

THE INDEX OF REFRACTION OF AIR IN THE  
PHOTOGRAPHIC INFRARED.

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ABSTRACT

The index of refraction of dry CO<sub>2</sub>-free air has been measured in the region 6500 A. to 11200 A. With 55 measurements at 30 wavelengths the following dispersion curve at standard conditions was calculated by the method of least squares:

$$(n_0 - 1) \cdot 10^7 = 2878.94 \pm 3.8 + \frac{24.74 \pm 4.5}{\lambda^2} - \frac{2.47 \pm 1.3}{\lambda^4}$$

The curve can be extrapolated further into the infrared, but not into the visible. Comparison with other observers indicates that the present results are higher than the others by 6 to 10 parts in 10<sup>7</sup>.

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INTRODUCTION

Since the invention of interferometers the index of refraction of air has been the object of many investigations. In order to justify the accurate measurements that can be made it was necessary to determine whether or not the index of refraction of normal air (0°C and 760 mm. pressure) could be expected to remain constant despite whatever fluctuations there may be in the content and density of normal air. Experiments performed since 1850 indicate that the percentage of oxygen in normal dry CO<sub>2</sub>-free air does not vary over a range of more than 0.2%.<sup>1</sup> This much change in the relative amounts of oxygen and nitrogen would amount to approximately 0.5 parts in 10<sup>7</sup> change in the value of the index of refraction. Thus it would seem that with sufficiently accurate methods one could determine the value of the index to one part in 10<sup>7</sup>.

But the variation in the values of the index given by different observers is of the order of magnitude of five parts in 10<sup>7</sup> (see Table V), and the variations among the data of each observer

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<sup>1</sup> M. S. Blanchard and S. F. Pickering, Bur. Stands. Sci. Papers, 21, p.167 (1926).

are probably somewhat greater.<sup>2</sup> The present work was undertaken with the hope of helping to clear up the discrepancies and of carrying the measurements farther into the infrared.

### APPARATUS

The method of obtaining the index of refraction in the present experiment was to observe the change in the order of interference on a Fabry-Perot interferometer when it was placed in a vacuum and when it was placed in dry CO<sub>2</sub>-free air. The interferometer and the vacuum chamber were situated between the collimator and the prism of the spectrograph. In order not to distort the fringes the windows were optical flats sealed to the brass vacuum chamber with pitch.

### Source and Photographic Plates

The source was a capillary neon tube with a bore of 1.5 mm. The tube, viewed from the side in order to obtain a long uniformly illuminated slit, was usually run at 25 milliamperes. These conditions, as well as the pressure of neon in the tube, conform to the requirements set by Jackson<sup>3</sup> for obtaining the neon secondary standards of wavelength. Run at these conditions the source was sufficiently intense to give most of the wavelengths out to 8800 Å.<sup>0</sup> on Eastman 144-P plates in one hour. But in order to obtain lines beyond this point with a reasonable exposure it was found advisable

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<sup>2</sup> e.g. see Meggers and Peters, who have tabulated their variations, Bur. Stands. Bulletin, 14, p.697-740 (1918).

<sup>3</sup> C. V. Jackson, Proc. Royal Society, 143A, p.124 (1933).

to run the tube at a higher current (52 milliamperes). Thus it was possible to obtain sharp lines in the neon group from 9100 A.<sup>0</sup> to 9700 A.<sup>0</sup> with an exposure of four hours on type 144-Q plates, but the group at 11000 A.<sup>0</sup> did not yield sharp lines even with twelve hour exposure on type I-Z plates. The inability to obtain sharp lines in these long wavelengths was due to a combination of circumstances. The lines are probably not as intense as the shorter wavelengths, thus necessitating a thin silver coat on the interferometer mirrors and consequently reduced resolving power, and the Z plates may not be as sensitive to this region as the Q plates are to the other. Unfortunately still one more effect contributed to the inaccuracy of the index for this region, namely the fact that the observed fringe shift was here at its minimum.

Only one line other than the neon lines was used and this was the intense mercury line at 10140 A.<sup>0</sup>. It was produced by an arc drawing 2.5 amperes and was obtained through a filter cutting out the visible lines in half hour exposures on either 144-Q or I-Z plates. The sharpness of this line was comparable with that of the neon lines in the group at 11000 A.<sup>0</sup>.

