FLOOD FREQUENCIES ON THE SAN GABRIEL RIVER

THESIS BY James M. Fox

Submitted in Partial Fulfillment of the Requirements
for the Degree of Master of Science in Civil Engineering.

California Institute of Technology
Pasadena, California
- 1936 -

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INTRODUCTION

The subject of flood-frequencies and forecasting has long been an important field of investigation among engineers, mathematicians and physicists. Numerous methods of analysis have been proposed, some of which are practical and in general use, and others which by reason of their complexity and unwarranted refinement have not met with popular approval.

The goal of all methods has been to determine the most universally suitable means of obtaining a graphical or mathematical relationship between the magnitude of flood flows on a given stream and the frequency with which they will be equaled or exceeded.

All are dependent upon the past record of a stream as an index to the future performance. In applying flood forecasting methods in engineering practice, one is assuming, in effect, that history will repeat itself; that the same magnitudes and frequencies of flood discharges which occurred during a given period will recur in similar magnitudes and frequencies in the future.

The longer the period of record, the more definitely is established the relationship upon which estimates of future flood events may be founded. The successful determination of the correct magnitude-frequency relationship of a stream would require a longer period of record than exists for all but a few streams. The accuracy of the study, then, is limited by the length of record available, and also, to some extant, by the judgement of the investigator, as will be illustrated.

In addition to the direct attack of flood-frequency problems, many other devices have been employed for the purpose of extending or correcting the data, checking the results, or assisting in their application to engineering problems.

The purpose of this paper is to utilize the data of the San Gabriel River in an examination of some of the more common of the flood-frequency and stream-flow methods.

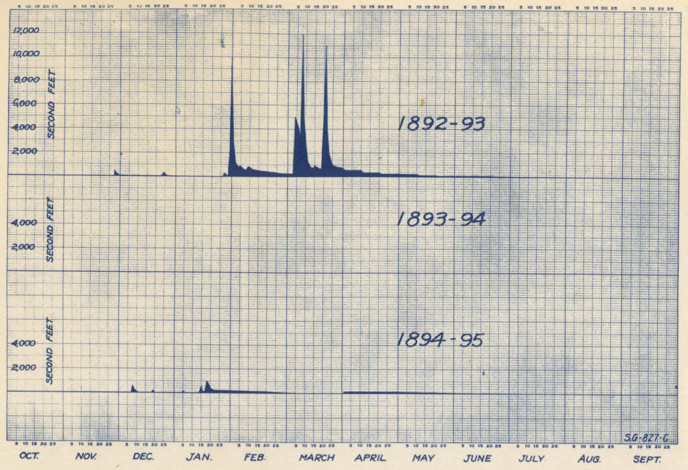
A. DESCRIPTION OF RIVER

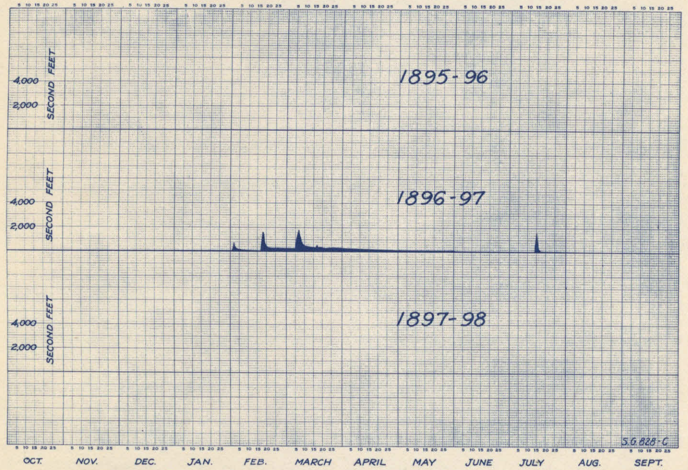
The San Gabriel River drains an area of 214 square miles, consisting almost entirely of mountainous territory. Due to its geographical situation, in the semi-tropical region of Southern California, a few miles north-east of Los Angeles, the stream is almost entirely rainfed. Only limited contributions from melting snow proceed from the highest peaks.

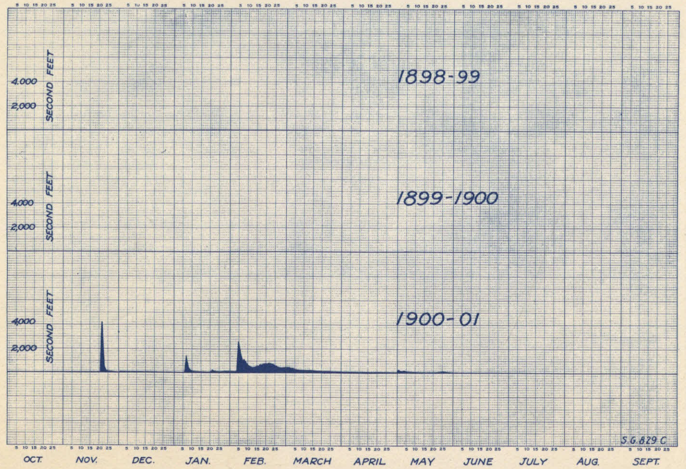
These conditions result in a stream of exceedingly varied discharge, ranging from a dry bed during several months of the year to peak flows of as high as 40,000 c.f.s. (January, 1916) Figure 1, donated by the Pasadena Water Department, charts the yearly hydrographs from 1892 to 1931, and illustrates clearly these characteristics of the stream. The height and duration of floods are closely, though not entirely, dependent upon the intensity and duration of the precipitation.

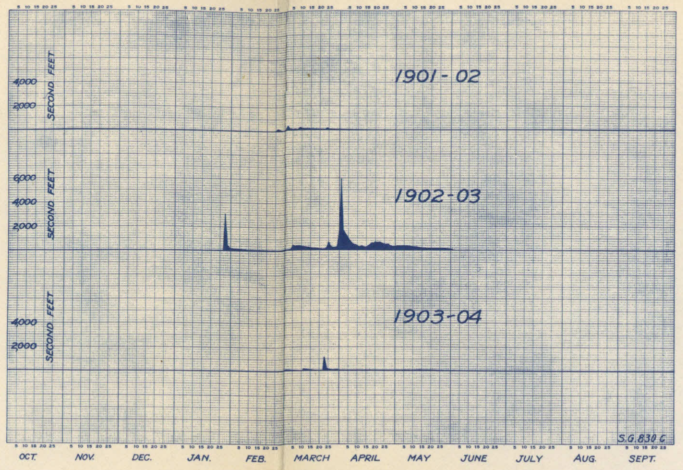
The recently completed Morris Dam was not placed in operation until 1934, and therefore had no influence on the data used. Two canals operated by the Southern California Edison Company are supplied by the stream a little above the gauging station whose records are used in this study. (Near Azusa, California) The data has been corrected, however, from the recorded flow in the canals. Thus, the data presented represent natural, undisturbed conditions of stream-flow on the San Gabriel River.

