AN ELECTRICAL METHOD OF MEASURING EARTH PRESSURES AGAINST STRUCTURES

THESIS

by

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Summary

In this thesis is described an instrument developed for the purpose of determining earth pressures in inaccessible places, such as, for example, pressures against footings and retaining walls of masonry structures. The method used is that of measuring the capacitance of a condenser formed by a fixed plate and a plate attached to an elastic diaphragm upon which the earth pressure acts. The spacing of the plates is a function of the earth force against the diaphragm, and the capacitance is in turn a function of the spacing. A scheme of measuring the capacitance has been developed whereby the effects of changes in the capacitance of the leads upon the reading has been eliminated. The electrical theory pertaining to the design of the device has been developed. Preliminary tests have been made and show that pressures can be determined with an accuracy in excess of that required in civil engineering practice.

Introduction

Owing to the great number of indeterminate factors involved it is practically impossible for civil and architectural engineers to calculate with any accuracy the pressures that will be exerted by the earth at various points of a structure, such as a dam or the foundation of a building. Since a knowledge of such pressures is a requisite for proper design, some method of measuring them is of considerable importance. In the past numerous attempts have been made, without much success, to devise instruments for this purpose. For some time, the only instrument available has been one in which a diaphragm is forced out against the pressure of the earth (by means of compressed air) sufficiently far to break an electric contact between a point on the diaphragm and a fixed point of the instrument. The air pressure at the instant of breaking of the contact is then assumed to be the pressure of the earth. Aside from the fact that the installation of this device is expensive because piping must be laid to each instrument to carry the compressed air, and a supply of the latter must be at hand, the measurements are inaccurate because of the fact that the actual pressure on the diaphragm is not measured, but rather the pressure after the diaphragm has moved an indeterminate distance against the earth. So far as the author is aware, until recently this was the only instrument available for measuring earth pressures in inaccessible places; other types had been devised, but these had to be accessible to the person taking the reading, or at least there had to be some mechanical connection from the instrument to the outside.

The device herein described is one in which the actual earth pressure against a diaphragm is measured without the necessity of any movement of the latter while the reading is being taken, and in which the total motivement of the diaphragm from no load to full load is about .002 inches. First merely the general scheme will be described and later each componant part will be taken up in detail.

This method is a modification of one first used by Whiddington (1) for the purpose of measuring extremely small deflections in the laboratory. A complete diagram of the electrical system and a schematic diagram of the mechanical system are shown in Figure 1. The force of the earth F which is to be measured acts on the circular steel diaphragm D to which is attached by means of insulators a circular plate P. Another plate P^1 is placed parallel to and very near the insulated plate P. A relay R breaks the connection between the line L and the

