IMPULSE VOLTAGES IN WINDINGS

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Foreword

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The author is indebted to those of his former associates of the Westinghouse Electric & Manufacturing Company who pioneered in the development of the technique of cathode ray oscillograph measurements in this country.

Acknowledgment is also due for information and inspiration derived from the analytical writings of L. V. Bewley and the classic papers of K. K. Palueff and his coworkers on transient phenomena in transformers.

The author expresses thanks to Mr. E. L. Bettanier of the Pasadena Municipal Light Department for the loan of three distribution transformers.

ABSTRACT

An outline of the technique of making impulse voltage measurements by cathode ray oscillograph is given. Circuits for operating, timing and synchronizing of oscillograph and impulse generator are discussed. Certain details of the technique of operating an oscillograph and of impulse testing are also mentioned.

The results of experimental determination of the effects of impulse voltages on distribution transformers are given in the form of oscillograms and conclusions derived therefrom. It may be said that, in general, the characteristics of such transformers are similar to those of power transformers.

A brief study of the literature of the subject is included.

A study of the mechanism of transient voltage production in transformers has led to development of a semi-empirical form of analysis which permits calculation of terminal transients after certain empirical constants are obtained by test. The method is essentially that of finding the response of the transformer to infinite rectangular applied wave from a few cathode ray oscillograph tests. By application of the superposition theorem, the response to any arbitrary wave can then be determined.

I

INTRODUCTION

One of the mose interesting branches of the study of lightning phenomena in electrical systems has been concerned with the reaction of transformers to such transient voltages. This is an old problem, but there are still many difficulties to be encountered in its solution.

Most of the work done on the problem has been in relation to design. This is, of course, of major importance, but methods of calculation requiring complete design data are of little use when such data are lacking.

Theoretical analysis has proved to be very difficult, and involves complexities that cannot be dealt with in many practical cases. Approximate methods of analysis have led to practical results for some of the major features of the transient effects, but often lack completeness and definiteness. A means of analysis that could be successfully applied to any transformer encountered, even though no design characteristics were known, would be desirable.

The greater portion of the work of detailed analysis of transient effects in transformers has been in connection with power type transformers. Of this work, the chief concern has been with the transient voltage distributions within the primary winding. Some work has been done on the transmission of impulse voltages through transformers, but this has also dealt primarily with large size transformers.

Since the type of winding construction of power and distribution type transformers differ, the former consisting of stacks of disc or pancake coils, whereas, the latter consist of wire wound homogeneous coils, it was felt that some differences in transient reactions might occur.

An investigation was, therefore, undertaken to clear up some of the points mentioned. A cathode ray oscillograph was available, but was without the auxiliary equipment necessary to apply it to impulse measurements. Furthermore, there was no equipment for generating impulse voltages. Construction of apparatus which would make complete impulse testing possible was, perforce, a part of the problem, and in point of time and effort expended, the major part.

The purpose of the investigation was eight-fold.

(1) To construct a surge generator for production of impulse voltages of any desired characteristics for engineering investigation requiring voltages up to a half million volts.

(2) To construct auxiliary apparatus suitable for the application of the cathode ray oscillograph to the measurement of impulse voltages and for synchronizing its operation with that of the surge generator.

(3) To develop and apply a suitable technique peculiar to the complete assembly of apparatus.

(4) To plan and execute an experimental investigation,

with the aid of the cathode ray oscillograph, of the effects of impulse voltages on distribution type transformers.

(5) To systematically prepare and study the resultant oscillographic data with the view of (a) reaching general conclusions as to the behavior of such transformers, (b) the checking of these conclusions with those of other investigators, and (c) the comparison of the impulse characteristics of distribution type transformers with those of the more thoroughly studied power transformers.

(6) To conduct a survey of the literature of the subject with particular reference to work dealing with analysis of fundamental causes.

(7) To study the mechanism of the transmission of surge voltage through a transformer and of the production of secondary winding terminal transients.

(8) To develop a semi-empirical method of calculation based on data secured from a few simple oscillograph tests, by which the complete transient voltages appearing at the secondary terminals and primary mid-tap could be quantitatively predetermined in all their complexity, for any wave shape of applied voltage or condition of connection.

II

THE MEASUREMENT OF HIGH SPEED TRANSIENTS

An experimental study of any kind must of necessity be concerned with the technique of making measurements. This is particularly true of a study of high speed transients whose total duration is only a few microseconds, because the technique of measuring such quantities is rather involved and the possibilities of error and misinterpretation of results are considerable.

A. Impulse Producing and Measuring Apparatus

The first problem to arise in the present investigation was that of providing sufficient apparatus with which to do the experimental work. A means of producing impulse voltages was the first requirement.

1. Surge Generator

The surge generator constructed is shown in Fig. 1. The condenser banks are made up of glass plates set in frames of maple impregnated with paraffin. The six condenser banks are mounted in a wooden framework in two vertical columns. Charging buses and discharge circuits are located in the space between the two columns, thus providing as short a discharge path as possible and with minimum inductance.

Condenser banks can be connected either in series or parallel. The series arrangement with six banks is capable of producing an impulse voltage with crest value close to

500,000 volts. This connection is the conventional Marx circuit shown diagramatically in Fig. 5. The arrangement of charging resistors in Fig. 5-b is used. The parallel arrangement can be operated at about 70 Kv. Capacitance of the six banks in parallel is about 0.25 microfarad.

The parallel arrangement can be represented by the simple circuit of Fig. 4-a. The wave form produced can be varied by using different numbers of condenser banks in parallel, by inserting inductance, or changing discharge resistance. A convenient form of inductance for this purpose is shown in Fig. 3-d. A variable discharge resistor having taps available is shown in Fig. 3-a.

Wave shapes can be determined if the constants of the surge generator and discharge circuit are known. The capacitance is easily determined by calculation, or measurement with a bridge, or by the charging current when a suitable alternating voltage is applied. The inductance is more difficult to determine, but can be found in two ways. The frequency of the oscillatory discharge of the surge generator when the discharge resistor is short circuited can be measured with a wave meter. From this natural frequency and the measured capacitance, the inductance can be calculated. A better method is to determine frequency from a cathode ray oscillogram taken under the same conditions. Such an oscillogram showing the natural frequency of oscillation of a surge generator after a gap breakdown is shown in Fig. 167-b of Appendix III. These methods are also mentioned by Foust, Kuehni and Rohats.[1]

The calculation of wave shape for the simple circuit of Fig. 4-a is not difficult. This is usually sufficiently accurate for the simple parallel arrangement of condensers. Refinements in wave shape calculation and development of impulse generator equivalent circuits have been treated by Bellaschi[2] and Thomason.[3]

2. Cathode Ray Oscillograph

The cathode ray oscillograph used in the main part of the investigation was one of the high voltage, cold cathode types made by the General Electric Company. This instrument has been fully described elsewhere,[4] so it will suffice to say that it was of the Dufour type. This is the type shown in Fig. 6-a in which no relay or electron block-

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[1] "Impulse Testing Technique," C. M. Foust, H. P. Kuehni and N. Rohats, General Electric Review, July 1932, p. 358.

[2] "Characteristics of Surge Generators for Transformer Testing," P. L. Bellaschi, A.I.E.E. Trans., Vol. 51, Dec. 1932.

[3] "Impulse Generator Circuit Formulas," J. L. Thomason, Electrical Engineering, Jan. 1934, p. 169.

[4] "Cathode Ray Oscillographs and Their Uses," E. S. Lee, General Electric Review, Vol. 31, August 1928.

ing device is used. The cathode beam cannot be continuously generated, but must be produced only during the time of photographing the transient.

Certain oscillograms to be shown representing features in the technique of operation and testing were obtained with a Dufour[1] oscillograph of early type. Others were taken with Westinghouse[2] Dufour or Norinder type instruments shown in Fig. 6, in addition to those from the General Electric oscillograph.

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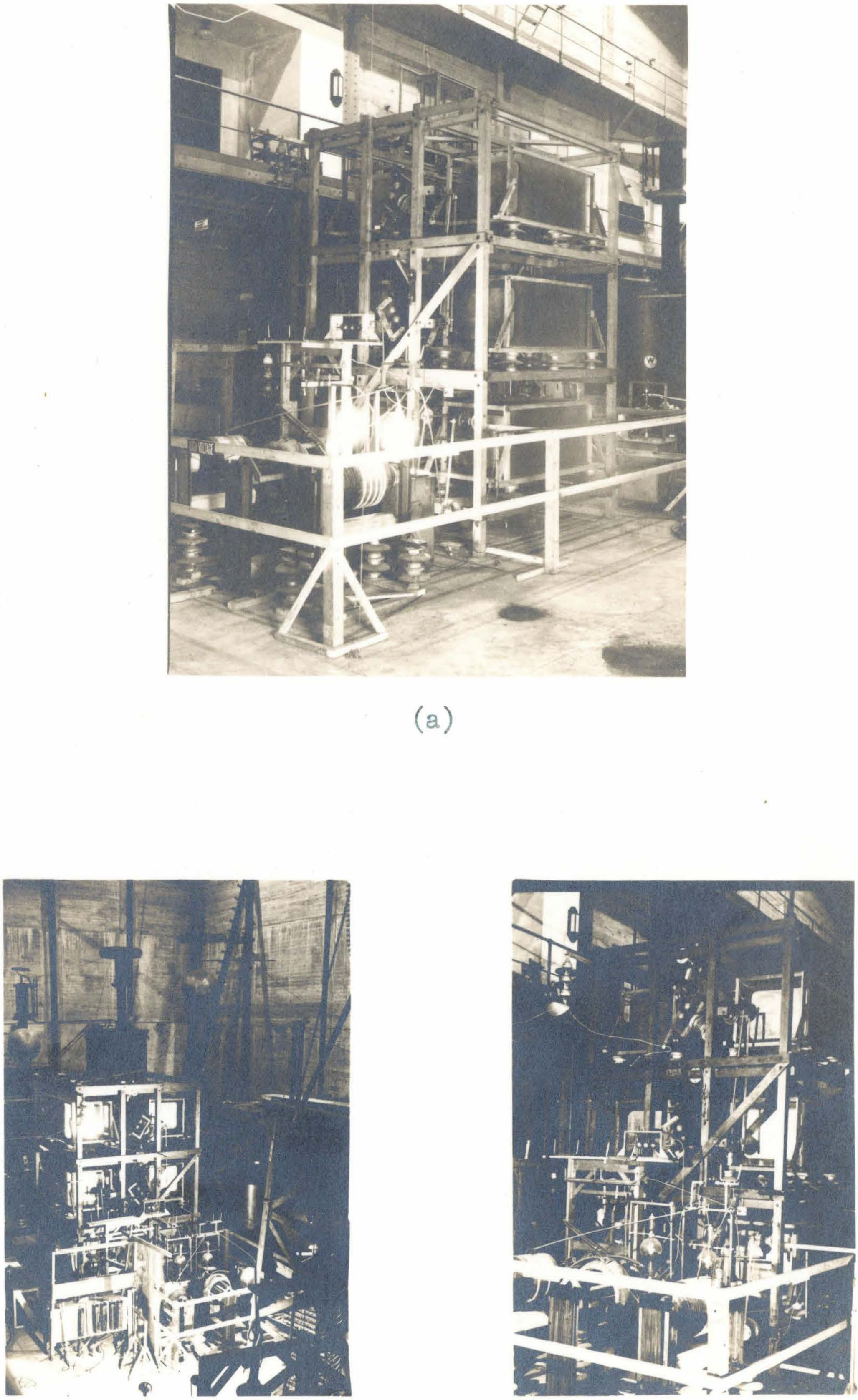
[1] "The Cathode Ray Oscillograph," A. B. Wood, Journal Institution of Electrical Engineers, Vol. 63, Nov. 1925, p. 1051.

See also A. Dufour in L'Onde Electrique, 1932, Vol. 1, pp. 638, 899, and 1923, Vol. 2, p. 19.

[2] "A Cathode Ray Oscillograph with Norinder Relay," O. Ackermann, A.I.E.E. Trans., Vol. 49, April 1930.

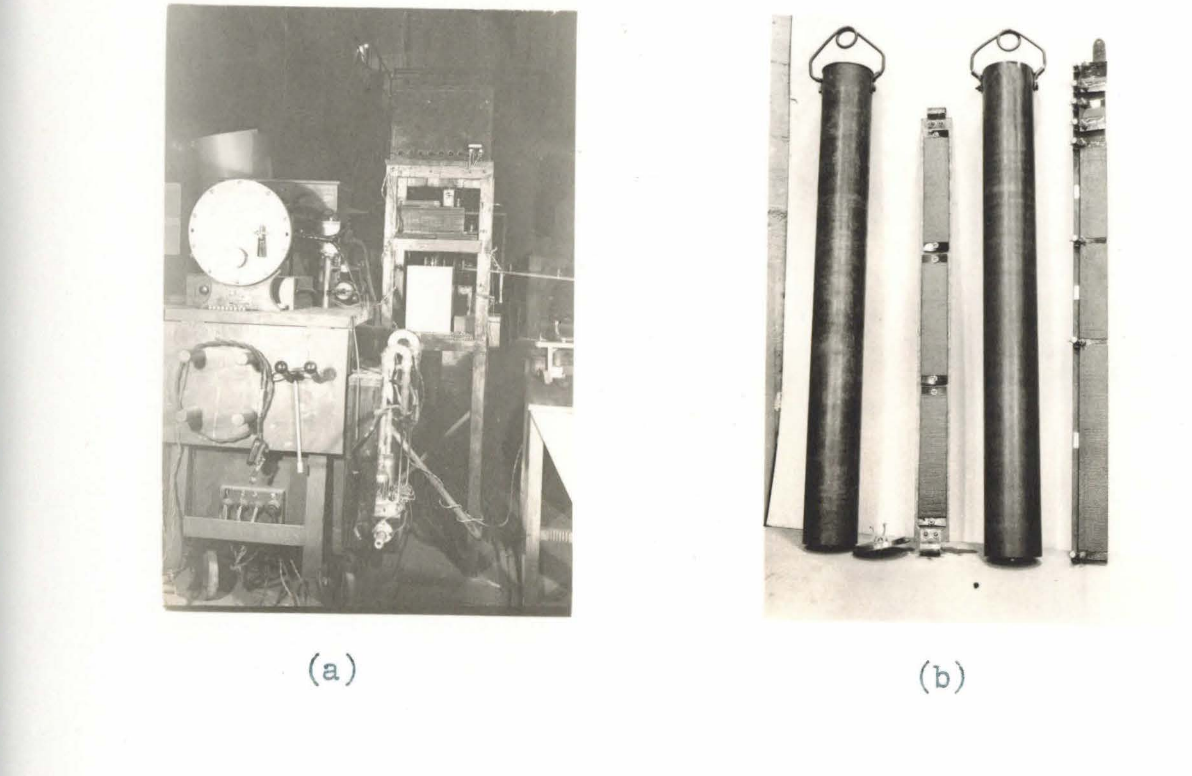


(a)

(b)

(c)

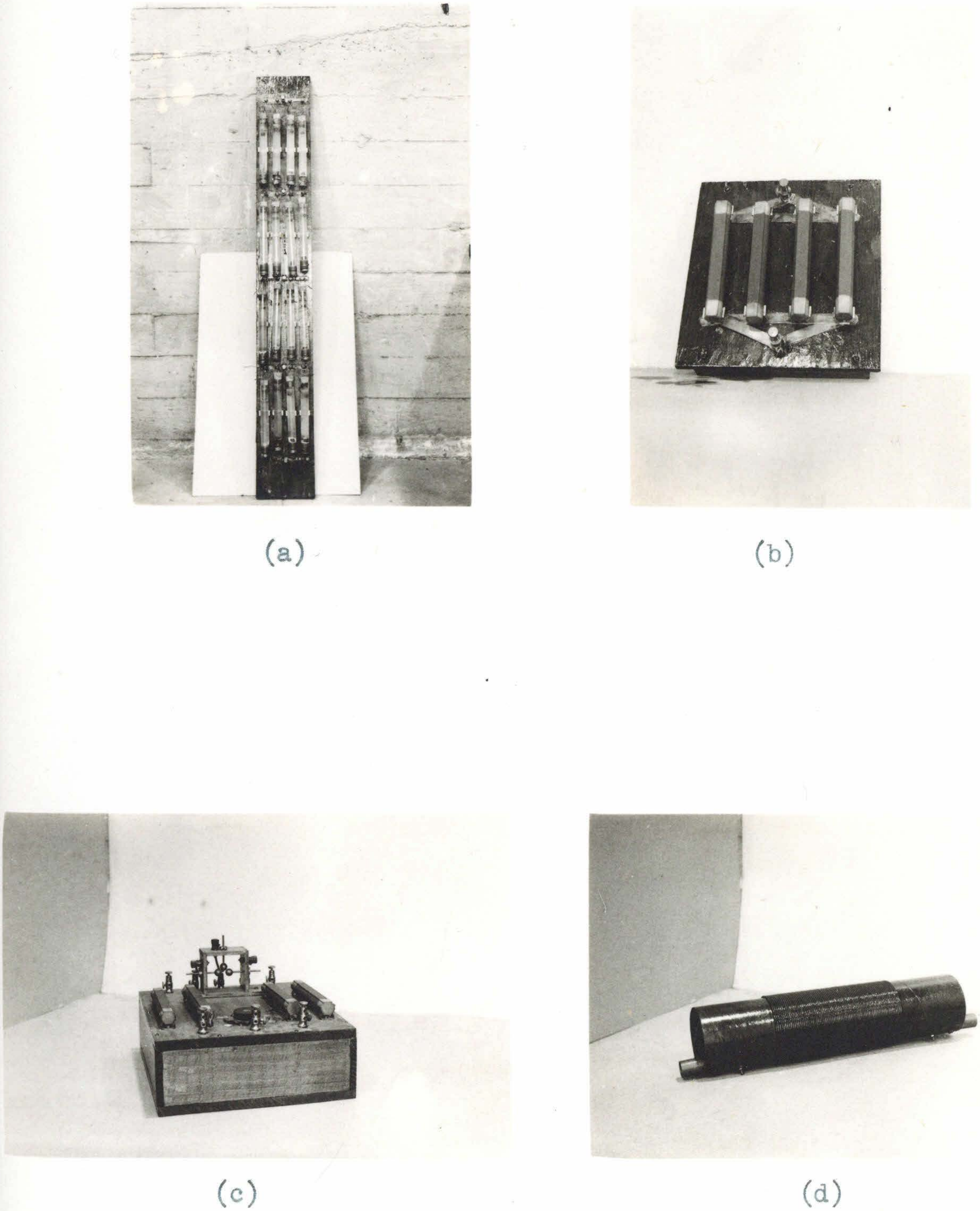
Fig. 1



(a) (b)

(c) (d)

Fig. 2



(a)

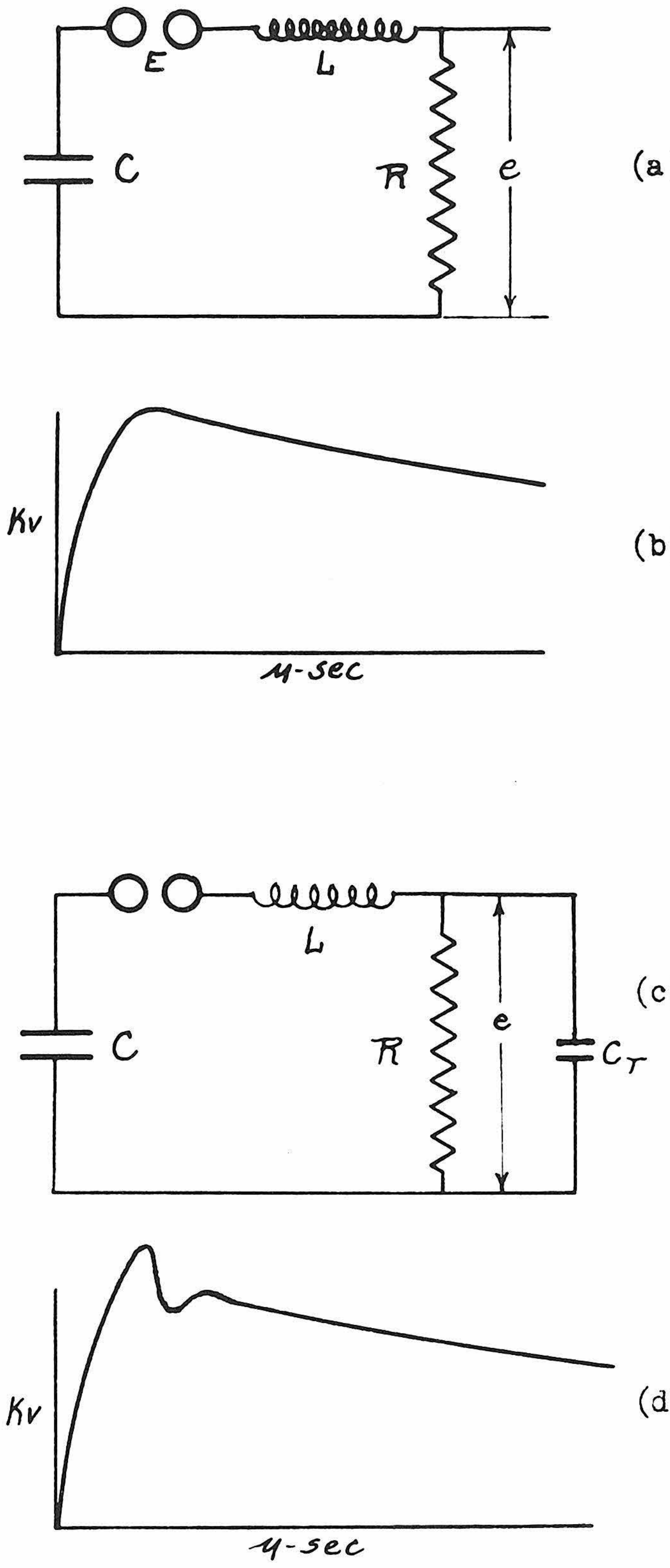
(b)

(c)

(d)

Fig. 3

SURGE GENERATOR CIRCUITS



(a) Elementary Surge Generator Circuit

(b) Voltage Impulse Produced by Simple Circuit

(c) Simple Surge Generator Circuit when Test has Capacity

(d) Voltage Impulse Showing Oscillation Caused by Test Capacity

Fig. 4

Surge Generator Circuits

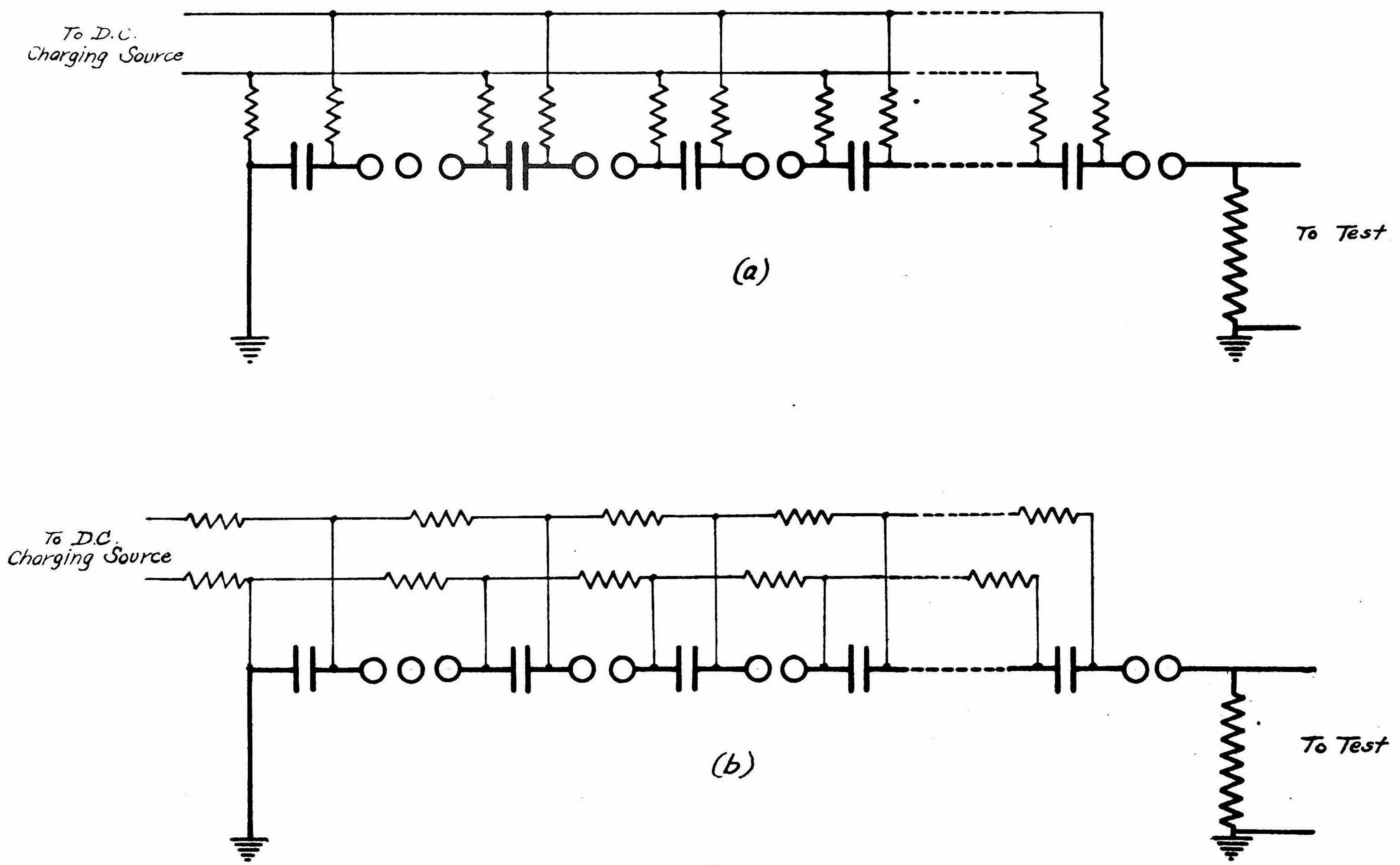
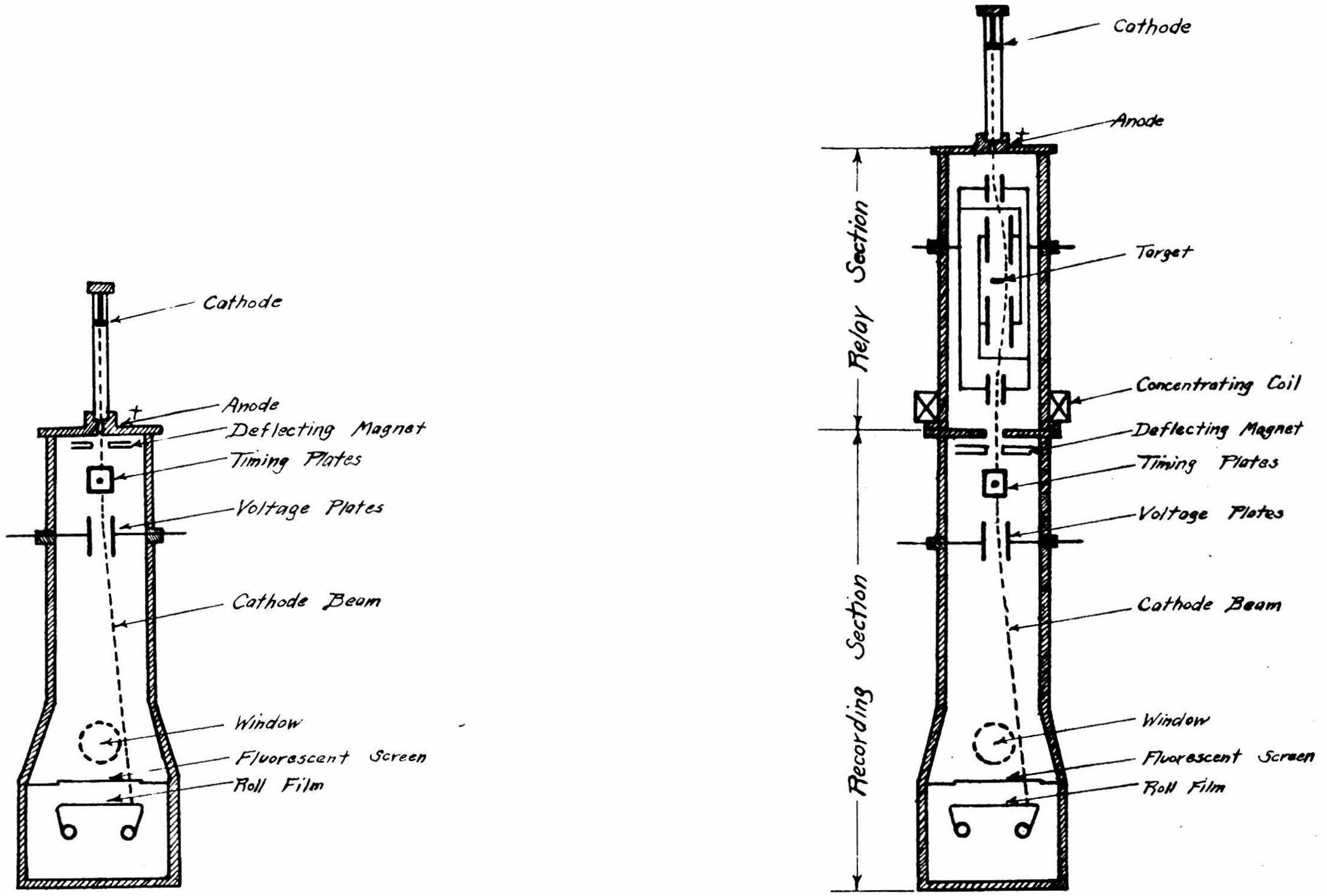


Fig. 5

Westinghouse Cathode Ray Oscillographs



Dufour Type
(a)

Norinder Type
(b)

Fig. 6

B. Synchronizing, Timing and Measuring Circuits

Successful application of the cathode ray oscillograph to surge studies depends upon the use of operating circuits that will accomplish the desired results. The chief problem to be met is that of synchronizing oscillograph, surge generator and timing operations. Numerous methods have been devised for doing this.

1. Operating and Synchronizing Circuits

For work requiring surge voltages up to 60 Kv., a circuit devised by Harrington and Opsahl[1] has proved entirely satisfactory. This circuit, (Fig. 7), for use with Dufour type oscillographs combines oscillograph and surge generator circuits and requires only one charging source. Its operation is as follows: condenser banks C_c, the cathode voltage supply, and C the surge generator capacitance, are charged up to a voltage which causes triple gap G_1 to break down. Breakdown of G_1 applies voltage to the oscillograph cathode and starts the cathode beam. This action also furnishes voltage which, after a short time delay, trips G_2 and applies the surge voltage to the test.

[1] "Technique of the Dufour Cathode Ray Oscillograph," A. M. Opsahl and G. F. Harrington, Electric Journal, August, 1927.

After the record has been obtained by the oscillograph, the charge on C_c is drained off through R_o until the cathode beam goes out. By choosing R_o of proper value, the beam can be stopped a few microseconds after the test impulse is over without affecting the oscillograph calibration during the measurement. The circuit of Fig. 7 is shown set up for taking volt-ampere curves. The problem of time measurement does not occur here. Time measurements can be made without additional synchronizing problems, if an oscillatory time scale is used. Timing systems will be described later.

Successful operation of the circuit depends largely upon the gap settings and values of resistances R_3 and R_4. The left side of G_1 is the master gap which determines the oscillograph calibration and surge voltage. The right side of G_1 should be set a little above one-quarter of the master gap voltage setting. If the master gap setting is 60 Kv., the left and right sides of G_2 should be approximately 40 and 20 Kv. respectively. The exact setting should be determined by trial until proper operation is obtained.

The ratio of resistances R_3 and R_4 is more important than their magnitudes. This ratio should be about 1 to 3 or 1 to 4. The smaller resistance of the two should not be less than 200,000 ohms. Representative values of other resistances in the circuit may be listed as follows, the proper value depending on the nature of the test:

Resistance	Ohms
R_0	20,000 to 100,000
R_1	40,000 to 150,000
R_2	2,000,000 to 10,000,000
R_3	200,000 to 1,000,000
R_4	700,000 to 3,500,000
R_5	10,000 to 20,000

The use of this circuit with a Marx circuit surge generator is shown in Fig. 8. The same type circuit with modifications for making impulse tests on windings with the Westinghouse Norinder type oscillograph, equipped with electron blocking relay, is shown in Fig. 9. This is the circuit used to take the oscillograms of power transformer primary transients included under Part III.

2. Timing Circuits

An additional problem in synchronizing arises when records showing time variation of voltage or current are desired. Since this is usually the case, a consideration of timing circuits is necessary.

a. Oscillatory Time Sweep. If an oscillatory voltage of high frequency were continuously impressed across the oscillograph time deflecting plates, a record of voltage or current against time would be obtained when the circuit of Fig. 7 was tripped. Examples of this type of record are shown in Fig. 27.

b. <u>Unidirectional</u> <u>Time</u> <u>Sweep</u>. Some of the requirements of a single time sweep for recording transients have been listed by Burch and Whelpton[1] as follows:

(A) Time sweep must be tripped by a conductively conveyed impulse with as little time lag as possible. Voltage and charge required for tripping should be as small as possible.

(B) The time scale should not be distorted by currents which come from the tripping impulse.

(C) Oscillograph time deflecting plates should preferably be equally and oppositely charged.

(D) The beam must not return across the film.

(E) The time sweep circuit must be non-oscillatory.

Several time sweep circuits are shown in Figs. 10 and 11. The timing circuit in Fig. 10-a and also in Fig. 9 is particularly valuable in the impulse testing of windings where it is desirable to record several voltages on a single film for comparison. To do this requires exact and unvarying synchronization of impulse voltage application and oscillograph time scale. Such a result is practically impossible to attain, if the starting of the time sweep depends on the breakdown of a gap. Examination of this circuit shows that exact synchronizing cannot be avoided.

Time sweep circuits utilizing a separate source of

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1. "Technique of the High Speed Cathode Ray Oscillograph," F. P. Burch and R. V. Whelpton, Journal Institution of Electrical Engineers, August, 1932, p. 380.

voltage and being tripped from the surge generator are shown in Fig. 10 and 11. These circuits are subject to the erratic time lags of the trip gaps, but have many advantages and are of most general use. In connection with the breakdown of the trip gaps, it may be well to mention that, although the normal time lag of a sphere gap is usually considered to be about 10^{-8} seconds, the time lag of a pre-stressed gap often seems to be quite large.

The arrangement of oscillograph, voltage divider, and time sweep apparatus is shown in Fig. 2-a. The timing resistor panel and trip gap is shown in Fig. 3-c.

3. Measuring Circuits

Measuring circuits will be considered in the section on voltage dividers.

Cathode Ray Oscillograph Circuit

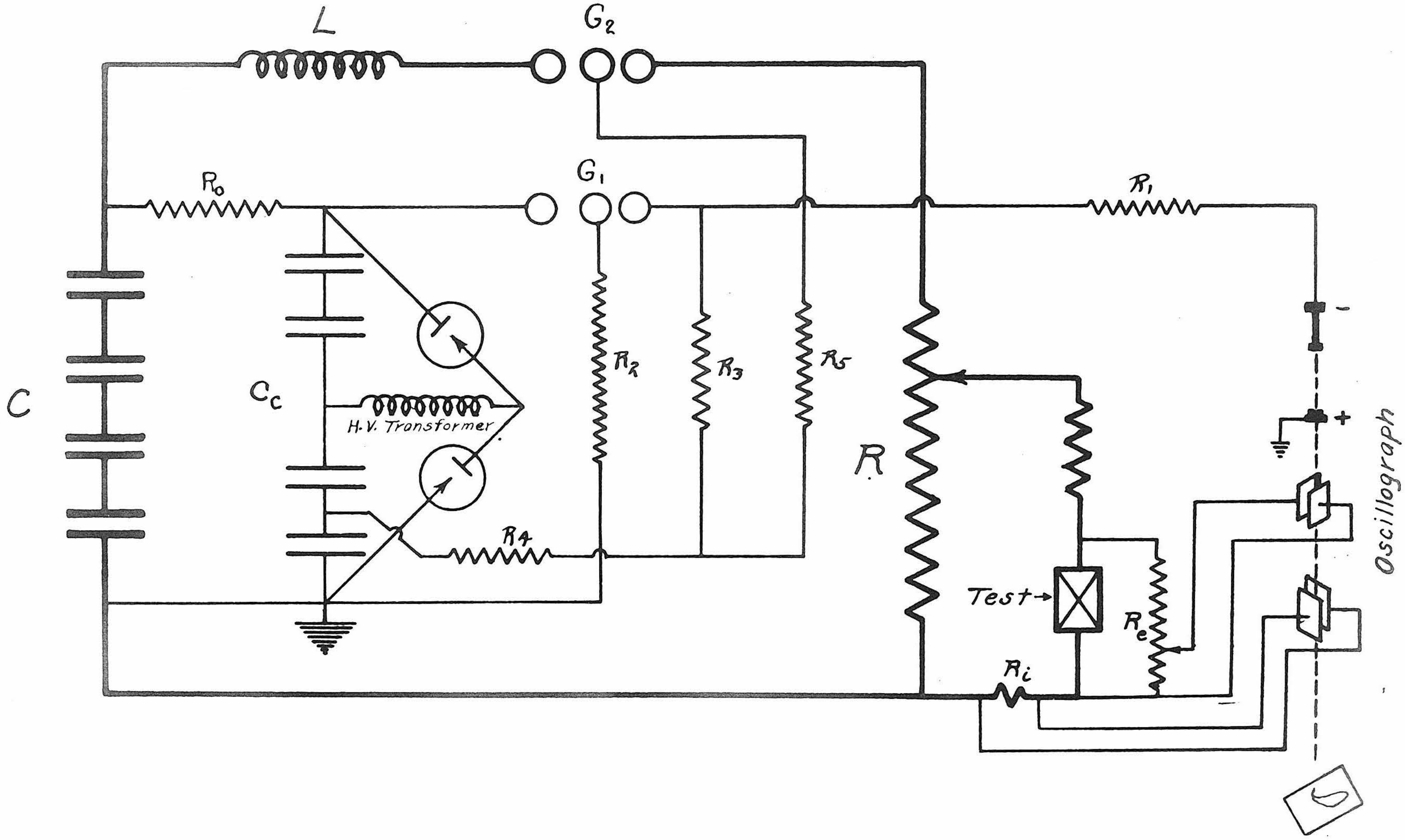


Fig. 7.

Cathode Ray Oscillograph Circuit

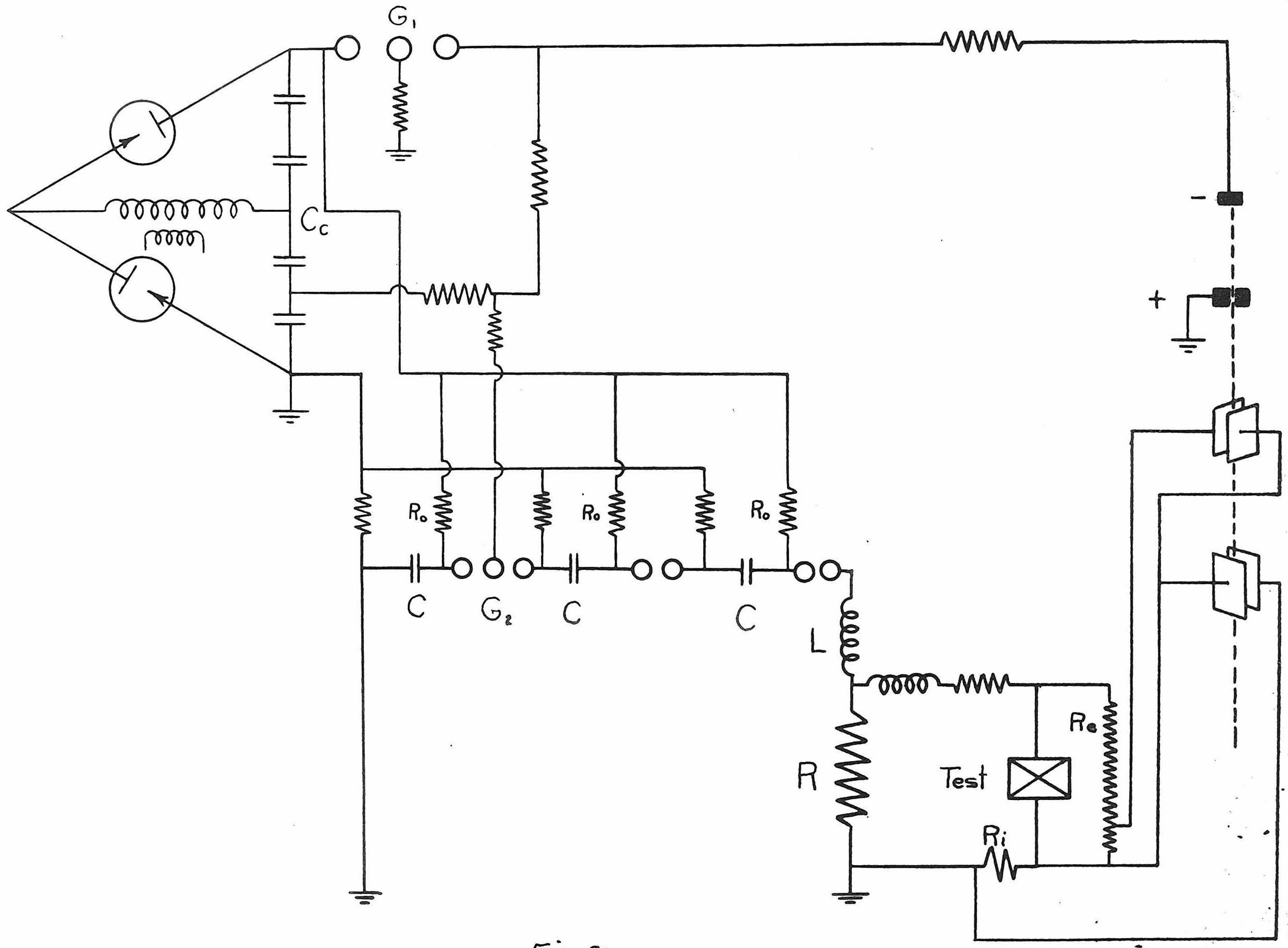


Fig. 8

Cathode Ray Oscillograph Circuit
For Impulse Voltage Tests on Windings

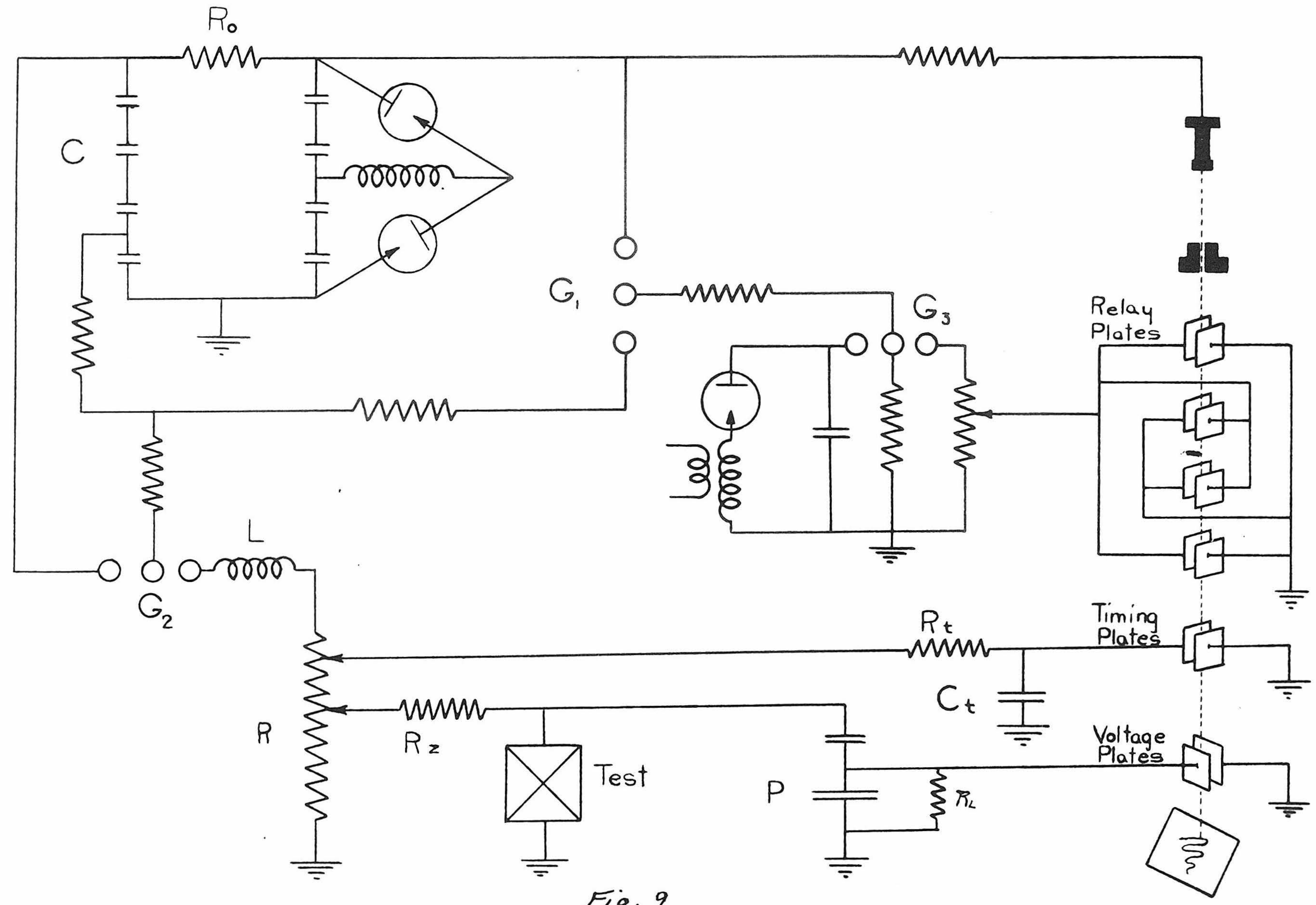
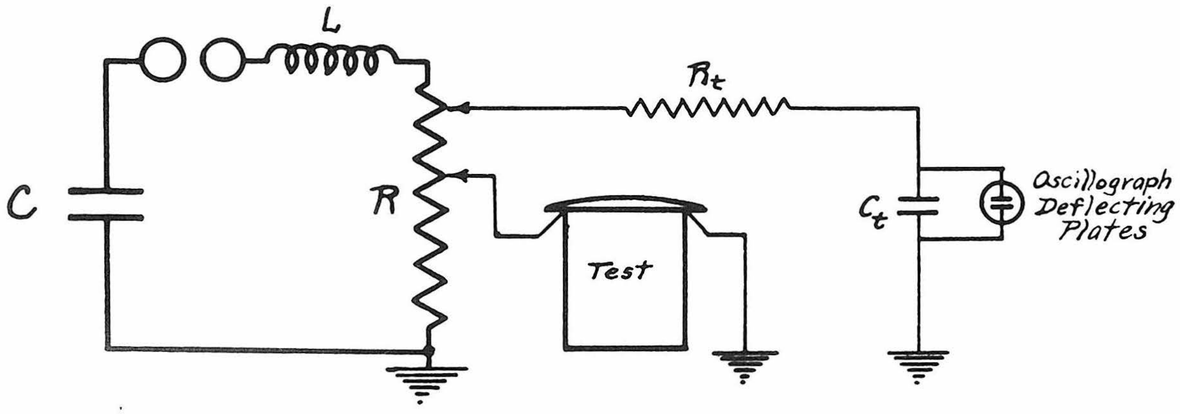
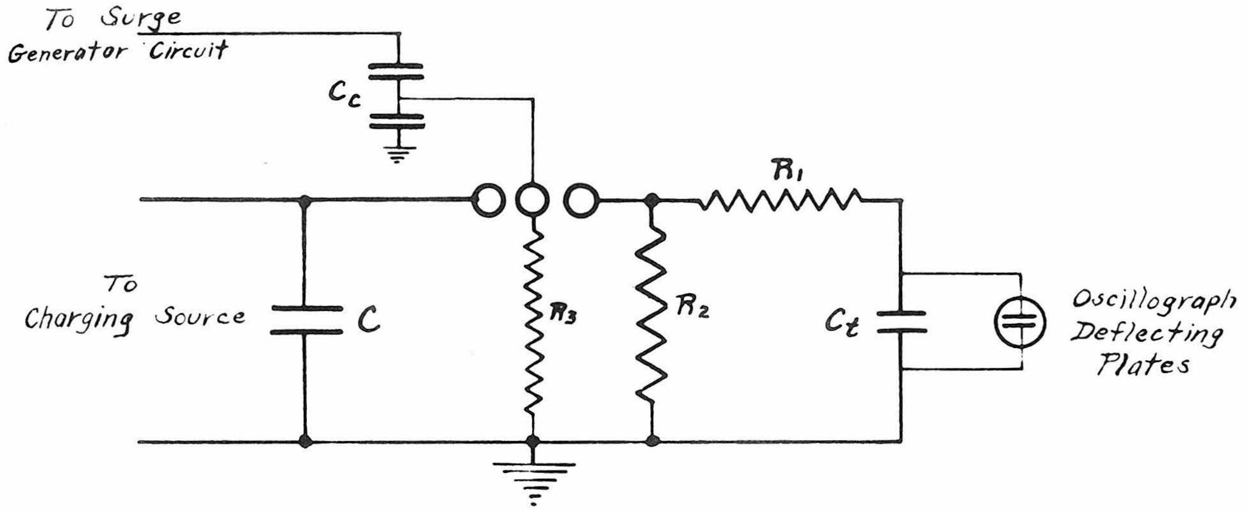


Fig. 9

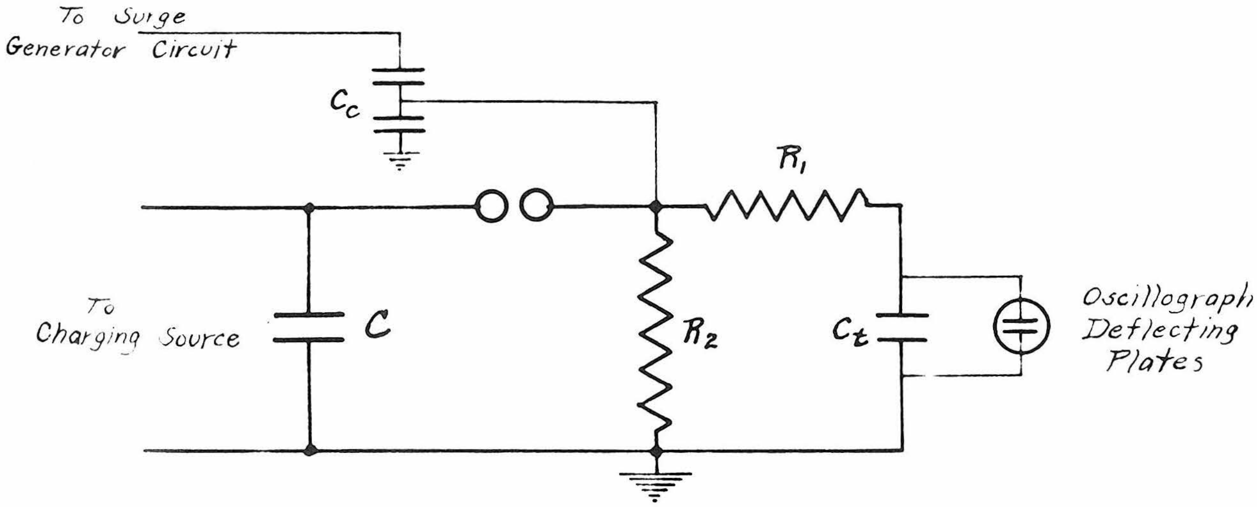
TIME SWEEP CIRCUITS



(a) Surge Derived Time Sweep



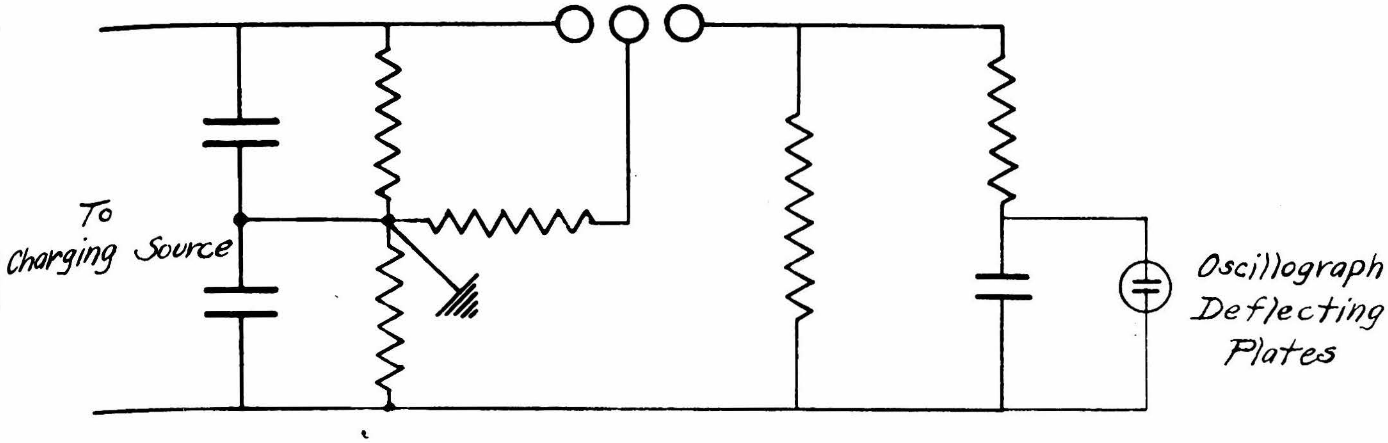
(b) Independent Time Sweep



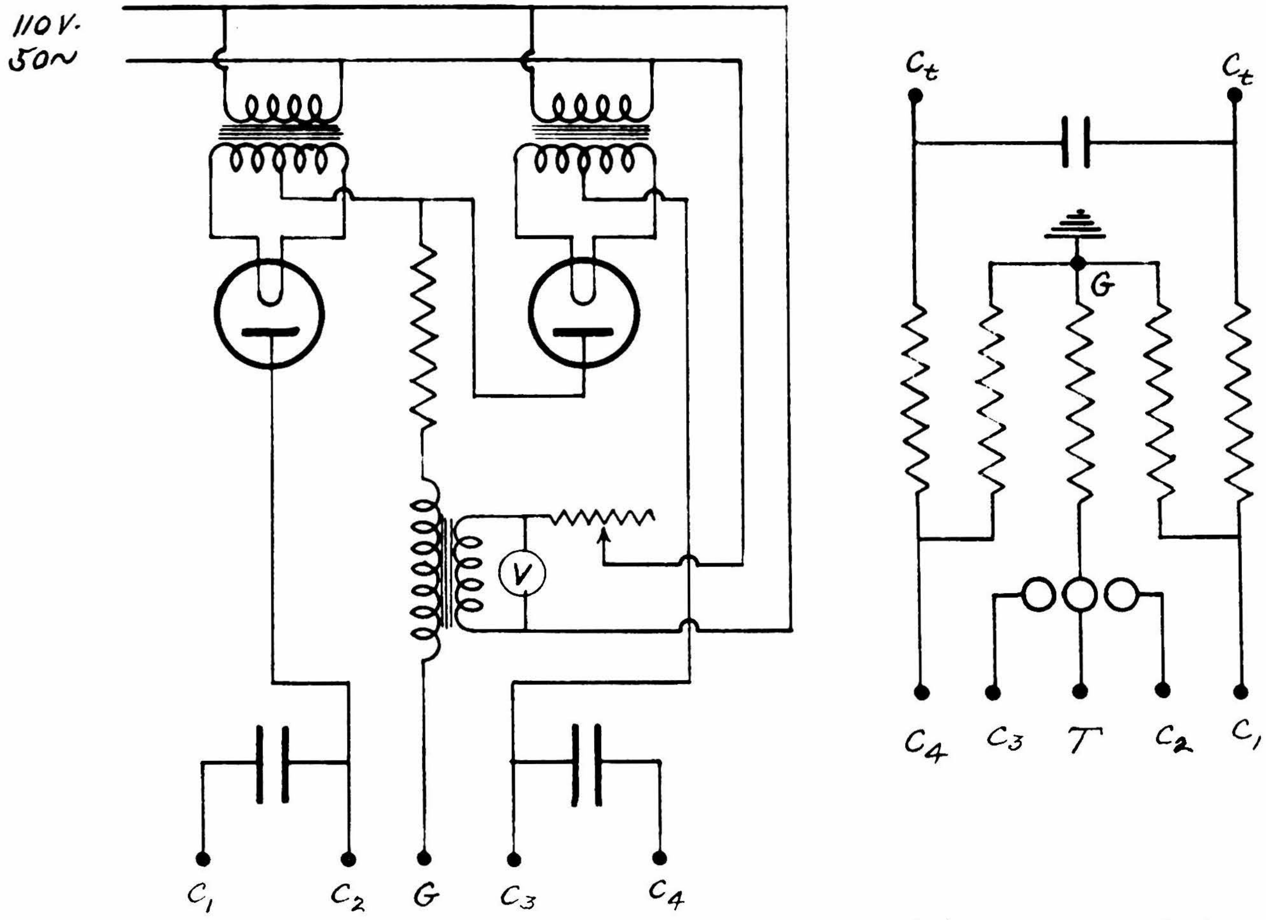
(c) Independent Time Sweep

Fig. 10

TIME SWEEP CIRCUITS



(a) Schematic Diagram of Unidirectional Time Sweep Circuit. Responds to Tripping Impulse of Both Polarities.



(b) Actual Connections Timing Rectifier

(c) Timing Resistor Panel

Fig. 11

C. Technique of Impulse Voltage Measurements

In connection with a discussion of technique, it is well to bring attention to the condition found by Peters, Blackburn and Hannen[1] who have stated that "in the literature of cathode ray oscillography there has been a disappointing lack of detail regarding the theory and technique of measurements."

Lack of time forbids more than a superficial treatment of the subject, but some remarks will be made concerning various features of oscillograph work. Some of the details mentioned may appear trivial, but they might prove puzzling to one unaccustomed to this type of work.

1. Operating Technique

a. Production of the Cathode Ray. It has been found necessary to provide a pre-ionizing potential on the cathode before the full cathode voltage is applied in order to eliminate erratic starting of the beam. This is the reason for the connection to the quarter point on the condenser bank

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[1] "Theory of Voltage Dividers and their use with Cathode Ray Oscillographs," M. F. Peters, G. F. Blackburn, P. T. Hannen, Bureau of Standards Journal of Research, July 1932, p. 114.

shown in Fig. 7. Erratic starting of the cathode beam and of the synchronizing circuit produces delay in initiation of the beam as shown in Fig. 19.

Under some conditions this initial voltage produces a fogging effect as shown in Fig. 20-a. This can be avoided if the time sweep starts off the film to the left. Use of lower operating pressure in the oscillograph also reduces this fogging.

Burning of the cathode by positive ion bombardment pr produces a wide fuzzy line as in Fig. 26-b or a triple line as in Fig. 18-c. This is due to a hollow beam with central core as shown in Fig. 18-b. Laszlo and Cosslett[1] have explained this as the image of the crater in the cathode produced by the burning. Fig. 18-a shows the effect of dirt or impurities in the glass of the cathode tube. Static sparks to such a spot produced the jagged line obtained.

Focussing of the beam in the oscillograph used was by the ionic method[2] depending on vacuum adjustment. Focus seems to be impaired by wide angle of deflection in an assymetrical field. This effect shows in Fig. 23-b.

b. <u>Operating Circuit Faults</u>. Failure of the synchronizing circuit to function properly results in delay of ca-

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[1] "Rectilinear Propagation and Diffraction of Electrons," Henry de Laszlo and V. E. Cosslett, Nature, Vol. 130, 1932. p. 59.

[2] "Measurements in Electrical Engineering by Means of Cathode Rays," J. T. MacGregor-Morris and R. Mines, Journal Institute of Electrical Engineering, November 1925.

thode beam as in Fig. 19-a. Improper values of resistances R_3 and R_4 of Fig. 7 is usually the cause. Erratic action of time trip gaps may be seen in oscillograms such as Fig. 26-a. Return of the beam across the film on the reverse time sweep occurs in Fig. 19-b. Proper adjustment of resistors in the timing and synchronizing circuits and proper vacuum adjustment will cause cut-off of the cathode beam before the reverse sweep reaches the film. This adjustment depends largely on resistor R_0 of Fig. 7.

c. Distortion. Distortion of recorded voltage comes from several sources. Drainage of charge from the cathode condenser bank changes the sensitivity of the oscillograph if this is allowed to take place before the measurement is completed. The increase in oscillator deflections in Fig. 19-b and Fig. 20-b is due to this.

Distortion of records also results from curved or non-parallel coordinates. Fig. 21-a and Fig. 20-b show such defects. These effects are most pronounced at maximum values of deflection and are caused largely by the electrostatic fields outside the region between the deflecting plates. Crude sketches of these fields as in Fig. 12 may indicate the nature of the distortion caused. For example, in Fig. 12-a an examination of the field shows that the coordinate lines would tend to converge as the beam moved from plate a to a point near plate b. This convergence would be intensified by decreased beam sensitivity due to the added velocity gained by the beam electrons falling through a greater potential change when near plate b than when near plate a. The latter effect has been considered by Roman and Whitehead.[1]

Another effect of the stray electrostatic fields in the oscillograph deflecting system is the initial reversal of time sweep. A particularly bad case of this is given in Fig. 22-a. In this case both voltage deflecting plates were at high potential above the oscillograph case. Again distortion is greatest when the time sweep starts from the extreme edge of the film. As would be expected, no distortion occurred when the beam started from the center of the film. Fig. 22-b proves this.

Distortion may also be caused by external fields. Electrostatic fields are easily eliminated by shielding. Magnetic fields are more difficult to eliminate, but are not so frequently encountered. Distortion from the magnetic effects of the surge generator discharge current are shown in Fig. 27-a and Fig. 27-b. The oscillator deflection is greater or less than normal on the wave front. This deflection should be the same as shown in Fig. 27-c and Fig. 27-d. This type of distortion is difficult to detect when a unidirectional time sweep is used.

There are also other causes of distortion such as the effect of the finite speed of the electrons in the beam,

[1] "The Cathode Ray Oscillograph - An Engineering Tool," E. R. Whitehead and W. G. Roman, Electric Journal, April 1934, p. 156.

and the time required to charge the capacitance of the deflecting plates through the surge impedance of the leads. These points have been thoroughly discussed by Mines[1] and by Klemperer and Wolff.[2]

2. Test Technique

a. Calibration. Voltage calibrations are most easily and directly made by means of a sphere gap. Accuracy is good if, (a) the spheres are polished with fine sandpaper; (b) the spheres are used at spacings not much greater than half a diameter; and, (c) the breakdown occurs on the flat crest of the wave. Oscillograms of such gap breakdowns are shown in Fig. 24-a and 24-d and in Fig. 26-a. Gaps shown in Fig. 2-c are suitable up to 60 Kv.

Calibration can also be made by applying a D.C. voltage to the deflecting plates and measuring the deflection as in Fig. 23-a. If the voltage divider ratio were known, the calibration of the whole could be determined. A resistance divider is generally the only one whose ratio is readily determined, however, so the D.C. method of calibration is limited to this type divider.

[1] "The Electron Jet as a Measuring Device," R. Mines, Journal Institution of Electrical Engineers, Vol. 63, Nov. 1925, p. 1096.

[2] "Die Verzerrungen im Kathodenoszillographen bei Hohen Messgeschwindigkeiten," H. Klemperer and O. Wolff, Archiv fur Electrotechnik, July 5, 1932, p. 495.

Calibrations of other type dividers can be made by comparison with a resistance divider of known ratio by taking an oscillogram of the same voltage with each divider connected to a pair of deflecting plates. The slope of the line gives the ratio of calibrations. Fig. 24-b shows such a comparative calibration.

Time sweep calibrations are made by applying a known high frequency to the voltage plates and making a record of the oscillatory voltage.

b. Voltage Measurements. Voltage dividers must be connected at the points between which voltage is desired. If the divider ground lead were connected to a different point on the ground system from the point where the test apparatus is grounded, an error may result. The surge generator discharge may raise the potential of the whole ground system. Voltages between two points on a ground bus a few feet apart may be quite high due to the inductive drop.

Spurious oscillations are always a source of trouble. These oscillations arise in the leads from test specimen to voltage divider and from divider to oscillograph. The chief remedy for such oscillations is in the use of the shortest possible leads.

Damping resistors may be used in some cases, but must not be used without careful checking of their effect. Damping resistors are unable to distinguish between spurious and genuine high frequency oscillations and may, therefore, eliminate part of the actual voltage which should be recorded. The effect of series damping resistors with a capacity divider, connected as in Fig. 13-b, can be seen in Fig. 25 and 26-a. Resistance voltage dividers are best suited for the use of damping resistors.

The test apparatus must also be shielded from the surge generator if it is subject to the effect of electrostatic fields.

c. Time Scales. The method of using the voltage across a charging condenser for a time sweep in general gives a logarithmic time scale. If the lower part of the charging curve is used the scale will be nearly linear, however. Fig. 14-a and 14-b show this.

A time scale with the condensed portion at the start is possible, if the timing condenser is charged by a voltage wave with slow front. Fig. 14-c shows how this may result and Fig. 99-a gives an oscillogram taken under these conditions.

d. Equivalent Test Circuits. It is customary to use lumped resistance in a test circuit to give the effect of line surge impedance. This is permissable under ordinary conditions. The conditions for equivalence and the approximations involved have been discussed by Boehne[1] and

[1] "Voltage Oscillations in Armature Windings under Lightning Impulses," E. W. Boehne, A.I.E.E. Trans., Vol. 49, October 1930. Appendix A, p. 1602.

Bewley.[1]

3. Voltage Dividers

a. Capacity Voltage Divider. The capacity divider must be used when drainage of charge from the test specimen is not permissable. Its capacity must be small compared with that of the test specimen. Such a divider must be well shielded from external electrostatic fields. For voltages below 50 Kv. a divider made up of glass plates in a shielded box, as in Fig. 2-d, is satisfactory.

The divider arrangement of Fig. 15-b has the advantage of producing a more symmetrical electrostatic field within the oscillograph deflecting system, but cannot be used on voltages such that the potential of deflecting plates is too high above that of the oscillograph case.

Differences in potential can be measured by use of two dividers of equal ratio connected as in Fig. 15-c. Equality of ratio can be checked by making records of the same voltage with each divider separately and then with the same voltage applied to both simultaneously. The results of such a test for equality of divider ratio is given in Fig. 23-b.

Capacity dividers usually require a leak resistor to prevent the isolated deflection plate from picking up a

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[1] "Discussion," L. V. Bewley, A.I.E.E. Trans., Vol. 50, March 1931. p. 67.

negative charge from the electron stream. Such a charge gives a shifting zero line as shown in Fig. 24-c. The leak must have a high enough resistance to prevent draining off charge due to the impulse to be measured.

The characteristics of capacity dividers and various forms of capacitance-resistance dividers are treated by Peters, Blackburn and Hannen[1] and by Rogowski, Wolff and Klemperer.[2]

b. Resistance Voltage Dividers. Resistance dividers are usually more satisfactory where drainage of charge is of no consequence. A satisfactory form of resistance divider is shown in Fig. 2-b. This consists of a wire wound resistor, wound on a thin strip of insulating material in a double layer, the parts of which are wound in opposite directions and connected in parallel. This forms a resistor which has negligible inductance.

The resistance of such a potentiometer must not be so high as to introduce errors due to the time required to charge the capacitance of the deflection plates. This limits the maximum resistance permissible to about 10,000 ohms for ordinary work.

[1] "The Theory of Voltage Dividers and their use with Cathode Ray Oscillographs," M. F. Peters, G. F. Blackburn, P. T. Hannen. U. S. Bureau of Standards Journal of Research, July 1932. p. 81.

[2] "Die Spannungsteilung bein Kathodenoszillographen," W. Rogowski, O. Wolff, H. Klemperer, Archiv fur Elektrotechnik, Vol. 23, 1929-30. p. 579.

Inductance in the divider can be detected by taking the volt-ampere characteristic of a water tube. If the record has a loop as in Fig. 21-b, the indication is that something is wrong in the measuring circuit.

The wire wound resistor has capacitance between turns and to ground so that it actually is a resistance-capacitance network as shown in Fig. 16-c. The response of such a network has been investigated by Bellaschi.[1]

c. Impedance-Cable Divider. The problem of synchronizing oscillograph, time sweep, and surge generator operations requires a time delay between tripping of oscillograph and time sweep by the surge generator and the arrival of the voltage at the deflecting plates. This may be accomplished by use of a delay cable in the measuring circuit. The resistance-cable divider of Gabor[2] is shown in Fig. 17-a. The resistance R_2 is not always used.

The cable divider has the advantage of allowing the oscillograph to be located at a place far enough from the surge generator to avoid trouble from external influences.

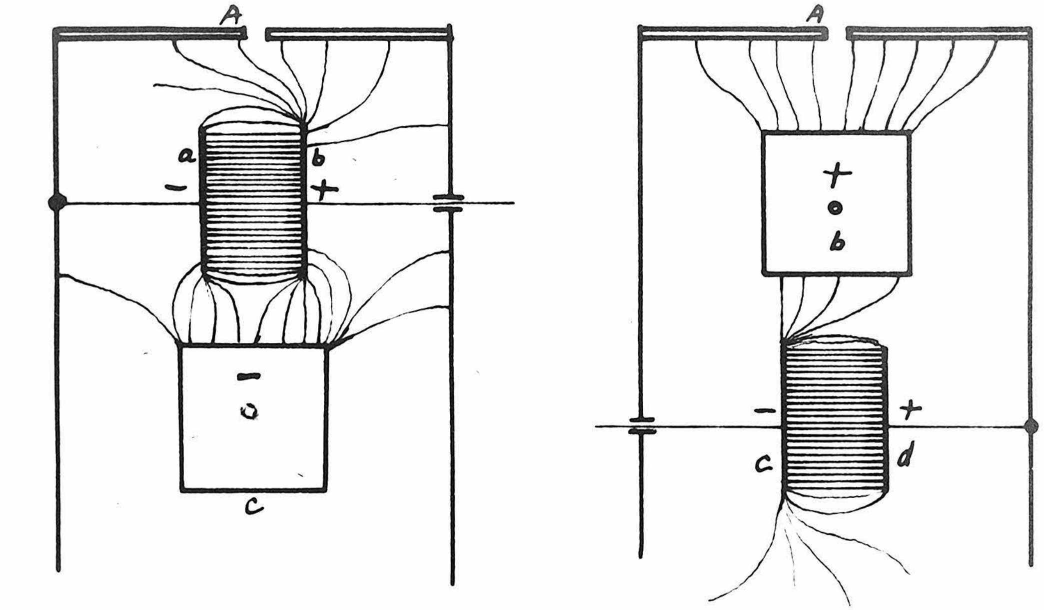
The characteristics of the cable type divider with terminal impedances, consisting of combinations of resistance and capacity, have been studied by Burch.[3]

[1] "The Measurement of High Surge Voltages," P. L. Bellaschi, A.I.E.E. Trans., Vol. 52, June 1933, p. 544.

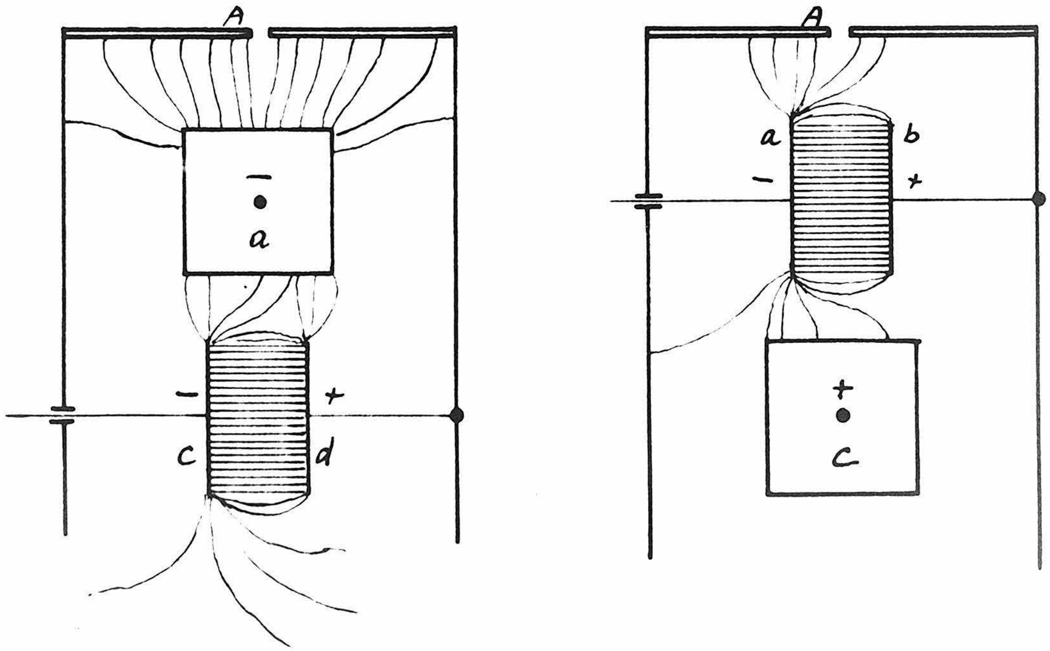
[2] "Forschungshefte der Studiengesellschaft fur Hochstspannungsanlagen," D. Gabor, September 1927, Heft 1.

[3] "On Potential Dividers for Cathode Ray Oscillographs," F. P. Burch, Philosophical Magazine, Series 7, Vol. 13, April 1932. p. 760.

DEFLECTING AND DISTORTING ELECTROSTATIC FIELDS

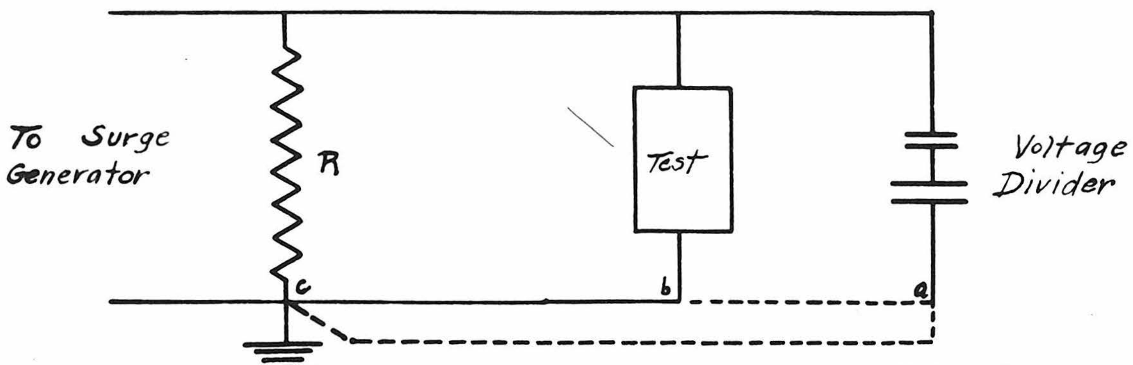


(a) Voltage Deflection on Upper Plates
Time Deflection on Lower Plates



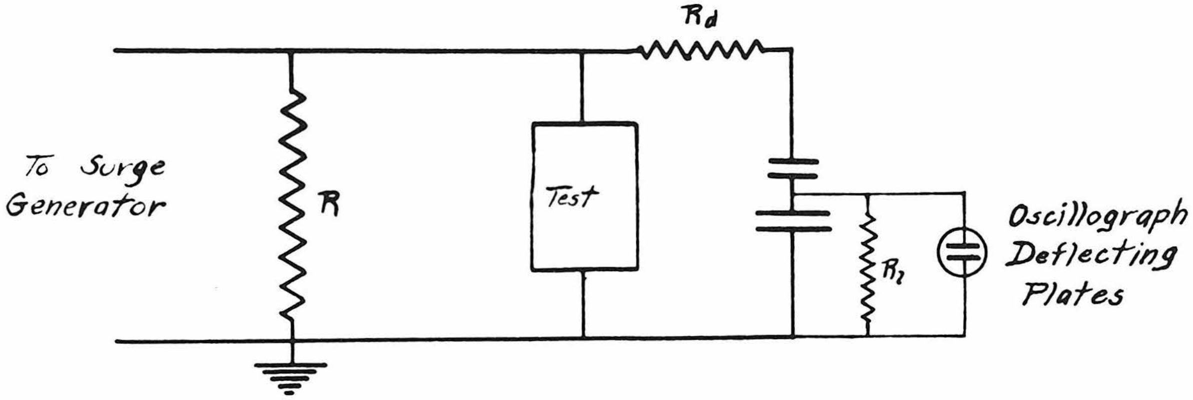
(b) Time Deflection on Upper Plates
Voltage Deflection on Lower Plates

Fig. 12

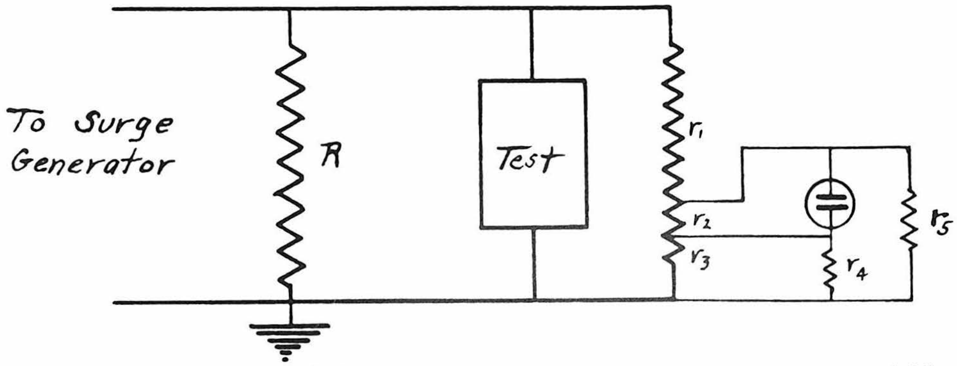


(a) Proper Voltage Divider Ground Lead Connection a-b.

Improper Connection a-c.



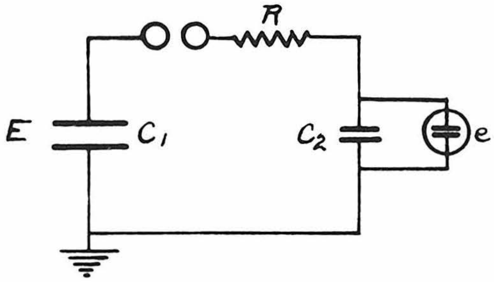
(b) Use of Damping Resistor R_d and Leak Resistor R_1 with Capacity Voltage Divider



(c) Use of Damping Resistors with Resistance Voltage Divider

Fig. 13

TIME SCALES



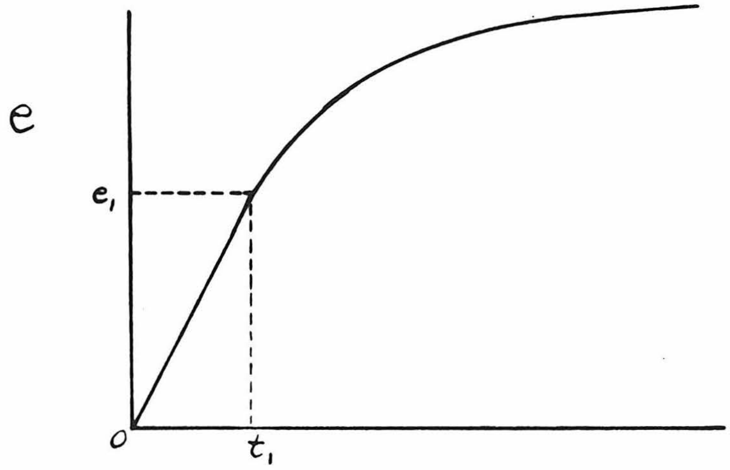
$$e = E(1-\epsilon^{-\frac{t}{RC}})$$

$$\text{If } C_2 < C_1 , \quad t < RC_2$$

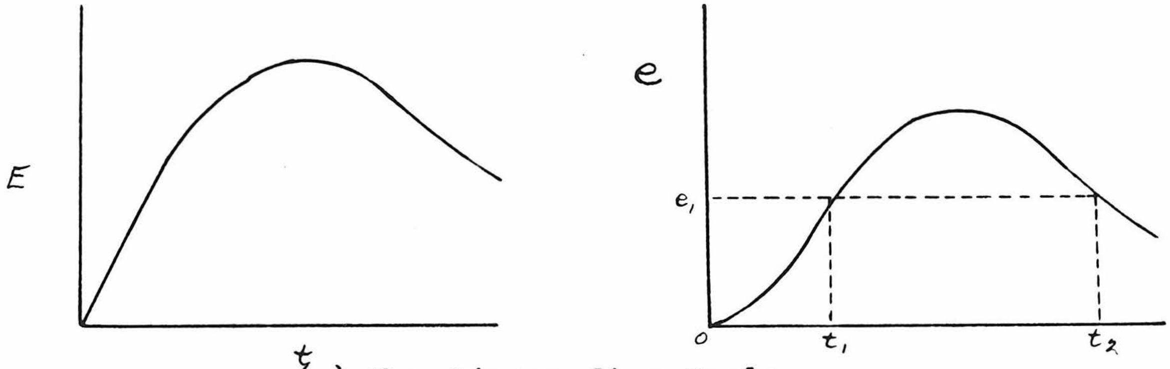
$$e = E\left[\frac{t}{RC} - \frac{t^2}{2R^2C^2} + \frac{t^3}{6R^3C^3} - \cdots\cdots\right]$$

$$e = \frac{E}{RC}t$$

(a) Simple Timing Circuit



(b) Voltage for Linear Time Scale



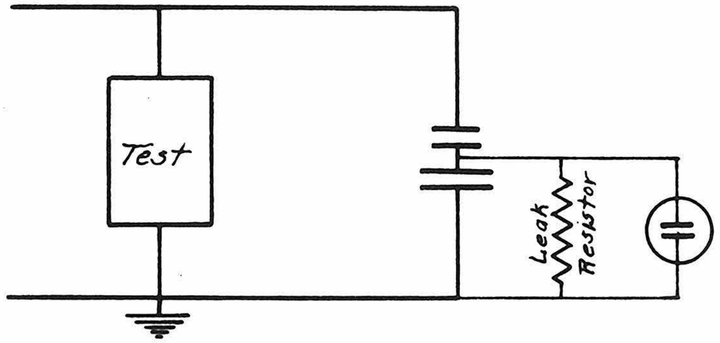
(c) Non-Linear Time Scale

Voltage Applied to Charge Timing Condenser

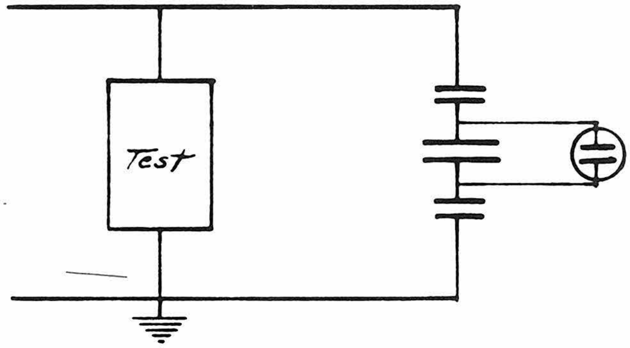
Voltage Rise Across Timing Condenser

Fig. 14

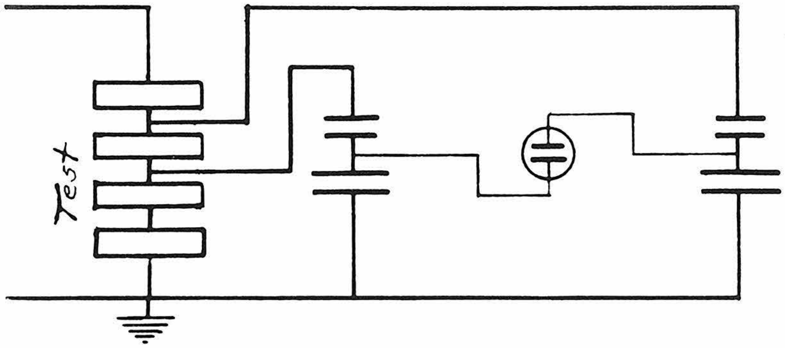
CAPACITY VOLTAGE DIVIDERS



(a) Simple Capacity Divider



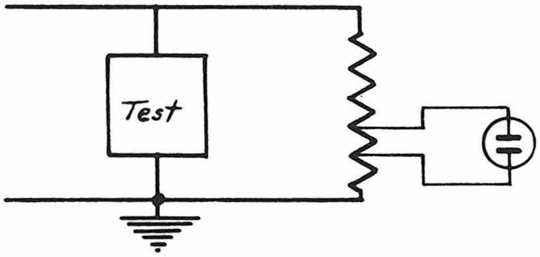
(b) Capacity Divider for Providing More Symmetrical Electrostatic Field in Oscillograph



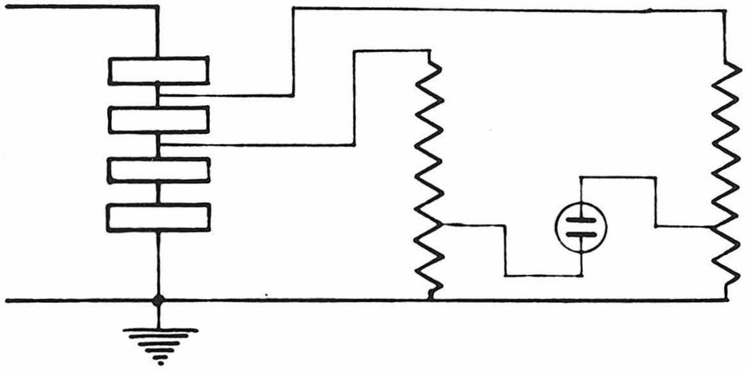
(c) Capacity Dividers for Measuring Difference Between Two Voltages Above Ground

Fig. 15

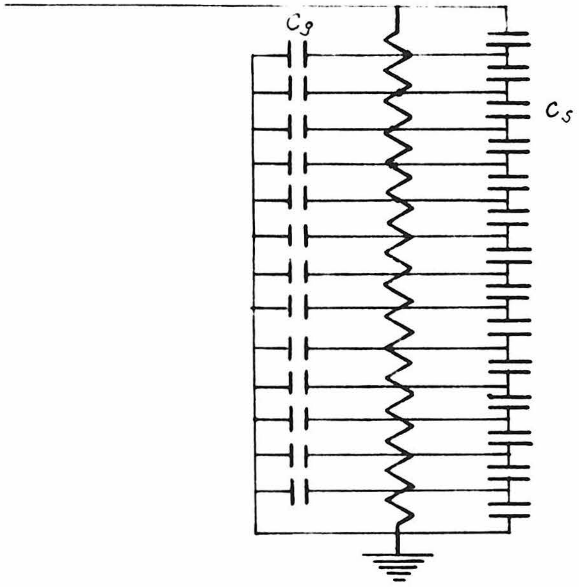
RESISTANCE VOLTAGE DIVIDERS



(a) Simple Resistance Divider



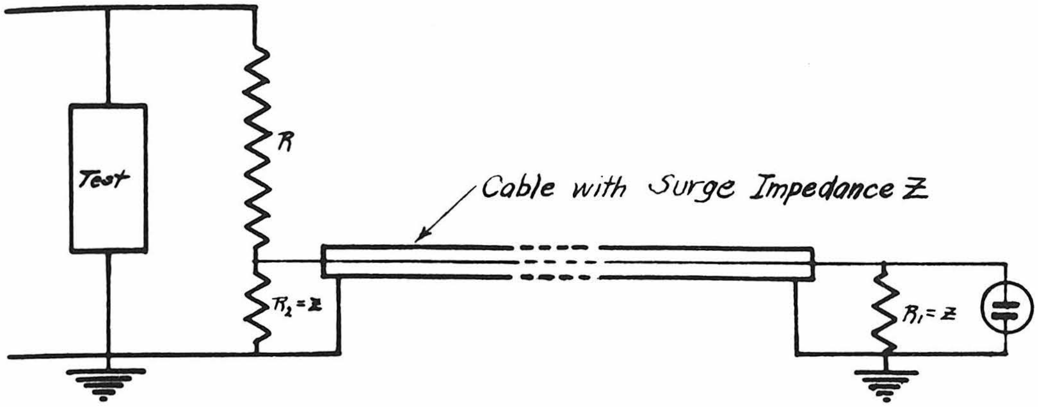
(b) Resistance Dividers for Measuring Difference Between Two Voltages Above Ground



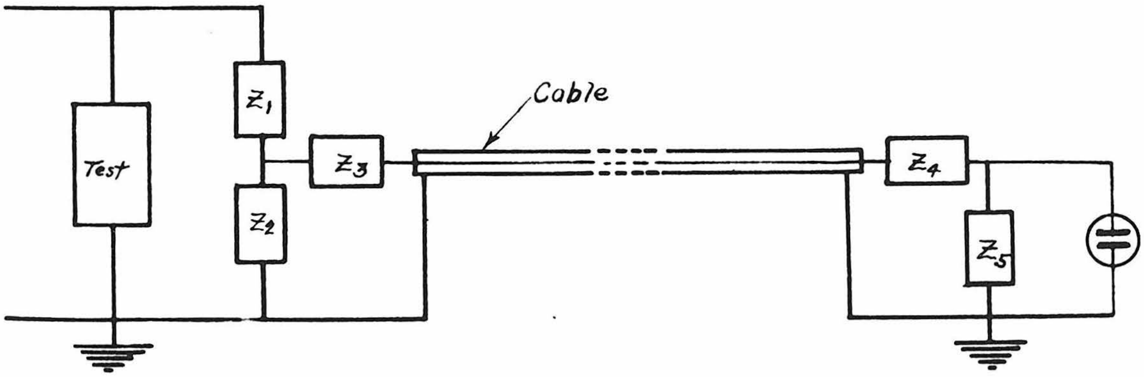
(c) Equivalent Resistance-Capacitance Network of a Resistance Voltage Divider

Fig. 16

IMPEDANCE-CABLE VOLTAGE DIVIDER



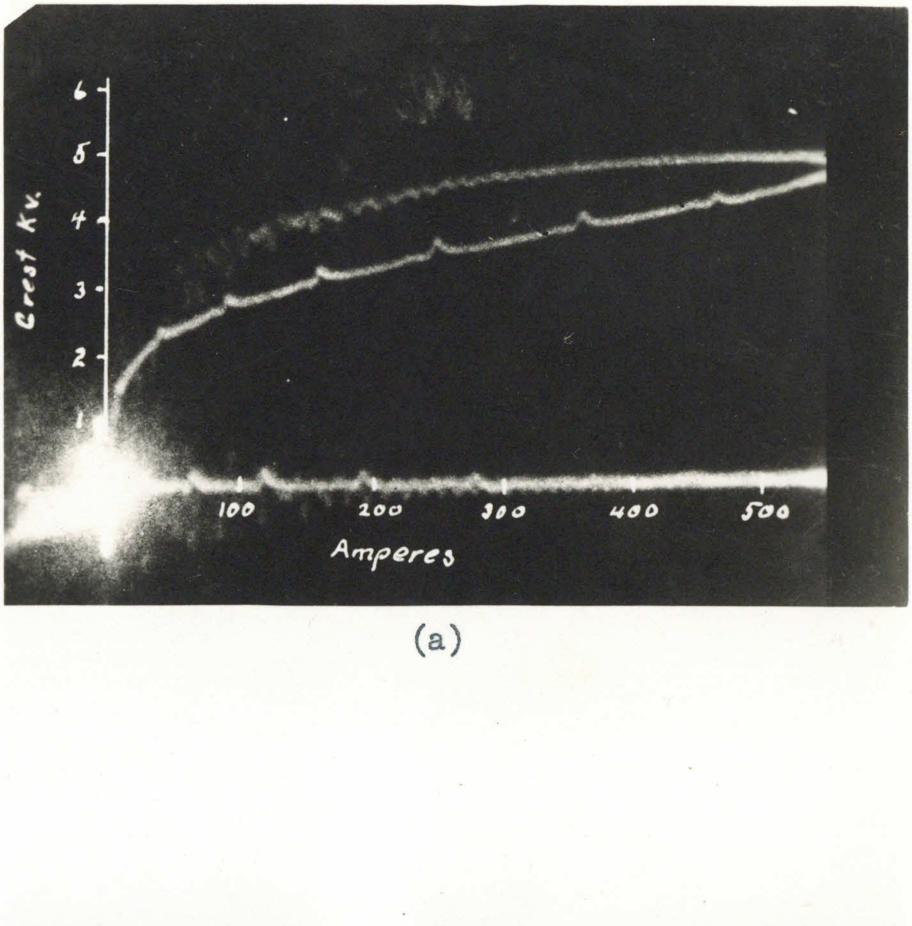
(a) Simple Resistance-Cable Voltage Divider



(b) General Impedance-Cable Voltage Divider

Fig. 17

4. Cathode Ray Oscillograms

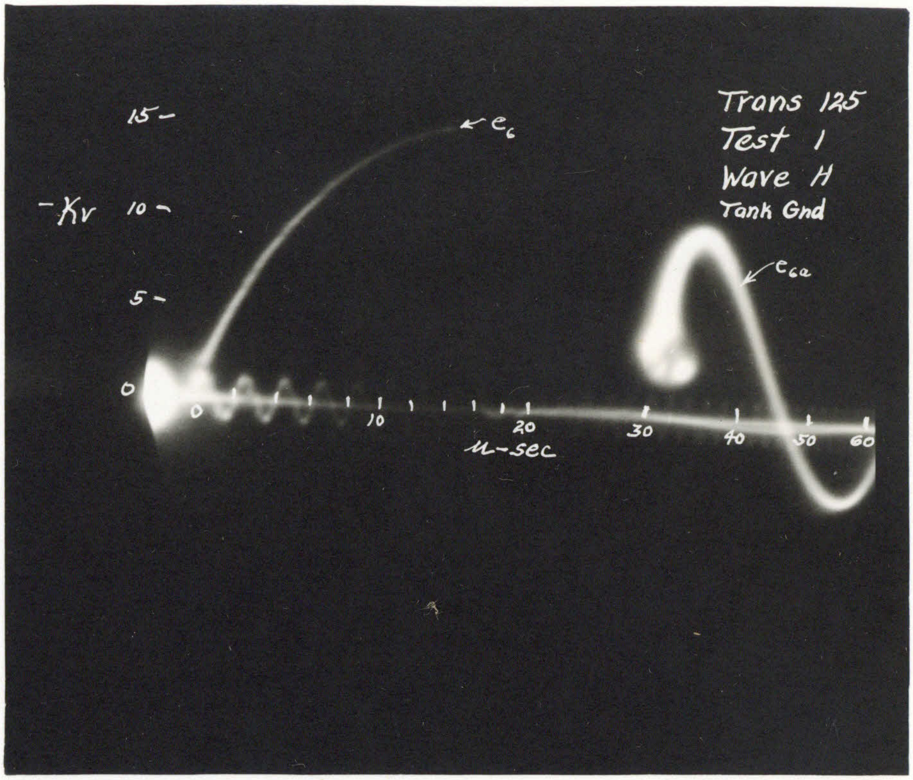


(a)

