A NEW *POWER* FACTOR BRIDGE AND ITS APPLICATION TO SYNCHRONOUS MACHINE TESTING

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OUTLINE

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SUMMARY AND CONCLUSIONS

A new power factor bridge intended for measuring the power factor, and thereby the losses, of synchronous machines operating as synchronous condensers is described. "Two-arm" and "three-arm" bridges of all types are considered in detail and some types, including those previously used for synchronous machine testing, are shown to be inaccurate. The development of a tuned circuit amplifier for a galvanometer is described. Tests were made on three synchronous machines. Tests of one machine with high load loss indicated that short-circuit or "load loss" decreased rapidly with increase in terminal voltage. This decrease is substantiated by tests similar to the present AIEE test for load loss, but made with enough external resistance to produce a small terminal voltage.

Conclusions are: the accuracy of the bridge method is limited by the stability of the synchronous machine being tested; accuracy greater than that of the present methods of test can, however, be obtained; and measurement of load loss under short-circuit conditions, as is now standard practice, gives a loss greater than the actual loss.

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I NTRODUCTION

Loss tests on synchronous machines have long been made by belting a small driving motor or "tool" to the machine to be tested, and operating the machine at open circuit and then at short-circuit. The loss at full voltage and full current is then assumed to be the sum of the open and short-circuit losses.

This procedure is objectionable: first, because of the trouble of belting a tool and because of belt and bearing loss the tool introduces; and second, because of the assumption that short-circuit losses exist at full load. Both of these objections are overcome by the test method of this thesis, since the measurement of power factor and KVA requires no belt and since the measurement of losses under synchronous condenser conditions is certainly more accurate than measurement under short-circuit conditions -- at least for synchronous condensers.

The presentation in this thesis deals largely with the subject of power factor bridge circuits and precautions necessary and corrections to be made in using them them, and with the subject of tuned amplifiers to be used with galvanometers.

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The subject of synchronous machine losses as measured by the method described is not pursued in detail, as it was not practical to make complete tests on large machines where load loss was appreciable. The one machine with large load loss that was tested was an induction motor operated as a synchronous machine. **The** results indicated that considerable change· in load loss can be expected in any machine at other than short-circuit conditions.

POWER FACTOR BRIDGE CIRCUITS

((for high voltage high current measurements)

The bridge circuits used, or considered for use, in this study were of two simple types. which will be identified as "two- arm" and "three-arm" power factor bridges. Due to the few number of "arms", one is able to list most of the possible circuits and then select the most suitable one.

"Two-arm" Bridges

In "two-arm"bridges, the voltage and current vectors from which the two bridge arms are built up are shown in Fig. 1. As there are only two arms, the problem is to obtain two parallel voltage vectors whose rotations from the above vectors are accurately known. *0* (*B0*°*f*₅ g 8[°]) *I* Fig. 1.

In order to obtain a voltage vector from the given current vector, recourse must be had to a resistance drop or to an inductive drop, since capacitance of sufficient magnitude to carry high currents at commercial frequencies is entirely impractical. Thus the current can be made to produce a voltage vector

in phase with the current, ninety degrees ahead of the current, or at any other desired angle in the quadrant preceding the current, as shown by the conventional circle diagram.

The phase relation between the current and the voltage vector obtained from it must be known to as small an angle of error as the angle of error allowable in the measurement of the power factor. If a power factor of 2%, for example, is to be known with an accuracy of $\frac{1}{2}\%$, the maximum allowable angle of error will be .005 $x cos^{-1}$.02 = .005 x 1° 9' = .345 minutes (=21 seconds of arc) = .0057 degrees. If the voltage vector is to be rotated *85°* from the current producing it, to continue the example, the angle must be 85° ±less than .0057 degrees. δ f $85^\circ \pm$ less than .0077% or $85^\circ \pm$ less than one part in 14, 000.

