A FREE QUARTZ PENDULUM

Electrostatically Driven
and
Photoelectrically Controlled
as a means of
Precision Timekeeping

Thesis Submitted
by
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SUMMARY OF REPORT

After making a careful study of the field of timekeeping the pendulum was found to be the best timekeeper. A lengthy investigation was made of all the factors that affect the period of a pendulum, and these are outlined in the report. Choosing a free pendulum in preference to the compensated compound pendulum, the period of one half second was taken, and the physical dimensions of a pendulum worked out. It was decided to make the pendulum entirely of quartz, and to operate it in a vacuum. An original electrostatic drive was adopted as the means of keeping the pendulum in motion. This drive required the design of a special circuit which has been worked out, and the constants of this circuit calculated with an assumed damping factor.

Construction was started, but much difficulty was encountered, especially in the making and fastening of a fine quartz spring to the rod. A strong mounting and the driving condenser have also been built. A good bibliography has been worked up for the aid of the person who carries on this work.

Statement of Problem

It is believed that by employing a quartz pendulum swinging in a vacuum and free from mechanical contacts a timekeeper could be made more accurate than any now existing. Because of the low internal friction of quartz it was planned to suspend the pendulum by a quartz fibre. To get away from all mechanical contacts it was thought that the pendulum could be driven by electrostatic or electromagnetic means and that the timing and regulation could be obtained by means of a light beam and a photoelectric cell.

With these rough ideas in mind the following detailed study was undertaken.

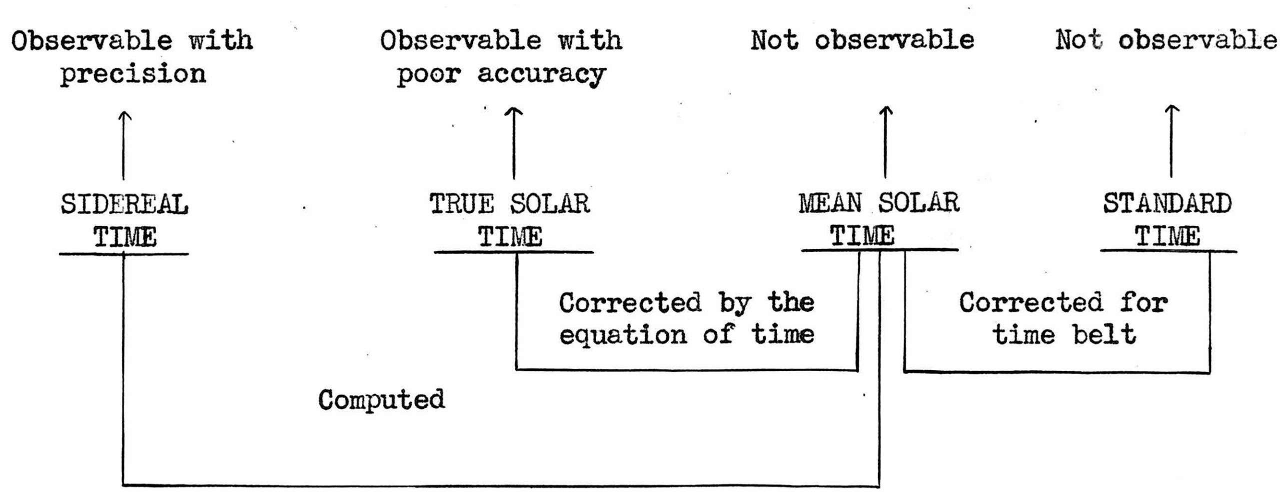
General Survey

At the start of this study an intensive reading program was undertaken in order that the author might become familiar with the general field of timekeeping. It is really quite fascinating to trace the history and development of the various methods of timekeeping from the early Roman water clocks and sand glasses to the precision Riefler and LeRoy clocks of the United States Naval Observatory. Even more interesting is to study the many and varied attempts to devise new and more accurate ways of keeping time, such as are being carried out in various laboratories in Germany, England and the United States.

Before going further into the study of timekeepers one must be familiar with what time is, and how we obtain it. The conception of time by itself means nothing, for it is inseparably tied up with space and motion. There are three natural periods for telling time, the day - (light and dark) as dependent upon the sun, the month - which is dependent upon the moon, and the year - as measured by the seasons. Since earliest time man has measured time by the sun, but this proved poor, as the length of the day varies erratically throughout the year; besides, it is not possible to make precision measurements of the sun's passage over the meridian. The next step was to measure the passage of the stars across the meridian, and this gave a good measurement of time, but one that did not coincide with the solar day. This - star time - is known as sidereal time. It is the only time that is now measured and is to be found in all time and astronomical observatories.

Unfortunately, sidereal time is not exactly precise, due to the fact that the rotation of the earth is not quite constant, and due more largely to the fact that the earth wobbles on it's axis as it spins. In fact, there are two distinct movements; one the precession of the equinoxes, which has a period of 26,000 years, and one that is called the Nutation, which has a period of 19 years. The first error is eliminated by taking the spring equinox as the datum point and the Nutation is merely averaged out over a period of 19 years. The other error, that of the slowing down of the earth's rotation due to tidal friction is not yet definitely known and hence cannot be compensated for; however, it is small and amounts to a few seconds in a century.

One pursuing the study of time should have a clear picture of the movements of the earth and sun. Any good astronomy textbook will give this picture and there might be mentioned a chapter in Baliie's book on Watches, which contains an excellent description of the basis of time. Below is given a chart which gives in brief form the relations of the various times now in use.



As one studies the various pieces of equipment that are used to keep precision time he is soon struck by the fact that practically every primary timekeeper is a pendulum. The pendulum is a good timekeeper, as it depends on one of the most constant forces that we know of, gravity. We shall shortly study the pendulum in considerable detail. There are, however, to be mentioned two other timekeepers of considerable importance, both of which depend upon elasticity as the restoring force. The first of these, the piezo-electric oscillator is an important newcomer to the timekeeping field, and one that has greater accuracy for short periods of time than the pendulum. Accuracies as great as 10^6 may be counted on for periods of less than a day. The second of these is the tuning fork, and old friend whose accuracy has been brought up as high as 10^6 by the use of vacuum tube circuits, but this limit seems to be almost unpassable, and for the time being the tuning fork has fallen into the class of a secondary standard.