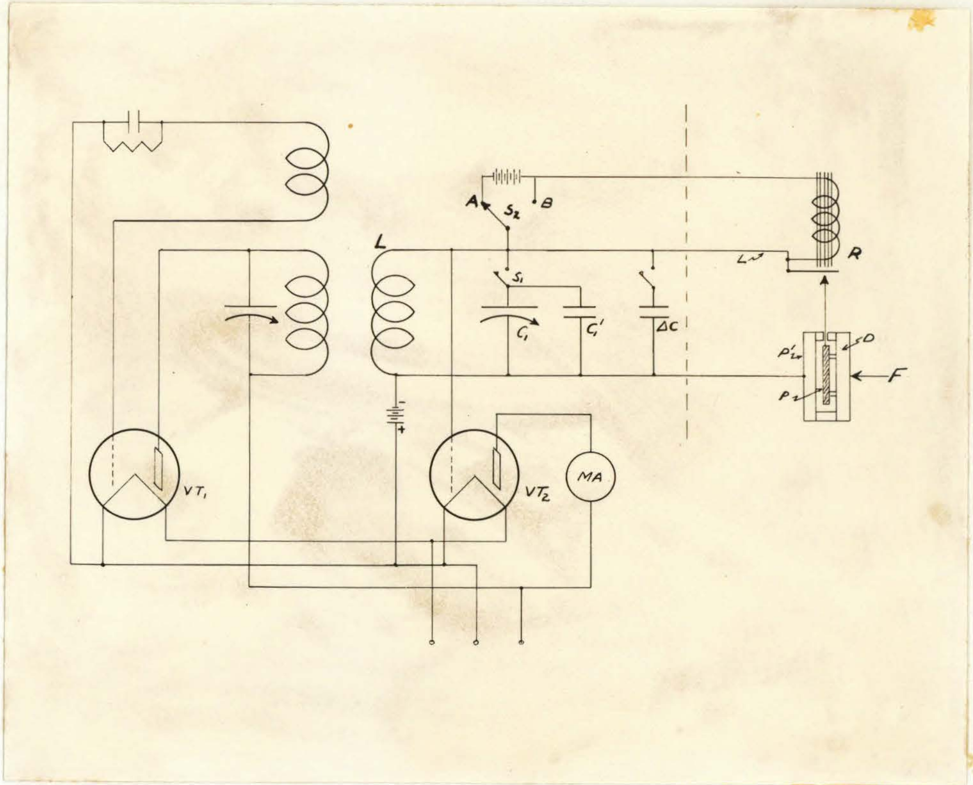

Figure 1   Electrical Circuit

movable plate P when the switch S_2 is in the position A as shown. The mechanical system consisting of the diaphragm, the condenser plate, and the supporting structure is called a "pressure cell". The leads are brought from the position of the pressure cell and relay to some accessible position where the reading can be taken. Readings of the capacitance of the condenser formed by P and P^1 (and hence readings of the force of the earth F) are taken by means of a so called "test set", consisting of the apparatus to the left of the vertical dotted line in Figure 1. A low frequency oscillator is coupled to the inductance L. C_1 is a variable condenser which can be thrown across the line at will by means of the switch S, and whose setting can be read by means of a dial.

Resonance in the circuit consisting of the inductance L and whatever capacitance may be in parallel with it is indicated by the vacuum tube voltmeter connected across the inductance.

The process of taking a reading is as follows. With S_2 in position B and S_1 open (i.e. with the variable condenser disconnected and the capacitance of the cell across the line) the oscillator is tuned to the resonant frequency of the circuit containing L. Then S_1 is closed and S_2 is placed in position B (with the result that the cell is disconnected from the line and C_1 is placed across it) C_1 is adjusted until the circuit is again in resonance with the oscillator frequency, and the reading of C_1 is taken. It will be shown later that after these adjustments have been made the capacitance of C_1 plus that of C_1' is to considerable precision the same as the capacitance of the pressure cell. From a previously determined calibration curve the magnitude of the earth force F may now be obtained.

Design of the Cell

A cross sectional drawing of the cell proper is shown in Figure 2 and a photograph of the assembled cell with external relay is given in Figure 3. Since all parts are circular in shape, the drawing of Figure 2 suffices to visualize the entire construction.. The metal parts of the cell are constructed entirely of cold rolled steel except for the movable condenser plate P which is brass. The plate P is insulated from the diaphragm D and the rest of the cell by means of a mica ring R and mica washers under the heads of the screws which hold the plate to the diaphragm. The diaphragm was turned to a thickness of approximately 0.12 inches, except for the ring M $\frac{1}{4}$ inches thick directly opposite the mica ring as shown. Into this projecting ring on the diaphragm are fastened the screws which hold the insulated plate P to the diaphragm. The plates P and P^{1} are separated by the steel ring S and the whole structure is held together by means of bolts. The stationary condenser plate P^{1} is protected from bending under load by the back plate P^{11} from which a circular cut has been taken leaving an air space as shown. When the cell is installed of course P^{11} rests against the structue upon which the earth is exerting its pressure. The face plate V, upon which the pressure