#### Temperature Control and Measurement

With such long exposures it was necessary to keep the temperature constant over long periods of time. A small celotex house about four feet high was built over the table enclosing the entire apparatus. The temperature of the air inside was controlled by a sensitive thermostat and constantly circulated by a fan. Also

the collimator, vacuum chamber, prism, and front half of the camera were enclosed in a celotex box supported about six inches above the table in order to allow free circulation of the air on all sides. Inside this box and between it and the vacuum chamber were placed as many metal cans filled with water as could be put in. This was to increase the thermal capacity of the box whose temperature it was desired to keep constant. The source was outside this box in the circulating air and was focused through a window onto the slit.

It was found necessary also to control the room temperature because the inside thermostat, although sensitive, was sluggish. The average temperature maintained by it and the heating coils varied slightly with the room temperature. The expanding substance in this thermostat was about 140 cubic centimeters of a light oil contained mostly in four thin walled brass tubes. The contact was made by means of mercury in a U-tube, one arm of which was a one millimeter capillary tube. Some oil was left above the mercury in the capillary tube in order to keep the surface from oxidizing at the contact. This was quite satisfactory with the current used to operate the relay, namely 50 milliamperes. The sensitivity of this thermostat was approximately 6 cm./degree.

With the room temperature varying less than a degree, the range of temperature of the air inside the house was constant and was about  $0.2^{\circ}$  C., rising and falling every few minutes. A thermometer with its bulb close to the vacuum chamber fluctuated no more than  $\pm 0.05^{\circ}$  C. in twenty-four hours. Usually during the

course of a run this thermometer reading varied inside a range of  $\pm 0.03^{\circ}$  C. The temperature of the interferometer and the air in the vacuum chamber probably varied much less than this. Readings of this thermometer just outside the vacuum chamber were taken to give the temperature of the air inside. It was a thermometer graduated in tenths of degrees and carefully checked with a similar one belonging to the chemistry department calibrated by the Bureau of Standards.

#### Pressure Measurement

The pressure of the air was measured on a carefully built manometer. The manometer tube, one centimeter in diameter, was heated under vacuum in order to clean it and to drive off as much adsorbed gas as possible before the mercury was distilled into it. The height of the column was measured with a cathetometer having a glass scale previously calibrated against a standard meter. Settings with the cathetometer could be repeated to less than 0.1 mm. The corrections applied to the reading of the height of the manometer were: (1) the meniscus correction which was small because of the fairly large tubing and the slight difference between the menisci on the two sides, (2) the correction for the error in the glass scale, (3) the correction due to the difference in the value of gravity from the standard value, and (4) the temperature correction for the mercury. The pressure of the air during the exposure for vacuum was probably about  $10^{-2}$  mm. of mercury, for exposures were made only when a discharge tube in the



system would take no current from the transformer used to excite the source.

#### Elimination of Vibration

Runs were always made by taking an exposure with the vacuum, then one with air at nearly atmospheric pressure, and then a third with the vacuum. In order to have the order of interference on the two vacuum exposures agree, it was found necessary to eliminate all sources of vibration. Consequently the fan inside was supported by springs from a framework, three pads of felt were placed under each leg of the table, and the Hyvac pump was placed on two pads of felt. Inside the vacuum chamber the interferometer was supported on rubber stoppers and had no other contact with the walls or bottom. The rubber stoppers, however, although they were coated with beeswax, did more harm than good. The vapors from them apparently caused the thin silver films to spoil in a very few days. For when the rubber stoppers were replaced by thin slabs of cork the silver films kept much longer.

Despite these precautions of keeping the temperature constant and of reducing the vibration, the zero readings failed in almost 80% of the runs to return satisfactorily to the same point. Only those plates on which the zero readings repeated within 0.0030 fringes on the average were used in the final results (except for wavelength above 10000 A.<sup>o</sup>).