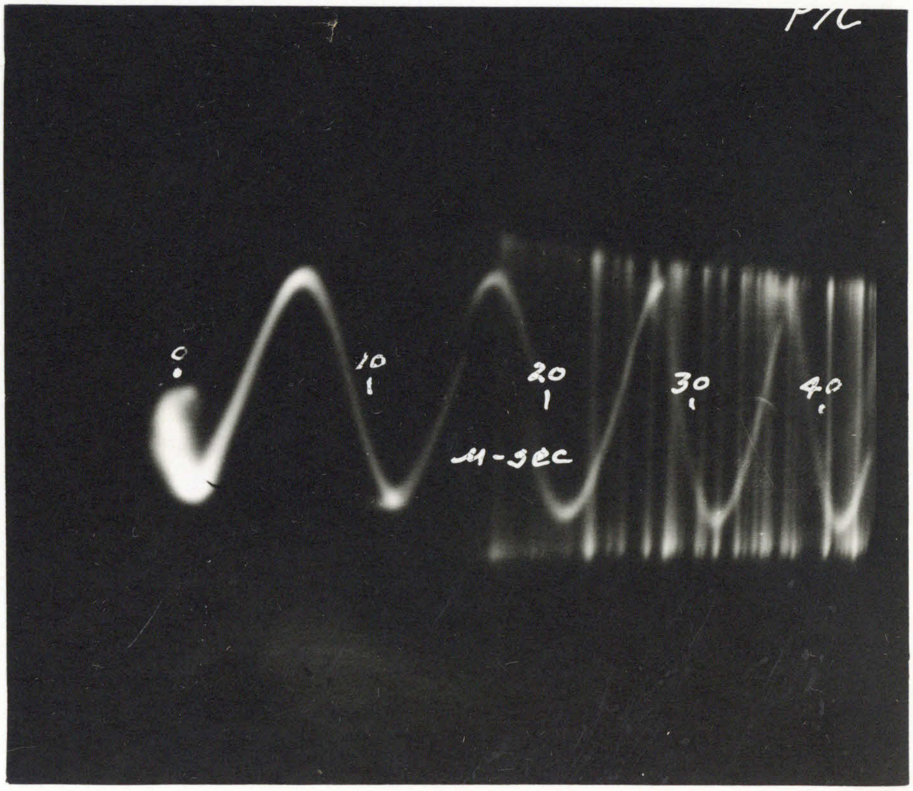
(b)

(c)

Fig. 18

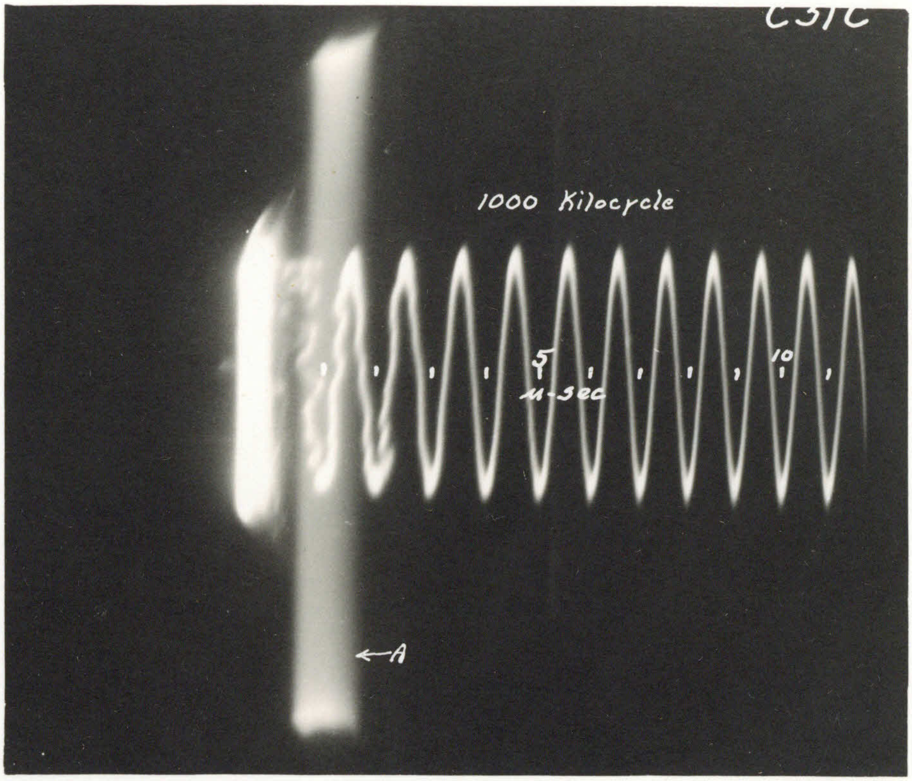

Trans 125
Test 1
Wave H
Tank Gnd
15
10
5
0
-Kv
e_c
e_{ca}
0
10
20
30
40
50
60
μ-sec

(a)

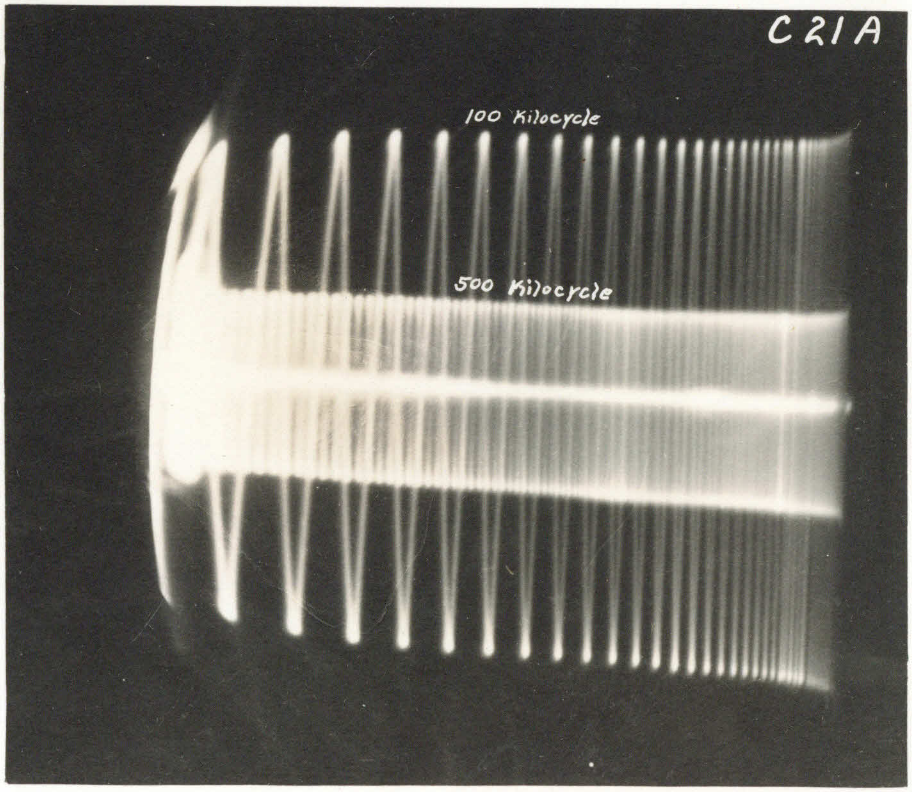

P/C
0
10
20
30
40
μ-sec

(b)

Fig. 19

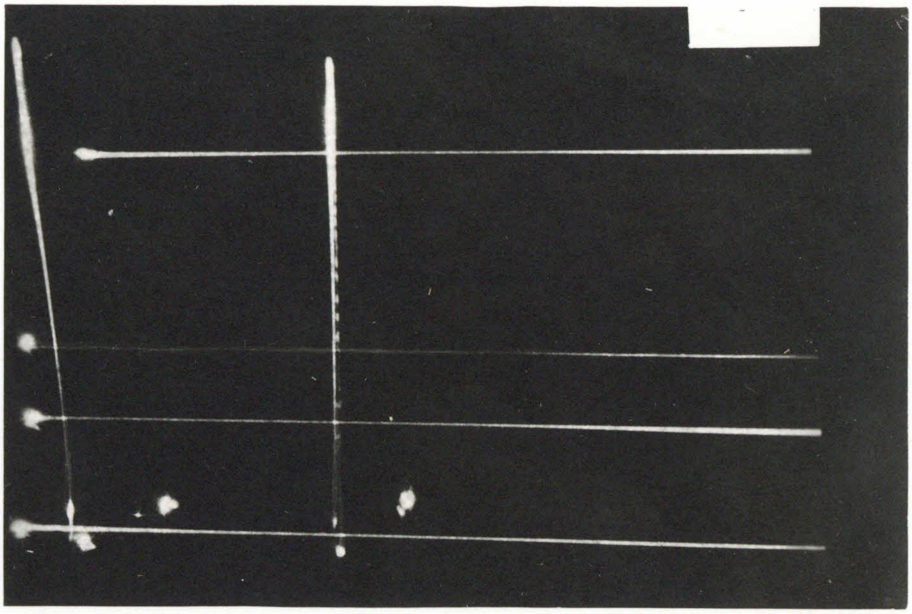

C31C
1000 Kilocycle
5
10
μ-sec
A

(a)

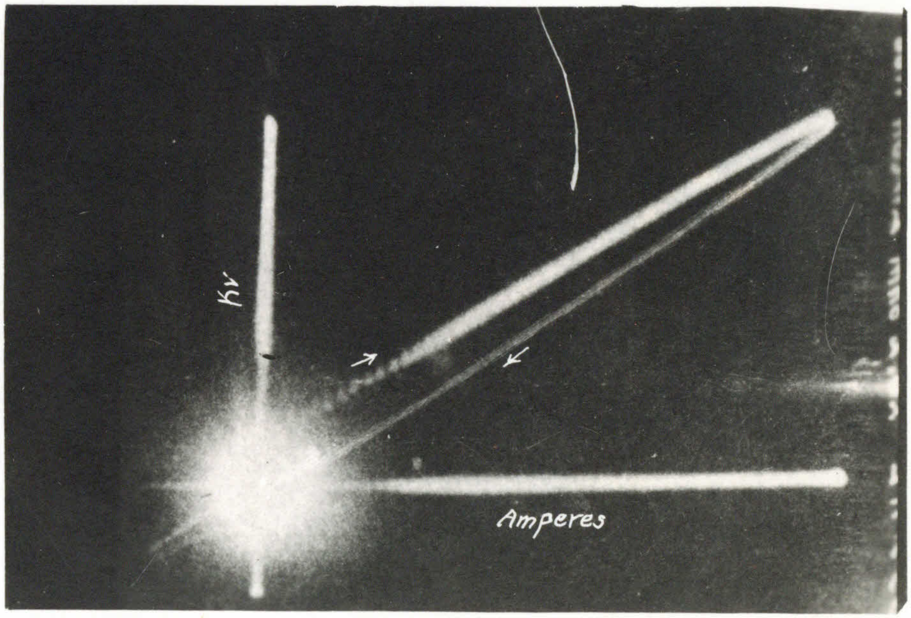

C21A
100 Kilocycle
500 Kilocycle

(b)

Fig. 20

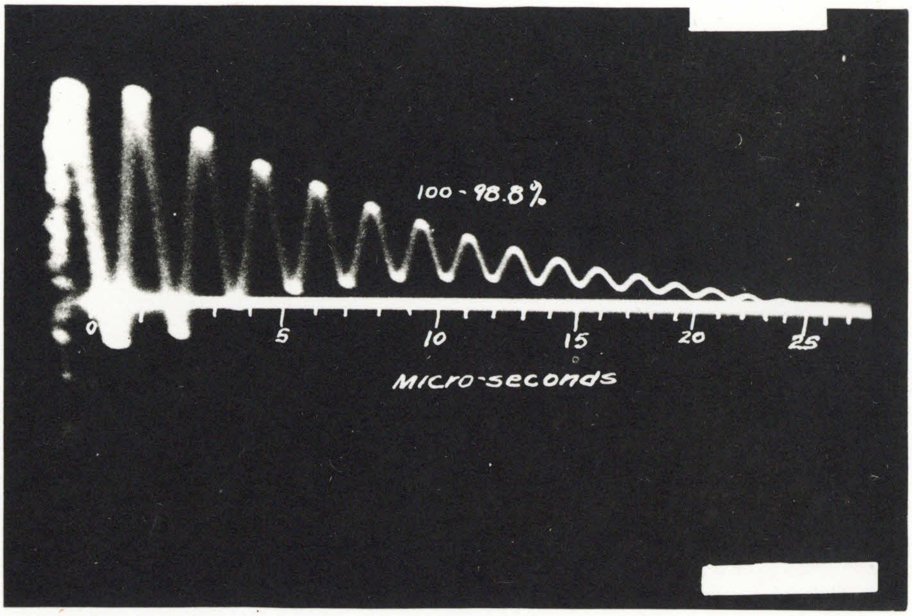


(a)

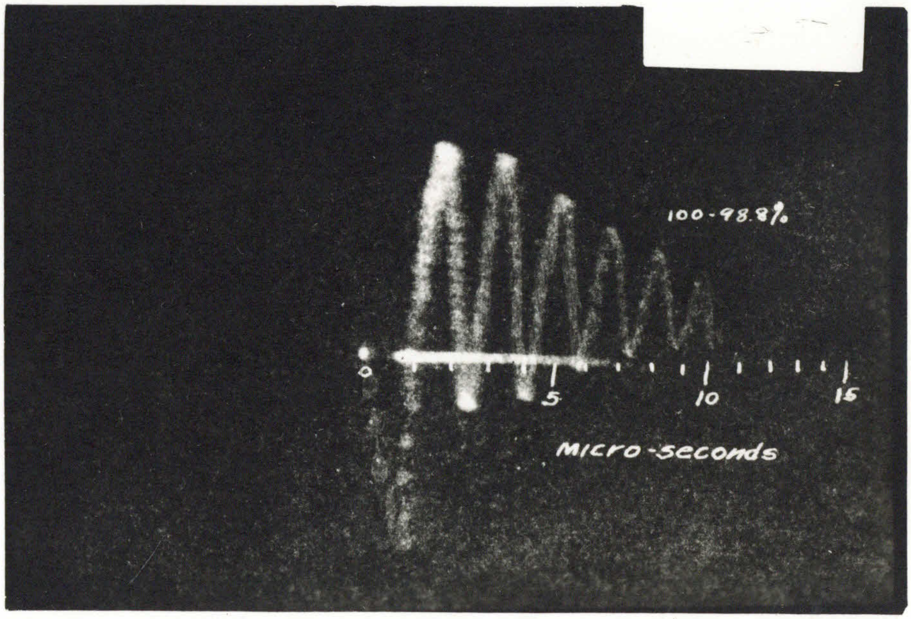


(b)

Fig. 21

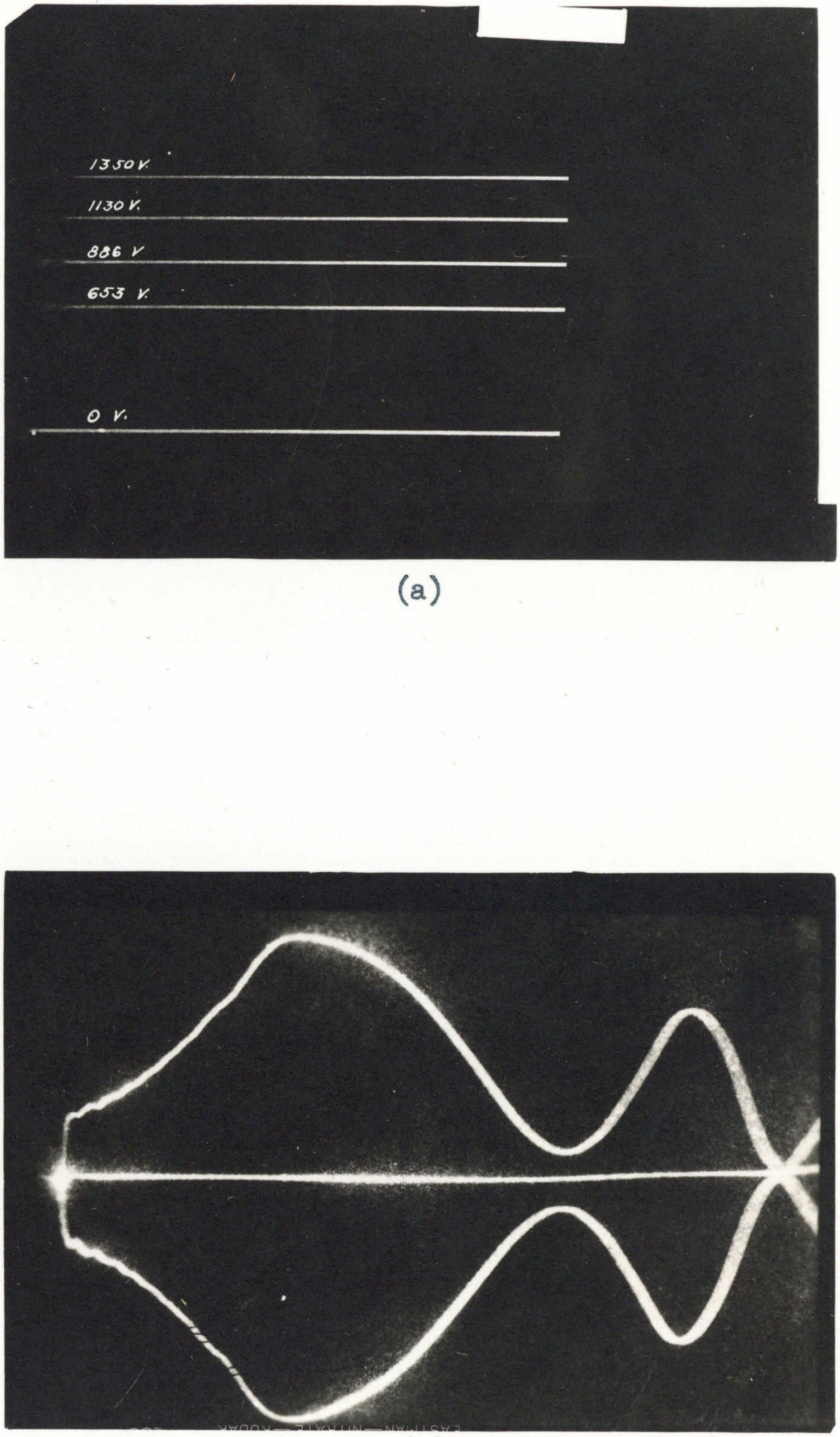

100 - 98.8%
0
5
10
15
20
25
Micro-seconds

(a)


100 - 98.8%
0
5
10
15
Micro-seconds

(b)

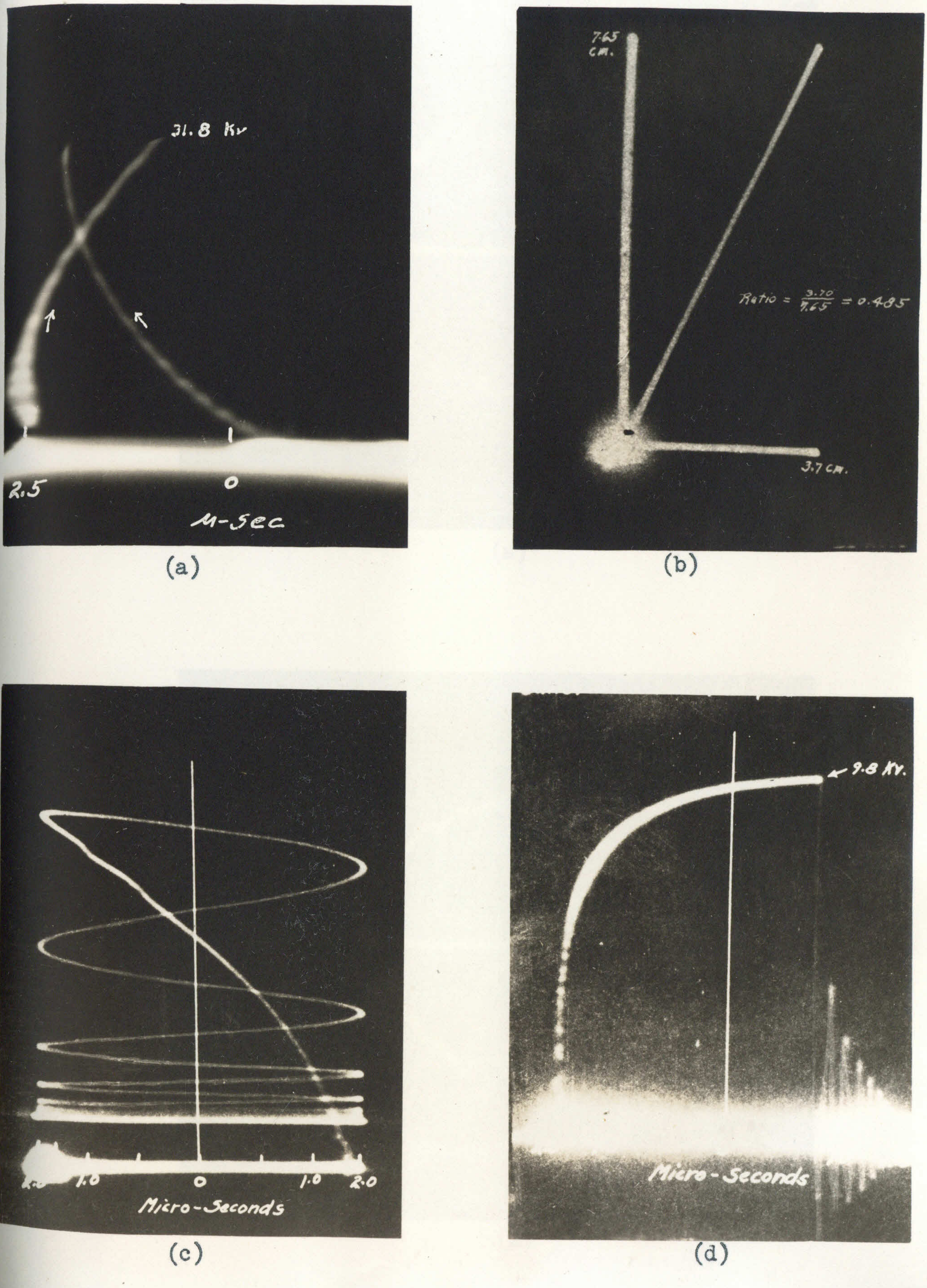
Fig. 22



(a)

(b)

Fig. 23


31.8 Kv
2.5
0
μ-sec
7.65 cm.
Ratio = 3.70/7.65 = 0.485
3.7 cm.
2.0
1.0
0
1.0
2.0
Micro-Seconds
9.8 Kv.
Micro-Seconds

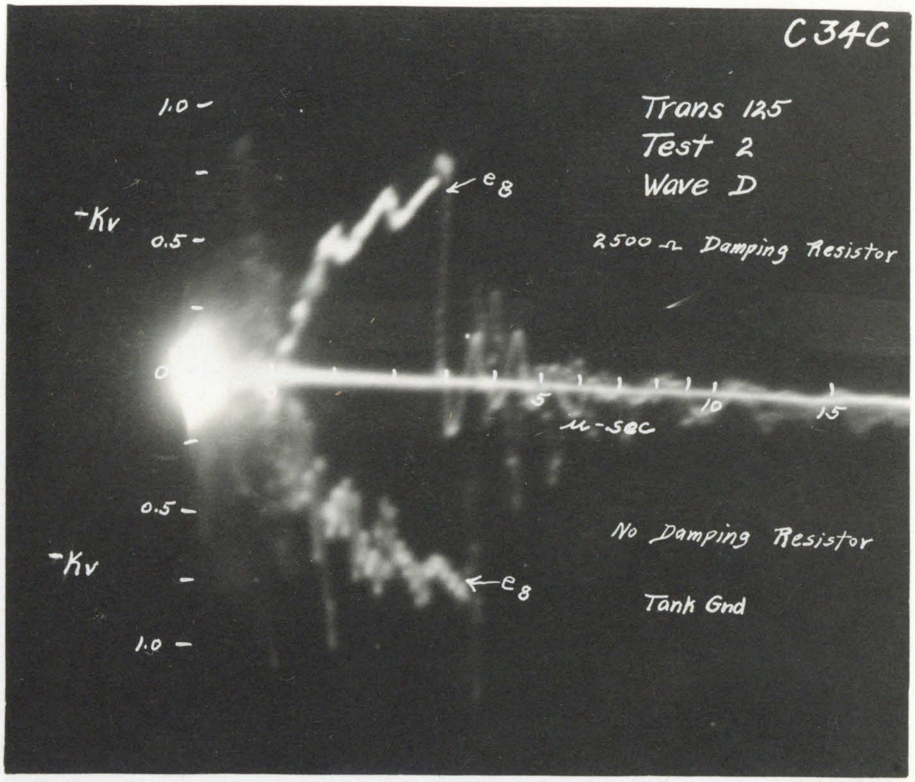
(a)

(b)

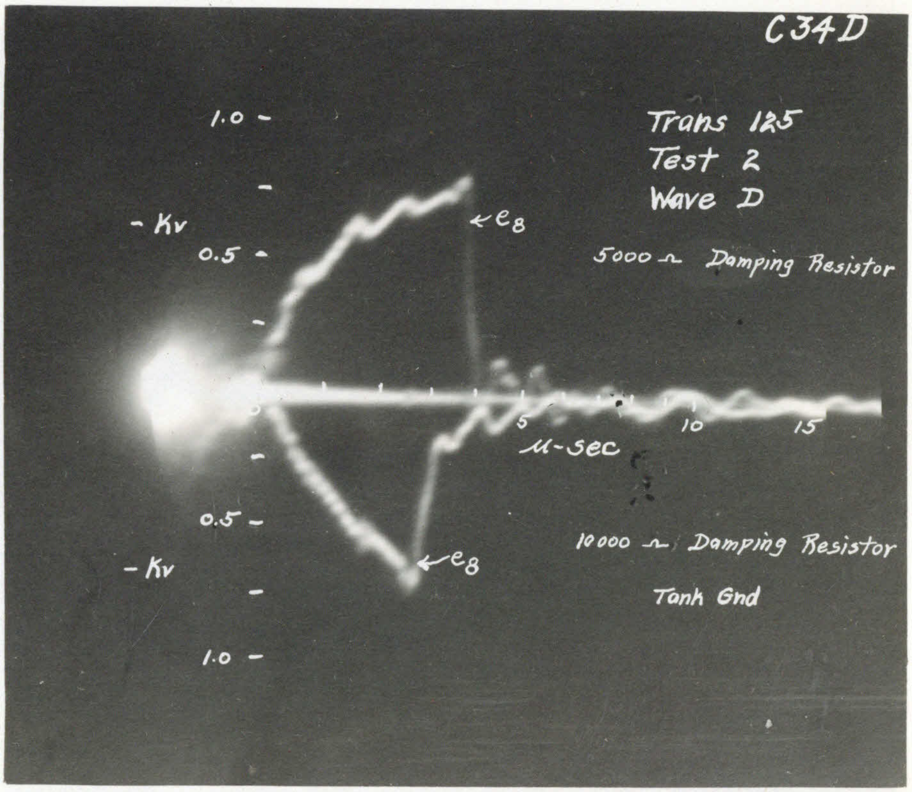
(c)

(d)

Fig. 24

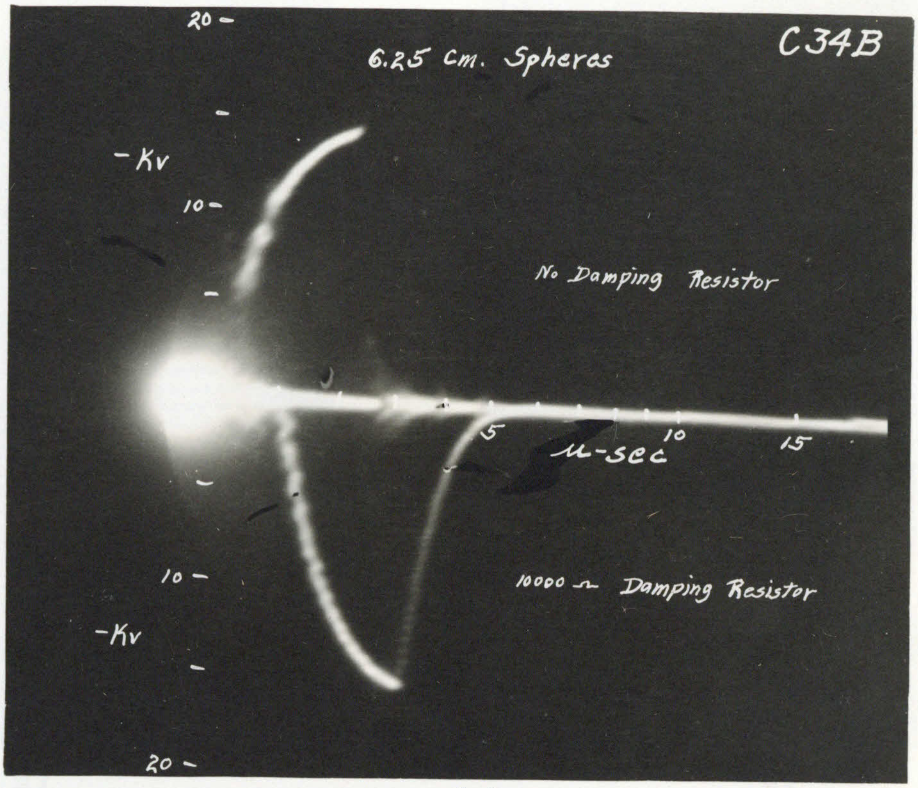

C34C
Trans 125
Test 2
Wave D
2500 Ω Damping Resistor
No Damping Resistor
Tank Gnd
1.0
0.5
-Kv
0
0.5
-Kv
1.0
e8
e8
5
10
15
μ-sec

(a)

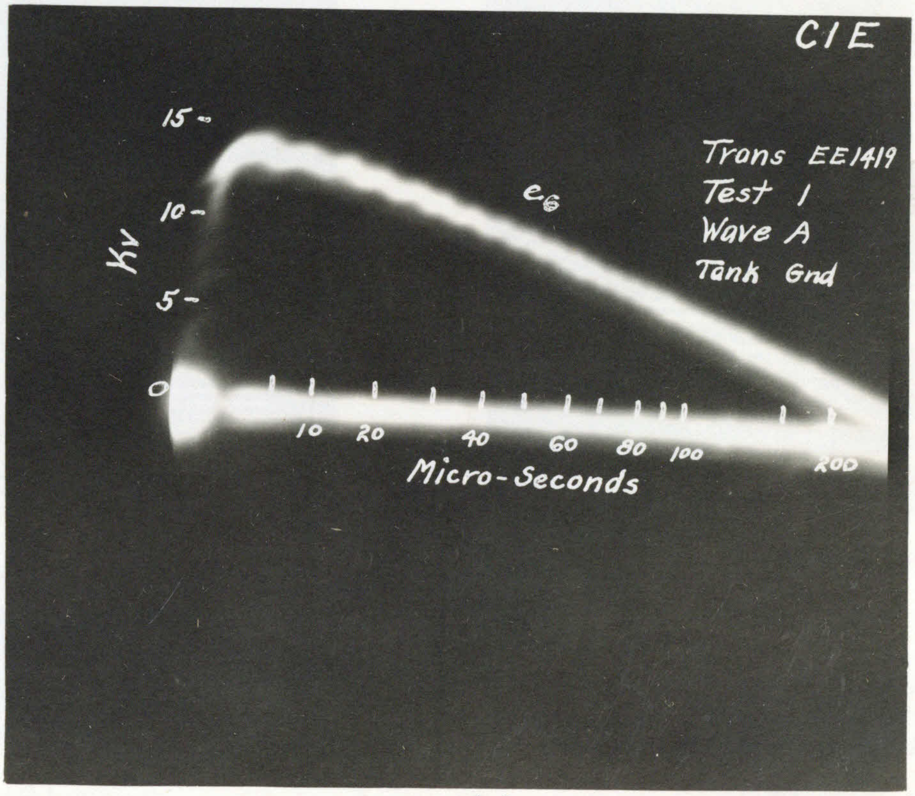

C34D
Trans 125
Test 2
Wave D
5000 Ω Damping Resistor
10000 Ω Damping Resistor
Tank Gnd
1.0
0.5
- Kv
0
0.5
- Kv
1.0
e8
e8
5
10
15
μ-sec

(b)

Fig. 25

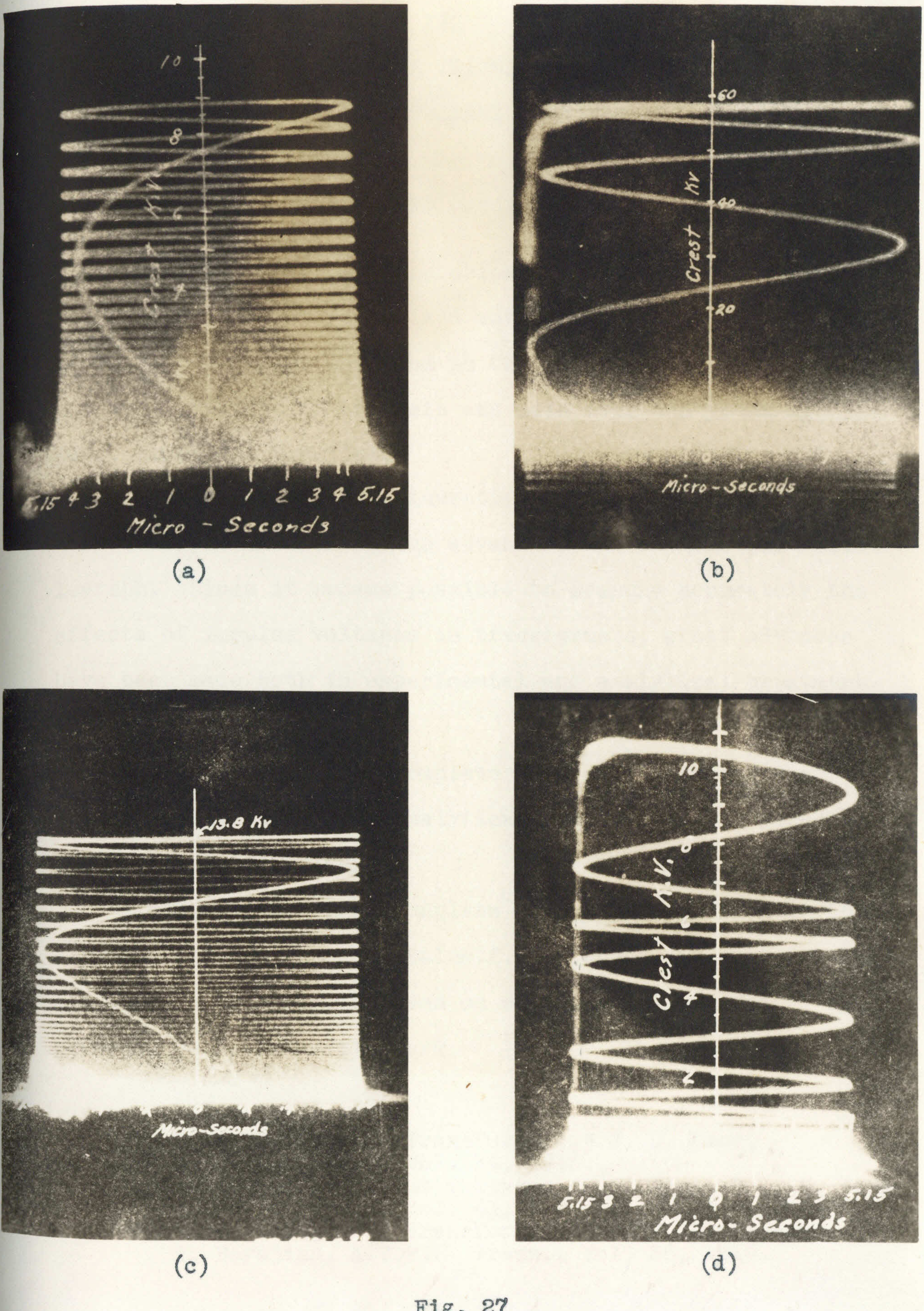

20
6.25 Cm. Spheres
C34B
-Kv
10
No Damping Resistor
5
10
15
μ-sec
10
10000 Ω Damping Resistor
-Kv
20

(a)


C1E
15
10
5
0
Kv
Trans EE1419
Test 1
Wave A
Tank Gnd
e_6
10
20
40
60
80
100
200
Micro-Seconds

(b)

Fig. 26


10
8
6
Crest Kv
4
5.15 4 3 2 1 0 1 2 3 4 5.15
Micro - Seconds
60
40
Crest Kv
20
0
Micro-Seconds
13.8 Kv
Micro-Seconds
10
8
Crest KV.
6
4
2
5.15 3 2 1 0 1 2 3 5.15
Micro-Seconds

(a) (b)

(c) (d)

Fig. 27

III

INTERNAL TRANSIENTS
in
TRANSFORMER PRIMARY WINDINGS

A. Theoretical Considerations

The importance of the problem of transient conditions in transformers was recognized several years ago. The fundamental principles involved in the problem were pointed out, and much progress in analysis was made over ten years ago by Weed, Blume and Boyajian.[1]

Lack of experimental investigation limited the advances that could be made until the advent of the cathode ray oscillograph. Since it became possible to measure accurately the effects of impulse voltages in transformers, great advances have been made both in experimental and analytical treatment of the phenomena.

The most recent and complete theoretical analyses are due to Bewley. Reference to analytical theory will, in general, mean to that of Bewley.

Similarly, the most complete series of experimental investigations are those of Palueff. Reference to experimental conclusions or analysis based on experimental data will, in general, be to Palueff's work.

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[1] "Abnormal Voltages in Transformers," J. M. Weed, A.I.E.E. Trans., Vol. 34, 1915.
"Prevention of Transient Voltages in Windings," J. M. Weed, A.I.E.E. Trans., Vol. 41, 1922.
"Abnormal Voltages in Transformers," L. F. Blume and A. Boyajian, A.I.E.E. Trans., Vol. 38, 1919.

1. Two Winding Theory

The rigorous solution of the complete transformer circuit including distributed winding constants and general primary, secondary and neutral impedances, has not been attempted. Mathematical analysis of transient oscillations in transformer windings has, therefore, been made on the basis of approximate equivalent circuits, the validity of which must finally rest on experimental verification.

The most complete analysis of the problem must consider both primary and secondary windings in their mutual relationship. Such an analysis, based on approximate equivalent circuits similar to those shown in Fig. 42, has been made by Bewley.[1]

The two winding theory has been found unnecessary, however, in dealing with the internal transients in primary windings. Consequently, a simpler single winding theory, which ignores the presence of secondary windings, has been developed for analysis of primary conditions. The chief value of the two winding theory is apparently that it provides a means of establishing the validity of the single winding theory and indicates that the reaction of the secondary winding produces no important effect in the primary.

It may be well to point out, however, that in most of

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[1] "Transient Oscillations of Mutually Coupled Windings," L. V. Bewley, A.I.E.E. Trans., Vol. 51, No. 2, June 1932.
"Traveling Waves on Transmission Systems," L. V. Bewley, John Wiley & Sons, Inc., 1933, Chap. XII.

the work that has been done the transformers considered have been step-down transformers, having a ratio considerably greater than unity, although this point is not mentioned specifically. If the situation were reversed and the impulse applied to the low voltage winding, the analysis of transient effects in the low voltage winding could not neglect the presence of the high voltage side. This will be dealt with in more detail later, where it will be shown that the reaction from the high voltage winding may predominate over all other effects.

2. Single Winding Theory

The major part of all published work deals only with the conditions in the high voltage primary winding. The analysis for this case has been worked out on the basis of a single independent winding with equivalent circuit similar to that shown in Fig. 42, except for the secondary and mutual coupling parts.[1] The analysis is thus based on a single winding, having series capacitances along the stack, capacitances to ground, and self and mutual inductance between turns.

a. Additional Approximations. In addition to neglecting the secondary winding, further approximations are made

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[1] "Transient Oscillations in Distributed Circuits with Special Reference to Transformer Windings," L. V. Bewley, A.I.E.E. Trans., Vol. 50, 1931.
"Traveling Waves on Transmission Systems," L. V. Bewley, John Wiley & Sons, Inc., 1933, Chap. XIII.

concerning capacity paths, mutual inductance and resistance. These matters are considered in detail by Blume & Boyajian.[1] All discontinuities in the winding, due to disc or pancake construction or interleaving, have been neglected. Thus, the conclusions of the theory may be applied to any winding in general and are found to check with semi-empirical analysis for distribution transformers, which will be described later.

b. Derivation of Equation. For the purpose of comparing the method of derivation of certain primary transients by the purely analytical method with the semi-empirical method mentioned, a brief resume of Bewley's methods and results will be given.

The solution involves the setting up of a fifth order partial differential equation for the equivalent circuit with uniformly distributed constants. Constants involved are self inductance, including certain partial interlinkages, mutual inductance between winding elements, series and shunt capacitance, series resistance, and shunt conductances to ground and along the winding. Variables are voltages to ground, currents in capacitances, inductances, and shunt conductances, flux density, flux linkages, mean length of turn, number of turns, length of leakage path, length of winding.

In order to calculate or estimate all the above quantities, the complete design data of the transformer are re-

[1] Loc. cit.

quired. The determination of many of the quantities is even then a matter of great difficulty and uncertain accuracy. These limitations are recognized by Bewley, who states, "It may be necessary to make arbitrary changes in the circuit constants, particularly of the inductance coefficient, to obtain accurate numerical agreement," with experiment. So it is seen that even in the simpler case of the single independent winding, purely theoretical reasoning is insufficient for a complete solution.

c. General Solution of Equation. The solution of the equation is obtained by trial and adjustment as commonly done. The initial voltage distribution can be obtained with a general impedance in the neutral, but a complete solution is practical only in the special cases of grounded or isolated neutral. After obtaining a general solution, it must, of course, be made to satisfy the boundary conditions at the ends of the winding and the initial and final voltage distributions. The general solution is in the form of an infinite series of space and time harmonics oscillating about the final distribution as an axis.

The initial voltage distribution depends, of course, only on the relative values of series and shunt capacitances of the winding. This is similar to the problem of distribution on an insulator string, except the capacitances are distributed instead of lumped.

The final distribution would depend only on resistance, since the applied wave was assumed to be an infinite rectangular wave.

The events in the transition period between these two conditions represent the transient conditions which it is desired to calculate.

d. Solution for Grounded Neutral. The general solution for the case of grounded neutral becomes, for the special conditions of no losses, mid-point of the winding, and fundamental space harmonic, a simple expression involving a constant term plus a cosine term. This same result will be shown to be derivable from the simple physical picture of the oscillation in a winding and constitutes the starting point from which the calculation of secondary terminal transients is made.

e. Effect of Losses. An interesting result of the analytical theory is the indication of the effect of losses on the transient oscillations. Three effects are shown to be present. These are damping of oscillations, lowering of frequency of oscillation, and shift of the axis of oscillation. These effects of losses may be observed in several oscillograms of the present investigation.

The decrement factors must be considered as empirical constants to be obtained by test, since their calculation depends upon knowledge of several other constants of the winding which are difficult to obtain. Thus, the analytical method again falls back on experiment. Bewley gives a value for damping in a power transformer of twenty per cent per half cycle of fundamental frequency. This damping increases in the higher harmonics about as the square of the order of the harmonic.

f. Mechanism of Oscillation. Three ways of viewing the mechanism of oscillation have been worked out. These are all obtainable from the general analytical solution, under somewhat different assumptions regarding certain circuit constants, chiefly the series capacitance and mutual inductance. The oscillations can be considered as being caused by (a) a fixed voltage distribution plus harmonic standing waves; (b) a fixed voltage distribution plus pairs of harmonic traveling waves; and (c) a simple reflecting traveling wave.

The first of these has been developed to the greatest extent by Palueff[1] in his work on the effect of transients on power transformer design. The traveling wave analysis has been perfected by Boehne[2] with particular regard to machine windings in slots where capacitance to ground is high, capacitance between turns negligible and mutual inductance very small. The relative merits of these theories and their proper sphere of application has been discussed at length by Bewley, Palueff and Boehne.[3]

3. Analytical Results for Arbitrary Wave Shapes

The foregoing discussion of the analytical theory, as worked out by Bewley, was for the case of a wave with verti-

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[1] "Effect of Transient Voltages on Power Transformer Design-I," K. K. Palueff, A.I.E.E. Trans., Vol. 48, July 1929.

[2] "Voltage Oscillations in Armature Windings Under Lightning Impulses," E. W. Boehne, A.I.E.E. Trans., Vol. 49, October 1930.

[3] Ibid. Discussion.

cal front and infinite tail applied to one transformer terminal. The extension of the theory to the case of applied voltages of arbitrary wave shape has also been carried out.[1] This, of course, involves the use of the superposition theorem.

To avoid the necessity of laborious calculation in the determination of amplitudes of oscillation for each new wave shape applied, Bewley has shown that curves of reduction factors for wave front and wave tail can be prepared, as functions of front or tail length and natural period of oscillation, from which the amplitude of oscillation can be estimated, if the value for the infinite rectangular wave is known.

Complete theoretical discussion of the effects produced by the impact of damped oscillatory wave trains is likewise available.[2]

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[1] L. V. Bewley. Op. cit.

[2] "Transformer Oscillations Caused by Damped Oscillatory Waves," L. V. Bewley, General Electric Review, Sept. 1931.

B. Previous Experimental Investigations

Complementary to the analytical methods just discussed, and inseparably connected with them, are the results of parallel experimental investigations. To complete the account of the present status of the work on transient phenomena in transformer primary windings some account of the experimental results is in order.

Experimental investigations of the internal transient oscillations have, in general, been made with the object of determining voltage stresses on insulation and in finding design changes which would eliminate oscillations, produce uniform voltage distribution at all times, and decrease insulation stress. Nearly all this work has been concerned with power transformers only. Little information is available on the internal conditions in small size distribution transformers, except protective studies, which will be mentioned in more detail in another section.

The experimental conclusions reached, therefore, apply directly only to power transformer windings. These windings are characterized by being built up of stacks of disc or pancake coils. This construction produces a winding having discontinuities in the circuit elements which may be a source of local oscillations not found in a more homogeneous structure.

1. Transformer with Grounded Neutral

Cathode ray oscillograph studies of the effects of surge generator voltages on grounded neutral windings have produced, in general, a very good agreement with the theory previously outlined. All the major effects called for by analytical theory have been found to exist. Discrepancies occur in the case of certain minor local oscillations, but this could be expected, due to the approximations made in developing the theory.

a. Mechanism of Oscillation. Oscillograms of internal oscillations and voltage distribution show excellent agreement with Palueff's theory of standing waves mentioned in the last section. Certain features of interest and importance concerning the harmonic standing waves he has listed as follows;[1] (a) the time frequency of the harmonics was found to be nearly proportional to the square of the order of the harmonic, the exact ratio depending chiefly on the series capacitance of the winding; (b) the higher the harmonic, the smaller its amplitude; (c) harmonics are symmetrically located with respect to the center of the winding; (d) the amplitude of oscillation depends on the voltage, steepness of front, and length of tail of the applied wave and on the initial electrostatic voltage distribution. These facts will be shown to have an important bearing on the voltage transferred to the secondary winding by capacity coupling.

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[1] "Lightning Studies of Transformers by the Cathode Ray Oscillograph," F. F. Brand and K. K. Palueff, A.I.E.E. Trans., Vol. 48, July 1929.

b. Effect of Wave Front. It has been found that a sloping wave front will produce oscillations of only those harmonics having periods of oscillation greater in length than the wave front. A wave with time to crest greater than the time of one cycle of the fundamental natural frequency of the transformer will produce no oscillation. Transformers have natural frequencies as low as 5,000 cycles or less which means that wave fronts as slow as 100 microseconds may produce oscillation. Most measured wave fronts due to natural lightning are shorter than this.

c. Effect of Length of Wave. Tests have shown that maximum amplitudes of oscillation will occur when the wave front is steep and its duration is greater than one-half cycle of fundamental frequency. Most natural lightning surges have been found to have durations ranging from 20 to 60 microseconds, which is within the range of half cycle periods of natural frequencies of many transformers.

d. Effect of Wave Tail. The wave tail appears to have an effect similar to that of the front, except in a negative sense. The resultant internal voltage would, then, depend on the steepness of wave tail and on the relation of the tail transient to the transient already started by the wave front. It is thus a case of two superimposed transient effects.

e. Effect of Transformer on Applied Wave. The effect of the capacitance of transformer and bushing in sloping wave fronts or reducing wave crests has been found negligible.

The surge impedance of a transformer is also too high to cause appreciable change in the applied wave. The primary terminal reaction can thus be neglected.

f. Effect of Damping. Losses in transformers are found to be insufficient to prevent oscillation or to appreciably reduce the amplitude of the first half cycle.

g. Effect of Type of Transformer. Much time and space has been devoted to the discussion of the relative merits of core and shell types of construction as regards internal transients. The difference depends mainly on the differences in characteristics of long, narrow stacks of coils and short, wide ones. One fact that stands out is that shell type windings are more likely to have local oscillations appearing within the winding. The major features of voltage distribution in the two types seems to depend largely upon which manufacturing company the investigator is employed by. However, both types are similar in that the windings are built up of stacks of disc or pancake coils. Little or no information has been given in the characteristics of small windings of homogeneous structure.

It may be remarked that the general conclusions regarding the effects of impulses as given by Palueff are also agreed to by Hodnette.[1]

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[1] "Effect of Surges on Transformer Windings," J. K. Hodnette, A.I.E.E. Trans., Vol. 49, Jan. 1930.

2. Transformer with Isolated Neutral

The general conclusions as to the effect of surge voltages in producing internal transients in transformers with grounded neutral can also be carried over to those with isolated neutral with the following changes:

(A) The initial voltage distribution which depends only on the capacitances is unaffected by neutral connection.

(B) The final voltage distribution becomes constant throughout the winding and is equal to the terminal voltage.

(C) Transient oscillations occur about the final distribution as an axis.

Studies of the transient conditions in windings when the neutral is grounded through various types of impedance have been made by Palueff[1] and by Vogel and Hodnette.[2]

3. Traveling Wave Theory

The existence of traveling waves within transformer windings is usually not mentioned when the mechanism of internal oscillations is discussed. The standing wave conception has so successfully explained all variations of voltage observed that there has been little reason to look for other explanations. However, for a complete understanding of transient phenomena in windings, it is well to ex-

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[1] "Effect of Transient Voltages on Power Transformer Design-II," K. K. Palueff, A.I.E.E. Trans., Vol. 49, July 1930.

[2] "Grounding Banks of Transformers with Neutral Impedances and the Resultant Transient Conditions in the Windings," F. J. Vogel and J. K. Hodnette, A.I.E.E. Trans., Vol. 50, March 1931.

amine all possible sources of voltage.

What has been described as a classic treatment of the subject of traveling waves in machine windings is due to Boehne.[1] Although his work was done with armature windings, the principles of traveling wave phenomena he develops would be the same for transformer windings.

a. Characteristics of Transformer and Machine Windings. The reason for the dominating effect of the traveling wave in machine windings and the apparent absence of it in transformer windings can easily be explained by the differences in construction. In the transformer winding, turn to turn capacitance is of great importance. Also, the capacitance between discs in the stack. In armature windings, turn to turn capacitance is nearly absent because the conductors are imbedded in separate slots. For the same reason, the turn to ground capacitance is very high compared with transformer windings. Similarly, the mutual inductance between turns and elements,which is a major feature in the transformer winding,is almost wholly absent in the armature winding.

These differences in circuit constants account for the difference in action of the two types of winding. In the transformer, by virtue of close electrostatic and electromagnetic coupling of the whole winding, the transient voltage appears simultaneously in all parts of the winding and

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1. "Voltage Oscillations in Armature Windings under Lightning Impulses," E. W. Boehne, A.I.E.E. Trans., Vol. 49, 1930.

the ensuing oscillations start everywhere at the same instant. In the armature, the impact of the applied wave is not felt everywhere in the winding due to lack of coupling between winding elements. The only way the wave can produce voltages at interior points is then to travel along the conductor. This explains the predominant influence of the traveling wave in machine windings.

An induction regulator possesses some of the features of both windings. The slotted construction and air gap between series and shunt windings would change capacity relations from those in either of the other type windings. Mutual inductance between elements would be similar to a transformer winding. Added complication would result from the reactions between the windings due to their mutual capacity, inductive and conductive coupling. The case of the induction regulator has not yet been adequately covered in the literature.

b. <u>Criteria of Traveling Wave Phenomena</u>. The seven characteristics of traveling wave phenomena have been listed by Boehne as follows:

(A) "The maximum voltage with the neutral open is twice that of the neutral grounded."

(B) "The frequency of the oscillations with the neutral grounded is twice that of the neutral open."

(C) "The time delay of any appreciable voltage rise increases in proportion to the distance from the line terminal at which the voltage is measured."

(D) "This time delay is the same with the neutral open or grounded."

(E) "The voltage oscillations measured at various points in the winding with either the neutral open or grounded are substantially in phase."

(F) "A certain flat top characteristic is evident in many oscillograms, it being predominant near the neutral of the open neutral winding and near the line terminal of the grounded neutral winding."

(G) "All records are distinctly unidirectional."

Some of these features can be recognized in oscillograms of transformer windings included in section III-C-5 and described in section III-C.

c. <u>Transformer Winding as a Transmission Line</u>. A winding may be considered as a finite transmission line by regarding it as an extension of the incoming line by another section of different surge impedance. A traveling wave reaching the junction point of the two lines would divide, part being reflected and part transmitted. The transmitted portion, which would appear as a traveling wave within the winding, would have a magnitude given by the usual relation

$$e = E\left(\frac{2Z_2}{Z_1 + Z_2}\right)$$

In this the surge impedance, Z_1, of the incoming line may be from 200 to 500 ohms, if it is an overhead line, or about 50 ohms, if it is a cable. The surge impedance, Z_2, of the winding may be as high as 5,000 ohms.

After entering the winding, the wave would be reflected at the terminals with polarity and magnitude depending on the nature of the terminal impedances. If there were discontinuities within the winding, there would be partial reflections at these points also.

Boehne[1] found that nearly everything appearing on oscillograms of surge tests on armature windings could be accounted for by the simple traveling wave theory. Successful explanations were thus found for such features as (a) relations between natural frequency and voltage in grounded, open and fractional windings, (b) phase displacements between points in the winding, (c) oscillation of voltage distribution curves about the final voltage distribution as standing waves, (d) damping of oscillations. The behavior of the internal traveling wave at line or neutral impedance was the same as outlined by Brune[2] for transmission line terminal impedances.

The simple traveling wave theory thus accounts for the major features of the transients appearing in machine windings and may also be expected to account for much of those appearing in transformer windings. It is not able to account for all transient effects found in transformers, however. The traveling wave theory recommends itself be-

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1. Loc. cit.
2. "Reflection of Transmission Line Surges at a Terminal Impedance," O. Brune, General Electric Review, May 1929, p. 258.

cause of the simplicity of the concept, but should be considered only one of the components in the mechanism of production of transient voltage oscillations.

4. Autotransformers

Autotransformers differ from ordinary transformers in their reaction to impulse voltages primarily in the connection of two points of the winding to separate transmission lines. Otherwise, the principles of internal transient oscillations are the same in both cases.

As has been shown analytically[1] and, also, experimentally,[2] the reaction to a voltage surge of a transformer, whose neutral is grounded through a resistance of four or five hundred ohms, is almost exactly the same as if the neutral were directly grounded. This is also indicated by the results obtained on small transformers in the tests to be described.[3] This would indicate that the effect of the second transmission line connected to an autotransformer would act as a ground with reference to a surge arriving on the first line. In other words, a wave coming in on the high voltage line would in effect find the other end of the series part of the winding grounded and the common part short circuited. Likewise, a wave coming in on the low voltage side would find the conditions

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[1] "Abnormal Voltages Within Transformer Windings," L. F. Blume and A. Boyajian, A.I.E.E. Trans., Vol. 38, 1919, p. 477.

[2] "Grounding Banks of Transformers with Neutral Impedances and the Resultant Transient Conditions in the Windings," F. J. Vogel and J. K. Hodnette, A.I.E.E. Trans., Vol. 50, March 1931, p. 63, Fig. 5.

[3] See Fig. 53, Test 5 and corresponding oscillograms, Figs. 126 to 129, of this thesis and compare with Test 2.

that of a ground on both ends of the whole winding.

Palueff[1] has again presented a thorough study of the transient reactions of autotransformers. He shows that the above relations hold only if the ratio of transformation of the autotransformer is considerably greater than unity. In the case where the ratio approaches unity, the series part of the winding between the high and low voltage lines consists of but few turns and, hence, acts as a small series inductance. The outgoing line would not remain at ground potential in this case, but would increase in voltage slowly compared to the other transient effects. Palueff thus divides the phenomenon into two parts; (a) the gradual rise and fall of secondary line voltage; (b) the internal oscillations which comprise the fast transient. The internal voltage stresses in parts of autotransformer windings are shown to be much higher than in corresponding transformer windings.

4. "Effect of Transient Voltages on Power Transformer Design-III," K. K. Palueff, A.I.E.E. Trans., Vol. 50, June 1931.

C. Certain Features of Power Transformer Primary Transients

The subject of transient oscillations in the primary winding of power transformers has been so thoroughly covered in the work briefly referred to in the preceding pages that no attempt has been made to appreciably extend that work in the experimental results here presented.

Some examples of the type of internal oscillations occurring in a normal type of high voltage power transformer are given to show graphically the type of phenomenon to be expected and to illustrate the previous discussion of such transients. These oscillograms show the complexity of the oscillations and indicate that the complete primary transient is not so simple as the analysis in Part IV might indicate. Reasons will be given as to why these complexities need not be considered in that particular analysis. Comparison of the center tap oscillations with those of distribution transformers described in Part IV is also of value.

A few oscillograms obtained under certain special or unusual conditions have been included for the purpose of illustrating some feature of interest or to indicate more clearly some point connected with the mechanism of transient voltage production.

All oscillograms in Figs. 30 to 41 were taken with a Westinghouse Cathode Ray Oscillograph of the Norinder type shown in Fig. 6-a.

The windings on which the tests were made were typical core type stacks of circular disc coils. The stacks were built up of twin coils composed of two discs connected together at the inside diameter. Connections between twin coils were made at the outside diameter. Taps were available between each coil and on the turns in the line coil.

The windings are described further in the following table.

Winding	No. Discs in Stack	Stack Length
A	56	48"
B	30	24"
C	20	24"
D	50	36"

Windings B and D were similar except for the number of coils in the stack. Taps are designated in per cent of the whole winding from the normally grounded end.

All measured voltages are voltages to ground unless otherwise specified.

1. Voltage Distribution

Typical transient oscillations in a grounded neutral winding are given in Fig. 30: (a) gives voltages to ground appearing on taps within the winding; (b) gives the same voltages for many taps throughout the winding. The first abrupt bend in the voltage curves before the oscillation starts represents the initial or capacity distribution.

The initial distribution is more sharply defined in Fig. 31 on winding B. Here the transition between initial distribution and oscillation is much slower due to the lower frequency of oscillation. It is interesting to note that the voltage difference between line terminal and any tap is greater at the time of the initial distribution than at any other time. This means that the distribution can be measured with a spark gap between line terminal and the various taps. However, this method should be used only as a check on the curve if oscillograms can be obtained.

The necessity of using a fast wave front in determining the capacity distribution can be seen in Fig. 35-a. With slow wave fronts, the sharp break in the curve disappears, and the proper point is difficult to locate.

The initial distribution for Winding C has been plotted from oscillograms in Fig. 28. The maximum distribution or envelope of maximum voltages is also shown.

The effect of isolated neutral on the capacity distribution can be seen by comparing Figs. 31 and 34-a.

An interesting test was made on Winding D which proves the capacity relations. In Fig. 33-a, the complete winding was used. The connections between coils were then cut, leaving each coil completely isolated. The same test was repeated without other change. For the first few microseconds the two cases are identical, thus giving experimental verification to the theory that capacitance plays the only part at the instant of impact of the surge.

2. Oscillations

Voltage distribution curves for various instants of time have been plotted from Fig. 30 to get the curves of Fig. 29. These curves show the oscillation of the voltage distribution about the final value as an axis.

In considering the internal oscillations appearing in the windings, both Figs. 30 and 31 show the 75% tap voltage rising to a value exceeding all others. The curves of voltage distribution in Fig. 29 show irregularities at both quarter points. These peculiarities are explained by the standing wave theory.

The oscillations of mid point and quarter points of Winding A are shown in Fig. 32-a and Fig. 39-a. The mid point oscillation appears to be a smooth sine wave, whereas the quarter point voltages evidently contain many harmonics.

Other interesting relations are shown in Fig. 32. When the center of the winding is grounded, the half length winding has exactly twice the frequency of oscillation as the whole stack. The free half of the winding, (Record 6), is seen to have half the frequency of the other half. With the whole winding isolated, the frequency is half that of the whole winding when grounded (Cf. Figs. 32-a and 34-b).

Examples of the local oscillations of the line coils are given in Figs. 35, 36 and 37. The effect of wave front on the magnitude of these oscillations can be seen in the test on Winding D, in which tap 1c is the 98.8% point located in the center of the first disc. This shows that the first disc acts independently of the rest of the winding, as though it were a separate winding by itself. The records of Fig. 36-a and 37-b differ in that they are the measured difference in voltage between line terminal and tap in the line disc, instead of voltage to ground. The voltage scale has been changed from the other tests on this winding.

The tests on windings with some interior tap grounded instead of the end of the winding were made to determine the characteristics of oscillation of short stacks. The oscillations in the isolated parts are also of interest.

3. Traveling Wave Analysis

Several oscillograms show with remarkable clarity the characteristics of traveling wave phenomena that have been previously given. The major features of the oscillations of Fig. 30 can be explained by the traveling wave concept. These oscillations show differences in time to the first crest at various points in the winding which might be construed as representing the progress of a wave propagated along the winding conductors. The fact that the voltage oscillations on taps near the ground reach their positive crest last and negative crest first would check with the idea of a wave passing the tap on its way to ground and then returning with reversed polarity after reflection at the ground terminal.

Perhaps the clearest picture of the existence of a traveling wave is to be seen in Fig. 40. Here the first coil was grounded and the remainder of the winding left isolated except for the ground connection at the first coil. An impulse was applied across the first coil. The chief characteristics of the voltage in the free end of the stack, which comprised 93.3% of the total length, are obviously due to a wave propagated along the winding. The shape of the voltage peaks and their time differences on various taps both call for a propagated wave.

The explanation of the pehnomena can be easily worked out. First, the surge applied, (Curve 1), to the first coil passes through that coil to ground. The time required is short, since only one coil is traversed. This initial wave is then reflected with reversed polarity and re-enters the winding, one wave going back through the first coil, and another entering the free end of the stack. The latter wave then proceeds along the stack until it reaches the open end, (Tap 16), where it doubles in value and is reflected with the polarity unchanged. The returning wave then proceeds back along the winding to the grounded point. At the grounded tap it is again reflected with reversed polarity and starts back along the winding towards the free end. This accounts for the sudden change in sign of the voltage at about forty microseconds. This shows that the time required for a wave to traverse the length of the winding was about 20 microseconds.

The prominence of the traveling wave in this case is no doubt due to the absence of some of the other causes of voltage normally present in a winding. The grounding of the lower disc of the first coil provides an electrostatic shield between the line disc and the free end of the stack. This would eliminate most of the capacity effects and the resulting oscillations due to initial capacity distribution of voltages. The electromagnetic effect from neutral inductance is, of course, still present. Tests can thus sometimes be devised to eliminate some of the variables of the problem and show more clearly the operation of remaining factors in the phenomenon.

Another point of interest appears in the voltage at the open end of an isolated neutral winding, (Fig. 34-a). This shows the "certain flat top characteristic" mentioned by Boehne. It will be shown that this effect is probably mainly due to losses in the iron core.

The preceding discussion of the traveling wave is not intended as an argument for discarding the standing wave theory of oscillations, but merely to show that there is experimental evidence that traveling waves do exist in transformer windings. It is felt that in this connection the idea of a wave propagated along the conductor has not generally been given the consideration it deserves.

4. Losses and Damping

The predictions of analytical theory as to the effects of losses on the transient oscillations have been previously stated to be threefold. Losses should cause damping of oscillations, lowering of frequency, and shift of axis of oscillation. The effect of the losses in the iron core in decreasing the frequency of oscillation may be seen by comparing the two oscillograms of Fig. 31. The maximum voltage due to the first half cycle of oscillation is not appreciably diminished by the losses, but the frequency is seen to be perceptibly changed. Similar decrease in frequency is evident on comparing Fig. 39-b and Fig. 41-a, and in the two oscillograms of Fig. 40. The damping of oscillations is also evident. The shift of axis of oscillation is clearly shown in Fig. 40. The presence of the iron core also seems to have a tendency to flatten the peaks of the oscillation. The shift of axis may also be seen in the test of a distribution transformer shown in Fig. 66-a.

If the voltages of Fig. 40 are considered as being due to a traveling wave, the effect of the iron would be to slow down the velocity of propagation. This is to be expected since the velocity of propagation of a wave is given by the relation $\frac{c}{\sqrt{K\mu}}$ where μ is the magnetic permeability and K the dielectric constant of the surrounding medium.

5. Cathode Ray Oscillograms of Transient Effects in Power Transformer Windings

Normal Connection of Winding[1]

Winding	Neutral Connection	Wave Applied	Fig. No.
A	Grounded	1.5/60	30-a, b
A	Grounded	1.5/60	32-a
A	Grounded	1.5/60	39-a
A	Isolated	1.5/60	34-b
B	Grounded	1.5/60	31-a, b
B	Grounded	0.5/-	37-a
B	Isolated	1.5/60	34-a
D	Grounded	0.3/60	33-a
D	Grounded	5/-	35-a
D	Grounded	1.5/60	35-b
D	Grounded	0.2/60	36-a
Double Shell	Grounded	1.5/60	41-b

---

[1] Normal connection means grounded or isolated neutral as the transformer would be used in practice.

A 1.5/60 wave means the voltage rises to crest value in 1.5 microseconds and falls to 50% of crest value on the tail in 60 microseconds.

Special Connections of Winding [1]

Winding	Tap Grounded	Applied Wave	Core	Fig. No.
A	50%	1.5/60	Air	32-b
A	50%	"	"	38-a, b
B	46.7%	"	"	39-b
B	46.7%	"	Iron	41-a
B	93.3%	"	Air	40-a
B	93.3%	"	Iron	40-b

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1. In these tests a tap in the interior of the stack was grounded and the surge applied between the 100% tap and ground. The portion between the grounded tap and the 0% end of the winding was left isolated.

Surge Voltage Distribution in Windings

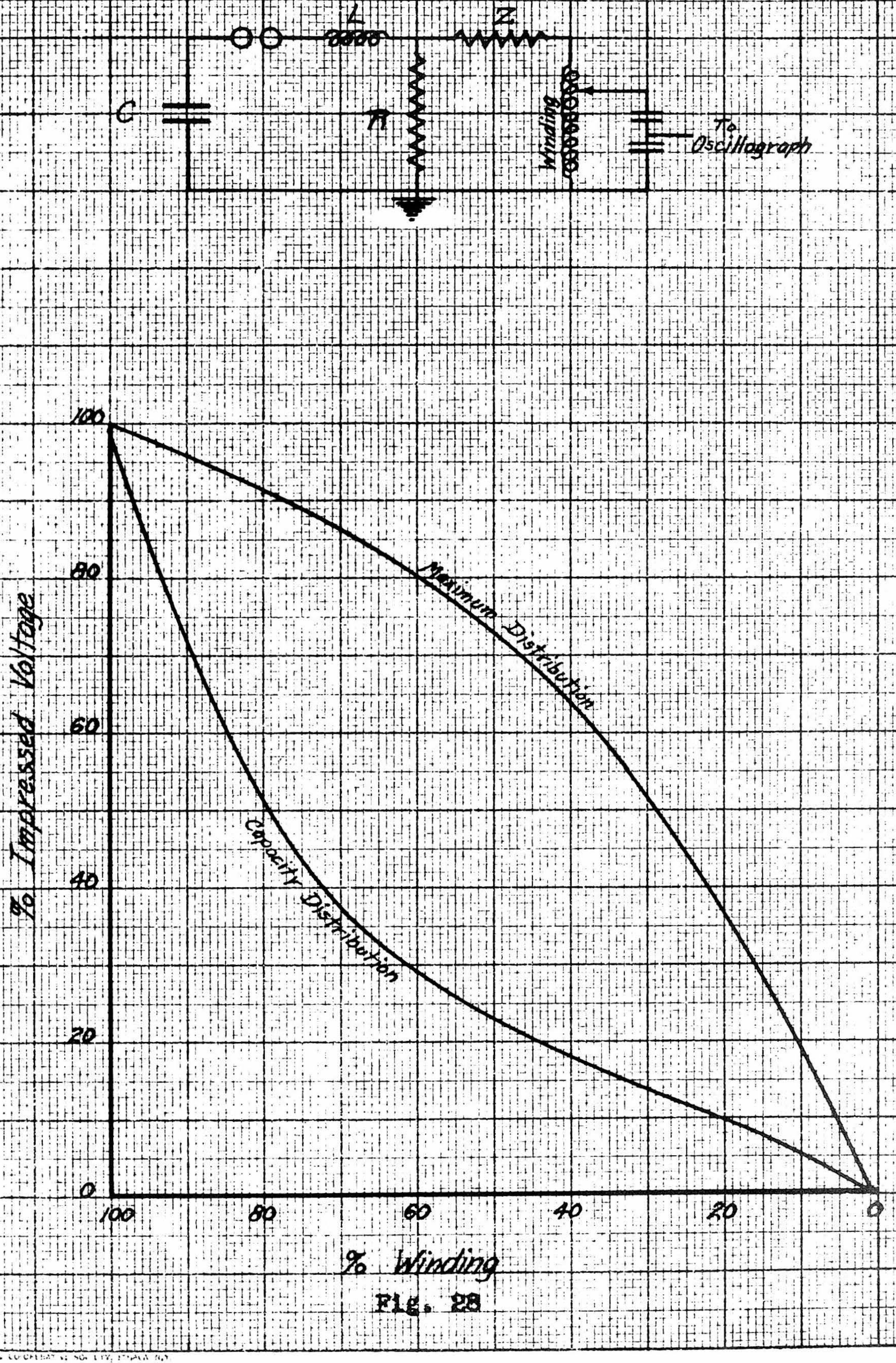


Fig. 28

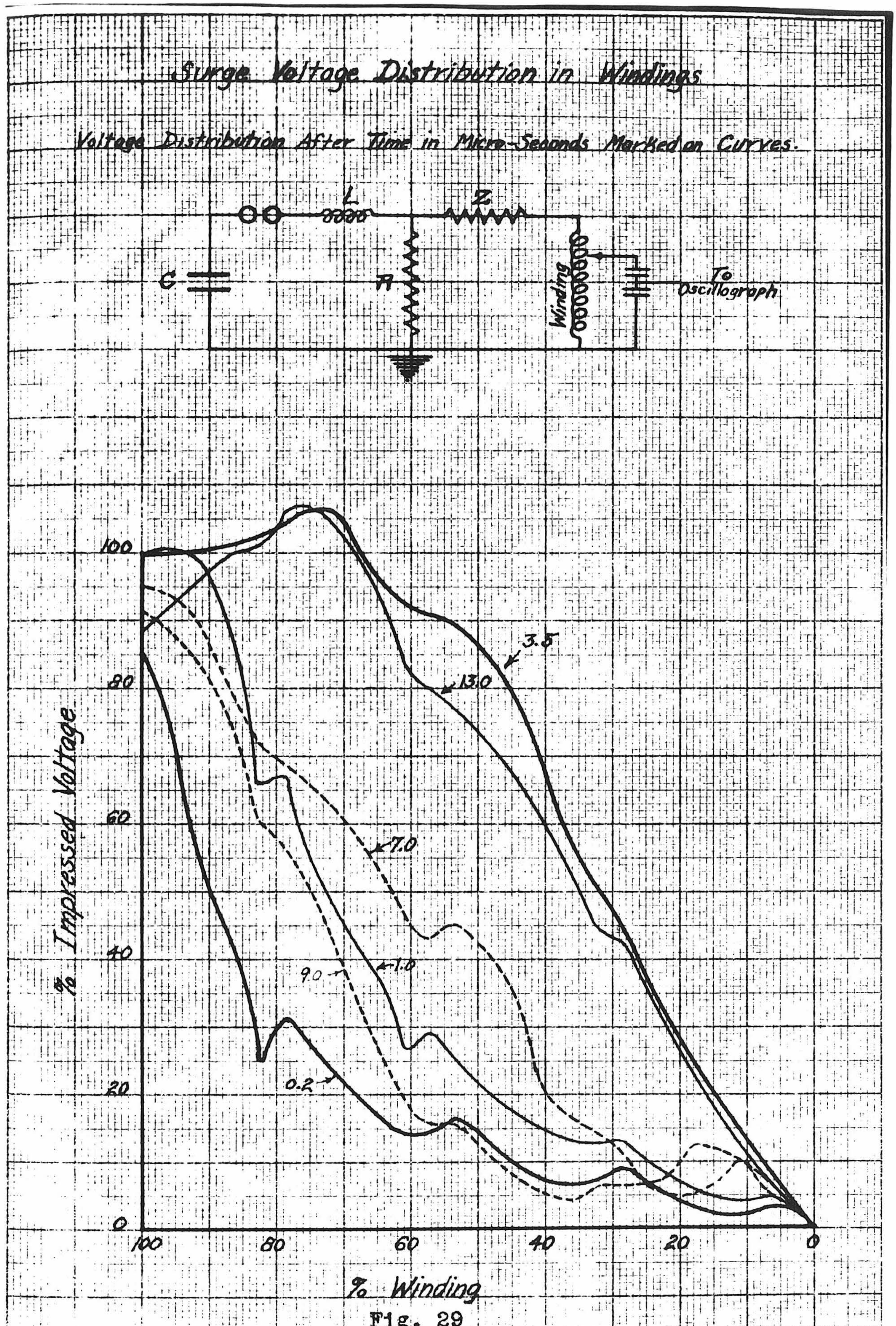
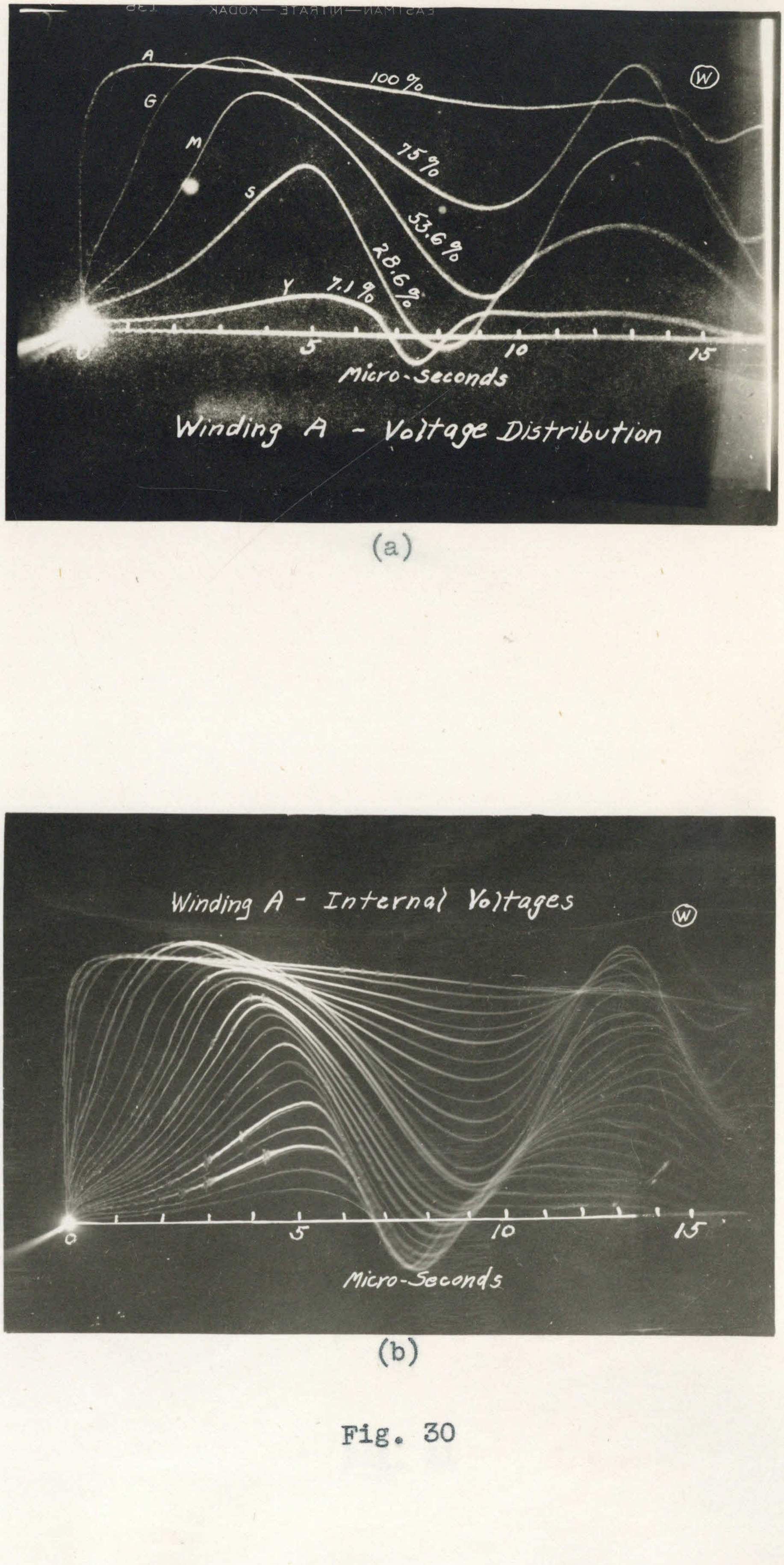

Surge Voltage Distribution in Windings
Voltage Distribution After Time in Micro-Seconds Marked on Curves.
L
Z
C
R
Winding
To Oscillograph
100
80
60
40
20
0
% Impressed Voltage
3.5
13.0
7.0
9.0
1.0
0.2
100
80
60
40
20
0
% Winding

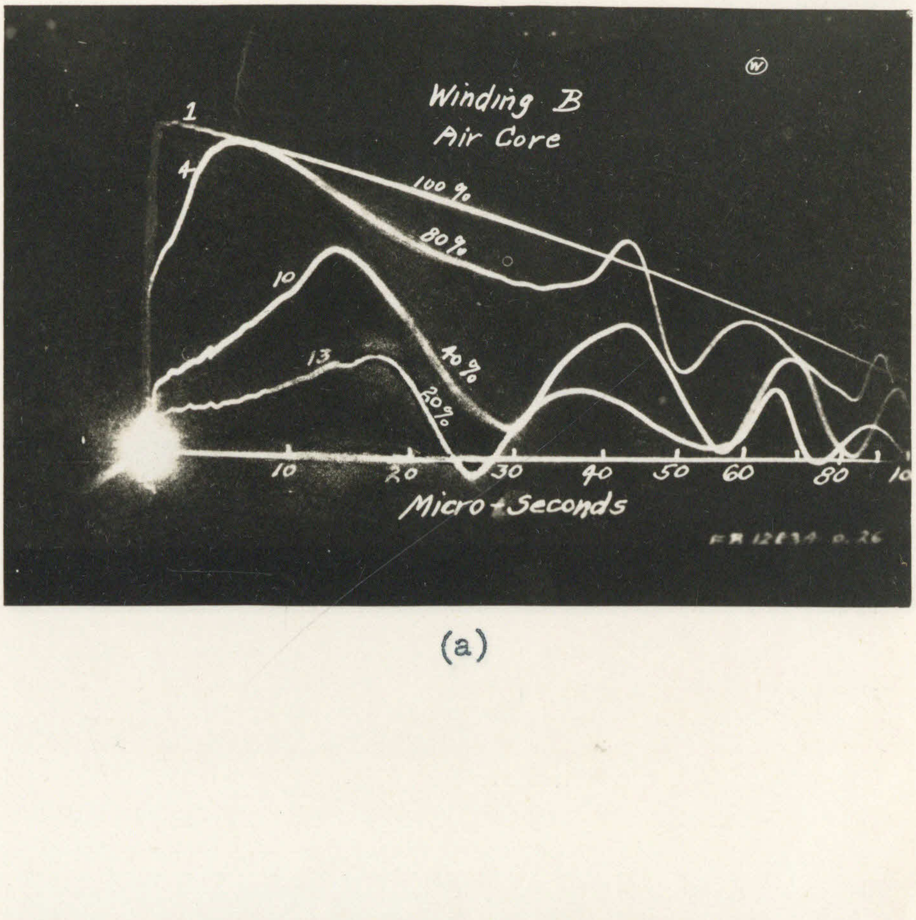
Fig. 29


A
G
M
S
Y
100 %
75 %
53.6 %
28.6 %
7.1 %
Ⓦ
0
5
10
15
Micro-Seconds
Winding A - Voltage Distribution
Winding A - Internal Voltages
Ⓦ
0
5
10
15
Micro-Seconds

(a)

(b)

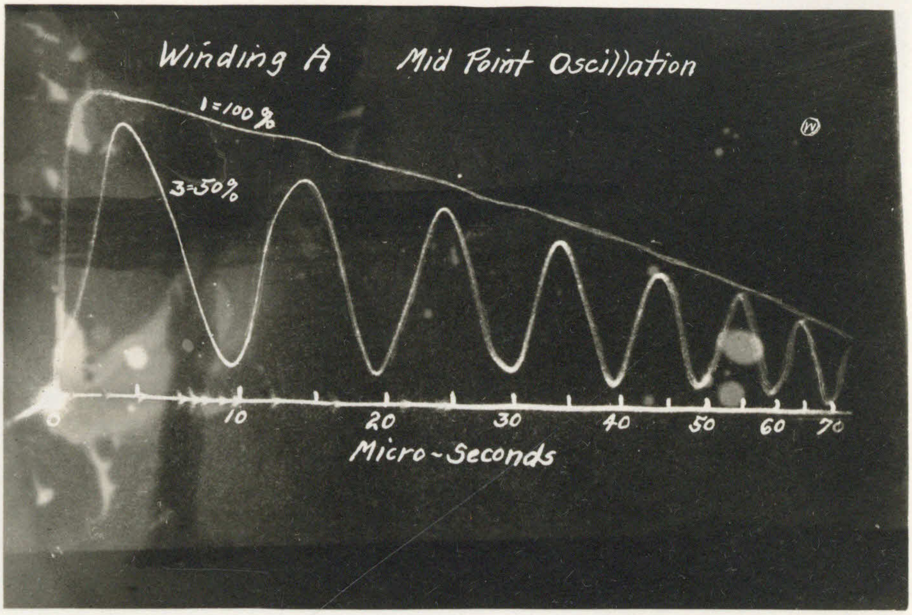
Fig. 30


Winding B
Air Core
1
4
10
13
100%
80%
40%
20%
10
20
30
40
50
60
80
Micro-Seconds
Winding B
Iron Core
1
4
10
100%
80%
40%
0
10
20
30
40
50
60
70
80
90
100
Micro-Seconds

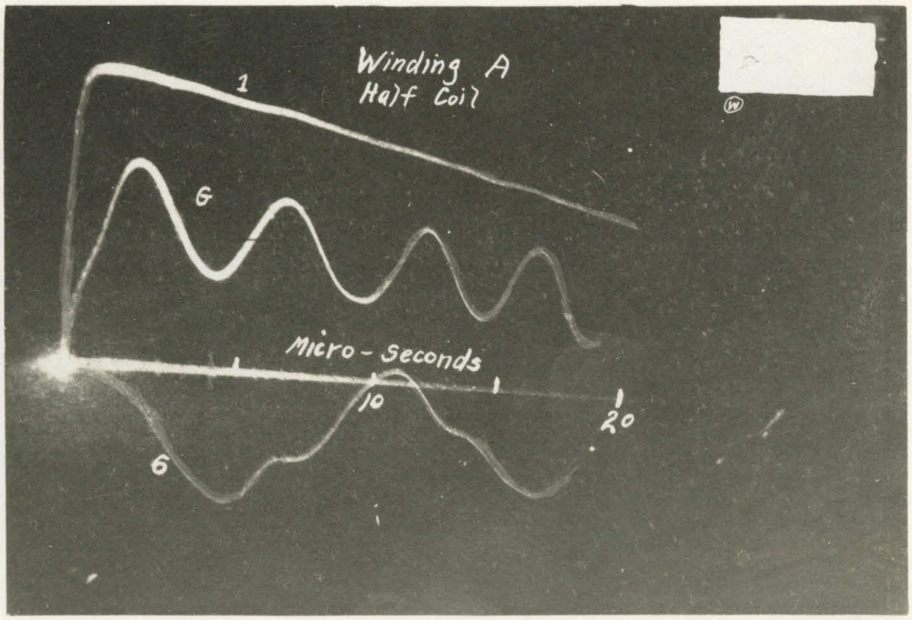
(a)

(b)

Fig. 31

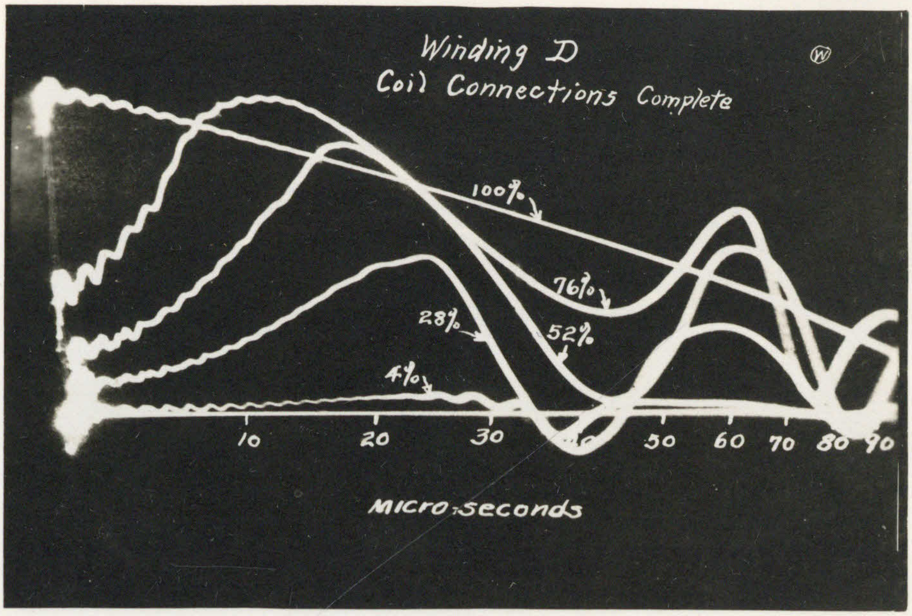

Winding A Mid Point Oscillation
1=100%
3=50%
0
10
20
30
40
50
60
70
Micro-Seconds

(a)

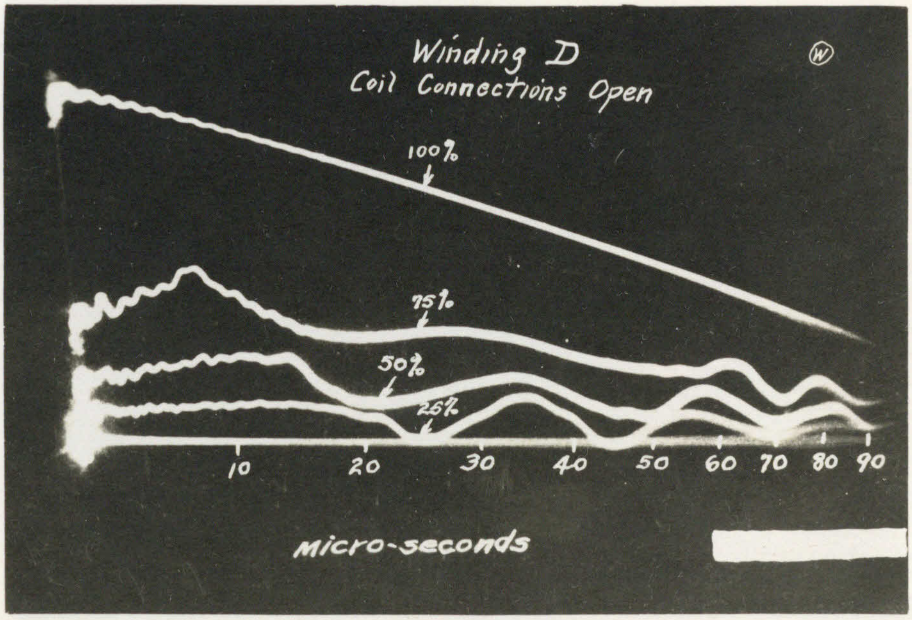

Winding A
Half Coil
1
G
6
Micro-Seconds
10
20

(b)

Fig. 32

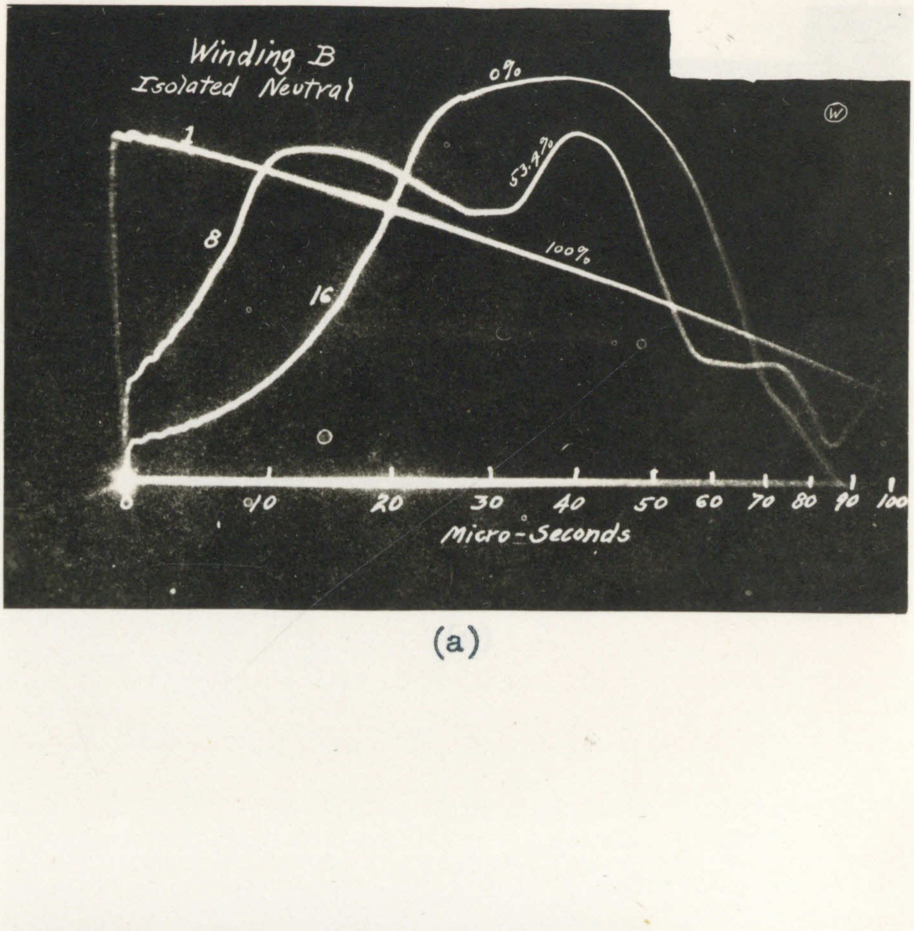

Winding D
Coil Connections Complete
Ⓦ
100%
76%
28%
52%
4%
10
20
30
40
50
60
70
80
90
micro-seconds

(a)


Winding D
Coil Connections Open
Ⓦ
100%
75%
50%
26%
10
20
30
40
50
60
70
80
90
micro-seconds

(b)

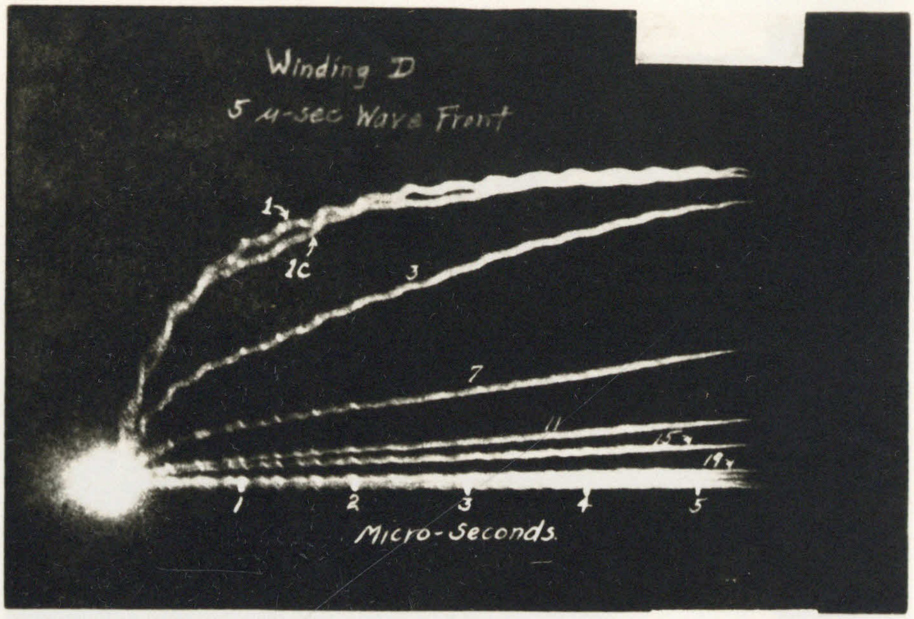
Fig. 33


Winding B
Isolated Neutral
0%
W
1
53.4%
8
16
100%
0
10
20
30
40
50
60
70
80
90
100
Micro-Seconds
Winding A
Isolated Neutral
3
75%
50%
G
100%
0
10
20
30
40
50
60
70
Micro-Seconds

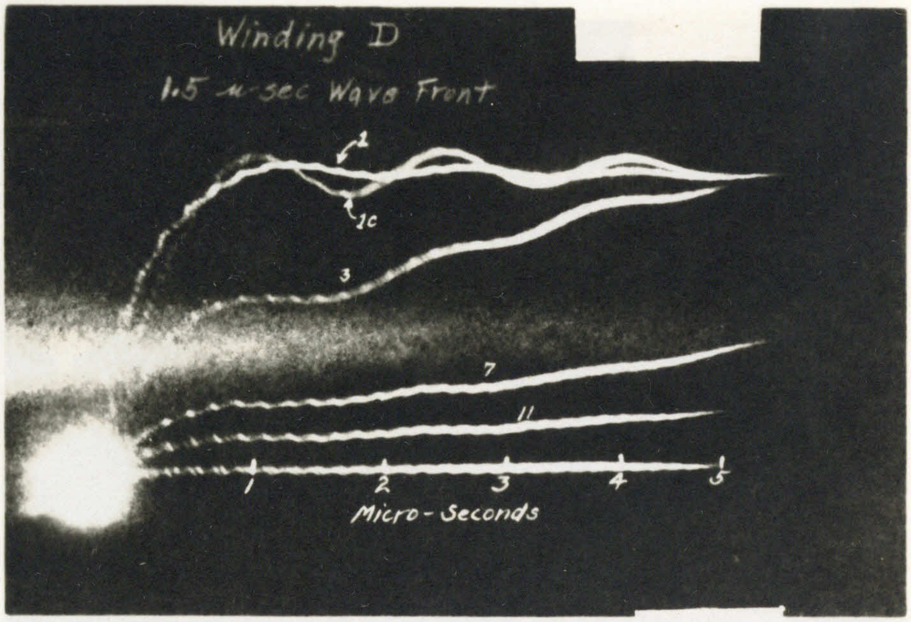
(a)

(b)

Fig. 34

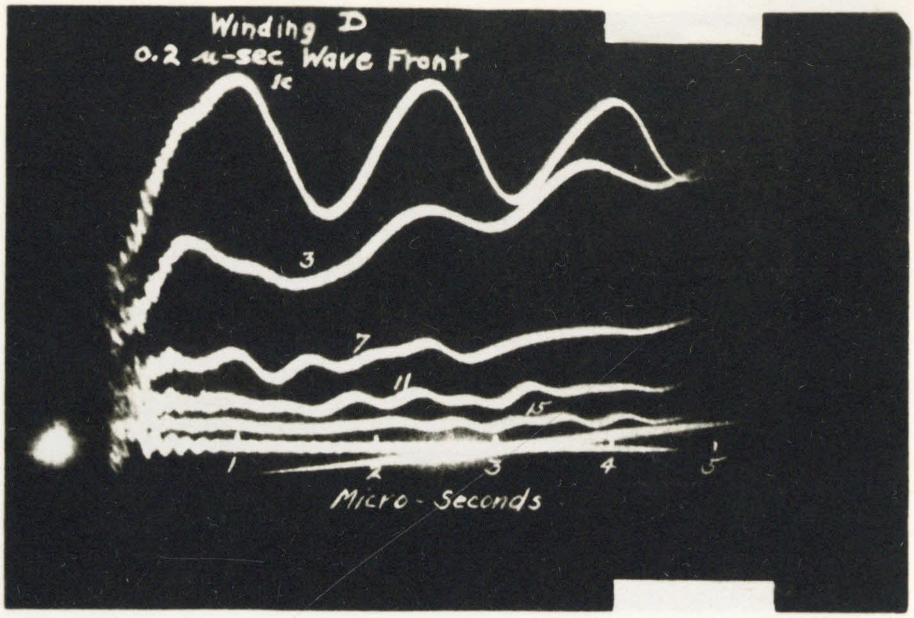

Winding D
5 μ-sec Wave Front
1
1c
3
7
11
15
19
1
2
3
4
5
Micro-Seconds.

