THE ECONOMICAL SPACING
OF
ROOF TRUSSES

Thesis by

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INTRODUCTION

The economical spacing of roof trusses is usually affected by a great deal of factors that it has been quite impossible to develop a theoretical formula giving the exact spacing for different spans. However, as a common practice and from the experiences of different engineers, the spacing of roof trusses usually varies from about 1/3 of the span for shorter trusses to about 1/5 of the span for the longer trusses.

The object of this problem has been, therefore, to try to confirm the statement above by actually designing trusses of different spans at different spacings, and to see what are the factors which enter into this question of economical spacing of roof trusses. In so doing, the author tried as much as possible to keep everything constant except the span and spacing.

The type of truss used by the author was the Fink steel truss with a pitch of 1/4 as shown in Fig. I. The roof covering consisted of corrugated steel supported on purlins. The specifications of the A.I.S.C. were used to govern the design of the members. Three different spans were tried at 40 feet, 60 feet, and 100 feet.

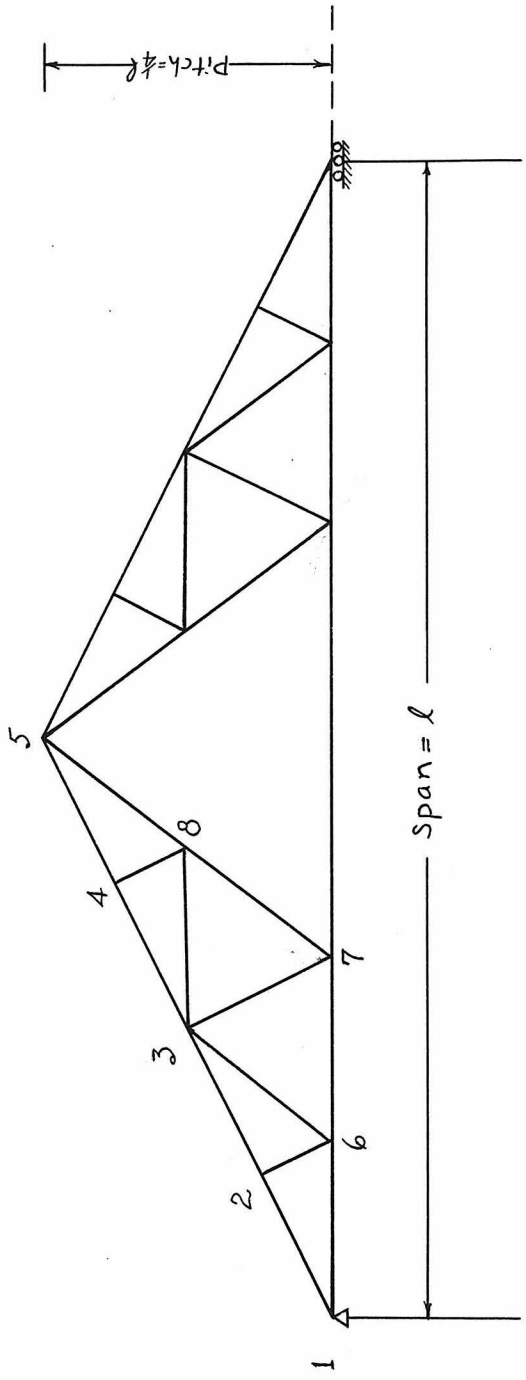
The spacings used were as follows:

(1) For 40' span, spacing at

8' = 1/5 span
10' = 1/4 span
12' = 3/10 span
15' = 3/8 span

(2) For 60' span, spacing at
10' = 1/6 span
12' = 1/5 span
15' = 1/4 span
18' = 3/10 span
20' = 1/3 span
22.5' = 3/8 span

(3) For 100' span spacing at
15' = 1/6.6 span
20' = 1/5 span
25' = 1/4 span
30' = 1/3.3 span



TRUSS DIAGRAM

Fig. 1

DISCUSSION.

In the discussion given in Hood and Johnson's "Handbook of Building Construction", Vol. 1, the theoretical spacing of trusses for least total cost of trusses, purlins, and roof covering depends upon the relative cost of the component parts. Let X = total cost of construction per square foot of roof.

(1) for truss: t = cost of truss per square foot of roof.
k = constant
s = spacing
$t = \frac{k}{s}$

(2) for purlin: p = cost of purlin per square foot of roof.
n = constant
$p = ns^2$

(3) for roof covering: c = cost of roofing per square foot of roof
m = constant
$c = ms$

then

$$X = t + p + c = \frac{k}{s} + ns^2 + ms$$

To obtain the minimum unit cost, with respect to different spacings,

$$\frac{dX}{ds} = 0 = -\frac{k}{s^2} + 2ns + m = 0$$

multiplying by s

$$-\frac{k}{s} + 2ns^2 + ms = 0$$

$$\therefore \qquad t = 2p + c$$

According to the equation above then, for the least cost, the spacing of trusses must be such that the above equation be satisfied. Hood and Johnson mentioned, however, that the relation given above can not be used directly for the determination of the truss spacing because the spacing does not appear in the equation.

