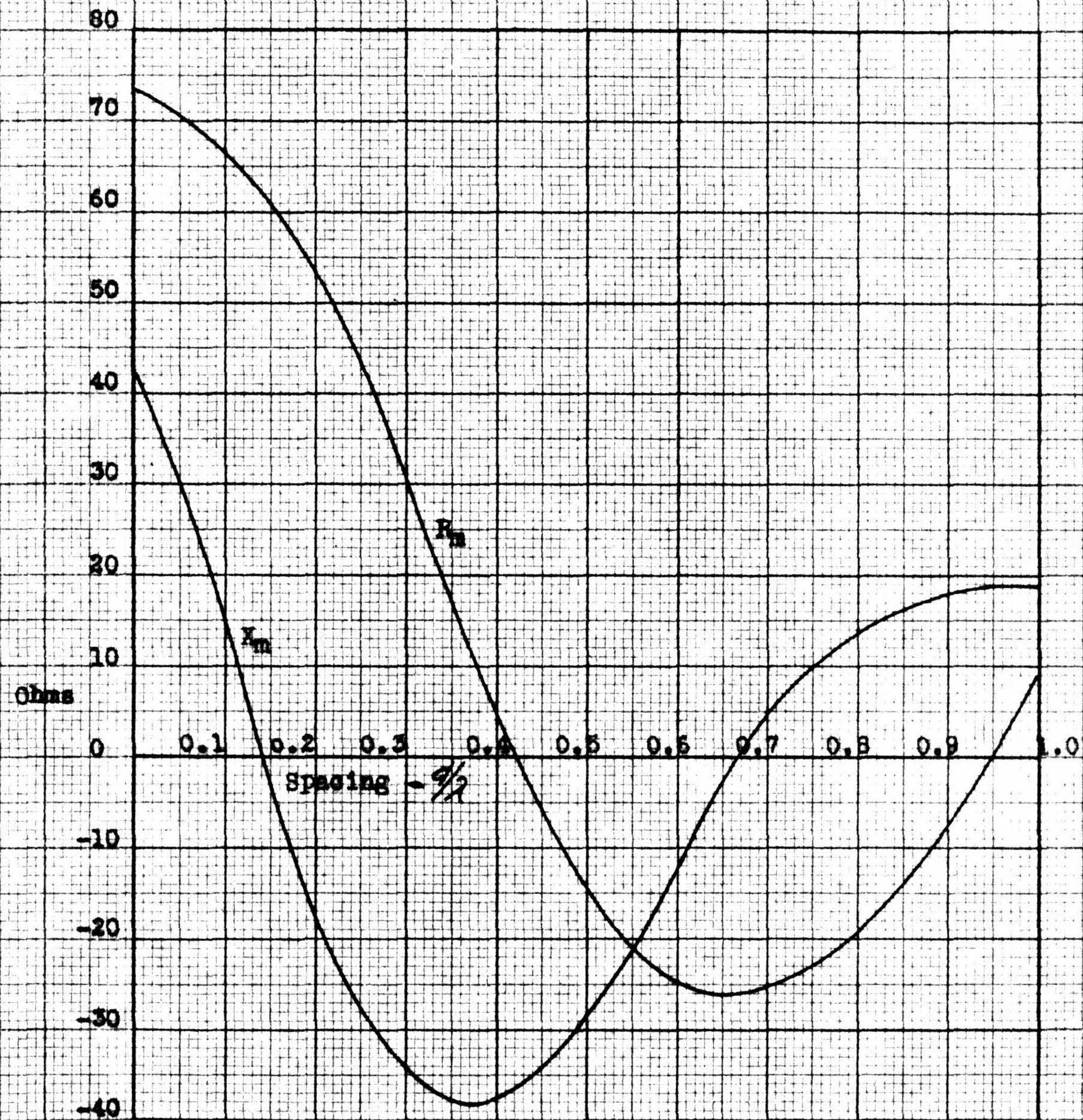


FIG. 18

The mathematical treatment of the self impedance of a single radiating antenna of arbitrary length was carried out in its entirety (Appendix A). After the solution was obtained it was founc that the entire analysis had been published in 1933[7]. However, the work has been presented here because it was an independent investigation.

A. Pasadena Antenna

The Pasadena antenna consists of two excited half wave radiators driven in phase with equal currents, and so spaced as to eliminate interference from the High Voltage Laboratory. The excited radiators are supplemented by two reflectors placed one-quarter wave length behind them.

The complete analysis of the directional characteristics of this antenna system is carried out in Appendix B. The resultant horizontal distribution pattern is shown in Fig. 19, in which unit field is that produced from a single half-wave radiator delivering the same power as the antenna array. This antenna array gives a signal gain of roughly two to one, or 6 db. in the direction of Palomar. This gain, coupled with the elimination of interference, gave much better reception at Pasadena than an ordinary half-wave antenna.

