A PRECISION COMPARISON OF LAUE AND BRAGG REFLECTION WITH THE DOUBLE CRYSTAL SPECTROMETER AND THE PHOTOGRAPHIC SPECTROMETER

Thesis by

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PART I. INTRODUCTION.

The fundamental problem underlying the present experimental investigation is one of considerable interest to present day physicists, namely, the explanation of the discrepancy between the measured values of x-ray wave-lengths as obtained by means of diffraction from crystals on the one hand and from ruled gratings on the other. This difference in wave-length measurements amounts to as much as three-tenths of one per cent of the wave-length, and has been repeatedly demonstrated by several observers using various types of apparatus, so that it cannot reasonably be regarded as the result of experimental error. (See references 1 to 8.) The continued existence of this unexplained discrepancy has been the cause of considerable disquietude in the minds of physicists, particularly as it may lead to errors in other measurements, such as those of the fundamental constants e and h.

Consider, for instance, the electronic charge e. The x-ray value of e determined by Bearden[6] and Bäcklin[9] is $4.8055 \pm .003 \times 10^{-10}$ e. s. units, as compared to the oil-drop value[10] of $4.768 \pm .005 \times 10^{-10}$ or the composite value determined by Birge[11], of $4.7668 \pm .0038 \times 10^{-10}$. Ruark[12] has recently shown that by employing the Bearden-Bäcklin value of e and the Shane-Spedding[13] value of e/m, his experimental determinations of the energies of x-ray photo-electrons[8] agree with those computed from x-ray wave-lengths, which may be interpreted as evidence for the correctness of the x-ray value of e. However inconclusive such evidence may be, its existence must be explained before order shall be established in the realm of our knowledge of the fundamental constants. For this purpose it is well to examine the assumptions underlying the evidence in question.

The x-ray value of e is obtained on the assumption of the correctness of the grating wave-lengths, and the consequent elimination of the discrepancy between grating and crystal wave-lengths by assigning to the crystal a lattice constant determined by the comparison of reflection angles in the two cases. This procedure leads, of course, to a value of the interplanar distance different from that obtained from density considerations, unless the value of Avogadro's number employed in the calculation is that determined by the x-ray value of e. It is in this manner that the question of the equality of the lattice constants effective at the surface and in the interior of the reflecting crystal has come to be intimately associated with the determination of the value of the fundamental constants.

As early as 1930, Zwicky[14] had explained the discrepancy between grating and crystal wave-lengths from consideration of his theory of the mosaic structure of crystals[15]. This theory predicted "that the mosaic structure of rock salt, for instance, is represented by a cubic frame-work of planes whose density is about 10 per cent larger than the density of the perfect blocks in between them. As the latter cause the interference of x-rays, their characteristic lattice constant d_o must be introduced into the Bragg formula. d_o is obviously larger than the distance d obtained by assuming that the crystal is of perfectly uniform density throughout." The difference between d and d_o predicted by Zwicky at that time was .3 of one per cent.[14]

Considerations such as these led to this attempt to determine whether the lattice constant effective in internal reflection was accurately equal to that effective in surface reflection. The problem was ultimately attacked by two different methods; the first utilized a double crystal spectrometer which was ready and available at the time the research was begun, the second required the construction of a special x-ray spectrograph. Each of these methods will be discussed separately.

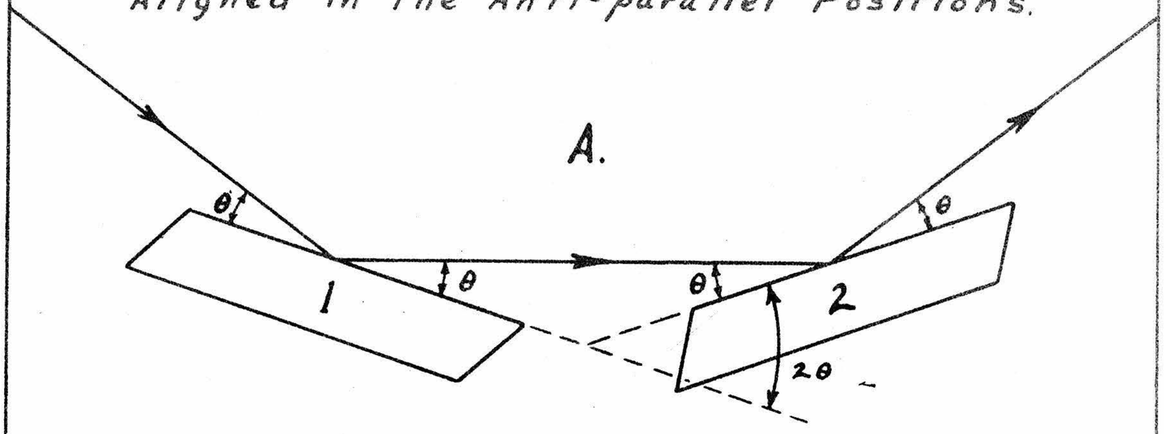
PART II. EXPERIMENTS WITH THE DOUBLE CRYSTAL SPECTROMETER.

A. THEORY OF THE FIRST METHOD OF INVESTIGATION.

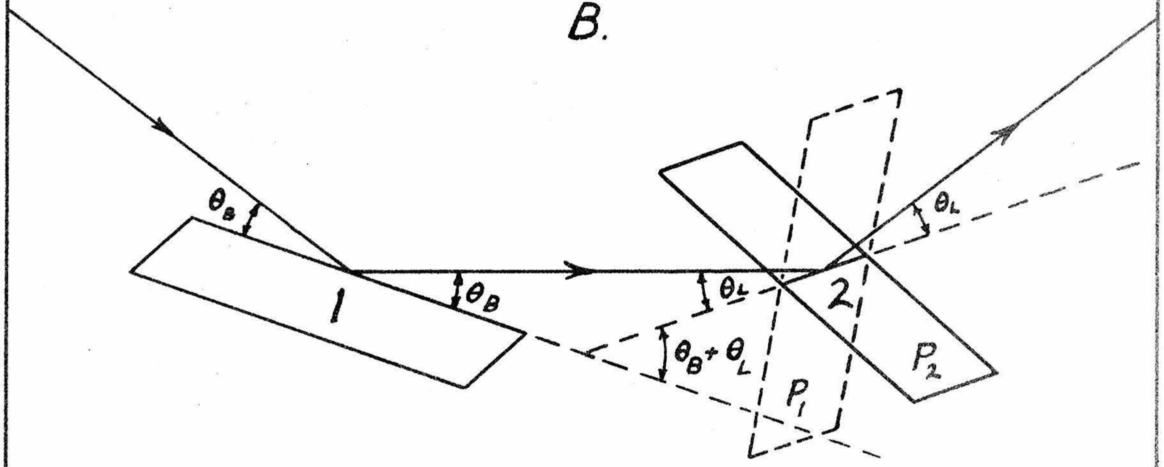
The experimental method first employed to investigate the equality of the grating constants of calcite effective in internal and surface reflection centered about the double crystal spectrometer.[16] In the usual form of this instrument, reflection of the x-ray beam occurs successively from the surface of two crystals of equal grating constants, as shown in Fig. 1 A. In order that a given wave-length component of the x-ray beam shall experience this double reflection, its wave-length λ must be that given by the Bragg equation, $n\lambda = 2\,d \sin\theta$, where θ is half the supplement of the dihedral angle between the reflecting surfaces of the crystals. To adapt this instrument to the problem at hand it was only necessary to replace the second calcite crystal by one so thin that when it was turned to the position shown in Fig. 1 B, the x-rays were not excessively absorbed and scattered but were reflected from the internal 100 planes. With this arrangement, the same restriction of wave-length to that given by the Bragg equation would hold, provided the grating constant effective in the internal reflection from the 100 planes of the second crystal were equal to that effective in the surface reflection from the 100 planes of the first crystal, it being understood, of course, that with the crystals in this position, the angle θ of the Bragg equation is half the supplement of the dihedral angle between the reflecting planes of the two crystals.

However, if the internal grating constant for the 100 planes

Crystals of the Double Spectrometer,
Aligned in the Anti-parallel Positions.



A. Crystals in the Bragg - Bragg Position.
X-rays are reflected successively from
the surfaces of crystals 1 and 2.



B. Crystals in the Bragg - Laue Position
X-rays are reflected from the surface of crystal
1 and from the internal planes of crystal 2.

Figure 1.

of calcite were not equal to the grating constant for the 100 planes effective in the surface reflection, then the double reflection of x-rays of a given wave-length would require a different dihedral angle between the reflecting planes than that which would be required for two surface reflections. A comparison of the dihedral angle for the internal or Laue type of reflection at the second crystal with that for the surface or Bragg reflection should, then, supply the desired information as to the magnitude of the internal grating constant of calcite. In practice, since the available spectrometer was not suited to the measurement of the absolute value of these angles, it was necessary to compare the angular separations of two chosen lines in the Bragg and Laue spectra.

Assuming that there is a difference between the two grating constants, we may proceed to investigate in greater detail the effect of this difference upon the angular separation of any two spectral lines. If the second crystal, initially identical with the first one, and having a grating constant d_B, were to be replaced by another of smaller grating constant d_L, then, in order to permit the double reflection of rays of the same wave-length as before, the second crystal would have to be turned through an angle $\Delta\theta$ with respect to the first crystal, where $\Delta\theta$ may be calculated by means of two Bragg equations, as follows:

$n\lambda = 2\, d_B \sin\theta$, and $n\lambda = 2\, d_L \sin(\theta + \Delta\theta)$.

Hence $\sin(\theta + \Delta\theta) = \frac{d_B}{d_L} \sin\theta$, or

$$\sin\theta \cos\Delta\theta + \cos\theta \sin\Delta\theta = \frac{d_B}{d_L} \sin\theta.$$

Replacing $\cos \Delta\theta$ by 1 and $\sin \Delta\theta$ by $\Delta\theta$ we have

$$\Delta\theta \doteq \left(\frac{d_B}{d_L} - 1\right) \tan \theta$$

If the first crystal were kept fixed then this equation should give to a high degree of accuracy the angle $\Delta\theta$ turned through by the second crystal, but in practice both crystals are rotated, and since the x-ray beam reflected by the first crystal rotates with it, the two crystals need be turned through only half the angle $\Delta\theta$ to again permit the same wave-length to be doubly reflected. This means that a given difference in grating constants should be twice as difficult to detect with a spectrometer of the type used in these experiments as it would be with an instrument on which the first crystal is stationary.

The expression $\frac{1}{2}\left(\frac{d_B}{d_L} - 1\right) \tan \theta$ gives the shift occurring in the angular position of a given spectral line when the second crystal is rotated from the Bragg to the Laue position, but the quantity actually measured in this investigation was the angular separation of two such spectral lines, and consequently it is the difference between two such expressions which will give the anticipated difference $\delta\theta$ in angular separations, as follows:

$$\delta\theta = (1/2)(\Delta\theta_\alpha) - (1/2)(\Delta\theta_\beta)$$

$$\delta\theta = (1/2)\left(\frac{d_B}{d_L} - 1\right)(\tan \theta_\alpha - \tan \theta_\beta)$$

This equation does not take into account the effect of the index of refraction.

It is of interest to employ this equation for $\delta\theta$ to compute the difference in the angular separation of the chosen spectral lines Mo $K\alpha_1$ and β_1 for the Laue and Bragg reflections, on the assumption of a difference of .30 of one per cent between the grating constants d_B and d_L.

Compton's value of θ_α, the Bragg angle for Mo $K\alpha_1$ reflected from calcite[17,30] is 6° 42' 35.9", and for θ_β, the Bragg angle for Mo $K\beta_1$, Larsson[18] obtained the value 5° 58' 42.5". Hence $\tan\theta_\alpha = .11765$ and $\tan\theta_\beta = .10415$, and so $\delta\theta = (1/2)(.0030)(.11765 - .10415) = 2.02 \times 10^{-5}$ radians, or 4.08 seconds of arc. Then since $(\theta_\alpha - \theta_\beta)$ equals approximately 43' 53.5", or 2633.5", it follows that $\delta\theta = .159$ per cent of the quantity being measured in this experiment, namely, the angular separation of $K\alpha_1$ and $K\beta_1$.

The magnitude of the effect of the refractive index for the case of surface reflection can be determined from the equation[19]

$$n\lambda = 2\,d\,\sin\theta\,\left(1 - \frac{1-\mu}{\sin^2\theta}\right),$$

where μ is the refractive index of the crystal for x-rays of wave-length λ. From this it follows that the angular deviation from the angle θ_0 predicted by the simple Bragg equation $n\lambda = 2\,d\,\sin\theta_0$ is[20] $(\theta - \theta_0) = (1-\mu) \div (\sin\theta\cos\theta)$. Using the values of θ_α and θ_β given above, one obtains $(\theta - \theta_0)_\alpha = (1-\mu)_\alpha\ (8.6174)$ and $(\theta - \theta_0)_\beta = (1-\mu)_\beta\ (9.6536)$. The refractive index of calcite has been determined for Mo K_{α_1} but not for Mo K_{β_1}. For $(1-\mu)_\alpha$ Hatley[21] obtained

$(2.03 + .1) \times 10^{-6}$, Compton[22], $(2.10 + .15) \times 10^{-6}$, and Pardue[23], $(2.001 + .009) \times 10^{-6}$. Assuming that both $(1 - \mu)_{\alpha}$ and $(1 - \mu)_{\beta}$ are 2.00×10^{-6}, we obtain for the increase in angular separation of $K\alpha_1$ and $K\beta_1$ due to the influence of the refractive index a value which, for the case of surface reflection, is $(\theta - \theta_0)_{\alpha} - (\theta - \theta_0)_{\beta} = (1.0352)(2.00 \times 10^{-6}) = 2.07 \times 10^{-6}$ radians = .426 seconds of arc.

For the case of internal reflection, the incidence is more nearly normal, and the effect of the refractive index would presumably be much less than that computed above, but even if the effect were entirely absent in this case, the error in the value of $\delta\theta$ introduced by completely neglecting the effect of the refractive index in this experiment would not exceed one-half of .426, or .213, seconds of arc. The factor of one-half is introduced, as in the calculation of $\delta\theta$, because the change from the Laue to the Bragg position occurs only at the second crystal.

An analysis of the effect for the Laue case was completed, however, largely as a matter of curiosity. The resulting value of the deviation from the angle θ_0 given by the simple Bragg equation $n\lambda = 2d \sin \theta_0$ was found to be

$$\theta - \theta_0 = \pm \tan^{-1} \frac{(1 - \mu) \sin 2\alpha}{2 \sin^2 \varphi}$$

where φ is the grazing angle of incidence on the cleavage surface of the crystal, and α is the dihedral angle of calcite[26], 74° 56.5', and the choice of plus or minus sign depends upon whether the second crystal is in the position P_2 or P_1, respectively, of Fig. 1B. For P_1, the position actually used, $\varphi_\alpha = 74^\circ\ 65.5' + 6^\circ\ 42.5' = 81^\circ\ 39'$, and $\varphi_\beta = 74^\circ\ 56.5' + 5^\circ\ 58.5' = 80^\circ\ 55'$. Assuming that $(1 - \mu)_\alpha = (1 - \mu)_\beta = 2.00 \times 10^{-6}$, as above, we obtain for the increase in angular separation of $K\alpha_1$ and $K\beta_1$ the value $(\theta - \theta_0)_\alpha - (\theta - \theta_0)_\beta = - \tan^{-1} (.5125 \times 10^{-6}) + \tan^{-1} (.515 \times 10^{-6}) = -.0005$ seconds of arc. On dividing by two, because the effect occurs at the second crystal only, we obtain finally a decrease in angular separation of $K\alpha_1$ and $K\beta_1$ of .00025 seconds of arc.

Thus it appears that the effect of the index of refraction is utterly negligible in the Laue case, and consequently the total correction to be applied to the observed difference in the angular separations of $K\alpha_1$ and $K\beta_1$ in the two cases, Laue and Bragg, is that for the Bragg case, which was shown to be approximately .213 seconds of arc. This correction is but .0084 of one per cent of the 2631.5 seconds between $K\alpha_1$ and $K\beta_1$, and is well beyond the limits of accuracy of the experiment. For the sake of precision, however, we may state that it increases the anticipated difference $\delta\theta$ in angular separations to 4.29 seconds of arc.

B. APPARATUS.

The apparatus employed in the first investigation of the problem consisted of a) the double crystal spectrometer, b) the intensity measuring apparatus, c) the x-ray tube, d) the source of high voltage, and c) appropriate slits and shields.

The particular spectrometer and ionization chamber used for this work were designed by Dr. J. W. M. DuMond and have been fully described by him in the literature.[16] In Figs. 2 and 3 the spectrometer is shown in place between the x-ray tube and ionization chamber. In Fig. 2 the crystals are seen to be aligned in the Bragg-Laue position; the first or entry crystal is in the Bragg position, the second or exit crystal is in the Laue position. The dihedral angle between the crystals is adjusted by turning the screw at the opposite end of the spectrometer so as to move equally and in opposite directions the two steel arms to which the crystal turntables are clamped. It follows that the angular position of the crystals as read from the cylindrical drum attached to the screw is actually the sine of the angle by which the position of the crystals differs from some arbitrarily chosen zero setting.

The intensity measuring system consisted of the ionization chamber[16] which was mechanically supported by a Hoffmann electrometer. The x-ray tube was of the water-cooled type having a molybdenum target; at first a Müller Metalix tube wrapped with leaded rubber was employed, but this was later replaced by a Victor tube supported in the leaded rubber housing shown in Fig. 3. Both the x-ray tube and the spectro-

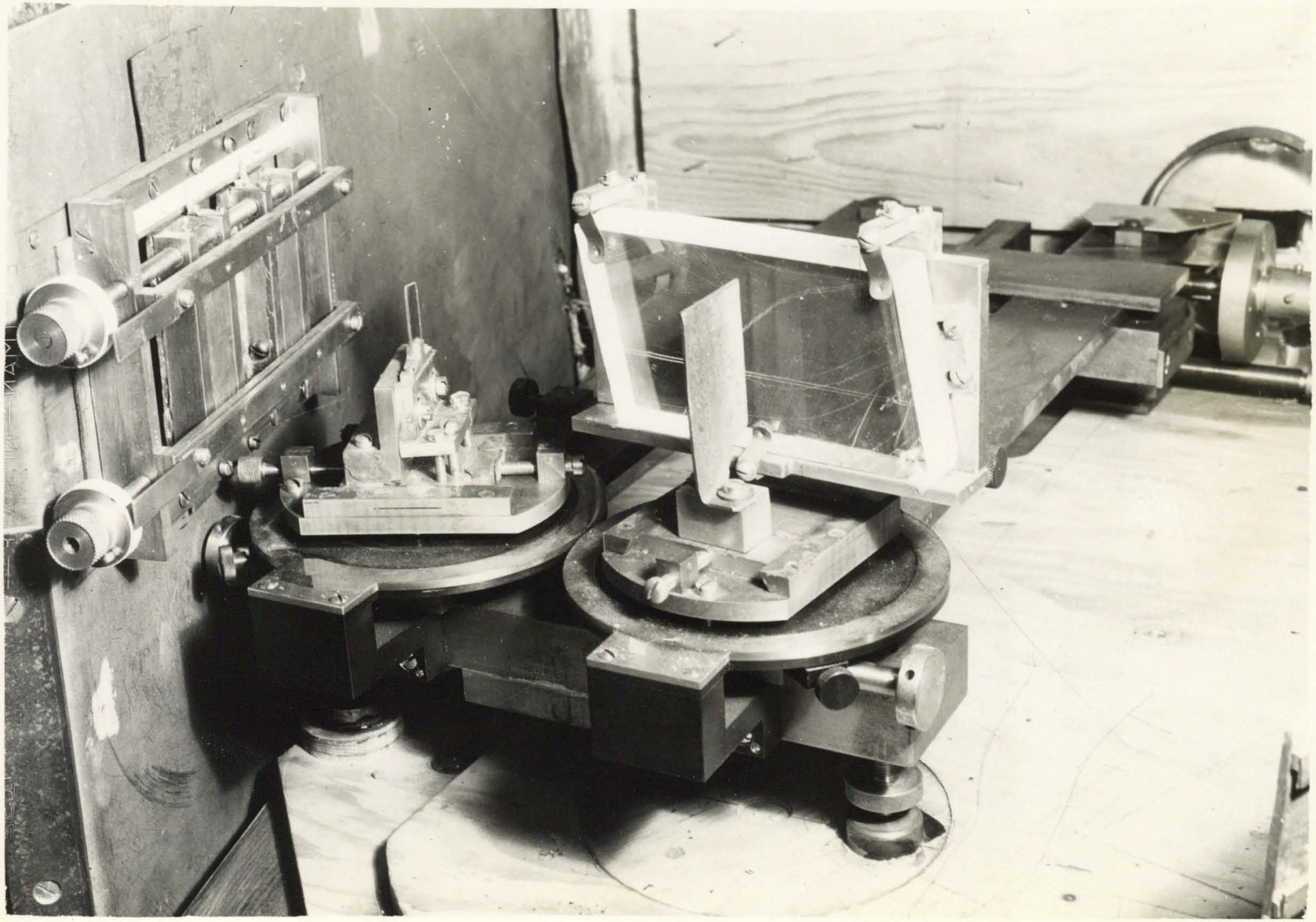


Figure 2.

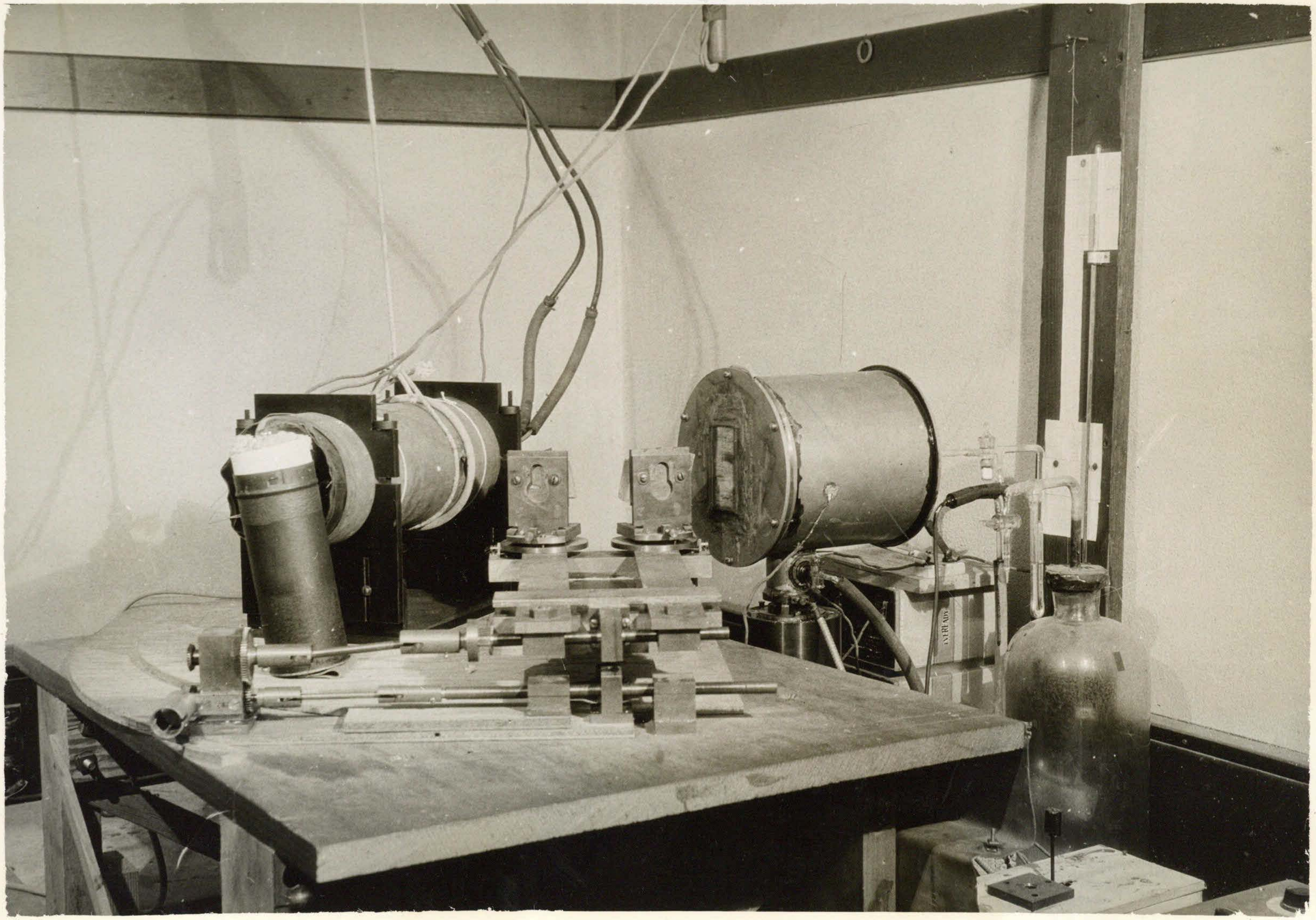


Figure 3.

meter were supported on rotating table-tops of ply-wood, which were so geared to the screw of the spectrometer that when the crystals were rotated through a given angle, the spectrometer rotated through twice this angle and the x-ray tube through four times this angle. This arrangement permitted the ionization chamber and electrometer to remain stationary.

The source of high voltage for the x-ray tube consisted essentially of a transformer, condensers, and two Kenetrons, arranged to give full-wave rectified and filtered current at potentials up to 50 kilovolts. Voltage regulation was accomplished by means of an auto-transformer in the primary circuit. As both terminals of the x-ray tube were at high potential, the cooling water was conducted to and from the tube through a glass water-column, and the storage batteries supplying the filament current to the tube were supported on a wooden insulating stand. Filament current was closely regulated by means of a mercury rheostat of special design[24], so as to accurately maintain the anode current of the tube at any desired value. The milliammeter which measured the anode current was mounted on the wooden insulating stand and was observed through a telescope.

As the slits and shielding were changed from time to time, their description may most conveniently be included in the following account of the experimental procedure.

C. EXPERIMENTAL PROCEDURE.

1. Initial Experiments with a Cleaved Crystal.

The investigational work was begun in January, 1933, when the first attempt was made to locate, by the ionization chamber method, the x-ray spectral lines reflected in the Laue manner from the internal planes of the second or exit crystal of the double crystal spectrometer. A theoretical analysis of this type of reflection was made by Dr. J. W. M. DuMond with the assistance of Mr. Carl Thiele, in an effort to determine the optimum thickness of calcite for which the two opposing features of reflection and absorption would be so proportioned as to give the maximum intensity of the reflected beam. This analysis indicated an optimum thickness given by

$$x_0 = \frac{1}{2\epsilon} \log \frac{2\epsilon + \alpha}{\alpha}$$

where ϵ and α are the coefficients of extinction and absorption, respectively.

For the case of transmission through calcite of radiation of wave-length 700 X. U., we have the following approximate values: $2\epsilon = 4000$, $\alpha = \mu = 39.4$, $\log \frac{2\epsilon + \alpha}{\alpha} = 4.6$, and therefore $x_o = .014$ mm. By means of the same theory it was shown that for a crystal 1/2 mm. thick the reflected intensity would be reduced relative to that for the optimum thickness in the ratio .357 to 1.

Later it was realized that the theory underlying the equation given above holds true only for ideally imperfect crystals, in which the phase relations of radiation selectively reflected from different small elements of the crystal is random, so that it is permissible to add intensities. In such a crystal the largest element within which perfect phase relationships could exist is small compared to the rate of variation of the intensities of incident and reflected beams. Calcite probably does not meet this requirement, but is more nearly of the perfect type.

The theoretical optimum thickness of the reflecting crystal was so small that an effort was made to obtain as thin as possible a specimen of cleaved calcite for use in the experimental investigation. The crystal finally chosen from among a number that had been cleaved with a pocketknife measured 20. x 14. x 2.5 mm. One of the 20. x 2.5 mm. faces was then silvered by means of the Brashear process, in order to facilitate the alignment of the crystal on the x-ray spectrometer. Into a brass plate $2\frac{1}{2}$ x 2 x $\frac{1}{4}$ inches in size was cut a window slightly smaller than the crystal. This mounting plate and the crystal were warmed, beeswax was melted along the edges of the window, the crystal was placed over the window, and then all were allowed to cool. The brass plate was then screwed to the adjustable mounting on the second or exit turn-table of the double crystal spectrometer.

For the number one crystal a large, well-cleaved calcite measuring $4\frac{1}{2}$ x $2\frac{1}{2}$ x $\frac{1}{2}$ inches was used. Its mounting consisted of a quarter-inch brass plate extending the entire length and breadth of the crystal, and fitted with two vertical screws upon which the crystal could rest. A piece of black paper was inserted behind the crystal in order to avoid the reflection of light from the brass plate during the process of alignment. This paper contained three holes, two at the bottom, near the supporting screws, and one at the top, in the center, through which the beeswax used to fasten the crystal in place might make contact between crystal and plate. The mounting plate and crystal were placed in a horizontal position, and heated, then allowed to cool, and the supporting screws tightened until they touched the surface of the crystal. The plate was then screwed to the mounting of the number one or entry turn-table. The two crystals were then ready for alignment in the Bragg-Laue anti-parallel position shown in Fig. 1B.

