CONCRETE MIX DESIGN: A SUMMARY

Thesis

by

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FOREWORD

Designing concrete mixes is a subject which has been considered by everyone from the wheelbarrow-pushing laborer to the systematic engineer. Arising from this variety of studies is a host of different concepts of what constitutes "good concrete." In an effort to collect, evaluate, and extract from this assortment of opinions, a usable, fundamentally correct method for the design of concrete mixes this study was undertaken, and herewith are presented the results.

This paper presumes the assumption that the accepted end in concrete design is the obtaining of strong, water-tight, homogeneous concrete at a minimum of expense.

SECTION 1

Maximum strength is developed by that concrete which is most dense. Countless tests have shown this to be a fact.

Sanford E. Thompson, writing in 1906 (Ref. 2, p. 184), outlines seven methods of designing such concrete (i.e., concrete with a minimum of voids):

(1′) Arbitrary selection, examples of which are,

(a) Use half as much sand as stone, as 1:2:4 or 1:3:6.

(b) Use volume of stone equivalent to the cement plus twice the volume of sand, as 1:2:5 or 1:3:7.

(2′) Determination of voids in the stone and in the sand, and proportioning of materials so that the volume of sand is equivalent to the volume of voids in the stone and the volume of the cement is slightly in excess of the voids in the sand.

(3′) Determination of the voids in the stone, and, after selecting the proportions of cement to sand by test or judgment, proportioning the mortar to the stone so that the volume of mortar will be slightly in excess of the voids in the stone.

(4′) Mixing the sand and stone and providing such a proportion of cement that the paste will slightly more than fill the voids in the mixed aggregate.

(5') Making trial mixtures of dry materials in different proportions to determine the mixture giving the smallest percentage of voids, and then adding an arbitrary percentage of cement, or else one based on the voids in the mixed aggregate.

(6') Mixing the aggregate and cement according to a given mechanical analysis curve.

(7') Making volumetric tests or trial mixtures of concrete with a given percentage of cement and different aggregates, and selecting the mixture producing the smallest volume of concrete; then varying the proportions thus found, by inspection of the concrete in the field.

Concerning these methods Thompson writes, "The most practical method....is by mechanical analysis of the aggregates,...." (method 6').

He continues, "Volumetric synthesis (method 7'), or proportioning by trial mixtures, is another method which is sometimes useful, and produces fairly scientific results."

To modernize this list there must be added two methods which were developed in an effort to simplify scientific design. These are:

(8') The surface-area method.

(9') The fineness modulus method.

These eight methods may be further summarized as being

(1) Arbitrary proportioning (includes 1')

(2) Proportioning by voids (includes 2', 3', 4', 5', 6', 7')

(3) Proportioning by surface-area (8')

(4) Proportioning by fineness modulus (9').

Of these four general methods it may be said as follows:

Arbitrary proportioning is an art acquired only by experience. Rules of thumb are often entirely adequate, but satisfactory only when accompanied by trained supervision. It seems well to point out here, however, that much excellent concrete has been placed which has been "designed" only by this method, and for many jobs, especially small ones, this method may prove the most generally advantageous. It should be noted, however, that complete proportioning designed on the basis of voids may be successfully applied in the field with the aid of only crude methods. Such operations are well described in the Portland Cement Company's booklet, Design and Control of Concrete Mixes.

In an article (Ref. 4) Mr. F. S. Besson discusses the remaining three methods, voids, surface-area, and fineness-modulus, so concisely that it seems well to quote him directly. He states:

"....There have been several methods of aggregate classification, including the surface-area and fineness-modulus methods, advocated as substitutes for the straight mechanical analysis (which pictures the granulometric composition of an aggregate) in attempts to obtain a method adapted directly to the solution of engineering problems. But both the fineness-modulus method and the surface-area method are unsuitable modifications of the straight mechanical analysis. Fineness-modulus arbitrarily gives undue weight to the larger particles making up the aggregate, while, on the other hand, the surface-area method gives exaggerated weight to the finer particles. The fineness-modulus method ignores the finest of all. These methods do not give true pictures of the material analyzed.

"Surface area was brought....to the fore by L. N. Edwards in 1918.... The method does not weight the particles in any logical manner, so that it may be stated that the value of an aggregate for concrete varies as an inverse function of the surface area, while (as used in cement specifications) the value of a cement varies as a direct function.

"Fineness-modulus dates back to Duff A. Abrams' 1919 report.... This report states that two aggregates having the same fineness-modulus will require the same quantity of water to produce a mix of the same plasticity, and will give concrete of the same strength, so long as the aggregates are not too coarse for the quantity of cement.

"Fineness-modulus is an attempt to measure the extent to which gradation and the coarser particles in an aggregate affect the concrete strength. A satisfactory way to (do this is as follows):"

For each cement to aggregate ratio make a series of samples

(1) keeping the consistency constant

(Note: use Burmister flow trough - most consistent method - detailed in M.W.D. Manual for Concrete Inspectors, p. 36 - see Ref. 6)

(2) and varying proportions of each aggregate as indicated on figure 3.

Plot strength tests of these samples against mix proportions, and obtain curves, as in figure 3, which indicate the desired relationships.

Mr. Besson makes an analysis of these curves which holds in the general case (Ref. 4).

"These curves may be analyzed as follows:

(a) The highest curve (maximum strength) represents combinations of coarse gravel and coarse sand.

(b) The lowest curve (minimum strength) represents combinations of fine gravel and fine sand.

(c) The gravel has more influence upon strength than the sand ....

(d) Substituting coarse sand for fine moves the peak of the strength curve toward larger proportions of sand.

(e) Mixes represented by the peaks of the curves are the best proportions for each of the four combined aggregates.

"An analysis such as (this) shows the exact influence of gradation and the importance of the coarser particles. These results cannot be obtained by the surface-area and fineness-modulus methods, which weigh the particles in accordance with arbitrary assumptions."

The justifiable conclusion seems to be that the voids method of design is preferable above all others. In view of this fact, the succeeding section outlines a design procedure on this basis.

SECTION 2

Procedure for the design of a concrete mix on the basis of minimum voids, with the aid of mechanical analysis of the aggregate may be outlined as follows. It is necessary to determine:

(1) Amount of cement per cubic yard of finished concrete.

(2) Water-cement ratio.

(3) Aggregate sizes and grading desired.

(4) Specific gravities (solid volume) of the several aggregates.

(5) Absorption of the several aggregates.

(6) Per cent of moisture in each of the several aggregates.

From these above data the following may be obtained, which three statements are the design quantities of the mix:

(1) Weight of cement per cubic yard.

(2) Weight of each aggregate per cubic yard.

(3) Amount of mixing water per cubic yard.

This method contemplates a finished product consisting of one cubic yard, twenty-seven cubic feet, of finished concrete. This yard of concrete has the three constituents named in the last paragraph, viz.: cement, water, and aggregate. The relative quantities of each are determined by subtracting from twenty-seven cubic feet, first, the volume of cement used, and, second, the volume of water used. The remainder is the volume of aggregate used.

But it is to be noted that this volume of aggregate is not the bulk volume, for in the mixed concrete the aggregate is completely surrounded (ideally) by

(1) The larger aggregate by the smaller

(2) The smaller aggregate by the cement

(3) The cement by the (unabsorbed) water.

Rather, this volume is the sum of the volumes of each individual particle. This quantity is known as the "solid volume."

Considering, one at a time, the quantities to be determined, there are:

(1) Cement

Information varies on the amount of cement to use under any particular circumstances. Current literature seems to assume one of three things:

(a) That the job foreman will strike a happy balance between the amount of cement used and the workability (an all-important item! For, of what virtue is dense concrete when filled with rock pockets and honey-combing?)

(b) That this proportion will have been determined experimentally beforehand, aggregate characteristics and water-cement ratio having been held constant, and the cement varied in trial batches.