After designing the 14 trusses, the author has made a study of the different factors which are involved in the determination of the value for t, p, and c. These factors as found seemed to vary a great deal with respect to local conditions and requirements, and to the time when and place where the structure is to be built.

The cost of steel structures may be divided into:

(1) cost of material
(2) cost of fabrication
(3) cost of erection, and
(4) cost of transportation.

Under these items there are the cost of drafting, mill details, shop labor, placing, bolting, hauling, falsework, etc.

In order to make a simple investigation, the author has neglected the minor costs and chosen the following ones to find t, p, and c.

A. t, the cost of truss per square foot of roof is made up of:

1. cost of material of truss	at 1.5 ¢/lb°
2. cost of shop labor--see table I	0.55 - 1.1 ¢/lb
3. cost of erection	at 0.35 ¢/lb
Total =	2.4 ¢/lb to 2.95 ¢/lb

B. p, the cost of purlin per square foot of roof is made up of:

1. cost of material--I beams and channels 3" to 15"	at 1.45 ¢/lb
2. cost of laying--punching holes and erection	at 0.35 ¢/lb
Total =	1.80 ¢/lb

C. c, the cost of roofing per square foot of roof is made up of

1. cost of material	at 2.0 ¢/lb
2. cost of laying	at 0.9 - 1.49 ¢/lb
Total =	2.9 ¢/lb to 3.49 ¢/lb

°These values were obtained from M. S. Ketchum's "Structural Engineer's Handbook".

The cost of shop labor was calculated for the structure as a whole. The following costs are based on an average charge of 40¢ per hour, and include detailing and shopwork.

In lots of at least six, the shop cost of ordinary riveted roof trusses in which the ends of the members are cut off at right angles is about as follows:

Total weight of truss	Cost in cents per pound
1,000 #	1.15 - 1.25 ¢
1,500	0.90 - 1.00 ¢
2,500	0.75 - 0.85 ¢
3,500 - 7,500	0.60 - 0.75 ¢

The values in Table I were obtained from the above figures

The cost of erection includes the cost of placing and bolting and was obtained from the following figures (40 ¢/hour basis).

Kind of structure	Cost of erection in dollars/ton
Mill building for ordinary condition	$6.00 - $8.00
Steel office building	$5.00 - $9.00
Steel bins	$10.00 - $15.00
Head frames	$12.00 - $18.00

For this particular problem the author chose the value of $7.00/ton for the cost of erection. This amounts to 0.35¢/lb. See Table I, on the following page.

Table I.

Values of t

(t = the unit cost of truss per pound)

Spacing	Total wt. of truss	Cost of Labor in ¢/#	Cost of Material in ¢/#	Cost of Erection in ¢/#	Span	t ¢/#
8'	1290 #	1.1 ¢/#	1.5 ¢/#	0.35 ¢/#		2.95¢/#
10'	1290	1.1	"	"		2.95
12'	1366	"	"	"	40'	2.95
15'	1393	"	"	"		2.95
10'	2250	0.85	"	"		2.70
12'	2273	0.85	"	"		2.70
15'	2513	0.80	"	"		2.65
18'	2625	0.80	"	"	60'	2.65
20'	3202	0.75	"	"		2.60
22.5'	3416	0.75	"	"		2.60
15'	6219	0.70	"	"		2.55
20'	7318	0.60	"	"	100'	2.45
25'	9246	0.55	"	"		2.40
30'	10295	0.55	"	"		2.40

Results of Design.

RESULT

The result of the design as shown in the tables below showed the economical spacings as follows:

Span	Economical spacing	Acceptable Economic spacing.
40'	10' = 1/4 span	12' = 3/10 span
60'	12' = 1/5 span	10' = 1/6 span
100'	15' = 1/6.6 span	20' = 1/5 span

Table II.

