HIGH VOLTAGE PRECISION MEASUREMENTS

Thesis by

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SUMMARY

The development of a sparkless sphere gap voltmeter as an absolute standard of high-voltage measurement is described in this thesis.

The force, due to electrostatic charges, existing between two conducting spheres, one of which is grounded, the other insulated from ground and maintained at a high potential, may be easily measured. From the force measurements the potential difference may be accurately and easily calculated from the fundamental laws of the static electric field. The force is a function only of the potential difference, the size of the spheres, the spacing between them, and the dielectric constant of their ambient medium.

Using one hundred centimeter spheres, potential differences up to eight hundred fifty thousand volts have been measured by this method. For convenience of comparison with present sphere gap voltmeter curves, voltages measured by the force method were applied to the one hundred centimeter sphere gap and sparking distances observed. The curve thus obtained differs considerably from the American Institute of Electrical Engineers standard spark-over curve now used for one hundred centimeter spheres.

The electrostatic-force method for measurement of high voltages possesses several advantages which are not found in the other systems now used. The more important of these advantages are:-

1. The force-voltage relationship may be analytically derived from known electro-static laws; hence, provides an absolute primary standard of high-voltage measurement.

2. The tests showed complete freedom from erratic readings so often attendant with the spark gap meter.

3. No empirical correction is necessary for the effects due to changes in temperature, humidity, and barometric pressure.

4. Since there is no electrical discharge associated with the measurement, high frequency surges are not produced in the circuit at the time of measurement.

5. The calibration when extraneous influences are avoided depends in no way on empirical data.

6. A resistance in series with the measuring apparatus to limit current flow is not necessary.

7. The erratic and little-known phenomena occurring with the electrical breakdown of air form no part of the operation of the meter.

8. The use of force measurements permits continuous voltage application and avoids the necessity for an opening of the test circuit at the time of measurement.

9. The relationship between force and voltage is equally applicable for direct and alternating current circuits.

10. The measurements are always R.M.S. or effective values regardless of wave form.

11. It is not required that the spheres be smooth or polished, or that they withstand electrical discharge.

12. The disturbing effects introduced by grounded laboratory walls, floor, and ceiling may be computed and are quite negligible under reasonable conditions of operation.

13. The sensitivity of the apparatus is such that the maximum error of measurement should be well within one per cent.

Although the apparatus as at present developed is essentially a laboratory type and of such form as to be used primarily for the calibration of secondary standard instruments, the fundamental principle involved appears to be readily applicable to a more portable and practical instrument for routine test measurements.

The results of this research have been discussed in a technical paper, "The Sparkless Sphere Gap Voltmeter", by Professor R. W. Sorensen, Simon Ramo, and the author of this thesis. The paper was written to be presented before the Mid-Summer Convention of the American Institute of Electrical Engineers at Ithaca, New York, June, 1935.

INTRODUCTION

Industrial development at the close of the nineteenth century required the transmission of electrical energy from convenient power sources to centers of commercial activity. As energy quantities and distances of transmission became greater, higher voltage transmission lines were found necessary for economical and efficient operation. To build and operate these lines, engineers were forced to find new methods of design and construction; to develop insulating materials; and to investigate the behavior of dielectrics in order to guard against electrical breakdown and to limit the corona power loss associated with high voltages. There was the closely related problem of protection against the damaging effects of lightning disturbances. The requirements of such problems called for new methods of measuring voltage because the instruments in general use could not be insulated for the higher voltages. Needle gaps have been used to measure quite satisfactorily potential differences not in excess of one hundred thousand volts by finding the breakdown voltage for two co-linear needles separated in air. Above that range stray fields which are always present in the laboratory may cause considerable deviations in measurement. Other disadvantages of the needle gap are: the inconsistency of flashover; the variation of performance with changes in air density, humidity, and air pressure; and the required empirical calibration.[1,2]*

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* See Bibliography for all references.

The sphere gap was introduced by F. W. Peek, Jr.,[3,4] of the General Electric Company, as an instrument of more extensive application for measuring high voltages. The flashover voltage of the sphere gap was found to be only slightly influenced by humidity changes, and was not affected by stray fields and adjacent grounded objects as much as the needle gap. By assumihg the breakdown gradient of air to be a constant value, Peek attempted to make a rigorous calibration of the sphere gap by calculations of the maximum field gradient. His attempts were unsuccessful because of the complicated character of the electrical breakdown of air. His proposed calibration curves, adopted by the American Institute of Electrical Engineers as standards for high voltage measurement,[5] were consequently based on an empirical calibration of small spheres by tertiary volt-coil measurements at the supply transformer. The calibration curves for twenty-five centimeters, fifty centimeters, seventy-five centimeters, and one hundred centimeters diameter spheres were obtained by extrapolation of the empirical curves obtained for six and one-quarter centimeter and twelve and one-half centimeter spheres. This extrapolation involved the questionable assumption that the influence of electrode size on breakdown voltage could be neglected. This assumption, with the empirical calibration, made the accuracy of the calibration curves doubtful.

The inconsistent behavior of the sphere gap, due in part to the series resistance required to limit current flow and to damp high frequency oscillations, and the inability to duplicate measurements either with the same apparatus or in different laboratories are perhaps the most objectionable features of the sphere spark gap.

Although the use of the spark gap has remained the standard method for measuring high potential differences, numerous other methods have been used. The bibliography of this thesis contains references to several of those methods. The corona voltmeter, tertiary volt-coil, vacuum tube crest voltmeter, charging current voltmeter, electrostatic force meter, and ionic wind voltmeter have been partially successful.

Two attempts have been made to develop an absolute standard of measurement. One method involved measuring the period of oscillation of a metallic ellipsoid suspended in a uniform electrostatic field.[6,7] The rotary voltmeter,[8,9] a more recent development, was based on the measurement of the rectified charging current of a condenser. The latter apparatus was developed by Kirkpatrick of the Leland Stanford University. Neither method was suitable for very high voltages because of insulation difficulties.

The development of an accurate, and preferably primary standard method of measurement has become very desirable, due to the recent interest in insulation coordination and surge investigation. Such interest was evidenced by the discussions of leading high voltage engineers at the Mid-Summer Convention of the American Institute of Electrical Engineers in 1934,(10,11,12,13) and by the activity of the Measurements and Standards Committee of the American Institute of Electrical Engineers for a re-calibration of the sphere gap.

It was this interest together with the evident dis-satisfaction with existing methods for high voltage measurement that created the desire for the development of a primary standard of measurement, by which other apparatus might be calibrated, and by the use of which many difficulties encountered with the spark gap might be avoided.

The following characteristics of performance were desired:-

1. The instrument should be a primary standard; that is, it should be capable of calibration directly from an analytical consideration of known fundamental laws of the electrostatic and electromagnetic field.

2. It should be readily applicable for measuring voltages up to one million volts and often higher.

3. The allowable error of measurement should be within one per cent and should lend itself to calculation.

4. The disturbances associated with the electrical breakdown of insulation material should not be present in the measurement since those effects are unknown and uncertain.

5. Measurements should be capable of duplication.

6. Temperature, air pressure, and humidity should have no influence on the operation of the meter.

7. There should be a minimum disturbance due to stray fields.

8. The instrument should be equally applicable to the measurement of continuous, power frequency, and surge potentials. It should also operate independently of the voltage wave form.

9. Polarity and time-lag effects should be entirely absent.

10. The size, cost, and ease of operating the instrument should be such as to make it useful in routine testing.

The first, fourth, and sixth characteristics directed this research to a consideration of calculating the existing electrostatic force as a function of the potential difference between electrodes of such form and dimensions as to permit accurate analysis of the electrostatic field. Parallel plates were first considered, but there were objectionable features of edge influence and the necessarily large size of plates.

Adjacent spheres were chosen as being suitable for investigation since edge effects were reduced to a minimum, and since the possibility of a distortion of the field due to corona formation was remote. Spheres of a suitable size were available in the Southern California Edison Company's High Voltage Laboratory at the California Institute of Technology. The force acting to reduce the gap between the spheres could be calculated as a function of sphere dimensions, spacing, and the potential difference between the two spheres. This calculated force is of such magnitude, varying from one hundred to four hundred grams, as to be easily and accurately measured. The spheres were mounted and tests were undertaken to determine the degree to which this sparkless sphere gap voltmeter would satisfy the requirements for a precision instrument of high voltage measurement.

