A Study of High Velocity Flow Around a Sharp Bend of a Rectangular Open Channel

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FORWARD

Unfortunately a thorough investigation of the problem of high velocity flow around sharp bends was not permitted due to lack of time. The time put in building the apparatus was much greater than that anticipated, with a result that only a pitifully short time was available for making runs. Actually only two runs were made.

A thorough investigation would include runs on at least three slopes (say 1.5%, 3.5% and 10%) with at least three or four different discharges each. Further, more angles should have been used, say 15°, 22½°, 30°, 37½°, and 45°. 22½° and 45° are commonly used in canal design, and would be especially valuable in that regard. Investigation should also include velocity distribution in the cross-sections of the channel. Also, it would have been interesting to plot contours of the water surface as related to the channel bottom. All these investigations would be necessary as super-critical velocity flow is peculiar unto itself.

Thanks and appreciation is hereby extended to Dr. Ippen for his many helpful hints, suggestions, and influence, to Warren Wagner for his invaluable aid, to the U. S. Soil Conservation group for their courtesy in extending the use of their tools, to the Mechanical Engineering Division for the use of their tools, and to Le Van Griffis, without whose help this manuscript would not have been completed.

Raymond Boothe

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A. STATEMENT OF PROBLEM

The problem under investigation concerns the conditions incident to water flowing at supercritical velocities encountering a sharp horizontal bend in an open channel of rectangular cross-section.

It is first necessary to define the critical velocity, and then place a limitation on the bend.

Critical velocity, with which we are dealing, is defined by the condition where discharge takes place with a minimum amount of energy, for a finite depth. This condition is best indicated by a wave of translation on the surface of water at a finite depth, the wave being relatively small. This critical velocity may be found by differentiating the specific energy E of a unit width of fluid with respect to its depth d. The specific energy E is given by eq. (1).

(1) $$E = d + \frac{v^2}{2g}$$

v is the velocity of the water, and g is the gravitational constant. Differentiation gives the result of the required critical velocity, in eq. (2).

(2) $$v = \sqrt{gd}$$

This same relationship can also be derived on a basis of momentum, but will not be done here for lack of space.

Now we come to the limitation on the bend, assuming the water flowing a supercritical velocities.

If there were a sheet of water flowing at a velocity above the critical, and a condition of free discharge (or disturbance) were set up, it is clear, that the resulting wave front would not travel in a direction opposite to the direction of flow, (assuming, of course, a relatively small wave of translation, and a stable condition of flow) but would assume some angle α with the direction of flow, as demonstrated in the diagram below:

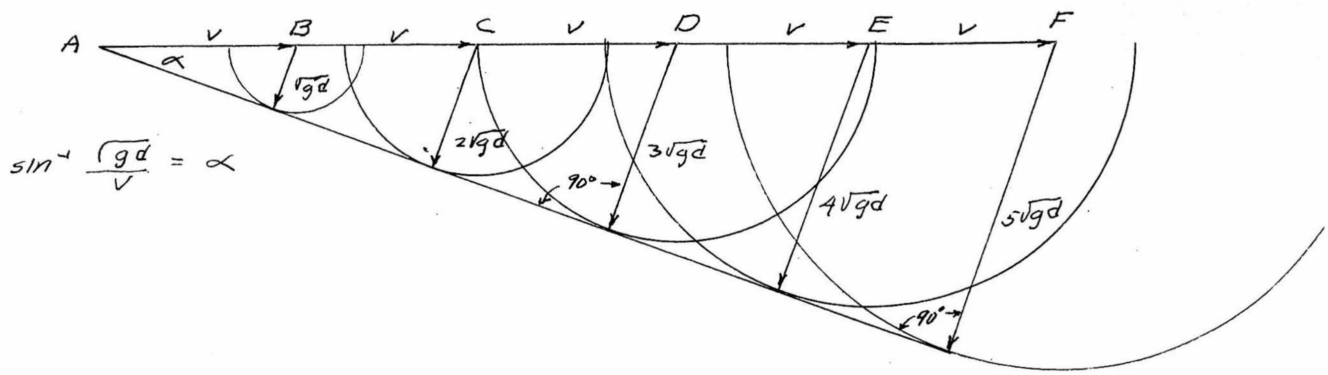


Fig. 1

Diagram of Wave Angle

It is clear from an examination of the diagram, that the angle α (between the direction of the flow of the sheet and wave front) is defined by

$$(3) \qquad \alpha = \sin^{-1}\frac{\sqrt{gd}}{v}$$

and where v is the velocity of the sheet.

The problem at hand is a consideration of what happens when water flowing at supercritical velocity down a rectangular channel encounters a bend at an angle greater than that defined by $\sin^{-1}\frac{\sqrt{gd}}{v}$ as indicated below.

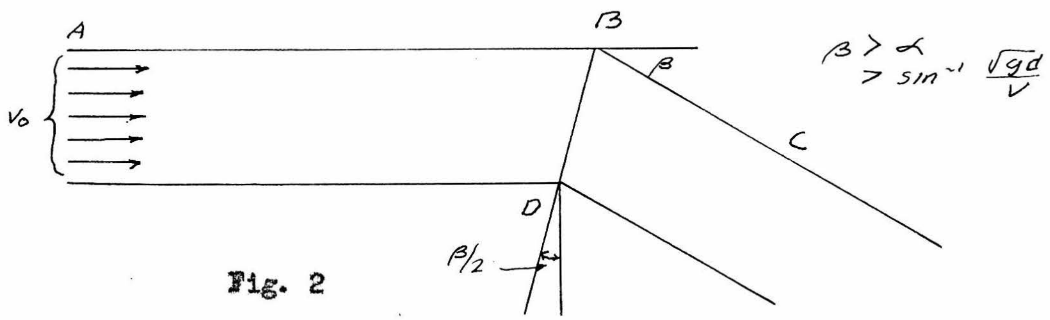


Fig. 2

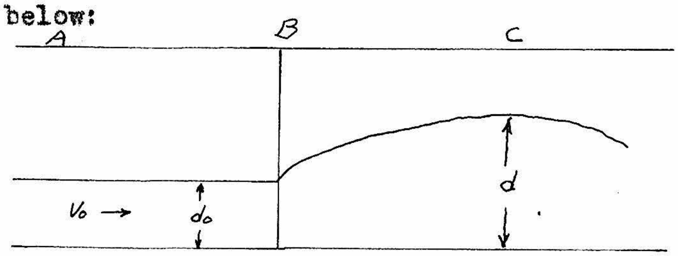
Plan of a Typical Channel

If β were less than α, it is clear that the first infinitesimal portion of water flowing along AB would create a wave front when it hit B which would travel back across the channel in a condition of discharge such that there would be a considerable decrease in the piling up along the outside edge than would normally be expected. The object of making $\beta > \alpha$ is to eliminate this lessening of the pile up. Thus we are able, more or less, to get the net effect of a sharp bend, as the first infinitesimal outside portion hitting

B is not able to set up a wave front.

It seems probable that the first outside element piling up, will, due to its increased height, help change the direction of its neighbor element due to a change in the momentum in the latter resulting from the differential pressure acting on it. It also seems evident that the water will continue to pile up to a certain maximum and then decrease. This maximum would be some distance downstream from the bend.

The preceding discussion is illustrated somewhat roughly by the diagrams below:



Profile Fig. 3

Pressure of a Buildup

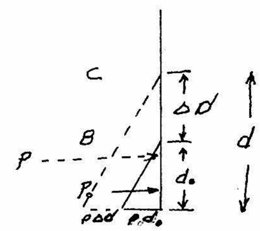


Fig. 4

Assuming in Fig. 2 that C is the point of maximum height, and drawing a profile along the outside wall, we would probably obtain a figure similar to Fig. 3. Fig. 4 illustrates the idea of a change in level producing a change in direction. P and P_o represent resultant pressures per unit width at C and A respectively, being the products of $p\,\frac{d^2}{2}$ and $p\,\frac{d_o^2}{2}$ where p is the density. The existence of a greater pressure tends to force neighboring elements to change direction without actually coming in contact with the wall.

Beyond this qualitative discussion, the problem of what goes on quantitatively is apparently quite complex. Where the maximum height will occur is another difficult problem. Were the increases in water level very small in relation to the depth, the problem would be very much simplified, but such is not the case. The height of water at C can easily be two or three times the depth at A.