In a three phase circuit, this error of one part in 14,000 could be produced by a coefficient of coupling of $\frac{l}{44,000}$ = one part in 8,000 between inductive shunts in the three phases. To obtain such low coupling between the phases with unshielded air core inductive shunts is not impossible, but is certainly quite difficult. If shielding were introduced, the losses in the shielding iron would be so indefinite

and variable that it would be almost impossible to determine the angle they introduced into the inductance. Consequently, although used in the first part of this investigation, air core inductive shunts and mutual inductance have been discarded as means of obtaining a voltage vector rotated from the current vector.

There remains the one possibility of using an iron core inductance, or a current transformer. Both of these involve not only calibration of error by laboratory tests, since iron losses cannot be accurately calculated, but also the assumption that magnetization of the iron and aging of the iron would not alter the calibration. To confirm the impracticability of this, the following quotation is made from a letter by A.S. McAllister, Acting Director of the Bureau of Standards. ".... inquiry concerning measurement of phase angle of current transformers to an accuracy of 5 seconds of arc." (.08 minutes or one part in $32,000$ of an angle of $85%$ "This accuracy is well be yond that which is commonly attained in testing work and we are inclined to doubt whether you will be able to find a transformer which will consistently repeat its phase angle to this precision for any long continued period."

Thus, at least with polyphase circuits, the only

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practical way to obtain a voltage vector with an accurately known relation to the current vector, is by use of a non-inductive shunt. This point has been gone into thoroughly because it was the reason for changing from the circuit originally used to the final circuit. Also the values of one part in 14,000 developed for maximum allowable phase angle error apply to any method of rotating the voltage vector **as** well as to any method of rotating the current vector, for which reason they will be used again.

To continue the discussion of "two-arm" bridges if the voltage of the "current" bridge arm is in phase with the current, the voltage of the "voltage" bridge arm must be rotated from the voltage vector by an angle equal to the power factor angle, and this angle of rotation must be known with less than the error of about one part in 14,000 as determined above. If rotation is to be produced by inductance of any type, therefore, the stray field linking the inductance is of vital importance. To illustrate this importance of average stray fields, a simple calculation will be made of the field concentration necessary in any air-core instrument to reduce stray field effects to less than one part in 14,000 when the instrument is used under average "test floor" conditions.

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Assume as average test floor conditions, that the stray field is that set up by a symmetrical three phase bus of considerable length carrying 500 amperes in each line. The field due to one bus at a distance "d" cm from that bus, will be:

$$
H = \left(\frac{2 \Gamma \left(\ln a \tan \varphi s\right)}{d}\right) \ln e \sin \varphi t
$$

= $\frac{2 \times 500}{d} \epsilon^{j \omega t} \quad | \text{ines/cm}^2$

The field at the same point due to the other busses will be, under the conditions of Fig. 2, approximately
 $\frac{2 \times 500}{d+0.6665} \epsilon^{j(\omega t + \frac{3\pi}{3})}$ and $\frac{.2 \times 500}{d+0.6665} \epsilon^{j(\omega t + \frac{4\pi}{3})}$ where "s" is the separation between busses. The total $H_p = 2 \times 500 \left[\frac{\epsilon^{j\omega t}}{d} + \frac{\epsilon^{j(\omega t + \frac{2\pi}{3})} + \epsilon^{j(\omega t + \frac{4\pi}{3})}}{d + .6665} \right]$ field will therefore be: = $2*500\left[\frac{\epsilon^{j\omega t}}{d} + \frac{\epsilon^{-j\omega t}}{d + .8665}\right]$ \overline{P} = $100 \frac{e^{j\omega t} \left| \frac{l}{d} + \frac{l}{d + .8665} \right|}{ }$ = $100 \epsilon^{j \omega t} \left[\frac{S}{d (d+.866S)} \right]$ $Fig. 2.$