The orientation of the crystals was accomplished by means of the method perfected by DuMond and Hoyt, and described by them in the literature.[14] In accordance with the requirements of this method of alignment, the large, number one calcite was so orientated, by means of an optical spectrometer fitted with a Gauss eyepiece, that its largest face was parallel to the axis of rotation of the turn-table and made a dihedral angle of 6° 20' with the plane of the two axes of rotation. The angle 6° 20' was chosen as being approximately midway between the calcite glancing angles for the $K\alpha_1$ and β_1 lines of the molybdenum spectrum, namely 6° 42' 36" and 5° 58' 43". Thus the first crystal was in a position to permit Bragg reflection of each of these lines in turn by proper adjustment of the spectrometer dial. Then, by means of an obvious extension of this same method of alignment, so as to make use of the reflection from the silvered face of the small crystal, this second calcite was so aligned that both its largest face and the smaller, silvered one were parallel to the axis of rotation, in order that reflection of x-rays might be obtained in the usual Bragg manner from the largest face, as well as in the Laue manner from the internal atomic planes parallel to the silvered face. In the final position of the small crystal, a reflection from its silvered face was obtained in the Gauss eyepiece of the telescope, which was set, as in the case of the largest crystal, at 6° 20'. This left the small crystal in the Laue position at the beginning of the x-ray work, with the possibility of turning it readily into the Bragg position at any time, by merely loosening the clamps on the turn-table.

The x-ray spectrometer was then set in place on the movable plywood table-top between the x-ray tube and ionization chamber, as shown in Fig. 3. The spectrometer was levelled, and its weight was removed from the rotating table-top by means of one of the counterweights previously mentioned. The lead shielding used at this time included a) a large screen between x-ray tube and spectrometer, provided with a small rectangular opening for the passage of the x-rays, b) a vertical knife-edge set on the turn-table of the large crystal at a distance of about 4 mm. from its face, c) lead foil slits on the mounting of the small crystal, so as to limit the effective size of the brass window, and d) a snout leading to e) the slits in f) the ionization chamber shield. In addition, there was protective shielding for the safety of the operators, consisting of three layers of leaded rubber wrapped directly around the x-ray tube, a glass tube wrapped with leaded rubber, leading from the window of the x-ray tube to that in shield a), and a large lead screen covering the entire front of the table, save for a small window through which the graduated drum of the spectrometer could be read with the aid of a thick lens fastened over this opening.

With the spectrometer adjusted as described, an attempt was made to locate the Mo Kα lines, but without success. With the x-ray tube operating at three milliamperes and 37.5 kilovolts, the range from 6° 0' to 6° 50' was carefully covered, taking readings every half minute of arc, but these readings of the time for ten-centimeter deflections of the ionization chamber beam indicated only a flat, featureless background such as might be attributed to scattered and fluorescent radiation coming from the second crystal.

This result led to an attack on the problem of the nature of the radiation comprising this heavy background, conducted by means of an experimental comparison between the hardness or penetrating power of this radiation and that of the beam incident on the first crystal. It was found that screens of either molybdenum or of aluminum reduced the intensity of the beam entering the ionization chamber but little more when placed in the entry beam (that incident on the first crystal) than when placed in the exit beam (that leaving the second crystal). Now the entry beam was presumably composed principally of continuous radiation, much of which would be more penetrating than the scattered molybdenum radiation or fluorescent calcium radiation comprising the exit beam, but in view of the fact that in the case of the Bragg reflection at the first crystal, the intensity of the scattered beam is negligible in comparison with that of the reflected beam, it follows that the only effective component of the entry beam was that capable of experiencing the Bragg reflection, namely, the characteristic radiation of molybdenum. It is also essential to note that the mass absorption coefficient[25] of molybdenum for its own $K\alpha$ radiation is 18.8 and for the $K\alpha$ radiation of calcium it is 17.2, - - nearly the same. For aluminum, however, the case is quite different. The mass absorption coefficient of aluminum for the $K\alpha$ radiation of molybdenum is 5.21 and for the $K\alpha$ radiation of calcium it is 433. Hence it was concluded that the similar results of the absorption tests obtained when using screens of molybdenum and of aluminum indicated that the troublesome background consisted principally of scattered molybdenum radiation. Moreover, if there had been any very considerable proportion of fluorescent radiation in the exit beam, the absorbing power of the aluminum screen would have been much greater in this position than when placed in the entry beam, a situation which is in direct contradiction to the experimental results.

This absorption test indicated that it would not be possible to employ a selective filter to eliminate the undesirable background. There seemed no alternative but to search again for the spectral lines. The range from 6° 40' 0" to 6° 45' 0" was covered in steps of 10", but without disclosing any lines. The small crystal was then rotated into the Bragg position, where no difficulty was experienced in locating both the $K\alpha$ and $K\beta$ lines. The failure to locate these lines in the Laue position by the use of the ionization chamber, in the face of later successes by means of the photographic method of preliminary adjustment, clearly demonstrated the superiority of the latter procedure.

At this time, however, the problem was attacked from another angle, namely the production of thinner crystals by etching.

2. Experiments with an Etched Crystal.

In order to obtain a calcite crystal more nearly approaching in thinness the theoretical ideal, it was decided to etch away one surface of a crystal with acid and thus obtain a specimen having one untouched face for use in Bragg reflection and also the thinnesss desirable for the Laue reflection. It was necessary, however, to keep at least one of the smaller faces of the crystal intact, to enable one to obtain a reflection by which to align the crystal on the spectrometer. It was then decided to preserve all four of these smaller surfaces intact, by painting them and one of the large faces with melted beeswax. The center of the single uncovered face was then eaten away by a minute jet of hydrochloric acid which flowed from a glass capillary tube at the end of a rubber hose that supplied acid from an elevated container. The result finally obtained was a calcite about half a millimeter thick, supported by a crystalline frame measuring roughly 30. x 25. x 3. mm. One of the 30. x 3. mm. faces was then silvered, and the crystal mounted with beeswax on the spectrometer, as before. The entire procedure of aligning the crystals and adjusting the spectrometer in place was carried out just as described for the preceding case. The first attempt to locate spectral lines in the Laue position was unsuccessful, but on a second occasion, when the operating conditions effecting the x-ray tube were maintained constant by an assistant, the $K\alpha_1$ and $K\alpha_2$

spectral lines were readily found within ten seconds of arc from their anticipated positions on the spectrometer dial.

The next problem was to improve the contrast ratio, by which is meant the ratio of the intensity of x-rays at the peak of some arbitrarily chosen line to the intensity at some point on the background. The initial adjustment of lead shields was such as to yield a contrast ratio for $K\alpha_1$ of but 1.7 to 1. By trial-and-error adjustments of the cheeks of the slit in front of the ionization chamber, the contrast ratio was increased to 7 to 1. A slightly wider slit, 4 mm. in width, such as would allow for a possible motion of the beam in shifting the spectrometer and x-ray tube from the $K\alpha_1$ to the $K\beta_1$ position, gave a contrast ratio of 6.1 to 1. With this adjustment it was then possible to definitely locate the $K\beta_1$ line. Finally, the contrast ratio for $K\alpha_1$ was increased to 13 to 1 by selecting the optimum separation between the vertical knife-edge and the face of the large crystal, which proved to be 3.4 mm.

Before attempting the determination of the exact angular separation of the lines $K\alpha_1$ and $K\beta_1$ as measured on the graduated drum of the spectrometer, it was deemed advisable to make a few preliminary runs over the peaks of these lines in order to determine whether they maintained a constant position on the graduated drum during successive runs. The procedure followed in taking these preliminary data was essentially the same as that employed in making later runs, as described below, save that no background readings were taken. The resulting plots of the peaks of $K\alpha_1$ and $K\beta_1$ were interesting in that both peaks showed peculiar irregularities of contour, similar to those seen in Figs. 4 and 5. It was believed that this phenomenon might be the manifestation of a mosaic structure in the small calcite crystal, and that as such it might prove of greater interest than the problem under investigation. Consequently it was decided to make complete and detailed runs with as great accuracy as possible over the entirety of the $K\alpha_{1,2}$ doublet and the $K\beta_{1,3}$ doublet, in order to study the structure of these lines.

The method of taking the readings in these and all succeeding measurements made with the double crystal spectrometer was essentially as follows. Preliminary readings taken at the peak of each line revealed the amount of electrometer binant voltage necessary in order to obtain a peak reading of from 25 to 30 seconds for a ten-centimeter deflection of the electrometer beam. The process of taking the data was then begun with a higher value of binant voltage, such as would give a practicable rate of deflection on the background, though it was still necessary to take readings for deflections of as little as two centimeters on the lowest parts of the background, in order to reduce the time required to a reasonable length. On ascending and descending the peak, at the point at which it was necessary to change binant voltage, readings were always taken on both voltages, in order to make possible the reduction of the background readings to the scale of the peak readings. Two readings of deflection times were taken at each point on the spectral lines, and the mean of these two readings is recorded in the tables that follow. In the later work, at all points at which the time for a ten-centimeter deflection exceeded 60 or 70 seconds one reading was deemed sufficient. The reciprocals of these deflection times provided a direct measure of the x-ray intensity, and these reciprocals plotted against angular position, as measured on the calibrated drum of the spectrometer screw, yielded the profiles of the spectral lines which are shown in Figs. 4 to 8, following the tables of data.

At the completion of these runs in the Laue position, the small, number-two calcite was rotated into the Bragg position, where no difficulty was experienced in locating the spectral lines. The changes made in the lead shielding were a) the introduction of a vertical knife-edge opposite the second crystal and at a distance of 2 mm. from it, and b) the covering of the edge of this crystal on the side toward the first crystal by means of a vertical strip of lead. Readings of x-ray intensity were then taken at a sufficient number of points to make possible the detailed outlining of the profiles of the $K\alpha_{1,2}$ and $K\beta_{1,3}$ lines, as shown in Figs. 6 and 7.

Finally, data were taken for the rocking curves of the crystals, with the number-two crystal set first in the Laue position and then in the Bragg position. Mo $K\alpha_1$ radiation was used in both cases. Both crystals were accurately aligned, by means of a microscope, so that the axes of rotation passed through their respective faces. For the Laue case, the unetched face of the second crystal was used as the entry face. The resulting curve is that shown in Fig. 8A. For the Bragg case, two rocking curves were obtained, one for which the entry face of the second crystal was shielded, as previously mentioned, and one for which the shielding was removed. These curves are shown in Figs. 8B and 8C. This completed the experimental investigation of the behavior of the etched crystal in the reflection of x-rays.

TABLE ONE.

DATA FOR THE PROFILE OF MO $K\alpha_{1,2}$ IN THE BRAGG-LAUE POSITION, AS PLOTTED IN FIGURE 4.

First crystal unetched and in the Bragg position.

Second crystal etched and in the Laue position.

X-ray tube operating at 7.0 milliamperes and 32. kilovolts.

Voltage on electrometer binants, 17.5 volts.

Date and time, 9 p.m. April 7, 1933, to 6 a.m. April 8.

Readings of angular position assume the central zero position of the arms of the spectrometer to be 6° 20'.

Readings of deflection time are for ten-centimeter deflections of the light beam of the Hoffmann electrometer.

Angular Position	Deflection Time	Reciprocal of Time
6° 40' 0"	431.5 sec.	2.32 x 10^{-3}
5"	450.65	2.22
10"	404.6	2.48
15"	406.35	2.46
20"	405.8	2.46
25"	366.75	2.72
30"	396.	2.52
35"	402.5	2.48
40"	395	2.53
45"	373.7	2.68
50"	355.	2.82
55"	344.	2.91
6° 41' 0"	315.5	3.17
5"	307.	3.26
10"	282.5	3.55
15"	261.	3.83
20"	244.	4.10
25"	213.	4.70
30"	175.5	5.71
35"	131.1	7.64
40"	86.8	11.51

Table One, Continued

Angular Position	Deflection Time	Reciprocal of Time
6° 41' 45"	49.15 sec.	20.35 x 10^{-3}
48"	37.75	26.5
50"	34.85	28.7
52"	33.4	29.95
54"	32.75	30.55
56"	33.05	30.25
58"	32.2	31.05
6° 42' 0"	31.0	32.25
2"	30.3	33.0
4"	29.7	33.7
6"	29.42	34.0
8"	28.9	34.6
10"	28.2	35.45
12"	27.4	36.5
14"	25.75	38.9
16"	23.5	42.5
18"	21.9	45.6
20.25"	19.9	50.2
22"	19.25	52.0
24"	19.23	52.0
26"	19.45	51.45
28.2"	21.1	47.4
30"	23.4	42.7
32"	29.66	33.65
34"	36.5	27.4
36"	46.7	21.4
38.2"	60.9	16.43
40"	70.8	14.11
42"	86.15	11.60
44"	102.0	9.80
46"	119.0	8.40
49"	134.4	7.44
52"	161.0	6.21
56"	192.6	5.21
6° 43' 0"	211.6	4.73
5"	233.0	4.29
10"	259.2	3.86
15"	267.5	3.74
20.2"	284.0	3.52
25"	299.5	3.34
30"	278.0	3.60
35.1"	291.5	3.43
40"	279.0	3.58
45"	271.0	3.69
50"	256.5	3.90
55"	211.5	4.74

Table One, Continued

Angular Position	Deflection Time	Reciprocal of Time
6° 44' 0"	185.5 sec.	5.40 x 10^{-3}
5"	125.25	7.99
10"	75.5	13.24
12"	67.85	14.75
14"	61.4	16.3
16"	60.2	16.6
18"	61.0	16.4
20"	60.2	16.6
22"	58.3	17.15
24"	56.7	17.6
26"	55.25	18.1
28"	54.5	18.35
30"	54.2	18.45
32"	53.4	18.73
36"	51.4	19.45
38"	47.4	21.1
40"	43.95	22.75
42"	40.75	24.55
44.6"	36.7	27.2
46"	35.8	27.9
48"	35.95	27.8
50"	36.85	27.1
52"	40.5	24.93
54"	44.65	22.4
56"	56.25	17.75
58.8"	74.5	13.4
6° 45' 0"	84.6	11.8
2"	102.6	9.75
4"	120.6	8.30
6"	149.2	6.71
10"	190.2	5.25
15"	246.0	4.07
20"	285.0	3.51
25"	319.0	3.14
30"	339.5	2.945
35"	368.5	2.71
40"	372.0	2.69

TABLE TWO.

DATA FOR THE PROFILE OF MO $K\beta_{1,3}$ IN THE BRAGG-LAUE POSITION, AS PLOTTED IN FIGURE 5.

First crystal unetched and in the Bragg position.

Second crystal etched and in the Laue position.

X-ray tube operating at 7.0 milliamperes and 32. kilovolts.

Voltage on electrometer binants, 20.5 volts.

Time and date, 2:30 to 6:30 p.m., April 9, 1933.

Angular Position	Deflection Time	Reciprocal of Time
5° 55' 0"	274.0 sec.	3.65×10^{-3}
20"	269.5	3.71
40"	280.0	3.57
5° 56' 0"	281.0	3.56
20"	270.5	3.70
40"	265.0	3.77
5° 57' 0"	262.5	3.81
20"	240.0	4.16
40"	162.7	6.15
51"	77.5	12.9
57"	70.0	14.3
5° 58' 0"	67.85	14.74
2.8"	63.4	15.76
4.0"	60.5	16.54
7.5"	54.2	18.43
9.1"	52.35	19.12
10.9"	51.6	19.4
12.0"	50.45	19.8
5° 58' 13.5"	49.5	20.2
15.4"	48.3	20.7
17.0"	47.9	20.86
18.4"	45.4	22.2
20.0"	43.1	23.2
22.8"	39.65	25.2
23.9"	38.1	26.25
26.0"	37.0	27.0
28.8"	38.1	26.25
30.0"	39.7	25.2
31.1"	41.35	24.2

Table Two, Continued.

Angular Position	Deflection Time	Reciprocal of Time
5° 58' 32.6"	43.8 sec.	22.85 x 10^{-3}
33.4"	45.65	21.9
35.1"	50.6	19.75
36.5"	55.6	17.98
38.1"	60.7	16.47
39.4"	62.6	15.97
42.0"	65.45	15.30
45.1"	66.5	15.04
47.9"	70.9	14.16
50.8"	82.1	12.18
54.0"	104.1	9.61
56.8"	127.0	8.13
59.8"	157.4	6.35
5° 59' 4.1"	190.0	5.26
8.0"	204.5	4.885
12.0"	236.0	4.24
20.5"	252.0	3.966
30.0"	252.5	3.960
40.3"	271.0	3.69
50.0"	278.0	3.60
6° 0' 0"	295.5	3.385
30.5"	304.0	3.29
6° 1' 0"	280.0	3.57
6° 2' 0"	317.0	3.106

TABLE THREE.

DATA FOR THE PROFILE OF MO $K\beta_{1,3}$ IN THE BRAGG-BRAGG POSITION AS PLOTTED IN FIGURE 6.

First crystal unetched and in the Bragg position.

Second crystal etched, but aligned with the unetched face in position for Bragg reflection.

X-ray tube operating at 7.0 milliamperes and 32. kilovolts.

Voltage on electrometer binants, 20.5 volts.

Time and date, 8 to 11 p.m., April 12, 1933.

Angular Position	Deflection Time	Reciprocal of Time
5° 55' 0"	201.3 sec.	4.97×10^{-3}
30"	202.8	4.93
5° 56' 0"	195.5	5.11
30"	182.0	5.49
5° 57' 0"	176.0	5.67
30"	149.0	6.70
48"	106.3	9.40
5° 58' 0"	64.2	15.6
4"	47.0	21.3
8"	37.9	26.4
12"	32.5	30.8
16"	29.8	33.6
18"	28.9	34.6
20"	28.7	34.8
22"	27.9	35.8
24"	27.3	36.6
26"	25.8	38.8
28"	25.2	39.7
30"	24.2	41.3
32"	23.3	42.9
34"	23.1	43.3
36"	23.1	43.3
38"	23.0	43.5
40"	22.8	43.8
42"	23.0	43.5
44"	23.0	43.5

Table Three, Continued

Angular Position	Deflection Time	Reciprocal of Time
5° 58' 44"	23.0 sec.	43.5×10^{-3}
46"	22.4	44.6
48"	22.9	43.6
50"	23.3	42.9
52"	23.8	42.0
54"	24.5	40.9
56"	26.8	37.3
58"	30.0	33.3
5° 59' 0"	33.6	29.8
2"	35.6	28.1
4"	38.2	26.2
6"	41.0	24.4
8"	43.3	23.1
10"	44.5	22.45
12"	47.0	21.25
14"	52.8	18.9
16"	58.8	17.0
18"	68.1	14.67
20"	78.6	12.70
22"	86.7	11.51
24"	100.6	9.94
28"	119.5	8.35
32"	134.8	7.41
40"	156.0	6.40
50"	172.7	5.80
6° 0' 10"	190.5	5.25
40"	199.5	5.01
6° 1' 30"	200.0	5.00

TABLE FOUR

DATA FOR THE PROFILE OF MO K$\alpha_{1,2}$ IN THE BRAGG-BRAGG POSITION AS PLOTTED IN FIGURE 7.

An immediate continuation of the readings taken for the profile of Mo K$\beta_{1,3}$. All conditions unaltered, except that the binant voltage, initially 20.5 volts, was changed to 14.5 volts at 6° 40' 40", and again, to 9.5 volts, at 6° 42' 4". The reciprocals of deflection times have been corrected so as to conform to the uniform scale used in plotting the data.

Time and date, 11 p.m., April 12, 1933, to 7 a.m., April 13.

Angular Position	Deflection Time	Reciprocal of Time
6° 40' 0"	152.5 sec.	6.56×10^{-3}
30"	127.0	7.88
6° 41' 0"	99.0	10.1
30"	64.52	15.5
40"	49.8[1]	20.1[1]
40"	100.6	20.1
50"	61.5	32.8
55"	44.5	45.4
6° 42' 0"	27.1	74.5
2"	23.3	86.5
4"	20.5[2]	98.5[2]
4"	40.67	98.5
6"	36.8	109.0
8"	33.9	118.5
10"	31.5	127.5
12"	30.8	130.3

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[1] The reciprocals beyond this point have been corrected by a multiplying factor of (100.6 - 49.8) = 2.02.

[2] The reciprocals beyond this point have been corrected by a multiplying factor of 2.02 x (40.67 - 20.5) = 4.01.

Table Four, Continued

Angular Position	Deflection Time	Reciprocal of Time
6° 42' 14"	30.6 sec.	131.3 $\times 10^{-3}$
16"	29.75	135.0
18"	30.35	132.3
20"	30.4	132.1
22"	30.63	131.0
24"	30.4	132.1
26"	29.9	134.1
28"	29.5	136.2
30"	29.0	138.6
32"	28.6	140.8
34"	28.3	142.0
36"	27.8	144.8
38"	27.2	147.9
40"	26.6	151.0
42"	26.47	151.6
44"	26.8	150.0
46"	27.2	148.0
48"	29.6	136.0
50.5"	35.2	114.0
52"	39.0	103.1
54"	47.8	84.0
56"	58.45	68.6
58"	74.7	53.6
6° 43' 0"	90.6	44.4
5"	127.3	31.6
10"	177.0	22.8
15"	210.0	19.2
20.5"	261.0	15.4
25"	279.3	14.4
30"	304.0	13.25
35"	320.2	12.6
40"	358.0	11.24
45"	346.0	11.63
50"	324.0	12.43
55"	308.0	13.08
6° 44' 0"	301.0	13.38
5"	277.0	14.53
10"	245.0	16.41
15"	207.0	19.46
20"	152.0	26.50
25"	107.0	37.6
30"	74.0	54.4
32"	67.3	59.7
34"	61.4	65.6
36"	59.0	68.2

Table Four, Continued

Angular Position	Deflection Time	Reciprocal of Time
6° 44' 38"	57.0 sec.	70.6×10^{-3}
40"	56.5	71.4
42"	56.7	71.0
44"	57.4	70.1
46"	57.9	69.6
48"	57.45	70.1
50"	57.1	70.5
52"	56.3	71.5
54"	55.9	72.0
56"	55.13	73.0
58"	54.6	73.6
6° 45' 0"	53.9	74.7
2"	53.2	75.5
4"	52.3	77.0
6"	51.9	77.6
8"	51.4	78.5
10"	52.1	77.3
12"	53.2	75.6
14"	58.2	69.2
16"	65.6	61.4
18"	78.1	51.5
20"	94.2	42.8
25"	154.3	26.0
30"	225.2	17.85
40"	356.0	11.3
6° 46' 0"	554.0	7.27

TABLE FIVE

DATA FOR THE ROCKING CURVE IN THE BRAGG-LAUE POSITION, AS PLOTTED IN FIGURE 8A.

First crystal unetched and in the Bragg position.

Second crystal etched and in the Laue position, with internal reflecting planes parallel to the reflecting face of the first crystal.

X-ray tube operating at 7.0 milliamperes and 32. kilovolts.

Voltage on electrometer binants, 20.5 volts for background and 9.5 volts for peak.

Time and date, 3 p.m., April 20, 1933.

Angular Position	Deflection Time	Reciprocal of Time
0' 0"	1562. sec.	.64 x 10^{-3}
35"	1610.	.62
1' 0"	1640.	.61
30"	1500.	.67
2' 0"	1490.	.67
30"	1330.	.75
3' 0"	857.	1.17
5"	709.	1.14
10"	405.2	2.48
15"	107.0	9.35
18"	84.4	11.84
20"	79.7	12.55
22"	74.8	13.40
24"	73.25	13.65
26"	66.47	15.05
28"	50.9	19.63
30"	50.75	19.70
32"	54.7	18.30
34"	57.1	17.50
36"	58.45	17.12
38"	54.5	18.36
40"	41.3	24.2
42"	32.0	31.25
44"	28.4	35.2
46"	23.2	43.15

Table Five, Continued

Angular Position	Deflection Time	Reciprocal of Time
3' 48"	20.3 sec.	49.25×10^{-3}
50"	24.45	40.9
52"	33.9	29.5
53"	43.2	23.2
54"	61.4	16.3
55"	84.2	11.9
56.5	121.6	8.23
58"	174.0	5.75
4' 0"	243.7	4.10
5"	417.5	2.39
10"	597.0	1.68
20"	926.0	1.08
40"	1290.	.78
5' 0"	1540.	.65
30"	1600.	.63
6' 0"	1595.	.63

TABLE SIX

DATA FOR THE FIRST ROCKING CURVE IN THE BRAGG-BRAGG POSITION, AS PLOTTED IN FIGURE 8B.

First crystal unetched and in the Bragg position.

Second crystal etched, but aligned with the unetched face parallel to the reflecting face of the first crystal.

Lead shielding removed from the entry face of the second crystal.

X-ray tube operating at 6.0 milliamperes and 25. kilovolts.

Voltage on electrometer binants, 17.5 volts for the background and 6.5 volts for the peak.

Time and date, 3 p.m., April 22, 1933.

Angular Position		Deflection Time	Reciprocal of Time
0'	0.0"	2660. sec.	.38 x 10^{-3}
	30.0"	2630.	.38
1'	0.0"	2310.	.43
	30.0	1890.	.53
2'	0.0"	1203.	.83
	20.0"	780.	1.28
	25.0"	715.	1.40
	30.0"	643.	1.55
	35.0"	542.	1.84
	40.0"	431.	2.32
	45.0"	336.	2.98
	48.0"	276.	3.62
	50.0"	241.	4.15
	51.0"	222.	4.50
	52.2"	197.	5.08
	53.0"	184.	5.44
	54.3"	162.3	6.15
	56.0"	133.3	7.50
	57.0"	116.5	8.60
	58.0"	101.5	9.85
	59.0"	88.0	11.4
3'	0.0"	72.0	13.9
	1.0"	57.2	17.5

Table Six, Continued

Angular Position	Deflection Time	Reciprocal of Time
2' 53.0"	184. sec.	5.44×10^{-3}
54.3"	162.3	6.15
56.0"	133.3	7.50
57.0"	116.5	8.60
58.0"	101.5	9.85
59.0"	88.0	11.4
3' 0.0"	72.0	13.9
1.0"	57.2	17.5
2.0"	47.9	20.9
3.0"	40.1	24.95
4.5"	35.6	28.1
6.0"	34.1	29.3
7.0"	33.9	29.5
8.0"	34.75	28.8
9.2"	36.0	27.8
10.5"	36.9	27.1
12.2"	39.3	25.45
13.1"	40.2	24.85
14.5"	40.6	24.65
16.0"	41.7	24.0
17.0"	42.35	23.6
18.5"	44.4	22.5
20.0"	49.35	20.3
21.0"	54.2	18.45
22.0"	58.55	17.1
24.0"	63.05	15.85
25.8"	60.36	16.6
27.9"	63.55	15.75
29.8"	68.0	14.7
32.0"	74.1	13.5
34.0"	83.3	12.0
36.0"	96.3	10.4
38.0"	107.0	9.35
40.0"	128.0	7.8
42.0"	157.4	6.35
44.0"	188.3	5.31
46.0"	215.2	4.65
48.0"	229.0	4.36
50.0"	252.0	3.97
55.0"	308.0	3.25
4' 0.0"	468.0	2.14
5.0"	801.0	1.25
10.5"	1088.	.92
40.0"	2090.	.48
5' 0.0"	2610.	.38

TABLE SEVEN

DATA FOR THE SECOND ROCKING CURVE IN THE BRAGG-BRAGG POSITION, AS PLOTTED IN FIGURE 8C.

All conditions the same as for the first rocking curve, of April 12, except that the small, silvered face of the second crystal was, in the present case, shielded by a vertical strip of lead.

Time and date, 7:30 to 11 p.m., April 29, 1933.

Angular Position		Deflection Time	Reciprocal of Time
0'	0"	601. sec.	1.667×10^{-3}
	20"	447.	2.236
	40"	239.	4.18
	50"	91.8	10.9
	55"	61.4	16.3
1'	0"	41.9	23.85
	2"	29.6	33.8
	4"	23.1	43.3
	6"	21.05	47.5
	7"	20.45	48.9
	8.5"	20.25	49.5
	10"	20.6	48.5
	12"	21.7	46.1
	14"	22.7	44.0
	16"	22.8	43.9
	18"	23.4	42.7
	20"	25.5	39.2
	22"	29.4	34.0
	23"	31.6	31.6
	24"	32.2	31.1
	25"	31.2	32.0
	26"	29.7	33.6
	27"	28.9	34.6
	28"	28.0	35.7
	29"	28.25	35.4
	30"	28.4	35.2
	33"	28.85	34.7
	36"	29.8	33.6
	39"	29.15	43.3
	40"	28.3	35.4

Table Seven, Continued.