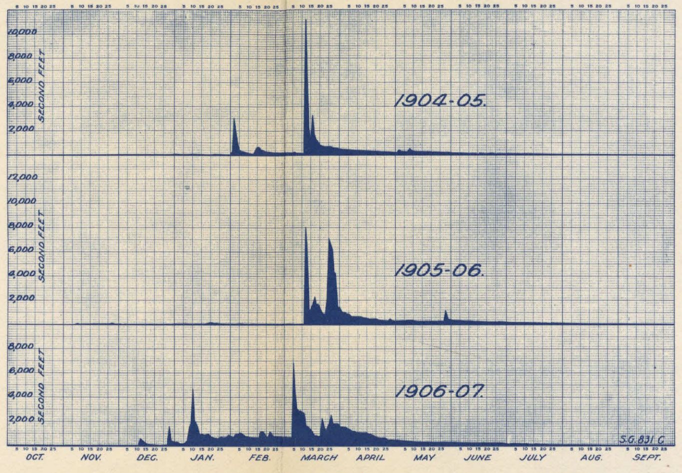
Figure 1

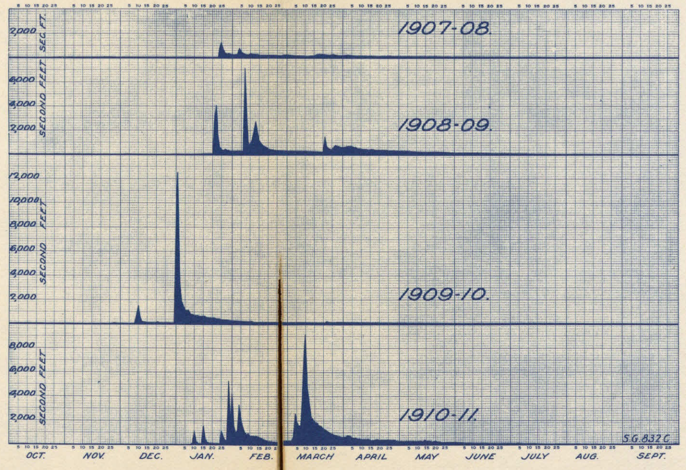


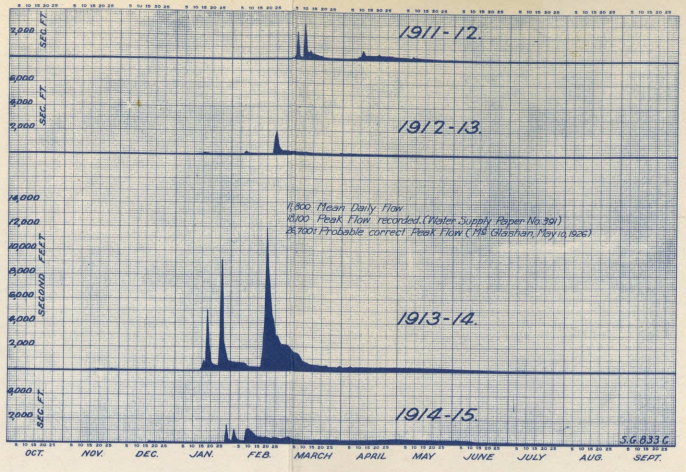


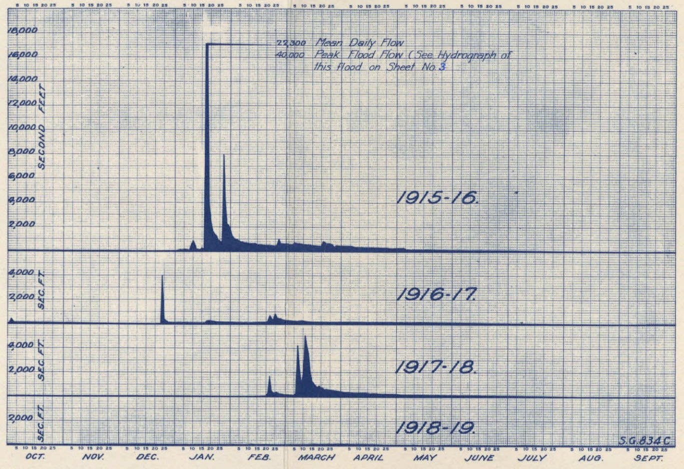


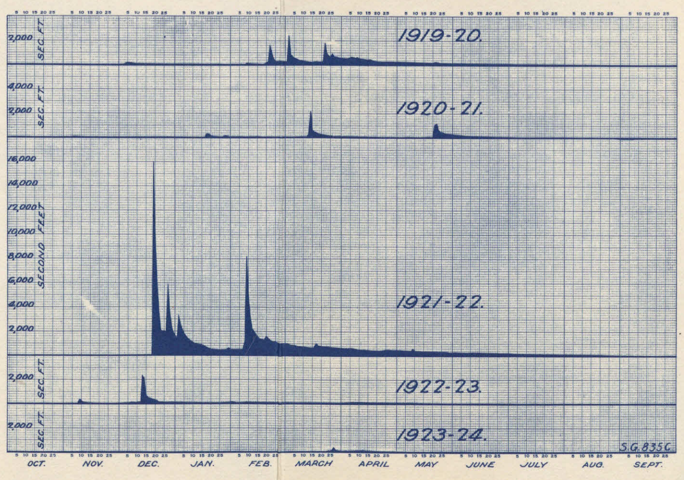


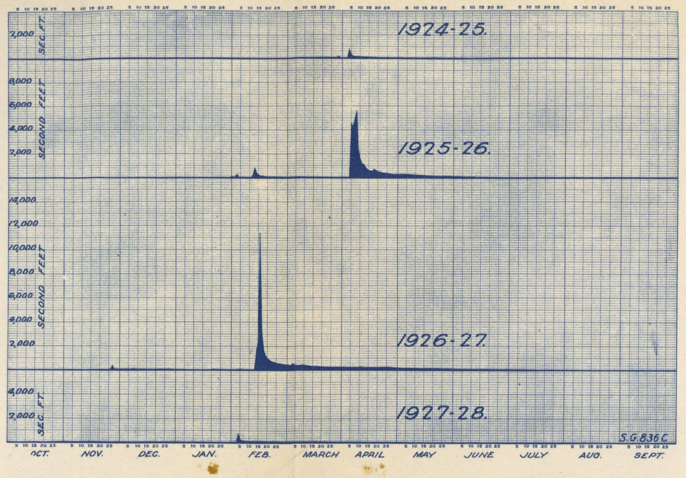


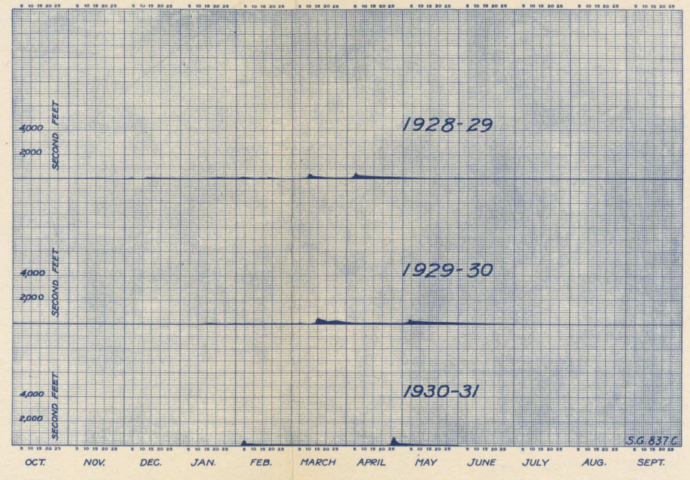












NOTE: Flows plotted are combined <u>mean daily discharge</u> of San Gabriel River and Edison conduit at mouth of canyon. Data compiled by U.S. Geological Survey. For example of relation between mean daily discharge and peak discharge see Sheet 3.

APPROVED:

Consulting Engineer

Consulting Engineer

No.	Date	Description	Made by	Checked	Approved

TABLE OF CHANGES USE ONLY PRINTS SHOWING LATEST CHANGES

PASADENA WATER DEPARTMENT
SAN GABRIEL PROJECT
PINE CANYON DAM

HYDROGRAPHS OF
SAN GABRIEL RIVER
1892-1931

Recommended C. E. Pearce. Designing Engineer

Approved Morris. Chief Engineer

SHEET No. 2 OF 52 SHEETS	
DATE	SCALE As shown
DRAWN BY	W.J.S. & G.W.O.
DESIGNED BY	
TRACED BY	W.J.S & G.W.O.
CHECKED BY	C.E.P.

FILE NUMBER
SG-312-P

B. FLOOD FREQUENCY DETERMINATION

I. Purpose and Methods

The purpose of flood-frequency studies, as mentioned before, is to establish a relationship between magnitude of floods and the frequency of their occurrence as a guide to the most economical design of structures coming directly or indirectly under their influence. The general assumption made, no matter what method of analysis is utilized, is that the stream will follow, with respect to magnitude and frequency, the same tendency as exhibited by past records.

The first procedure in making a flood-frequency plot is to collect the annual flood-flows on the stream under consideration. The annual one-day flood is that usually selected, defined as the maximum average 24-hour flow occurring each year. The total length of the record then, represents 100% on the time scale, and each individual flood flow occurred $\frac{1}{n}x100$ percent of the time, where n is the total number of items in the record. It follows that if the floods are arranged in descending order of magnitude and numbered consecutively from 1 to n, the percent of time any flood was equalled or exceeded is $\frac{m}{n}x100$, where m is the number corresponding to the flood.

In plotting, however, the actual percentages are not used. Instead it is assumed that each flood has an equal chance of falling anywhere within the range of time it repesents, and therefore it is plotted at the mid-point of the interval or at $p = \frac{(2m - 1)}{2n}x100$. This procedure has been followed, using a

50 year period of record, in Table I.

In plotting, either the actual floods or their ratio to the mean flood are taken as the ordinates and the time scale as the abscissa. All standard types of paper, as well as specially constructed paper, are used for plotting. Until about 20 years ago, arithmetic or logarithmic paper were used, but the trend since then has been toward the use of what is termed "hydraulic probability paper". On this paper, the ordinate is either arithmetic or logarithmic, and the time-scale abscissa is taken from the integral of the probability curve. If plotted on this paper data following the normal law of error will fall on a straight line.

Stream-flow data does not follow the normal law of error, but approaches it closely enough that a plot on probability paper has the combined effect of "straightening out" the curve and magnifying the ends of the time-scale.

If it is desired to project the curve beyond the range of the data, this is done with more certainty with a straight line than with an empirical curve. The use of probability paper acheives this result.

It is possible, though rarely necessary, to determine mathematical expressions which approximate the empirical data. The straight line which closely approximates the 29 year curve in Figure 4, is of the form $Q = 31{,}000/e^{.0485p}$, where Q is the flood discharge in c.f.s., p the percent of time Q is equalled or exceeded, and e the base of natural logarithms.

The method now in general use was developed mainly by Fuller Foster and Hazen. It involves obtaining, through the use of three coefficients and a set of empirical logarithmic factors,

TABLE I.