Various factors which might enter into consideration are (1) The possibility that the vertical acceleration of the water have a marked effect

(2). The effect of turbulence. (3). The effect of changes in viscosity due to changes in temperature. (4). Consideration of the negative front set up at D in Fig. 2. (5). The changes in longitudinal velocities which would probably occur. These changes are indicated when one substitutes different depths in the Chezy formula ($V = c\sqrt{ms}$, c being a constant, m being the hydraulic depth or radius, s being the slope) and the Specific Energy equation. ($V = \sqrt{2g(E-d)}$, notation as previously used) (6). The effect of friction. (7) The increase in energy which occurs because of drop in the channel.

Reflection leads one to believe that numbers 2 and 3 could be eliminated. It is also probable that number 5 could be eliminated, because, as Dr. Ippen indicated, the velocity tends to increase with increase of depth, referring to the Chezy formula, while the velocity, considering the Specific Energy equation, tends to decrease, the two effects probably canceling each other. It also seems likely that numbers 6 and 7 could be eliminated. The increase in energy due to drop in the channel will partly (if not wholly) overcome the effects of friction and turbulence. This is dependent, of course, on the slope, and whether the velocity is increasing as it goes down the channel.

It is seen, however, that even if these simplifying assumptions are made, we still have a very complex problem if we are to get the theoretical relations that exist.

One very helpful characteristic of supercritical flow is the velocity distribution. Water in subcritical flow has a distribution as indicated in Fig. 5, while that of supercritical flow is shown in Fig. 6.

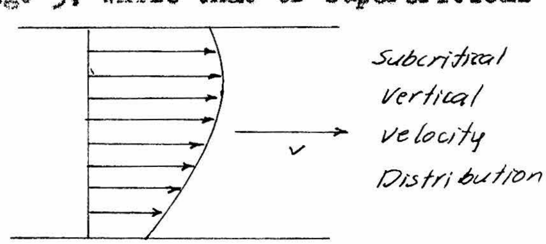


Fig. 5

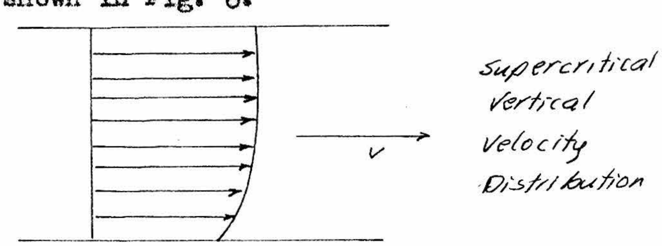
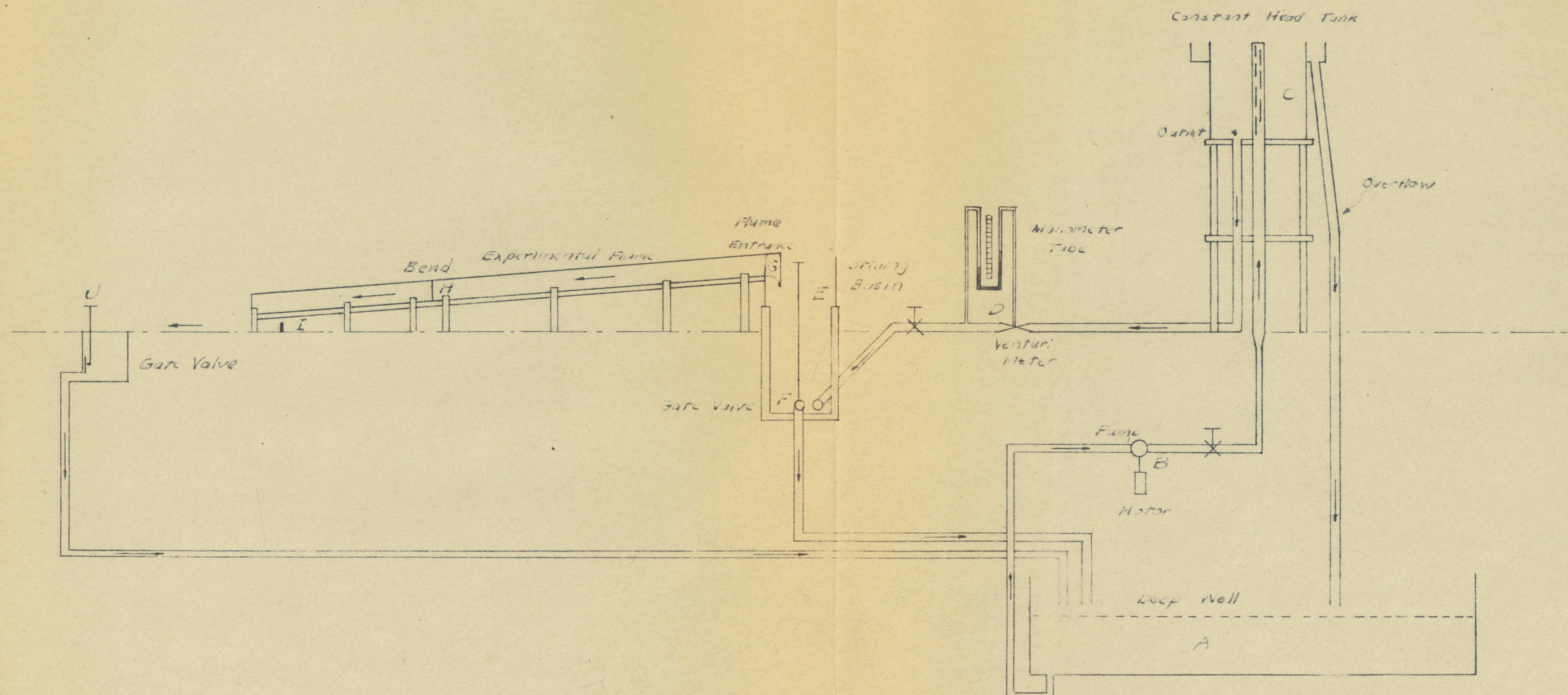


Fig. 6

Hence, it is clear that the velocity may be treated as constant throughout the entire corss-section without appreciable error. This is a very great simplifying assumption.