The Tuning Fork

Considerable research was done by Dye in England in an attempt to use a tuning fork as a precision time standard. He, as have many other investigators, came to the conclusion that it could not be used as a primary time standard, although it must be said that he succeeded in obtaining accuracies as high as one part in 10^6. The principal limiting features in the case of the tuning fork are as follows. The largest factor is that of temperature, which changes the coefficient of elasticity; then there is the irregular and little known effects of ageing and fatigue in the materials of which the tuning fork is made. Again there is the effect of friction and hysteresis loss in the material. These factors combine to form a limit which, to the present time, has kept the obtainable accuracy below 10^6.

Quartz Oscillators

Advances in the telephone industry necessitated the use of a high precision standard of frequency and so in 1928 several members of the Bell Laboratory set about the task of developing a primary standard of frequency. Marrison and Lack were eminently successful in this undertaking and developed a ring shaped quartz crystal that was cleverly mounted in an aluminum case whose temperature and pressure were closely controlled. This crystal oscillated at 100,000 cycles, but by means of a submultiple generator and a synchronous motor they obtained frequencies as low as ten cycles.

The accuracy obtained from this set-up could be depended upon to one part in 10^6 at all times, and for short durations of time was considerably better than that. Connected to the synchronous motor driven by the crystal is a clock which of course gives excellent time. Loomis, in his laboratory at Tuxedo Park, has compared this as a timekeeper with several Schott clocks that he has in operation. He has found that for short periods of time, less than a day, and especially for very short periods of time, that the quartz crystal is superior as a time keeper. However, for periods of time longer than eight or ten hours the pendulum is far the better time keeper. The quartz oscillator is better for shorter times, as it depends directly on a very short time interval, and can be read directly to one part in 100,000. Again, the pendulum has small errors due to slight changes in gravity, irregularities of amplitude, and inability to read to high precision. In the long run, however, these errors are all compensating, whereas in the quartz crystal they are accumulative; hence the pendulum is much the superior timekeeper over long periods of time.

The Pendulum

It is seen from the foregoing that the pendulum is still reigning supreme in the field of precision timekeeping, except for the territory of short time intervals, where the quartz crystal has superseded it. In this paper we are interested in the design and construction of a quartz pendulum. Consequently we shall devote considerable space to the factors controlling and errors entering into a pendulum.

Almost everyone is familiar with the equation of a simple pendulum, which is a point weight hung by a weightless string, and the period is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

This is an ideal case that can never be realized in practice, although the above formula will give approximate results for the case of a heavy weight hung by a string. The correct formula for a compound pendulum is given by

$$T = 2\pi\sqrt{\frac{I_o}{u'}} = 2\pi\sqrt{\frac{I_o}{\frac{mgs\ \sin a}{a}}}$$

In the above "T" is the period, "I" is the moment of inertia about the point of support, "m" the weight, "g" gravity, "s" the distance from the center of gravity to the point of support, and "a" the angle between the vertical and the inclination of the pendulum. With the use of Stiner's equation the above may be written in terms of the moment of inertia about the center of gravity.

$$T = 2\pi\sqrt{\frac{I_g\ a}{mgs\ \sin a} + \frac{sa}{g\sin a}}$$

Following are listed and discussed the most essential factors that affect the period of a pendulum.

1. Temperature

   The greatest single error arises from the change in length caused by changes in temperature. There are three ways of getting around this, as listed below.

   a. Compensation, by using metals of different coefficients of expansion.

   b. The use of materials of extremely low coefficient of expansion. The two most notable substances and their coefficients are -

Invar (36% Ni) Steel	$.90 \times 10^{-6}$
Fused Quartz	$.25 \times 10^{-6}$

   c. Control of temperature by means of a thero-couple and electric heating units (light bulbs) all located in a good heat insulated case.

2. Pressure

   The effect of pressure is quite pronounced, but is one that is easily eliminated.

   a. Barometric compensation. A difficult and not very satisfactory method.

   b. By keeping a constant pressure in a vault. The method that is most generally used.

   c. Operating the pendulum in a vacuum, the best method, but one that is only applicable to pendulums whose amplitude is controlled, otherwise instability results.

3. Wear of Bearings

   As an ordinary second pendulum has to make 86,400 complete swings a day, it is seen that this wear soon mounts up.

   a. The general method is to mount the pendulum on jewel knife edges. A variation of this, to get away from wear, is to fix the knife edges and mount the plate on the pendulum.

   b. To suspend the pendulum by means of a small spring.

c. The compensated pendulum. This consists essentially of a compound pendulum mounted by a knife edge that is located at a point in it's length that gives the minimum change in period due to change in distance from point of support to center of gravity. This is worked out and well shown in the curve on page ten.

4. Amplitude Control

Examination of the equation of the pendulum shows that the period depends on the factor sin a divided by a. As long as the amplitude is constant, but the variations in amplitude will cause small changes in the period, hence the amplitude must be controlled.

a. Physical contact of electrodes. Loomis has secured good results with this method.

b. Photoelectric contact is the best means and has been employed by Thomilson.

5. Damping

It is desirable to make this factor as low as possible. For the smaller the driving force the less error will irregularities in input power cause. Decreased damping may be accomplished by:

a. Operating in a vacuum.

b. Decreasing the friction of bearing or suspension spring.

6. Driving Force

The shape of the driving force has an effect though of relatively minor importance. The best would be to apply a sinusodial force at all times, but if we had such a thing we would not need a pendulum. Good results are obtained by applying the force just after the pendulum has passed through the center of it's swing. Various means of doing this are

a. Mechanical push - used on most astronomical clocks.

b. Electromagnetic drive.