(a)

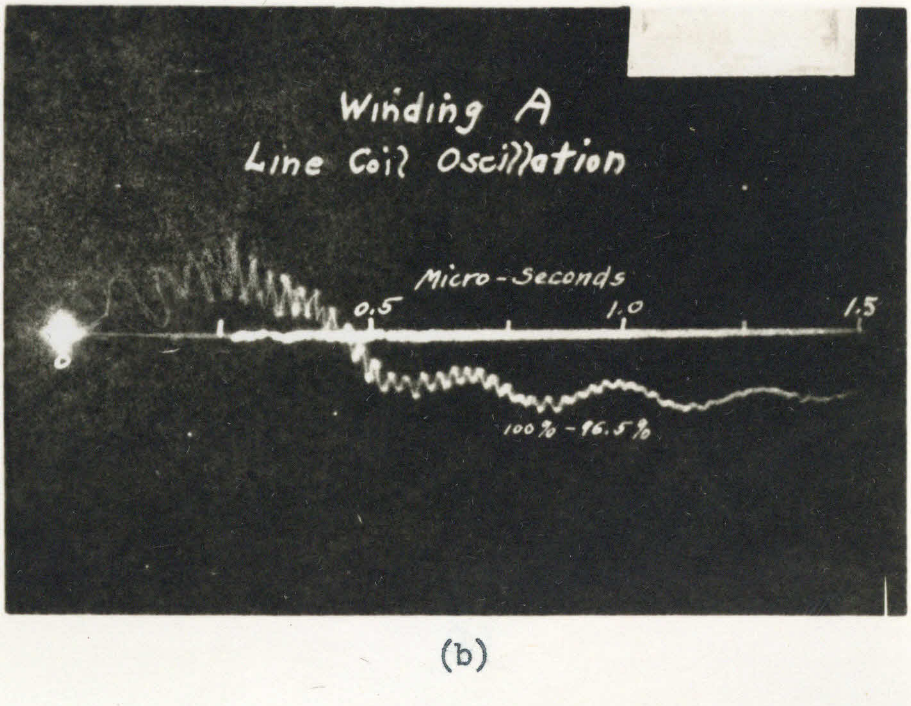

Winding D
1.5 μ-sec Wave Front
1
1c
3
7
11
1
2
3
4
5
Micro-Seconds

(b)

Fig. 35

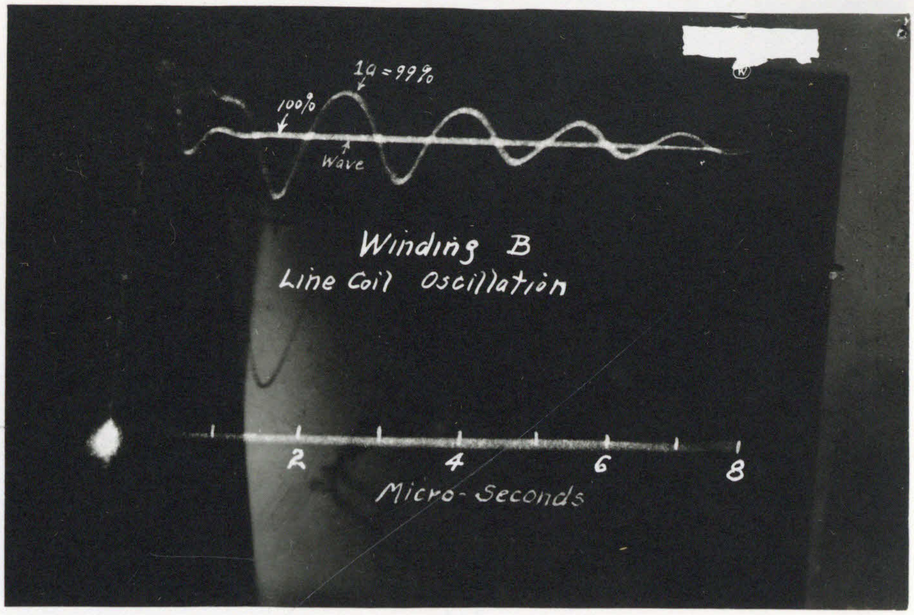

Winding D
0.2 μ-sec Wave Front
1c
3
7
11
15
1
2
3
4
5
Micro-Seconds

(a)

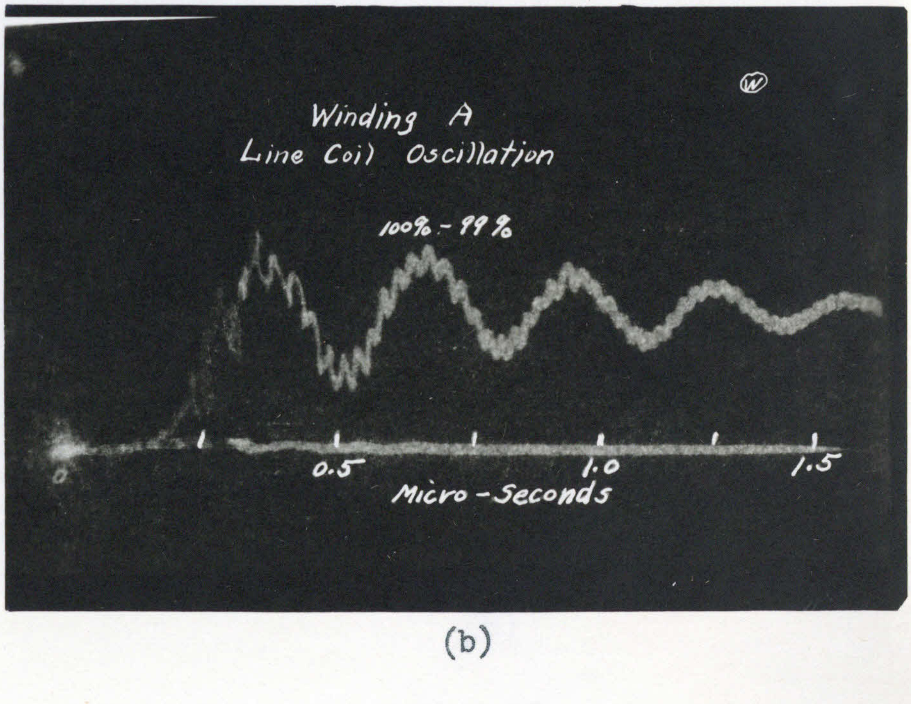

Winding A
Line Coil Oscillation
Micro-Seconds
0
0.5
1.0
1.5
100% - 96.5%

(b)

Fig. 36

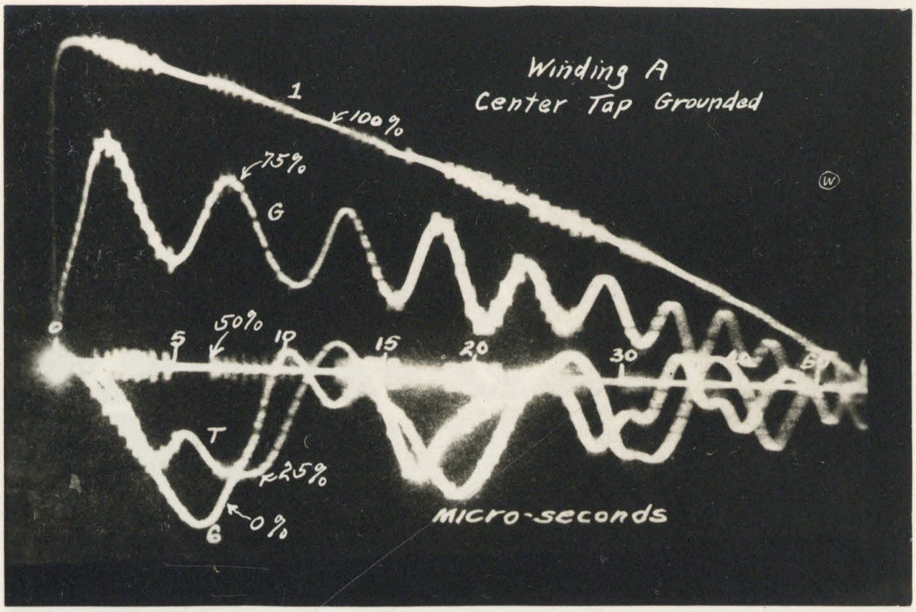

1a = 99%
100%
Wave
Winding B
Line Coil Oscillation
2
4
6
8
Micro-Seconds

(a)

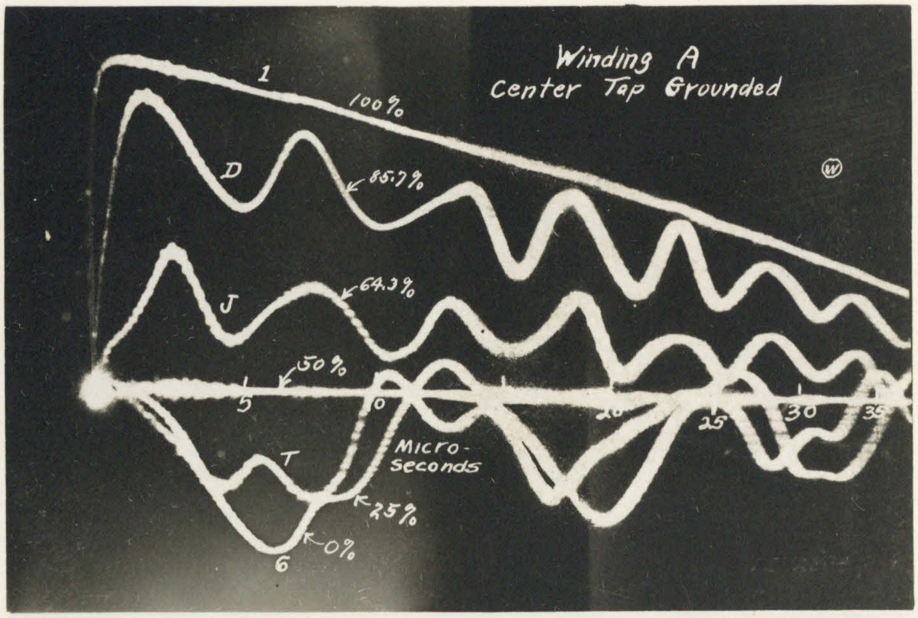

Winding A
Line Coil Oscillation
100% - 99%
0
0.5
1.0
1.5
Micro-Seconds

(b)

Fig. 37

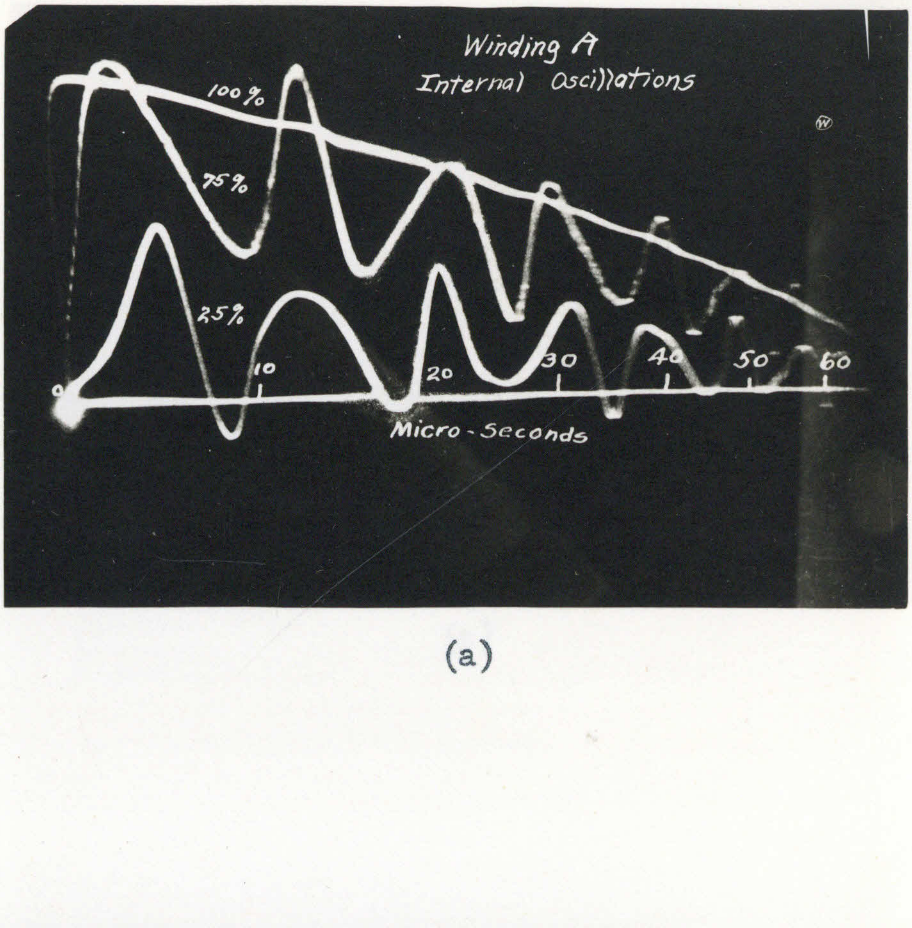

Winding A
Center Tap Grounded
1
100%
75%
G
50%
T
25%
0%
6
0
5
10
15
20
30
Micro-seconds
W

(a)


Winding A
Center Tap Grounded
1
100%
D
85.7%
J
64.3%
50%
T
25%
0%
6
5
10
20
25
30
35
Micro-seconds
W

(b)

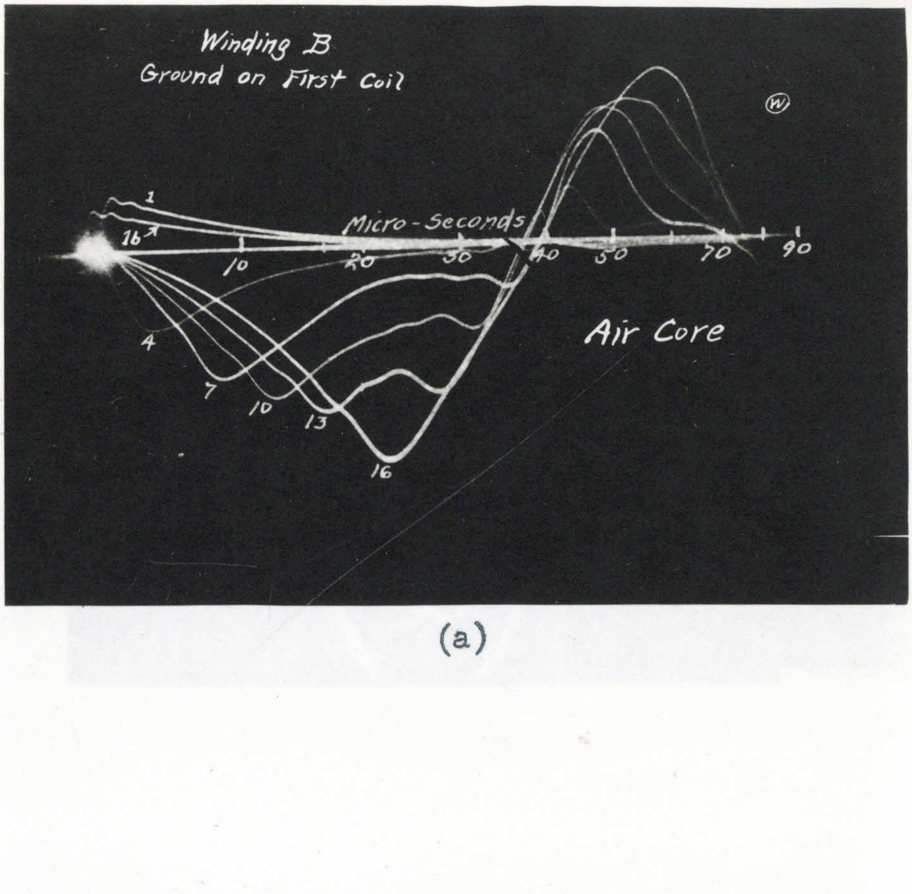
Fig. 38


Winding A
Internal Oscillations
100%
75%
25%
0
10
20
30
40
50
60
Micro-Seconds
Air Core
Winding B
Internal Ground
2
↖93.3%
4
↖80%
6
↖73.4%
8
↖53.4%
↖46.7% Ground
10
↖40%
12
↖26.7%
14
↖13.3%
16
↖0% Open
10
20
30
40
50
60
80
Micro-Seconds

(a)

(b)

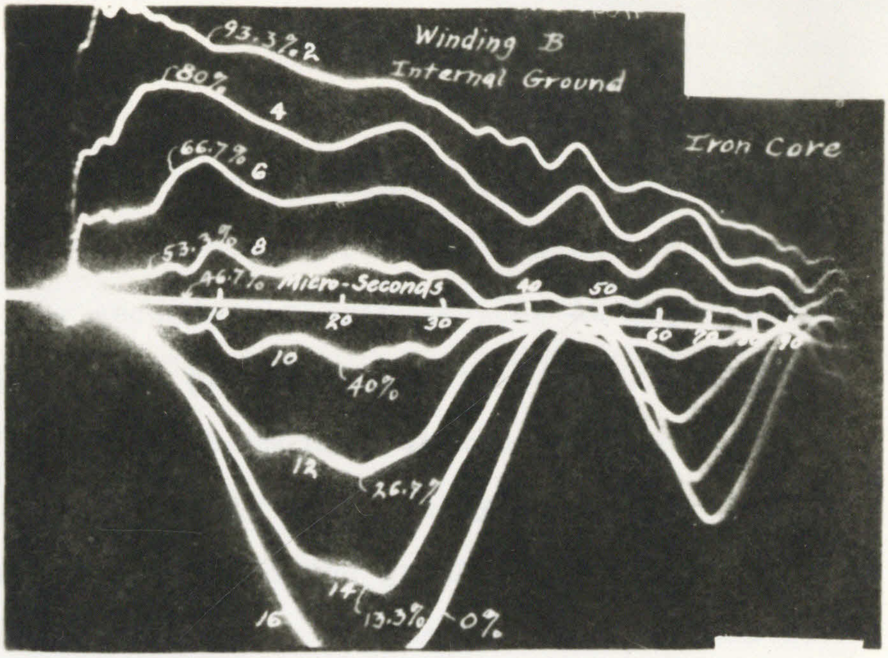
Fig. 39


Winding B
Ground on First Coil
Micro-Seconds
1
1b
4
7
10
13
16
10
20
30
40
50
70
90
Air Core
Winding B
Ground on First Coil
Micro-Seconds
1
1b
20
30
40
10
60
80
4
7
10
13
16
Iron Core

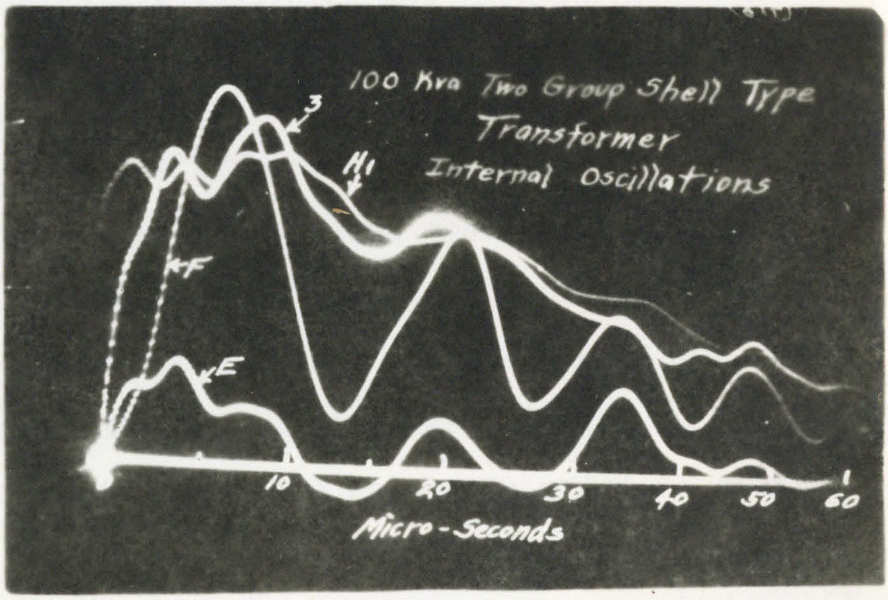
(a)

(b)

Fig. 40


Winding B
Internal Ground
Iron Core
93.3% 2
80% 4
66.7% 6
53.3% 8
46.7%
Micro-Seconds
10
20
30
40
50
60
70
80
90
10
40%
12
26.7%
14
13.3%
0%
16

(a)


100 Kva Two Group Shell Type
Transformer
Internal Oscillations
3
H1
F
E
10
20
30
40
50
60
Micro-Seconds

(b)

Fig. 41

IV

TERMINAL TRANSIENTS OF TRANSFORMER SECONDARY WINDINGS

Another phase of the problem of transient conditions in transformers is the study of the transient voltages appearing on the secondary terminals when an impulse voltage is applied to the primary. Progress in the solution of this part of the problem has not kept pace with that of the internal transients in the primary winding. Perhaps this has been due partly to the greater importance of the primary phemomena, but it is also due to the greater complexity of the problem of secondary transients, and to the consequent difficulty involved in devising methods of predicting those transients. Although this problem has not been solved as completely as the problem of the primary winding, nevertheless something has been accomplished.

A. Theoretical Considerations

1. General Analytical Theory

Bewley's general two winding theory, based on the solution of the equivalent circuit of a transformer shown in Fig. 42, has been previously discussed with the view of finding the reaction of the secondary winding on the primary internal transients. It will now be in order to examine this theory to see what successes it has had in determining the nature of the transients in the secondary winding.

An outline of the general procedure followed in developing the theory will be of value. This is, of course, the common way of attacking any such problem mathematically, but was first stated with reference to the transients in windings by Wagner[1] and has been re-stated by Bewley.[2] The steps involved in the process may be listed as follows:

(A) An equivalent circuit is developed.

(B) The differential equation of the circuit is derived and the terminal conditions specified.

(C) Initial voltage distribution for impact of an infinite rectangular wave is determined.

(D) Final voltage distribution or axis of oscillation is obtained.

(E) Complete solution is obtained using the relation that the initial distribution equals the final distribution plus the initial transient.

(F) Solutions for any arbitrary wave shape can be obtained from the above solution by use of Duhamel's integral.

The equation derived for the equivalent circuit turns out to be an eighth order partial differential equation.

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1. "Das Eindringen einer Elektromagnetischen Welle in eine Spule mit Windungskapazitat," K. W. Wagner, Electrotechnik und Maschinenbau, Feb. 1915, S. 89.
2. "Traveling Waves on Transmission Systems," L. V. Bewley, John Wiley & Sons, 1933, p. 209.

Solution of this equation, after initial and final distributions have been obtained, is in terms of an infinite Fourier expansion of space and time harmonics, and involves the determination of sixteen integration constants, a wave length constant, and two frequency constants. The sixteen integration constants are further restricted by the relations imposed by two auxiliary fourth order partial differential equations. These facts are mentioned in order to give some idea of the difficulty which must be overcome in making an analytical attack on the problem.

Solutions have been found possible only for grounded or isolated terminal conditions. Even with these limitations, certain useful general conclusions have been reached. The initial distributions and electrostatic components of the terminal voltages are seen to be determined entirely by capacitances of the windings and the terminal impedances. The final distributions depend on inductances of the winding, final electrostatic field, and terminal impedances. Internal transient oscillations have been seen to consist of an infinite series of space and time harmonics whose amplitudes depend on the difference between initial and final voltage distributions.

2. Approximate Analysis

The difficulties encountered by the purely analytical method, and the practical limitations of the general theory, have led to attempts to devise approximate methods of analysis which would account for the more important features of the phenomenon and, at the same time, avoid the complexities entailed by the use of more rigorous methods. Such an approximate method of analysis has been devised by Palueff[1] in his fourth paper on transients in transformers.

A different and more complete method of analysis will be developed herein.

a. Components of Secondary Terminal Transient Voltage. Both theory and experiment indicate that the transient secondary terminal voltages can in general be divided up into four component voltages, each of these components being produced from a different cause. These four component voltages are: (a) an electrostatic voltage from the applied primary wave, induced by capacity coupling between the two windings; (b) a primary frequency component arising from the voltage produced by the natural oscillation of the high voltage winding and induced in the secondary by capacity coupling; (c) an oscillatory component arising from the free oscillation of the secondary winding; (d) a unidirectional component produced by electromagnetic induction between the windings.

The relative magnitudes of these components depend on the transformer characteristics, terminal impedances, and shape of applied wave. The electrostatic component obviously

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[1] "Effect of Transient Voltages on Power Transformer Design-IV," K. K. Palueff and J. H. Hagenguth, A.I.E.E. Trans., Vol. 51, Sept. 1932.

does not depend on turn ratio, but only on effective capacitances. The primary frequency component is also independent of turn ratio and depends on capacitances which may differ from those effective for the electrostatic component. This point will be considered in detail later. The secondary frequency component depends on the distributed constants of the secondary winding and its amplitude on the turn ratio and applied wave shape. The electromagnetic component is directly proportional to turn ratio and is independent of capacity relations.

Some disagreement exists between the present writer and Palueff as to mechanism of production of the oscillatory components. Palueff states[1] with regard to the primary frequency component that, "The induction is accomplished by means of the electrostatic and <u>electromagnetic</u> <u>fields</u> of these harmonics, and is dependent upon the distributed constants and <u>turn</u> <u>ratio</u> of the windings." It is undoubtedly true that voltages are induced in the secondary by electromagnetic means from the primary oscillation. However, a consideration of the directions of the oscillatory current in elements of the primary winding, and of the magnetic fields produced by them, will show that the voltages induced by such means will have components equal and opposite, and will, therefore, cancel as far as terminal effect is con-

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[1] Loc. cit.

cerned. This can be seen in the relations between the harmonic currents and magnetic fields as shown by Palueff himself.[1]

Differences may also exist in connection with the mechanism of secondary oscillation. Palueff states[2] that in the secondary free oscillation, "The resulting voltages depend upon the distributed constants of the low voltage winding." He mentions no effect due to turn ratio. The initial electrostatic voltage distribution in the secondary would tend to produce an oscillation whose initial magnitude would depend on the difference between the initial and final distributions. It must also be true that a steep front or chopped tail on the electromagnetic component would be productive of oscillations since this would cause a displacement of voltage from the equilibrium distribution. If the initial electrostatic voltages are the same in all parts of the low voltage winding, no oscillation from this source would be generated. An examination of many of the oscillograms presented in Figs. 60 to 165 of this thesis show that this situation exists and that no oscillation from this source would be present. This reduces the cause of oscillation to the electromagnetic component whose steepness of front and tail Palueff has shown to depend on turn ratio and leakage reactance.

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1. "Lightning Studies of Transformers by the Cathode Ray Oscillograph," F. F. Brand and K. K. Palueff, A.I.E.E. Trans., Vol. 48, July 1929, p. 1001, Figs. 4 and 5.
2. Ibid.

b. Palueff's Approximate Analysis.[1] A search for a simpler, more practical approach to the problem of predicting secondary terminal transients has led to a method of computation which gives with engineering accuracy the most important component of secondary voltage under ordinary conditions. This is the electromagnetic component.

An examination of the relative importance of the four components of voltage when the secondary winding is grounded, or connected through impedances to ground of a few hundred ohms, has indicated that the electromagnetic component dominates. This will also be shown in the oscillograms of the secondary voltages of small transformers included in this work.

Palueff shows that the electromagnetic component can be calculated by considering the transformer replaced by a series inductance between primary and secondary lines. The value of this inductance can be taken as the short circuit inductance of the transformer. This value must, of course, be that of the equivalent one to one ratio transformer.

This method results in the conclusion that the electromagnetic component may be considered as a copy of the applied primary wave, reduced by the turn ratio, and with front sloped to T_2 microseconds where

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[1] "Effect of Transient Voltages on Power Transformer Design-IV," K. K. Palueff and J. H. Hagenguth, A.I.E.E. Trans., Vol. 51, Sept. 1932.

$$T_2 = \frac{3L_s}{Z_1 + Z_2}$$

T_2 = Time to 95% of crest value.

L_s = Equivalent short circuit inductance.

Z_1 = Surge impedance of primary line.

Z_2 = Surge impedance of secondary line.

This relation assumes a grounded neutral connection. A more rigorous solution for this component has been presented by Hagenguth[1] and a method derived by the general analytical method by Bewley.[2]

The electrostatic component may in some cases exceed in magnitude the electromagnetic component. This is nearly always the case if the secondary winding is open and isolated. Even in a grounded secondary connected to a line, this component may reach high initial values, but will have short duration. This will also be seen in the oscillograms of small transformers included herein.

Palueff's treatment of this component is based on considerations of the total energy contained in the primary and secondary waves. This method apparently leads to correct general conclusions as to the behavior of this component, but does not seem to lend itself to definite numerical calculation.

The two oscillatory components are shown to be of probably minor importance in most cases.

[1] Ibid.
[2] Ibid. See discussion.

It must be said that the analysis described is intended to give approximately correct results for the major components of secondary transient voltages in transformers under usual service conditions. Within these limitations, it appears to adequately fulfill the requirements.

B. Previous Experimental Investigations

The published experimental work dealing with secondary terminal transients is not very extensive. With one notable exception, most of this work has been concerned with the protection of distribution transformers from lightning voltages.

1. Accuracy of Approximate Analysis

The only important paper dealing with the mechanism of secondary transient production that has come to the attention of the author is that by Palueff and Hagenguth. This has just been discussed as an analytical work, but it might just as well have been considered an experimental one. The analysis presented therein is based on, and arose from, an experimental background.

Actual experimental data included in the form of curves or oscillograms are disappointingly meagre. A few examples of the application of Palueff's method of analysis to the calculation of specific cases have been given.[1] These show good agreement between calculated and measured curves, if allowances are made for the approximations and limitations of the method. It should be noted that all examples given are for high voltage power type transformers.

2. Protective Studies

Cathode ray oscillograph studies of secondary terminal voltages have been made with the object of determining the

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[1] Palueff and Hagenguth, Op. cit., Discussion p. 619. Discussion by J. H. Hagenguth, A.I.E.E. Trans., Vol. 49, July 1930, p. 1192.

effectiveness of protective measures. Published accounts of such work gives little information as to the character of the voltages found other than crest values of voltage.

Some examples of oscillograms of secondary voltages are to be found, which show the effect of grounded or isolated tank, effect of lightning arrester protection, and effect of method of connection.[1] Passing mention of the effect of mid point ground and terminal impedance has also been made by Treanor and Cooney.[2] These papers give little, however, which is of much value in an analysis of the nature of secondary transients.

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[1] Symposium on "Lightning Protection for Distribution Transformers," A.I.E.E. Trans., Vol. 51, March 1932.

[2] "Insulation Coordination of Distribution Transformers," E. D. Treanor and W. H. Cooney, A.I.E.E. Trans., Vol. 51, Dec. 1932.

C. Impulse Tests on Distribution Transformers

The experimental part of the present investigation was undertaken to obtain records of the characteristics of secondary terminal transients under a wide variety of conditions. Distribution type transformers were used for three reasons: first, such transformers were more easily available than other types and more convenient to handle on test; second, there is an essential difference in type of construction of windings in distribution and power type transformers which would affect the characteristics of the equivalent transient network, thus making comparison with published results on power types desirable; third, it was felt that a method developed for the analysis of the transient effects in small transformer windings could also be applied to any other winding with suitable modifications. The tests can be separated roughly into three groups: (a) voltage impulse was applied to primary; (b) voltage impulse was applied to secondary winding; (c) primary and secondary windings were connected together to simulate an autotransformer.

1. Impulse Applied to Primary

The normal condition to be found in practice is where the surge arrives on the primary line. Complete tests were made under these conditions, using four different transformers and a variety of wave shapes and secondary conditions. A few of the outstanding characteristics of the secondary transients found in these tests may be listed as follows:

(A) The capacity components of voltage predominate when the secondary is isolated and open circuited.

(B) The effect of isolating the transformer tank is to increase greatly the capacity component of voltage on the secondary when it is isolated and open circuited.

(C) Secondary terminal impedances to ground lower than 500 ohms eliminate most of the oscillatory components and reduces greatly the electrostatic component.

(D) Electromagnetic component predominates when secondary is grounded directly or through terminal impedances less than 500 ohms.

(E) A grounded secondary has no primary frequency component.

(F) Duration of the electrostatic component with secondary grounded is less than one microsecond and may be as little as two-tenths of a microsecond. Magnitude of the electrostatic component may be as great as the electromagnetic component under these conditions, however.

2. Impulse Applied to Secondary

Only a few tests of this type were made, because of time limitations. This condition is important, however, since it is not impossible for surges to arrive on the secondary lines in service. Similar situations exist when impulses are applied across the series windings of induction regulators or autotransformers. Many of the characteristic effects observed when impulses are applied to the low voltage winding have been observed in induction regulators.[1] Some of the general results of this type of test follow:

(A) Applied secondary surge is increased in crest value by the turn ratio.

(B) Reaction of high voltage winding produces the major part of the transient voltage appearing in the low voltage winding.

(C) Duration of applied impulse voltage is very short. This voltage is similar in appearance to the secondary electrostatic component, but is the inductive voltage across the low voltage winding.

(D) Transient in both high and low windings is characterized by the long period oscillation of the high voltage winding.

The above characteristics may be seen in the oscillograms of Figs. 135 to 140.

3. Autotransformer Connection

The ratio of the transformers connected as autotransformers was too near untiy to show clearly the effects given in the previous discussion of Palueff's work. Some information of value concerning the component voltages can be obtained from these tests, however.

Those tests (Test 21), where the ratio was higher, show in striking fashion the transient characteristics ob-

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[1] "Surge Tests on Induction Regulators," M. E. Gainder, Unpublished Engineering Memorandum No. 865 of the Westinghouse Electric & Mfg. Co., July 1930.

served in the preceding tests in which the surge was applied to the secondary winding. Similar results are found in induction regulators.

4. General Conclusions

Most of the general conclusions mentioned in the previous discussions of transformer transient characteristics have been found in the distribution transformer tests. The general transient effects seem to be the same for the type winding used in small transformers as in the type used in power transformers. The chief differences probably lie in the character of the local oscillations. The center tap only of the high voltage winding was available in these tests so the presence of local oscillations could not be detected. It may be seen, however, that the transient oscillation at the center point of the winding is a sine wave of fundamental frequency in both power type and distribution type transformers. This is fortunate, since the fundamental frequency is the only one transmitted to the secondary in appreciable magnitude. The analysis of the primary frequency component of secondary transient given in the next sections will then be applicable to either type transformer.

Some of the general conclusions which may be drawn are the following:

(A) Free oscillations of either or both windings may be caused by impact of surge voltage.

(B) No oscillations occur, if wave front of applied surge is slower than the period of oscillation.

(C) Duration of applied surge greater than one-half cycle of oscillation produces highest amplitudes.

(D) Steep wave fronts produce maximum amplitudes of oscillation.

(E) Oscillations are damped by losses in the dielectric and iron.

(F) Losses reduce amplitude of first half cycle of oscillation but little.

(G) Transmission of applied surge through a transformer with crest changed by amount of the turn ratio is to be expected.

(H) Natural frequencies of windings vary over a wide range. Values from 5,000 cycles to nearly 1,000,000 cycles were found.

(I) Electrostatic distribution of voltage in the windings occurs at the instant of impact.

(J) Transient voltages appearing in windings are the result of several factors, the predominating influence changing with time after the instant of application of the impulse wave front.

Many more similar conclusions could be listed. Some data taken from the oscillograms of Test 1, with secondary winding isolated and open circuited, is given in Tables I to VI.

This brief treatment of the experimental results obtained in the present investigation is not to be construed as an indication of its lack of importance. The oscillograph tests constitute the most vital part of the work. Time was not available to extract and tabulate all of the information that was available in the approximately 700 oscillograph records at hand.

It is felt that, to some extent, the oscillograms can be left to speak for themselves, since the reader can interpret the results in the light of the discussions of transient characteristics that have been presented.

D. Calculation and Test of Distribution Transformer Primary Transients

A method of calculation, by which the transient reactions of transformers to applied impulse voltages can be calculated, is, of course, to be desired. There are several reasons for this. A quantitative analysis would lead to a much better understanding of the phenomena observed. It would be possible to predict what would happen under any given conditions. It would be an invaluable aid to the design, application and protection of transformers. It would be a valuable advance in the problem of insulation coordination of power systems. It would be another step toward the mastery of transient conditions in systems as a whole.

The limitations of the general analytical method of attack is evident, because of its complexity, impossibility of obtaining a general solution, and the practical impossibility of evaluating accurately the constants of the equivalent circuit from which the solution was derived, even though complete design data were available.

The limitation of Palueff's approximate analysis lies in the inadequate treatment of the minor components of transient voltage.

These limitations apply chiefly to the calculation of secondary transients, since the primary transients are more easily handled. In any case, it seems to be necessary to have recourse to experiment in order to get some part of the data required.

It would seem desirable to devise a means of analysis that would be more complete and free from the limitations of the other methods, but that would at the same time make practical numerical calculations possible. Since recourse must be made to experiment anyway, the analysis might as well be made a semi-empirical one derived from experimentally determined relations. The results of an attempt to make such an analysis will now be considered.

1. General Method of Calculation

Since the calculation of primary internal transients has been fairly successful by other methods, the calculation of secondary terminal transients will be the primary object. To determine the secondary terminal transients, it is necessary to know the transient voltage conditions at the center of the primary winding. For this reason, the calculation of that voltage will be considered.

a. Production of Oscillations. Voltage oscillations in windings are produced when a force or potential is applied which causes a displacement of voltage conditions from the equilibrium distribution. If the potential of a part of the winding were suddenly changed, oscillations would ensue during the return to equilibrium. If the potential of the whole winding were changed suddenly, no oscillation would occur because the voltage distribution would remain unchanged.

This point should be kept in mind.

It can be seen from the oscillograms of primary mid-tap voltage that the impact of the surge at the primary terminal produces an initial voltage distribution differing greatly from uniformity. The return of the voltage to the uniform distribution, which is the equilibrium condition, causes the oscillations. The initial amplitude of this oscillation would be the difference between the initial and the normal potentials of the mid point of the winding. This simple relation is all that is required to determine the amplitude. The frequency of the oscillation, of course, depends on the distributed constants of the windong.

b. Determination of Initial Distribution. The initial voltage distribution can be determined from an oscillogram of the applied voltage and the mid tap voltage. A wave with steep front and long tail is desirable for this test. This will give the initial amplitude of oscillation.

This value could also be obtained with a simple spark measurement of the maximum voltage between line terminal and 50% tap, as mentioned in Part III-c. This is the first constant required.

c. Frequency and Damping. Frequency and damping constants must be obtained from oscillograms. Both these can be obtained from the oscillogram of mid point voltage from which the initial distribution was found. The rate of damping can be more easily obtained from a record obtained by chopping the applied wave at the crest with a spark gap. The oscillations then appear without any other component of voltage to complicate the record. It has been found that a value of damping of about 30% per half cycle is representative of the distribution transformers tested. The time scale on the oscillogram should be such as to give at least two cycles of oscillation to show the damping. These are the second and third constants required.

d. Mathematical Calculation of Center Tap Voltage for Arbitrary Applied Wave. The complete determination of the mid tap transient can be calculated for any shape of applied wave, if the three constants required have been obtained.

The equation of the applied wave can usually be written in the form of the difference of two exponentials. The general equation of most lightning and impulse generator waves is then

$$e = E\left(\epsilon^{-at} - \epsilon^{-bt}\right) \qquad (1)$$

The equation of the indicial admittance or response of the mid tap to the application of an infinite rectangular wave (Heaviside's Unit Function) can immediately be written down. The normal equilibrium voltage of the center tap is, of course, half the terminal voltage. The resultant voltage is the sum of this normal voltage and the oscillating voltage. The general equation of the indicial admittance is thus

$$e_{6a} = A + B\cos(\omega t + \theta) \qquad (2)$$

if no damping is assumed. Damping can be accounted for by multiplying the cosine term by an exponential. Center point voltage is designated as e_{6a} to correspond to the lettering on the oscillograms.

In passing, it may be remarked that Bewley's general analytical solution reduces to the form of equation (2) for the special case of center of winding and fundamental space wave.[1]

The response for any applied wave can now be obtained by using equations (1) and (2) in Duhamel's integral. The integral can be evaluated formally, since the equations involve only exponentials and trigonometric terms which are the easiest forms to handle. Most of the integrals involved have been evaluated and are available in tables.

e. Graphical Calculation. The calculation could also be made graphically, or rather arithmetically, by dividing the wave shape and response curves into a suitable number of steps and adding up the components for each step. This method makes the applied voltage a stepped wave composed of small rectangular waves. The response to each step is known and the effect of the whole determined by adding the various responses in proper time relation.

2. Response to Rectangular Waves

a. Response to Infinite Rectangular Wave. The response of center tap to rectangular waves is simple, but typical, and will be considered in more detail. Three sets of curves

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[1] "Traveling Waves on Transmission Systems," L. V. Bewley, John Wiley & Sons, 1933, p. 243, Eq. (43).

have been prepared showing the effect of infinite and finite rectangular waves on the center point of the winding. The data used is the actual data from transformer No. 125 of the tests. (See Part IV-f.) The data for this transformer was found from oscillograms of Fig. 60-a and 66-a to be as follows:

Initial distribution	17%
Natural period	34 microseconds per cycle
Damping	30% per half cycle

Amplitude of oscillation is then 50%-17% or 33% of the impressed voltage. The mid tap voltage can then be plotted as in Fig. 43 where Curve A is the impressed wave, B. the normal mid tap voltage which acts as the axis of oscillation, C the undamped oscillation, and D the final response to infinite rectangular wave, if damping is considered.

b. Response to Finite Rectangular Wave. The effect of finite waves can be found by chopping the infinite tail of the applied wave. This can be accomplished by applying a similar infinite rectangular wave of opposite polarity at the time chopping is desired. The response of the center tap to this second wave is the same as to the first, except opposite in polarity and displaced in time. The resulting center tap voltage is the sum of the two effects. These relations are shown in the curves of Fig. 43.

This brings out the important point of the phase relations between the component oscillations caused by wave front and tail. It can be seen from the curves of Figs. 43

to 45 that the two oscillatory components can be practically out of phase or in phase or with any other intermediate phase relation, depending on the frequency and point at which the first wave is chopped. The resulting amplitude is then dependent on this phase relation as is the time of crest of the oscillation.

c. Wave Front and Wave Tail Components of Primary Oscillation. As has been pointed out, any wave can be divided up into a stepped wave. The front is divided into a number of small infinite rectangular waves, thus giving a slowly rising front and flat top. The tail is accounted for by applying another stepped wave of opposite polarity. All the oscillations caused by the stepped front of the first oscillation could be added to give a resultant wave front component. Similarly, a resultant wave tail component of oscillation could be found. The two components add to give the final oscillation.

We have thus simply explained the whole effect of wave shape on the internal oscillations,and the voltage in the oscillograms immediately becomesclear.

3. Equations of Wave Shape

The equations of most waves that are used in practice may be represented by the difference of two exponentials. Other forms of waves may often be obtained by adding a number of simple shapes.[1]

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[1] See for example "Traveling Waves on Transmission Systems," L. V. Bewley, Wiley 1933, pp. 19-24.

The general approximate equation of a practical wave may be written as

$$e = E\left(\epsilon^{-at} - \epsilon^{-bt}\right) \qquad (1)$$

The values of the constants a and b are most easily found from general curves such as given by Rorden.[1] The data required are the time to maximum and the time to half value on the tail of the wave.

The two exponential components of a wave rising to maximum in 15 microseconds and falling to half value in 47 microseconds are shown in Fig. 46-b.

The equation of a chopped wave may be obtained in two parts. First, the equation obtained for the full wave which will hold up to time t_1. A second wave equal and opposite the first at every point beyond time t_1 must then be added. The sum of the equations will be zero for all values of t greater than t_1.

Actually, since the original wave is made up of two components, the second wave is also made up by chopping two exponentials. Fig. 46-a shows the relations between an exponential and the second chopping wave. It is clear that the second wave will reduce the first to zero everywhere after t_1 since

$$-E\,\epsilon^{-at_1}\,\epsilon^{-a(t-t_1)} = -E\,\epsilon^{-at}$$

---

[1] "The Solution of Circuits Subjected to Traveling Waves," by H. L. Rorden, A.I.E.E. Trans. Vol. 51, No. 3, Sept. 1932, p. 825, Fig. 3.

The four components of the chopped wave equation for the specific case of the 15/47 wave are plotted in Fig. 46-c.

4. Response to Practical Full Wave

As an example, the method outlined for calculation of the voltage at the center of the primary winding will be applied to an actual case. The transformer No. 125 in the tests to be described was a 10 Kva, 2200 volt Moloney transformer. In accordance with designations of voltages on the oscillograms, the impressed surge voltage will be denoted as e_6, and the voltage of the mid point of the primary winding to ground as e_{6a}. The impressed wave chosen is the test wave G, reaching its crest in 15 microseconds and falling to half value in 47 microseconds.

a. Wave Shape Equation. Upon evaluating the constants in the general wave shape equation (1), the equation is found to be

$$e_6 = 205\left(\epsilon^{-0.03t} - \epsilon^{-0.1275t}\right) \qquad (3)$$

where time t is expressed in microseconds and instantaneous voltage e_6 is expressed in per cent of the crest voltage of the wave.

b. Indicial Admittance. The general equation of the indicial admittance from (2) is,

$$e_{6a} = A + B\cos(\omega t + \theta) \qquad (2)$$

The value of A is 0.50, since it represents the uniform distribution voltage which would be 50% of terminal voltage.

The value of B can be determined from the initial distribution of voltage. The initial voltage for this transformer was found to be 17%; the amplitude of oscillation is then 50-17 = 33%; the oscillation starts from a negative maximum (assuming positive applied wave) so that the cosine should be -1 when t is zero. This requires θ to be π. The period was found to be 34 microseconds, hence w would be

$$\omega = \frac{2\pi}{34} = 0.184$$

The equation for applied unit function then becomes

$$e_{6a} = 0.50 + 0.33 \cos (0.184 t + \pi) \tag{4}$$

c. Damping. The damping was found to be about 30% per half cycle. This can be inserted in the equation by multiplying the cosine term by an exponential ϵ^{-ct} which reduces to 0.7 at t = 17. This can be determined as follows:

$$\epsilon^{-17c} = 0.7$$

$$-17c = \log 0.7$$

$$c = -\frac{1}{17} \log 0.7$$

$$c = 0.021$$

This damping factor actually gives decreasing amplitudes as follows:

Half Cycle	% Decrease
1	30
2	21
3	15
4	10

The final equation with damping included is therefore:

$$e_{6a} = 0.50 + 0.33\,\epsilon^{-0.021t} \cos(0.184t + \pi) \qquad (5)$$

All data was obtained from oscillograms of Figs. 60-a and 66-a.

d. Use of Duhamel's Integral. The response to the actual applied wave can be obtained by means of some form of Duhamel's integral.[1] The following form will be used.

$$e = e(o)A(t) + \int_0^t A(\lambda)\, e'(t-\lambda)\, d\lambda \qquad (6)$$

In this expression, e(o) is given by equation (2) for t = o. The first term is, therefore, zero. Under the integral $A(\lambda)$ is given by equation 5 with t replaced by λ ; $e'(t-\lambda)$ is obtained by replacing t by $t-\lambda$ in equation (2) and differentiating with respect to t. Upon making the above substitutions, equation (6) becomes

$$e_{6a} = 0 + 205 \int_0^t \left[0.50 + 0.33\,\epsilon^{-0.021\lambda} \cos(0.184\lambda + \pi)\right]\left[-0.03\,\epsilon^{-0.03(t-\lambda)} + 0.1275\,\epsilon^{-0.1275(t-\lambda)}\right] d\lambda$$

Re-arranging terms

$$e_{6a} = -2.05\,\epsilon^{-0.03t} \int_0^t \left[1.5\,\epsilon^{0.03\lambda} + 0.99\,\epsilon^{0.009\lambda} \cos(0.184\lambda + \pi)\right] d\lambda$$

$$+ 2.05\,\epsilon^{-0.1275t} \int_0^t \left[6.37\,\epsilon^{0.1275\lambda} + 4.22\,\epsilon^{0.1065\lambda} \cos(0.184\lambda + \pi)\right] d\lambda$$

---

[1] "Operational Circuit Analysis," V. Bush, John Wiley & Sons, 1929, Chap. V, p. 68.

After integrating and simplifying, this becomes

$$\begin{aligned} e_{6a} = & -11.0\,\epsilon^{-0.021t}\sin(0.184t + 182.8^\circ) \\ & + 40.5\,\epsilon^{-0.021t}\sin(0.184t + 210.1^\circ) \\ & + 11.0\,\epsilon^{-0.03t}\sin(182.8^\circ) - 40.5\,\epsilon^{-0.1275t}\sin(210.1^\circ) \\ & + 102.5\,(\epsilon^{-0.03t} - \epsilon^{-0.1275t}) \end{aligned} \qquad (7)$$

or

$$\begin{aligned} e_{6a} = & -11.0\,\epsilon^{-0.021t}\sin(0.184t + 3.19) + 101.9\,\epsilon^{-0.03t} \\ & + 40.5\,\epsilon^{-0.021t}\sin(0.184t + 3.67) - 78.1\,\epsilon^{-0.1275t} \end{aligned} \qquad (8)$$

From Equation (7), the value of e_{6a} is zero at $t = o$ and $t = \infty$ as it is known to be. The oscillatory components are of different magnitudes and phase, but have the same damping. This equation should give the voltage appearing at the mid point of the primary winding when the full wave G is impressed on the line terminal.

The normal axis of oscillation would be the line of 50% impressed voltage and would thus be a wave of the same shape, but half the magnitude. This component is given in the last term of the equation, since it is equation (2) reduced to half value. The remaining two exponential terms are interesting in that they represent a shift in the axis of oscillation due to the damping term. Such a shift of axis may be seen on some of the oscillograms of chopped waves where the normal axis of oscillation is known to be the zero axis.

e. Check between Experimental and Calculated Values. The voltages calculated from equation (8) and that replotted from Fig. 36-a are given in Fig. 47. The various components which make up this voltage are also shown.

The oscillogram indicates greater damping on the tail and less on the front than calculated. This may mean that the damping is not exponential, but more nearly uniform.

The accuracy of the check between the two curves is within the range of errors in reading oscillograms and of slide rule computation.

5. Calculation of Response to Chopped Wave

The calculation of the mid point voltage in case of a chopped wave is similar to that of the full wave, except that it is necessary to divide the problem into two parts. The response to the application of the full wave must first be calculated as in the preceding case. This will give the voltage appearing up to the time the wave is chopped at $t = t_1$.

The reduction of impressed voltage to zero at $t = t_1$ and maintenance of zero voltage thereafter can be accomplished analytically by impressing a wave of opposite polarity to the transformer terminal at $t = t_1$. This wave must have a vertical front equal to the value of the full wave at the time of chopping and a tail which has the same value at all points as the tail of the full wave.

The response of the mid point to the application of the new wave is determined by use of Duhamel's integral as in the previous case, but with time reckoned from the time of chopping. The final voltage is then calculated from the first equation up to time t_1 and from the sum of the two equations after t_1.

For the case of the wave which was used in the preceding calculation, the voltage up to the time of chopping is given by equation (7) or (8). The response to the second wave of opposite polarity must be calculated in addition.

The equation of the second wave can be found by the method outlined in Section D-3. From Equation (3), the equation up to time of chopping is

$$e_6 = 205 \left(\epsilon^{-0.03t} - \epsilon^{-0.1275t} \right) \qquad (3.)$$

If the wave be chopped at 8 microseconds, the chopping equation becomes

$$e_6' = -205 \left[0.787\, \epsilon^{-0.03(t-8)} - 0.361\, \epsilon^{-0.1275(t-8)} \right] \qquad (9)$$

The equation of the indicial admittance is given in (5)

$$e_{6a} = 0.50 + 0.33\,\epsilon^{-0.021\,t}\left[\cos\left(0.184t + \pi\right)\right] \qquad (5)$$

The response equation is found from (9) and (5) by use of the superposition theorem, equation (6).

$$e_{ca} = -43.7 - 28.6\,\epsilon^{-0.021(t-8)}\cos\left[0.184(t-8) + \pi\right]$$
$$+205\int_0^{t-8}\left[0.50 + 0.33\,\epsilon^{-0.021\lambda}\cos\left(0.184\lambda + \pi\right)\right]\left[0.0236\,\epsilon^{-0.03(t-8-\lambda)}\right.$$
$$\left. -0.046\,\epsilon^{-0.1275(t-8-\lambda)}\right]d\lambda$$

$$e_{6a} = 2.42\,\epsilon^{-0.03(t-8)}\int_0^{t-8}\epsilon^{0.03\lambda}\,d\lambda$$
$$+1.60\,\epsilon^{-0.021(t-8)}\epsilon^{-0.009(t-8)}\int_0^{t-8}\epsilon^{0.009\lambda}\cos(0.184\lambda + \pi)\,d\lambda$$
$$-4.71\,\epsilon^{-0.1275(t-8)}\int_0^{t-8}\epsilon^{0.1275\lambda}\,d\lambda$$
$$-3.115\,\epsilon^{-0.021(t-8)}\epsilon^{-0.1065(t-8)}\int_0^{t-8}\epsilon^{0.1065\lambda}\cos(0.184\lambda + \pi)\,d\lambda$$
$$-43.7 - 28.6\,\epsilon^{-0.021(t-8)}\cos\left[0.184(t-8) + \pi\right]$$

Integrating and reducing, this becomes

$$\begin{aligned} e'_{6a} = {} & 8.7\,\epsilon^{-0.021(t-8)} \sin\left[0.184(t-8)+\pi+0.049\right] \\ & -8.7\,\epsilon^{-0.03(t-8)} \sin\left(\pi+0.049\right) \\ & -14.63\,\epsilon^{-0.021(t-8)} \sin\left[0.184(t-8)+\pi+0.526\right] \\ & +14.63\,\epsilon^{-0.1275(t-8)} \sin\left(\pi+0.526\right) \\ & -\left[80.6\,\epsilon^{-0.03(t-8)} - 36.9\,\epsilon^{-0.1275(t-8)}\right] \\ & -28.6\,\epsilon^{-0.021(t-8)} \cos\left[0.184(t-8)+\pi\right] \end{aligned} \qquad (10)$$

or

$$\begin{aligned} e'_{6a} = {} & 8.7\,\epsilon^{-0.021(t-8)} \sin\left[0.184(t-8)+3.19\right] \\ & -80.2\,\epsilon^{-0.03(t-8)} + 29.6\,\epsilon^{-0.1275(t-8)} \\ & -14.63\,\epsilon^{-0.021(t-8)} \sin\left[0.184(t-8)+3.67\right] \\ & -28.6\,\epsilon^{-0.021(t-8)} \cos\left[0.184(t-8)+\pi\right] \end{aligned} \qquad (11)$$

Equation (10) shows all the components due to the chopping wave. Equation (11) has been condensed for easier calculation. The final equation would be (8) up to time t_1 and the sum of (8) and (11) after time t_1.

The calculated curves from the above equations and the re-plotted curves from Fig. 87-b are given in Fig. 48.

The agreement in this case is not so good. Examination of other oscillograms of the same voltage, such as Fig. 89-b, shows some distortion in the oscillation of Fig. 87-b not present in other oscillograms. The cause of this has not been determined, but it is felt that this effect causes the discrepancy.

6. Experimental Results

It will be well to re-state the experimental requirements of the method of calculation just outlined. First, the transformer must have a tap available at the mid-point of the high voltage winding. Most distribution transformers have this. Second, the following data are required.

(A) Initial voltage distribution.

(B) Natural period of oscillation.

(C) Damping constant.

This information may be obtained from three records on one oscillogram or from a spark gap measurement and one oscillograph record. If the natural frequency could be obtained by other means, no oscillograph records would be necessary, since damping can be taken as 30% per half cycle without much error.

No additional data would be required to calculate the case of a transformer in a delta bank with surges coming in on both lines at once. This case could be calculated by considering line 2 grounded and finding the effect of a surge on line 1; line 1 would then be considered grounded and the effect of a surge on line 2 determined. The sum of the two effects would give the desired solution with no additional experiment necessary.

A tabulation of the impulse characteristics of the primary windings of the transformers tested is given in Table I. These values were taken from the oscillograms in Part IV, Section F.

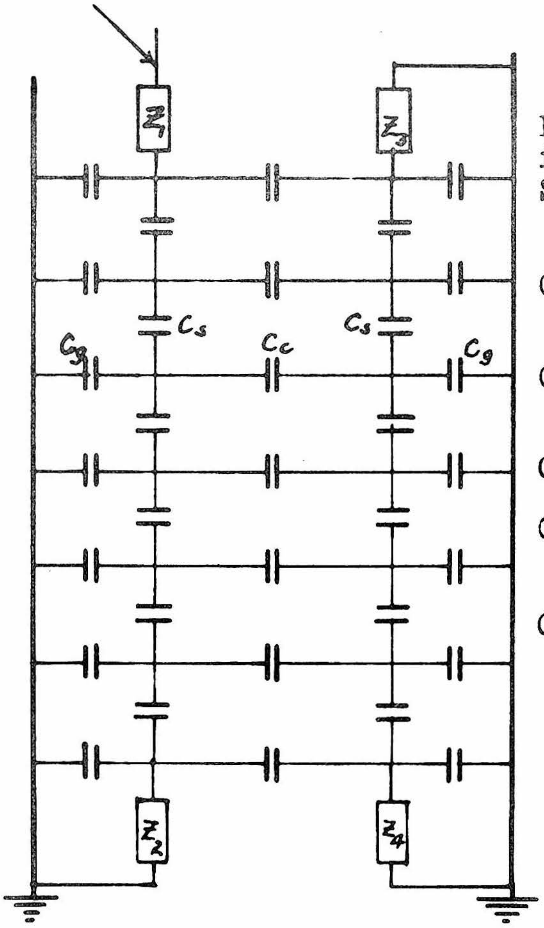
Table I

Surge Characteristics of Distribution Transformer Primary Windings

Transformer Number	Wave	50% Tap Voltage[1] Initial	Maximum	Natural Period
125	A	17	80	34
473	A	26	57	120
E1129	A	42	48.5	100
EE1419	A	*	*	200
125	B	17	63	34
473	B	26	27	120
125	C	17	80	34
E1129	C	41	56	100
125	D	17	20	34

1. Columns 3 and 4 give voltages in per cent of applied wave crest. Column 5 gives time of one cycle of oscillation in microseconds. Characteristics of waves are given in Table IX and curves of Figs. 58 and 59.

* Center tap on primary was not available.



(a)

Equivalent Circuit of a Transformer at Instant of Impact of Surge

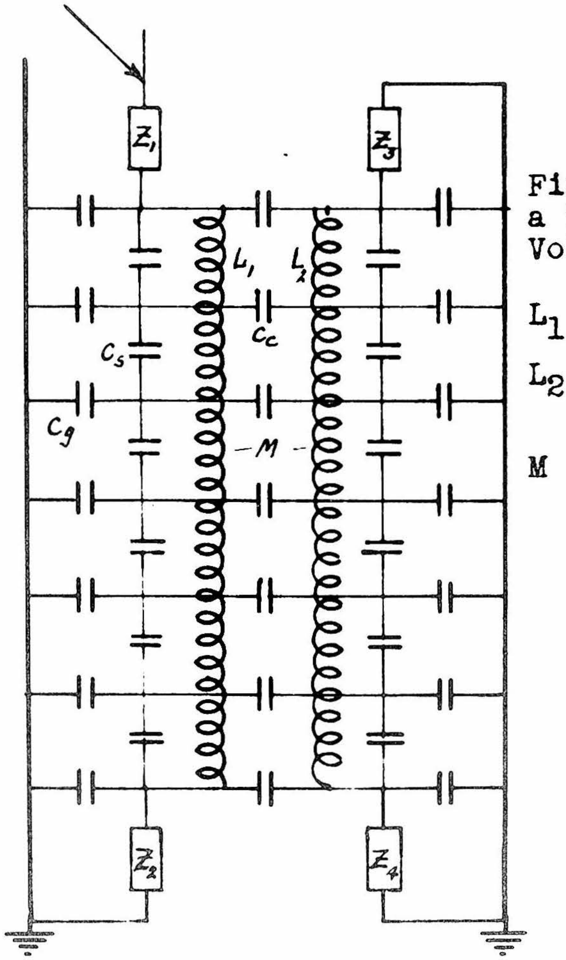
C_{g1} - Primary Capacity to Ground

C_{g2} - Secondary Capacity to Ground

C_{s1} - Primary Series Capacity

C_{s2} - Secondary Series Capacity

C_c - Capacity Between Windings



(b)

Final Equivalent Circuit of a Transformer for Surge Voltages

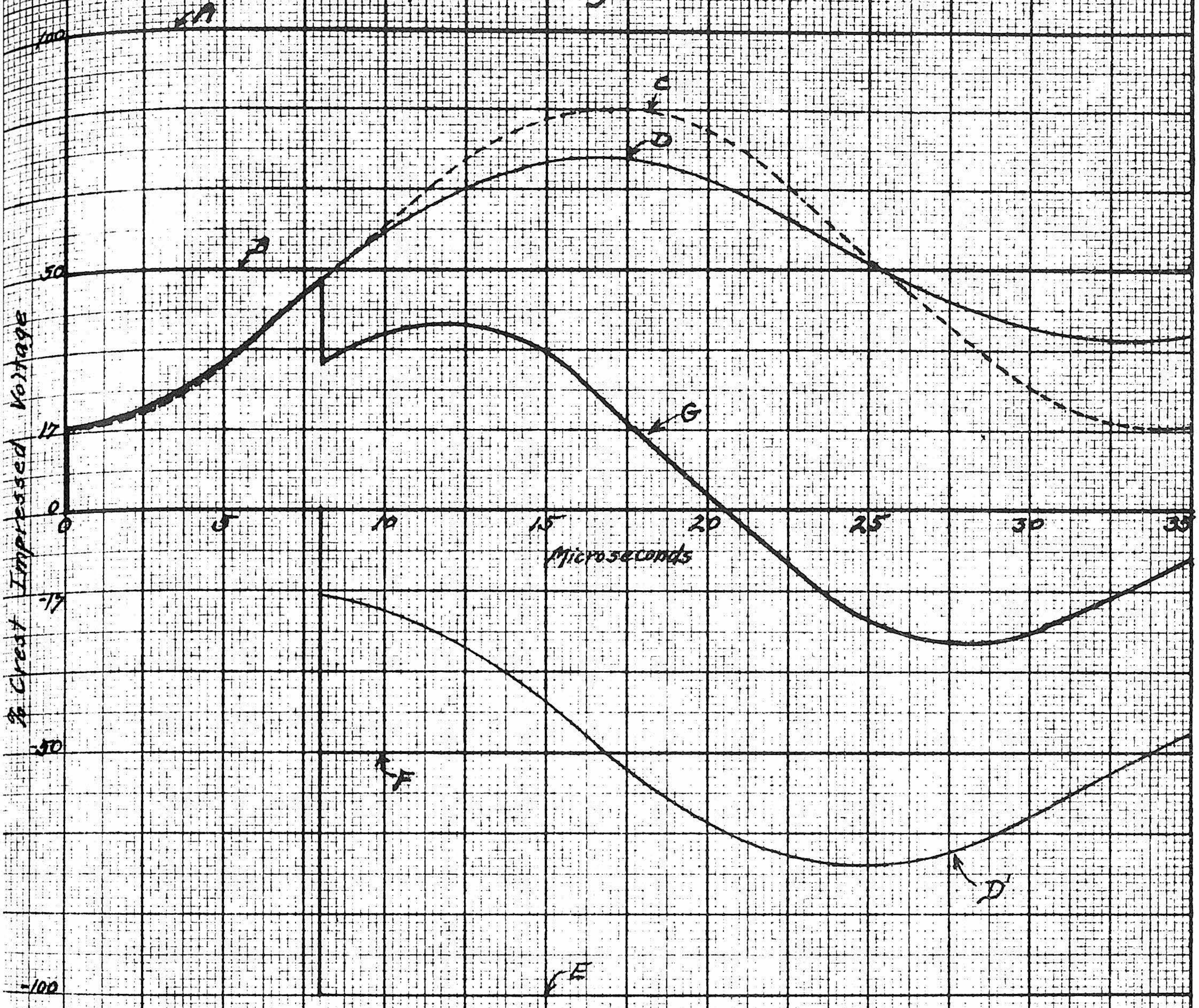
L_1 - Primary Self Inductance

L_2 - Secondary Self Inductance

M - Mutual Inductance

Fig. 42

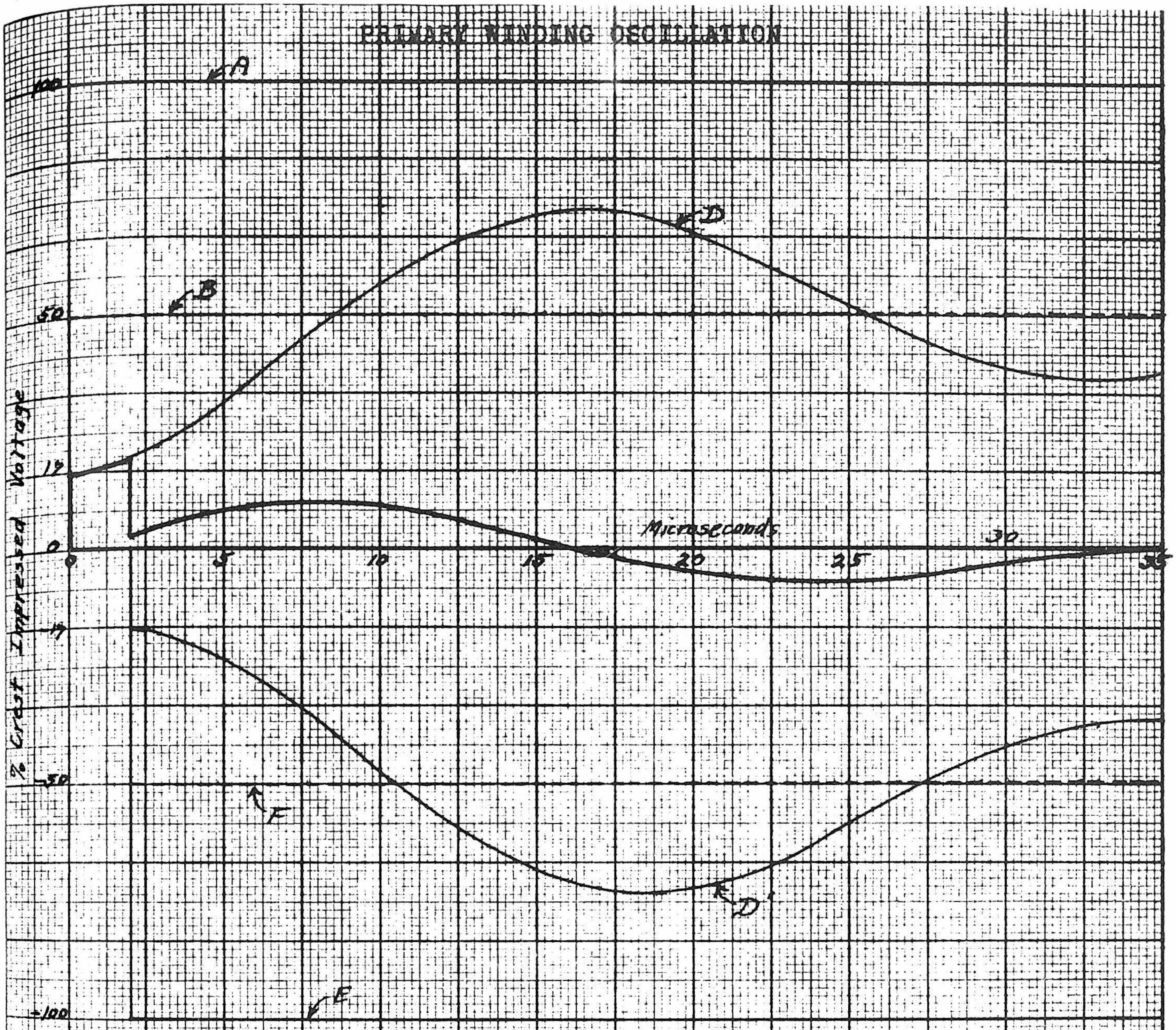
Primary Winding Oscillation



Finite Rectangular Wave
Transformer 125

A - Impressed Infinite Rectangular Wave
B - Voltage at 50% Tap for Uniform Voltage Distribution
C - Undamped Oscillation About B As Axis
D - Oscillation C with Damping of 30% Per Half Cycle
E - Impressed Wave of Reversed Polarity
F - Voltage for Reversed Polarity Wave Corresponding to B
G - Resultant Voltage at 50% Tap

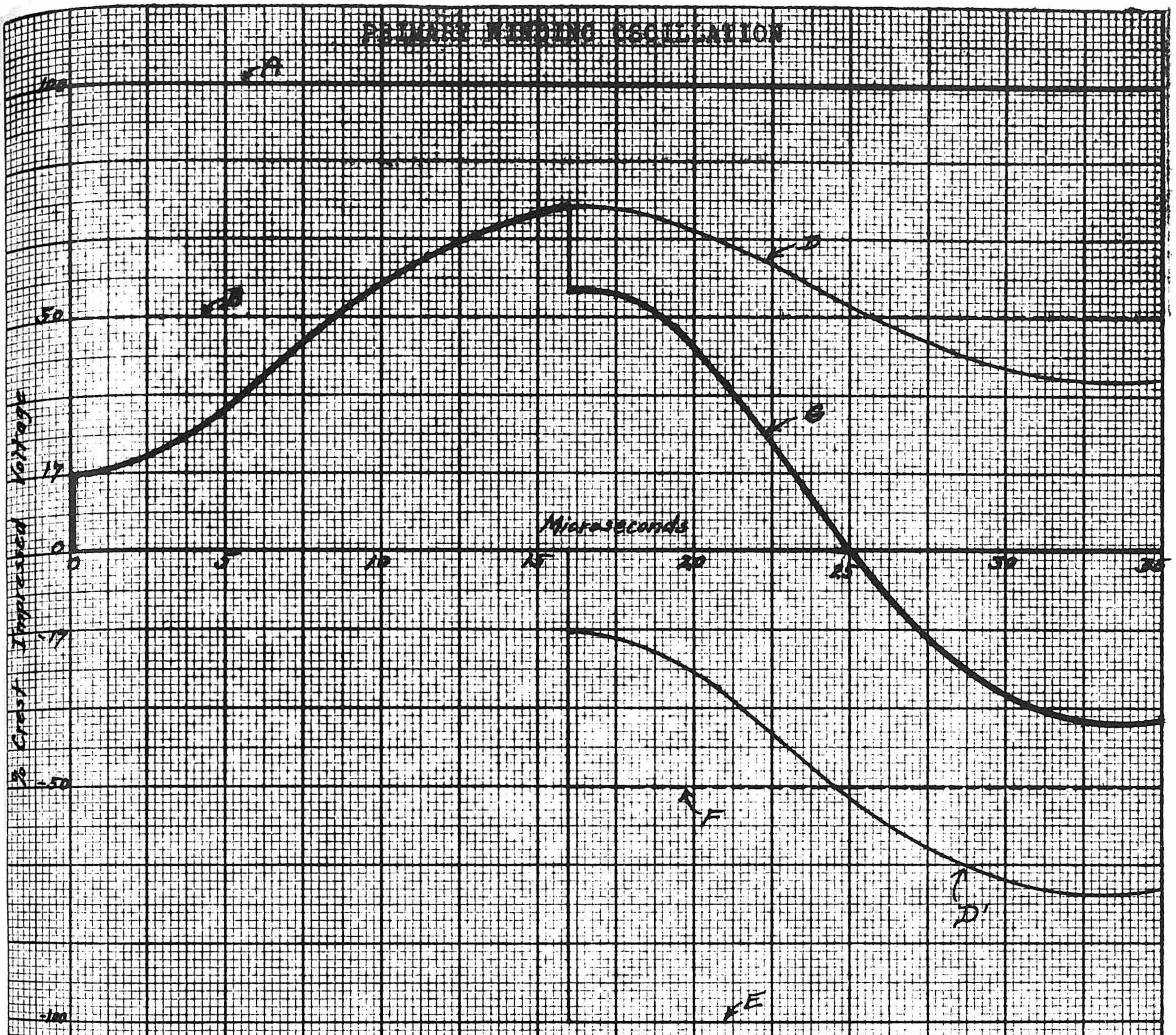
Fig. 43



Finite Rectangular Wave
Transformer 125

A - Impressed Infinite Rectangular Wave
B - Voltage at 50% Tap for Uniform Voltage Distribution
D - Oscillation C with Damping of 30% per Half Cycle
D' - Oscillation for Reversed Polarity Wave Corresponding to D
E - Impressed Wave of Reversed Polarity
F - Voltage for Reversed Polarity Wave Corresponding to B
G - Resultant Voltage at 50% Tap

Fig. 44

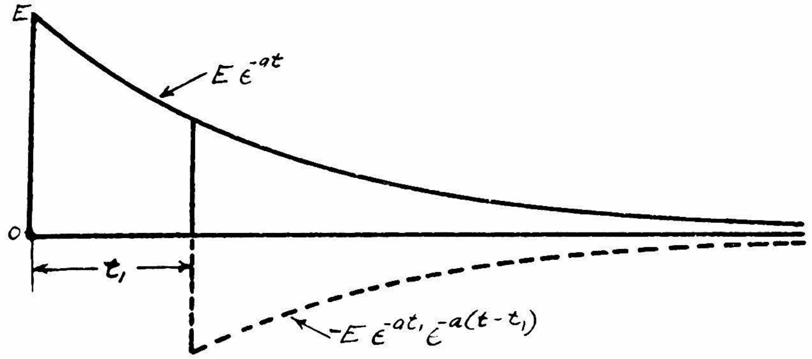


Finite Rectangular Wave
Transformer 125

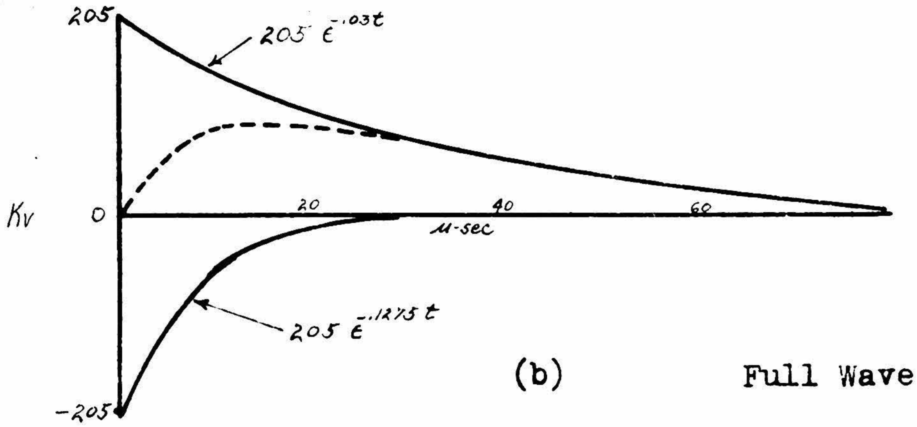
A - Impressed Infinite Rectangular Wave
B - Voltage at 50% Tap for Uniform Voltage Distribution
D - Oscillation C with Damping of 30% per Half Cycle
D' - Oscillation for Reversed Polarity Wave Corresponding to D
E - Impressed Wave of Reversed Polarity
F - Voltage for Reversed Polarity Wave Corresponding to B
G - Resultant Voltage at 50% Tap

Fig. 45

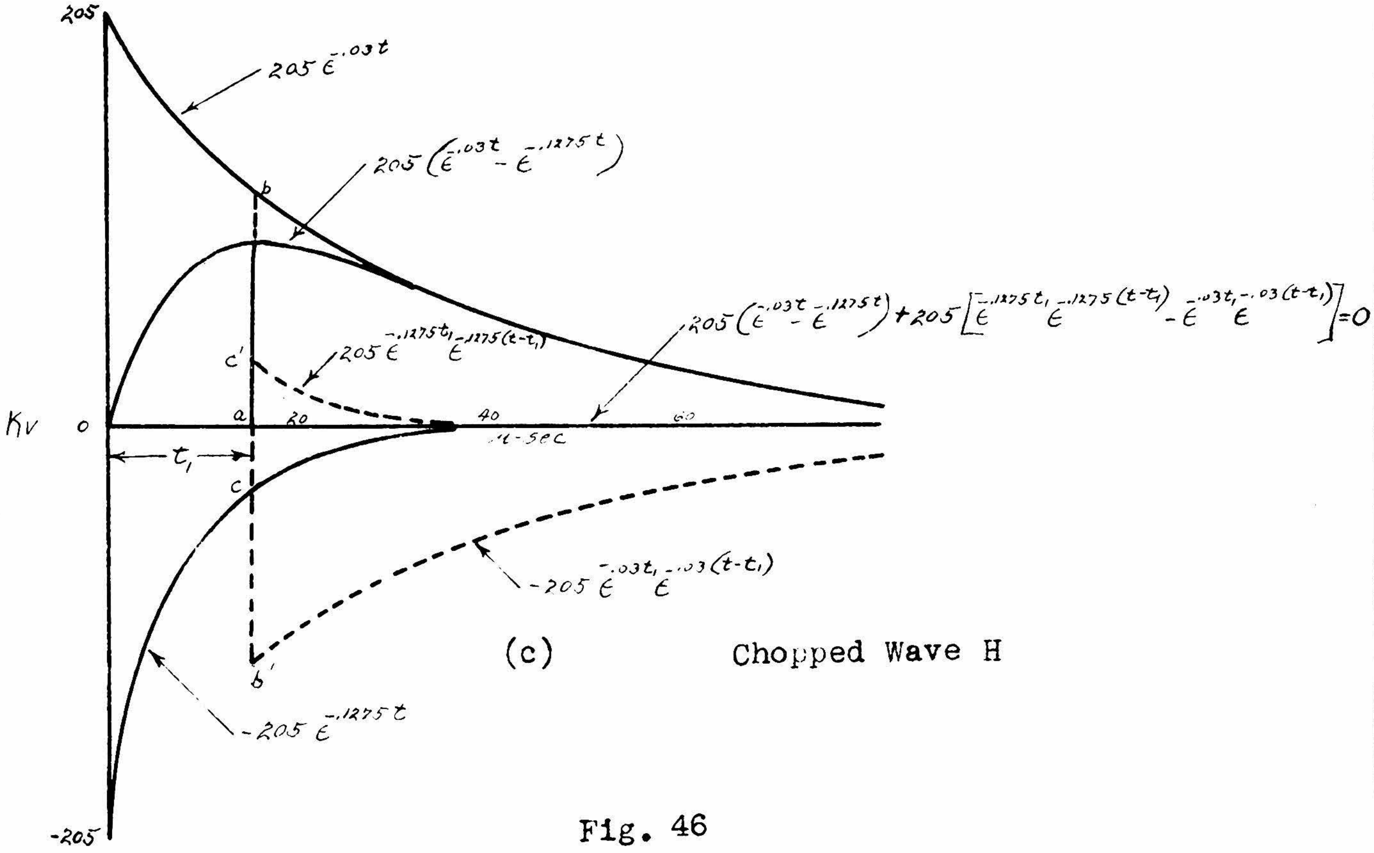
EXPONENTIAL COMPONENTS OF TEST WAVES



(a) Chopped Exponential Curve



(b) Full Wave G



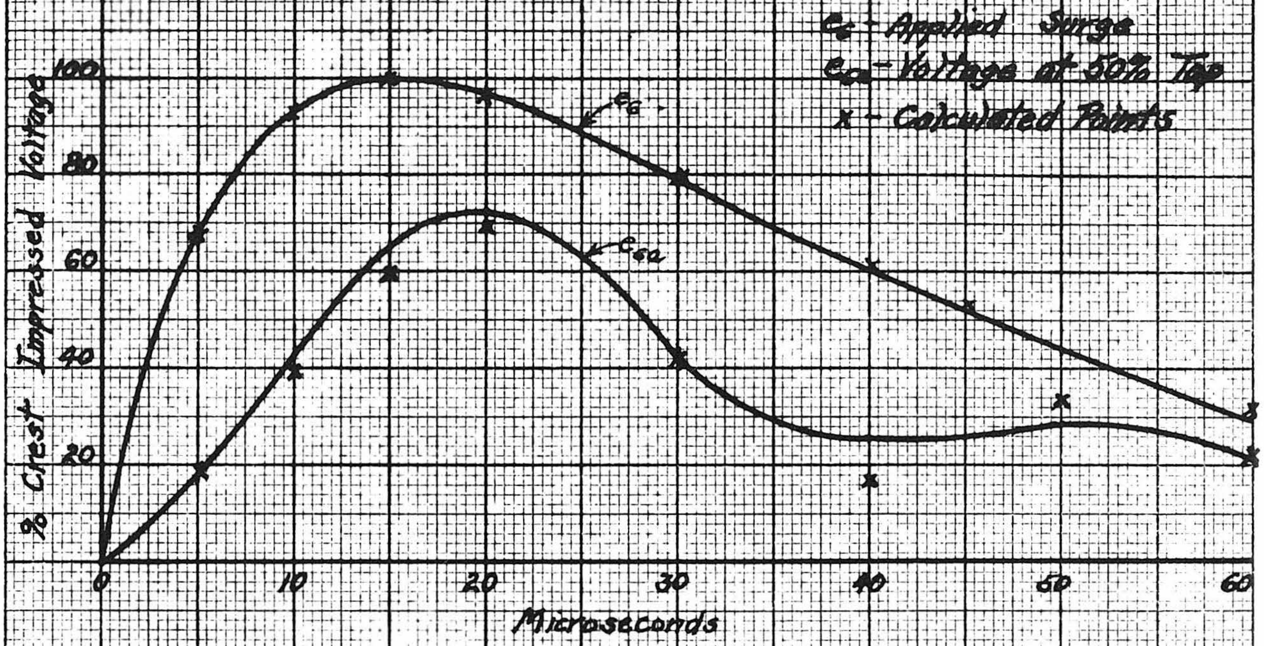
(c) Chopped Wave H

Fig. 46

Primary Winding Oscillation

Transformer 125 Wave G

Curves Replotted from Oscillogram C39A



Calculated Components of e_{ca}

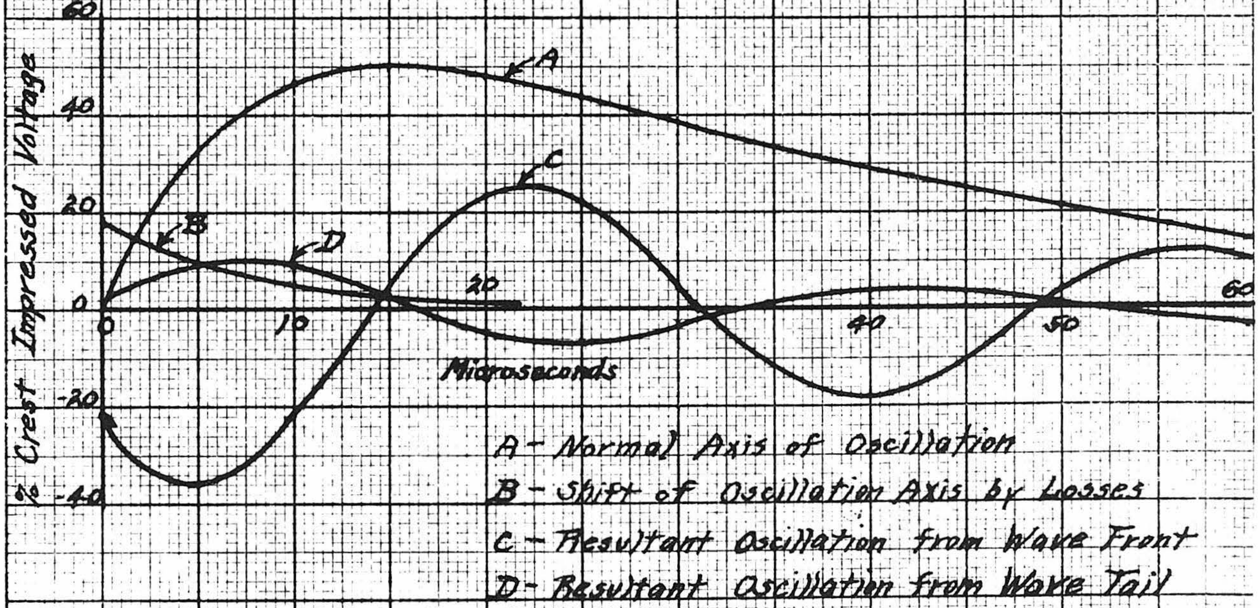


Fig. 47

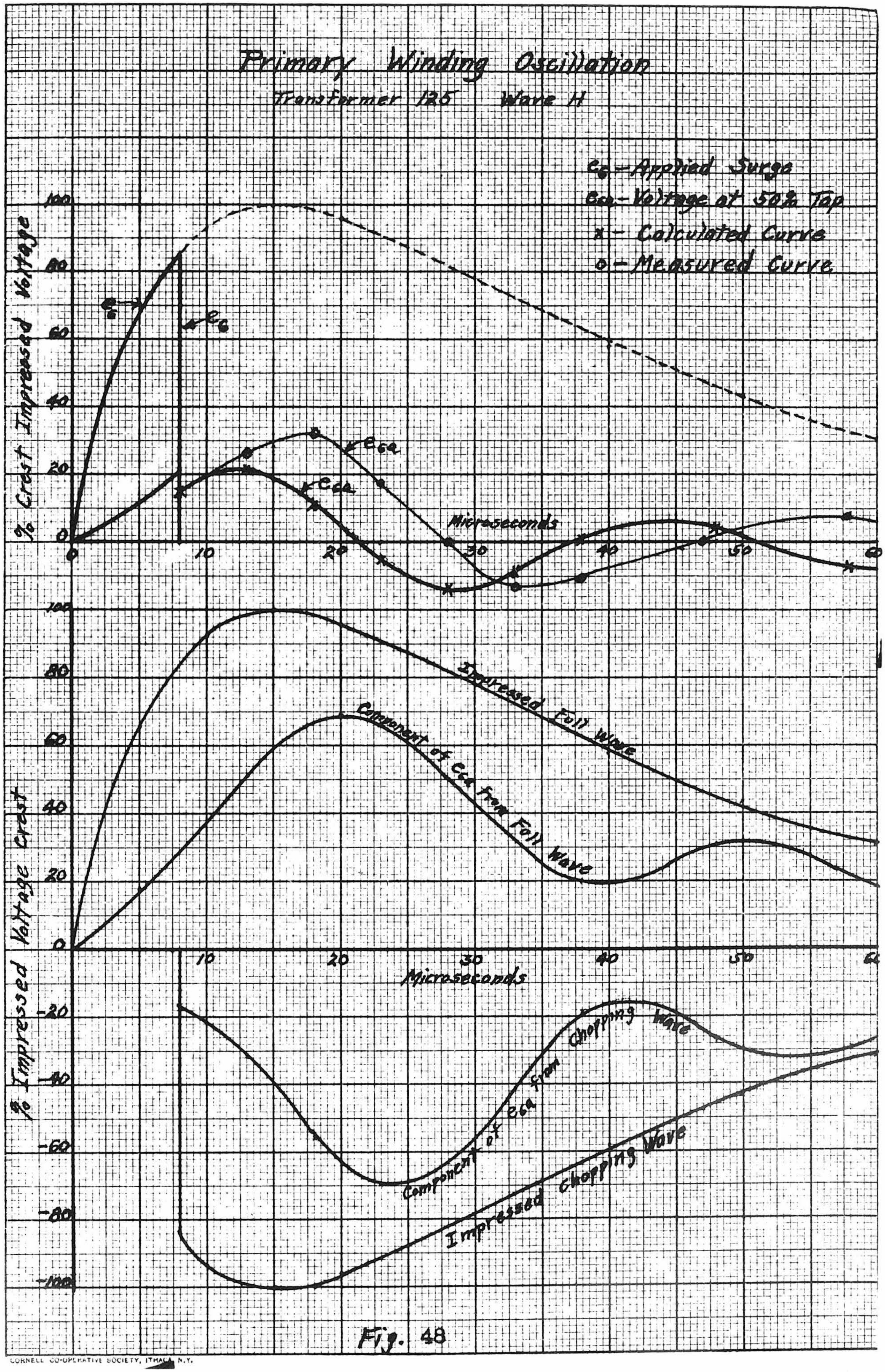

Primary Winding Oscillation
Transformer 125 Wave H
e_c – Applied Surge
e_ca – Voltage at 50% Tap
x – Calculated Curve
o – Measured Curve
% Crest Impressed Voltage
100
80
60
40
20
0
e_c
e_c
e_ca
e_ca
Microseconds
0
10
20
30
40
50
60
% Impressed Voltage Crest
100
80
60
40
20
0
-20
-40
-60
-80
-100
Impressed Full Wave
Component of e_ca from Full Wave
10
20
30
40
50
60
Microseconds
Component of e_ca from Chopping Wave
Impressed Chopping Wave

Fig. 48

E. Calculation of Secondary Voltages

The calculation of the primary winding center tap transient just described is essential to the calculation of secondary terminal transients under certain conditions. The calculation of the secondary transients will now be undertaken by a continuation of the same method of procedure used for the primary.

The object of the method is to use experiment as freely as necessary, but to keep the number of empirical constants required to as small a value as possible. The aim is, therefore, to combine experiment and analysis to get complete results.

1. General Method of Analysis

The general method of attack is the same as in the case of the primary transient. The primary center tap transient must be first determined as in the preceding section. Additional experimental data is required before the secondary calculation can be made. The new empirical constants to be determined depend on the nature of the secondary connection and terminal impedances.

a. Electromagnetic component. The computation of this component is often unnecessary, since it is nearly enough a reproduction of the applied wave as to make its use as such permissible, reduced, of course, by the turn ratio. If calculation is necessary to determine the sloping of the voltage rise, the method of Palueff should be used. This component is not affected by capacity coupling or effect of grounded or ungrounded tank. Terminal impedance would affect the magnitude if it were too low. This condition can also be treated in the same manner. Calculation would require short circuit inductance to be determined by test.

b. Electrostatic component. The capacity coupling factor between primary and secondary can only be determined by taking an oscillogram. Capacity relations depend on effective values under impulse conditions and are not necessarily the same as measured at low frequency with a bridge.

Capacity coupling factors are best expressed as a percentage based on the ratio of magnitudes of secondary to primary components. The secondary component can then be obtained from the primary by simple multiplication. This simple relation would not hold without consideration of additional factors, if the secondary were not completely isolated.

It is convenient to consider the whole of the primary mid-tap voltage to be transferred to the secondary. The equation of the primary voltage must first be obtained and this multiplied by the appropriate coupling factor. This, of course, means the transfer of each component of primary voltage. The electrostatic component will be taken to mean that part of the voltage transferred which is directly attributable to the applied wave. The primary frequency component is not considered a part of this.

The primary center point voltage is made up of a component the same as the applied wave and half the magnitude, which can be called the normal center point wave, and an oscillatory component. The first of these contributes to the electrostatic component of the secondary. The second contributes the secondary primary frequency component which will be considered later.

It will be shown in the specific case to be considered later that the whole electrostatic component of secondary voltage is not accounted for by transfer of the normal center point wave. There is an additional component which must be considered. This additional component can be accounted for by another coupling factor to be multiplied by the impressed wave. This is a convenient way of treating the matter, since it would be awkward to consider the normal center point wave and the primary oscillatory wave as having different coupling factors. The division of electrostatic coupling factors is arbitrary and largely a matter of convenience, since a single factor could be used. It would seem quite probable that the effective capacitance between windings would differ for the radically different voltage distributions which would exist for voltages at line terminal and center point.

If the secondary were connected to ground through terminal impedances, the effect of this connection on the electrostatic component would have to be accounted for by introduction of an exponential term, the value of which would have to be found from oscillograph tests.

Experimental data required for determination of this component can be obtained from an oscillogram of secondary terminal voltage when a steep front, long tail wave is applied to the primary. One oscillogram would be sufficient.

c. Primary Frequency Component. This component is produced by capacity coupling between the windings as just mentioned in connection with the electrostatic component. The absence of such voltage arising from electromagnetic mean has already been pointed out in the discussion of Palueff's work. The coupling factor may be determined from the same oscillograms required for the electrostatic component.

It might be well to inquire into the possibility of the appearance of primary harmonic oscillation frequencies in the secondary. In general, the magnitudes of these harmonics decrease about as the square of their order so that the amplitudes of higher harmonics would be small. These values would be further reduced by the transfer to the secondary winding. In addition, voltages would be induced from both positive and negative half cycles of the primary standing waves, thus canceling all effect from even harmonics and most of that from the odd harmonics.

The effect of secondary terminal impedance could be accounted for as in the case of the electrostatic component.

Oscillograms required are of primary impressed wave and center tap voltage, and of secondary terminal voltages. These four records can be placed on one film, if desired. It may be pointed out that several records with the same wave and transformer connection can be taken as easily as one.

d. Secondary Oscillation. The secondary oscillation can be calculated in a manner similar to that of the primary oscillation. Magnitude would depend on the electromagnetic component, but this can be calculated also. It has been pointed out that the initial electrostatic distribution produces essentially equal voltages throughout the secondary winding and, hence, causes no oscillation. If the oscillograms did show such difference in initial distribution, the oscillation from this cause could also be calculated.

Frequency and damping must be obtained by experiment. Damping is greater with the higher frequencies. All information can be obtained from the same oscillograms required for the other components. Since the secondary frequencies are usually high, it is necessary to use the steepest wave possible to excite such oscillation. Chopped waves will often cause such oscillation.

It is interesting to see why the secondary frequency does not appear in the primary. In the first place, the maximum voltage of secondary oscillation is small compared to the voltage of the high voltage winding. In the second place, this oscillatory voltage would be reduced still further by capacity transfer to primary, since the capacity coupling factor is the same whichever direction the voltage is transferred.

e. Effect of Grounded and Isolated Tank. The effect of isolating the transformer tank is to change radically the capacity relations existing with the tank grounded. The effects of this change may be found from an additional oscillograph record, if desired.

The chief effect on secondary voltage is in the case where the secondary winding is isolated and open. The secondary winding takes essentially the same potential as that of the tank. The tank potential is an average of the voltage over the primary winding. In general, therefore, under these conditions, the tank and secondary winding will assume a potential about half that of the impressed primary surge. The capacity relations are shown in Fig. 49. Actual capacitance is not lumped as shown, but an equivalent lumped capacitance might be used. Capacity bridge measurements cannot be assumed to give proper values for use. Table VI gives tabulated data on the effect of grounded or isolated tank.

2. Calculation for Isolated Secondary - Full Wave

An example of the application of the foregoing analysis to the calculation of a specific case will now be given. Since the primary voltage has already been calculated in the preceding section, the secondary terminal voltages to ground will be calculated for the same case. This is for transformer No. 125 with test wave G, a 15/47 full wave impressed. Transformer ratio is 2200/220 or 10 to 1, which makes 10% of primary voltage between terminals or 5% from terminal to ground.

a. <u>Electromagnetic Component</u>. This can be considered a replica of the impressed wave reduced to 5% of its primary terminal value. Sloping of wave front is negligible when transformer leakage is low and terminal impedance infinite. The equation of the primary impressed wave was found to be

$$e_6 = 205\left(\epsilon^{-0.03t} - \epsilon^{-0.1275t}\right) \qquad (3)$$

The secondary electromagnetic component is 5% of this on each terminal, or

$$e_{1a} = 10.25\left(\epsilon^{-0.03t} - \epsilon^{-0.1275t}\right) \qquad (12)$$

This appears on both terminals with the same value, but with opposite polarity. For a subtractive polarity transformer, it is added to e_5 and subtracted from e_1. Designations e_1 and e_5 are those in the diagram and oscillograms of Test 1.

b. <u>Electrostatic Component</u>. This has the same shape as the primary wave, but reduced in magnitude by the electrostatic coupling factor. This component can be arbitrarily divided into two parts as previously explained.