ANNUAL FLOODS ON THE SAN GABRIEL RIVER

Chronologically		In Order of Magnitude		Percentage of Time	
Year	Flow c.f.s.	No.	Flow c.f.s.	Actual	Plotting
1885-1886	8,000*	1	22,300	2 %	1 %
1886-1887	785*	2	20,000*	4	3
1887-1888	3,000*	3	16,900	6	5
1888-1889	5,200*	4	14,500*	8	7
1889-1890	20,000*	5	12,500	10	9
1890-1891	1,500*	6	11,800	12	11
1891-1892	830*	7	11,400	14	13
1892-1893	14,500*	8	10,000*	16	15
1893-1894	270*	9	9,400	18	17
1894-1895	4,000*	10	9,160	20	19
1895-1896	230*	11	8,200	22	21
1896-1897	2,900*	12	8,000*	24	23
1897-1898	185*	13	7,100	26	25
1898-1899	200*	14	6,810	28	27
1899-1900	290*	15	5,910	30	29
1900-1901	5,200*	16	5,530	32	31
1901-1902	350*	17	5,500*	34	33
1902-1903	5,500*	18	5,200*	36	35
1903-1904	245*	19	5,200*	38	37
1904-1905	10,000*	20	5,030	40	39
1905-1906	9,430	21	4,000*	42	41
1906-1907	6,810	22	3,020	44	43
1907-1908	1,240	23	3,000*	46	45
1908-1909	7,100	24	2,900*	48	47
1909-1910	12,500	25	2,600*	50	49
1910-1911	9,160	26	2,400	52	51
1911-1912	3,020	27	2,380	54	53
1912-1913	1,960	28	1,960	56	55
1913-1914	11,800	29	1,690	58	57
1914-1915	1,430	30	1,500*	60	59
1915-1916	22,300	31	1,430	62	61
1916-1917	820	32	1,240	64	63
1917-1918	5,030	33	984	66	65
1918-1919	146	34	830*	68	67
1919-1920	2,400	35	820	70	69
1920-1921	16,900	36	785*	72	71
1921-1922	8,200	37	754	74	73
1922-1923	199	38	668	76	75
1923-1924	332	39	492	78	77
1924-1925	668	40	476	80	79
1925-1926	5,530	41	350*	82	81
1926-1927	11,400	42	332	84	83
1927-1928	754	43	290*	86	85
1928-1929	492	44	270*	88	87
1929-1930	476	45	245*	90	89
1930-1931	984	46	230*	92	91
1931-1932	5,910	47	200*	94	93
1932-1933	1,690	48	199	96	95
1933-1934	2,380	49	185*	98	97
1934-1935	2,600*	50	146	100	99

a theoretical frequency curve which fits the actual plotted data. These coefficients are:

Flood Coefficient = mean flood/$A^{0.8}$, where A is the drainage area of the stream-basin.

Coefficient of Variation = CV = $\sqrt{\frac{\Sigma d^2}{n - 1}}$ where d = (flood - mean flood)/mean flood and n = length of record.

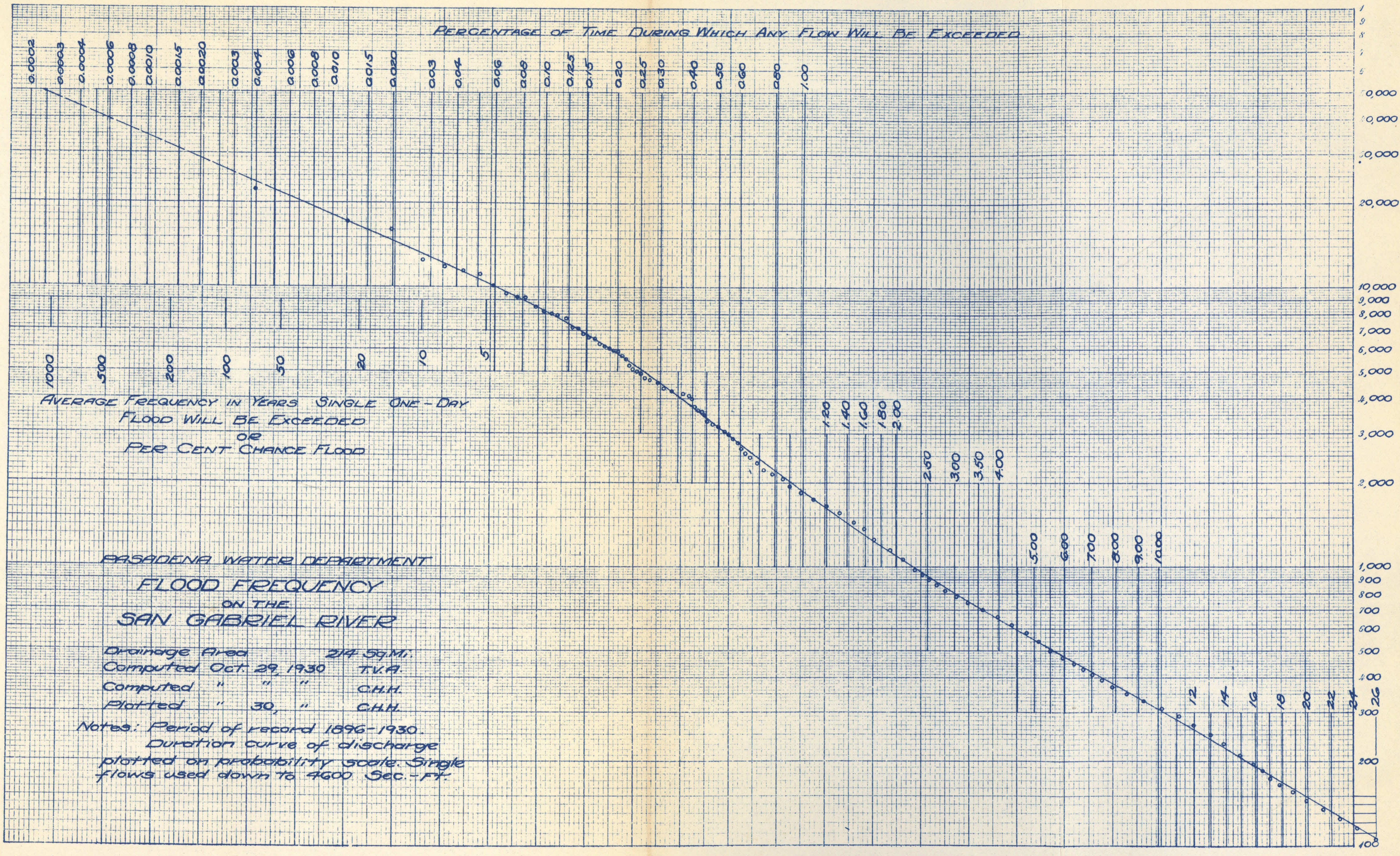
Coefficient of Skew, C.S. = $\frac{\Sigma d^3}{(n - 1) \times CV^3}$. CS is further adjusted by means of an empirical factor $f = (1 + \frac{8.5}{n})$. The CV is a measure of the variation of the data from the mean and determines the slope of the curve. CS is a measure of the assymetry of the curve and determines the point at which the curve intersects the ordinate of mean flood.

II. Frequency Curves

Figures 2, 3, 4, and 5 are all various forms of frequency curves derived from the data of the San Gabriel River. Figure 2 was furnished through the courtesy of Mr. Sopp of the Pasadena Water Department. It represents a flow-duration curve on which all flows for the period 1896 - 1930 are plotted on probability paper. The plotting points are determined in the same manner as outlined above, but on this scale each flood or flow represents only one day during the period, rather than one year. A special scale is superposed on the chart locating the yearly or percent chance floods for various intervals. The conversion is simply made. For example, the 100 year, or one percent chance flood, on this chart occurs $\frac{1}{365 \times 100} \times 100$ = .00274 percent of the

Comp. 464
Sheet 6

Figure 2



PASADENA WATER DEPARTMENT

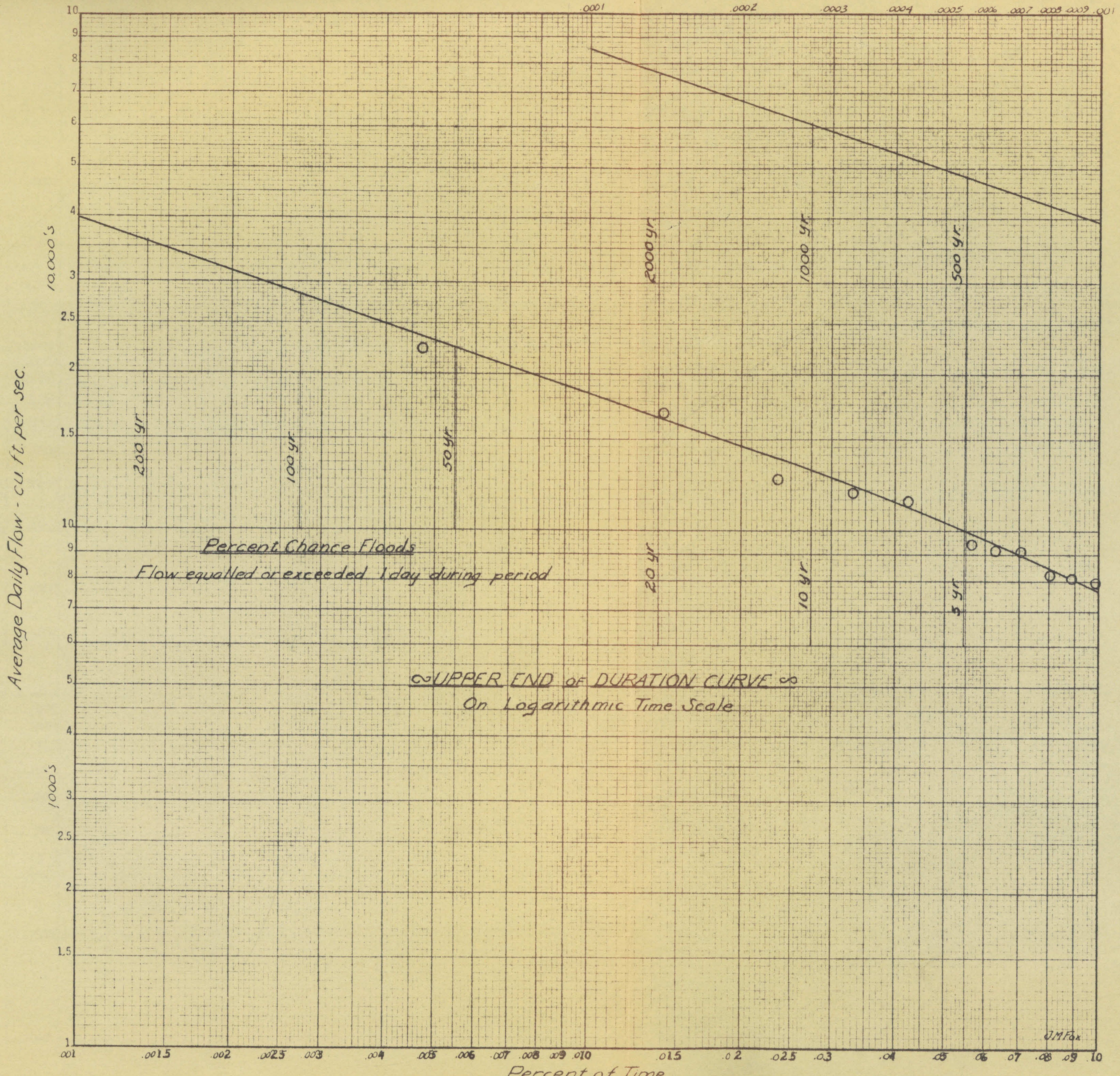
FLOOD FREQUENCY
ON THE
SAN GABRIEL RIVER