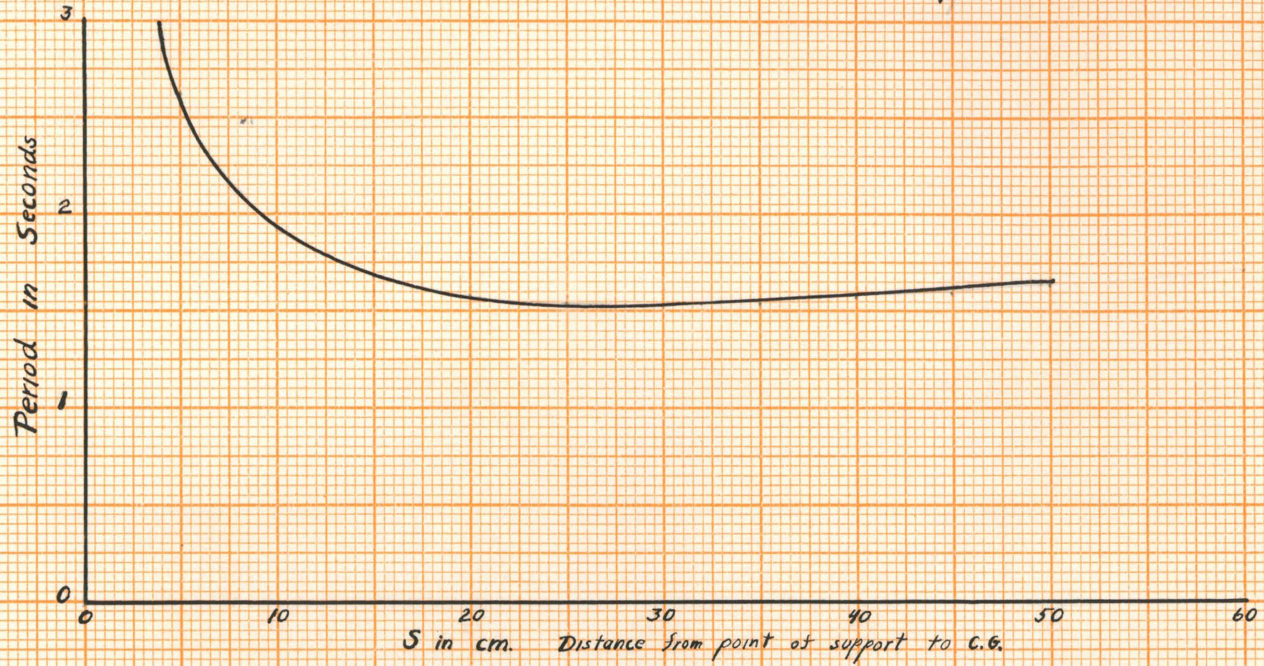
c. Electrostatic drive.

7. Recording the Time

a. Mechanical means - by a trip signal, or gear and wheel.

b. Electrical means - by an Hg contact, or mere contact points.

Compensation Effect found in the Compound Pendulum



As an example we will take the simple case of a rod which is 100 cm. long and ½ cm. in diam. Now

$$T = 2\pi\sqrt{\frac{I_g + ms^2}{\frac{mgs \sin\alpha}{\alpha}}}$$

For small angles $\sin\alpha = \alpha$ so

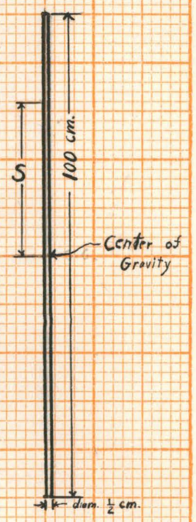
$$T = 2\pi\sqrt{\frac{I_g}{msg} + \frac{s}{g}}$$

Now I_g for a rod $= m\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$ Hence

$$T = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{l^2+3r^2}{12s} + s}$$

Subistuting the values in

$$T = .2\sqrt{\frac{8334}{s} + s}$$



Using this formula the points for the curve above were ploted. It is seen that in the vacinity of s = 27 the curve is flat which shows that for a considerable change in S there will be a very small change in T.

c. Photoelectric means - beam of light on mirror, or a light beam passing through a slit. Much the best.

8. Ageing of Materials

This is a long time effect for which, at the present time, there is very little information available. In some clocks a stretching or permanent elongation has been observed.

A careful consideration of all of the above factors, and wide reading of experimental investigations on precision timing, leads one to the conclusion that there are two vital methods of attack open. The first is a free pendulum that is mounted by means of quartz fibres or ribbon. The second method is to use knife edges and make use of the compensation effect of the compound pendulum.

This second method is being worked on by M. Shuler in Germany, and he is obtaining very excellent results. However, it must be remembered that while this effect reduces the error due to wear on bearings, it does not eliminate it.

The author believes that a free pendulum suspended by a quartz fibre or ribbon will give better results than a compensated pendulum, as it is seen that the error due to wear of bearings is completely eliminated when the pendulum is suspended by a quartz spring. The only possible error would be that the quartz might show fatigue effects and stretch. However, this is quite unlikely and would be extremely small.

Another advantage that we hope to gain by the use of the free pendulum is that due to the low internal resistance of quartz it would be possible to reduce the damping further than by any other means. These reasons lead the author to choose the free pendulum for his design, the details of which are presented on the following pages.

Design of Pendulum

Choice of Length and Period

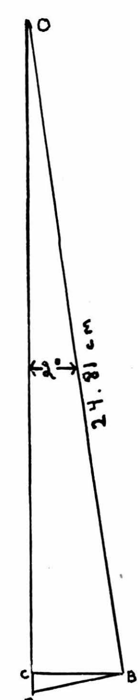
The first problem that confronts us is what period we wish the pendulum to operate at, using the equation of a simple pendulum we have

$$l = \left(\frac{T\sqrt{g}}{2\pi}\right)^2$$

If we wish a pendulum that will beat seconds then T = 2 and substituting in the above and solving we get l = 99.25 cm. This is rather long, and a little awkward to work with, and as there seemed, at least in experimental work, no particular reason why we should not work with a shorter interval, a period of T = 1 was chosen. From the above equation we find the length to be l= 24.81 cm. and this was the value chosen for our design.

Calculation of Energy

It was found that in most precision clocks the pendulum was permitted to swing through an angle of 2°. From the figure the horizontal and vertical distances are calculated.



$CB = l\sin\theta = 24.81 \times .0349$

$CB = 0.867$ cm.

$CA = l\,(1 - \cos\theta) = 24.81 \times (1 - .99939)$

$CA = .01513$ cm.

Therefore the energy of the pendulum is equal to

$$WgCA = .01513 \times 979.6 \times w = 14.84\ W \text{ ergs}$$

where w is equal to the weight of the pendulum.

Now let us consider the various factors that affect a pendulum, which were outlined in an earlier part of this report. We shall consider each case, and determine what is the best procedure to follow.

Temperature

First of all, the pendulum is to be built entirely and completely of quartz, a rather difficult procedure, but one that can be accomplished. Quartz has the lowest coefficient of expansion of any usable substance, it being only 4.2×10^{-7} in the range from 0 to 30° C.