Weight of Trusses

40'-span

		8'spac.	10'	12'	15'
	Total number of pieces	wt.in #	wt.in #	wt. in #	wt. in #
Chord members:					
1-5	4 Ls @ 22.5'	369.00	369.00	442.00	442.00
1-7	4 Ls @ 13.0'	166.00	166.00	166.00	166.00
7-7'	2 Ls @ 15.0'	95.50	95.50	95.50	95.50
2-6, 4-8	4 Ls @ 2.7'	34.50	34.50	34.50	34.50
3-7	2 Ls @ 5.3'	52.00	52.00	52.00	52.00
3-8, 3-6	4 Ls @ 5.9'	75.20	75.20	75.20	75.20
5-7	4 Ls @ 12.0'	153.00	153.00	153.00	153.00
Sag tie	1 L @ 9.0'	28.70	28.70	28.70	28.70
Weight of members:		974.00#	974.00#	1047.00#	1047.00#
Connection Factor:					
Joint 1	2 gusset pls.	67.40	67.40	67.40	90.00
" 2,4	4 " "	12.00	12.00	12.80	12.80
" 3	2 " "	40.80	40.80.	40.80	40.80
" 5	1 " "	57.50	57.50	60.00	64.50
" 6,8	4 " "	17.90	17.90	17.90	17.90
" 7	2 " "	20.40	20.40	20.40	20.40
Shoe L, shoe	plates, rivets etc.	100.00	100.00	100.00	100.00
Total connection:		316.00#	316.00#	319.30#	346.40#
Total Truss Weight		1290.00#	1290.00#	1366.30#	1393.00#

Table III.

Weight of Trusses

60'-span

		10'	12'	15'	18'	20'	22.5'
		wt. in #	wt. in #	wt. in #	wt. in #	wt. in #	wt. in #
Members:							
1-5	4 ∠s	1974.0	1974.0	1110.0	1325.0	1528.0	1580.0
1-7	4 ∠s	244.0	246.0	282.0	354.0	388.0	488.0
7-7'	2 ∠s	120.0	120.0	126.0	136.0	184.5	184.5
2-6, 4-8	4 ∠s	58.0	58.0	58.0	58.0	58.0	58.0
3-7	4 ∠s	144.0	144.0	144.0	144.0	144.0	144.0
3-8, 3-6	4 ∠s	115.0	115.0	115.0	115.0	115.0	115.0
5-7	4 ∠s	227.0	227.0	227.0	227.0	227.0	227.0
Sag tie	1 ∠	45.3	45.3	45.3	45.3	45.3	45.3
Weight of meml		1927.3	1929.3	2117.3	2404 .3	2689 .3	2841.8
Connection Factor:							
Jt. 1		63.8	76.4	112.0	112.0	153.0	168.0
" 2,4		13.7	13.7	16.7	16.3	16.3	18.4
" 3		36.7	36.7	36.7	40.0	40.0	58.0
" 5		57.2	64.5	72.6	83.0	99.0	108.6
"6,8		20.4	20.4	20.4	20.4	20.4	27.2
" 7		27.6	27.6	33.0	33.0	36.7	46.0
shoe ∠, shoe pl., rivets etc.		104.6	104.6	104.6	125.8	146.6	148.3
~~wt. of~~ Conn. Factor		322.0	344.0	395.0	420.5	512.0	574.4
~~Total weight~~ of Truss		2249.0#	2273.0#	2513.0#	2625.0#	3202.0#	3416.0#

Table IV.

Weight of Trusses

100'-span

		15'	20'	25'	30'
		wt. in #	wt. in #	wt. in #	wt. in #
Members:					
1-5		2925.00	3830.00	5400.00	6040.00
1-7		906.00	978.00	1252.00	1551.00
7-7'		539.00	539.00	539.00	576.00
2-6,4-8		200.00	200.00	200.00	200.00
3-7		416.00	416.00	416.00	416.00
3-8,3-6		426.00	426.00	426.00	426.00
5-7		100.00	100.00	100.00	100.00
Sag tie		176.00	176.00	176.00	176.00
wt. of members		5688.00#	6665.0 #	8509.00 #	9485.00#
Connection Factor:					
Joint 1		137.00	151,00	181.00	186.00
" 2,4		40.80	40.80	40.80	40.80
" 3		84.00	84.00	84.00	84.00
" 5		97.50	121.50	153.00	191,00
" 6,8		32.60	32.60	32.60	32.60
" 7		33.60	33.60	33.60	43.00
		177.60	190.00	212.00	232.40
wt. of Conn. Factor		603.10#	653.40 #	737.00#	809.80#
Total wt.	of Truss	6291.00#	7318.40#	9246.00 #	10,294.00#

Table VIII.

Comparison of Cost

40'-span

	wt.per sq.ft. #	unit cost ¢/#	cost per sq.ft.	2p	2p + c	t	Spac-ing
Roofing	2.46	3.49 ¢	8.60 ¢				
Purlin	1.64	1.80	2.95	5.90	14.5	11.9	8'
"w"	4.03	2.95	11.90				
Cost per sq.ft.of roof =			23.45				
Roofing	2.46	3.49	8.60				
Purlin	2.16	1.80	3.89	7.78	16.38	9.5	10'
"w"	3.22	2.95	9.50				
Cost of Truss per sq.ft.of roof =			21.99 ✓	Economical Spacing			
Roofing	2.46	3.49	8.60				
Purlin	3.00	1.80	5.40	10.80	19.4	8.38	12'
"w"	2.84	2.95	8.38				
Cost of Truss per sq.ft. of roof =			22.38				
Roofing	2.46	3.49	8.60				
Purlin	4.00	1.80	7.20	14.40	23.00	6.85	15'
"w"	2.32	2.95	6.85				
Cost of Truss per sq.ft. of roof =			22.65				

Table IX.

