Appendix A

HOMOGENIZED STIFFNESS MATRIX OF PARALLEL TESSELLATION

The equivalent in-plane stiffness matrix for equally spaced parallel truss elements oriented at an angle α to the x-axis is given by the expression [4]:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \frac{EA}{l} \begin{bmatrix} \cos^4\alpha & \sin^2\alpha\cos^2\alpha & \sin\alpha\cos^3\alpha \\ \sin^2\alpha\cos^2\alpha & \sin^4\alpha & \sin^3\alpha\cos\alpha \\ \sin\alpha\cos^3\alpha & \sin^3\alpha\cos\alpha & \sin^2\alpha\cos^2\alpha \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{A.1}$$

where E, A, and l represent the Young's modulus, cross-sectional area, and spacing between the elements, respectively. The terms N_x, N_y, and N_{xy} denote the force per unit width stress resultants, while ϵ_x, ϵ_y, and γ_{xy} are the corresponding in-plane strain components.

The homogenized in-plane stiffness matrix for the complete truss tessellation is obtained by summing the stiffness matrices for each set of parallel trusses.

For instance, the truss tessellation composed of equilateral triangles, as shown in Fig. 2.19(b), is divided into three sets of parallel trusses with inclination angles of $\alpha = 0°$, $60°$, and $120°$. The spacing between two adjacent parallel trusses is $\sqrt{3}L/2$, while the Young's modulus and cross-sectional area of the trusses are denoted as E_n and A_n, respectively. Applying Eq. (A.1), the constitutive equation for the truss tessellation is expressed as:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \frac{3\sqrt{3}E_nA_n}{4L} \begin{bmatrix} 1 & 1/3 & 0 \\ 1/3 & 1 & 0 \\ 0 & 0 & 1/3 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}. \tag{A.2}$$

Appendix B

MODIFICATION OF CABLE STIFFNESS IN ABAQUS/CAE

This method allows for independent control over the constitutive behavior of multiple cables in the system, with each cable following its unique trajectory of material transition.

Consider a system with a single cable and two different constitutive behaviors, CB_1 and CB_2. The transition is governed by field variables (FV), where the cable is assigned its own FV to control the behavior shift. The cable material properties will change smoothly between CB_1 to CB_2 as the FV is ramped between the corresponding values across all nodes, as shown in Table B.1.

Table B.1: Material definition for a single cable

Constitutive Behavior	Field Variable Value
CB_1	Value 1
CB_2	Value 2

In this study, when the FV value of a cable with a specific extension is shifted from 0 to 1, the cable force transitions from that corresponding to zero stiffness behavior to non-zero stiffness behavior, described in Section 4.2.2 and depicted in Fig. 4.4. This change reflects the *activation* of the cable, enabling it to contribute to the structure's overall stiffness.

As the number of cables increases, so do the dependencies on the FV, making the system more complex and requiring precise control over the transitions between constitutive behaviors. Table B.2 presents the material definition for cable i in a system containing n cables, each exhibiting two distinct constitutive behaviors.

Table B.2: Material definition for cable i in a multi-cable system

Constitutive Behavior	FV1	FV2	.....	FVi
CB_1	0	0	0	0
CB_2	1	1	1	1

In this system, the dependencies on FV1 increase with the addition of each cable, from cable 2 to cable n. Hence, each cable is assigned a unique FV (e.g., FVi for cable i), enabling independent control over the transition of each cable between the two behaviors.

A PREDEFINED FIELD is established for each cable at the initial simulation step, where the FV number is assigned, and the magnitude is set to the value corresponding to the desired behavior at the start of the simulation. The magnitude of the FV defined in this manner can be modified to facilitate the transition between behaviors at later specific simulation steps.