This minimizes the error due to temperature changes, but it will also be necessary to control the temperature and this is to be done by building a heat insulating case in which the temperature will be maintained at a slightly higher value than the average temperature surrounding the case. This is done by the use of a thermo-couple and electric light bulbs for heating units. A highly accurate thermometer should be installed in the case in such a position that it can be read periodically. It should be possible, however, to keep the temperature within one one-hundredth of a degree centigrade at all times. For example, if the temperature changes 10° C the clock will be in error by 4.32 seconds a day.

Now $$T = 2\pi\sqrt{\frac{l}{g}}$$

$$dT = \frac{2\pi}{\sqrt{g}}\frac{1}{2\sqrt{l}}\ dl$$

But $dl = a\, l dt$

$$\therefore \quad dT = \frac{\pi}{g}\sqrt{l}\ a\ dt$$

$$dT = 2.1 \times 10^{-7}\, dt$$

It is seen from the foregoing that by keeping the temperature to within one one-hundredth of a degree, which is not difficult with a good thermo-couple, the resulting error will be only 2.1 in 10^9 , or expressed in another way, it will be 0.066 seconds per year. As it is hoped to obtain an accuracy of one part in 10^8 it is therefore essential that the temperature be kept to within one one-hundredth of a degree.

Pressure

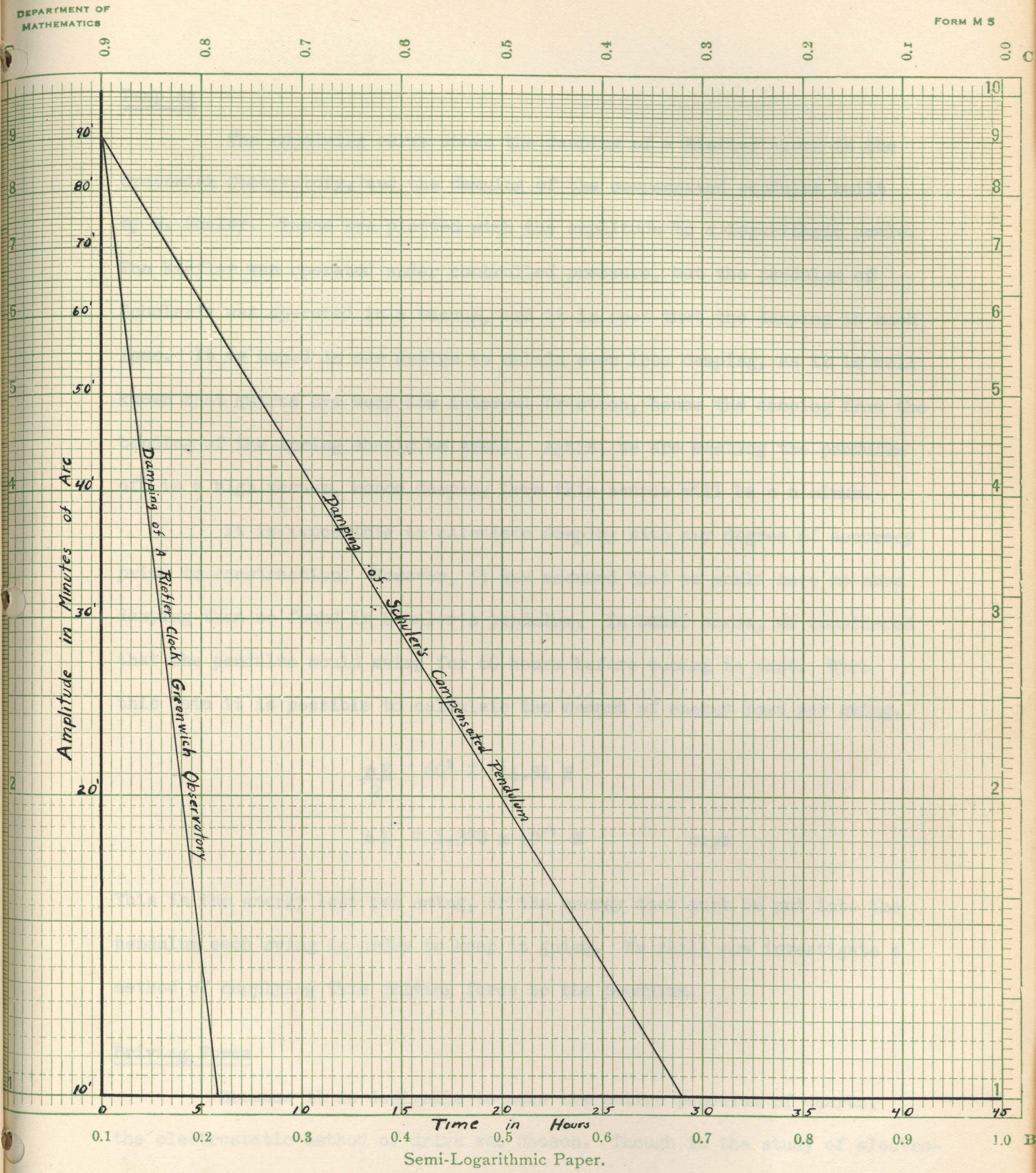
The error introduced by variations in pressure is easily taken care of, as we plan to operate the pendulum in a good vacuum. Not only from the pressure standpoint is it desirable to have a high vacuum, but it also greatly reduces the damping, and hence it will require less energy to keep it in operation. The drawings call for the pendulum being mounted in a pyrex glass tube, and sealed off in a high vacuum.

A high vacuum has one bad effect, in that it produces a slight instability. At least, such has been the observation of Tholminson and of Loomis. This objection does not apply, however, when a close amplitude control is maintained, and as it is necessary to use amplitude control anyway, the objection need not be contended with.

Wear on Bearings

This factor is eliminated altogether by the use of a fine quartz ribbon as a suspension spring. However, there is one factor to be watched for, and that is the creeping or shape-changing effect that quartz seems to have after it has been fused. It is felt that this will be small, and will not continue after the quartz has been sufficiently aged.

DAMPING


DEPARTMENT OF MATHEMATICS
FORM M 5
Amplitude in Minutes of Arc
90'
80'
70'
60'
50'
40'
30'
20'
10'
Damping of A Riefler Clock, Greenwich Observatory
Damping of Schuler's Compensated Pendulum
0
5
10
15
20
25
30
35
40
45
Time in Hours
Semi-Logarithmic Paper.