#### Interferometer

The interferometer had plates  $4\frac{1}{4}$  inches in diameter. They

were held apart by separators made of invar posts held in a steel ring. Adjustment is secured by means of three springs the compression in which is regulated by screws. Once adjusted, the interferometer would stay in adjustment for many weeks and almost no inconvenience was caused by being unable to adjust it without taking the apparatus apart. The separators used were either 14 mm. or 21 mm.

#### Air Purification

In order to obtain dry  $\text{CO}_2$ -free air several procedures were tried. Many of the runs were taken with air treated as follows: after first drying the air over calcium chloride and phosphorous pentoxide it was passed through two coils each immersed in liquid air. This method will definitely remove the  $\text{CO}_2$  and the water, but it was at one time thought that cooling the air so much might also remove a fraction of the oxygen. To investigate this, results were obtained when the  $\text{CO}_2$  was removed by a different process. The air was bubbled through a six normal solution of potassium hydroxide and then dried. In order to assure the air sufficient contact with the solution to remove the  $\text{CO}_2$  without using too high a column of liquid, the second container with the solution, instead of being a bottle with glass beads, was a long glass tube about one inch in diameter held at a small angle ( $6^\circ$ ) with the horizontal. It took the bubbles nearly seven seconds to travel up this tube, and for the most part they did not increase to more than one half centimeter in diameter. The results of using either of these two

methods were the same.

Still another arrangement was used. It was thought at one time that vapors from the phosphorous pentoxide might be causing the silver films to spoil and so it was replaced by anhydrous magnesium perchlorate. But since this drying reagent loses most of its drying power when it changes to the tri-hydrate form (without much change in appearance), liquid air was again employed. Now, however, it was shielded from direct contact with the coils by a cylindrical copper jacket. Never has there been a trace of  $\text{CO}_2$  or of water that could be seen in the coils when the liquid air and the copper shield were removed, although the temperature of the coils was well below  $-110^\circ \text{C}$ . This method, used in the latter half of the runs, gave results which could not be distinguished from those of the other methods.

#### Procedure for a Run

As was previously mentioned, each run consisted of at least three exposures alternately with vacuum and with air. In order to assure temperature equilibrium the air was let in slowly (especially if it was bubbling through the potassium hydroxide solutions) and then the exposure started after 30 to 50 minutes had elapsed. The temperature was recorded at the beginning and at the end of each exposure (and once or twice in between on the long exposures). The air in the interferometer chamber was not at full atmospheric pressure and consequently the stop-cock connecting it to the outside was closed. It was found that it did not matter when the pressure was measured, but usually it was read about midway during the exposure.

REDUCTION OF OBSERVATIONS

The index of refraction was calculated directly from the orders of interference on the plates. For if  $M_0$  is the order of interference in the vacuum and if  $M$  is the order of interference at some pressure and temperature, then the index,  $n$ , is given by:

$$n = \frac{\lambda_{vac}}{\lambda_{air}} = \frac{M}{M_0}$$

Method for Obtaining the Fractional Order at the Center of the Plate.

To find  $M$  and  $M_0$  it was first necessary to obtain the fractional order of interference at the center of the plate for each wavelength. This was done by measuring on each line eight fringes on both sides of the center with a comparator graduated in thousandths of millimeters. The fractional order was calculated from these eight diameters by a least squares method originally developed by Dr. W. V. Houston. Since it has not been published it is given here. If  $D_n$  is the diameter of the  $n$ <sup>th</sup> fringe and if  $\delta$  is the fractional order of interference at the center, then

$$\delta = \frac{D_{n+1}^2}{D_{n+1}^2 - D_n^2} - n$$

The results given by the different values of  $D_n$  will vary. Since  $(D_{n+1}^2 - D_n^2)$  is supposed to be constant, let us say that

the least squares value of this difference is  $a$ . If  $D_{n+1}^2 \equiv O_n$  we will obtain a set of values of  $\delta$  from

$$\delta_n = \frac{O_n}{a} - n$$

Let us consider that we have  $N$  values of  $O_n$  running from  $n=\alpha$  to  $n=\beta$ . If we call  $\delta$  the least squares value of

the fractional order of interference, then we can obtain  $a$

and  $\delta$  from  $\frac{\partial X}{\partial a} = \frac{\partial X}{\partial \delta} = 0$ , where