(c) That the amount of cement to be used per yard will have been specified by someone higher in authority than the mix designer.

Published papers seem either to deal with specific special cases, or to be so complete and varied as to permit of no generalization. Reference 2 has a very complete set of tables dealing with this matter.

The amount of cement varies

(a) with the particular aggregate (due to shape and maximum size)

(b) with strength desired.

Simply as an illustration the following table is included (Ref. 6):

28 Day Strength	Approx. bbl./cu.yd.	Max. W/C
4000#/sq.in.	1.75	0.80
3000#/sq.in.	1.50 1.375	0.90
2500#/sq.in.	1.375 1.25	1.00 0.95
2000#/sq.in.	1.125	1.05
1500#/sq.in.	1.00	1.10

The extent to which job conditions may vary from these figures may be seen by noting that the 4000-pound concrete was placed with 1.60 bbl./cu.yd. with about the same W/C, and that the 1500 was run at the specified cement content, but with W/C equal to about 1.40.

(2) Water-cement ratio

Although subject to the specific conditions found on the job, this may be estimated from several curves available, one of which is reproduced here (figure 4) from Reference 3.

On the job the apparent ratio will be found to vary with the temperature and humidity of the atmosphere, among other things.

(3) Aggregate sizes

Aggregate size (maximum) may be determined by the material available. In any case, it must be small enough to pass between any reinforcing steel in the structure so as to allow reasonable workability of the concrete in placing.

The aggregate should be of good quality, strong and non-friable (see A.S.T.M. specifications for details).

The desirable proportioning of the various sizes of aggregate may be definitely determined, within small tolerances, from curves which have been established by experiment. Several of these "ideal" curves are given in figure 5.

The aggregate should be screened into as many different sizes as is necessary to obtain the desired ideal curve.

The proportions necessary to duplicate this grading curve may be found readily by choosing arbitrary percentages (known as "bin percentage") of each aggregate size, calculating the resulting curve, estimating a correction to the trial bin percentages, and repeating until the desired results are obtained.

It may be convenient to determine from the ideal curve the ideal per cents of each aggregate size which will be present in the final aggregate mixture (or "ideal mix"). This allows the explicit calculation of bin percentages, directly, by solving a set of simultaneous equations, or by successive approximations (which last is very rapidly done by experienced men).

For sake of completeness: the grading analysis, showing the percentage of each sieve size contained in each bin of aggregate, is obtained by shaking a sample of the material from each bin through a set of screens having known openings which vary uniformly in clearance.

The aggregate size and proportioning, then, has now been determined. It is now necessary to obtain the

(3) Specific gravities

The quantity referred to here is the "solid volume" specific gravity, which may be defined as the ratio of the weight of any single, solid particle of the aggregate to the weight of an equal quantity of water. This may be obtained by

(a) Filling container of known volume, V_1, with dry aggregate of known weight, W.

(b) A known weight, W_3, and, hence, volume, of water is then added, taking time to let water displace all air. Subtract from ~~Add to~~ W_3 the absorption (see (4), following) and call W_2.

(c) Calculation then gives d, the solid volume specific gravity:

V_1 is first converted to the weight, W_1, of an equal volume of water.

Then:

$$\frac{W}{W_1 - W_2} = d$$

This quantity must be obtained for each aggregate, but once accurately obtained, it may be considered constant for that aggregate and need only be checked occasionally.

(4) Absorption

This is determined by evaporating damp aggregate to surface dryness, weighing, (W_1), then evaporating to constant weight (W).

$$100 \cdot \frac{W_1 - W}{W} = \text{Absorption \%, G.}$$

(5) Moisture in aggregates

This is determined for each aggregate, at whatever intervals necessary by weighing moist sample, (W_1) then evaporating to dryness, (W). Then:

$$100 \cdot \frac{W_1 - W}{W} = \text{total moisture, F.}$$

The necessary unknowns are now determined, and the mix may be calculated on the principles outlined previously.

This example will illustrate, and indicate a convenient form for computations. The figures are those for a day's run on the M.W.D. Distribution System.

Given: Cement/cu.yd. = 1.69 bbl.

Water-cement ratio = 0.78

Solid volume density of cement = 196#/cu.ft.

Loose volume density of cement = 94#/cu.ft.

1 bbl. = 4 cu.ft. = 376# of cement

From which is obtained:

Volume of cement: $1.69 \cdot \frac{376}{196} = 3.24$ cu.ft.

Volume of water: $(1.69 \cdot \frac{376}{94})(0.78) = 5.28$ cu.ft.

Sum, volume water plus volume cement: 8.52 cu.ft.

Since remainder of yard must be aggregate:

27.00 - 8.52 = 18.48

or solid volume of aggregate = 18.48 cu.ft.

By sieving each of the three aggregates in use we get the "screen analysis" shown in table at left on sheet C.

Experience suggests "ideal mix" percentages of 34.5% sand, 15.0% pea gravel, and 50.5% rock.

Successive estimates, trials, and corrections indicate that the ideal "bin" percentages are 44.1% from the sand bin, 6.6% from the pea gravel bin, and 49.3% from the rock bin. It may be noted that these percentages are determined by weight. But these same figures may be taken as per cents of solid volume because the small differences in density may be neglected here.

Knowing already the total solid volume of aggregate, and knowing now the individual per cents of solid volume for each aggregate, the remaining calculations are readily carried out as indicated on sheet D.

Additional data, obtained by methods outlined:

	Sand	Pea	Rock
Specific gravities:	2.62	2.68	2.68
Today's moistures (total):	6.4%	1.5%	2.2%
Absorptions:	0.6%	0.2%	0.2%

Note: In seeking to obtain any particular strength with concrete it must be observed, and may well be noted here, that curing has a very great effect upon this item. Moist curing for a minimum of seven days is essential, and twenty-eight days is desirable.

COMPUTATION OF SCREEN ANALYSIS OF MIXED AGGREGATE

	Screen analysis % each size			"Bin" % each size in mix					
Retained on sieve size	-1/8	-1/2	-1	-1/8 44.1	-1/2 6.6	-1 49.3	Total % each size	Total % re-tained	Agg. Grading required
0	1.4			0.6			0.6	100	
200	2.6			1.1		0.1	1.2	99.4	
100	11.7	3.0	1.0	5.2		0.1	5.3	98.2	
									34.5
48	21.2			9.4	0.1		9.5	92.9	
28	22.5			9.9		0.1	10.0	83.4	
14	17.4			7.7	0.1	0.1	7.9	73.4	
8	18.6	3.0	0.4	8.2	0.2	0.2	8.6	65.5	
									15.0
4	4.6	60.7	0.8	2.0	4.0	0.4	6.4	56.9	
3/8		33.3	46.9		2.2	23.2	25.4	50.5	
									50.5
3/4			46.5			22.9	22.9	25.1	
1			4.4			2.2	2.2	2.2	
Totals	100.0	100.0	100.0	44.1	6.6	49.3	100.0		

COMPUTATION OF CONCRETE BATCH QUANTITIES

Aggregate	- 1/8"	-1/2"	- 1"	Mixed Aggregate
B - "Bin" % each size by solid volume	44.1	6.6	49.3	100.0
C - Solid volume per concrete batch, cu.ft.	8.14	1.22	9.12	18.48
D - Specific gravity, solid volume	2.62	2.68	2.68	
E - Wt.of dry agg./batch 62.4·C·D lbs.	1332	204	1526	3062
F - Total moisture, %	6.4	1.5	2.2	
G - Absorption, %	0.6	0.2	0.2	
H - Surface moisture, % F - G	5.8	1.3	2.0	
J - Surface moisture wt.in batch $\frac{E \cdot H}{100}$ lbs.	77	3	31	111
K - Total moisture wt. in batch $\frac{E \cdot F}{100}$ lbs.	85	3	34	122
L - Gross wt. of agg. for batch E K	1417	207	1560	3184

Now deduct surface moisture on aggregate from mixing water to get water to be added:

$$5.28 - \frac{111}{62.4} = 3.50 \text{ cu.ft.}$$

Mixing time (specified) not less than 1 1/2 minutes. The mix is now designed. Information sheet for mixer operator reads (for one yard batch):

Cement	Sand	Pea	Rock	Water
635#	1417#	207#	1560#	3.50 cu.ft.