Comparison of Cost

60'-span

Spacing		wt. per sq. ft. #	unit cost ¢/#	cost per sq. ft.	2p	2p c	t
	Roofing	2.46	3.49	8.60¢			
10'	Purlin	2.28	1.80	4.10	8.20	16.80	10.12
	"w"	3.75	2.70	10.12			
Cost of Truss per sq. ft. of roof =				22.82			
	Roofing	2.46	3.49	8.60			
12'	Purlin	3.08	1.80	5.55	11.10	19.70	8.54
	"w"	3.16	2.70	8.54			
Cost of Truss per sq. ft. of roof =				22.69 ✓ Economical Spacing			
	Roofing	2.46	3.49	8.60			
15'	Purlin	4.00	1.80	7.20	14.40	23.00	7.42
	"w"	2.80	2.65	7.42			
Cost of Truss per sq. ft. of roof =				23.22			
	Roofing	2.46	3.49	8.60			
18'	Purlin	5.00	1.80	9.00	18.00	26.60	6.45
	"w"	2.43	2.65	6.45			
Cost of Truss per sq. ft. of roof =				24.05			
	Roofing	2.46	3.49	8.60			
20'	Purlin	6.12	1.80	11.00	22.00	30.60	6.94
	"w"	2.57	2.60	6.94			
cost of Truss per sq. ft. =				26.54			
	Roofing	2.46	3.49	8.60			
22.5'	Purlin	7.36	1.80	13.25	26.50	35.10	6.58
	"w"	2.53	2.60	6.58			
Cost of Truss per sq. ft. =				28.43			

Table X.

Comparison of Cost

100'-span

Spacing		wt. per sq.ft. #	unit cost ¢/#	cost per sq.ft.	2p	2p e	t
		3.36#	2.90¢	9.75¢			
		3.67	1.80	6.60	13.2	22.95	10.70
		4.20	2.55	10.70			
Cost of Truss per sq. ft. of roof =				27.05 ✓ Economical Spacing			
		3.36	2.90	9.75			
		4.90	1.80	8.82	17.64	27.39	8.95
		3.65	2.45	8.95			
Cost of Truss per sq. ft. of roof =				27.52			
		3.36	2.90	9.75			
		6.00	1.80	10.80	21.60	31.35	8.85
		3.69	2.40	8.85			
Cost of Truss per sq. ft. of roof =				29.40			
		3.36	2.90	9.75			
		8.64	1.80	15.54	31.08	40.83	8.25
		3.44	2.40	8.25			
Cost of Truss per sq. ft. of roof =				33.54			

Discussion of Results.

DISCUSSION OF RESULTS

Mr. M. S. Ketchum in his "Structural Engineer's Handbook" stated that he tried this problem of economical spacing for a 60' span and found that the least weight was at a spacing of 10'. The author's result showed that this value was the second value for the economical spacing of a 60' span truss.

As it can be noted from Tables VIII, IX, and X, the results did not check with the theoretical condition in the formula $t = 2p + c$ by Hood and Johnson.

The estimated value of c was much too great to give the value of $2p + c$ a much larger value than that of t. But in every case except the 60' span, the economical spacing occurred when $t > 2p$, having a surplus for the value of c. However, c was kept a constant value and thus did not affect the equation:

$$X = t + p + c$$

to give a different economical spacing. We can say then in this particular case that the economical spacing of trusses occurs when the value of 2p is just less than the value of t.

Besides the different factors affecting the cost of the structure, there are minor factors in the design of the component parts themselves. Roof covering costs vary with the nature

of the covering. In the design of purlins and members of the truss, if it were possible to obtain rolled sections which would provide exactly the required areas for all members, the result of the problem would have come out more accurate. But as could be seen from the designs above, the sizes of many members were determined by the specifications andyby the requirements of standard practice. For the web members such as 2-6, 4-8, 3-6, and 3-8, the designed members provided much larger areas than were required. The author therefore made a study of this factor, which he called "specification factor". He picked the members which were affected by it; the percentage weight of these members to the total weight of the whole truss is tabulated in the following tables: Tables XI, XII, and XIII.

