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 $\mathbf{B} \mathbf{y}$ 

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Candidate for the degree cf Master of Science **in**  Civil Engineering Presented to Adviser for approval  $\frac{1}{2}$  =  $\frac{1}{2}$  =  $\frac{1935}{2}$ .

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Presented to Dean of the Graduate School for  $approx1$  \_ \_ \_ \_ \_ \_ 1935.

 $\Delta$ pproved by \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_

 $\mathcal{L}_{\rm{eff}}$ 

#### LETTER OF TRANSMITTAL

California Institute of Technology

Pasadena, California

May 18, 1935

Dr. Richard c. Tolman

Dean of the Graduate School

California Institute of Technology

Pasadena, California

Dear Sir:

In accordance with the regulations of the Department of Civil Engineering of California Institute of Technology, I hereby submit this thesis as a partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

The title of this thesis is Economical Design of Building Frames.

Respectfully submitted,

(Candidate for the Degree) of M. S. in C. E.

T *A* B L E O F C O N T E N T S

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 $\frac{1}{2} \frac{d}{dt}$ 



#### ECONOMICAL DESIGN OF BUILDING FRAMES

#### CHAPTER I

#### INTRODUCTION

As a result of accurate experimental and research works carried on to determine the actual conditions of stress distribution relative to its assumptions, engineers are now abled to know within a fairly reasonable margin of this actual stress which will exist in the finished structure. Since such variations from the ideal conditions are so closely approximated, this exactness of results has paved the agitation in favor of increasing the allowable working unit stresses used in designs in order to develop as economical a structure as possible and with sufficient degree of safety.

On the other hand, many experiments give results of considerable difference between these two stresses; therefore, it must be borne in mind that conditions do vary and not necessarily according to any specific law. Even the most intricate is not generally the most precise solution. The complexity of the analysis involved the consideration of all possible factors that will exert any influence upon the finished structure, which will necessarily provide reasonable assurance to the safety of the design when the laws of nature act accordingly and exclude factors beyond human comprehension. The main consideration in any structural analysis is to have the laws of statics being fulfilled and to be sure there are no forces being overlooked. Having all these considerations fulfilled to satisfaction, one may conclude that the most

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economical method of analysis will no doubt be one for which the least amount of labor and material are expended. At times one will be forced to sacrifice precision for time, not exactly at the expense of safety, but for structures of less importance which do not justify too enormous an amount of time and labor **to**  be spent.

In the past years, for buiidings of moderate height, the analysis of wind stresses is not of great importance in many cases. However, one cannot now deny that such stresses will deserve a careful and thorough analysis, because of the growing tendency toward building of sky-scrapers, especially in populous cities where land values are enormously high, there exists a greater desire to tax a greater return from a unit area of land in order to justify its utilization. The question naturally arises as to which method of solution to apply in order to obtain reliable results and still permits greatest economy of labor and materials. Among the various methods that may serve this purpose, the approximate solutions find greater application than those "exact" ones. mainly because of the increased labor of computation by these latter methods. Although these exact solutions sometimes yield resulting stresses representative of the ideal conditions, it should be remembered that such closeness of agreement between the assumed and actual stresses rest greatly upon the assumptions of the wind load rather than wholly on the differential of "exactness" and "approximation".

Experiments have shown that wind velocities vary with the altitude; furthermore, it shows that with a given wind velocity, the

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average pressure is less on a large surface than on a small surface. This information, however, is not sufficient to enable the engineer to know just what approximate pressure to provide for on buildings of different height, mass, and exposure. Consequently, the word "exact" solution will be exact to the extent that the above conditions are correctly provided for.

With these various methods for wind stress analysis, it is the writer's intention to show by examples that the approximate methods will permit a more economical design of building frames than the slope deflection method which is the exact solution under consideration in all these discussions. Additional examples will offer a comparison of the relative economy of the cantilever and Porter methods. Included in this discussion of economy in design will be an example of the design of continuous haunch beams by the exact and the moment distribution method.

#### CHAPTER II

#### SLOPE DEFLECTION METHOD

#### SYMMETRICAL BENT WITH BAYS OF UNEQUAL WIDTH

The exact method considered here is the slope deflection method which takes into account the relative stiffnesses of the columns and girders and that the point of contraflexure of each column and girder is not fixed at the midpoint which in this respect differs greatly from the assumption made by the approximate methods. The assumptions upon which the theory of slope deflection is based are as follows:

1. The connections between the columns and girders are perfectly rigid.

2. The change in length of a member, due to direct stress in it, is negligible.

3. The length of the members are the distance between the intersections of their neutral axis.

4. The deflection of a member, due to internal shear is negligible.

5. The wind load is resisted entirely by the steel frame.

In order to show the importance of the distribution of moments at any joint in direct proportions to their stiffnesses, the writer has chosen the symmetrical three bay tenstory bent of unequal width as shown in figure 1. Since the cantilever and portal methods are approximate solutions and in order to exaggerate the difference of results, no attempt is here made to readjust the column and girder moments by

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considering its relative stiffnesses. The cross-sectional areas of the columns are taken to be equal which in this particular case is very nearly so. The assumptions upon which the cantilever metnod is based are as follows:

1. A bent of a frame acts as a cantilever.

2. The point of contraflexure of each column is at its mid-length.

3. The point of contraflexure of each girder is at its mid-length.

4. The direct stress is directly proportional to the distance from the column to the neutral axis of the bent.

5. The wind load is resisted entirely by the steel frame.

The assumptions on which the portal method is based are as follows:

1. A bent of a frame acts as a series of independent :portals.

2. The point of contra-flexure of each column is at its mid-height of the story.

3. The point of contra-flexure of each girder is at its mid-length.

4. The horizontal shear on any plane is divided equally between the number of aisles. *An* outer column thus takes but one-half of the shears of an interior column.