Drainage Area	214 Sq.M.	
Computed Oct. 29, 1930		T.V.A.
Computed " " "		C.H.H.
Plotted " 30, "		C.H.H.

Notes: Period of record 1896-1930.
Duration curve of discharge plotted on probability scale. Single flows used down to 4600 Sec.-Ft.

SEC. 4 ~ PLATE 6

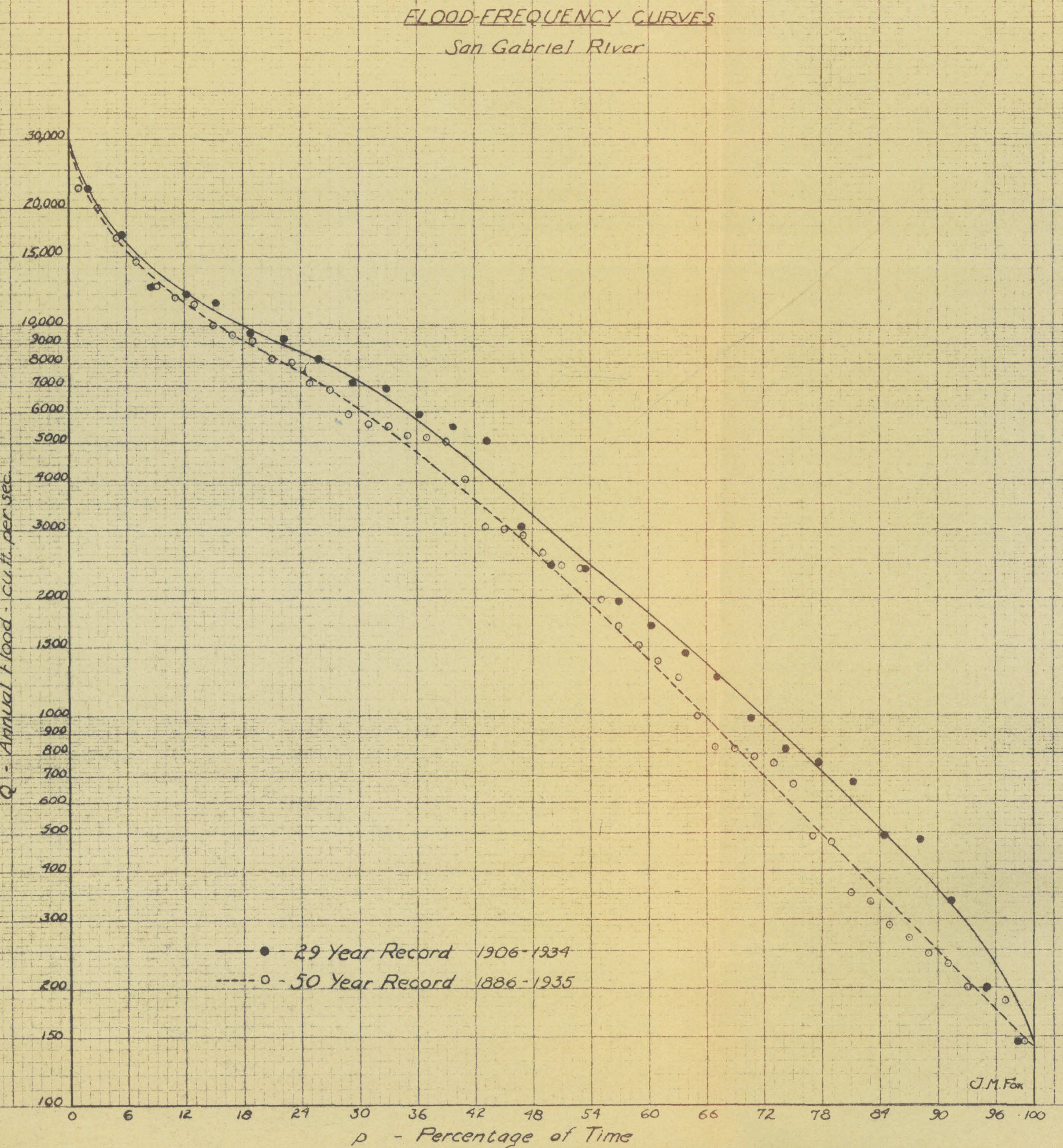
Figure 3



~UPPER END OF DURATION CURVE~

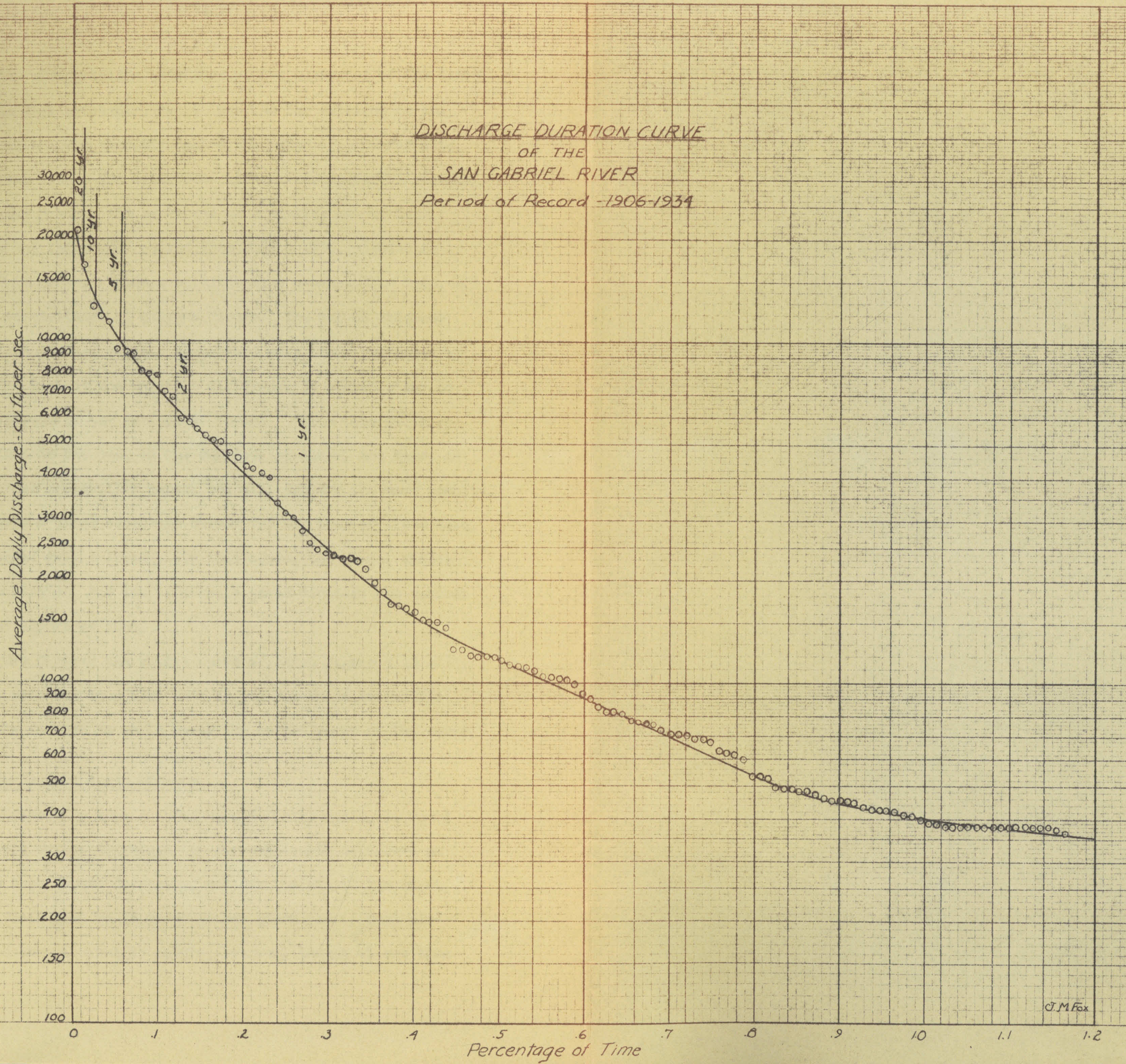
On Logarithmic Time Scale

Figure 4



FLOOD-FREQUENCY CURVES
San Gabriel River

Figure 5



DISCHARGE DURATION CURVE
OF THE
SAN GABRIEL RIVER
Period of Record - 1906-1934

time. The straight-line extension of the upper end of the curve seems somewhat arbitrary, but unless the law followed by the variations changes abruptly beyond the range of observed data, the extreme floods are determined with a fair degree of accuracy.