Angular Position		Deflection Time	Reciprocal of Time
1'	42"	27.7 sec.	36.1×10^{-3}
	44"	26.9	37.2
	46"	26.5	37.8
	48"	26.7	37.5
	50"	31.75	31.5
	52"	53.15	18.8
	54"	84.0	11.9
	56"	115.7	8.65
	58"	145.5	6.87
2'	1"	198.0	5.05
	5"	289.0	3.46
	10"	480.0	2.08

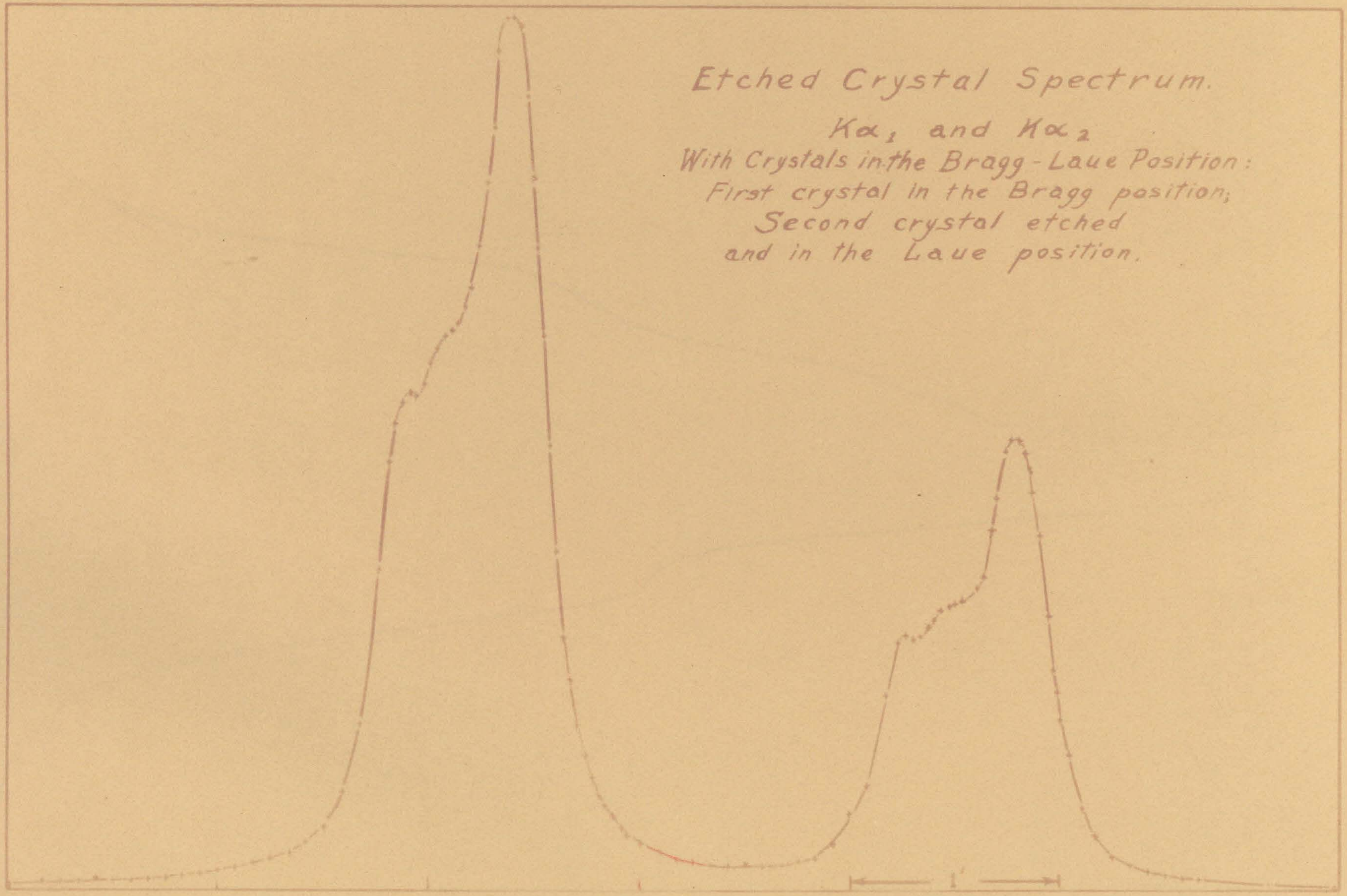

Etched Crystal Spectrum.
$K\alpha_1$ and $K\alpha_2$
With Crystals in the Bragg - Laue Position:
First crystal in the Bragg position;
Second crystal etched
and in the Laue position.
1'

Figure 4.

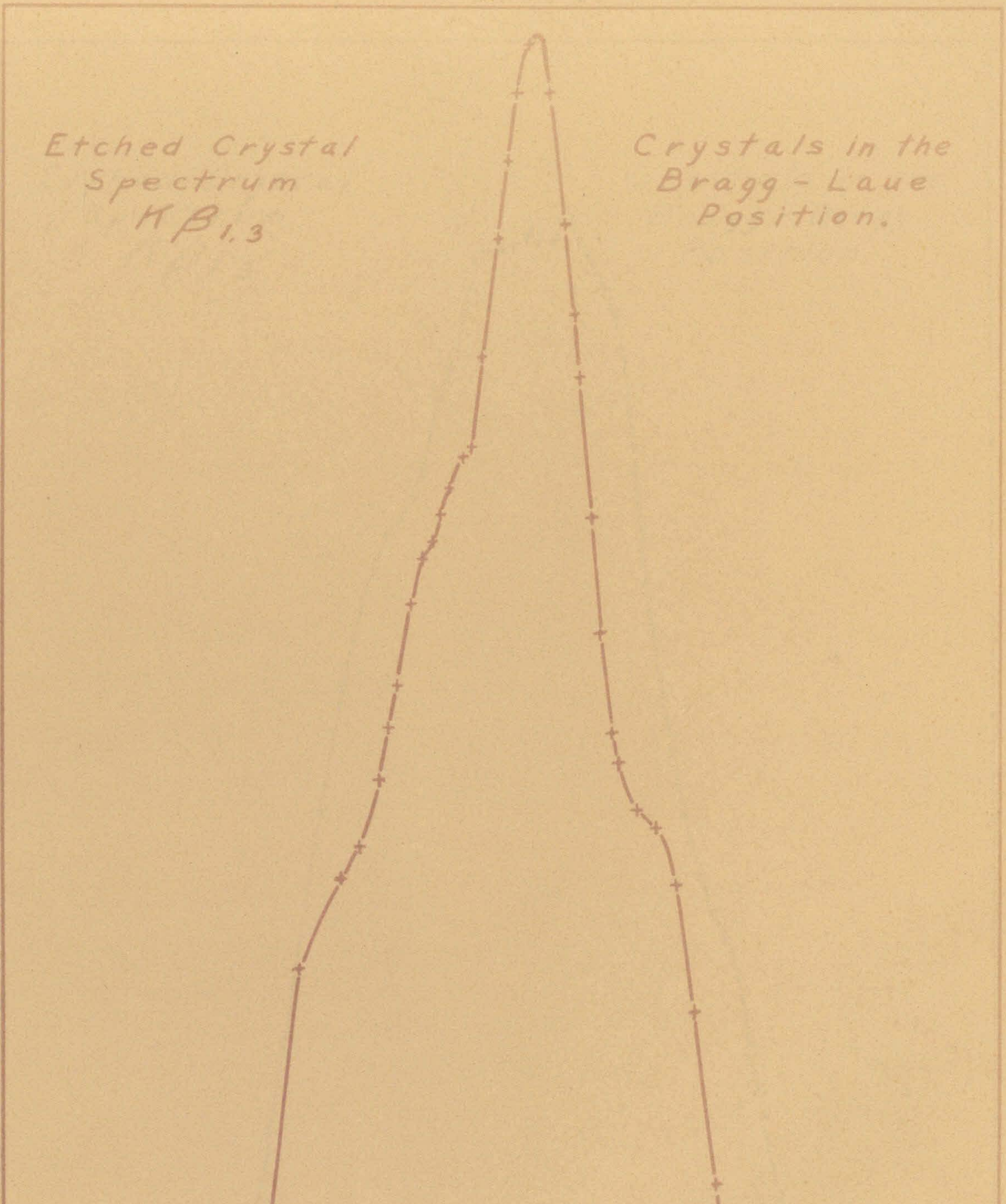

Etched Crystal
Spectrum
$K\beta_{1,3}$
Crystals in the
Bragg - Laue
Position.
←10″→

Figure 5.

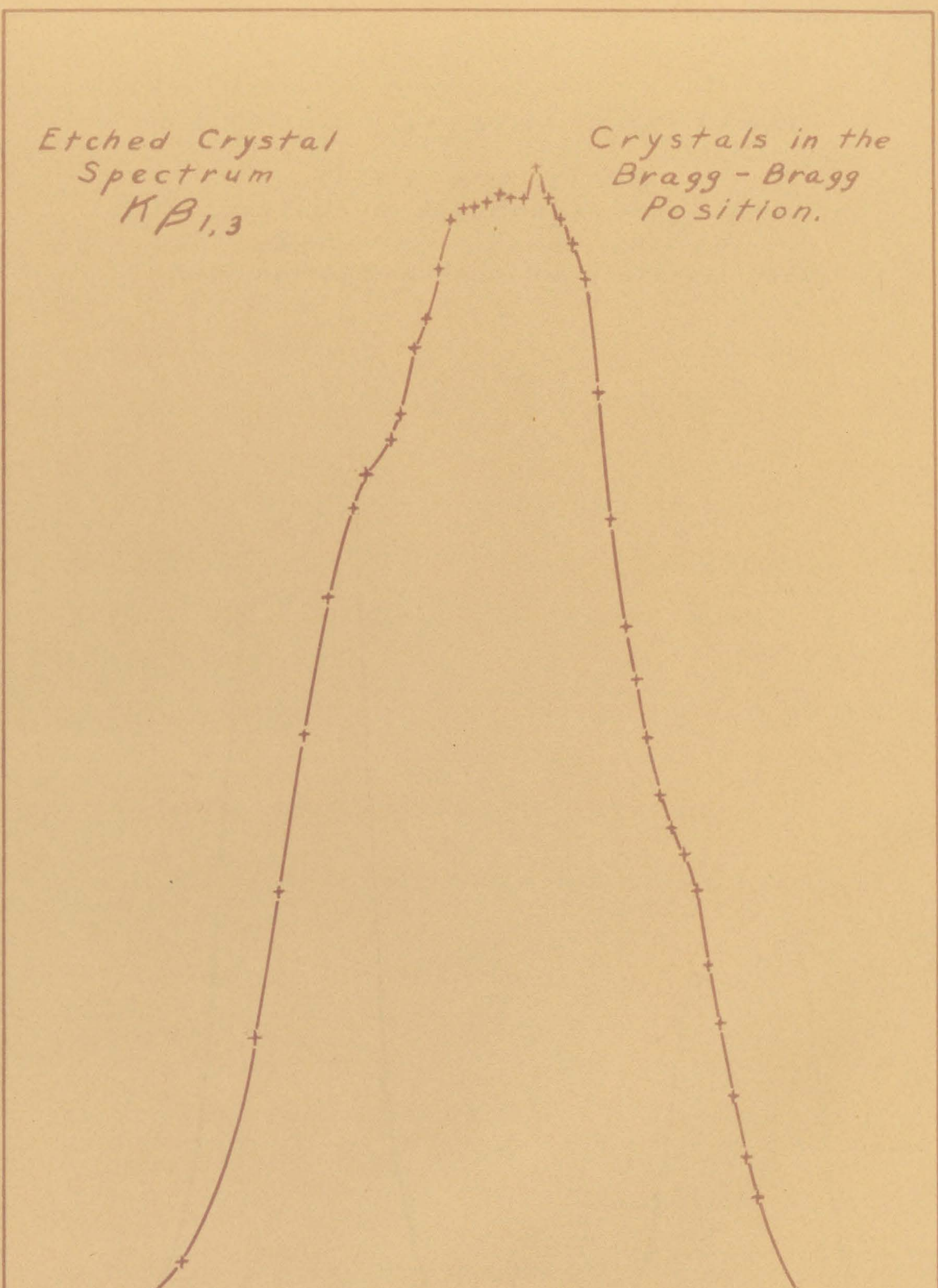
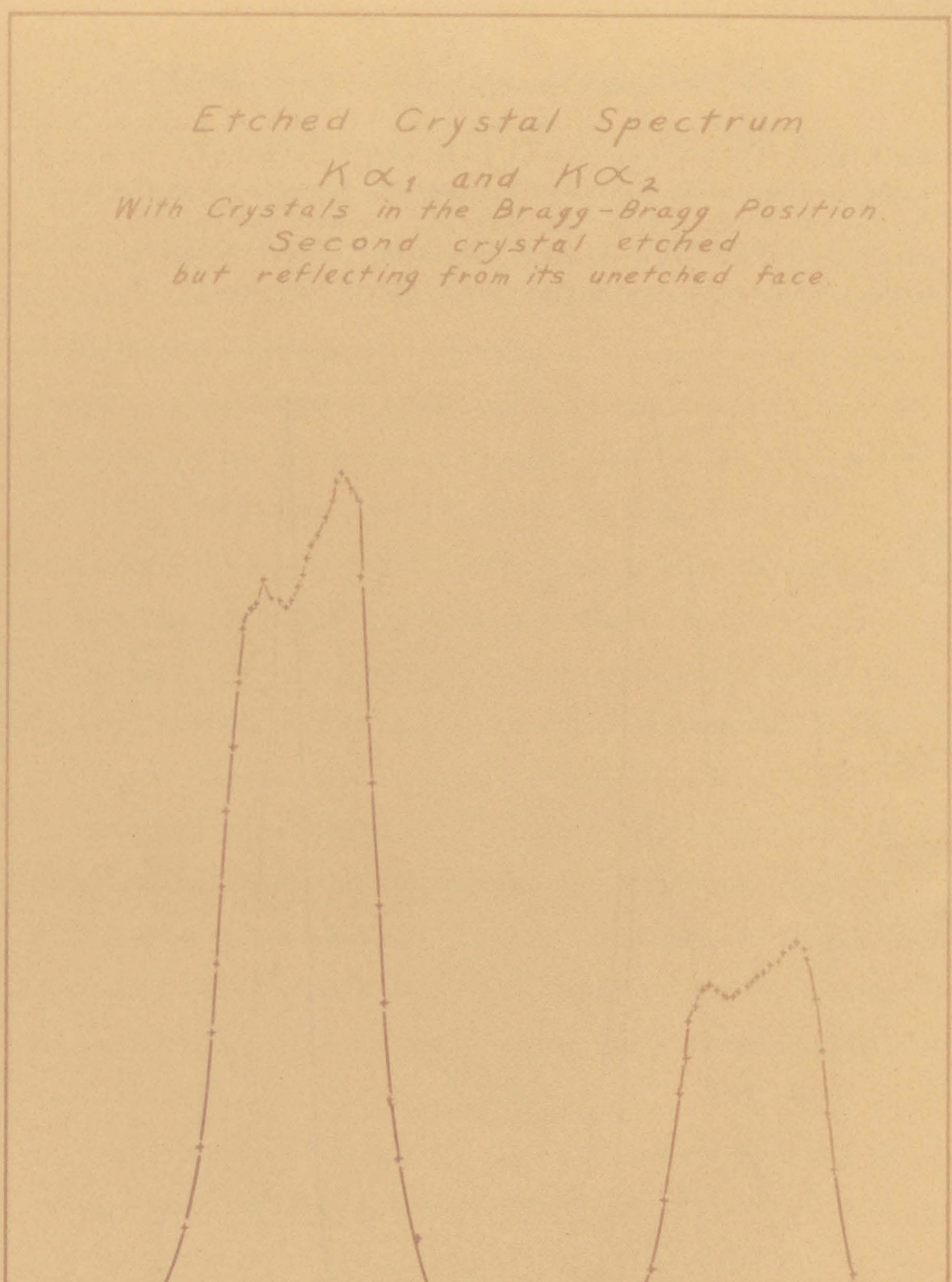

Etched Crystal
Spectrum
$K\beta_{1,3}$
Crystals in the
Bragg - Bragg
Position.
10"

Figure 6.



Etched Crystal Spectrum
$K\alpha_1$ and $K\alpha_2$
With Crystals in the Bragg-Bragg Position.
Second crystal etched
but reflecting from its unetched face.

Figure 7.

Etched Crystal Rocking Curves.

A. Crystals in the Bragg-Laue (1,-1) position.
B. and C. Crystals in the Bragg-Bragg (1,-1) position.
B. Second crystal etched but reflecting from unetched face.
C. Same position of crystals as in B. Entry edge of second crystal shielded from x-rays. Slits readjusted to insure reflection from entire face of second crystal.

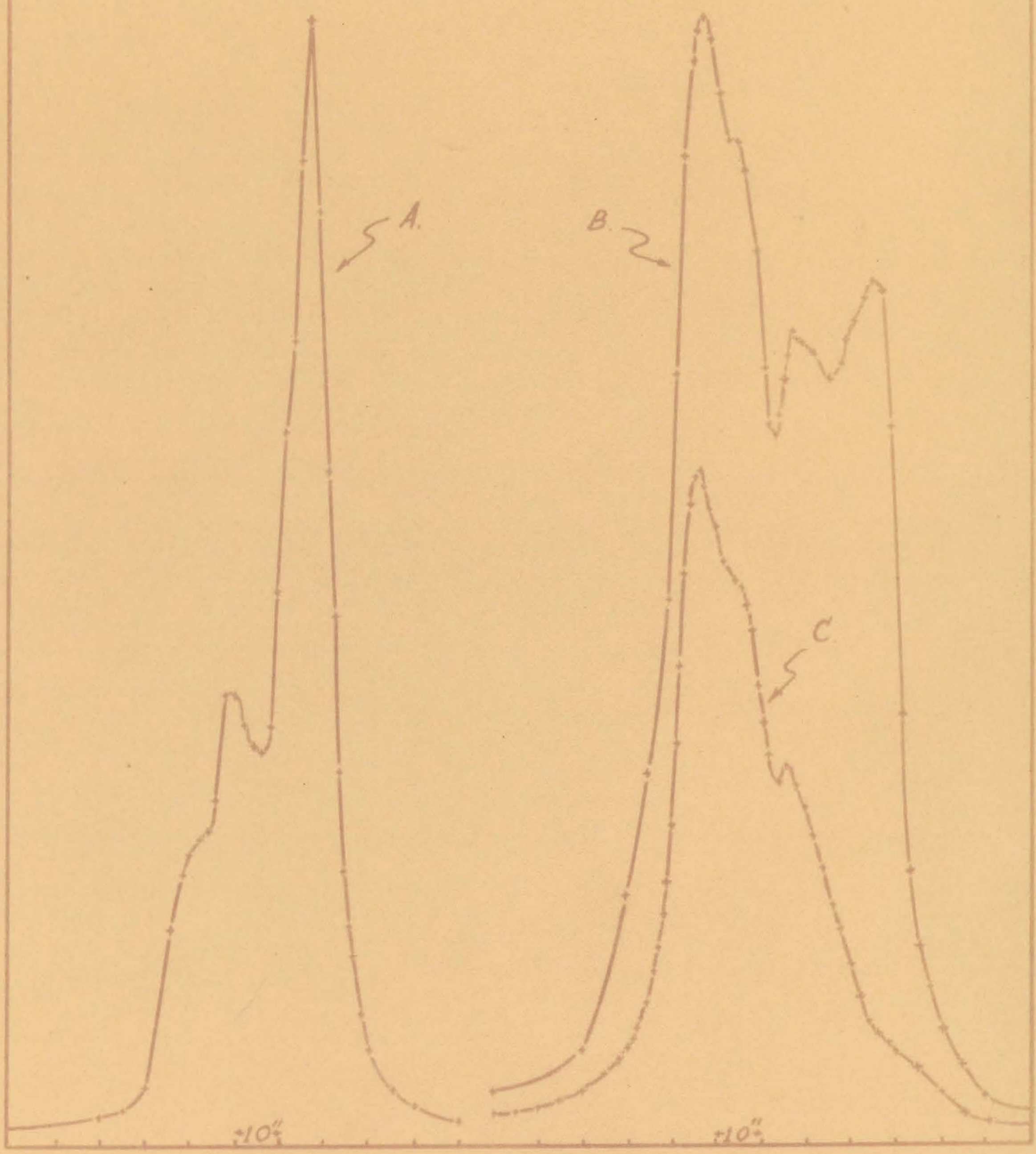


Figure 8.

3. Experiments with a Thinner Cleaved Crystal.

During the discussion of the results of the experimental investigation of the etched crystal, it was learned that Mr. Julius Pearson had developed to such a degree the ability to cleave calcite crystals as to be able to obtain specimens roughly one-third as thick as the cleaved crystal used in our first investigation. From among several of these thinner crystals, one measuring .932 x 15. x 25. mm. was chosen for use as the second or exit crystal on the spectrometer. One of the 25 mm. edges was silvered, and the crystal was then mounted on a block of brass, 1/4 inch thick, by means of melted beeswax extending the full length of the lower 15 mm. edge.

This crystal and the same large crystal previously used as the first or entry crystal were aligned on the spectrometer by the usual optical method. With this pair of crystals set in the Bragg-Laue position, no difficulty was experienced in locating the Mo Kα and β spectral lines. After adjusting the various lead screens so as to improve the contrast ratio, readings were taken for the plotting of the profiles of Mo K$\alpha_{1,2}$ and K$\beta_{1,3}$. These profiles were found to be without any of the irregularities peculiar to the curves obtained with the etched crystal.

The crystals were then set in the Bragg-Bragg position, and the adjustments and readings made as before. Sufficient data were taken to enable the plotting of a number of curves, each showing the peaks of Mo Kα_1 and β_1. There was found to be considerable variation in the angular separation of these lines, and various steps were under-taken in an effort to eliminate this difficulty.

In the course of the search for the cause of the inconsistency of the readings, the small calcite was accidentally knocked off its mounting, thus concluding the research work for the academic year 1932-33. During the summer new angle-scales were engraved for the turn-tables of the spectrometer. These scales, shown in Fig. 2, were engraved on rings of monel metal which were inlaid in the brass disks beneath the crystals. The scales were divided into degrees, with vernier readings to two minutes of arc. Tests made by the engraver indicated that the scales were accurate to one minute of arc, though it is to be understood that this statement has no bearing whatever upon the accuracy of the results of this experiment, as these angle-scales were of use merely as a convenience in the process of orientating the crystals and locating the spectral lines, when changing from the Laue to the Bragg position, and vice versa.

Another improvement made during the summer of 1933 was the lubrication of the ply-wood table-tops. These rotating supports for the x-ray tube and the spectrometer were suspected of binding, so as to prevent the x-ray tube and spectrometer from following the motion of the spectrometer-screw and angle-drum as closely as might be desired. Here again it is to be understood that the smoothness and accuracy of these adjustments are not to be regarded as a measure of the precision with which the angular separation of the spectral lines could be measured by means of the spectrometer. This precision is determined primarily by the reproducibility of the settings of the dihedral angle between the crystals. Only in the extreme case in which the x-ray tube and spectrometer were so far out of line as to cause part of the x-ray beam to be absorbed by the lead shielding or otherwise deflected from its normal path into the ionization chamber could the accuracy with which the tube and spectrometer follow the motion of the angle-drum be expected to effect the results of this experiment. In order to eliminate the possibility of any such extreme variations in angular position of the tube and spectrometer, the table-tops were lubricated with powdered graphite. This treatment caused the mechanism to run much more smoothly than before.

Another improvement in apparatus made at this time was the construction of a convenient and readily adjustable form of slit to be placed in front of the ionization chamber. The jaws of the original slit were lead strips held in place by the pressure of machine screws. The jaws of the improved slit were constructed of lead strips mounted on brass blocks which were drilled and tapped for the passage of two threaded rods of steel. By turning knobs attached to these rods, as shown in Fig. 2, the jaws could be moved separately into any desired position with a certainty of better than .01 inch. The ease with which this adjustment could be made greatly facilitated the alignment of the slit and shields by the photographic method, as described below.

On August 21, 1933, the spectrometer was re-aligned, using the same pair of crystals that were used in the last experiment. Both crystals were set in the Bragg position at first, in order to facilitate the adjustment of the lead shielding. In the course of these adjustments the electrometer was observed to behave in a peculiar manner. The rate of deflection for a number of background readings taken successively would decrease steadily, which was interpreted as being caused by leakage of charge over the surface of the insulators supporting the electrometer needle and the ion collector.[16] In an effort to remedy this difficulty, the electrometer was re-evacuated, and the methyl-bromide and the drying-agent in the ionization chamber were renewed. Then it was discovered that the empire cloth insulation of the ionization chamber[16] had broken down under the action of the methyl-bromide, and showed plainly the signs of an electric arc. This experience led to the replacement of the empire cloth by a ring of micanite, and as a precautionary measure a Littel fuse was connected in series with the batteries supplying the collecting potential, to prevent their destruction in case of another short-circuit in the ionization chamber. The electrometer, however, continued to behave capriciously. At times the light beam was observed to move in a direction opposite to that for a normal deflection. Finally it was discovered that there was a loose connection inside the Littel fuse. With the insertion of a new fuse the intensity measuring apparatus was again in working order.

After the adjustment of the shielding and the location of the spectral lines had been accomplished in the Bragg-Bragg position, the second crystal was turned to the Laue position. The spectral lines were readily located, but it soon became evident that their position on the angle-drum was changing from day to day, at a more or less steady rate of 2 1/2 seconds of arc per day. This gradual drift in position continued in spite of all efforts to eliminate it by the tightening of the various clamping and mounting screws on the spectrometer, and to discover its cause by placing melted wax on the various joints suspected of slipping. Finally it was decided that the difficulty must lay in the mounting of the crystals, and that there was no alternative but to attempt to remedy the trouble by remounting both crystals.

When the large crystal was removed from the brass mounting-plate it was discovered that during the months when the air in the laboratory had been cooled with water, the black paper between the crystal and mounting-plate had absorbed sufficient moisture to cause it to become corrugated. It was believed that the paper, in swelling, might have gradually turned the crystal in its mounting, and thereby shifted the apparent angular position of the spectral lines.

In remounting the large crystal, the arrangement of springs and pins shown in Fig. 2 was employed. The three drops of melted beeswax of the first mounting were replaced by steel pins protruding through the mounting-plate, two pins at the sides, near the bottom, and one at the center, near the top. The crystal was held firmly against these pins by bronze springs, one opposite each pin.

It was perhaps unfortunate that the small crystal was remounted; certainly the choice of this same method of mounting for the small crystal was an unhappy one. Three small steel balls were set solidly in a brass plate, along the edge of a window slightly smaller than the crystal, and opposite these ball bearings were three bronze springs, each tipped with a hemispherical boss. The adjustment of the position of the point at which each spring pressed against the crystal was made by placing a small piece of glass, of the same thickness as the crystal, between each ball and spring in turn, and shifting the spring until the glass remained parallel to the plate when the latter was gently tapped. It was hoped that by this means the alignment would be sufficiently good to insure that there would be no warping of the crystal by unbalanced pressure of the springs.