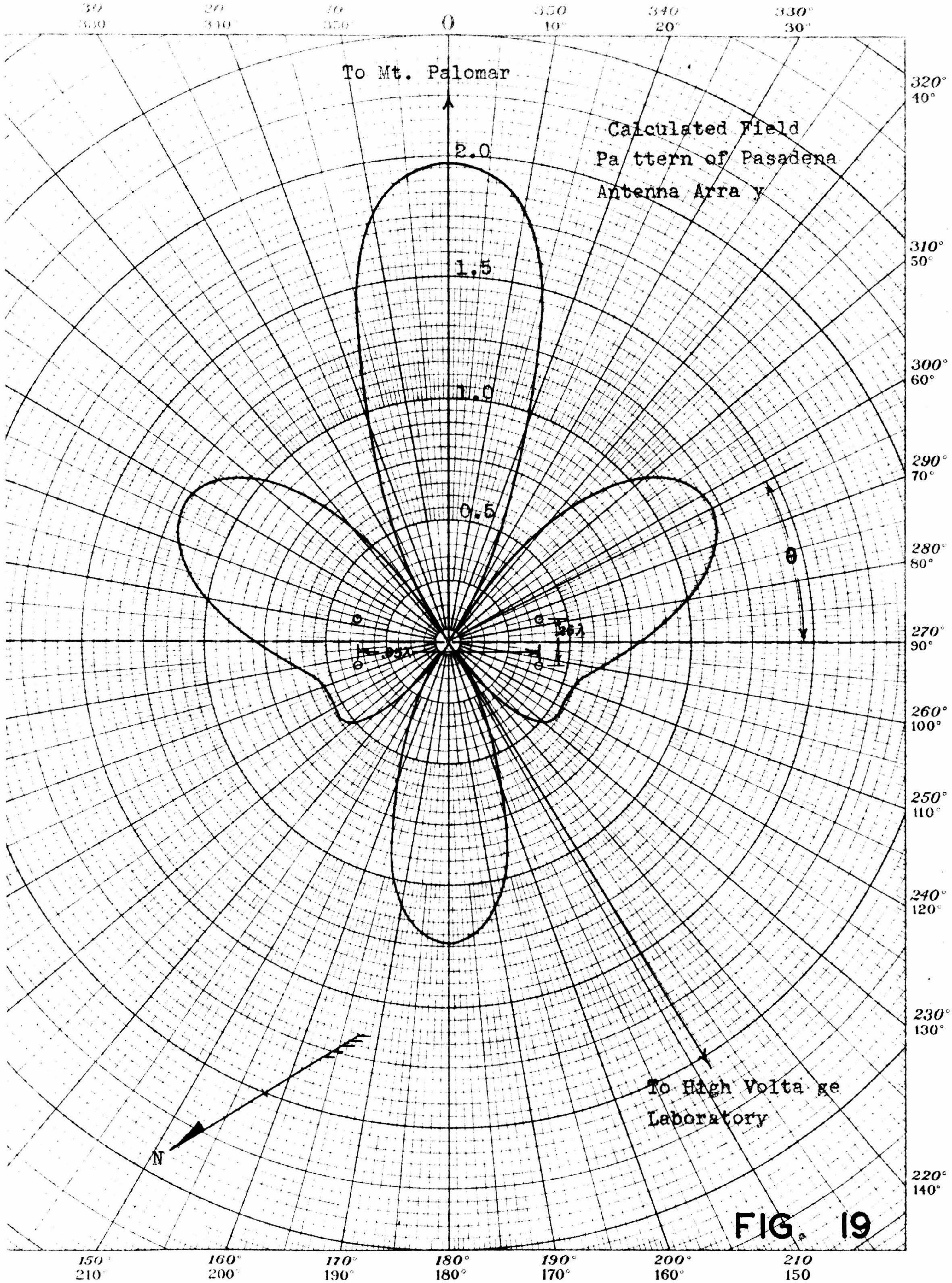
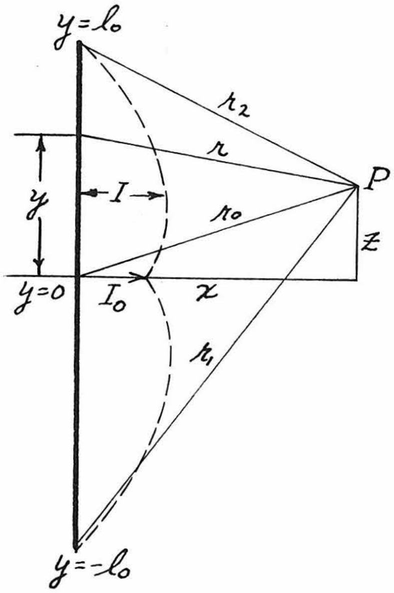