THE RELATION BETWEEN FORCE AND VOLTAGE

Two adjacent conducting spheres are assumed to be isolated in free space, with no disturbing electrical fields or objects near them. Sphere A, Fig. 1, is maintained at a potential V while sphere B is at ground potential. A series of inverted images may be used conveniently for the calculation of the force acting on each sphere.(14,15)

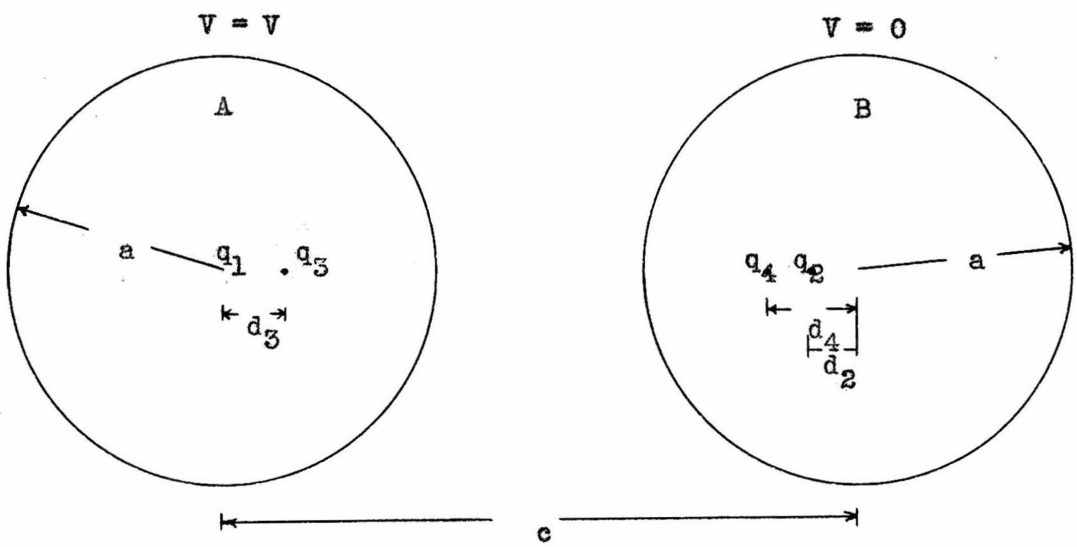


Figure 1.

The field about an isolated sphere A at a potential V is that due to a charge $q_1 = aV$ at the center of the sphere, if "a" is the radius of the sphere. If a grounded sphere, B, of the same radius is brought to a position in the vicinity of A with a distance "c" between sphere centers, the original field will be distorted due to the presence of B. However, a charge of $q_2 = -\frac{a}{c} q_1$ at a distance $d_2 = \frac{a^2}{c}$ from the center of the grounded sphere B, and lying on the line of centers, will give zero potential over the surface of sphere B in the presence of charge q_1. These two charges will not, however, satisfy the condition that the surface of sphere A be an equi-potential. To satisfy this condition a charge q_3 is introduced on the line of centers in sphere A at a distance d_3 from the sphere center, such that

$$q_3 = -\frac{a}{c - d_2} q_2 \qquad d_3 = \frac{a^2}{c - d_2}$$

q_3 is then the inverted image of q_2, and the resultant potential of sphere A due to q_2 and q_3 is zero, so that the total potential of sphere A is the desired value V due to q_1 alone. To keep B at zero potential under the influence of q_1, q_2, and q_3, the charge q_3 must be imaged by a charge q_4 in B at a distance of d_4, where

$$q_4 = -\frac{a}{c - d_3} q_3 \qquad d_4 = \frac{a^2}{c - d_3}$$

Thus a doubly infinite series of images is created in order to maintain A at a potential V, and B at zero potential. The field at any point outside the spheres is the resultant field due to these charges. All of the image charges in sphere A will have the same sign as the original charge q_1, while those in B will have the opposite sign.

In general:

$$q_n = - \frac{a}{c - d_{n-1}} q_{n-1} \qquad\qquad d_n = \frac{a^2}{c - d_{n-1}}$$

The total force acting on sphere B tending to move the sphere toward A is the sum of forces acting on each charge in B due to all the charges in A. The force of attraction between two charges q_s and q_t, separated by a distance f in a medium of dielectric constant k is:

$$F = \frac{q_s \, q_t}{k \, f^2}$$

We shall consider the spheres to be surrounded by air, hence k is very near unity.

The total force acting on B may be expressed as a double summation over s and t,

$$F = \sum_{\substack{s \\ 1 \\ \text{(odd)}}}^{\infty} \sum_{\substack{t \\ 2 \\ \text{(even)}}}^{\infty} \frac{q_s \, q_t}{(c - d_s - d_t)^2}$$

Since c is always greater than 2a, the nth image charge is much smaller than the (n -1)th charge and the series represented by the above summation is rapidly convergent. The inclusion of more pairs of images contributes rapidly diminishing amounts to the total force. To evaluate the series, it is necessary to take only as many images as are required to make certain that the total force of all neglected images will be less than the allowable error.

A calculation for 100 cm. spheres with a distance between centers of 130 cm. (30 cm. gap spacing) will illustrate the method. The first four pairs of images and the distance of each from the center of the sphere in which it is imaged are given in Table I.

TABLE I.

Calculated Images

a = 50 cm., Spacing = 30 centimeters

Sphere A (potential V)		Sphere B (potential zero)	
$q_1 = Va$	$d_1 = 0$ cm.	$q_2 = -.385\ Va$	$d_2 = 19.2$ cm.
$q_3 = .174\ Va$	$d_3 = 22.6$ cm.	$q_4 = -.086\ Va$	$d_4 = 23.2$ cm.
$q_5 = .0378\ Va$	$d_5 = 23.4$ cm.	$q_6 = -.0177\ Va$	$d_6 = 23.45$ cm.
$q_7 = .00833\ Va$	$d_7 = 23.5$ cm.	$q_8 = -.00393\ Va$	$d_8 = 23.55$ cm.

The force, due to these four pairs of charges only, is then:-

$$F = \frac{q_1\ q_2}{(130 - d_2)^2} + \frac{q_3\ q_2}{(130 - d_2 - d_3)^2} + \frac{q_5\ q_2}{(130 - d_2 - d_5)^2} + \frac{q_7\ q_2}{(130 - d_2 - d_7)^2}$$

$$+ \frac{q_1\ q_4}{(130 - d_4)^2} + \frac{q_3\ q_4}{(130 - d_4 - d_3)^2} + \frac{q_5\ q_4}{(130 - d_4 - d_5)^2} + \frac{q_7\ q_4}{(130 - d_4 - d_7)^2}$$

$$+ \frac{q_1\ q_6}{(130 - d_6)^2} + \frac{q_3\ q_6}{(130 - d_6 - d_3)^2} + \frac{q_5\ q_6}{(130 - d_6 - d_7)^2} + \frac{q_7\ q_6}{(130 - d_6 - d_7)^2}$$

$$+ \frac{q_1\ q_8}{(130 - d_8)^2} + \frac{q_3\ q_8}{(130 - d_8 - d_3)^2} + \frac{q_5\ q_8}{(130 - d_8 - d_5)^2} + \frac{q_7\ q_8}{(130 - d_8 - d_7)^2}$$

If V is expressed in statvolts, a in centimeters (a for this case is equal to 50 centimeters), and F in dynes, the equation reduces to:

$F = .137\ V^2$ dynes.

In general, $F = S\,V^2$ dynes where V is expressed in statvolts and S is a constant depending only on the sphere radii and the distance between sphere centers.

Then $V = \sqrt{\frac{F}{S}}$ statvolts if F is expressed in dynes.

If F is expressed in grams, and V in volts:

$$V = 9405\sqrt{\frac{F}{S}} \text{ volts}$$

This equation is the relation used to calculate from force measurements the potential differences between the spheres of the sparkless sphere gap voltmeter.

In the calculated example of Table I, with charges (7) and (8), neglected, the value of S would be reduced about one per cent. Since the contribution of charges (7) and (8) to the total force is less than half the contribution of charges (5) and (6), the error made by neglecting all charges beyond (8) cannot exceed one per cent.

Sir William Thomson (Lord Kelvin) in his "Papers on Electrostatics and Magnetism"[(14)], page 96, has tabulated the value of S as a function of radius and spacing for two spheres of equal radii. The values, correct to five significant figures, are given in Table II. as a function of gap spacing in centimeters (the distance between adjacent sphere surfaces) for 100 cm. spheres. This table may be used directly for spheres of other dimensions, the only requirement being that the two spheres have equal radii. For

TABLE II.

COMPUTED VALUE OF THE SPACING FACTOR S

(Sir W. Thomson - "Papers on Electrostatics and Magnetism" p. 96.)

