THE EFFECT OF OPENINGS ON THE LATERAL STIFFNESS OF WALLS BETWEEN CONTINUOUS, RIGID FLOORS

Thesis by

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THE EFFECT OF OPENINGS ON THE LATERAL STIFFNESS OF WALLS BETWEEN CONTINUOUS, RIGID FLOORS

1. INTRODUCTION: The subject of this thesis was suggested by Professor R. R. Martel as a topic of immediate intrest and value in connection with earthquake-proof design.

After a short consideration of the problem, it became apparent that the nature of the floor systems above and below any wall in question would greatly influence its resistance to lateral deflections. The various kinds of floor systems were therefore divided into three groups as follows.

1. Continuous and rigid:- Systems composed of floors which deflect laterally as planes, without warping. Such floor systems would fix the tops and bottoms of columns and walls against rotation.

2. Continuous.- Systems composed of floors sufficiently rigid so that, in case of lateral motion, the vertical members between them will have approximately the same deflection but not necessarily the same end rotation. A floor of such a system will warp when lateral motion takes place but each element of the floor will move approximately the same distance horizontally. (the case of torsion between floors excluded)

3. Discontinuous.- Systems composed of floors which are not continuous from room to room throughout the entire story and which are not capable of forcing equal horizontal deflections of the vertical members between them.

If walls be investigated for each of the three different floor systems mentioned above, enough information will probably be obtained to enable one to interpolate for intermediate cases.

In regard to the relative importance of the three floor systems mentioned above it may be said, that in earthquake proof construction the continuous system would be by far the most important. Structures with continuous and rigid floor systems (or nearly so) would be very massive and in most cases not practical, while structures with discontinuous floor systems would be unsafe in earthquake regions.

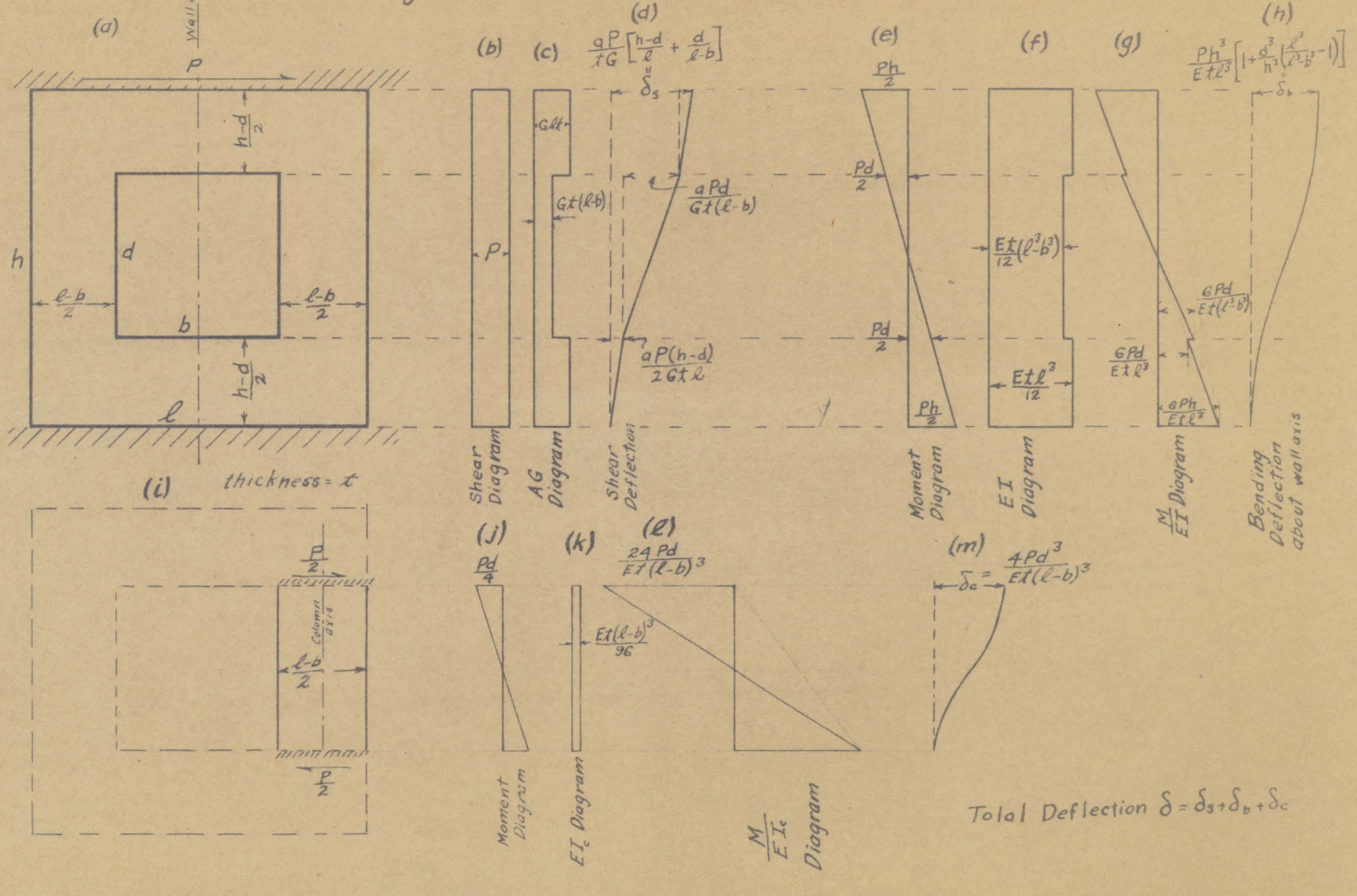
This investigation deals with walls which are between floors clasified above as continuous and rigid. This class was chosen as the first to study because it is easier to duplicate the loading conditions experimentally. After the development in technique and method of experimentation coming from the investigation of this first class we will be more competent to extend the research to the more important and complicated cases.

2. METHOD IN GENERAL OF OBTAINING DEFLECTION EQUATIONS:

The equations for the deflection of walls with openings will be developed by the use of the moment area method and certain assumptions which will be mentioned when they are made. These equations will contain certain undetermined constants which will be evaluated experimentally.

Fig. 1

Diagrams for Moment Area Method



Tolal Deflection $\delta = \delta_s + \delta_b + \delta_c$

3. ANALYTICAL SOLUTION: Consider the total deflection of the wall (see Fig. .1 (d), (h) and (m)) to be composed of three parts:

δ_s = the shear deflection of the entire wall

δ_b = the bending deflection of the wall about the axis of the wall

δ_c = the bending deflection of the two vertical strips on the sides of the opening which will be referred to hereafter as the columns

The method for finding δ_s can be seen easily from figure .1 (b), (c) and (d)

$$\delta_s = \frac{a\,P}{t\,G}\left[\frac{h-d}{l} + \frac{d}{l-b}\right]$$

(a)[1] is a constant whose value depends upon the shear distribution. For rectangular sections with parabolic shear distribution $a = \frac{3}{2}$, for uniform shear distribution $a = 1$

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1. See S. Timoschenko, Strength of Materials, Part I, Page 186; also see Naito, Design for Earthquake Stresses (translation from Japanese, Page 77 or Naito, Bulletin of Seismological Soc. of America, Vol. 17-18, 1927-28.

We obtain the bending deflection δ_b about the axis of the wall by taking the statical moment of the $\frac{M}{E\,I}$ diagram about the top of the wall.

Let Fig. .2 represent the $\frac{M}{E\,I}$ diagram, in figure .1 (g)

Consider the triangles in the following order:

moment of △ BCD

moment of △ ADB

moment of △ fgh

moment of △ ehf

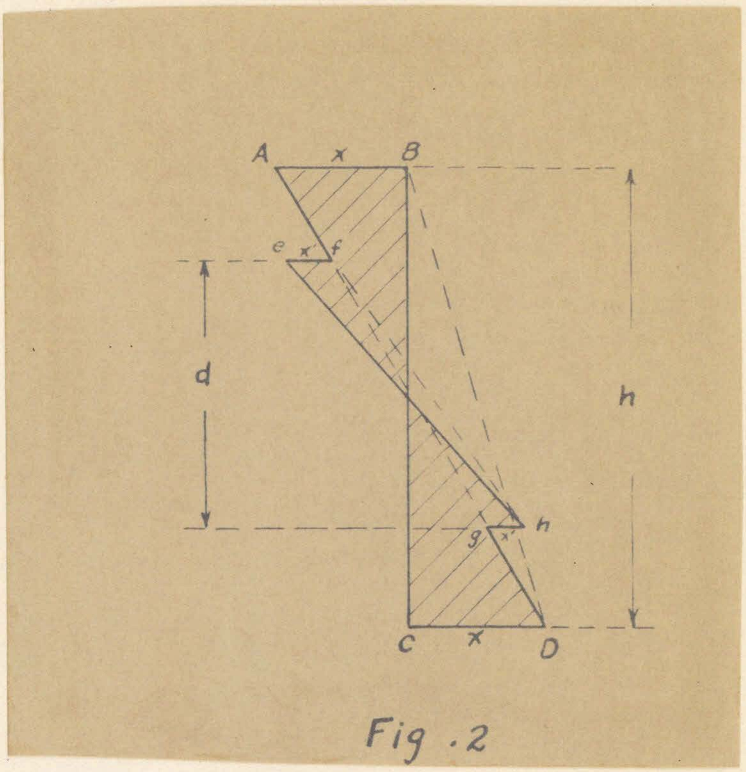

Fig. .2

$$\delta_b = x\frac{h}{2}\cdot\frac{2h}{3} - \frac{xh}{2}\cdot\frac{h}{3} + \frac{x'd}{2}\cdot\left[\frac{h-d}{2} + \frac{2d}{3}\right] - \frac{x'd}{2}\cdot\left[\frac{h-d}{2} + \frac{d}{3}\right]$$

$$\delta_b = \frac{xh^2}{6} + \frac{x'd}{2}\cdot\frac{d}{3}$$

$$\delta_b = \frac{xh^2}{6} + \frac{x'd^2}{6}$$

substituting $\frac{6\ Ph}{Etl^3}$ for x and $\left[\frac{6\ Pd}{Et\ (l^3 - b^3)} - \frac{6\ Pd}{E\ t\ l^3}\right]$ for x'

we have

$$\delta_b = \frac{P\ h^3}{E\ t\ l^3} + \frac{P\ d^3}{E\ t}\left[\frac{1}{l^3 - b^3} - \frac{1}{l^3}\right]$$

The second term of the right hand member in the above equation is very small compared to the first term when the opening in the wall is small. When the opening is large this term is very small compared to the value of δ_c (determined later), so that this term may be dropped without introducing appreciable error.

From the moment area diagrams it can be seen that:

$$\delta_c = \frac{4\ P\ d^3}{E\ t\ (l - b)^3}$$

The total deflection then is:

$$\delta = \delta_s + \delta_b + \delta_c$$

$$\delta = \frac{a\ P}{t\ G}\left[\frac{h-d}{l} + \frac{d}{l-b}\right] + \frac{P\ h^3}{E\ t\ l^3} + \frac{4\ P\ d^3}{E\ t\ (l-b)^3}$$

$$\delta = \frac{P}{E\ t}\left\{a\ \frac{E}{G}\left[\frac{h-d}{l} + \frac{d}{l-b}\right] + \frac{h^3}{l^3} + \frac{4\ d^3}{(l-b)^3}\right\} \qquad (1)$$

In the derivation of this equation it was assumed that the material immediately above and below the opening was just as effective as the rest in resisting lateral load.

As a matter of fact this material is ineffective in resisting lateral load when the floors above and below the wall are rigid.

If the shaded areas are not so effective in giving rigidity then the value of d in the equation will need to be modified in order to obtain the proper deflections.