SCHEMATIC DIAGRAM OF CIRCULATION SYSTEM

Figure 7



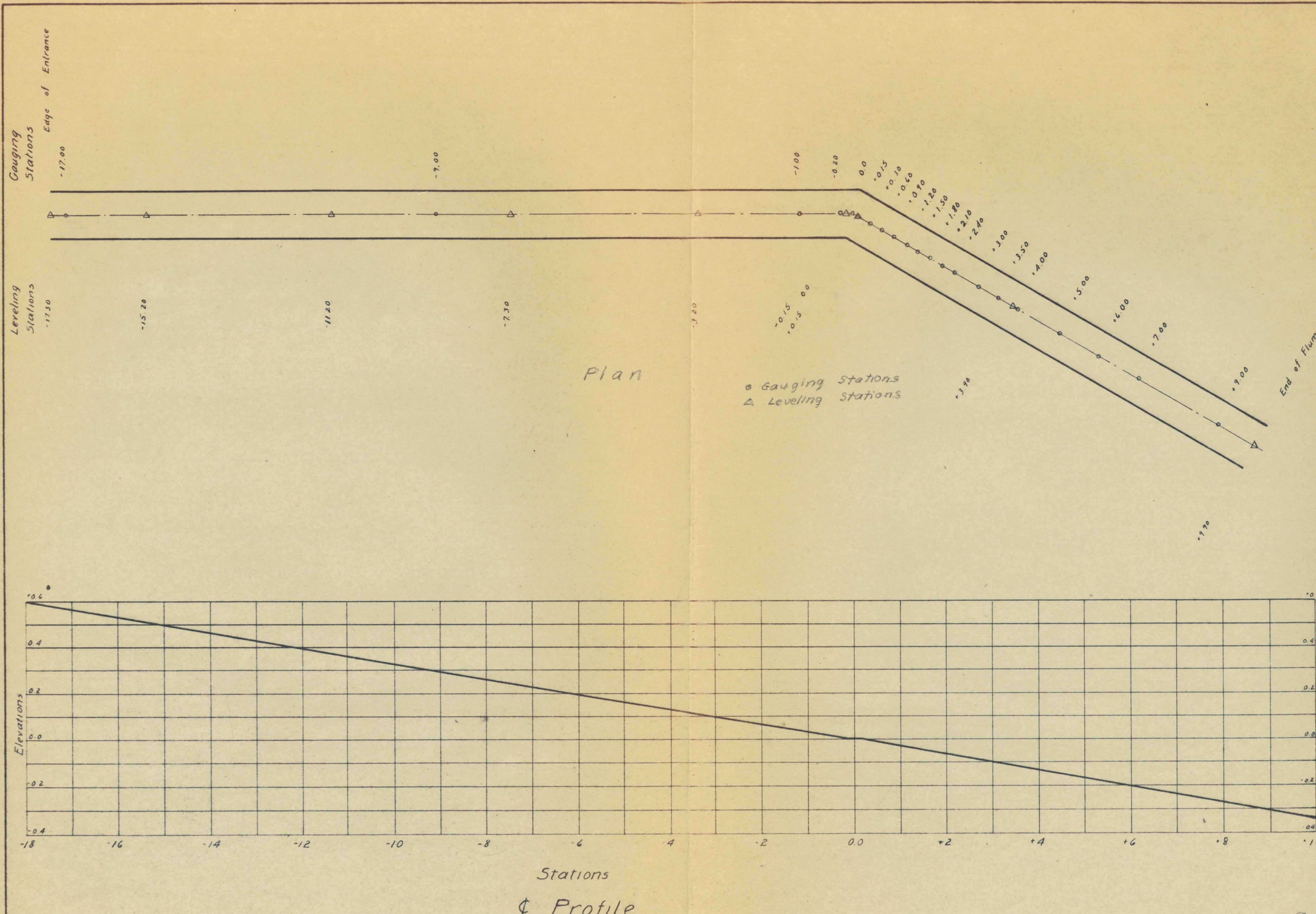

Gauging Stations
Edge of Entrance
Leveling Stations
Plan
Gauging Stations
Leveling Stations
End of Flume
Elevations
Stations
℄ Profile

Figure 8

DESCRIPTION OF APPARATUS

On the preceding pages will be found a schematic diagram of the circulation of water system. No attempt was made to picture the exact relationships, only the relative ones. Discussion will be taken up in order of the letters on the sheet.

The water was drawn from a storage reservoir (A) of about 1400 cubic feet capacity by a centrifugal pump (B) to a constant head tank (C) which was 5' high, and about 6' in diameter. The schematic diagram indicates how it works. When the central tank is full, it overflows into the outer basin, which, as is seen, has a pipe connected which leads back to the well. A condition of overflow was maintained during the runs.

The water from the constant head tank was taken through a 10" pipe then through a 8" - 3½" Venturi meter by which the inflow was measured into the flume. The measuring device was a mercury manometer (D). It was calibrated in feet of mercury. There were several connections at the throat and full cross-sections, thus insuring an accurate reading. The manometer was equipped with valves which allowed the line to be bled before each reading, further insuring accuracy. On the following sheet is a calibration curve for this particular Venturi meter. During the second run, for some unknown reason, the discharge changed from a reading corresponding to 1.265 c.f.s. to 1.235 c.f.s. This is a change of -2.4%, but, considering the accuracy with which the cross-sections could be read (after the bend) the variation was so small it could be, and was, neglected. A picture of the box containing the manometer is seen below the constant head tank in Fig. 20.

After leaving the Venturi meter, the water went through the 8" pipe into the bottom of the stilling basin, (E). The stilling basin was necessary in order that turbulent conditions would not be present at the flume entrance. The discharge into the basin was accomplished by a long pipe which lay the

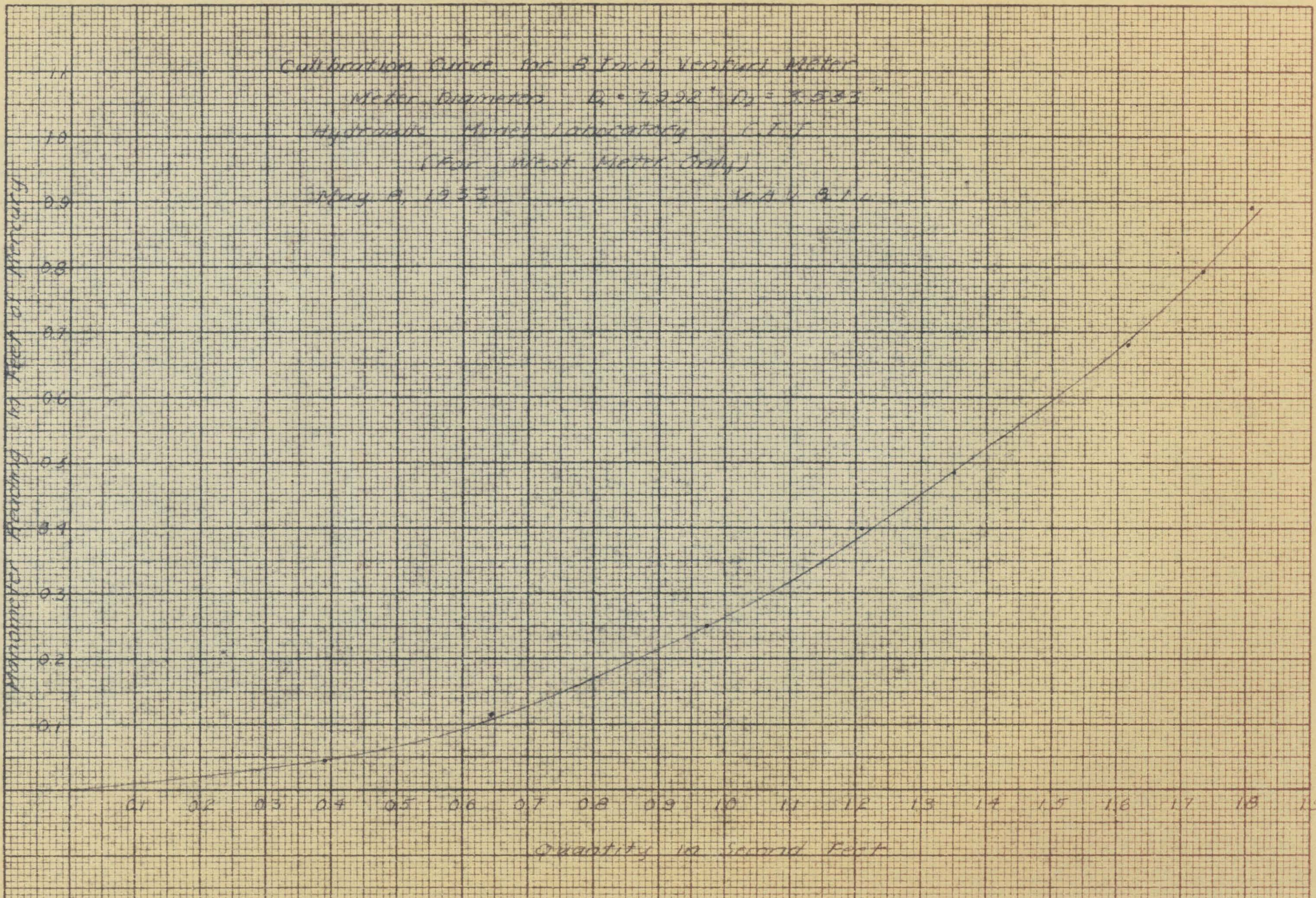

Calibration Curve for 8 Inch Venturi Meter
Meter Diameters D_1 = 7.992" D_2 = 3.533"
Hydraulic Model Laboratory C.I.T.
(For West Meter Only)
May 8, 1933
Manometer Reading in Feet of Mercury
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
Quantity in Second Feet