It was during the process of aligning the spectrometer with the crystals mounted in the manner just described that the photographic method of adjusting the shields and slits was first tried. With the small crystal in the Laue position, a photographic film was placed beside the ionization chamber slits in such a position as to be exposed to the beam of x-rays reflected from the first crystal and passing undeviated through the second. The resulting photograph, Fig. 9A, showed that the reflected beam was well centered with respect to the window in the mounting of the second crystal. The effective size of this window was then reduced by means of a lead screen mounted on the turntable of the second crystal and provided with an opening resembling a bay window, so as to fit close to the crystal without touching it. The position of this bay window was adjusted by means of photographs of the kind reproduced in Fig. 9B. This particular film showed that the singly reflected beam passed to the left of the center of the opening, as seen from the ionization chamber, so that the doubly reflected beam, also visible on this film, might be expected to pass reasonably near the center, as desired. During the exposure of the film shown in

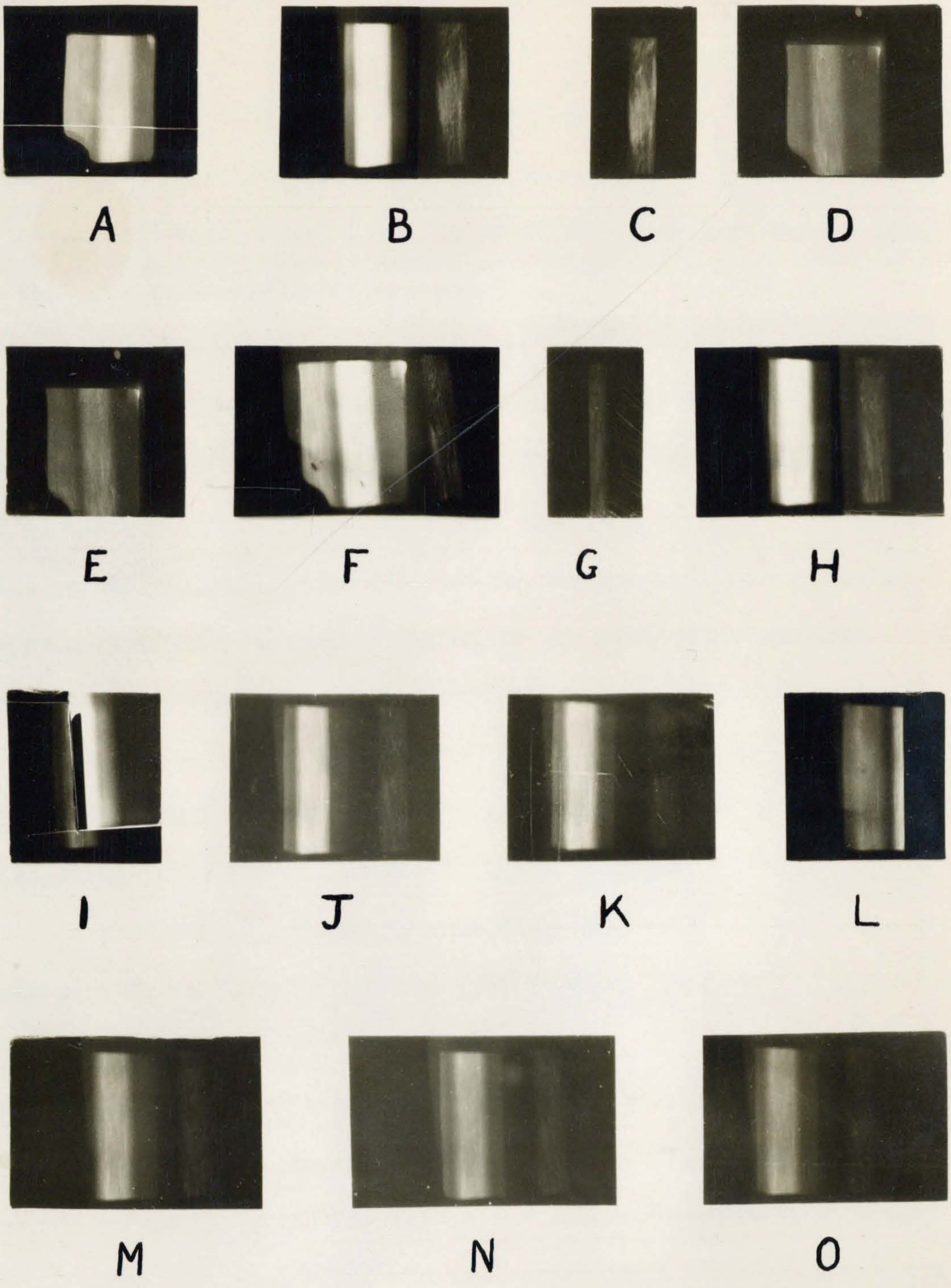


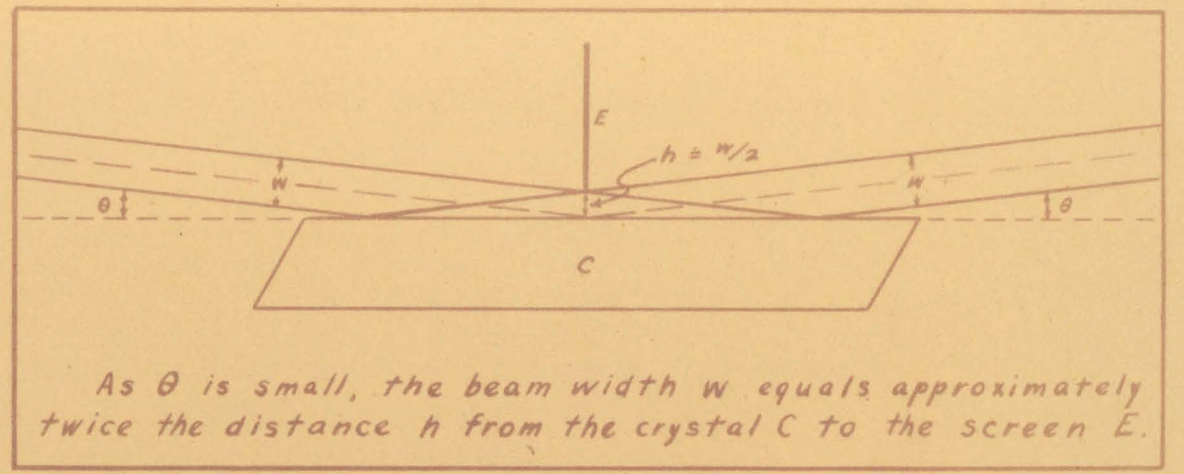
Figure 9.

Fig. 9B, another film placed behind the ionization chamber slit was also exposed. One of the jaws of the slit was purposely set in such a position as to cut into the doubly reflected beam, as shown in Fig. 9C, in order that the position of the beam relative to the slit might be revealed. Then, knowing the width of the beam, the correct position of the slits was readily determined.

A knowledge of the width of the beam was also useful in the adjustment of the vertical knife-edge facing the first crystal. The central position of the knife-edge relative to the beam was determined by taking a series of measurements of the x-ray intensity as the knife-edge was moved parallel to the face of the crystal, then plotting a curve of intensity against position of the knife-edge, and choosing that position which seemed most central on this curve. Then the knife-edge was placed at a distance from the face of the crystal slightly greater than half the width of the x-ray beam, in order that the entire beam might pass through, as shown in the following illustration, Fig. 10.

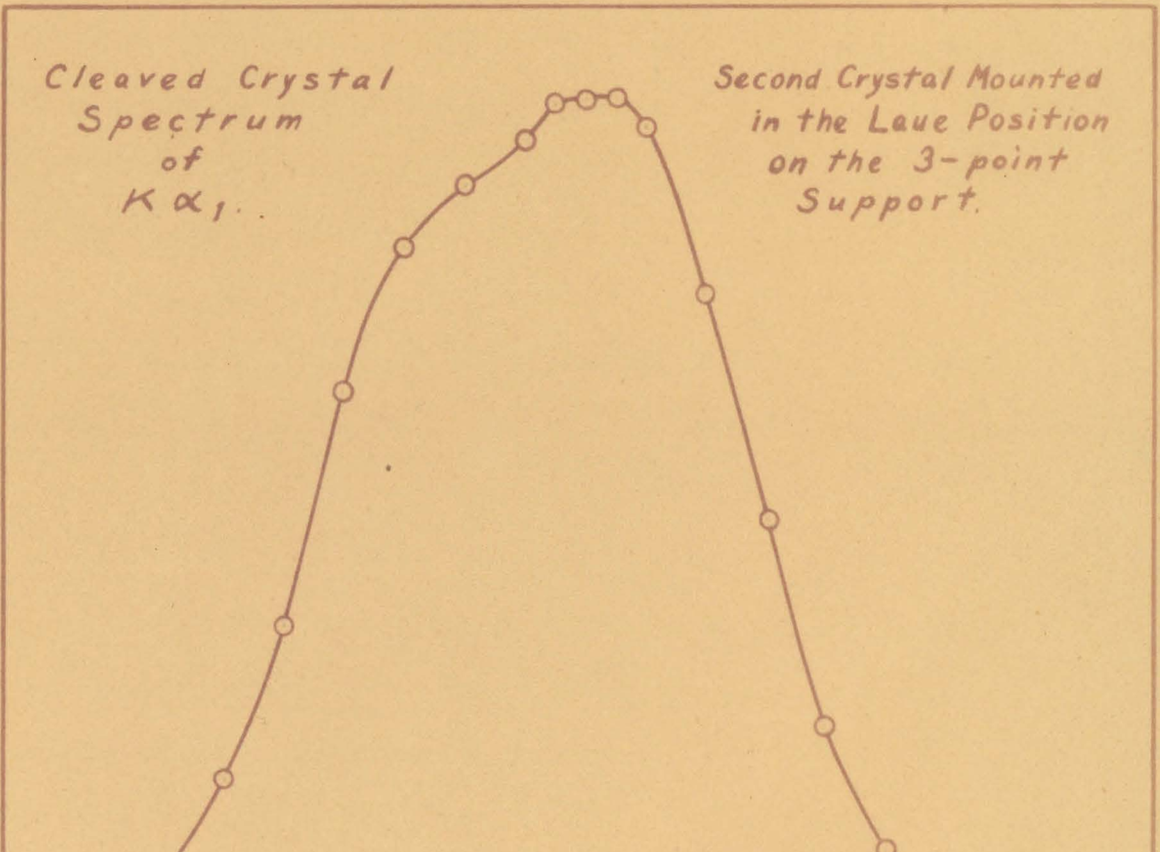
When the adjustment of shielding and slits had been completed, data were taken for plotting the profile of Mo $K\alpha_1$ for the Bragg-Laue position. The resulting curve, shown as Fig. 11, was decidedly asymmetrical, and in view of the precautions taken in adjusting the slits and shields, it was decided that this asymmetry must be attributed to warping of the small crystal by the action of the three-point mounting.

The next step taken was the remounting of the small crystal in a support designed to eliminate the possibility of warping the crystal. This support was simply a block of brass, soldered to the brass plate



As θ is small, the beam width w equals approximately twice the distance h from the crystal C to the screen E.

Figure 10.



Cleaved Crystal Spectrum of $K\alpha_1$.

Second Crystal Mounted in the Laue Position on the 3-point Support.

Figure 11.

of the discarded three-point mounting at a point just below the window, and provided with a vertical slot in which the crystal was fastened with beeswax. As the block extended over only one-third of the width of the crystal, it was believed that no considerable warping of the crystal would occur.

When the crystals had been re-aligned in the Bragg-Laue position, a photograph, shown as Fig. 9D, disclosed the fact that the reflected beam was not centered with respect to the second crystal. The x-ray tube was then moved through a small angle relative to the spectrometer, to shift the reflected beam to the central position, as seen in Figs. 9E and F. Then the films of the type shown in Figs. 9G and H, taken simultaneously and with the bay window shield in place at the second crystal, determined the correct location of this shield and of the ionization chamber slit.

The small crystal was then turned to the Bragg position, and a photographic film was placed perpendicular to the singly reflected beam and just behind the second crystal. The resulting photograph is seen in Fig. 9I, where it is superposed on a film showing the full width of the singly reflected beam. In order to determine whether the small crystal was accurately aligned in the center of this beam, it was necessary to compute the projected width of the crystal in a direction normal to the beam. This projected width of 1/16 inch found by computation, proved to be precisely that shown on the photograph, which indicated that the full projected width of the crystal had been photographed on the film, a fact that had not previously been obvious, as the edge of the shadow cast by the crystal happened to coincide with the edge of the film. With these data at hand, it became evident that the small crystal was well centered relative to the singly reflected beam.

As a final precaution, the two photographs shown in Figs. 9J and K were taken with the small crystal reset in the Laue position. In the case of Fig. 9J the vertical knife-edge at the first crystal was set at a distance of 2 mm. from the face of the crystal, where, as is seen in the photograph, it intercepted part of the x-ray beam. When the separation was increased to 3 mm., however, the entire beam passed through, as shown in Fig. 9K. The latter adjustment was, therefore, retained during the process of measuring the angular separation of the Mo $K\alpha_1$ and $K\beta_1$ spectral lines.

Largely as a matter of curiosity, the photograph shown as Fig. 9L was taken with the spectrometer adjusted to reflect the peak of Mo $K\beta_{1,3}$. Although this spectral line was situated at the opposite end of the useful range of angles from the $K\alpha_1$ line by means of which the adjustments of shielding had been made, the film showed that the singly reflected $K\beta_{1,3}$ line was still properly centered in the bay-window. A portion of the singly reflected $K\alpha_1$ line can also be seen on this film, at the right edge of the exposed region.

4. Procedure Followed in Measuring the Angular Separation of $K\alpha_1$ and $K\beta_1$.

On October 28, 1933, readings were taken for the first complete set of data for the measurement of the angular separation of $K\alpha_1$ and $K\beta_1$ in the Bragg-Laue position. The procedure employed in taking the readings and plotting the data was the same as that described above for the case of the etched crystal. This set of data was followed by another, after which the small crystal was turned to the Bragg position, where two complete sets of data were obtained. Then, when the small crystal was reset in the Laue position, it was necessary to re-align the shield containing the bay window, by means of the photographs shown in Figs. 9M, N, and O, which represent the three successive steps taken in arriving at the final adjustment, shown as Fig. 9-O. At this time three additional sets of readings were taken in the Bragg-Laue position, and these were followed by two sets taken in the Bragg-Bragg position. Thus there were in all, five sets of readings taken in the Bragg-Laue position and four taken in the Bragg-Bragg position, by means of which it was possible to compare the angular separation of the $K\alpha_1$ and $K\beta_1$ lines in the two cases. These data are listed in Tables 8 to 25, and the profiles of the lines are shown in Figs. 12 to 15.

While taking the readings for the first set of data, it became evident that it would be advisable to compute the magnitude of the shift in angular position of the spectral lines to be expected as a consequence of the thermal expansion of the spectrometer screw under the influence of the heat radiated by the angle-drum light. The effective length of this steel screw was 5.00 inches, so that it would expand 5.00 x .000012 = .00006 inch per degree centigrade. There were 20 threads per inch, and each thread corresponded to a rotation of the crystals of 10 minutes of arc, so that one inch corresponded to 200 minutes, or 12,000 seconds, of arc. It followed that there would be a shift in the apparent angular position of any given spectral line of

.00006 x 12,000 = .72 second of arc per degree centigrade rise in temperature of the screw relative to the rest of the spectrometer.

In order to minimize the possibility of an error arising from this source, the angle-drum light of 40 watts used in taking the initial set of data was later replaced by one of 10 watts which was turned on only while taking readings. A thermometer was placed at the center of the spectrometer to measure the rise in temperature of the instrument as a whole, but to detect differences in temperature at the two ends of the spectrometer, a special battery of thermocouples was required. The thermocouples were constructed from 28 wires of chromel and alumel, and were divided into two batteries, one under each arm of the spectrometer, and extending practically the full length of the instrument.

The thermoelectric potential arising in the entire battery of thermocouples was .56 millivolts per degree centigrade. Readings of this potential, made with a Rawson Multimeter and, later, with a galvanometer of high sensitivity, showed that the screw end of the spectrometer was ordinarily about .1 degree hotter than the other end, but the variation in this temperature gradient as the crystals were turned from the $K\beta_1$ to the $K\alpha_1$ position was negligible in all cases except for the data taken on December 21, when the variation itself was .1 degree. Even in this worst case, the shift in the apparent angular position of $K\alpha_1$ relative to $K\beta_1$ would be but .1 x .72 = .07 seconds of arc, which is beyond the limit of reproducibility of the spectrometer readings, as it corresponds, for instance, to a variation in the thickness of the film of oil on the spectrometer screw of but six millionths of an inch. Moreover, since the angular separation of $K\alpha_1$ and $K\beta_1$ was found to be 2632. seconds of arc, a variation of .07 second would introduce an error of but .0027 of one percent. Thus it was finally concluded that the experimentally observable differences in temperature were not great enough to give rise to any appreciable error in the measurements in question.

TABLE EIGHT.

DATA FOR THE PROFILE OF MO $K\beta_{1,3}$ IN THE BRAGG-LAUE POSITION, AS PLOTTED IN FIGURE 12A.

First crystal in the Bragg position, second crystal in the Laue position. Shielding aligned as in Fig. 9K.

X-ray tube operating at 6.0 milliamperes and 35. kilovolts.

Voltage on electrometer binants, 13.5 volts.

Angle-drum light of 40 watts left burning throughout entire time.

Time and date, 9 a.m. to 12 m., October 28, 1933.

Readings of angular position assume the central zero position of the arms of the spectrometer to be 6° 20'.

Readings of deflection time are for ten-centimeter deflections of the light beam of the Hoffmann electrometer.

Angular Position	Deflection Time	Reciprocal of Time
6° 10' 0"	440.0 sec.	2.2×10^{-3}
6° 11' 0"	444.0	2.25
50"	433.2	2.31
6° 12' 30"	404.0	2.47
6° 13' 0"	400.1	2.50
20"	374.0	2.67
36"	340.0	2.94
46"	296.6	3.37
54"	248.8	4.02
6° 14' 0"	201.2	4.975
4"	167.6	5.96
6"	148.8	6.74
8"	134.4	7.44
10"	117.6	8.51
12"	102.8	9.73
14"	87.2	11.47
16"	73.7	13.56
18"	59.0	17.0
20"	47.7	20.9
22"	39.2	25.5

Table Eight, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 14' 24"	33.4 sec.	29.9 x 10^{-3}
26"	30.2	33.7
30"	31.7	31.5
32"	36.2	27.6
34"	41.9	23.85
36"	47.2	21.2
38"	50.7	19.7
40.5"	51.2	19.5
42"	49.5	20.5
44"	47.7	20.94
46"	47.6	21.0
48"	49.7	20.1
50"	55.6	17.97
52"	65.2	15.33
54.1"	81.2	12.3
56"	100.2	9.98
58"	121.8	8.20
6° 15' 0"	141.2	7.08
2"	160.4	6.24
4"	185.1	5.40
7"	221.5	4.52
10"	262.5	3.91
14"	278.5	2.59
20"	314.6	3.18
28"	346.7	2.885
38"	379.0	2.64
54"	413.0	2.42
6° 16' 16"	427.5	2.34
46"	444.5	2.25
6° 17' 30"	446.0	2.24
6° 18' 30"	471.5	2.12

TABLE NINE.

DATA FOR THE PROFILE OF MO $K\alpha_1$ IN THE BRAGG-LAUE POSITION, AS PLOTTED IN FIGURE 13A.

Conditions as stated in Table Eight, except that the binant voltage was changed to 22.5 volts for the background and 7.5 volts for the peak.

Time and date, 2:40 p.m., October 28, 1933.

Angular Position	Deflection Time	Reciprocal of Time
6° 54' 0"	1198. sec.	.835 x 10^{-3}
6° 55' 0"	1124.	.889
6° 56' 0"	1080.	.926
6° 57' 0"	865.	1.156
20"	758.	1.320
30"	696.	1.436
38"	544.	1.838
45"	431.	2.32
50.2"	339.6	2.945
54"	279.	3.58
57"	230.8	4.33
6° 58' 0"	186.4	5.36
2"	151.8	6.59
4"	128.5	7.78
6.3"	100.	10.0
8"	85.2	11.74
10"	65.3	15.32
12"	50.8	19.68
14"	42.0	23.8
16"	35.8	27.9
18"	32.6	30.7
20"	31.5	31.7
22"	33.8	29.6
24.1"	38.8	25.8
26.2"	48.5	20.6
28"	58.1	17.2
30"	73.0	13.7
32"	90.75	11.1
34"	113.6	8.80
36"	142.5	7.01

Table Nine, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 58' 38"	169.5 sec.	5.90 x 10^{-3}
40"	196.8	5.08
43"	242.5	4.12
46"	292.5	3.42
50"	351.5	2.836
55"	420.5	2.376
6° 59' 2"	512.0	1.952
10"	624.0	1.602
20"	714.0	1.400
32"	731.0	1.368
43"	768.5	1.30
55"	744.0	1.344
7° 4' 0"	1235.	.810
7° 5' 0"	1262.	.792

TABLE TEN

DATA FOR THE PROFILE OF MO $K\beta_{1,3}$ IN THE BRAGG-LAUE POSITION, AS PLOTTED IN FIGURE 12B.

First crystal in the Bragg position, second crystal in the Laue position. Shielding aligned as in Fig. 9 K.

X-ray tube operating at 6.0 milliamperes and 35. kilovolts.

Voltage on electrometer binants, 22.5 volts for the background and 10.5 volts for the peak of the line.

Angle-drum light of 40 watts turned on only while reading the angle-drum.

Time and date, 8:40 a.m. to 11:30 a.m., November 4, 1933.

Angular Position	Deflection Time	Reciprocal of Time
6° 10' 0"	626.5 sec.	1.595×10^{-3}
6° 11' 0"	654.	1.528
50"	606.	1.65
6° 12' 30"	597.	1.675
6° 13' 0"	549.	1.82
20"	526.	1.90
36"	481.5	2.074
46"	441.5	2.363
6° 14' 0"	283.	3.535
4"	233.	4.29
6"	203.8	4.91
8"	188.2	5.31
10.2"	158.7	6.30
12.1"	139.8	7.15
14"	120.	8.33
16"	94.1	10.62
18"	79.9	12.52
20"	63.4	15.76
22"	53.5	18.2
24"	46.7	21.4
26"	43.8	22.8
28"	44.2	22.6
30"	48.2	20.7

Table Ten, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 14' 32.1"	55.8 sec.	17.93×10^{-3}
34"	64.4	15.53
36"	72.0	13.89
38"	74.8	13.37
40"	75.2	13.30
42"	71.5	13.98
44"	68.8	14.54
46"	69.6	14.36
48"	77.15	12.95
50"	86.8	11.52
52"	104.4	9.57
54"	127.6	7.84
56"	155.6	6.435
58"	189.6	5.27
6° 15' 0"	220.	4.55
2"	252.	3.97
4"	278.	3.59
7"	325.	3.08
10"	362.	2.76
14"	410.	2.44
20"	459.	2.175
28"	487.5	2.05
38"	535.	1.87
54"	568.	1.76
6° 16' 16"	598.	1.622
46"	627.	1.593
6° 17' 30"	629.	1.598
6° 18' 30"	661.	1.513

TABLE ELEVEN

DATA FOR THE PROFILE OF MO $K\alpha_1$ IN THE BRAGG-LAUE POSITION, AS PLOTTED IN FIGURE 13B.

Conditions as stated in Table Ten, except that the binant voltage was 22.5 volts for the background and 7.5 volts for the peak.

Time and date, 11:30 a.m. to 3:30 p.m., November 4, 1933.

Angular Position	Deflection Time	Reciprocal of Time
6° 54' 0"	1231. sec.	$.8125 \times 10^{-3}$
6° 55' 0"	1116.	.896
6° 56' 0"	1077.	.929
6° 57' 0"	854.	1.171
20"	729.	1.372
30"	618.5	1.617
38"	524.	1.908
45"	424.	2.36
50"	343.	2.92
54"	297.	3.37
57"	241.2	4.144
6° 58' 0"	195.8	5.11
2'	168.0	5.95
4"	140.8	7.11
6.1"	113.4	8.82
8"	90.2	11.09
10"	72.0	13.9
12"	57.3	17.45
14"	46.7	21.4
16"	38.4	26.0
18.1"	33.7	29.7
20"	32.0	31.25
22.2"	34.1	29.3
24"	37.2	26.88
26"	44.6	22.4
28"	56.5	17.7
30"	70.8	14.13
32"	87.5	11.42
34"	110.0	9.09
36"	135.8	7.36
38"	165.0	6.06
40"	190.0	5.26

Table Eleven, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 58' 43"	237.2 sec.	4.22×10^{-3}
46"	287.0	3.484
50"	343.0	2.916
55"	417.5	2.393
6° 59' 2"	525.0	1.905
10"	616.0	1.624
20"	696.5	1.436
32"	745.5	1.341
42"	766.0	1.306
46"	782.5	1.277
50"	769.0	1.300
7° 4' 0"	1222.	.818
7° 5' 0"	1257.	.795

TABLE TWELVE.

DATA FOR THE PROFILE OF MO $K\beta_{1,3}$ IN THE BRAGG-BRAGG POSITION, AS PLOTTED IN FIGURE 14A.

Both crystals in the Bragg position.

X-ray tube operating at 6.0 milliamperes and 35. kilovolts.

Voltage on electrometer binants, 22.5 volts for the background and 13.5 volts for the peak.

Angle-drum light of 10 watts turned on only while taking readings.

Temperature at center of spectrometer rose from 20.4° to 21.8° C.

Time and date, 9:00 a.m. to 1:30 p.m., November 9, 1933.

Angular Position	Deflection Time	Reciprocal of Time
6° 10' 0"	488. sec.	2.05×10^{-3}
6° 11' 0"	503.	1.988
50"	477.	2.096
6° 12' 30"	446.	2.240
6° 13' 0"	406.	2.463
20"	383.	2.610
36"	346.	2.89
46"	317.	3.155
54"	285.	3.51
6° 14' 0"	243.	4.12
4"	203.	4.925
6"	185.7	5.39
8"	166.5	6.005
10"	149.2	6.70
12"	134.4	7.44
14"	116.0	8.62
16"	98.4	10.16
18"	85.2	11.74
20.1"	65.8	15.2
22"	53.6	18.66
24"	42.1	23.76
26"	34.9	28.66
28"	31.7	31.55
30"	31.4	31.85

Table Twelve, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 14' 32"	33.55	29.80 × 10^{-3}
34"	39.0	25.64
36"	45.5	21.98
38"	51.3	19.49
40"	54.0	18.52
42"	54.1	18.48
44"	51.2	19.53
46"	48.2	20.75
48"	48.35	20.68
50"	52.25	19.14
52"	60.2	16.61
54"	72.5	13.79
56"	91.7	10.90
58"	108.5	9.22
6° 15' 0"	131.5	7.60
2"	152.2	6.57
4"	172.8	5.79
7"	201.7	4.96
10"	238.0	4.20
14"	270.0	3.70
20"	314.5	3.18
28"	357.4	2.80
38"	398.6	2.51
54"	427.0	2.34
6° 16' 16"	448.5	2.23
46"	465.0	2.15
6° 17' 31"	466.0	2.15
6° 18' 30"	485.0	2.06

TABLE THIRTEEN

DATA FOR THE PROFILE OF MO $K\alpha_1$ IN THE BRAGG-BRAGG POSITION, AS PLOTTED IN FIGURE 15A.

Conditions as stated in Table Twelve, except that an aluminum filter was inserted in the x-ray beam entering the ionization chamber in order to reduce the intensity sufficiently to insure deflection times of at least 30 seconds at the peak of the line when using the minimum practical binant potential of 4.5 volts. Binant potential for background was 22.5 volts, as before.

Temperature at center of spectrometer rose from 21.8° to 22.4° C.

Time and date, 1:30 to 4:30 p.m., November 9, 1933.

Angular Position	Deflection Time	Reciprocal of Time
6° 54' 0"	1830. sec.	.546 x 10^{-3}
6° 55' 0"	1667.	.600
6° 56' 0"	1777.	.563
6° 57' 0"	1020.	.980
20"	870.	1.149
30"	754.	1.327
38"	638.	1.567
45"	473.	2.114
50"	415.	2.410
54"	337.	2.97
57"	277.2	3.61
6° 58' 0"	222.0	4.505
2"	187.7	5.33
4"	157.5	6.35
6"	128.8	7.76
8"	102.3	9.76
10"	79.0	12.66
12"	62.4	16.03
14.2"	46.95	21.30
16.2"	38.6	25.90

Table Thirteen, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 58' 18"	34.3 sec.	29.15×10^{-3}
20"	32.65	30.63
22"	34.8	28.74
24"	40.0	25.00
26"	48.7	20.53
28"	61.1	16.37
30"	76.7	13.04
32"	95.0	10.53
34"	117.7	8.495
36"	144.8	6.905
38"	171.0	5.85
40"	183.0	5.46
43"	240.6	4.155
46.2"	320.	3.125
50"	359.	2.785
55"	485.	2.06
6° 59' 2"	608.	1.645
10"	723.	1.38
20"	820.	1.22
30"	886.	1.13
40"	918.	1.09
46"	931.5	1.07
54"	918.	1.09
7° 4' 0"	1880.	.53
7° 5' 0"	1900.	.52

TABLE FOURTEEN

DATA FOR THE PROFILE OF MO K$\beta_{1,3}$ IN THE BRAGG-BRAGG POSITION, AS PLOTTED IN FIGURE 14B.

Conditions as stated in Table Twelve.

Temperature at center of spectrometer rose from 20.6° to 21.6° C.

Time and date, 8:50 to 11:40 a.m., November 18, 1933.

Angular Position	Deflection Time	Reciprocal of Time
6° 10' 0"	475. sec.	2.11×10^{-3}
6° 11' 0"	476.	2.10
50"	471.	2.12
6° 12' 30"	421.	2.375
6° 13' 0"	394.	2.54
20"	367.	2.73
36"	345.	2.86
46"	312.	3.205
54"	268.	3.73
6° 14' 0"	231.	4.32
4"	196.	5.155
6"	178.4	5.605
8"	162.0	6.17
10"	141.0	7.09
12"	127.7	7.82
14.1"	109.7	9.115
16"	97.0	10.31
18"	81.4	12.28
20"	66.2	15.11
22"	53.0	18.87
24"	41.9	23.87
26"	33.8	29.59
28"	30.1	33.22
30"	29.8	33.56
32"	32.2	31.06
34.1"	37.15	26.92
36"	43.25	23.12
38.1"	49.2	20.33
40"	52.0	19.23
42.1"	51.6	19.42
44.1"	48.6	20.58
46"	46.1	21.69

Table Fourteen, Continued

Angular Position	Deflection Time	Reciprocal of Time
6° 14' 48"	46.6 sec.	21.46 x 10^{-3}
50"	50.25	19.90
52"	57.8	17.30
54"	69.25	14.44
56"	85.0	11.76
58"	104.7	9.55
6° 15' 0"	124.3	8.045
2"	146.0	6.85
4"	166.2	6.02
7"	199.7	5.01
10"	222.2	4.50
12"	241.3	4.14
14"	265.7	3.76
20"	302.0	3.31
28"	346.5	2.885
38"	373.0	2.68
54"	410.6	2.44
6° 16' 16"	439.0	2.28
46"	458.5	2.18
6° 17' 30"	463.0	2.16
6° 18' 30"	475.5	2.10

TABLE FIFTEEN

DATA FOR THE PROFILE OF MO Kα_1 IN THE BRAGG-BRAGG POSITION, AS PLOTTED IN FIGURE 15B.