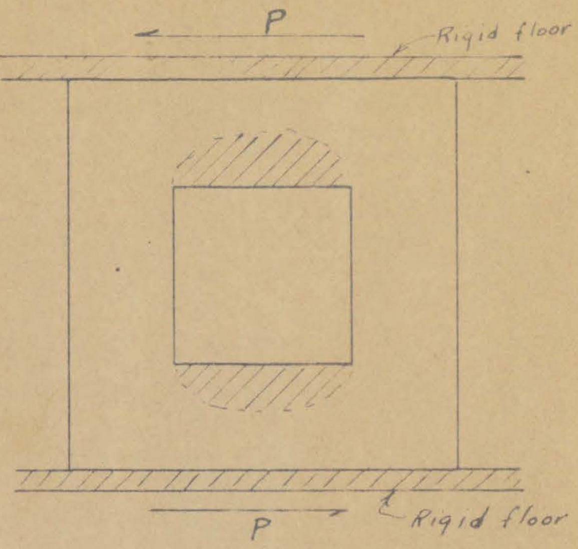

Fig .3

We will say that the effective height of the opening

d' is: $d' = d + c\,b$

Substituting d' for d in equation (1), we have the general equation for deflection of walls with openings.

$$\delta = \frac{P}{E\,t}\left\{ \frac{dE}{G}\left[\frac{h-d'}{l} + \frac{d'}{l-b}\right] + \frac{h^3}{l^3} + \frac{4(d')^3}{(l-b)^3}\right\} \quad (2)$$

$\frac{E}{G}$[2] will vary according to the material of which the wall is made

---

[2] See S. Timoschenko, Strength of Materials (Part 1, Art.14,15,16
(Part 2, Art. 46

Since $\frac{E}{2(1 + w)} = G$ and[3] the limits of w are 0 and $\frac{1}{2}$, the value of $\frac{E}{G}$ must be between 2 and 3.

The value of[4] $(a\,\frac{E}{G})$(as found by experiments described later)[5*] is 3, for solid walls at least. Substituting this in equation (2), we have:

$$\delta = \frac{P}{E\,t}\left\{ 3\left[\frac{h-d'}{1} + \frac{b'}{1-b}\right] + \frac{h^3}{1^3} + \frac{4(d')^3}{(1-b)^3}\right\} \quad (2a)$$

$$d' = d + c\,b$$

$\frac{E}{G} = 2.5$ and $a = 1.2$ are reasonable values.[5]

The graph below gives C as a function of the percentage of the wall remaining. The equation can now be solved.

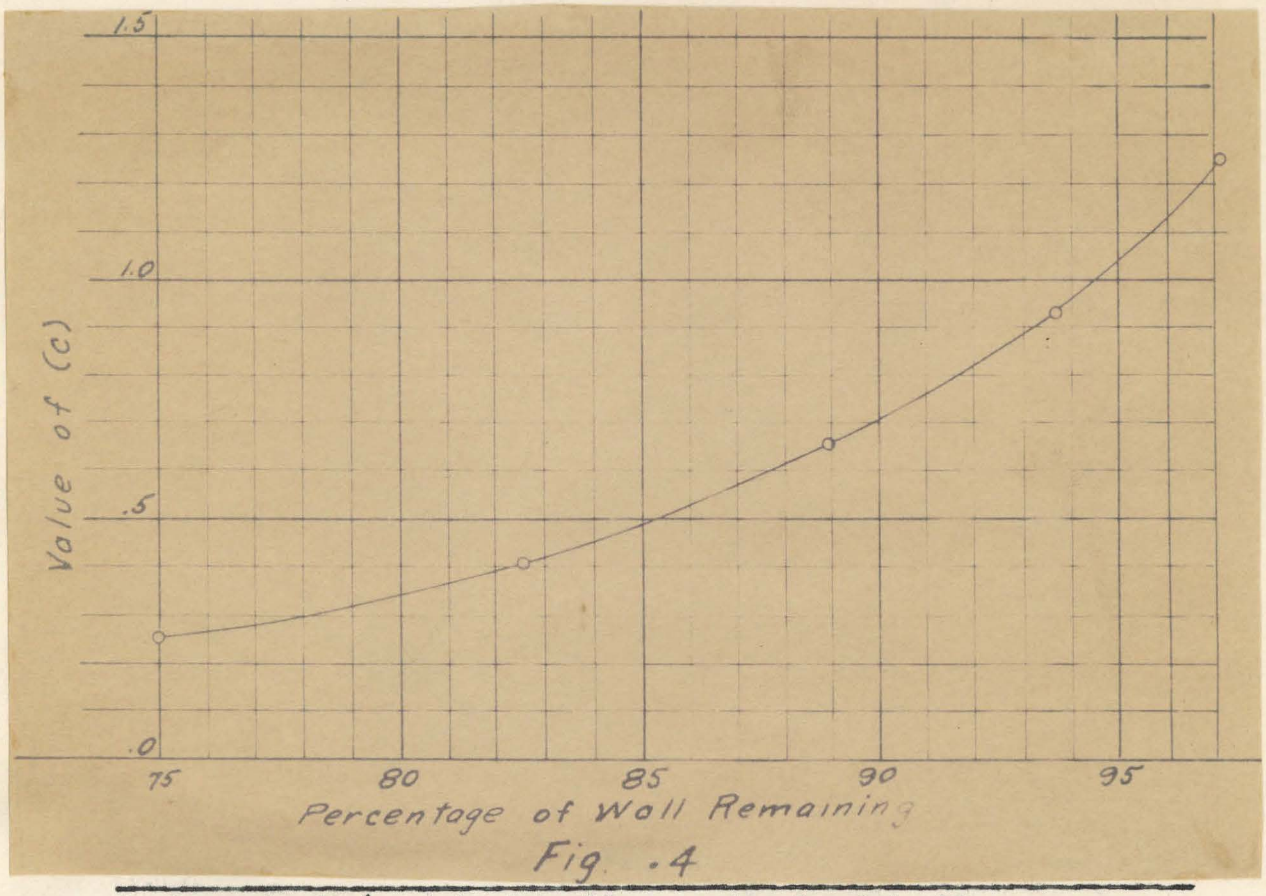

Fig. .4

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[3] See note 2. (part 1, Art. 16.   [4] See Art. 12

[5] See Naito, Design for Earthquake Stresses , translation from Japanese, Page 77, Vol. 2, Page 296.

[5*] See Art. 12

EXPERIMENTAL TESTS

4. **PROCEDURE:** Twenty-one celluloid models[6] 6" square and .015" thick were used in this series of tests. The program for testing the models follows:

The model was mounted as shown in Fig. 6, with initial tension forces sufficient to assure that all parts of the model would remain in tension after the lateral test forces were applied.

Vectors P and F represent the forces measured in the tests. The force F was used only to make the loading bar remain parallel to the base when it was so desired. In all the tests the force P was made just large enough to give a lateral deflection of the point $_p$ of .0081 inches. For each particular model of a windowed wall four values of P were taken:

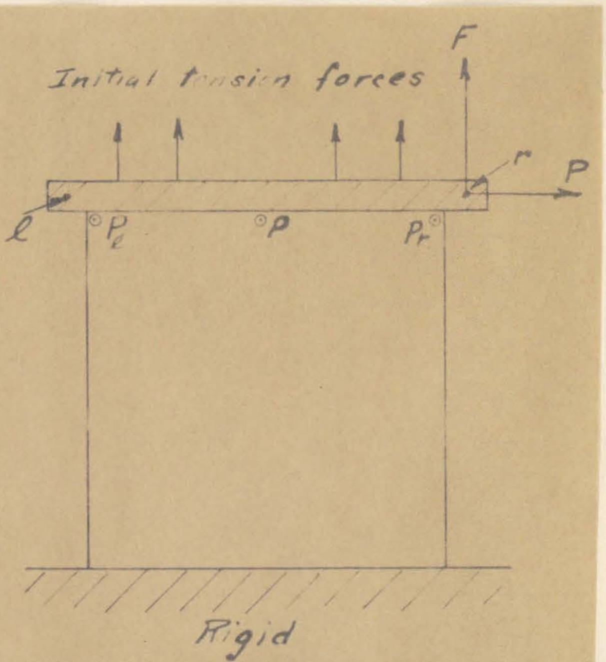

Fig. .6

1. The force at (r) required to deflect the wall .0081 in. to the right with F not acting.
2. The force at the same point with the same deflection with F acting. (loading bar parallel to base)
3. The force at (1) required to deflect the wall .0081 in. to the left with F not acting
4. The force at the same point with the same deflection with F acting. (loading bar parallel to base)

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[6] The commercial name of the celluloid was Pryolin, made by DuPont Viocoloid Compand. For physical properties see International Critical Tables, Vol. 2, Page 296

Fig. 1 shows the twenty-one models and the number and kind of openings represented by each. Altogether ninety-five different openings were represented. Copies of the data sheets are found in the appendix. The method of numbering the particular models is taken from the area, position, and shape of the opening. Thus the first window cut from model 4 (see Fig. 1) would be numbered

1 - 01 - S

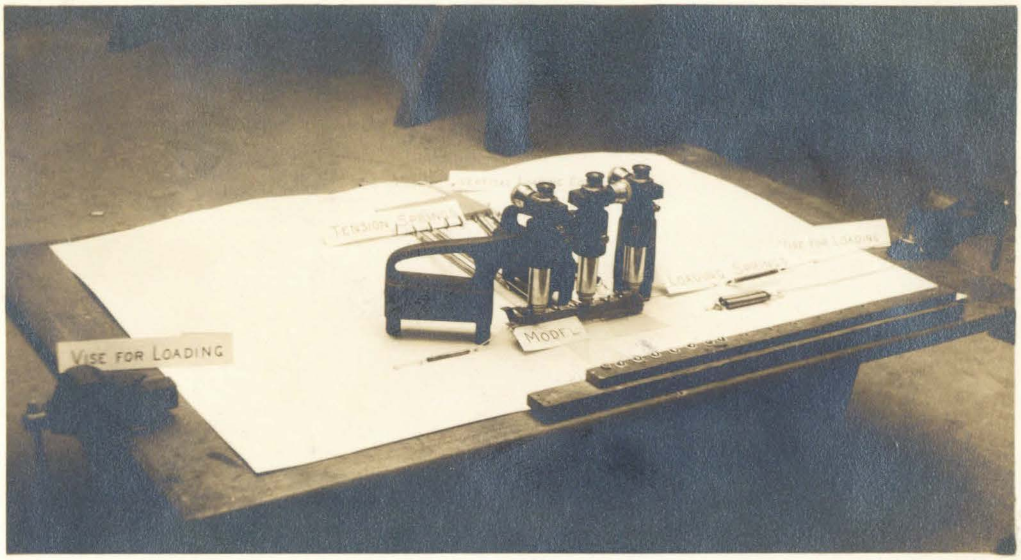
meaning:

Area of window = 1 sq. inch

Position of window center $= \left(\begin{array}{l} x = 0 \text{ in.} \\ y = 1 \text{ in.} \end{array} \right.$

Shape of window square

5. FORCE MEASUREMENTS: Small well made springs were calibrated and used to measure the forces , (P and F). By means of long wire hooks one end of such a spring was attached to the loading bar and the other end to the movable part of a small steel vise. By turning the vise screw the forces could then be gradually varied or maintained constant. Figure .25 is a photograph of the apparatus.



Springs of different tension values were used and ~~so~~ calibrated accurately enough so that the force measurements of the P forces are not in error more than one percent. The values for the F forces are less accurate but they have not been used.

6. <u>DEFLECTIONS</u>: All lateral deflections of the point P (Fig.2) were made the same .0081 inches. Observations were made by a standard Biggs Deformeter microscope. When the F force was also applied, two other similar microscopes were used to determine when the loading bar was parallel to the base. The error in these deflection observations is less than three percent.

7. <u>SPEED</u> <u>OF</u> <u>LOADING</u>: As is well known the rate of deflecting models has an effect on the force readings. By extreme variations

in this rate it was possible to get 10 percent error; such extremes were avoided and errors of not more than 3 per cent are to be expected from this source.

8. BUCKLING: In the case of the larger openings buckling was noted immediately above and below the opening. In several tests these regions were supported on ball bearings and loaded from above to prevent the buckling. Such support of these regions did not greatly stiffen the models and no great error is believed to have been introduced by such buckling.