5. The wind load is resisted entirely by the steel frame.

The bent of figure 1 was first analysis for moments and shears in columns and girders by the slope deflection method

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and as a comparison of the economy as resulted from the approximate solutions, the same bent was analyzed by the Cantilever method. The method of stress analysis by the slope deflection is outlined here in general as follows:

Consider a strip of the bent one foot wide in a direction perpendicular to the plane of fig. l; the loads are due to a horizontal wind pressure of 30 lb. per sq. ft. The columns are assumed to be fixed at the base. Arbitrary values of  $K$  for the columns and girders are shown. The change in slope of the members meeting at the joint is represented by  $\Theta$  and the deflection angles by  $\mathcal{L}_{\bullet}$ . From the condition of symmetry the following relation holds:



The fundamental equations used to determine moments in girders being  $M = 2EK(2\Theta_0 + \Theta_f)$ 

and for columns,  $M = 2EK(2\Theta_r + \Theta_f - 3\mathcal{L})$ 

Where  $M =$  moment in inch pounds  $E$  = modulus of elasticity  $K =$  stiffness factor =  $I/L$  $\theta_n$  = twist angle near joint  $\Theta_f$  = twist angle at opposite joint  $\alpha$  = deflection angle

By application of the above formulas, the joint equations are as follows:

Joint No. AlO:

 $2(12.5+22.21)\theta_{40}$  +12.50so +22.210se -3X22.21d<sub>1</sub>= 0

 $69.42\theta$ az +12.5 $\theta$ sio +22.21 $\theta$ as -66.63 $\alpha_{0} = 0$ 

Joint No. BlO:

 $2(12.5+11.11+22.21)\theta$ sir +12.50m +11.110sir +22.210ss -3X22.21d<sub>is</sub> 0

 $102.75\theta$ en +12.5 $\theta$ an +22.21 $\theta$ es -66.63 $\epsilon$  = 0

The complete equations for the other joints are found in table 1.

In addition to the joint equations, the additional independent equations for the determination of the d values are obtained by considering the equilibrium of a portion of the frame formed by cutting sections across the frame just below the top and just above the bottoms of all columns in any story. The equations thus obtained are:

For the tenth story:

 $22.21(\Theta_{40}$  + $\Theta_{49}$  + $\Theta_{80}$  + $\Theta_{89}$  ) - $4X22.21\leq 21\leq 330X14X12/12E$ 

 $133.26(\Theta_{4/6} + \Theta_{49} + \Theta_{40} + \Theta_{69} + \Theta_{69}) - 535.04\frac{1}{200} = -55.440/2E$ For the ninth story:

 $22.21(\theta_{49} + \theta_{48} + \theta_{89} + \theta_{88} + 0) - 4X22.21 = -750X168/12E$ 

 $135.26(\theta_{49} + \theta_{48} + \theta_{69} + \theta_{68}) - 535.04 = -126,000/2E$ The complete equation for the other stories are given in Table 1.

The solution of the equation are shown in Table 2. The desired stresses are found directly after these unknown quantities are solved. The moments in the columns and girders are obtained by the two formulas given above. The shears in columns and girders are equal to the sum of the end moments divided by its length. The resulting values are shown in Table 4 and 5.

Following are the numerical values of the change in slopes and deflection angles found from the elimination of the 30 equations:

$$
73-74
$$
 -1.4281  $\theta_{A/O} = 308.549$   

$$
\theta_{A/O} = 216.05
$$

73)  $\theta_{\text{Bio}} - 7.26 = 152.00$ 

$$
\theta_{\text{Bve}} = 159.26
$$

70) 
$$
\mathcal{L}_{\rho} = 0.2995(159.26) \times 0.2775(216.05) \times 308.340
$$

$$
= 47.693 \times 59.954 \times 308.340 = 415.99
$$

67) 
$$
\theta_{0.9} = 0.7233(415.99) - .0051(159.26) - 0.2352(216.05) - 233.867
$$
  
= 300.893-0.812-50.827+233.867 = 483.12

65) 
$$
\theta_{\infty} = 0.0210(483.12) -0.1749(159.26) +0.5245(415.99) +189.112
$$
  