Figure 3 shows the upper end of a similar set of data, covering the fifty-year period from 1886 - 1935, plotted on logarithmic paper. Again a straight-line extension is made, and carried to positions of extreme floods. The values obtained from this curve are closely comparable with those indicated by Figure 2. Most of the existing difference is probably due to the difference in the periods of record. From this it is apparent that in the case of the San Gabriel River, log paper is as usable as probability paper, and will achieve the same results.

Figure 4 plots frequency curves for two sets of flood data, as indicated in the legend. On this curve, the annual floods only are plotted, and the time-scale gives directly the percent chance of any flood flow.

Figure 5 gives the entire curve of which Figure 3 is a portion, with the exception that in the former an arithmetic time-scale is used so as to cover the entire range of the curve.

III. The Cycle and its Effects

If there were such a thing as a definite cycle, it would be a simple matter to choose the correct length of record on which to base future forecasts, and even to choose the exact size of flood that would occur in any given year. Such simplicity does not exist in nature, however, though at one time it was supposed that all streams had a fairly well-defined cycle of runoff.

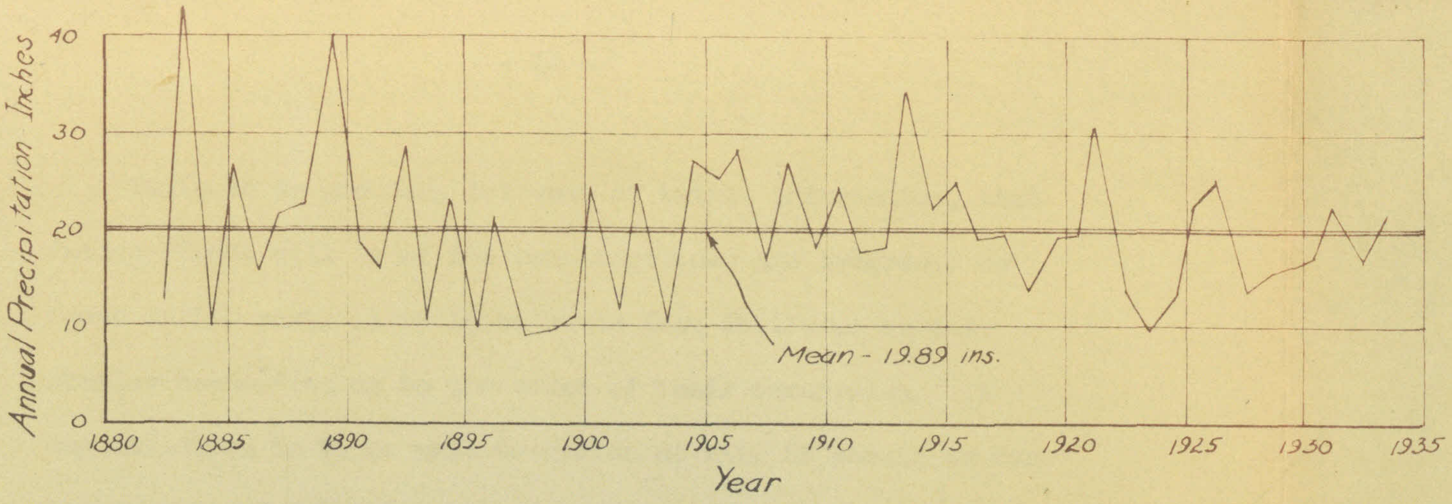
As yet, however, records of sufficient length to establish the positive truth of this supposition do not exist. It is generally known that there are fluctuations from high to low extremes over a period of years, but these changes are not regular enough, within the time of recorded data, to admit of successful analysis.

That such fluctuations do occur on the San Gabriel is readily seen from Figure 6, on which are plotted hydrographs for rainfall, average annual discharge, and yearly floods for the period 1883-1934. The record is not long enough, nor is the cycle definite enough to warrant any alteration of the flood frequency methods so as to include cyclic effects.

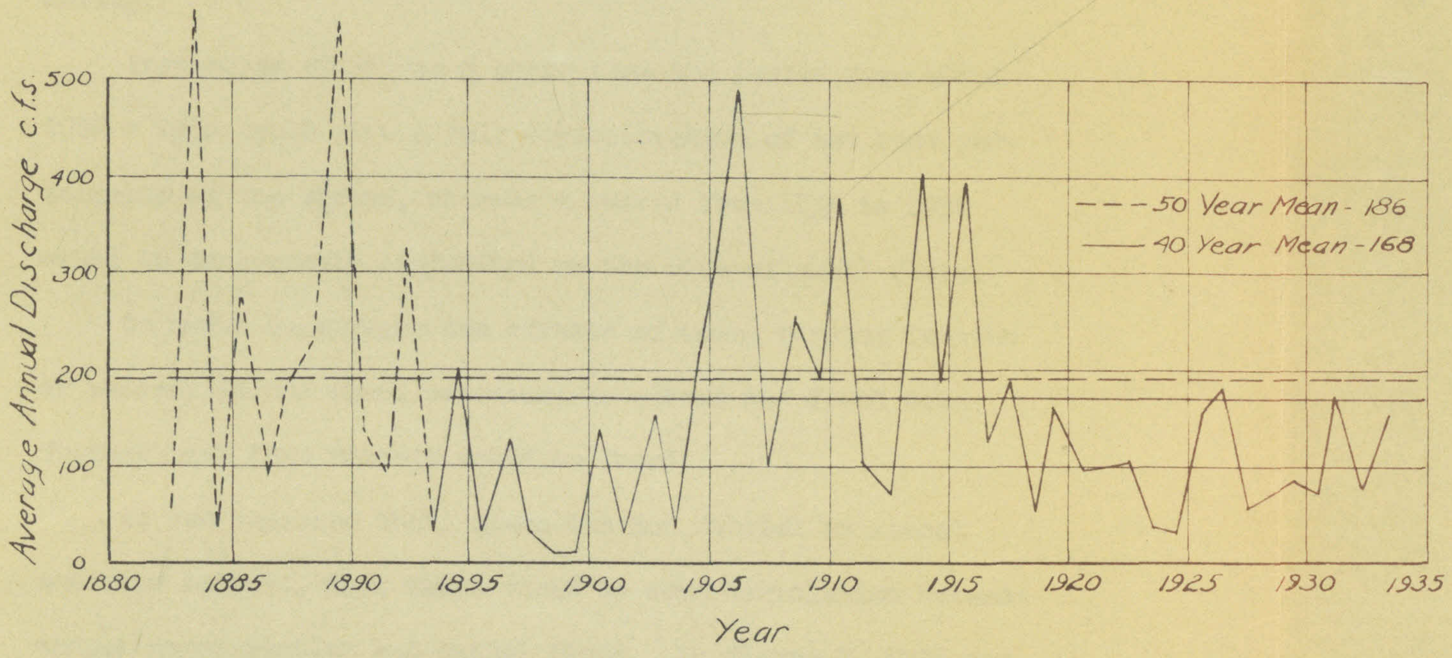
Selecting a period of record for flood frequency study requires the exercise of considerable judgement, due to the alternating tendencies of the stream. From a record of ten years length one may obtain a value for the maximum flood to be expected once in the next 50, 100, or 1000 years. This is obviously a fallacy. Forecasts based on record periods of less than 20 or 30 years are very likely to lead to erroneous estimates. Records of shorter length have not covered a cycle of the flow and may therefore indicate vastly different tendencies than are actually the case.

The same danger exists, to a less degree, in any longer set of data which is "lop-sided", that is, which covers more data on one side of the true mean than on the other.

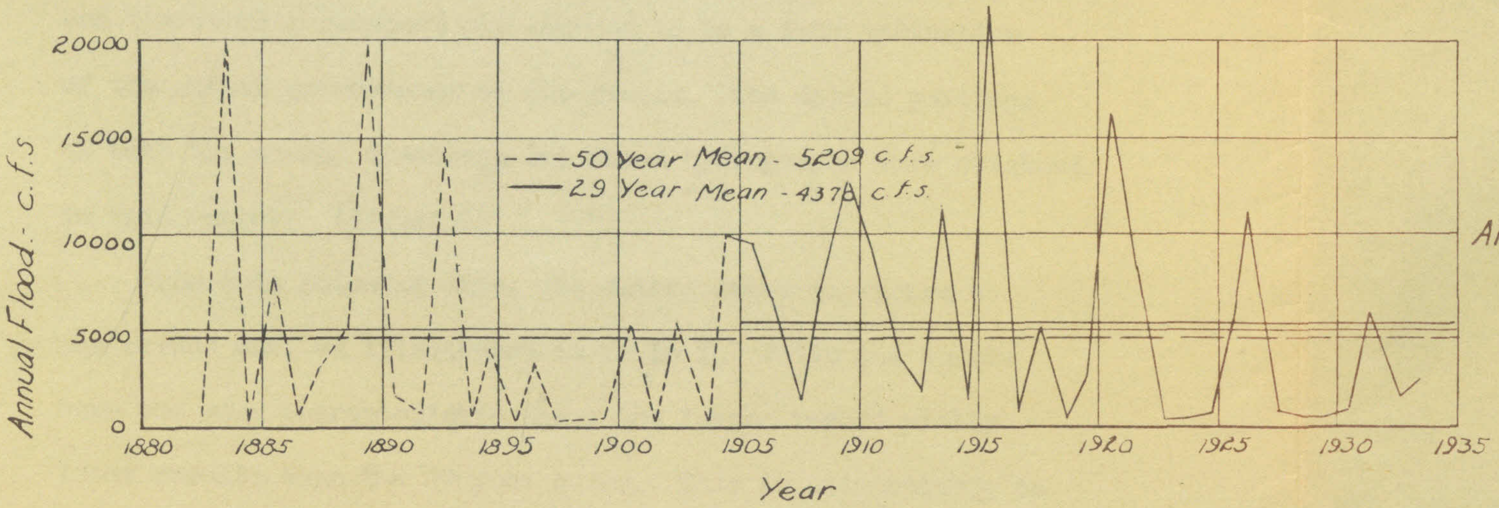
Figure 6



RAINFALL



ANNUAL DISCHARGE



ANNUAL FLOOD

J.M Fox

While it is assumed, for want of better information, that future floods will be of the same magnitude and frequency as those in the past, it would be worse than futile to make a similar assumption as to the order of their occurrance. If any attention is to be paid to cycles at all, it should be for the purpose of determining the correct period of record upon which to base future forecasts with the greatest degree of safety.