TEST DATA ANALYSIS

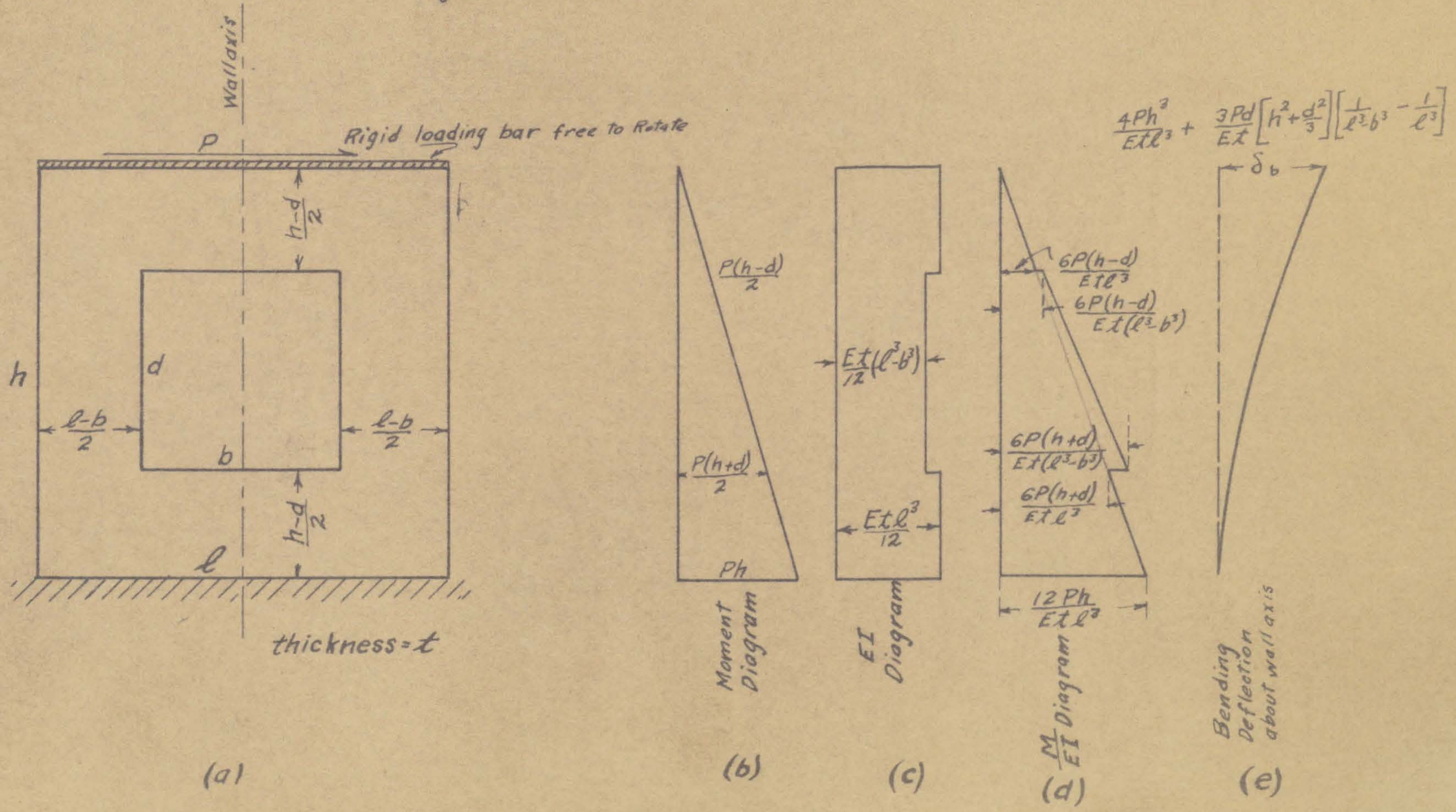
9. **EFFECT OF SIZE OF OPENING:** The effect that the size of the opening has upon the stiffness of a wall can be seen by an inspection of Fig. 2 and 1 . In all cases the stiffness gets smaller as the per centage of area left gets smaller, but that is about the only general thing which can be said for all types of openings. Any formula which expresses the stiffness as a function of only the per centage area left would indeed by a crude one.

10. **EFFECT OF THE ECCENTRICITY OF THE OPENING:** To obtain a picture of this effect examine in connection with the first eleven models of Fig. 1 the corresponding lines for the same models in Figures 2 and 10. Note two things; the separation of the white and black dots for each particular model and also the variation of the lines for the different models.

In general it seems that eccentricity has very little effect except in the case where the opening approaches very near one of the boundaries of the wall. The case of openings near to and cutting the boundaries needs more investigation. Examination of lines for models 3,5,7,9, and 11 will disclose irregularities not accounted for by this thesis.

11. **EFFECT OF THE SHAPE OF THE OPENING:** The shape of the opening is a very important factor in determining the stiffness of a wall. In connection with Fig. 1 examine Figures 3,4,5,6,7 and 11,12,13, 14, 15. All these Figures show the same thing but for openings of different eccentricity. The first series are for the (P) loadings

Fig .5


Wallaxis
P
Rigid loading bar free to Rotate
h
d
b
$\frac{h-d}{2}$
$\frac{h-d}{2}$
$\frac{\ell-b}{2}$
$\frac{\ell-b}{2}$
ℓ
thickness = t
(a)
$\frac{P(h-d)}{2}$
$\frac{P(h+d)}{2}$
Ph
Moment Diagram
(b)
$\frac{Et}{12}(\ell^3-b^3)$
$\frac{Et\ell^3}{12}$
EI Diagram
(c)
$\frac{6P(h-d)}{Et\ell^3}$
$\frac{6P(h-d)}{Et(\ell^3-b^3)}$
$\frac{6P(h+d)}{Et(\ell^3-b^3)}$
$\frac{6P(h+d)}{Et\ell^3}$
$\frac{12Ph}{Et\ell^3}$
$\frac{M}{EI}$ Diagram
(d)
$\frac{4Ph^3}{Et\ell^3} + \frac{3Pd}{Et}\left[h^2+\frac{d^2}{3}\right]\left[\frac{1}{\ell^3-b^3}-\frac{1}{\ell^3}\right]$
δ_b
Bending Deflection about wall axis
(e)

only (loading bar free to rotate) and the second are for the (P) and (F) combined loadings (loading bar parallel to base). It can be seen in general that broad openings greatly reduce the stiffness. Not only because of the slender columns produced on each side, but also because of the relatively large amount of material above and below the opening which is ineffective.

12. DETERMINATION OF THE CONSTANT ($\frac{aE}{G}$) IN THE DEFLECTION EQUATION:

In a manner similar to the method of Article 3 the deflection equation for a model represented by Fig. 5 may be determined. In this case the loading bar is free to rotate about the wall axis while in the previous case the top and bottom connections of the wall were forced to remain parallel by force F.

As before the total deflection δ is given by

$$\delta = \delta_s + \delta_b + \delta_c$$

The first and third terms of the right hand member are the same as before (equation 1) and δ_b is determined by taking the statical moment of the $\frac{M}{E\,I}$ diagram about the top of the wall.

Total moment $=$ moment of large triangle $+$ moment of trapezoid

$$\delta_b = \frac{4\,Ph^3}{Etl^3} + \frac{3Pd}{Et}\left[h^2 + \frac{d^2}{3}\right]\left[\frac{1}{l^3 - b^3} - \frac{1}{l^3}\right]$$

The moment of the trapezoid may be neglected because it is relatively very small.

$$\delta_b = \frac{4\ Ph^3}{E\ t\ l^3}$$

Adding this to the values of δ_s (approx.) and δ_c we have:

$$\delta = \frac{P}{E\ t}\left\{\frac{a\ E}{G}\left[\frac{h-d}{l} + \frac{d}{l-b}\right] + \frac{4\ h^3}{l^3} + \frac{4\ d^3}{(l-b)^3}\right\} \qquad (3)$$

If we replace d by d'[7] we have:

$$\delta = \frac{P}{E\ t}\left\{\frac{a\ E}{G}\left[\frac{h-d'}{l} + \frac{d'}{l-b}\right] + \frac{4\ h^3}{l^3} + \frac{4\ d'^3}{(l-b)^3}\right\} \qquad (4)$$

This is the general equation for the deflection of a wall with an opening where the top of the wall is free to rotate but rigid against warping.

If equations (2) and (4) are applied to solid walls (where d' and b are zero), we have:

(2) becomes: $$\delta_p = \frac{P_p}{E\ t}\left\{\frac{a\ E}{G}\ \frac{h}{l} + \frac{h^3}{l^3}\right\} \qquad (4a)$$

(4) becomes: $$\delta_f = \frac{P_f}{E\ t}\left\{\frac{a\ E}{G}\ \frac{h}{l} + \frac{4\ h^3}{l^3}\right\} \qquad (4b)$$

Let the subscripts ${}_p$ and ${}_f$ mean loading bar parallel to base and free to rotate respectively. In the experiments (where all the deflections were made the same), 42 values were obtained for P for each of the two types of loading represented by the equations above. (see data sheets in the appendix) From these data we obtain an average value of:

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[7] Art. 3 , page 6

$$\frac{P_f}{P_p} = \frac{5.79}{10.16} = .57 \text{ or } \frac{4}{7} \qquad (5)$$

From the equations above we have:

$$\frac{P_f}{P_p} = \frac{\frac{a\,E}{G} \times \frac{h}{l} + \frac{h^3}{l^3}}{\frac{a\,E}{G} \times \frac{h}{l} + \frac{4\,h^3}{l^3}} = \frac{\frac{a\,E}{G} + 1}{\frac{a\,E}{G} + 4} \qquad (6)$$

Equating the right hand members of equations (5) and (6), we have:

$$\frac{4}{7} = \frac{\frac{a\,E}{G} + 1}{\frac{a\,E}{G} + 4}$$

from which:

$$\frac{a\,E}{G} = 3^{7*}$$

This value was substituted into equation (2) to obtain equation (2a), Art. 3.

It is recognized that the shear distribution factor (a) can and probably does change when openings are cut in the wall. However the above value of $\frac{a\,E}{G}$ is kept constant in equations (2a) and (4) and will be shown by experimental data to be satisfactory.[8]

13. DETERMINATION OF THE CONSTANT C IN EQUATION $d' = d - cb$.[9]

The value of this constant (c) will be taken as that value which will make equation (2a) fit the average line for $\frac{d}{b} = 1$, formed by superimposing figures 11, 12, 13, 14, and 15. Figure 15

---

[8] See Artical 14

[9] See artical 3 and equations (2a) and (4)

7*. See note 5*, page 7

is the average formed by superimposing the above five figures.

The inverse ratio of the deflections due to a unit load on a windowed wall to the deflection unit caused by load on a solid wall will be equal to the stiffness ratio (R) of a windowed wall to the solid wall.

$$R = \frac{\dfrac{1}{\text{deflection of windowed wall due to unit load}}}{\dfrac{1}{\text{deflection of solid wall due to unit load}}}$$

$$R = \frac{\text{deflection of solid wall due to unit load}}{\text{deflection of windowed wall due to unit load}}$$

From equations (4a) and (2a) we have:

$$R = \frac{\dfrac{P}{E\,t}\left\{\dfrac{3a\,E}{G} \times \dfrac{h}{l} + \dfrac{h^3}{l^3}\right\}}{\dfrac{P}{E\,t}\left\{3\left[\dfrac{h-d'}{l} + \dfrac{d'}{l-b}\right] + \dfrac{h^3}{l^3} + \dfrac{4\,d'^3}{(l-b)^3}\right\}}$$

From the dimentions of the models we may substitute:

1 for $\frac{h}{l}$

$(d + cb)$ for d'

6 for l

3 for $\frac{a\,E}{G}$

and we have:

$$R = \frac{4}{3\left[1 - \dfrac{d+cb}{6} + \dfrac{d+cb}{6-b}\right] + 1 + \dfrac{4\,(d+cb)^3}{(1-b)^3}}$$

$$R = \frac{4}{4 + (d+cb)\left(\dfrac{3}{6-b} - \dfrac{1}{2}\right) + 4\left(\dfrac{d+cb}{6-b}\right)^3} \quad (7)$$

Equation (7) is the expression for the ordinate values of the lines of figure 15.1. We will take the 6 open points on the line where $\frac{d}{b} = 1$, in order to determine the value of c as a function of the per centage of solid wall remaining. The following table gives the values of c which satisfy equation (7) when the six R values from the middle $\frac{d}{b}$ line of figure 15.1 are substituted in the equation.