To Mt. Palomar
Calculated Field Pattern of Pasadena Antenna Array
2.0
1.5
1.0
0.5
θ
To High Voltage Laboratory
N
0
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°
120°
130°
140°
150
160°
170°
180°
190°
200°
210°
320°
310°
300°
290°
280°
270°
260°
250°
240°
230°
220°
210
200°
190°
180°
170°
160°
150°
350
340
330°

FIG. 19

APPENDIX A

The Calculation of the Self Impedance of a Straight Antenna of Arbitrary Length

The field from the antenna is calculated by the method introduced by Brown.[4]



The current in the antenna may be represented by the expression

$$I = I_0\left[\cos ky - \beta \sin ky\right]\sin \omega t, \quad y > 0 \tag{1}$$

where I_0 is the amplitude of the current at the center of the antenna, and

$$\beta = \cot k l_0\,, \quad k = \frac{2\pi}{\lambda}\,. \tag{2}$$

The antenna current may also be written in the form

$$I = I_0\, \varepsilon^{j\omega t} \cos k(y+\delta) \tag{3}$$

where

$$\tan k\delta = \beta\,. \tag{4}$$

The retarded vector potential will have a component in the z direction only, which, for the upper half of the antenna, is given by the expression

$$A_{1z} = \int_0^{l_0} \frac{[i]_{t-\frac{r}{c}}}{r}\, dy\,. \tag{5}$$

Upon substituting the retarded value of I from (3) the vector potential for the upper half of the antenna becomes

$$A_{1z} = \frac{I_0}{2}\, \varepsilon^{j\omega t}\left\{ \varepsilon^{jk(z+\delta)} \int_0^{l_0} \frac{\varepsilon^{-jk(r-y+z)}}{r}\, dy + \varepsilon^{-jk(z+\delta)} \int_0^{l_0} \frac{\varepsilon^{-jk(r+y-z)}}{r}\, dy \right\}. \tag{6}$$

Making the following substitutions

$$u = k(r+y-z) \qquad u_2 = k(r_2+l_0-z)$$
$$v = k(r-y+z) \qquad \text{etc.} \tag{7}$$

this expression becomes,

$$A_{1z} = \frac{I_0}{2}\varepsilon^{j\omega t}\left[\varepsilon^{-jk(z+\delta)}\int_{u_0}^{u_2}\frac{\varepsilon^{-ju}}{u}\,du - \varepsilon^{jk(z+\delta)}\int_{v_0}^{v_2}\frac{\varepsilon^{-jv}}{v}\,dv\right]. \tag{8}$$

Letting ϕ represent the azimuth angle as measured in the plane normal to the antenna, and since $\bar{H} = \bar{\nabla}\times\bar{A}$, the component of the magnetic field normal to the paper, H_ϕ, is

$$H_{1\phi} = -\frac{\partial A_{1z}}{\partial x}$$

where the subscript indicates that the upper half only of the antenna is being considered.

Equation (8) may be differentiated with respect to x by use of the relation

$$\frac{\partial}{\partial x}\int_{u_0}^{u_2}\frac{\varepsilon^{-ju}}{u}\,du = \frac{\varepsilon^{-ju_2}}{u_2}\frac{\partial u_2}{\partial x} - \frac{\varepsilon^{-ju_0}}{u_0}\frac{\partial u_0}{\partial x}.$$

The vector potential for the lower half of the antenna may be obtained in a similar manner, and after taking its curl as indicated above the total magnetic field normal to the paper becomes

$$H_\phi = -\frac{I_0}{2}\varepsilon^{j\omega t}\left\{\left[\frac{kx}{r_2}\varepsilon^{-jkr_2}\left(\frac{v_2\varepsilon^{-j\theta} - u_2\varepsilon^{j\theta}}{v_2u_2}\right) - \frac{kx}{r_1}\varepsilon^{-jkr_1}\left(\frac{v_1\varepsilon^{j\theta} - u_1\varepsilon^{-j\theta}}{u_1v_1}\right)\right]\right. \tag{9}$$

$$\left. -j\beta\left[\frac{kx}{r_2}\varepsilon^{-jkr_2}\left(\frac{v_2\varepsilon^{-j\theta} + u_2\varepsilon^{j\theta}}{v_2u_2}\right) + \frac{kx}{r_1}\varepsilon^{-jkr_1}\left(\frac{v_1\varepsilon^{j\theta} + u_1\varepsilon^{-j\theta}}{u_1v_1}\right) - \frac{2kx}{r_0}\varepsilon^{-jkr_0}\left(\frac{u_0+v_0}{u_0v_0}\right)\right]\right\}$$

where (10)

$$\theta = kl_0.$$

From Maxwell's equations for free space the following expression is obtained for the electric field

$$\frac{1}{c}\frac{\partial \bar{E}}{\partial t} = \nabla \times \bar{H}.$$

Ordinarily this equation cannot be solved for $\bar{E}$ in terms of $\bar{H}$, but since sinusoidal currents only are involved, and since complex notation is being used, the value of $\bar{E}$ becomes

$$\bar{E} = \frac{c\nabla \times \bar{H}}{j\omega} = \frac{1}{jk}\nabla \times \bar{H}.$$