SECTION 3

Last year it was the author's good fortune to be in the employ of the Metropolitan Water District of Southern California as a concrete inspector. In this capacity time was spent both in the Rochester precast concrete pipe plant and in the field on some line structures. Through the kindness and cooperation of Mr. R. B. Diemer, Engineer of the Distribution System, of Mr. Hugh Jones, Assistant Engineer, Metropolitan Water District, and of Mr. W. H. Spear, Assistant Engineer, Metropolitan Water District, the author has been permitted to make use of results obtained by the District on its pipe lines, as well as of results of a study, conducted by Mr. Spear, which will be treated in Section 4 of this paper.

At present it is the purpose to compare with "standard," or "ideal" curves, two curves of aggregate analyses used by the District in the manufacture of its pipe. These two curves are average curves, the data being compiled from mixes used for pipe from January 26, 1937 through April 20, 1937, for one curve, and from April 21, 1937 to July 24, 1937, for the other curve. (For data, see sheets A and B.)

Throughout the former period considerable difficulty was had with the mix. Despite all efforts made to improve it by mixture changing, it remained harsh.

The second period marks a series of smooth mixes brought about by the use of blending sand to increase the fines. From

MIX DESIGN DATA - SCHEDULE 3P - ROCHESTER PLANT - M.W.D.

Date of Design 1937	Ideal Mix	% each size retained on Tyler Standard Sieves										
		Pan	#200	#100	#48	#28	#14	#8	#4	3/8"	3/4"	1"
1-26	35-11-54	0.6	1.0	3.7	9.3	11.4	9.0	7.6	3.4	22.5	27.4	4.1
2-17	35-11-54	0.6	1.0	3.5	9.4	11.5	9.0	7.1	3.9	22.3	27.2	4.5
2-26	35-11-54	0.6	1.1	3.6	9.3	11.5	8.9	7.1	3.9	21.7	28.1	4.2
3- 2	35-12-53	0.6	1.1	3.6	9.3	11.6	8.8	7.3	4.7	22.7	27.0	3.3
3- 3	35-11-54	0.6	1.1	3.6	9.3	11.6	8.8	7.3	3.7	22.9	27.8	3.3
3- 5	35-15-50	0.5	0.9	3.6	9.4	11.8	8.8	7.6	7.4	20.7	25.4	3.9
3- 8	35-15-50	0.5	0.9	3.6	9.7	11.8	8.5	7.4	7.6	22.1	24.7	3.2
3- 9	35-14-51	0.6	0.9	3.4	9.4	11.7	9.0	7.3	6.7	24.6	23.6	2.8
3-11	36-12-52	0.5	0.9	3.7	9.8	12.0	9.1	7.5	4.5	25.5	23.9	2.6
3-15	35-15.5-49.5	0.5	0.7	3.7	9.2	11.7	9.1	8.4	7.1	17.0	27.5	5.1
3-17	35-15.5-49.5	0.4	1.0	4.0	9.4	11.9	8.3	7.2	8.3	21.8	25.2	2.5
3-20	35-15.5-49.5	0.6	0.9	4.0	9.6	11.5	8.4	8.0	7.5	21.8	23.9	3.8
3-25	35-15.5-49.5	0.6	1.0	3.9	9.7	11.3	8.5	8.1	7.4	24.8	22.5	2.2
4- 1	35-15.5-49.5	0.4	1.1	3.8	9.6	11.7	8.4	8.2	7.3	25.1	21.9	2.5
4- 9	35-15.5-49.5	0.5	1.1	3.9	9.7	11.3	8.5	7.8	7.7	25.2	21.7	2.6
* 4-16	35-15.5-49.5	0.5	1.1	3.9	9.8	11.4	8.3	7.7	7.8	25.8	21.2	2.5
4-19	35-15.5-49.5	0.5	1.1	4.6	11.0	9.9	7.9	7.9	7.6	23.1	22.9	3.5
4-20	35-14-51	0.3	0.8	4.4	11.0	10.5	8.0	9.1	4.9	23.2	24.1	3.7
Totals		9.4	17.7	68.5	173.9	206.1	155.3	138.6	111.4	417.8	446.0	60.3
Average		0.5	1.0	3.8	9.7	11.4	8.6	7.7	6.2	23.1	24.7	3.3
Total %) Retained)		100.0	99.5	98.5	94.7	85.0	73.6	65.0	57.3	51.1	28.0	3.3

* Changed from 1.69 to 1.60 bbl./yd. of cement (strength higher than needed).

Date of Design 1937	Ideal Mix	% each size retained on Tyler Standard Sieves										
		Pan	#200	#100	#48	#28	#14	#8	#4	3/8"	3/4"	1"
** 4-21	35-14-51	0.6	1.7	6.5	11.5	8.2	6.5	7.2	6.8	23.7	23.7	3.6
4-27	35-14-51	0.5	1.5	5.8	10.8	9.3	7.1	8.1	5.9	24.2	23.4	3.4
5- 1	35-14-51	0.8	1.7	5.1	10.0	10.3	7.1	6.9	7.0	25.9	22.3	2.9
5- 4	35-14-51	0.8	2.0	6.1	9.9	9.6	6.6	6.3	7.7	25.9	22.1	3.0
5-10	35-14-51	0.9	2.2	6.1	9.6	9.5	6.7	6.3	7.7	26.3	21.8	2.9
5-12	35-14-51	1.0	3.7	6.9	9.3	7.9	6.2	5.5	8.5	26.5	21.6	2.9
5-13	35-13-52	0.9	2.7	7.4	9.5	8.4	6.1	7.1	5.9	23.8	24.2	4.0
5-18	35-13-52	0.8	2.7	6.6	9.3	8.9	6.7	6.1	6.9	24.7	24.5	2.8
5-21	35-12-53	0.9	2.7	6.5	9.2	9.2	6.5	5.7	6.3	27.2	22.2	3.6
5-28	35-12-53	0.8	2.3	5.9	9.5	9.0	7.5	6.7	5.3	29.8	20.4	2.8
6-10	35-13-52	1.1	2.7	6.3	9.2	8.4	7.3	5.9	7.1	27.7	22.6	1.7
6-14	35-12-53	1.2	2.5	6.3	8.7	8.5	7.8	6.2	5.8	26.9	22.5	3.6
6-24	35-12-53	1.0	2.5	5.8	9.1	9.2	7.4	6.4	5.6	25.5	23.9	3.6
Totals		11.3	30.9	81.3	125.6	116.4	89.5	84.4	86.5	338.1	295.2	40.8
Average		0.9	2.4	6.2	9.7	9.0	6.9	6.5	6.6	26.0	22.7	3.1
Total %) Retained)		100.0	99.1	96.7	90.5	80.8	71.8	64.9	58.4	51.8	25.8	3.1

** Started using blending sand to raise fines. Workability improved greatly.

15% to 20% of the sand was this blend, of which a typical analysis is:

Retained on	%
0	7.0
200	23.1
100	46.3
48	19.2
28	4.4
	100.0

While on this point of fines in the sand, it is of interest to note the curves of figures 6 and 7, contributed by Mr. Bert J. Soderblom.

These curves are among the observations made by Mr. Soderblom during his residence at the Fan Hill Plant, Thousand Palms Tunnel No. 1, M.W.D.

One curve shows a very critical point for strength with respect to the per cent of aggregate passing the 100 mesh screen. The other curve shows how the strength continues to drop, as the very fine particles, silt, increase in percentage.