$\frac{d}{b} = 1$

b	d	Per centage area left	Experimental value of R	c	Calculated Values of R
0	0	100	1.00		1.00
1	1	97.2	.915	1.25	.873
1.5	1.5	93.7	.715	.92	.723
2.0	2.0	88.9	.560	.65	.567
2.5	2.5	82.6	.410	.40	.432
3.0	3.0	75.0	.290	.25	.291

Fig. .7

These values of c may now be plotted as ordinates against the per centage area of wall left as abscissa giving a curve from which values of c may be taken. (Figure .4 Art. 3)

14. EXPERIMENTAL CHECK ON THE GENERAL DEFLECTION EQUATIONS: In order to determine the c values, the points on the middle $\frac{d}{b}$ line of Fig. 15.1, were used. In order to check the accuracy of equations (2a) and (4) we have all the points on the five remaining $\frac{d}{b}$ lines of Figures 15.1 and 8. Points corresponding to these experimental points have been computed by using the already determined c values in equations (2a) and (4). The results may be seen by inspecting Figures 15.1 and 8.

15. GRAPHICAL SUMMARY OF EXPERIMENTAL DATA: Experimental charts 15.1 and 8 have been enlarged and more lines interpolated between those already existing (See Figs. 9 and 17). The method of this interpolation can be understood by inspecting Figures 8a, 8b, 15.2, and 15.3. These charts may be used to obtain the wall stiffness for square or almost square walls where the eccentricity of the opening is not more than $x = \frac{1}{6}$ and $y = \frac{h}{6}$.

16. CONCLUSIONS:

(a). As the size of openings are increased in a given wall, it's stiffness decreases.

(b). Small eccentricities of the opening does not greatly affect the stiffness.

(c). When an opening approaches very near the wall boundary or cuts it, there is a noticable decrease in stiffness.

(d). The shape of openings is an important factor in determining the wall stiffness. Horizontal openings have a greater weakening effect than square or vertical openings of equal area.

(e). The material immediately above and below an opening is not as effective as the rest of the wall in lending stiffness.

The amount of this ineffective material probably increases when the opening is near to the boundary of the wall.

(f). Rectangular openings in rectangular walls may not be the most efficient shape for admitting light. Diamond shaped or eliptical openings may be better.

Diagrams of Models

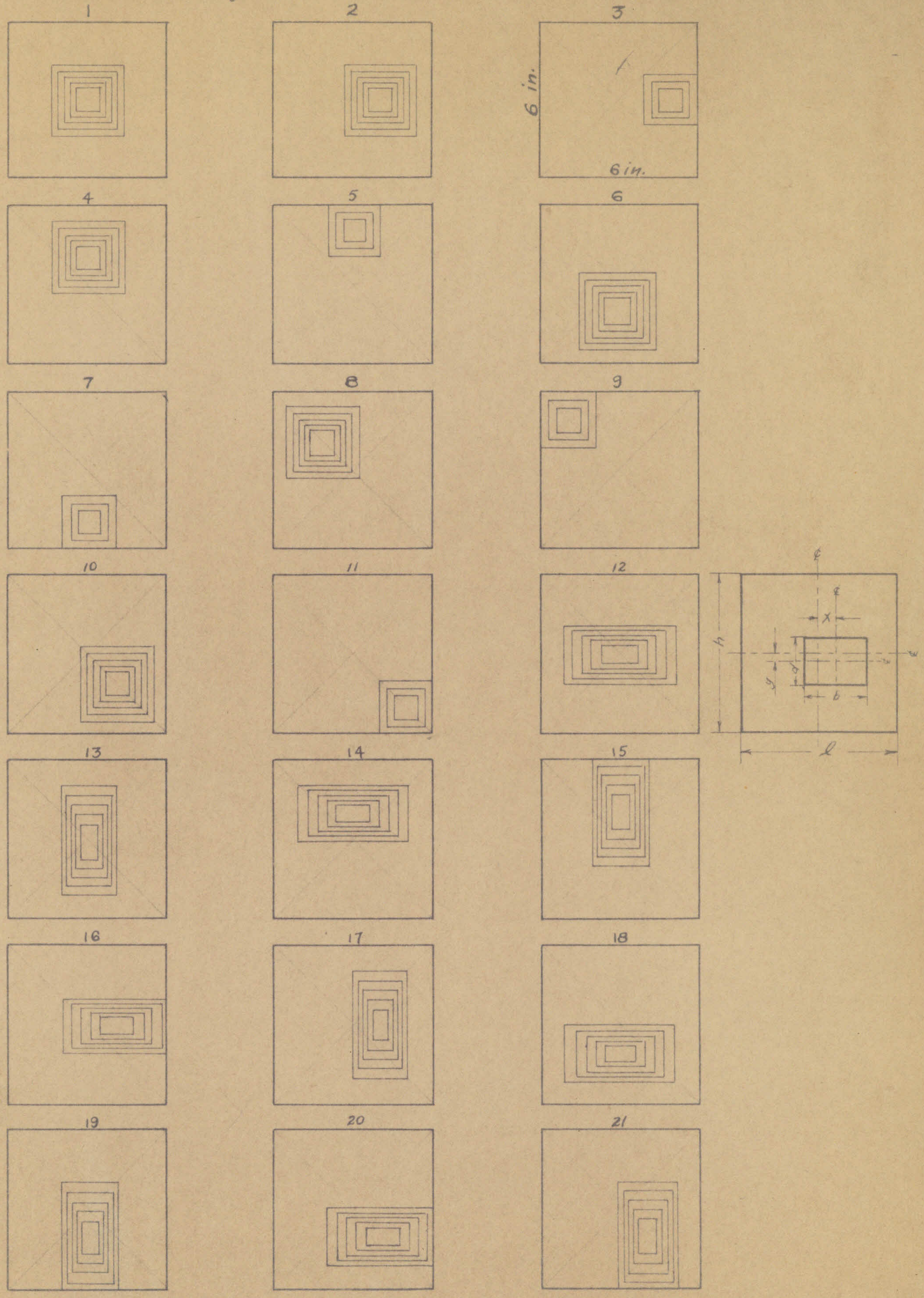


Fig 1

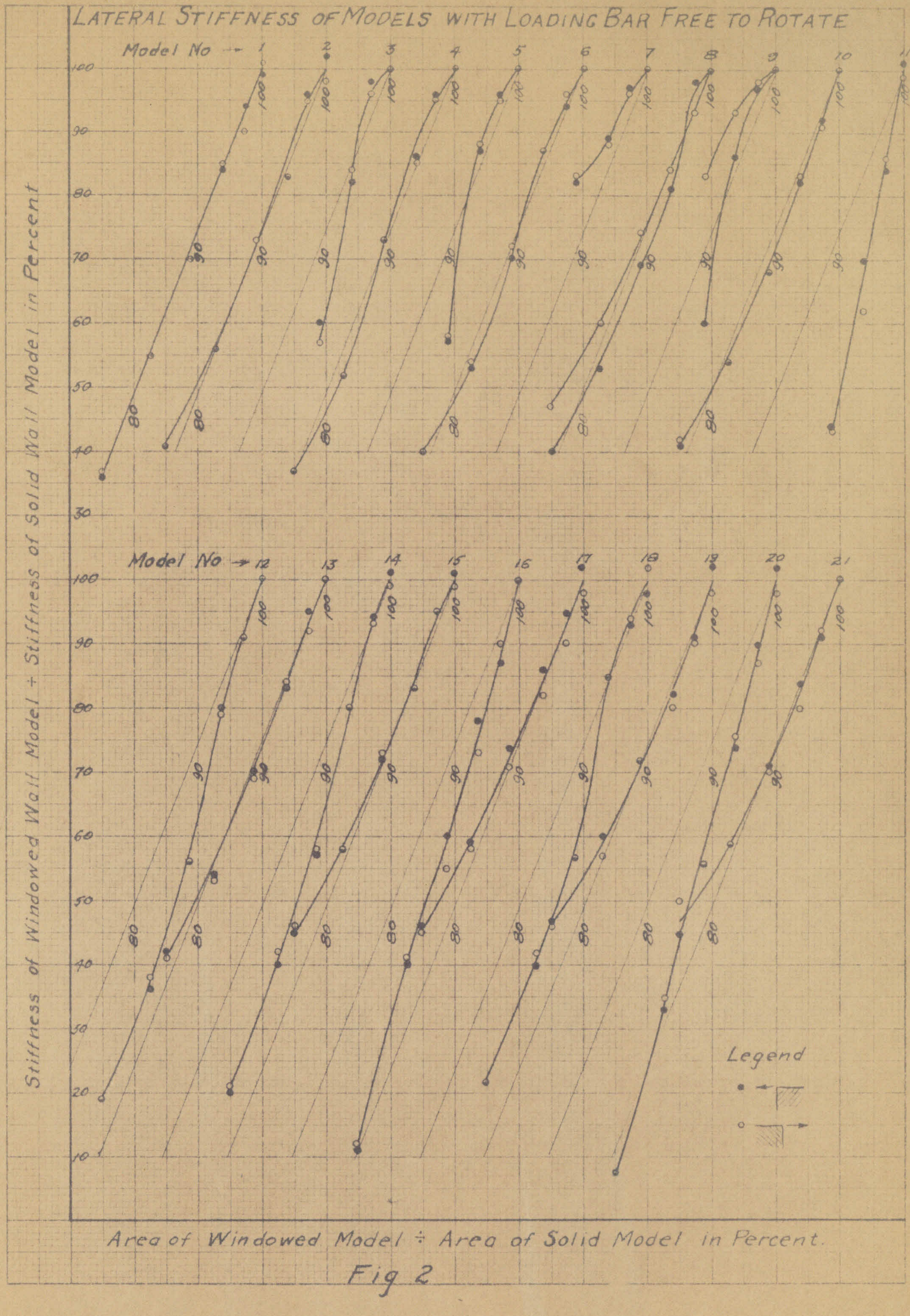
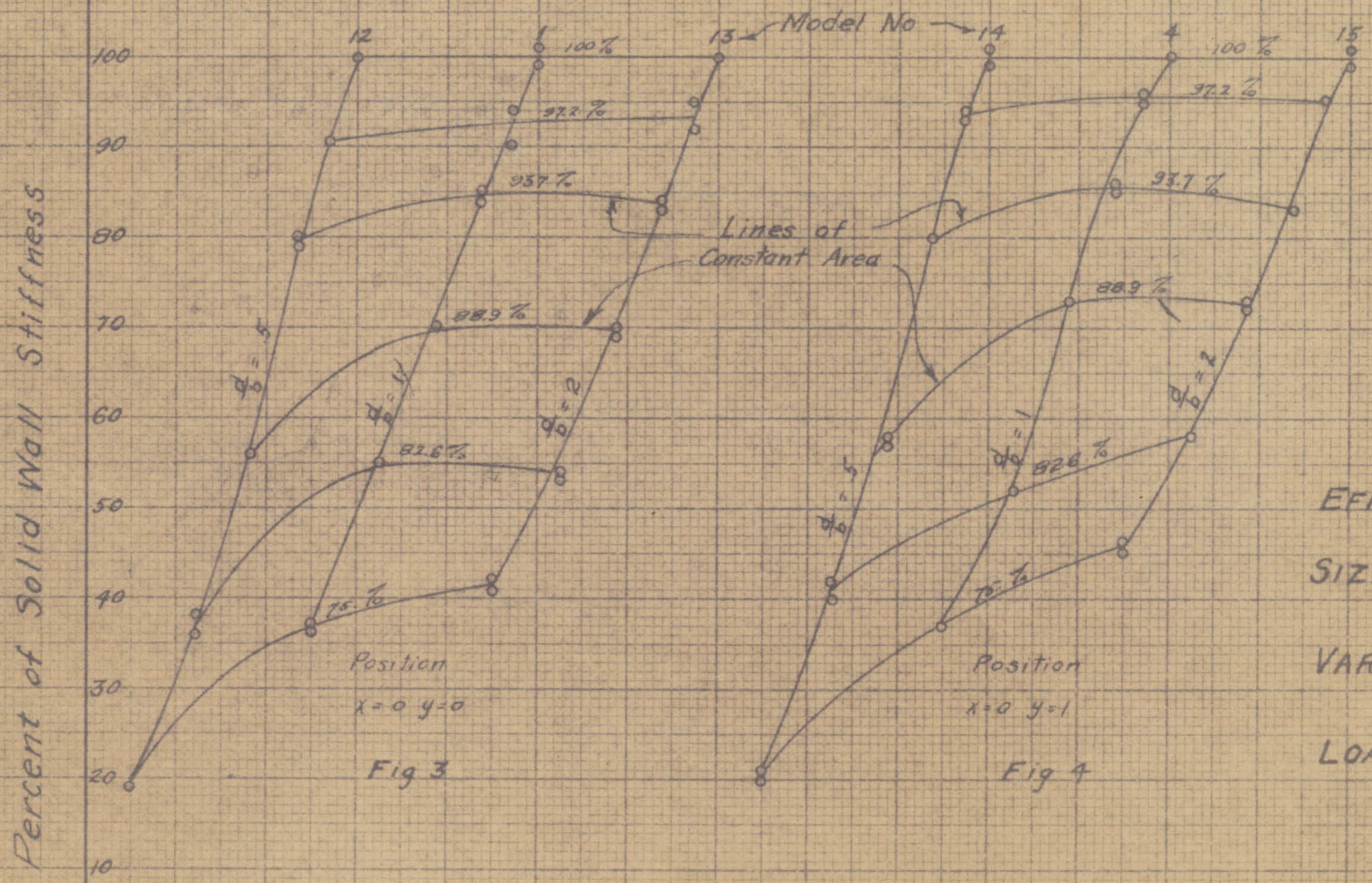