If $.866s = 2$ feet = 61cm and $d = 20$ feet = 610cm,

$$
H_p = 100 e^{j\omega t} \left[\frac{6!}{6! \rho (6! \rho + 6!)} \right] = \frac{100}{67} = \frac{1}{67.1} \text{ /me } \left/ \text{cm}^2 = \frac{1}{10.4} \text{ /ine } \right/ \text{ in}^2
$$

 $\frac{1}{47}$ lines/cm² must be less than one part in And if 14,000 of the total flux, the flux through the mutual $\frac{14,000}{h^{7.1}}$ = 209 lines/cm² inductance must be

The ampere turns needed in a coil one inch in radius

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to obtain density at the center of 209 lines/ cm^2 is given by $H = \frac{2\pi M}{r} = 209 = \frac{2\pi N I}{2.54}$

$$
\therefore NI = \frac{209 \times 2.54}{.2\pi} = 850 \text{ Ampere turns.}
$$

This, with 500 $Amps/in^2$ means a copper cross section of 1.7 square inches. The proportions of such a cioil

are shown in Fig. 3. Thus, if the stray field assumed above is to be negligible, any air core instrument or mutual in-

ductance must be of entirely

impractical size and proportions. If shielding is to be resorted to, the shielding will concentrate the stray fields outside and will allow the inside field to produce iron loss. Such a shield would be cumbersome and heavy, and the result would be hard to calculate, as any tests would have to be made at a series of currents to evaluate saturation effects,

It has been shown then, that the obtaining of a voltage vector displaced by a very accurately known angle from either the current or the voltage of the machine cannot be satisfactorily accomplished by means of an air core self or mutual inductance without prohibitive shielding and calibration. The possibility of using an iron core inductance is eliminated by the same considerations that have already been taken up

in connection with the "current arm" of the bridge i.e. aging of the iron core.

Thus the only remaining possibility is the use of capacitance to provide a rotated voltage. The consideration of possible defects in condensers will cover: first, dielectric loss (or power factor); second, stray electrostatic field effects; and third, leakage effects.

Dielectric loss. Since condensers with ordinary dielectrics have power factors of the order of magnitude of $\frac{1}{2}$ to 1%, and since the desired error is .0059 degrees, any ordinary condenser used would have to be very accurately tested to determine the value and constancy of its power factor. Mica condensers have lower loss than any others using liquid or solid dielectrics, but compared with the desired accuracy of .0059 degrees, even their power factor is high and would have to be calibrated. The only remaining possibility is, then, an air condenser.

Stray electrostatic field effects. Since any condenser used could or would be shunted by a resistance of the order of magnitude of one megohm, the stray electrostatic field effect would be entirely negligible compared with the leakage effect.

Leakage effect. With the condenser shunted by approximately a megohm, and with a desired error of less than one part in 14,000, the leakage resistance

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between the ungrounded side of the condenser and any source of out of phase potential must be 14.000 x one megohm x the ratio of the condenser voltage to the out of phase voltage x the sin of the angle between the two voltages = $140,000,000$ for $10,000$: 1 voltage ratio. But the order of magnitude of the resistance of insulators is 1,000,000 to 1,000,000,000 megohms per cm. cube. so that a square inch of dielectric only .01 inch thick would have 10,000 to 10,000,000 megohms resistance, which is more than necessary. Therefore the surface leakage is all that can be of importance, and as this is variable with dirt and moisture conditions. it cannot be calculated. However, by careful attention to this item, it can be made negligible. as it can be made to approach the resistance of any dielectric itself.

If, then, air condensers can be made to provide sufficiently pure reactance without calibration or correction factor. they are the logical ones to use unless their inherently small capacitance makes such use prohibitive.

Consideration of Final Circuit

There are many ways in which a "two- arm " bridge may be connected using air-condensers, that will provide wholly practical apparatus for the purpose, but we will confine our discussion simply to the circuit used in order that it may be quite fully considered. The circuit is shown in Fig. 4.