Conditions as stated in Table Thirteen.

Temperature at center of spectrometer rose from 2.18° to 22.4° C.

Time and date, 12:50 to 3:30 p.m., November 18, 1933.

Angular Position	Deflection Time	Reciprocal of Time
6° 54' 0"	1830. sec.	.55 x 10^{-3}
6° 55' 0"	1618.	.62
6° 56' 0"	1477	.68
6° 57' 0"	983.5	1.02
20"	860.	1.16
30"	731.	1.37
38"	520.	1.92
45"	406.4	2.46
50"	337.0	2.98
54"	321.6	3.11
57"	270.0	3.70
6° 58' 0"	216.0	4.63
2"	181.7	5.50
4"	134.0	7.46
6"	120.2	8.32
8"	86.2	11.60
10"	72.6	13.77
12"	58.9	16.98
14"	45.55	21.95
16"	38.1	26.25
18.3"	32.4	30.86
19.9	32.0	31.25
22"	34.15	29.28
24"	39.4	25.38
26.1"	49.5	20.20
28"	60.65	16.49
30"	75.6	13.23
32"	94.4	10.59
34"	116.0	8.62
36"	140.2	7.13
38"	173.2	5.77
40"	201.5	4.97

Table Fifteen, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 58' 43"	257.4 sec.	3.88×10^{-3}
46"	309.0	3.24
50"	383.0	2.61
55"	484.0	2.07
6° 59' 2"	600.5	1.66
10"	716.0	1.34
20"	798.0	1.25
30"	879.0	1.14
40"	920.5	1.09
46"	928.0	1.08
52"	883.0	1.13
7° 4' 0"	1820.	.550
7° 5' 0"	1905.	.525

TABLE SIXTEEN

DATA FOR THE PROFILE OF MO $K\beta_{1,3}$ IN THE BRAGG-LAUE POSITION, AS PLOTTED IN FIGURE 12C.

First crystal in the Bragg position, second crystal in the Laue position. Shielding aligned as in Fig. 9-M.

X-ray tube operating at 6.0 milliamperes and 35. kilovolts.

Voltage on electrometer binants, 22.5 volts for the background and 13.5 volts for the peak of the line.

Angle-drum light of 10 watts turned on only while reading the angle-drum.

Temperature at center of spectrometer rose from 20.8° to 22.0° C.

Time and date, 9:00 a.m. to 12:10 p.m., November 25, 1933.

Angular Position	Deflection Time	Reciprocal of Time
6° 10' 0"	455.1 sec.	2.20×10^{-3}
6° 11' 0"	456.6	2.19
50"	442.2	2.26
6° 12' 30"	430.5	2.32
6° 13' 0"	417.4	2.40
20"	403.2	2.48
36"	344.4	2.90
46"	327.0	3.06
54"	276.0	3.62
6° 14' 0"	236.6	4.22
4"	201.4	4.96
6.2"	177.6	5.63
8.3"	161.1	6.20
10"	148.2	6.75
12"	130.3	7.67
14"	114.6	8.73
16"	96.3	10.37
18"	81.86	12.22
20.2"	64.7	15.45
22"	53.7	18.60
24"	44.3	22.55

Table Sixteen, Continued

Angular Position	Deflection Time	Reciprocal of Time
6° 14' 26"	37.15 sec.	26.90×10^{-3}
28"	32.3	30.94
30"	30.7	32.55
32.6"	32.25	31.00
34.2"	35.35	28.28
36"	40.00	25.00
38.2"	47.5	21.05
40"	51.8	19.30
42"	53.27	18.76
43.9"	52.85	18.90
46"	50.6	19.76
48"	49.1	20.37
50"	49.4	20.24
52"	53.9	18.55
54"	62.8	15.92
56"	75.9	13.17
58"	94.5	10.58
6° 15' 0"	114.0	8.77
2"	136.8	7.31
4"	160.8	6.23
6"	182.4	5.48
8"	208.3	4.80
12"	262.7	3.80
18"	289.5	3.45
26"	339.6	2.95
36"	376.8	2.65
50"	409.2	2.44
6° 16' 11"	429.6	2.33
40"	433.5	2.31
6° 17' 30"	453.6	2.20
6° 18' 30"	478.8	2.09

TABLE SEVENTEEN

DATA FOR THE PROFILE OF MO $K\alpha_1$ IN THE BRAGG-LAUE POSITION, AS PLOTTED IN FIGURE 13C.

Conditions as stated in Table Sixteen, except that the binant voltage for the peak of the line was in this case 7.5 volts.

Temperature at center of spectrometer rose from 22.3° to 22.8° C.

Time and date, 1:10 to 3:55 p.m., November 25, 1933.

Angular Position	Deflection Time	Reciprocal of Time
6° 54' 0"	1260. sec.	$.79 \times 10^{-3}$
6° 55' 0"	1194.	.84
6° 56' 0"	1155.	.87
6° 57' 0"	912.	1.10
20.5"	794.	1.26
31"	681.	1.47
38"	613.	1.63
45"	507.5	1.97
50"	422.6	2.37
54"	353.5	2.83
56"	325.4	3.07
58"	290.0	3.45
6° 58' 0"	252.6	3.96
2"	222.2	4.50
4"	191.3	5.23
6"	160.8	6.21
8"	131.5	7.61
10"	106.0	9.43
12"	86.0	11.63
14"	66.25	15.10
16"	54.0	18.52
18"	43.5	22.97
20"	37.35	26.75
22"	33.15	30.15
24"	32.0	31.25
26"	33.5	29.85
27.8"	38.6	25.91
30.2"	49.4	20.24
32"	60.0	16.67
34"	74.12	13.48

Table Seventeen, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 58' 36"	96.16 sec.	10.40×10^{-3}
38"	117.2	8.53
40"	141.5	7.07
42.2"	178.5	5.60
44"	205.0	4.87
46"	231.5	4.32
48"	262.6	3.81
52"	330.0	3.03
56.2"	396.0	2.52
6° 59' 2"	488.0	2.05
8"	565.0	1.77
16"	607.0	1.65
26"	730.0	1.37
36"	749.0	1.33
46"	781.0	1.28
52"	778.0	1.28
7° 4' 0"	1323.0	.75
7° 5' 0"	1290.0	.77

TABLE EIGHTEEN

DATA FOR THE PROFILE OF MO K$\beta_{1,3}$ IN THE BRAGG-LAUR POSITION, AS PLOTTED IN FIGURE 12D.

Conditions as stated in Table Sixteen, except that the shielding was re-aligned, as shown in Fig. 9-O.

Temperature at center of spectrometer rose from 20.1° to 20.8° C.

Time and date, 9:00 a.m. to 12:30 p.m., December 2, 1933.

Angular Position	Deflection Time	Reciprocal of Time
6° 10' 0"	471. sec.	2.12 x 10^{-3}
6° 11' 0"	470.	2.13
50"	462.4	2.16
6° 12' 30"	438.5	2.28
6° 13' 0"	420.5	2.34
20"	400.0	2.50
36"	360.8	2.77
46"	315.5	3.17
52"	292.8	3.41
58"	240.4	4.16
6° 14' 2"	204.5	4.89
4"	185.2	5.40
6"	166.8	5.99
8"	145.2	6.88
10"	130.0	7.69
12"	112.1	8.92
14"	94.4	10.59
16"	78.6	12.73
18.15"	63.3	15.8
20.05"	51.8	19.3
22.1"	41.6	24.04
24"	35.85	27.14
26"	32.25	31.00
28"	31.2	32.05
30"	32.8	30.49
32"	36.15	27.65
34"	43.35	23.07
36"	48.95	20.42
38"	53.9	18.55
40"	55.3	18.08

Table Eighteen, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 14' 42"	53.6 sec.	18.66×10^{-3}
44"	51.3	19.49
46"	50.2	19.92
48"	51.5	19.42
50.1"	58.25	17.17
52"	68.0	14.70
54"	81.8	12.20
56"	103.0	9.71
58"	126.2	7.92
6° 15' 0"	148.6	6.73
2"	176.5	5.69
4"	193.5	5.17
6"	218.8	4.56
8"	239.0	4.185
10"	258.0	3.87
14"	286.7	3.49
20"	325.0	3.08
28"	365.5	2.73
38"	394.6	2.53
50"	422.5	2.36
6° 16' 10"	441.0	2.27
40"	472.5	2.11
6° 17' 30"	471.0	2.12
6° 18' 30"	482.5	2.07

TABLE NINETEEN

DATA FOR THE PROFILE OF MO $K\alpha_1$ IN THE BRAGG-LAUE POSITION, AS PLOTTED IN FIGURE 13D.

Conditions as stated in Table Seventeen.

Temperature at center of spectrometer rose from 21.0° to 21.7° C.

Time and date, 1:00 p.m. to 4:30 p.m., December 2, 1933.

Angular Position			Deflection Time	Reciprocal of Time
6°	54'	0"	1269. sec.	.79 x 10^{-3}
6°	55'	0"	1205.	.83
6°	56'	0"	1138.	.88
6°	57'	0"	1090.	.92
		20"	806.5	1.24
		30"	708.0	1.41
		36"	612.0	1.63
		44"	492.0	2.03
		48"	426.4	2.34
		52"	356.6	2.80
		54"	322.0	3.11
		56"	285.4	3.50
		57.9"	252.4	3.96
6°	58'	0"	218.1	4.58
		2"	187.0	5.35
		4"	153.25	6.53
		6"	128.20	7.80
		8.2"	99.14	10.08
		10"	81.82	12.23
		12"	64.78	15.44
		14"	51.60	19.38
		16"	42.45	23.54
		18"	35.75	27.94
		20"	32.95	30.34
		22"	32.95	30.34
		24"	35.30	28.32
		26"	41.00	24.39
		28"	51.80	19.30
		30"	64.20	15.58
		32"	81.50	12.27
		34.15"	105.40	9.48
		36"	126.20	7.93
		38"	156.25	6.40

Table Nineteen, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 58' 40"	186.7 sec.	5.36 x 10^{-3}
42.2"	222.4	4.49
44"	248.6	4.02
46"	281.5	3.55
48"	313.2	3.19
50"	349.5	2.86
52"	375.0	2.67
56"	434.0	2.30
6° 59' 0.2"	502.0	1.99
6"	574.0	1.74
16"	677.5	1.47
26.3"	742.	1.35
36"	788.	1.27
46"	780.	1.28
55"	786.	1.27
7° 0' 4"	732.	1.36
7° 4' 0"	1257.	.79
7° 5' 0"	1303.	.77

TABLE TWENTY

DATA FOR THE PROFILE OF MO $K\beta_{1,3}$ IN THE BRAGG-LAUE POSITION, AS PLOTTED IN FIGURE 12E.

Conditions as stated in Table Eighteen.

Temperature at center of spectrometer rose from 20.2° to 21.3° C.

Time and date, 8:50 a.m. to 12:30 p.m., December 21, 1933.

Angular Position	Deflection Time	Reciprocal of Time
6° 10' 0"	483. sec.	2.07 x 10^{-3}
6° 11' 0"	477.	2.10
50"	454.	2.20
6° 12' 30"	431.	2.32
6° 13' 0"	416.	2.40
20"	399.5	2.50
36"	351.0	2.85
46"	318.5	3.15
52"	286.5	3.49
58"	236.5	4.23
6° 14' 2"	200.0	5.00
4"	183.5	5.46
6"	165.2	6.05
8"	148.9	6.71
10"	130.9	7.64
12"	115.7	8.64
14"	100.0	10.00
16"	83.59	11.96
18"	68.00	14.71
20"	55.30	18.08
22"	46.30	21.60
24.05"	37.55	26.63
26.4"	31.60	31.65
28"	30.90	32.36
29.95"	31.00	32.26
32.05"	35.50	28.17
34"	40.50	24.69
36"	46.05	21.71
38"	51.50	19.42
40"	53.95	18.53
42"	53.20	18.80
44"	51.20	19.53
46"	49.65	20.14

Table Twenty, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 14' 48"	49.70 sec.	20.12×10^{-3}
50"	53.60	18.66
52"	62.45	16.01
54"	75.40	13.26
56"	91.90	10.88
58"	113.6	8.80
6° 15' 0"	136.9	7.30
2"	159.1	6.28
4"	181.5	5.51
6.1"	204.0	4.90
8"	218.7	4.57
10"	241.6	4.14
14"	272.4	3.67
20"	304.0	3.29
28"	336.8	2.97
38"	374.3	2.67
50"	402.5	2.48
6° 16' 10"	418.0	2.39
40"	433.6	2.31
6° 17' 30"	450.0	2.22
6° 18' 30"	447.5	2.23

TABLE TWENTY-ONE.

DATA FOR THE PROFILE OF MO Kα_1 IN THE BRAGG-LAUE POSITION, AS PLOTTED IN FIGURE 13E.

Conditions as stated in Table Eighteen.

Temperature at center of spectrometer rose from 21.4° to 21.6° C.

Time and date, 1:05 to 4:30 p.m., December 21, 1933.

Angular Position	Deflection Time	Reciprocal of Time
6° 54' 0"	1122. sec.	.89 x 10^{-3}
6° 55' 0"	1069.	.93
6° 56' 0"	1002.5	1.00
6° 57' 0"	844.0	1.18
20"	702.0	1.42
30"	629.0	1.59
36"	574.0	1.74
44"	455.0	2.20
48"	392.6	2.55
52"	333.2	3.00
54"	302.2	3.31
56"	274.5	3.64
58"	243.0	4.11
6° 58' 0"	208.8	4.78
2"	179.75	5.56
4.15"	146.00	6.85
6"	122.60	8.15
8"	99.83	10.02
10"	81.21	12.31
12"	64.10	15.60
14"	50.20	19.92
16"	41.87	23.89
18"	35.60	28.10
20"	32.55	30.72
22.2"	32.30	30.96
24"	34.90	28.65
26"	40.50	24.69
28.1"	51.85	19.28
30"	64.25	15.56
32"	81.60	12.25
34"	101.21	9.88

Table Twenty-one, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 58' 36"	124.8 sec.	8.01×10^{-3}
38"	150.8	6.63
40"	179.0	5.59
42"	210.3	4.75
44"	240.7	4.15
46.1"	273.4	3.66
48"	301.6	3.32
50"	334.6	2.99
52"	359.2	2.78
56"	409.0	2.44
6° 59' 0"	470.0	2.13
6"	547.5	1.83
16"	633.0	1.58
26"	695.0	1.44
36"	726.0	1.38
46"	732.5	1.36
7° 4' 0"	1128.	.89
7° 5' 0"	1195.	.84

TABLE TWENTY-TWO.

DATA FOR THE PROFILE OF MO K$\beta_{1,3}$ IN THE BRAGG-BRAGG POSITION, AS PLOTTED IN FIGURE 14C.

Both crystals in the Bragg position. Conditions as stated in Table Twelve.

Temperature at center of spectrometer rose from 20.8° to 21.4° C.

Time and date, 9:30 a.m. to 1:00 p.m., January 5, 1934.

Angular Position	Deflection Time	Reciprocal of Time
6° 10' 0"	423. Sec.	2.36×10^{-3}
6° 11' 0"	433.	2.31
50"	420.	2.38
6° 12' 30"	397.	2.52
6° 13' 0"	375.	2.67
20"	343.4	2.91
36"	321.4	3.11
46"	296.4	3.37
52"	284.0	3.52
58"	260.0	3.85
6° 14' 2"	237.6	4.21
4"	226.4	4.41
6"	206.0	4.85
8"	194.0	5.15
10"	177.0	5.65
12"	159.6	6.26
14"	146.1	6.84
16"	131.1	7.63
18"	113.2	8.83
20"	98.15	10.19
22"	84.30	11.86
24"	69.70	14.35
26.1"	54.10	18.48
28"	44.80	22.32
30"	35.42	28.23
32"	30.65	32.62
34"	28.85	34.66
36"	30.50	32.79
38"	34.40	29.01

Table Twenty-two, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 14' 40"	40.10 sec.	24.94 x 10^{-3}
42"	46.15	21.67
44"	50.50	19.80
46"	51.52	19.40
48"	49.72	20.11
50"	46.80	21.37
52"	45.70	21.88
54"	47.40	21.10
56.2"	54.60	18.31
58"	62.60	15.97
6° 15' 0"	77.90	12.84
2"	95.40	10.48
4"	112.95	8.85
6"	134.6	7.43
8"	155.0	6.45
10"	171.2	5.84
14"	205.0	4.88
20"	255.0	3.92
28"	300.6	3.33
38"	312.0	3.20
50"	374.5	2.67
6° 16' 10"	385.8	2.69
40"	414.0	2.41
6° 17' 30"	430.0	2.33
6° 18' 30"	435.0	2.30

TABLE TWENTY-THREE

DATA FOR THE PROFILE OF MO $K\alpha_1$ IN THE BRAGG-BRAGG POSITION, AS PLOTTED IN FIGURE 15C.

Conditions as stated in Table Twelve.

Temperature at center of spectrometer remained constant at 21.4° C.

Time and date, 1:00 to 4:30 p.m., January 5, 1934.

Angular Position	Deflection Time	Reciprocal of Time
6° 54' 0"	1597. sec.	.63 x 10^{-3}
6° 55' 0"	1555.	.64
6° 56' 0"	1447.	.69
6° 57' 0"	944.0	1.06
20"	854.5	1.17
30"	760.0	1.32
36"	712.5	1.40
44"	588.0	1.70
48"	534.5	1.87
52.2"	448.5	2.23
54"	420.5	2.38
56"	377.0	2.65
58"	352.0	2.84
6° 58' 0.4"	299.0	3.34
2"	282.0	3.55
4"	234.0	4.27
6"	205.5	4.87
8"	173.5	5.76
10"	144.8	6.91
12"	115.8	8.63
14"	93.00	10.75
16.1"	70.67	14.15
18"	55.10	18.15
20"	44.10	22.68
22"	36.50	27.40
24"	32.20	31.06
26"	31.80	31.45
28"	34.40	29.07
30"	40.60	24.63
32"	49.80	20.08
34"	62.30	16.05

Table Twenty-three, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 58' 36"	79.00 sec.	12.66×10^{-3}
38"	98.00	10.20
40"	118.8	8.42
42"	143.7	6.96
44"	174.4	5.73
46"	205.0	4.88
48"	235.7	4.24
50"	275.3	3.64
52"	306.0	3.27
54.1"	348.6	2.87
56"	382.0	2.62
6° 59' 0"	458.0	2.18
6"	576.0	1.74
14"	666.5	1.50
24"	758.5	1.32
34"	833.0	1.20
44"	874.8	1.14
54"	864.0	1.16
7° 0' 0"	855.0	1.17
7° 4' 0"	1646.	.61
7° 5' 0"	1689.	.59

TABLE TWENTY-FOUR.

DATA FOR THE PROFILE OF MO $K\beta_{1,3}$ IN THE BRAGG-BRAGG POSITION AS PLOTTED IN FIGURE 14D.

Conditions as stated in Table Twelve.

Temperature at center of spectrometer rose from 19.1° to 20.7° C.

Time and date, 9:15 a.m. to 1:00 p.m., January 12, 1934.

Angular Position	Deflection Time	Reciprocal of Time
6° 10' 0"	438.0 sec.	2.28×10^{-3}
6° 11' 0"	451.0	2.22
50"	434.4	2.30
6° 12' 30"	413.0	2.42
6° 13' 0"	365.2	2.74
20"	353.6	2.83
36"	328.0	3.05
46"	312.5	3.20
52"	293.0	3.41
58"	262.8	3.80
6° 14' 2"	243.0	4.11
4"	234.5	4.26
6"	219.3	4.56
8"	203.7	4.91
10"	187.0	5.35
12"	169.0	5.92
14"	153.3	6.52
16"	137.0	7.30
18"	122.2	8.18
20"	108.0	9.26
22"	93.1	10.74
24"	78.65	12.71
26"	63.45	15.76
28"	49.80	20.08
30"	39.90	25.06
31.9"	33.84	29.53
34"	30.25	33.05
36"	30.80	32.47
38"	33.53	29.82
40"	39.10	25.58
42"	45.65	21.90
44"	50.92	19.63
46"	52.80	18.92

Table Twenty-four, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 14' 48"	51.93 sec.	19.25×10^{-3}
50"	49.00	20.41
52"	46.95	21.30
54"	47.80	20.92
56"	52.70	18.98
58"	61.60	16.23
6° 15' 0"	74.83	13.36
2"	92.40	10.82
4"	110.20	9.07
6"	130.25	7.68
8"	148.0	6.76
10"	168.2	5.94
14"	207.3	4.82
20"	251.3	3.98
28"	267.3	3.74
38"	310.5	3.22
50"	363.0	2.75
6° 16' 10"	397.5	2.52
40"	414.0	2.41
6° 17' 30"	423.0	2.36
6° 18' 30"	430.5	2.32

TABLE TWENTY-FIVE

DATA FOR THE PROFILE OF MO $K\alpha_1$ IN THE BRAGG-BRAGG POSITION AS PLOTTED IN FIGURE 15D.

Conditions as stated in Table Thirteen

Temperature at center of spectrometer rose from 20.7° to 21.4° C.

Time and date, 1:00 to 4:30 p.m., January 12, 1934.

Angular Position	Deflection Time	Reciprocal of Time
6° 54' 0"	1632. sec.	$.61 \times 10^{-3}$
6° 55' 0"	1502.	.67
6° 56' 0"	1482.	.68
6° 57' 0"	994.	1.00
20"	865.	1.16
30"	781.	1.28
36"	718.	1.39
44"	599.	1.67
48"	531.2	1.88
52"	483.0	2.07
54"	441.3	2.27
56"	406.5	2.46
58.1"	359.3	2.78
6° 58' 0"	328.0	3.05
2"	286.3	3.49
4"	252.0	3.97
7"	197.6	5.06
10"	147.0	6.80
12"	120.0	8.33
14.15"	93.33	10.71
16"	75.60	13.23
18"	58.50	17.09
20"	45.40	22.03
22"	37.20	26.88
24"	33.00	30.30
26"	32.50	30.77
28.9"	38.32	26.10
30"	41.75	23.95
32"	51.75	19.32
34"	54.10	15.60
36"	80.80	12.36
38"	101.3	9.87

Table Twenty-five, Continued.

Angular Position	Deflection Time	Reciprocal of Time
6° 58' 40"	125.1 sec.	7.99×10^{-3}
42"	149.0	6.71
44"	179.2	5.58
46"	213.0	4.70
48"	257.0	3.89
50"	283.7	3.53
52"	322.5	3.10
56"	397.7	2.51
6° 59' 0"	476.0	2.10
6"	597.0	1.68
16"	735.0	1.36
26"	821.0	1.22
36"	892.0	1.12
46"	907.5	1.10
56"	919.0	1.09
7° 0' 6"	845.0	1.18
7° 4' 0"	1891.	.53
7° 5' 0"	1945.	.51

TABLE TWENTY-SIX

ANGULAR SEPARATION OF THE SPECTRAL LINES AS PLOTTED IN FIGURE 16.

I. Crystals in the Bragg-Bragg position:

A	α_1	6° 58' 19.8"
	β_1	6° 14' 29.1"
		43' 50.7"
B	α_1	6° 58' 19.8"
	β_1	6° 14' 29.4"
		43' 50.4"
C	α_1	6° 58' 25.5"
	β_1	6° 14' 34.15"
		43' 51.35"
D	α_1	6° 58' 25.1"
	β_1	6° 14' 35.0"
		43' 50.1"

Mean separation for the Bragg-Bragg position, 43' 50.64"

II. Crystals in the Bragg-Laue position:

A	α_1	6° 58' 19.4"
	β_1	6° 14' 27.4"
		43' 52.0"
B	α_1	6° 58' 20.2"
	β_1	6° 14' 26.7"
		43' 53.5"
C	α_1	6° 58' 23.7"
	β_1	6° 14' 30.4"
		43' 53.3"
D	α_1	6° 58' 21.1"
	β_1	6° 14' 28.0"
		43' 53.1"
E	α_1	6° 58' 21.2"
	β_1	6° 14' 28.4"
		43' 52.8"

Mean separation for the Bragg-Laue position, 43' 52.94"

$\therefore$ The difference between the experimental values of the mean angular separation of Mo Kα_1 and Kβ_1 in the Bragg-Laue and Bragg-Bragg cases is 2.30 seconds of arc.

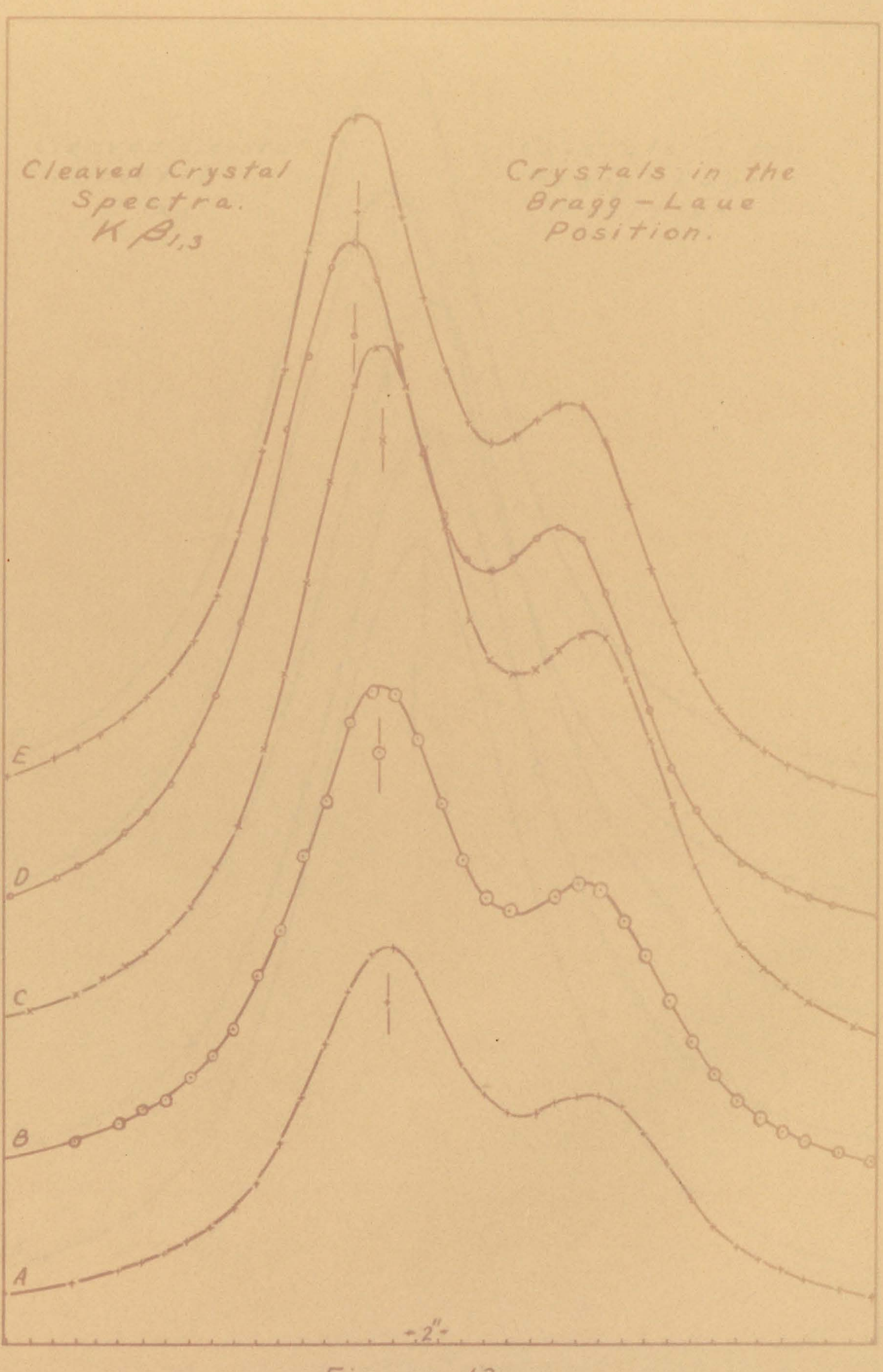

Cleaved Crystal Spectra. Kβ1,3
Crystals in the Bragg-Laue Position.
E
D
C
B
A
←2″→

Figure 12.