LATERAL STIFFNESS OF MODELS WITH LOADING BAR FREE TO ROTATE
Model No →
1
2
3
4
5
6
7
8
9
10
11
Model No →
12
13
14
15
16
17
18
19
20
21
100
90
80
70
60
50
40
30
20
10
Stiffness of Windowed Wall Model ÷ Stiffness of Solid Wall Model in Percent
Area of Windowed Model ÷ Area of Solid Model in Percent.
Legend

Fig 2



Position
x = 0 y = 0

Fig 3

Position
x = 0 y = 1

Fig 4

EFFECT OF SHAPE
AND
SIZE OF OPENING
AT
VARIOUS POSITIONS
WITH
LOADING BAR FREE
TO
ROTATE

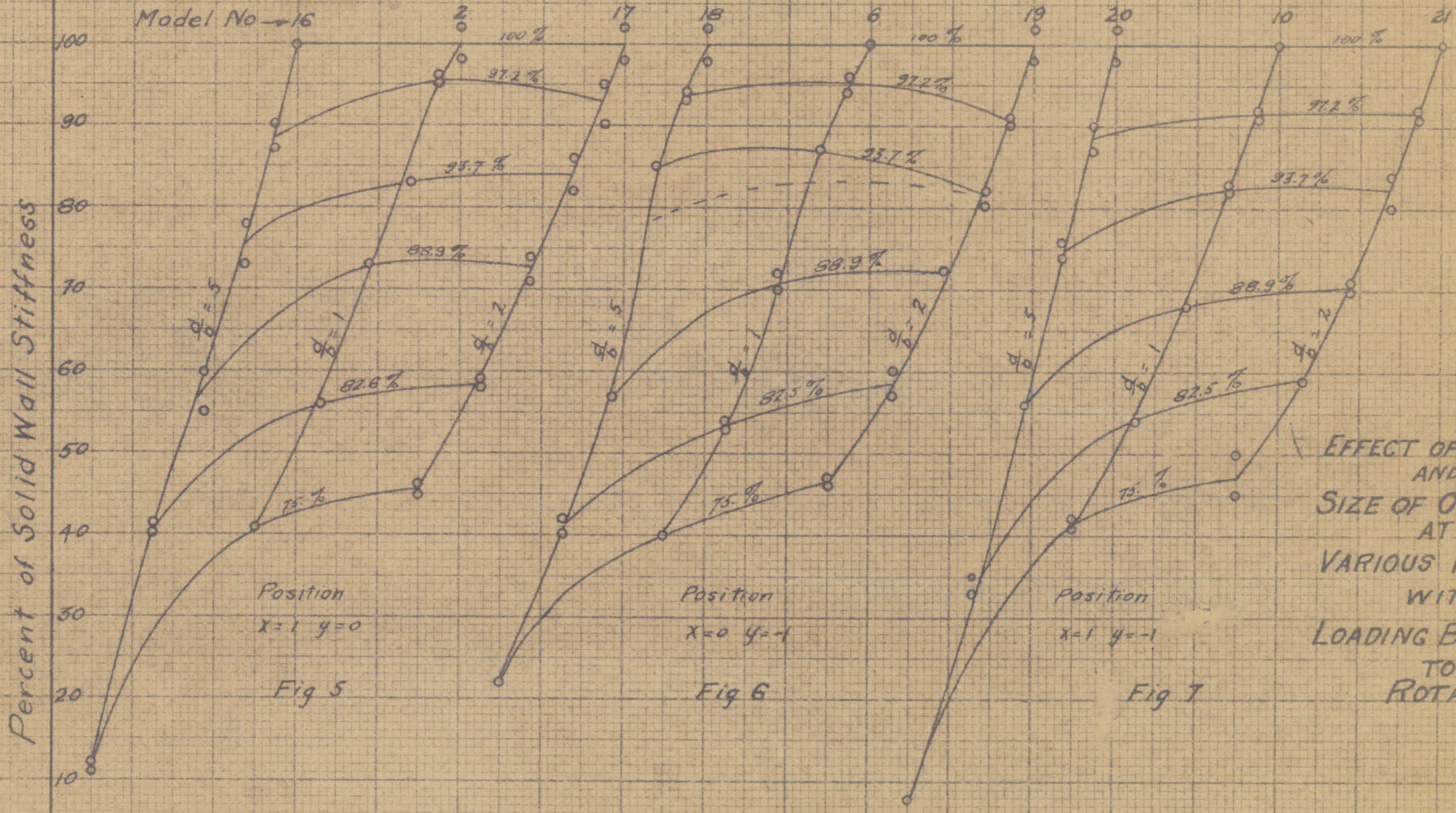


Fig 5

Fig 6

Fig 7

EFFECT OF SHAPE
AND
SIZE OF OPENING
AT
VARIOUS POSITIONS
WITH
LOADING BAR FREE
TO
ROTATE

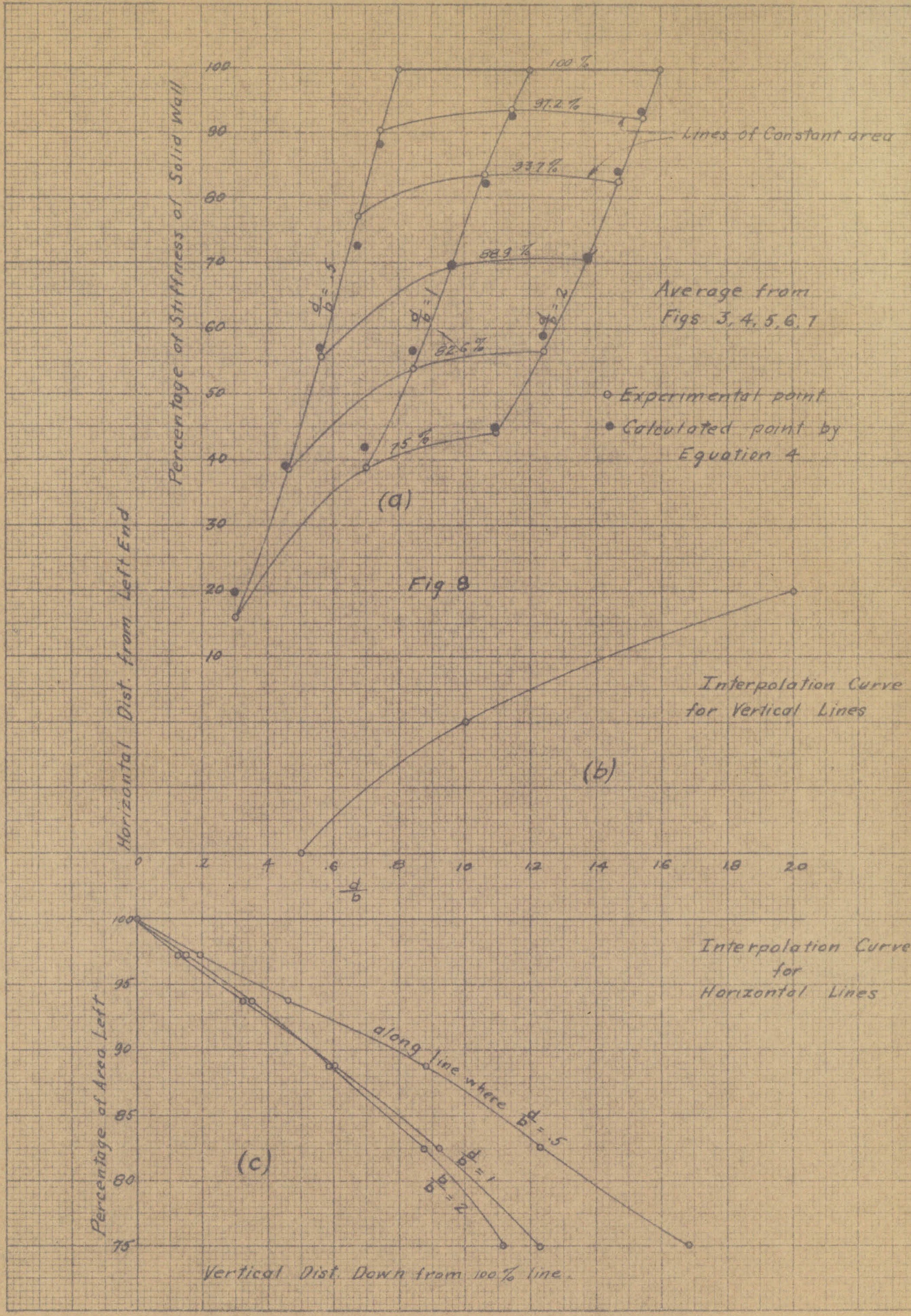

100 %
97.2 %
93.7%
88.9 %
82.6 %
75 %
Lines of Constant area
$\frac{d}{b} = .5$
$\frac{d}{b} = 1$
$\frac{d}{b} = 2$
Average from
Figs 3, 4, 5, 6, 7
○ Experimental point
● Calculated point by
Equation 4
(a)
Percentage of Stiffness of Solid Wall
100
90
80
70
60
50
40
30
20
10
Horizontal Dist. from Left End
Interpolation Curve
for Vertical Lines
(b)
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
$\frac{d}{b}$
Interpolation Curve
for
Horizontal Lines
along line where $\frac{d}{b} = .5$
$\frac{d}{b} = 1$
$\frac{d}{b} = 2$
(c)
Percentage of Area Left
100
95
90
85
80
75
Vertical Dist. Down from 100 % line.