= 10.145-27.855+218.192+189.112 = 389.59

62) 
$$
\omega_{\mathbf{a}} = 0.3021(389.59) + 0.2800(483.12) + 551.699
$$

 $=$  117.695 $\star$ 135.276 $\star$ 551.699 = 804.66

$$
\Theta_{AB} = 0.7079(804.66) - .0052(389.59) - 0.2349(483.12) + 297.254
$$
  
= 569.700-2.026-113.485+297.254 = 751.44

$$
67) \qquad \theta_{\theta\theta} = 0.0218(751.44) + 0.5247(804.66) - 0.1749(389.59) + 247.742
$$

$$
= 16.382 + 422.205 - 68.141 + 247.742 - 618.19
$$

54)  $\mathcal{L}_8 = 0.2989(618.19) + 0.2775(751.44) + 709.320$ 

$$
= 184.777 \times 208.525 \times 709.320 = 1102.6
$$

$$
\Theta_{AT} = 0.7195(1102.6) - 0.0059(618.19) - 0.2343(751.44) + 383.275
$$
  
= 793.321-3.65-176.062+383.275 = 996.88

49) 
$$
\theta_{67} = 0.0220(996.88) \neq 0.5249(1102.6) - 1749(618.19) \neq 319.785
$$
  
= 21.932+578.351-108.198 = 811.87

46) 
$$
\alpha_7 = 0.296(811.87) + 0.275(996.88) + 900.00
$$

 $= 240.31 + 274.16 + 900.00 = 1414.5$ 

45) 
$$
\theta_{46} = 0.7190(1414.5) - 0.0052(811.87) - 0.2343(996.88) + 45945
$$
  
= 1017.025-4.222-233.58' +459.45 = 1238.7

41) 
$$
\Theta_{96} = .0224(1238.7) - 0.1745(811.87) + 0.5236(1414.5) + 381.033
$$

$$
= 27.747 - 141.671 + 740.632 + 381.033 = 1007.7
$$

$$
38) \qquad \alpha_6 = 0.2941(1007.7) + 0.2735(1238.7) + 1042.18
$$

$$
= 296.365 \times 338.784 \times 1042.18 = 1677.3
$$

$$
\theta_{\text{as}} = 0.6663(1677.3) - 0.0042(1007.7) - 0.2177(1238.7) \times 520.565
$$
  
= 1117.585-4.232-269.665 \times 520.585 = 1364.3

$$
\Theta_{\beta s} = 0.0185(1364.3) - 0.4769(1677.3) - 0.1590(1007.7) + 424.345
$$
  
= 25.240-799.904-160.224+424.345 = 1089.3

$$
\mathcal{L}_{\zeta} = 0.2983(1089.3) \star 0.2764(1364.3) \star 1158.97
$$
  
= 324.938 \star 377.093 \star 1158.97 = 1861.0

$$
= 324.938 \times 377.093 \times 1158.97 = 1861.0
$$

27) 
$$
\theta_{44} = 0.7104(1861.0) - 0.0039(1089.3) - 0.2328(1364.3) + 575.517
$$
  
= 1322.125-4.248-317.609+575.517 = 1575.8

$$
\theta_{\mu} = 0.0170(1575.8) \star 0.5168(1861.0) - 0.1722(1089.3) \star 481.522
$$
  
= 26.789 \star 961.765 - 187.577 \star 481.322 = 1282.3

22) 
$$
\mathcal{L}_4 = 0.2992(1282.3) + 0.2760(1575.8) + 1370.71 = 2189.3
$$

19) 
$$
\theta_{A3} = 0.0009(1282.3) + 0.7173(2189.3) - 0.2395(1575.8) + 704.945
$$
  
= 1.152+1570.385-377.404+704.945 = 1899.07

17) 
$$
\theta_{63} = -0.0030(1899.1) -0.1758(1282.3) \neq 0.5274(2189.3) \neq 600.043
$$
  
=  $-5.697 - 225.428 \neq 1154.637 \neq 600.043 = 1523.6$ 

14) 
$$
\alpha_3 = 0.293(1523.6) + 0.2708(1899.1) + 1954.13
$$

$$
= 446.414 \div 514.268 \div 1954.13 = 2914.8
$$

11) 
$$
\theta_{A2} = 0.6493(2914.8) + 0.0021(1523.6) - 0.2185(1899.1) + 967.970
$$

$$
= 1892.580 + 3.200 - 414.947 + 967.970 = 2448.8
$$

9) 
$$
\theta_{\theta2} = -0.0095(2448.8) + 0.463(2914.8) - 0.1543(1523.7) + 812.965
$$
  
= -23.264+1349.552-235.107+812.965 = 1904.1

 $\label{eq:12} \frac{1}{\lambda} \frac{1}{\lambda} \frac{1}{\lambda}$ 

6) 
$$
\mathcal{L}_{2} = 0.2683(2477.6) + 0.2910(1904.1) + 2384.92
$$

$$
= 664.740 + 554.093 + 2384.92 = 3603.7
$$

 $\theta_{a}$  = 0.6228(3603.7)-0.212(2448.8)+0.0040(1903.9)+1060.89  $3)$  $=$  2244.74-519.14+7.61+1060.89 = 2794.1

1) 
$$
\Theta_{6} = -0.0194(2794.1) + 0.4369(3603.7) - 0.1459(1904.1) + 744.109
$$

$$
= -54.205 + 1574.456 - 277.808 + 744.109 = 1986.5
$$

A) 
$$
\alpha_1 = +0.2498(1986.5) +0.2498(2794.1) +1909.827
$$
  
= 496.228-697.966-1909.827 = 3104.0

Following are the calculations for the moments in column A by direct substitution into column equations:

M10-9 = 22.21 2X216.05+483.12-3X415.99

 $= 22.21(-1248.0+915.2) = -7390$ 

 $M9-10 = 22.21(2X483.12 \div 216.05 - 3X415.99$ 

 $= 22.21(-1248.0 \times 1182.3) = -1460$ 

 $M9-8 = 22.21(2X483.12+751.44-3X804.66)$ 

 $=$  22.21(-2414.0+1727.6) = -15.240

 $M8-9 = 22.21(2X751.44+483.12-3X804.66)$ 

 $= 22.21(-2414.0+1986.0) = -9.530$ 

- $M8-7 = 22.21(2X751.44+996.9-3X1102.6)$ 
	- $= 22.21(-3307.8+2499.8) = -17,500$
- $M7-8 = 22.21(2X996.93+751.95-3X1102.83)$

 $= 22.21(-3308.49+2745.81) = -12,500$ 

 $M7-6 - 22.21(2X996.93+1238.7-3X1414.5)$ 

 $= 22.21(-4243.5+3232.56) = -1010.9X22.21 = -22.500$ 

 $M6-5 = 22.21(2X1238.7+1364.3-3X1677.3)$ 

 $= 22.21(-503.19+3841.7) = -26.420$ 

 $(10)$ 



 $(11)$ 

 $\tilde{\mathbf{z}}$ 

 $\frac{3}{J}$ 

#### Moments in the B-Columns



 $= 25.0(-6567.9+4088.3) = -61,990$ 

( 12)

 $M3-2 = 21.42(2X1523.7+1904.1-3X2914.8)$  $= 21.42(-8744.4+4951.5) = -81.250$  $M5-4 = 25.0(2X1523.7+1282.5-3X2189.5)$  $= 25.0(-6567.9+4329.7) = -55.960$  $M2-3 = 21.42(2X1904.1+1523.7-3X2914.8)$  $= 21.42(-8744.4+5331.9) = -73,200$  $M2-1 = 20.82(2X1904.1+1986.5-3X3603.7)$  $= 20.82(-10811.1+5794.7) = -104,300$  $M1-2 = 20.82(2X1986.5+1904.1-3X3603.7)$  $= 20.82(-10811.1+5877.1) = -103,000$  $M1-0 = 18.52(2X1986.5-3X3104.0)$  $= 18.52(-9312.0+3973.0) = -99,000$  $MO-1 = 18.52(1986.5 - 3X3104.0)$  $= 18.52(-9312.0+1986.5) = -136,000$ 

#### Moments in the Girders  $(Bay AB)$





 $\overline{\mathcal{A}}$ 

Moments in Girders BB'



 $\Bigg)$ 

# SLOPE DEFLECTION METHOD ANALYSIS OF WIND STRESSES SYMMETRICAL TEN-STORY THREE BAY BENT

WIND LOAD DUE TO PRESSURE OF 30 LBS. PER SQ. FT. ACTING ON A VERTICAL STRIP I FT. WIDE



 $16.0'' - 18.0'' - 16.0''$ 

 $(r_{ig.}1)$  $(15)$ 



 $\vec{5}$ 

TABLE NO !



SLOPE DEFLECTION METHOD GENERAL EQUATIONS

TABLE / (CONTINUED)



EQUATIONS FOR ELIMINATION OF UNKNOWNS

TABLE NO. 2.





### SLOPE DEFLECTION METHOD EQUATIONS FOR ELIMINATION OF UNKNOWNS TABLE NO. 2 (CONTINUED)





EQUATIONS FOR ELIMINATION OF UNKNOWNS

TABLE 2 (CONTINUED)





EQUATIONS FOR ELIMINATION OF UNKNOWNS

TABLE 2 (CONTINUED)





EQUATIONS FOR ELIMINATION OF UNKNOWNS TABLE 2 (CONTINUED)





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EQUATIONS FOR ELIMINATION OF UNKNOWNS TABLE 2 (CONTINUED)





Ι

EQUATIONS FOR ELIMINATION OF UNKNOWNS TABLE 2 (CONTINUED)





EQUATIONS FOR ELIMINATION OF UNKNOWNS TABLE 2 (CONTINUED)



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VALUES OF CHANGE IN SLOPE AND DEFLECTION ANGLES TABLE NO. 3.



MOMENTS IN COLUMNS AND GIRDERS

TABLE NO. 4.



## VALUE OF SHEAR AND DIRECT STRESSES



TABLE NO. 5.

#### CHAPTER III

#### CANTILEVER METHOD

#### SYMMETRICAL BENT WITH BAYS OF UNEQUAL WIDTH

In analyzing the ten-story bent shown in fig. 3, the writer has found it very feasible to determine the constants\* for horizontal shear, direct stress, and moments in the columns by the use of fig. 2. The method of doing this is to apply a unit load at joint A and take moment at a distance 1 foot below  $\mathbb{A}$  . Thus this is equilvalent to assuming the point of contraflexure to be 1 ft. below A and the total shear in that story is then 1 pound. The solution of the bent is outlined in general as follows:

Assuming area of columns equal, the moment of inertia of the bent =  $(1(y)^{2} + 1(25)^{2}) = 1412$  ft.