Oscillograms of Fig. 86 show that 37% of the primary center tap voltage appears by capacity coupling on both secondary terminals. They also show that 14% of the impressed wave appears as initial secondary voltage. This initial 14% may be considered as made up of one part arising from 37% reduction of primary mid-tap initial voltage and another part from the impressed wave itself. That part of the 14% coming from initial primary mid-tap voltage would be about 6%; this leaves nearly 8% to be accounted for as coupling direct with primary line terminal.

The part coming from normal center point wave would be 37% of half the impressed wave or 18.5%. To this the above direct coupling factor of 7.7% should be added, making a total coupling factor with primary impressed wave of 25.2%. From equation (3), this gives the total electrostatic component as

$$e_{1b} = 53.7 \left(\epsilon^{-0.03t} - \epsilon^{-0.1275t} \right) \qquad (13)$$

c. Primary Frequency Component. Oscillograms of Fig. 86 show a coupling factor of 37% for primary oscillation. The components of primary oscillation were found in Equation (8) to be

$$-11.0\,\epsilon^{-0.021t}\sin(0.184t+3.19)$$

$$+40.5\,\epsilon^{-0.021t}\sin(0.184t+3.67)$$

This component should also include the exponential terms representing shift of axis of oscillation due to damping. These are from Equation (7).

$$11.0\,\epsilon^{-0.03t}\sin(182.8^\circ)-40.5\,\epsilon^{-0.1275t}\sin(210.1^\circ)$$

After reduction by capacity coupling, these appear on both secondary terminals with equal value and some polarity, becoming

$$\begin{aligned} e_{1c} = &-4.07\,\epsilon^{-0.021t}\sin(0.184t+3.19) \\ &+15.0\,\epsilon^{-0.021t}\sin(0.164t+3.67) \\ &-0.22\,\epsilon^{-0.03t}+7.1\,\epsilon^{-0.1275t} \end{aligned} \qquad (14)$$

d. Secondary Oscillation. Since the wave front and tail are slow compared to the natural period of secondary oscillation, there will be no secondary frequency component in this case. This could be shown by going through the calculation, but the general experimental conclusions show this to be unnecessary.

e. Complete Solution. The calculation then reduces to transferring the whole primary mid-point voltage to the secondary side by capacity coupling and adding another electrostatic component and an electromagnetic component.

Summing up the various component voltages, we obtain the resulting terminal voltages. The equations of these are, from Equations (12), (13), and (14)

$$\begin{aligned} e_5 = {} & 64.0\left(\epsilon^{-0.03t} - \epsilon^{-0.1275t}\right) - 0.22\,\epsilon^{-0.03t} \\ & + 7.1\,\epsilon^{-0.1275t} - 4.07\,\epsilon^{-0.021t}\sin(0.184t + 3.19) \\ & + 15.0\,\epsilon^{-0.021t}\sin(0.184t + 3.67) \end{aligned} \tag{15}$$

$$\begin{aligned} e_1 = {} & 43.4\left(\epsilon^{-0.03t} - \epsilon^{-0.1275t}\right) - 0.22\,\epsilon^{-0.03t} \\ & + 7.1\,\epsilon^{-0.1275t} - 4.07\,\epsilon^{-0.021t}\sin(0.184t + 3.19) \\ & + 15.0\,\epsilon^{-0.021t}\sin(0.184t + 3.67) \end{aligned} \tag{16}$$

The calculated voltage from Equation (15) and the re-plotted curve from oscillogram of Fig. 86-b are shown along with component voltages in Fig. 50.

3. Calculation for Isolated Secondary - Chopped Wave

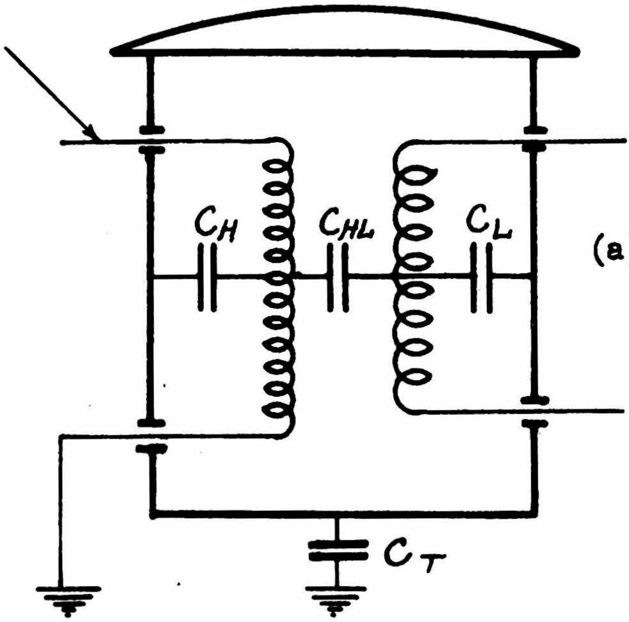
The calculation for the case of a chopped wave is made in exactly the same manner. For the specific case calculated for Wave G, the 15/47 microsecond wave, chopped at 8 microseconds, the equation for the fall wave holds up to the time of chopping. Thus, from 0 to 8 microseconds, Equations (15) or (16) should be used.

After the impressed wave is chopped, both the electrostatic component from the applied wave, and the electromagnetic component disappear. This leaves only the voltage produced by capacity coupling with primary mid-tap and a possible secondary oscillation. From equation (11) and the 37% coupling factor, we obtain as the secondary terminal voltage after time of chopping at 8 microseconds the equations

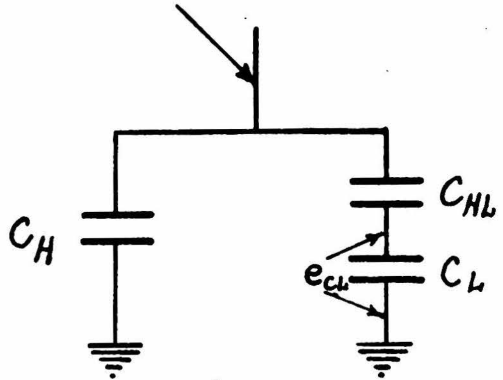
$$
\begin{aligned}
e_s = e_1 = {} & 3.2\,\epsilon^{-0.021(t-8)} \sin\left[0.184(t-8)+3.19\right] - 29.6\,\epsilon^{-0.03(t-8)} \\
& - 5.4\,\epsilon^{-0.021(t-8)} \sin\left[0.184(t-8)+3.67\right] + 11.0\,\epsilon^{-0.1275(t-8)} \\
& - 10.6\,\epsilon^{-0.021(t-8)} \cos\left[0.184(t-8)+\pi\right] \\
& - 4.1\,\epsilon^{-0.021t} \sin(0.184t+3.19) + 37.8\,\epsilon^{-0.03t} \\
& + 15.0\,\epsilon^{-0.021t} \sin(0.184t+3.67) - 28.9\,\epsilon^{-0.1275t}
\end{aligned}
$$

The calculated voltage and voltage re-plotted from oscillogram of Fig. 88 are given in Fig. 51. The two curves are not directly comparable, since the calculated curve was for chopping at 8 microseconds, whereas, the measured one was for chopping at 16 microseconds. The general agreement is evident, however.

WINDING CAPACITY RELATIONS

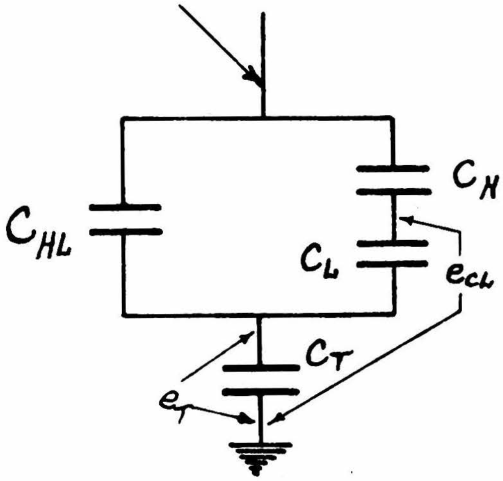


(a) Simplified Capacity Relations for a Transformer



(b) Equivalent Circuit for Grounded Tank

Voltage of Secondary to Ground Depends on Relative Effective Values of C_{HL} and C_L.



(c) Equivalent Circuit for Isolated Tank

Transformer Case Assumes Potential Nearly the Average Potential of the Primary Winding Since C_T is Very Small.

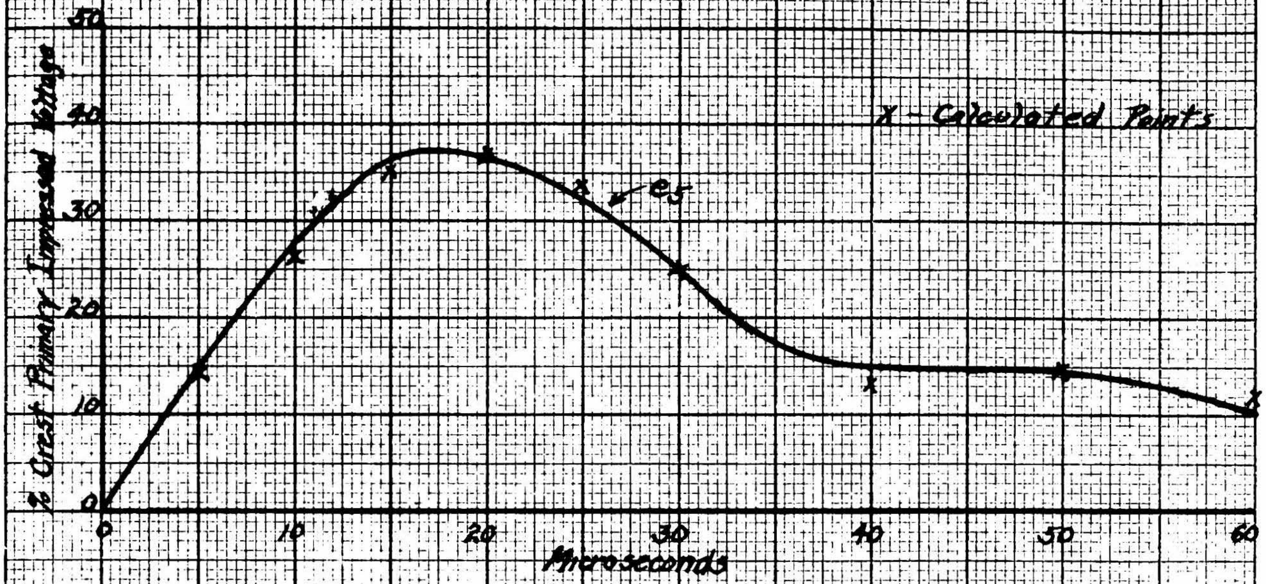
Secondary Assumes Potential of the Case.

Fig. 49

Secondary Terminal Voltage

Transformer 125 Wave G

Curve Replotted from Oscillogram C39B



Calculated Components of C_5

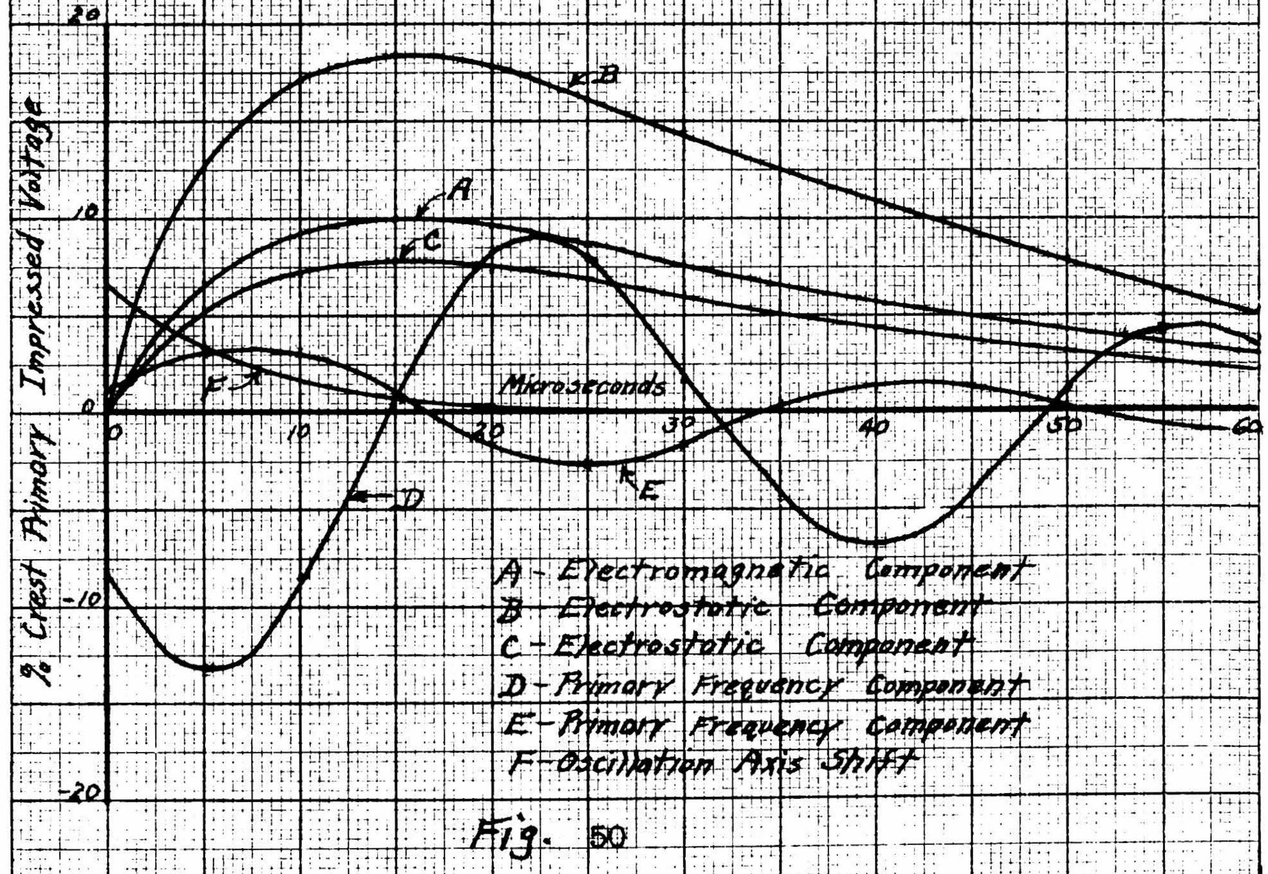


Fig. 50

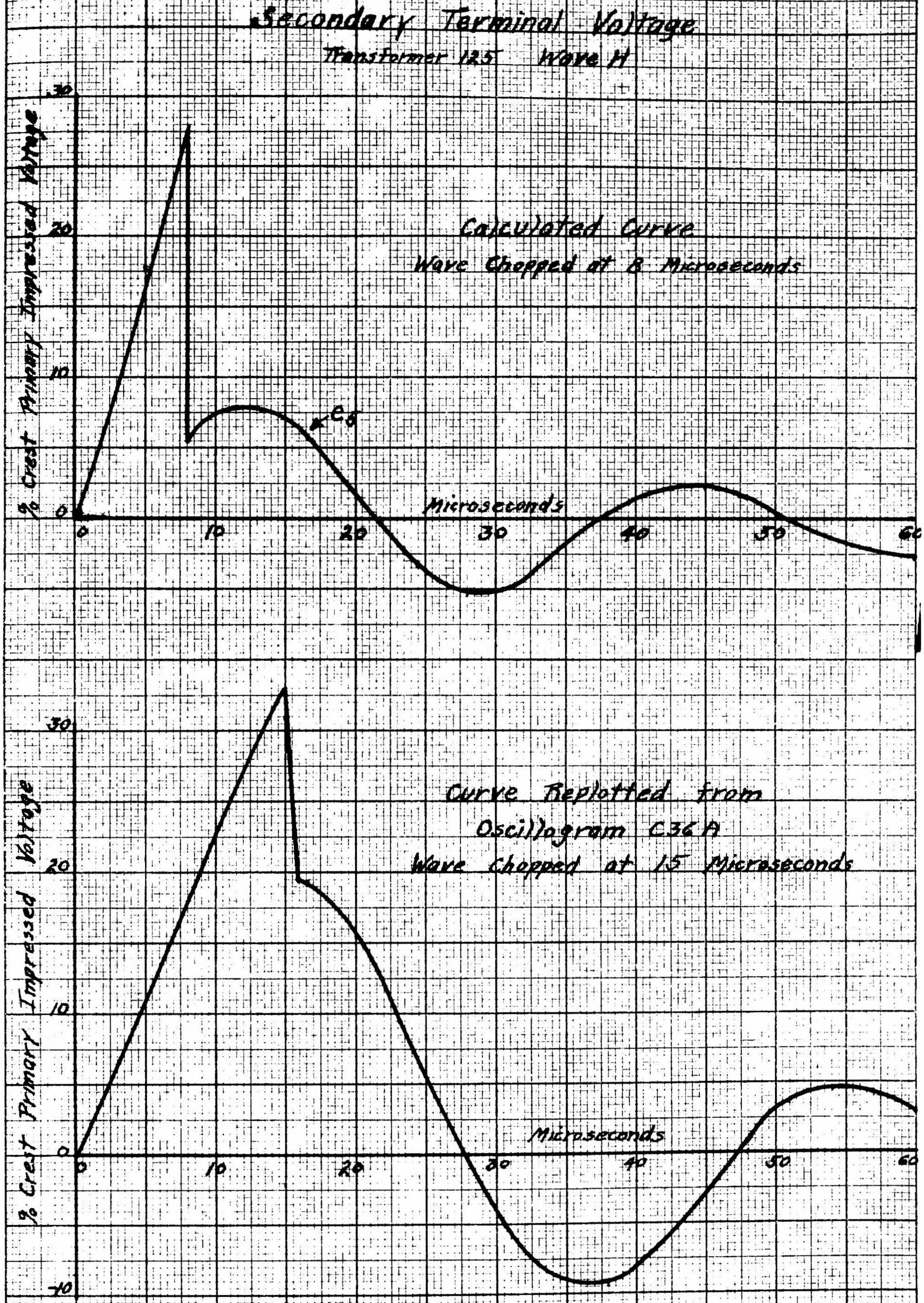

Secondary Terminal Voltage
Transformer 125 Wave H
% Crest Primary Impressed Voltage
30
20
10
0
0
10
20
30
40
50
60
Microseconds
Calculated Curve
Wave Chopped at 8 Microseconds
e_5
% Crest Primary Impressed Voltage
30
20
10
0
-10
0
10
20
30
40
50
60
Microseconds
Curve Replotted from
Oscillogram C36A
Wave Chopped at 15 Microseconds

Fig. 51

F. Experimental Results

The major portion of the experimental results from the cathode ray oscillograph study of the effects of impulse voltages on distribution transformers are included in this section. Approximately 275 cathode ray oscillograms were obtained with records of between 700 and 800 transient voltages.

1. Test Circuits

The four transformers listed in Table VII were tested.

Diagrams of transformer test connections are given in Figs. 52 to 57. Terminal impedances shown were in the form of lumped resistance. All voltages were those from point numbered to ground. Arrows indicate point of application of impulse voltage from the surge generator.

2. Test Waves

The characteristics of the test waves applied are listed in Table IX. The waves are plotted for comparison in Figs. 58 and 59.

3. Cathode Ray Oscillograms

Some of the oscillograms taken during the investigation are shown in the following Figs. 60 to 165. All oscillograms in this group were obtained in the High Voltage Laboratory of the California Institute.

Certain features of the oscillograms may require explanation as an aid in interpreting the records.

a. Timing Wave. A constant amplitude oscillation along the zero voltage line is a known frequency oscillator wave for time sweep calibration.

b. Reversal of Oscillograph Polarity. The impressed surge voltage was in all cases negative. In order to make more efficient use of film, the oscillograph deflection polarity was reversed to obtain records on both upper and lower halves of the film. More records could thus be placed on one film without confusion. This was practical, of course, only for unidirectional impulses.

Normally, a negative voltage to ground would give a deflection upwards. The voltage is then marked -Kv. When deflection downward is marked -Kv, the oscillograph polarity was reversed. When a deflection is marked Kv, the oscillograph polarity was normal and the voltage to ground was positive.

c. Voltage Scales. Voltage scales differ on upper and lower records on a film, if the test voltages differed greatly in magnitude.

d. Time Scales. In nearly all cases, the time scales were the same for all records on one film.

TABLE II

SECONDARY TERMINAL TRANSIENTS
Test 1
SECONDARY WINDING ISOLATED
TERMINAL IMPEDANCE INFINITE

Transformer Number	Wave	Tank*	Maximum Volts to Ground**		
			e_1	e_3	e_5
125	A	G	32.2	37.2	42.2
125	A	I	47.0	51.5	57.0
473	A	G	21.8	21.0	20.1
473	A	I	43.0	42.2	41.5
1129	A	G	33.0	30.0	26.0
1129	A	I	-	50.0	49.0
1419	A	G	16.4	17.1	20.7
1419	A	I	43.0	41.0	41.0
125	B	G	26.5	-	36.5
125	C	G	33.0	-	44.0
1129	C	G	41.0	-	35.0
125	D_1	G	10.0	14.0	19.0
125	D_1	I	44.0	49.0	33.0
125	D_2	G	10.5	16	-
125	E	G	33.0	38.0	45.0
125	F	G	20.0	26.0	31.0
125	G	G	32.0	34.0	36.0
473	G	G	24.0	22.2	20.5
473	H	G	14.0	12.0	10.0
125	H	G	27.0	32.0	37.0

* G - Transformer tank grounded. I - Transformer tank isolated.

** Voltages to ground in per cent of primary impressed wave crest. e_1 and e_5 are secondary terminal volts to ground; e_3 is secondary mid-point voltage to ground.

TABLE III

SECONDARY TERMINAL TRANSIENTS
Test 1
SECONDARY WINDING ISOLATED
TERMINAL IMPEDANCE INFINITE

Transformer Number	Wave	Tank*	Maximum Volts** Across Winding	Electro-static Component	Electro-magnetic Component
125	A	G	10.0	15.0	10.0
125	A	I	10.0	51.0	10.0
473	A	G	7.0	10.6	3.3
473	A	I	3.0	40.0	3.3
1129	A	G	16.2	26.2	10.0
1129	A	I	16.2	48.0	10.0
1419	A	G	12.0	5.0	6.7
1419	A	I	12.0	35.0	6.7
125	B	G	10.0	14.0	10.0
125	C	G	10.0	14.0	10.0
1129	C	G	-	-	-
125	D_1	G	14.0	14.0	9.0
125	D_1	I	49.0	49.0	9.0
125	D_2	G	10.0	14.0	11.0
125	E	G	12.5	-	12.5
125	F	G	10.5	13.5	10.5
125	G	G	9.0	-	9.0
473	G	G	3.6	-	3.6
473	H	G	4.0	-	4.0
125	H	G	10.0	-	10.0

* G - Transformer tank grounded. I - Transformer tank isolated.
** All voltages are given in per cent of primary impressed wave crest.

TABLE IV

SECONDARY TERMINAL TRANSIENTS
Test 1
SECONDARY WINDING ISOLATED
TERMINAL IMPEDANCE INFINITE

Transformer Number	Wave	Tank*	Primary Frequency Component	
			Period**	Amplitude[1]
125	A	G	34	37
125	A	I	0	0
473	A	G	120	37
473	A	I	120	small
1129	A	G	100	55
1129	A	I	0	0
1419	A	G	-	-
1419	A	I	-	-
125	B	G	34	38
125	C	G	34	31
1129	C	G	100	50
125	D_1	G	34	32
125	D_1	I	0	0
125	D_2	G	34	30
125	E	G	34	38
125	F	G	34	39
125	G	G	34	41
473	G	G	120	40
473	H	G	120	40
125	H	G	34	62

* G - Transformer tank grounded. I - Transformer tank isolated.

** Time of one cycle of oscillation in microseconds.

[1] Amplitude in terms of per cent of amplitude of primary mid-tap oscillation.

TABLE V

SECONDARY TERMINAL TRANSIENTS
Test 1
SECONDARY WINDING ISOLATED
TERMINAL IMPEDANCE INFINITE

Transformer Number	Wave	Tank[1]	Secondary Oscillation Period[2]	Secondary Oscillation Amplitude[3]
125	A	G	0	0
125	A	I	0	0
473	A	G	2.5	5.0
473	A	I	2.5	5.0
1129	A	G	4.0	7.0
1129	A	I	4.0	7.0
1419	A	G	16.5	12.8
1419	A	I	16.5	12.8
125	B	G	0	0
125	C	G	0	0
1129	C	G	-	6.0
125	D_1	G	0	0
125	D_1	I	0	0
473	G	G	0	0
473	H	G	2.5	1.9*
125	H	G	0	0

1. G - Transformer tenk grounded. I - Transformer tank isolated.
2. Time of one cycle of secondary frequency oscillation.
3. Maximum voltage across secondary winding due to secondary oscillation expressed in per cent of primary impressed wave crest.

* Oscillation caused by chopping; no oscillation on wave front.

TABLE VI

SEDONDARY TERMINAL TRANSIENTS
Test 1
SECONDARY WINDING ISOLATED
TERMINAL IMPEDANCE INFINITE

Transformer Number	Wave	Tank*	Tank Potentials** to Ground	to H. V.	to L. V.
125	A	G	0	100	37.2
125	A	I	48	51.5	0
473	A	G	0	100	21
473	A	I	45	54	1
1129	A	G	0	100	30
1129	A	I	48	52	0
1419	A	G	0	100	17.1
1419	A	I	41	57	0
125	B	G	0	100	31
125	C	G	0	100	38
1129	C	G	0	100	38
125	D_1	G	0	100	14
125	D_1	I	-	-	-
125	D_2	G	0	100	16
125	E	G	0	100	38
125	F	G	0	100	26
125	G	G	0	100	34
473	G	G	0	100	22
473	H	G	0	100	12
125	H	G	0	100	32

* G - Transformer tank grounded. I - Transformer tank isolated.
** All voltages are in per cent of primary impressed wave crest.

Table VII

Transformer Data

No.	Make	Kva Rating	H.V.	L.V.	Polarity
125	Moloney	10	2200	220	Negative
473	Westinghouse	10	6600	220	Positive
E1129	G.E.	5	2080	208	Positive
EE1419	G.E.	1	6600	440	Positive

Table VIII

Transformer Capacitances

Transformer Number	Capacitances in Microfarads*		
	H.V. to Case	H.V. to L.V.	L.V. to Case
125	0.00163	0.0035	0.00204
473	-	-	-
E1129	0.000895	0.00222	0.00110
EE1419	-	-	0.00125

*Capacitance as measured by bridge with 1,000 cycles.

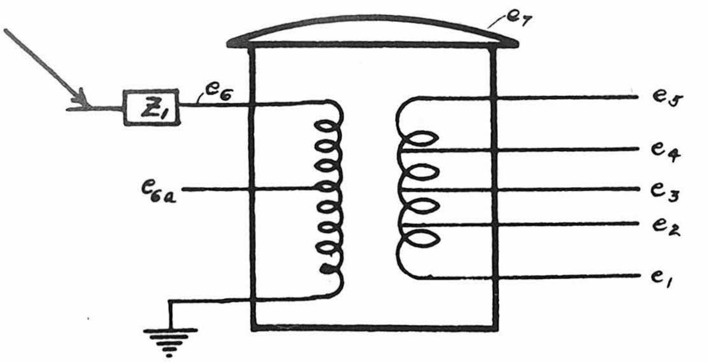
Table IX

Test Wave Characteristics

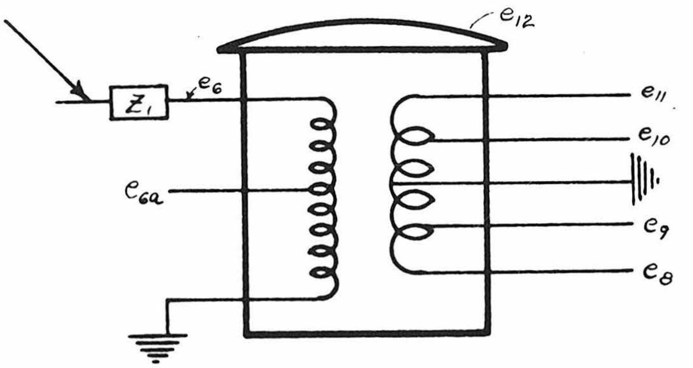
Wave	Time to 90% Crest	Time to 100% Crest	Time to 50% Crest on Tail
A	0.75	2.0	75
B	0.70	1.8	20
C	0.75	2.0	340
D_1	0.70	1.8	0.4
D_2	0.70	1.8	0
E	3.5	8.0	60
F	3.5	8.0	0.4
G	7	15	47
H	8	13	0.4

All values of time are in microseconds.

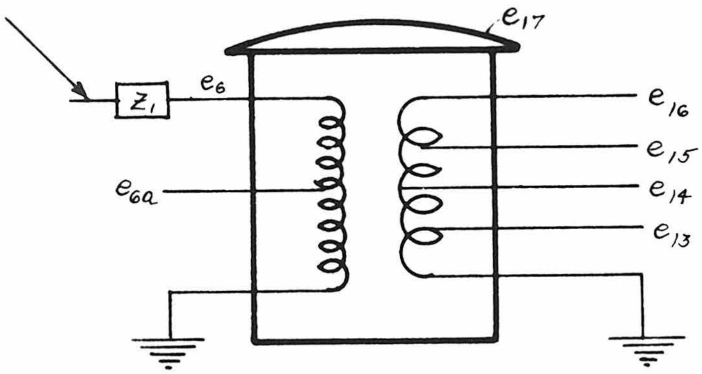
TRANSFORMER TEST DIAGRAMS



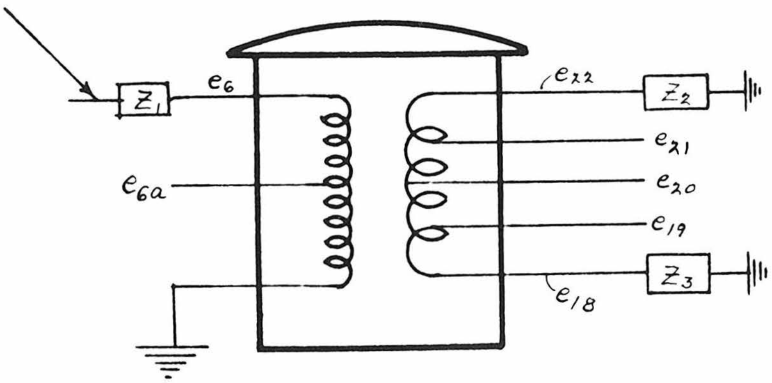
Test 1



Test 2



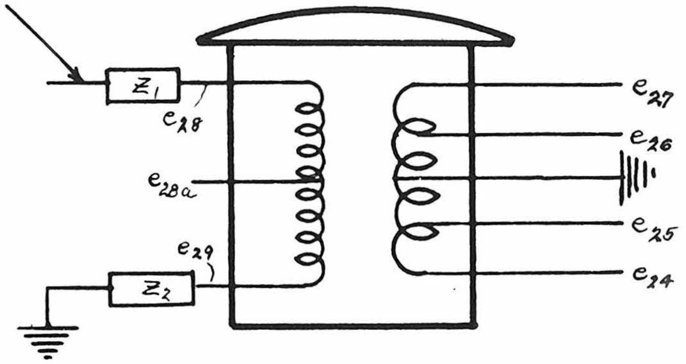
Test 3



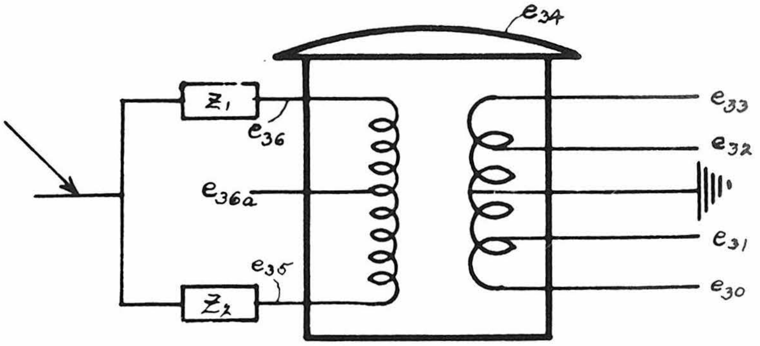
Test 4

Fig. 52

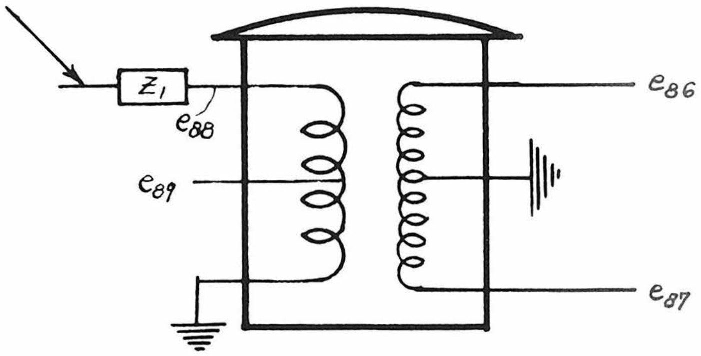
TRANSFORMER TEST DIAGRAMS



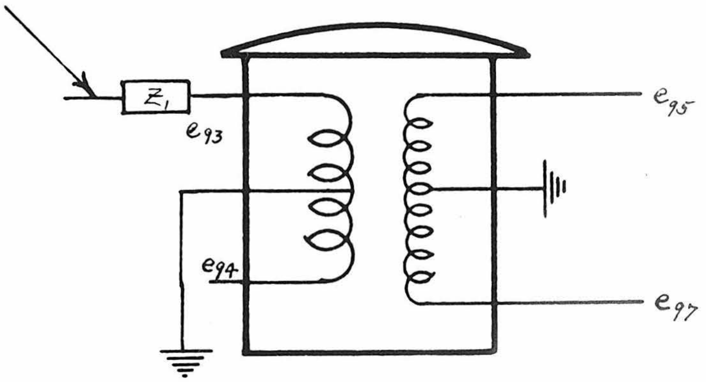
Test 5



Test 6



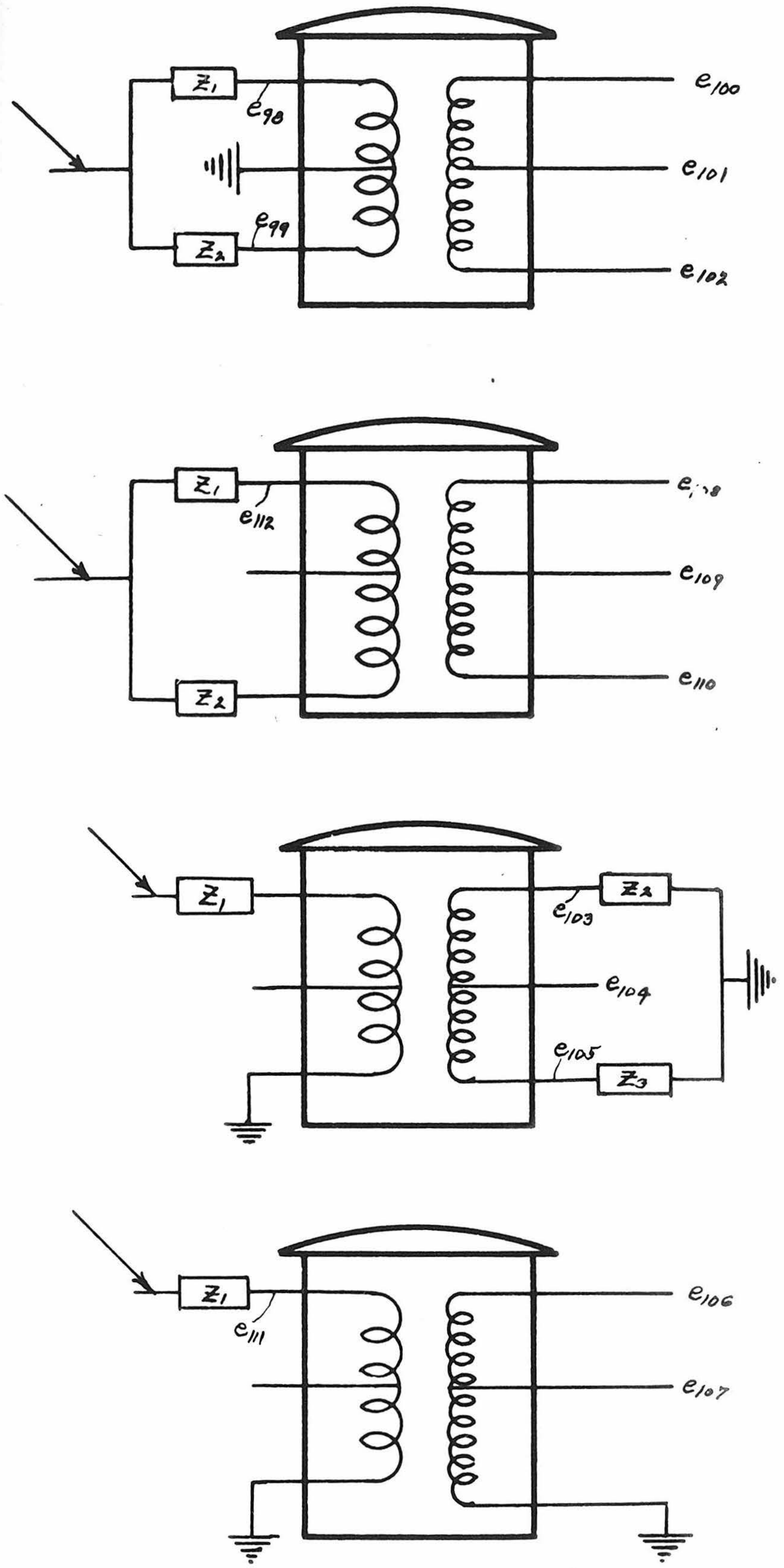
Test 7-A



Test 8

Fig. 53

TRANSFORMER TEST DIAGRAMS



Test 9

Test 10

Test 11

Test 12

Fig. 54

TRANSFORMER TEST DIAGRAMS

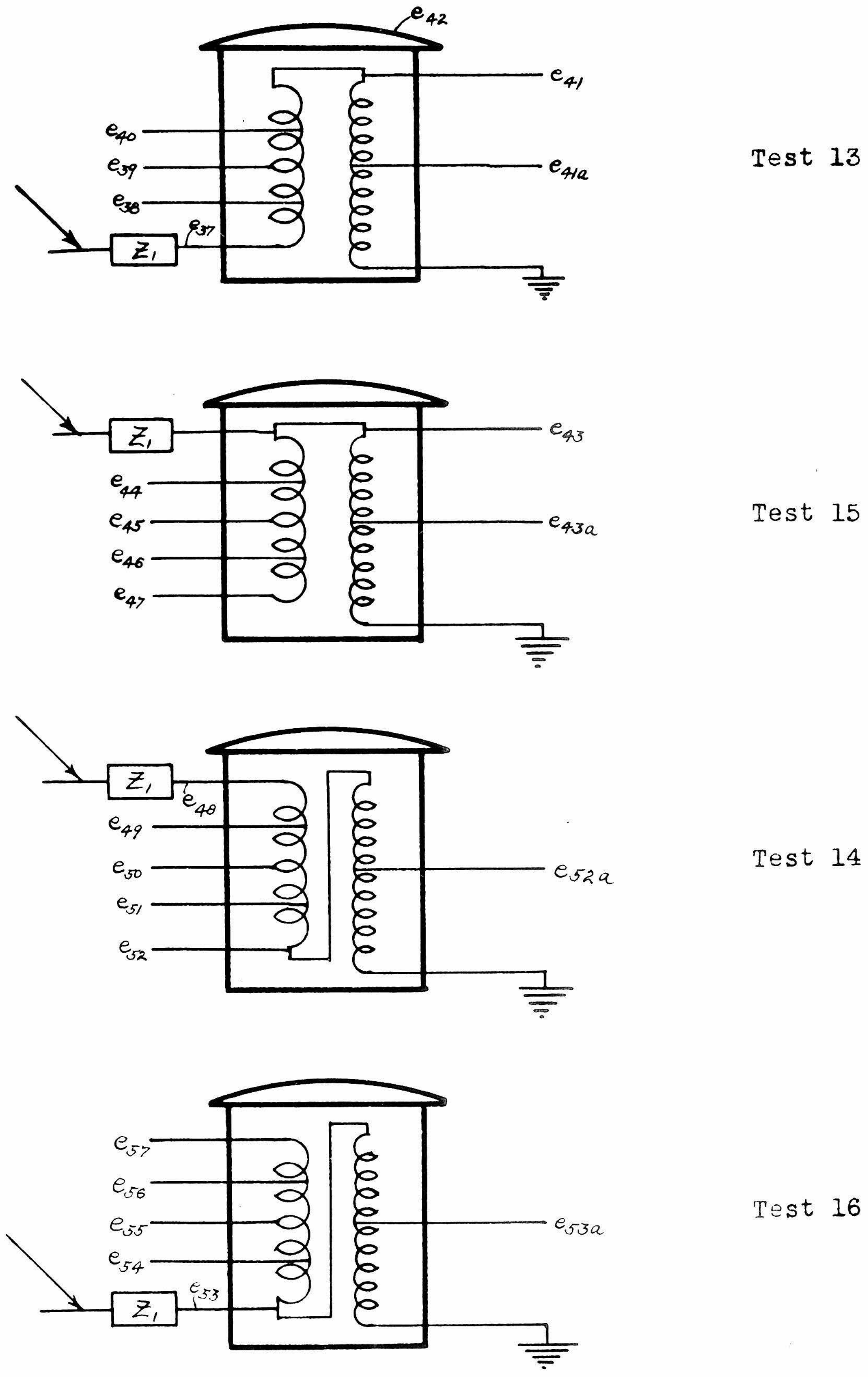
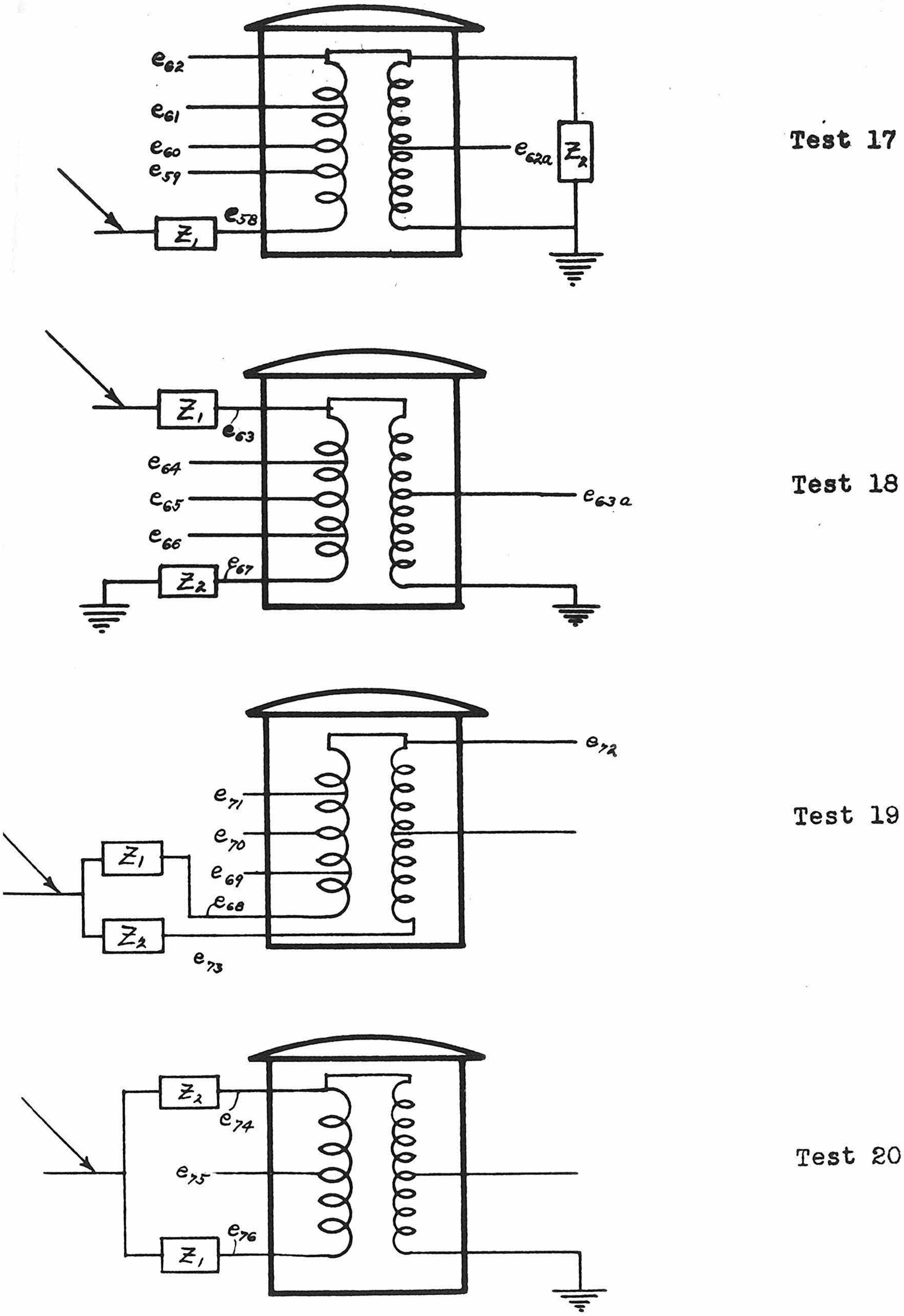


Fig. 55

TRANSFORMER TEST DIAGRAMS



Test 17

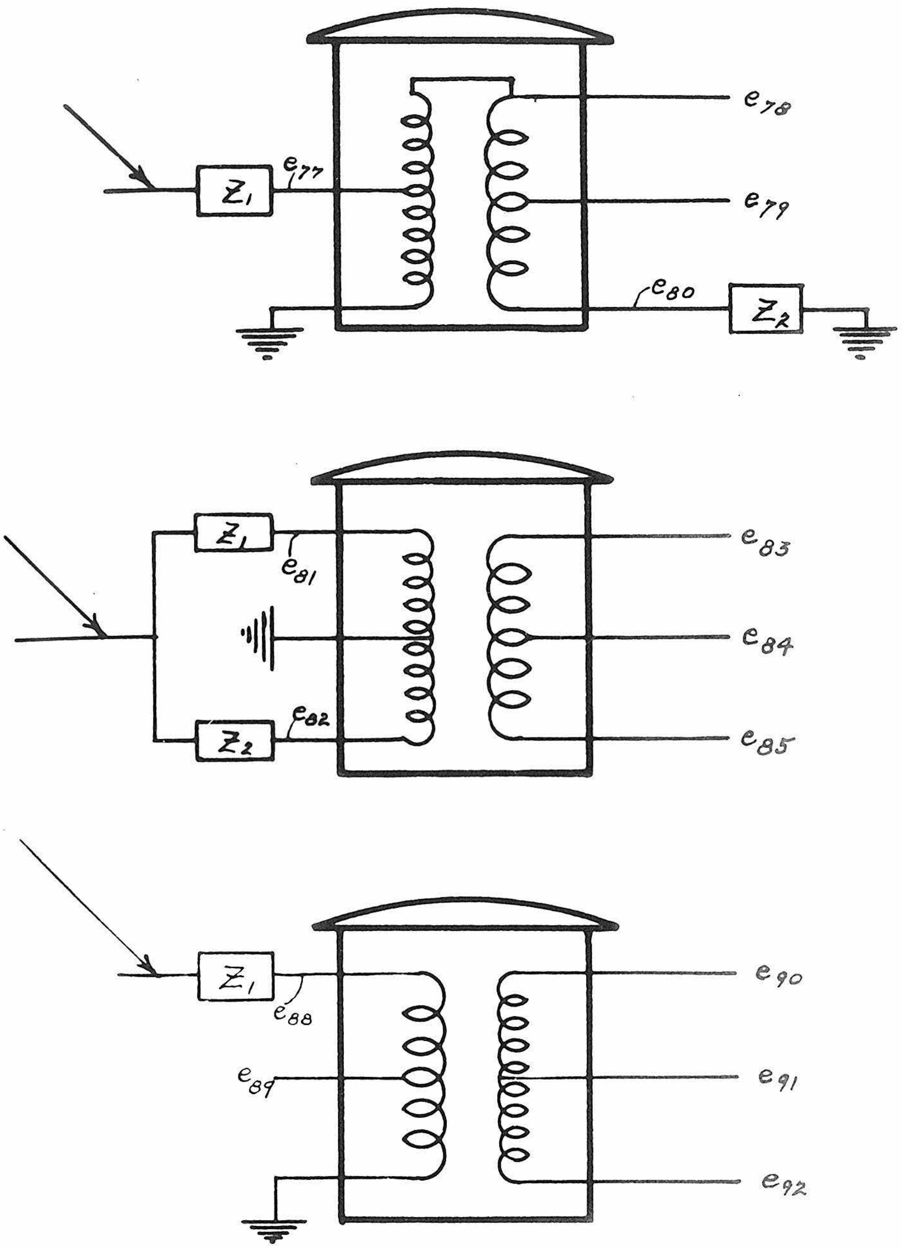
Test 18

Test 19

Test 20

Fig. 56

TRANSFORMER TEST DIAGRAMS



Test 21

Test 22

Test 7

Fig. 57

Test Waves

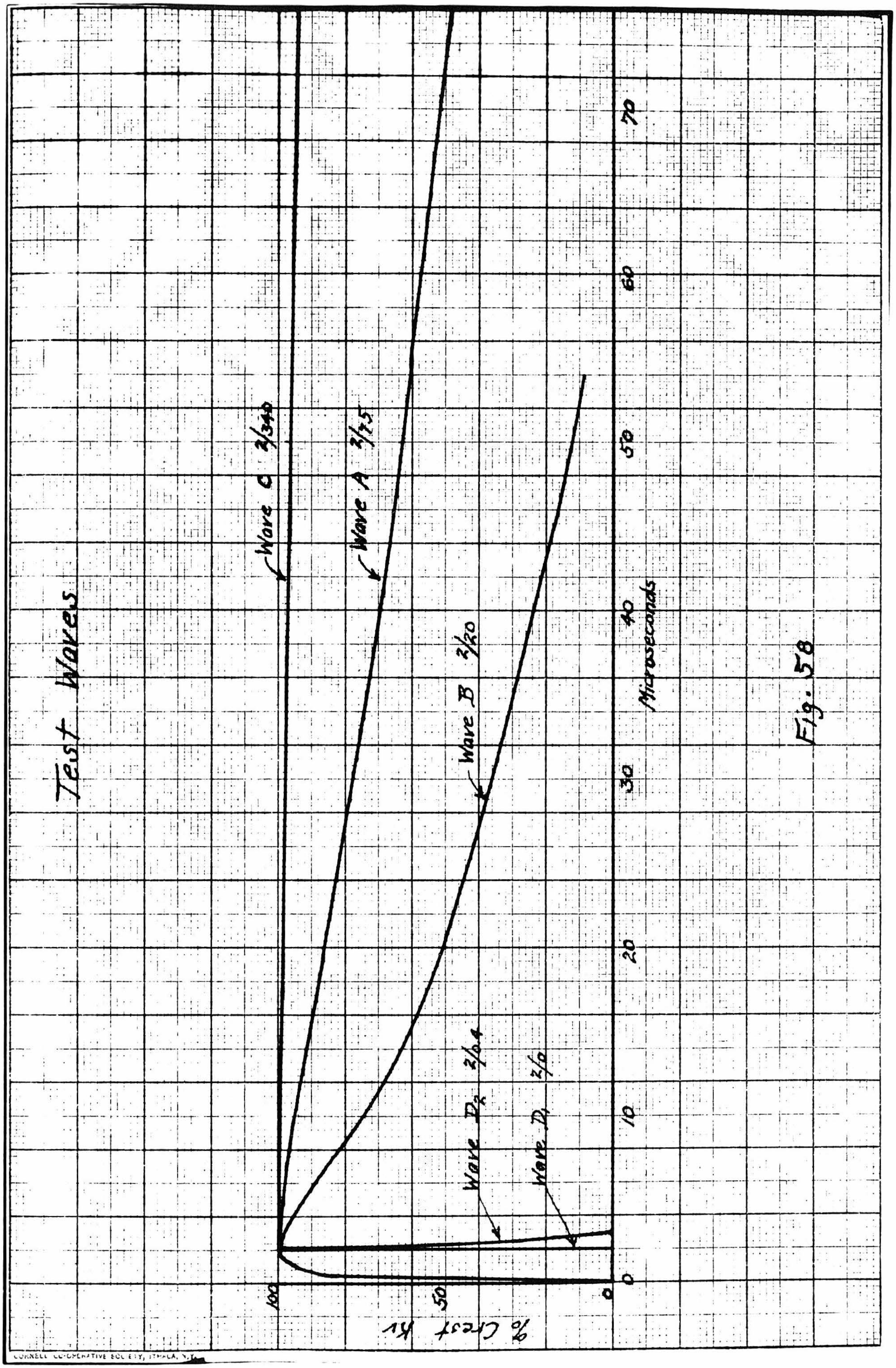


Fig. 58

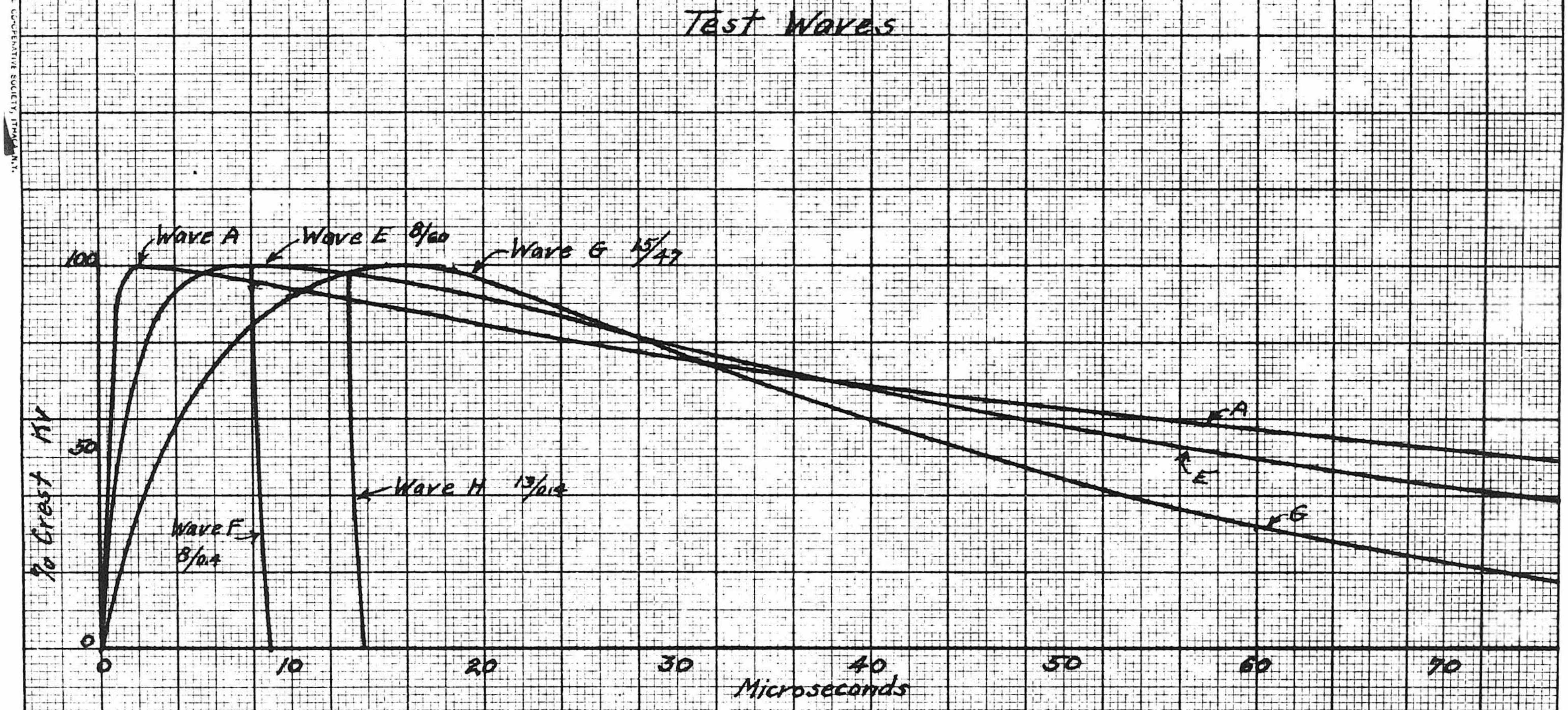
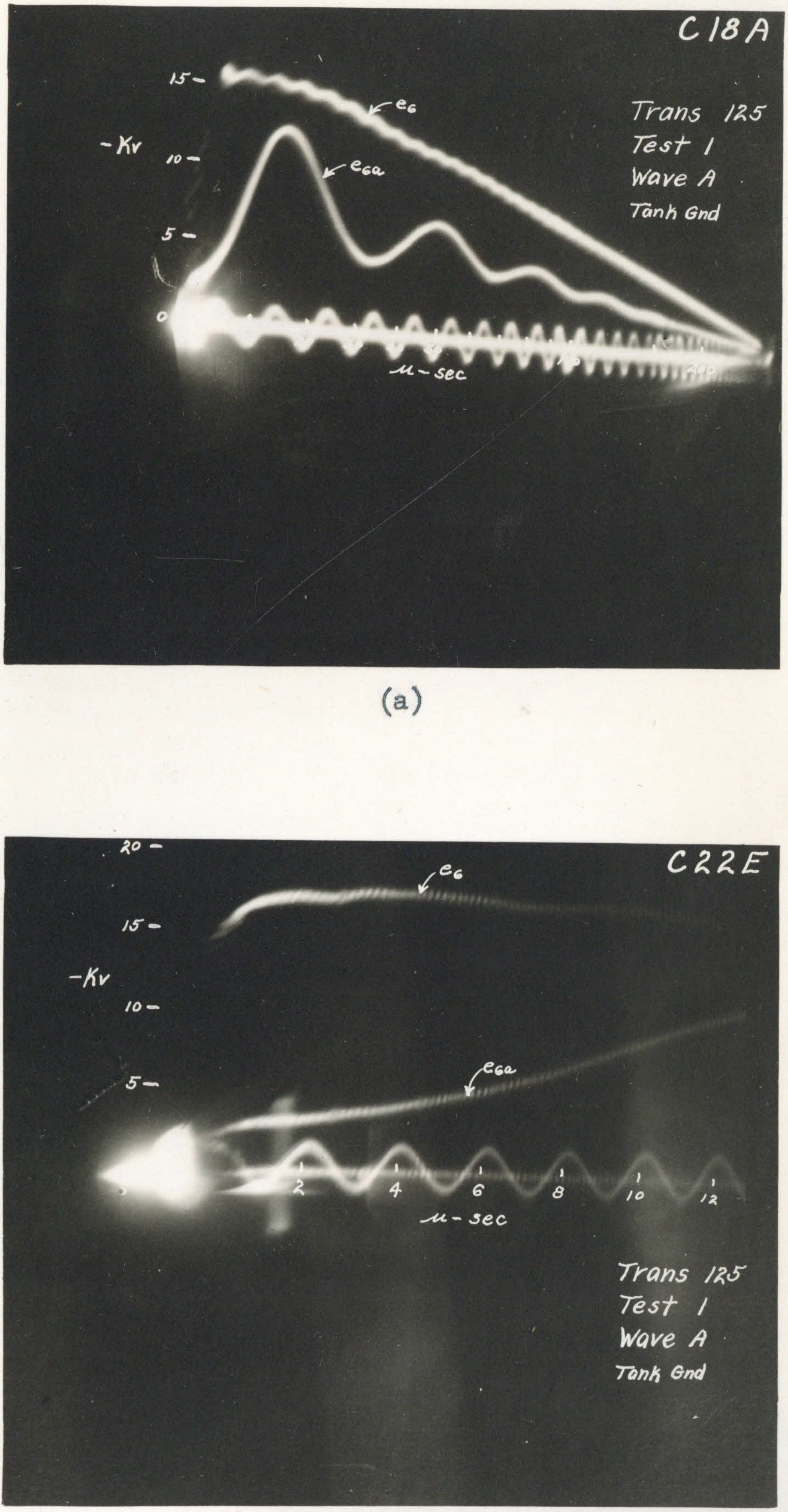

Test Waves

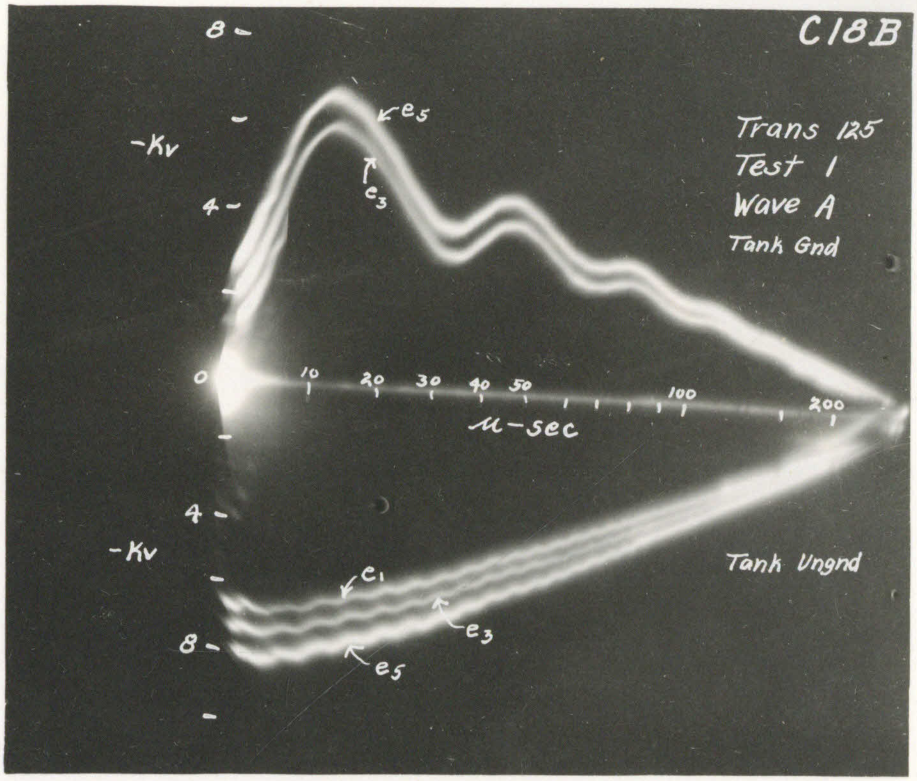
Fig. 59


C18A
15
e6
-Kv
10
e6a
5
0
Trans 125
Test 1
Wave A
Tank Gnd
µ-sec
C22E
20
e6
15
-Kv
10
e6a
5
2
4
6
8
10
12
µ-sec
Trans 125
Test 1
Wave A
Tank Gnd

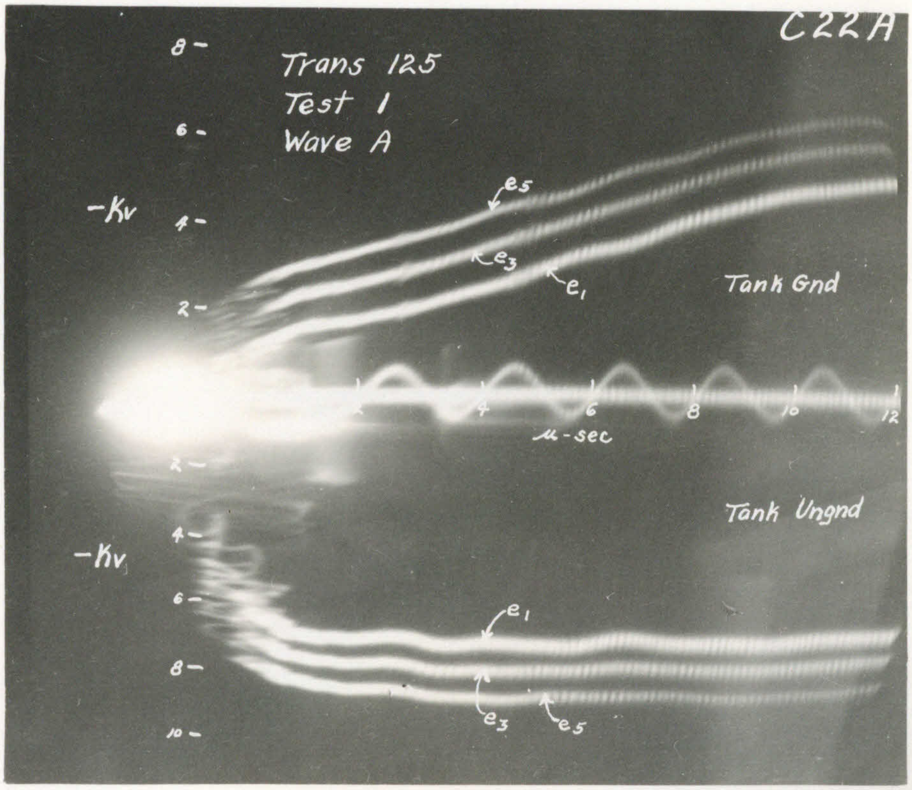
(a)

(b)

Fig. 60

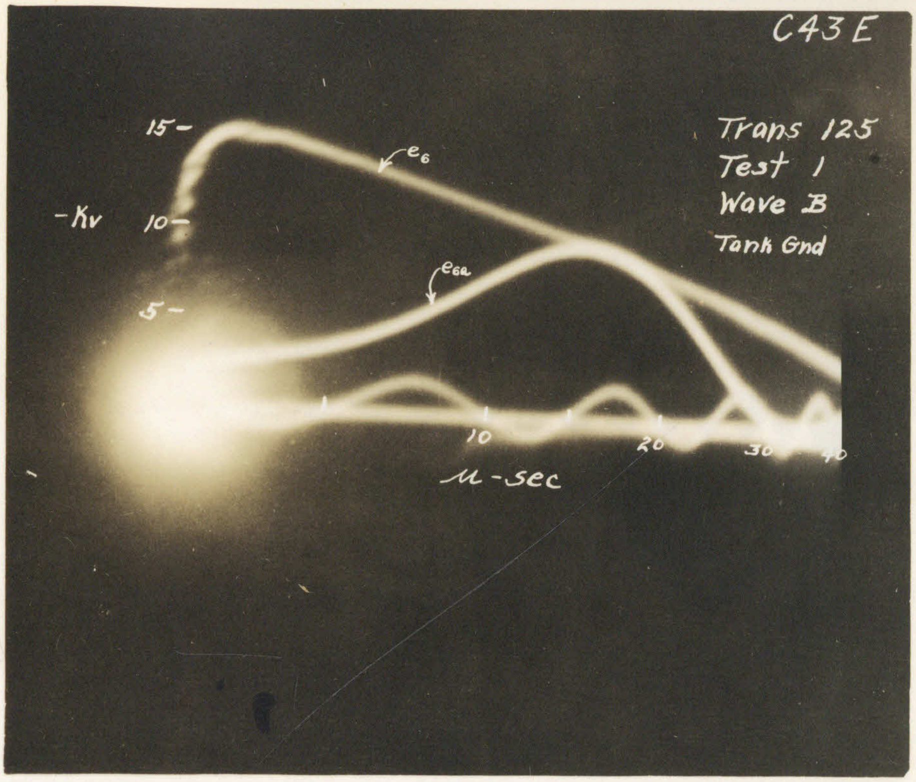

C18B
Trans 125
Test 1
Wave A
Tank Gnd
Tank Ungnd
-Kv
-Kv
8
4
0
4
8
10
20
30
40
50
100
200
μ-sec
e5
e3
e1
e3
e5

(a)

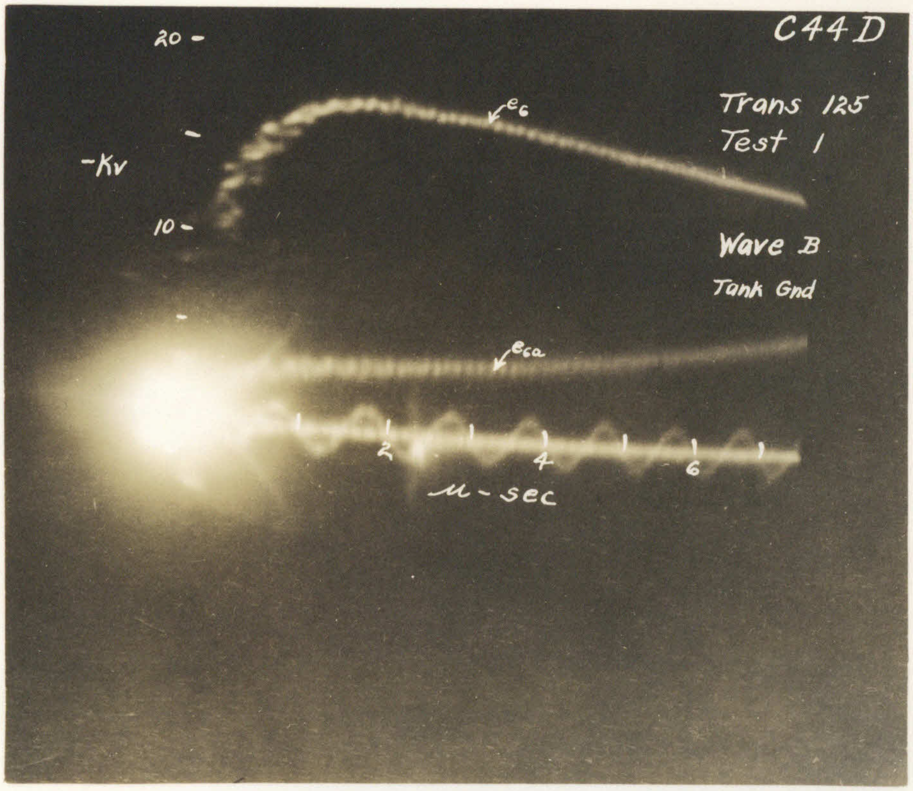

C22A
Trans 125
Test 1
Wave A
Tank Gnd
Tank Ungnd
-Kv
-Kv
8
6
4
2
2
4
6
8
10
4
6
8
10
12
μ-sec
e5
e3
e1
e1
e3
e5

(b)

Fig. 61

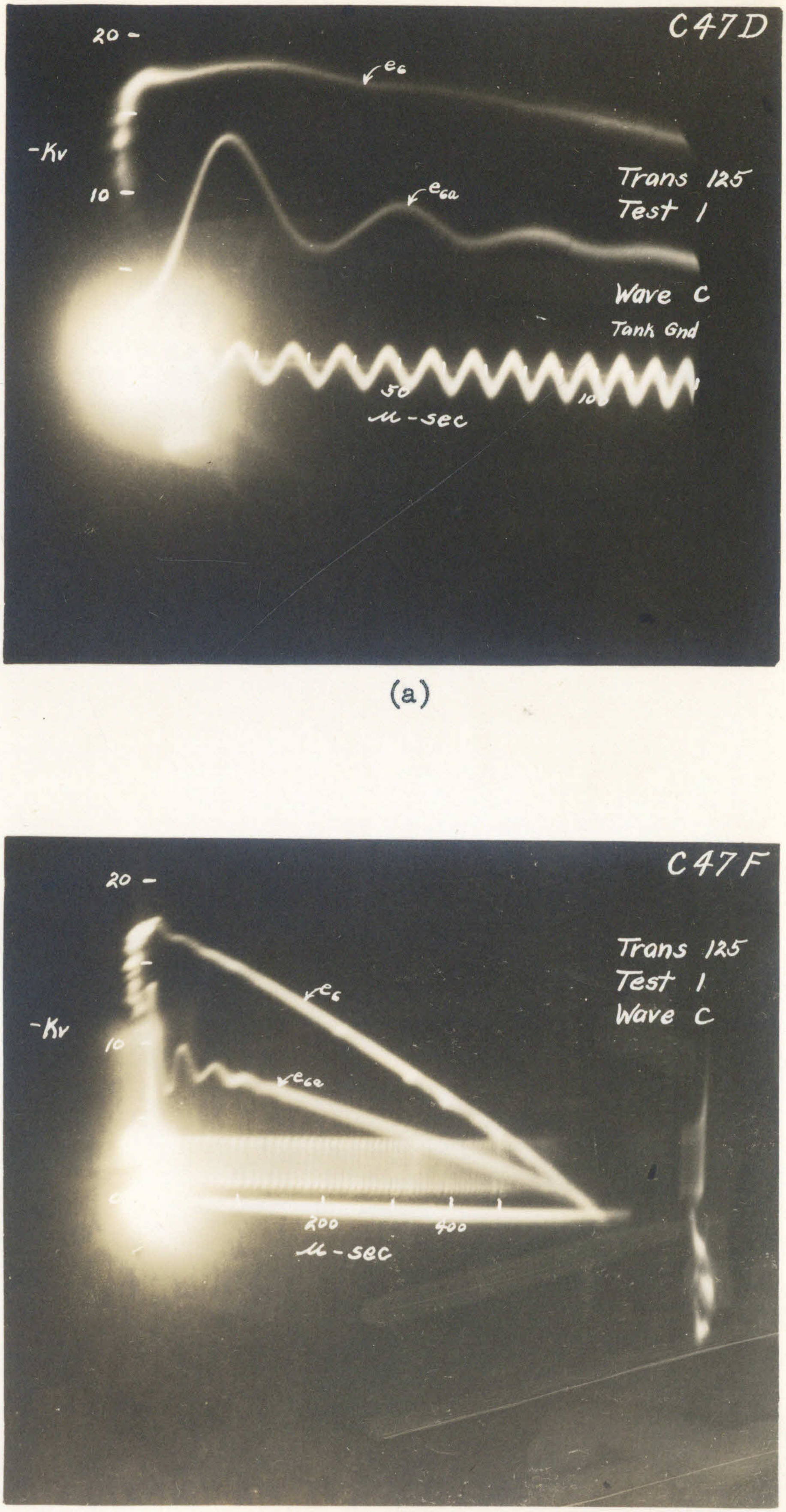

C43E
Trans 125
Test 1
Wave B
Tank Gnd
15
-Kv
10
5
e_6
e_{6a}
10
20
30
40
μ-sec

(a)


C44D
Trans 125
Test 1
Wave B
Tank Gnd
20
-Kv
10
e_6
e_{6a}
2
4
6
μ-sec

(b)

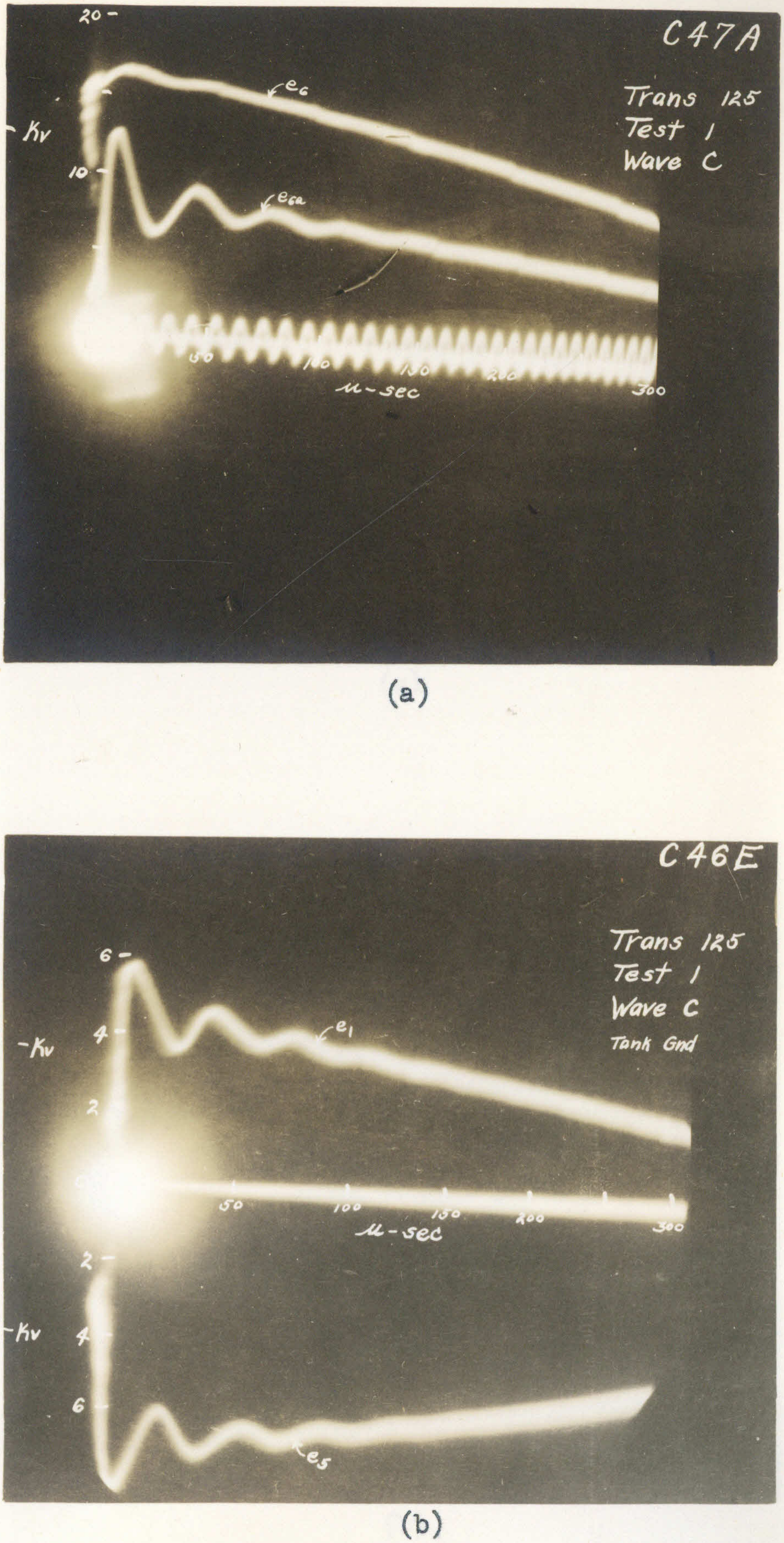
Fig. 62


C47D
20 -
-Kv
10 -
e_6
e_{6a}
Trans 125
Test 1
Wave C
Tank Gnd
50
100
μ-sec
C47F
20 -
-Kv
10
e_6
e_{6e}
Trans 125
Test 1
Wave C
200
400
μ-sec

(a)

(b)

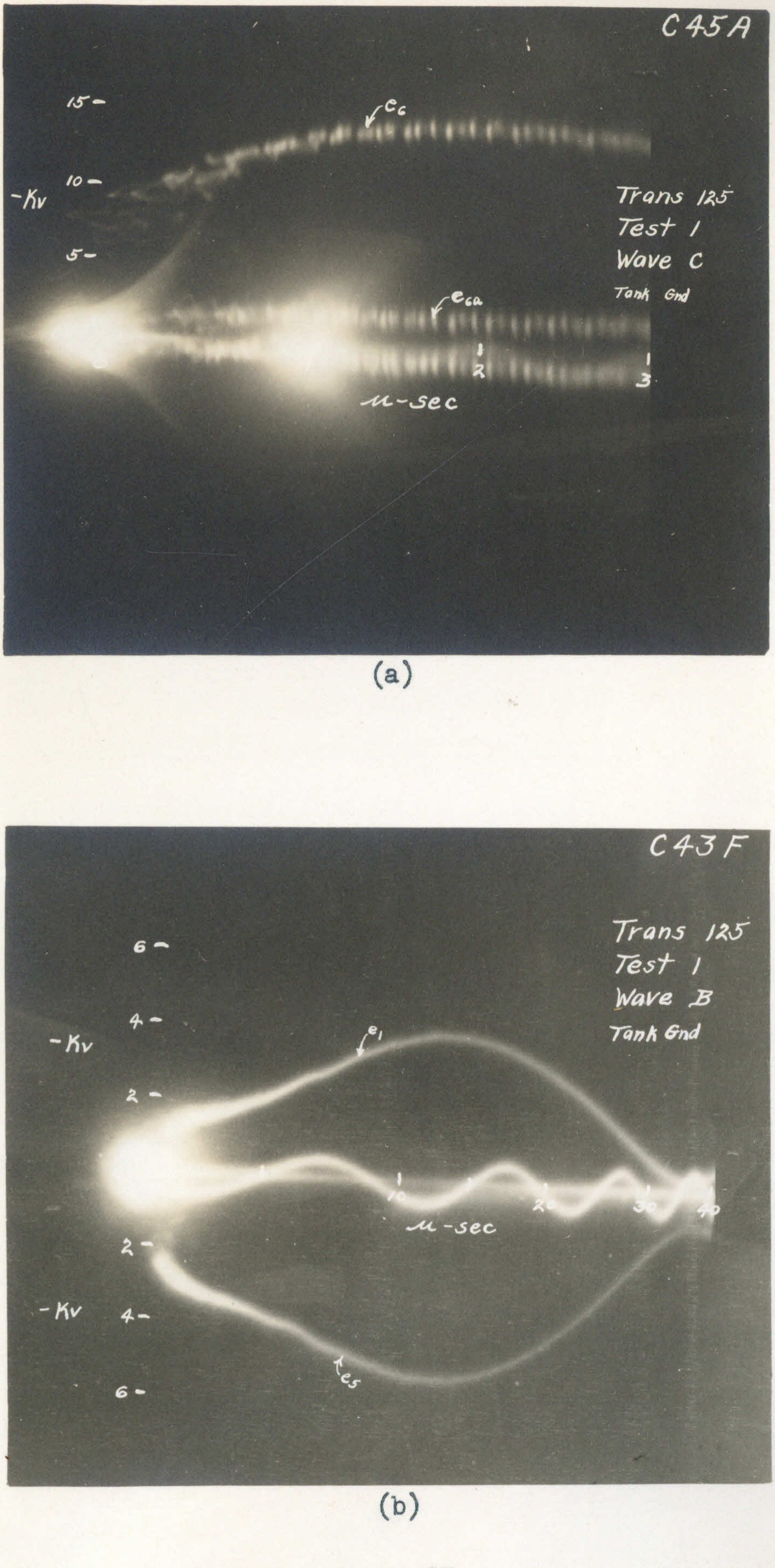
Fig. 63


C47A
Trans 125
Test 1
Wave C
20
Kv
10
e6
e6a
50
100
150
200
300
µ-sec
C46E
Trans 125
Test 1
Wave C
Tank Gnd
6
4
2
-Kv
e1
50
100
150
200
300
µ-sec
2
-Kv
4
6
e5

(a)

(b)

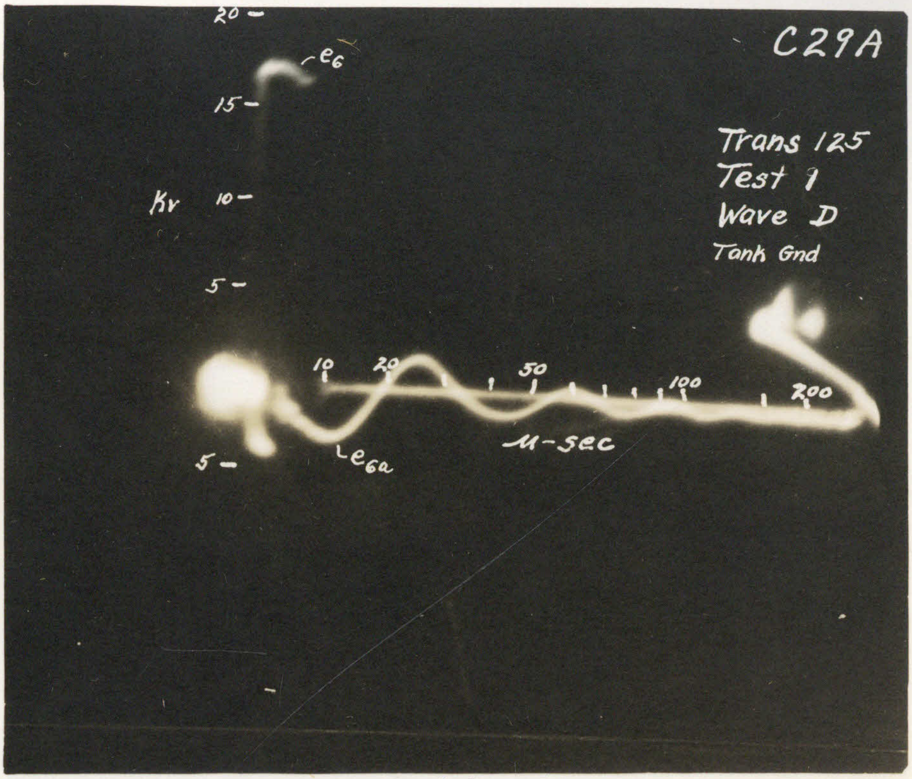
Fig. 64


C45A
15
10
5
-Kv
e_c
e_ca
Trans 125
Test 1
Wave C
Tank Gnd
1
2
1
3
μ-sec
C43F
Trans 125
Test 1
Wave B
Tank Gnd
6
4
2
-Kv
e_1
10
20
30
40
μ-sec
2
-Kv
4
6
e_s

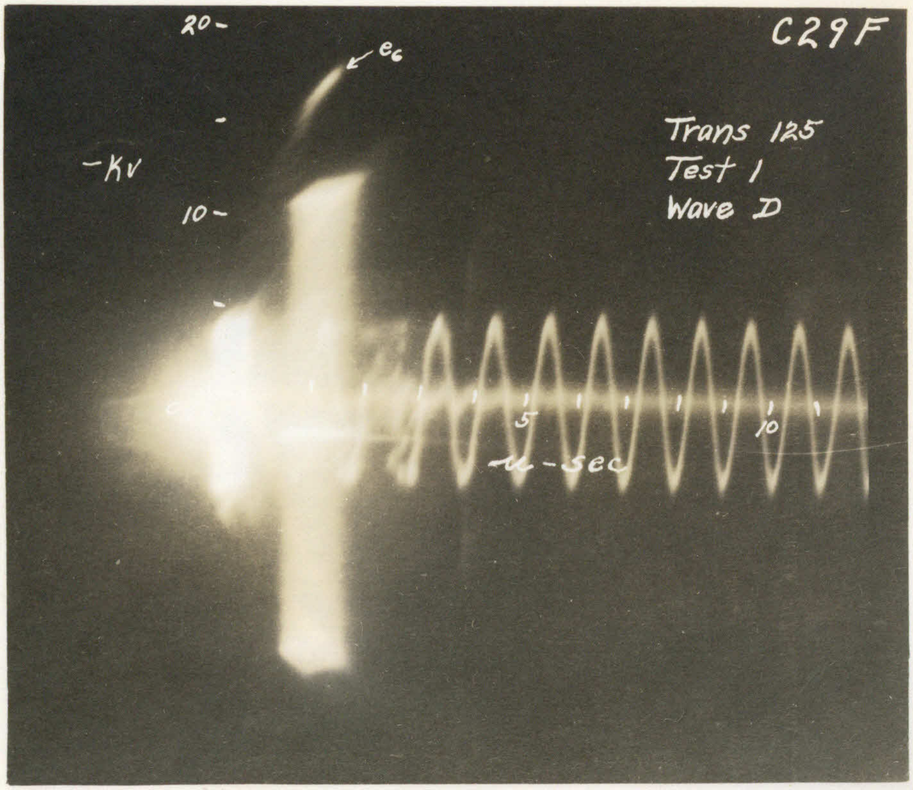
(a)

(b)

Fig. 65

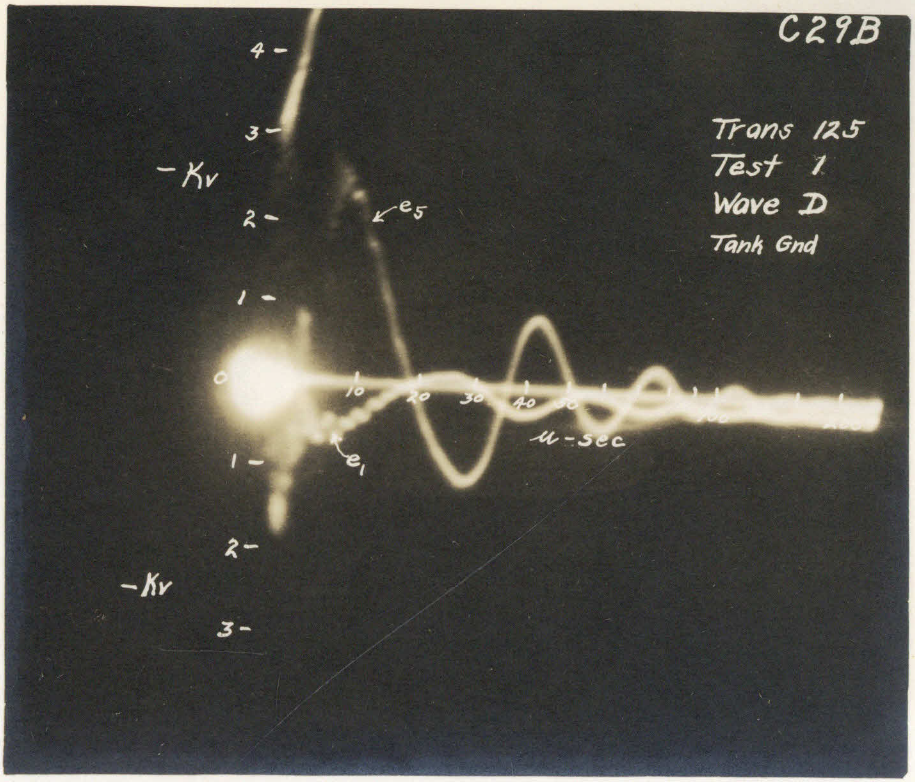

C29A
Trans 125
Test 1
Wave D
Tank Gnd
20
15
Kv
10
5
5
e6
e6a
10
20
50
100
200
μ-sec

(a)

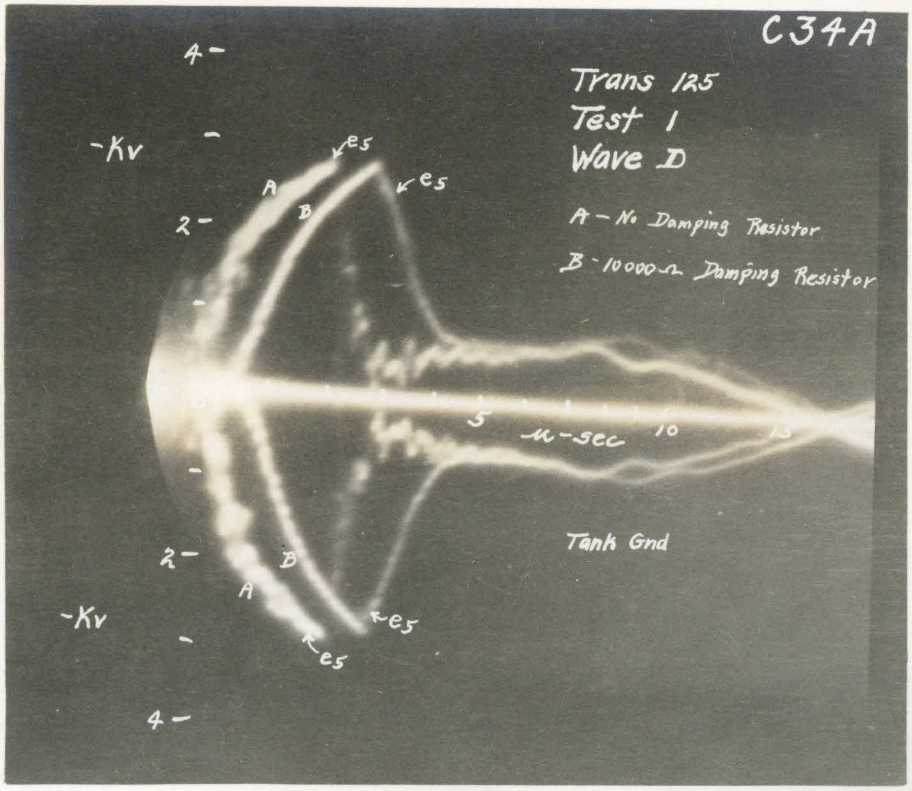

C29F
Trans 125
Test 1
Wave D
-Kv
20
10
e6
5
10
μ-sec

(b)

Fig. 66

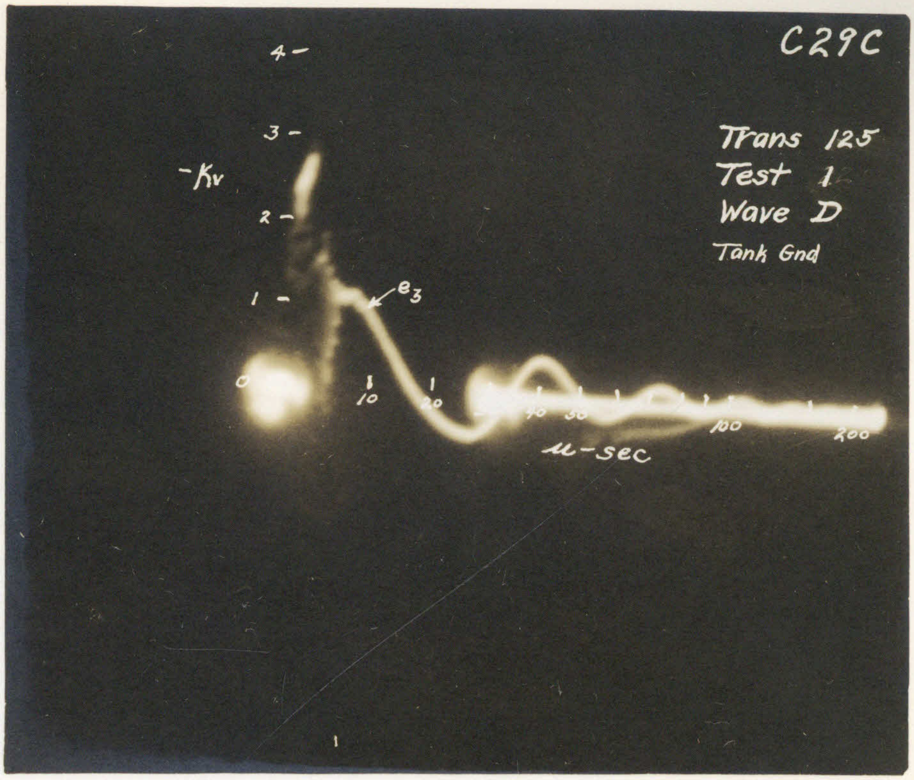

C29B
Trans 125
Test 1
Wave D
Tank Gnd
4
3
-Kv
2
e5
1
0
10
20
30
40
50
100
200
u-sec
e1
1
2
-Kv
3

(a)