Figure 9

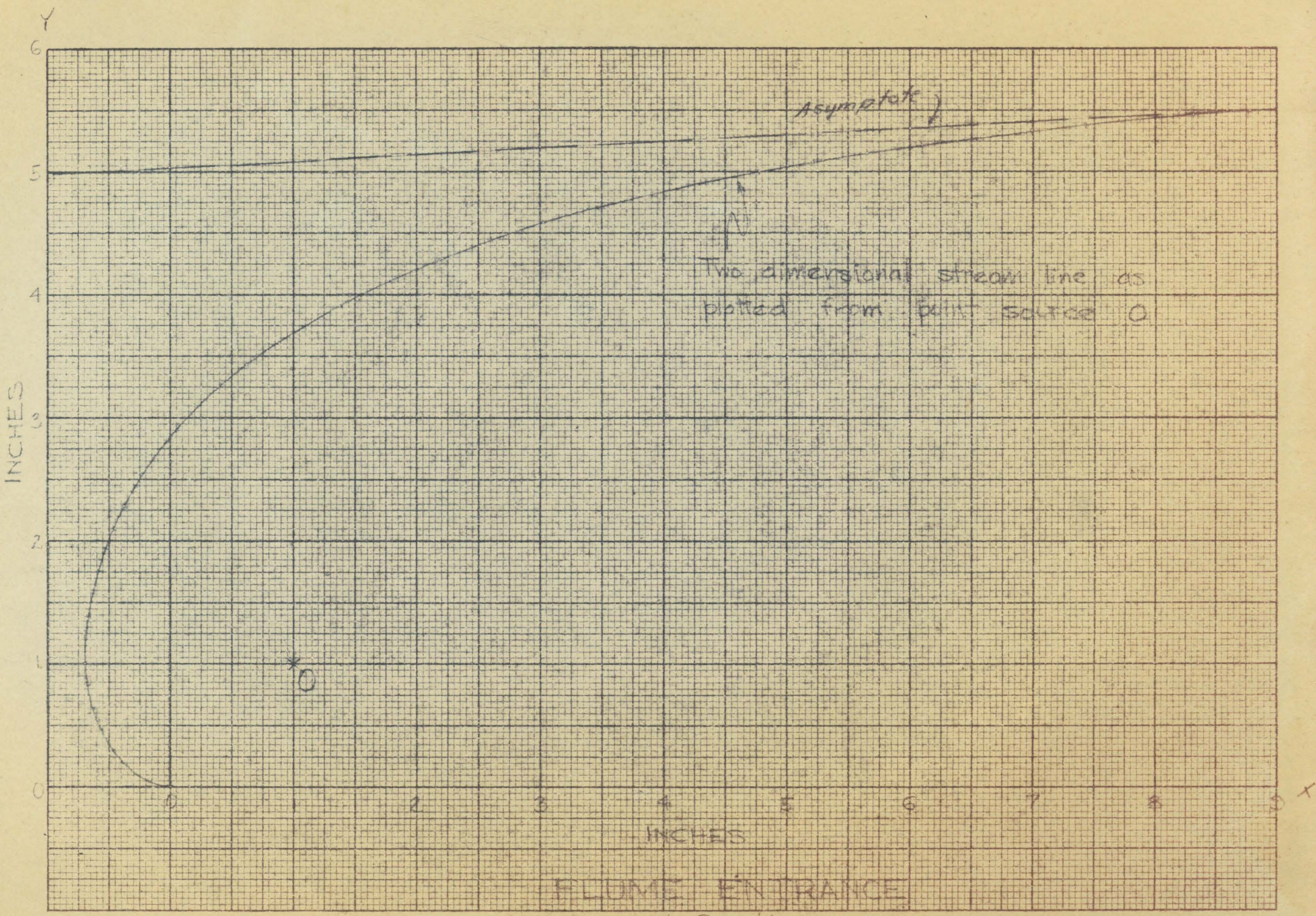
length of the stilling basin, and was perforated throughout. At (F) is indicated a gate valve, which facilitated emptying the stilling basin.

Considerable difficulty was had in constructing this basin, and consumed most of the time spent on the project. The chief trouble was its location and the materials used. It would have been easier to have made the basin walls out of concrete, but this was barred for various reasons. Essentially it was a sheet metal addition or topping on concrete. Waterproofing the juncture was accomplished through the use of tar paper, mastic, and plaster. There were no leaks of measurable quantities in it, however, when it was finished.

After run number 1, it was noticed that the water surface dropped, probably due to leakage through the gate valve F. The amount of drop was 2.6 cm. in four minutes. Since the surface of the basin was 2.82' x 14.20' or 400 sq. ft., the leakage corresponded to a flow of about 0.142 second feet, which value was subtracted from the discharge obtained from the manometer reading. In the second run, there was no leakage whatever. The stilling basin was very satisfactory.

As a check on operations in general, stage readings were taken in one corner of the stilling basin. The variation obtained in the first run was 1.2 millimeters, and 18 millimeters in the second. This amount of variation would have a negligible effect on the quantity of water carried by the channel.

The water entered the flume by a specially designed stream-line entrance (G). This precaution was necessary in order to avoid turbulent flow in the channel, so that decent results could be obtained. This entrance is shown in Fig. 18. The basis of the entrance design was a two dimensional streamline as plotted from a point source. The actual curve is shown on the following page. It was necessary to draw an asymptote to the curve as shown, because the streamline would not have become parallel to the walls of the straight



X-Section

Figure 10

section of the channel (assuming the channel walls parallel to the x axis) until an infinite distance out. In the making of the entrance, the sheet metal was worked until it was parallel to the x axis. It was felt necessary to extend the end of the sheet metal beyond the theoretical end opposite point O on the x axis. This portion of the curve was put in by eye.

Since the curve merely represents the cross-section as obtained from a plan or side view, it was necessary to make a development of the two dimensional stream line to fit three dimensions. A development was made corresponding to the arris as laid out on a plane. This development is shown in Figure 12. From this development was made a template which in turn was used to cut out the sheet metal shapes. These sheet metal plates were curved according to the two dimensional stream line, and soldered together to form the entrance. Flanges were added of course. It was noted with satisfaction that the entrance worked perfectly.

The flume was a sheet metal rectangular channel 1' x 11" in cross-section, supported at frequent intervals (about every 4' - 5') throughout its length. Joints between sections as shown in Figure 11. Gaskets were made of tar paper, which were covered with axle grease. Bolts were used in joining the sections.

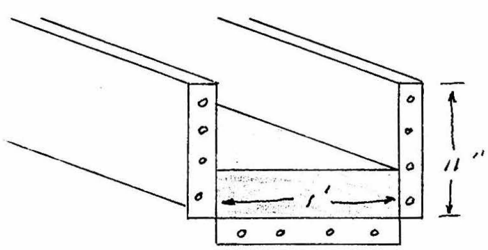


Fig. 11
Detail of Joint

The flume was set approximately in place, and then set accurately with the use of a precise level. The elevations were determined by taking 3½% of the length of the channel, relative to the beginning and end of the bend section. The error in this procedure would amount to only 3.5% x versine ($\sin^{-1}.035$) x length or 0.000577' in the entire length which was closer than the flume could be set anyway. At each leveling section or