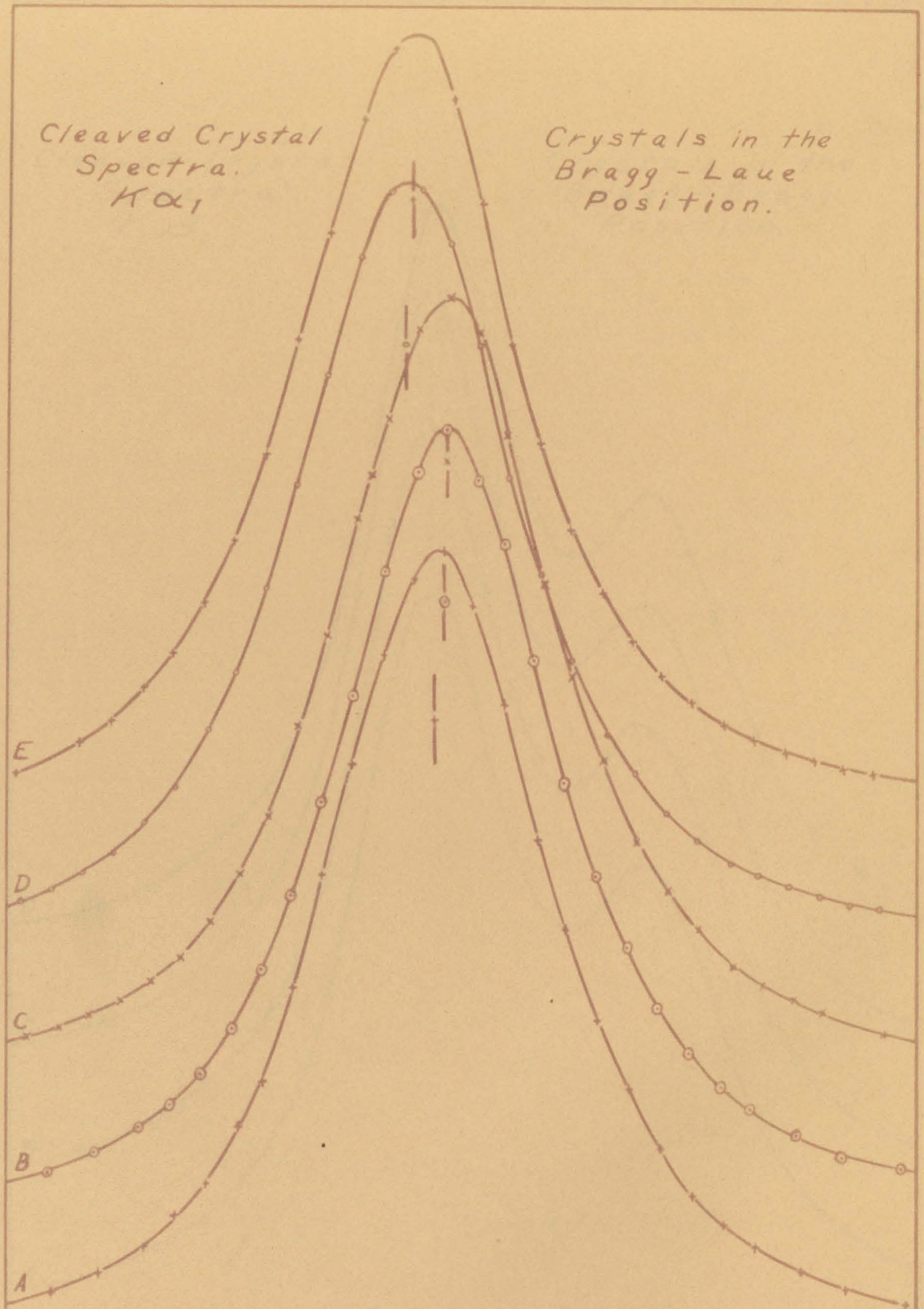
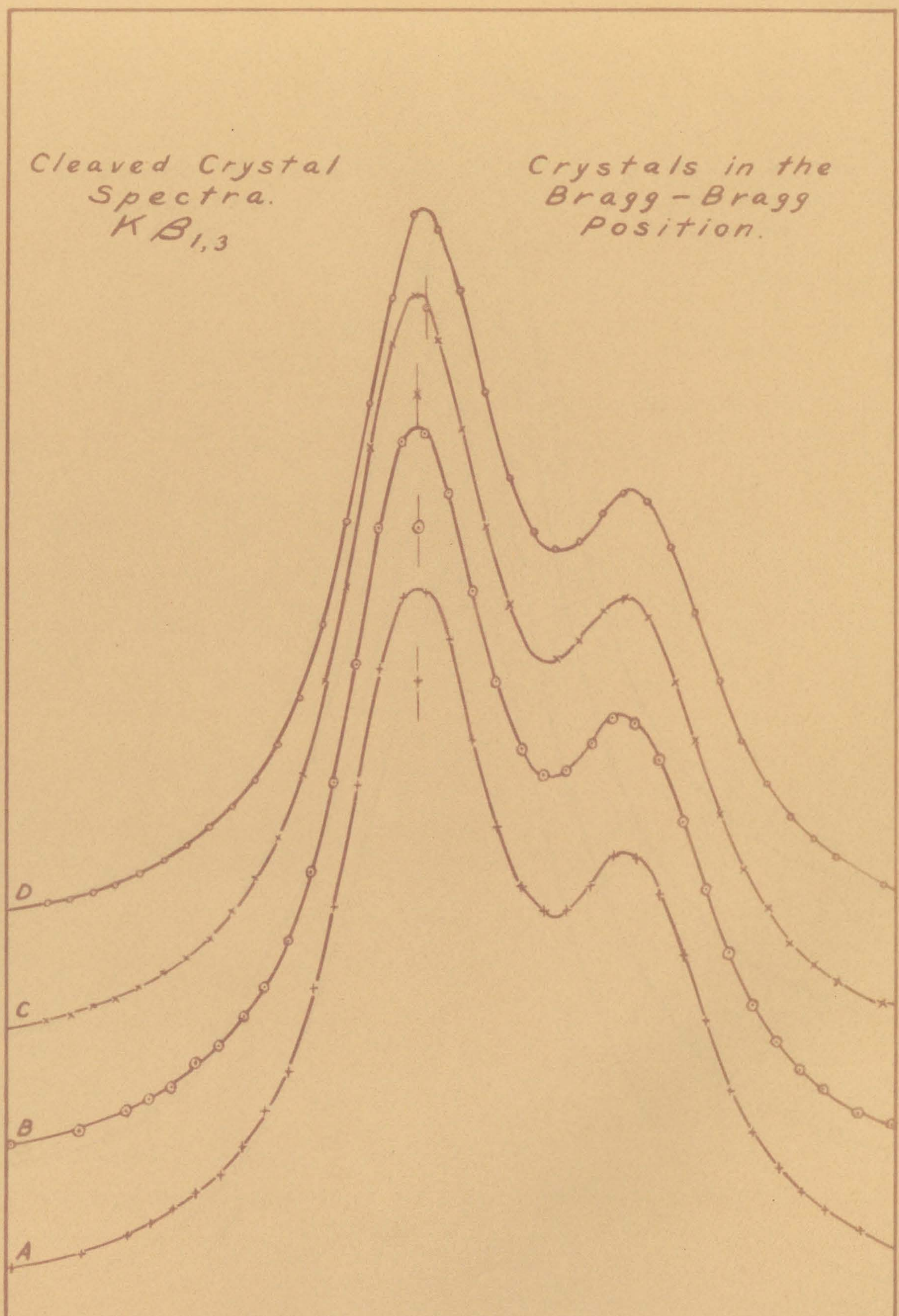

Cleaved Crystal
Spectra.
$K\alpha_1$
Crystals in the
Bragg - Laue
Position.
E
D
C
B
A
2"

Figure 13.



Cleaved Crystal Spectra.
$K\beta_{1,3}$
Crystals in the Bragg-Bragg Position.
D
C
B
A
2"

Figure 14.

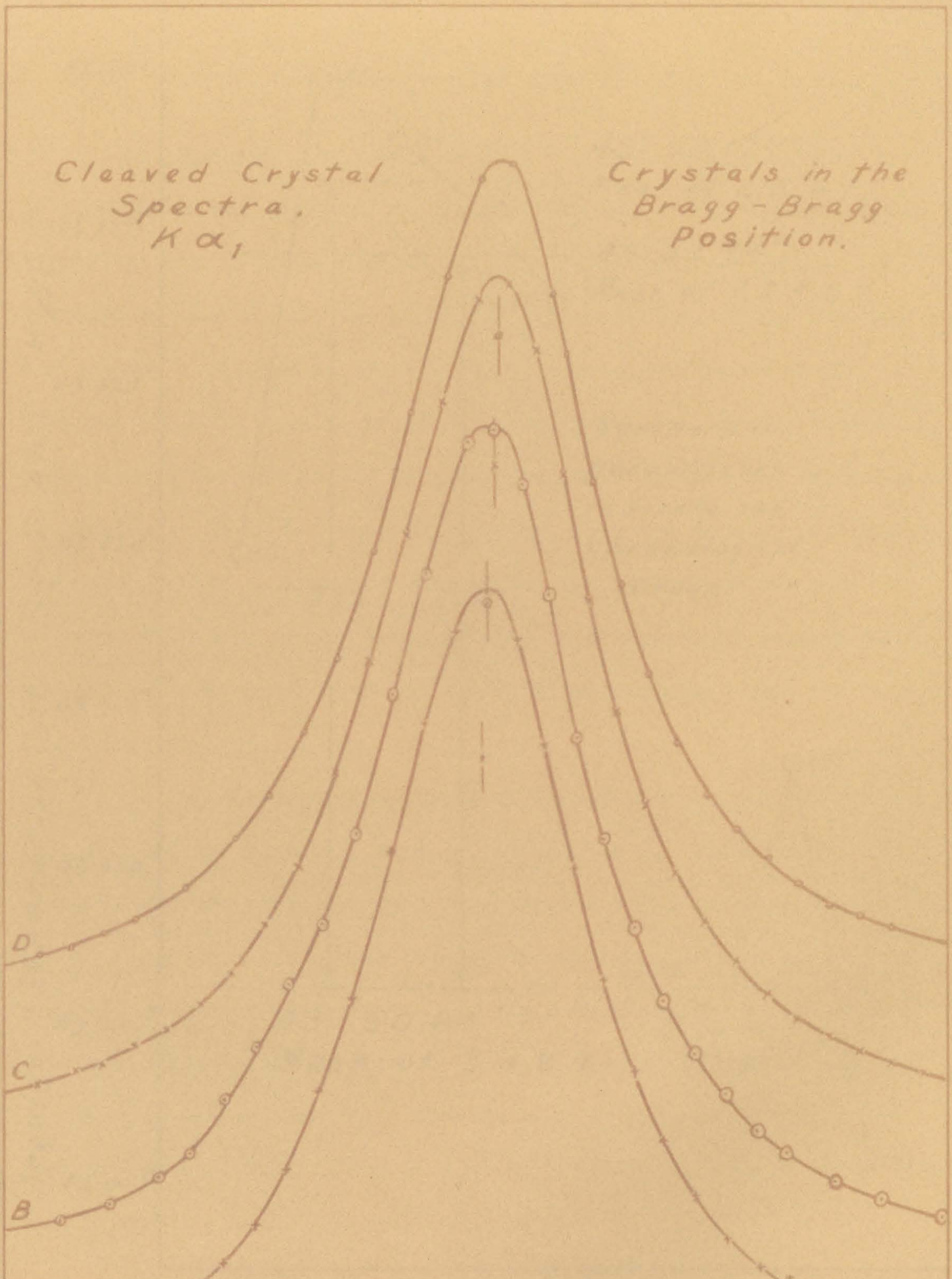

Cleaved Crystal
Spectra,
Kα1
Crystals in the
Bragg-Bragg
Position.
D
C
B
A
2"

Figure 15.

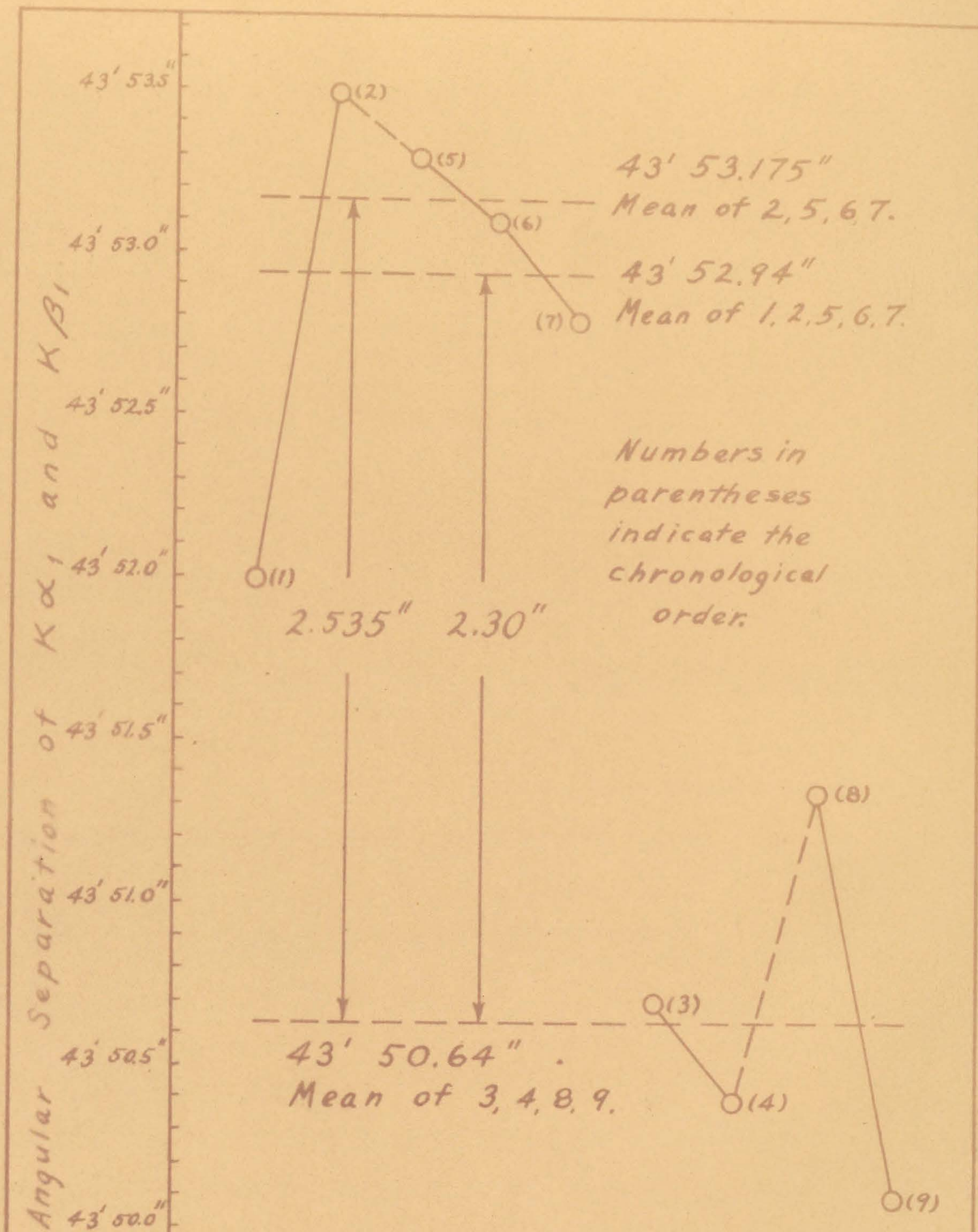

Angular Separation of Kα₁ and Kβ₁
43' 53.5"
43' 53.0"
43' 52.5"
43' 52.0"
43' 51.5"
43' 51.0"
43' 50.5"
43' 50.0"
(1)
(2)
(5)
(6)
(7)
(3)
(4)
(8)
(9)
43' 53.175"
Mean of 2, 5, 6, 7.
43' 52.94"
Mean of 1, 2, 5, 6, 7.
2.535"
2.30"
43' 50.64".
Mean of 3, 4, 8, 9.
Numbers in parentheses indicate the chronological order.
A(1)
B(2)
C(5)
D(6)
E(7)
A(3)
B(4)
C(8)
D(9)
Bragg - Laue Position.
Bragg - Bragg Position.

Figure 16.

D. DISCUSSION OF RESULTS.

In examining the graphs of Figs. 12 to 15, it is well to have in mind the method employed in plotting the data. The curves appearing in any one of these figures can be divided into two groups, lettered A,B and C,D,E, for which the zero positions of the crystals were slightly different, so that there is no significance to the apparent shift in angular position of one group relative to the other; consequently the two groups have been aligned together for convenience in plotting. It is to be noted, however, that the angular position of the Laue spectra as actually measured on the angle-drum was made to coincide with that of the Bragg spectra to within eight seconds of arc, so that the same portion of the spectrometer screw was used for both measurements.

An inspection of the graphs discloses that for any particular alignment of the crystals, that is, within the A,B group or the C,D,E group of curves, there may be a considerable shift in angular position of the lines from one set of data to the next. Moreover, in the comparison of curve B with curve A for the Laue position, it is readily seen that the α_1 line shifted in the direction of increasing angles, the β_1 line in the direction of decreasing angles, and the measured angular separation of α_1 and β_1 for one or possibly both of these cases must therefore be considerably in error.

Such considerations led to the conclusion that the data are not sufficiently accurate to merit a numerical calculation of the center of gravity of the spectral lines. This conclusion has subsequently been verified by the results of more accurate experiments, to be described below, which have disproved the results of this experiment and explained one of the principal sources of error in these measurements. The cause of the shifting of the spectral lines, however, is still unknown.

In order to measure the separation of the α_1 and β_1 lines to a degree of accuracy comparable to that of the data, the center-lines of the upper portions of the plotted curves were drawn. When the center-line of $K\beta_1$ was drawn, the first sign of departure from linearity was interpreted as being caused by the inception of $K\beta_3$, so that a point well within the limits of the straight, vertical portion of the center-line was chosen as representing the position of the spectral line. These points for the β_1 lines, as well as those for the α_1 lines, are shown on the graphs. The resulting angular separations are listed in Table Twenty-six and plotted in Fig. 16.

An inspection of Fig. 16 shows that with the exception of the data for points (1) and (2), measurements in either position became successively shorter. The remarkable exception indicated by points (1) and (2) may possibly have been caused by some unexplained thermal effects, for the 40 watt light at the angle-drum was replaced by one of 10 watts after the first set of data had been obtained. If, therefore, point (1) is disregarded, the mean angular separation for the Bragg-Laue case becomes 43' 53.175", and the difference between this value and that for the Bragg-Bragg case is 2.535 seconds, as compared to 2.30 seconds obtained by retaining all the data. At the time the experiment was performed, the fact that this larger value was of the same order of magnitude as the 4.29 seconds predicted on the basis of a .3 per cent difference in grating constant led to the conclusion that there was a difference in the grating constants of calcite effective in the two cases. This result was not considered conclusive, however, because of the lack of reproducibility in the measurements of the angular separation. It was decided that the problem should next be attacked from another angle, by means of a photographic spectrometer to be designed and built for this particular problem. These later experiments are described below, after which the present results are discussed in the light of more accurate data.

PART III. EXPERIMENTS WITH THE PHOTOGRAPHIC SPECTROMETER

A. OBJECT OF THE SECOND PART OF THE EXPERIMENTAL RESEARCH.

The object of the second part of the experimental investigation, to be achieved by means of the photographic spectrograph, was to verify and render more conclusive the evidence obtained by means of the double crystal spectrometer. The latter instrument had not been particularly well suited to the problem under consideration in that it was not adapted to the measurement of the entire glancing angle, but made possible, instead, a mere comparison of glancing angles over a relatively narrow range. The effect sought with the double spectrometer was not, therefore, a shift in the angular position of a certain spectral line, but a difference in the shifts in the angular position of two spectral lines, as made manifest in the difference in the angular separation of these lines in the Laue and Bragg cases. Not only was the angular separation of the chosen lines less than one ninth as great as the glancing angle for Mo Kα_1 , but in view of the fact that both lines should shift in the same direction and through almost the same angle, the difference in angular separation, as measured with the double spectrometer, should be but six per cent of the difference in the absolute value of the glancing angle of any certain line, such as would be measured by a photographic spectrograph.

B. THEORY UNDERLYING THE DESIGN OF THE SPECTROGRAPH.

These considerations led to the construction of a photographic spectrometer designed especially for the measurement of the absolute value of the glancing angles of Mo Kα and Kβ with the crystal in the Laue position. The instrument consisted essentially of a thin calcite crystal placed against a sharply defined slit through which the beam of x-rays, reflected from the internal planes of the crystal, passed to the photographic plate, as shown in Figs. 17 A and C. The vertical, transverse internal planes of the crystal were aligned perpendicular to the photographic plate, so that by changing the position of the spectrograph relative to the x-ray tube, as shown in the figures, any specified spectral line could be photographed in two positions, P and P', at opposite ends of the plate. By measuring the distance PP' between the lines and the distance ON from slit to plate, the Bragg angle θ effective in these internal reflections could be determined. A comparison of this value of the Bragg angle with the accepted value of the Bragg angle for this same spectral line reflected at the surface of calcite provided a direct comparison of the lattice constants effective in these two cases, once the possibility of a difference in departure from the simple Bragg equation, $n\lambda = 2\, d \sin\theta$, due to the difference in the effect of the index of refraction in the two cases, had been taken into account.

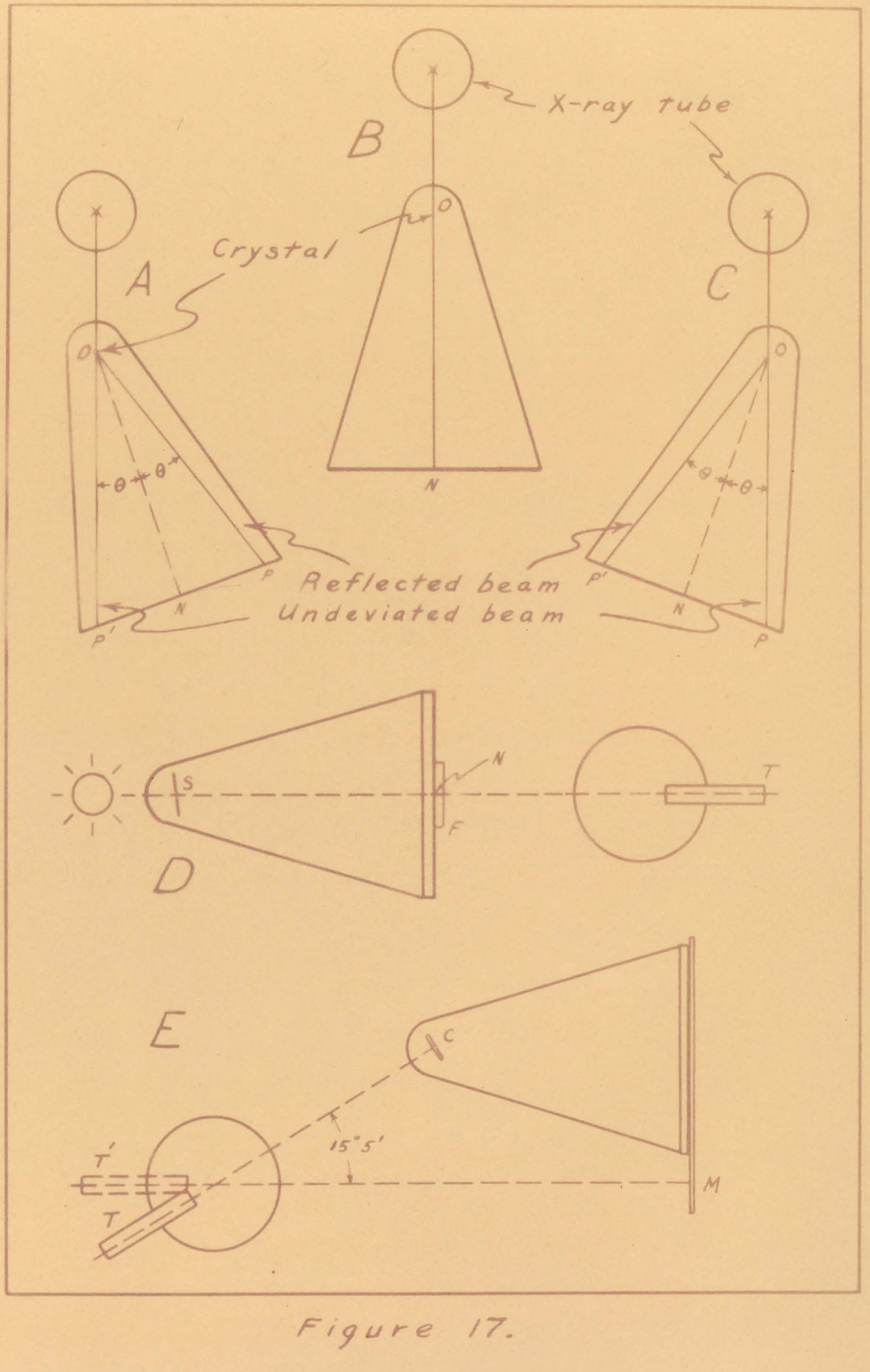

X-ray tube
B
O
N
Crystal
A
O
θ
θ
C
O
θ
θ
Reflected beam
Undeviated beam
P
P'
N
P'
P
N
D
S
N
F
T
E
C
15° 5'
T'
T
M

Figure 17.

C. DESCRIPTION OF THE SPECTROGRAPH.

When completed the spectrograph appeared as shown in Fig. 18. The base plate was a steel casting, 29 1/4 inches long, 8 1/2 inches wide at one end and tapering to 3 1/2 inches at the other. The holder for the photographic plate was also of cast steel, and so machined that the vertical face was accurately perpendicular to the base. It was provided with a lead shield to enable half the plate to be screened from the undeviated radiation while the other half was being exposed to the reflected spectral lines, as shown in Fig. 18. The holder for the crystal and slit was turned from brass and mounted on a conical pivot of steel which fit snuggly in the conical journal of a brass cylinder screwed to the base plate. The jaws of the slit were machined from type metal and the two edges effective in defining the slit were very sharp. The slit-holder, like the crystal-holder, was mounted on a sliding block by means of four screws, three being in tension and surrounding the central one which was under pressure, so that both the slit and the crystal could be adjusted and held in any desired position.

A lead screen of pyramidal shape was arranged to extend from the plate-holder to the slit, in order to protect the photographic plate from stray radiation. The photographic plate was protected from light by means of a) a velvet-lined box which fitted closely over the plate-holder, from the rear, and b) a piece of black paper cemented over the window in the plate-holder. When in position, the photographic plate was backed by a steel plate, which was faced with felt and held firmly against the photographic plate by means of two clamping screws.

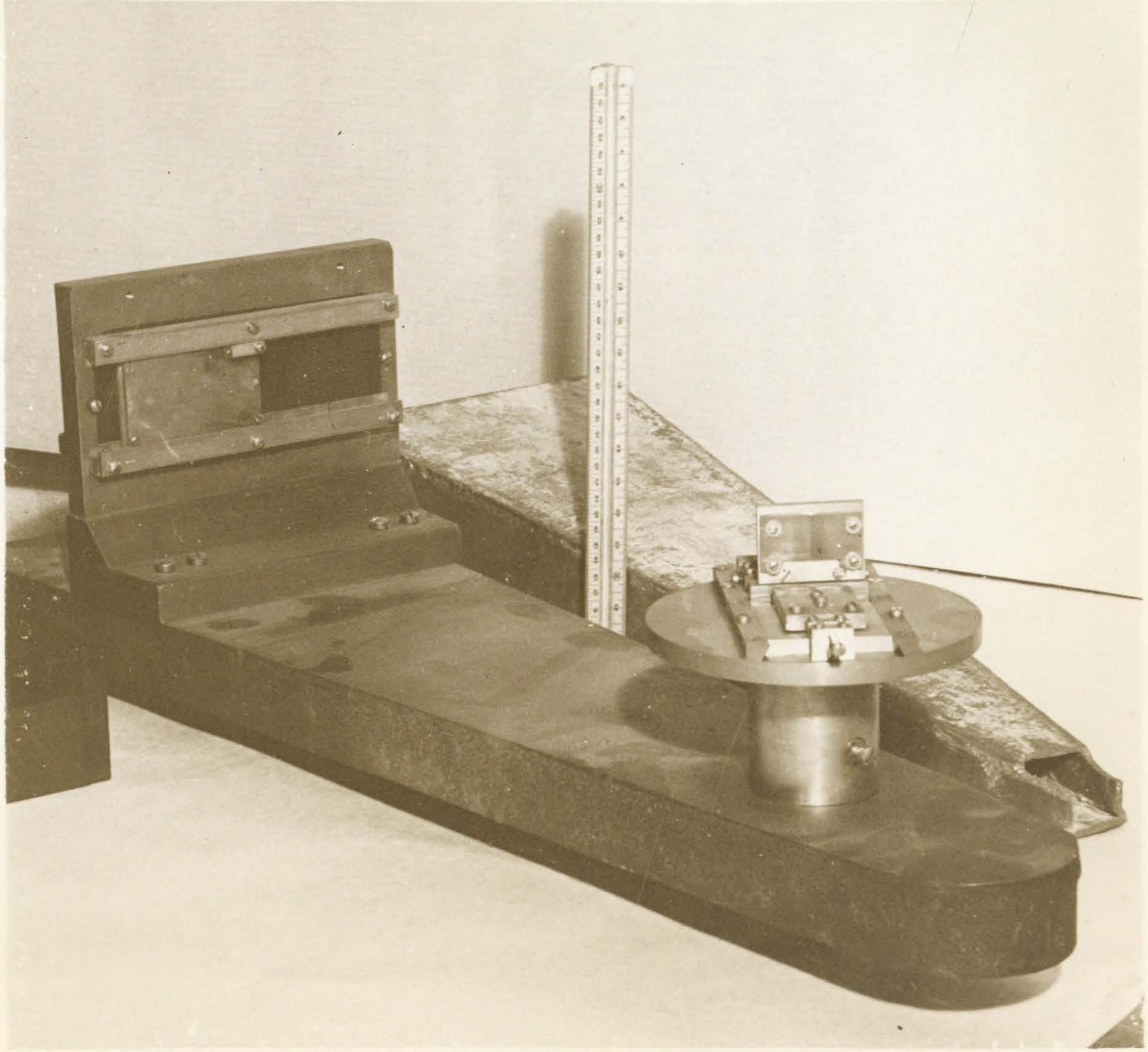


Figure 18.