Sphere Gap Setting	S
0	∞
5	1.13844
10	.52852
15	.32917
20	.23159
25	.17432
30	.13696
35	.11082
40	.09174
45	.07720
50	.06592
55	.05693
60	.04963
65	.04363
70	.03863
75	.03441
80	.03084
85	.02775
90	.02509
95	.02278
100	.02075

example, if the radius of each sphere is equal to a centimeters, then the sphere gap spacings given in Table II. are multiplied by the factor $\frac{a}{50}$.

The tabulated value of S in Table II is plotted as a function of gap spacing in Figure 2. It is interesting to note that the value obtained for S in the example of Table I by considering only eight images is .137 as compared with .13696 from the table obtained by considering many more image charges.

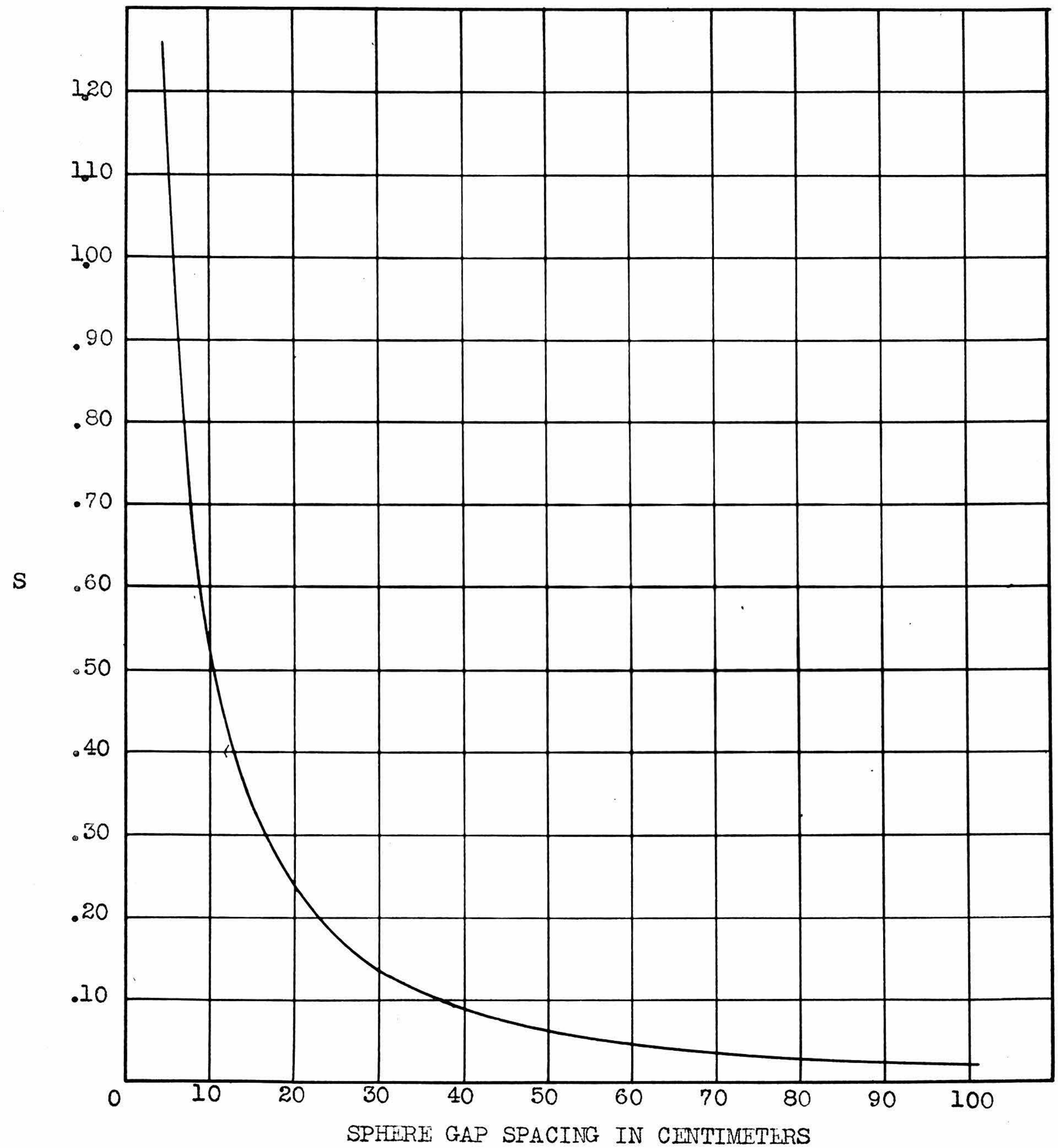
The relation between force and potential difference could have been obtained from energy considerations. In terms of the Maxwell capacity coefficients (See "Electricity and Magnetism", Sir James Jeans, pages 92-95), the charge, Q_1, existing on sphere A when it is maintained at a potential V_1 and sphere B is maintained at a potential V_2 is:- (15)

$$Q_1 = q_{1-1} V_1 + q_{1-2} V_2$$

Similarly, the charge, Q_2, existing on sphere B is:-

$$Q_2 = q_{2-1} V_1 + q_{2-2} V_2$$

where q_{1-1}, $q_{1-2} = q_{2-1}$, and q_{2-2} are coefficients depending only on the dimensions of the system. The coefficients are defined thus:- q_{1-1} is the charge existing on sphere A when it is raised to unit potential with sphere B grounded, q_{2-2} is the charge existing on Sphere B when it is raised to unit potential with sphere A grounded, and q_{1-2} is the charge induced on sphere A when it is grounded and sphere B is raised to unit potential.



1.20
1.10
1.00
.90
.80
.70
S
.60
.50
.40
.30
.20
.10
0
10
20
30
40
50
60
70
80
90
100
SPHERE GAP SPACING IN CENTIMETERS

Fig. 2.

In terms of the capacity coefficients and the potentials of the two spheres, the energy of the system is:-

$$W = \frac{1}{2}\left(q_{1-1} V_1^2 + 2\, q_{1-2} V_1 V_2 + q_{2-2} V_2^2\right)$$

If c is the coordinate of distance between sphere centers, the force tending to increase the separation of the spheres is:-

$$F = -\frac{\partial W}{\partial c} = -\frac{1}{2}\left(\frac{\partial q_{1-1}}{\partial c} V_1^2 + 2\,\frac{\partial q_{1-2}}{\partial c} V_1 V_2 + \frac{\partial q_{2-2}}{\partial c} V_2^2\right)$$

The negative sign indicates that there is an attractive force between the spheres. If sphere B is maintained at ground potential, the equation for force reduces to:-

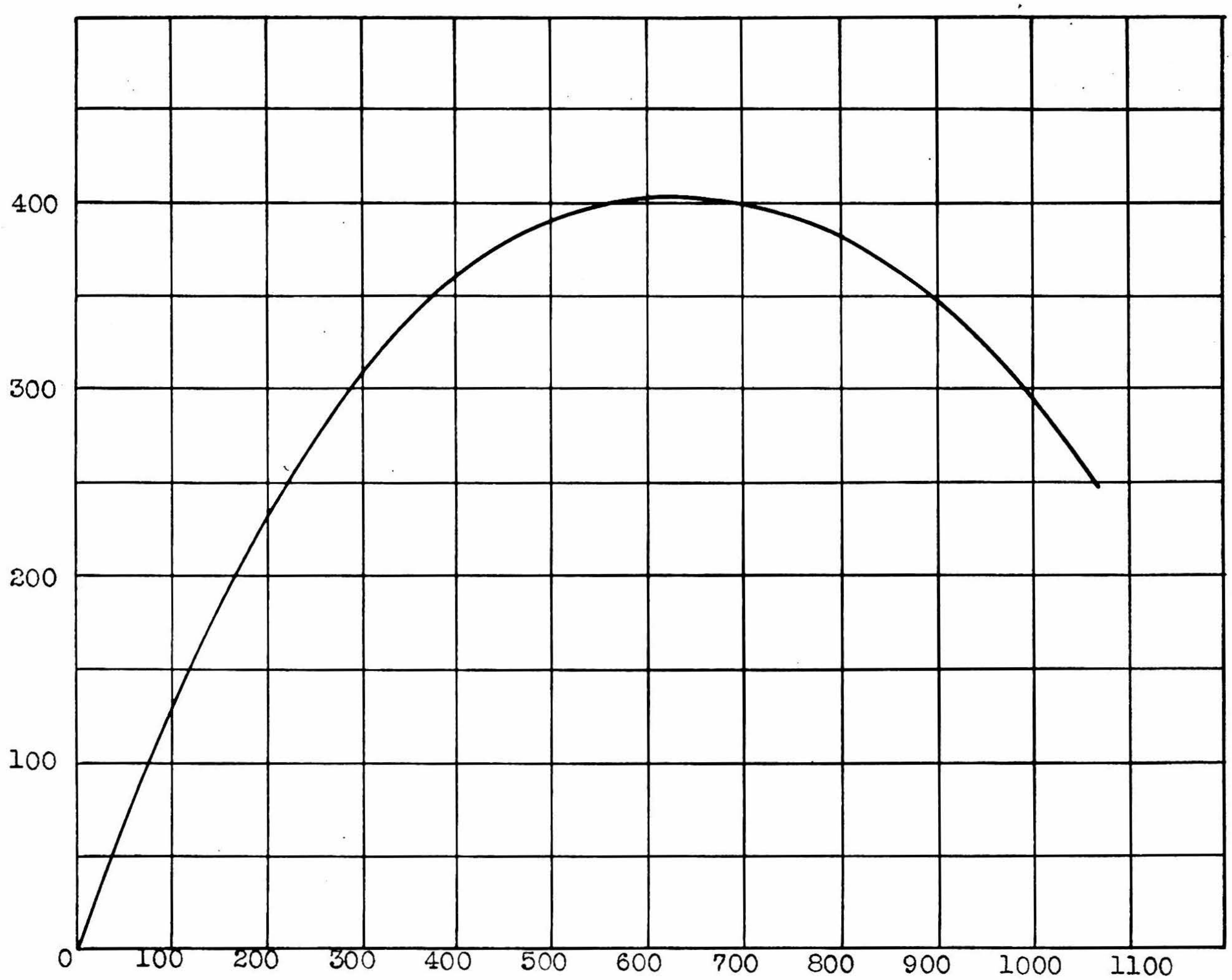
$$F = -\frac{1}{2}\,\frac{\partial q_{1-1}}{\partial c}\, V_1^2$$