Returning to the distribution pipe, it is worthy of note that, with the control established, and with reasonably uniform rock plant output, it was possible to hold the strengths developed at a very steady figure. In addition, the mixing water remained a practically constant quantity over a day's run. Where aggregate moistures and grading were erratic, these two items were very jumpy.

Before leaving the pipe plants, a notation of the control methods and equipment used seems in order.

A description of the concrete equipment at the Rochester pipe plant, owned and operated by the American Concrete and Steel Pipe Company, under contract to M.W.D., is a typical installation.

Aggregate is delivered to the plant by trucks, dumped in a pit, and carried to elevated storage bins by a bucket chain.

Cement is unloaded from freight ~~storage~~ cars by one man operating a small motor-driven scraper. It is stored in a large silo, and elevated to the mixer supply tank as needed.

Materials are batched consecutively into a single weighing hopper, a Fairbanks-Morse scale. Each aggregate can be weighed automatically within five to ten pounds, plus or minus, depending upon circumstances, by the automatic photo-cell equipment. The aggregates can also be delivered by operating manually controlled air valves. Feed from the aggregate bunkers is through open gates onto short conveyor belts. The belts are motor driven, and each is equipped with a jog for close final weighing. Cement feed is accomplished by use of a screw. Water is measured into a one-batch storage tank by an automatic shut-off meter reading to one-hundredths of a cubic foot. A release valve is provided for excess water.

Mixer capacity is about 3 1/2 yards.

Well water is used in mixing.

The same plant is used for mixing dry batch for hauling to ditch.

SECTION 4

In preparing this paper, two methods of mix design have been encountered which deal with calculations based on the aggregate particles themselves.

One of these is a method of proportioning described by Mr. Archie A. Smith in Concrete for May, 1936. Its purpose is the calculation of a mix, having given the screen analysis, and water-cement ratio. The procedure is given in detail. After becoming accustomed to the method, one would probably find it as rapid as any other. It should be noted, however, that, because of assumptions of symmetrical size distribution in the method, some aggregates can throw the results into appreciable error.

The other method of using the aggregate particles, is the method, already referred to, developed by Mr. Spear for the calculation of particle interference. This affords a mathematical device, based upon screen analysis, for showing up immediately any flaws in the design. It is based upon the space between particles of any given size available to receive the particles smaller.

Mr. Spear's work is reproduced, on the following pages, verbatim, save for one or two explanatory lines.

The curve of this aggregate is plotted on figure 5, and the mis-matchings indicated by the table on the 200 mesh, the 16, the 3/8, and the 3/4 are apparent. The excess in the 3/8 size will be particularly apparent, for this central part of the ideal curve seems most sensitive to variations. Movements which will make a mix impossible here, are only permissible variations at either end.

CONCRETE DESIGN

CALCULATION OF PARTICLE INTERFERENCE

By W. H. Spear

Given: a particular screen analysis of combined aggregates.

Consider the solid volume of the total mixed concrete as 100%, and compute the percentages of each size of aggregate according to screen analysis on the design sheet.

When mixing has taken place the average clear distance apart of two adjacent particles of any size "n" may be called "t", and should be large enough so that the space will accomodate the next smaller size.

The solid volume "S" of the size "n", divided by its natural volume "V", in a dry rodded state may be called "d_o", i.e., "d_o" would be 60% of a material having 40% voids.

The solid volume "S" of the size "n", divided by its expanded volume "E", when the particles have been pushed apart by smaller particles, cement and water (forgetting these as though not present to affect density) will be called "d_a", or RELATIVE DENSITY.

The average diameter of that size "n" will be called "D_n", and is the average distance between centers of particles before mixing.

The average distance between centers of particles in the same group size after mixing will be called "D_m", and "D_m" equals "D_n" + "t".

The following derivation determines the formula to be used in calculating particle interference in any mix design:

The natural volume "V" of the size "n" will be:

$$V = N \times g \times D_n^3$$

Where N number particles in the unit of volume

Where g function of the shape of the particles.

The expanded volume "E" of the size "n" when this size is added to a smaller size "m", plus cement and water will be:

$$E = N \times g \times D_m^3$$

Where N and g are identical with above, in both cases the same solid volume being considered.

It will be noted that these factors cancel in the following analysis:

$$d_o \times V = S \quad \text{or} \quad d_o = \frac{S}{V}$$

$$d_a \times E = S \quad \text{or} \quad d_a = \frac{S}{E}$$

$$\frac{d_o}{d_a} = \frac{S/V}{S/E} = \frac{E}{V} = \frac{N \times g \times D_m^3}{N \times g \times D_n^3} = \frac{D_m^3}{D_n^3}$$

$$\text{or} \quad \frac{D_m}{D_n} = \sqrt[3]{\frac{d_o}{d_a}} \qquad \text{or} \quad \frac{D_n + t}{D_n} = \sqrt[3]{\frac{d_o}{d_a}}$$

$$\text{or} \quad \frac{D_n}{D_n} + \frac{t}{D_n} = \sqrt[3]{\frac{d_o}{d_a}}$$

Solving this for t:

$$t = \left[\sqrt[3]{\frac{d_o}{d_a}} - 1 \right] D_n$$

Where:

d_o = determined experimentally and is according to natural voids.

d_a = computed from design quantities (item 7 on sample calculation sheet).

Data assumed in following sample calculation:

1. Assume water-cement ratio = 0.78.
2. Assume 6 3/4 sacks of cement per yard, or 6.75 c.f. (loose volume) for concrete yielding 4,400 to 5,000 lbs. per square inch strength.

Therefore Water = 6.75 x 0.78 = 5.28 c.f. per cubic yard.

F

In applying the formula to practical use set up in tabular form as follows: (Actual test run on Distribution System)

Water Cement Ratio=.78 Water=5.28 c.f. Cement=3.24 c.f. Total Aggregate=18.48 c.f., or 68.4% of Total Unit

Item												
(1)	Screen	0	200	100	50	30	16	8	4	3/8"	3/4"	1"
(2)	Analyses %	.6	1.2	5.3	9.5	10.0	7.9	8.6	6.9	25.4	22.4	2.2
(3)	Max.Diam. m.m.(screen)	0	.074	.147	.295	.589	1.168	2.362	4.699	9.423	18.85	26.67
(4)	Avg.Diam. m.m. = D_n	.037	.110	.221	.442	.878	1.765	3.530	7.061	14.181	22.76	29.14
(5)	Solid Vol. = Item (2) x % of Total Aggregate	.004	.008	.036	.065	.069	.054	.059	.047	.174	.153	.015
(6)	Subtract Item (5) from unity; begin at right	.320	.328	.364	.429	.498	.552	.611	.658	.832	.985	1.000
(7)	d_a = Item (5) ÷ Item (6)	.012	.024	.099	.150	.139	.098	.097	.072	.209	.156	.015
(8)	Exper. d_o	.550	.600	.630	.650	.650	.650	.650	.650	.650	.650	.650
(9)	= Item (8) ÷ Item (7)	45.8	25.0	6.6	4.3	4.7	6.6	6.7	9.1	3.1	4.2	43.2
(10)	= $\sqrt[3]{\text{Item (9)}}$	3.58	2.93	1.88	1.63	1.68	1.88	1.89	2.10	1.46	1.62	3.51
(11)	= Item (10) - 1	2.58	1.93	.88	.63	.68	.88	.89	1.10	.46	.62	2.51
(12)	= Item (11) x Item (4) = "t" *	.096	.212	.194	.278	.596	1.550	3.14	7.78	6.52	14.12	73.2
(13)	= Item (4): - Set 1 column to right		.037	.110	.221	.442	.88	1.76	3.53	7.06	14.18	22.8

* "t" must be greater than Average "D" of next smaller size.

The amount greater depends on richness of cement, varying from 2 x .051 m.m. to 2 x .025, i.e., twice the thickness of cement film.