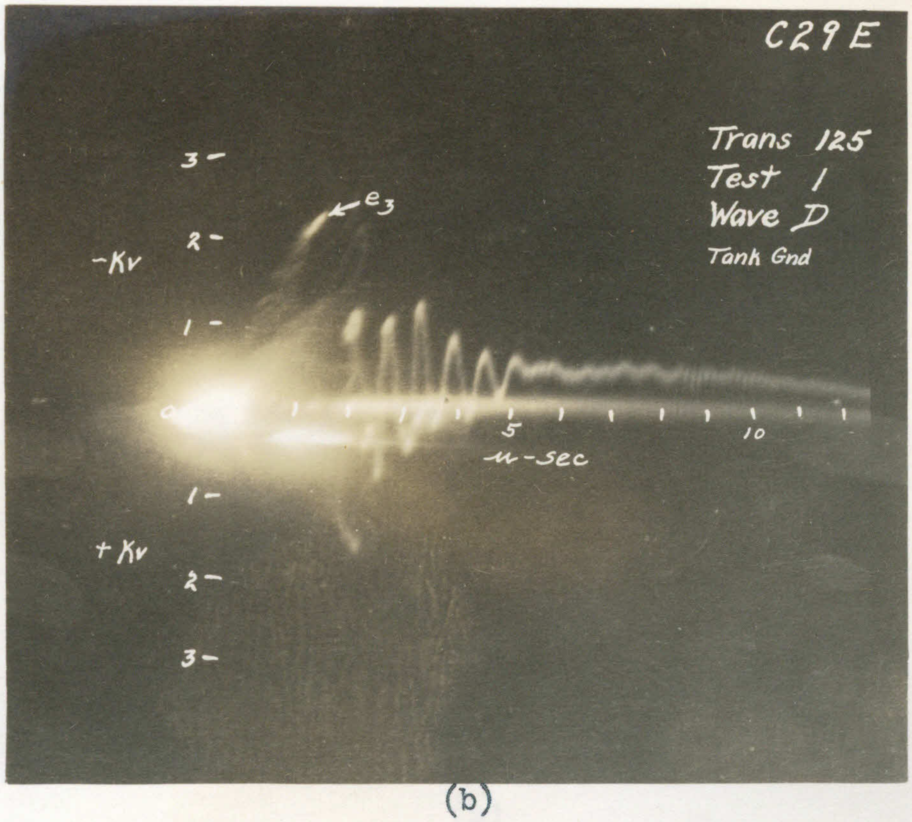

C34A
4
Trans 125
Test 1
Wave D
-Kv
e5
A
e5
B
2
A - No Damping Resistor
B - 10000 Ω Damping Resistor
5
u-sec
10
15
Tank Gnd
2
B
A
-Kv
e5
e5
4

(b)

Fig. 67

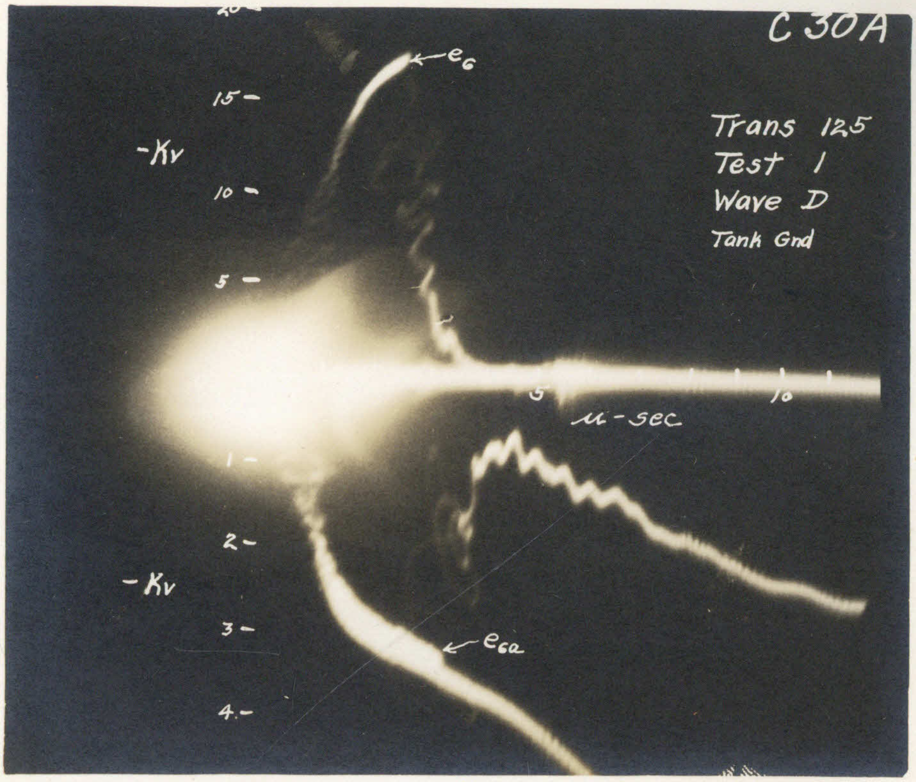

C29C
Trans 125
Test 1
Wave D
Tank Gnd
-Kv
4
3
2
1
0
e3
10
20
40
50
100
200
u-sec

(a)

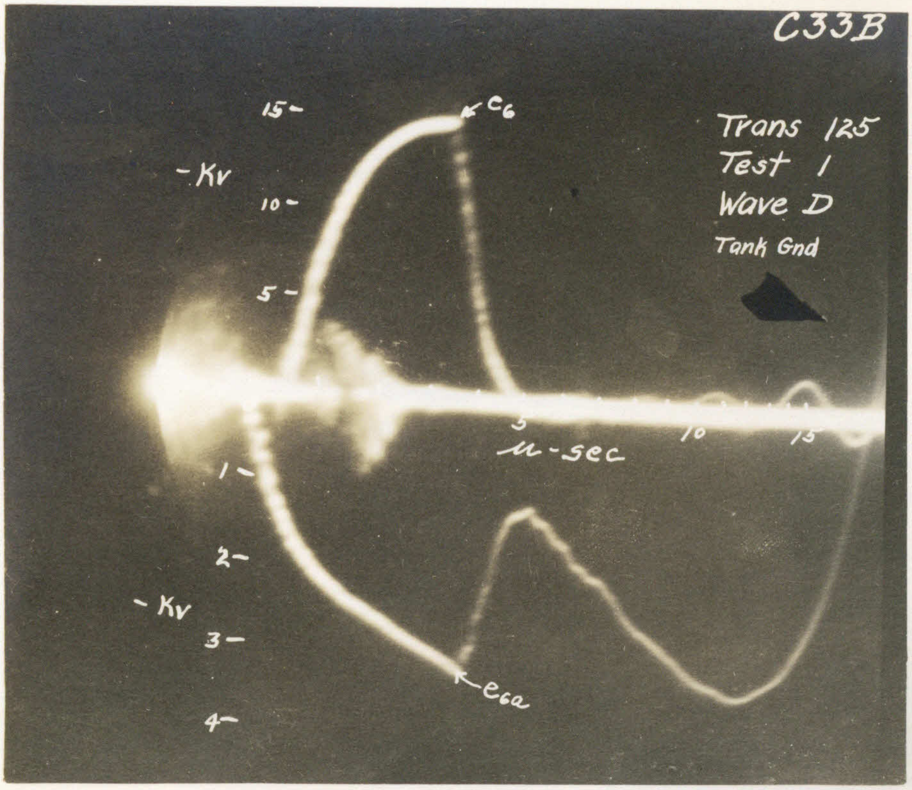

C29E
Trans 125
Test 1
Wave D
Tank Gnd
-Kv
+Kv
3
2
1
0
1
2
3
e3
5
10
u-sec

(b)

Fig. 68

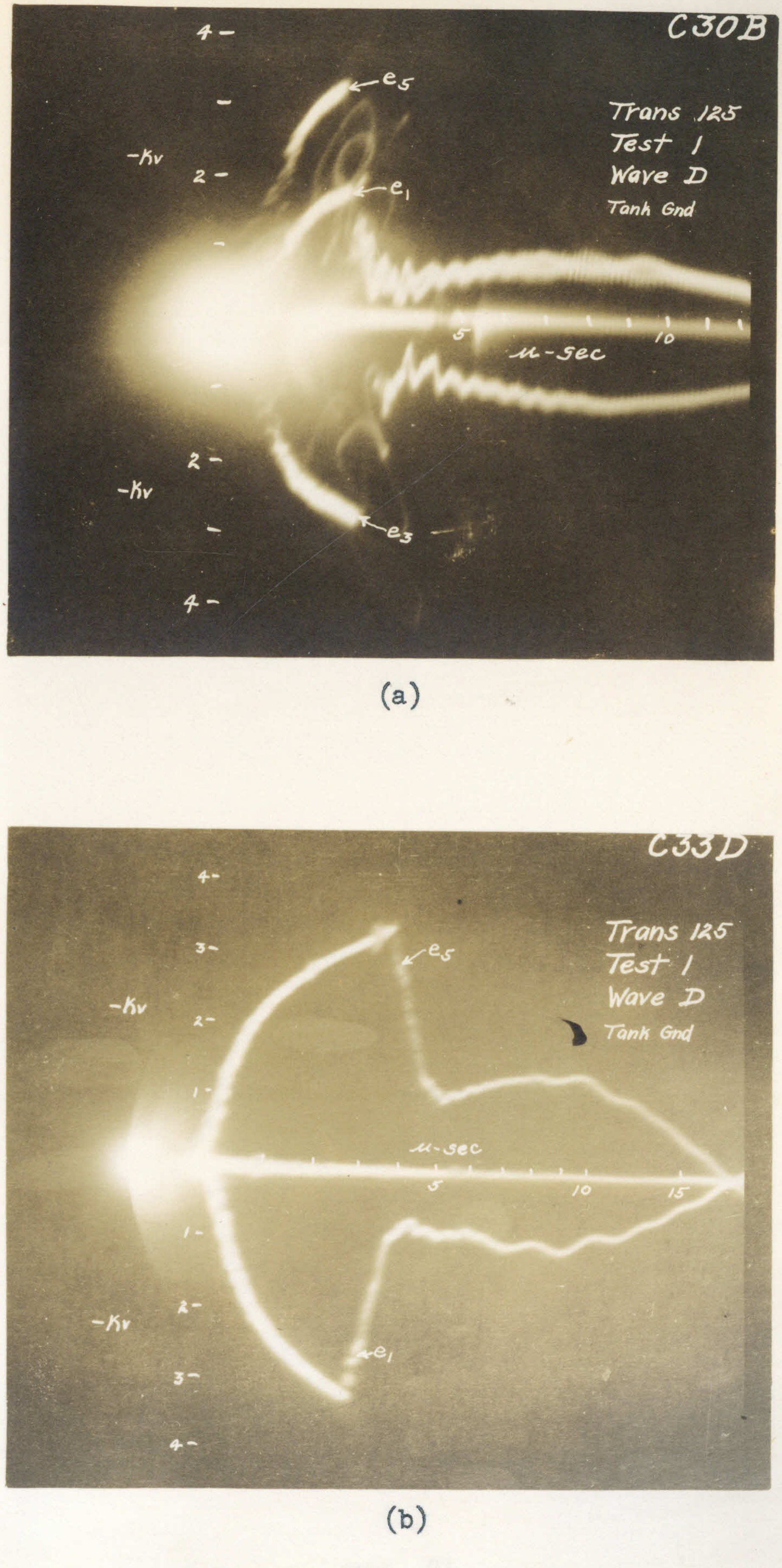

C 30A
Trans 125
Test 1
Wave D
Tank Gnd
-Kv
15
10
5
1
2
3
4
-Kv
e6
e6a
5
10
μ-sec

(a)


C33B
Trans 125
Test 1
Wave D
Tank Gnd
-Kv
15
10
5
1
2
3
4
-Kv
e6
e6a
5
10
15
μ-sec

(b)

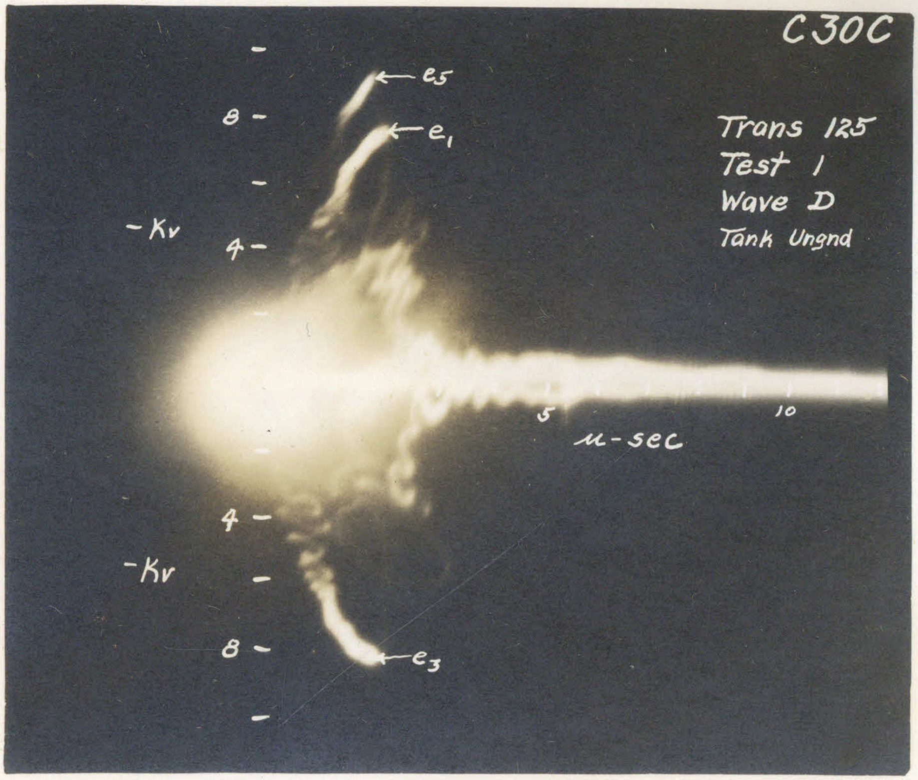
Fig. 69


C30B
4
2
-Kv
e5
e1
Trans 125
Test 1
Wave D
Tank Gnd
5
10
μ-sec
2
-Kv
e3
4
C33D
4
3
2
1
-Kv
e5
Trans 125
Test 1
Wave D
Tank Gnd
μ-sec
5
10
15
1
2
-Kv
e1
3
4

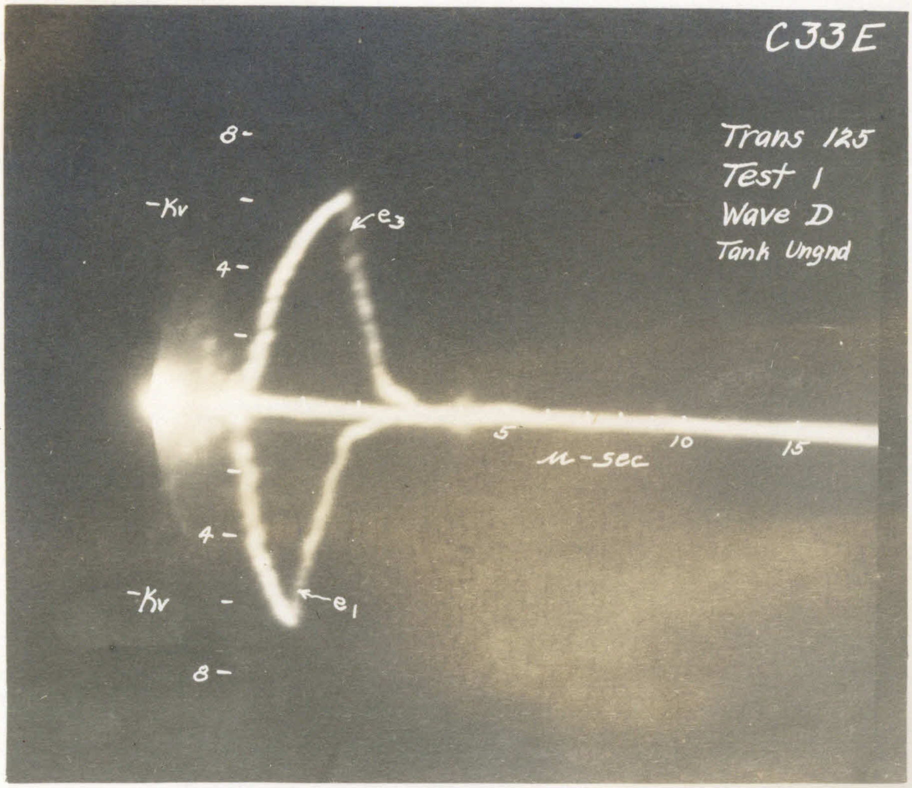
(a)

(b)

Fig. 70

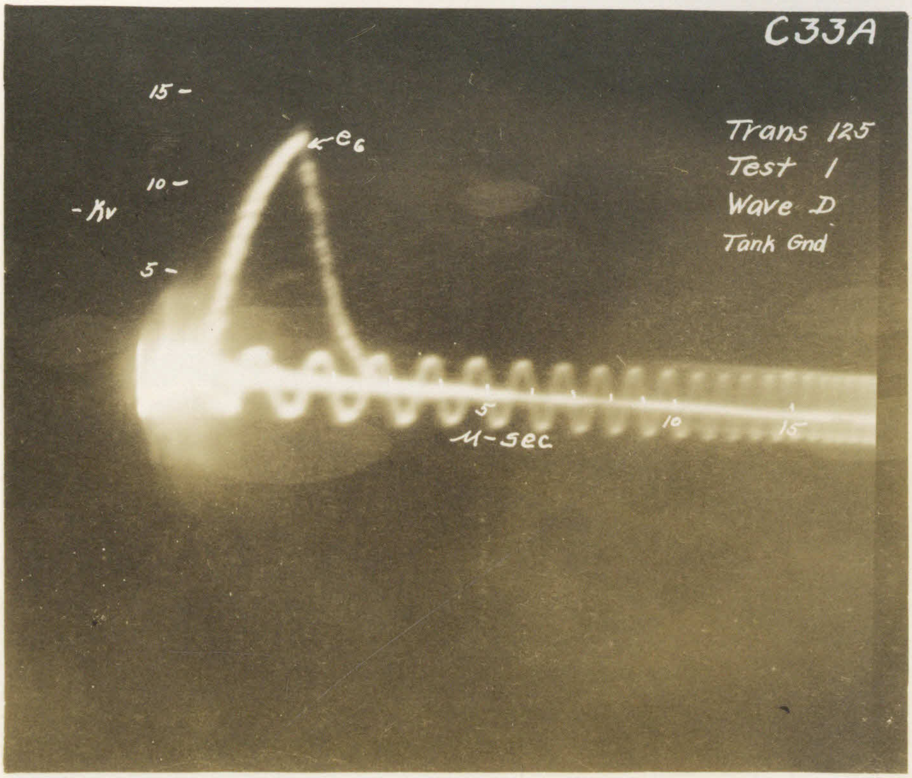

C30C
Trans 125
Test 1
Wave D
Tank Ungnd
e5
e1
e3
8
4
4
8
-Kv
-Kv
5
10
μ-sec

(a)

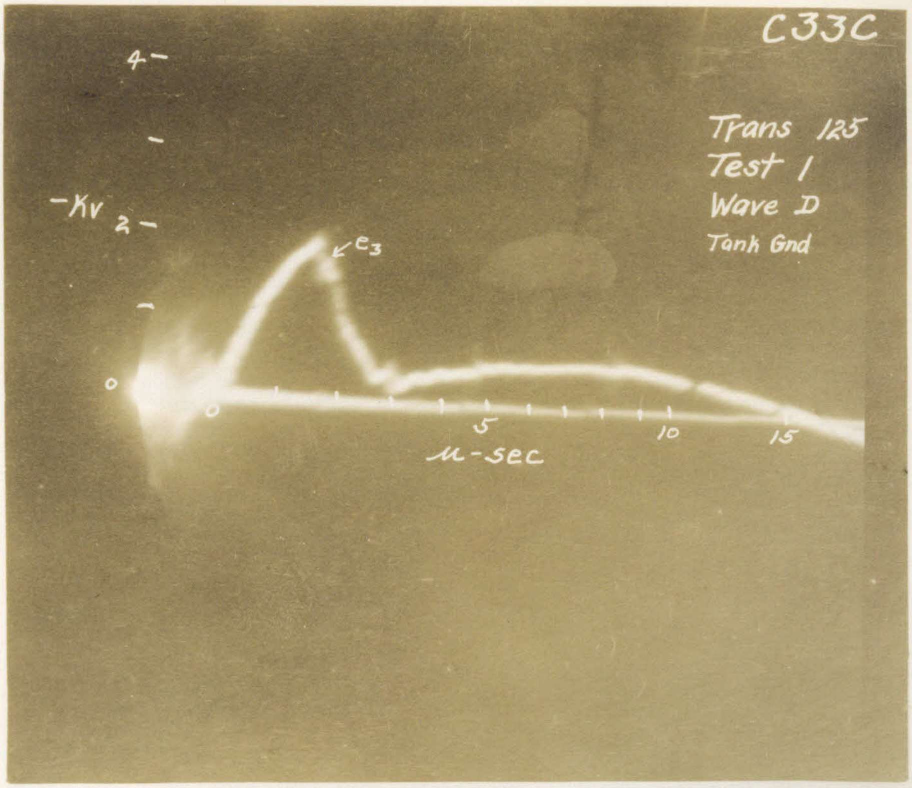

C33E
Trans 125
Test 1
Wave D
Tank Ungnd
e3
e1
8
4
4
8
-Kv
-Kv
5
10
15
μ-sec

(b)

Fig. 71

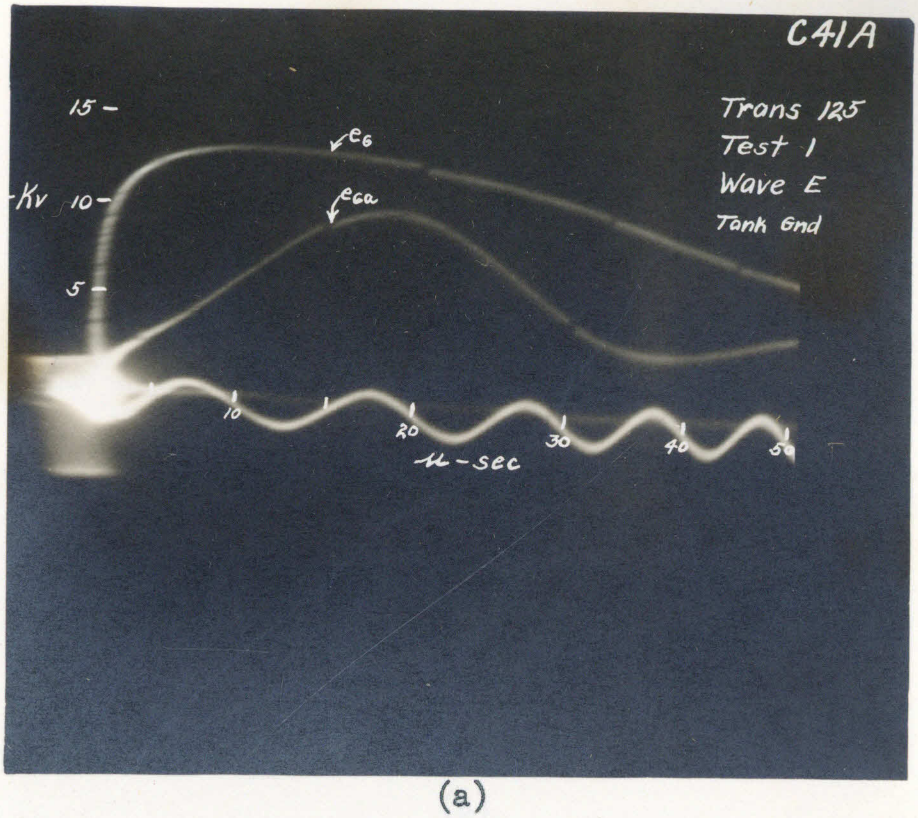

C33A
15 -
10 -
5 -
- Kv
e6
Trans 125
Test 1
Wave D
Tank Gnd
5
10
15
μ-sec

(a)

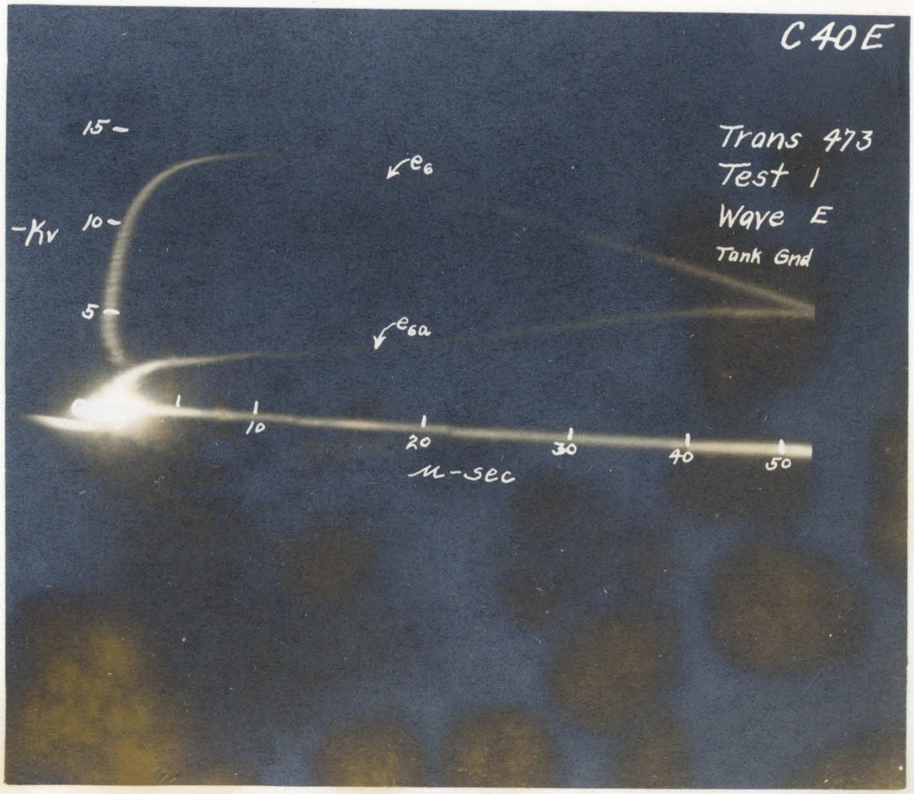

C33C
4 -
-Kv 2 -
0
e3
Trans 125
Test 1
Wave D
Tank Gnd
0
5
10
15
μ-sec

(b)

Fig. 72

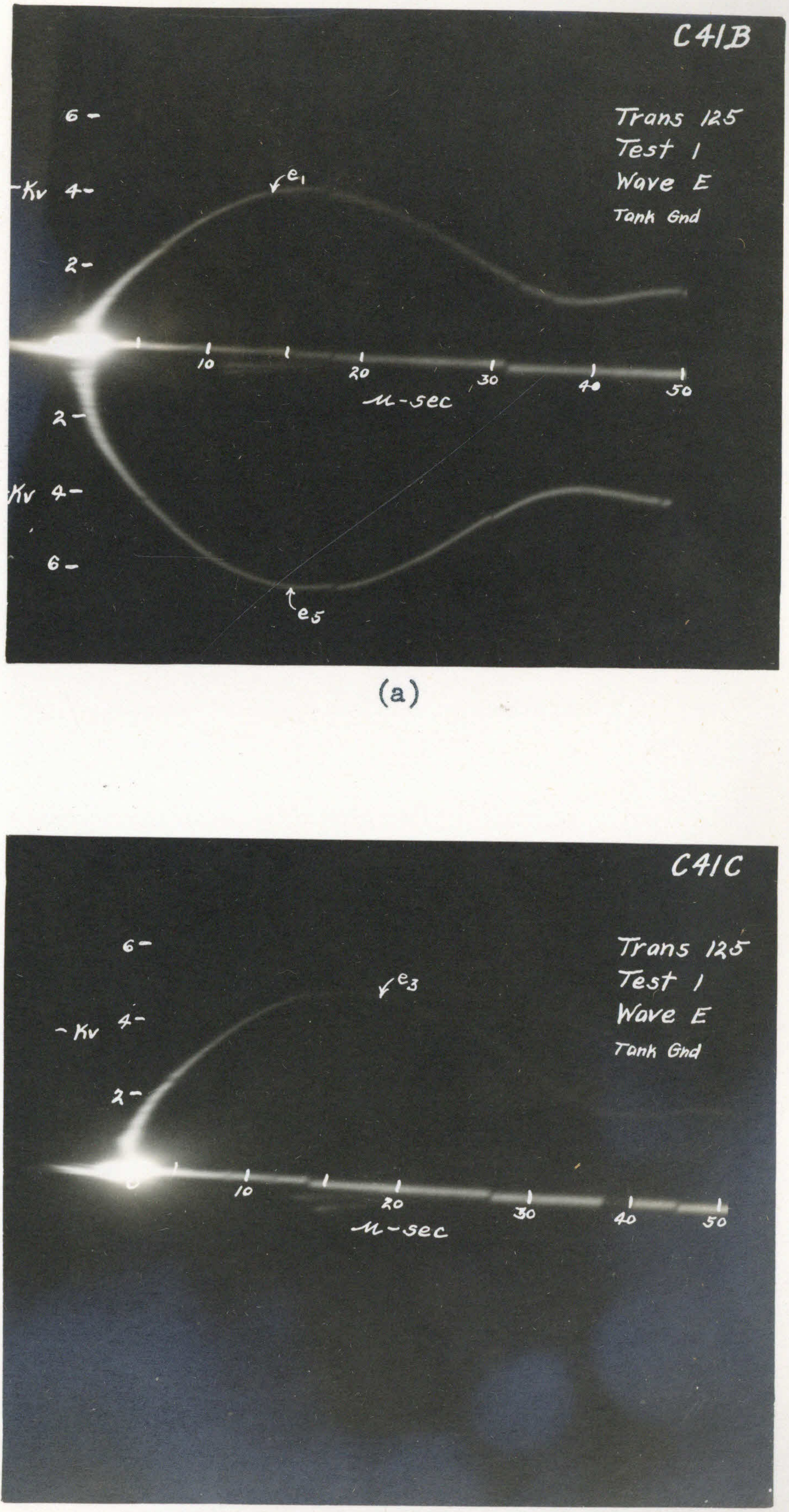

C41A
Trans 125
Test 1
Wave E
Tank Gnd
15
10
5
-Kv
e_6
e_{6a}
10
20
30
40
50
μ-sec

(a)


C40E
Trans 473
Test 1
Wave E
Tank Gnd
15
10
5
-Kv
e_6
e_{6a}
10
20
30
40
50
μ-sec

(b)

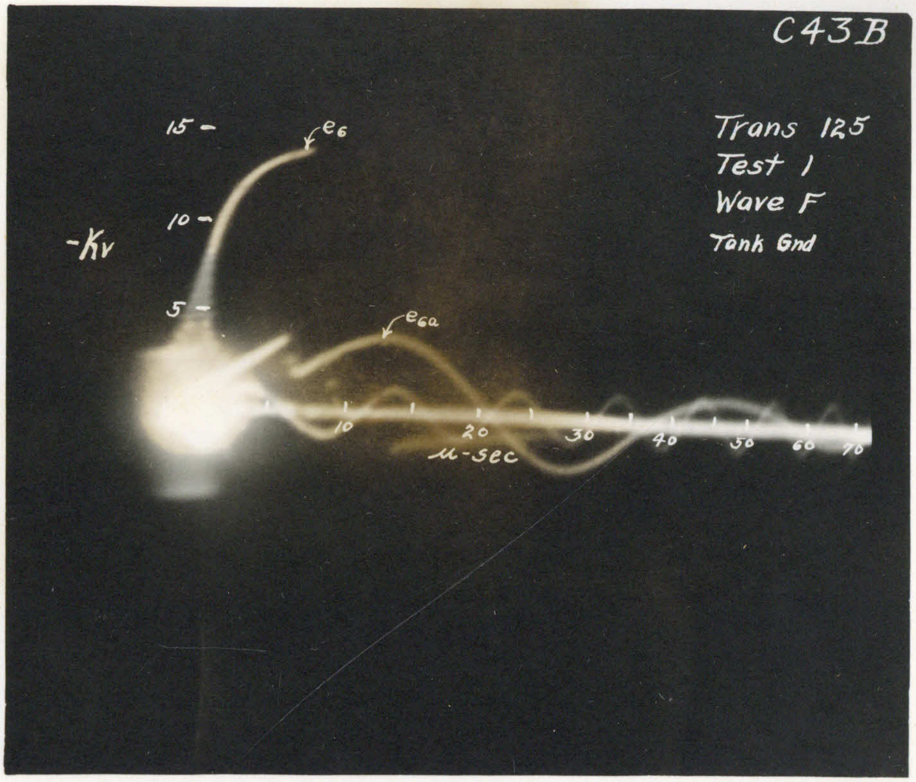
Fig. 73


C41B
Trans 125
Test 1
Wave E
Tank Gnd
6
-Kv 4
2
e_1
10
20
30
40
50
μ-sec
2
-Kv 4
6
e_5
C41C
Trans 125
Test 1
Wave E
Tank Gnd
6
e_3
-Kv 4
2
10
20
30
40
50
μ-sec

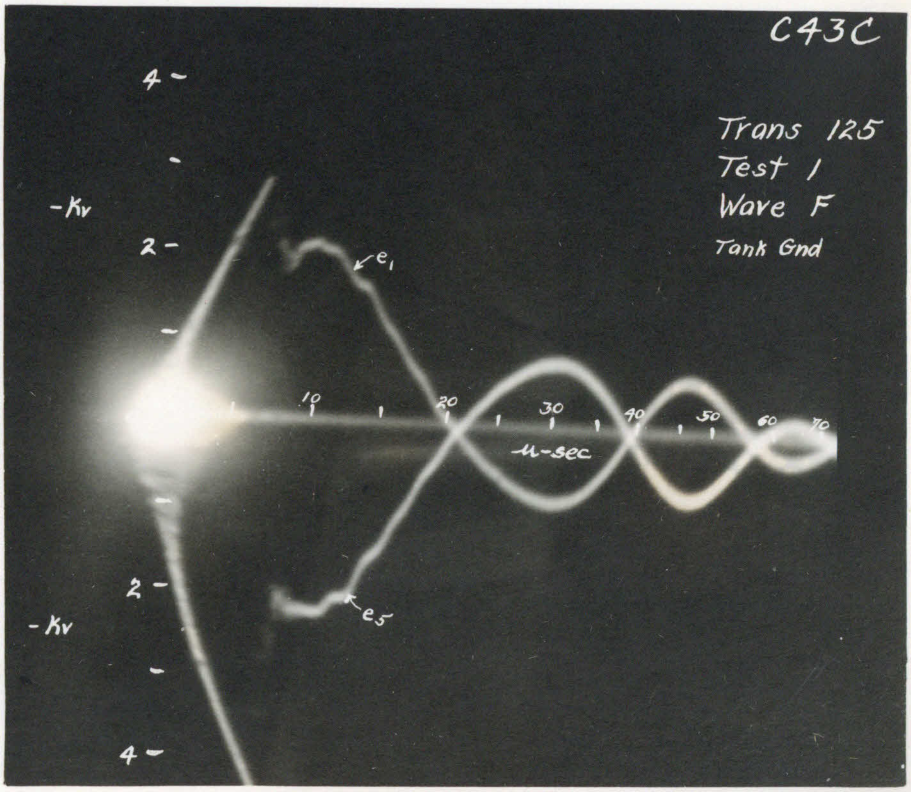
(a)

(b)

Fig. 74

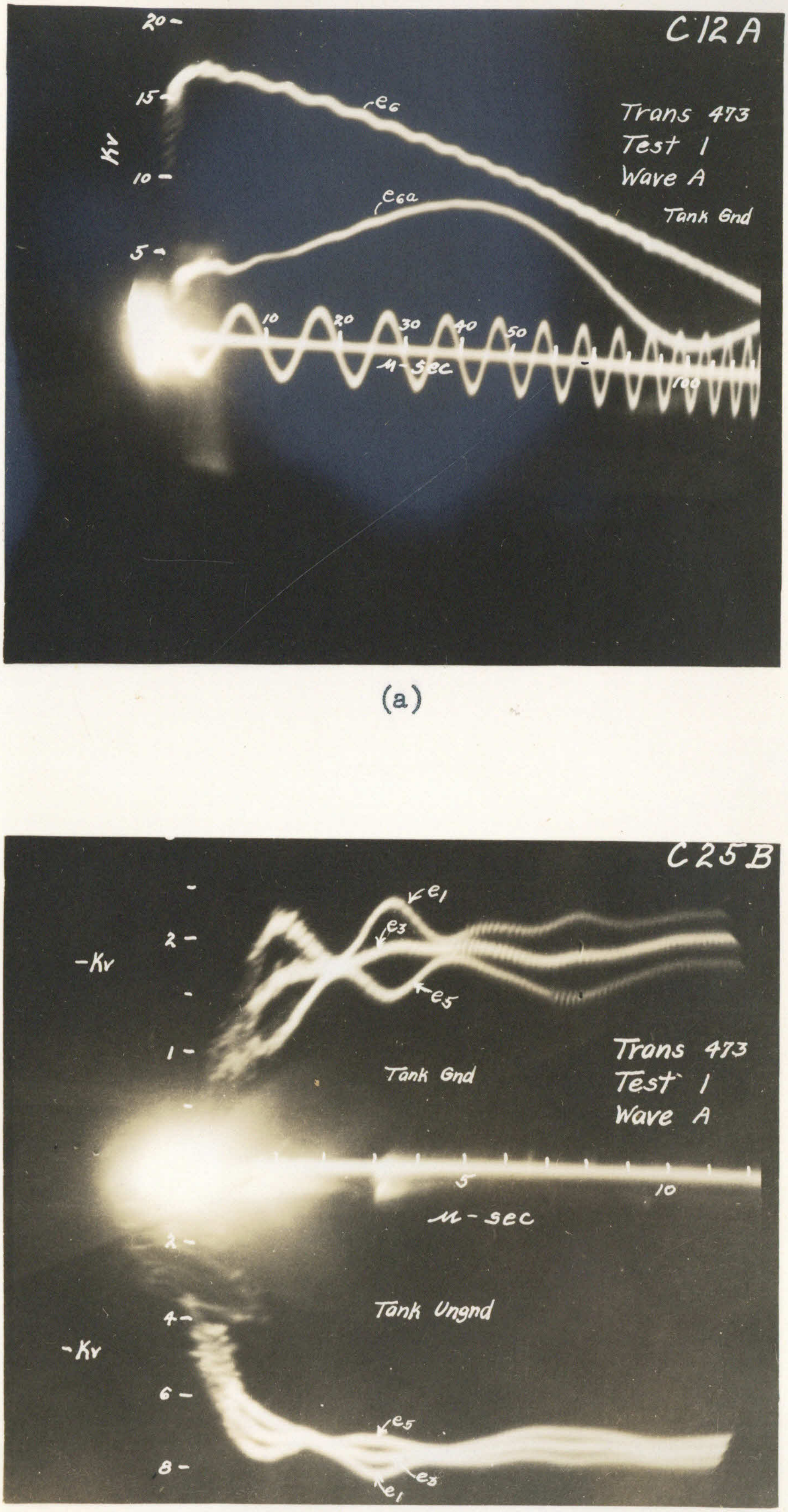

C43B
15
10
5
-Kv
e6
e6a
10
20
30
40
50
60
70
μ-sec
Trans 125
Test 1
Wave F
Tank Gnd

(a)


C43C
4
2
-Kv
e1
10
20
30
40
50
60
70
μ-sec
2
-Kv
e5
4
Trans 125
Test 1
Wave F
Tank Gnd

(b)

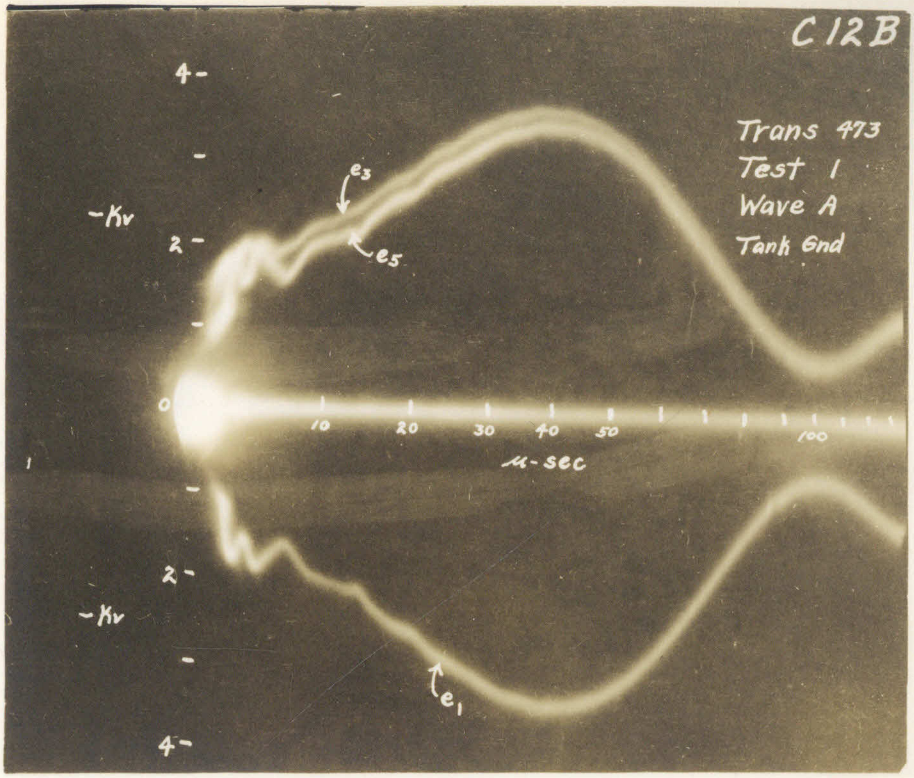
Fig. 75


C12A
20
15
10
5
Kv
e6
e6a
Trans 473
Test 1
Wave A
Tank Gnd
10
20
30
40
50
100
μ-sec
C25B
2
1
-Kv
e1
e3
e5
Tank Gnd
Trans 473
Test 1
Wave A
5
10
μ-sec
2
4
6
8
-Kv
Tank Ungnd
e5
e3
e1

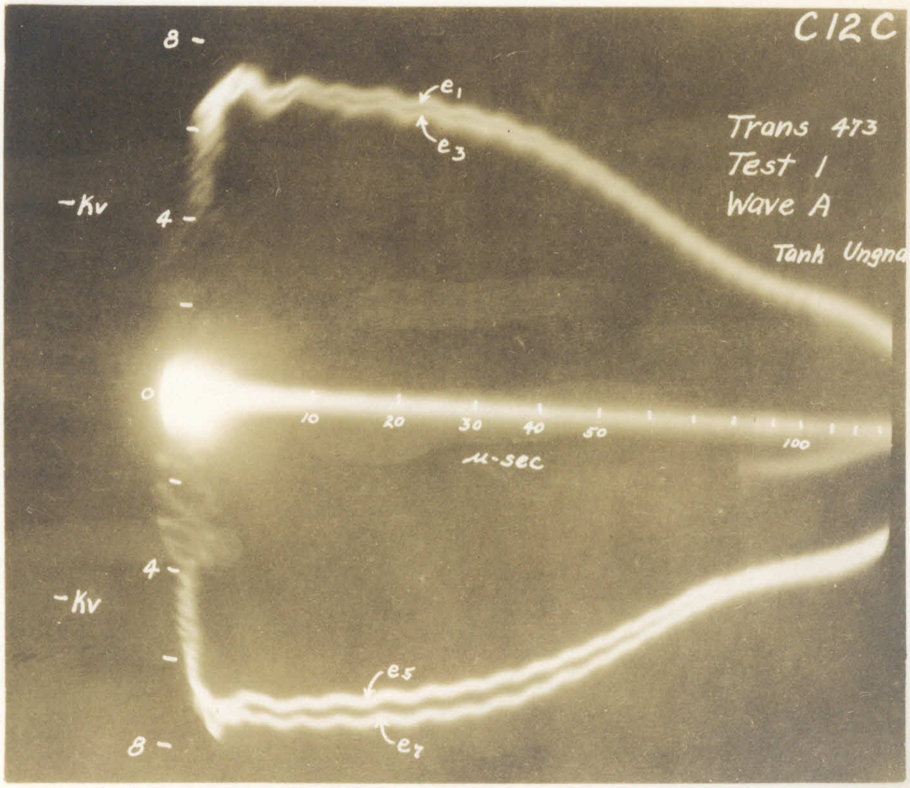
(a)

(b)

Fig. 76

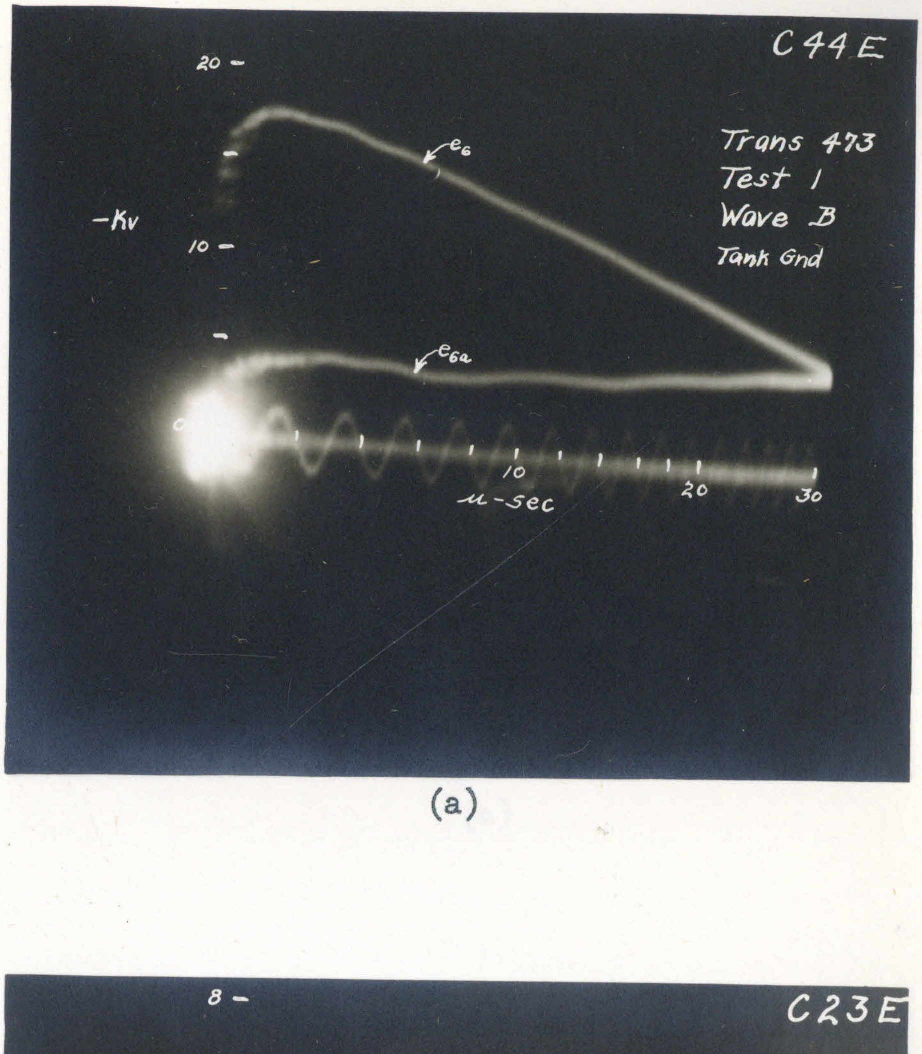

C12B
Trans 473
Test 1
Wave A
Tank Gnd
4
2
0
2
4
-Kv
-Kv
e3
e5
e1
10
20
30
40
50
100
μ-sec

(a)


C12C
Trans 473
Test 1
Wave A
Tank Ungn
8
4
0
4
8
-Kv
-Kv
e1
e3
e5
e7
10
20
30
40
50
100
μ-sec

(b)

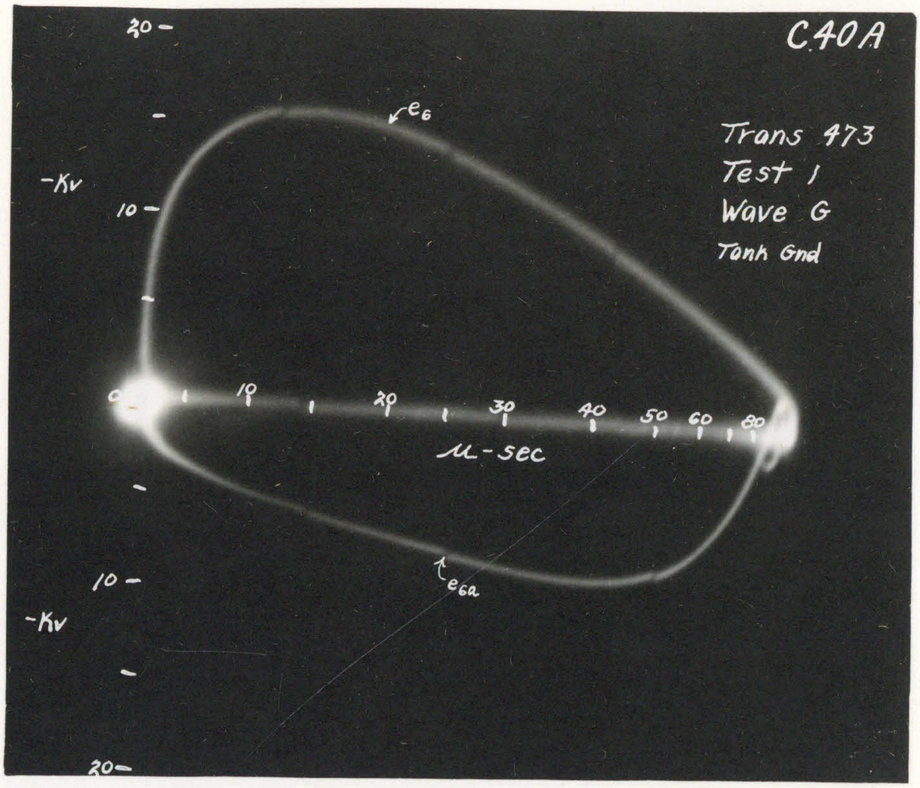
Fig. 77


C44E
20
10
-Kv
0
e6
e6a
Trans 473
Test 1
Wave B
Tank Gnd
10
20
30
u-sec
C23E
8
6
4
2
-Kv
e1
e3
e5
Trans EE1129
Test 1
Wave A
Tank Gnd
5
10
u-sec
Tank Ungnd
4
6
8
10
-Kv
e5
e3
e1

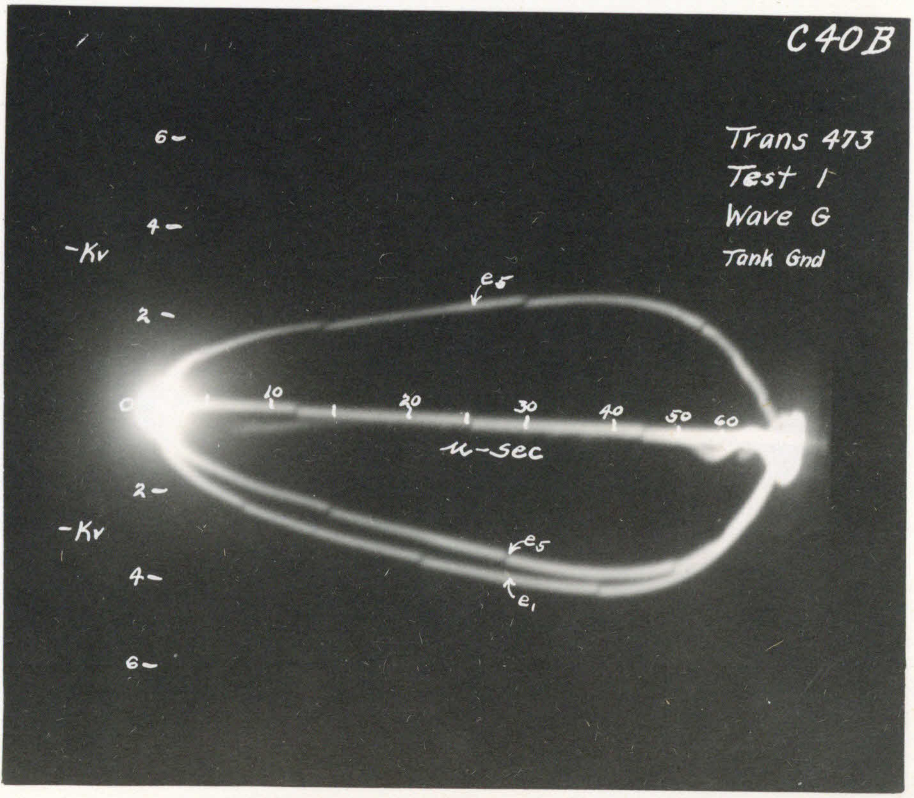
(a)

(b)

Fig. 78

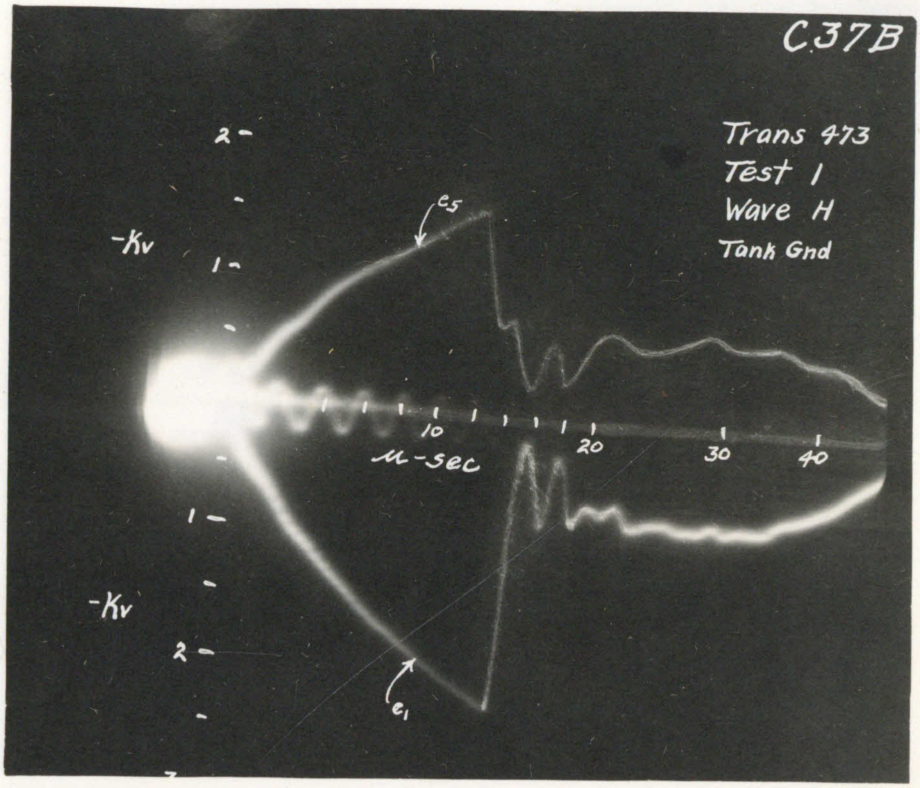

C40A
Trans 473
Test 1
Wave G
Tank Gnd
20
e_6
-Kv
10
0
10
20
30
40
50
60
80
μ-sec
e_{6a}
10
-Kv
20

(a)

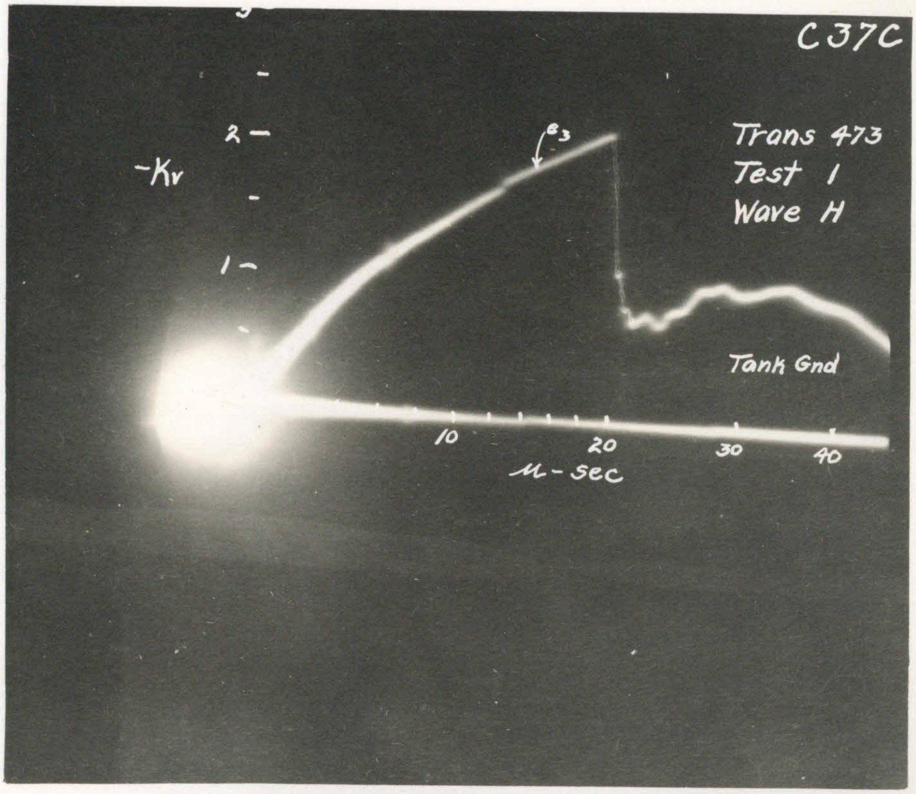

C40B
Trans 473
Test 1
Wave G
Tank Gnd
6
4
-Kv
e_5
2
0
10
20
30
40
50
60
μ-sec
2
-Kv
e_5
4
e_1
6

(b)

Fig. 79

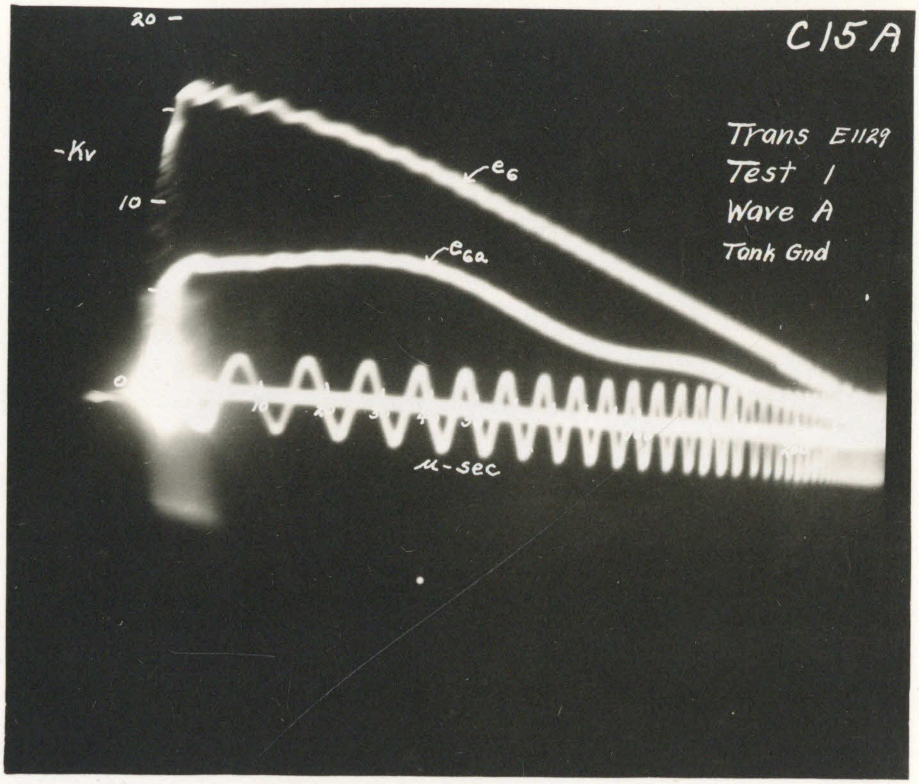

C37B
Trans 473
Test 1
Wave H
Tank Gnd
-Kv
2
1
1
-Kv
2
e5
e1
10
20
30
40
μ-sec

(a)

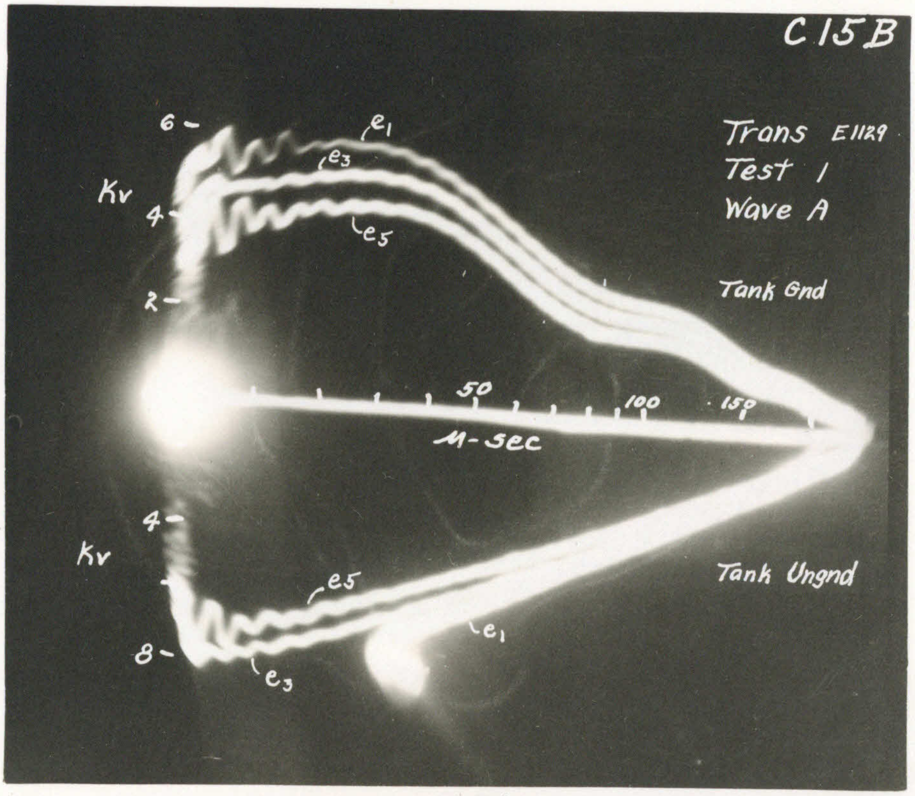

C37C
Trans 473
Test 1
Wave H
Tank Gnd
-Kv
2
1
e3
10
20
30
40
μ-sec

(b)

Fig. 80

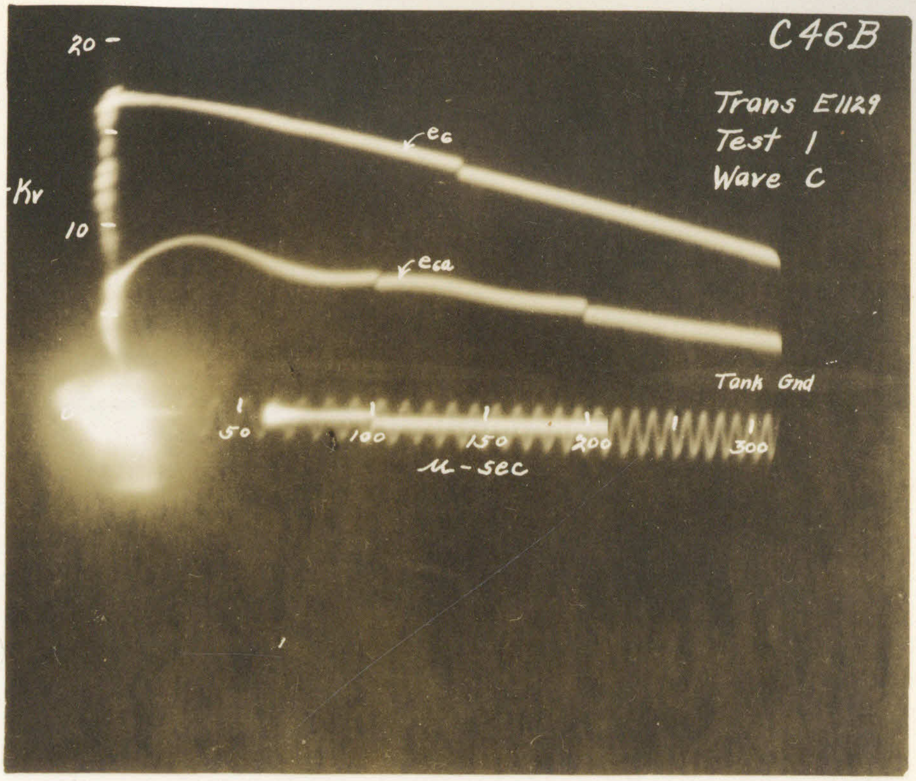

C15A
20
-Kv
10
0
e6
e6a
μ-sec
Trans E1129
Test 1
Wave A
Tank Gnd

(a)

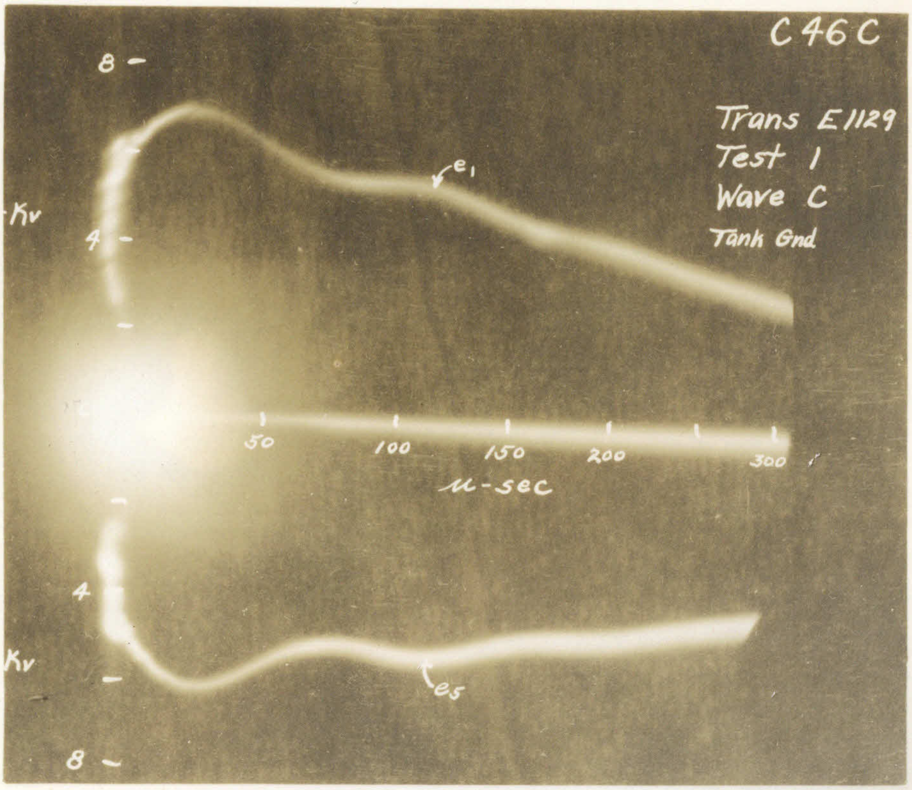

C15B
6
Kv
4
2
e1
e3
e5
50
100
150
μ-sec
4
Kv
8
e5
e1
e3
Trans E1129
Test 1
Wave A
Tank Gnd
Tank Ungnd

(b)

Fig. 81

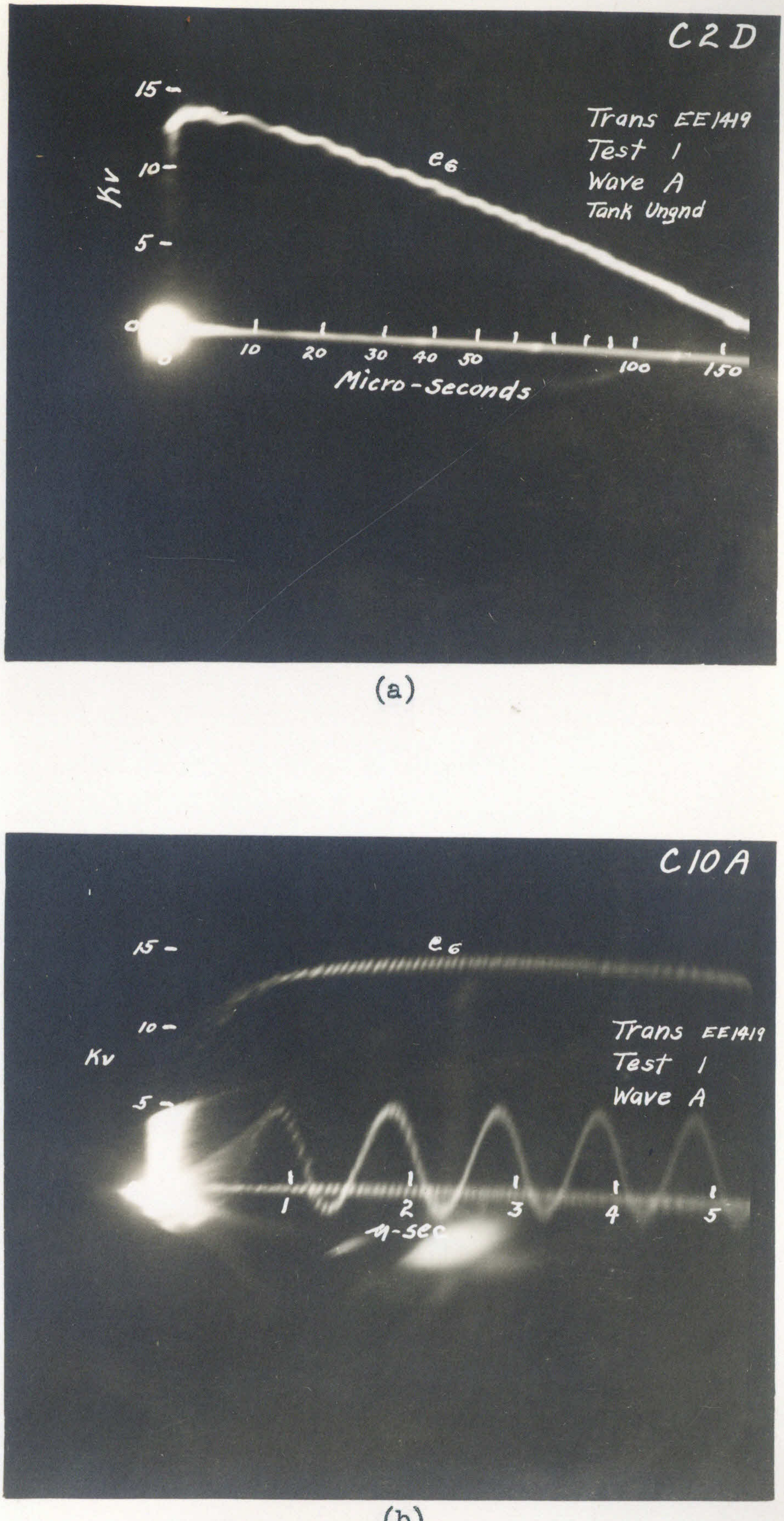

C46B
Trans E1129
Test 1
Wave C
20
10
Kv
e6
e6a
Tank Gnd
50
100
150
200
300
u-sec

(a)


C46C
Trans E1129
Test 1
Wave C
Tank Gnd
8
4
Kv
e1
50
100
150
200
300
u-sec
4
Kv
e5
8

(b)

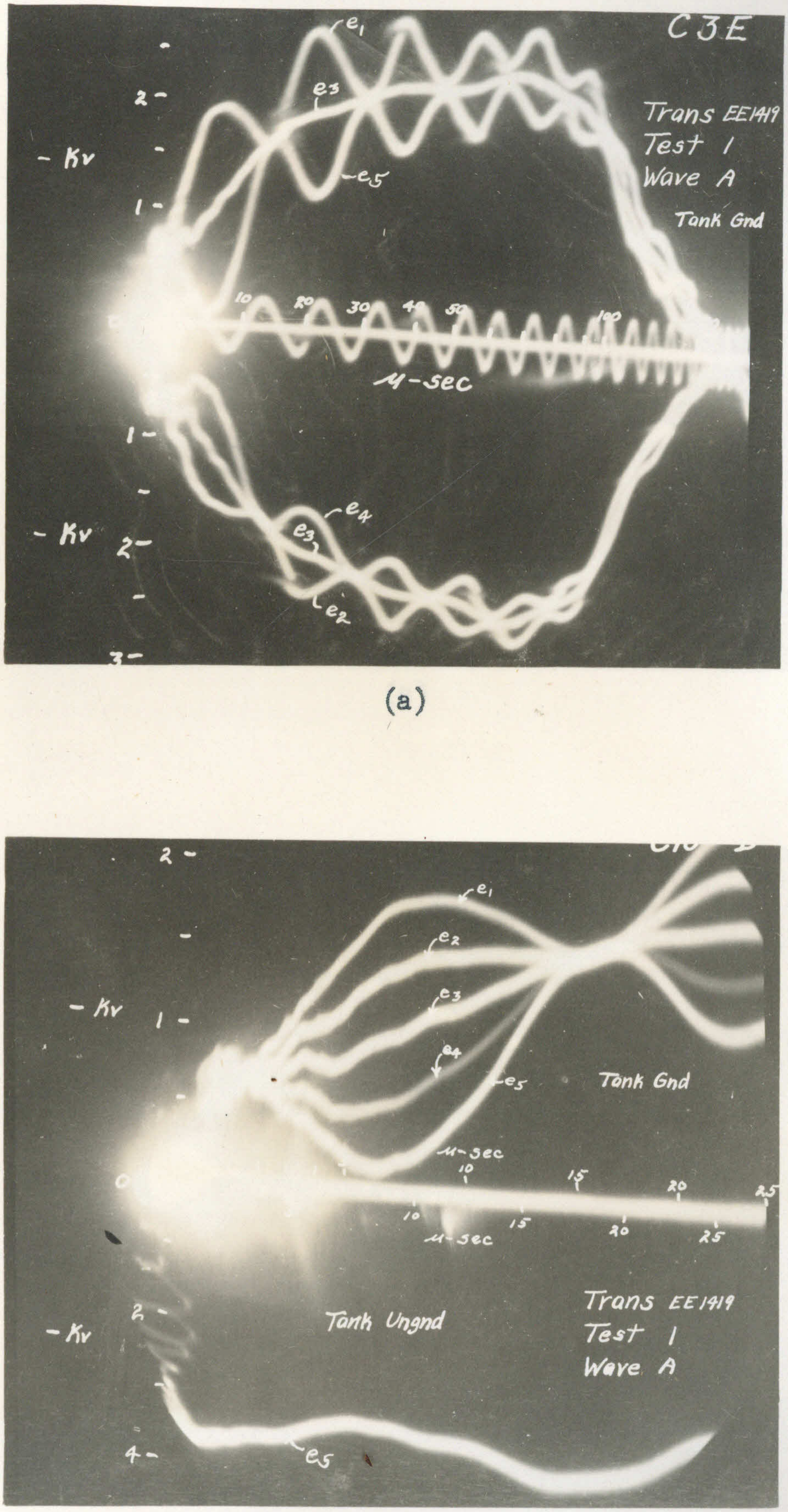
Fig. 82


C2D
15
10
5
0
Kv
e6
Trans EE1419
Test 1
Wave A
Tank Ungnd
0
10
20
30
40
50
100
150
Micro-Seconds
C10A
e6
15
10
5
Kv
Trans EE1419
Test 1
Wave A
1
2
3
4
5
m-sec

(a)

(b)

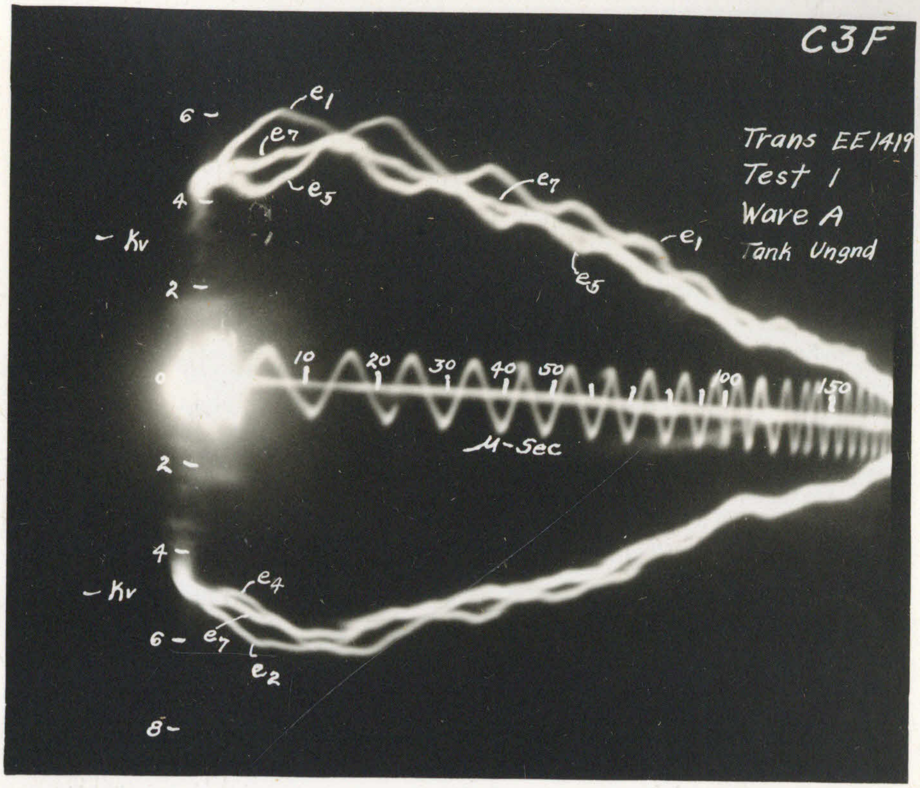
Fig. 83


C3E
Trans EE1419
Test 1
Wave A
Tank Gnd
- Kv
2
1
0
10
20
30
40
50
100
μ-sec
e1
e3
e5
e4
e2
- Kv
1
2
3
Tank Gnd
Tank Ungnd
Trans EE1419
Test 1
Wave A
e1
e2
e3
e4
e5
μ-sec
10
15
20
25
- Kv
2
1
0
4

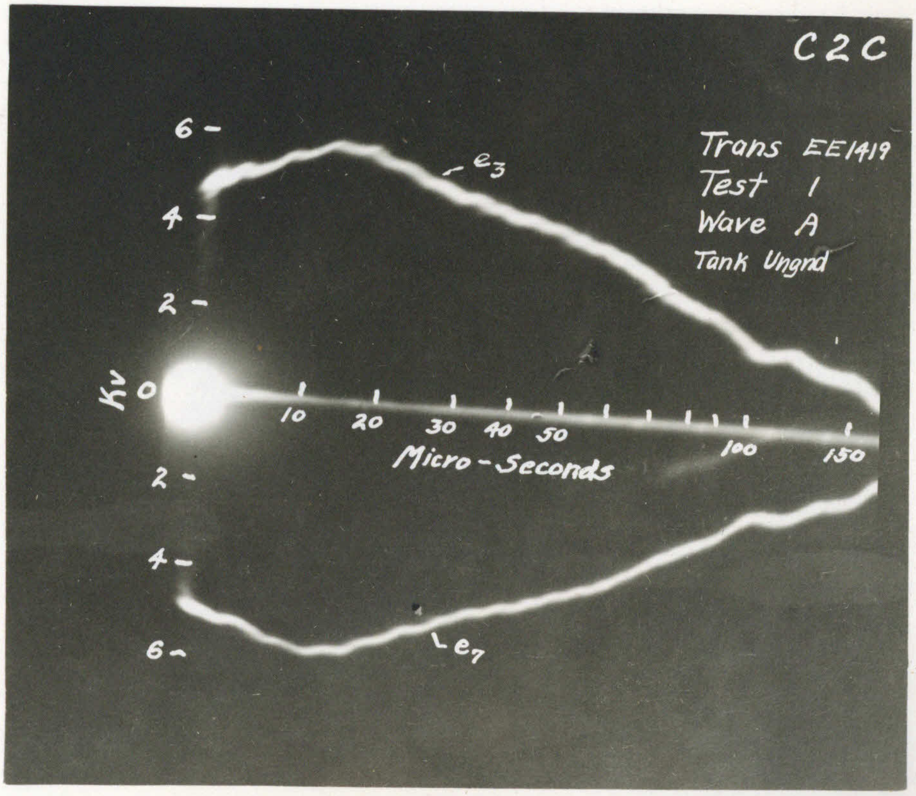
(a)

(b)

Fig. 84

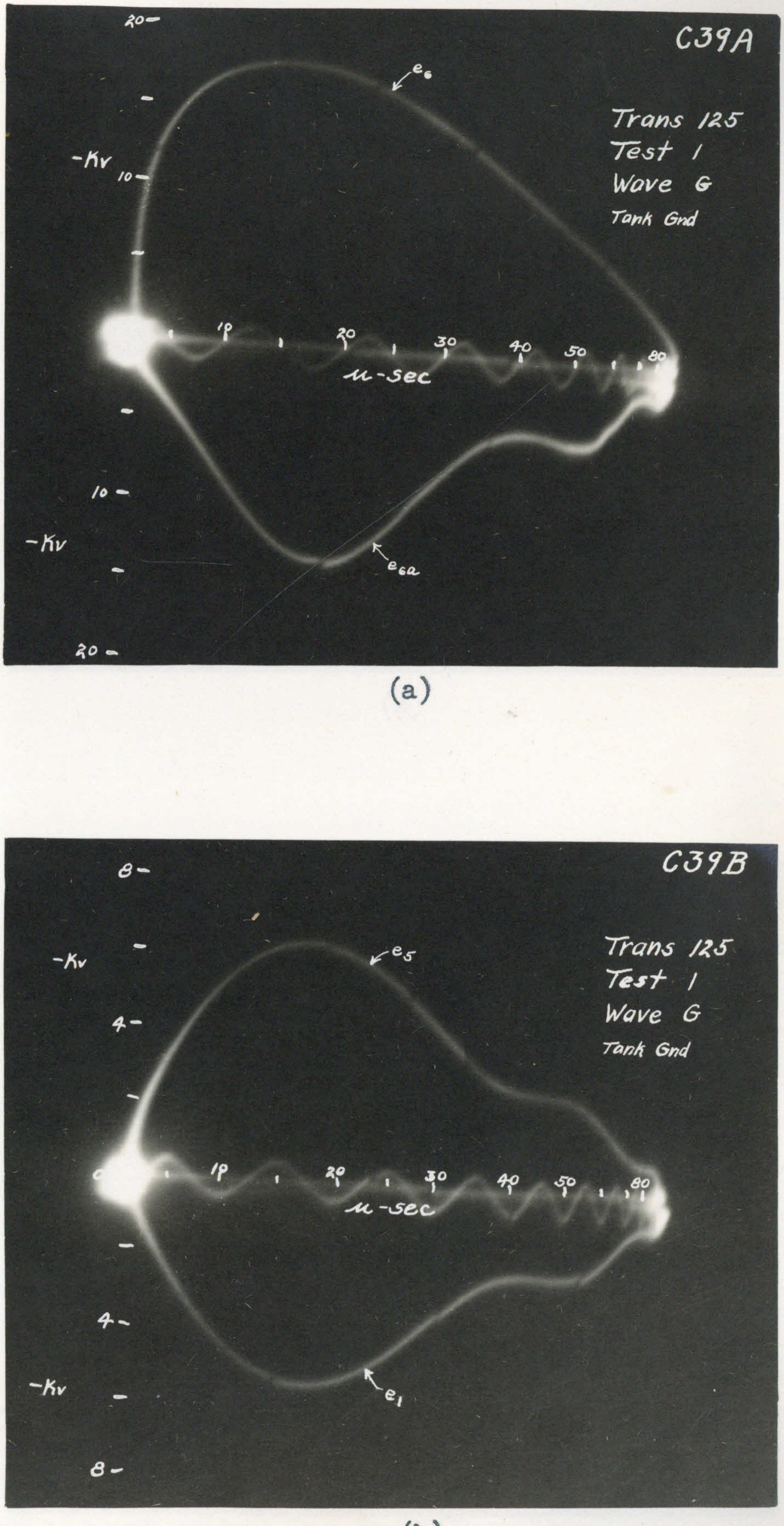

C3F
Trans EE1419
Test 1
Wave A
Tank Ungnd
6
4
Kv
2
0
2
4
Kv
6
8
e_1
e_7
e_5
e_7
e_1
e_5
e_4
e_7
e_2
10
20
30
40
50
100
150
μ-Sec

(a)


C2C
Trans EE1419
Test 1
Wave A
Tank Ungnd
6
4
2
Kv
0
2
4
6
e_3
e_7
10
20
30
40
50
100
150
Micro-Seconds

(b)

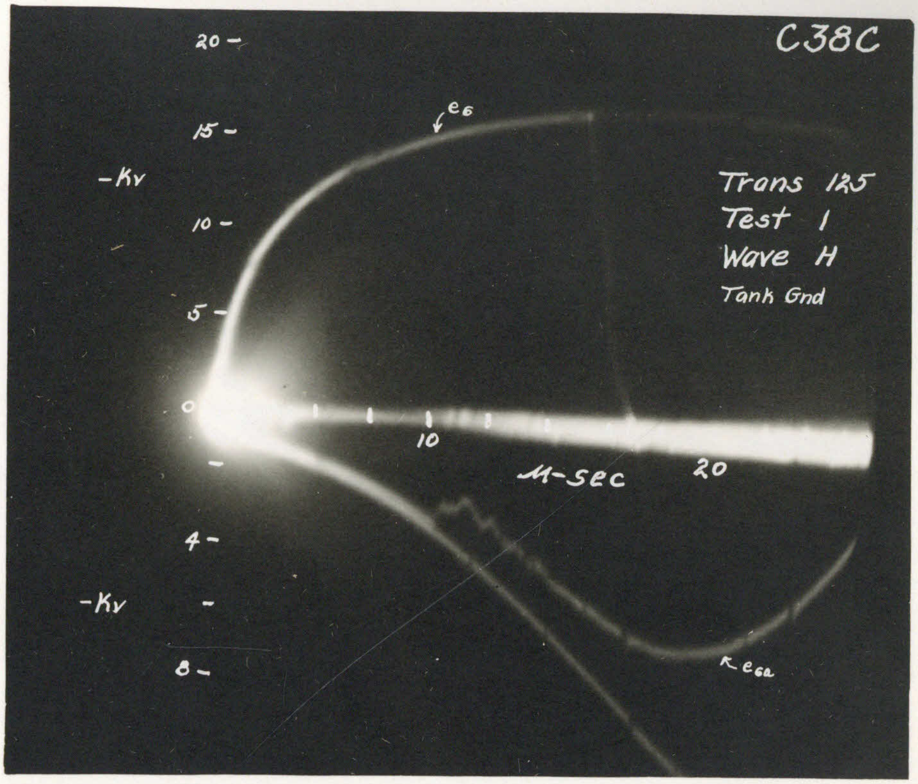
Fig. 85


C39A
Trans 125
Test 1
Wave G
Tank Gnd
20
-Kv
10
10
-Kv
20
e_6
e_{6a}
10
20
30
40
50
80
μ-sec
C39B
Trans 125
Test 1
Wave G
Tank Gnd
8
-Kv
4
4
-Kv
8
e_5
e_1
0
10
20
30
40
50
80
μ-sec

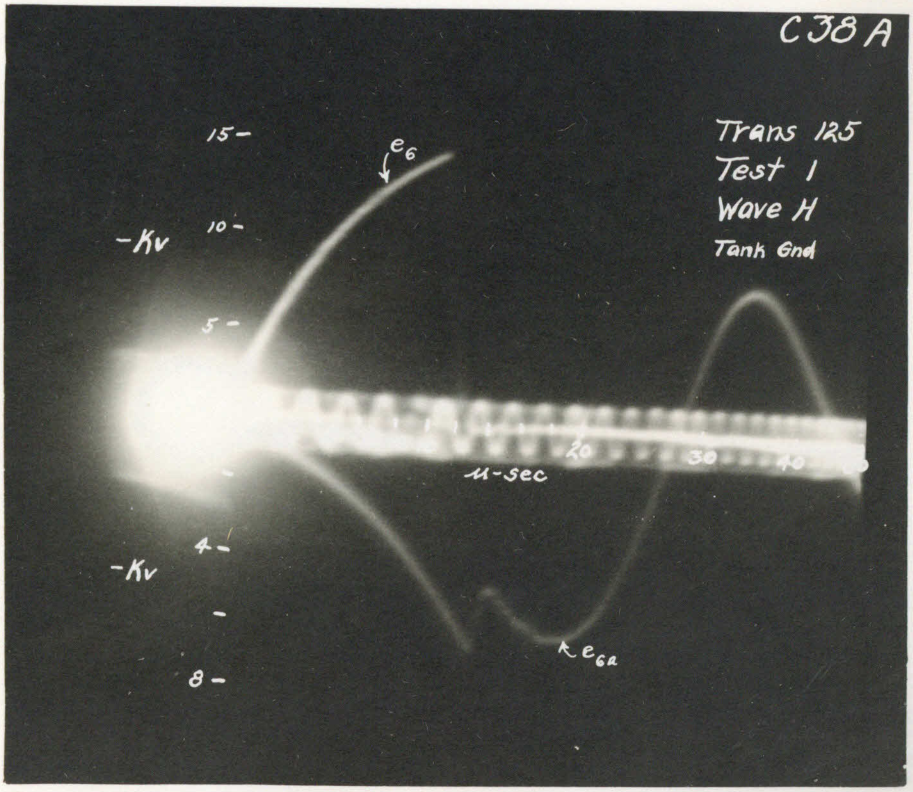
(a)

(b)

Fig. 86

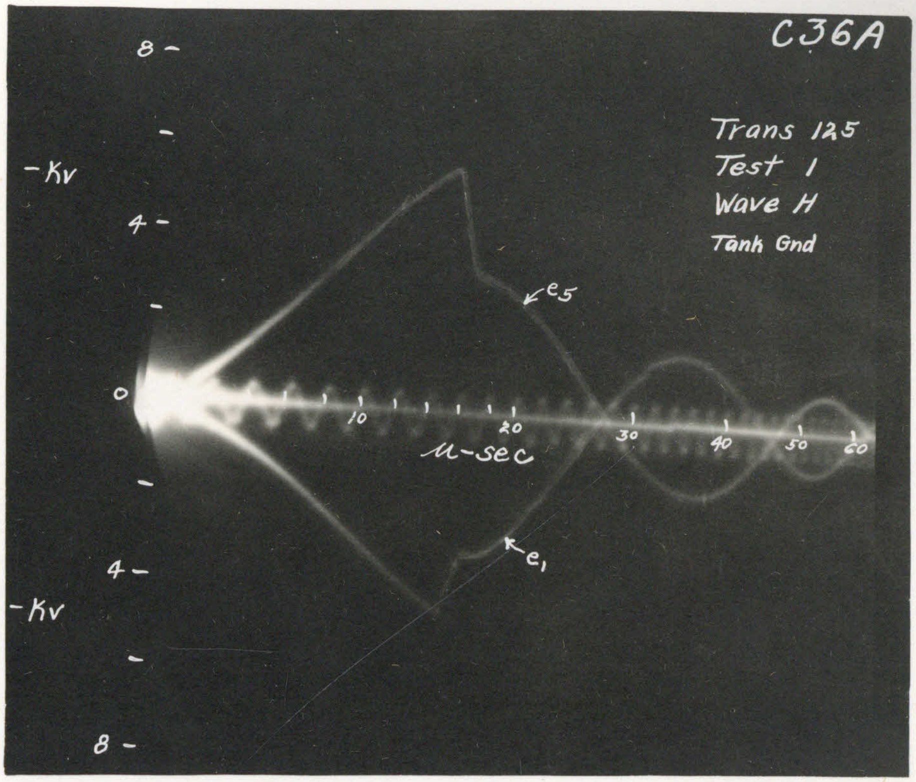

C38C
20
15
10
5
0
4
8
-Kv
-Kv
e_6
e_{6a}
10
20
μ-sec
Trans 125
Test 1
Wave H
Tank Gnd

(a)

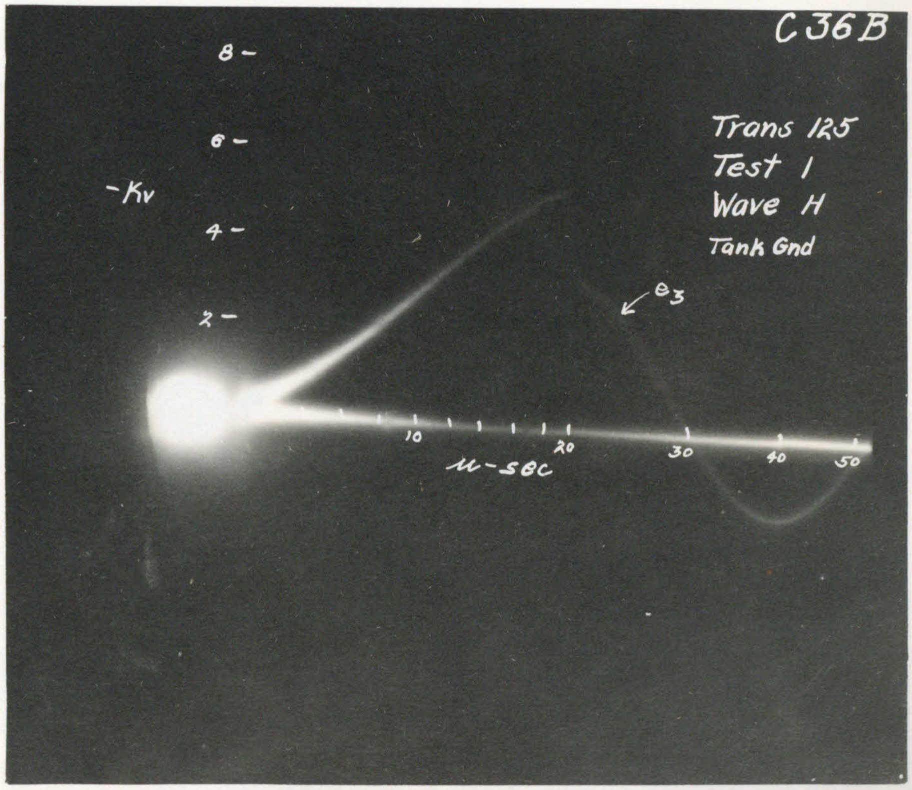

C38A
15
10
5
4
8
-Kv
-Kv
e_6
e_{6a}
20
30
40
μ-sec
Trans 125
Test 1
Wave H
Tank Gnd

(b)

Fig. 87

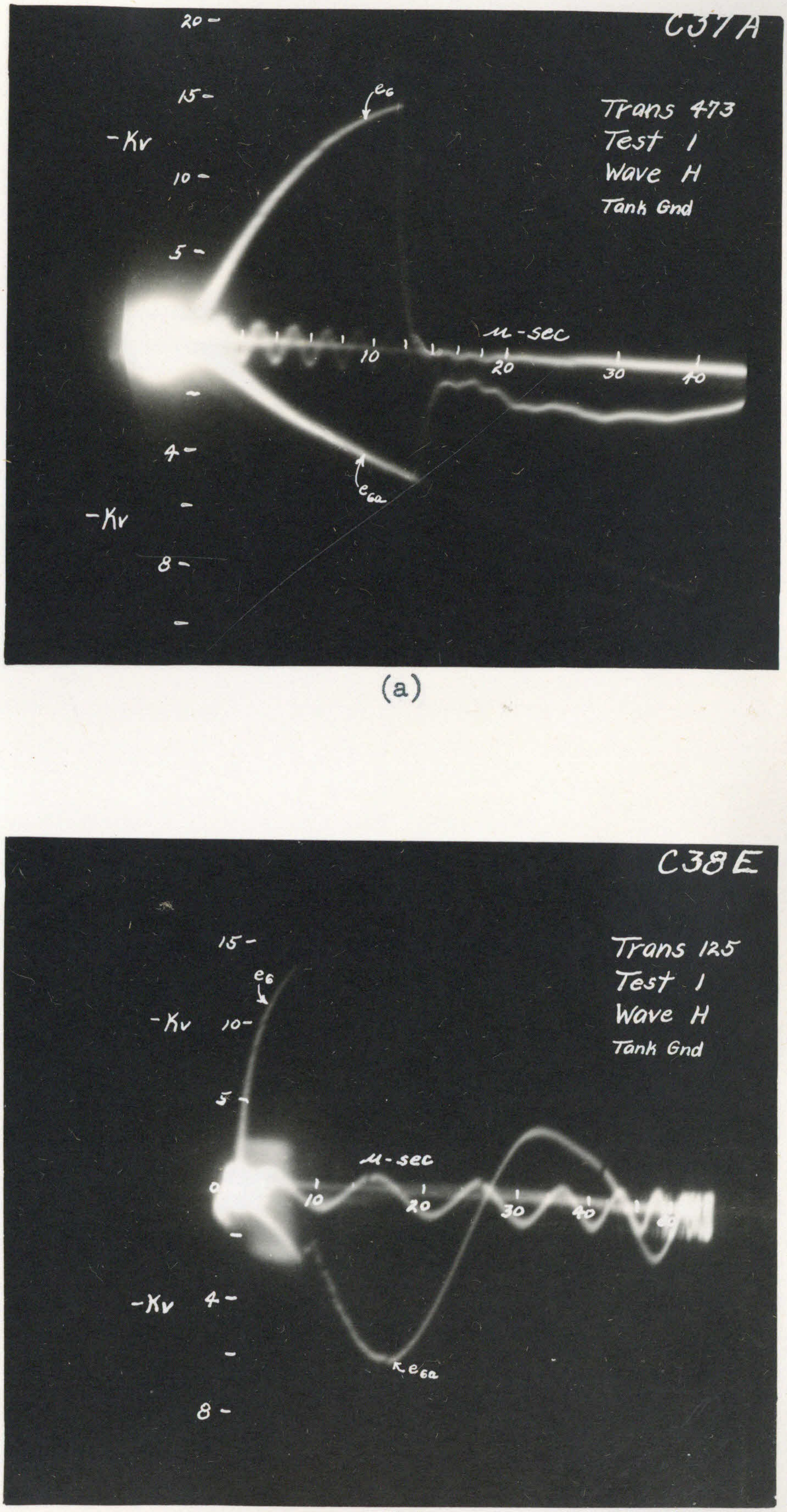

C36A
8
-Kv
4
0
4
-Kv
8
Trans 125
Test 1
Wave H
Tank Gnd
e5
e1
10
20
30
40
50
60
u-sec