In the system of coordinates chosen the vertical component of the electric field is then

$$E_z = \frac{1}{jkx}\frac{\partial}{\partial x}(xH_\phi). \tag{11}$$

By use of (7) and (9) the expression for xH_ϕ takes the simple form

$$xH_\phi = \frac{jI_0\varepsilon^{j\omega t}}{\sin\theta}\left\{\left[\varepsilon^{-jkr_2} + \varepsilon^{-jkr_1}\right] - 2\cos\theta\,\varepsilon^{-jkr_0}\right\}. \tag{12}$$

After simplification the value of E_z as determined by equation (11) is

$$E_z = \frac{-jI_0\varepsilon^{j\omega t}}{\sin\theta}\left[\frac{\varepsilon^{-jkr_2}}{r_2} + \frac{\varepsilon^{-jkr_1}}{r_1} - 2\cos\theta\frac{\varepsilon^{-jkr_0}}{r_0}\right]. \tag{13}$$

If I_0 is chosen as the r.m.s. value of the current at the center of the antenna, and if the current is chosen as the reference vector in the ordinary electrical engineering use of complex notation, the r.m.s. value of the field is given by the expression

$$E_z = \frac{-30j\, I_o}{\sin\theta}\left[\frac{\varepsilon^{-jkr_2}}{r_2} + \frac{\varepsilon^{-jkr_1}}{r_1} - 2\cos\theta\,\frac{\varepsilon^{-jkr_o}}{r_o}\right] \tag{14}$$

where the current is measured in amperes and the field is in volts per cm.

Brown's paper[4] suggested the possibility of obtaining a mathematical solution for the radiation resistance and reactance of an antenna which is not a half wave length long. Pistolkors[5] first showed that the power radiated from an antenna could be obtained from the relation

$$P = -\int_{-l_o}^{+l_o} I\bar{E}\cdot d\bar{l} \tag{15}$$

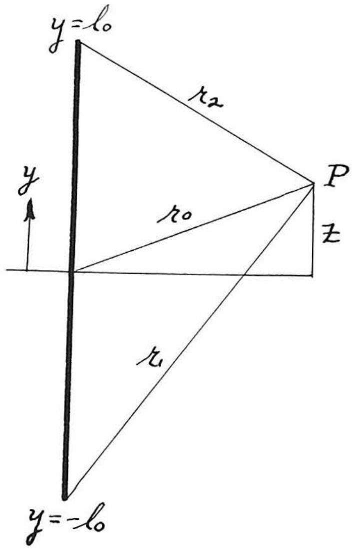
where $\bar{E}$ = Instantaneous field intensity at the conductor

I = Instantaneous current in the conductor.

When sinusoidal quantities are involved E and I in general will not be in phase, and, in analogy to ordinary circuits, a so-called reactive power will be obtained. When complex notation is adopted, and if the r.m.s. values of the current and voltage are used with the current as the reference vector, equation (15) will become

$$-\int_{-l_o}^{+l_o} I\bar{E}\cdot d\bar{l} = I_o^2(R_{oo} + jX_{oo}) \tag{16}$$

where I_o is the r.m.s. value of the current at the center of the antenna, R_{oo} corresponds to the radiation resistance of the antenna, and X_{oo} may be called the radiation reactance of the antenna. Both R_{oo} and X_{oo} are referred to the current at the center of the antenna.



It is only necessary to evaluate the integral for the upper half of the antenna, since from symmetry both halves give the same contribution.

The r.m.s. value of the current may be obtained from (1), and the value of the field is obtained from (14). If the radius of the antenna conductor is ρ ,

$$\begin{aligned} r_2' &= \sqrt{(l_0-y)^2+\rho^2} \\ r_1' &= \sqrt{(l_0+y)^2+\rho^2} \\ r_0' &= \sqrt{y^2+\rho^2} \end{aligned} \tag{17}$$

and the vertical component of the field at the antenna becomes

$$\begin{aligned} E_y = \frac{-30 I_0}{\sin\theta}\Bigg\{&\left[\frac{\sin kr_2'}{r_2'}+\frac{\sin kr_1'}{r_1'}-2\cos\theta\frac{\sin kr_0'}{r_0'}\right] \\ &+j\left[\frac{\cos kr_2'}{r_2'}+\frac{\cos kr_1'}{r_1'}-2\cos\theta\frac{\cos kr_0'}{r_0'}\right]\Bigg\}. \end{aligned} \tag{18}$$