Fig. 4

Circuit originally used but discarded due to stray field effects described on Page |O

Fig. 5

 $Fig. 6$

Circuit used by Churcher and Dannatt¹, but shown in this thesis to have important error due to stray field. (See Page (0) .

Accuracy

As there are only two "arms" in the final bridge used, and as the detector was of the vacuum type, the consideration of residual errors at balance is much simplified.

Also see B Hague, "AC Bridge Methods." Pitman and Sons, London.

^{1.} B.G. Churcher, "Alternating current bridge methods. Their application to electrical engineering problems with special reference to the testing of synchronous condensers." Electrician, Vol. 101, pp518-520, 545- $547(1928)$.

The residual inductance of all the "non-inductive" resistors is one important source of error. These residual inductances are calculated in Appendix A, and are found to produce 50 cycle ratios of $\frac{X}{R}$ of $\frac{b}{10,000}$ 4 10,000

Since R_3 is used in a voltage divider, its phase error has no effect on the balance. The angular defects of R_2 and R_1 , however, add and subtract directly from the power factor angle being measured, as shown in

Fig. 7.

Fig. 7. Fortunately, therefore, they partly cancel each other.

The only other error, besides those due to phase

defects of the re sistors and the capacitor (which last is negligible), is that due to the condenser current, I_c , in Fig. 8, flowing through the resistance, R_{a} , of the voltage. divider. As the drop

across R_0 is fixed at about one volt by the sensitivity of the detector, I_c is fixed at $\frac{1}{R_2}$ amperes. Also the drop across R_p is fixed, as it is approximately equal to $\Gamma_c X_c = \Gamma_c \frac{X_c}{R_c} R_c = \Gamma_c (\omega t \theta) R_c$

= approx $\frac{1}{PF}$ = 10 to 50 V approx.

Thus the tangent of the angle \propto of Fig. 8 is $\frac{L_c R_a}{L_L R_c}$

 $\frac{\Gamma_c R_a}{50} = \frac{\frac{1}{R_a} \cdot R_a}{50} = \frac{R_a}{2R}$ And if $tan \alpha$ is to be kept below $\frac{1}{1000}$, $\frac{R_a}{R_a}$ will have to be $\frac{1}{500}$. Another way to eliminate error from this source is to make the voltage divider of condensers or reactors rather than of resistors, so that the loading current I_n will produce an in phase voltage between A and C.

Measurement of magnitude of resistances will be assumed to be readily accomplished with negligible errors.

Balancing

One unusual characteristic of "two-arm" bridges of the type described, is the peculiar procedure necessary for balancing.

If, for example, it is desired to make the drop E_g of Fig. 9 identical with the drop E_1 , it is necessary to increase E_h and

 $Fig. 4$

decrease $E₂$ alternately. But whereas, in the usual

bridge, varying of one voltage does not affect the balance of the other voltage, the varying of \mathbb{E}_b will change the balance of E_2 , and vice versa, so that the method adopted was that illustrated in Fig. 10,

where the series of dotted "V-curves" are taken in order \div to determine the main "Vcurve." The bottom of the Galyanometer main "V-curve" is the final balance point. The bottoms $F(q,10)$.

of the dotted **"V-** curves" can be determined by eye or by plotting, depending on the desired accuracy.

"Three-arm"Bridges

All that has been said regarding "two-arm" bridges applies to "three-arm" bridges except the method of balancing. This is true because the third arm is always small and at approximately a right angle with respect to the other two arms, so that its presence does not appreciably alter the position or magnitude of the two main bridge arms, but only assists in balancing them. The easiest method of introducing the third arm in the particular bridge used, is to add a shunt resistor, since a voltage in phase with the supply voltage is approximately at right angles with the current, and if the third arm is kept small enough, a phase defect in it of 10% is not prohibitive.