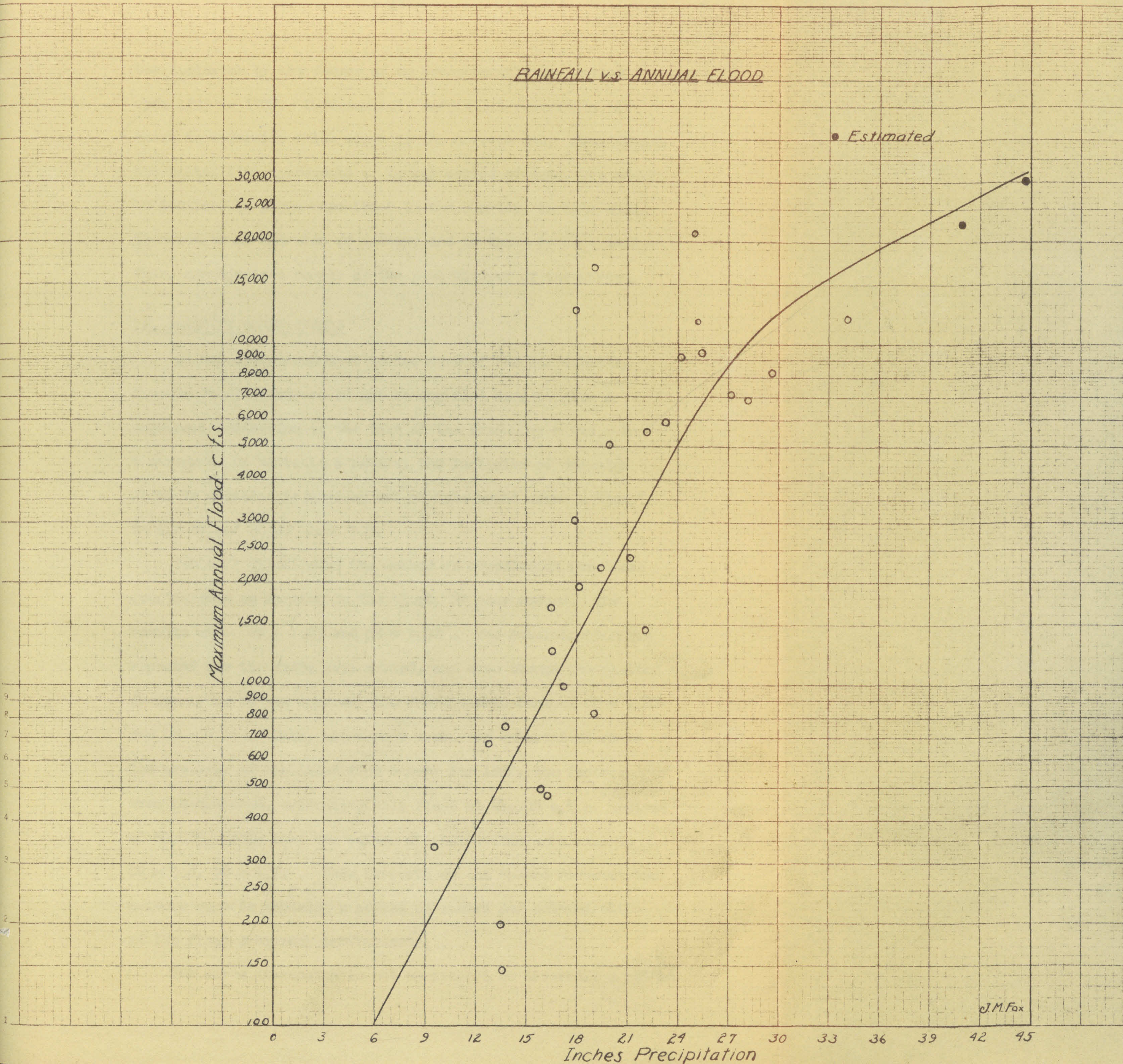
Inspection of Figure 6 shows that the period from about 1906 - 1935 would give a fair representation of the past performance of the stream, whereas a record from 1893 to 1935 would be dangerously prejudiced on the side of small flows.

In order to compare the effects of using varying lengths of record, it was found necessary to extend the flood data further back than records could be found.

It was supposed that, since the San Gabriel is almost entirely rainfed, that there would be some correlation between annual precipitation and annual flood. In Figure 7, this was attempted. While there is considerable error for individual floods, for a long record there is a tendency to compensate, and the results obtained are assumed to be a fair indication of the actual performance of the stream. The dotted portions of both the annual discharge and flood hydrographs were obtained in this manner. (Figure 6)

From this extended data, the dotted curve on Figure 4 was worked out, as illustrated in Table I. While the curves have the same characteristic form, the longer record yields lower results than the 29 year curve. This is attributable to

Figure 7


RAINFALL v.s. ANNUAL FLOOD
● Estimated
Maximum Annual Flood - C.f.s.
30,000
25,000
20,000
15,000
10,000
9,000
8,000
7000
6,000
5,000
4000
3,000
2,500
2,000
1,500
1,000
900
800
700
600
500
400
300
250
200
150
100
0
3
6
9
12
15
18
21
24
27
30
33
36
39
42
45
Inches Precipitation
J.M. Fox

the inclusion by the former of an exceptionally dry period from 1893 to 1904. Unfortunately sufficient records do not exist to determine which curve is the correct one. Even though the dotted line represents a longer period of data, one would be inclined to place more trust in the shorter, in this case. It would be on the side of safety, and is known to have been a fair, unprejudiced sample of the past history of thes tream.

IV. HAZEN'S COEFFICIENTS

It was discovered that in the case of the San Gabriel, Hazen's Coefficients could not be depended upon to give a permanent indication of the form of the frequency curve. In a stream of so variable a nature, the inclusion of one high flood in addition to a given set of data would cause a "jump" in the CV and CS of about 30%.

Table II illustrates the method of computation for these coefficients as applied to the short, 29 year record. The results are: CV = 1.07 and CS = 1.72 . The determination was repeated for the fifty year period, and even though the plotted frequency curve was lowered, the coefficients became: CV = 1.42 and CS = 2.11 . Then, since some doubt was entertained as to the accuracy of the flood flow chosen for 1889, the factors were re-computed, estimating this flood at 40,000 c.f.s. instead of 20,000. This time the values of the coefficients rose to CV = 1.6, CS = 3.71 . This property of the method necessitates extreme care in choosing a period of record for determination of the flood-frequency coefficients.

The above discussion is not an attempt to discredit a method

TABLE II

STANDARD VARIATION AND COEFFICIENT OF SKEW

29 Year Period from 1906 to 1934

(1) No.	(2) Ratio to Mean	(3) Variation	(4) (3) squared	(5) (3) cubed +	(6) (3) cubed -
1	4.28	+ 3.28	+ 10.730	35.400	
2	3.25	+ 2.25	5.070	11.400	
3	2.40	+ 1.40	1.960	2.740	
4	2.27	+ 1.27	1.610	2.050	
5	2.00	+ 1.00	1.000	1.000	
6	1.81	+ .81	.656	.530	
7	1.74	+ .74	.550	.407	
8	1.57	+ .57	.325	.185	
9	1.46	+ .46	.211	.097	
10	1.31	+ .31	.096	.030	
11	1.14	+ .14	.020	.003	
12	1.06	+ .06	.004	.000	
13	.96	- .04	.002		.000
14	.58	- .42	.176		.074
15	.46	- .54	.291		.157
16	.45	- .55	.302		.166
17	.38	- .62	.384		.233
18	.32	- .68	.462		.314
19	.28	- .72	.518		.373
20	.24	- .76	.578		.440
21	.19	- .81	.656		.531
22	.16	- .84	.705		.593
23	.14	- .86	.740		.636
24	.13	- .87	.751		.655
25	.09	- .91	.826		.752
26	.09	- .91	.826		.752
27	.06	- .94	.883		.830
28	.04	- .96	.923		.886
29	.03	- .97	.940		.913
		Totals	32.195	53.842	8.305

$$\text{Coefficient of Variation} = \sqrt{\frac{32.195}{29 - 1}} = \sqrt{1.15} = \underline{1.07}$$

$$\text{Coefficient of Skew} = \frac{53.842 - 8.305}{(29 - 1)\text{x}1.07^3} = \frac{45.537}{28\text{x}1.23} = \underline{1.325}$$

$$\text{Adjustment Factor} = (1 + \frac{8.5}{29}) = 1.293$$

$$\text{Adjusted Coefficient of Skew} = 1.293 \text{ x } 1.325 = \underline{1.72}$$

accepted by eminent engineers, but it serves to illustrate the fact that for streams of the peculiar nature of the San Gabriel, the process is far from fool-proof, and should not be used without the added precaution of checking by some other means. A statement made in the UnitedStates Geological Survey Water Supply Paper No. 771 testifies to the fact that some uncertainty in the use of the above process still exists. The reccommendation is made that if a well-fitting curve is not obtained using the derived coefficient of skew, a new coefficient of skew should be selected, and the curve again constructed, repeating until an accurate fit is obtained.