This expression is identical to that obtained from the consideration of image charges. The constant S now becomes:-

$$S = -\frac{1}{2}\,\frac{\partial q_{11}}{\partial c}$$

The relation between force and the square of the potential difference between the spheres could also have been derived by integrating over one sphere the force acting on each element of that sphere due to the electrostatic field. The constant S of proportionality may be evaluated more easily, however, from the forces existing between image charges than by either of the latter methods.

It is of course necessary that the spheres be separated by a distance greater than that at which sparkover would occur for the

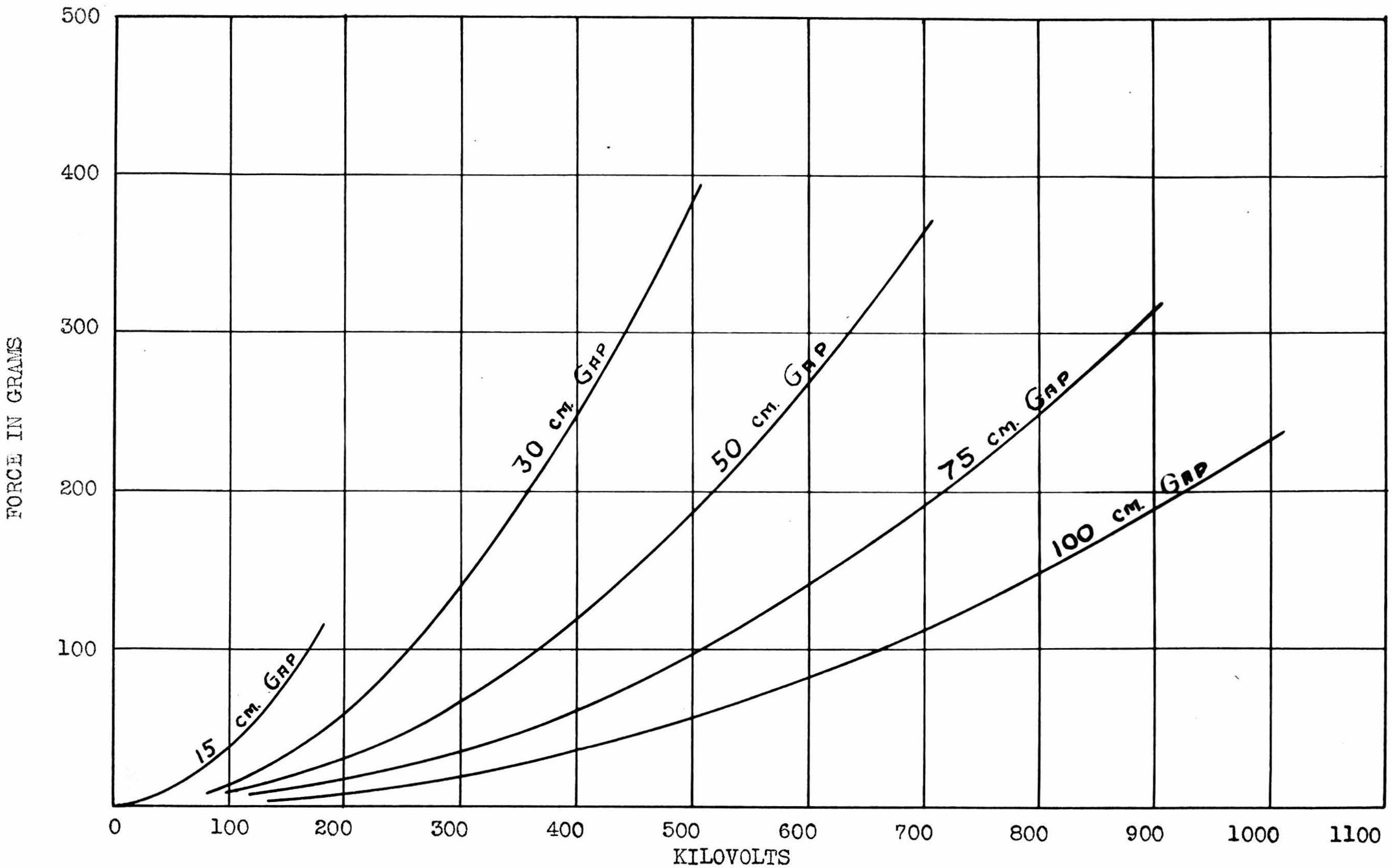


VARIATION OF FORCE WITH VOLTAGE WITH THE 100 cm. GAP AT SPARKING DISTANCE

Fig. 3.

voltage to be measured. The force acting between the spheres is increased as the gap spacing is decreased for a given potential difference between the spheres; hence, the maximum force which may be obtained for a particular potential difference is that corresponding to a gap spacing slightly larger than the critical sparkover distance. Using the spark-over distance corresponding to the insulated sphere potential of a 100 cm. sphere gap, with one sphere grounded, given by F. W. Peek[16], the maximum force which may be measured between the spheres of such a gap was calculated as a function of the potential of the insulated sphere. The data thus obtained are plotted on the curve of Fig. 3. It may be observed from this curve that the force increases with the potential difference until at a voltage of about 625 kilovolts the increased gap spacing has more effect in reducing the force than has the increased potential in increasing it.

The variation of force with potential difference for the 100 cm. sphere gap at constant spacings of 15 cm., 30 cm., 50 cm., 75 cm., and 100 cm. is shown on the curves of Fig. 4. Since large forces may be measured with small relative error, the curves illustrate the importance of maintaining the gap spacing as small as sparkover will permit.



VARIATION OF FORCE WITH VOLTAGE FOR CONSTANT SPACINGS OF THE 100 cm. GAP

Fig. 4.

TEST APPARATUS AND TEST PROCEDURE

Two polished cast-aluminum spheres, 100 cm. in diameter, were available in the High Voltage Laboratory at the California Institute of Technology. The spheres are mounted as shown in Figures 5, 6, 7, and 10 to form the sparkless sphere gap voltmeter. The right-hand sphere assembly of Figures 5 and 7 is suspended from the roof of the laboratory by two suspension insulator strings. The center of the sphere is twenty feet above the floor. An iron pipe shank twenty feet long supporting the sphere is adapted to slide horizontally in wooden bearings mounted on the supporting beam. The motion is obtained by a screw mechanism driven by means of an insulating rope belt from a motor on the laboratory floor, shown in Fig. 6. The assembly is insulated from ground for more than one million volts. Four ropes connected to the supporting beam through porcelain "goose-egg" insulators are stretched horizontally to the corners of the laboratory to prevent any motion of the frame. The arrangement of these ropes may be seen from the photographs of Fig. 6 and Fig. 7.