 $(fig. 2)$ 

#### (\*See Schneider's Practical Wind Bracing)  $(28)$

Direct stress in columns A and  $A' = 25X1 = 0.01768$  lb. Direct stress in columns B and B' =  $\frac{1412}{1412}$  = 0.00636 Vertical shear in  $AB = 0.01768$  lb<sub>p</sub>.  $t$  in BB' -  $(0.0176840.00636)$  - 0.02404 lb.  $\text{in } B'A' = (01768 - 00000) = 0.01768 \text{ lb.}$ Moment in AB and  $A' B' = 0.01768X8 = 0.14144$  ft. lb.  $BB' = 0.02404X9 = 0.21636$  ft. lb. columns A and  $\mathbf{A}^{\prime} = 0.14144$  ft. lb.  $I = I = I$   $I = B$  and  $B' = 0.35780$  ft. 1b. The direct stress in the roof beams are:  $AB = (1-0.14144) = -0.85856$  lb.  $AB' = (0.85856 - 0.35780) = +0.50076$  lb.

 $B'A' = (0.50076 - 0.35780) = +0.14296$  lb.

With these constants found, the stresses in the columns and girders are readily solved. Thus the shear in column A and A' of the tenth story is  $0.14144X330 = 47$  lbs., for B and B' is  $0.35780X$  $330$   $\pm$  118 lbs. The moment in column *A* and *A*' is then  $\pm$  7X47  $\pm$ 329 ft. lbs., and for columns B and  $B' = 7X118 = 826$  ft. lbs. The direct stress in columns  $A$  and  $A' = 0.01768X2310 = 41$  lbs., and that for B and B' =  $0.00636X2310$  = 15 lbs. The vertical shear in girders AB and  $A^{\dagger}B^{\dagger}$  = the direct stress of column A and A', while that for BB' = vertical shear of AB plus the direct stress of column B which is  $41-15 = 56$  lbs. Similarly the other stresses are obtained.

Inspection of the resulting direct column stresses will immediately reveal the economy of this method. It will be noted that the column moments are quite less than that obtained by the

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slope deflection method, thereby resulting a saving of material in regard to bracing and connections. This difference in the column moment and direct stresses are due wholly to the difference in assumption of the point of contraflexure at the columns and girders by these two methods. The resulting stresses will have closer agreement had the stiffnesses of the columns and girders as used in the slope deflection been taken equal.

# CANTILEVER METHOD SYMMETRICAL TEN-STORY BENT



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#### CHAPTER IV

#### CANTILEVER METHOD

#### SYMMETRICAL BENT WITH UPPER STORIES SET BACK

The bent is shown in fig. 6, with the upper five stories of bent set back. The analysis for wind stresses for these stories were made similiar to that mentioned in the preceding chapter. Since the same wind pressure as well as the same dimension were used, the constant of fig. 2 thus applies here. The stresses for these upper five stories are taken directly from fig. 2.

Now by the use of fig. 4 below, the direct stress at the sixth floor line will be calculated and carry down the columns immediately below as reactions.



The vertical shear in the 6th. floor beams are for AB, the direct stress in column A at the 6th. floor line, minus the direct stress in the same column at the 6th. story, or  $(1223-1025)$ = 198. For beam BB' it is equal to the algebraic sum of the direct stresses in the same columns at the 6th. story, or (1223+ 4391)-(1025-378} = 259, and that for *A'B'* is the same as AB

The moment in the 6th. floor beams are  $198X8 = 1584$  ft. lbs. for beams AB and  $A'B'$ ; and equal to  $259X9 = 2331$  ft. lbs. for beam BB'. These are the moments necessary to balance the moments in the 6th. story column.

The constant factors are next determine for the bent below the 6th. floor as shown in fig. 5 below.



Assume a horizontal load of one pound applied at the 6th. floor line of figure above and assume point of inflection of the columns to be one foot below the 6th. f'loor line. The total wind load is then one pound and a unit overturning moment in foot-pound.

The center of gravity coincides with the center line as the bent is symmetrical. Assume the cross-sectional areas of columns to be all equal. Then  $I = (1(9)^{z} + 1(25)^{z} + 1(41)^{z}Z = 4774 \text{ ft}$ .