The small calcite crystal that had been used on the double spectrometer was remounted for use on the spectrograph. The new mounting was similar to the old one in that it consisted of a brass block provided with a slot in which the crystal was held with beeswax, but in this case the block extended the entire width of the crystal.

D. EXPERIMENTAL PROCEDURE

1. Method of Aligning the Spectrograph.

The procedure used in aligning the spectrograph was as follows: 1) The slit was rendered parallel to and coincident with its axis of rotation by means of a micrometer microscope. 2) The point N where the perpendicular ST from the slit to the plate-holder intersected the latter was located by the arrangement of optical instruments shown in Fig. 17 D. The telescope T was moved laterally until a position was found in which the slit was coincident with the vertical cross-hair of the Gauss eyepiece of the telescope, and at the same time the line of sight was shown to be normal to the surface of the plate-holder by means of the reflection of the image of the vertical cross-hair to the Gauss eyepiece from the surface of the optical flat F. The point N was then marked on the plateholder. 3) A knife-edge was placed across the plate-holder at N and the distance SN from slit to plate-holder was measured by means of a cathetometer constructed by Mr. Fred Hensen and equipped with a Brown and Sharpe scale. This measurement was repeated five times by two observers, and the mean value computed. 4) The slit was then removed from the turn-table, and the spectrograph was leveled so that the axis of rotation of the turn-table was accurately vertical.

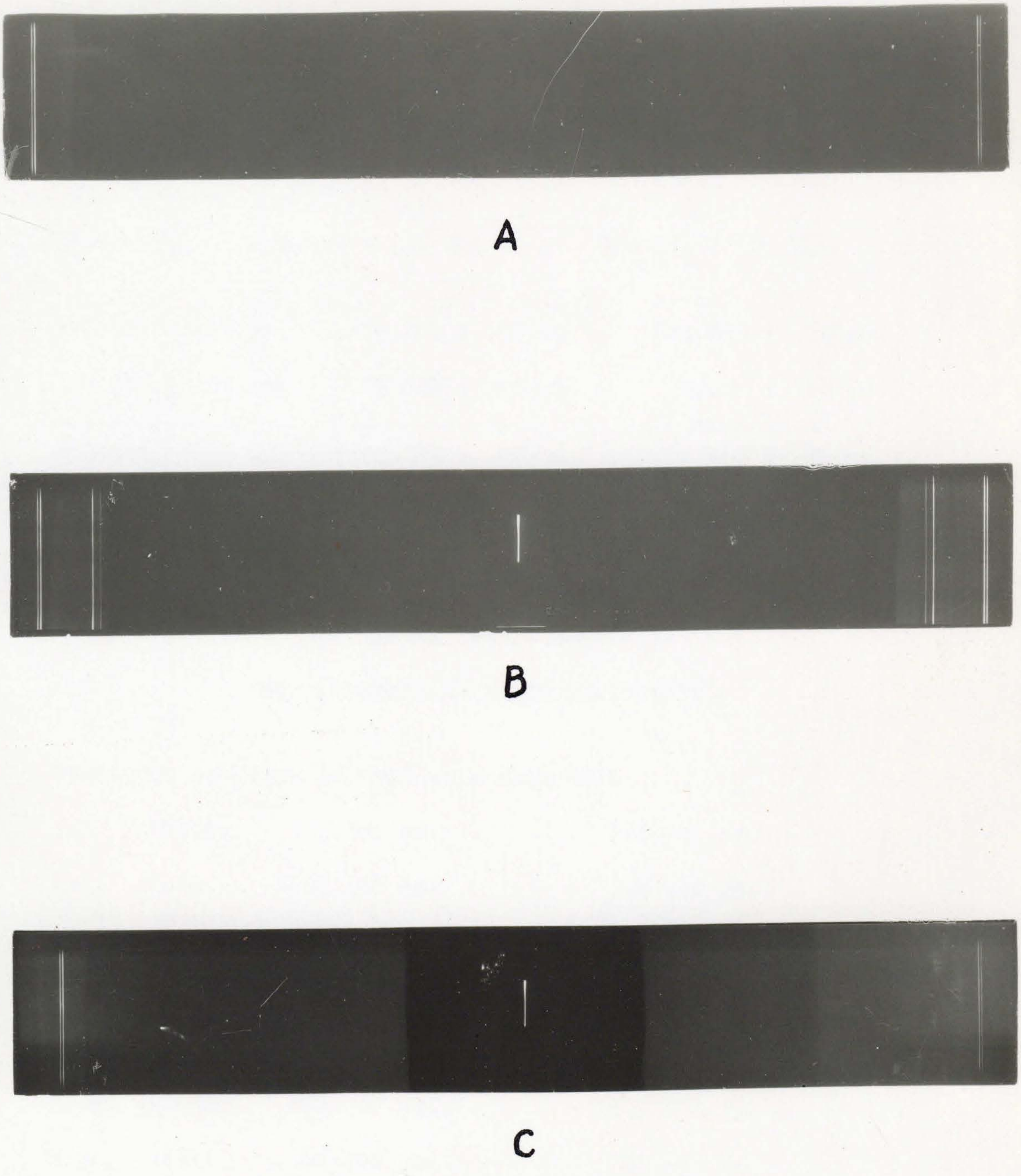
5) The calcite crystal was mounted on the turn-table, and the level checked. 6) An optical spectrometer was placed in front of the spectrograph, as shown in Fig. 17 E, so that the telescope T, which was provided with a Gauss eyepiece, could be sighted first at the crystal C and then at the mirror M by turning through an angle of 15° 5', which is the complement of the smaller projected dihedral angle of calcite.[26] 7) The optical spectrometer was then leveled so as to make its axis of rotation accurately vertical. 8) The line of sight of the spectrometer telescope was rendered accurately normal to the axis of rotation of the crystal turn-table by simultaneous adjustments of the crystal-holder and the telescope, splitting the difference in adjustment on every 180° rotation of the crystal, until the cross-hairs in the Gauss eyepiece coincided with their image reflected from either surface of the crystal. 9) The telescope was then sighted on the mirror M which was flush against the plate-holder, and from this reference position the telescope was turned through the calcite angle of 15° 5', so as to sight on the crystal. The optical spectrometer was then in its final state of adjustment, with its line of sight normal to the axis of rotation of the crystal turn-table and at the desired angle with reference to the plate-holder. 10) The crystal-holder was removed from the turn-table and the slit was mounted in place. 11) The slit was re-aligned in the axis of rotation and parallel to it, as in 1). 12) The crystal-holder was slid back into place on the turn-table, and the crystal was adjusted until both its faces and its narrow silvered edge reflected the image of the cross-hairs into the telescope in coincidence with the

cross-hairs themselves. 13) Then the crystal-holder was slid toward the slit-holder until the distance between crystal and slit was reduced to a few thousandths of an inch. 14) Finally the turn-table was rotated until the image of the cross-hairs reflected from the flat face of the crystal returned in coincidence, which signified that the crystal was in its desired position, with the narrow silvered edge, and the internal reflecting planes, normal to the plate-holder. The turn-table was then clamped in place, thereby completing the adjustment of the various parts of the spectrograph.

The next step was the alignment of the spectrograph relative to the x-ray tube. The spectrograph was first placed in the position shown in Fig. 17A, in order to reflect the Mo $K\alpha$ lines to the right end of the plate, at P. From considerations of symmetry it was seen that when the spectrograph was properly situated relative to the x-ray tube, the undeviated beam should meet the plate-holder at the same point P' to which the spectral lines would be reflected when the spectrograph was in the alternative position shown in Fig. 17C. Since this point P' could be found by calculations, it was possible to align the spectrograph visually by means of a fluorescent screen. A vertical wire of lead was placed at P', and the spectrograph was rotated about the axis O until the shadow of the lead wire divided the undeviated beam into two equal parts. This position of the spectrograph was marked and the process repeated with the instrument in the alternative position, as in Fig. 17C. The alignment of the apparatus was then complete.

2. Procedure Followed in Taking the Photographs of the Spectra.

On March 1, 1934 the first photograph was taken. The time of exposure was three hours for each end of the plate. The resulting photograph, reproduced in Fig. 19A, showed the lines of the Mo $K\alpha$ doublet. Although the measurements indicated a value of the Bragg angle which was accurately equal to the accepted value for surface reflection, a second photograph was taken, for the purpose of including on the plate the position of the center-line. This was accomplished by placing a narrow slit in front of the plate in coincidence with the previously determined center-line, and exposing the slit and plate for 15 minutes to the radiation transmitted directly through the crystal when the spectrograph was in the position of Fig. 17B. Then each of the reflected spectral lines, Mo $K\alpha_{1,2}$ right, $K\beta_{1,3}$ right, $K\alpha_{1,2}$ left, and $K\beta_{1,3}$ left, was photographed for six hours. The resulting plate, reproduced in Fig. 19B, showed $K\beta_2$ also. The distances between corresponding pairs of lines were carefully measured on a comparator, and recorded in Table Twenty-nine, which is included in the following section of data.



A

B

C

Figure 19.

TABLE TWENTY-SEVEN.

MEASUREMENT OF DISTANCE FROM SLIT TO PLATE.

Positions of slit and plate as read on Brown and Sharpe cathetometer-scale.

Slit,	683.550 ± .05 mm,	683.525 ± .05 mm.
Plate,	77.025 ± .05 mm.	77.050 ± .05 mm.
Distance,	606.525 ± .05 mm.	606.475 ± .05 mm.

678.825 ± .05 mm.	678.900 ± .05 mm.	678.900 ± .05 mm.
72.350 ± .05 mm.	72.325 ± .05 mm.	72.400 ± .05 mm.
606.475 ± .05 mm.	606.575 ± .05 mm.	606.500 ± .05 mm.

Mean distance from slit to plate, 606.51 ± .05 mm.

TABLE TWENTY-EIGHT

MEASUREMENTS OF FIRST PHOTOGRAPH.

Positions of lines as read on comparator.

$K\alpha_1$ right,	192.307 mm.	192.301 mm.
$K\alpha_1$ left,	49.582 mm.	49.588 mm.
Distance,	142.725 mm.	142.713 mm.

Mean distance between $K\alpha_1$ lines, 142.719 mm.

$K\alpha_2$ right,	192.732 mm.	192.735 mm.
$K\alpha_2$ left,	49.152 mm.	49.147 mm.
Distance,	143.580 mm.	143.588 mm.

Mean distance between $K\alpha_2$ lines, 143.584 mm.

TABLE TWENTY-NINE

MEASUREMENTS OF SECOND PHOTOGRAPH.

Kα_1	right,	179.944 mm.	Kα_1	left,	37.226 mm
		179.944			37.233
		179.947			37.226
	Mean	179.945 mm.		Mean,	37.228 mm.

Mean distance between Kα_1 lines, 142.717 mm.

Kα_2	right,	180.382 mm.	Kα_2	left,	36.796 mm.
		180.384			36.796
		180.386			36.800
	Mean	180.384 mm		Mean,	36.797 mm.

Mean distance between Kα_2 lines, 143.587 mm.

K$\beta_{1,3}$	right,	172.114 mm.	K$\beta_{1,3}$	left,	45.055 mm.
		172.117			45.055
		172.124			45.055
	Mean,	172.118 mm.		Mean,	45.055 mm.

Mean distance between K$\beta_{1,3}$ lines, 127.063 mm.

Kβ_2	right,	170.951 mm.	Kβ_2	left,	46.204 mm.
		170.956			46.223
		170.953			46.210
	Mean,	170.953 mm.		Mean,	46.212 mm.

Mean distance between Kβ_2 lines, 124.741 mm.

Table Twenty-nine, Continued.

Center-line, 109.151, 109.151, 109.152, 109.155, 109.153,
109.148, 109.149, 109.150, 109.156, 109.150.

Mean position of center-line, 109.151 mm.

Distance from $K\alpha_1$ left, to center-line,	71.923 mm.
Distance from $K\alpha_1$ right, to center-line,	70.794 mm.
Difference,	1.129 mm.

COMPUTATIONS.

Calculation of the anticipated distance between the $K\alpha_1$ lines on the photographic plate, assuming a Bragg angle equal to the generally accepted value of the glancing angle for the case of surface reflection from calcite, as discussed on the next page.

Let x = one half the distance between the $K\alpha_1$ lines on the plate.

m = the distance from slit to plate,

= 606.51 mm.

θ = accepted value of the Bragg angle for surface reflection of Mo $K\alpha_1$ from calcite.

= $6^\circ\ 42'\ 35.5''$

Then $x = m \tan\theta$, so $\log x = \log m + \log\tan\theta$.

$$\log m = \log 606.51 = 2.78284$$

$$\log\tan 6^\circ\ 42'\ 35.5'' = \underline{9.07059 - 10}$$

$$\log x = 1.85345 \qquad \text{so } x = 71.356\bar{7} \text{ mm.}$$

Hence the distance between the $K\alpha_1$ lines = $2x = 142.713\bar{4}$ mm.

E. RESULTS AND CONCLUSIONS.

Comparison of the Bragg Angle for Internal Reflection with the Accepted Value for Surface Reflection.

The Bragg angle for surface reflection from calcite of the $K\alpha_1$ spectral line of molybdenum has been determined experimentally to a high degree of accuracy. The following table, quoted from an article by Bearden,[32] shows the results of these measurements.

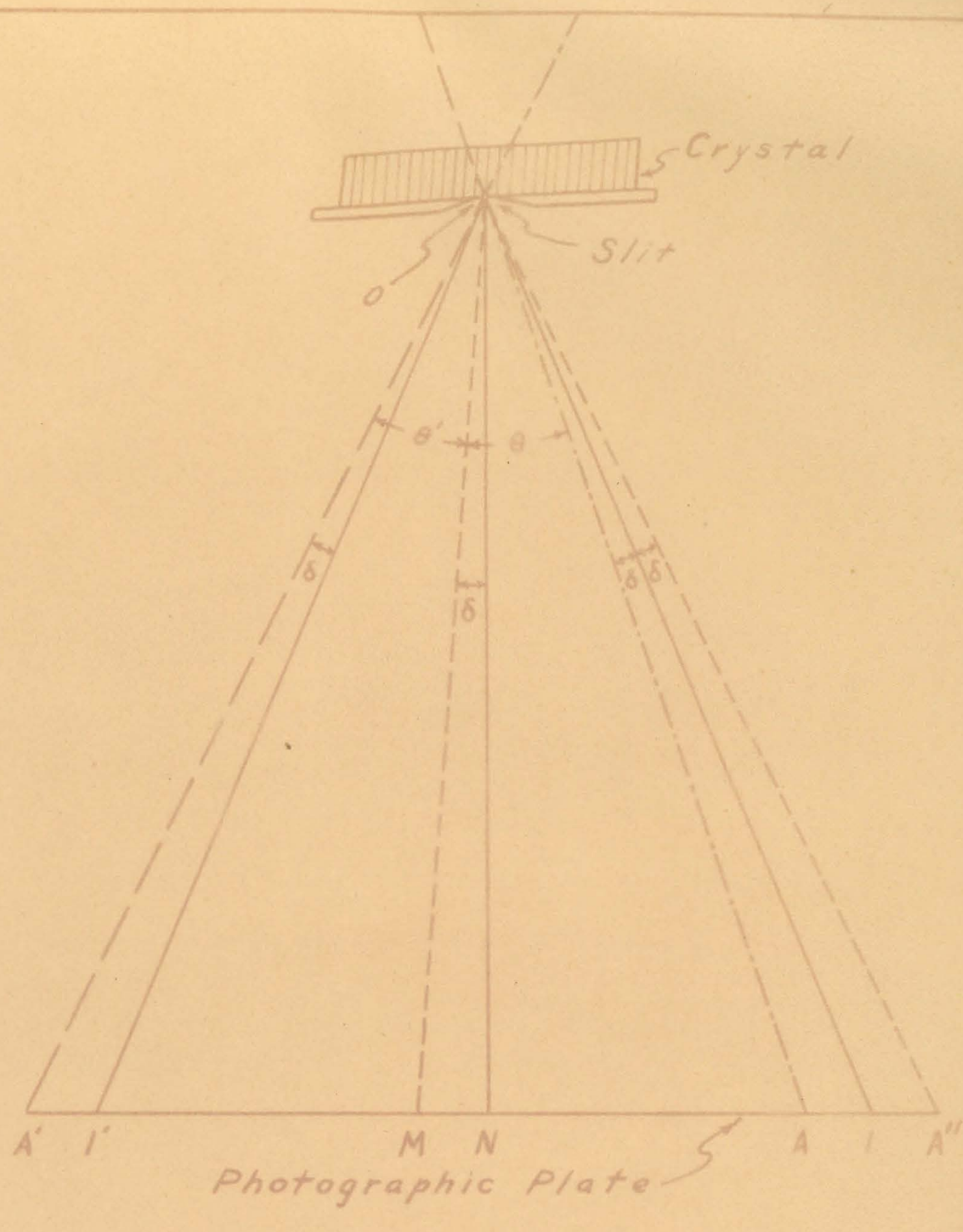
Observer	Date	Bragg Angle
Leide[27]	1925	6° 42' 34.3"
Allison and Armstrong[28]	1925	6° 42' 33.3"
Larsson[29]	1927	6° 42' 35.3"
Compton[30]	1931	6° 42' 35.9"
Tu[31]	1932	6° 42' 35.5"
Bearden[32]	1933	6° 42' 35.4"

The mean of the last four values is 6° 42' 35.5", which we have taken as the generally accepted value of the Bragg angle of Mo $K\alpha_1$ for surface reflection from calcite, for purposes of comparison with the results of this experiment.

Using the accepted value of the Bragg angle, the distance between the $K\alpha_1$ lines on the plate has been computed as shown in the section on computations. The distance thus computed is 142.713 mm. This is to be compared with the value 142.719 mm. obtained from the first photographic plate, and 142.717 mm. obtained from the second.

The difference of .005 mm. between the computed value for surface reflection and the mean of the two experimental values for internal reflection is but .0035 of one per cent. It is to be noted, however, that no account has yet been taken of either a) the possible error in the measurement of the Bragg angle caused by the deviation of the internal planes of the crystal from their assumed position normal to the photographic plate, or b) the difference of the effect of the index of refraction in the two cases, which would influence conclusions as to the equality of the effective lattice constants in the two cases.

The magnitude of the effect of the imperfect alignment of the crystal may be accurately determined in the following manner. According to the data given in Table Twenty-nine, there was a difference of 1.129 mm. in the distances from the center-line to the $K\alpha_1$ lines on opposite ends of the photographic plate. If the location of the center-line is assumed to be correct, then this difference is to be attributed to the fact that the internal planes of the crystal were not accurately normal to the plate-holder, so that each of the $K\alpha_1$ lines was shifted .5645 mm. from the ideal position of symmetry about the center-line. The situation is represented diagrammatically in Fig. 20, in which I and I' are the ideal symmetrical positions of the spectral lines from which the actual positions A and A' depart by .5645 mm. because of the deviation δ in the alignment of the crystal. The angle δ is seen to be approximately $.5645 \cos \theta \div 610.7 = 9.21 \times 10^{-4}$ radians. A more lengthy and exact calculation gave $\delta = 9.179 \times 10^{-4}$ radians, and by means of this, two values of the actual experimental Bragg angle,



Dimensions of Spectrograph and Spectra :

First Experiment. Reflection from 100 planes.	Second Experiment. Reflection from $1\bar{1}0$ planes.
$A'A = 142.717$ mm.	$A'A = 138.6145$ mm.
$A'N = 71.923$ mm.	$A'N = 69.6542$ mm
$AN = 70.794$ mm.	$AN = 68.9603$ mm.
$AA'' = 1.129$ mm.	$AA'' = .6939$ mm.
$ON = 606.51$ mm	$ON = 482.55$ mm

Figure 20.

namely θ and θ', were computed as follows:

$$\theta' = \text{angle } A'OM = \text{angle } A'ON - \text{angle } MON = \tan^{-1}\left(\frac{71.923}{606.51}\right) - \delta$$

$$= 6^\circ\ 45'\ 46.1'' - 3'\ 9.5'' = 6^\circ\ 42'\ 36.6''$$

$$\theta = \text{angle } AOM = \text{angle } AON + \text{angle } MON = \tan^{-1}\left(\frac{70.794}{606.51}\right) + \delta$$

$$= 6^\circ\ 39'\ 27.2'' + 3'\ 9.5'' = 6^\circ\ 42'\ 36.7''$$

The remarkable agreement between these two values of the Bragg angle is an indication of the correctness of the assumption underlying their calculation, namely, that the center-line was accurately located at the foot of the perpendicular from the slit to the plate-holder.

A comparison of the mean of these two values of the Bragg angle, namely, 6° 42' 36.65" or 24156.65", with the accepted value for the case of surface reflection, which is 6° 42' 35.5" or 24155.5" shows that the experimental values of the Bragg angle for internal and surface reflection differ by 1.15", or .00475 of one per cent. On comparing this with the difference of .0035 per cent obtained by neglecting the deviation of the crystal from perfect alignment, it is seen that the improvement in accuracy obtained by taking the deviation into account is negligible. The more accurate calculation is essential, however, in demonstrating the perfection of the alignment of the crystal and the accuracy in locating the center-line.

It is to be noted at this point that in view of the fact that neither of the experimental values of the Bragg angle under comparison has been corrected for the effect of index of refraction, it is not permissible to conclude that the grating constants effective in the two cases are equal, to the same degree of accuracy as are the angles themselves. As was explained on page 9, the index of refraction causes a decrease in the apparent Bragg angle for the case of internal reflection of Mo $K\alpha_1$ from calcite which is given by $(\theta_0 - \theta)_\alpha = \tan^{-1}(.5125 \times 10^{-6}) = .1058$ seconds. Compton and Allison[33] have computed the increase in apparent Bragg angle for the case of surface reflection of Mo $K\alpha_1$ from calcite, and found it to be 3.60 seconds. Consequently the corrected angles θ_0 for use in the simple Bragg equation, $n\lambda = 2d \sin \theta_0$, are 6° 42' 36.76" and 6° 42' 31.9" for the Laue and Bragg cases, respectively. The difference is 4.86 seconds, or .0201 per cent of the Bragg angle. Then, from consideration of the equation $n\lambda = 2 d_B \sin \theta_B = 2 d_L \sin \theta_L$, we may conclude that the grating constants d_B and d_L effective in the two cases are equal, to an accuracy of .02 per cent.

Summary of Conclusions.

The comparison of the experimental value of the Bragg angle of Mo $K\alpha_1$ reflected internally from calcite with the accepted experimental value of the Bragg angle for surface reflection indicated a difference of but .0201 of one per cent. The measurements involved in this experiment were reliable to at least this degree of accuracy, because the least accurate of the measurements, namely that of the distance from slit to photographic plate, was obtained with an accuracy estimated at .01 of one per cent. This experiment indicates, then, that to an accuracy of .02 per cent, the Bragg angles for internal and surface reflection of x-rays from the 100 planes of calcite are equal.

Finally it is to be concluded that the lattice constant for the 100 planes of calcite effective in the internal reflection of x-rays by transmission through the crystal is equal to the lattice constant for the 100 planes effective in the ordinary Bragg reflection at the surface of the crystal, to an accuracy of .02 per cent.

PART IV. SUPPLEMENTARY EXPERIMENT ON SECONDARY STRUCTURE.

After the K spectrum of molybdenum had been photographed by reflection from the 100 planes of the calcite crystal, a similar photograph was made by reflection from the $1\bar{1}0$ planes. This experiment was suggested by Dr. Fritz Zwicky, who believed that the mean effective grating constant for the $1\bar{1}0$ planes might prove to be less than the interplanar distance predicted by a geometrical calculation based on the experimental value of the interplanar distance for the 100 planes. The possibility of the occurence of an experimentally demonstrable diminution of the effective grating constant was to be expected from consideration of Dr. Zwicky's theory of secondary structure.[14,15]

A. EXPERIMENTAL PROCEDURE.

In order to locate the plate-holder of the spectrograph at the proper distance from the crystal, it was necessary to know the value of the Bragg angle of the chosen spectral line, Mo $K\alpha_1$, when reflected from the $1\bar{1}0$ planes. This was computed, as shown in the section on computations, from a knowledge of a) the Mo $K\alpha_1$ Bragg angle for the 100 planes, and b) the dihedral angle of calcite.[26] The resulting value of $8^\circ\ 10'\ 24''$ indicated that the plate-holder should be moved some 12 cm. nearer the crystal, in order that the $K\alpha_1$ lines would not fall beyond the ends of the plate. This change was made and the spectrograph was aligned as before. Each end of the photographic plate was exposed for 11 1/2 hours, while the x-ray tube was operating at 20 milliamperes and 48 kilovolts. The center-line was exposed for 15 minutes, at 10 milliamperes. The pyramid-shaped lead shield was too long for use with the plate-holder in its new position, so that the photographic plate, reproduced in Fig. 19C, showed some general fogging caused by scattered x-rays. The distances between spectral lines and between slit and plate were measured as in the previous experiment, and the results were recorded in the following tables.

TABLE THIRTY

DATA RELATING TO THE THIRD PHOTOGRAPH.

Measurements of the Distance from Slit to Plate.

Slit,	596.375 ± .05 mm.	596.400 ± .05 mm.
Plate,	113.825 ± .05 mm.	113.850 ± .05 mm.
Distance,	482.550 ± .05 mm.	482.550 ± .05 mm.

Mean distance from slit to plate, 482.55 ± .05 mm.

Measurements of the Distance between $K\alpha_1$ Lines.

$K\alpha_1$	right,	179.696 mm.	$K\alpha_1$	left,	41.078 mm.
		179.692			41.082
		179.695			41.080
		179.698			41.082
	Mean,	179.6950 mm.		Mean,	41.0805 mm.

Mean distance between $K\alpha_1$ lines, 138.6145 mm.

Measurements of the Distance between $K\alpha_2$ Lines.

$K\alpha_2$	right,	180.127 mm	$K\alpha_2$	left,	40.664 mm.
		180.128			40.660
		180.128			40.662
		180.129			40.662
	Mean,	180.1280 mm.		Mean,	40.6620 mm.

Mean distance between $K\alpha_2$ lines, 139.4660 mm.

Table Thirty, Continued.

Location of Center-line.

Center-line, 110.736 mm., 110.740 mm., 110.735 mm., 110.728 mm.

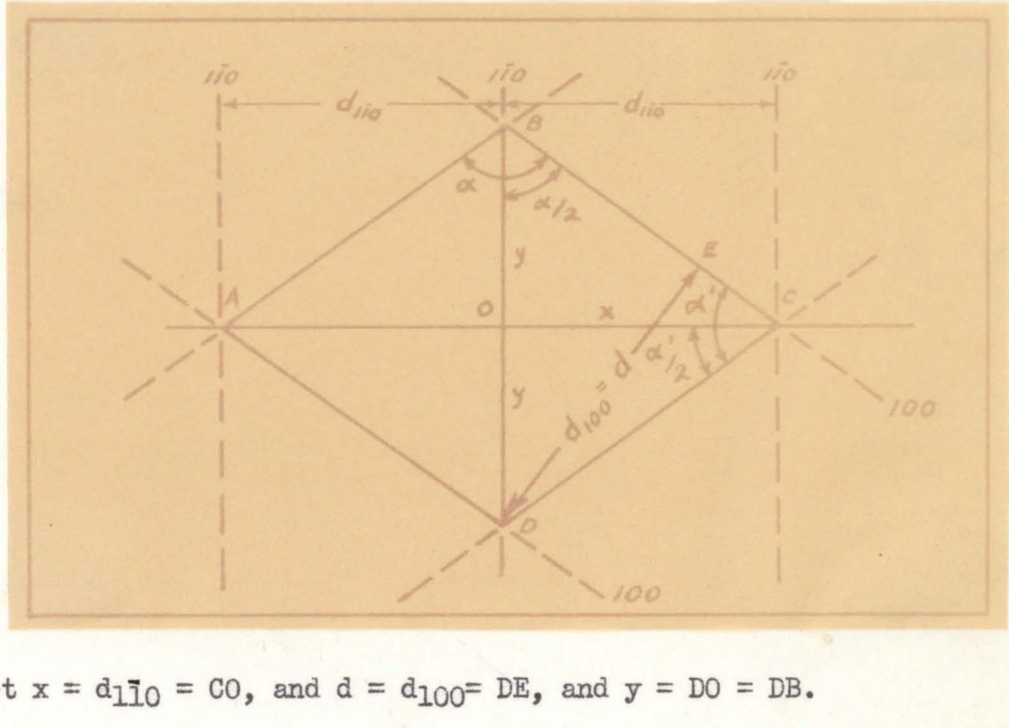
Mean position of center-line, 110.7347 mm.