The other sphere, fastened to an iron pipe shank fifteen feet long, is suspended with the shank horizontal and coaxial with the shank of the insulated sphere. The suspension is made from the roof by four ropes arranged in pairs, a pair attached at either end of the shank and forming the letter V from the shank to the laboratory roof. The V formation may be seen by a close inspection of

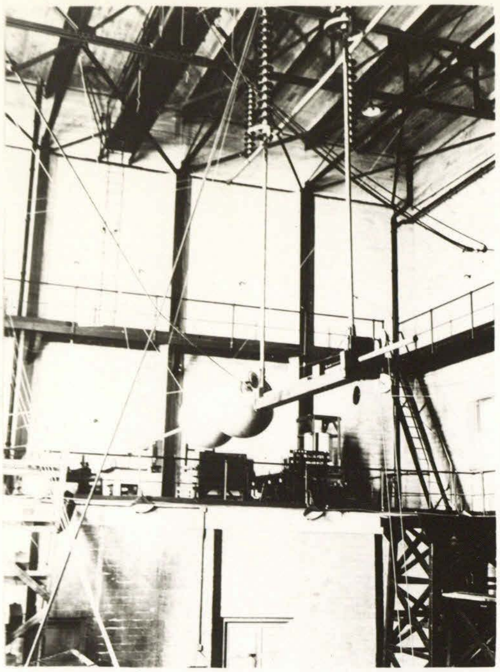


FIG. 5

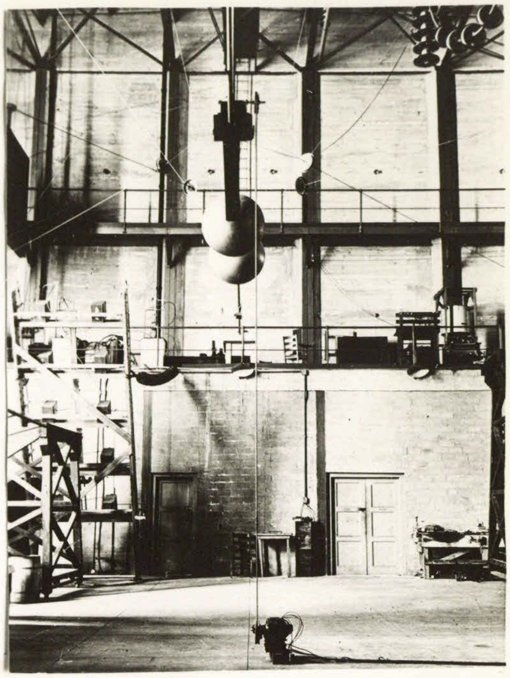


FIG. 6

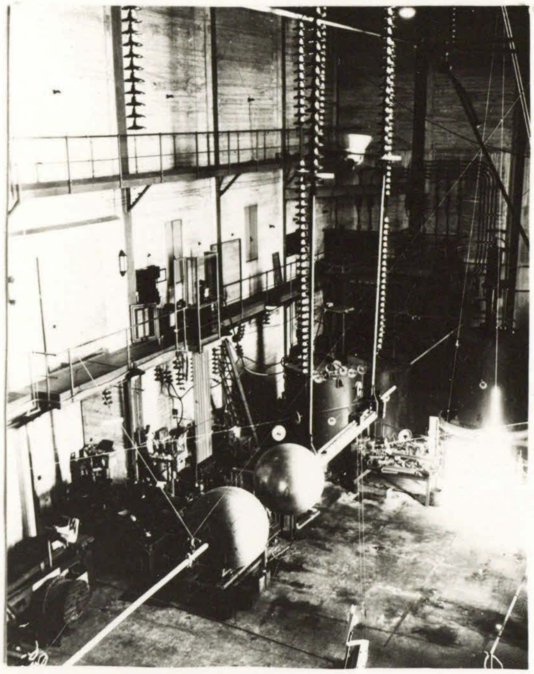


FIG. 7

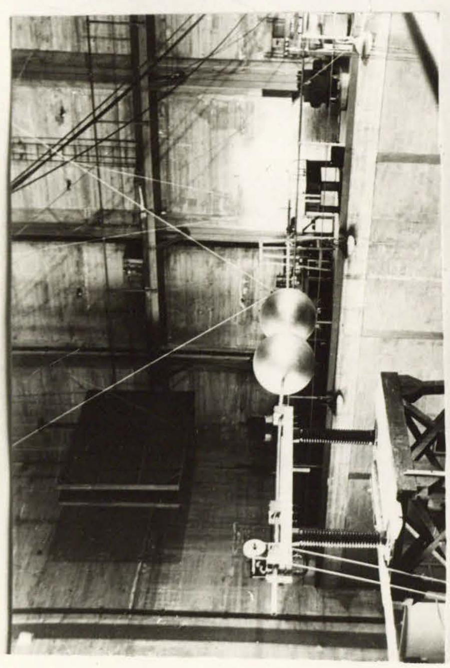


FIG. 8

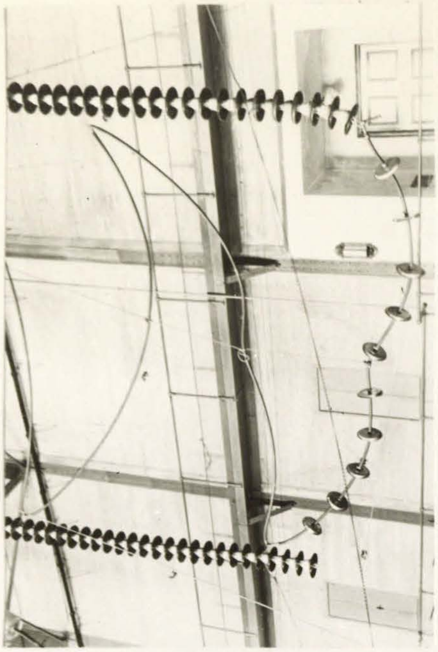


FIG. 9

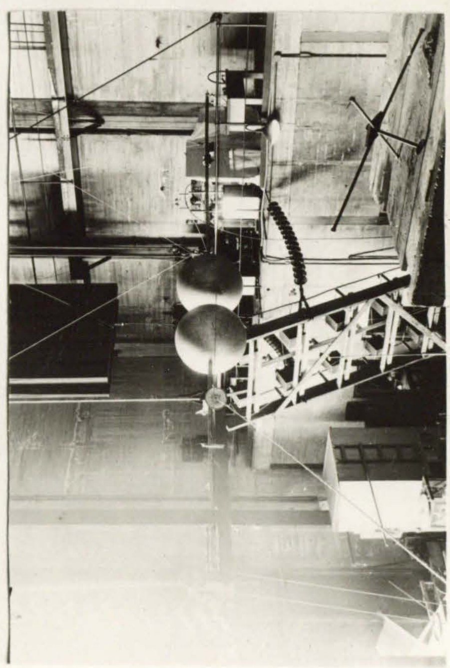


FIG. 10

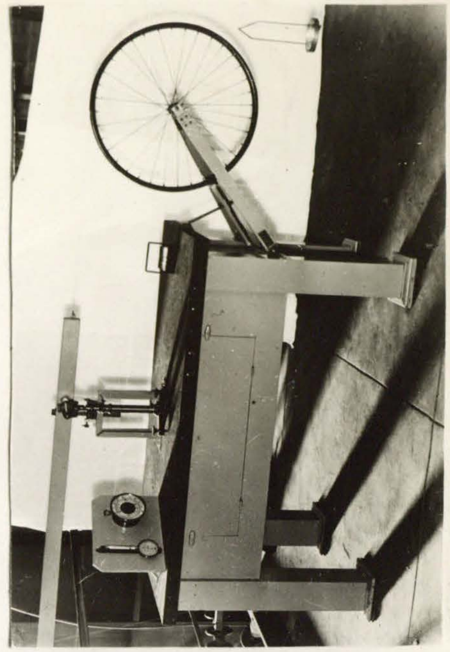


FIG. 11

the photographs. This method of suspension prevents lateral motion of the sphere but permits motion in the longitudinal direction along a horizontal line through the sphere centers. The shank and sphere are grounded by flexible connections.

A light cord attached to the end of the shank of the grounded sphere passes over a ball-bearing bicycle wheel used as a pulley, and supports a weight pan, shown in Fig. 11. Near the end of the shank is mounted an illuminated hair line. A cathetometer telescope focused on this hair line is used to observe any motion of the grounded sphere assembly. A short square rod, projecting at right angles to the shank, moves between adjustable stops which are fastened to the weighing table on the laboratory balcony and limits (See Fig. 11) the horizontal motion. The apparatus is so sensitive that the motion resulting from a weight of less than one-half gram on the pan may be easily observed with the cathetometer. The inherent damping of the rppes prevents oscillation and makes unnecessary any special provision for added damping.