These tables also give the percentage of the "Connection Factor", which includes the weight of connection plates, shoe plates, etc. The sum of the "Specification Factor" and the "Connection Factor" therefore gives the percentage in the roof construction that is fixed by specifications and standard practice. Subtracting the above sum from 100% gives the percentage that the designer has a right to vary the sizes and forms of the members. The relationship between these percentages is shown in Table XIV. As can be seen, the values of the "Specification Factor" varies with the spacing, decreasing as the spacing is

increased. The "Connection Factor" seemed to be quite a constant percentage ranging from:

23.4% - 25.7% for 40' span truss

14.3% - 16.8% for 60' span truss

7.88% - 9.6% for 100' span truss.

Tables below give values of the "Specification Factor" and the"Connection Factor" in percentage of the Total Weight of Truss.

Table XI. 40'-span

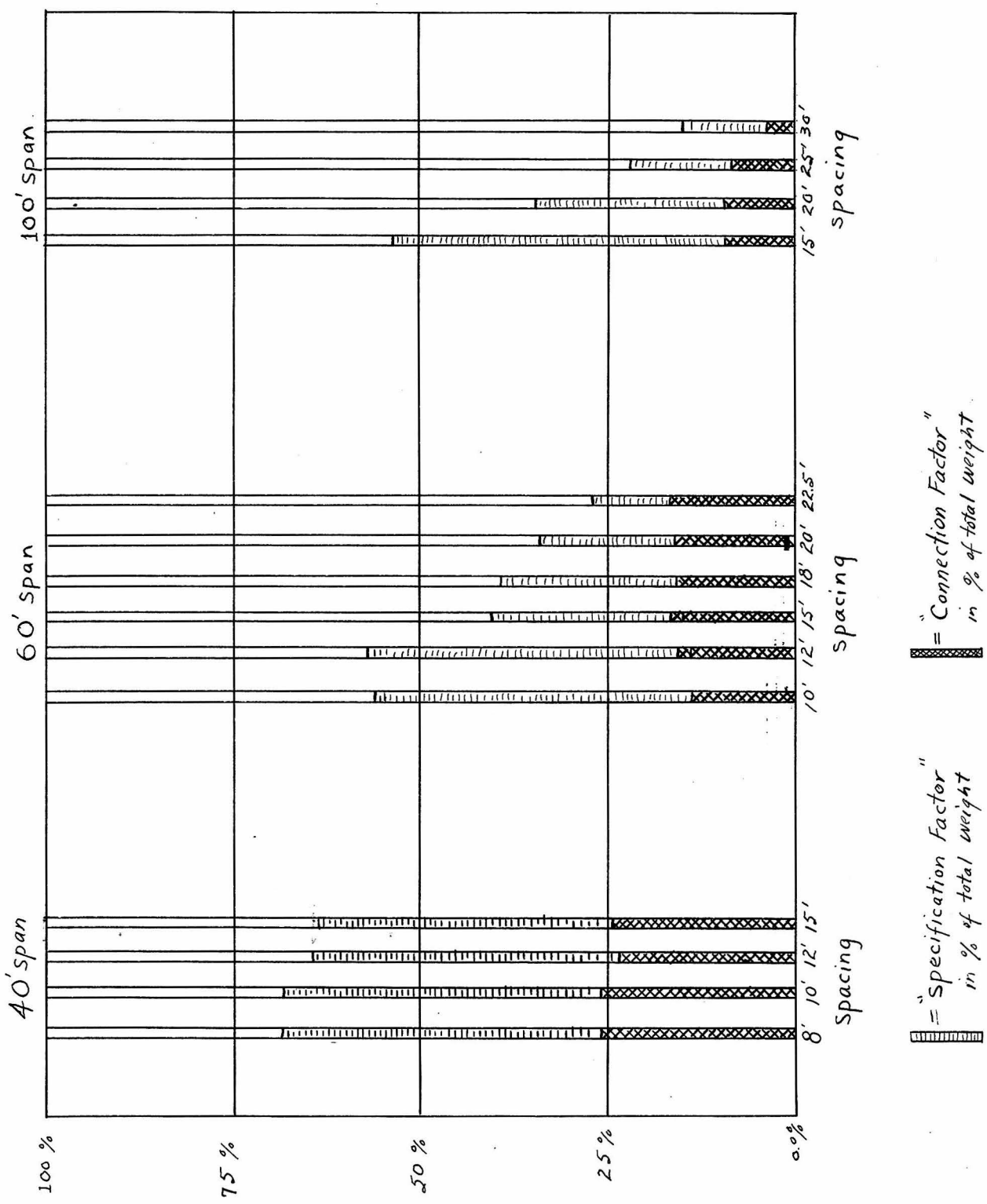
	8'		10'		12'		15'	
	wt.	%	wt.	%	wt.	%	wt.	%
Weight of Truss	1290#	100%	1290#	100%	1366#	100%	1393#	100%
Spec. Factor	553	42.8	553	42.8	553	40.5	553	39.7
Conn. Factor	316	25.7	316	25.7	319	23.4	346	24.8