Developement - Flume Entrance Section

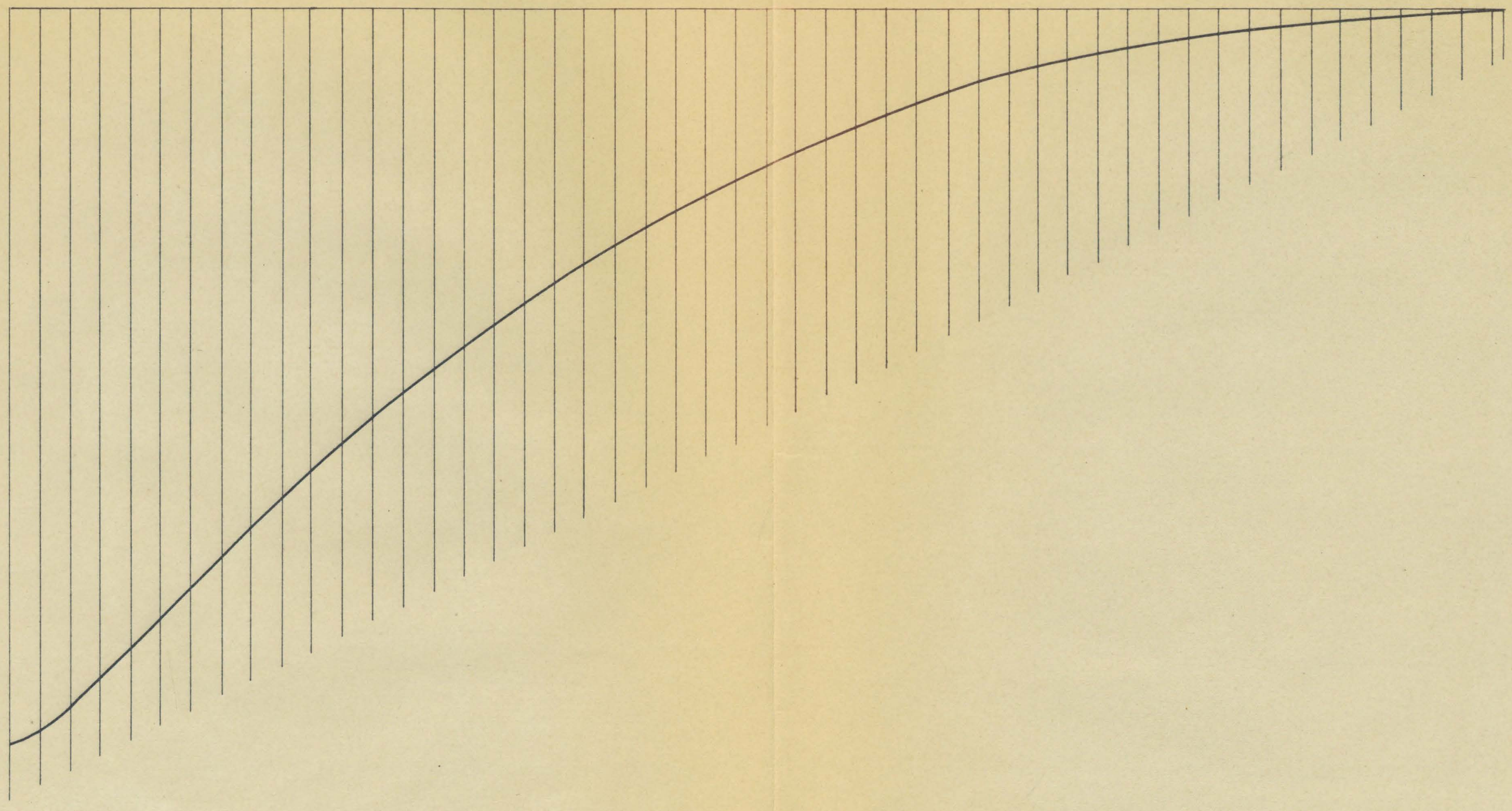


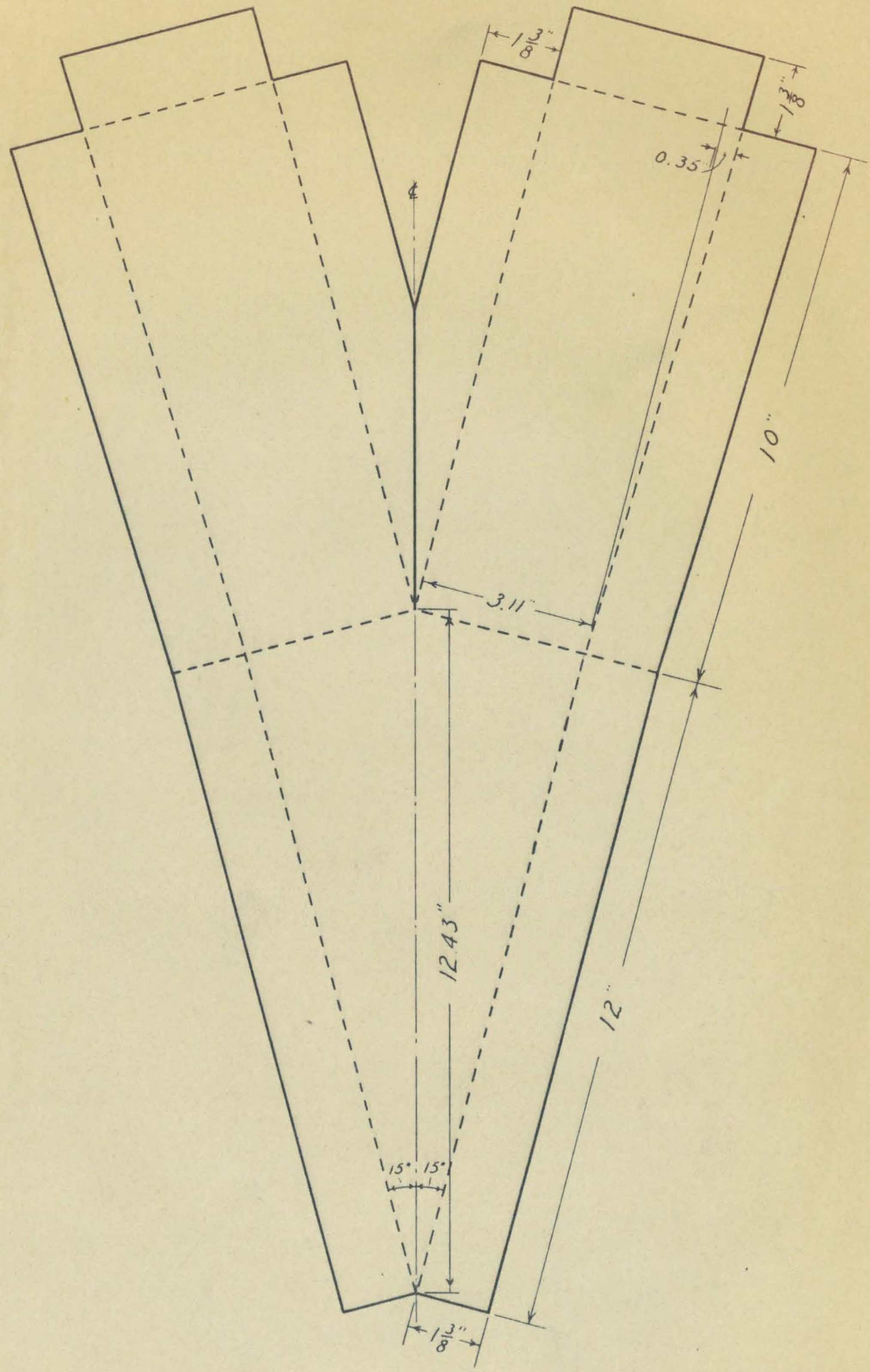
Figure 12

station, shims were placed under the supports to make the flume level. Both sides were leveled. The accuracy of these elevations was not more than 0.005' off, and the level of each side to within 0.002' of the other. Good pictures of the flume are shown in Figs. 17, 22, 23, and 24. A diagram showing the plan and profile of the flume is shown in Fig. 8 on page 9 .

At (H) is shown the bend. A drawing of the template of this angle is shown in Fig. 13. It will be noticed that it is so made (referring to the fold lines on the vertical wall) that the floor of the bend will be level when in place. This is necessary to avoid skewing the channel near the bend, as a little reflection will show. If the channel with the bend were laid out level, and then the plane on which it were laid tilted to a $3\frac{1}{2}\%$ slope, either the upstream section of the channel, or the downstream section, or both sections would not have a horizontal cross-section, which is untenable in this experiment. Skewing should have to be resorted to in order to get them level, which is undesirable. It was thought best to make the floor of the bend level in order that the channel could have a horizonatal cross-section throughout its length, even if it did add some complications to the conditions giving rise to the standing waves. For this reason, the angle floor was made as small as possible. A 30° angle was used.

The water was discharged about 9.9' downstream from the bend onto a concrete surface. The water was kept in its place by a small dam shown at (I). The course which this water took in going to its ultimate destination of the gate valve at (J) was extremely interesting in itself, because of the way it shifted about for no apparent reason. However, such consideration was outside the small scope of the experiment. From the valve at (J), the water returned to the well, thus completing the circuit of water.

Outside of the one leak mentioned in connection with the stilling basin, there were no leaks of sufficient magnitude to even consider, and hence they



Template of 30° Bend

Figure 12

were neglected. The whole outfit held water very well considering the difficulties in its construction.

In order to make measurements of the cross-section and profiles of the water surface, stations were established different from those used to level the flume. The schematic diagram of these stations is shown in Fig. 7. It was thought necessary to establish stations above the bend, in order to indicate the condition of the water entering the bend in regard to cross-section, and also whether it had reached a stable velocity as indicated by the area of cross-section. The stations at and below the bend were put at such intervals as thought necessary to give a fairly complete study of the problem. Below about 4' or 5' from the channel, bend, most of the phenomena of importance had occurred, and so stations were placed more sparingly from there on.

The layout of these stations was referred to the intersections of the center lines of the upstream and downstream sections of the channel. The upstream stations were measured in feet and marked minus. Those downstream were marked plus. The elevations were made relative to the intersection just mentioned. At each station was established a grid iron. These places were made at every inch across the channel, and were marked to facilitate making the cross-section measurements. It is interesting to note that these marks were of no particular good in the second run because they couldn't be seen. A ruler across the top had to be resorted to.