APPLICATIONS TO SYNCHRONOUS MACHINES

(of Power Factor Bridge)

All synchronous machines are now tested under short circuit conditions and then under open circuit conditions, l.e. first with no current and second with no voltage, and the iron loss with full voltage and full current is assumed to be the sum of the no-load iron loss plus the short circuit losses. A more simple and more accurate way of obtaining not only the iron losses, but all the "annature input" losses, (F & W, LL, Core L, and Arm $I^{2}R$) is to measure the power factor when the machine is operating as a synchronous condenser, that is, drawing full KVA but delivering no mechanical power. For synchronous condensers, this method of measuring losses gives exactly the desired loss, but for machines operating at other than zero power factor, there is some slight difference between "condenser" losses and the desired loss, due to the shift of the axis of demagnetization with power factor. This effect of change of loss with power factor will, however, be taken up later when short circuit losses are considered. Another advantage of the determining of armature losses by the determination of power factor is the ease of making the determination - no

belts or auxiliary driving motors being required. The only theoretical objection to the method is that any measurement that is made must be limited to fundamental frequency components of current and voltage, and that if the loss due to harmonic currents is to be taken into account, separate measurements of the power factors of the different harmonic currents and voltages will have to be made.

The necessary accuracy is not great, as the accuracy of the conventional method is not great. Eight per cent error in the total loss would not be prohibitive, and three per cent error would be very satisfactory, judging by the accuracy of the present methods. One factor that would help the accuracy considerably is the lack of belt loss and increase in bearing loss that goes with it.

The range of power factors to be measured by the bridge method would be from a minimum of one per cent for a machine with one per cent armature loss, 7 up to a maximum of ten or fifteen per cent, at which point a wattmeter reading full scale with twenty per cent power factor would be easier to use and sufficiently accurate if calibrated.

The maximum accuracy was found to be limited by machine instability in all the tests made for this study. It was first thought that this instabilityy

was due to variation of line frequency and voltage, but after trying storage batteries as a power supply (thru a motor generator set), and for excitation, it was found that almost as much was accomplished by using storage batteries for field excitation as was acc omplished by using storage batteries for the entire power supply, and that even with a storage battery supply enough instability remained to limit accuracy of balance to about one per cent. Later it was found that a direct connected exciter was quite satisfactory. The displacement of the test points from the final curves is a good indication of the amount of instability that remained. Fig. 17 shows a curve taken with storage battery power supply and Fig. 18 a curve taken with one motor generator set for excitation and one for power supply.

As mentioned before, a tuned galvanometer circuit was used in all tests so that only the fundamental of current and voltage was effective. The neglecting of the harmonics of voltage and current is due to the close approximation of the voltage and current waves to sine waves, as shown in Figs. 24 and 25. Even if one harmonic of both the current and voltage waves were 5% , and the harmonic power factor were 30%, the consequent loss would be only .05 $x.05 x.30 = .00075$ of the machine KVA, or about $\frac{.00075}{10}$ = .0075 = $\frac{3}{4}$ % of the machine KW.

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This question of harmonics has, however, potential importance. as it would be valuable in design calculations to know which harmonics represent loss and which do not.

Test Results

The significant test results are shown in Figures 18, 19, and 23. Those figures which have not been discussed are practically self explanatory. The alignment of points indicates the accuracy of balance; the comparison of the tests with tests by the standard AIEE method indicates the expected agreement; and the consistency of phase differences in Fig. 18 indicates accuracy over the full range of voltages.

Three different machines were tested: a standard lOKVA four pole GE laboratory a.J.ternator; a standard 15KVA six pole Westinghouse laboratory alternator; and a lOHP eight pole Westinghouse wound rotor induction motor operated as a synchronous condenser. The first had 15% load loss, the second had 5% load loss, and the last had 40% load loss when used in this manner, which 40% load loss is the reason for the induction motor's having been tested.

The points near the lower end of the curves of Fig. 23 were taken by the same method as the short circuit point, except that external drop was produced by a resistor, and its I^2R loss was subtracted from the measured loss.