Damping

The adjoining curve shows the damping of a Riefler clock in the Greenwich Observatory, and the damping of the compensated pendulum built by M. Shuler. These are plotted with the amplitude to a logarithmic scale. The Riefler was operated under controlled pressure, but the pendulum of Schuler's was operated in a vacuum, and it is seen that the damping is much less. It is hoped in our design to obtain even less damping, as it is well known that quartz has very low internal friction, hence the damping from the bending of the spring should be small. Again, we are sealing the pendulum off in a high vacuum, hence damping from this source will be a minimum.

It is impossible to calculate damping with any degree of accuracy prior to construction. However, it was assumed that we would have no more damping than we found in Schuler's pendulum. In other words, we considered that the pendulum would swing for 60 hours before coming to rest. With this data it is possible to calculate the amount of energy lost per swing.

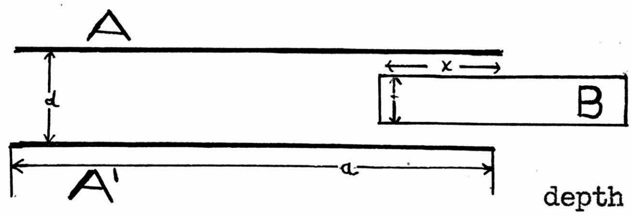
$$\frac{\Delta E}{2} \quad 60^3 = 14.84 \text{ W}$$

$$\Delta E = 1.370 \times 10^{-4} \text{ W} \qquad \text{ergs}$$

This is the energy lost per swing, or the energy that must be put into the pendulum each swing in order to keep it going. We shall now investigate a method of supplying this driving force to the pendulum.

Driving Force

Because it is desirable to make the entire pendulum of quartz, the electrostatic method of drive was chosen. Though in the study of electrodynamics it has long been known that a dielectric placed in the field of two parallel condenser plates is pulled into the center of them by the electrostatic field, as far as is known this fact has never before been made use of to drive a pendulum. This method just suited our needs, as we could build the pendulum entirely of quartz, which would not have been the case with a magnetic drive. In construction we were able to build the quartz pendulum and then seal it off in a pyrex glass tube, and apply the driving force from the outside by means of parallel condenser plates. On the following pages the derivation of the equations for the calculation of these forces are worked out.



depth of plates B

Capacity before inserting plate B

$$C = \frac{ab}{d}\,p \qquad \text{where} \quad p = 8.84 \times 10^{-14} \text{ for vacuum}$$

Now, if a metal plate B is inserted a distance x between the condenser plates A, A'

$$C = \frac{x\,b\,p}{d-t} + \frac{(a-x)\,b\,p}{d}$$

$$= \frac{p\,x\,b\,d + (d-t)\,(a-x)\,bp}{d\,(d-t)}$$

$$= \frac{pxbd - bpda - bxpd - atbp - txpb}{d\,(d-t)}$$

$$= bp\,\frac{(ad - at + xt)}{d\,(d-t)}$$

$$= \frac{abp}{d} + \frac{tbp}{d(d-t)}\,x$$

It is seen from the above equation that the capacity will increase directly with the displacement x.

Now, the energy of a condenser is equal to

$$W = \frac{C\,E^2}{2}$$

If the voltage is kept constant the change in energy will be

$$DW = \frac{E^2}{2}\,dc$$

But we have already found from above that by inserting the plate B

$$dc = \frac{tbp}{d(d-t)}\,x$$

Hence $$dw = \frac{E^2}{2}\,\frac{tbp}{d(d-t)}\,x \text{ joules increase in stored energy}$$

If we make the width of the plate B equal to $\frac{a}{2}$ then the energy delivered is

$$dw = \frac{E^2}{4} \frac{abpt}{d(d-t)} \qquad \text{joules}$$

From previous calculation we have that the damping for each swing is equal to

$$\Delta E = 1.370 \times 10^{-4} \; W \qquad \text{ergs}$$

But

$$W = \frac{a}{2} bt \;\; 2.65 \qquad \text{for quartz at room temperature}$$

Hence

$$\Delta E = (1.370 \times 10^{-4}) \; (1.325 \; abt) \; 10^{-7} \quad \text{joules}$$

$$\Delta E = 1.815 \; abt \times 10^{11} \quad \text{joules}$$

By equating these two we can derive an equation that will tell us what voltage is necessary to supply the lost energy.

$$\frac{E^2}{4} \; \frac{abt \times 8.85 \times 10^{-14}}{d(d-t)} = 1.815 \times 10^{-11} \; abt$$

$$E^2 = \frac{7.261 \quad 10^{-11}}{8.84 \quad 10^{-14}} \; d(d-t)$$

$$E^2 = 823 \;\; d(d-t) \qquad \text{volts}$$

or

$$E = \sqrt{823 \; d(d-t)}$$

This gives us an equation for voltage in terms of the distance between the plates. It is to be observed that the equation is only approximate, as all end effects were disregarded.

The next problem that confronted us was to choose the dimensions of the pendulum. In order to do this intelligently the following table was made up. Notations are those used in the previous figure.

	a	b	t	d	E^2	E	W
	cm	cm.	cm.	cm.	volts ^{2}	volts	grams
Max.	1	15	2	3	2480	49.9	39.8
Min.	0.1	1	.2	1	640	25.3	.0265
Av.	.5	8	1	2	1650	40.6	5.3
	.4	14	1	2	1650	40.6	7.41
	.6	10	1.2	2.2	1815	42.6	9.55
	.7	10	1	2	1650	40.6	9.28
	.6	2	1.2	2.2	1815	42.6	1.91
	.6	10	1	2	1650	40.6	7.96
Final	.6	11.33	1	2.5	3080	55.5	8.77

The last value in the table was the one chosen and the drawings were made up on this basis. It is seen that in the neighborhood of 55 volts will be required to keep the pendulum in operation. On the following pages are the detailed drawings from which the pendulum was constructed.