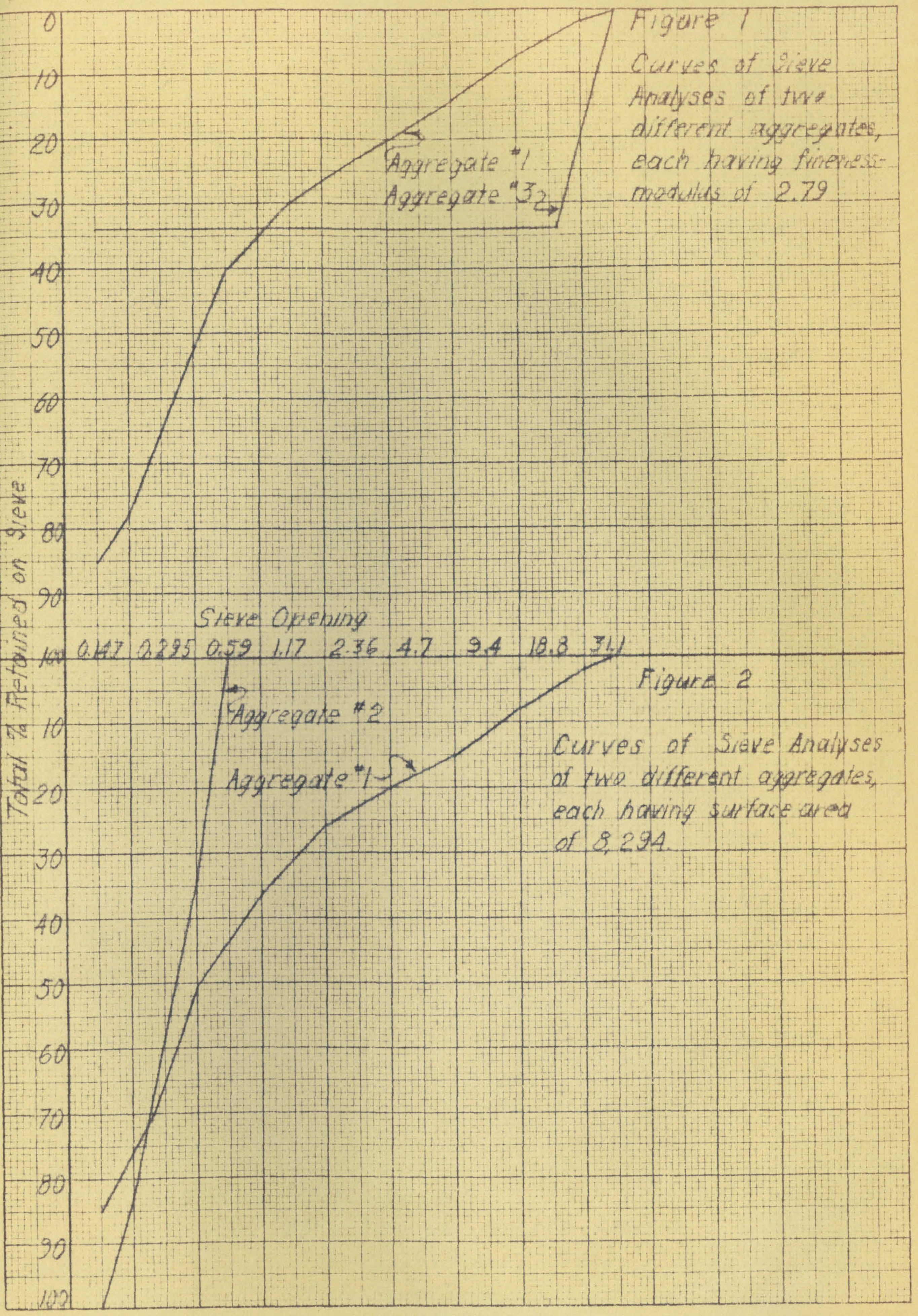


Figure 1
Curves of Sieve Analyses of two different aggregates, each having fineness-modulus of 2.79
Aggregate #1
Aggregate #3
Sieve Opening
0.147
0.295
0.59
1.17
2.36
4.7
9.4
18.8
37.7
Total % Retained on Sieve
Figure 2
Curves of Sieve Analyses of two different aggregates, each having surface area of 8,294.
Aggregate #2
Aggregate #1