$$X = \sum_{n=\alpha}^{\beta} [O_n - a(n + \delta)]^2$$

The two equations are:

$$a \sum_{n=\alpha}^{\beta} n^2 + 2a\delta \sum_{n=\alpha}^{\beta} n + a\delta^2 \sum_{n=\alpha}^{\beta} 1 - \sum_{n=\alpha}^{\beta} n O_n - \delta \sum_{n=\alpha}^{\beta} O_n = 0$$

and

$$a \sum_{n=\alpha}^{\beta} n + a\delta \sum_{n=\alpha}^{\beta} 1 - \sum_{n=\alpha}^{\beta} O_n = 0$$

The first of these equations can be simplified by subtracting  $\delta$  times the second one and the pair thus obtained can be readily solved giving:

$$Na = \frac{\sum_{n=\alpha}^{\beta} n O_n - \frac{\alpha+\beta}{2} \sum_{n=\alpha}^{\beta} O_n}{\frac{1}{12}(N^2-1)}$$

and

$$\delta = \frac{\sum_{n=\alpha}^{\beta} O_n}{Na} - \frac{\alpha+\beta}{2}$$

In practice  $Na$  was the first calculated and  $\delta$  obtained from

the second formula. This of course gives no measure of the differences among the various  $\delta_n$ . If these differences are desired each  $\delta_n$  will have to be calculated. It is worth pointing out that  $\delta$  is the arithmetic average of the  $\delta_n$ .

Method for Obtaining the Integral Order of Interference.

With the fractional orders of interference at the center of the plate thus obtained, the complete orders of interference were then calculated. If the integral orders are  $p_1, p_2, p_3, \dots$  and the fractional orders  $\delta_1, \delta_2, \delta_3, \dots$  for a series of wavelengths  $\lambda_1, \lambda_2, \lambda_3, \dots$  on the same plate, then twice the distance between the interferometer plates is given by:

$$2d = (p_1 + \delta_1) \lambda_1 = (p_2 + \delta_2) \lambda_2 = (p_3 + \delta_3) \lambda_3 = \dots$$

Now we choose some particular line as the key line for the calculations, e.g.  $\lambda_1$ , and write:

$$(p_1 + \delta_1) \frac{\lambda_1}{\lambda_2} = p_2 + \delta_2$$

$$(p_1 + \delta_1) \frac{\lambda_1}{\lambda_3} = p_3 + \delta_3$$

• • •

Then we calculate from the known wavelengths the ratios  $\lambda_1/\lambda_i$ . It is upon the accuracy of these ratios that the method depends and fortunately in the case of the neon lines the wavelengths are given with sufficient accuracy. After estimating by knowing approximately the separation of the interferometer plates,

we can by trial and error find the particular integer,  $\rho_1$ , which gives from the above equations the set of values  $\delta_2', \delta_3', \delta_4' \dots$  most closely agreeing with  $\delta_2, \delta_3, \dots$ . The correct integral orders for the lines  $\lambda_2, \lambda_3, \dots$  are also given by this calculation. We simply add these calculated integers to the observed fractional orders to obtain the complete orders of interference.

#### Reduction of the Values of the Index to Standard Conditions

The values of the index were in general obtained at some pressure between 700 mm. and 740 mm. of mercury and at some temperature between 25° C. and 31° C. To change the observed value to standard conditions the formula  $(n-1)/d = \text{constant}$  ( $d =$  density of air) was used. The ratio of the densities at the two conditions was assumed to be that of a perfect gas. Both Meggers and Peters<sup>4</sup> and Perard<sup>5</sup> have given values for the temperature coefficient (see Table I) of the index. Meggers and Peters find that it is slightly larger in this region than the reciprocal of the absolute zero and that it decreases with increasing wavelength. Perard, on the other hand, states that the temperature coefficient is independent of wavelength and gives a value higher than that of the Bureau.

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<sup>4</sup> Meggers and Peters, see footnote 2, page 1.

<sup>5</sup> Perard, Jour. de Physique et le Radium, 6, Ser.6, p.217-227, (1925).