Distance from $K\alpha_1$ left, to center-line,	69.6542 mm.
Distance from $K\alpha_1$ right, to center-line,	68.9603 mm.
Difference,	.6939 mm

COMPUTATIONS.

Calculation of the Bragg Angle for Reflection of Mo Kα_1 from the $1\bar{1}0$ Planes of Calcite, for Comparison with the Experimental Value.

According to Bearden's precise experimental determination,[25] the dihedral angle of calcite is 105° 3' 29". The accepted value of the Bragg angle for surface reflection of Mo Kα_1 is 6° 42' 35.5". Knowing these, the angle of reflection from the $1\bar{1}0$ planes may be obtained from consideration of the unit cell of calcite, as follows.



Let $x = d_{1\bar{1}0} = CO$, and $d = d_{100} = DE$, and $y = DO = DB$.

Then $x = y \cot(\alpha'/2) = y \tan(\alpha/2)$.

But $y = \dfrac{d}{2 \sin(\alpha/2)}$ so $x = \dfrac{d \tan(\alpha/2)}{2 \sin(\alpha/2)} = \dfrac{1}{2} \dfrac{d}{\cos(\alpha/2)}$

Or $d_{1\bar{1}0} = \dfrac{1}{2} \dfrac{d_{100}}{\cos(\alpha/2)}$

But $n\lambda = 2\, d_{100} \sin \theta_{100} = 2\, d_{1\bar{1}0} \sin \theta_{1\bar{1}0}$

Hence $\dfrac{\sin \theta_{1\bar{1}0}}{\sin \theta_{100}} = \dfrac{d_{100}}{d_{1\bar{1}0}} = 2 \cos (\alpha/2)$

and therefore $\sin \theta_{1\bar{1}0} = 2 \cos (\alpha/2) \sin \theta_{100}$

where $(\alpha/2) = (105^\circ\ 3'\ 29'') \div 2 = 52^\circ\ 31'\ 44.5''$

and $\theta_{100} = 6^\circ\ 42'\ 35.5''$

So $\log \sin \theta_{1\bar{1}0} = \log 2 + \log \cos (\alpha/2) + \log \sin \theta_{100}$

$$\begin{array}{ll}
\log 2 = & .30103 \\
\log \cos 52^\circ\ 31'\ 44.5'' = & 9.78416 - 10 \\
\log \sin 6^\circ\ 42'\ 35.5'' = & \underline{9.06761 - 10} \\
\log \sin \theta_{1\bar{1}0} = & 9.15280 - 10
\end{array}$$

Hence $\theta_{1\bar{1}0}$, the Bragg angle for reflection of Mo Kα_1 from the $1\bar{1}0$ planes of calcite is 8° 10' 23.85". The neglect of the effect of the index of refraction introduces an uncertainty of .015 of one per cent in this result.[33]

B. RESULTS AND CONCLUSIONS.

Comparison of the Bragg Angle for Internal Reflection from the $1\bar{1}0$ Planes with the Value Calculated from the Case of the 100 Planes.

Having obtained the experimental data necessary for determining the Bragg angle for the reflection of Mo $K\alpha_1$ from the $1\bar{1}0$ planes of calcite, it was next necessary to determine the effect of the deviation of the reflecting planes from the ideal position normal to the plate-holder. Referring again to Fig. 20, the angle of deviation $A'OI'$ or δ is seen to be approximately equal to $(.6939 \div 2) \cos \theta \div 482.55 = 7.15 \times 10^{-4}$ radians. A lengthy calculation gave $\delta = 7.172 \times 10^{-4}$ radians, or 2' 28.0". Consequently the two values θ and θ' of the Bragg angle are as follows:

$$\theta = \text{angle } AOM = \text{angle } AON + \text{angle } MON = \tan^{-1}\left(\frac{68.9603}{482.55}\right) + \delta$$

$$= 8^\circ\ 7'\ 58.0'' + 2'\ 28.0'' = 8^\circ\ 10'\ 26.0''$$

$$\theta' = \text{angle } A'OM = \text{angle } A'ON - \text{angle } MON = \tan^{-1}\left(\frac{69.6542}{482.55}\right) - \delta$$

$$= 8^\circ\ 12'\ 49.2'' + 2'\ 28.0'' = 8^\circ\ 10'\ 21.2''$$

Hence the mean experimental value of the Bragg angle is 8° 10' 23.6".

As was shown in the section on computations, the value of the Bragg angle obtained by a direct trigonometrical calculation, based upon the known geometrical form of calcite and the accepted value of the Bragg angle for surface reflection from the 100 planes, was 8° 10' 23.85". This agrees with the experimental value to an accuracy greatly exceeding the uncertainty of .015 per cent introduced by the limitations of accuracy in measuring the distance from slit to plate and by the neglect of the effect of the index of refraction.

When the accepted value of the Bragg angle for surface reflection from the 100 planes is replaced by the value of the Bragg angle for internal reflection from the 100 planes that was obtained in the first experiment with the spectrograph, the calculated value of the Bragg angle for the $1\bar{1}0$ planes becomes 8° 10' 24.5", which differs from the experimental value of 8° 10' 23.5" by .9" or .003 of one per cent. It is to be noted, however, that the uncertainty in the measurement of the distance from slit to plate was of the order of .01 per cent, and the uncertainty introduced by neglecting the effect of the refractive index was of the order of .015 per cent.

The inescapable conclusion to be reached from consideration of these results is that the anticipated diminution of the mean grating constant effective in the internal reflection of x-rays from the $1\bar{1}0$ planes of calcite is certainly not greater than .015 per cent, if it exists at all.

PART V. SUBSEQUENT EXPERIMENTS WITH THE DOUBLE CRYSTAL SPECTROMETER.

This thesis would not be complete without some attempt to explain the apparent contradiction between the rather inconclusive results of the comparison of the Bragg and Laue spectra by means of the double spectrometer and the conclusive results obtained by means of the photographic spectrograph. This explanation is facilitated by the experimental work performed by Mr. V. L. Bollman subsequent to the completion of the experiments with the spectrograph as described above.

Experience had already shown that the small calcite crystal could easily become sufficiently warped to cause the spectral lines obtained in the Bragg-Laue position to be decidedly asymmetrical. (See Fig. 11.) With this in mind, the crystal was remounted on the spectrometer in such a way as to permit Bragg reflection from either face, and Laue reflection with the beam entering either face. The angular separation of the lines Mo $K\alpha_2$ and $K\beta_2$ was measured with the small crystal set in each of these four positions in turn. The α_2 and β_2 lines were preferred to α_1 and $\beta_{1,3}$ because a) their angular separation is the largest obtainable in the K spectrum, b) they are of more nearly equal intensity than are α_1 and $\beta_{1,3}$ and thus they permit the use of the same voltage on the binants of the electrometer for both lines, and c) β_2 is more symmetrical than $\beta_{1,3}$ and so may be more readily and definitely located, by determining the centroid of area of the entire line rather than that of the upper portion only, as was done with $\beta_{1,3}$.

The results obtained by Mr. Bollman are shown graphically in

Fig. 21 . Eight complete sets of data were taken, two for each of the four positions of the crystal, and as is seen in the illustration, in three of the four cases the two sets of data were in such good agreement both as to intensity and position of the spectral lines on the angle-drum that it was not deemed significant to draw two curves through the dots and crosses used to represent the two sets of data.

The figure also shows that although there is a difference between the angular separation of the lines in the Laue and Bragg cases when either of the two faces of the crystal is considered separately, nevertheless when the angular separations for both faces are averaged together, the difference between them for Bragg and Laue cases is only .225 seconds of arc, or .0071 per cent of the angular separation, in perfect agreement with the results obtained by the photographic method.

The inequality of the angular separation of the spectral lines reflected in the Bragg manner from opposite faces of the crystal, or in the Laue manner with the x-ray beam entering opposite faces, was believed to have been caused by a combination of two factors, namely, a) warping of the thin crystal by strains at the point of support, and b) a slight shift in the position of the x-ray beam in passing from one spectral line to the other. The simultaneous existence of these two imperfections in the apparatus would require that in order to permit double reflection of x-rays of a specific wave-length, it would be necessary to adjust the angle between the crystals to a value depending upon whether the x-ray beam was incident upon the concave or the convex face of the crystal. This may be understood from an examination of the

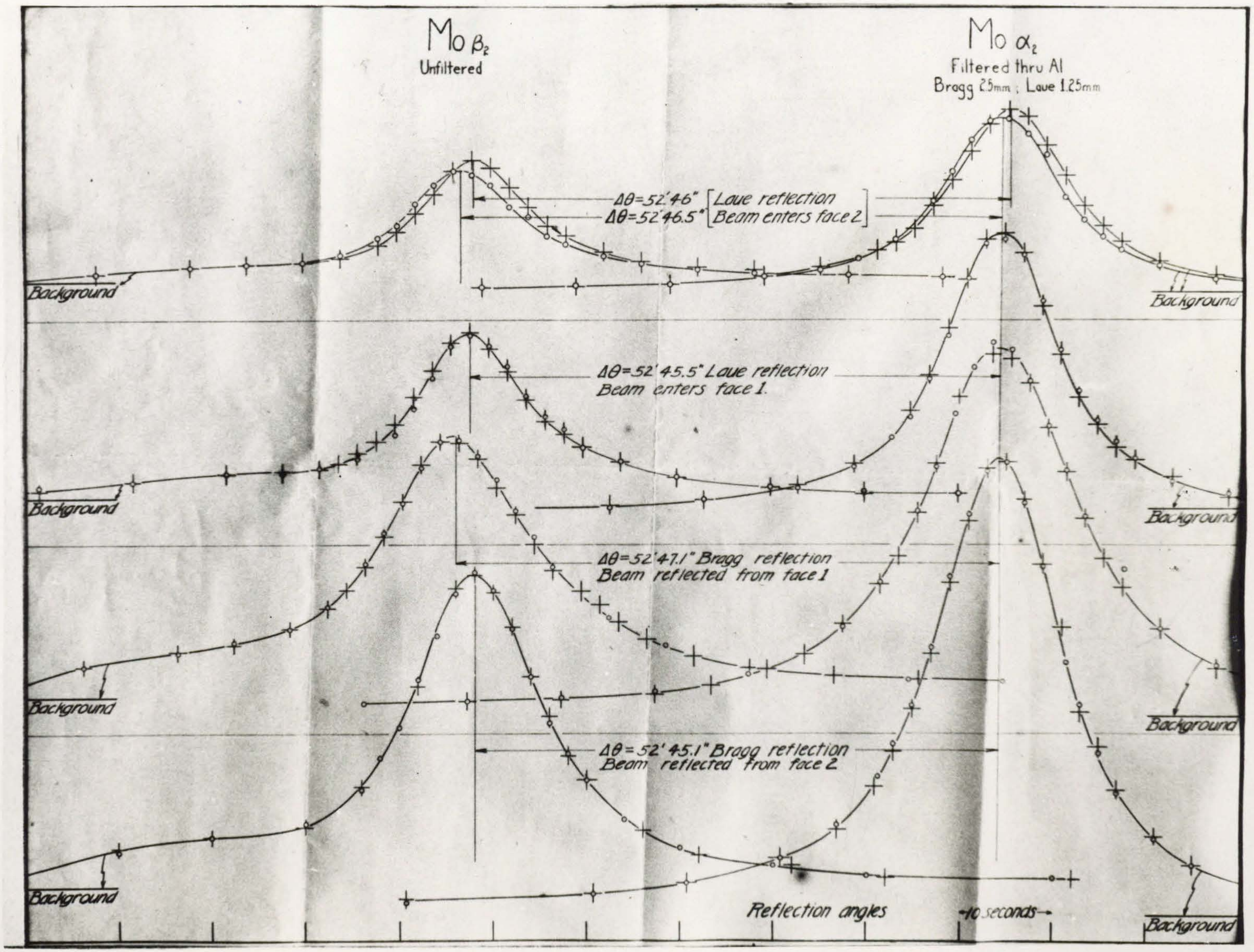
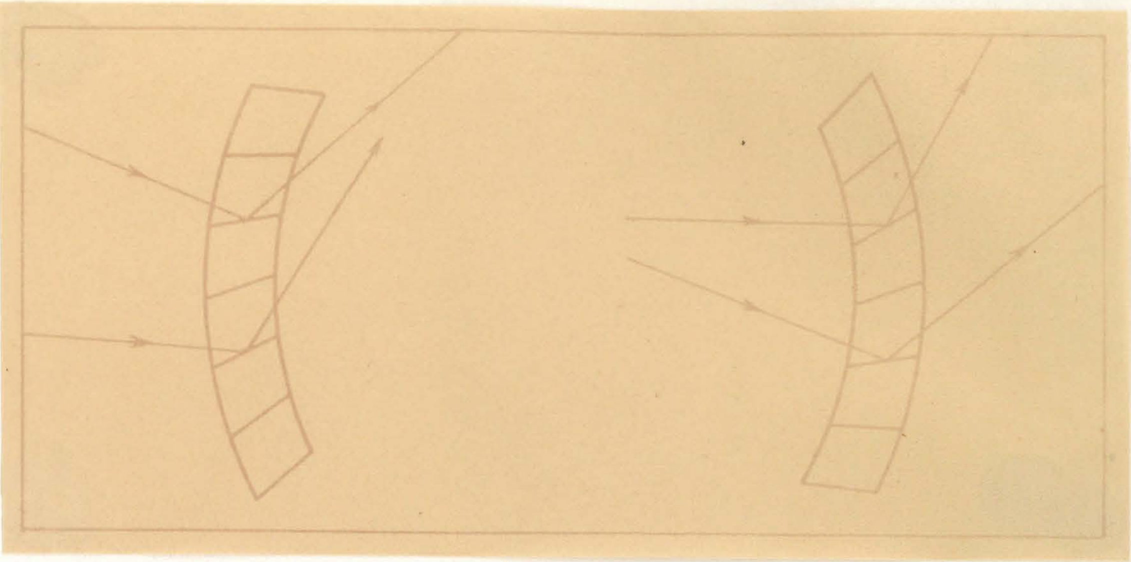

Mo β_2
Unfiltered
Mo α_2
Filtered thru Al
Bragg 2.5mm ; Laue 1.25mm
$\Delta\theta = 52'46''$ [Laue reflection
$\Delta\theta = 52'46.5''$ [Beam enters face 2
$\Delta\theta = 52'45.5''$ Laue reflection
Beam enters face 1.
$\Delta\theta = 52'47.1''$ Bragg reflection
Beam reflected from face 1
$\Delta\theta = 52'45.1''$ Bragg reflection
Beam reflected from face 2
Background
Background
Background
Background
Background
Background
Background
Background
Reflection angles
←10 seconds→

Figure 21.

following figure.



In order to determine whether the thin crystal was actually warped, Mr. Bollman made a series of rocking curves in the parallel position, (1,-1) order, for both Bragg and Laue reflection, using various regions of the small crystal across its breadth. The Laue rocking curves reproduced in Fig. 22 were obtained by restricting the effective portion of the second crystal to the regions shown in the illustration, by means of a movable lead port placed close behind this crystal. The fact that the peaks were located at slightly different positions on the angle-drum proves that the different regions investigated were not mutually parallel. An examination of the curves shows that as the exploring port was moved across the face of the thin crystal the position of the peak shifted in one direction, except at one point, where the peak was apparently double. When the beam was reflected from a region near the edge of the crystal where there could not possibly be any strain from the support, the rocking curve was sharp and symmetrical. Furthermore, in the Bragg

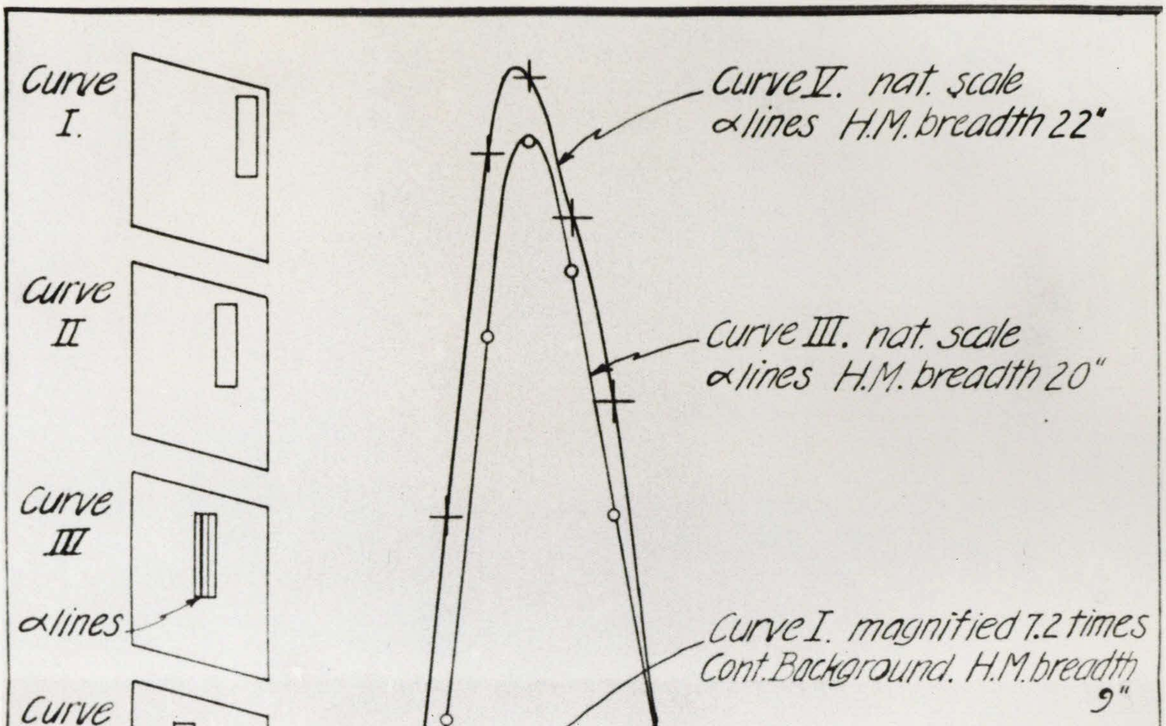

Curve I.
Curve II
Curve III
αlines
Curve IV
Curve V
αlines
Curve V. nat. scale
αlines H.M. breadth 22"
Curve III. nat. scale
αlines H.M. breadth 20"
Curve I. magnified 7.2 times
Cont. Background. H.M. breadth 9"
Curve II. magnified 7.2 times
Cont. Background. H.M. breadth 14"
Curve IV. mag. 7.2 times
Cont. Background.
10"

Figure 22.

rocking curves, shown in Figs.23 and 24, the peak is double only for reflection from one certain face of the crystal; for the other face the peak is symmetrical but broader than a good unwarped crystal would give. From consideration of these data, it was concluded that the crystal was probably curved as shown in the lower left corner of Fig. 22, with a radius of curvature of twenty or thirty meters, and that a small flake not conforming to the curvature of the rest of the crystal was situated somewhere a little to one side of the center of one face.

The exact nature and magnitude of the deformation of the crystal may perhaps best be considered as problematical, but even assuming it definitely known, no irrevocable conclusion can be formed concerning the cause of the inequality in the measured values of the angular separation for Bragg and Laue reflection that were obtained originally, because the crystal had been remounted since those first measurements were made. However, it does seem most probable that the inequality in question is to be attributed to the curvature of the crystal, inasmuch as the angular separations for Bragg and Laue reflection proved to be equal in both a) the experiment in which the effect of curvature was eliminated by employing reflections from both sides of the crystal, and b) the photographic experiment, in which a slight curvature would not affect the results.

Finally, it is to be concluded that the proof of the equality of the grating constants of calcite effective in Bragg and Laue reflection that was demonstrated in the results of the photographic experiment is strengthened, if possible, by the removal of the feeble voice of dissent arising in the contradictory results originally obtained with the double spectrometer.

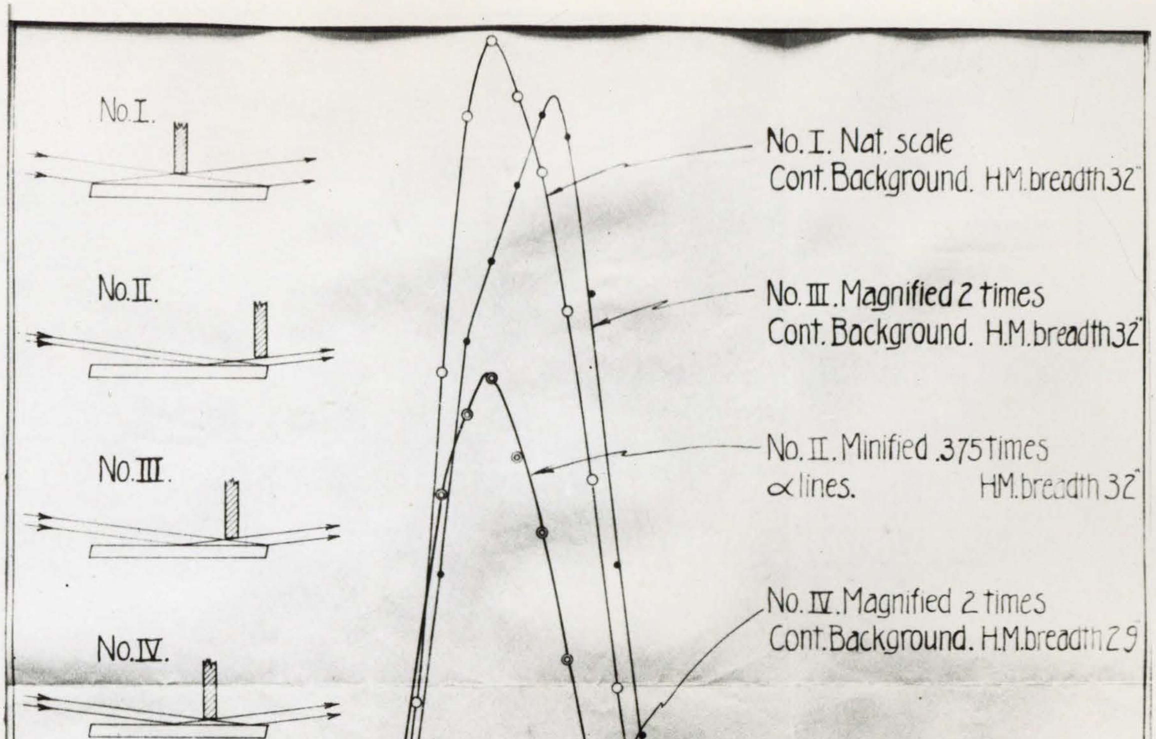

No. I.
No. II.
No. III.
No. IV.
No. V.
No. VI.
No. I. Nat. scale
Cont. Background. H.M. breadth 32"
No. III. Magnified 2 times
Cont. Background. H.M. breadth 32"
No. II. Minified .375 times
α lines. H.M. breadth 32"
No. IV. Magnified 2 times
Cont. Background. H.M. breadth 29"
No. V. Magnified 2 times
Cont. Background. H.M. breadth 36"
No. VI. Magnified 2 times
Cont. Background. H.M. breadth 36"
←10"→

Figure 23.

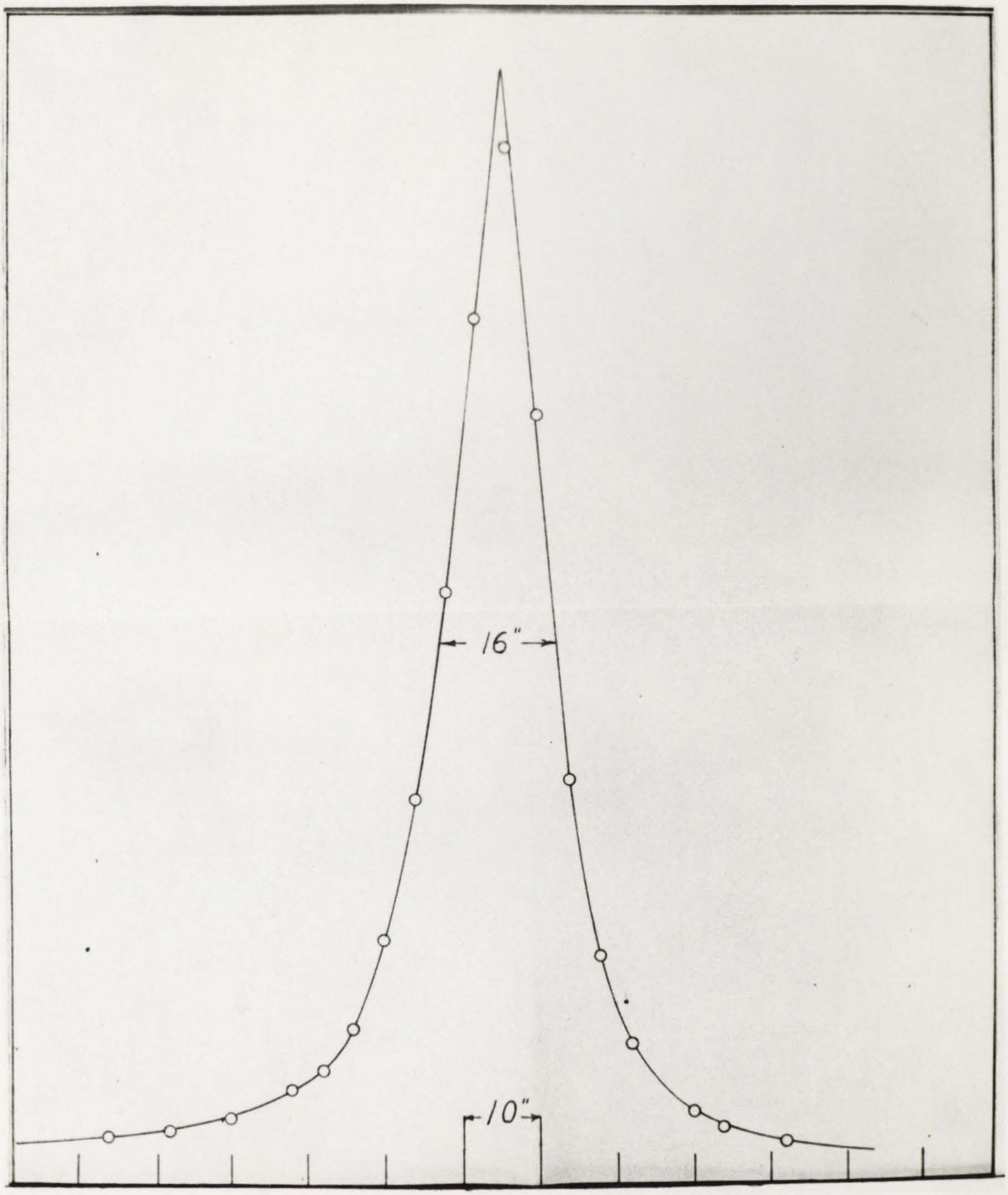


Figure 24.

PART VI. GENERAL SUMMARY OF RESULTS.

1. The comparison, by means of the spectrograph, of the experimental value of the Bragg angle of Mo $K\alpha_1$ reflected internally from calcite with the accepted experimental value of the Bragg angle for surface reflection indicated that to an accuracy of .02 per cent, the Bragg angles for internal and surface reflection of x-rays from the 100 planes of calcite are equal.

2. This result is verified by the experiments with the double crystal spectrometer, which demonstrated the equality of angular separations of the spectral lines Mo $K\alpha_2$ and $K\beta_2$ in internal and surface reflection from calcite.

3. Consequently it is to be concluded that the lattice constant for the 100 planes of calcite effective in the internal reflection of x-rays by transmission through the crystal is equal to the lattice constant for the 100 planes effective in the ordinary Bragg reflection at the surface of the crystal, to an accuracy of .02 per cent.

4. The comparison, by means of the spectrograph, of the Bragg angle of Mo $K\alpha_1$ reflected internally from the $1\bar{1}0$ planes of calcite with the value computed by means of a direct trigonometrical calculation based upon the known geometrical form of calcite and the accepted value of the Bragg angle for surface reflection from the 100 planes, indicated that the anticipated diminution of the lattice constant effective in the internal reflection of x-rays from the $1\bar{1}0$ planes of calcite is certainly not greater than .015 per cent, if it exists at all.

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