Fig 8

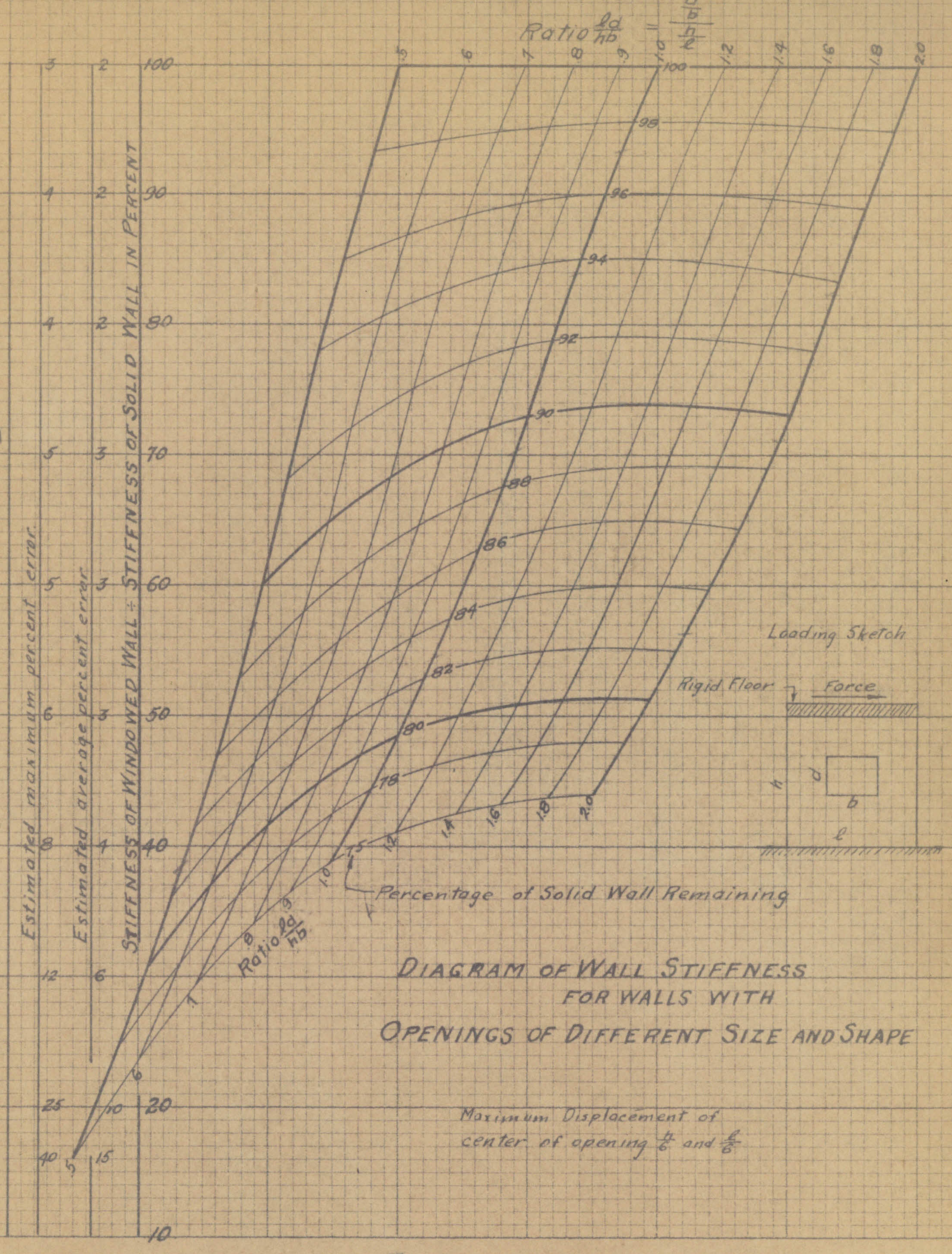


DIAGRAM OF WALL STIFFNESS FOR WALLS WITH OPENINGS OF DIFFERENT SIZE AND SHAPE

Fig 9

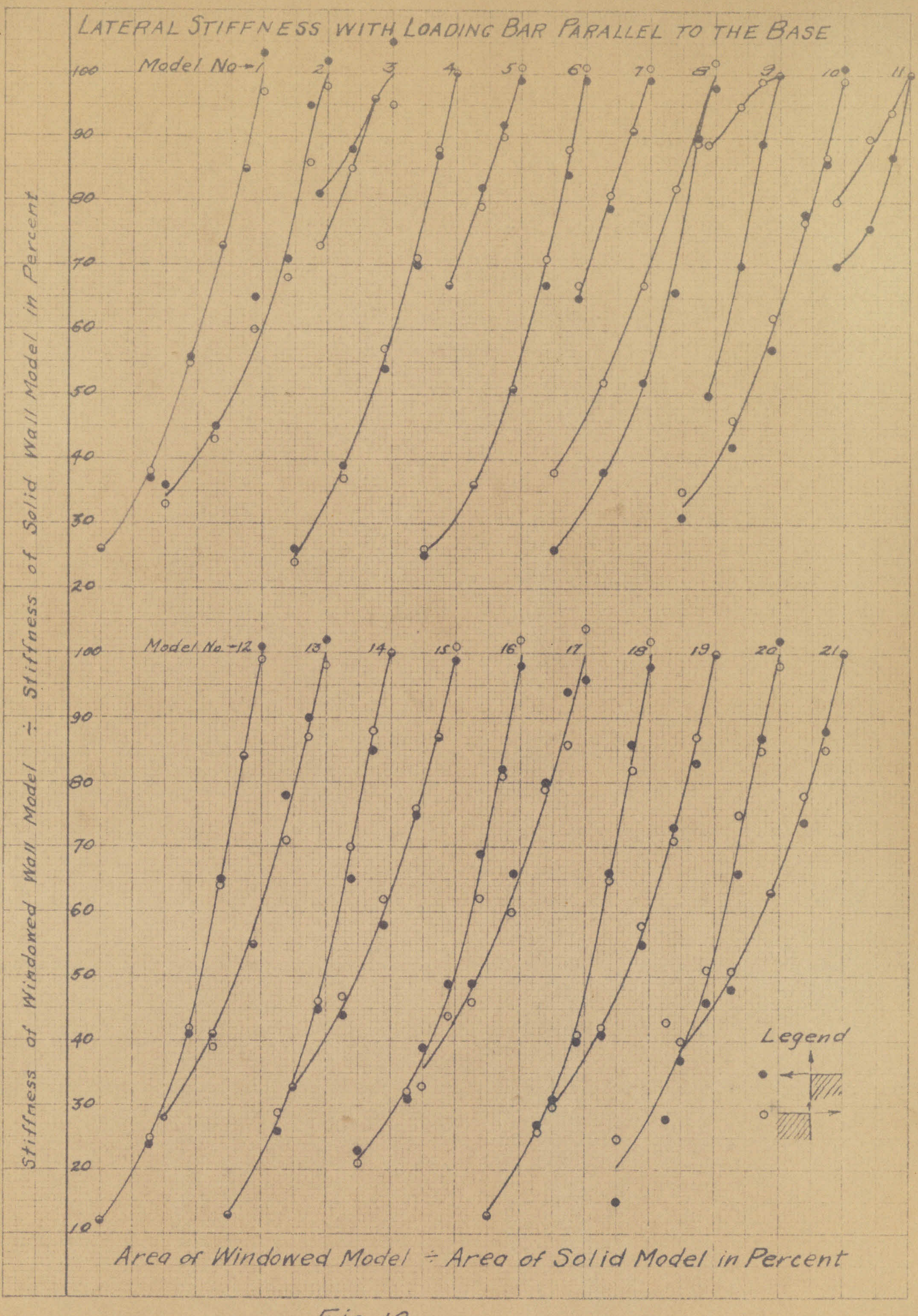

LATERAL STIFFNESS WITH LOADING BAR PARALLEL TO THE BASE

Fig 10

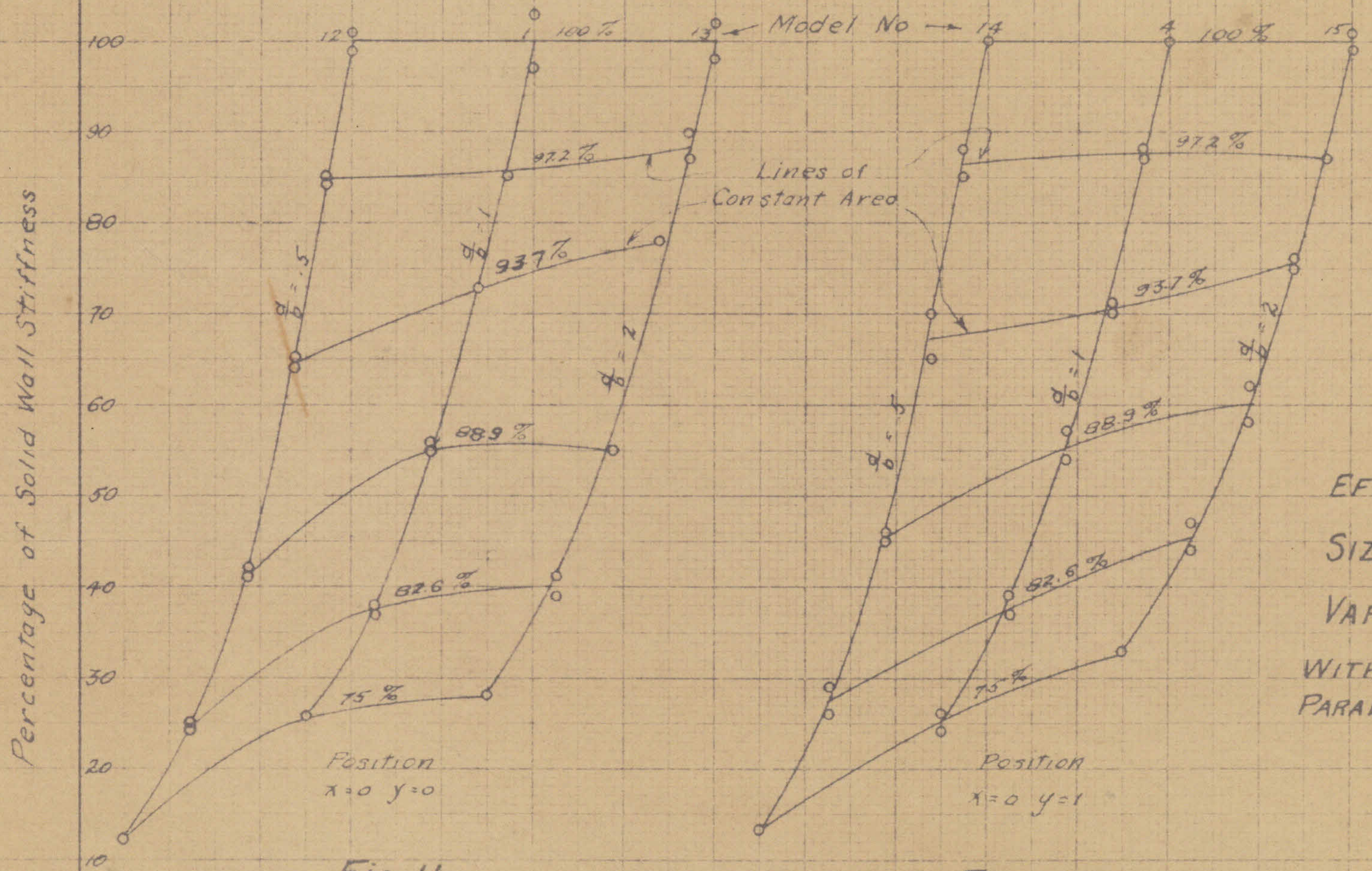


Fig 11

Fig 12

EFFECT OF SHAPE
AND
SIZE OF OPENING
AT
VARIOUS POSITIONS

WITH THE LOADING BAR
PARALLEL TO THE BASE

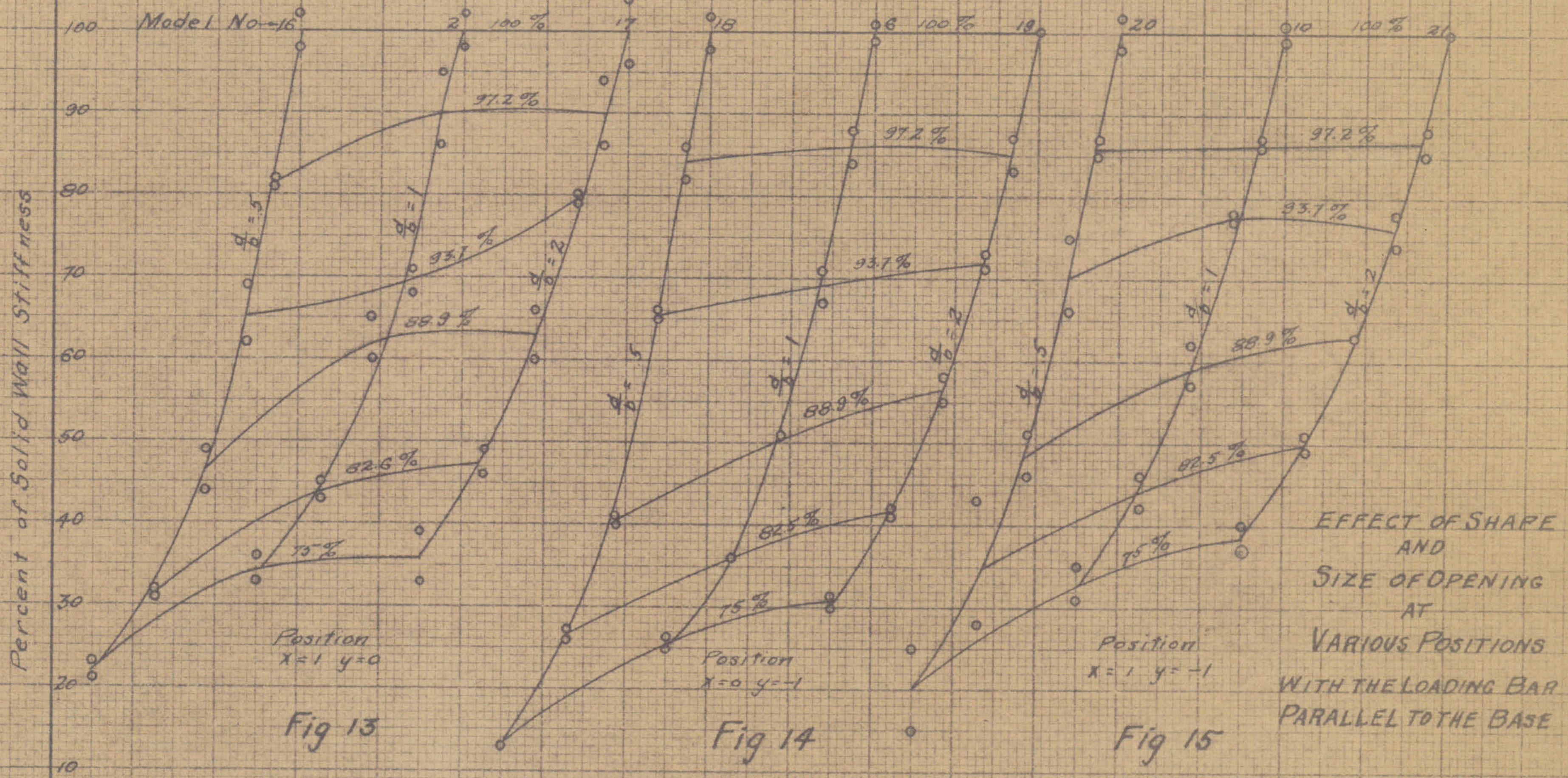


Fig 13

Fig 14

Fig 15

EFFECT OF SHAPE
AND
SIZE OF OPENING
AT
VARIOUS POSITIONS
WITH THE LOADING BAR
PARALLEL TO THE BASE

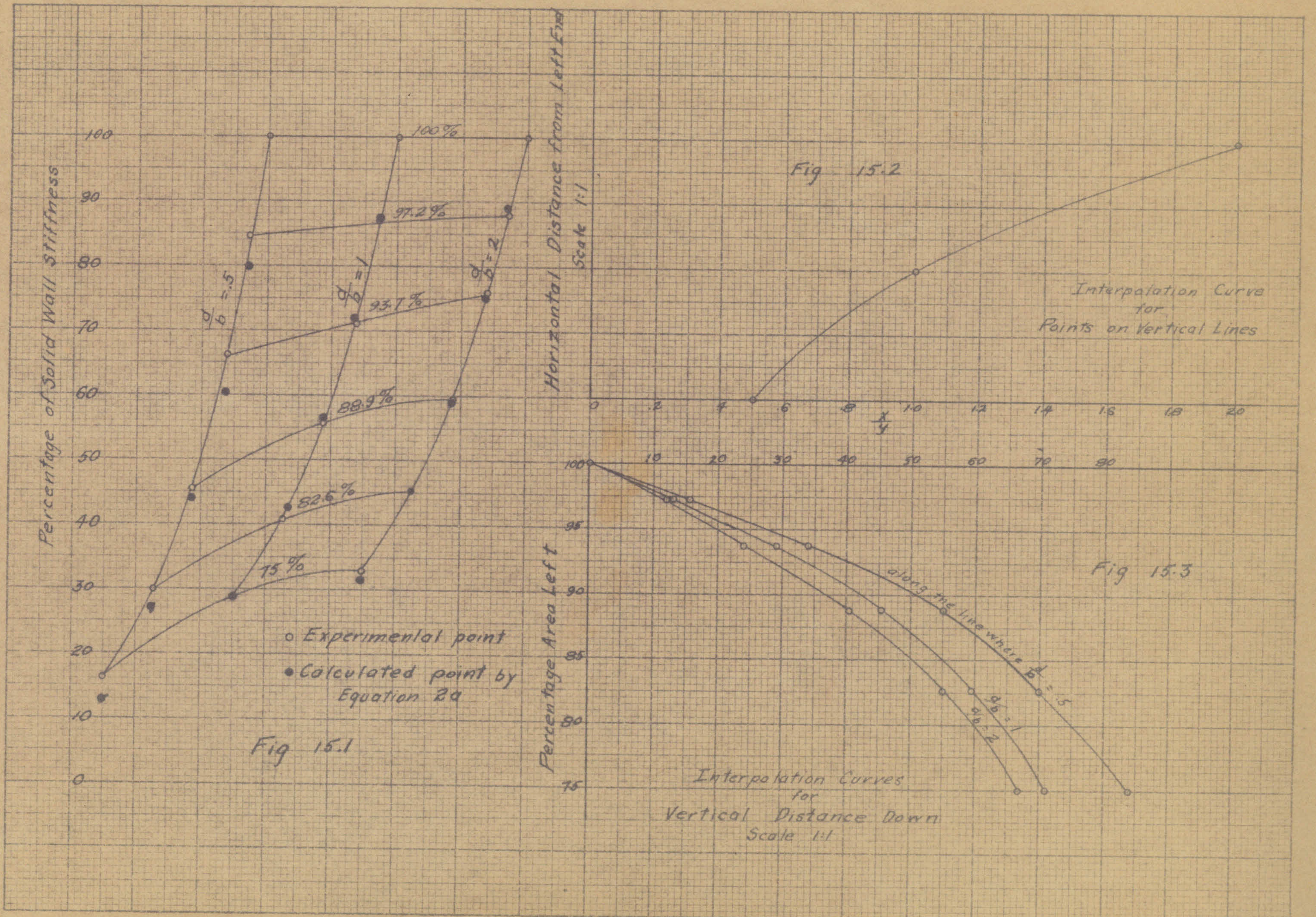
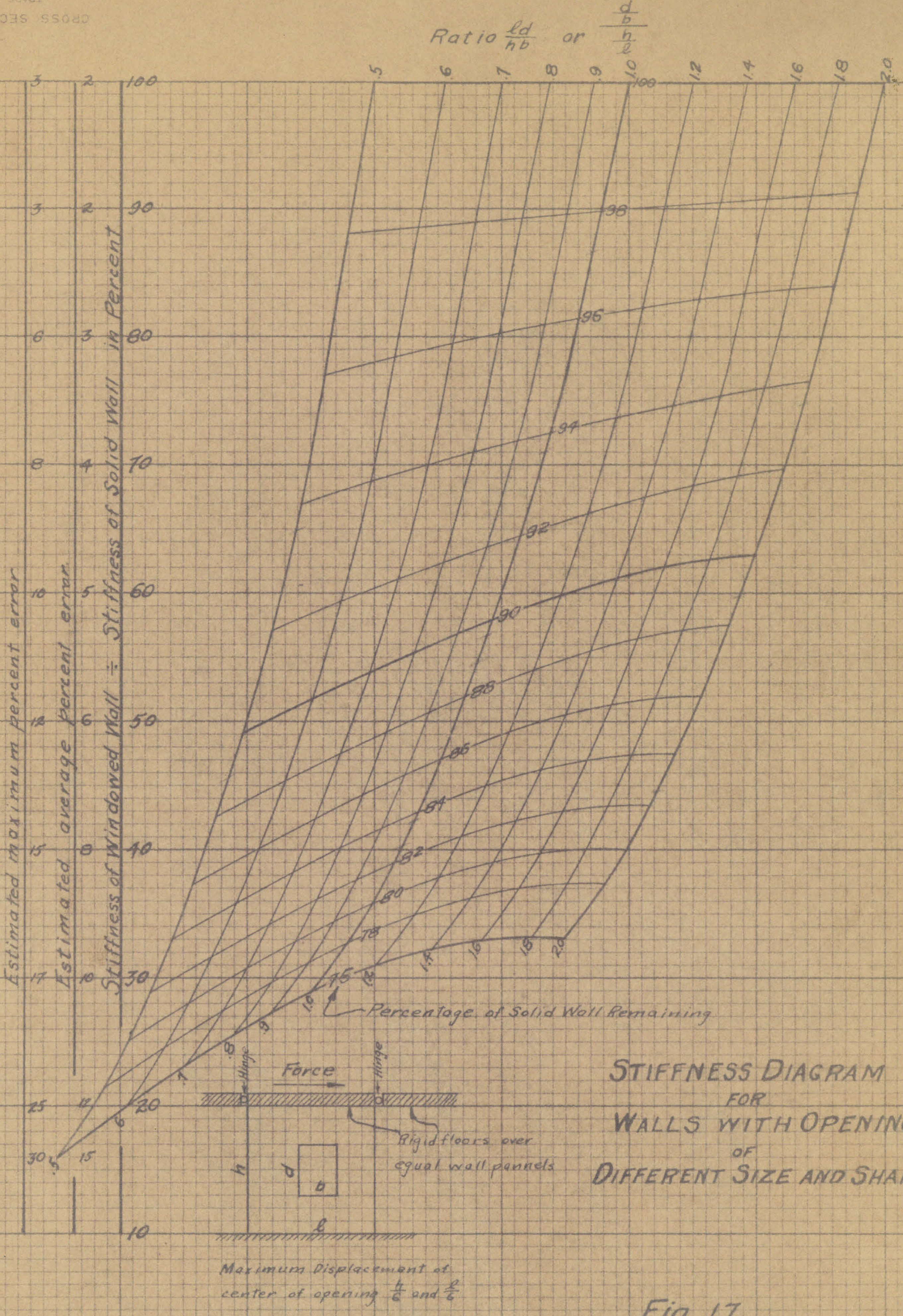
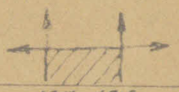

Percentage of Solid Wall Stiffness
100
90
80
70
60
50
40
30
20
10
0
100%
97.2%
93.7%
88.9%
82.6%
75%
$\frac{d}{b}=.5$
$\frac{d}{b}=1$
$\frac{d}{b}=2$
○ Experimental point
● Calculated point by Equation 2a
Fig 15.1
Horizontal Distance from Left End
Scale 1:1
Fig 15.2
Interpolation Curve for Points on Vertical Lines
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
$\frac{x}{y}$
10
20
30
40
50
60
70
80
100
95
90
85
80
75
Percentage Area Left
Fig 15.3
along the line where $\frac{d}{b}=.5$
$d/b=1$
$d/b=2$
Interpolation Curves for Vertical Distance Down
Scale 1:1

Fig 16



STIFFNESS DIAGRAM
FOR
WALLS WITH OPENINGS
OF
DIFFERENT SIZE AND SHAPE

Fig 17

Model No.	Opening				Deflection = .0081 in. Required Force lbs.				Percentage Left $\frac{36-Area}{36}\times100$	Percentage of Stiffness of Solid Wall Lateral force Only		Percentage Stiffness of Solid Wall with loading bar moving horizontally	
	Area (in²)	Position**		Shape									
		x	y										
1	0				5.5	5.6	10.7 10.4	10.0 9.7	100	99	101	103	97