Figure 3 Trial Mix Strength Curve

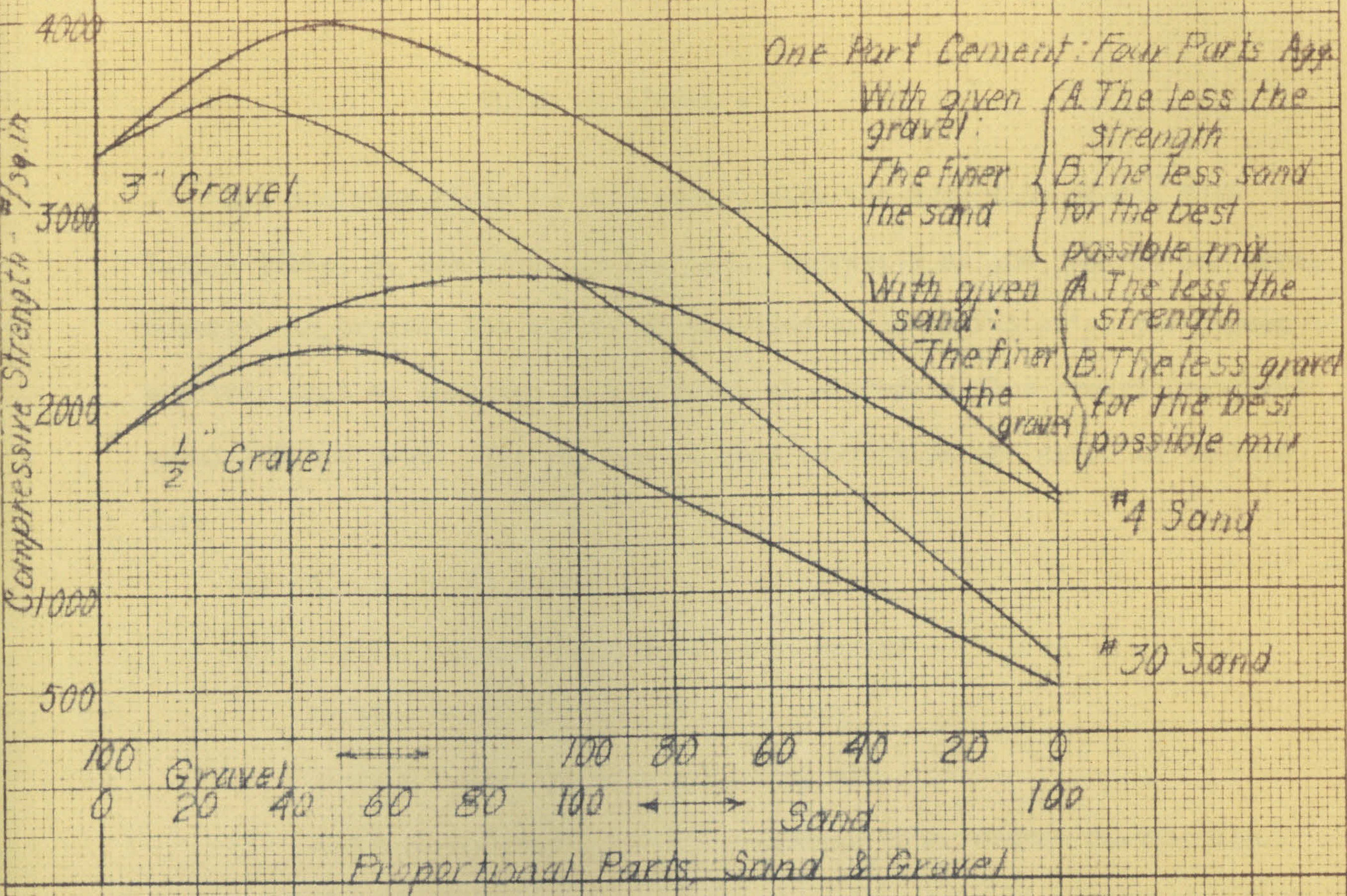
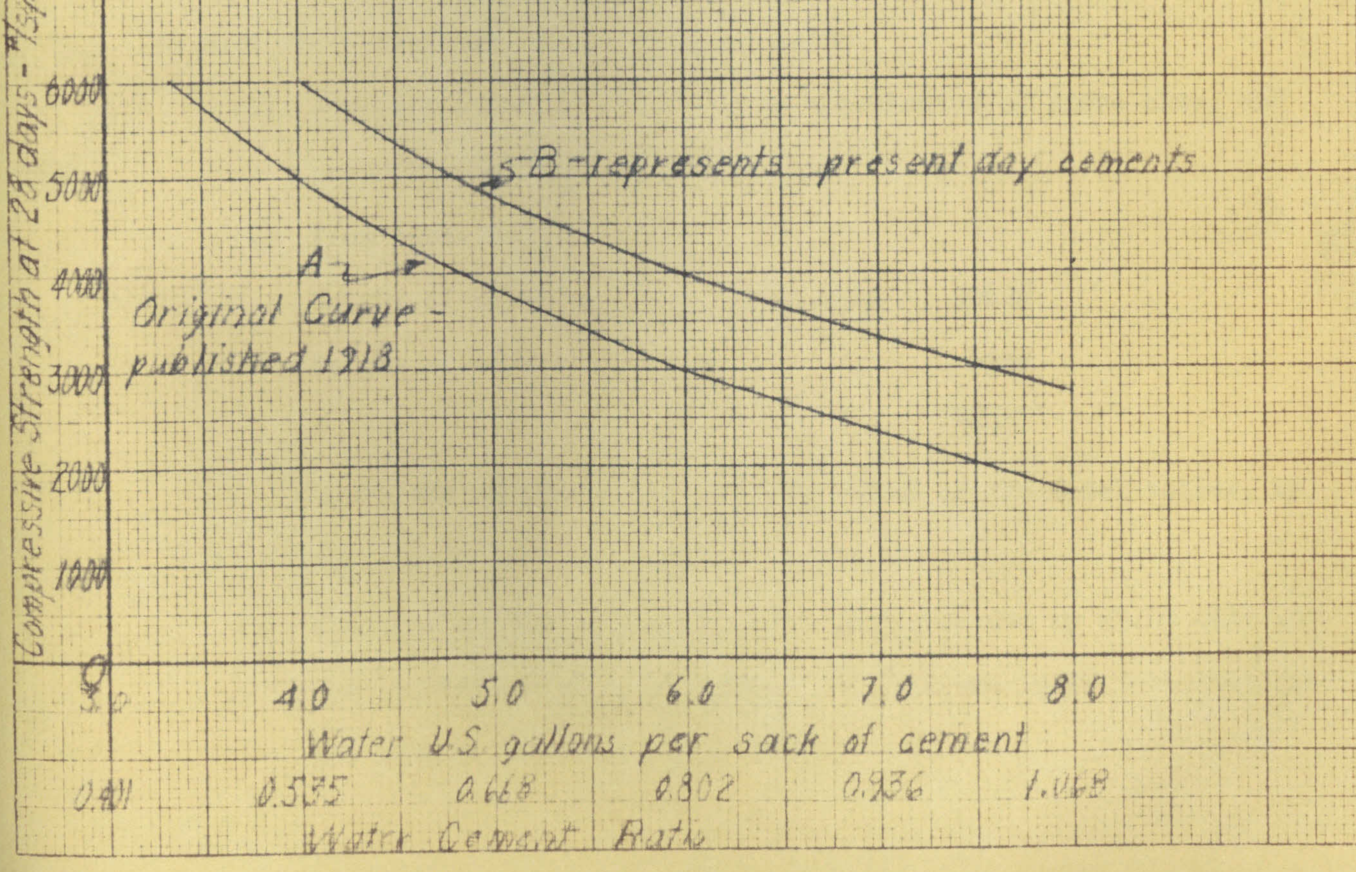


Figure 4 Strength-W/C Curves



NOTES ON FIGURE 5

All analyses given in this paper, with the exception of the following table, E, are based on the total aggregate as one hundred per cent. However, to present the curves on the same basis as Fuller's, the screen analysis must be computed on the basis of aggregate plus cement (weights) equal 100 per cent. The results of this conversion are given in table E for:

(1) M.W.D. ideal curve

(2) M.W.D. average-field-conditions curves -
curve A being for satisfactory mix
curve B being for unsatisfactory mix.

(3) Curve of aggregate used in particle interference calculation.

Also tabulated there are the coordinates, under the same conditions, for:

(4) Fuller's curve of maximum density - practical mix.

To make the conversion, it is observed that

(1) Analysis curves are plotted on coordinates of sieve size, and proportion of the total by weight.

(2) From table D:

Weight of total aggregate per cu.yd. = 3062#

Weight of cement per cu.yd. = 635#

3687

Hence the aggregate is $\frac{3062}{3687}$ or 83.2% of the total solids, by weight.

(3) Multiplying the aggregate previously recorded by 83.2% will give the percentage of total solids, the desired basis for plotting for direct comparison with Fuller.

NOTES ON FIGURE 5 (continued)

(4) The cement is $\frac{635}{3687}$, or 17.2% of the total solids. It will be assumed that all of the cement passes the 200 mesh sieve.

TABLE E

Retained on Tyler Sieves:

Curve	Pan	200	100	48	28	14	8	4	3/8	3/4	1"
A	82.8	82.4	82.0	78.9	70.6	61.3	54.1	47.6	42.5	23.3	2.7
B	82.8	82.5	80.5	75.3	67.2	59.7	54.0	48.5	43.1	21.4	2.5
C (M.W.D.ideal)	82.8	82.2	81.1	76.5	68.6	60.8	54.1	49.5	45.0	26.2	2.1
D (particle interference)	82.8	82.6	81.8	77.2	69.8	61.0	54.5	47.4	41.6	20.5	1.8
E (Fuller)	93.0	89.0	85.0	81.8	76.8	72.2	66.1	60.0	47.4	25.0	8.9