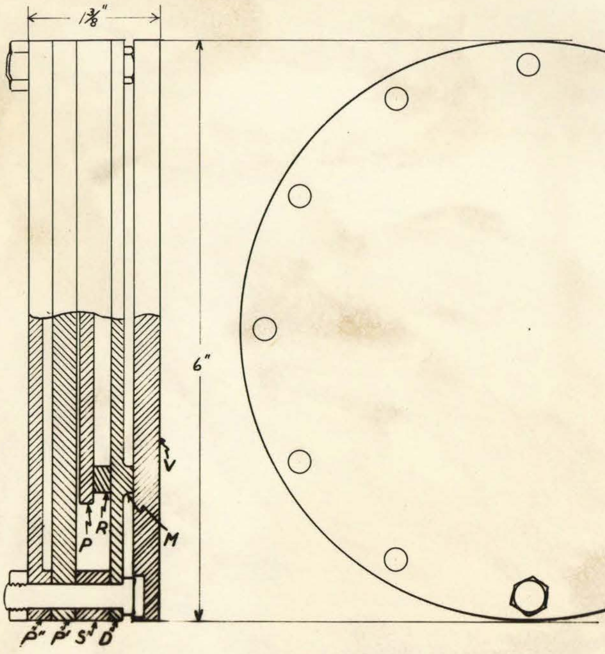

Figure 2 Drawing of Cell

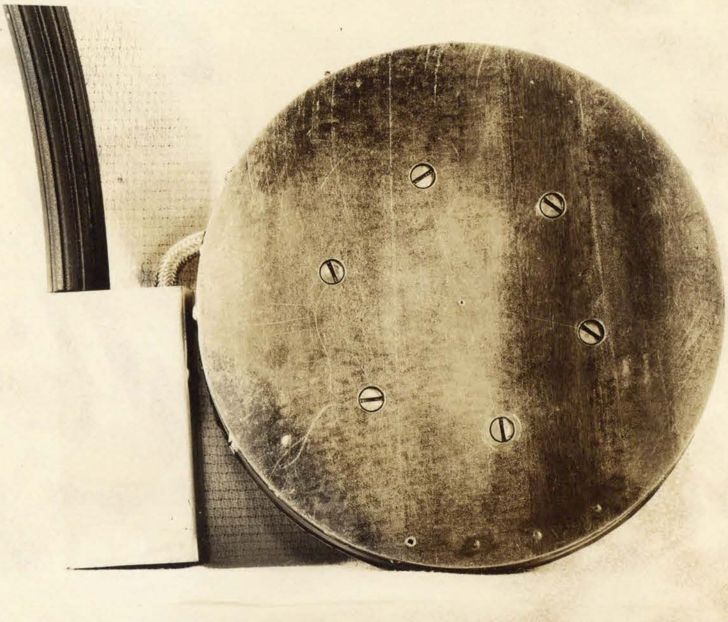

Figure 3 Assembled Cell, showing cable and external relay

of the earth acts, rests upon the turned ring M and is, like plate P, fastened to the ring M by means of screws.

The lead to the movable plate is brought into the cell through a hole in the steel separating ring S as shown. So far in testing the instrument the relay has been placed directly outside the cell as indicated in Figure 2, because a relay small enough to be inserted in the cell could not be found, and will have to be constructed unless the system of placing the relay in an external water-tight case is adopted. The construction os such a relay should present no difficulty, because it does not have to be sensitive nor does it have to break any appreciable current.

When the cell is assembled the spacing between the condenser plates is .004 inches, and the diaphragm was designed to have a total deflection with full load of .002 inches. The diameter of the movable plate P is 3.6 inches, which gives at .004" separation of the plates a capacitance of .0005 microfarads. Hence the range of capacitance of the cell is theoretically from .0005 microfarads to .001 microfarads. The actual range proved to be from .0005 to .0009 microfarads approximately.

The method of obtaining flat surfaces of the condenser plates and the correct spacing between them is as follows. The plate P is fixed to the diaphragm and the latter is fastened rigidly to the separating ring S by means of screws between the bolt holes. The face of the ring S and of the condenser plate are then machined to the same level and the surface made flat by polishing it by means of carborundum and oil on a plate glass surface. One side of the fixed plate P^1 is polished in a similar manner. All rolled, pressed, and drawn materials, such as cold rolled steel and ordinary brass, contain internal stresses which make it vitually impossible to turn pieces of any size to a perfectly flat surface because of warping - thus the necessity of grinding. It takes only a short time to grind the surfaces flat to less than .0005". The flatness can be tested by laying the edge of a good steel rule across the surface and noting any spacing between the surface and the rule. After all traces of chips of metal are removed by washing the parts in clean gasoline, the two ground surfaces are placed together with a shim of the correct thickness (.004") between the ring S and the plate P^1, the back plate P^{11} is put in place, and the parts are bolted together. The face plate V can now be fastened to the diaphragm.