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 TABLE I. Temperature Coefficient
 

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	7000 A.	100000 A.
Meggers and Peters	0.003673	0.003670
Perard	0.003716	
Perfect gas	0.003661	

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At 7000 A. the error introduced by using the perfect gas coefficient over a temperature change of 30 C. is according to the Meggers and Peters value 0.9 parts in  $10^7$ . In view of these conflicting reports it seemed best to use the perfect gas coefficient.

#### Corrections

Since the force on the separators of the interferometer was produced by springs it was felt that the change in length of the invar posts caused by the different pressures would be appreciable. A careful survey of the existing measurements on the elastic constants of invar and other steels indicated that the bulk modulus of invar was about  $12 \cdot 10^{11}$  dynes/sq.cm. This correction as applied to the index at standard conditions increased its value by 2.8 parts in  $10^7$ .

In order to be sure that the phase change at the metallic reflection was the same in air as in vacuum, separators of two sizes, 14 mm. and 21 mm., were used. The results with both separa-



tors were the same and so this effect was neglected.

### Estimation of Errors and Weighting of Observations

Altogether 55 measurements were made at 30 different wavelengths. Each observation was weighted separately on the basis of experimental errors in order to obtain the most probable observed value at each wavelength. As it was mentioned above, only those runs were used in which the absolute value of the algebraic average of the differences in the order of interference at zero pressure was less than 0.0030 fringes. But in order to calculate the weight of each observation the actual difference ( $d_1$ ) was used. This gave a measure of the way in which the zero value varied but not specifically a measure of the accuracy of the fractional order itself.

In all cases two separate measurements were made on the fractional order of interference at the high pressure. The difference between these two ( $d_2$ ) was taken as a measure of the accuracy of the measurement of the order of interference. In Table II are given these quantities for a sample plate.

The probable error ( $\rho_1$ ) was calculated simply on the basis of  $d_1$  and  $d_2$  by using

$$\rho_1 = \frac{.6745}{2} \sqrt{\frac{d_1^2 + d_2^2}{D^2}} [(n_0 - 1) \cdot 10^7]$$

where  $D$  is the total fringe shift.

TABLE II. Errors and Weights for a Sample Plate.

Wavelength	Total Fringe Shift	$d_1$	$d_2$	Probable Error in $(n_0-1) \times 10^7$	Weight
7488	8.6	- 0.0011	0.0004	0.13	3.8
7535	8.5	+ 0.0004	0.0003	0.05	4.8
8136	7.4	+0.0031	0.0016	0.40	1.2
8300	7.1	+0.0029	0.0024	0.44	1.0
8853	6.2	+0.0049	0.0015	0.61	0.6

The weight was obtained by the formula:

$$W = \frac{C}{\rho_1^2 + \rho_2^2}$$

and involves an estimation of the error ( $\rho_2$ ) in the measurement of the temperature and pressure. This error was taken to be 0.23 parts in  $10^7$  of the index and was considered constant for all plates. The constant,  $C$ , was so chosen that unit weight corresponds to a probable error of 0.5 parts in  $10^7$ .

The above estimations of error and weighting gave the observed average values of the index with their probable internal errors<sup>6</sup> at each wavelength. Now the agreement between the various values at one wavelength, which measures the external errors, showed that

<sup>6</sup> For discussion of external and internal errors, see R. T. Birge, Phys. Review, 40, p.207-227, (1932).

the estimation of the internal errors gave entirely too small results for the probable error. Nevertheless the values obtained with these weights were thought to be the best obtainable. Then in weighting the value of the index at each wavelength in order to calculate the dispersion curve the external probable error was used. These errors then become the internal errors of the dispersion curve. Where only one measurement was made at a wavelength its final weight was estimated from the behavior of the internal and external errors at other wavelengths. The distribution of the total weight among different wavelength regions is given in Table III.

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TABLE III. Distribution of Weighting Used  
in Calculating the Dispersion Curve.

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Wavelength range	Weighting
6500-7600 A.	7.18
7600-8900 A.	6.61
8900-9700 A.	1.27
Beyond 9700 A.	0.18
Total weight	<u>15.24</u>

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The grouping chosen for this table is that of the lines themselves, as was mentioned above in connection with the source and plates. The difficulty of obtaining lines in the far infrared is reflected in this table. This distribution of weights leaves

something to be desired for the calculation of the dispersion curve, but due to the nature of the curve the one obtained is satisfactory throughout the region of observation.