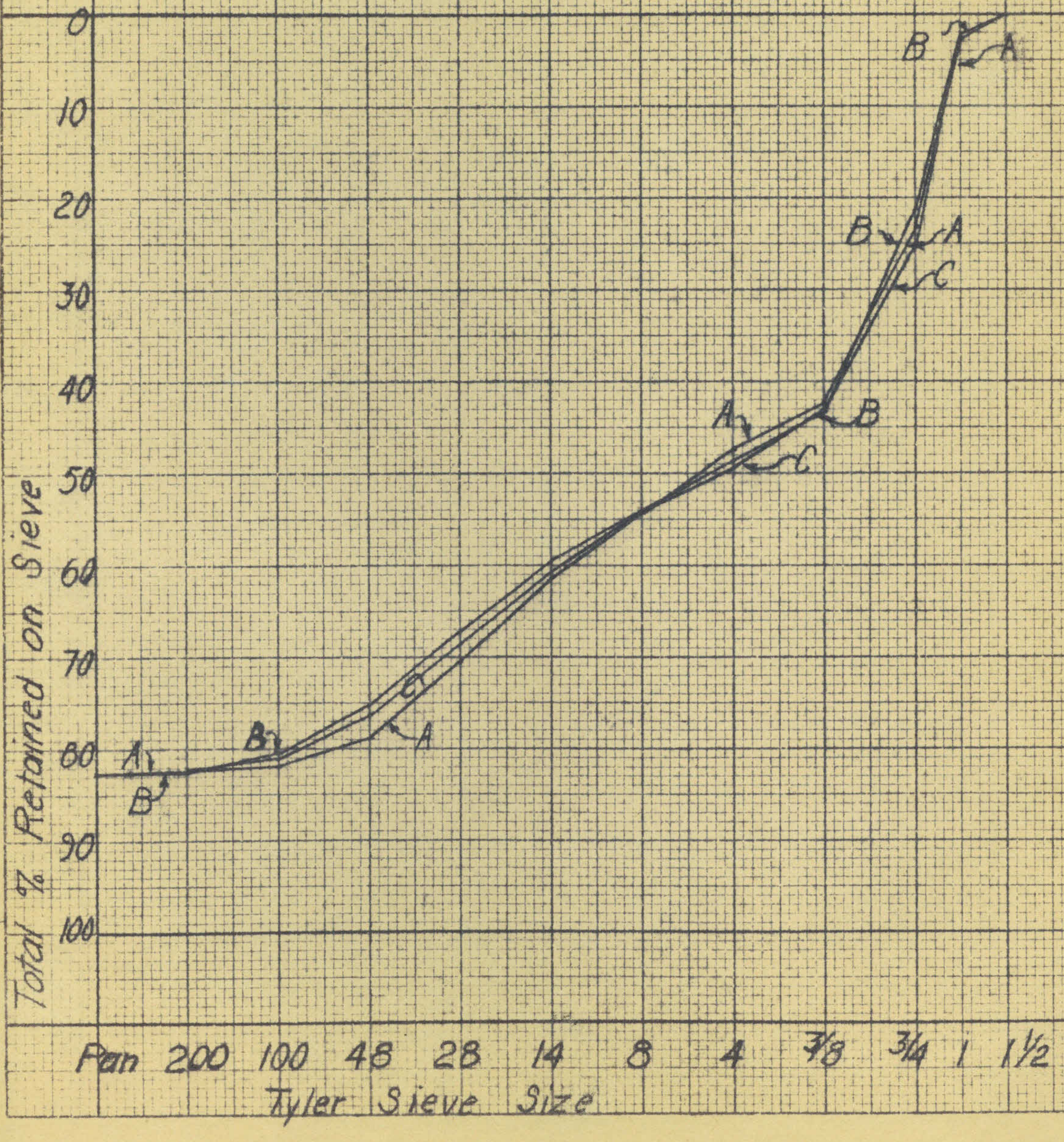
Figure 5a

Comparison of Analyses Curve

Curve A (from table A) gave harsh mix

Curve B (from table B) gave harsh mix

Curve C – is ideal curve, as determined by MWD men



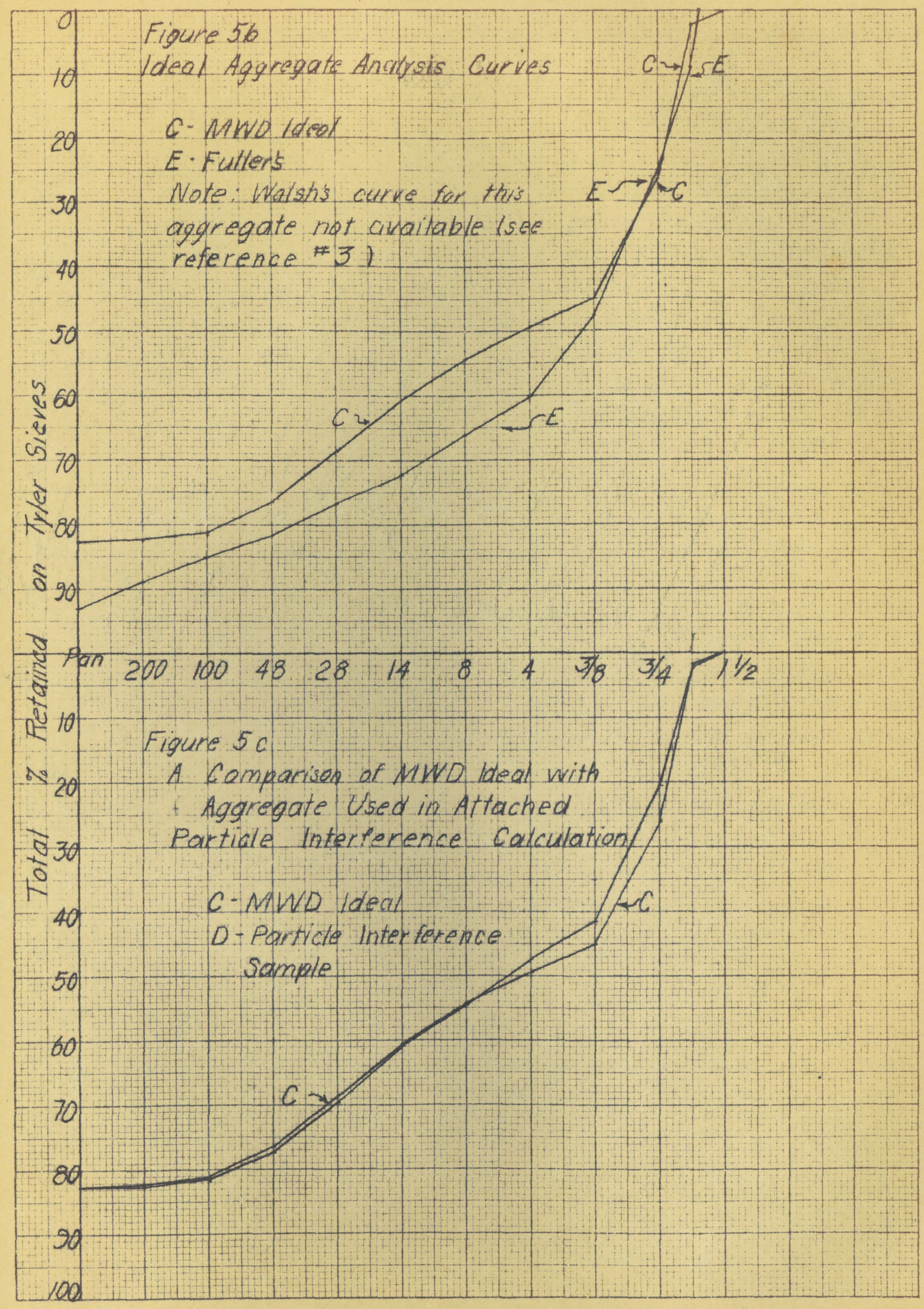
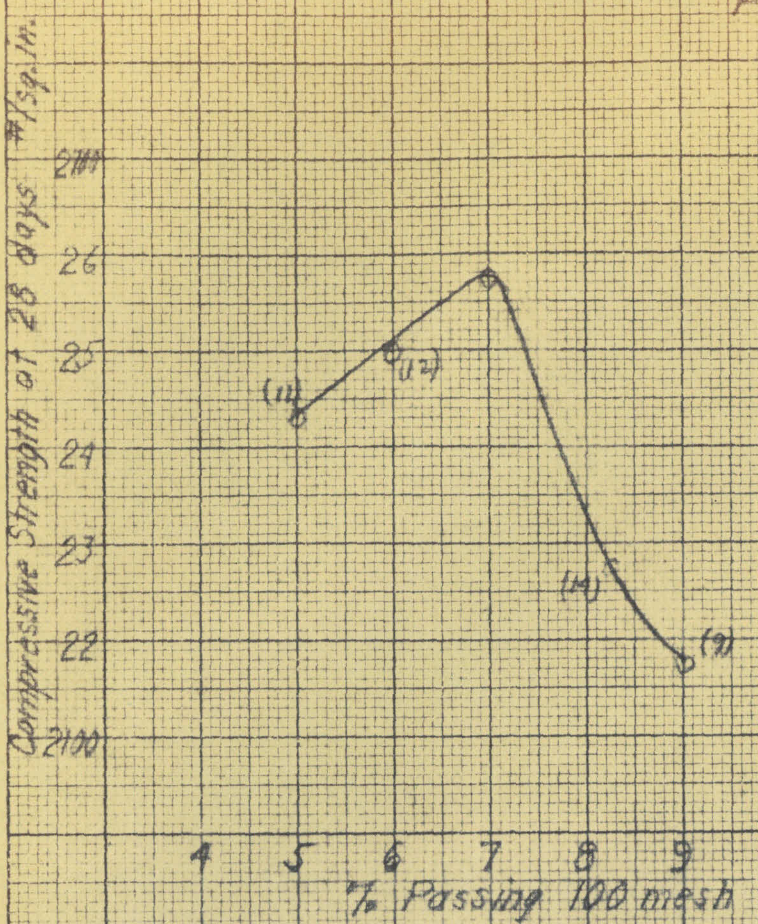