The actual measurements of the cross-section were made by means of a point gauge which is shown in Figure 22. It was simply a pointer which moved up and down relative to a vernier on the cross piece. A scale in centimeters was attached to the pointer. In order to move the pointer horizontally, the whole gauge had to be moved. The readings obtained were changed to feet before plotting them.

Although the pointer could be read to the nearest 1/100 of a centimeter by virtue of the vernier, actual measurements were made only to the nearest 1/10 of a centimeter, or one millimeter. Below the bend, this accuracy was hardly justified, because of the difficulty in telling just exactly where the surface of the water was, as there were mild fluctuations, usually less than ½ centimeter. However, an honest attempt was made to get accurate readings.

Due to the fact that the flanges of the channel on which the cross-piece of the gauge rested were not always level, although the bottom was, or even a constant height above the bottom, it was necessary to take a reading on each point of the bottom for any given cross-section, and the reading thus made would have the reading to the surface subtracted from it in order to get the depth. (Actually the reverse, as regards subtraction was necessary, because the reading increased as the pointer was raised). In reading the point gauge, the attempt was made to get the distance perpendicular to the bottom of the channel. Consideration of the versine ($\sin^{-1} .035$) shows that the error is not worth troubling over, being about .0006136 times the length to the point.

In making measurements, it was found to be expedient to make measurements at other than the planned points on the cross-section, as other changes in the water surface than were anticipated occurred.

It was thought that a value of n of 0.007 could be used as the coefficient of roughness in calculations of the channel in using the Kutter equation of flow.

The maximum flow which was available to the flume was about 0.5 second feet, although, at the base of the tower, about 2.5 second feet could have been obtained. This difference is explained mainly by friction in the pipes and outlet in the stilling basin.

DISCUSSION OF RESULTS

The following discussion is concerned with the physical appearance of the deformation of surface, as there is no mathematical theory at the hand of the writer by which the results can be checked. Although an attempt was made to find a mathematical relationship, no success was had.

The reason for the rather long channel before the sharp bend was to give the water sufficient time to reach a condition of constant velocity before striking the bend. An examination of the cross-sections of run number 2 give the following data:

Station	-17.00	-9.00	-1.00	-0.20	+0.30	+1.20	+5.00	+9.00
Area in Ft.^{2}	.355	.210	.192	.192	.244	.206	.201	.196
Av. Velocity ft/sec.	3.48	5.88	6.43	6.43	5.05	6.00	6.15	6.30

It is apparent that, by the time the bend was reached, the water had attained a more or less constant velocity, which was what was sought.

It is next to impossible, in the light of present knowledge, to correlate this rather sudden velocity change (after the bend) with turbulence or vertical component. The major portion may be ascribed, it seems, to the momentum change.

It is noticed in the cross-section at station 0.00 (Figure 15) on the outside, a raising of the water surface, which had commenced about 0.12' above the bend proper. It is rather hard to explain this, because there ought not to be any build up before the bend. It seems likely that it is a case of hydraulic jump. As was noted in the description of apparatus, it was necessary to have the bottom of the bend level, in order to have the channel make a bend without being skewed. It seems probable that the sudden change of 3½% slope to level was sufficiently large to cause this jump, and especially at the edge, where there are two retarding surfaces of friction, with resulting lower velocities near to them.

It was also noticed that there was a quite rapid dying out of the disturbances set up, downstream from the bend. The writer has no explanation for this. In the experiments of Dr. Ippen on curved channels, these waves were of relatively large size even quite a ways downstream. Altho the pattern of wave fronts is clearly discernible in the pictures, their magnitude is not very large, as can be seen by consulting the cross-section drawings.

Another interesting thing is a small backwater wave which was at station +1.20. It was about one half the width, and extended from the inside edge. It was perpendicular to the direction of flow. No absolute explanation can be given, but it again seems to be a matter of hydraulic jump. It will be noticed that at station 0.90, that the surface of the water on the inside edge is very low. It may be that friction on the bottom was sufficiently great to slow the velocity down to the critical velocity at which it would jump. The critical velocity at which jump occurs is not the same as that for the velocity of a wave. It is given by equation (4).

(4) $v_c = \sqrt[3]{gQ}$ (4a) $d_c = \sqrt[3]{\frac{Q^2}{g}}$

where dc is the critical depth, vc is the critical velocity, and Q is the discharge.

Another interesting comparison is to plot the profiles along the walls as is done in Figure 16. A remark will be made which it is well to bear in mind: Instead of being as shown, the outside edge (LEFT WALL) is actually 0.6' longer than the inside wall. These profiles are plotted in reference to the stations as laid down along the centerline of the channel. For convenience in plotting, the profiles were laid out according to the stations. At any rate, 0.6' of a foot is hardly noticeable on the horizontal scale selected. The vertical exaggeration is 12.5 times the horizontal.

Leading up to the bend, is noticed the characteristic "down drop" curve which is encountered in open channels on a down slope. It seems apparent in both runs, that the increase in friction due to increased velocity resultin g from the drop in the channel, was matched by the rate of increase of specific energy, resulting in a stable condition.

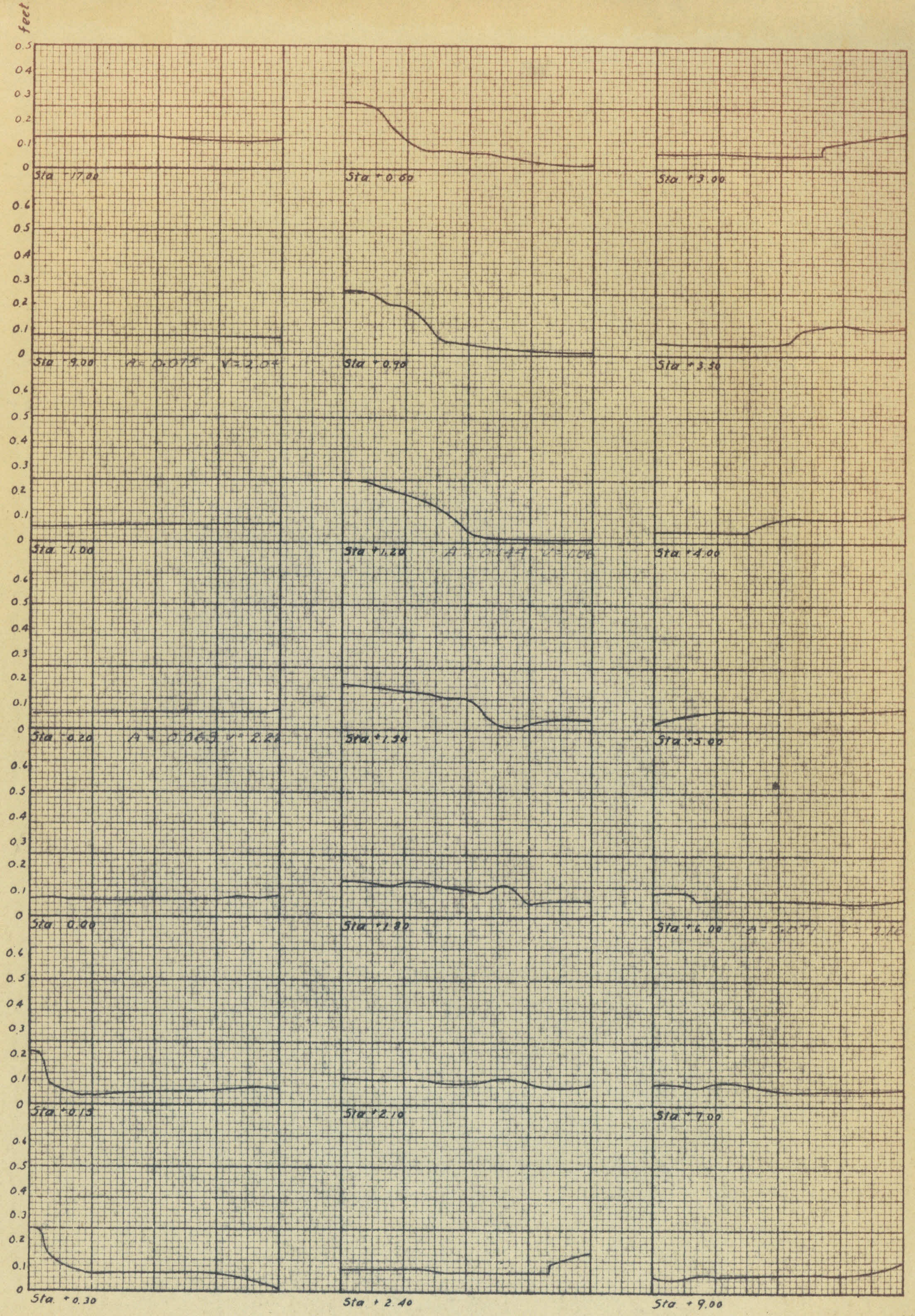
The theoretical wave angle of run number 1 should be $\sin^{-1} \frac{5.67\sqrt{d}}{V} = \sin^{-1} \frac{5.67\sqrt{.069}}{2.22}$ $42° 13'$. The wave angle for run number 2 should be $\sin^{-1} \frac{5.67\sqrt{.193}}{6.43}$ or $22° 50'$.