Upon integration the values of R_{00} and X_{00} as given by equation (16) are

$$\begin{aligned} R_{00} = \frac{60}{\sin\theta}\Bigg[&\int_0^{l_0}\frac{\sin kr_2'\cos ky}{r_2'}\,dy-\beta\int_0^{l_0}\frac{\sin kr_2'\sin ky}{r_2'}\,dy+\int_0^{l_0}\frac{\sin kr_1'\cos ky}{r_1'}\,dy \\ &-\beta\int_0^{l_0}\frac{\sin kr_1'\sin ky}{r_1'}\,dy-2\cos\theta\int_0^{l_0}\frac{\sin kr_0'\cos ky}{r_0'}\,dy+2\beta\cos\theta\int_0^{l_0}\frac{\sin kr_0'\sin ky}{r_0'}\,dy\Bigg] \end{aligned} \tag{19}$$

and $$X_{00} = \frac{60}{\sin\theta}\left[\int_0^{l_0} \frac{\cos kr_2' \cos ky}{r_2'}\,dy - \beta\int_0^{l_0}\frac{\cos kr_2' \sin ky}{r_2'}\,dy + \int_0^{l_0}\frac{\cos kr_1' \sin ky}{r_1'}\,dy \right.$$

$$\left. - \beta\int_0^{l_0}\frac{\cos kr_1' \sin ky}{l_0 + y}\,dy - 2\cos\theta\int_0^{l_0}\frac{\cos kr_0 \cos ky}{r_0'}\,dy + 2\beta\cos\theta\int_0^{l_0}\frac{\cos kr_0' \sin ky}{r_0'}\,dy\right]. \tag{20}$$

If it is assumed that the radius of the conductor is small all of the integrands in the expression for R_{00} remain finite as $\rho \to 0$, and after considerable manipulation equation (19) becomes

$$R_{00} = 60\left\{(1-\beta^2)\int_0^{2\theta}\frac{\sin^2\varphi\,d\varphi}{\varphi} + 2\beta\left[\int_0^{2\theta}\frac{\sin\varphi\cos\varphi}{\varphi}\,d\varphi - 2\int_0^{\theta}\frac{\sin\varphi\cos\varphi}{\varphi}\,d\varphi\right] \right. \tag{21}$$

$$\left. + 4\beta^2\int_0^{\theta}\frac{\sin^2\varphi\,d\varphi}{\varphi}\right\}.$$

At this point the sine and cosine integral functions are introduced as defined by Jahnke and Emde[6].

Definitions:

$$Ci(a) = -\int_a^{\infty}\frac{\cos\varphi}{\varphi}\,d\varphi \tag{22}$$

$$Si(a) = \int_0^{a}\frac{\sin\varphi}{\varphi}\,d\varphi \tag{23}$$

By expansion of (22) into a power series the value of $Ci(\varepsilon)$ for small values of ε may be obtained in the form

$$Ci(\varepsilon) = \gamma + \log\varepsilon \tag{24}$$

where γ = Euler's constant.

To express R_{00} in terms of these functions it is first necessary to examine the integral

$$M=\int_0^{\theta}\frac{\sin^2\varphi}{\varphi}\,d\varphi.$$

Let $$M=\lim_{\varepsilon\to 0}\int_{\varepsilon}^{0}\frac{\sin^2\varphi}{\varphi}\,d\varphi=\lim_{\varepsilon\to 0}\frac{1}{2}\left[\log\frac{\theta}{\varepsilon}-\int_{\varepsilon}^{\theta}\frac{\cos 2\varphi}{\varphi}\,d\varphi\right]$$

then $$M=\lim_{\varepsilon\to 0}\left[\log\frac{\theta}{\varepsilon}-\int_{\varepsilon}^{\infty}\frac{\cos 2\varphi}{\varepsilon}\,d\phi+\int_{\theta}^{\infty}\frac{\cos 2\varphi}{\varphi}\,d\varphi\right]$$

or

$$M=\lim_{\varepsilon\to 0}\left[\log\frac{\theta}{\varepsilon}+Ci(2\varepsilon)-Ci(2\theta)\right].$$

Using (24) this becomes

$$M=\frac{1}{2}\left[\log 2\theta+\gamma-Ci(2\theta)\right].$$

By similar manipulations the value of (21) becomes

$$R_{00}=30\Big\{(1-\beta^2)\left[\log 4\theta+\gamma-Ci(4\theta)\right]+2\beta\left[Si(4\theta-2\,Si(2\theta))\right] \quad (25)$$
$$+4\beta^2\left[\log 2\theta+\gamma-Ci(2\theta)\right]\Big\}.$$

This expression gives the radiation resistance referred to the current at the center of the antenna. If the antenna is longer than a half wave length it is customary to refer the radiation resistance to a current node. The value of R_{00} then becomes

$$R_{00}'=30\sin^2\theta\Big\{(1-\beta^2)\left[\log 4\theta+\gamma-Ci(4\theta)\right]+2\beta\left[Si(4\theta)-2Si(2\theta)\right] \quad (26)$$
$$+4\beta^2\left[\log 2\theta+\gamma-Ci(2\theta)\right]\Big\}$$

for $l_0 \geqq \frac{\lambda}{4}$.