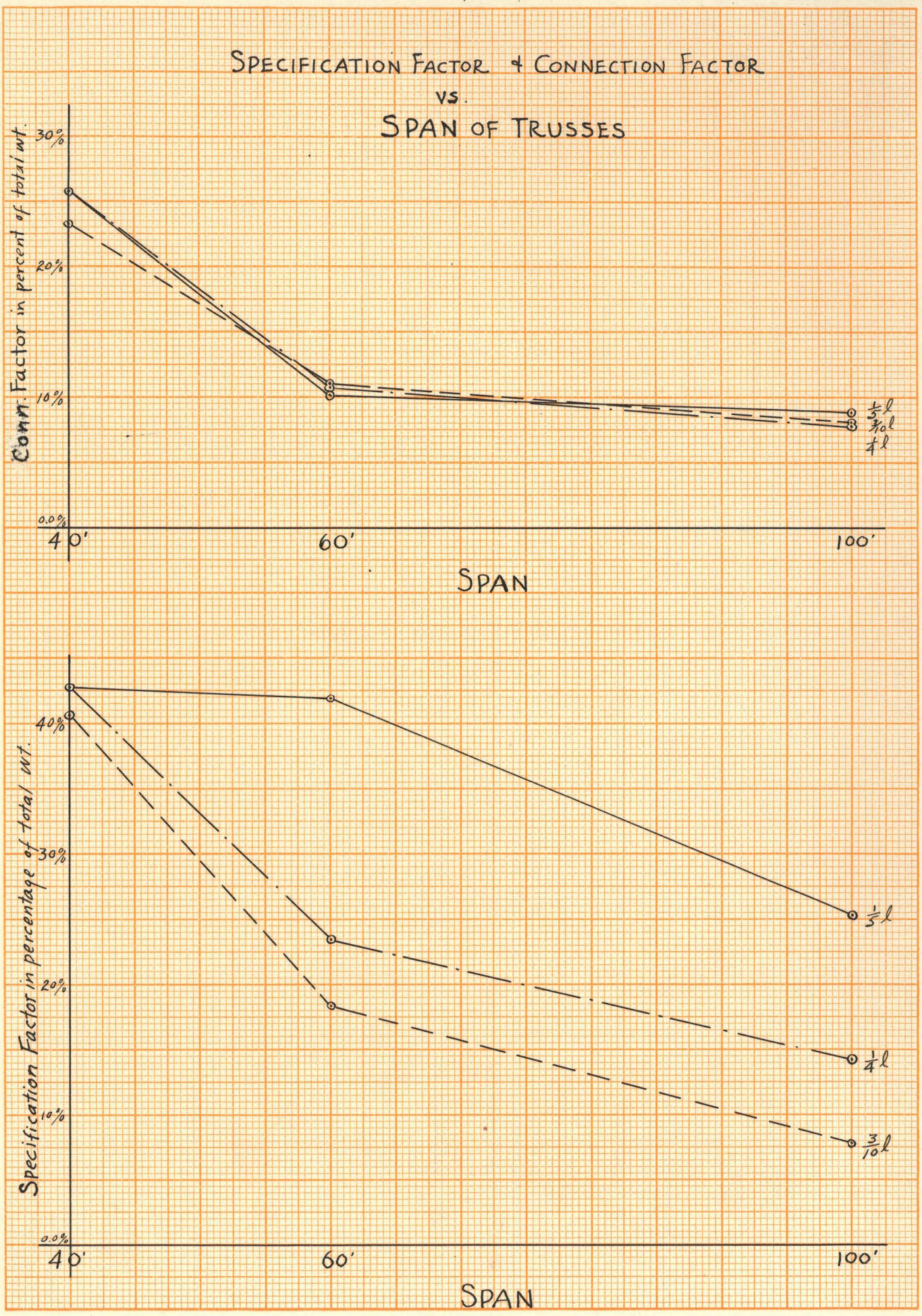
Table XII. 60'-span

	10'		12'		15'		18'		20'		22.5'	
	wt.	%	wt.	%	wt.	%	wt.	%	wt.	%	wt.	%
Truss wt.	2249#	100%	2273#	100%	2513#	100%	2625#	100%	3201#	100%	3416#	100%
Spec. Factor	953	42.4	953	42.0	589	23.5	589	22.5	589	18.4	362	10.6
Conn. Fac.	322	14.3	344	15.1	396	15.8	420	16.0	512	16.0	574	16.8

Table XIII. 100'-span

	15'		20'		25'		30'	
	wt.	%	wt.	%	wt.	%	wt.	%
Truss wt.	6291#	100%	7318#	100%	9246#	100%	10295#	100%
S pec. Fac.	2763	44.0	1857	25.4	1318	14.25	802	7.8
Conn. Factor	603	9.6	653	8.93	737	7.97	810	7.88