	1	0	0	*S	5.2	5.0	9.3 8.6	8.2 8.5	97.2	94	90	85	85
	2.25	0	0	S	4.65	4.75	8.0 7.4	8.0 7.3	93.8	84	85	73	73
	4.00	0	0	S	3.90	3.90	6.3 5.65	6.0 5.5	88.9	70	70	56	55
	6.25	0	0	S	3.03	3.05	4.3 3.10	3.8 3.8	82.7	55	55	37	38
	9.00	0	0	S	2.00	2.06	2.9 2.6	3.0 2.6	75.0	36	37	26	26
2	0				5.7	5.5	10.0 9.8	10.2 9.5	100	102	98	102	98
	1	1	0	S	5.4	5.35	9.8 9.2	8.4 8.3	97.2	96	95	95	86
	2.25	1	0	S	4.7	4.7	7.0 6.3	6.2 6.5	93.8	84	84	71	68
	4.00	1	0	S	4.1	4.7	7.2 6.3	5.9 5.75	88.9	73	84	65	60
	6.25	1	0	S	3.15	3.15	5.9 4.4	3.5 4.1	87.7	56	56	45	43
	9.00	1	0	S	2.30	2.30	5.1 3.45	2.4 3.2	75.0	41	41	36	33
3	0				5.75	5.75	11.6 11.1	9.6 10.0	100	100	100	105	95
	1	2	0	S	5.6	5.5	11.5 10.1	10.3 10.1	97.2	98	96	96	96
	2.25	2	0	S	4.7	4.8	11.8 9.3	8.0 9.0	93.8	82	84	88	85
	4.00	2	0	S	3.45	3.25	12.6 8.6	5.9 7.7	88.9	60	57	81	73
4	0				5.70	5.75	11.0 10.1	10.2 10.2	100	100	100	100	100
	1	0	1	S	5.5	5.45	9.4 8.8	9.2 8.9	97.2	96	95	87	88
	2.25	0	1	S	4.95	4.85	7.4 7.1	7.25 7.25	93.8	86	85	70	71
	4.00	0	1	S	4.2	4.2	6.8 5.85	5.7 5.5	88.9	73	73	57	54
	6.25	0	1	S	3.0	3.0	3.9 3.8	3.9 4.0	82.7	52	52	37	39
	9.00	0	1	S	2.13	2.11	3.7 2.7	2.8 2.45	75.0	37	37	26	24
5	0				5.75	5.75	11.4 10.4	11.5 10.6	100	100	100	99	101
	1	0	2	S	5.5	5.45	11.4 9.7	10.0 9.4	97.2	96	95	92	90
	2.25	0	2	S	5.00	5.05	9.6 8.6	8.7 8.3	93.8	87	88	82	79
	4.00	0	2	S	4.3	4.3	7.7 7.0	7.6 7.0	88.9	57	58	67	67
6	0				5.75	5.70	10.7 10.3	11.0 10.6	100	100	100	99	101
	1	0	-1	S	5.4	5.5	98 8.8	9.8 9.2	97.2	94	96	84	88
	2.25	0	-1	S	5.0	5.0	7.2 7.0	7.6 7.4	93.8	87	87	67	71
	4.00	0	-1	S	4.0	4.1	5.0 5.3	4.9 5.3	88.9	70	72	51	51
	6.25	0	-1	S	3.04	3.05	3.9 3.8	3.8 3.8	82.7	53	54	36	36
	9.00	0	-1	S	2.28	227	2.95 2.55	3.10 2.70	75.0	40	40	25	26