Referring to Figure 24, it is seen that the wave angle is equal or less than 30°. There is an obvious inconsistency here, as the photograph was made of run number 1. Considering the large losses connected with the stilling basin in run number 1, it is probably best to refrain from making any definite conclusions from run number 1. Unfortunately, no angle measurements were taken on run number 2.

It is rather interesting, however, to note the very close correspondence between the profile on the two runs on both the outside and inside walls.

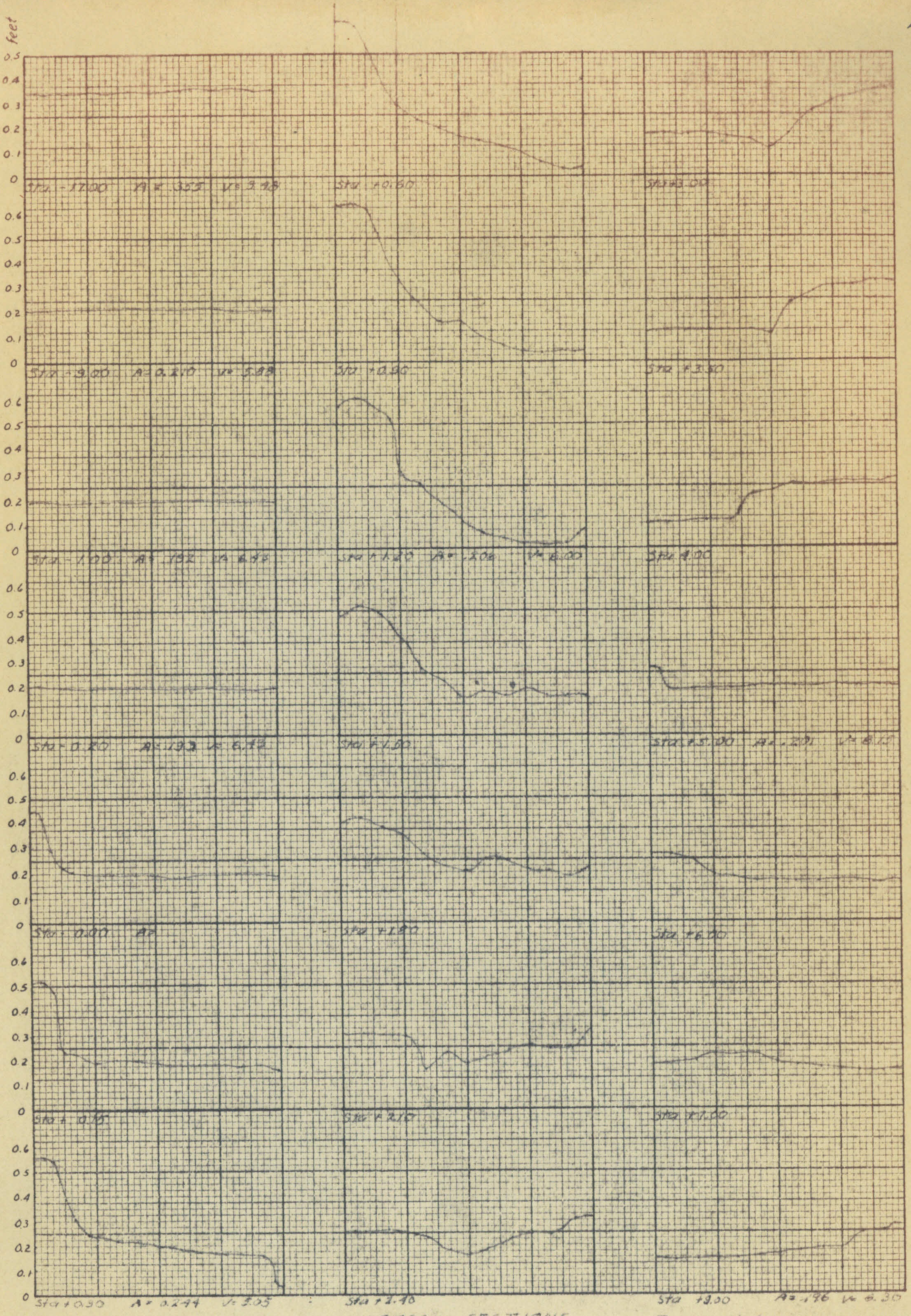
Another interesting point is the rather rapid decrease in the magnitude of the standing wave as it progresses down the channel. It is rather unfortunate that the channel was not longer on the downstream end, so that the wave pattern could be further studied.

The photographs tell their own story, and will not be further discussed here.



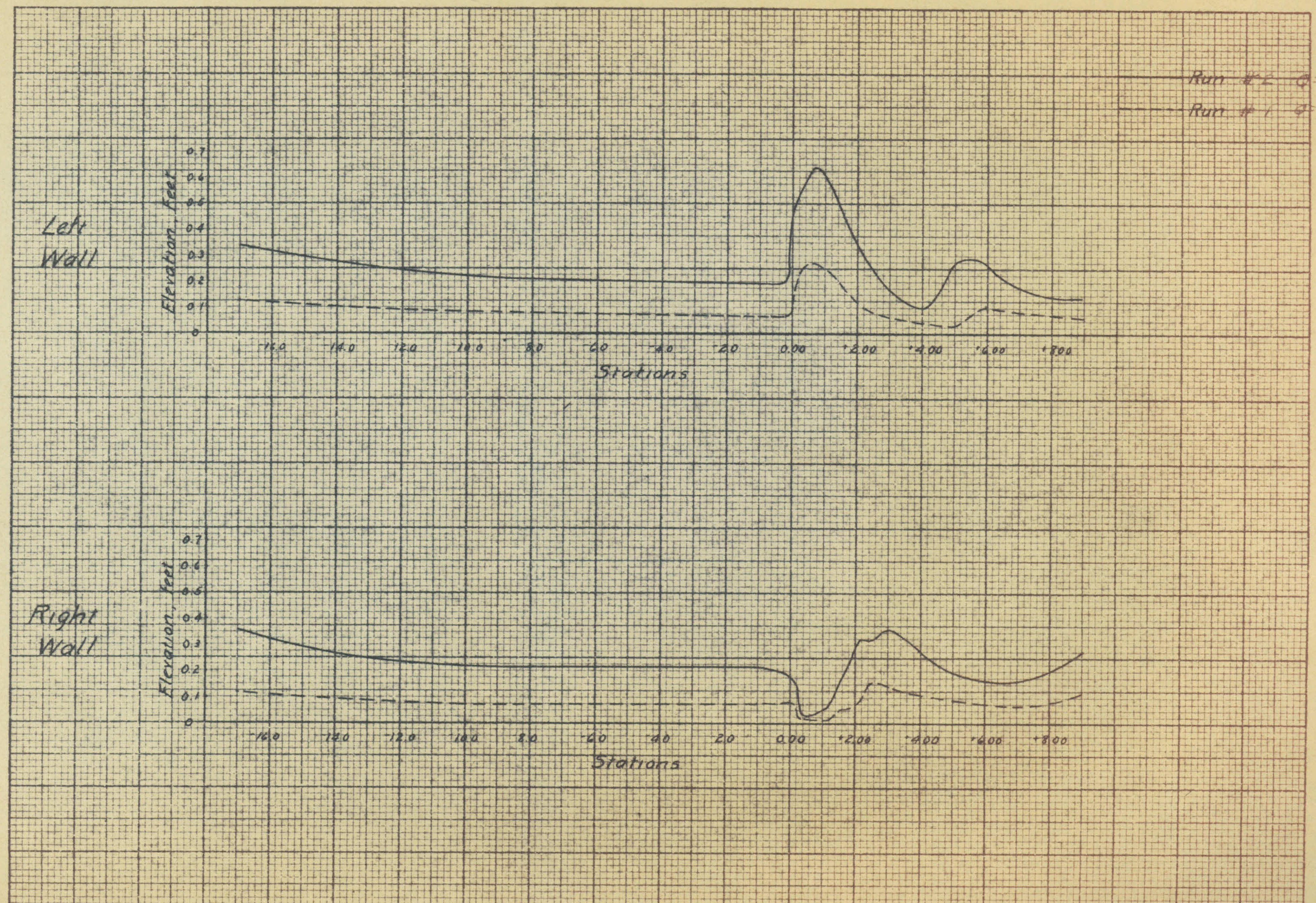
CROSS - SECTIONS Run #1 Q = 0.153 ft³/sec