The two ground surfaces before assembly are shown in Figure 4.

In calculating the thickness of the diaphragm requisite for .002" deflection under 500 pounds load, the diaphragm was assumed to be fixed at the outside edge and loaded at the ring M. Under these conditions the expression for the deflection W of the ring is (2)

$$W = \frac{P}{8\pi D}\left[-2b^2\log\frac{r}{b} + \frac{1}{2}\left(1+\frac{b^2}{r^2}\right)(r^2-b^2)\right.$$

$$D = \frac{Et^3}{12\left(1-\frac{1}{m^2}\right)}$$

where E is the modulus of elasticity (29 x 10^6), $\frac{1}{m}$ is Poisson's ratio, r is the radius of the diaphragm, and b is the radius of the load ring. Foa a deflection of .002" under 500 lbs. load the thickness t of the diaphragm should be approximately 0.12".

In the stress analysis of the diaphragm the assumption was made that the stress due to loading on a circular ring is the same as the stress due to uniform loading of magnitude sufficient to give the same deflection at the load ring. For uniform loading

$$W = \frac{Q}{64D}(r^2-b^2)$$

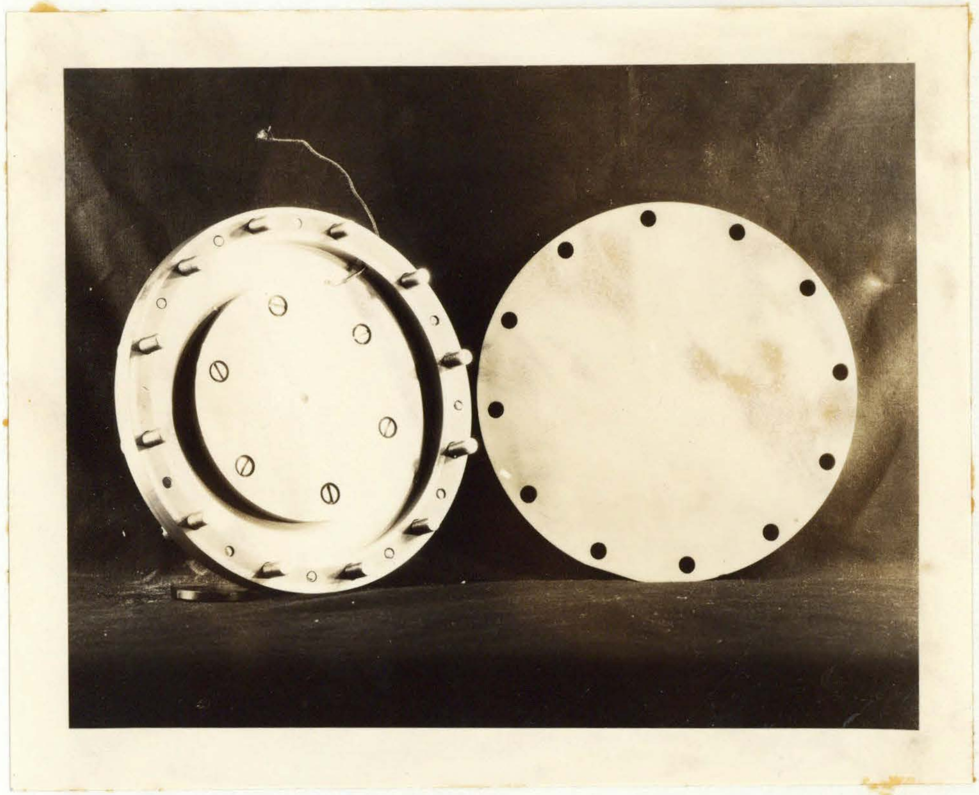

Figure 4   View showing ground surfaces of condenser plates

where Q is the pressure of the load in pounds per square inch. From this expression the value of Q necessary to give .002" deflection at the ring may be calculated. The expression for maximum tensile stress in pounds per square inch is then

$$p = \frac{3Qr^2}{4t^2}$$

The maximum stress of the diaphragm was calculated in this manner to be 18,500 lbs. per square inch. The elastic limit of coled rolled steel is reached at about 35,000 pounds per square inch, and hance the calculated stress under full load is about half the elastic limit. In view of the assumptions and uncertainties involved the accuracy of this value is a question, though it is probably somewhat greater than the actual stress; at any rate, when the cell was loaded with a force of 1000 pounds there was no evidence that the elastic limit of the diaphragm had been exceeded.