(a)


C36B
8
6
-Kv
4
2
Trans 125
Test 1
Wave H
Tank Gnd
e3
10
20
30
40
50
u-sec

(b)

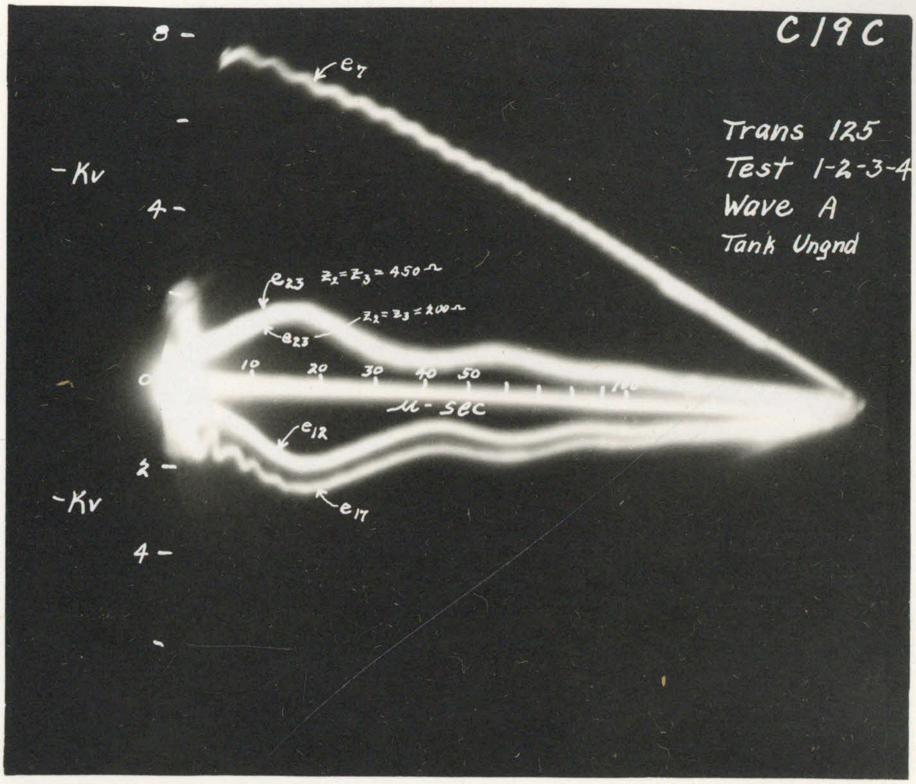
Fig. 88


C37A
Trans 473
Test 1
Wave H
Tank Gnd
-Kv
20
15
10
5
4
8
e6
e6a
μ-sec
10
20
30
40
C38E
Trans 125
Test 1
Wave H
Tank Gnd
-Kv
15
10
5
0
4
8
e6
e6a
μ-sec
10
20
30
40

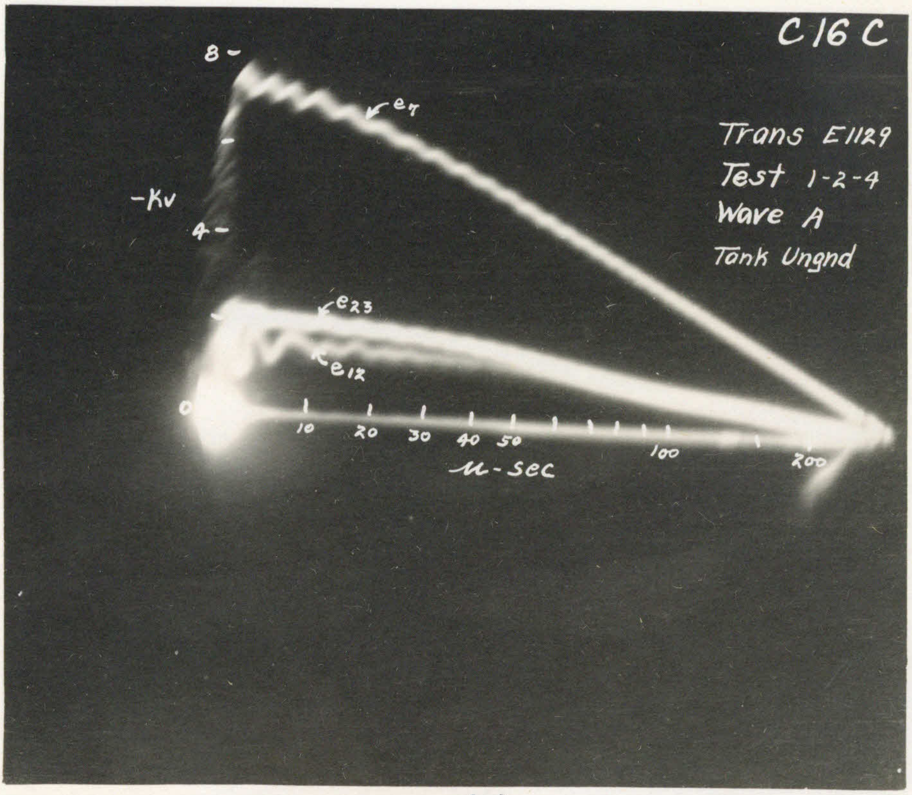
(a)

(b)

Fig. 89

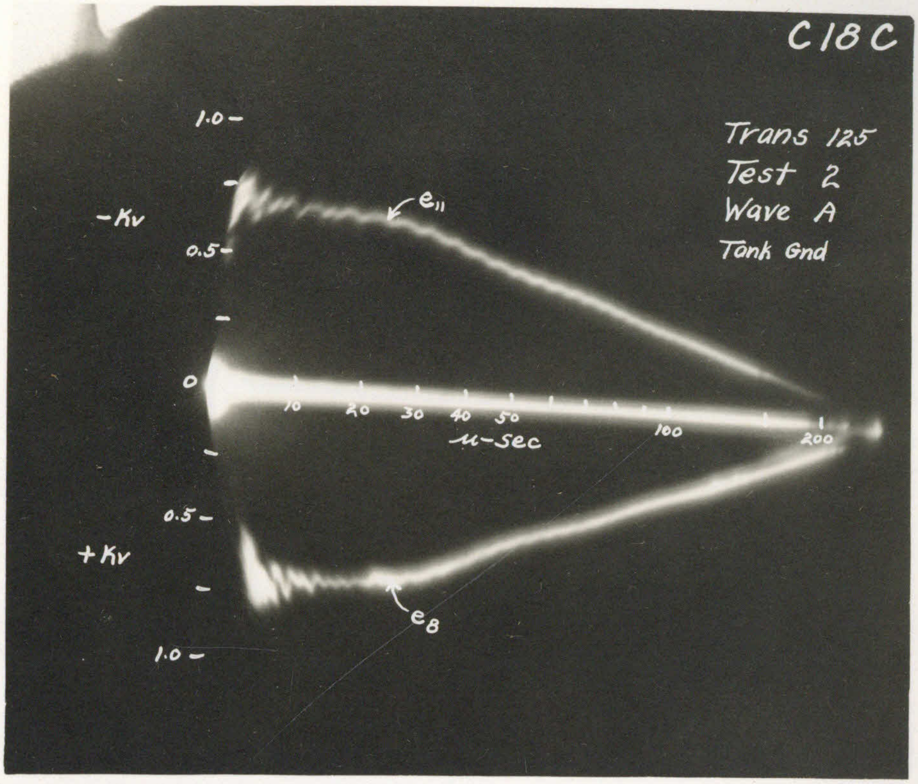

C19C
8 -
-Kv
4 -
e7
Trans 125
Test 1-2-3-4
Wave A
Tank Ungnd
e23 Z2=Z3=450Ω
Z2=Z3=200Ω
e23
0
10
20
30
40
50
100
μ-sec
e12
2 -
-Kv
e17
4 -

(a)

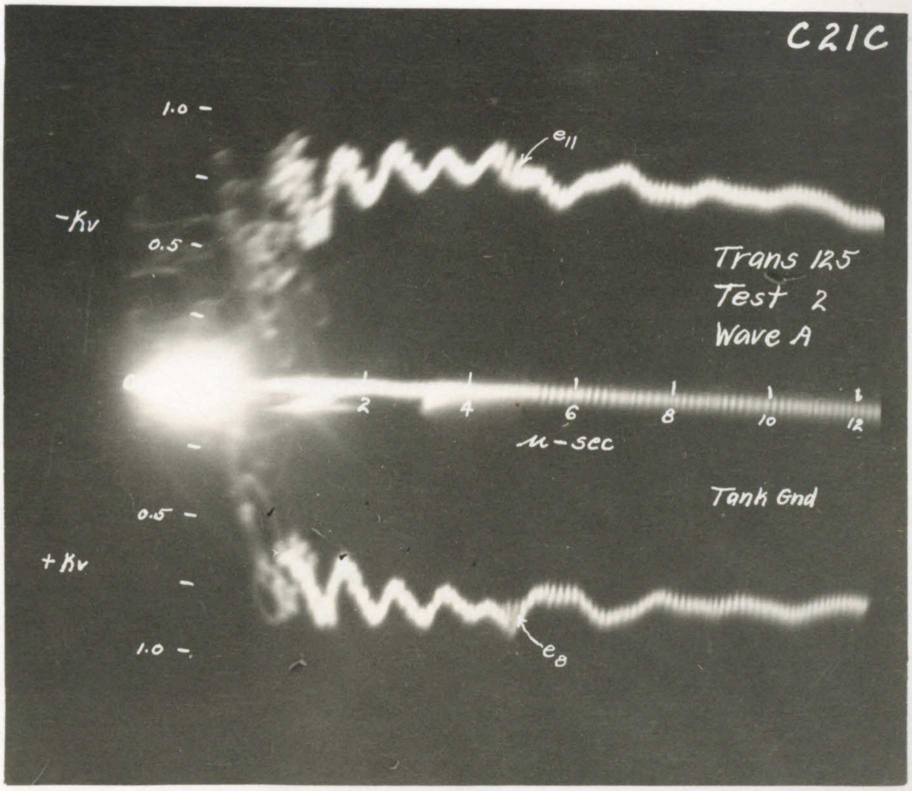

C16C
8 -
e7
-Kv
4 -
Trans E1129
Test 1-2-4
Wave A
Tank Ungnd
e23
e12
0
10
20
30
40
50
100
200
μ-sec

(b)

Fig. 90

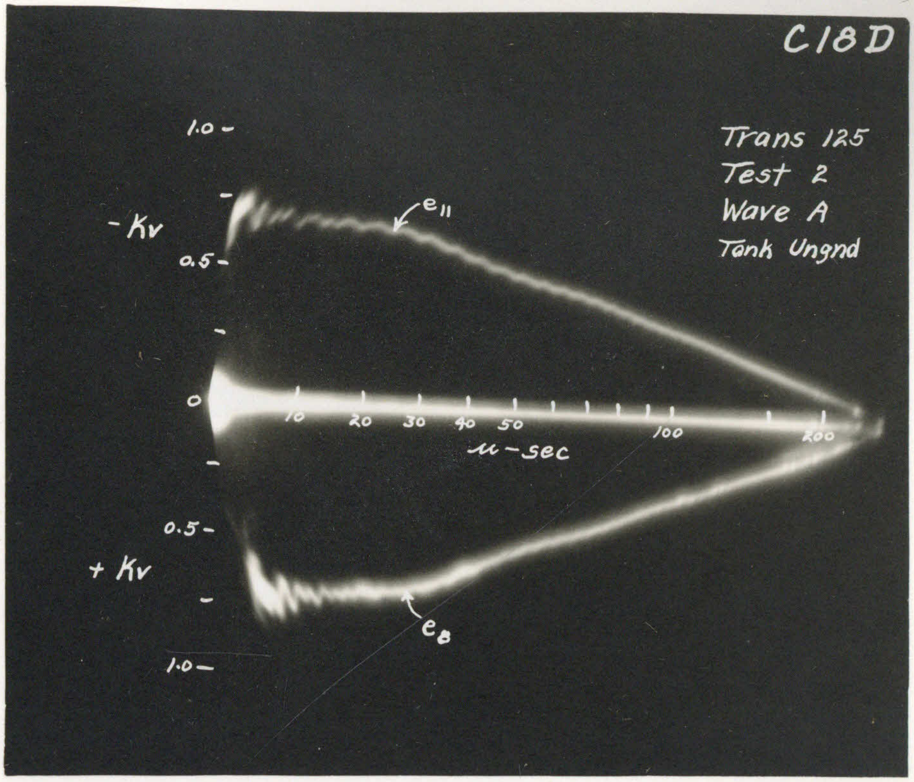

C18C
1.0
-Kv
0.5
0
10
20
30
40
50
100
200
μ-sec
0.5
+Kv
1.0
e11
e8
Trans 125
Test 2
Wave A
Tank Gnd

(a)

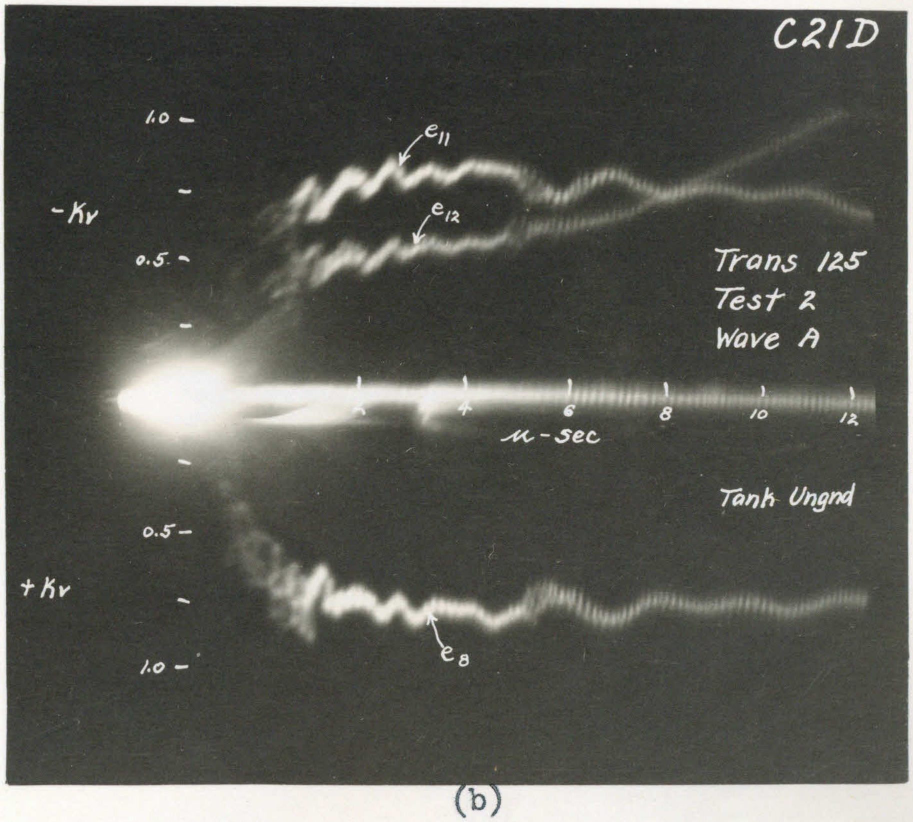

C21C
1.0
-Kv
0.5
0
2
4
6
8
10
12
μ-sec
0.5
+Kv
1.0
e11
e8
Trans 125
Test 2
Wave A
Tank Gnd

(b)

Fig. 91

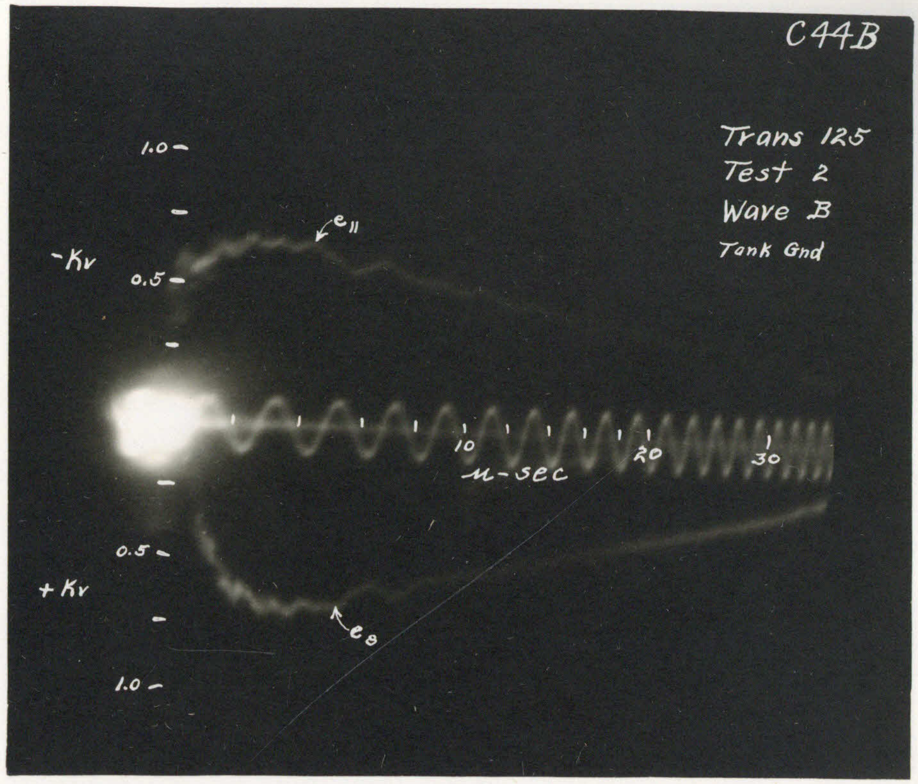

C18D
1.0
-Kv
0.5
0
+Kv
0.5
1.0
e11
e8
10
20
30
40
50
100
200
µ-sec
Trans 125
Test 2
Wave A
Tank Ungnd

(a)

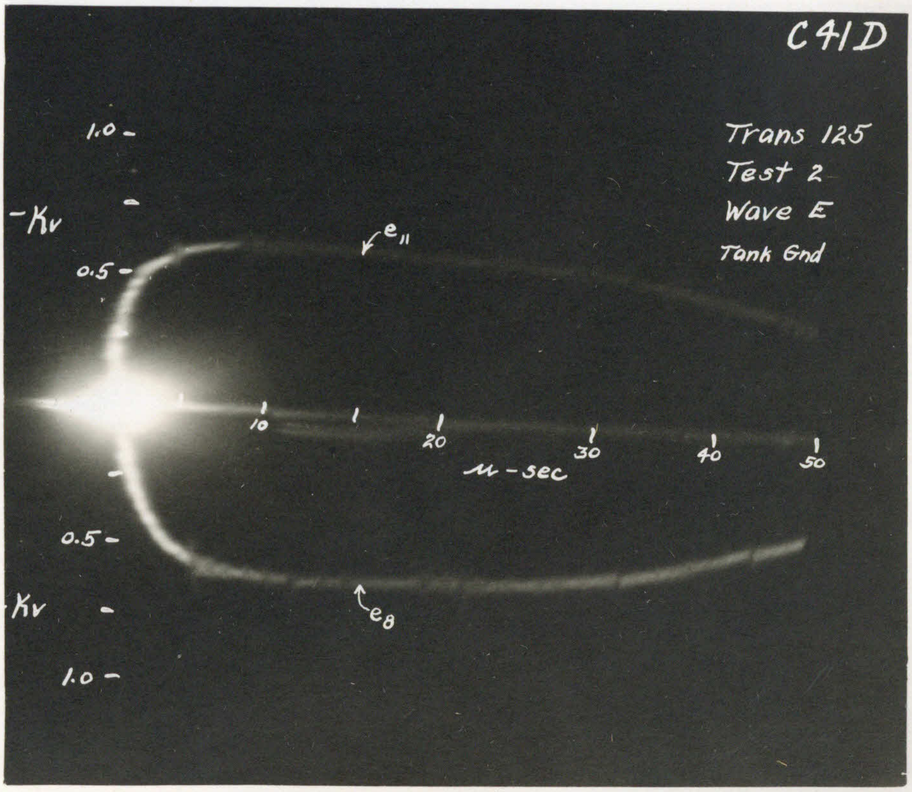

C21D
1.0
-Kv
0.5
+Kv
0.5
1.0
e11
e12
e8
2
4
6
8
10
12
µ-sec
Trans 125
Test 2
Wave A
Tank Ungnd

(b)

Fig. 92

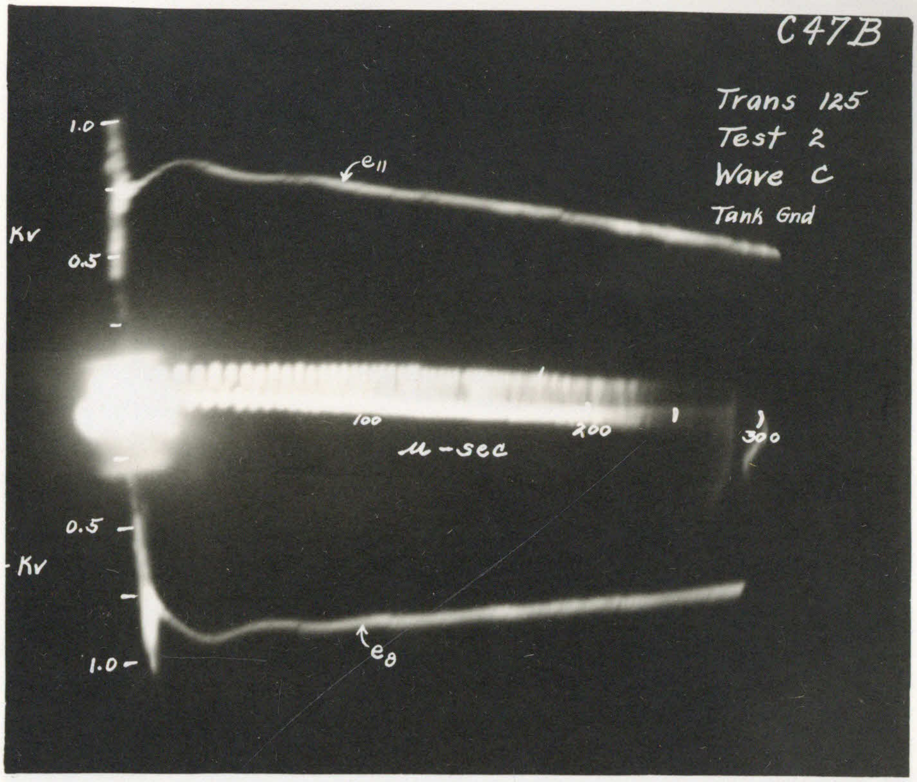

C44B
Trans 125
Test 2
Wave B
Tank Gnd
1.0
-Kv
0.5
e_{11}
10
20
30
μ-sec
0.5
+Kv
e_8
1.0

(a)

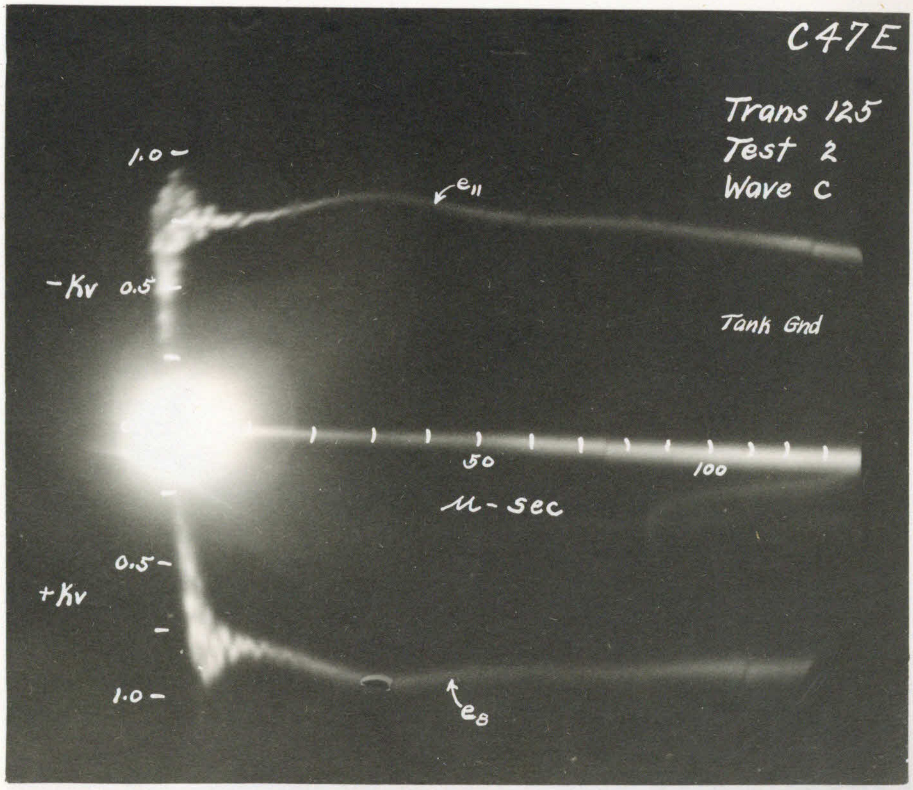

C41D
Trans 125
Test 2
Wave E
Tank Gnd
1.0
-Kv
0.5
e_{11}
10
20
30
40
50
μ-sec
0.5
Kv
e_8
1.0

(b)

Fig. 93

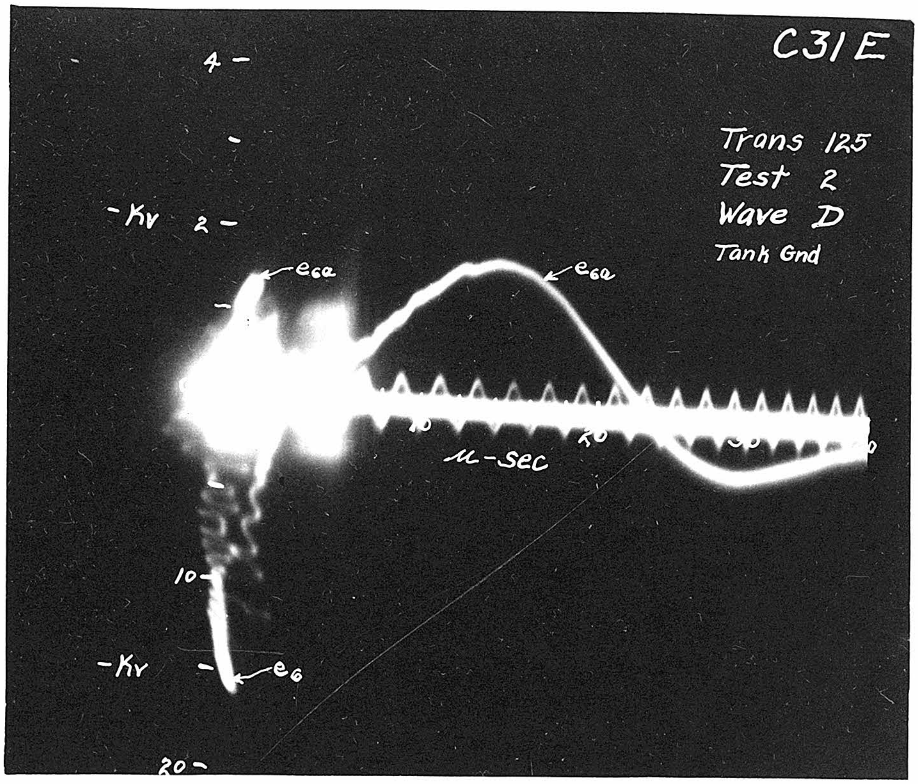

C47B
Trans 125
Test 2
Wave C
Tank Gnd
1.0
0.5
Kv
e_{11}
100
200
300
μ-sec
0.5
-Kv
1.0
e_8

(a)

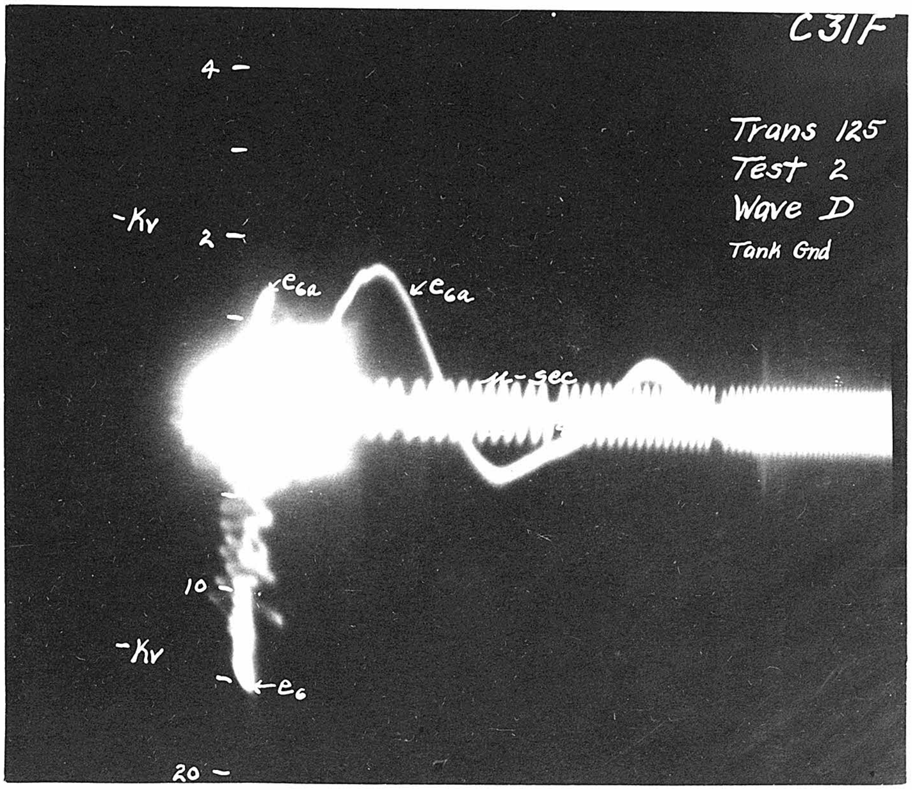

C47E
Trans 125
Test 2
Wave C
1.0
e_{11}
-Kv 0.5
Tank Gnd
50
100
μ-sec
0.5
+Kv
1.0
e_8

(b)

Fig. 94

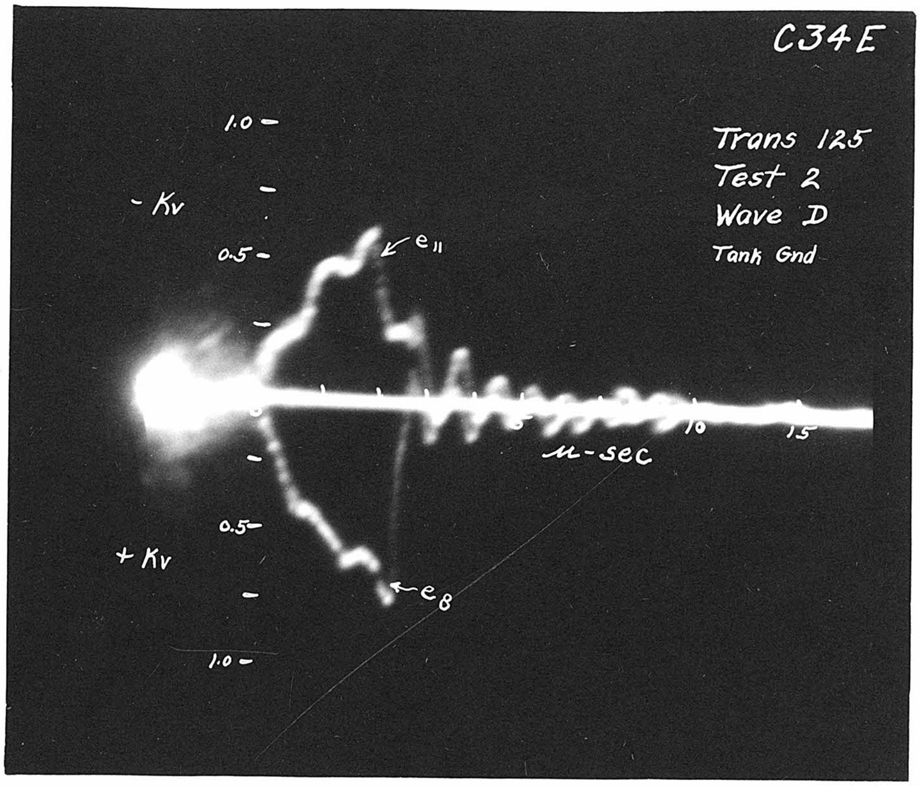

C31E
4
-Kv
2
e6a
e6a
Trans 125
Test 2
Wave D
Tank Gnd
10
20
μ-sec
10
-Kv
e6
20

(a)

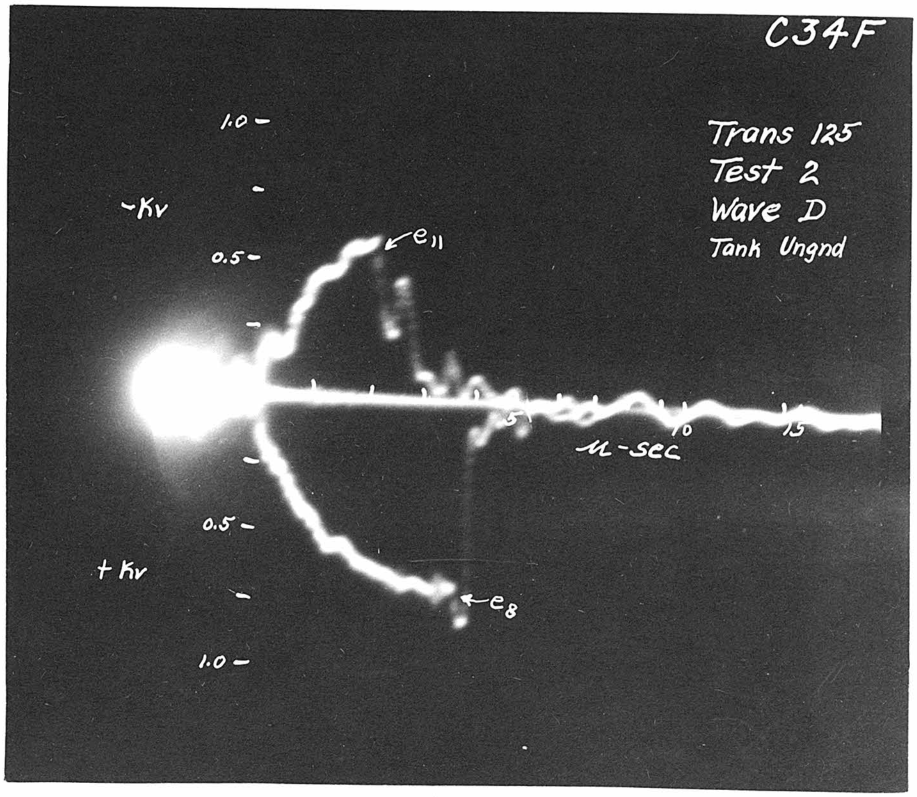

C31F
4
Trans 125
Test 2
Wave D
Tank Gnd
-Kv
2
e6a
e6a
μ-sec
10
-Kv
e6
20

(b)

Fig. 95

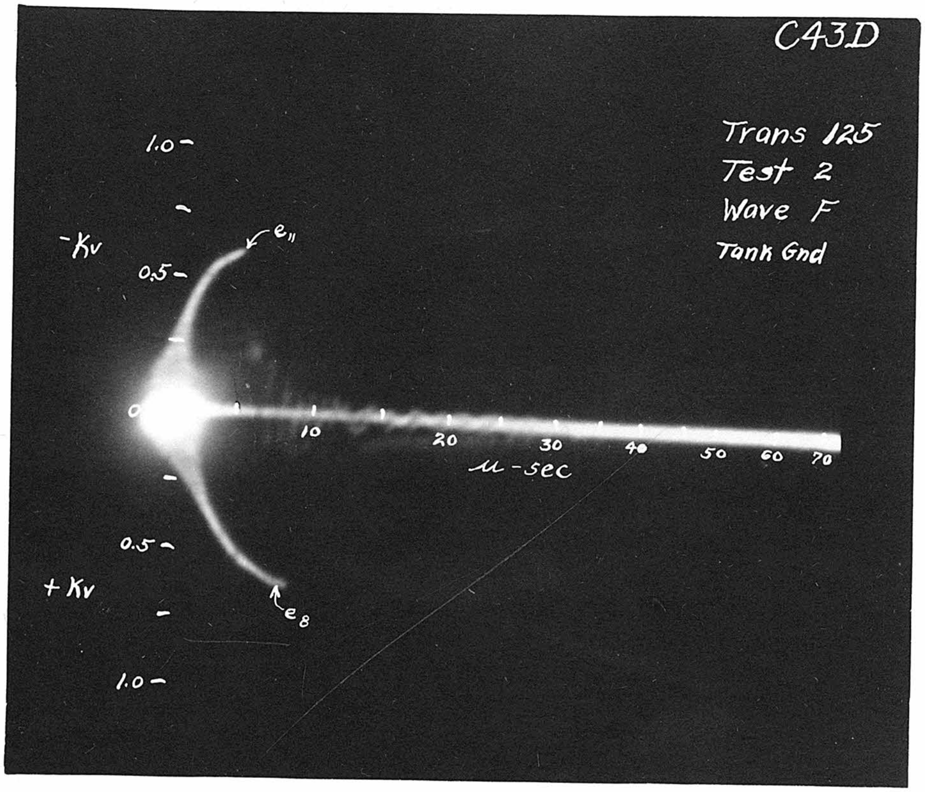

C34E
Trans 125
Test 2
Wave D
Tank Gnd
1.0
-Kv
0.5
e11
0.5
+Kv
e8
1.0
5
10
15
μ-sec

(a)

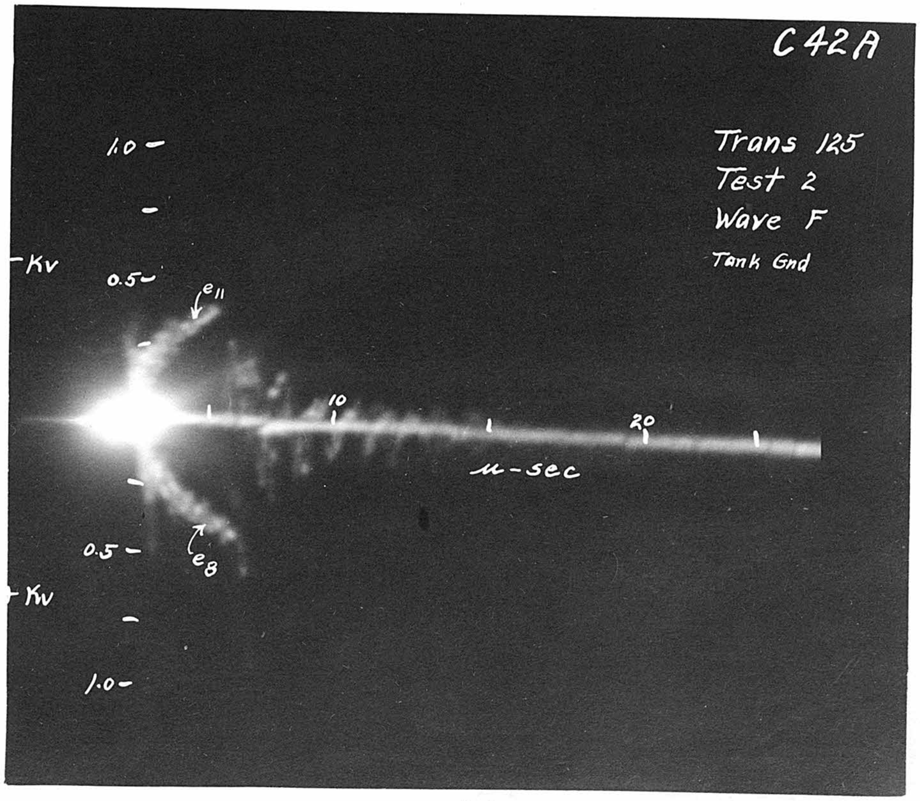

C34F
Trans 125
Test 2
Wave D
Tank Ungnd
1.0
-Kv
0.5
e11
0.5
+Kv
e8
1.0
5
10
15
μ-sec

(b)

Fig. 96

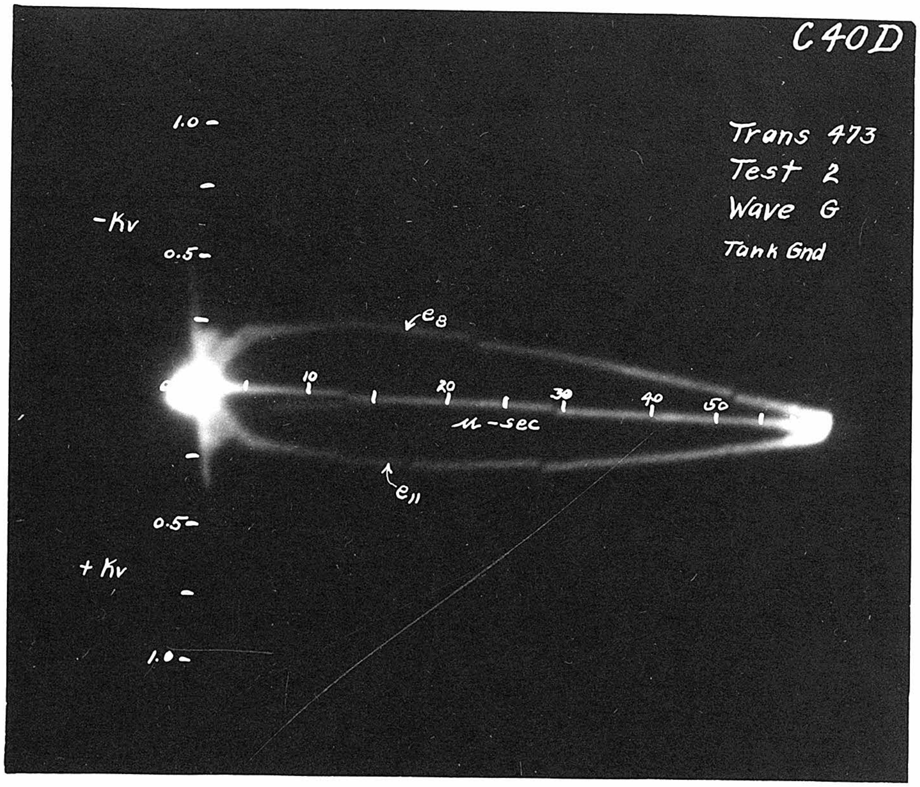

C43D
Trans 125
Test 2
Wave F
Tank Gnd
1.0
-Kv
e_{11}
0.5
0
10
20
30
40
50
60
70
μ-sec
0.5
+Kv
e_8
1.0

(a)

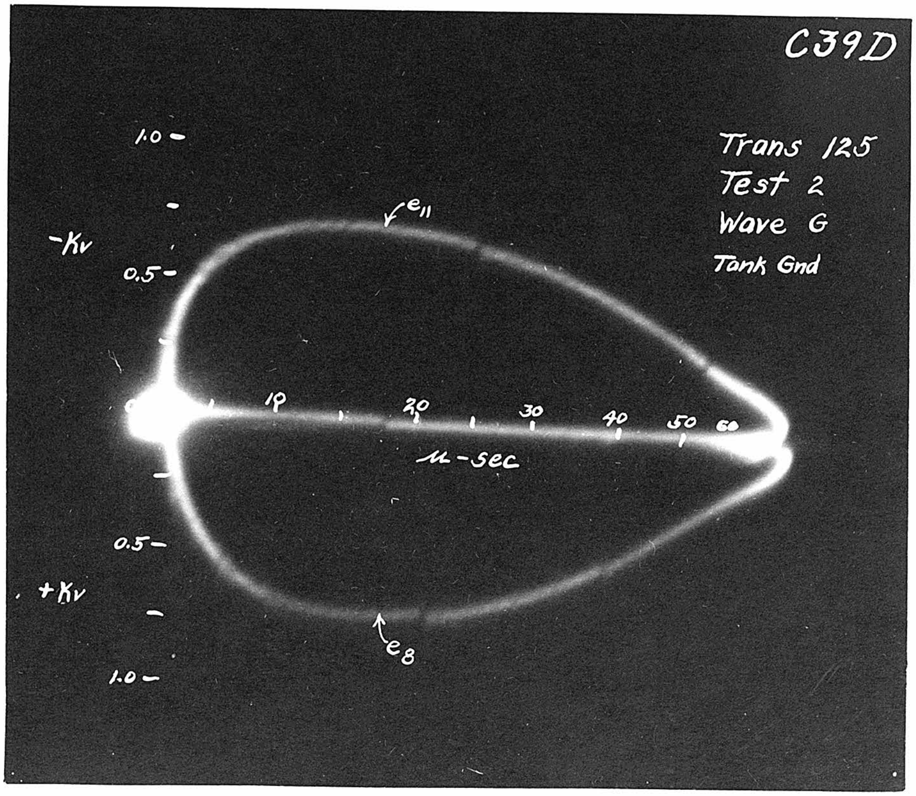

C42A
Trans 125
Test 2
Wave F
Tank Gnd
1.0
-Kv
0.5
e_{11}
10
20
μ-sec
0.5
e_8
+Kv
1.0

(b)

Fig. 97

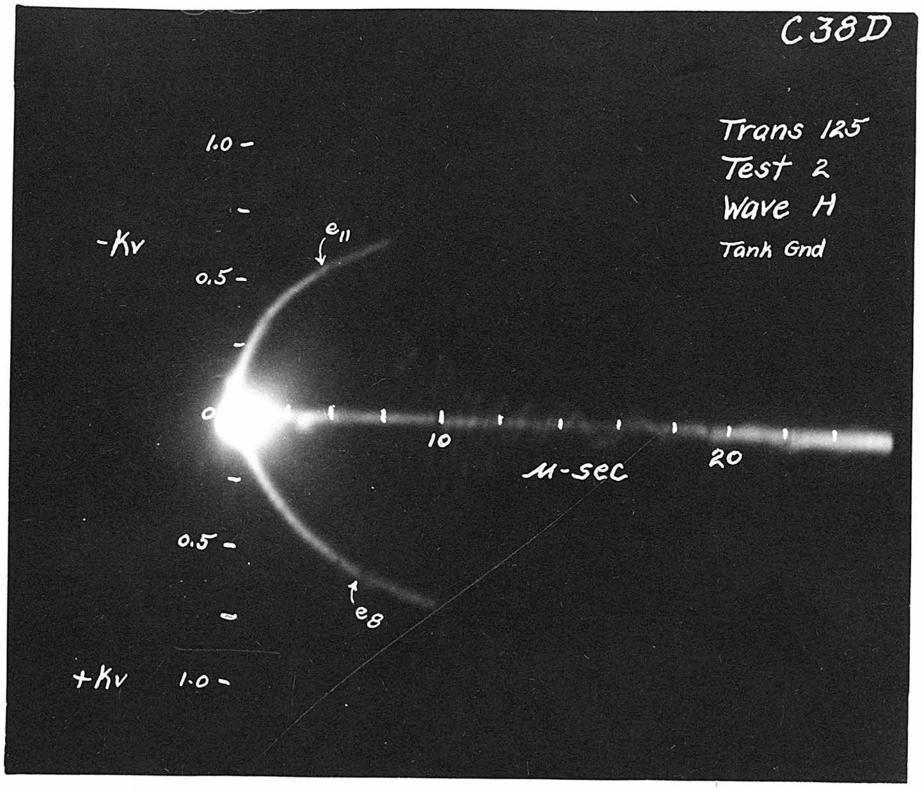

C40D
Trans 473
Test 2
Wave G
Tank Gnd
1.0
-Kv
0.5
e8
10
20
30
40
50
μ-sec
e11
0.5
+Kv
1.0

(a)

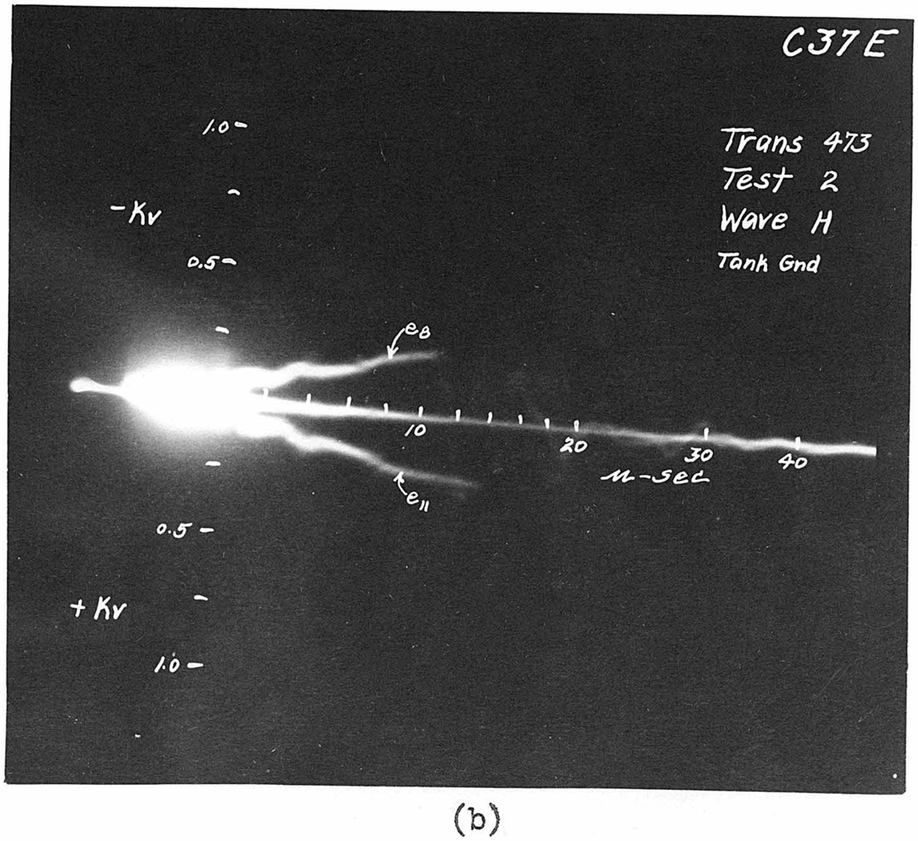

C39D
Trans 125
Test 2
Wave G
Tank Gnd
1.0
-Kv
0.5
e11
10
20
30
40
50
60
μ-sec
0.5
+Kv
e8
1.0

(b)

Fig. 98

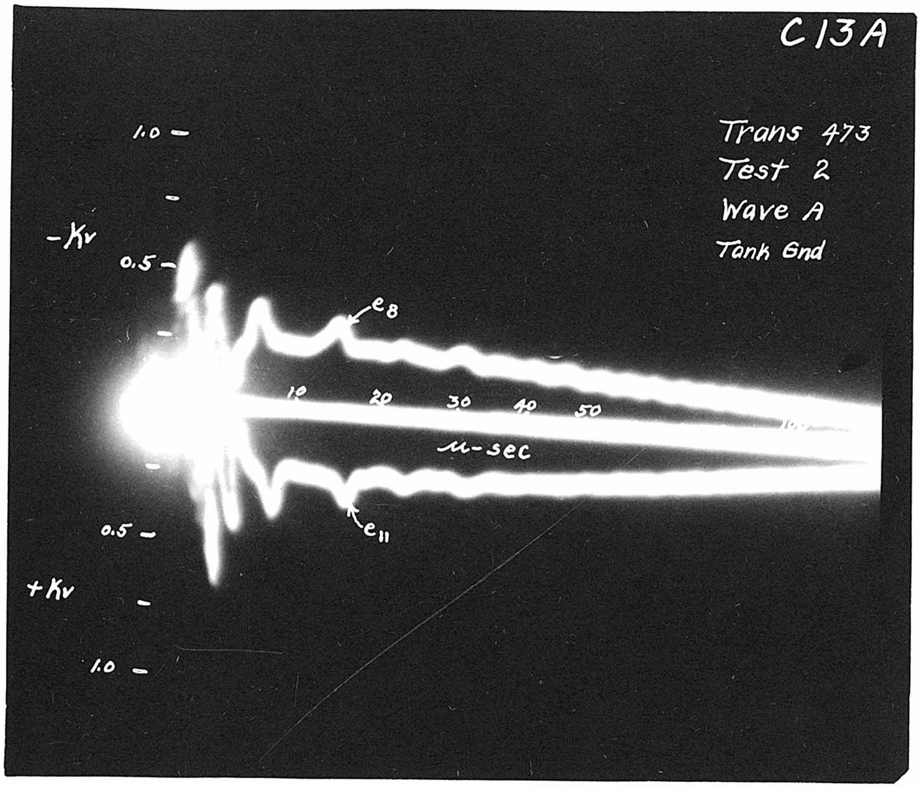

C38D
Trans 125
Test 2
Wave H
Tank Gnd
-Kv
+Kv
1.0
0.5
0
0.5
1.0
e11
e8
10
20
μ-sec

(a)

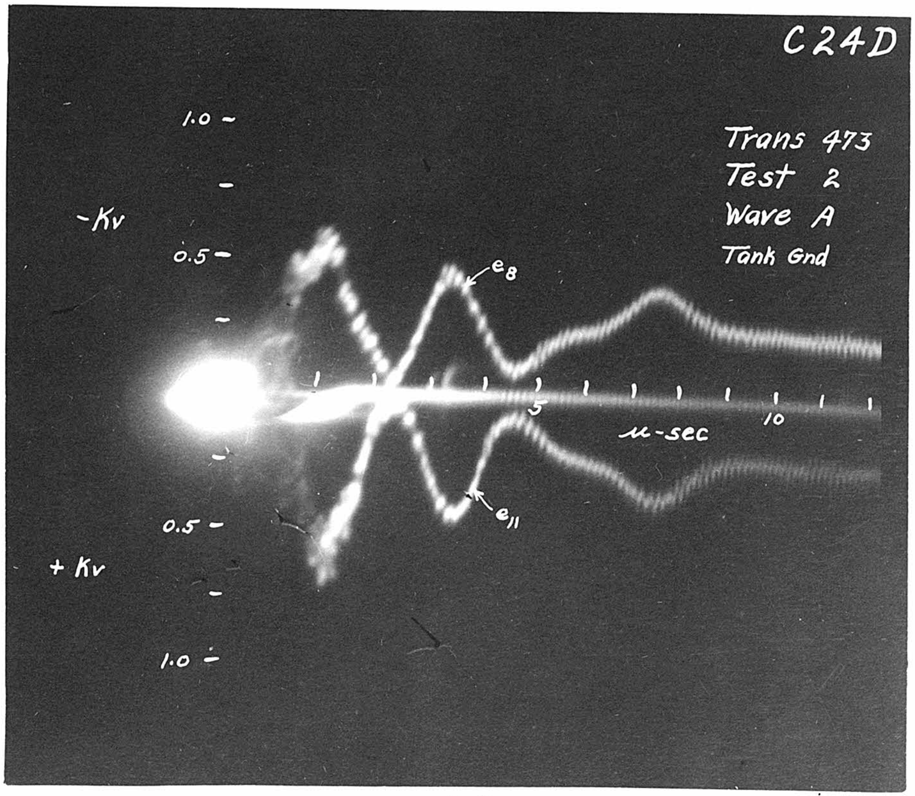

C37E
Trans 473
Test 2
Wave H
Tank Gnd
-Kv
+Kv
1.0
0.5
0.5
1.0
e8
e11
10
20
30
40
μ-sec

(b)

Fig. 99

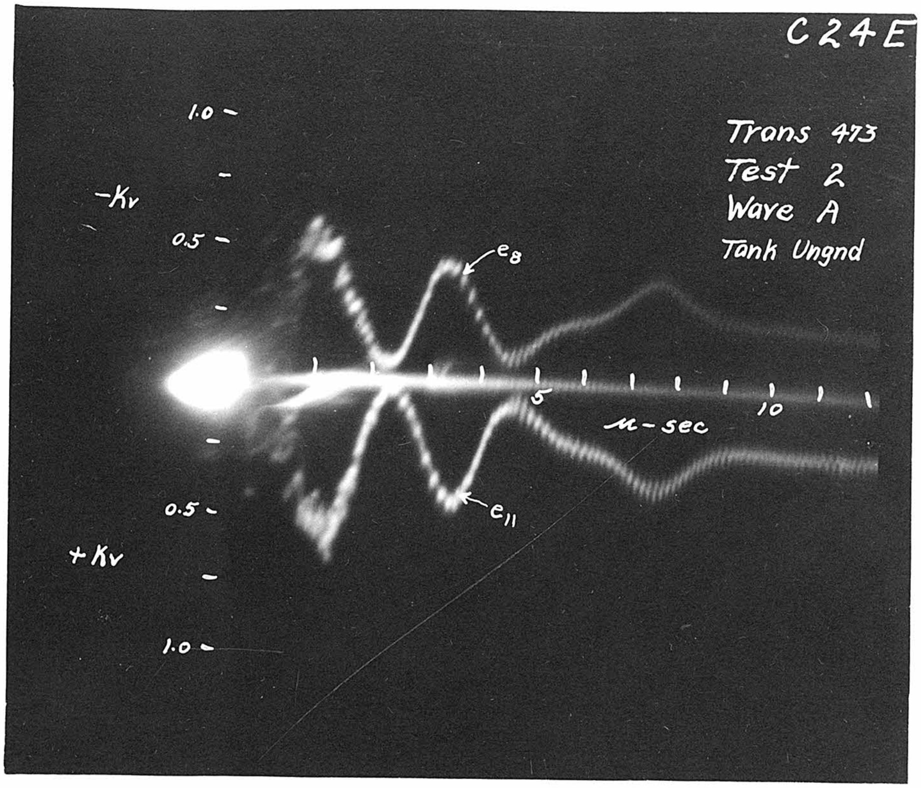

C13A
Trans 473
Test 2
Wave A
Tank Gnd
1.0
-Kv
0.5
e8
10
20
30
40
50
μ-sec
e11
0.5
+Kv
1.0

(a)

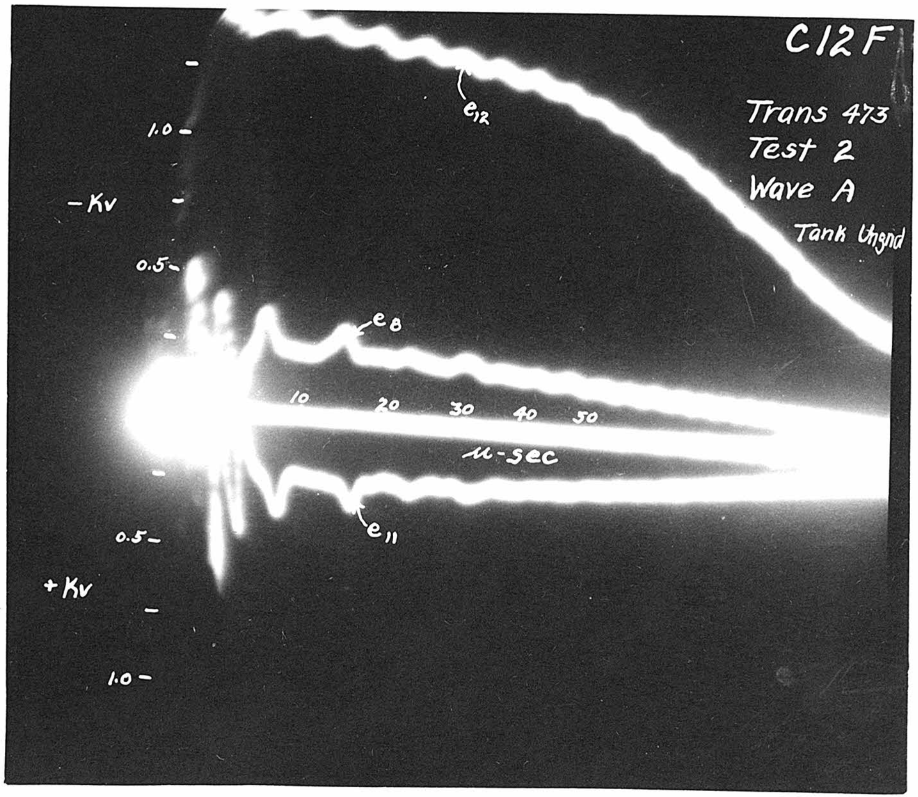

C24D
Trans 473
Test 2
Wave A
Tank Gnd
1.0
-Kv
0.5
e8
5
10
μ-sec
e11
0.5
+Kv
1.0

(b)

Fig. 100

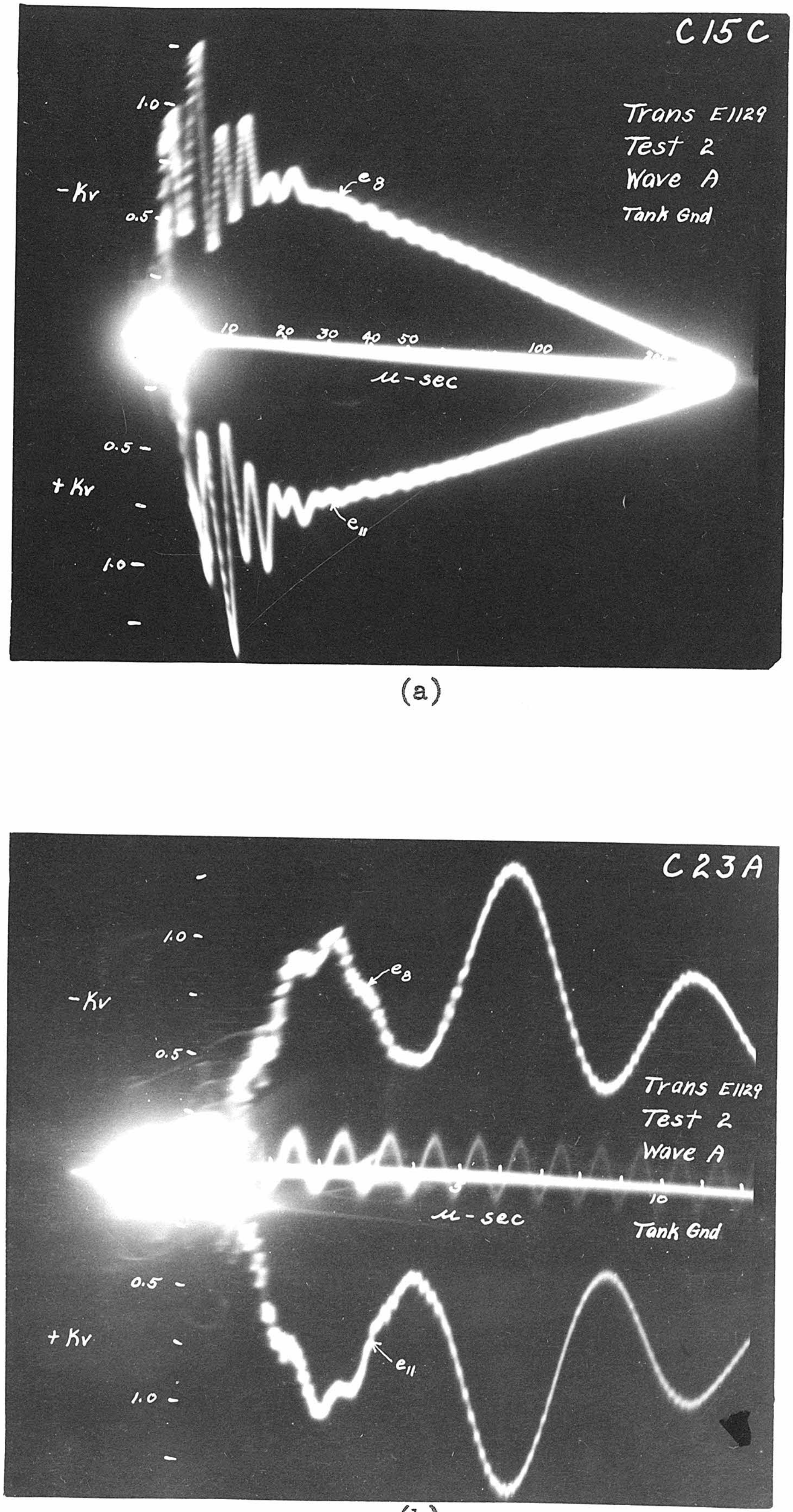

C24E
Trans 473
Test 2
Wave A
Tank Ungnd
1.0
-Kv
0.5
e8
5
10
μ-sec
0.5
e11
+Kv
1.0

(a)


C12F
e12
Trans 473
Test 2
Wave A
Tank Ungnd
1.0
-Kv
0.5
e8
10
20
30
40
50
μ-sec
e11
0.5
+Kv
1.0

(b)

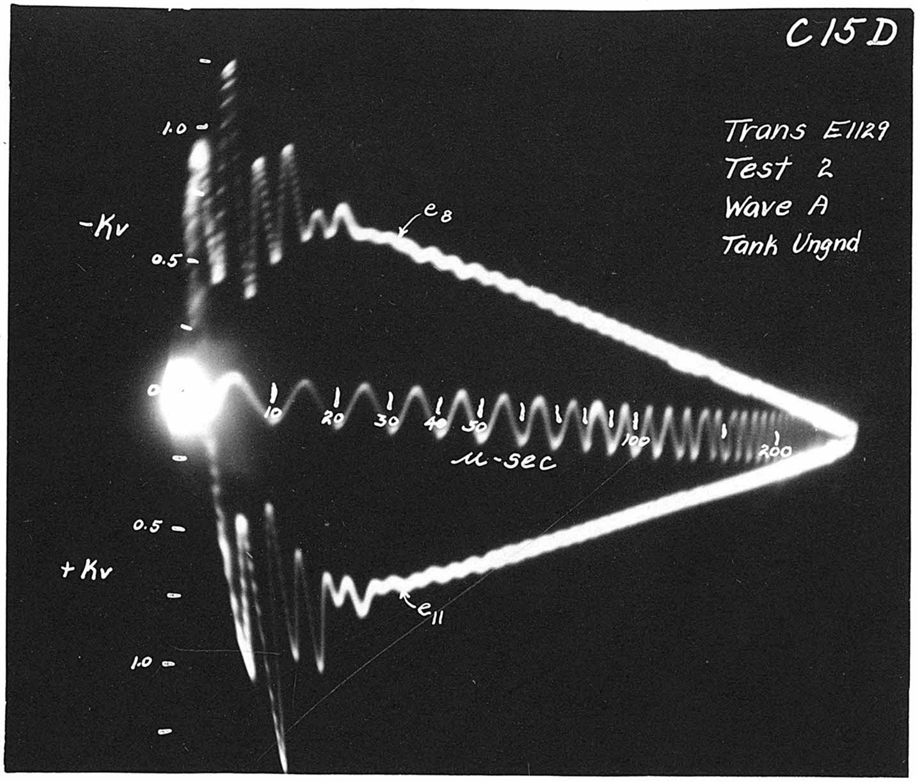
Fig. 101


C15C
1.0
-Kv
0.5
e_8
Trans E1129
Test 2
Wave A
Tank Gnd
10
20
30
40
50
100
µ-sec
0.5
+Kv
e_{11}
1.0
C23A
1.0
e_8
-Kv
0.5
Trans E1129
Test 2
Wave A
5
10
µ-sec
Tank Gnd
0.5
+Kv
e_{11}
1.0

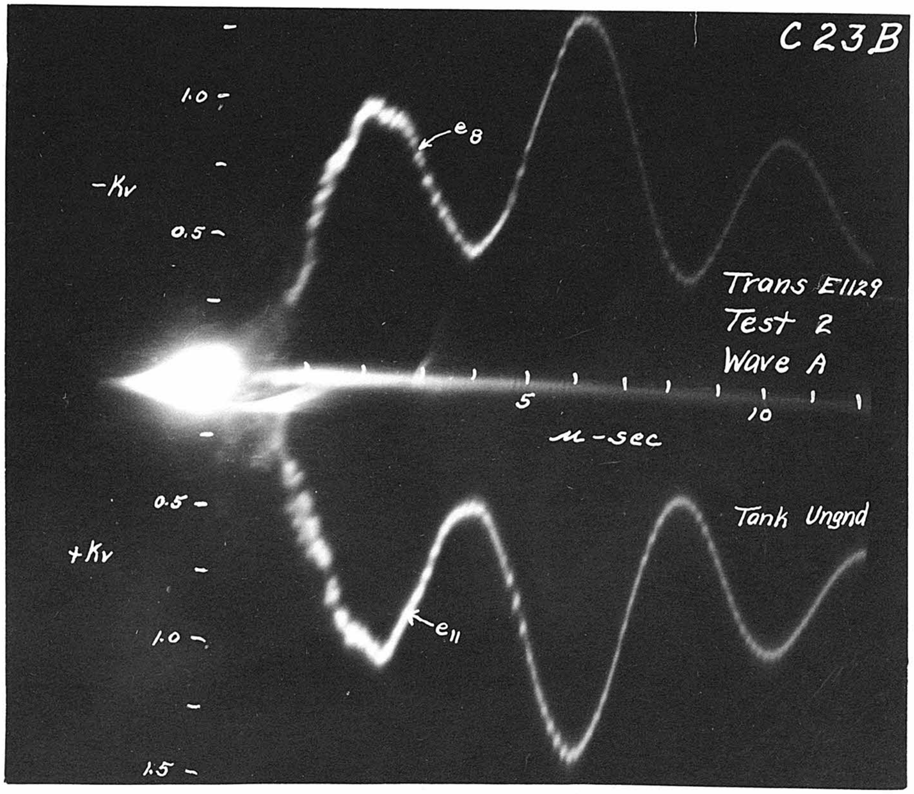
(a)

(b)

Fig. 102

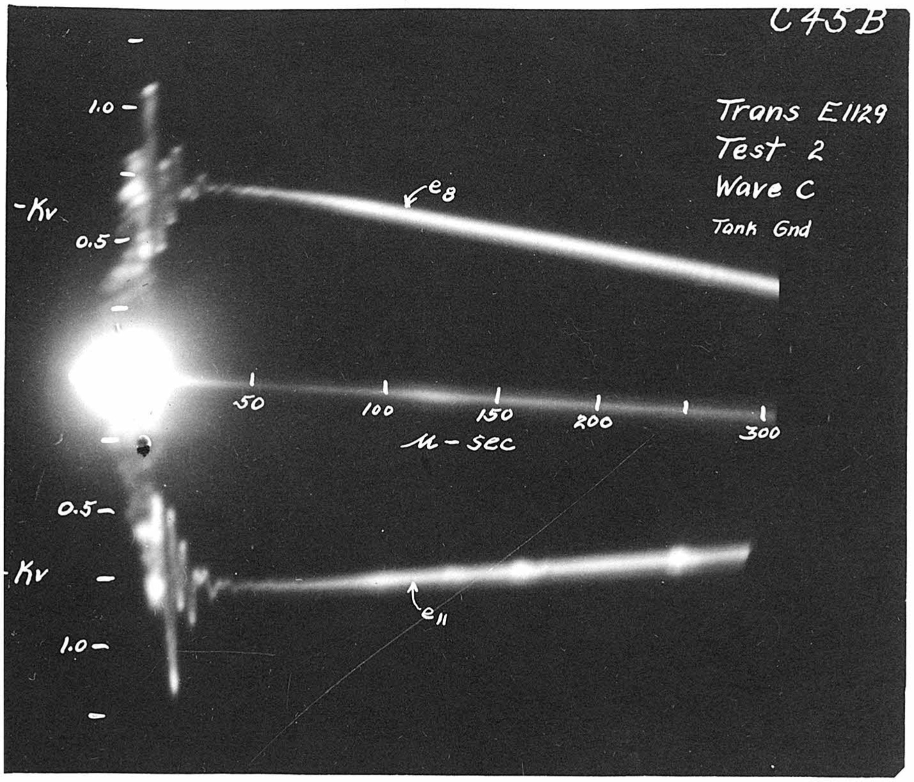

C15D
Trans E1129
Test 2
Wave A
Tank Ungnd
-Kv
+Kv
1.0
0.5
0.5
1.0
e8
e11
10
20
30
40
50
100
200
μ-sec

(a)

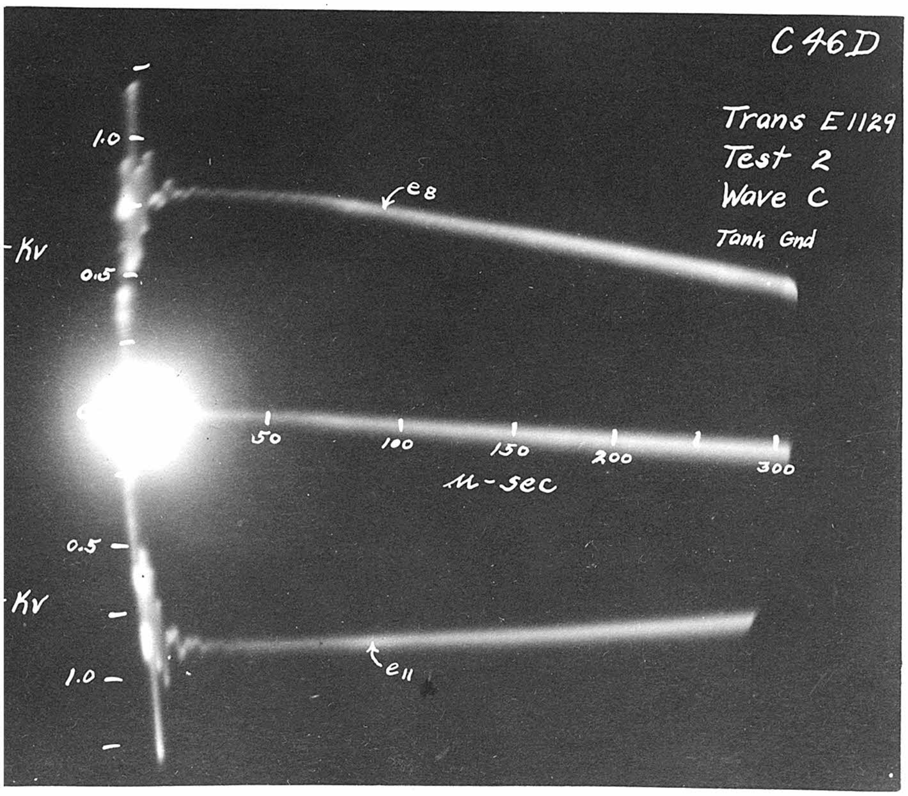

C23B
1.0
-Kv
0.5
0.5
+Kv
1.0
1.5
e8
e11
5
10
μ-sec
Trans E1129
Test 2
Wave A
Tank Ungnd

(b)

Fig. 103

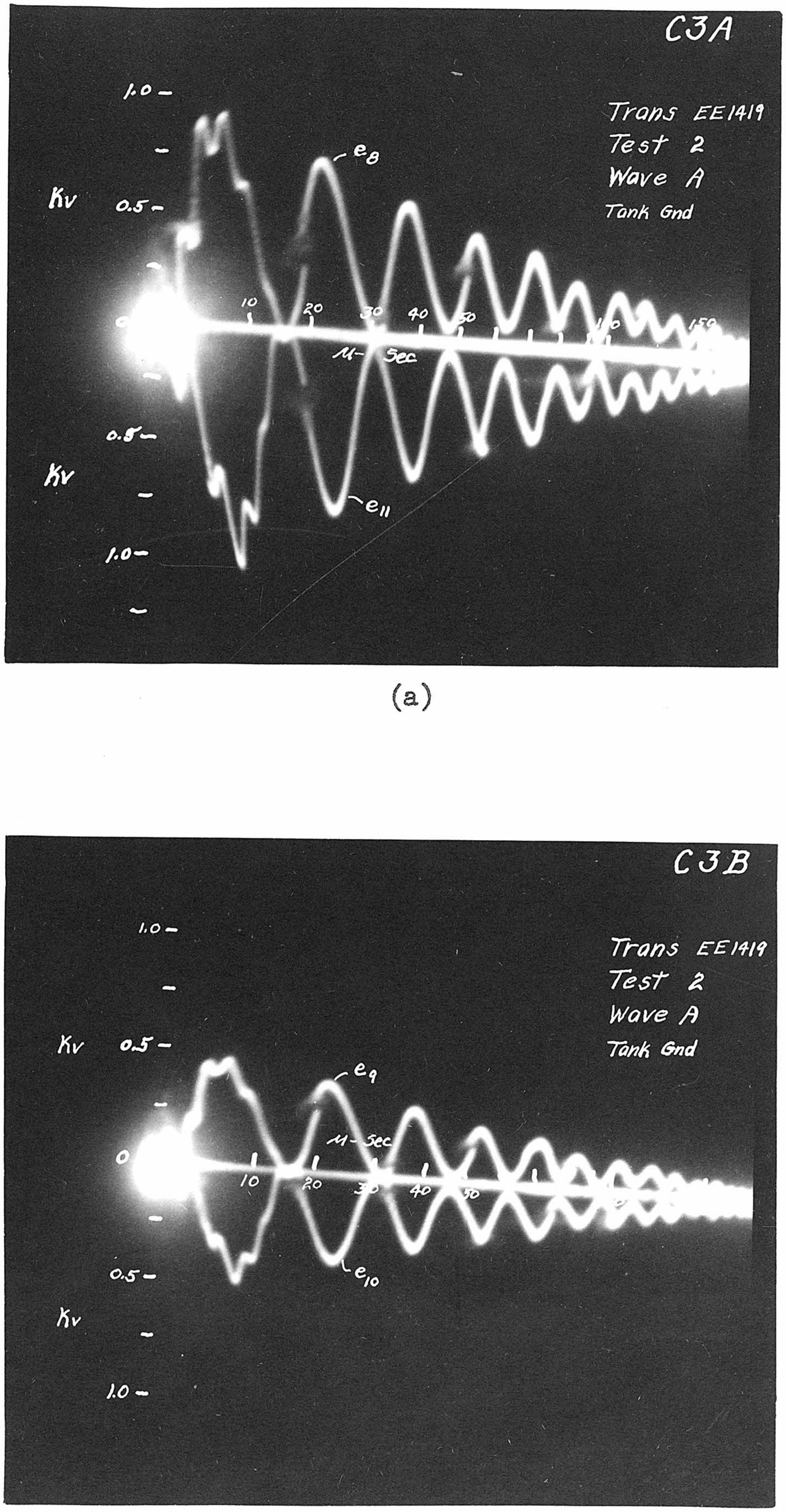

C45B
Trans E1129
Test 2
Wave C
Tank Gnd
1.0
0.5
-Kv
e8
50
100
150
200
300
μ-sec
0.5
-Kv
1.0
e11

(a)


C46D
Trans E1129
Test 2
Wave C
Tank Gnd
1.0
0.5
-Kv
e8
50
100
150
200
300
μ-sec
0.5
-Kv
1.0
e11

(b)

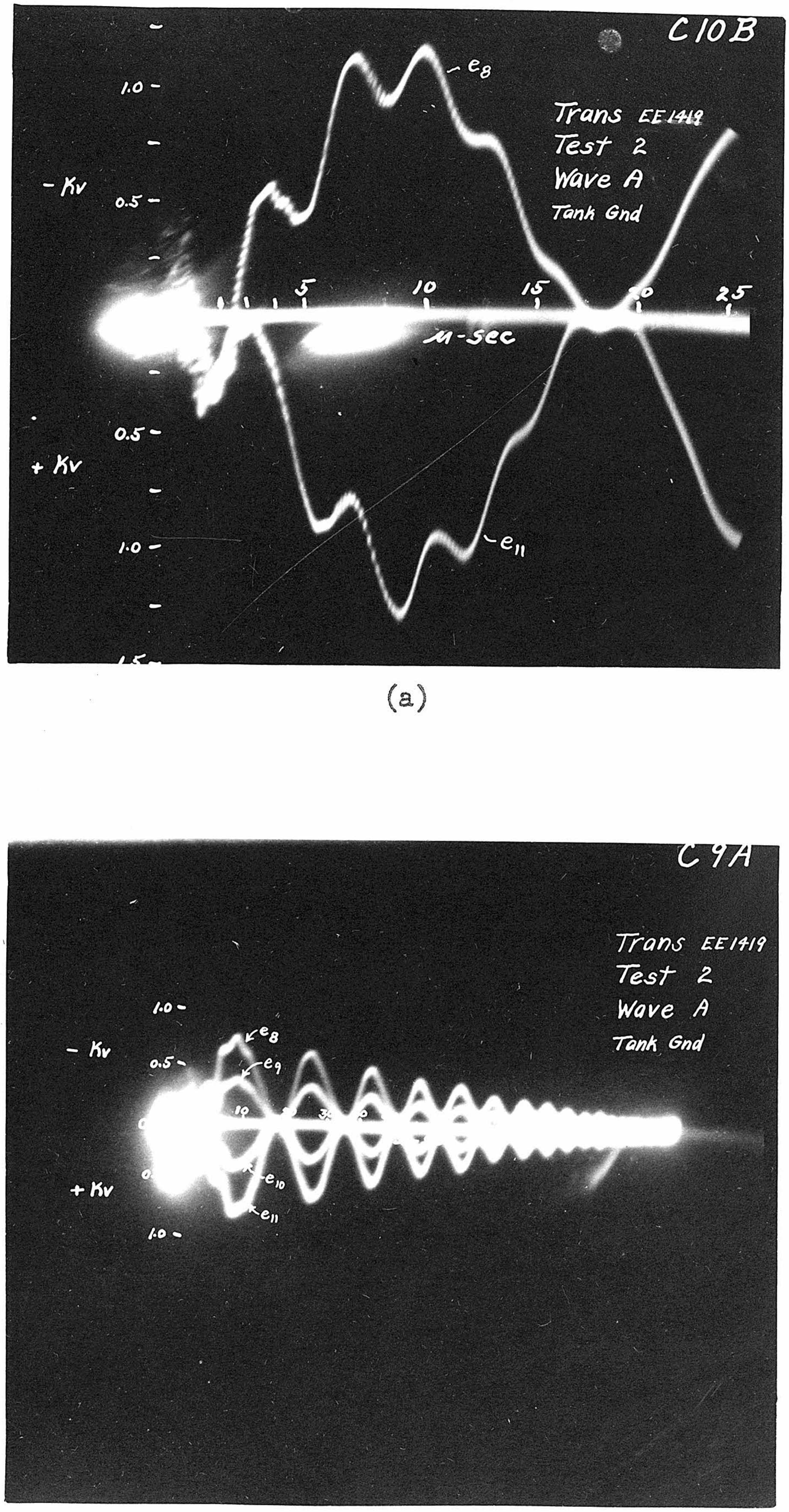
Fig. 104


C3A
Trans EE1419
Test 2
Wave A
Tank Gnd
1.0
Kv
0.5
0
0.5
Kv
1.0
e_8
e_{11}
10
20
30
40
50
100
150
μ-Sec
C3B
Trans EE1419
Test 2
Wave A
Tank Gnd
1.0
Kv
0.5
0
0.5
Kv
1.0
e_9
e_{10}
μ-Sec
10
20
30
40
50

(a)

(b)

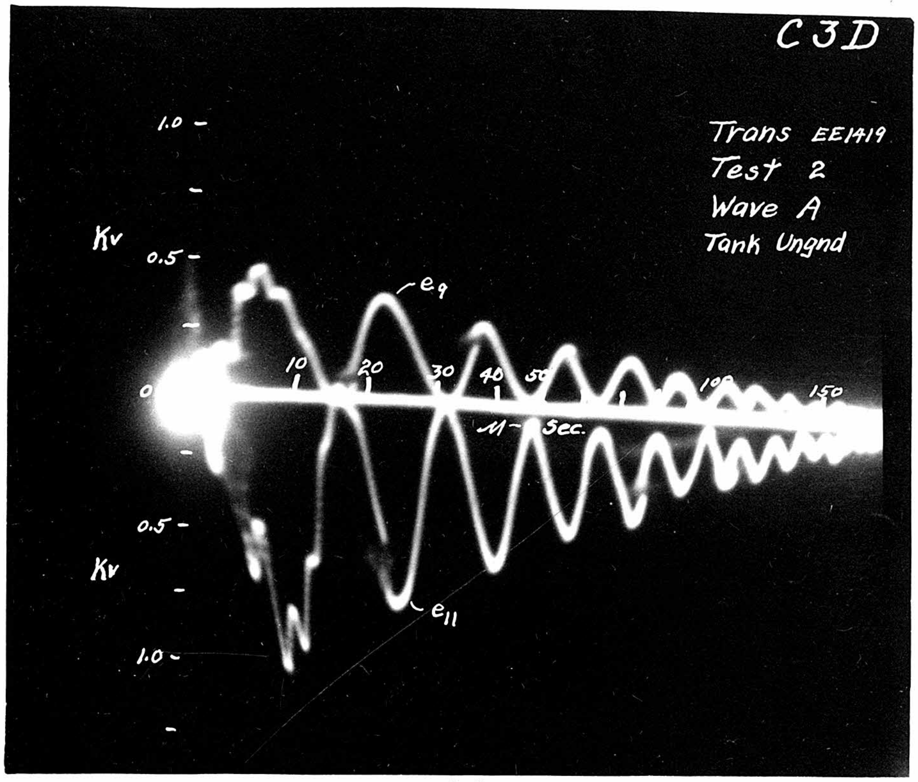
Fig. 105


C10B
1.0
0.5
- Kv
+ Kv
0.5
1.0
e8
e11
5
10
15
20
25
µ-sec
Trans EE1419
Test 2
Wave A
Tank Gnd
C9A
Trans EE1419
Test 2
Wave A
Tank Gnd
1.0
0.5
- Kv
+ Kv
1.0
e8
e9
e10
e11
10

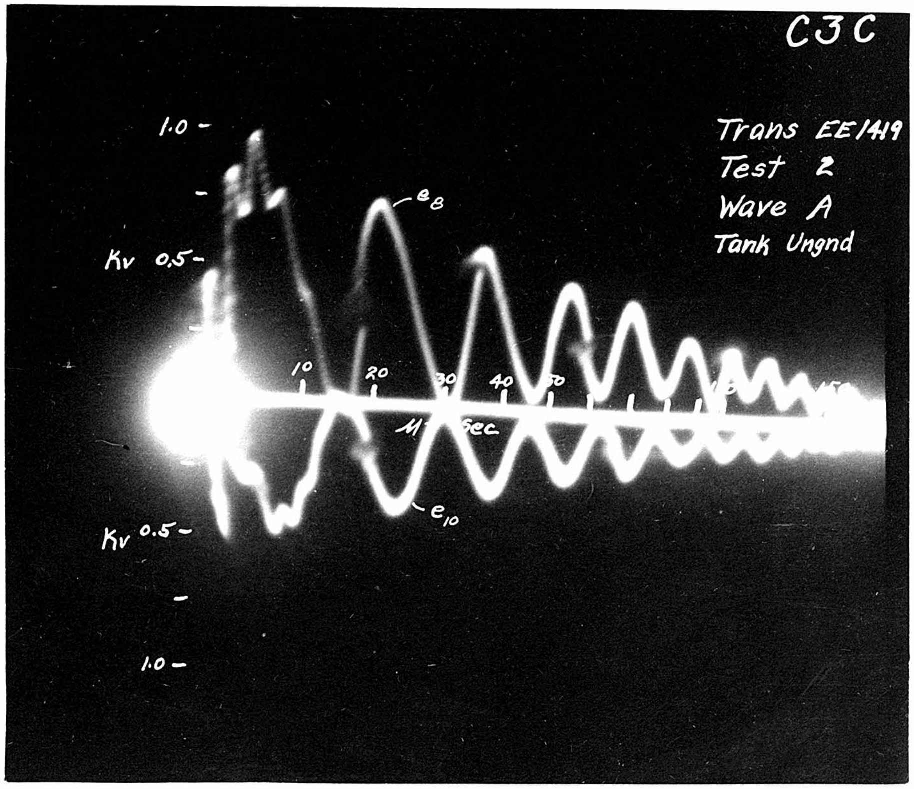
(a)

(b)

Fig. 106

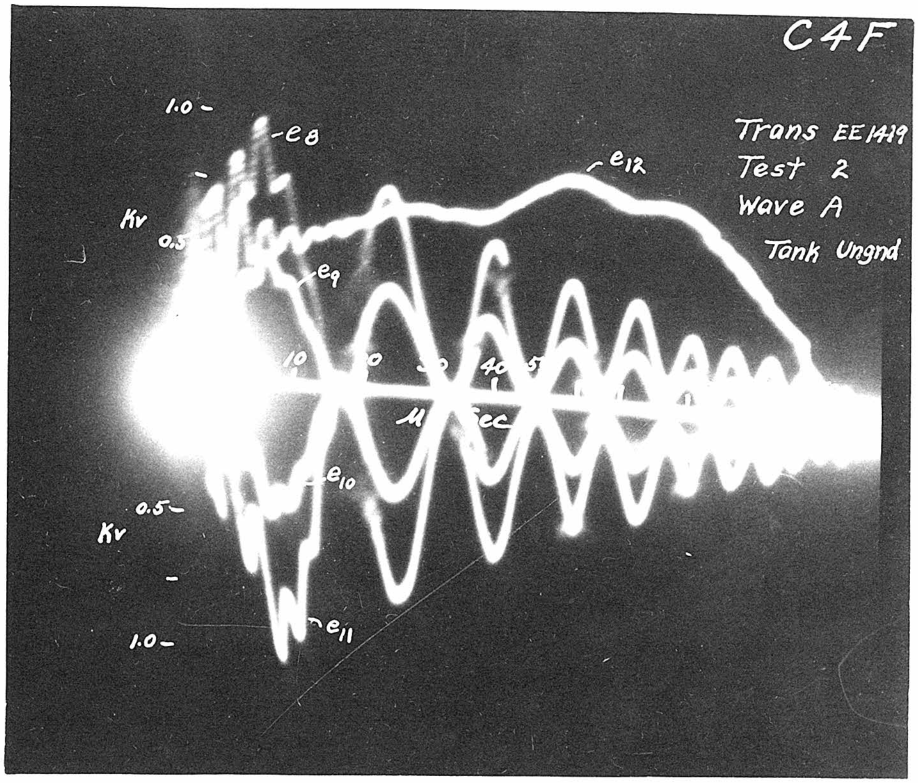

C3D
Trans EE1419
Test 2
Wave A
Tank Ungnd
Kv
1.0
0.5
0
0.5
1.0
Kv
e9
e11
10
20
30
40
50
100
150
μ-Sec.

(a)

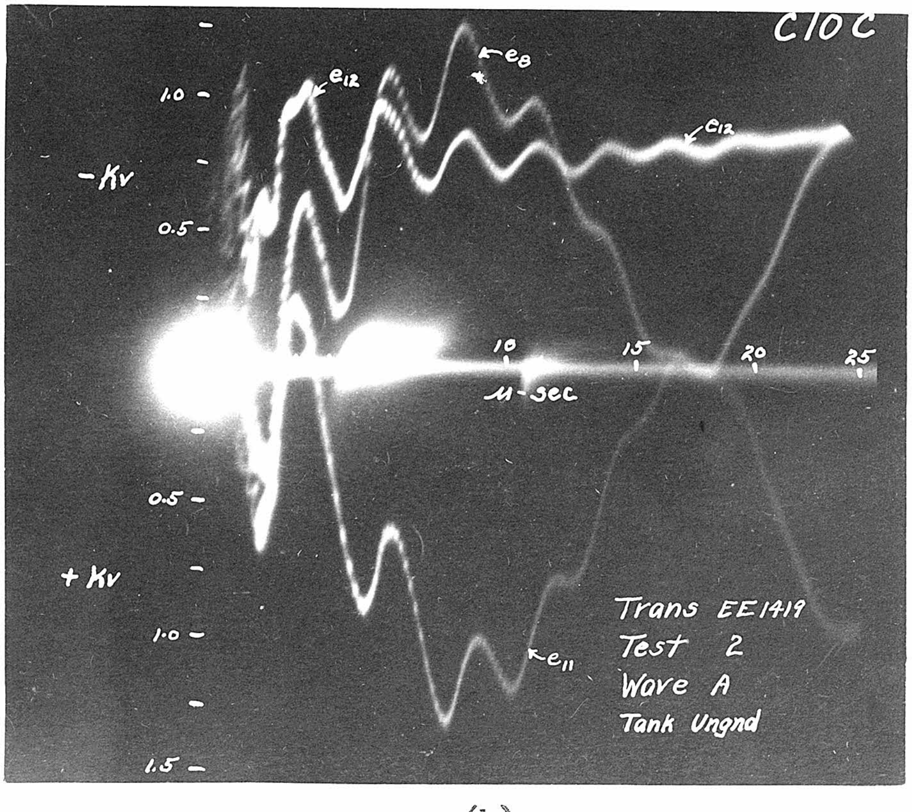

C3C
Trans EE1419
Test 2
Wave A
Tank Ungnd
1.0
Kv 0.5
Kv 0.5
1.0
e8
e10
10
20
30
40
50
μ-Sec.