Calculation of Size and Strength of Quartz Spring

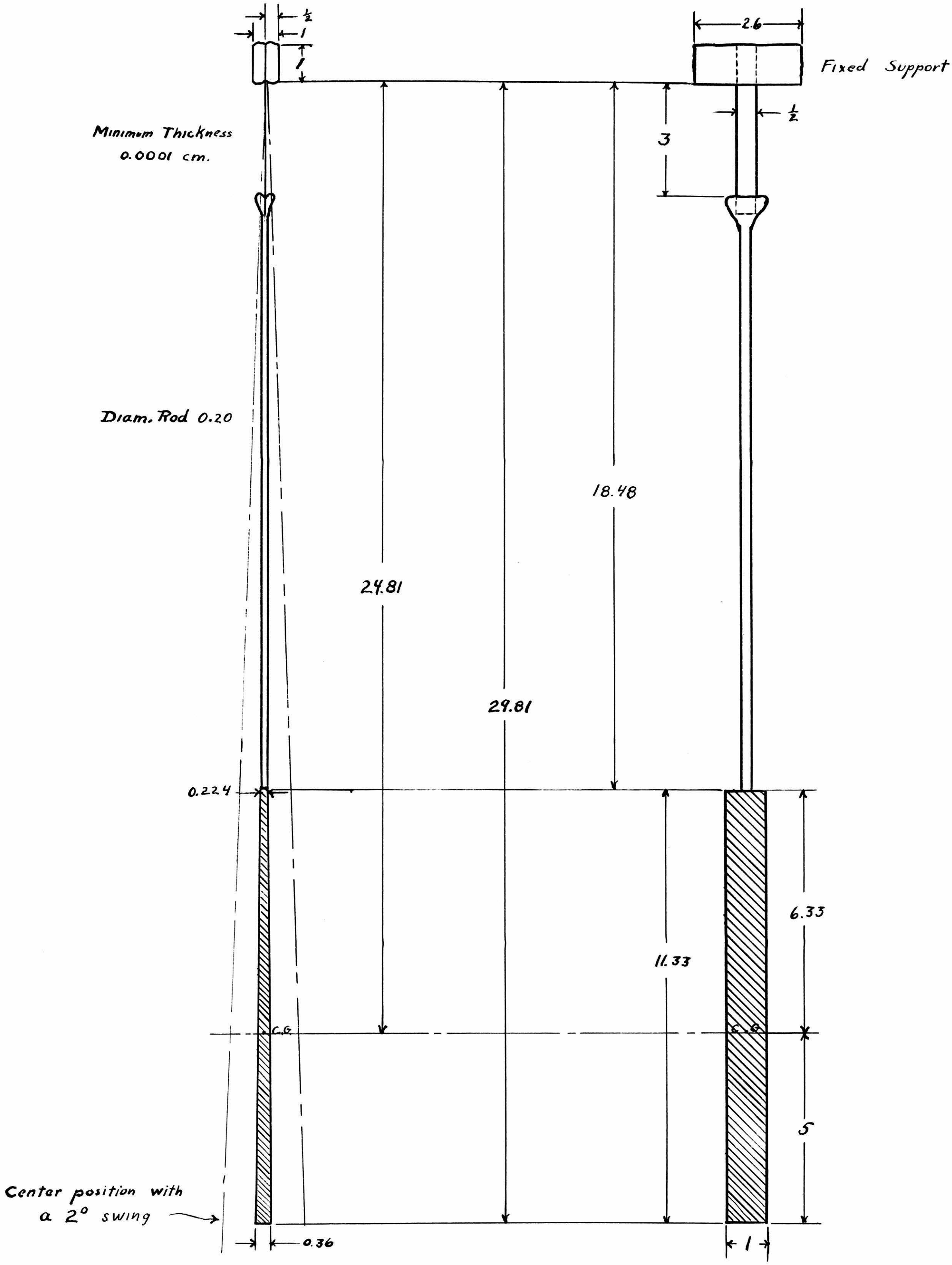
From the previous table the weight to be held up is 8.77 grams. From standard tables we obtain the tensile strength of fused quartz, which is 1.1×10^9 dynes per square cm. Now

$$\frac{9 \times 980}{1.1 \times 10^9} = 8 \times 10^{-6} \text{ sq. cm.}$$

is the minimum area to hold up nine grams. If we allow a factor of safety of 6.25 then the area becomes 5×10^{-5} sq. cm. Next let us assume a spring one half cm. wide, then the thickness will be

$$t = 1 \times 10^{-4} \text{ cm. or } \frac{1}{10,000} \text{ cm. thick}$$

The rotation of the earth causes the pendulum to progress around in a circle, or if we have a perfectly free pendulum it will swing in a plane, and the earth will turn with respect to this plane. This is known as the Foucault effect. To make the pendulum follow the earth's rotation and to prevent it from swinging in an eliptical path, great care was used in the design of the pendulum so that it would be rather rigid in all but it's normal path of swing. Consequently a rather heavy rod (0.20 cm. diam.) was used to suspend the bob, and the spring was made one-half cm. in width to prevent any sideways motion.



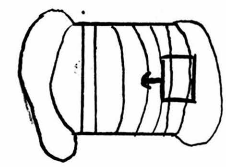
DESIGN OF QUARTZ PENDULUM

Made entirely of fused quartz

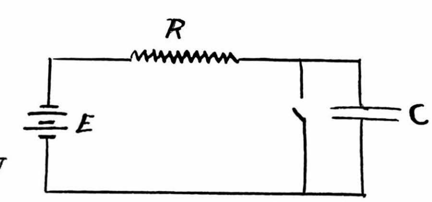
All demensions in cm. Scale 1" = 4 cm.

DESIGN OF THE ELECTRIC CIRCUIT

As has already been mentioned, the pendulum is to receive its driving force from the effect that the bob will be pulled in towards the center of a parallel plate condenser. It will be readily seen that if the voltage is maintained on the condenser plates the bob will be held in the center position. It is, therefore, essential to cut off, or short-circuit the voltage just as the condenser reaches the center position. Also, the voltage should be kept down for a sufficient length of time to permit the bob to swing away from the field of the condenser, without a retarding force.