In case the antenna is exactly a half wave length long the radiation resistance takes the familiar value

$$R_{00} = 30\left[\log 2\pi + \gamma - Ci(2\pi)\right] = 73.2 \text{ OHMS.}$$

In the determination of X_{00} it is found that the assumption $\rho \to 0$ causes some of the integrals to diverge. After manipulation all of these infinite integrals may be reduced to the form

$$I = \int_0^{l_0} \frac{\cos ku \cos k\sqrt{u^2+\rho^2}}{\sqrt{u^2+\rho^2}}\, du.$$

If it is assumed that $\rho \ll l_0$, this integral reduces to

$$I = \log\frac{2u_1}{\rho} + \int_{u_1}^{l_0} \frac{\cos^2 ku}{u}\, du$$

where

$$\rho \ll u_1 \ll l_0.$$

This expression may then be reduced to

$$I = \frac{1}{2}\left[\log\frac{2l_0}{k\rho^2} - \gamma + Ci(2kl_0)\right]$$

by the use of (22), (23), and (24).

After repeated operations the expression for X_{00} becomes

$$X_{00} = -30\Big\{2\beta\left[\log\frac{l_0}{k\rho^2} - \gamma + 2\,Ci(2\theta) - Ci(4\theta)\right] \tag{27}$$

$$+ (\beta^2-1)\left[Si(4\theta) - 2\,Si(2\theta)\right] - 2(\beta^2+1)\,Si(2\theta)\Big\}.$$

For a half wave antenna the reactance becomes

$$X_{00} = 30\, Si(2\pi) = 42.5 \text{ ohms.}$$

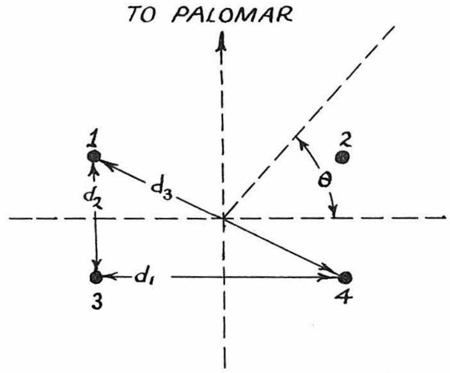
This result shows that a half wave antenna, excited at the center, is not a resonant circuit, but that it has an inductive reactance of 42.5 ohms compared to a resistance of 73.2 ohms.

It was intended to carry the investigation much further, and to find the antenna length to give zero reactance, the nature of the variation of radiation resistance and reactance with length, etc. However, after completing the above work a similar treatment was found published in German[(7)]. This made it of no particular value to continue the investigation. The above work is presented since it was an independent investigation.

APPENDIX B

Calculation of the Horizontal Field Pattern of the Pasadena Antenna

The antenna may be considered as made up of 4 half-wave radiators, spaced as shown. Elements 1 and 2 are excited by transmission lines, while elements 3 and 4 act as parasitic reflectors.



If we now consider the four antenna elements as four simple circuit elements with self impedances and mutual impedances, the following equations are obtained:

$$V_1 = I_1 Z_{11} + I_2 Z_{21} + I_3 Z_{31} + I_4 Z_{41} \quad (1)$$

$$V_2 = I_1 Z_{12} + I_2 Z_{22} + I_3 Z_{32} + I_4 Z_{42} \quad (2)$$

$$V_3 = I_1 Z_{13} + I_2 Z_{23} + I_3 Z_{33} + I_4 Z_{43} \quad (3)$$

$$V_4 = I_1 Z_{14} + I_2 Z_{24} + I_3 Z_{34} + I_4 Z_{44} \quad (4)$$

where the currents are measured at the center of the radiators, ~~and~~ the voltages are the exciting voltages applied there, and complex notation is used. From symmetry it may be assumed that:

$$Z_{11} = Z_{22} = Z_{33} = Z_{44} = Z_{00}$$

$$Z_{13} = Z_{31} = Z_{24} = Z_{42} = Z_2$$

$$Z_{14} = Z_{41} = Z_{32} = Z_{23} = Z_3$$

$$Z_{12} = Z_{21} = Z_{34} = Z_{43} = Z_1$$

Since the two excited antennas are excited from identical feeders it may be assumed that