C. PERTINENT RELATIONSHIPS

I. Rainfalland Annual Discharge

Inspection of the precipitation and average annual discharge hydrographs, Figure 6, yields a few interesting facts which would prove helpful in anticipating future discharges.

Due to the lag induced by ground-storage, there is not a positive agreement between the fluctuations of rainfall and annual runoff. The principles involved are obvious. For example, though the precipitation in the years 1899 - 1900 and 1901 - 2 was above normal, the flow, due to previous depletion of storage, was well below average, while in the period 1904 - 7, three moderately wet years resulted in cumulatively higher discharges, culminating in an unusually high "peak" in 1907.

II. Annual Floods

The annual floods are also more or less definite in their relation to other hydrological data. Figure 7 illustrates the function of rainfall vs. annual flood. In the case of annual runoff against annual flood, while there is certainly a tendency for the maximum flood to occur during a year of high flow, there is no distinct numerical comparisson, and the relationship seems independed of height of flow. Roughly speaking, the maximum yearly flood varies from 9 to 48 times the rate of average annual discharge, with some indication of grouping about a straight line expressing the maximum flood as 28 times the average annual discharge. The function is not plotted in the report.

III. Peak Flows

While the 24 hour flood is that normally used in flood frequency investigations, in many cases the maximum flow of the stream for short intervals is the vital factor in application of the results to design.

The height of this momentary peak depends upon several factors; shape and size of drainage area, topographical characteristics, or slope, and direction of travel of storm, besides, of course, the actual intensity of the precipitation. In general the relative height of the peak flow varies inversely as the area of the stream-basin.

Attempts were made to determine, in the case of the San Gabriel, some relation between height of average discharge and height of peak, and also between height of peak and time of occurrance, without success. These are apparently entirely independent elements. From available data, it was determined that the momentary peak, on the average, was 2.13 times the flow averaged over the corresponding 24 hour interval.

While actual flow records were not available previous to 1906, it was possible from two or three sources to discover estimates of the highest peaks occurring on the San Gabriel. There is some controversy as to whether the flood of 1884 or 1889 was the largest. In the Report of the Board of Engineers Flood Control to the Board of Supervisors, Los Angeles County, in the section prepared by H. Hawgood, estimates were given varying between 40,000 and 50,000 c.f.s.

Lon F. Chapin, in his memoirs, "Thirty Years in Pasadena" describes the floods and the damage incurred. All evidence points to the conclusion that these floods were the highest of which any quantitative record has been kept, thought they were closely approached by the peak of 40,000 c.f.s. in 1916.

A peak of 50,000 c.f.s., corresponding to about 23,000 c.f.s. average 24 hour flow, is seen from the frequency curves to represent approximately the one hundred year, or one percent chance flood.

IV. Attempts at Forecasting Floods

Numerous attempts have been made to determine some accurate means of forecasting floods, i.e. anticipating the flood flow for a particular year. Of course, prediction over a period of years is impossible, but some hope is still entertained of being able to forecast a flood a few months or a year in advance. So far little success has been attained, and it is improbable that with the present limited knowledge of meteorology and allied subjects any practical, certain means will be discovered for some time to come.

Dr. Ford A. Carpenter, in a pamphlet "Sunspots, Cycles, and Seasonal Rainfall", suggests the use of the rainfall of a certain month, as November, as an index to the precipitation to be expected during the ensuing water year. He states that in a number of California streams, using a record fifty-two years in length, a reasonably close correlation was obtained. 36 times of the fifty-two, the November rainfall approximately indicated the amount of the next year's precipitation.

It was hoped, in view of the dependence of the size of flood on rainfall, that a relation involving November rainfall and the coming flood could be deduced. The attempt failed completely. Apparently Dr. Carpenter's relation does not hold in Pasadena and vicinity.

There exists, on the San Gabriel, a well-defined 2 or 3 year cycle which might be of assistance in forecasting the expected trend of the hydrograph for the succeeding year, though any effort toward an actual quantitative prediction would be useless and dangerous. The annual flood hydrograph on Figure 5 illustrates this point.

In general, it may be stated that while sound engineering judgement coupled with complete statistical data may yield correct forecasts in some cases, the determining factors are too varied to admit of definite solution. This is especially true in the case of the San Gabriel River.

D. APPLICATIONS OF RESULTS

I. General

The purpose of the frequency studies has been to discover some basis for the economic design of structures related to streams and runoff. Without definite knowledge of the habits of the particular stream, a design may be made unsafe under even average flow conditions, or it may be exaggerated on the side of safety to an uneconomic extent.

With a record of usable length, and the assistance of the flood frequency devices, it is possible to eliminate uncertainty to a great extent.

In designing on the basis of expected floods, there are two conditions to be considered, that of maximum, momentary peak, and that of flow-duration. For bridges, highways, levees culverts, spillways, etc., it is the height of the flood crest which must be anticipated and provided for. The momentary peak is, of course, the deciding factor.

The use of dams and reservoirs to "iron out" the destructive high points of flood flows requires an examination of flow-durationdata. Any property situated below a reservoir receives protection to the extent of the controlling ability of the dam. Structures and property lying above the protective influence of controlled storage are safe only as long as they remain above the high-water stage of the stream.

In allowing for expected floods, an arbitrary expected damage loss is assigned to a flood exceeding the designed capacity. This damage may occur to the structure itself, or to dependent areas, and may range from assumed complete destruction of the structure to a small percentage of its cost.

If human life depends upon the safety of the structure under any circumstances, it is designed for the most extreme condition which may reasonably be expected during its lifetime. If the life of human beings is not endangered by its failure, then some design is chosen, based upon the judgement of the engineer in charge, and upon available funds, which it is hoped will not be subjected to an extreme flood until it is paid for.

II. Theoretical Economical Design

In view of the intensive research on the subject of flood-frequencies, it seems justifiable to carry the cause of refinement a step further, into the field of economic design. As a flood exceeding the capacity of a structure causes damage, and financial loss, and yet to build a structure entirely free from danger of extreme floods entails prohibitive cost, there should be some happy medium, at which the yearly cost of upkeep, retirement and repair would be a minimum.

From the mathematical point of view, the annual cost of a structure is $C_a = M + Ir + A$, where M is the yearly maintenance, I the initial cost of the structure, r the rate of interest and A the annuity necessary to create a replacing sum at the end of the life of the structure. A can be written in terms of I and r: $A = \frac{Ir}{(r + 1)^n - 1}$ where n is the assumed length of life. Then

$$C_a = M + Ir + \frac{Ir}{(r + 1)^n - 1}$$

$$= M + Ir(1 + \frac{1}{(r + 1)^n - 1})$$

$$(1) \qquad = M + Ir(\frac{(r + 1)^n}{(r + 1)^n - 1})$$

For a constant value of I, C_a decreases continually with an increase in n.

In practice, however, the length of life of a structure, or at least the length of the interval between damaging, and therefore expensive, floods, is dependent upon the capacity of the structure. As I increases with the size of the structure, and as the interval n also increases with the size, I becomes an increasing function of n and may be written in the form,

$$I = A + Bn^c$$

where A, B, and c are constants depending upon the cost function of the particular type of structure.

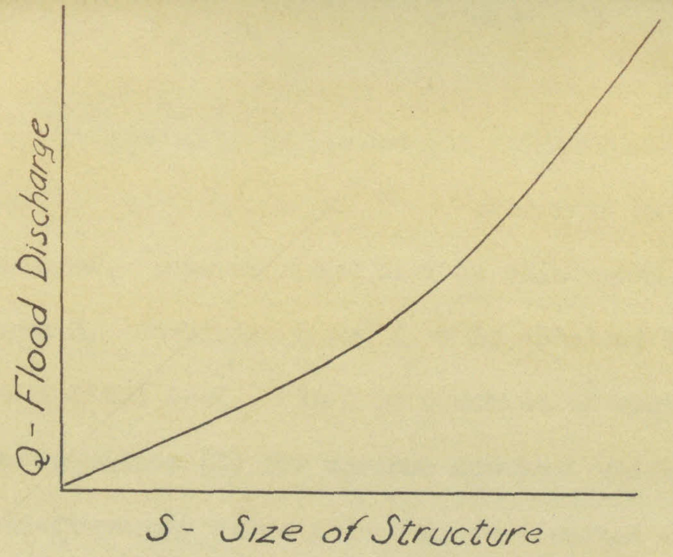
Formula (1) may then be re-written

$$(2) \qquad C_a = M + (A + Bn^c)\frac{r(r + 1)^n}{(r + 1)^n - 1}$$

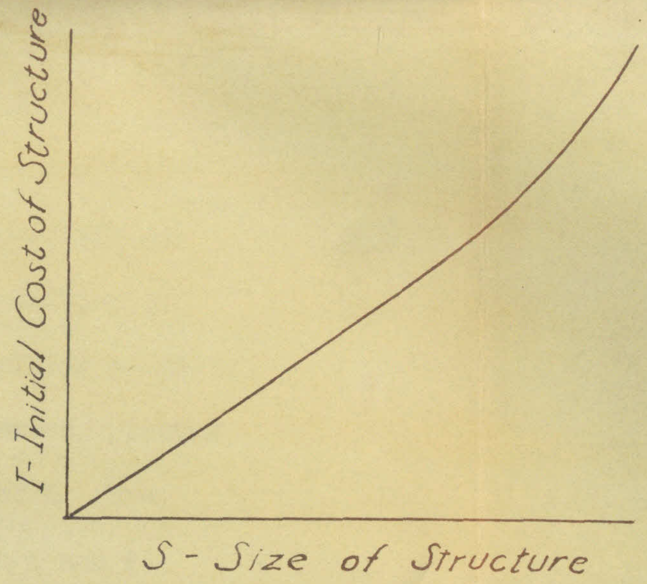
(2) has a minimum point for some value of n.