AMPLIFIER

Calculation of Necessary Detector Characteristics

The Power Factor Bridge circuit is shown in Fig. 11. The type of detector needed will depend on the wave shape of the current and voltage, and on the accuracy of balance desired. For the purpose of the following calculations, 10% voltage and current harmonics and 1% accuracy of balance will be assumed. Also the voltage across R_1 will be assumed to be one volt.

Then the third harmonic current through R_{\odot} (fundamental current through R_2) $x \frac{R_2 + jX_0}{R_2 + j\frac{1}{2}X_0}$ (fundamental $I_{R_2} \mathbf{0} \left(\frac{1 + 120}{1 + 16.7} \right)$ = (fundamental of line current) x approximately 3.

Therefore the third harmonic across R_0 = approximately 3 x (% magnitude of harmonic) x fundamental = $3 \times (.1) \times (fundamental) = .3 \times fundamental.$

Next, the third harmonic acrosssR₁ = . I x fundamental, since 10% current harmonic is assumed.. But, referring to the voltage triangle OCD E of Fig. 12, it is seen that the power factor is the ratio of CD to OC, and if the power factor is to be measured to within 1% , the length CB must must $\begin{bmatrix} L_L R_{\rm i} & \text{and} & L_L R_{\rm n} \end{bmatrix}$ *C I* be known to within 1% . If OC (vol tage across R₁) el volt, Fig. 12. then the voltage to be measured by the detector at balance is .01 x CD \leq .01 x power factor x OC(= .01 x.05 x 1 volt $\ddot{=}$.0005 volts $\ddot{=}$.0005 x fundamental. Therefore the maximum ratio of the third harmonic to measurable fundamental will be

$$
\frac{.1 + .3}{.0005} = \frac{.4}{.0005} = 800:1
$$

For .01 power factor and .001 accuracy, the ratio = $\frac{\text{third}}{\text{fundamental}}$ = $\frac{4}{.00001}$ = 40,000 : 1

And if the third harmonic must be less than $\frac{1}{5}$ of the fundamental in order to cause no interference and to give a sharp balance, these ratios become 4,000: 1 and 200,000: 1.

For .01 accuracy and .65 power factor the minimum fundamental to be detected : power factor x accuracy x voltage across $R_1 = .05 x .01 x 1 = .5 MV$

For .002 accuracy and .02 power factor, the minum fundamental to be detected $\tilde{=}$.01 x.001 x 1 = 10 microvolts = .04 MV.

Consideration of Effects of Stray Fields

Due to the ordinary stray fields present on a test floor, it was found that air core inductances were entirely unsatisfactory, even after the last step of amplification. As an example, **a** 100 henry **air core** inductance with approximately three pounds of #36 wire and a diameter of twelve inches was found to pick up a voltage of ten millivolts when four feet from a small lOKVA laboratory alternator.

It was also found that almost any iron core inductance was affected too greatly by stray fields to be acceptable. The only exception was a coil (Western Electric 25A Repeater Coil) which apparently had no air gap or non-overlapping joints in the magnetic circuit. This coil could be used before amplification if properly shielded by an auxiliary heavy sheet metal case. The absence of inductance coils in the final circuit was due to the desire to eliminate with certainty the effects of stray fields.

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Vibration Galvanometer vs. DC Galvanometer

One disadvantage of the condenser and resistance tuning was that the detector was more sensitive to frequencies below fundamental than to fundamental, but

Fig. 13

enough subsynchronous voltage harmonic was found to exist in all machines tested to cause some trouble.

This harmonic is shown in the oscillogram2 **of** Fig. 13, and its effect is shown in Fig. 14, where the narrow V-curve was taken with a vibration galva-

Fig. 14 nometer relatively insensitive to low frequencies, and the rounded V-curve was taken with the DC galvanometer with rectifiers, which instrument was more sensitive to sub-synchronous harmonics than to fundamental.