(b)

Fig. 107

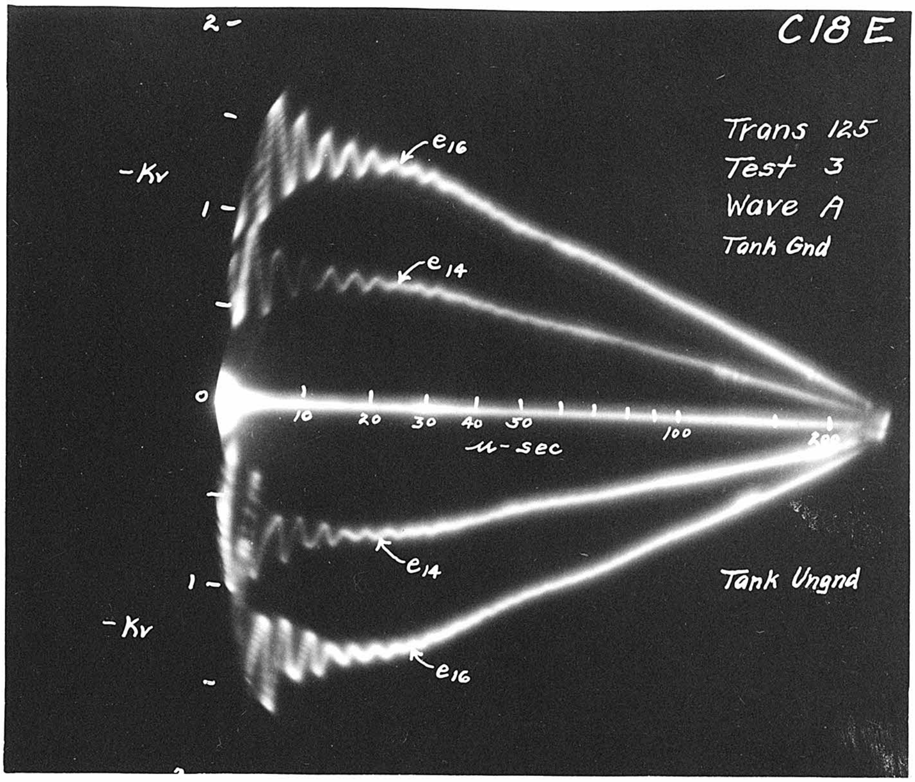

C4F
Trans EE1419
Test 2
Wave A
Tank Ungnd
1.0
Kv
0.5
e8
e9
e12
e10
e11
10
20
30
40
50
μ-sec
0.5
Kv
1.0

(a)

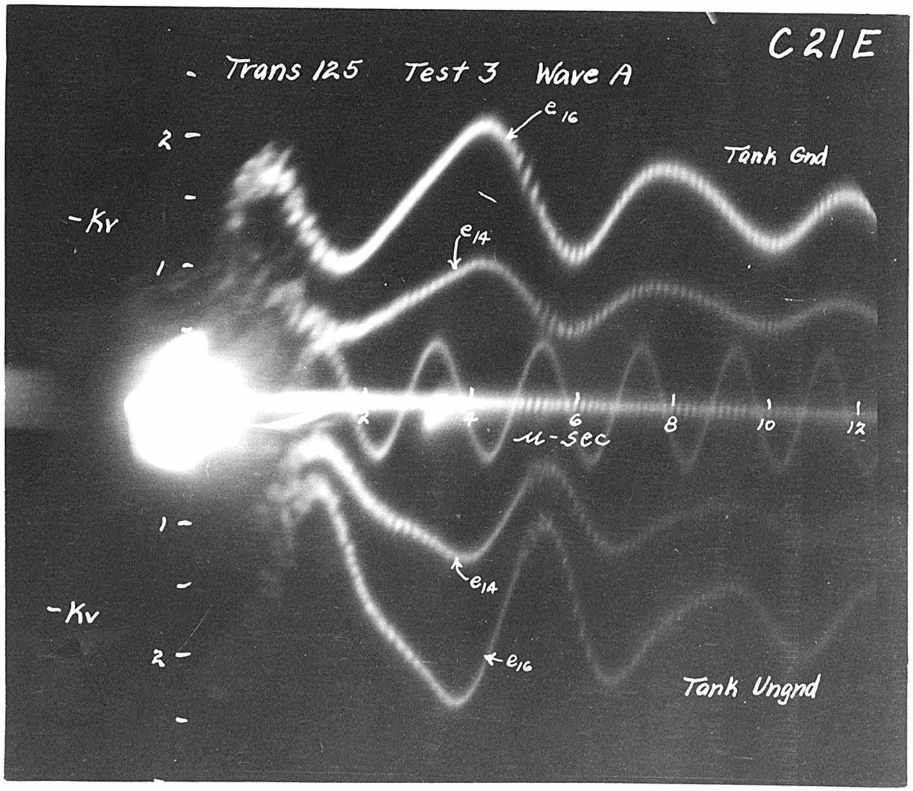

C10C
1.0
-Kv
0.5
e12
e8
e12
10
15
20
25
μ-sec
0.5
+Kv
1.0
1.5
e11
Trans EE1419
Test 2
Wave A
Tank Ungnd

(b)

Fig. 108

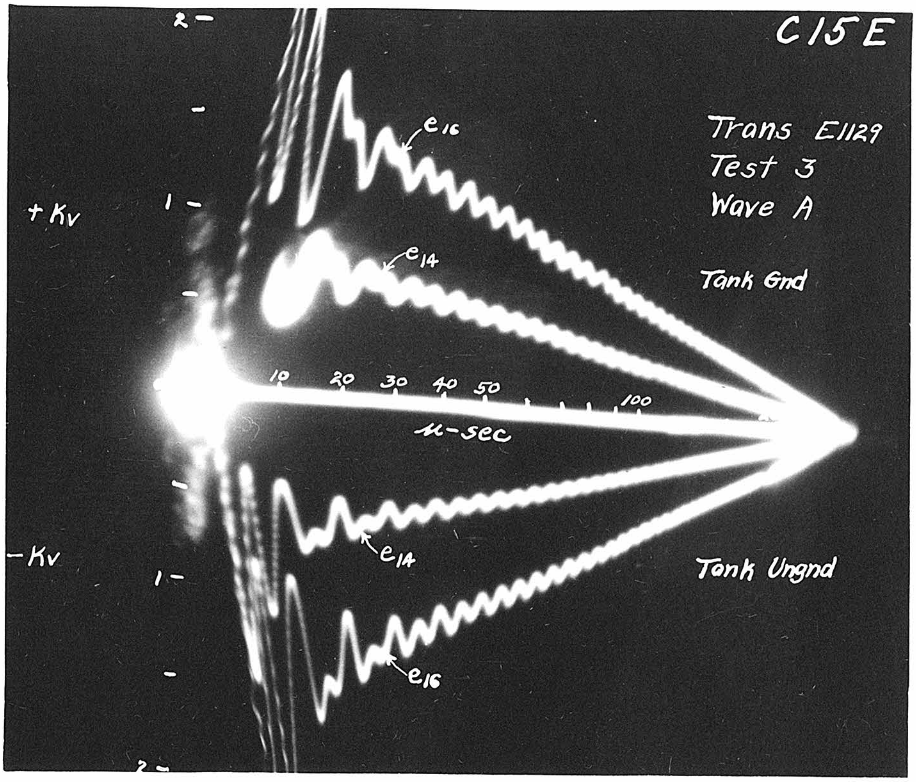

C18 E
Trans 125
Test 3
Wave A
Tank Gnd
-Kv
2
1
0
e16
e14
10
20
30
40
50
100
200
u-sec
e14
e16
1
-Kv
Tank Ungnd

(a)

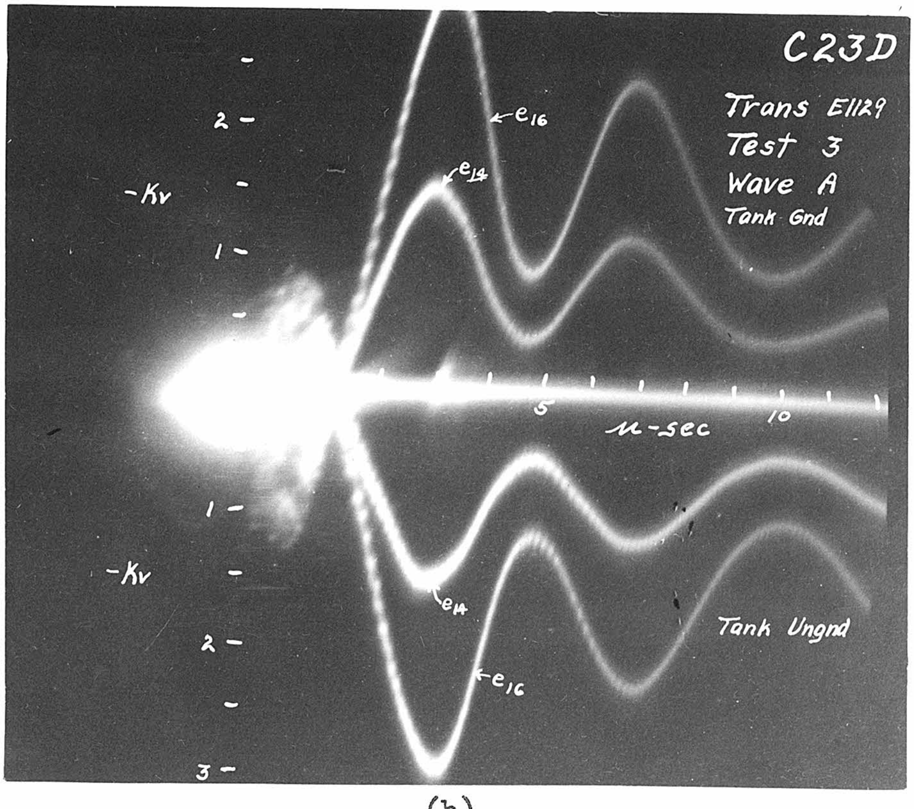

C21E
Trans 125 Test 3 Wave A
e16
Tank Gnd
2
-Kv
1
e14
2
4
6
8
10
12
u-sec
1
e14
-Kv
2
e16
Tank Ungnd

(b)

Fig. 109

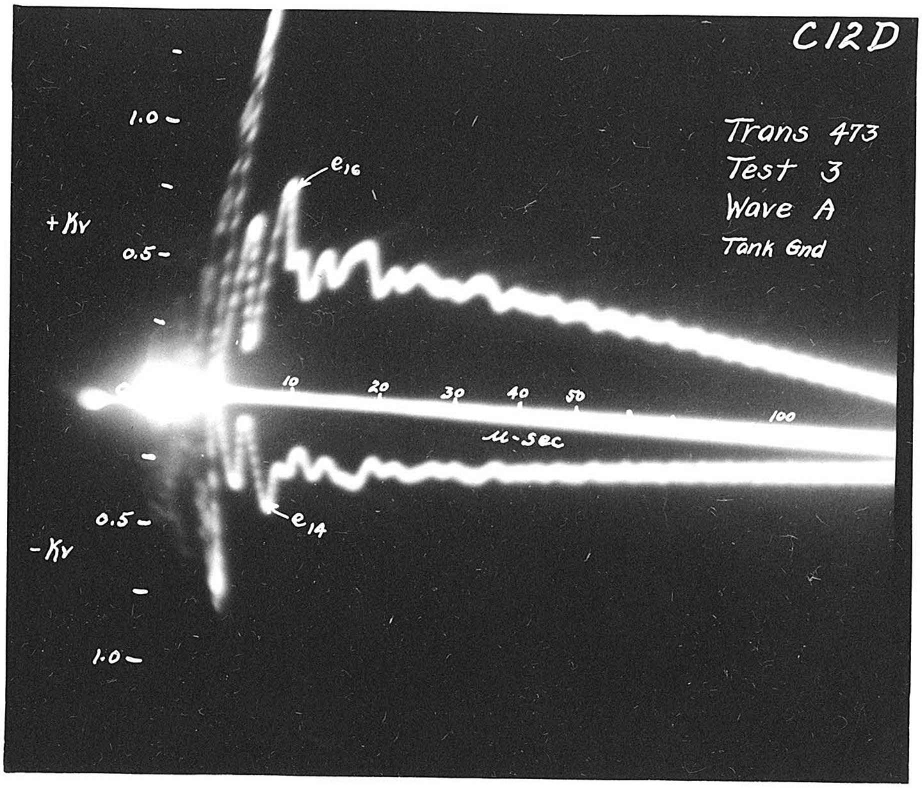

C15E
Trans E1129
Test 3
Wave A
Tank Gnd
Tank Ungnd
+Kv
-Kv
2
1
1
2
e16
e14
e14
e16
10
20
30
40
50
100
μ-sec

(a)

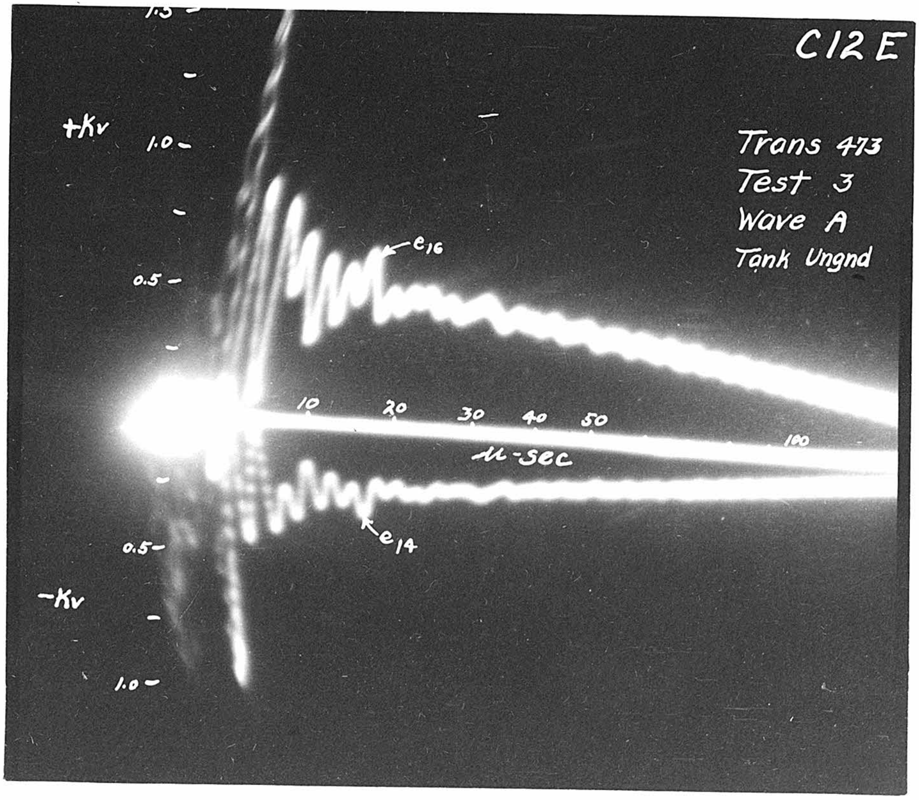

C23D
Trans E1129
Test 3
Wave A
Tank Gnd
Tank Ungnd
-Kv
-Kv
2
1
1
2
3
e16
e14
e14
e16
5
10
μ-sec

(b)

Fig. 110

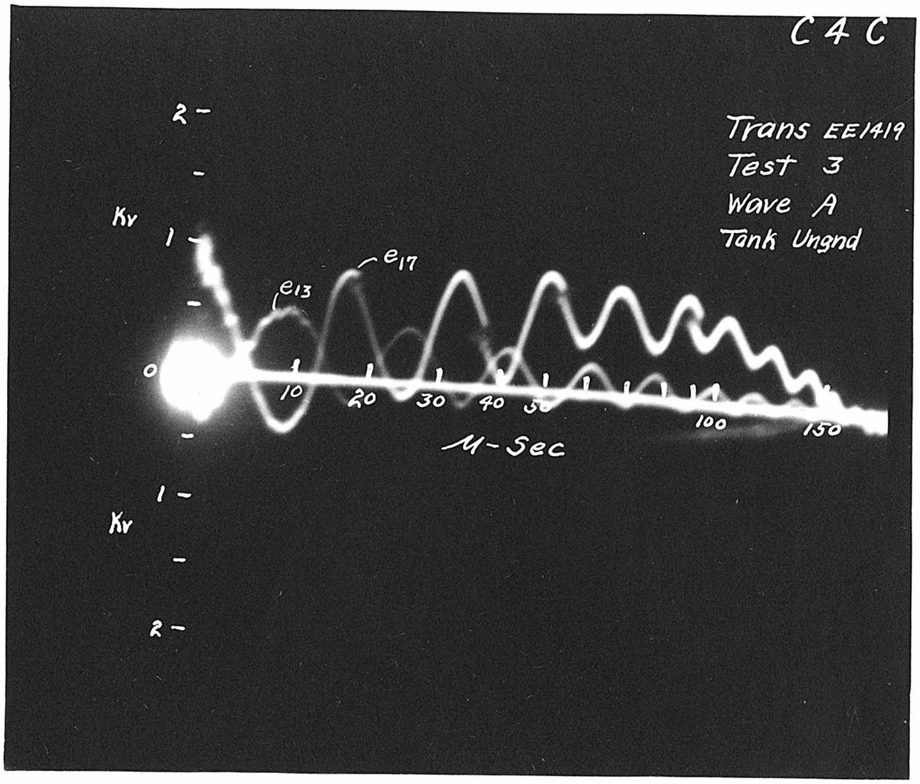

C12D
Trans 473
Test 3
Wave A
Tank Gnd
+Kv
-Kv
1.0
0.5
0.5
1.0
e16
e14
10
20
30
40
50
100
µ-sec

(a)

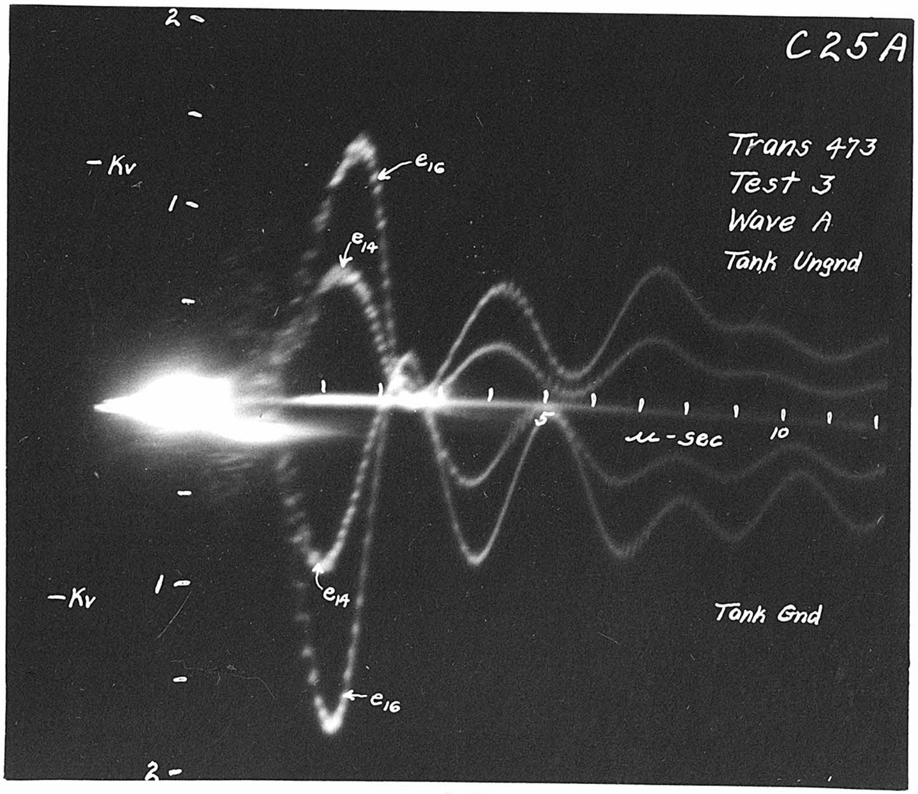

C12E
Trans 473
Test 3
Wave A
Tank Ungnd
+Kv
-Kv
1.5
1.0
0.5
0.5
1.0
e16
e14
10
20
30
40
50
100
µ-sec

(b)

Fig. 111

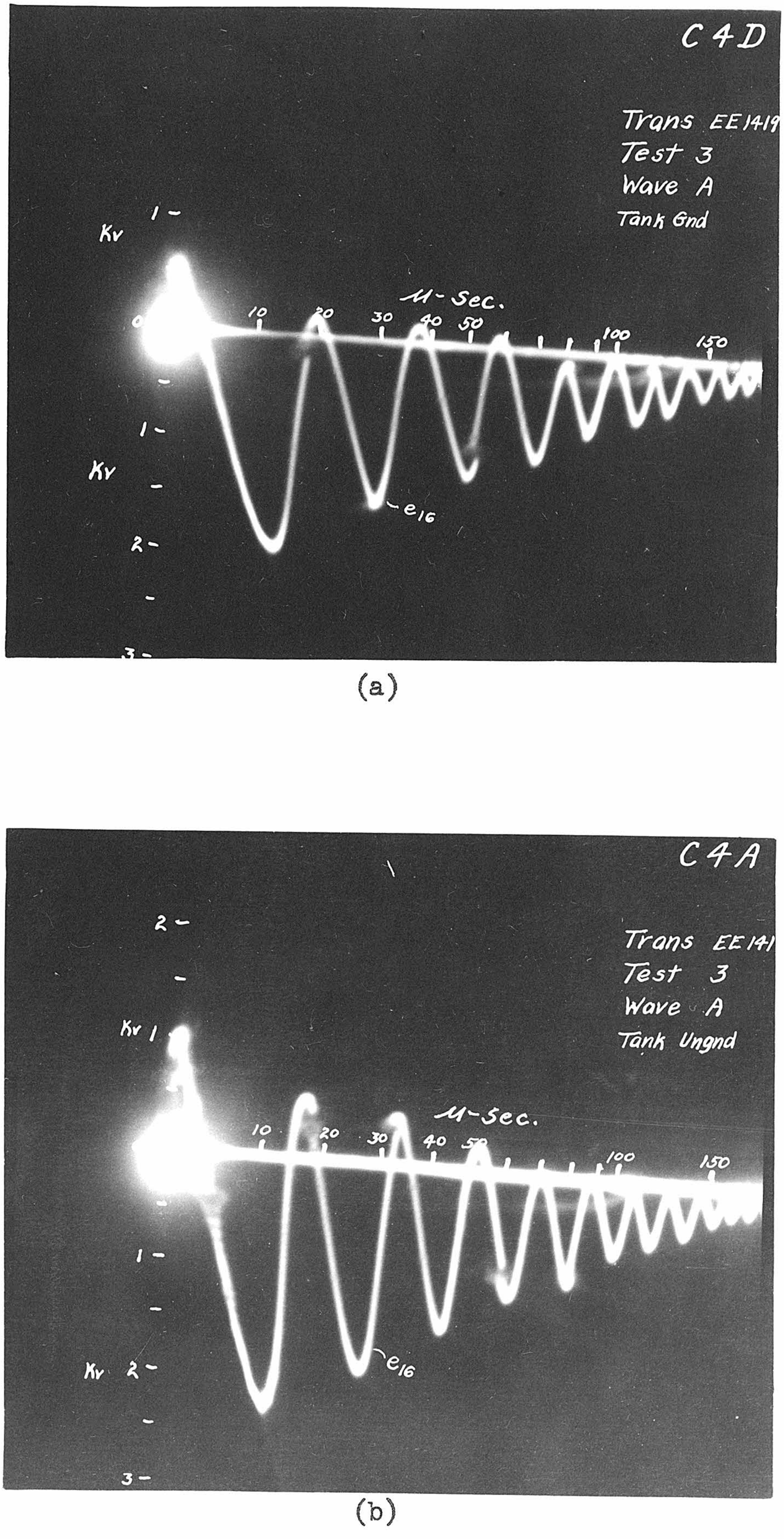

C 4 C
Trans EE1419
Test 3
Wave A
Tank Ungnd
Kv
2
1
0
1
Kv
2
e13
e17
10
20
30
40
50
100
150
μ-Sec

(a)


C25A
Trans 473
Test 3
Wave A
Tank Ungnd
-Kv
2
1
e16
e14
5
10
μ-sec
-Kv
1
2
e14
e16
Tank Gnd

(b)

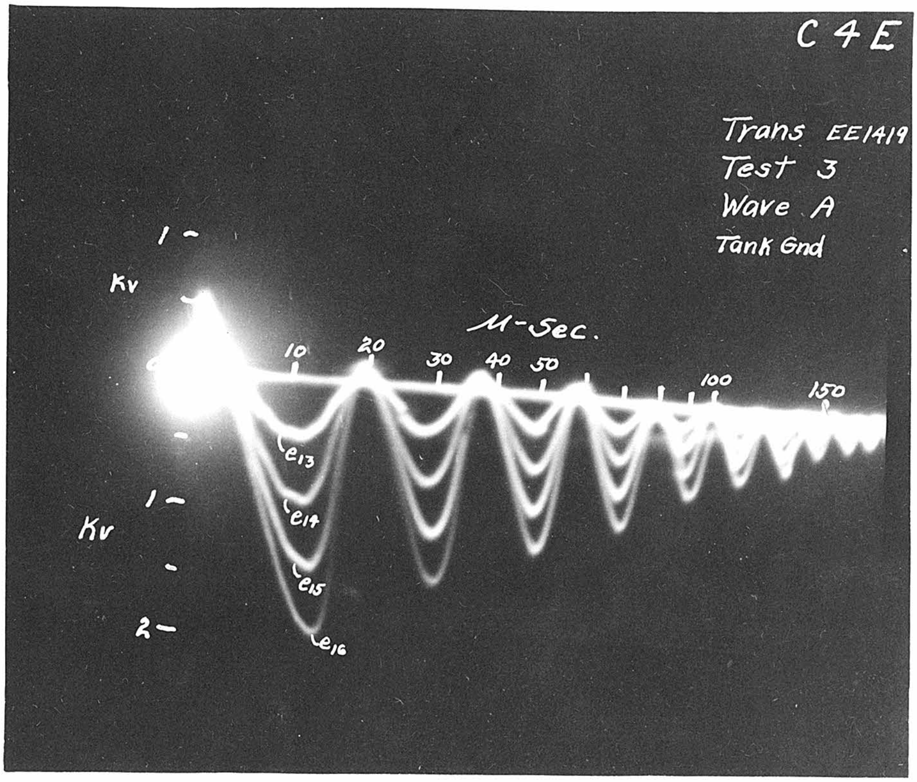
Fig. 112


C 4 D
Trans EE1419
Test 3
Wave A
Tank Gnd
1
Kv
0
1
Kv
2
3
μ-Sec.
10
20
30
40
50
100
150
e16
C 4 A
2
Trans EE141
Test 3
Wave A
Tank Ungnd
Kv 1
μ-Sec.
10
20
30
40
50
100
150
1
Kv 2
e16
3

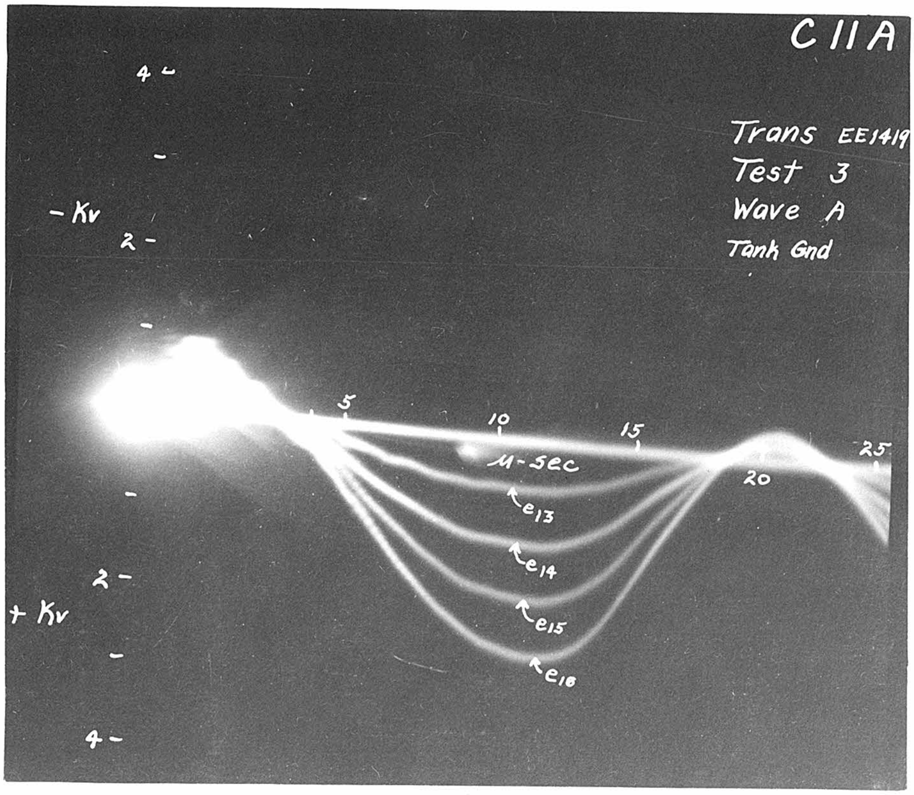
(a)

(b)

Fig. 113

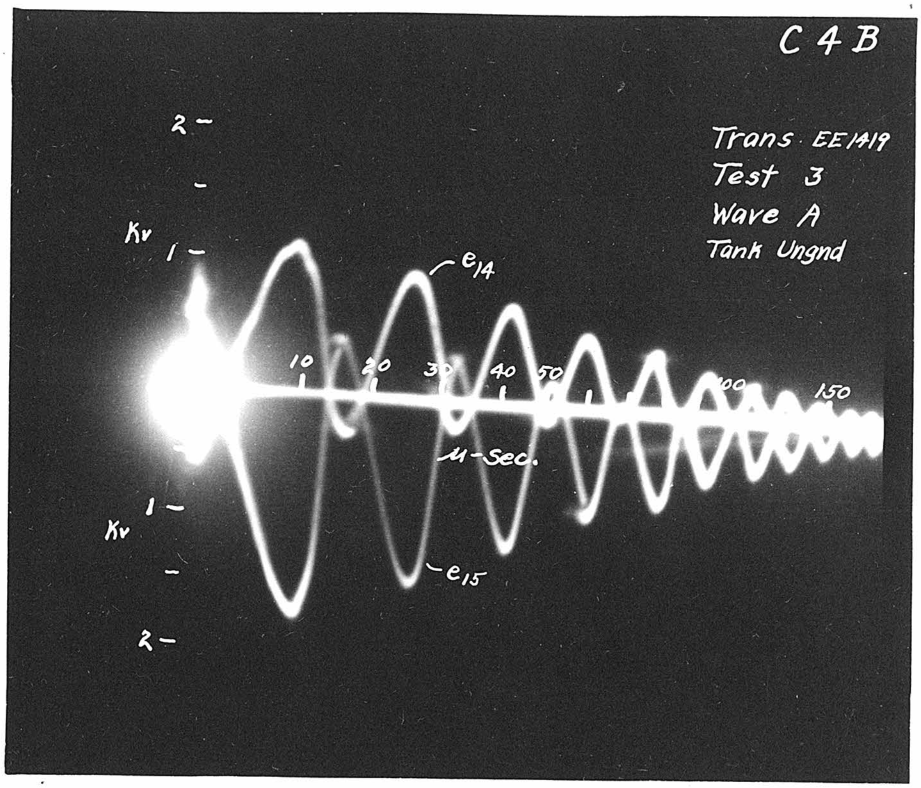

C 4 E
Trans EE1419
Test 3
Wave A
Tank Gnd
1
Kv
0
1
Kv
2
μ-Sec.
10
20
30
40
50
100
150
e13
e14
e15
e16

(a)

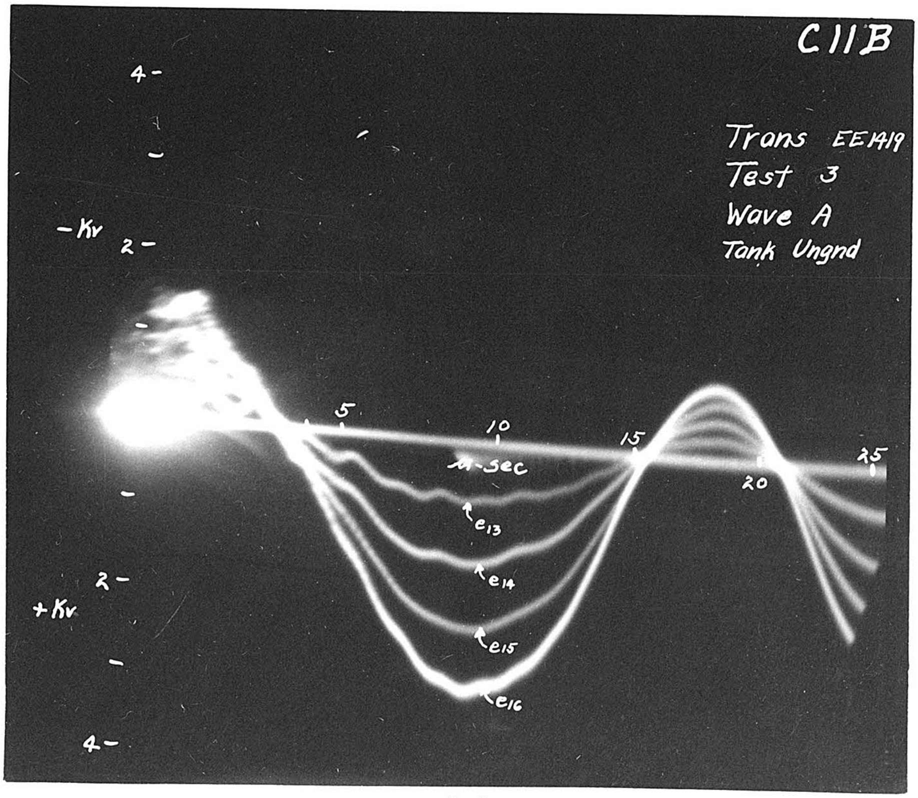

C 11 A
Trans EE1419
Test 3
Wave A
Tank Gnd
4
- Kv
2
2
+ Kv
4
5
10
15
20
25
μ-Sec
e13
e14
e15
e16

(b)

Fig. 114

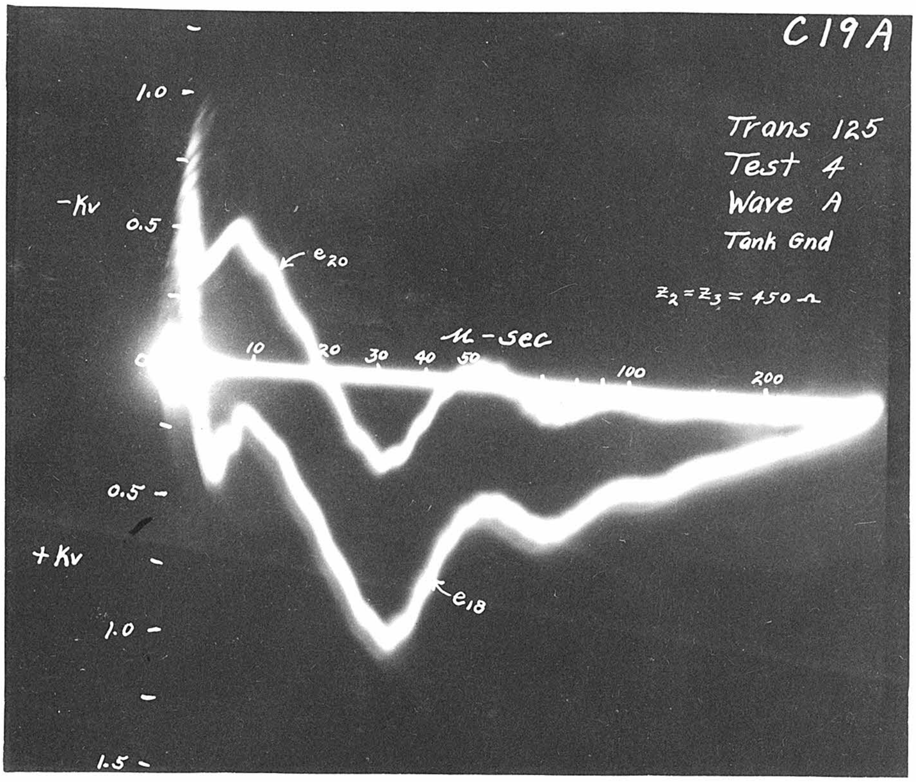

C 4 B
Trans EE1419
Test 3
Wave A
Tank Ungnd
2
Kv
1
e14
10
20
30
40
50
100
150
μ-sec.
1
Kv
e15
2

(a)

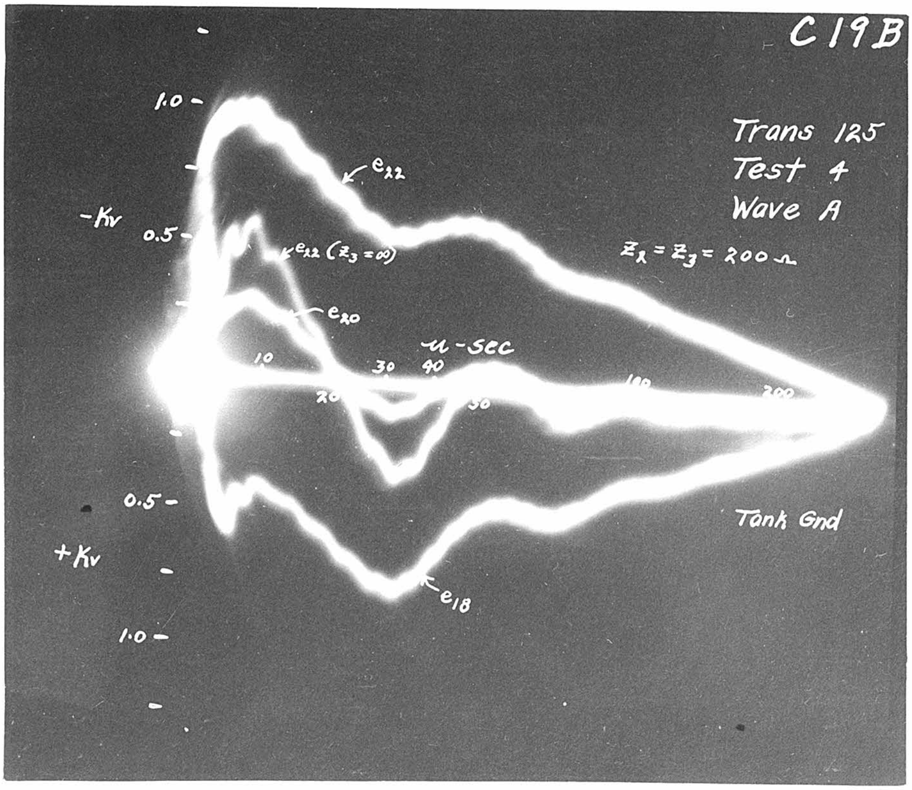

C11B
Trans EE1419
Test 3
Wave A
Tank Ungnd
4
-Kv
2
5
10
15
20
25
μ-sec
e13
e14
e15
e16
2
+Kv
4

(b)

Fig. 115

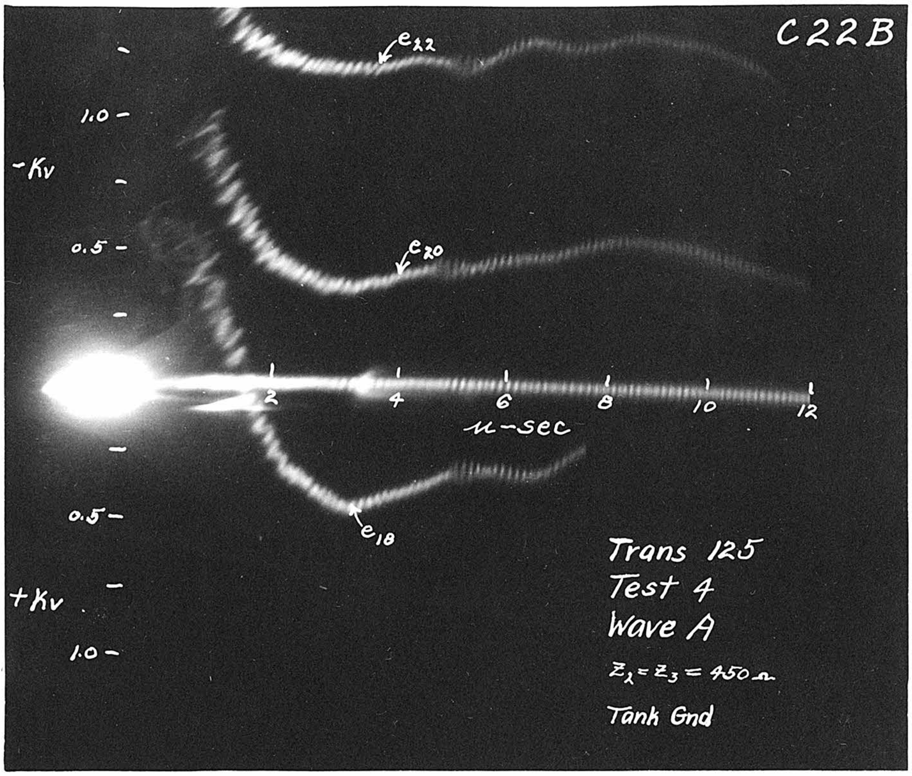

C19A
Trans 125
Test 4
Wave A
Tank Gnd
$Z_2 = Z_3 = 450\ \Omega$
1.0
0.5
-Kv
+Kv
0.5
1.0
1.5
e_{20}
e_{18}
μ-sec
10
20
30
40
50
100
200

(a)

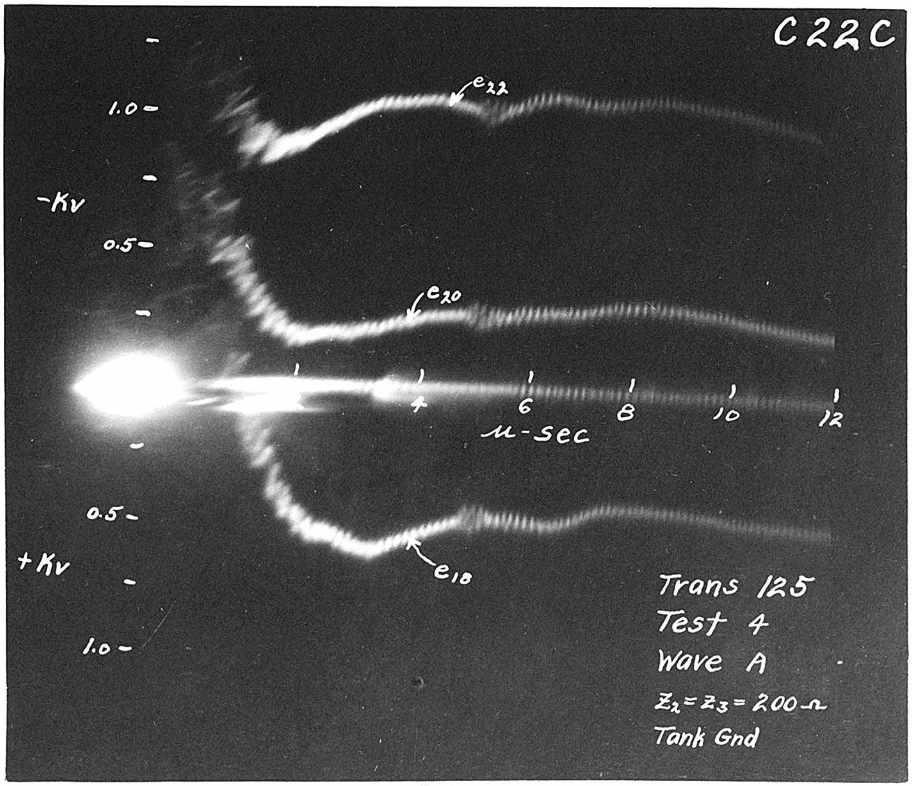

C19B
Trans 125
Test 4
Wave A
$Z_2 = Z_3 = 200\ \Omega$
Tank Gnd
1.0
0.5
-Kv
+Kv
0.5
1.0
e_{22}
e_{22} ($Z_3 = \infty$)
e_{20}
e_{18}
μ-sec
10
20
30
40
50
100
200

(b)

Fig. 116

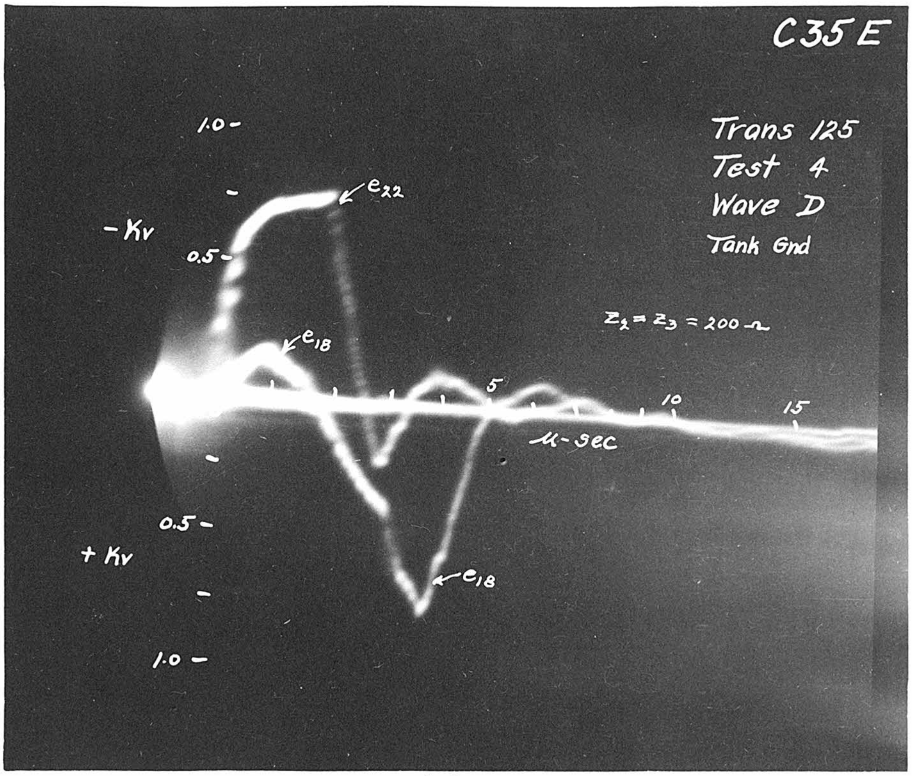

C22B
e_{22}
e_{20}
e_{18}
1.0
-Kv
0.5
0.5
+Kv
1.0
2
4
6
8
10
12
μ-sec
Trans 125
Test 4
Wave A
$Z_2 = Z_3 = 450\,\Omega$
Tank Gnd

(a)

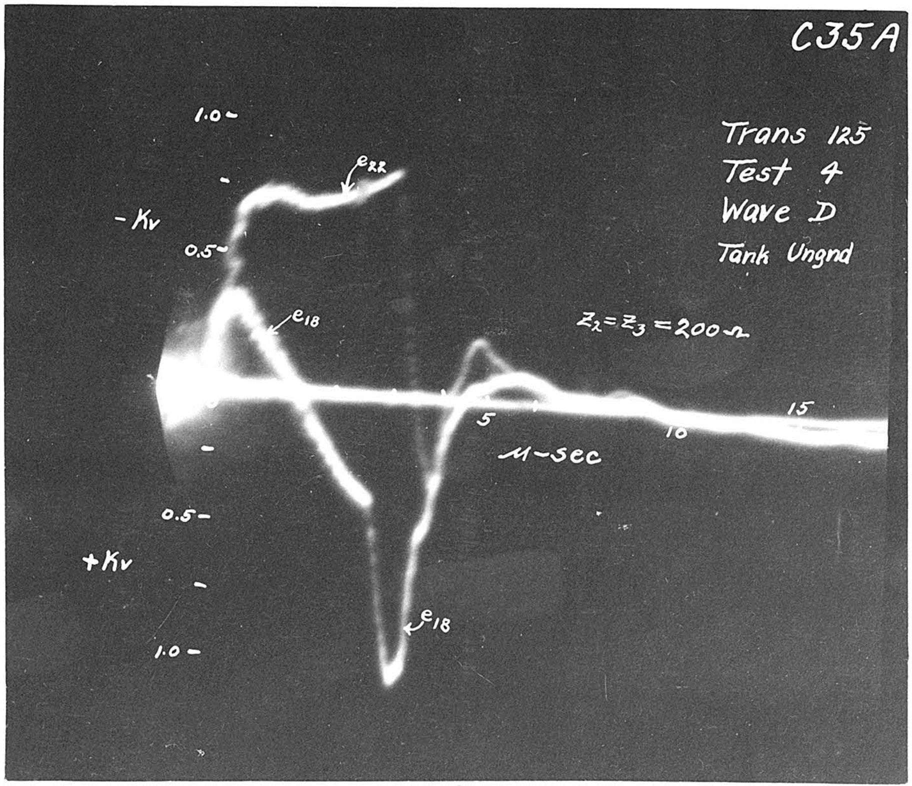

C22C
e_{22}
e_{20}
e_{18}
1.0
-Kv
0.5
0.5
+Kv
1.0
4
6
8
10
12
μ-sec
Trans 125
Test 4
Wave A
$Z_2 = Z_3 = 200\,\Omega$
Tank Gnd

(b)

Fig. 117

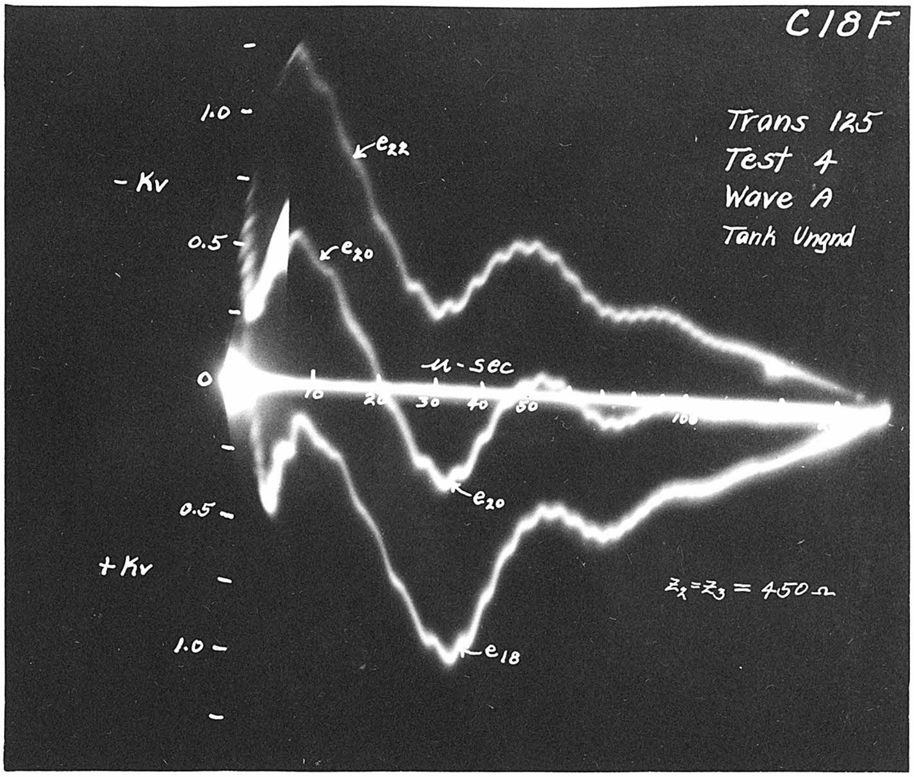

C35E
Trans 125
Test 4
Wave D
Tank Gnd
Z₂ = Z₃ = 200 Ω
1.0
0.5
-Kv
+Kv
0.5
1.0
e22
e18
e18
5
10
15
μ-sec

(a)

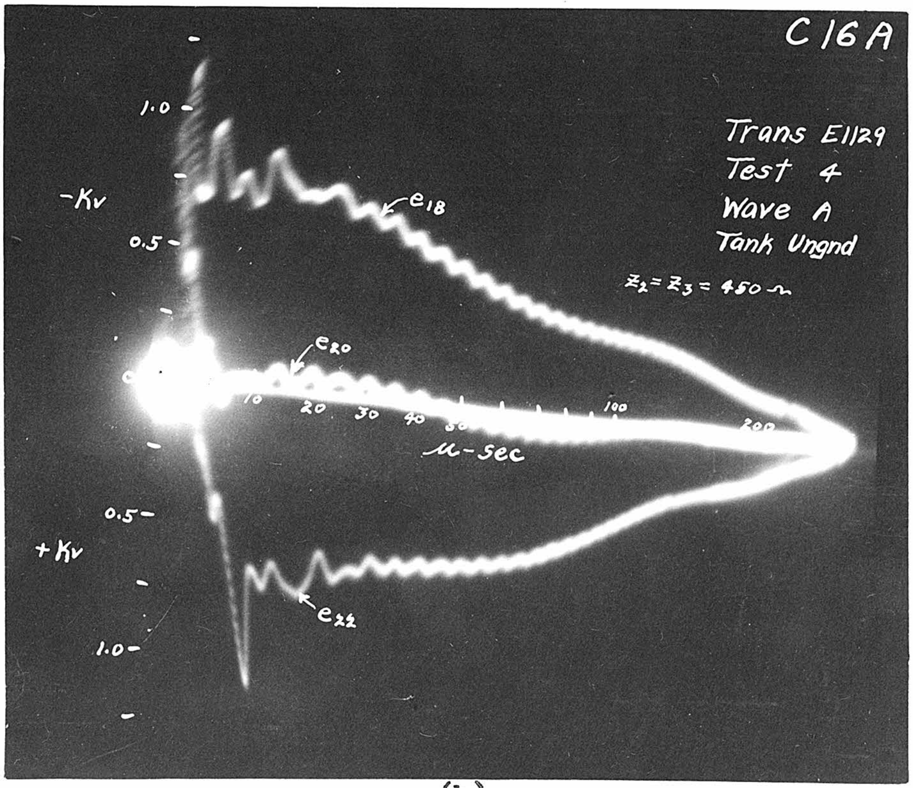

C35A
Trans 125
Test 4
Wave D
Tank Ungnd
Z₂ = Z₃ = 200 Ω
1.0
0.5
-Kv
+Kv
0.5
1.0
e22
e18
e18
5
10
15
μ-sec

(b)

Fig. 118

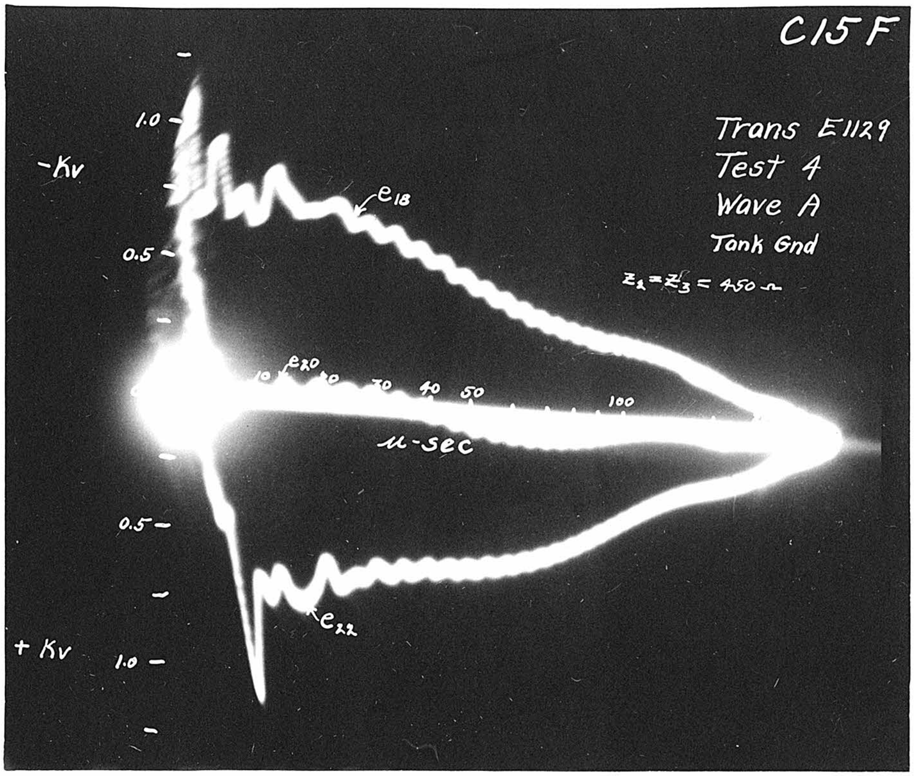

C18F
Trans 125
Test 4
Wave A
Tank Ungnd
1.0
-Kv
0.5
0
0.5
+Kv
1.0
e22
e20
μ-sec
10
20
30
40
50
100
e20
e18
Z2=Z3 = 450 Ω

(a)

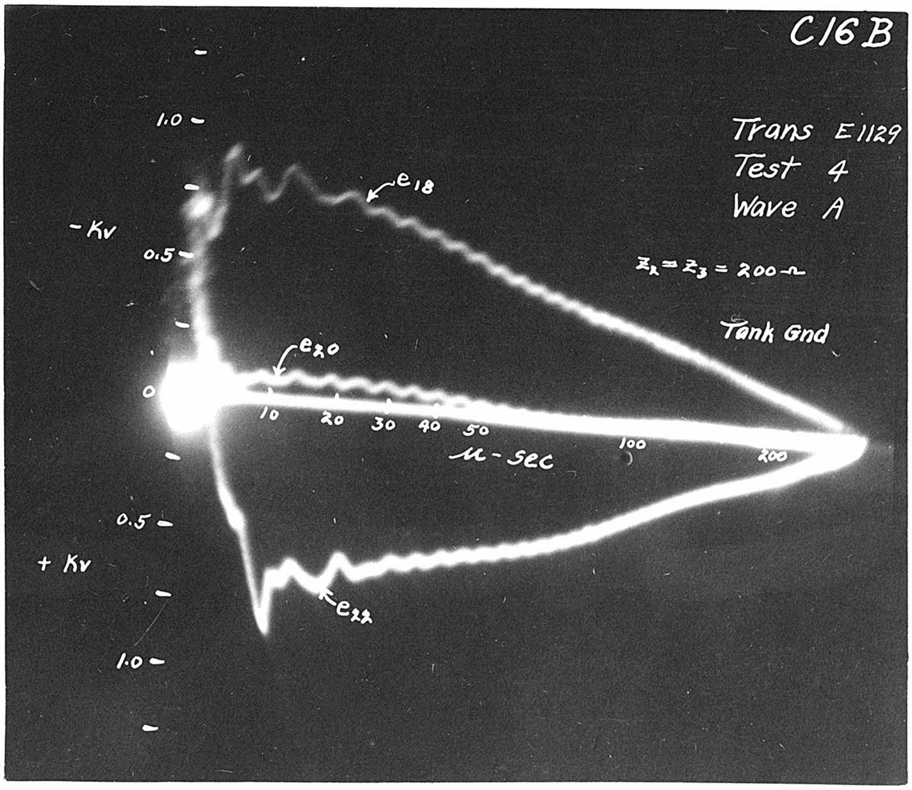

C16A
Trans E1129
Test 4
Wave A
Tank Ungnd
Z2=Z3 = 450 Ω
1.0
-Kv
0.5
e18
e20
10
20
30
40
50
100
200
μ-sec
0.5
+Kv
1.0
e22

(b)

Fig. 119

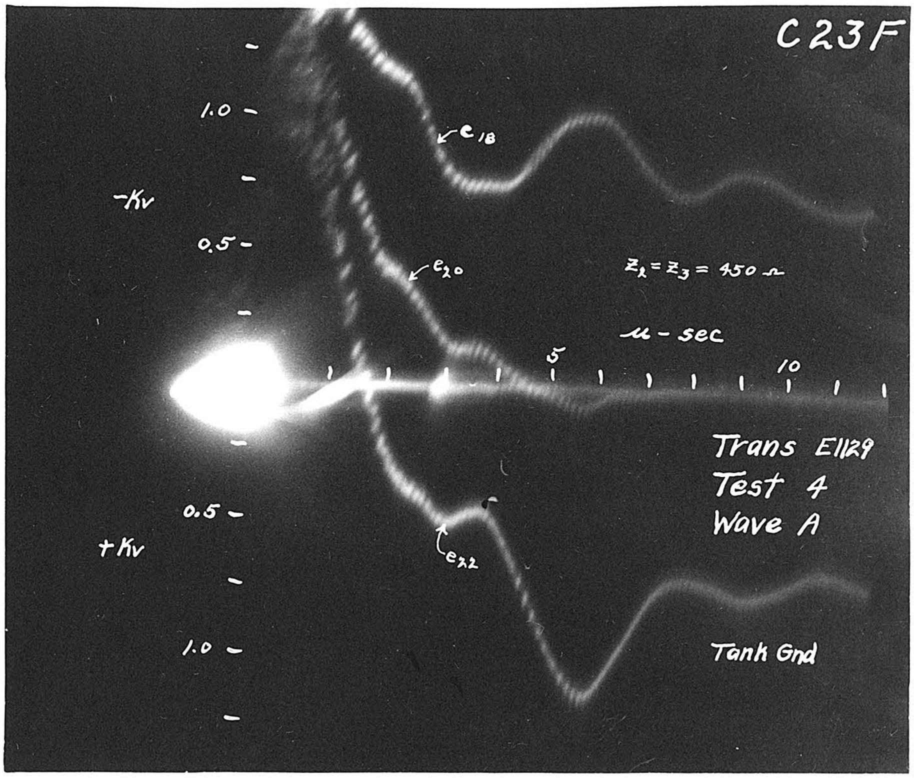

C15F
Trans E1129
Test 4
Wave A
Tank Gnd
$Z_2 = Z_3 = 450\,\Omega$
-Kv
+Kv
1.0
0.5
0.5
1.0
e_{18}
e_{20}
e_{22}
10
20
30
40
50
100
μ-sec

(a)

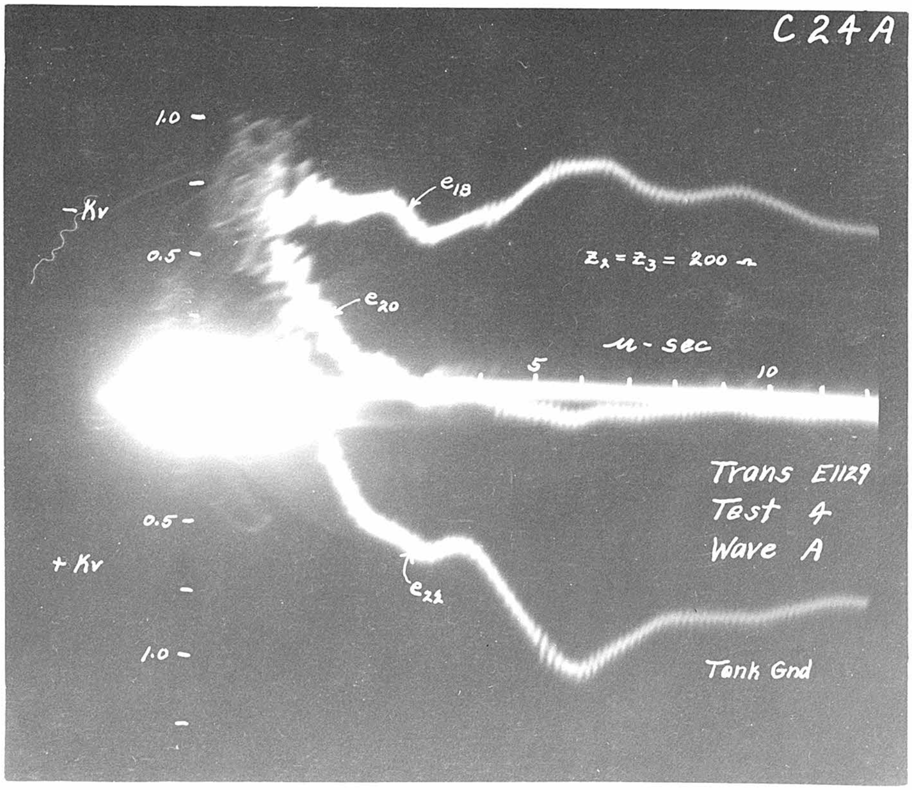

C16B
Trans E1129
Test 4
Wave A
$Z_2 = Z_3 = 200\,\Omega$
Tank Gnd
-Kv
+Kv
1.0
0.5
0
0.5
1.0
e_{18}
e_{20}
e_{22}
10
20
30
40
50
100
200
μ-sec

(b)

Fig. 120

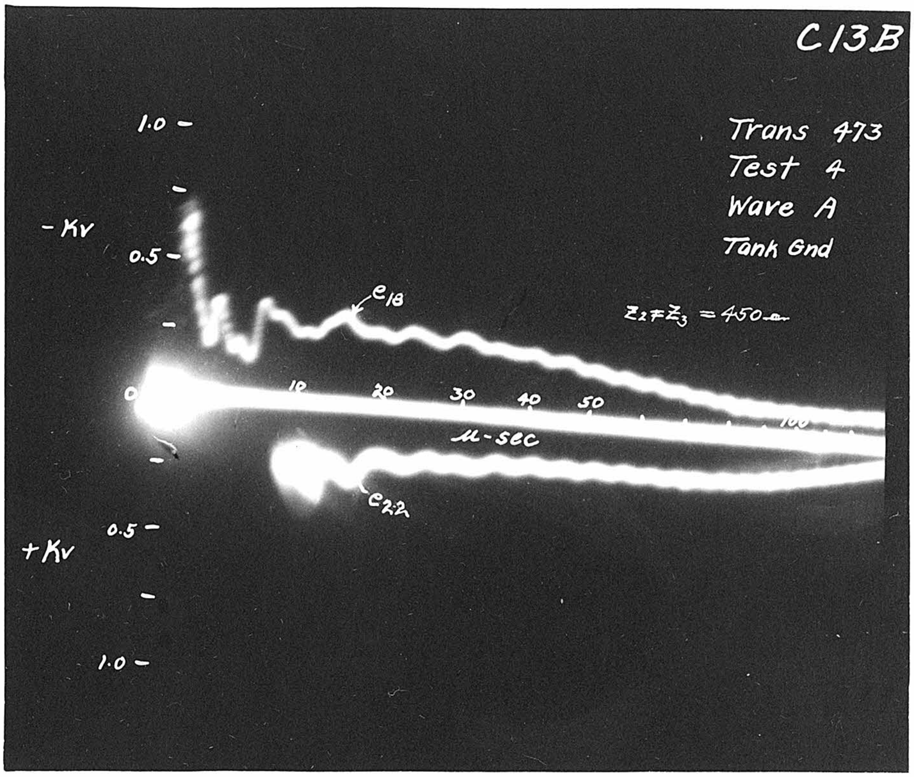

C23F
1.0
-Kv
0.5
e18
e20
Z2 = Z3 = 450 Ω
μ-sec
5
10
Trans E1129
Test 4
Wave A
0.5
+Kv
e22
1.0
Tank Gnd

(a)

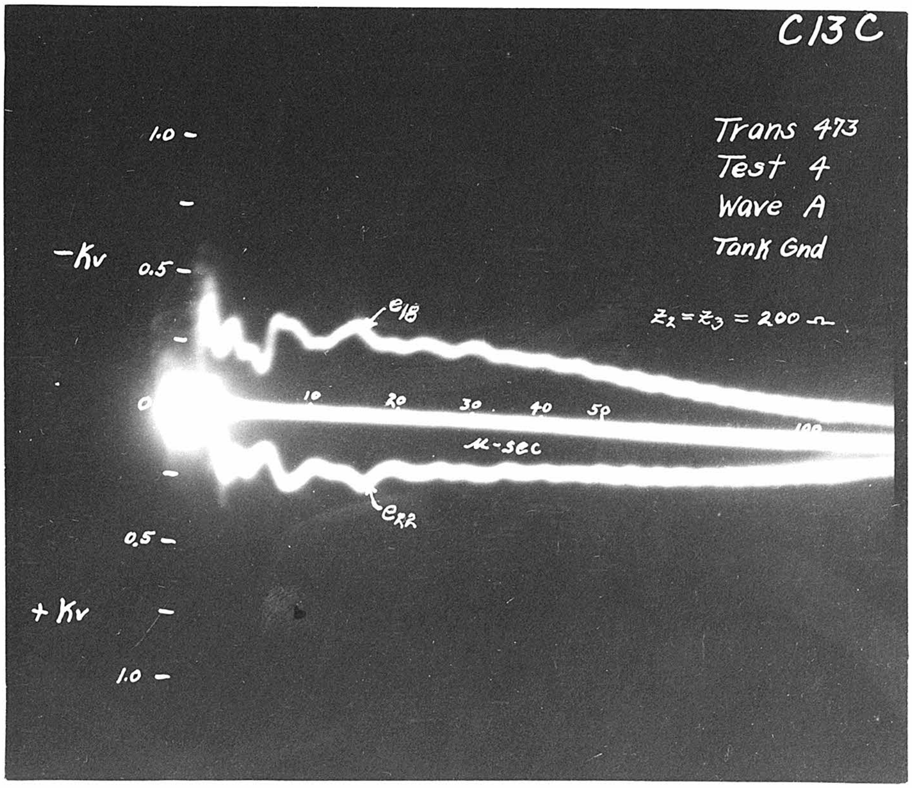

C24A
1.0
-Kv
0.5
e18
Z2 = Z3 = 200 Ω
e20
μ-sec
5
10
Trans E1129
Test 4
Wave A
0.5
+Kv
e22
1.0
Tank Gnd

(b)

Fig. 121

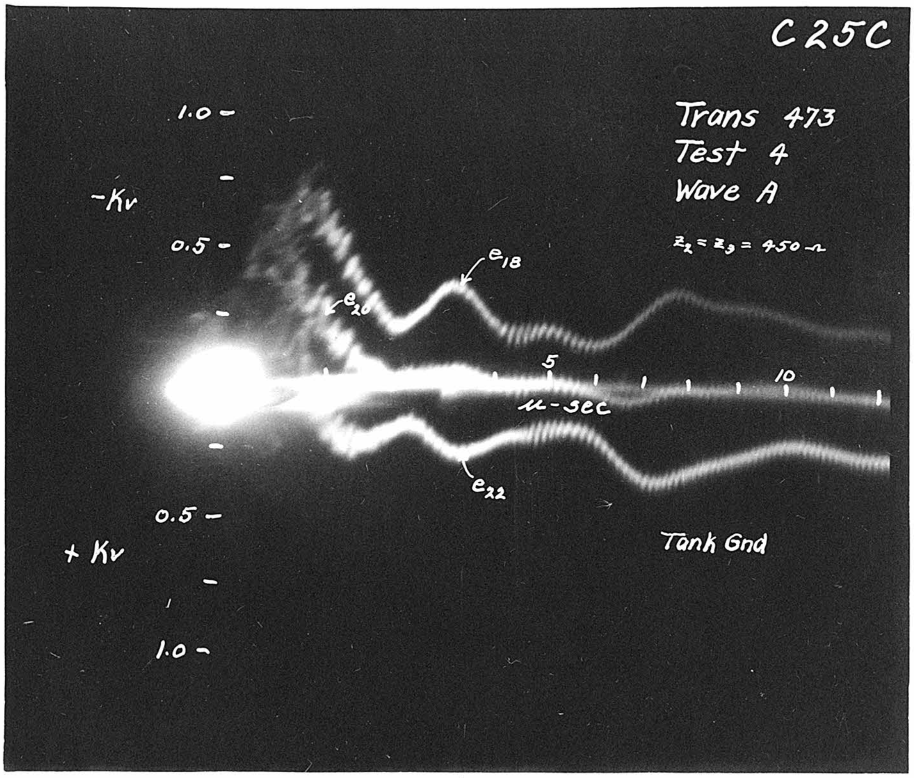

C13B
Trans 473
Test 4
Wave A
Tank Gnd
$Z_2 \neq Z_3 = 450 \Omega$
1.0
0.5
0
0.5
1.0
-Kv
+Kv
e_{1B}
$e_{2.2}$
10
20
30
40
50
100
μ-sec

(a)

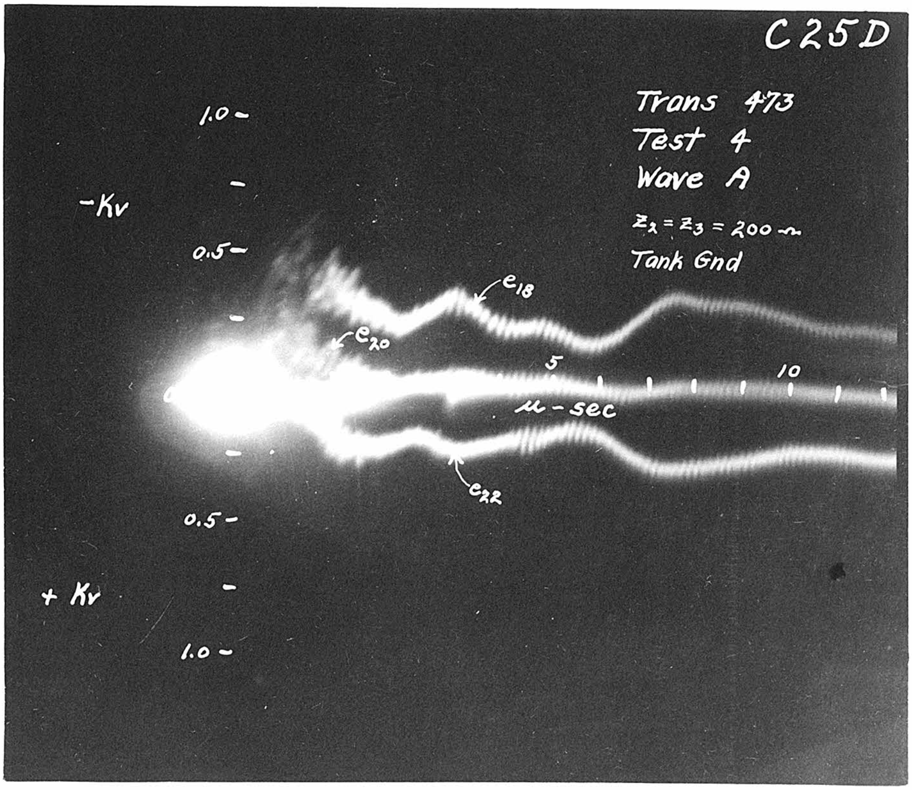

C13C
Trans 473
Test 4
Wave A
Tank Gnd
$Z_2 = Z_3 = 200 \Omega$
1.0
0.5
0
0.5
1.0
-Kv
+Kv
e_{1B}
$e_{2.2}$
10
20
30
40
50
100
μ-sec

(b)

Fig. 122

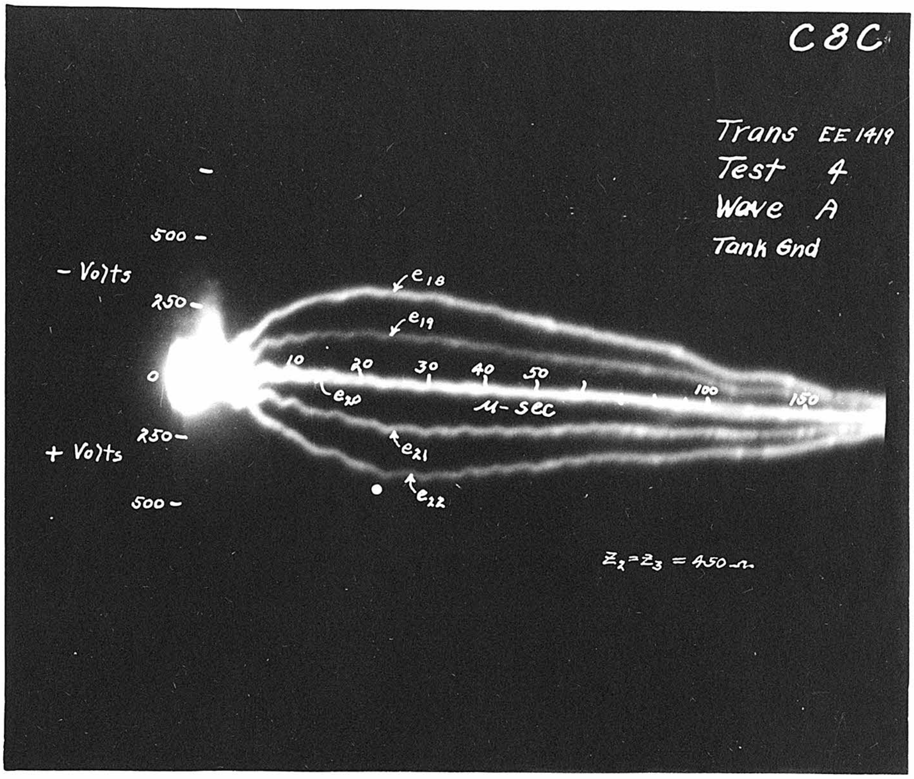

C25C
Trans 473
Test 4
Wave A
$Z_2 = Z_3 = 450\ \Omega$
1.0
-Kv
0.5
e_{18}
e_{20}
5
10
μ-sec
e_{22}
0.5
+Kv
1.0
Tank Gnd

(a)

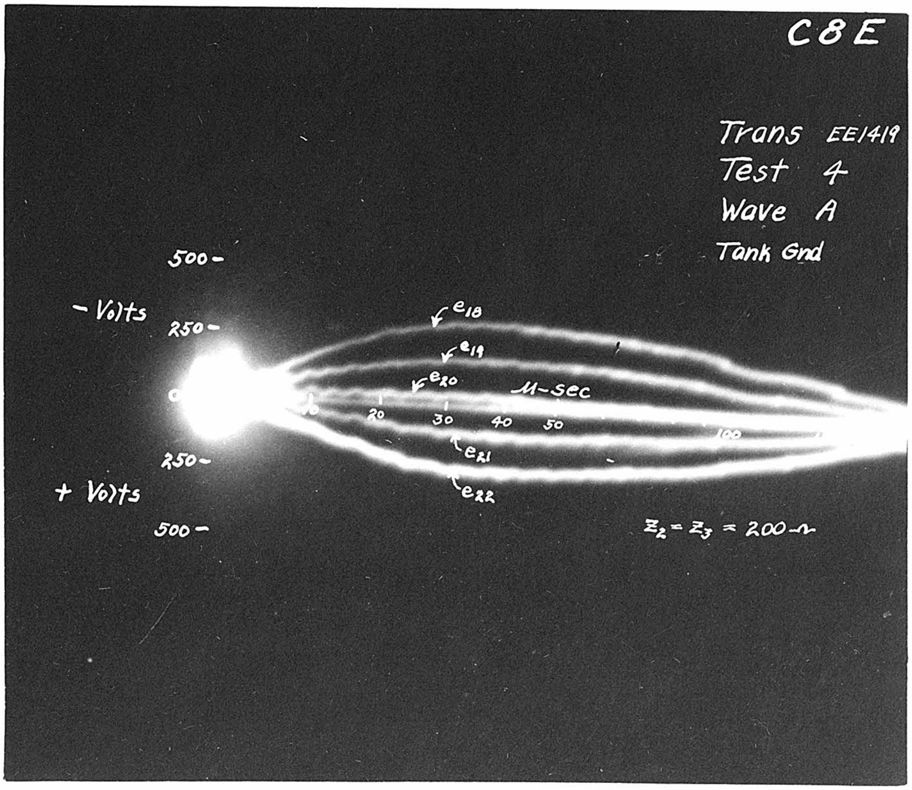

C25D
Trans 473
Test 4
Wave A
$Z_2 = Z_3 = 200\ \Omega$
Tank Gnd
1.0
-Kv
0.5
e_{18}
e_{20}
5
10
μ-sec
e_{22}
0.5
+Kv
1.0

(b)

Fig. 123

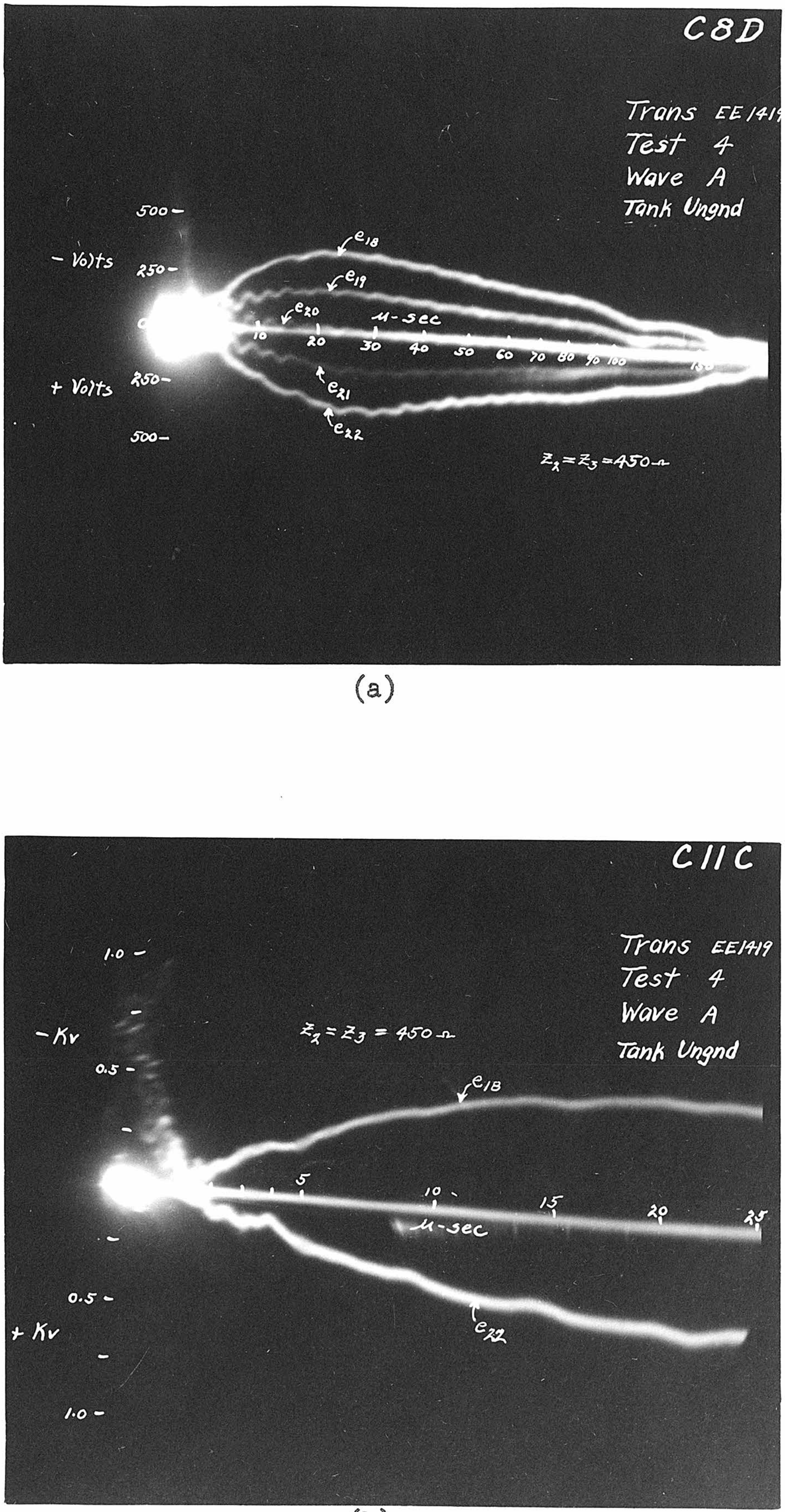

C8C
Trans EE 1419
Test 4
Wave A
Tank Gnd
500
- Volts
250
0
250
+ Volts
500
e18
e19
e20
e21
e22
10
20
30
40
50
100
150
μ-sec
$Z_2 = Z_3 = 450\Omega$

(a)


C8E
Trans EE1419
Test 4
Wave A
Tank Gnd
500
- Volts
250
0
250
+ Volts
500
e18
e19
e20
e21
e22
μ-sec
20
30
40
50
100
$Z_2 = Z_3 = 200\Omega$

(b)

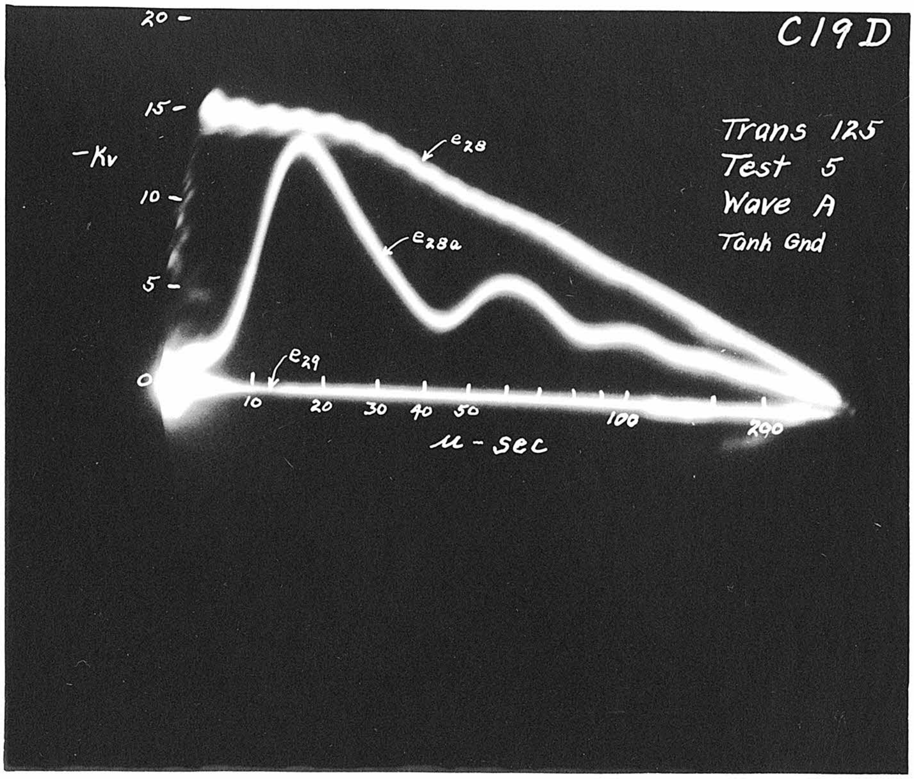
Fig. 124


C8D
Trans EE1419
Test 4
Wave A
Tank Ungnd
500
- Volts
250
0
+ Volts
250
500
e_{18}
e_{19}
e_{20}
e_{21}
e_{22}
μ-sec
10
20
30
40
50
60
70
80
90
100
150
$Z_2 = Z_3 = 450\Omega$
C11C
Trans EE1419
Test 4
Wave A
Tank Ungnd
1.0
- Kv
0.5
0.5
+ Kv
1.0
$Z_2 = Z_3 = 450\Omega$
e_{18}
e_{22}
5
10
15
20
25
μ-sec

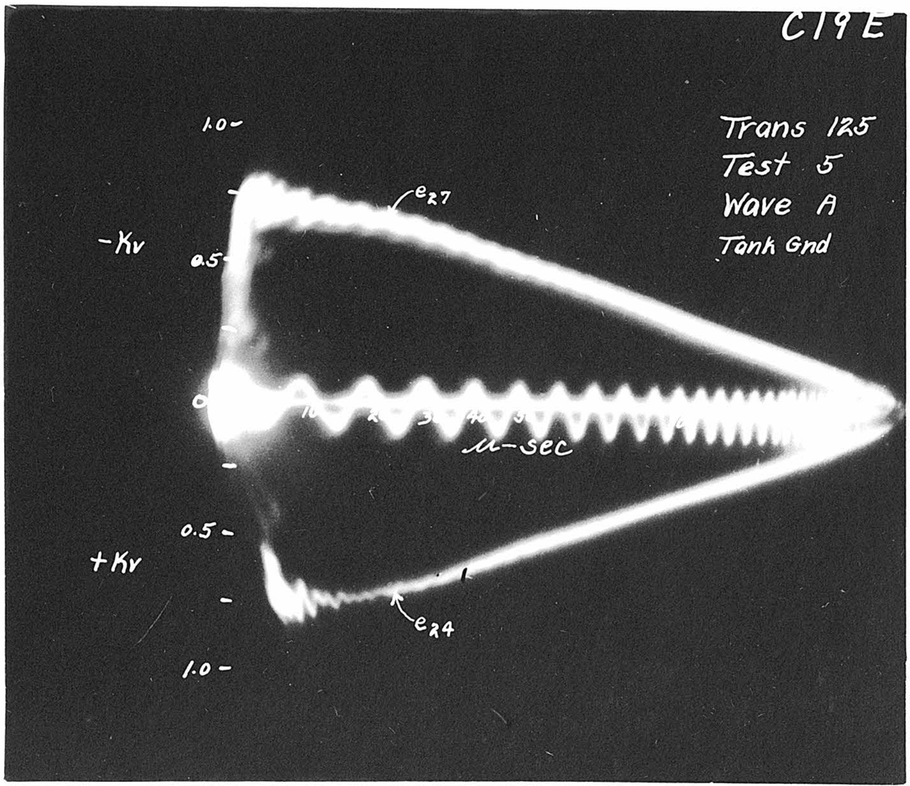
(a)

(b)

Fig. 125

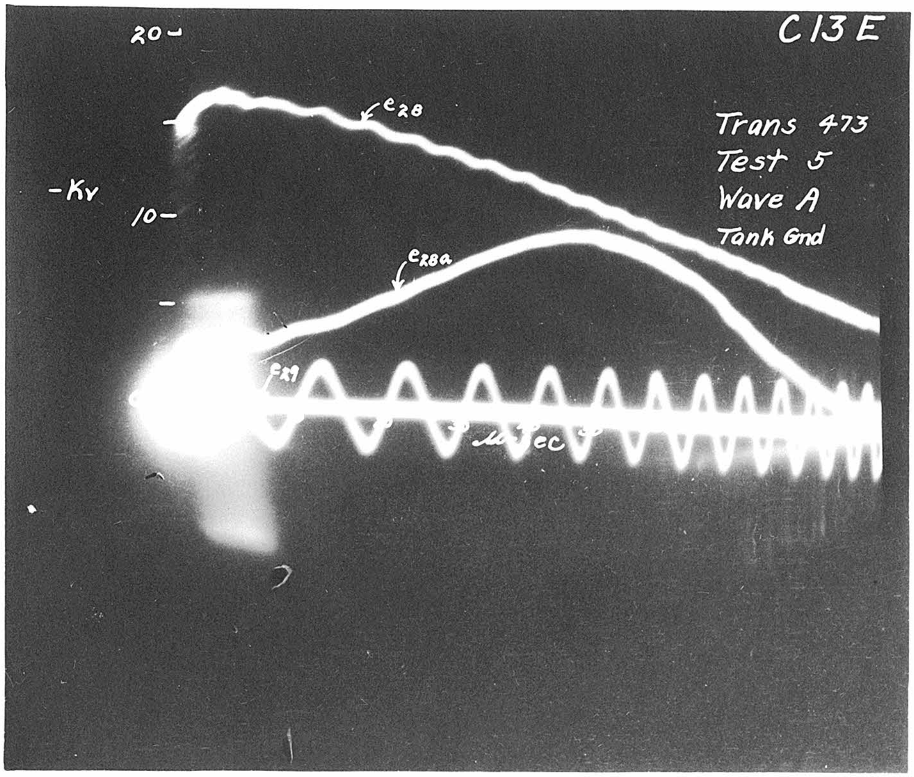

C19D
Trans 125
Test 5
Wave A
Tank Gnd
-Kv
20
15
10
5
0
e28
e28a
e29
10
20
30
40
50
100
200
µ-sec

(a)

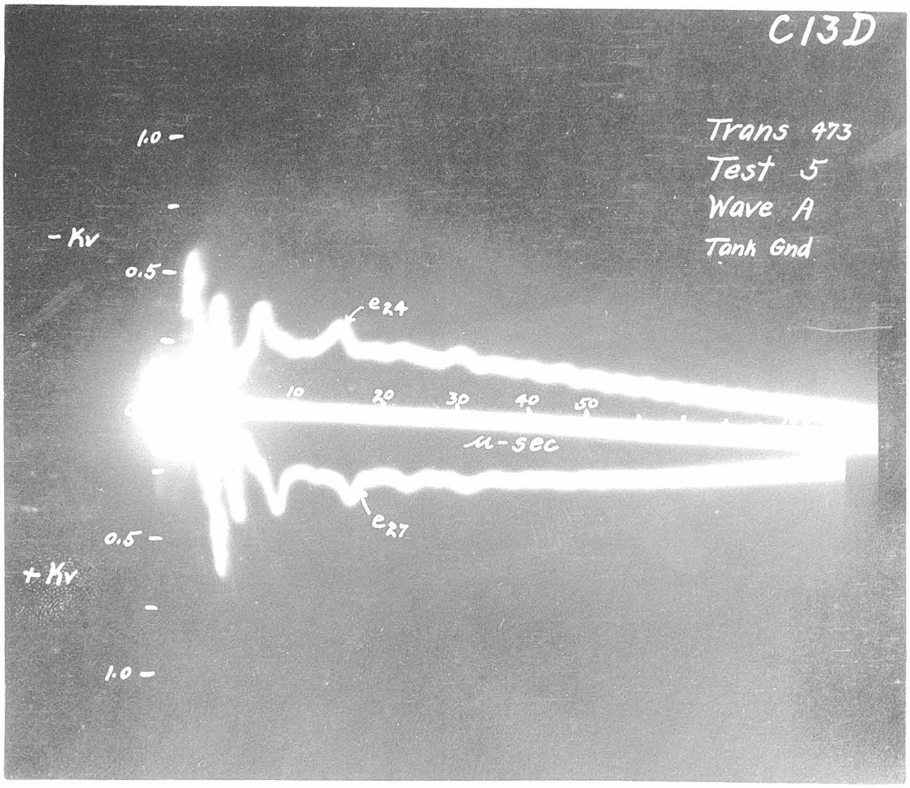

C19E
Trans 125
Test 5
Wave A
Tank Gnd
-Kv
+Kv
1.0
0.5
0
0.5
1.0
e27
e24
10
20
30
40
50
100
µ-sec

(b)

Fig. 126

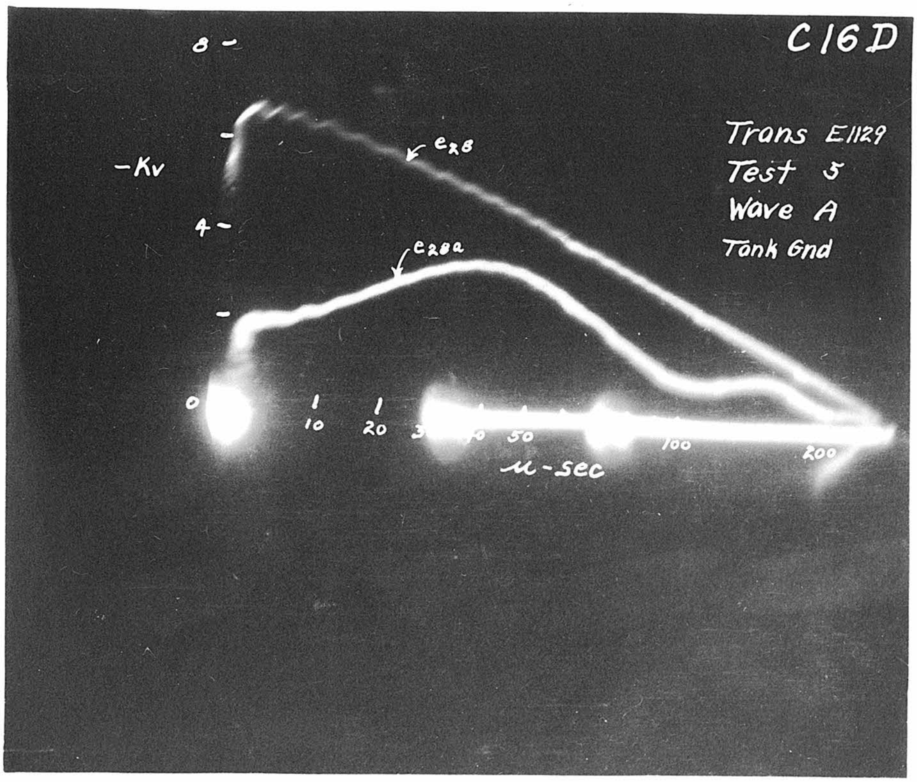

C13E
20
10
-Kv
e28
e28a
e29
μ-sec
Trans 473
Test 5
Wave A
Tank Gnd

(a)

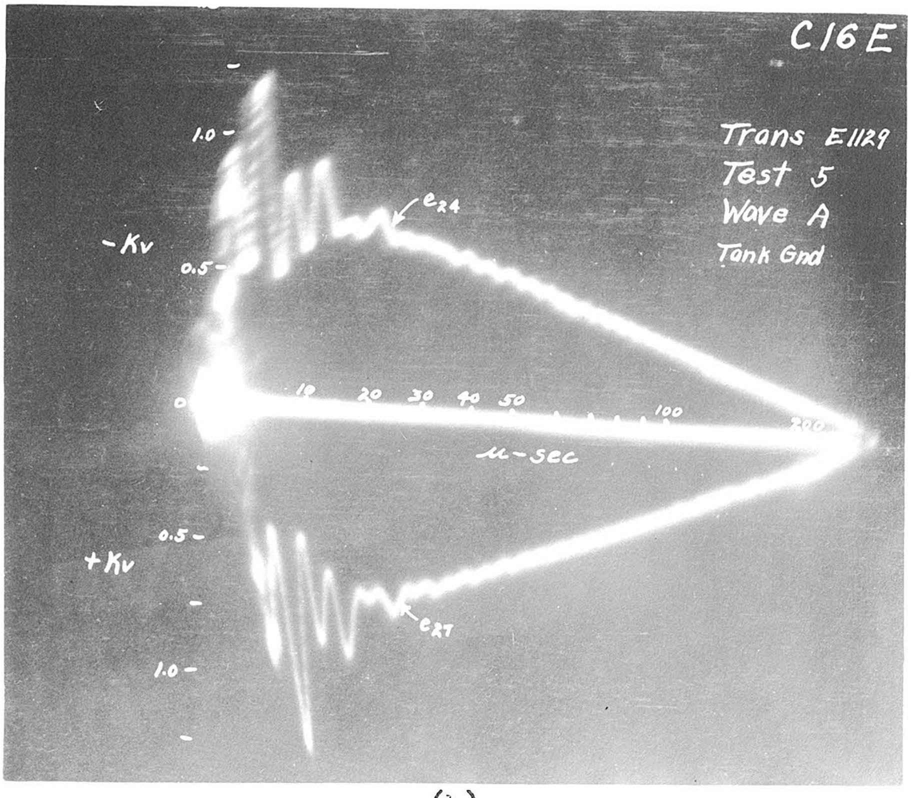

C13D
1.0
0.5
-Kv
e24
10
20
30
40
50
μ-sec
e27
0.5
+Kv
1.0
Trans 473
Test 5
Wave A
Tank Gnd