Figure 14



CROSS - SECTIONS
Figure 15

Run #2 Q = 1.235 ft³/sec



PROFILES ALONG WALLS

Figure 16

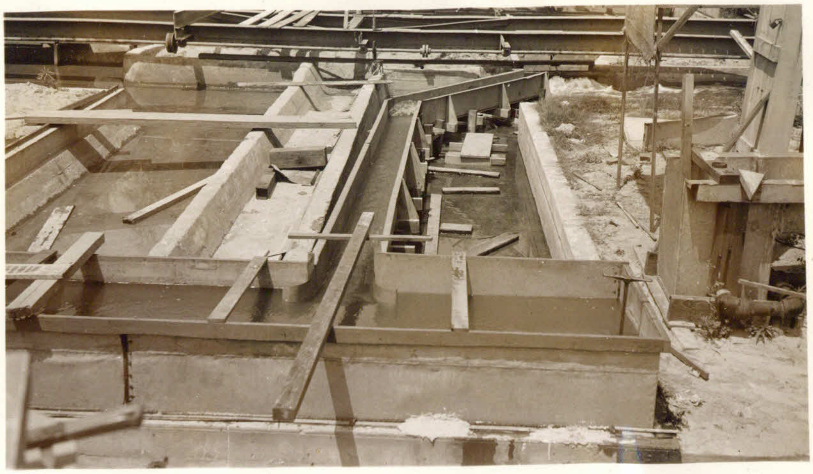


Fig. 17 General View of Experimental Setup

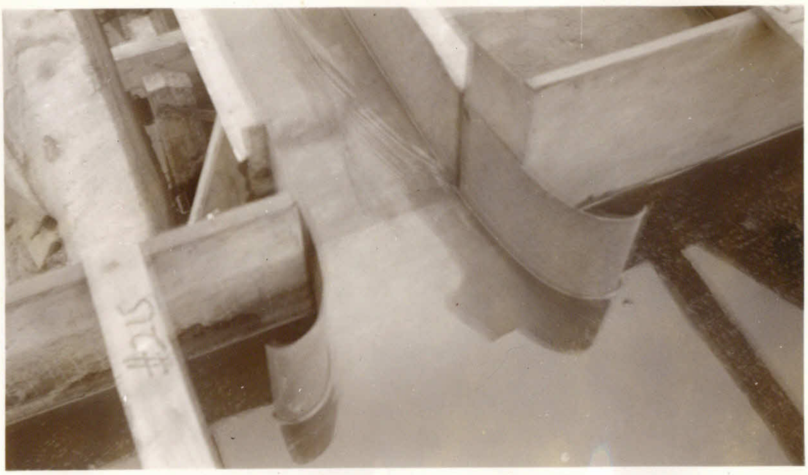


Fig. 18 View of Flume Entrance

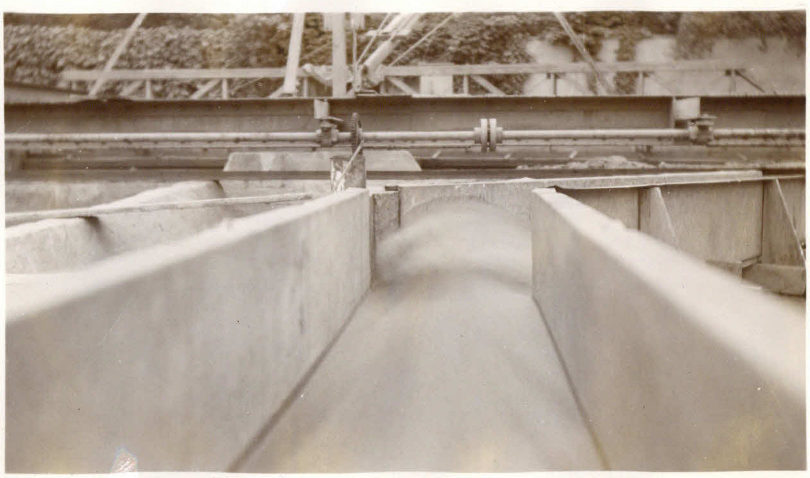


Fig. 19 View of First Maxima

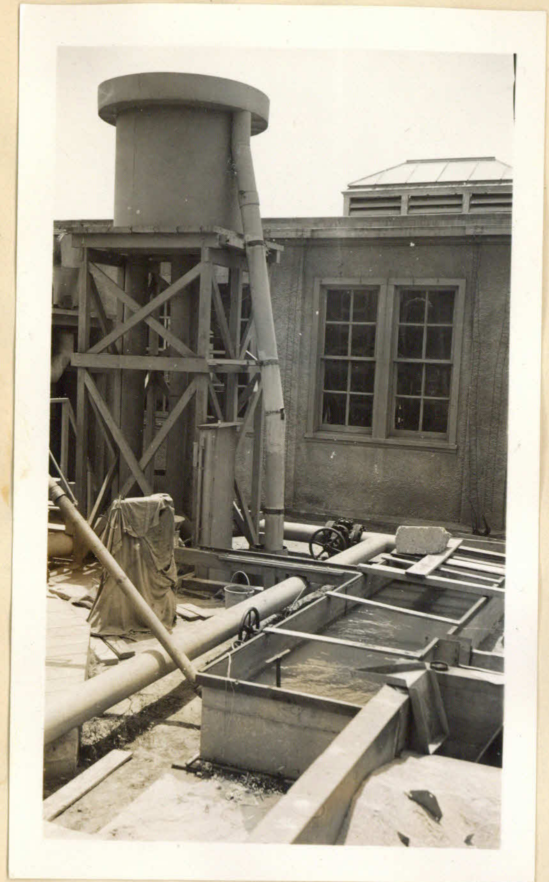


Fig. 20. Constant Head Tank

Fig 21. View of Bend

Fig 22. Point Gauge

Fig 23. Closeup of the standing wave

Fig. 24 View of Wave Pattern

D. LIST OF WORKS CONSULTED

Burgers, J. M.

On Waves Produced in a Channel by a Moving Transversal Wall

Zeitschrift für Angewandte Mathematik und Mechanik, Vol. 13, April, 1933 pgs. 66-71

This gave little help on the question due to its more or less fundamental difference with problem considered.

Bakhmeteff, Boris A.

The Hydraulic Jump and Related Phenomena.

A very good book, but does not deal with turns in the channel other than vertical.

Lamb, Horace

Hydrodynamics

Cambridge - at the University Press, 1895.

Excellent discussion of fundamentals that were over the head of the writer, and may have contained all the elements of a complete analysis of the problem at hand.

Reich, Fredrich

Umlenkung eines freien Flüssigkeitsstrahles an einer senkrecht Zur Strömungsrichtungstehenden ebenen Platte

Heft 290 V.D.I.F., 1926 fur V.D.I. - Mitglie der 7.20

This revealed an interesting analysis of a jet hitting a plate normal to the direction of flow, but had no particular relation to problem under consideration.

Havelock

Theory of Wave Resistance.

A good book on the subject, but no apparent relation found between the account and the problem.

Gibson:

Hydraulics

Very good discussion on waves, but little help on the particular problem.

Ippen:

Study of High Velocity Flow in Curved Sections of Open Channel.

Excellent discussion of closely related, but not sufficiently related phenomena. Much helpful matter obtained, however.