2. This oscillogram was taken with the original bridge circuit shown in Fig. 5, but the effect would be identical with the final bridge circuit.

Voltage Supply

The use of rectified AC for the plate supply was not possible, as the amplifier was tuned to be most sensitive to low frequencies, and it was necessary to obtain zero output with zero input. The use of AC for the filaments, which were of the heater type, was tried but found to introduce line frequency into the output. The use of rectified AC for the filaments was not tried, because a storage battery was easier to obtaini However, even rectified AC would introduce some coupling with the AC source, and consequently much greater care would have to be exercised in grounding the amplifier to the machine being tested.

Because of the critical nature of the screen grid tube, it was found that a separate plate voltage source was essential for the tube preceding it in order to prevent "motor-boating."

Amplification Control

Due to the fact that the fundamental components of the two voltage arms of the bridge were opposed to each other, leaving very little fundamental but almost all the harmonics to impress on the amplifier, the voltage input to the amplifier appeared very much as in Fig. 15. It was essential that the input be

 $-27-$

connected directly to a grid, as any current drawn would unduly distort the bridge circuit used. If, however,

the first tube produced much Fig. 15 amplification, it might amplify the harmonic voltages so much as to block the grid of the next tube and reduce its sensitivity to the fundamental voltage. The scheme followed was therefore to use a three element tube, with an amplification constant of only five or six, in the first step, followed by enough tuning to prevent blocking of the next grid by the harmonics.

As it was desirable that the sensitivity of the amplifier be extremely variable, in order that it could be progressively increased as the balancing of the bridge progressed, the output of the first tube was fed into a potentiometer, which gave variation from zero to maximum sensitivity. This arrangement was very satisfactory and was found to be very convenient to use. The necessity for some such device is due to the fact that the ordinary expedient used in balancing bridge circuits, i.e. observing the direction of deflection of the galvanometer needle to determine which side of balance the bridge is on, could not be used with either the vibration galvanometer or the DC galvanometer with rectifying circuit.

Galvanometer Construction

The AC vibration galvanometer used was originally built, for reasons of economy, from the magnet of a standard GE oscillograph vibrator. It was later found, however, that the magnetic circuit of a commercial vibration galvanometer was so sensitive to stray fields that it was necessary to provide considerable extra shielding to obtain a zero reading whenever the instrument was within several feet of a small lOKVA laboratory alternator. The oscillograph magnet, however, was sufficiently compact and of such a material that it was not influenced by the same fields that affected the commercial instrument.

The rectifier used with the DC galvanometer was one of the standard instrument type of full wave copper oxide rectifier. It was supplied by Westinghouse. The resistance curve for this type of rectifier is shown in Fig. 16, as such infonnation was almost essential in properly matching the output of the amplifier. Curves are also shown for a crystal detector using a galena crystal and one or five catwhiskers. An attempt was made to change the characteristics of the copper oxide rectifier.when operated at about one micro-ampere by reducing the area of contact between adjacent oxide

and copper surface, but no appreciable change could be effected.

Amplification. The calculations made under the heading "Calculations of Necessary Detector Characteristics" showed that the minimum input voltage to the detector might be as low as 40 microvolts. From the curve of Fig. 16, it is seen that the necessary output voltage with a DC galvanometer requiring one micro-ampere is .04 volts. Therefore the amplification in the detector must be $.04$ or 10,000.