( 33 )

Direct stress in columns *A* and  $A' = \frac{1X25}{4774} = .00523$  lb. **r r r r B** and **B'** =  $\frac{1X9}{4774}$  = 0.00188 lb. **<sup>n</sup>**" **ff n.** C and C' = 1X41 = .00858 lb. 4774 Vertical shear in *CA* = 0.00858 lb. **fl tt tt** AB -( .00858+.00523) = 0.01381  $BB'' = (.00858+.00523-.00188) = 0.01569$ **tt it**  $B'A' = (.0085 \times .00523 \times .00188 - .00188) = 0.01381$  $r \t n \t n \t A' C' = 0.00858$ Moment in CA and  $C'A' = 8X0.00858 = 0.06864$  ft. 1b.  $^{\circ}$   $^{\circ}$  **AB** and  $B'A' = 8X.01374 = 0.11048$  $^{\text{r}}$   $^{\text{r}}$   $^{\text{B}}$   $^{\text{B}}$   $\approx$   $9\text{X.01569}$   $\approx$  0.14121 Moments in columns C and C' =  $0.06864$ **ti ti ti A** and  $A' = (0.06864+.11048) = 0.17912$  $\mathbb{I}$  **IT**  $\mathbb{I}$  **B** and  $B' = (.11048+.14121) = 0.25169$ Horizontal shear in columns **C** and C' = 0.06864 lb.  $\mathbf{r}$  **ii**  $\mathbf{r}$  **A**  $\mathbf{r}$  **A** Ħ  $^{\circ}$   $^{\circ}$   $^{\circ}$   $^{\circ}$   $^{\circ}$   $^{\circ}$  B and B' = 0.25169 Direct stresses in beam:  $AC = (1.0000 - .06864) = 0.93136$  $AB = (.93136-.17912) = 0.75224$  $BB' = (.75224-.25169) = 0.50055$  $B'A' = (0.50055 - 0.25169) = 0.24886$  $A'C' = (0.24886 - 0.17912) = 0.06974$ The resulting stresses in the columns and girders of the bent are shown in fig. 6. The direct column stress in the first

floor are for C and  $C' = \pm 1625$  lbs., for *A* and  $A' = \pm 2214$  lbs., and for B and  $B' = \pm 796$  lbs.

(34)

### CANTILEVER METHOD WIND STRESS ANALYSIS SYMMETRICAL TEN-STORY SET BACK BENT



#### CHAPTER V

#### PORTAL METHOD

#### SYMMETRICAL BENT WITH UPPER STORIES SET BACK

For symmetrical bent with equal bays, the Portal method possesses a unique feature in that the interior columns of the bent will not be subjected to direct stress. Since the bent under consideration are of unequal width, the moment in the joints will not balance. Therefore, in order to overcome this complication, use is made of the solution as proposed by Professcr Smith. The scheme utilized is outline here in general as follows:

The four columns with their girders which form the bent of the upper five stories are assumed as three separate portals with a load of  $1/3$  pound applied at each joint. The horizontal shear in each column of each portal is equal to one-sixth of  $a$ . pound. The moment in each column a unit distance below the joint is one foot pound and is shovm in fig. 7 below.



 $(fig. 7)$ 

The direct stress in columns  $\lambda$  and B of the first portal and the columns  $A'$  and  $B'$  of the third portal =  $1X0.3333$  or **16** 

 $0.0208$  lbs. The direct stress in columns B and B' of the second portal  $-1X0.3333$  or 0.0185 lbs.

The vertical shear in the girders of each portal equals the direct stress in its respective columns which are 0.0208. 0.0185 and 0.0208 lbs. for AB, BB' and B'A' respectively.

The moment for girders are  $8x.0208 = 0.1666$  ft. lb. for AB; 9X.0185 = 0.1666, for girder BB'; and 8X0.0208 = 0.1666 for  $B^{\dagger}A^{\dagger}$ .

These stresses are then combined algebrically to be used as the constants for the upper five stories of the bent. The resulting stresses thus combined are shown in fig. 8 below:



The direct stress for columns  $A$  and  $A'$  remained the same; but for column  $B' = -0.0208 \div 0.0185 = -0.0023$  lb. and for column  $B = +0.0208-0.0185 = +0.0023$  lb.

The direct stress for  $AB = 1.00-0.1667 = 0.8333$  lb.; for BB' = 1.00-0.1667-0.3333 = 5.000 lb.; and for  $B^{\dagger}A^{\dagger} = 1.00$ -0.1667  $-0.3333-0.3333 = 0.1666$  lb.

The moments and vertical shears for the beams remained the same after combining. The moment for columns B and B' =  $0.1667 \star$ 0.1666 - 0.3333 and for columns A and A' it is the same as before.

 $(57)$ 

Having these constants, the bent is mechanically solved and the resulting stresses are shown in fig. 11. Starting with the fifth story, a new set of constants are obtained by means of fig. 9.

The proceedure in obtaining these constants is precisely that outlined above. Referring to fig. 9, the six columns with their girders are considered as five portals; therefore, the wind load at each portal is 0.2 lb. applied at the joint. The shear in the columns of all portals are 0.10 lb.



The direct stress in the columns of all portals excepting the center one is  $0.20 = 0.0125$  lb. The direct stress for column B and B' =  $0.20 = 0.0111$  lb.

The vertical shear in each portal equals its respective column direct stress which is 0.0111 1b. for BB' and 0.0125 1b. for the other girders.

These portals stresses are then combined by adding algebraically the results shown in the above figure from which the constants are obtained for fig. 10 below:



 $(fig. 10)$ 

This algebriac summation is carried out exactly as before mentioned. Then before proceeding to apply these constants to the bent, it is necessary to calculate the overturning moment at the floor of the sixth story from which the direct stress at the floor line may be carried down the columns of the floor immediately below as reaction. This is explained in detail in the cantilever method and will not be repeated here.