40' span
60' span
100' span
8' 10' 12' 15'
Spacing
10' 12' 15' 18' 20' 22.5'
Spacing
15' 20' 25' 30'
Spacing
100 %
75 %
50 %
25 %
0.0%
= "Specification Factor" in % of total weight
= "Connection Factor" in % of total weight



In the calcuh tion of stresses, w was obtained using Howe's formula:

$$w = \frac{pL}{150 + 5L + ps/3}$$

There are, however, different formulas to determine the value of w. Among them are the following ones taken from the discussion of Mr. R. Fleming on "Weight of steel Roof Trusses by Empirical Formulas"—Eng. News Recd., March 20, 1919.

T = wt. of truss

$w = \frac{\text{wt.}}{\text{sq. ft. horiz. area}}$

s = spacing

L = span

P = loading of truss in $\frac{\text{lb.}}{\text{sq. ft. horiz. area}}$

Cambria Steel Co., for spans of 75' or less:

$$T = 5 \cdot Ls$$

Carnegie Steel Co., 1917, for loads of 40# or more per sq. ft.

$$w = \frac{P}{40} \cdot \frac{1}{5} (\sqrt{L} + \frac{L}{8})$$

Fowler, 1909, for Fink trusses up to 200' span

$w = 0.06 \cdot L + 0.6$ for heavy loads

$w = 0.04 \cdot L + 0.04$ for light loads

Johnson, Bryan, and Turneaure

$$w = \frac{L}{25} + 4.0$$

Jones and Laughlin, 1916

$$w = \frac{P}{40}\left(\frac{L}{20} + \frac{12}{s}\right)$$

Ketchum, 1916

$$w = \frac{P}{45}\left(1 + \frac{L}{5\sqrt{s}}\right)$$

Maurer, 1903

$$w = \frac{L}{25} + 1$$

Merriman, 1911

$$T = \tfrac{3}{4}Ls\left(1 + \frac{L}{10}\right)$$

Ricker, 1907

$$w = \frac{L}{25} + \frac{L^2}{6000}$$

Ricker, 1912

$$w = \frac{L}{25} + \frac{L^2}{12{,}600}$$

The values calculated for these different trusses are shown in Table XV on the following page.

The author has taken out Howes', Jones and Laughlins', and Ketchum's formulas and compared the values obtained by these formulas with the actual designed values of w, the curves on pages (27) showing the relationship.

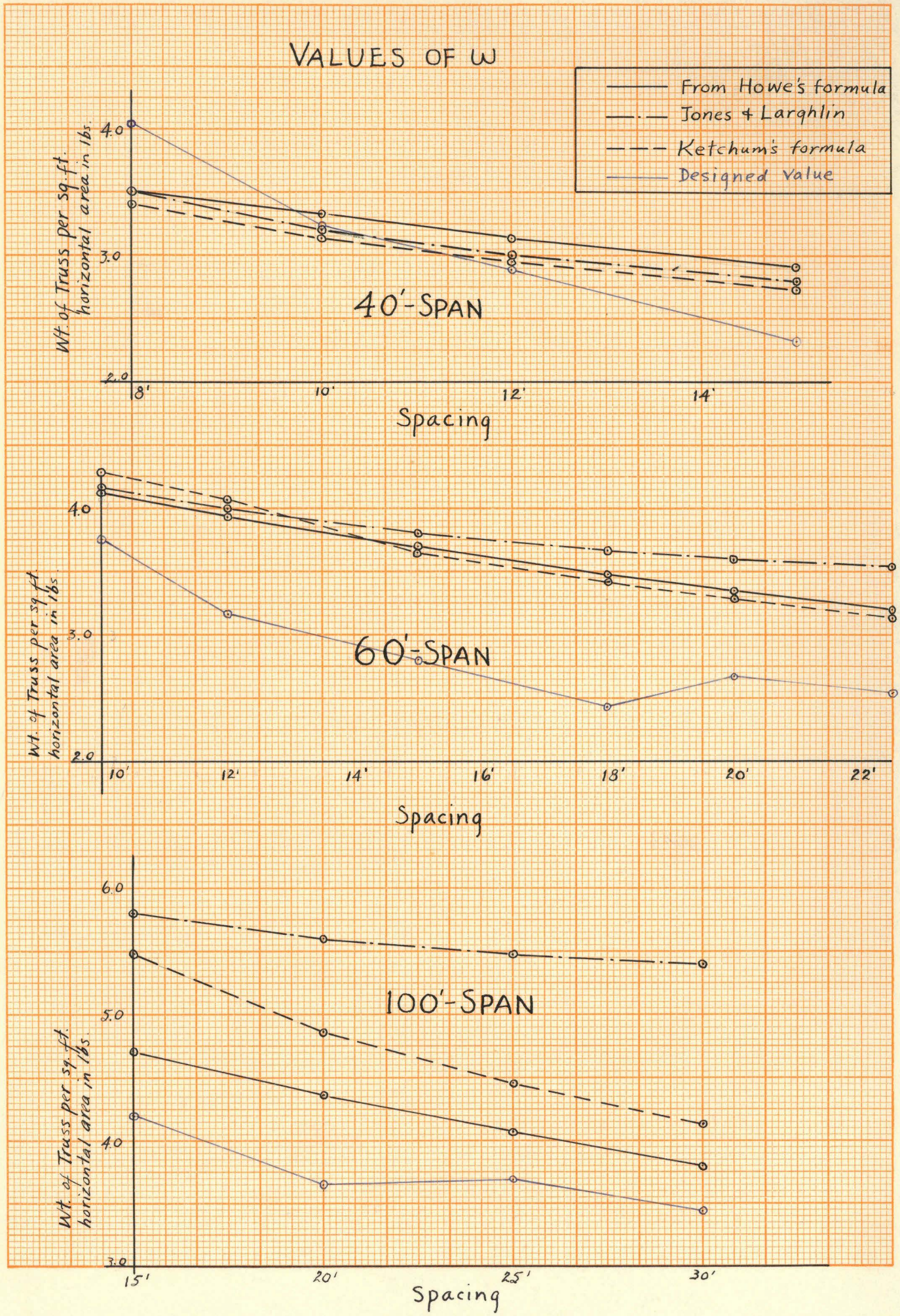
In every case the actual w is below the calculated values from the formulas as shown.