In setting up for readings the grounded sphere is permitted to assume its natural position due to gravity forces, and the hair line of the shank is then centered in the field of vision of the cathetometer. The insulated sphere is moved very slowly toward the grounded one until the spheres are incontact. The position of contact may be better observed by watching the cross hair with the cathetometer than by the usual electrical means of determining contact. The position of a hair line carried by the shank of the movable sphere, relative to a meter scale on the supporting beam, is noted as the reading of zero gap spacing. The insulated sphere may be moved with the motor driven mechanism to vary the gap spacing from zero to 150 centimeters.

Voltage is applied to the insulated sphere from the million volt transformer supply through a series resistance shown in Fig. 9. This resistance of about two and one-half megohms consists of water running through sixty feet of three-quarter inch garden hose. The water is sprayed from the ceiling into a tub at the top of the hose, flows through the hose, and runs from the lower end into a tank on the laboratory floor. Pie tins are inserted in the hose at eighteen inch intervals in the upper and lower sections to prevent corona formation on the rubber hose. It has been found impossible to use the rheostat for more than 850,000 volts, since for such high voltages the voltage gradient along the hose, when sparkover of the gap occurs, causes external flashover. A longer hose of larger cross section would give the same series resistance but would permit the application of higher voltages.

The ample clearance around the spheres is shown by Figures 5, 6, and 7. Fig. 8 is a photograph showing the insulated sphere assembly supported on a wooden tower, the first construction made for test purposes. Fig. 10 illustrates the location of the measuring apparatus relative to the sphere gap.

After carefully leveling the grounded sphere shank; moving the insulated sphere in to find the position of zero gap spacing; and after moving it out again to the desired gap spacing, voltage measurements are made. Gap distances are measured by noting the position of the insulated sphere shaft relative to the meter scale on the wooden beam, seen in Fig. 7. The spheres are now adjusted for the desired spacing with the hair line carried by the grounded sphere coincident with that of the cathetometer.

With these adjustments made voltage is applied to the insulated sphere and held constant during the test by means of the tertiary coil voltmeter at the control desk. As the voltage increases up to the test value, the force acting on the grounded sphere tends to move it toward the other sphere, but the stops on the measuring table are adjusted so the sphere can move only a short distance, about one-fourth inch, from the rest position. With the voltage held constant and at the final value, small shot is put on the weight pan until the grounded sphere moves back to the rest position, as observed when the hair line on the shank is aligned with that of the cathetometer. The gap spacing is now the desired value and the weight on the pan is the force acting on the grounded sphere. The shot is carefully weighed in grams and the voltage, applied to the insulated sphere, is calculated from the relation

$$V = 9405 \sqrt{\frac{F}{S}} \text{ volts}$$

where the value of S corresponding to the gap spacing is taken from Table II.

In order to compare the voltage measurement thus obtained with that which would have been obtained with the spark gap meter using the calibration curve of Peek for 100 cm. spheres, the voltage is held constant after the force measurement. The grounded sphere is clamped in the rest position by the thumb-screw stops on the measuring table, and the insulated sphere is slowly moved with the motor driven mechanism to decrease the gap until sparkover occurs. The sparkover distance is read from the meter scale on the insulated beam and the corresponding voltage is obtained from Peek's curve. The latter measurement requires that air pressure, temperature, and relative humidity be known. These values are read from instruments on the table and the proper corrections are applied to adjust the sparkover distance to that corresponding to standard atmospheric conditions (760 mm. of mercury pressure, 20° centigrade, and 60% relative humidity).

EXPERIMENTAL RESULTS

More than three hundred test measurements of voltage, ranging from 0 to 850,000 volts, have been made with the sparkless sphere gap voltmeter. It has been found possible to reproduce under all test conditions the force measurement for any potential applied to the insulated sphere within the limit of accuracy of observing the tertiary-winding voltmeter. In accordance with the experience of other observers and with the statement made in the Introduction, the sparkover distance corresponding to a given potential difference has been found inconsistent. It should be remembered, however, that sparkover observations form no part of the voltage measurement with the sparkless sphere gap voltmeter, and were made only to compare the results with the calibration curve of the sphere spark gap.

Measurements selected at random covering the test range are given in Table III. The fifteen sets of data are sufficient to determine the calibration curve of the sphere spark gap. From the data of columns A and B, the air density correction factor, δ, is calculated using the standard relation:

$$\delta = \frac{.392\ b}{273 + T}$$

where b is the barometric pressure in millimeters of mercury, recorded in column B; and T is the temperature in degrees Centigrade,

TABLE III. - EXPERIMENTAL DATA

Reading	A Temp. °C.	B Press mm. Hg.	C Rel. Hum. %	D δ	E Zero Gap Distance	F Force Distance cm. Scale	F Force Distance cm. Distance	G Tertiary Voltmeter Reading	H Force (Grams)	I Calculated Voltage	J Sparkover Distance cm. Scale Reading	J Sparkover Distance cm. Distance	J Sparkover Distance cm. Distance Corrected
1	11	743	65	1.025	16.0	26.0	10.0	53	186	176,000	25.1	9.1	9.5
2	18	744	65	1.001	12.5	32.5	20.0	85	207	280,000	27.91	15.41	15.4
3	18	745	65	1.002	13.6	38.6	25	119	296	388,000	35.3	21.7	21.7
4	11	743	65	1.025	33.5	63.5	30	120+	273	420,000	56.90	23.40	24.0
5	14.5	747	65	1.018	12.9	47.9	35	146	320	505,000	42.0	29.1	29.7
6	13	742	65	1.015	13.5	48.5	35	152	353	531,000	44.80	31.30	31.8
7	12	742.5	66	1.025	15.8	55.8	40	170	362	590,000	51.50	35.30	36.2
8	12	742.5	66	1.03	15.8	60.8	45	180	350	632,000	54.80	39.00	40.2
9	11	752	65	1.039	33.5	83.5	50	190	351	685,000	77.40	43.90	45.5
10	13	747	65	1.023	13.6	68.6	55	195	336.8	724,000	62.5	48.9	50.6
11	13	747	65	1.023	13.6	68.6	55	202	362.5	750,000	65.3	51.7	53.7
12	12	742.5	66	1.033	15.8	75.8	60	210	330	767,000	69.6	53.8	55.7
13	13	747	65	1.023	13.6	78.6	65	220	338.9	829,000	76.3	62.7	65.4
14	13	747	65	1.023	13.6	83.6	70	224	309.1	841,000	79.4	65.8	68.4
15	13	747	65	1.023	13.6	83.6	70	225	320	857,000	827	69.1	72.1

recorded in column A. δ is used to correct the sparkover distance to that of standard atmospheric conditions in this manner: The scale reading of sparkover distance recorded in column J, less the zero gap distance of column E, gives the sphere gap spacing at sparkover. This value, appearing in the middle column of J, is used with Peek's curve to find the corresponding voltage. The voltage thus obtained is multiplied by the correction factor, and the sparkover distance, corresponding to the corrected voltage is again obtained from Peek's curve.

The voltage recorded in column I is calculated from the relation:

$$V = 9405\sqrt{\frac{F}{S}}$$

where F is the force in grams from column H, and S is the spacing factor from Table II, corresponding to the gap spacing of column F. The calculated voltages are R.M.S., or effective, values since the force is a function of the square of the potential difference across the gap.

The experimental curve of Fig. 12 has been drawn from the data of calculated voltage plotted against the corrected sparkover distance. The points on the curve sheet are plotted to show the erratic behavior of the gap at sparkover. Peek's curve has been drawn to the same scale for comparison with the calibration curve for the 100 cm. sphere spark gap as obtained from force measurements.

The breakdown voltage of a sphere gap is dependent on the maximum rather than the effective value of the applied voltage wave. For direct comparison with Peek's curve, the voltage wave must be sinusoidal. Fig. 13 shows two wave forms; the lower curve is that of the supply voltage at the transformer, and the upper curve is the voltage wave applied to the insulated sphere.