The Electrical System

The oscillator used in the test set is, as can be seen from the circuit diagram in Figure 1, of very simple form. The only requirement with regard to frequency stability is that it keep essentially the same frequency during the few seconds while the reading is being taken. In order to obtain a sharp peaked resonance curve (high Q) and at the same time a high voltage input to the vacuum tube voltmeter, a low oscillator frequency was found to be most satisfactory, although the choice is by no means critical. The frequency that has been used is in the neighborhood of 17 kilocycles. As will be seen later, a low frequency is also desirable because it results in eliminating any effects of the leads to the cell upon the readings.

Although resonance can be determined by noting when the milliammeter M.A. (Figure 1) reads maximum current, a method described by Terman (3) has been used because it is much more precise. Resonance is indicated by the fact that connecting or disconnecting the small condenser ΔC produces no change in the reading of MA. By making the value of ΔC such that the steepest portion of the resonance curve is utilized, the resonance point can be found with very great precision.

The tube of the voltmeter (VT_2) has its grid bias voltage such as to give plate detection. Attempts have been made to eliminate this tube by use of a meter in the plate circuit of the oscillator tube and by putting a crystal detector and D.C. meter directly in the resonant circuit; however, neither method was nearly so satisfactory in giving a sharp indication of resonance as the detector system because closer coupling between the oscillator and the inductance L was required to deflect the meter in both cases. If these methods were used a much more sensitive and expensive current indicating device than the milliammeter would be required.

C_1 is a calibrated variable condenser having a capacitance of .005; it is shunted with C_1', a fixed condenser also of capacitance .005. This combination can then be varied to include the range of variation of the capacitance of the pressure cell, at the same time spreading the readings over most of the range of the condenser dial

The process of disconnecting the leads by the relay is of course intended to prevent their capacitance from affecting the measurements of the capacitance of the cell. The question will at once arise as to whether the fact that in one case the condenser is at the far end of the transmission line while in the other case it is at the near end, will make the constants of the line affect the measurement to any extent. In other words, after adjustments have been made in the manner previously described will the condensers c_1 and c_1' have together the same capacitance as the pressure cell, or will they have some different value depending upon the line constants? This question can be answered by an analysis of the circuit. Denoting as E the vector EMF generated in the inductance L by the oscillator, the solution for the current in L when only the far-end condenser (cell) is connected is given by

$$I_L = \frac{E}{R + j\omega L + \dfrac{\cosh\theta + j\omega C_0 Z_0 \sinh\theta}{\dfrac{\sinh\theta}{Z_0} + j\omega C_0 \cosh\theta}} \quad ---(1)$$

and the solution for the current in the inductance with only the near-end condensers connected is

$$I_L' = \frac{E}{R + j\omega L + \dfrac{1}{j\omega C_2}\left(\dfrac{1}{1 - \dfrac{j\sinh\theta}{Z_0 \omega C_2 \cosh\theta}}\right)} \quad ---(2)$$

where C_0 is the capacitance of the cell, C_2 is the sum of C_1 and C_1', θ is the hyperbolic angle of the line, R is the resistance of the inductance L, Z_0 is the line's characteristic impedance and ω is 2π times the frequency of E. The fact that θ is a function of ω makes the process of finding the resonant frequencies in the two cases (by minimizing the denominators of the fractions) a difficult process. However, the constants of the cable are such that for 100 feet of line (the longest length expected to be used with this instrument), and at a frequency of 17 kilocycles, the hyperbolic angle θ has a value of approximately $0.012 + j\,0.010$. Now

$$\cosh(u + jv) = \cosh u \cos v + j \sinh u \sin v$$

and when u and v are both small, even though different in magnitude, $\cosh(u + jv)$ approximates the real value 1. From tables it can be seen that to a high order of precision $\cosh\theta = 1$. (Its actual value is $.99999 + j.00012$). Also

$$\frac{\sinh(u + jv)}{u + jv} = \frac{\sinh u \cos v + j \cosh u \sin v}{u + jv}$$