The final stresses are shown in fig. 11; the direct stress at the first floor columns are for C and C'  $=$   $\neq$  2366 lbs., for A and A' is £1438 lbs., and for B and B' it is £424 lbs.

### PORTAL METHOD WIND STRESS ANALYSIS SYMMETRICAL TEN-STORY SET BACK BENT



 $(40)$ 

#### MODIFIED PORTAL METHOD

In order to retain one of the features of the Portal method, the writer has slightly modify **the** assumption to arrive at this desired result. The assumption as modified here being to assume the shear as distributed in proportion to the unit volume of each bay in the bent and not equally among the bays irrespectively of its width. The basis for this assumption may be erratic in character, nevertheless, the stresses obtained will not differ greatly from that of the Portal method. In the design, the wind load is assumed to act on the total surface of the building in the windward side, but it must be remembered that the building is not a surface, and that the wind could blow in any direction; thus under these circumstances, the above assumption is justified.

The procedure of calculation of the stresses is exactly as outlined for the Cantilever and Portal methods. The only step which need to be mentioned here will be the distribution of the columns shears. Thus for columns A and A' the shear is  $1/2$  X 16/50 X 330 = 53 lbs. and for columns B and B' the shear is  $1/2(18/50 + 16/50)330 = 112$  lbs. from which the respective moments are 7X53 or 371 ft. lbs. and 7Xll2 or 784 ft. lbs. Proceeding from here, the shearing and compressive stress for the girders are obtained as mentioned in the previous chapters.

The resulting stresses are shown in fig. 11-A. For this particular bent, there is hardly any variation in the results

(41)

obtained, but for bents with outer bayers whose width are proportionately greater than the center bay, the moments in the outer columns will be greater than that got by the original portal method and vice versa.

### PORTAL METHOD (MODIFIED) SYMMETRICAL TEN-STORY SET BACK BENT



### CHAPTER VI CONTINUOUS HAUNCH BEAMS

Haunched beams have recently come into extensive use because it offers two essential features not characterized by ordinary straight beams; namely, the artistic appearance it presents to the structure and the relative economy of material. This reduction of material at the center reduces the moment due to dead load of the girder, and in effect to decrease the moment at the centec while increasing the negative moments at the supports. Since such beams occur in building frames very frequently, the writer in this connection have chosen such a continuous beam over four supports to illustrate the possible economy in material as required by the calculations of the slope deflection method and that of the moment distribution method. The columns which suppose to form the frame work are purposely omitted and substitute for it with simple supports.

The general slope deflection equations used for the calculation of the moments are: (1) for uniform loadings:

> M, = AE <u>I</u>, (BO,  $\neq$ CO<sub>2</sub> -3.c) +CF  $M_z = AE \frac{I_z}{I_z} (B\theta_z + C\theta_r - 3\epsilon) - CF$  $\overline{\ell}$

and (2) for concentrated loads, the term F in the above equation is replaced by  $\sum D_r P_\ell$  and  $\sum D_z P_\ell$ , where D, and D<sub>z</sub>are coefficients depending upon the position of the concentrated load P. A, B, and Care coefficients depending upon the proportions of the haunch and F is the fix end moment due to uniform load and  $=$ 1 W . See fig. 14, for the dimension and loading of this beam. 12





 $D = D2$  $M_{AB} = A_1 E K_1 (C_1 \theta_B) + C_1 F$  $F = \frac{1}{12} W/L^2$ = 3.66 x 0.0185 x1.47 E OB +1.147 x 43.2 = 0.0678 ×1.147E  $\theta$ 8 + 49.6  $MBA = A_1 E K_1 (B_1 \theta B) - G F$  $= 0.0678 \times 1.81908 - 49.6$  $M_{BC}$  =  $AzEKz(Bz\theta_{B}+Cz\theta_{C})+CzF$ = 3.04x.033E (1.895 88 + 1.105 Oc) + 1.105 x50 = 0.102 E (1.895  $\theta$ 8) +1.105  $\theta$ c) + 55.25  $MCB = AzEK_2(B_2B_8 + G_2B_6) - C_2F$ =  $0.102E(A.895\theta c+1.105\theta R)$  $M_{CD} = A_3 E K_3 \Gamma(B_3 + C_2)(B_3 - C_3) \theta_6 / B_3 + C_3 (H - C_3/ B_3)$ = 3.66X 0.0416 E [1.852 +1.148) (1.852-1.146) Oc |1.852] + 1.148x1.618x0.147x128  $= 0.152E(1.098 \theta_{c})+35.2$  $M_{DC} = 0$ 

Joint Equations for the solution of Os.  $B)$  + 0.1234  $\theta_8$  + 49.60 +0.193  $\theta_8$  + 0.1127  $\theta_6$  + 55.25 = 0  $0.3164$   $\theta_8$  +  $0.1127$   $\theta_6$  + 5555=0  $C$  + 0.1127 $\theta_8$  + 0.36  $\theta_6$  - 20.05 = 0