Figure 5b
Ideal Aggregate Analysis Curves
C - MWD Ideal
E - Fuller's
Note: Walsh's curve for this aggregate not available (see reference #3)
Figure 5c
A Comparison of MWD Ideal with Aggregate Used in Attached Particle Interference Calculation
C - MWD Ideal
D - Particle Interference Sample
Total % Retained on Tyler Sieves
Pan
200
100
48
28
14
8
4
3/8
3/4
1 1/2
0
10
20
30
40
50
60
70
80
90
100
C
E

Figure 6

Strength- Fines Curve

Figures 6 & 7 are based on mix of average proportions 29-7-30-34, 5" flow. Compiled at Banning test Lab by MWD staff. Figures by points indicate number of tests on which that point is based.



Note: Because of limited data on which these curves are based, they should not be regarded as general.

Figure 7

Strength- Silt Curve

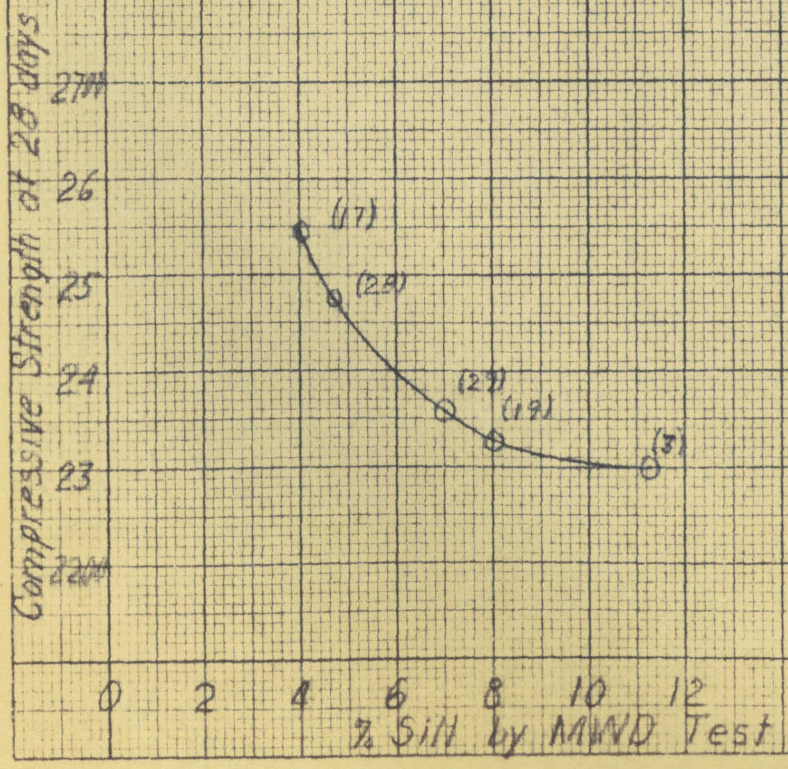
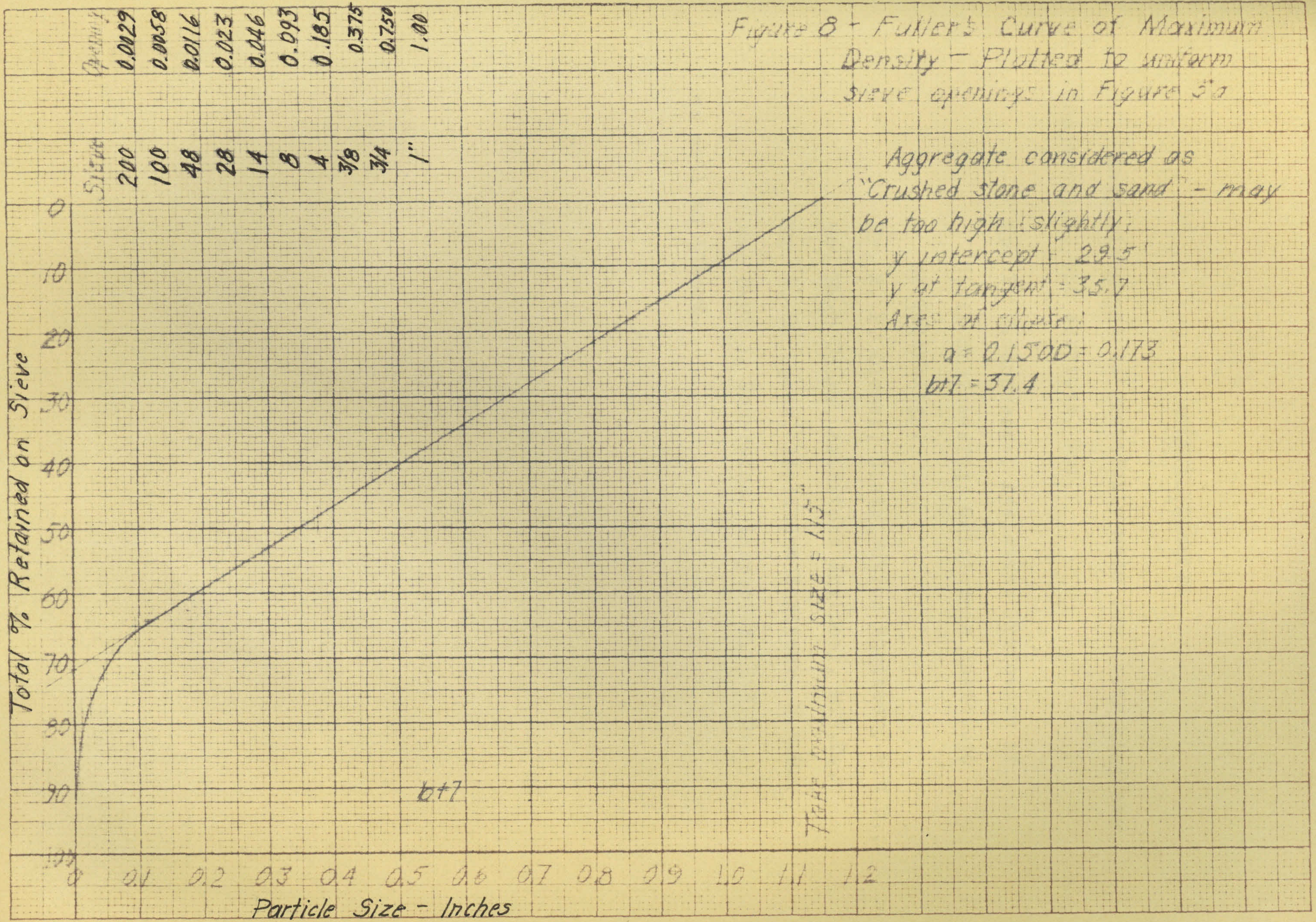


Figure 8 – Fuller's Curve of Maximum Density – Plotted to uniform sieve openings in Figure 5a



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