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APPROXIMATE RESISTANCE

OF RECTOX METER RECTIFIER

 $Fig 16$

 $-32-$

 $-39-$

APPENDIX

Calculation of Residual Inductance of Non-Inductive Resistances

I. Residual Inductance of R, of Fig. 4

Fig. 26. A formula for the inductance of such a shaped coil could not be obtained, so that it was necessary to develop an expression from the formulas for the self and mutual inductance between two parallel conductors of

finite length. The total inductance for several pairs of parallel wires carrying current in alternate directions is:

> $L_0 = 10L_1 - 2M_{12} - 2M_{14} - 2M_{16} - 2M_{18} \ldots$ $+2M_{1.3}$ + $2M_{1.5}$ + $2M_{1.7}$ $-2M_{23} - 2M_{25} \cdots$ $+2M_{24} + 2M_{26}$ $-2M_{34} - 2M_{36} \cdots$ $+2M_{35}$ + $-2M_{45}...$ etc. = $16L_1 - 2M_{1-16}$ + $4M_{1-15}$ - $6M_{1-14}$ + $8M_{1-13}$... $\cdots + 28M_{1-3} - 30M_{1-2}$

To calculate L and M, formulas 135 and 174 in the Bureau of Standards Circular 74, "Radio Instruments and Standards," are used. Thus where distances are in centimeters.

From these, $L = .1072 \text{ }\mu \text{ H}$

Therefore L_{residual} = 7.124 -6.933 = .191 μ H
and, at 50 cycles, $\frac{X}{R}$ = .191 x50 x27 x10⁻⁶ = $\frac{6}{10,000}$

II. Residual Inductance of R₂ of Fig. 4.

This resistor was a wire-wound $1\frac{1}{2}$ megohm resistor, of the dimensions shown in Fig. 27.

Fig. 27

From formula 158 of the U.S. Bureau of Standards circular #74, the self inductance of one coil is:

 L_u = .01257an² $\left[2.303(1+\frac{b^2}{32a^2}+\frac{c^2}{96a^2})\log_{10}\frac{8a}{d} - \gamma + \frac{c^2}{16a}r\right]\mu$ $\gamma_1 = f_{n_1}(\frac{b}{c}) = .83$ and $\gamma_2 = f_{n_2}(\frac{b}{c}) = .72$ where

 \therefore L_u = 660,000 μ H = 2/3 henries

And from formula 188, the mutual inductance is $M = n^2$ Fa where F = fn₃ $\left(\frac{D}{\sqrt{D^2 + 4a^2}}\right)$ $= .0140$ for coils $1-2$ 2.007 for $\pmb{\sharp} \pmb{\sharp}$ $1 - 3$ $\pmb{\text{ii}}$ $.004$ $\pmb{\text{ii}}$ $1 - 4$ $\pmb{\mathfrak{y}}$ $\pmb{\ast} \pmb{\mathfrak{g}}$ $= .0021$ $1 - 5$ $.0016$ 68 Ħ $1 - 6$ $\,$ 11 $\,$ $= .0011$ \mathbf{H} $1 - 7$ $\mathfrak{g} \mathfrak{g}$ $\frac{2}{3}$.0007 $\pmb{\theta} \pmb{\theta}$ $1 - 8$

But, as for the inductance of Fig. 26, the total inductance is:

 $L = 8L_u -14M_{12} +12M_{1-3} -10M_{1-4} +8M_{1-5} -6M_{1-6} +4M_{1-7} -2M_{1-8}$ $=$.66 x8-14 x.347+12 x.173-10 x.10+8 x.052-6x.04

$$
+4 x.03-2 x.02
$$

= 1. 8 henries

Therefore at 50 cycles, $\frac{X}{R} = \frac{2 \pi x 50 \times 1.8}{1.5 \times 10^{6}} = \frac{4}{10,000}$ approx.

III. Residual Reactance of R3 of Fig. 4.

The reactance of this resistor was covered by formula #167 of the U.S. Bureau of Standards circular $#74$, but since the formula was found to be incorrect, and as a bridge suitable for measuring the reactance was available, tests of the reactance were made. These tests showed reactance of about .4 millihenries one scale division **on** the bridge inductance being one mill ihenry.

Consequently the 50 cycle ratio of $\frac{X}{R}$

 $\frac{2 \pi x 50 x.4}{1250}$ = $\frac{1}{10,000}$

Dimensions of R_3 of Fig. 4 Fig. 28

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