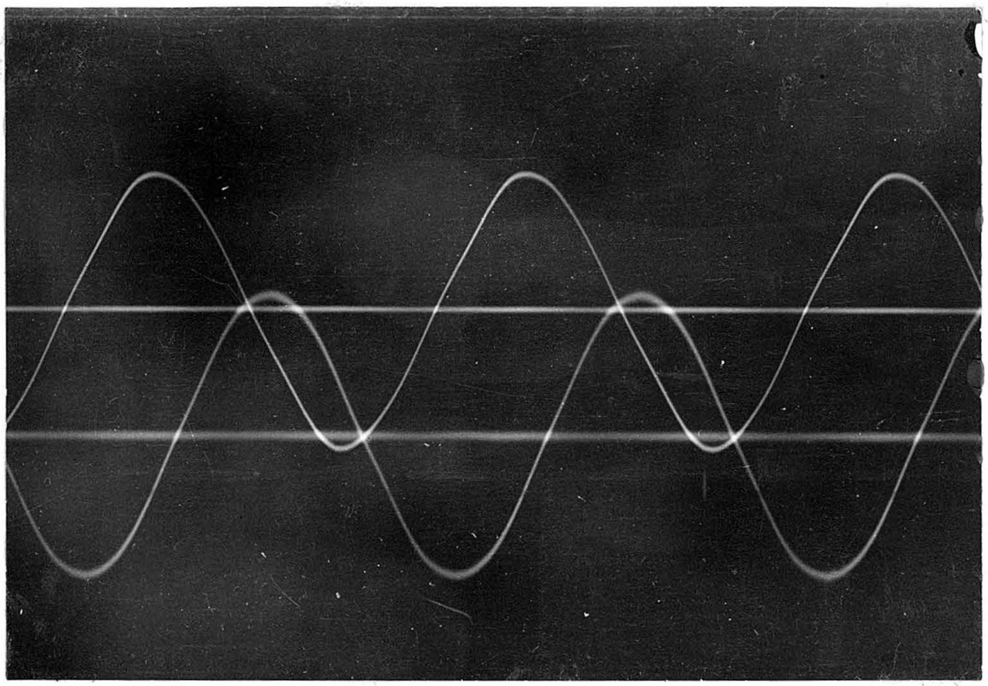


Fig. 13.

The oscillogram of output wave form was obtained by inserting a water hose resistance of five megohms between the insulated sphere and ground with one element of a General Electric mechanical oscillograph connected in series with the resistance. Another element of the oscillograph was connected across the supply voltage line to obtain the lower wave. An analysis of the voltage wave applied to the spheres (the upper wave of Fig. 13) using fifty ordinates has been madė. The effective value of the wave has been found to be 99.25% that of a true sine wave having the same crest value.

The disturbing influencesof the end wall of the laboratory and the laboratory floor on the field between the spheres, have been investigated analytically by considering images of the original infinite set of image charges obtained for the case of isolated spheres. The spheres were imaged behind the end wall and underneath the floor in each case, a distance equal to that of the real spheres from these grounded planes. Each of the original series of charges then caused the creation of an infinite series of images to maintain the disturbing planes at zero potential and the two real spheres as equipotential surfaces.

Slide- rule computations for the meter spheres set at 25 cm. gap spacing, to determine the disturbing effect of grounded planes, were made, one plane 20 feet from the gap parallel to the line of the sphere centers represented the laboratory floor; and the other 20 feet from the gap perpendicular to the line of the sphere centers, represented an end wall. Since only an approximate indication of the disturbance was desired, image charges less than one per cent of the original charge were neglected; and charges located within a few centimeters of each other were grouped in a mean position when considering the forces exerted on them by charges several hundred centimeters distant. The result of these calculations indicated the error due to the assumption of isolated spheres to be of the order of 1/2 per cent. The error may be ascertained as accurately as desired by considering more image charges.

Grounded test planes, six feet in diameter, were placed as near as possible to the spheres without causing sparkover from spheres to plane when a potential was applied to the insulated sphere. The change of force acting on the grounded sphere due to the presence of the test planes was very small.

A model has been made representing to scale the mounting of the spheres in the laboratory. 12.5 cm. spheres were used, and planes representing the laboratory walls, floor, and ceiling were set at varying distances from the sphere gap. The influence of these planes was investigated by observing the change in the force acting on the grounded sphere as the planes were moved into position while holding the applied potential constant. The force, for a 7.5 cm. spacing of the gap and the applied potential, was about 12.5 grams. The voltage variation of the transformer supply has been such that the change of force due to this variation, during the test, completely masked the change due to the presence of the ground planes. No further investigations with the model have been made to date, although efforts are being made to secure a constant voltage supply and the observations will be continued in the near future. The disturbing effect, if any, of the conducting sphere shanks will also be investigated with the model.

A further analytical study of the influence of grounded objects in the vicinity of the sparkless sphere gap voltmeter on force measurements will be made for each spacing of the gap when the apparatus has been permanently mounted in the laboratory. The influence is small and the computations necessarily involved are very tedious, so that a more complete study is thought to be unnecessary until the apparatus has been permanently located.

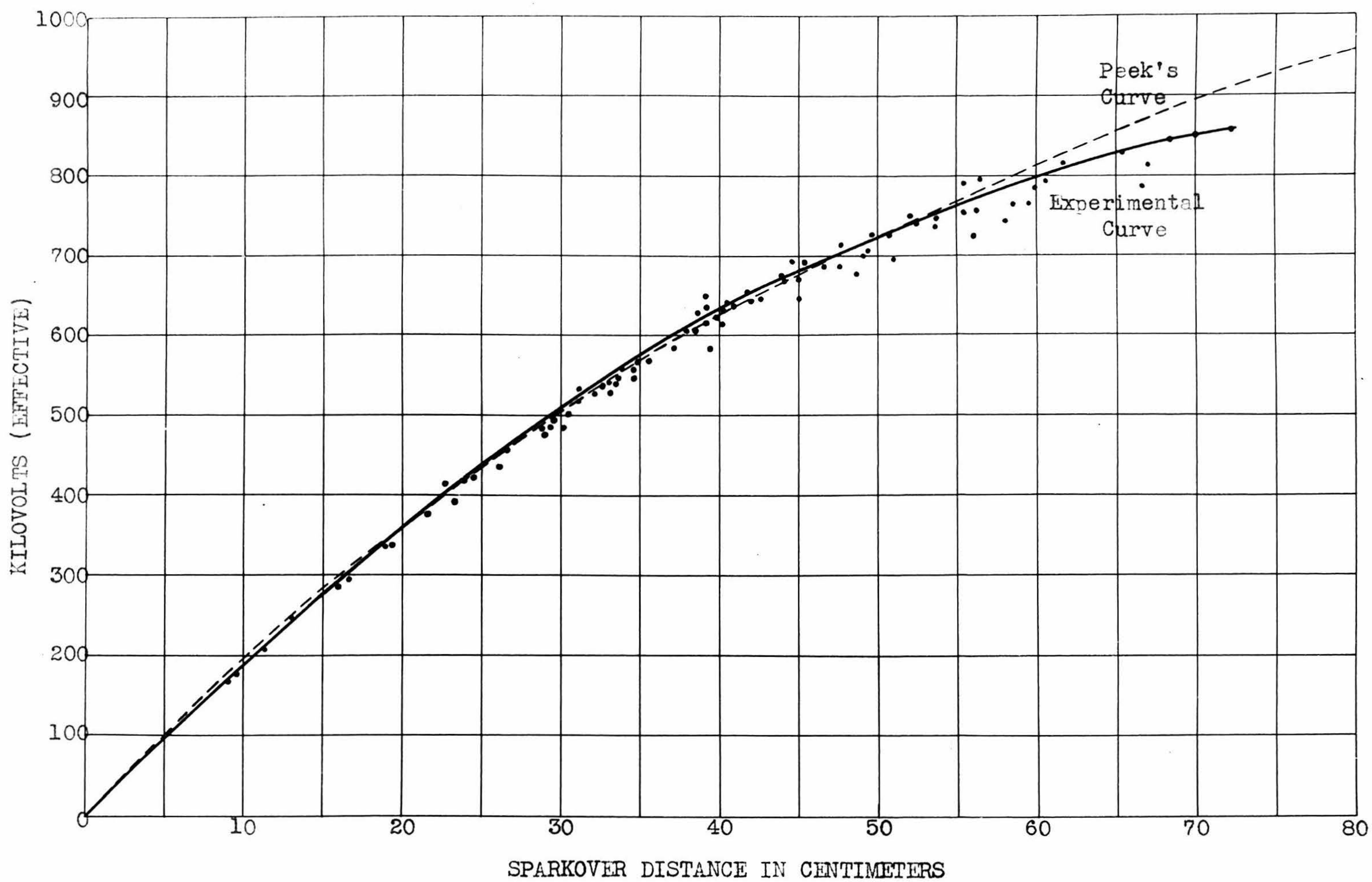

Peek's Curve
Experimental Curve
KILOVOLTS (EFFECTIVE)
1000
900
800
700
600
500
400
300
200
100
0
10
20
30
40
50
60
70
80
SPARKOVER DISTANCE IN CENTIMETERS

Fig. 12.