$$V_1 = V_2 = V$$

$$I_1 = I_2 = I$$

Since elements 3 and 4 are parasitic reflectors there is no external excitation applied to them, and

$$V_3 = V_4 = 0$$

Upon solving the above system of equations the results are:

$$V = I\left\{(Z_o + Z_1) + \frac{(Z_2 + Z_3)\left[Z_1(Z_2 + Z_3) - Z_o(Z_3 + Z_2)\right]}{Z_o^2 - Z_1^2}\right\} \quad (5)$$

$$I_4 = I_3 = \frac{I\left[Z_1(Z_2 + Z_3) - Z_o(Z_3 + Z_2)\right]}{Z_o^2 - Z_1^2} \quad (6)$$

A half wave antenna has a reactance of 42.5 ohms (see Appendix A). In practice most antennas are made about 95% of the theoretical length in order to make this reactance zero. Such a choice gives

$$Z_{oo} = 73.2 + j0$$

for the self impedance of each radiator, since the shortening has no appreciable effect upon the radiation resistance.

The shortening has little effect upon the mutual impedance values, and Fig. 18 may be used for their determination.

Substitution into equations 5 and 6 of the numerical values for these impedances yields

$$V = I(72.7 + j\,19.3)$$

$$I_3 = I_4 = -I(.517 - j\,0.29)$$

Since the total power radiated is twice that radiated by each excited radiator,

$$I = \sqrt{\frac{P_T}{2(72.7)}} \qquad (7)$$

and

$$I_3 = I_4 = -\sqrt{\frac{P_T}{2(72.7)}}\left(0.517 - j\,0.29\right) \qquad (8)$$

The above expressions give the currents at the center of the half wave radiators of the array. Knowing this, it is a relatively simple matter to calculate the field pattern.

It is well known that the vertical field intensity from a half wave antenna in free space in the horizontal plane through the center of the antenna is given by the expression*

$$E = \frac{60\,I}{r}\,\varepsilon^{-jkr}$$

* This expression may be obtained from equation (14) in Appendix A by placing $l_0 = \frac{\lambda}{4}$, and $r_2 = r_1 = r$.

where $k = \frac{2\pi}{\lambda}$

$\lambda =$ wave length in cm.

$I =$ current at center of antenna (r.m.s. value)

$E =$ field in volts per cm. (r.m.s. value)

$r =$ distance from antenna in cm.

The field at a great distance, r, may then be calculated as a function of the azimuth angle, θ , by the use of this expression in conjunction with the values of the currents from equations 7 and 8 .

Since the radiation resistance of a single half wave antenna is 73.2 ohms, the r.m.s. value of the field from such an antenna delivering P_T watts is

$$E_o = \frac{60}{r_o} \sqrt{\frac{P_T}{73.2}} .$$

The use of this expression and the above work yields a relation between the ratio of the field intensity of the antenna array to that of a simple dipole delivering the same power. The result is:

$$\frac{E}{E_o} = \sqrt{2} \sqrt{\frac{73.2}{72.7}} \left\{ \sqrt{2} \left| \cos \left(\frac{kd_1}{2} \cos\theta \right) \right| \right\} \sqrt{A^2 + B^2}$$

Where:

$$A = \left[1 - 0.517 \cos (kd_2 \sin\theta) + 0.29 \sin (kd_2 \sin\theta) \right]$$

$$B = \left[0.517 \sin (kd_2 \sin\theta) + 0.29 \cos (kd_2 \sin\theta) \right]$$

This expression is shown as a function of the azimuth angle in Fig. 19.

APPENDIX C

Sample Data Sheet and Reduction to db. above 1 mv./meter

The sample data chosen was recorded from 9:30 A.M. to 10:30 A.M. on May 12. The resultant curve appears in Fig. 15.

Using the substitution method the field strength meter was tuned to the incoming signal and the grid bias adjusted to give $I_p = 0.6$ ma. The antenna was then disconnected and the signal introduced from the standard signal generator. Without changing the field strength meter the attenuator was adjusted to give $I_p = 0.6$, while the signal generator current was held constant at 60 ma. The reading, A, of the attenuator was recorded.

Data:

May 12

Time	A
9:30	109.6
9:40	108.8
9:50	108.8
10:00	108.7
10:10	108.9
10:20	109.1
10:30	108.8

The antenna current as announced from Palomar was constant all morning at 1.12 ampere.

Ultimately the standard field generator was placed a half wave length in front of the center of the antenna, and the set was found to respond at a grid bias voltage corresponding to $A = 66.6$, using the calibration curves in Fig. 5. The thermogalvanometer in the radiating loop was held at the full scale reading of 115 ma.