Solving

 $\begin{cases} 0.3164 \text{ } \theta_8 + 0.1127 \text{ } \theta_6 + 5.55 = 0 \\ 0.1127 \text{ } \theta_8 + 0.360 \text{ } \theta_6 - 20.05 = 0 \end{cases}$ 

$$
0.0127 \theta c + 0.626 = 0
$$
\n
$$
0.1140 \theta c - 6.360 = 0
$$
\n
$$
0.1013 \theta c = 6.986
$$
\n
$$
\theta c = 6.986
$$

$$
\begin{cases} a.1140 \, 66 + 2.00 = 0 \\ 0.013 \, 66 - 2.27 = 0 \end{cases}
$$
  
a.101 \, 66 = -4.27  

$$
\theta_8 = -42.3
$$

Values of the moments:  
\n
$$
Mass = 0.0678811147(-42.3) + 49.6
$$
  
\n $= -3.14 + 49.6 = +46.5 \times 4.4.4$   
\n $MSA = +0.0678 \times 1.819(-42.3) - 49.6$   
\n $= -5.35 - 49.6 = -54.9 \times 1.44$   
\n $disc = 0.1932(-42.3) + 0.113(169) + 55.25$   
\n $= -8.35 + 7.0 + 55.25 = +54.7 \times 1.44$   
\n $McB = 0.1932(169) + 0.1127(-42.3) - 55.25$   
\n $= +13.3 - 4.77 - 55.25 = -46.7 \times 1.44$   
\n $McD = 0.152(1.098)(169) + 35.2$   
\n $= +11.5 + 35.2 = +46.7 \times 1.44$ 

H

The moments are calculated by the aid of constants from Turneaure and Maurer's Concrete Construction Book.

The complete solution by the moment distribution method is shown in table 6. Fig. (6) shows the  $1/I$  curve and fig. (C) shows the moment diagram.

The notations used are as follows:

- L = length of the 1 curve for each span<br>
= depth of beam at centroid of the 1/I curve  $q = L/d^{3}$  $X =$  distance from centroid of the  $1$  curve of I the span  $\dot{L}_o = \frac{1}{2} d \mathbf{L}^2$ 12  $I = \overline{\dot{b}}_0$  *ro* $X^2$ m • moment from moment diagram of fig. (C)
	- $\overline{X}$  = kern point of the haunched beam
- $S =$  stiffness factor of the members
	- = carry-over factor
- $M'$  = fixed end moment

The resulting moments obtained by the two methods show little variations from each other. Nevertheless, the moment distribution method will seem more feasible though not a marked saving of material will be affected. As the continuity of the structure becomes more complicated, this method will offer a considerable economy with corresponding reduction in labor of computation.

## CONTINUOUS HAUNCH BEAM

 $BY$ 

MOMENT DISTRIBUTION INETHOD







 $(48)$ 



 $(49)$ 

#### CHAPTER VII

#### CONCLUSION

As the maximum moment is the criterion in determining the column cross-sections, the maximum economy is obtained when the bending moment at each end of the column is equal; i.e. when the point of inflection falls at the center of the columns. Since this is one of the assumptions upon which these approximate methods are based, it will be seen that a greater economy is obtained by using these methods for wind analysis than by using the slope deflection method. From actual experiments conducted in tall buildings for the location of the contraflexure point of columns, it was found that this assumption holds true for all columns in the mid-stories. This point of zero moment being less than the mid-height of the column as measured from the bottom for those columns of the stories close to the top of the building, and is sufficiently greater than the mid-length of the columns for those at the base of the building.

For the girders, the approximate methods likewise offer greater economy as these inflection points are also taken to be at the center. For the middle bay due to symmetry, this condition also holds for the deflection method, but as these moment must hold the colwnn moments of any joint in equilibrium, it will be greater than those obtained by either approximate method.

Since the direct stresses in the columns are a function

(50)

of its moments existing, it is also greater than those obtained by the approximate methods.

Solving the same bent by assuming all stiffnesses of the columns and girders to be the same and cross-sectional areas of the columns as equal, it was found that the resulting column moments and direct stress are still greater than those obtained by the approximate method, thus showing that it is less economical for the simple reason mentioned above.

As to the comparative economy of the two approximate methods, the portal method will be the more economical since it distributes the greater part of the burden of resisting the external moments to the outside columns, and all direct stresses are then taken by the outside columns for symmetrical bent with bays of equal width. For this particular bent analyzed, the direct stresses obtained by the portal method shows quite a distinct difference from those obtained by the cantilever method. If the outer bays had been chosen very wide relative to the center bay, which condition generally occurs, there is practically no economical advantage over the cantilever method. Precisely, such theoretical economy is a marked feature as the height of the building increases.

It seems to the writer that the Po~tal method will be the most favorable for wind stress analysis in practically all cases for symmetrical bents, because of the economy in labor and its underlying assumption affords a greater possibility of economy of material as well. This method will meet with

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complications when the bent becomes more perplex especially if it is treated for architectural value.

Therefore, it must be concluded, that in order to arrive at the maximum economy, arrangement of the building space as well as the method of solution should be carefully considered. Furthermore, with such an impossibility of knowing just exactly what wind load should be assumed, it remains doubtful as to what degree of precision such as supposely exact solution will yield.

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