and when u and v are both small this fraction likewise approximates 1. But

$$\frac{\sinh\theta}{Z_0} = \frac{\sinh\theta}{\theta}Y,$$ where Y is the

total shunt admittance of the line. Since the conductance can be totally neglected Y is equal to $j\omega C_L$, where C_L is the total capacitance of the line. Hence for this small value of θ,

$$\frac{\sinh\theta}{Z_0} = j\omega C_L,$$

very nearly. Similarly, even with the largest capacitance C_0 that the pressure cell ever has, the quantity $j\ C_0Z_0\sinh\theta$ is negligible in comparison to $\cosh\theta$. Making these substitutions in equations (1) and (2) one obtains the simple forms

$$I_L = \frac{E}{R + j\omega L + \frac{1}{j\omega(C_0 + C_L)}}$$

and

$$I_L' = \frac{E}{R + j\omega L + \frac{1}{j\omega(C_2 + C_L)}}$$

This shows that the line merely acts as an additional capacitance in parallel with the inductance L, and that its constants do not affect readings taken by the method given above.

This conclusion was born out experimentally by the fact that no difference could be detected in the resonant frequency when a given capacitance was changed from one end of the line to the other for any length of line up to 100 feet. Of course it is assumed that the capacitance of the line remains constant while a reading is being taken. For this reason it is necessary to have the double pole switch S_2 (Figure 1) in order that the two ends of the relay leads will always be shorted. Otherwise the capacitance of the line would change during the process of taking a reading.

With the assumption that $C_2 = C_1 + C_1' = C_o$ the equation for the earth force can be derived as follows. Let the area of the condenser plates of the pressure cell be A square centimeters, their separation in centimeters be X, their separation when the earth force F is zero be X_o, the force constant of the diaphragm be k, and the small non-varying capacitance of the cell be a.

Then $X = X_o - kF$

and $C_2 = \dfrac{A}{X_o - kF} - a$ electrostatic units ---- (3)

or $F = \dfrac{X_o}{k} - \dfrac{A}{k\,(C_2 - a)}$

The force-capacitance curve is thus seen to be an hyperbola. If the variable condenser plates were so shaped that

$$\frac{1}{C_2 - a} = k'\theta$$

where θ is the reading (or angular position) of the variable condenser and k' is a constant, there would of course be a straight line relationship between F and the reading. The condenser which has been used is of the straight line wavelength variety; it gives a calibration curve which is nearly straight.

The rate of change of capacitance with respect to force might be taken as a measure of the sensitivity of the cell. From equation 3,

$$\frac{dC_2}{dF} = \frac{AK}{(X_0 - KF)^2}$$

The sensitivity is seen to decrease with the separation of the plates and to increase with the applied force.

It is desirable to keep the line capacitance as low as possible for two reasons. In the first place the line is effectively a non-varying capacity in parallel with the cell and thus it decreases the accuracy with which the reading can be taken. Secondly, since the line represents rather a poor condenser with high losses it absorbs considerable energy from the oscillator if its capacitance is high, with the result that the voltage input to the voltmeter is low and the decrement of the circuit is high. Since ordinary cable has high distributed capacitance, a special cable was constructed by pulling two #24 insulated wires through 3/4" composition rubber tubing. These two wires connect to the relay, and the ground connection from the case of the cell is made by a heavy wire laid beside the rubber tubing. The shunt capacitance of the cable when in water (a condition similar to its being surrounded by moist earth or concrete) was found to be about .0012 microfarads per hundred feet. Ordinary rubber covered cable (such as, for example, #18 U.S. Royal Cord) has under the same conditions a capacitance of roughly five times this value. The cost of the tubing is approximately the same as that of the cable.