The field from the radiating loop was calculated by the following equation[1]:

$$E = \frac{240\,\pi^2 A I}{\lambda^3} (1.05)$$

where E = r.m.s. value of field in volts per meter

A = area of loop in square meters

I = current in loop in amperes

λ = wave length in meters.

Upon substitution of the numerical values the field strength from the loop is obtained as,

E = 18.5 millivolts per meter.

But 18.5 mv./meter is

$$20 \log_{10} \left(\frac{18.5}{1}\right) = 25.4 \text{ db.}$$

above 1 mv./meter, and by subtracting $(66.6 + 25.4) = 92.0$ from the values of A the field strength is obtained in db. below one mv. per meter.

If the signal strength is desired in db. above 1 mv. per meter it may be obtained from the expression,

$$-(A - 92)$$

Actually, however, the antenna current was 1.12 amperes or, $20 \log_{10}\left(\frac{1.12}{1}\right) = 1$ db. above the standard of one ampere. Therefore the above data may be referred to the signal with one ampere of antenna current by subtracting 1 db. from the measured values. The final value of the field intensity to be used in comparison with other data may be obtained from the expression

- (A - 91)

The data for plotting then become:

Time	Field Str. Db.above 1 mv./m. 1.0 amp.std.
9:30	-18.6
9:40	-17.8
9:50	-17.8
10:00	-17.7
10:10	-17.9
10:20	-18.1
10:30	-17.8

See Fig. 5 for the curve corresponding to these data.

The average field is -17.9 db. above 1 mv./meter, which corresponds to 0.139 mv./meter, or approximately 140 microvolts per meter.

BIBLIOGRAPHY

1. J.C. Schelleng, ,C.R. Burrows, and E.B. Ferrell, "Ultra-Short-Wave Propagation", Proc. I.R.E., vol. 21, p.p. 427-463; March (1933).

2. D.E. Harnett and N.P. Case, "The Design and Testing of Multirange Receivers", Proc. I.R.E., vol. 23, pp. 578-593, June (1935).

3. Byerly, Textbooks on Bessel functions, Fourier series, etc.

4. G.H. Brown, "Directional Antennas", Proc. I.R.E., vol. 25, pp. 78-145; January (1937).

5. A.A. Pistolkors, "The Radiation Resistance of Beam Antennas", Proc. I.R.E., vol. 17, pp. 562-628; March (1929).

6. E. Jahnke and F. Emde, "Funktionentafeln mit Formeln und Kurven".

7. J. Labus, "Rechnerische Ermittlung der Impedanz von Antennen", "Hochfrequenztechnik und Elektroakustik", pg. 17, 1933.

8. R.C. Colwell and A.W. Friend, "Ultra-High-Frequency Wave Propagation Over Plane Earth and Fresh Water", Proc. I.R.E., vol. 25, pp. 32-37; January (1937).

9. L.F. Jones, "A Study of the Propagation of Wavelengths Between Three and Eight Meters", Proc. I.R.E., vol. 21, pp. 349-386; March (1933)

10. B. Trevor and P.S. Carter, "Notes on Propagation of Waves Below Ten Meters in Length", Proc. I.R.E., vol. 21, pp. 387-426; March (1933).

11. C.R. Englund, A.B. Crawford, and W.M. Mumford, "Some Results of a Study of Ultra-Short-Wave Transmission Phenomena", Proc. I.R.E., vol. 21, pp. 464-492; March (1933).

12. H.O. Peterson and D.R. Goddard, "Field Strength Observations of Transatlantic Signals, 40 to 45 Megacycles", Proc. I.R.E., vol. 25, pp. 1291-1299; October (1937).

13. P. von Handel and W. Pfister, "Ultra-Short-Wave Propagation Along the Curved Earth's Surface", Proc. I.R.E., vol. 25, pp. 346-363; March (1937).

14. C.R. Burrows, "The Surface Wave in Radio Propagation Over Plane Earth", Proc. I.R.E., vol. 25, pp. 219-229; February (1937).

15. H.H. Beverage, "Some Notes on Ultra High Frequency Propagation", R.C.A. Review, vol. 1, pp. 76-87, January (1937).

16. C.R. Burrows, A. Decino, and L.E. Hunt, "Ultra-Short-Wave Propagation over Land", Proc. I.R.E., vol. 23, pp. 1507-1535; December (1935).

17. H.G. Houghton, "Transmission of Visible Light Through Fog" and "Theoretical Investigation of the Transmission of Visible Light through Fog", Publications from the Massachusetts Institute of Technology, Serial No. 74; September, 1931.

18. H.G. Houghton, "The Size and Size Distribution of Fog Particles", Publications from the Massachusetts Institute of Technology, Serial No. 82; January, 1933.