### The Dispersion Curve

The usual Cauchy dispersion formula was used to describe the results at standard conditions.

$$(n_0 - 1) \cdot 10^7 = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

The observed value of the index at each wavelength was weighted as described above and used in the customary manner to form the normal equations determining  $A$ ,  $B$ , and  $C$  by the method of least squares. The result of this calculation was:

$$(n_0 - 1) 10^7 = 2878.94 \pm 3.8 + \frac{24.74 \pm 4.5}{\lambda^2} - \frac{2.47 \pm 1.3}{\lambda^4}$$

$\lambda$  is expressed in thousandths of millimeters ( $\mu$ ).

The probable errors given here were calculated by the formulas 6 & 7

$$\rho_A = .6745 \sqrt{\frac{A_{11}}{\Delta}} \sqrt{\frac{\sum w_i v_i^2}{s - q}}$$

$$\rho_B = .6745 \sqrt{\frac{A_{22}}{\Delta}} \sqrt{\frac{\sum w_i v_i^2}{s - q}}$$

$$\rho_C = .6745 \sqrt{\frac{A_{33}}{\Delta}} \sqrt{\frac{\sum w_i v_i^2}{s - q}}$$

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<sup>6</sup> Birge, see footnote, page 15.

<sup>7</sup> Whittaker and Robinson, The Calculus of Observations, second edition, page 241.

$\Delta = |a_{pq}|$  is the determinant of the coefficients in the normal equation.  $A_{pq}$  is the minor of  $a_{pq}$ .

$S$  is the number of equations (30 in this case).

$q$  is the number of constants determined (3).

$w_i$  is the weight of each observation.

$v_i$  is the residual.

With such large probable errors for  $B$  and  $C$ , it is not advisable to extrapolate this curve to shorter wavelengths. This is apparent from Table IV. For longer wavelengths these terms become smaller and the extrapolation in this direction is much more trustworthy. In fact the probable error in the calculated values of the index decreases continuously with increasing wavelength. The observed values, their probable errors, the calculated values and the differences are given in Table IV.

TABLE IV. The Observed Values of the Index

Wavelength (A.)	$(n_0 - 1) 10^7$ (observed)	$(n_0 - 1) 10^7$ (calculated)	(obs.) - (calc.)
6506	$2923.2 \pm 0.5$	2923.6	-0.4
6532	$22.8 \pm 0.9$	23.3	-0.5
6717	$21.4 \pm 1.4$	21.6	-0.2
6929	$19.2 \pm 0.4$	19.7	-0.5
7173	$18.5 \pm 1.0$	17.6	+0.9
7245	$15.5 \pm 0.5$	17.1	-1.6
7438	$17.6 \pm 1.2$	15.6	+2.0

TABLE IV. (continued):

Wavelength (A.)	$(n_o - 1) 10^7$ (observed)	$(n_o - 1) 10^7$ (calculated)	(obs.) - (calc.)
7488	$15.6 \pm 0.6$	15.2	+0.4
7535	$14.7 \pm 0.4$	14.8	-0.1
7943	$12.1 \pm 3.5$	11.9	+0.2
8136	$09.5 \pm 1.6$	10.7	-1.2
8300	$10.4 \pm 0.6$	09.6	+0.8
8377	$08.8 \pm 0.9$	09.2	-0.4
8495	$08.2 \pm 0.4$	08.5	-0.3
8591	$07.9 \pm 0.7$	07.9	0.0
8780	$07.8 \pm 0.3$	06.9	+0.9
8783	$10.6 \pm 0.7$	06.8	+3.8
8853	$05.1 \pm 0.6$	06.5	-1.4
9148	$03.3 \pm 0.7$	05.0	-1.7
9201	$00.2 \pm 5.0$	04.7	-4.5
9220	$2897.2 \pm 3.5$	04.6	-7.4
9300	$2903.6 \pm 0.8$	04.2	-0.6
9425	$00.2 \pm 3.5$	03.6	-3.4
9534	$04.2 \pm 2.5$	03.2	+1.0
9665	$2899.8 \pm 0.8$	2902.6	-2.8
10140	$95.0 \pm 1.4$	00.7	-5.7
10798	$84 \pm 5.0$	2898.3	-14
11143	$2901 \pm 5.0$	97.3	+4
11177	$2892 \pm 2.7$	2997.2	-5

TABLE V. Values of  $(n_0-1) 10^7$  Calculated from the Dispersion Formulae of Different Observers.