Results of Tests

In order to calibrate and test the cell it was loaded by means of the lever arrangement shown in Figure 5. Accurately placed knife edges resting on flat pieces of metal formed the fulcrums. The effective force of the beam was calculated from its weight. Since the ratio of the lengths of the lever arms was ten to one, a weight of less than fifty pounds was required at the end of the beam in order to place full load (500 lbs.) on the cell. The point of application of the load on the face plate of the cell could be changed at will in order to study the effect of eccentric loads. The general results of tests on the instrument may be summarized as follows. The cell exhibits a definite elastic hysteresis effect - that is, when loads are plotted as abscissae and readings as ordinates the loading curve is below the unloading curve. The separation of these curves increases with increasing load. At first there was noted a gradual increase of the reading when the cell was left under full load. At that time a fiber insulating ring was used, and when this was replaced by mica (which has much better elastic properties than fiber) and at the same time the insulated plate was more firmly damped to the diaphragm, this shift of the readings entirely dis-

Figure 5 Method of loading cell

appeared, although the hysteresis effect still remained and had the same magnitude as before. The curves repeat with great precision; that is, if a curve is taken as the cell is loaded, then the load is removed and another loading curve taken, the corresponding readings on these two curves will be alike to less than one half per cent of the total range of the readings. The same thing holds true for the unloading curves. If the loads are increased to some fraction of full load and then decreased a small hysteresis loop is formed inside the larger one, and this likewise repeats. Loading the cell at the edge of the face plate rather than uniformly over its surface causes a slight decrease of each reading, the amount of this decrease varying with the load and never amounting to more than two percent of the range of the readings. It might be mentioned that when a cell were actually installed, soil would be packed carefully around it to insure a fairly uniform distribution of pressure. As a matter of fact, if the pressure varied appreciably over the surface of the cell it would certainly vary over the surface of the structure against which the earth is acting, and hence an exact reading of the pressure at any particular point would be of little value. Increase in temperature decreases the force constant of the diaphragm and hence increases the readings, but, considering the fact that the cell is to be installed underground, where the temperature is nearly constant, this effect is negligible.

Figure 6 shows a calibration curve which is typical of about twenty-five such curves taken over a period of two weeks. Dial readings of the variable condenser C_1 (Figure 1) are plotted against the total force on the cell in pounds. It will be seen that if the true calibration curve is assumed to be the dotted line between the upper (unloading) curve and the lower (loading) curve, the measured value of the force on the cell will never be in error by more than 15 pounds in approximately 600, and usually by less than this amount. Now civil engineers say that there is no point to reading the forces with a greater accuracy than 20 pounds in 500, and so the accuracy of the cell is within the requirements. In practice, however, the loading curve would doubtless be the one used, since it is seldom that the earth pressures decrease with time.

This hysteresis effect is believed to be a characteristic of the material of which the

Figure 6. Calibration Curve.

diaphragm is made rather than to some mechanical effect such as slipping of the diaphragm at the bolts. If so construction of the diaphragm of phospher bronze or spring steel would probably assist greatly in eliminating it.

There has not been sufficient time to thoroughly test the behavior of the cell under continuous loads. There may, of course, be a gradual yield of the diaphragm which, though imperceptable over the short period of time that the cell has been under test, would amount to considerable error in a matter of months. Tests will of course have to be made to determine this.

The characteristics of the cell would be improved and any tendency of the diaphragm to slowly yield to the pressure would be diminished by decreasing the stresses. This could of course be done by decreasing the deflection, and it is believed that it may be possible to reduce the total deflection to less than .001". Also use of a multiple diaphragm consisting of two or more thin diaphragms bolted together at the center would likewise help to reduce the stress.

Conclusion

Since the work described in this thesis was started there has been announced a method of measuring earth pressures (4) based upon measuring the deflection of an elastic member by determining the frequency of vibration of a stretched wire whose tension varies with the deflection. This latter system, developed in Germany, has been rather incompletely described and nothing is known as to its performance. It seems to the author quite certain that smaller displacements can be measured, and hence smaller diaphragm deflections employed, with the method described in this paper than with the stretched wire method.

The instrument described in this thesis, while still in the laboratory stage, has behaved in a way to indicate that it will survive the rather severe treatment that it must undergo in actual use, and that it will have eventual success. As soon as a sturdy and convenient test set with the cell enclosed in a water tight case of copper sheeting can be constructed the device will be given to a group of civil engineers for use in making field tests.

The author wishes to express his gratitude to Mr. W. H. Saylor of the Metropolitan Water District of Southern California both for his suggesting the need of such a device and for his interest and help in the experimental work, and to Mr. Hugo Benioff of the Carnegie Institution of Washington for the loan of the electrical apparatus used in making the tests.

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