DISCUSSION

Since the force measuring apparatus is very sensitive, and since force measurements of voltage may be reproduced at will, the scattering of test results shown in Fig. 12 must be due to the inconsistent behavior of the sphere gap at sparkover. The inconsistency is due to several factors; namely, changing atmospheric conditions, changing states of ionization of the gap, dust and moisture deposits on the sphere surfaces, and restrictions of energy available for electrical breakdown because of the high series resistance needed to protect the sphere surfaces from damage. The erratic behavior is more pronounded for the higher voltages applied to the gap, and is perhaps the most effective argument which may be used against the spark gap as a standard of measurement.

If the spark gap is to remain in general use as the measuring instrument for high voltages, engineers are agreed that a re-calibration is necessary; particularly for the 50 cm., 75 cm., 100 cm., and 200 cm. gaps. The sparkless sphere gap voltmeter is more suitable for a calibrating instrument than any other yet proposed. It would be necessary for only two very carefully constructed laboratory meters to be used for calibrating the spark gaps used in this country for experimental and industrial measurements, - one sparkless sphere gap meter to be used as a check against the other. The accuracy of measurement by these two meters could be determined to within a small fraction of one per cent.

It would be more desirable if the sparkless voltmeter should replace entirely the spark gap for measuring purposes. That this would be possible is evidenced by the construction and use of a voltmeter in the Darmstadt laboratory by E. Heuter (18) which depends for its operation upon the movement of one sphere of a sphere gap against the resisting force of a spring when voltage is applied. The development was made simultaneously and independently of that made by the author and other workers in the Edison Company's Laboratory. No attempt was made by Heuter to express the resulting force in terms of the measured voltage; consequently, his voltmeter is a secondary standard instrument. The development of a more portable and compact voltmeter, using the principle employed in the sparkless sphere gap meter, is to be attempted during the following months. The meter will probably not be as accurate as the primary standard laboratory type, but will be much more satisfactory for general use than would the spark gap meter even if a re-calibration of the latter instrument were accurately made.

Before the sparkless sphere gap voltmeter is to be used as a precision instrument, further investigations of the influence of the sphere shanks and other objects disturbing the field between the spheres should be made both analytically and using the test model.

CONCLUSIONS

The theory of operation and the laboratory tests of the sparkless sphere gap voltmeter make evident the following advantages of the meter as an instrument for the precision measurement of high voltages:

1. The force voltage relationship may be analytically derived from known electrostatic laws; hence, provides an absolute primary standard of high voltage measurement.

2. The measuring apparatus has been found to be of such sensitivity that the experimental error is always less than one-half of one per cent.

3. There is complete freedom from the erratic readings so often attendant with the spark gap meter.

4. The operation of the meter is in no way dependent on temperature, humidity, and barometric pressure.

5. The calibration of the meter is not dependent on empirical data.

6. Measurements are always made in terms of the effective value of the applied voltage wave.

7. The relationship between force and voltage is equally applicable for direct and alternating voltages of any frequency.

8. The use of the sparkless gap permits continuous voltage application and avoids the necessity for an opening of the test circuit at the time of measurement.

9. It is not required that the spheres be smooth and polished or that they withstand electrical discharge.

10. A resistance in series with the measuring apparatus to limit current flow is not necessary.

11. The method may be used for any range of voltage measurement. The only change necessary is that the size of the spheres used be such as to give forces which may be readily measured.

12. Measurements of voltage may be duplicated at any time.

13. The effects of field distortion may be accurately calculated.

14. Polarity and time lag effects are entirely absent.

15. The instrument with some modification as to mounting is equally suitable for a laboratory primary standard or for a more portable secondary standard for general use.

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BIBLIOGRAPHY

1. Peek, F. W., Jr., Discussion, Trans. A.I.E.E. v. 32, p. 812, 1913.
2. Chubb & Fortescue, "Calibration of the Sphere Gap Voltmeter", Trans. A.I.E.E., v. 32, p. 739, 1913.
3. Peek, F. W., Jr., "The Sphere Gap as a Means of Measuring High Voltage", Trans. A.I.E.E., v. 33, p. 923, 1914.
4. Peek, F. W, Jr., "The Sphere Gap as a Means of Measuring High Voltage", G.E. Review, v. 16, p. 286, 1913.
5. A.I.E.E. Standards No. 4, 1928.
6. Thornton, "High Voltage Precision Measurements", I.E.E. Jl. v. 69, 1931.
7. Thornton & Thompson, "Absolute Standard of Measurement of High Electrical Pressures", I.E.E. Jl., v. 71, n. 426, 1932.
8. Kirkpatrick, P., "The Rotary Voltmeter", A.I.E.E. Jl. v. 51, n. 12, 1932.
9. Kirkpatrick & Miyoka, "Further Development of Rotary Voltmeter", Rev. Scientific Instruments, v. 3, n. 8 & v. 3, n. 1, 1932.
10. Meador, J. R., "Calibration of the Sphere Gap", Electrical Engineering, v. 35, p. 942-8, 1934.
11. Bellaschi & McAuley, "Impulse Calibration of Sphere Gaps", Electric Journal, June, 1934.
12. Bellaschi, P. L., "The Measurement of High Surge Voltages", A.I.E.E. Trans., v. 52, p. 544, 1933.
13. Discussion, Electrical Engineering, v. 35, p. 1652, 1934.

14. Sir W. Thomson (Lord Kelvin), "Papers on Electrostatics and Magnetism", Macmillan Co., 1884.

15. Sir J. Jeans, "Electricity and Magnetism", Cambridge University Press, 1927.

16. Peek, F. W., Jr., "Dielectric Phenomena in High Voltage Engineering", Third Edition, McGraw-Hill Book Co., 1929.

17. Carroll, J. S. & Cozzens, B., "Sphere-Gap and Point-Gap Arc-over Voltage", Trans. A.I.E.E., p. 1, 1929.

18. Heuter, E., "Uber die Messungeffektiver Spannungswerte mittels der Kugel funkenstrecke", Electrotechnische Zeitschrift, p. 833, 1934.

Additional References on High Voltage Measurement

19. Hendricks, A. B., Jr., "The Voltmeter Coil in Testing Transformers", Trans. A.I.E.E., v. 35, p. 117, 1916.

20. Edgcumbe & Ockenden, "Some Recent Advances in A. C. Measuring Instruments", I.E.E. Jl. v. 65, p. 553, 1927.

21. Sharp & Doyle, "Crest Voltmeters", Trans. A.I.E.E., v. 35, p. 99, 1916.

22. Chubb, L. W., "The Crest Voltmeter", Trans. A.I.E.E., v. 35, p. 109, 1916.

23. Miner, D. F., "Crest Voltmeters and Their Characteristics", Elec. Jour. v. 22, p. 571, 1925.

24. Work, W. R., "Notes on the Measurement of High Voltage", Trans. A.I.E.E., v. 35, p. 119, 1916.

25. Ryall, L. E., "Design and Construction of a Simple Neon-tube High Tension Crest Voltmeter", I.E.E. Jl., v. 69, p. 891, 1931.

26. Whitehead & Costellain, "Sphere Gap Calibration", I.E.E. Jl., v. 69, p. 898, 1931.

27. Davis, Bowdler, Standring, "Measurement of High Voltages with Special Reference to Measurement of Peak Voltages", I.E.E. Jl., v. 68, p. 1222, 1930.

28. Peters, J. F., "The Klydonograph", Elec. World, April 19, 1924.

29. Thornton, "The Ionic Wind Voltmeter and Thermo-Electrostatic Relay", I.E.E. Jl., v. 69, p. 533, 1931.

30. Crozier, B., "The Design, Construction, and Test of a Vacuum Tube Crest Voltmeter", M.S.E.E. Thesis, Purdue University, June, 1933.

31. "Electrostatic Voltmeters of Hartmann and Braun", Archiv. fuer Technisches, v. 1, n. 7, 1932.

32. Starke and Schroeder, "Electrostatic High Tension Voltmeter", Archiv. fuer Electrotechnik v. 26, n. 4, 1932.

33. Plummer, "High Voltage Measurements", Elec. Rev. (London) v. 107, n. 2766, 1930.

34. de la Jorce, "Measurement of High Voltages", Societe Francaise des Electriciens, Bul. v. 6, n. 62, 1927.

35. Palm, A., "High Voltage Voltmeters", Elektrotechnische Zeitschrift, v. 47, n. 30-31, 1926.

36. Rayner, "A Precision Form of the Kelvin Electrostatic Voltmeter", I.E.E. Jl., v. 59, n. 297, 1921.

37. Hobson, J. E., "An Outline for a Textbook in High Potential Measurements", M.S. Thesis, Purdue University, 1933.