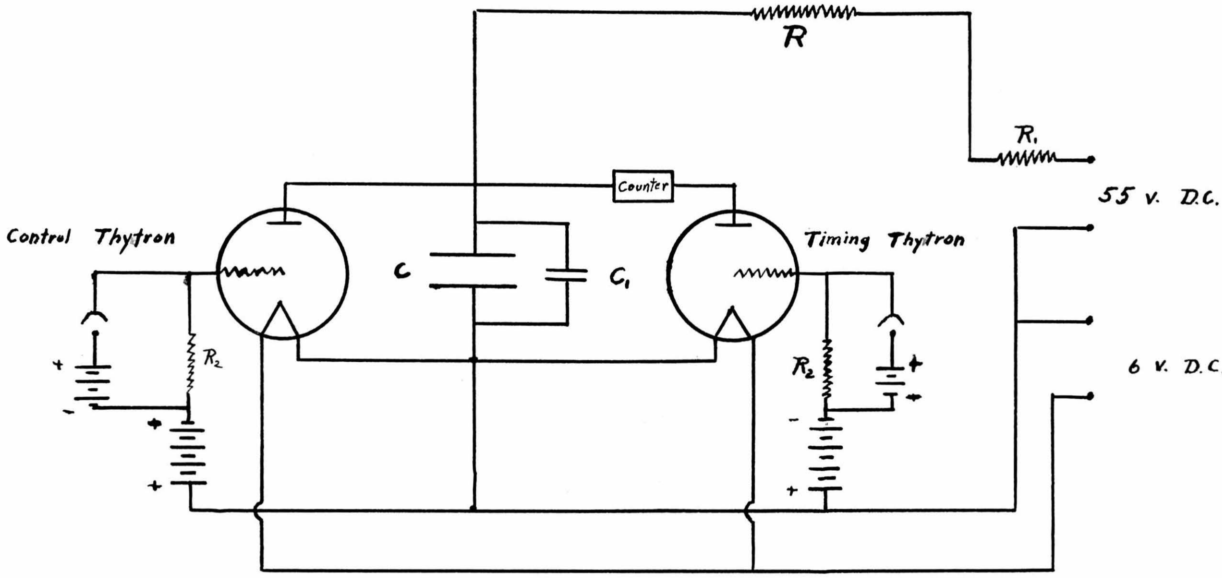
As no circuit was known that fulfilled the conditions we were searching for, it was necessary to design a new one. The nearest approach to our requirements that could be found in the literature was that used by Tomlinson in his work. His had several objectional features, however, and so a circuit was designed to suit our needs. The circuity is essentially as shown, consisting of a constant potential which charges the condenser through a righ resistance. The condenser is then shorted just as the bob passes through the center position. It is desirable to have the switching done as fast as possible so as to prevent any retarding force, consequently it was planned to short the condenser through a thyratron, which has a tripping time of only 10^{-5} seconds, which is faster than any mechanical relay.



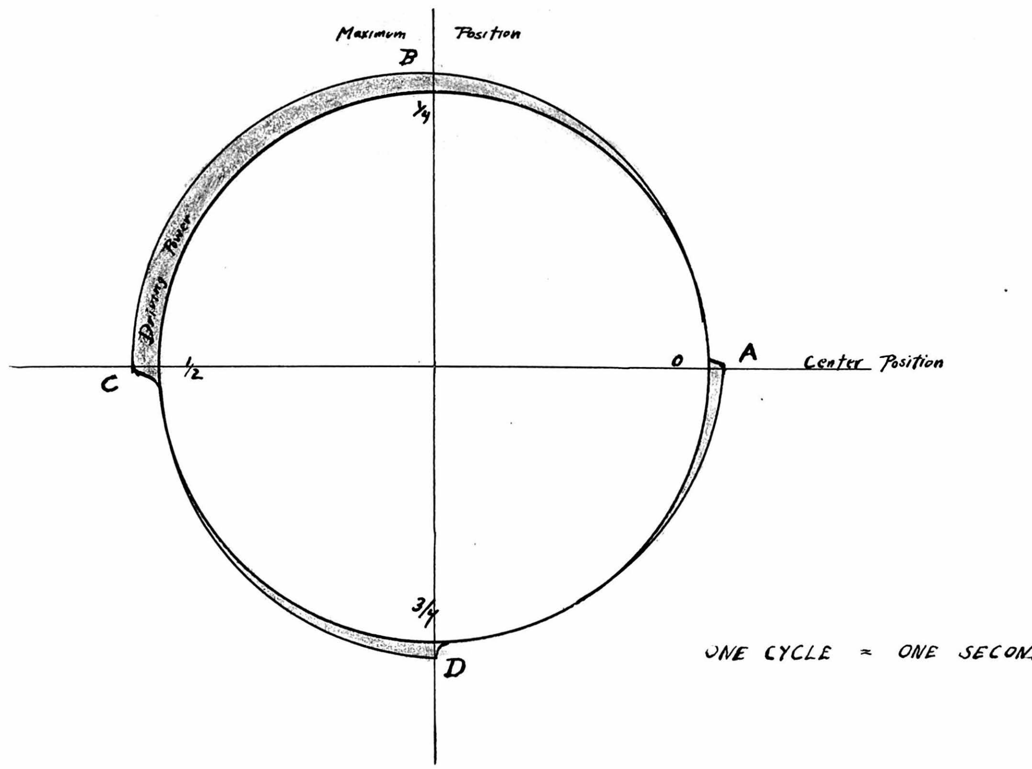
The actual circuit as it was developed is best understood by studying the drawing to be found on the following page. The condenser C is the one that drives the bob, and it consists of two long thin brass plates placed just outside the pyrex container of the pendulum. As we desired more capacity than is contained by this condenser, another larger condenser C_1 was placed in parallel with it. The timing thyratron shorts the condenser each time the pendulum passes through the center position, that is, each half second. There is, therefore, placed in the plate circuit of this thyratrom a solenoid counter, which will record the number of times the pendulum passes through the center position. In a permanent installation this half second impulse would be used to move the hands of a clock.

It is seen from the circuit that the thyratrom is actuated by a photocell which receives an impulse from a beam of light that is reflected by a small oscillograph mirror mounted on the rod of the pendulum. This same mirror reflects another beam of light at the point of maximum amplitude, and this beam falls on the photocell in the grid circuit of the control thyratron. Whenever the pendulum swings far enough out to trip this circuit the condenser is shorted and hence the next impulse given to the pendulum will be very small. By proper adjustment of the forces the amplitude may be maintained to comparatively close limits by this means.

On the following page is a drawing of the cycle of operation. At point A the timing thyratron has just shorted so the pendulum swings out freely; as it swings out the charge on the condenser slowly builds up, and as it comes to rest and starts back on its return trip it is pulled into the condenser until it reaches the point C where the timing thyratron is again shorted by



CIRCUIT FOR DRIVING CONDENSER

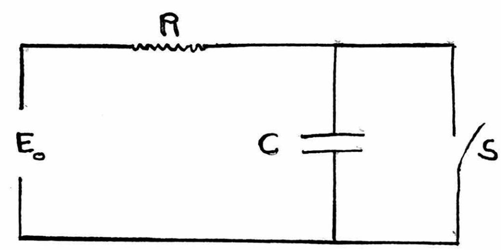


PROPOSED CYCLE OF OPERATION

the light beam, permitting the bob to swing out freely. This time we will assume that it swings a little too far, and so trips the control circuit, which shorts the condenser, hence when the bob next enters the condenser it will receive only a small part of its normal pull. The voltage applied to the condenser should be such that it causes the control circuit to trip once every four or five swings. Calculations show this voltage to be in the neighborhood of 55 volts, but only by test will the exact value be found, consequently a variable resistance R_1 is shown in the circuit for the purpose of regulating the voltage.