It would be possible, if the mathematical expressions for all the factors were determined, to obtain by differentiation a minimum value for C_a. The resulting expression, however, would be so complex, and the derivation of the other relationships mathematically so tedious that a graphical or tabular method, illustrated by Figure 8, would accomplish the same result with less effort.

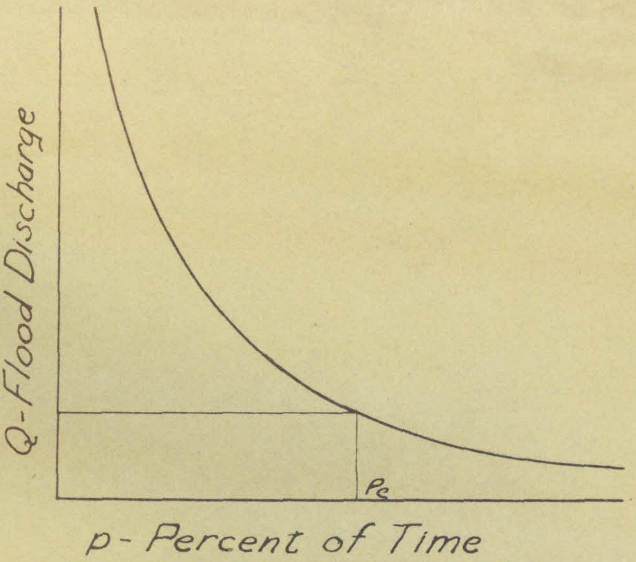
Figure 8



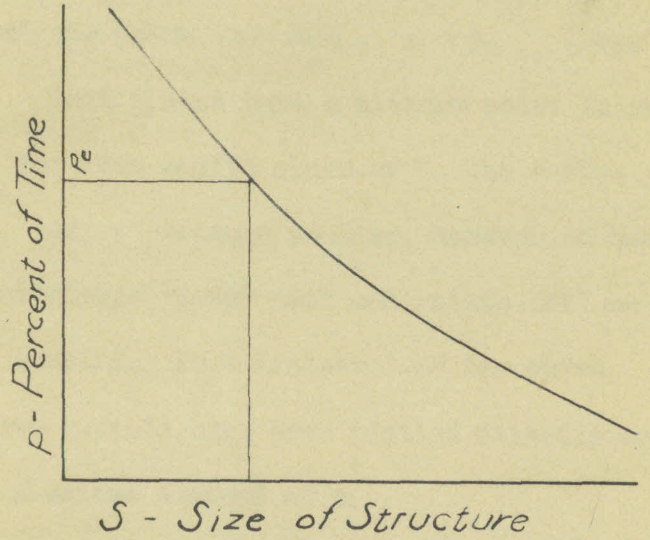
(a) Source - Rating Curve - etc.



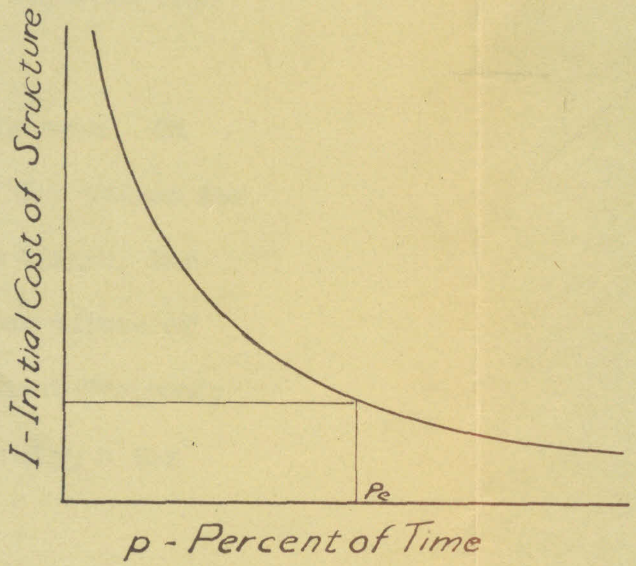
(b) Source - Cost Records



(c) - Source - Frequency Curve



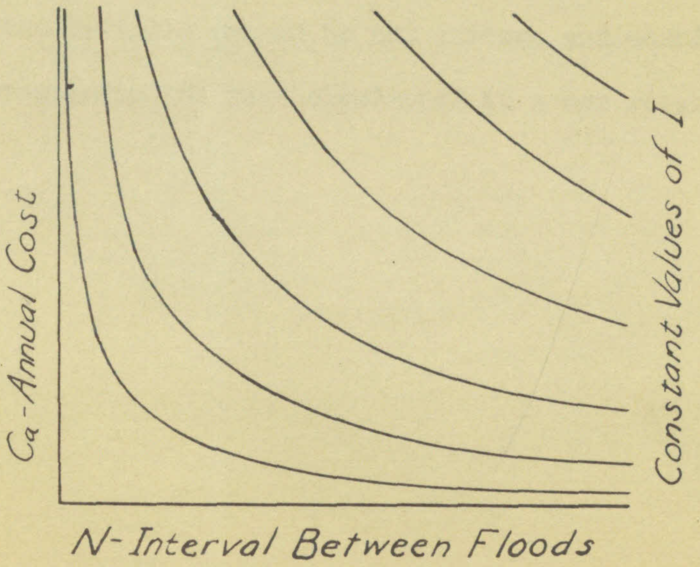
(d) - Source - (a) and (c)



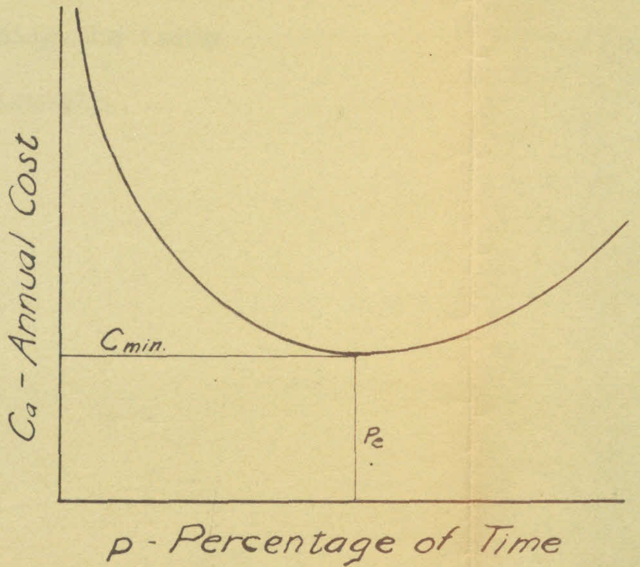
(e) Source - (b) and (d)

GRAPHICAL SOLUTION

Economical Period of Design



(f) Computed



(g) - Source - (e) and (f)

Curves a, b, and c are all constructed from available data, c being the actual flood frequency curve already determined. Between a and c, Q is eliminated, resulting in curve d. Combining b and d, e is obtained involving p and I, the initial cost. In f is a series of cost curves, plotted from equation (2) for various constant values of I. The cost-frequency curve g is then constructed from e and f. Choosing various values of p, these values are laid off on the abscissa in g and the ordinates for C_a are selected from f at the point $n = \frac{100}{p}$, $I = I_p$. (From e).

That g must have a minimum point is readily seen. On f for even small values of I, the curves give high values for C_a. As p becomes smaller, however, n becomes larger, the cost curves "level out" and points fall on higher values of I, resulting in a "raising" of the curve. The cost-frequency curve g could have been plotted directly on f, using n for an abscissa instead of p.

The process can be performed either graphically or in a tabular form; in either case the operation involves the removal of the variable common to two curves, and combining the terms corresponding to that eliminated in a new relationship.

E. SUMMARY

The results of this paper may be summarized as follows: The San Gabriel River has a fairly well-defined flood-frequency relationship. It plots equally well and is equally usable on either logarithmic or probability paper. The use of Hazen's coefficients is uncertain on such streams, and should not be relied upon without some form of check. It is impossible to forecast with any certainty the size or time of occurrance of any particular flood event. The results obtained through flood frequency methods are expressions of the average expectancy of floods, and if applied generally will accomplish the desired purpose, but can not be relied upon to hold for any individual design. It is possible to determine the economical percent chance flood for which to design, but this procedure also is worth while only when applied over a sufficient number of investigations to allow the law of averages to compensate for any individual errors.

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