For shorter spans, the three chosen formulas do not seem to deviate from each other very much. For longer spans, Howe's formula seemed to give values closest to the actual designed values of w.

TABLE XV

Values of w from different formulas compared with actual w.

	40' span				60' span						100' span			
	8'	10'	12'	15'	10'	12'	15'	18'	20'	22.5'	15'	20'	25'	30'
actual	4.03	3.22	2.84	2.32	3.75	3.16	2.795	2.43	2.67	2.53	4.2	3.65	3.69	3.44
Howe	3.5	3.31	3.14	2.91	4.12	3.94	3.70	3.48	3.35	3.2	4.7	4.36	4.07	3.8
Cambria	5.0				5.0									
Carnegie	2.265				3.05						4.50			
Fowler	3.0				4.2						6.60			
T. B.T.	5.6				6.4						8.00			
Jones + Larghlin	#3.5	#3.2	3.0	2.8	4.2	4.0	3.8	3.66	3.6	3.54	5.8	5.6	5.48	5.4
Ketchum	3.4	3.14	2.95	2.73	4.27	4.07	3.65	3.41	3.28	3.14	5.49	4.87	4.45	4.13
Maurer	2.6				3.4						5.0			
Merriman	3.75				5.25						8.25			
Ricker, 1907	1.87				3.0						5.66			
Ricker, 1912	1.73				2.69						4.8			
Trautwine	2.6				3.9						6.5			
Tyrrell	3.5													
Carnegie Handbook	2.264				3.05						4.5			



In calculating stresses, the weight of the truss is usually so small compared with the weight of the covering and the snow and the wind, that an error in its assumption is negligible. Table XVI on the following page shows that the actual weight of truss to the total load on the truss for the determination of stresses in different members varied from 7.55% to 10.6%.

For different loadings the variation in weight is usually from 25 to 75% of the variation in loading.°

It should be noted that the personal equation of the designer and many factors entering into the weight of roof trusses may cause a variation of 5 to 25% in the same truss. Thus the designer must base on his own judgment and experience.

°Eng. News. Recd., March 20, 1919.

TABLE XVI

The Ratio of Truss Weight to Total Load*

	Panel loads	Spacing 8'	10'	12'	15'	18'	20'	22.5'	25'	30'
40' span	Truss wt.	127.5#	166.#	188.5#	218.0#					
	Total vert. load	1,456.	1,965.	2,262.	2,884.					
	Truss wt. as % of tot.load	8.75%	8.45%	8.34%	7.55%					
60' span	Truss wt.		309.#	355.#	416.#	470.#	502.#	540.#		
	Total load		3098.	3774	4792	4860	6653	7690		
	Truss wt. as % total load		10.%	9.4 %	8.69%	9.65 %	7.55 %	7.02%		
100' span	Truss wt.				882.		1090		1270	1425
	Total load				8327		11,325	~~14,428~~.	14,428	18,175
	Truss wt. as % total load				10.6%		9.62 %		8.8 %	7.85%

*Total load = Dead + 1/2 snow + Wind (vertical)