The grid of the thyratron quickly regains control after the short-circuit, as when the thyratron is shorted the voltage falls so low that it is unable to maintain the arc. This takes place in approximately 10^{-5} seconds. After this the condenser slowly regains its charge, being almost completely charged one half second later, although at the one-quarter second point it is only about half charged. As the force applied to the pendulum varies as the square of the voltage it is seen that the force when the control thyratron is tripped is only one-fourth of the usual applied force.

Calculation of Circuit Constants



The thyratron which is shown as the switch S is shorted, but the voltage almost instantly falls below the value necessary to maintain the arc, and hence the grid will regain control, which in our figure is the same as opening the switch S.

Now

$$i = \frac{E_o}{R + \frac{1}{pc}} = \frac{PCE_o}{RCP\ 1} =$$

$$i = \frac{E_o}{R}\ \varepsilon^{-\frac{t}{RC}}$$

Hence

$$E_c = E_o - i\,R$$

$$= E_o - E_o\ \varepsilon^{-\frac{t}{RC}}$$

$$= E_o\ (1 - \varepsilon^{-\frac{t}{RC}})$$

We wish E_c to take $\frac{1}{2}$ second to build up to a reasonable value.

When $\frac{t}{RC} = 1$ $\qquad E_c = .632\ E_o$

$\frac{t}{RC} = \frac{1}{2}$ $\qquad E_c = .393\ E_o$

and $RC = \frac{1}{2}$

or if $R = .5$ megohms
$C = 1$ mfd.

It will be noticed from this calculation that there will be a regulating effect for any changes in the time t. Hence if the voltage E_o can be kept constant and if R remains constant there will be good automatic amplitude control.

AMPLITUDE CONTROL

It will be recalled that in the calculations for the period of the pendulum for the factor $\frac{\sin a}{a}$ was considered equal to unity for small angles. It is, of course, very close to unity, but it is obvious that small variations in the angle will cause variations in the factor $\frac{\sin a}{a}$, hence for precision work it is necessary to keep the angle a constant.

This was accomplished by means of a control thyratron and photoelectric cell. When the pendulum swung out to a certain maximum amplitude a mirror on the rod of the pendulum reflected a beam of light which trips the photocell causing the thyratron to short the driving condenser. This means that the following driving impulse will be only one-quarter as strong as usual. By so regulating the applied voltage that this tripping will take place only every four or five swings very close amplitude control is maintained.

RECORDING

The pendulum has been designed to beat half seconds; however, in construction this cannot be realized exactly. Hence, for experimental work and precision work it is simpler and more accurate to determine a multiplying factor. In our set-up the time is to be found by inserting a solenoid counter in the plate circuit of the timing thyratron. This will count the number of times the pendulum passes the center point and so this number multiplied by the factor will give the correct ttime in seconds.

To compare the time of the pendulum accurately with a standard such as the signals sent out by the United States Naval Observatory in Washington, D.C. the impulse from the plate circuit of the timing thyratron is lead to a chronograph or to an oscillograph. The time signals are sent out corrected to an accuracy of one-hundredth of a second. This means that in a days run the pendulum can be checked one part in 8.64×10^6 which is pretty high accuracy.

For this work a high precision chronograph is not necessary as the time signals are accurate to only one one-hundredth of a second. A study of a chronograph has not been undertaken, but several suggestions can be offered. A little reed attached to a piece of iron in front of a magnet that is actuated by the impulse could be used to make a mark on blackened piece of paper, or on a soot covered phonograph record. Or again, the impulse voltage could be raised and a spark made to pierce a paper strip as it moved uniformly past a pair of electrodes. There are many possibilities and the details are left to whoever carries on this work.

THE LIGHT CIRCUIT

As the thyratrons will require about six volts d.c. it was thought an excellent thought than an automobile head light bulb would make an excellent source of light. This should be focused and passed through a slit onto the mirror which is mounted on the rod of the pendulum. From here it is reflected into a photocell. There naturally arises the question of the sharpness of defination that can be obtained by this method.

For our work it was decided that the use of an oscillograph mirror which was only 2 x 3 thousands of an inch in size, together with a narrow slit would give us sufficient light and sharpness of action for the work. However, if after experimentation it is thought that greater light and greater sharpness of defination are desirable the method of a series of slits which was developed and worked out by Tomlinson in his report should be used.

FUTURE DEVELOPMENTS

As is seen from the foregoing work considerable theoretical investigation has been carried on regarding the problem of developing a better precision pendulum. A free quartz pendulum was chosen and designed. At the present time such a pendulum is nearing completion in the Physics Department of the Institute. Great difficulty has been encountered in fastening the fine quartz spring to the heavier quartz rod. It has been found that we are unable to fuse them as was originally hoped, but have had to resort to making plates and cementing the spring between them with silver chloride. Once this is accomplished the pendulum still remains to be evacuated and then set up in a solid mounting. This mounting had already been constructed, as has the condenser.

By offsetting the condenser and applying a voltage the pendulum may be started. The first test to be made is that of damping, which will consist of giving the pendulum a certain amplitude and then permitting it to swing freely and come to rest. The amplitude should be measured every hour or so and a curve plotted similar to those shown in this report. After the amount of damping has been determined accurately the calculations of the voltage necessary to drive the pendulum will have to be made over again. Also, the constants of the electric circuit will have to be calculated. Having accomplished this the timing circuit should be constructed and the pendulum be placed in operation. Obtain as constant a voltage as possible and make a long test to see if variations in amplitude are going to affect it much. If they seem to, the amplitude control should be built and connected.

The theoretical work done has attempted to include all the factors that affect a pendulum, but as the work on construction progresses and tests are made, some other things are likely to show up. However, it is believed that all the important factors have been taken care of. The work remaining to be done is principally construction and testing, all of which should be done with the greatest of care. Tests should be made over a considerable length of time, so as to be significant.

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