Observer	5000 A.	7000 A.	9000 A.	11000 A.
M. & P. <sup>4</sup>	2935.4	2904.7	2892.9	2887.1
S. <sup>9</sup>	36.6	04.9	2891.8	95.2
P. <sup>5</sup>	44.3	11.6	99.1	92.6
K. & L. <sup>8</sup>	43.7	10.2	97.0	90.3
B. <sup>10</sup>	38.4	19.1	2905.7	97.6

DISPERSION FORMULAE:

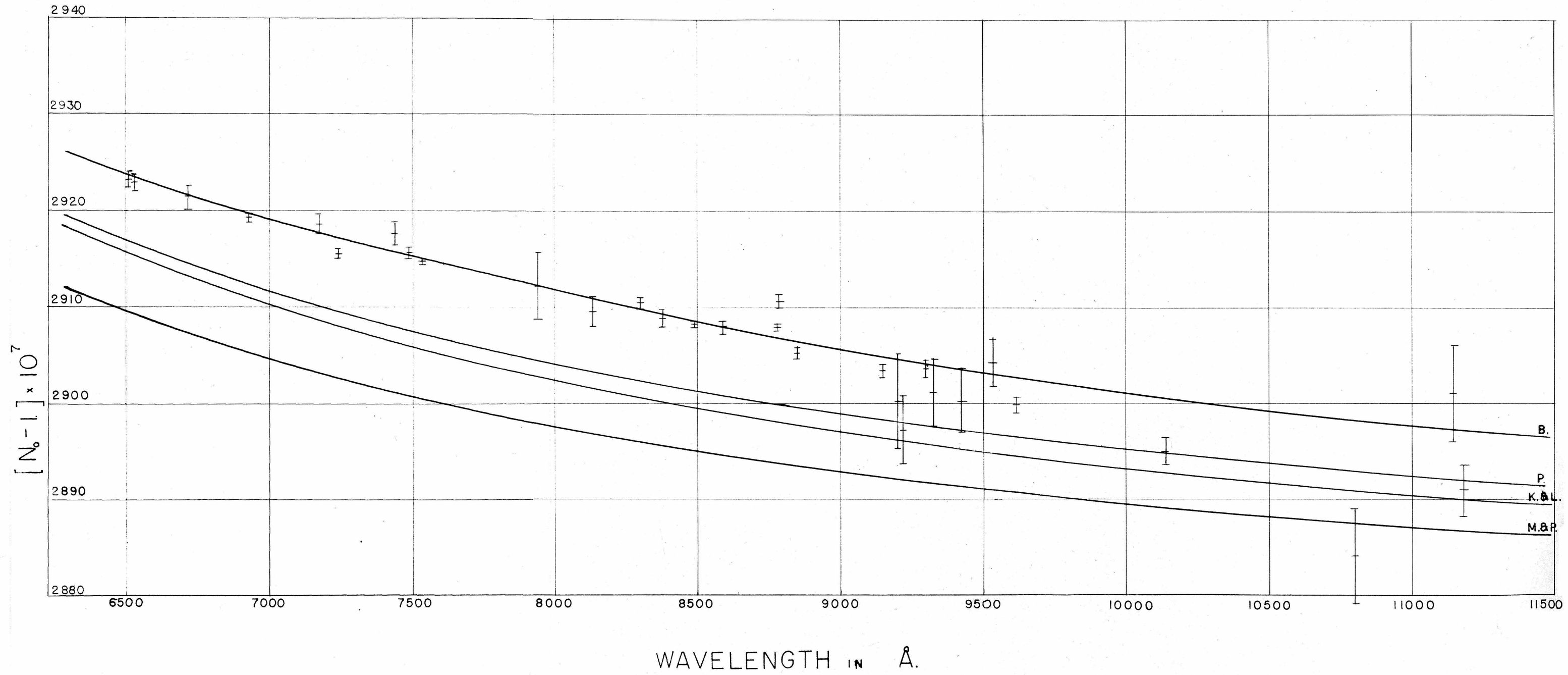
$$\begin{aligned}
 \text{M. \& P.}^4 & \quad 2875.66 + \frac{13.412}{\lambda^2} + \frac{0.3777}{\lambda^4} \\
 \text{S.}^9 & \quad 2871.87 + \frac{16.17000}{\lambda^2} \\
 \text{P.}^5 & \quad 2880.2 + \frac{14.748}{\lambda^2} + \frac{0.316}{\lambda^4} \\
 \text{K. \& L.}^8 & \quad 2877.10 + \frac{15.84}{\lambda^2} + \frac{0.1935}{\lambda^4} \\
 \text{B.}^{10} & \quad 2878.94 + \frac{24.74}{\lambda^2} - \frac{2.47}{\lambda^4}
 \end{aligned}$$