* Square

** x and y are given in inches from the center of the model

Model No.	Opening				Deflection ≐ .0081 in Required Force lbs.		Percentage Left $\frac{36-\text{Area}}{36}\times 100$	% of Stiffness of Solid Wall Lateral force only	% Stiffness of Solid Wall with loading bar moving horizontally
	Area (in)²	Position x	Position y	Shape	← →	↑ ↑ ← →			
7	0				5.75 5.75	11.2 10.5 10.0 10.2	100	100 100	99 101
	1	0	-2	S	5.60 5.50	10.6 9.0 9.2 9.2	97.2	97 96	91 91
	2.25	0	-2	S	5.10 5.10	8.9 8.0 8.0 8.2	93.8	89 88	79 81
	4.00	0	-2	S	4.75 4.75	7.2 7.2 6.6 6.8	88.9	82 83	65 67
8	0				5.80 5.80	10.3 10.8 9.8 10.3	100	100 100	98 102
	1	-1	+1	S	5.70 5.40	10.0 9.4 9.0 9.0	97.2	98 93	90 89
	2.25	-1	+1	S	4.7 4.9	6.7 8.5 6.6 8.2	93.8	81 84	66 82
	4.00	-1	+1	S	4.0 4.3	5.5 6.7 5.25 6.7	88.9	69 74	52 67
	6.25	-1	+1	S	3.05 3.45	3.85 5.7 3.8 5.2	82.7	53 60	38 52
	9.00	-1	+1	S	2.30 2.65	2.5 4.8 2.65 3.8	75.0	40 47	26 38
9	0				5.9 5.9	11.0 10.6 10.1 10.2	100	100 100	100 100
	1	-2	+2	S	5.75 5.8	9.0 9.5 9.0 10.1	97.2	97 98	89 99
	2.25	-2	+2	S	5.10 5.5	6.0 10.0 7.1 9.7	93.8	86 93	70 95
	4.00	-2	+2	S	3.55 4.9	2.95 9.4 5.1 9.0	88.9	60 83	50 89
10	0				6.0 6.0	11.0 11.0 10.4 10.2	100	100 100	101 99
	1	1	-1	S	5.5 5.45	10.0 9.4 8.9 9.0	97.2	92 91	86 87
	2.25	1	-1	S	4.9 5.0	9.5 7.9 8.0 7.9	93.8	82 83	78 77
	4.00	1	-1	S	4.1 4.1	6.6 6.6 5.9 6.4	88.9	68 68	57 62
	6.25	1	-1	S	3.25 3.25	5.65 4.3 4.35 4.8	82.7	54 54	42 46
	9.00	1	-1	S	2.45 2.55	4.85 3.25 3.20 3.60	75.0	41 42	31 35
11	0				6.0 5.9	11.1 11.5 10.3 10.3	100	101 99	100 100
	1	2	-2	S	5.0 5.1	11.0 10.0 9.0 9.7	97.2	84 86	87 94
	2.25	2	-2	S	4.15 3.7	10.5 10.0 7.85 9.3	93.8	70 62	76 90
	4.00	2	-2	S	2.60 2.55	13.3 8.5 7.2 8.2	88.9	44 43	70 80

Model No.	Opening: Area in²	Opening: Position x	Opening: Position y	Opening: Shape	Deflection = .0081 in Required Force# 	% Left $\frac{36-\text{Area}}{36} \times 100$	% of Stiffness of Solid Wall Lateral Loads Only	% of Stiffness of solid wall with loading bar moving horizontally
12	0				11.0 10.2 5.95 6.0 10.4 10.1	100	100 100	101 99
	1	0	0	H*	9.7 9.0 5.4 5.45 8.65 8.65	97.2	91 91	84 84
	2.25	0	0	H	7.5 6.8 4.8 4.7 6.7 6.55	93.8	80 79	65 64
	4.00	0	0	H	4.65 4.50 3.35 3.35 4.25 4.35	88.9	56 56	41 42
	6.25	0	0	H	2.85 3.05 2.15 2.25 2.50 2.60	82.7	36 38	24 25
	9.00	0	0	H	1.4 1.5 1.13 1.12 1.20 1.22	75.0	19 19	12 12
13	0				10.7 10.0 5.8 5.8 10.1 9.75	100	100 100	102 98
	1	0	0	V**	9.7 9.3 5.5 5.3 8.9 8.6	97.2	95 92	90 87
	2.25	0	0	V	8.5 7.5 4.8 4.85 7.7 7.0	93.8	83 84	78 74
	4.00	0	0	V	5.9 6.2 4.05 4.00 5.5 5.45	88.9	70 69	55 55
	6.25	0	0	V	4.2 3.8 3.15 3.10 4.1 39	82.7	54 53	41 39
	9.00	0	0	V	3.10 3.10 2.45 2.40 2.80 2.85	75.0	42 41	28 28
14	0				10.7 10.7 5.9 5.8 10.2 10.3	100	101 99	100 100
	1	0	1	H	8.8 9.4 5.5 5.45 8.7 9.0	97.2	94 93	85 88
	2.25	0	1	H	7.0 7.0 4.7 4.7 6.7 7.2	93.8	80 80	65 70
	4.00	0	1	H	5.0 4.7 3.35 3.40 4.6 4.7	88.9	57 58	45 46
	6.25	0	1	H	2.80 3.2 2.35 2.45 2.70 2.95	82.7	40 42	26 29
	9.00	0	1	H	1.7 1.7 1.20 1.22 1.34 1.36	75.0	20 21	13 13
15	0				10.5 10.5 5.85 5.75 10.2 10.4	100	101 99	99 101
	1	0	1	V	11.0 8.5 5.5 5.5 9.0 895	97.2	95 95	87 87
	2.25	0	1	V	8.6 8.4 4.8 48 7.75 7.85	93.8	83 83	75 76
	4.00	0	1	V	6.3 7.2 4.2 4.25 5.95 6.4	88.9	72 73	58 62
	6.25	0	1	V	5.60 5.70 3.35 3.35 4.50 4.85	82.7	58 58	44 47
	9.00	0	1	V	4.35 4.75 2.65 2.60 3.35 3.35	75.0	45 46	33 33
16	0	1	0	H	10.6 10.6 5.8 5.8 10.0 10.4	100	100 100	98 102
	1	1	0	H	9.0 7.5 5.05 5.25 8.35 8.3	97.2	87 90	82 81
	2.25	1	0	H	7.2 6.7 4.55 4.25 7.0 6.35	93.8	78 73	69 62
	4.00	1	0	H	6.2 3.7 3.45 3.20 4.95 4.50	88.9	60 55	49 44
	6.25	1	0	H	4.8 2.5 2.40 2.30 3.2 3.3	82.7	40 41	31 32
	8.85	1	0	H	5.70 1.5 .64 .67 2.34 2.12	75.5	11 12	23 21

* H = horizontal opening $\frac{\text{Width}}{\text{height}} = 2$

** V = Vertical " " $= \frac{1}{2}$

Model No.	Opening			Deflection = .0081 in Required Force lbs.		% Left 36-Area / 36 x 100	% of Stiffness of Solid Wall Lateral Loads only	% of Stiffness of Solid Wall with loading bar moving horizontally
	Area in.²	Position x | y	Shape	←→				
17	0			5.9 6.7	9.4 10.4 9.4 10.2	100	102 98	96 104
	1	1 0	V	5.5 5.25	10.2 8.45 9.25 8.45	97.2	95 90	94 86
	2.25	1 0	V	5.00 4.75	8.4 8.0 7.85 7.7	93.8	86 82	80 79
	4.00	1 0	V	4.30 4.10	7.3 5.6 6.50 5.90	88.9	74 71	66 60
	6.25	1 0	V	3.45 3.35	5.8 3.6 4.85 4.5	82.7	59 58	49 46
	9.00	1 0	V	2.65 2.60	4.8 3.05 3.8 3.25	75.0	46 45	39 33
18	0			5.2 5.4	9.8 10.3 9.55 10.0	100	98 102	98 102
	1	0-1	H	4.95 5.0	9.0 8.5 8.4 8.05	97.2	93 94	86 82
	2.25	0-1	H	4.5 4.5	6.5 6.5 6.5 6.35	93.8	85 85	66 65
	4.00	0-1	H	3.0 3.0	3.70 4.15 3.95 4.0	88.9	57 57	40 41
	6.25	0-1	H	2.13 2.25	2.70 2.45 2.55 2.65	82.7	40 42	26 27
	9.00	0-1	H	1.14 1.17	1.04 1.35 1.23 1.26	75.0	22 22	13 13
19	0			6.0 5.75	11.4 10.4 10.3 10.3	100	102 98	100 100
	1	0-1	V	5.35 5.30	9.0 10.0 8.6 9.0	97.2	91 90	83 87
	2.25	0-1	V	4.8 4.7	7.7 7.8 7.55 7.35	93.8	82 80	73 71
	4.00	0-1	V	4.2 4.25	6.0 6.0 5.7 6.0	88.9	72 72	55 58
	6.25	0-1	V	3.5 3.35	4.20 4.15 4.25 4.35	82.7	60 57	41 42
	8.85	0-1	V	2.75 2.70	3.25 3.35 3.25 3.15	75.5	47 46	31 30
20	0			6.0 5.7	10.7 10.0 10.4 9.9	100	102 98	102 98
	1	1-1	H	5.3 5.1	9.4 8.8 8.8 8.6	97.2	90 87	87 85
	2.25	1-1	H	4.3 4.45	8.1 8.5 7.5 7.65	93.8	74 76	66 7.5
	4.00	1-1	H	3.25 3.30	6.3 5.25 4.65 5.20	88.9	56 56	46 51
	6.25	1-1	H	1.92 2.06	5.15 3.80 2.80 4.40	82.7	33 35	28 43
	8.85	1-1	H	.44 .42	5.05 1.8 1.50 2.5	75.5	7.5 7.2	15 25
21	0			5.95 6.0	11.0 10.2 10.2 10.3	100	100 100	100 100
	1	1-1	V	5.45 5.5	10.0 8.5 9.0 8.7	97.2	91 92	88 85
	2.25	1-1	V	5.00 4.75	8.3 7.8 7.55 7.95	93.8	84 80	74 78
	4.00	1-1	V	4.25 4.20	6.6 6.2 6.45 6.5	88.9	71 70	63 63
	6.25	1-1	V	3.55 3.55	6.1 5.1 4.9 5.2	82.7	59 59	48 51
	8.85	1-1	V	2.70 3.00	4.7 4.0 3.75 4.05	75.5	45 50	37 40