<sup>8</sup> Phys. Zeit., 35, p.223 (1934).

<sup>9</sup> Stoll, Ann. d. Physik, 69, p.81 (1922). This curve was not plotted on the graph because of its proximity to that of M. & P.

<sup>10</sup> The writer.

# DISPERSION CURVES





DISCUSSION OF RESULTS

The dispersion curves, Table V (page 20), and the set of dispersion formulae (page 20) exhibit the principal differences among observers. The dispersion formula obtained in the present experiment can be seen to differ considerably in the last two terms from that of other observers. This could be due to several reasons. In the first place the Cauchy dispersion formula may not be valid in this region due to the presence of an absorption band nearby. This would involve a fourth term in the formula of the form  $-D\lambda^2$ . The calculation of such a curve was not carried out because of the lack of accuracy in the long wavelengths. Also, since the second and third terms are small in the infra red, the calculation of the curve from the present data should not be expected to give these terms as accurately as the first term. The only possibility of a systematic error seems to be the change in the methods of air purification, but, as it was noted above, no difference was apparent between the results of the various methods.

It is apparent that the discrepancies between observers have only been increased by the present experiments. In order to compare results it would be useful if all observers gave the probable error in their dispersion curves, because these quantities seem to give a better measure of the validity of the results than any other quantities. In the present experiments these probable errors, especially in the first coefficient, turned out

larger than expected. It seems probable that other observers would find errors of the same magnitude in view of the differences between their observed and calculated values.

As described above a correction amounting to 2.8 parts in  $10^7$  was made in the observed values of the index because of the expansion of the invar separators in the vacuum. Meggers and Peters, whose interferometer was similar to the one used in the present experiment, did not make this correction. They estimated that the bulk modulus of invar was such as to cause a change in length of 2 parts in 3,000,000 and then stated that this would cause the value of the index to be "about one unit too small in the seventh decimal place". With this change in length the index will be nearly seven units, instead of one unit, too small in the seventh decimal place. A correction of this order of magnitude would remove much of the difference between the results of Meggers and Peters and those of Perard and of Kosters and Lampe. (see Dispersion Curves).

The method used by Stoll involved the change in optical path through glass tubes which, having a smaller bulk modulus than invar, change more in length. This correction would increase the values given by Stoll by 7 to 9 parts in  $10^7$  depending upon the actual value of the bulk modulus. The substance used by Perard as having constant length in air and in vacuum was a platinum-iridium alloy having a bulk modulus larger than that of invar by a factor of about two. Thus the correction to Perard's values would be only 1 or 2 parts in  $10^7$ . Kosters and Lampe

give no experimental details and so no change is made in their value. Table VI gives the results when the above corrections are made.

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TABLE VI.	Results when the Bulk Moduli Corrections are Made.		
	7000 A.	9000 A.	11000 A.
M. & P.	2908	2896	2890
S.	12	99	92
P.	13	2901	94
K. & L.	10	2897	90
B.	19	2906	98

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