(b)

Fig. 127

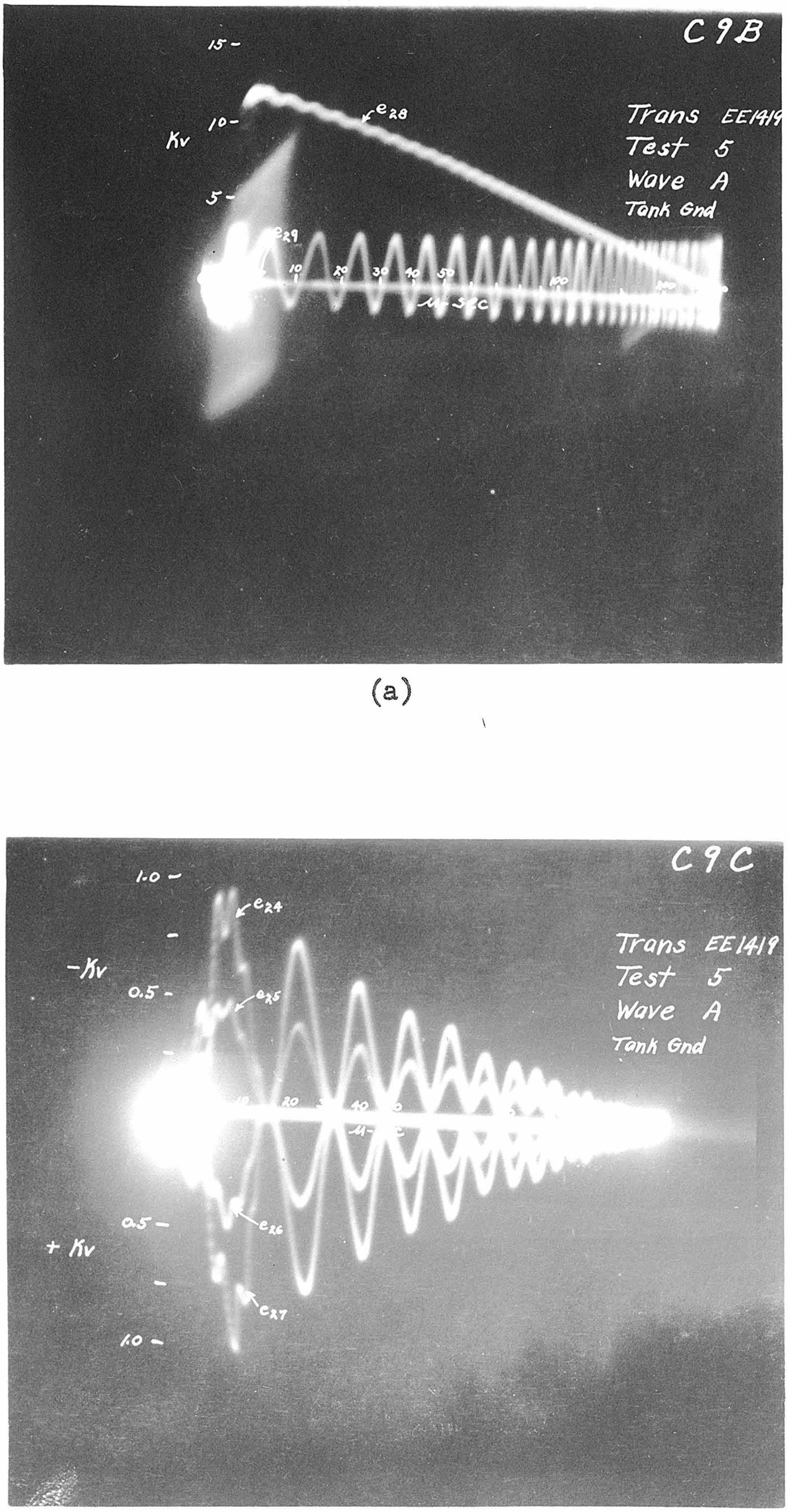

C16D
8
-Kv
4
0
e28
e28a
10
20
30
40
50
100
200
μ-sec
Trans E1129
Test 5
Wave A
Tank Gnd

(a)


C16E
1.0
-Kv
0.5
0
e24
10
20
30
40
50
100
200
μ-sec
0.5
+Kv
1.0
e27
Trans E1129
Test 5
Wave A
Tank Gnd

(b)

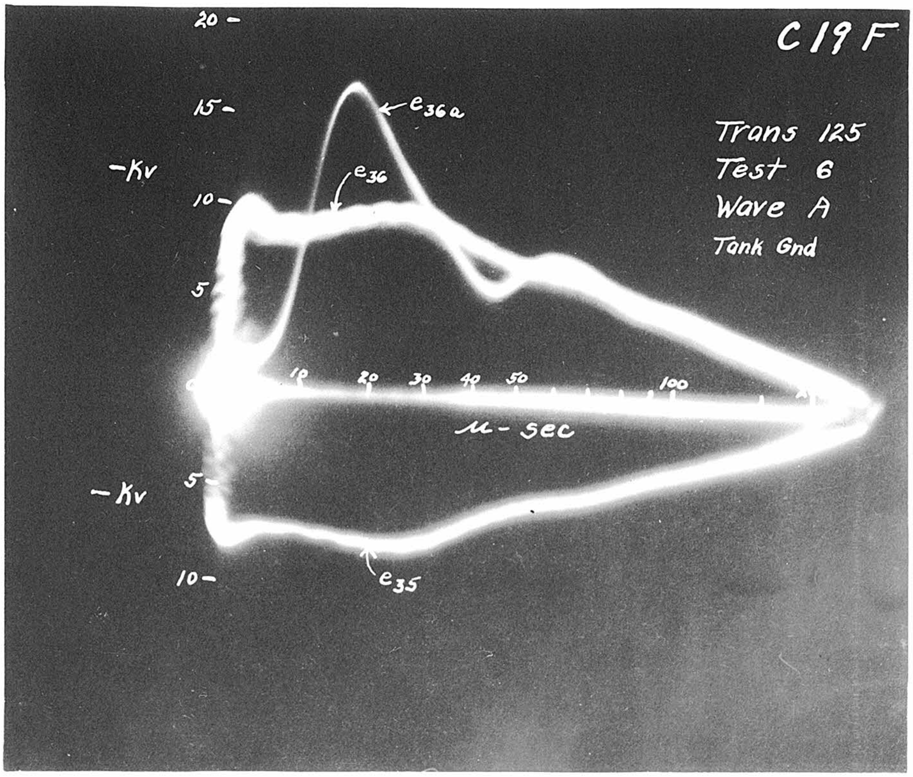
Fig. 128


C 9 B
15 -
10 -
Kv
5 -
e28
e29
10
20
30
40
50
100
μ-sec
Trans EE1419
Test 5
Wave A
Tank Gnd
C 9 C
1.0 -
-Kv
0.5 -
e24
e25
10
20
30
40
μ-sec
0.5 -
+ Kv
1.0 -
e26
e27
Trans EE1419
Test 5
Wave A
Tank Gnd

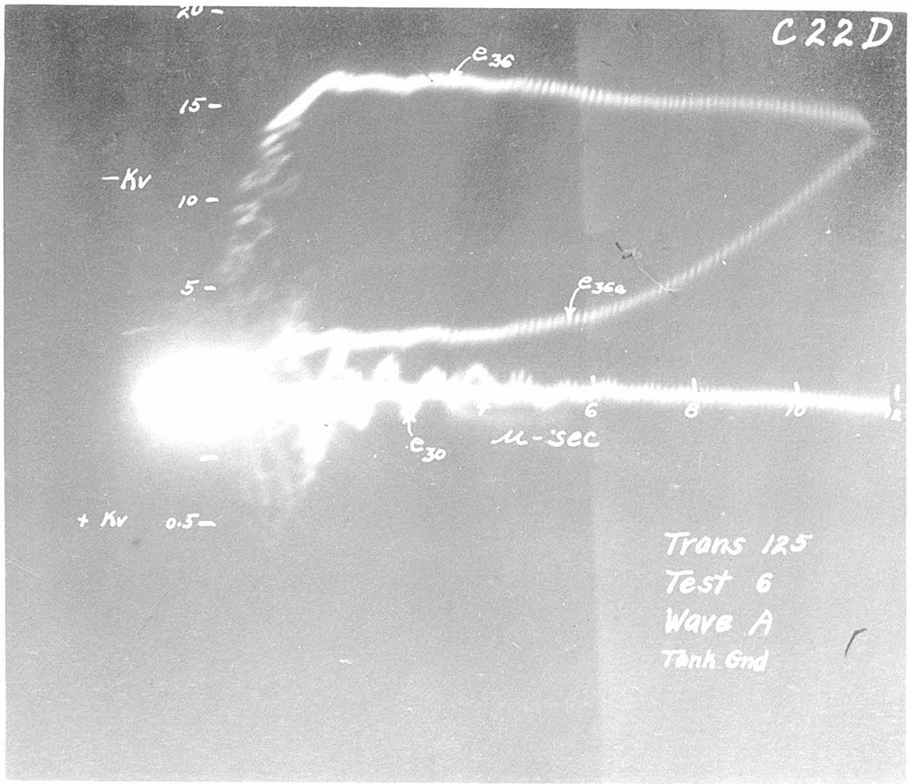
(a)

(b)

Fig. 129

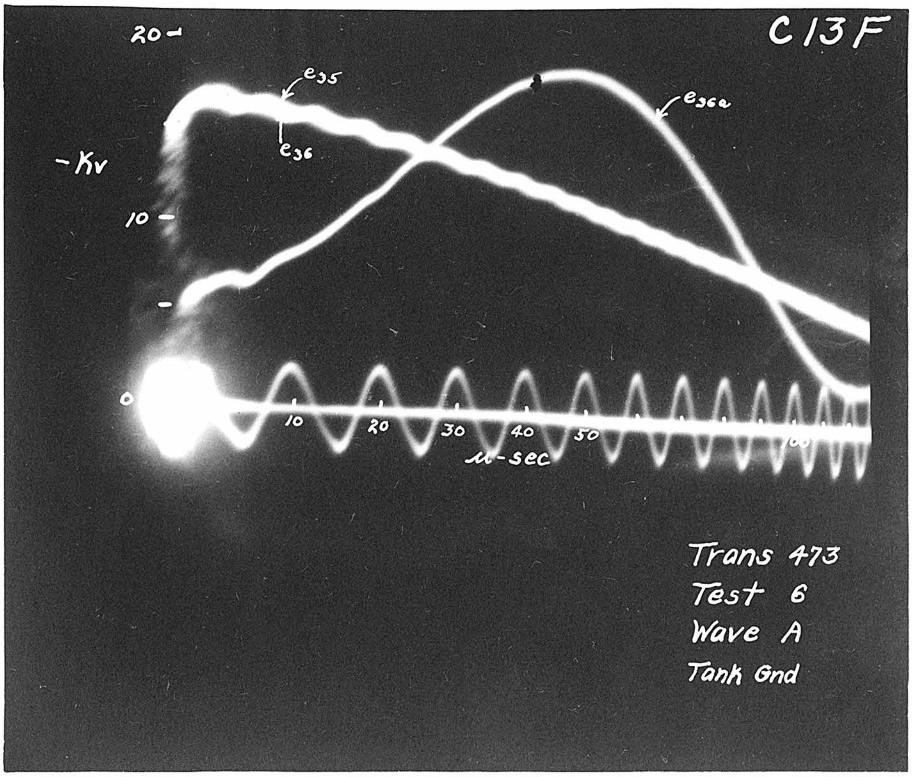

C19F
20
15
10
5
-Kv
e_{36a}
e_{36}
Trans 125
Test 6
Wave A
Tank Gnd
10
20
30
40
50
100
μ-sec
-Kv
5
10
e_{35}

(a)

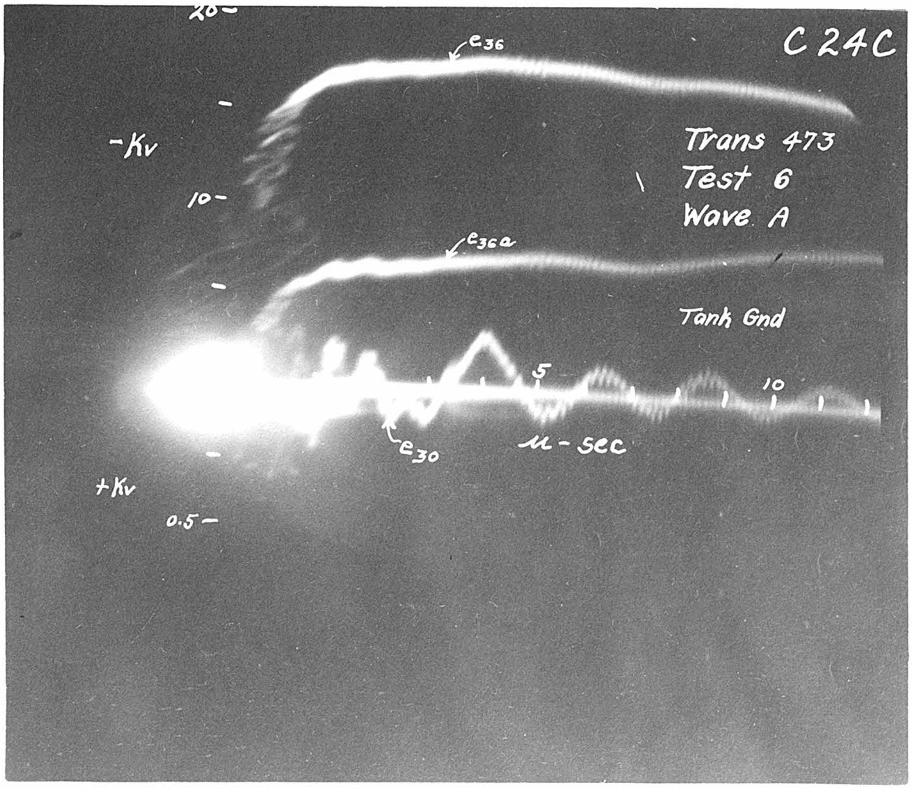

C22D
20
15
10
5
-Kv
e_{36}
e_{36a}
6
8
10
12
μ-sec
e_{30}
+Kv
0.5
Trans 125
Test 6
Wave A
Tank Gnd

(b)

Fig. 130

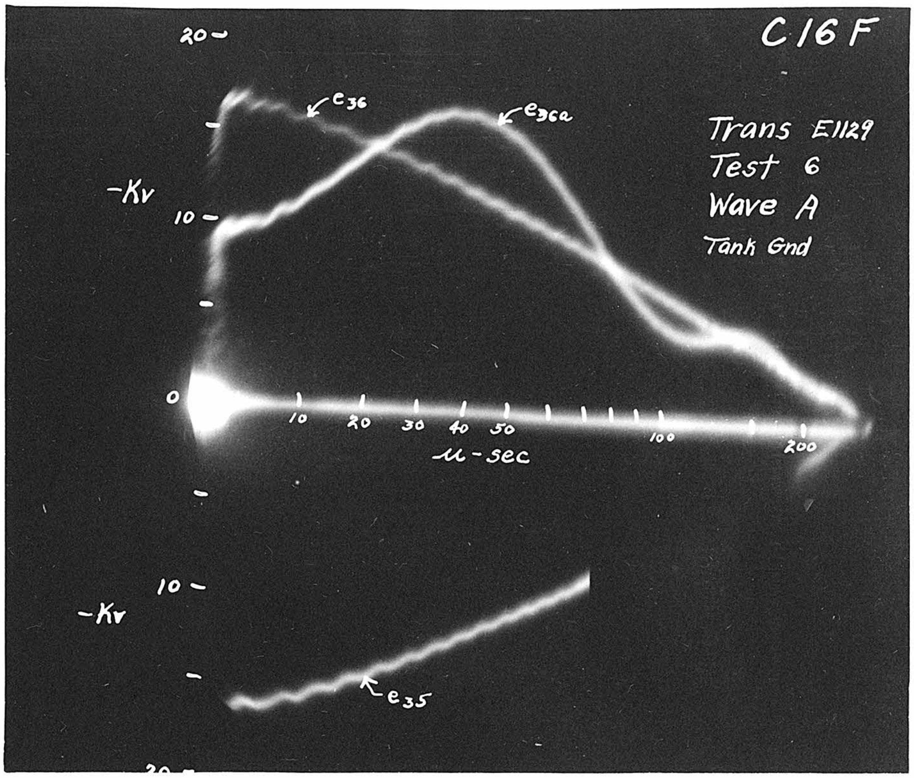

C13F
20
-Kv
10
0
e_{35}
e_{36}
e_{36a}
10
20
30
40
50
100
μ-sec
Trans 473
Test 6
Wave A
Tank Gnd

(a)

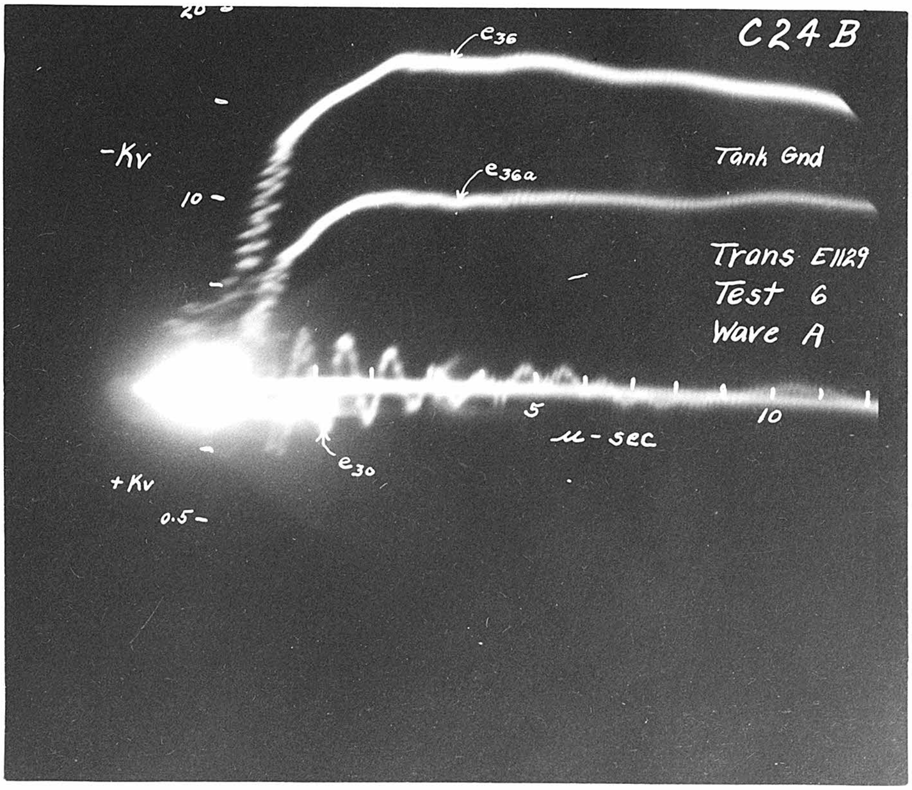

20
C24C
e_{36}
-Kv
10
Trans 473
Test 6
Wave A
e_{36a}
Tank Gnd
5
10
μ-sec
e_{30}
+Kv
0.5

(b)

Fig. 131

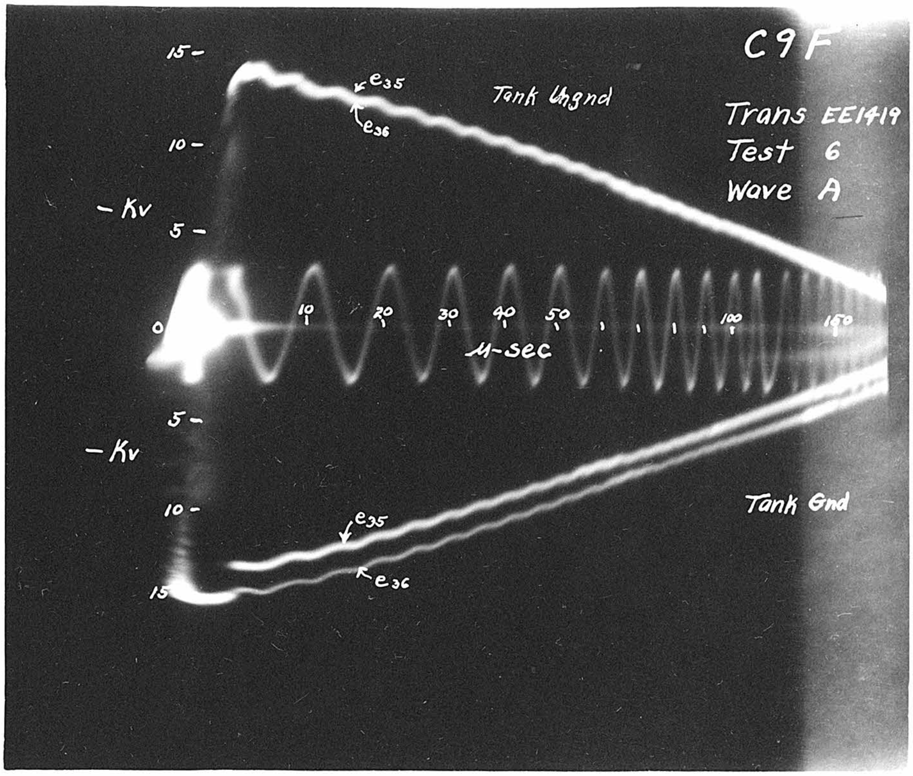

C16F
20
-Kv
10
0
e36
e36a
Trans E1129
Test 6
Wave A
Tank Gnd
10 20 30 40 50 100 200
μ-sec
10
-Kv
e35

(a)

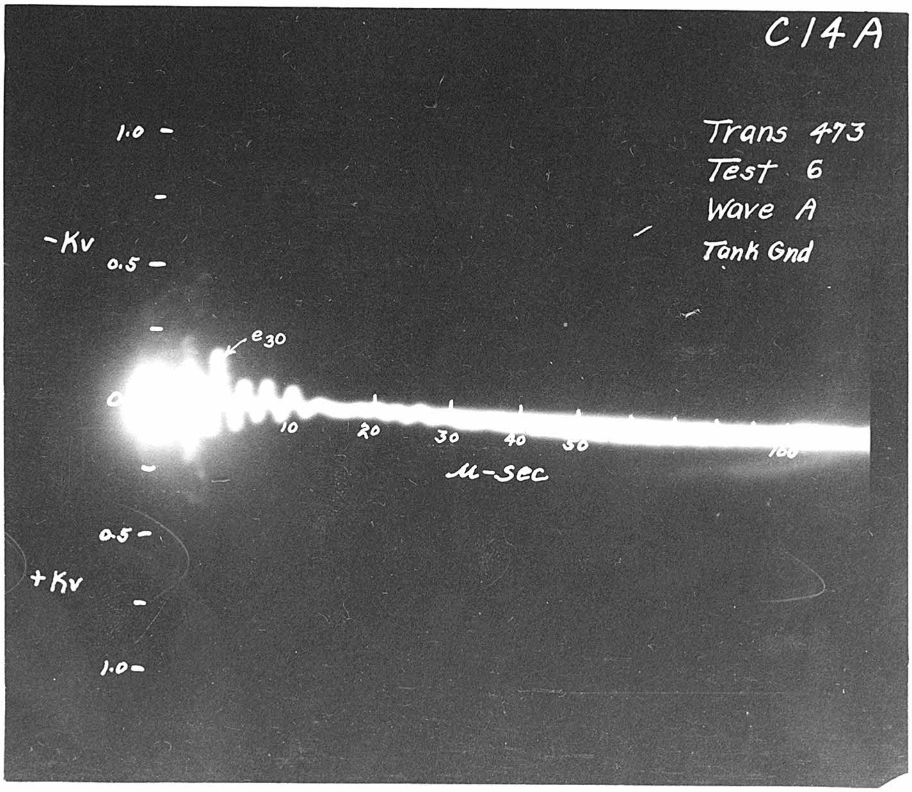

C24B
20
-Kv
10
e36
e36a
Tank Gnd
Trans E1129
Test 6
Wave A
5
10
μ-sec
e30
+Kv
0.5

(b)

Fig. 132

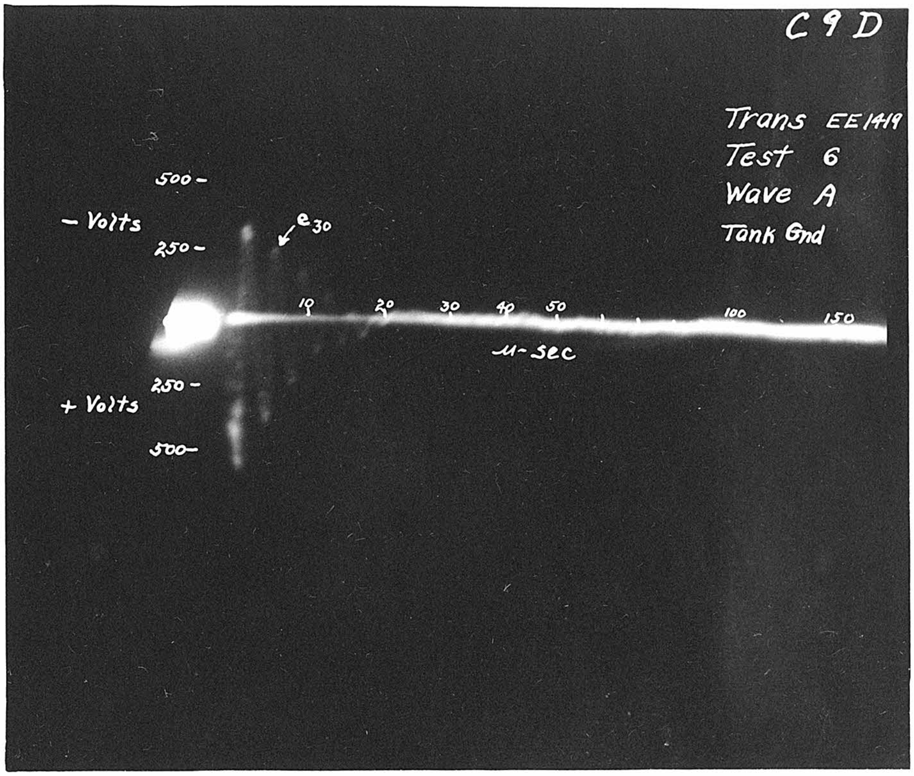

C9F
Trans EE1419
Test 6
Wave A
Tank Ungnd
Tank Gnd
e_{35}
e_{36}
e_{35}
e_{36}
15
10
5
0
5
10
15
-Kv
-Kv
10
20
30
40
50
100
150
μ-sec

(a)

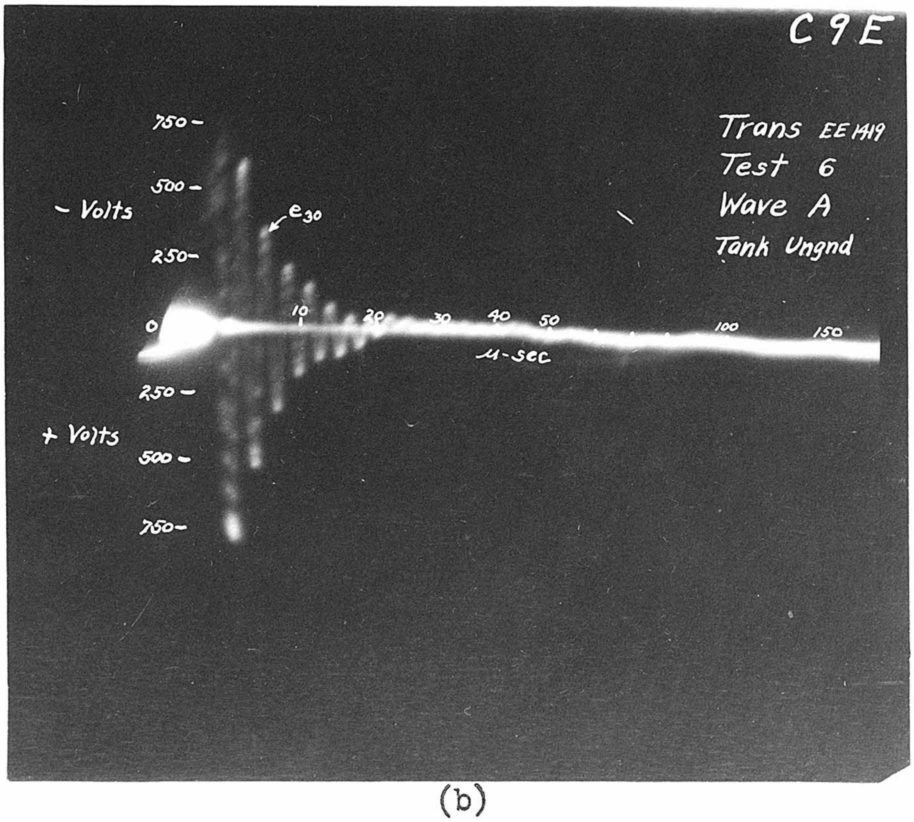

C14A
Trans 473
Test 6
Wave A
Tank Gnd
e_{30}
1.0
0.5
0
0.5
1.0
-Kv
+Kv
10
20
30
40
50
100
μ-sec

(b)

Fig. 133

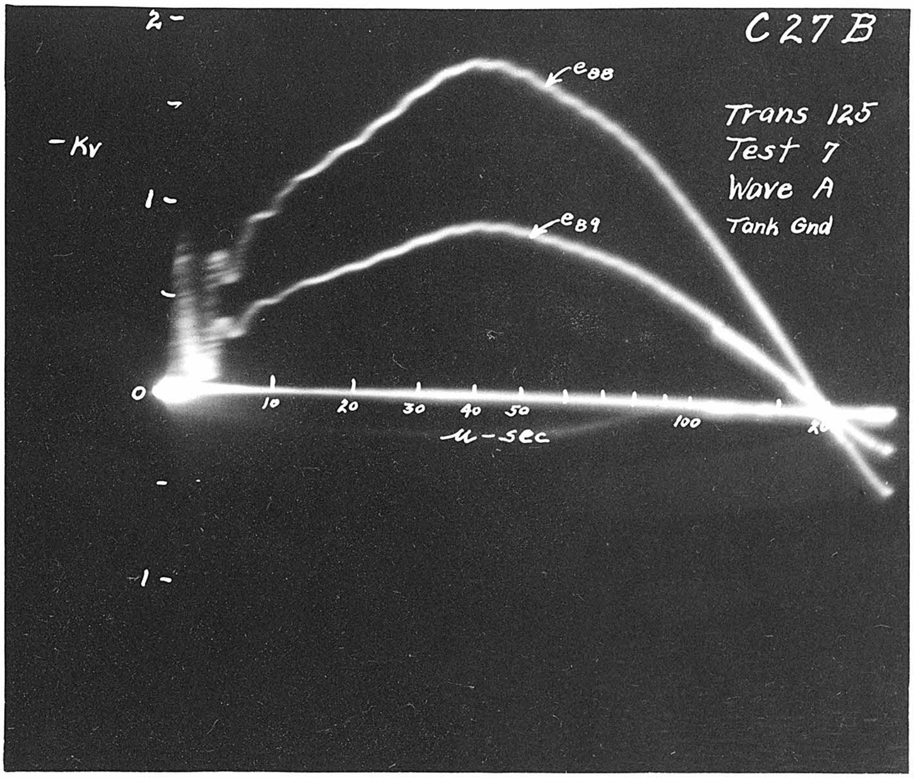

C 9 D
Trans EE 1419
Test 6
Wave A
Tank Gnd
500 -
- Volts
250 -
e_{30}
10
20
30
40
50
100
150
μ-sec
250 -
+ Volts
500 -

(a)

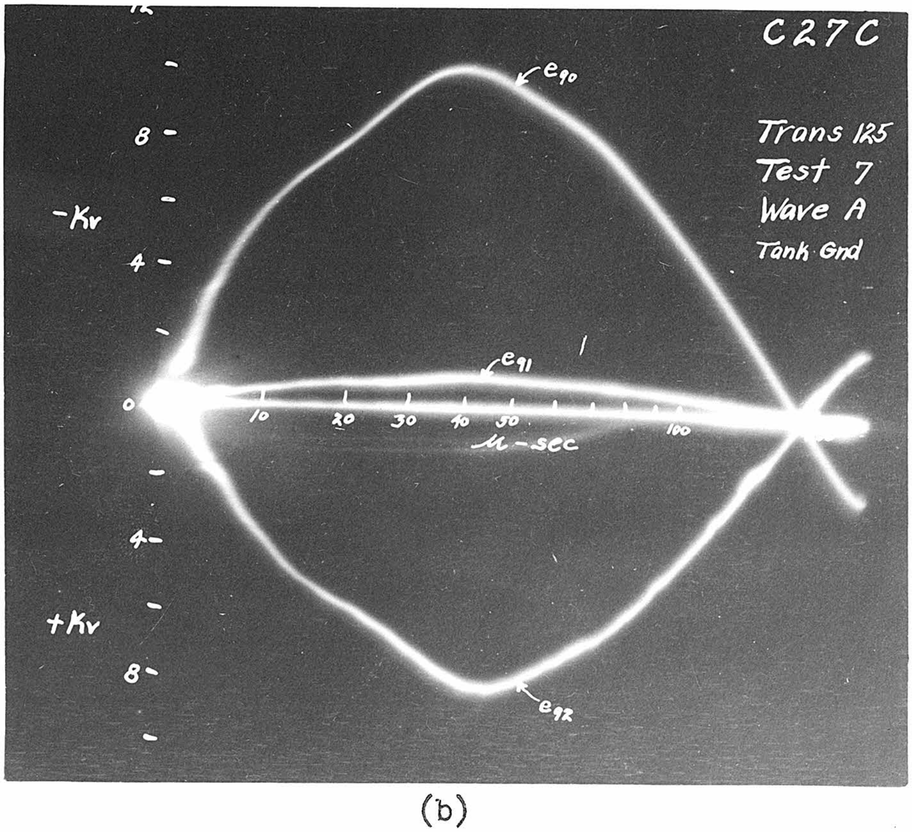

C 9 E
Trans EE 1419
Test 6
Wave A
Tank Ungnd
750 -
500 -
- Volts
e_{30}
250 -
0
10
20
30
40
50
100
150
μ-sec
250 -
+ Volts
500 -
750 -

(b)

Fig. 134

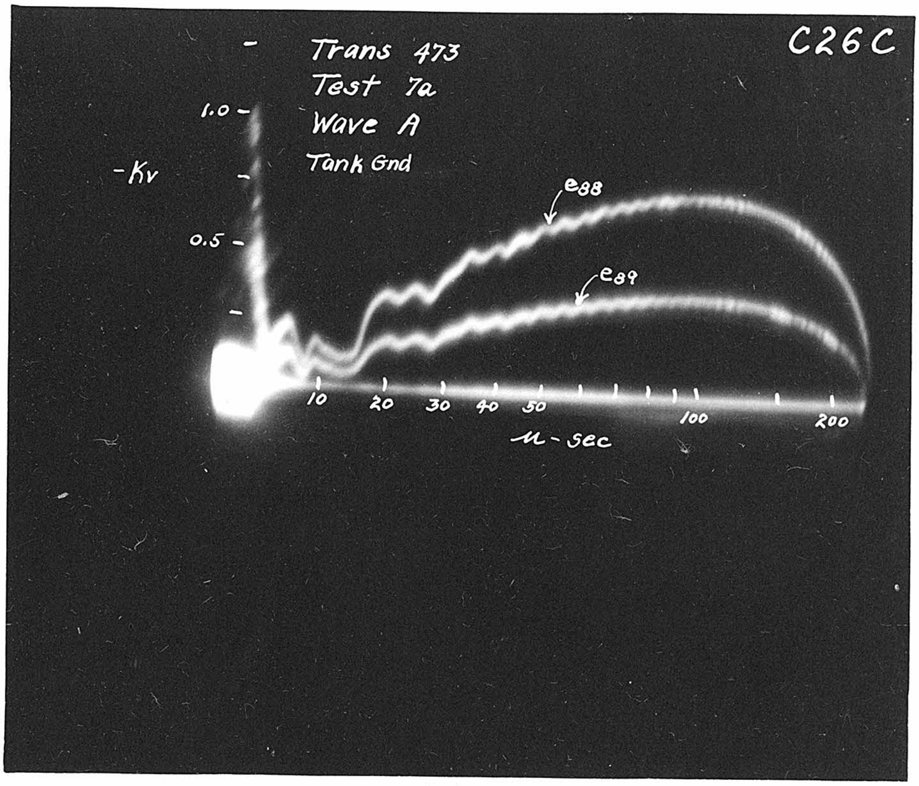

C27B
Trans 125
Test 7
Wave A
Tank Gnd
-Kv
2
1
0
1
e_{88}
e_{89}
10
20
30
40
50
100
200
μ-sec

(a)

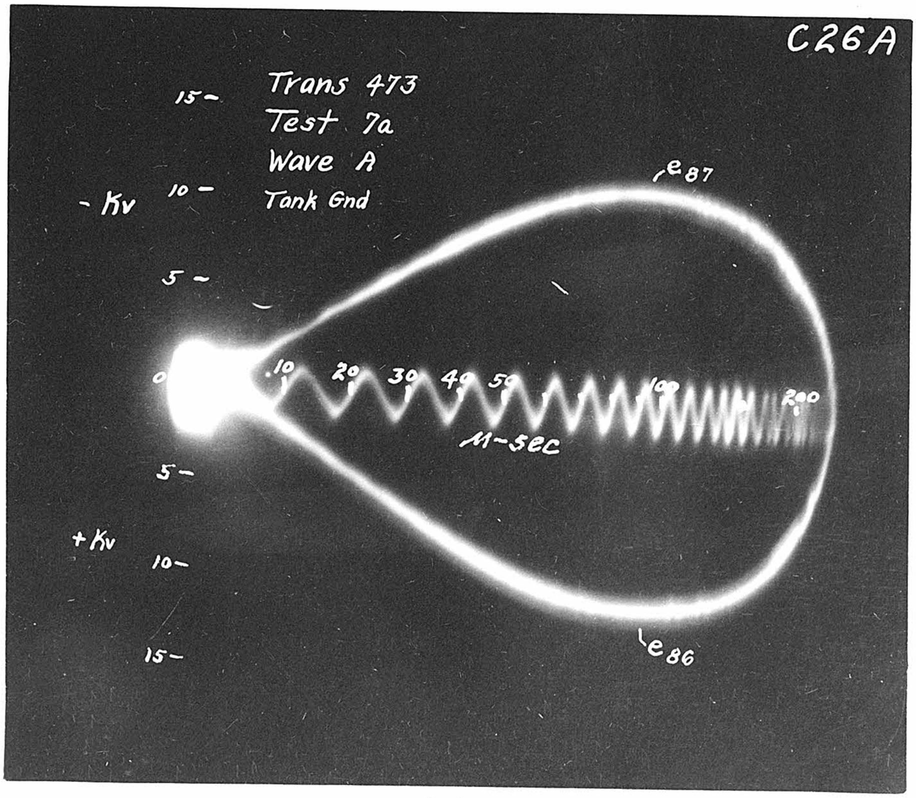

C27C
Trans 125
Test 7
Wave A
Tank Gnd
-Kv
+Kv
12
8
4
0
4
8
e_{90}
e_{91}
e_{92}
10
20
30
40
50
100
μ-sec

(b)

Fig. 135

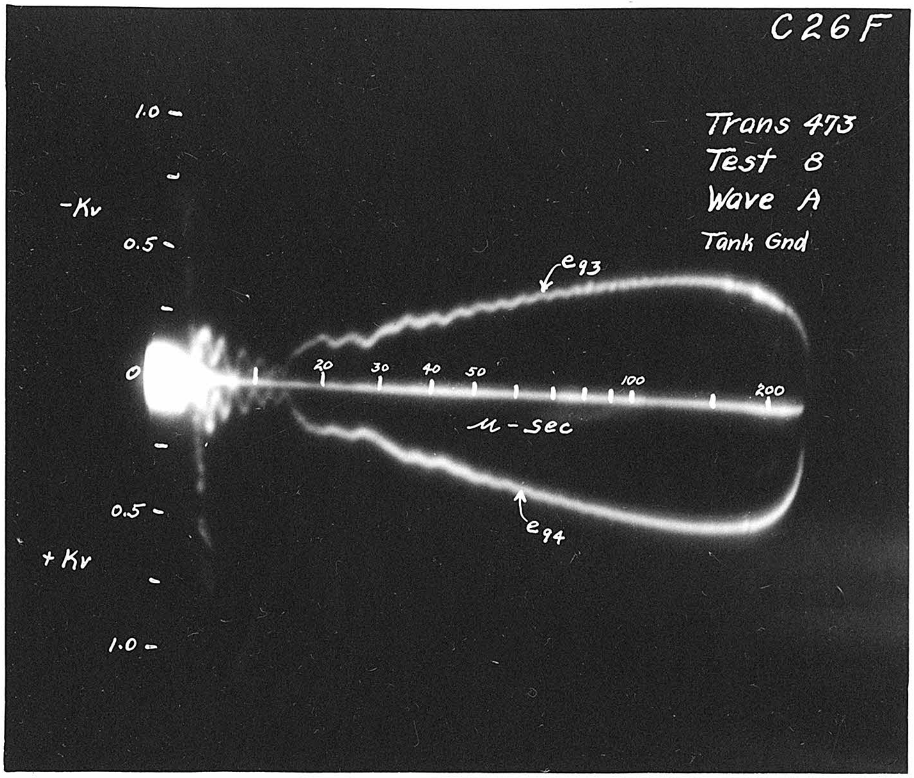

C26C
Trans 473
Test 7a
Wave A
Tank Gnd
-Kv
1.0
0.5
e88
e89
10
20
30
40
50
100
200
μ-sec

(a)

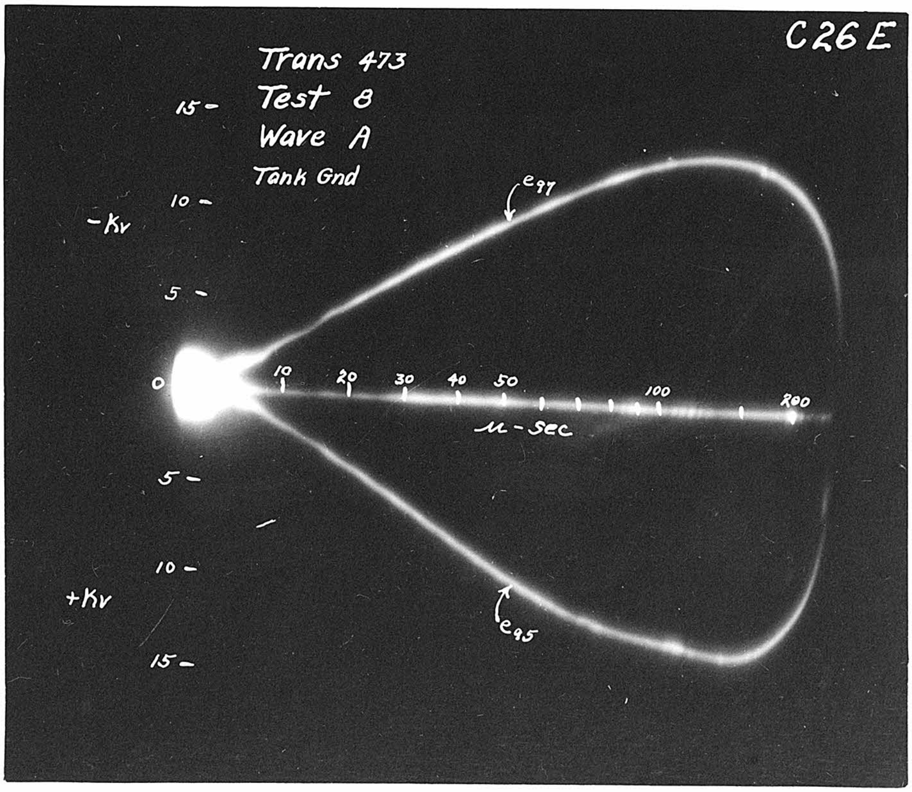

C26A
Trans 473
Test 7a
Wave A
Tank Gnd
-Kv
15
10
5
0
5
10
15
+Kv
e87
e86
10
20
30
40
50
100
200
μ-sec

(b)

Fig. 136

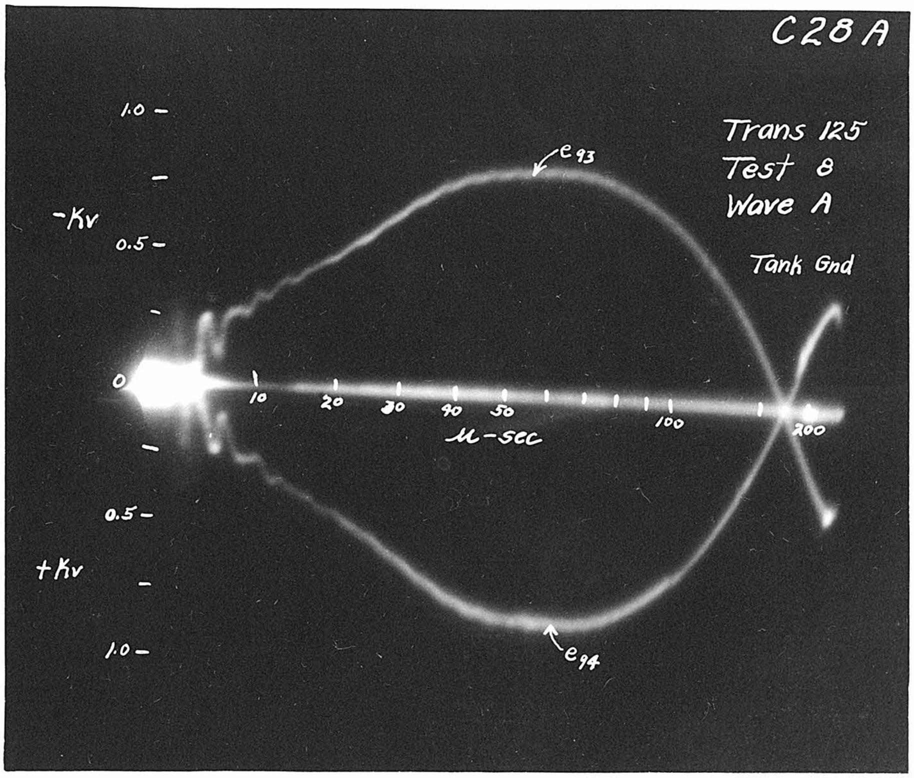

C26F
Trans 473
Test 8
Wave A
Tank Gnd
1.0
-Kv
0.5
0
0.5
+Kv
1.0
e93
e94
20
30
40
50
100
200
μ-sec

(a)

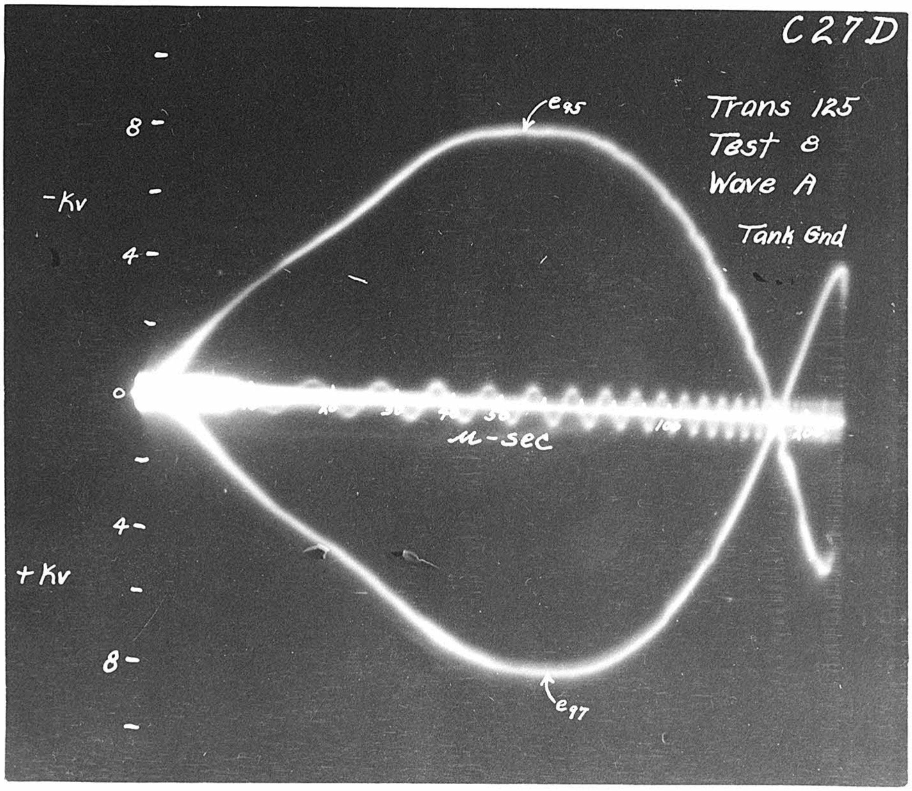

C26E
Trans 473
Test 8
Wave A
Tank Gnd
15
10
-Kv
5
0
5
10
+Kv
15
e97
e95
10
20
30
40
50
100
200
μ-sec

(b)

Fig. 137

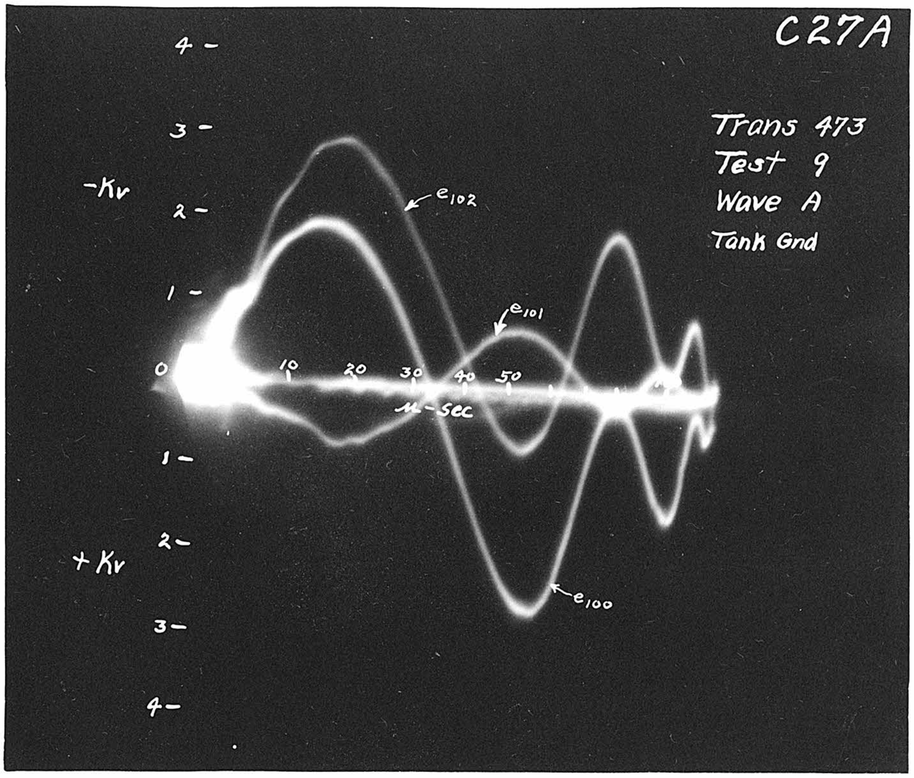

C28A
Trans 125
Test 8
Wave A
Tank Gnd
1.0
-Kv
0.5
0
0.5
+Kv
1.0
e_{93}
e_{94}
10
20
30
40
50
100
200
μ-sec

(a)

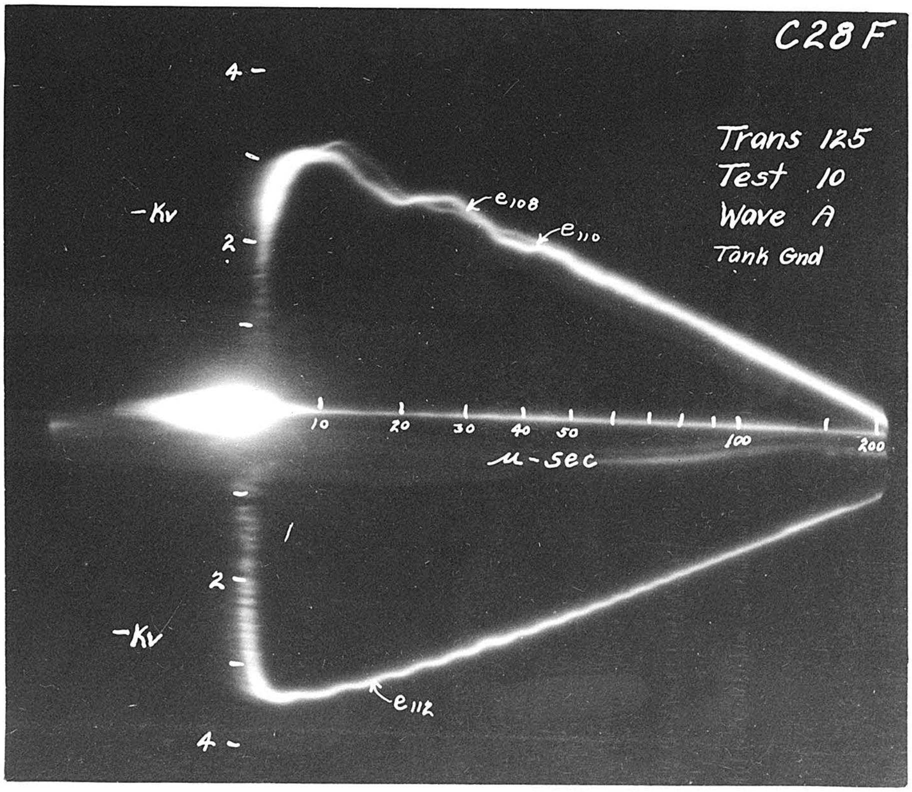

C27D
Trans 125
Test 8
Wave A
Tank Gnd
8
-Kv
4
0
4
+Kv
8
e_{95}
e_{97}
μ-sec

(b)

Fig. 138

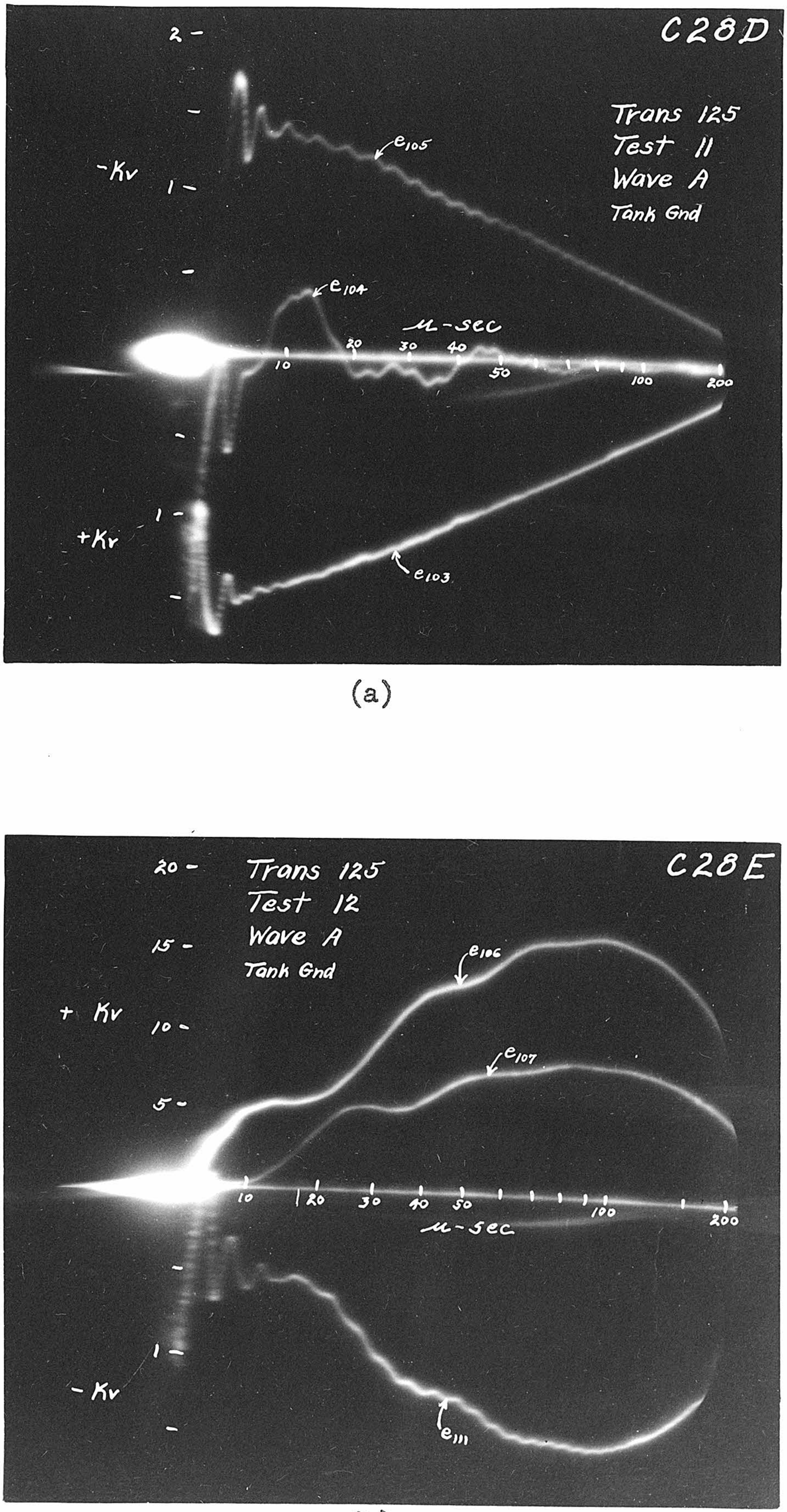

C27A
Trans 473
Test 9
Wave A
Tank Gnd
4
3
-Kv
2
1
0
1
2
+Kv
3
4
e102
e101
e100
10
20
30
40
50
μ-sec

(a)


C28F
Trans 125
Test 10
Wave A
Tank Gnd
4
-Kv
2
e108
e110
10
20
30
40
50
100
200
μ-sec
1
2
-Kv
e112
4

(b)

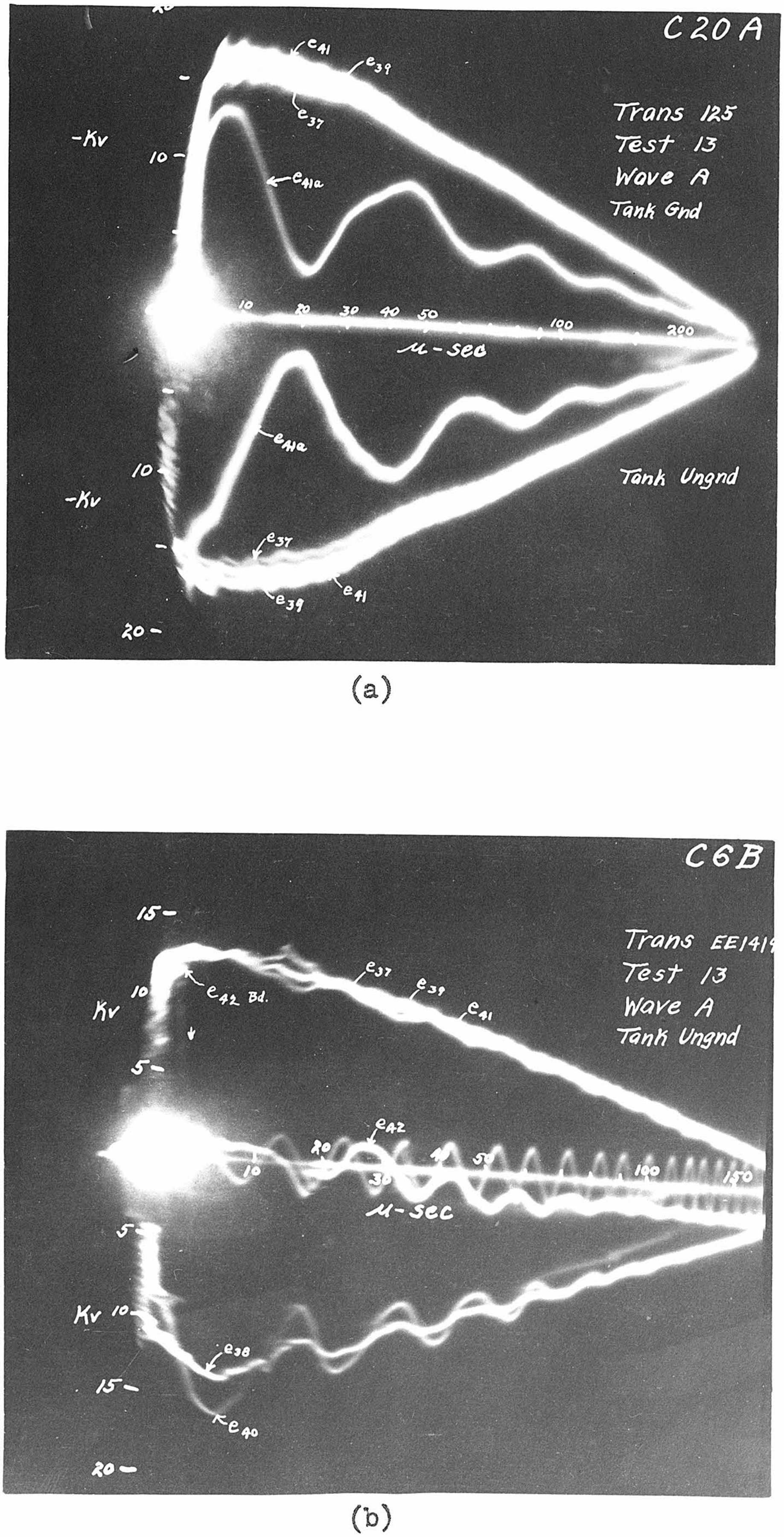
Fig. 139


C28D
Trans 125
Test 11
Wave A
Tank Gnd
2
-Kv
1
e_{105}
e_{104}
μ-sec
10
20
30
40
50
100
200
1
+Kv
e_{103}
C28E
Trans 125
Test 12
Wave A
Tank Gnd
20
15
+ Kv
10
5
e_{106}
e_{107}
10
20
30
40
50
100
200
μ-sec
1
- Kv
e_{111}

(a)

(b)

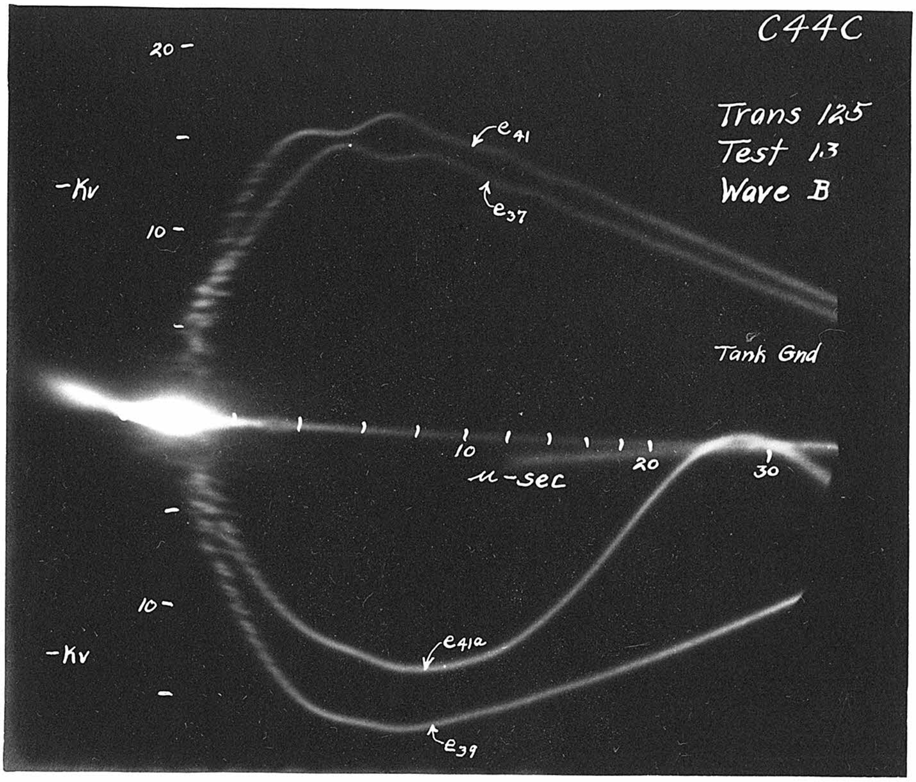
Fig. 140


C20A
Trans 125
Test 13
Wave A
Tank Gnd
Tank Ungnd
-Kv
10
20
e41
e39
e37
e41a
10
20
30
40
50
100
200
μ-sec
C6B
Trans EE1419
Test 13
Wave A
Tank Ungnd
Kv
15
10
5
20
e37
e39
e41
e42 Bd.
e42
e38
e40
150

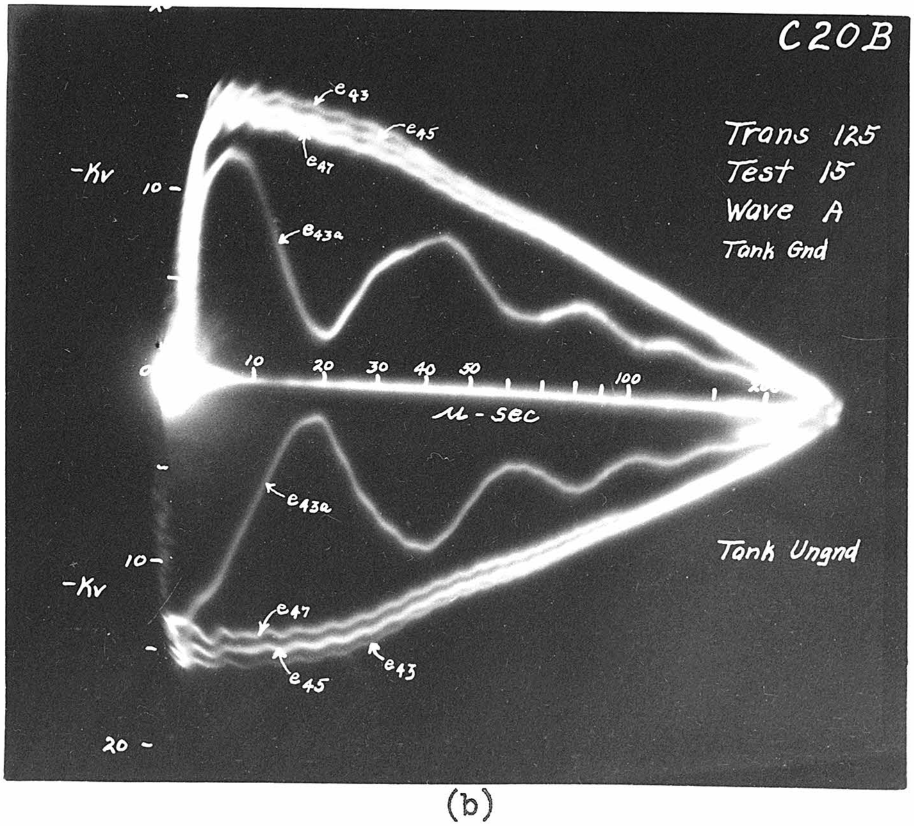
(a)

(b)

Fig. 141

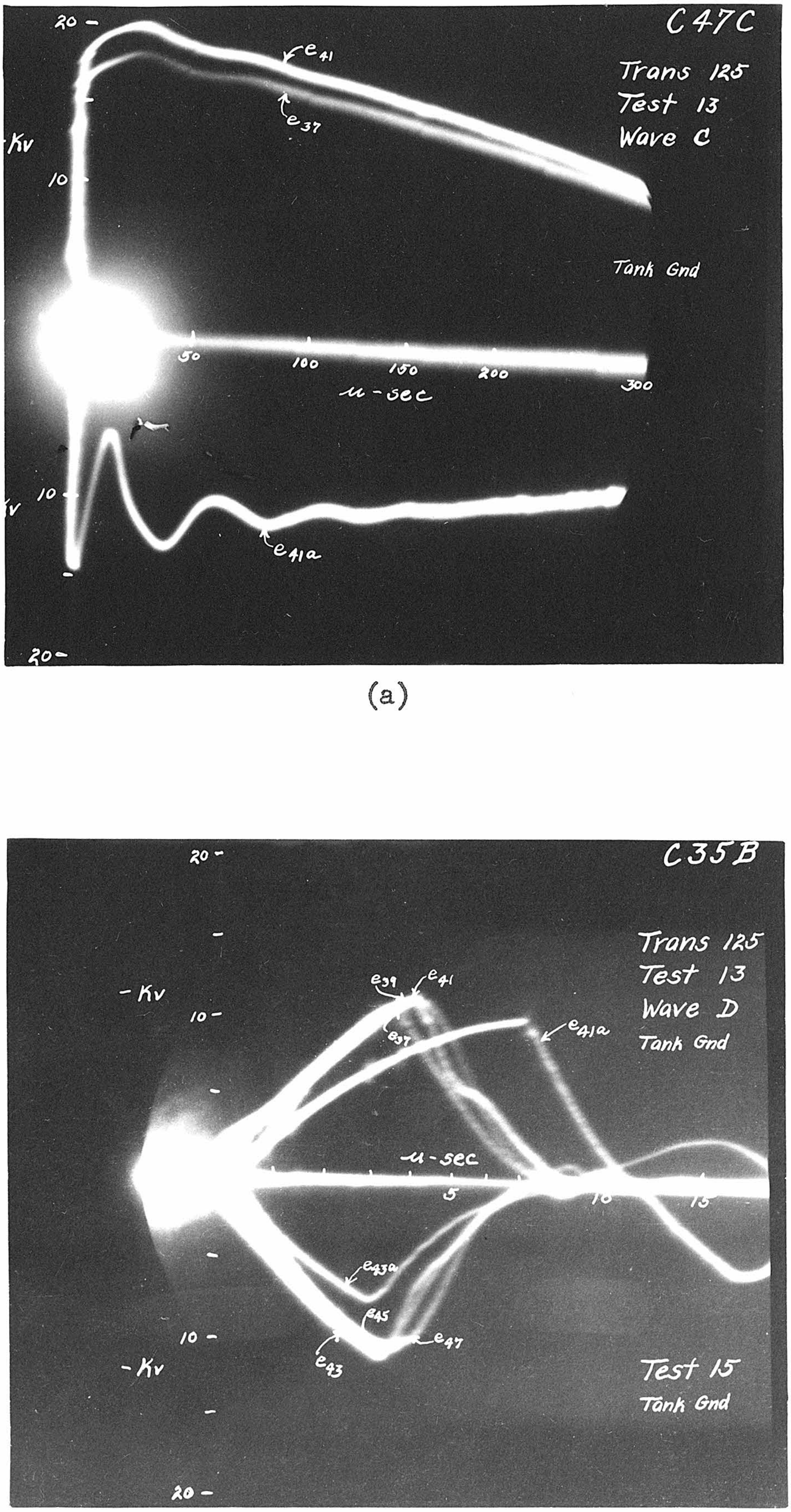

C44C
20 –
Trans 125
Test 13
Wave B
e41
e37
-Kv
10 –
Tank Gnd
10
20
30
μ-sec
10 –
e41a
-Kv
e39

(a)


C20B
e43
e45
e47
Trans 125
Test 15
Wave A
Tank Gnd
-Kv
10 –
e43a
0
10
20
30
40
50
100
μ-sec
e43a
Tank Ungnd
10 –
-Kv
e47
e45
e43
20 –

(b)

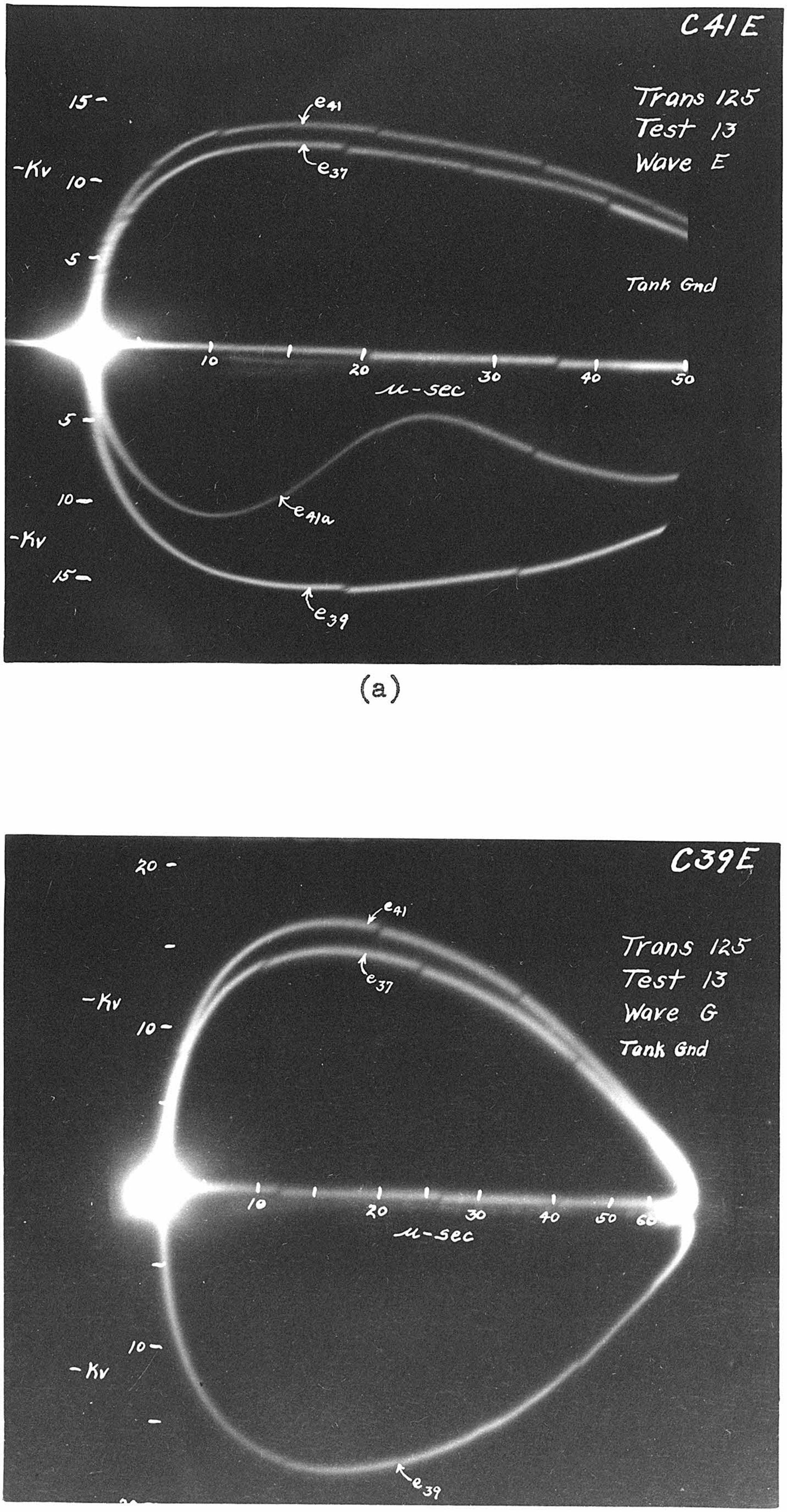
Fig. 142


20
C47C
e41
e37
Trans 125
Test 13
Wave C
Kv
10
Tank Gnd
50
100
150
200
300
u-sec
10
Kv
e41a
20
20
C35B
Trans 125
Test 13
Wave D
Tank Gnd
-Kv
10
e39
e41
e37
e41a
u-sec
5
10
15
e43a
e45
e47
e43
10
-Kv
Test 15
Tank Gnd
20

(a)

(b)

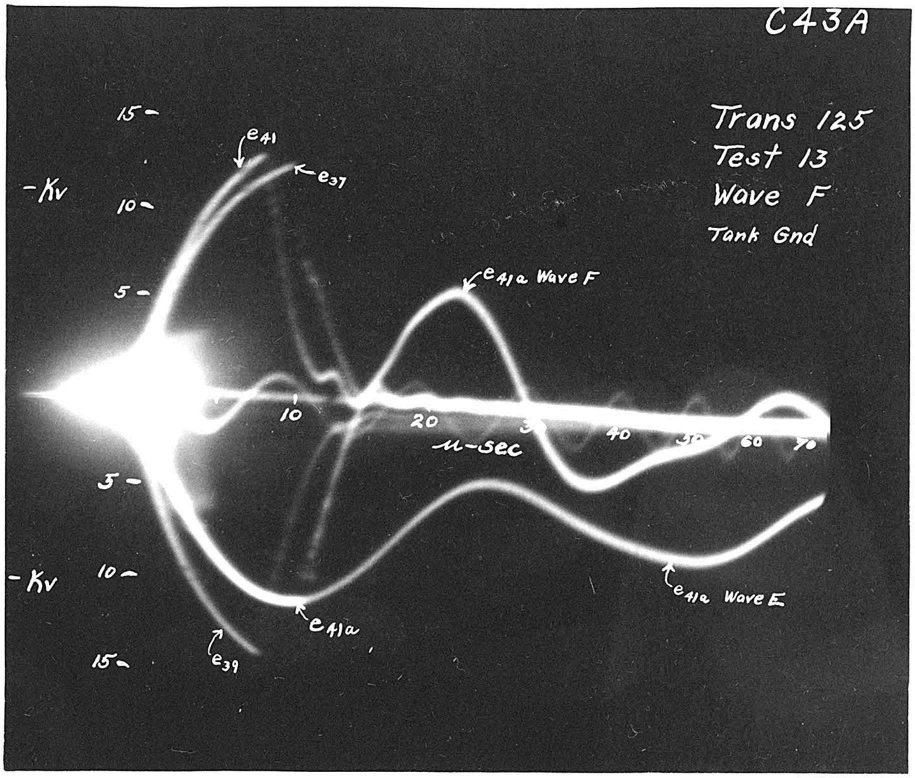
Fig. 143


C41E
Trans 125
Test 13
Wave E
Tank Gnd
15
10
5
-Kv
5
10
-Kv
15
e41
e37
e41a
e39
10
20
30
40
50
μ-sec
C39E
20
-Kv
10
Trans 125
Test 13
Wave G
Tank Gnd
e41
e37
10
20
30
40
50
60
μ-sec
10
-Kv
e39

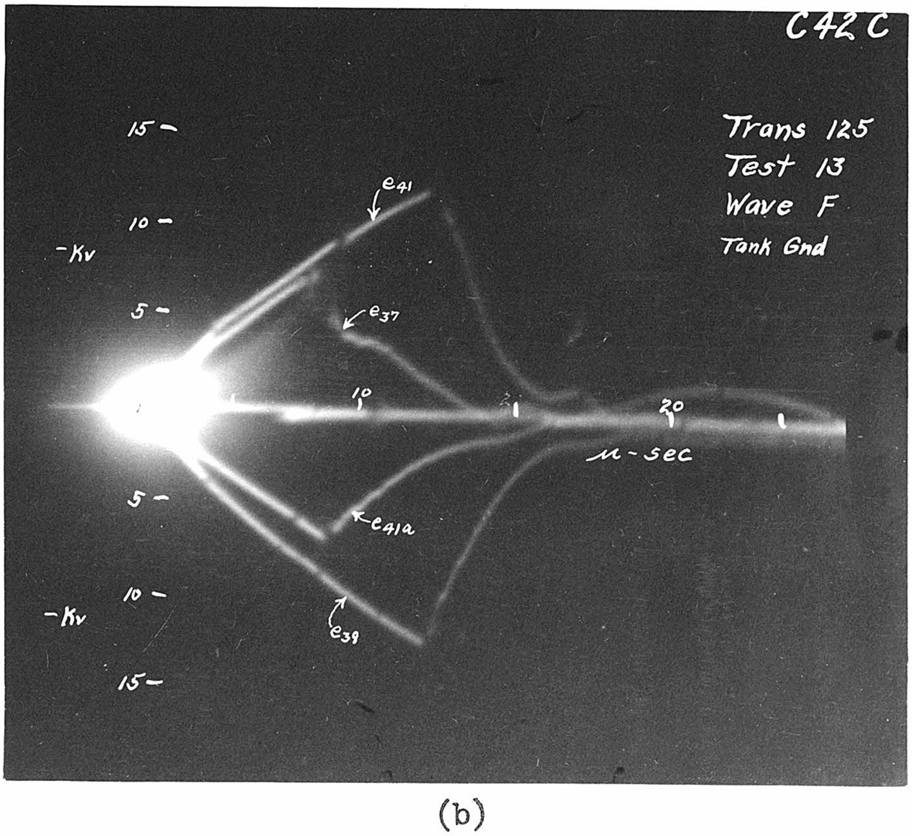
(a)

(b)

Fig. 144

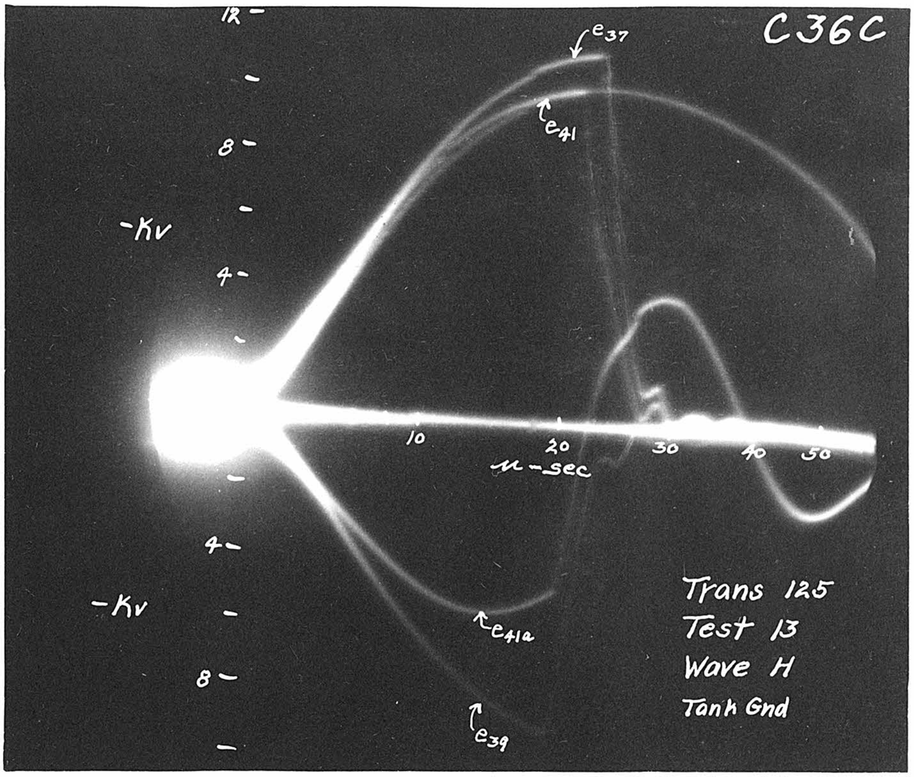

C43A
Trans 125
Test 13
Wave F
Tank Gnd
15
10
5
5
10
15
-Kv
-Kv
e41
e37
e41a Wave F
e41a
e39
e41a Wave E
10
20
30
40
50
60
70
μ-sec

(a)

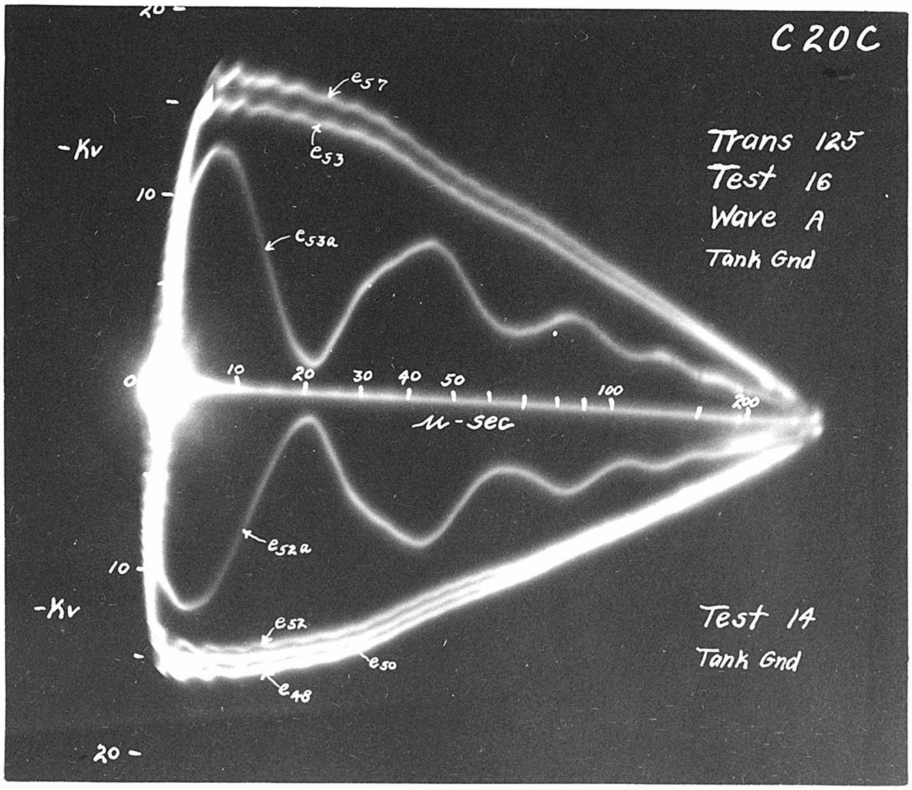

C42C
Trans 125
Test 13
Wave F
Tank Gnd
15
10
5
5
10
15
-Kv
-Kv
e41
e37
e41a
e39
10
20
μ-sec

(b)

Fig. 145

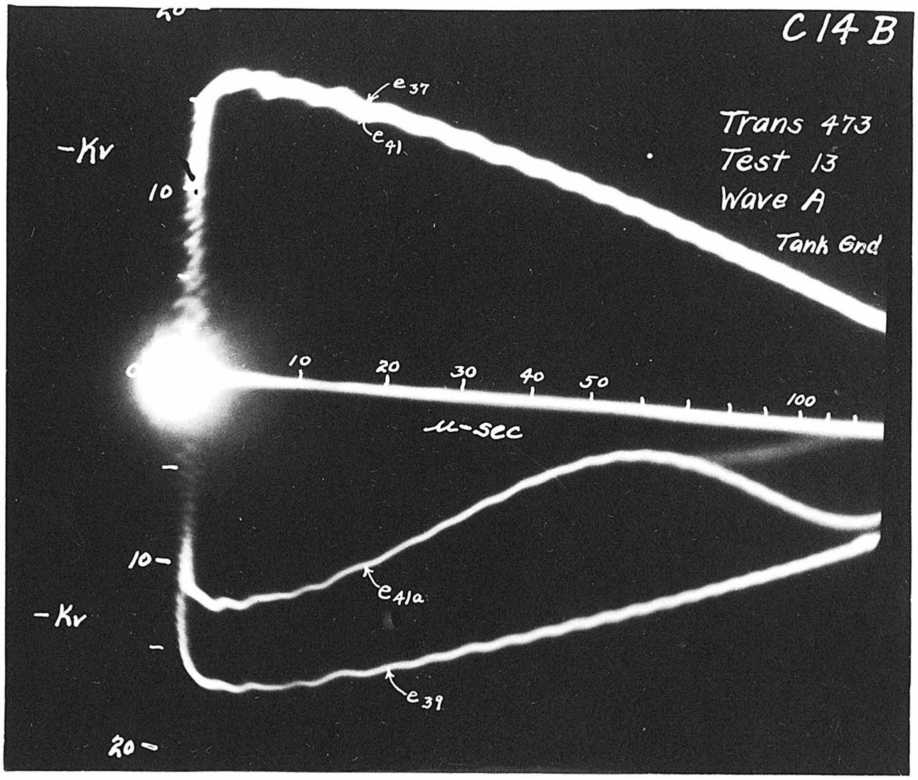

C36C
12
8
-Kv
4
4
-Kv
8
e_{37}
e_{41}
e_{41a}
e_{39}
10
20
30
40
50
µ-sec
Trans 125
Test 13
Wave H
Tank Gnd

(a)

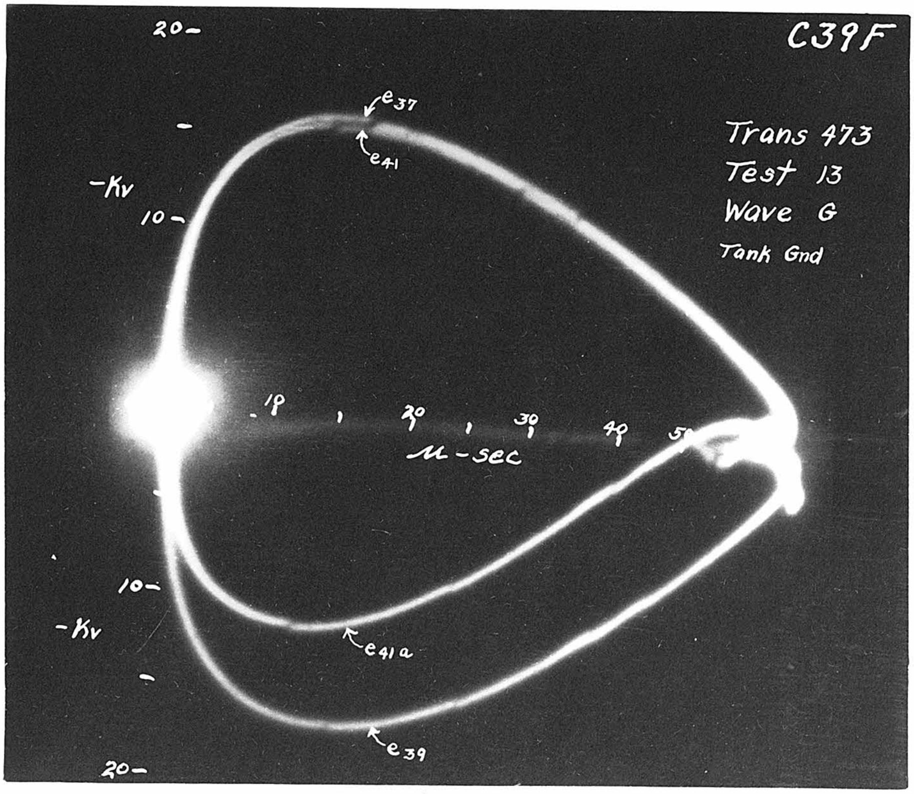

C20C
20
-Kv
10
0
10
-Kv
20
e_{57}
e_{53}
e_{53a}
e_{52a}
e_{52}
e_{50}
e_{48}
10
20
30
40
50
100
200
µ-sec
Trans 125
Test 16
Wave A
Tank Gnd
Test 14
Tank Gnd

(b)

Fig. 146

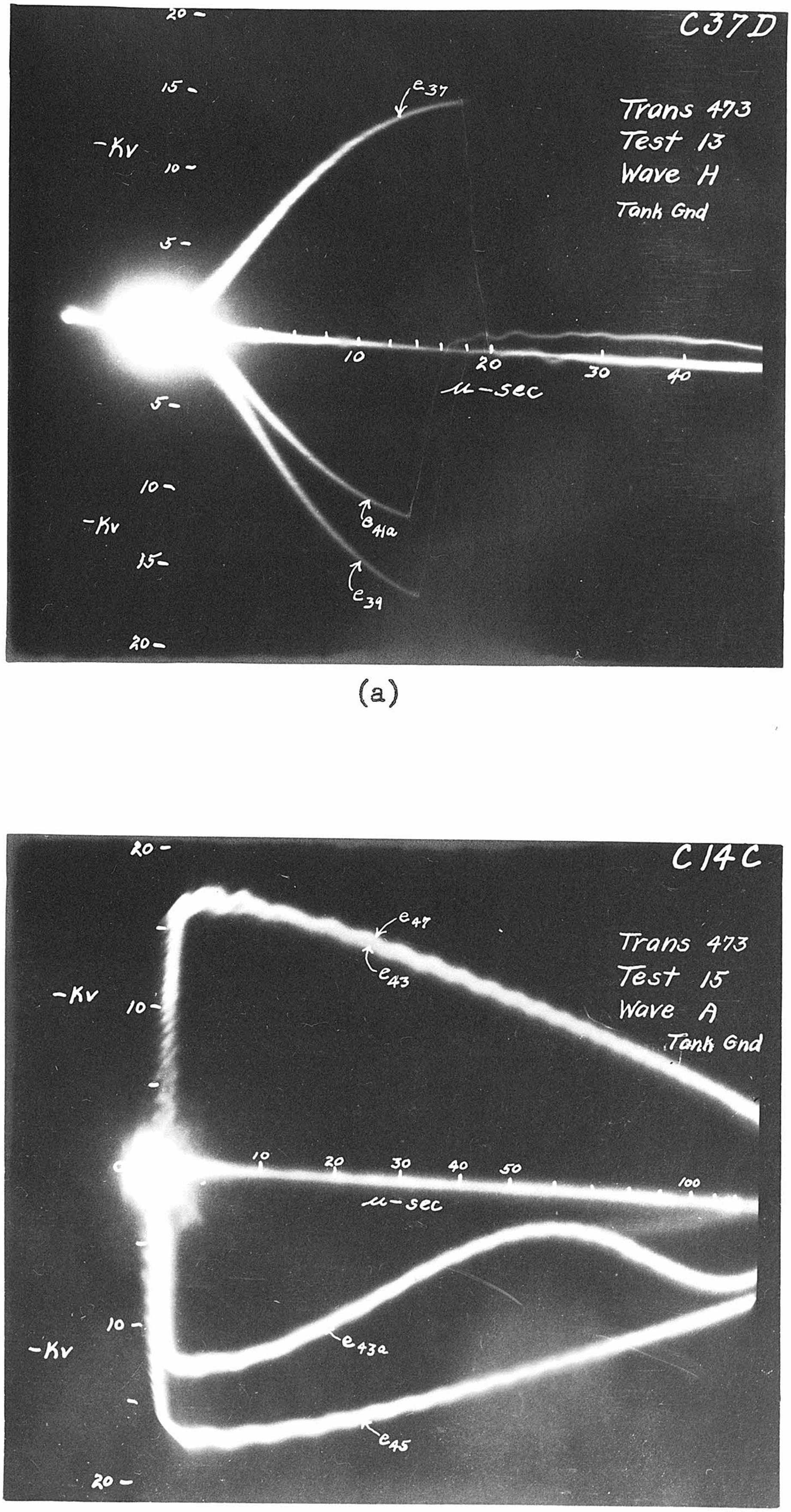

C14B
Trans 473
Test 13
Wave A
Tank Gnd
-Kv
10
e_{37}
e_{41}
10
20
30
40
50
100
μ-sec
10
-Kv
20
e_{41a}
e_{39}

(a)


C39F
Trans 473
Test 13
Wave G
Tank Gnd
20
-Kv
10
e_{37}
e_{41}
10
20
30
40
50
μ-sec
10
-Kv
20
e_{41a}
e_{39}

(b)

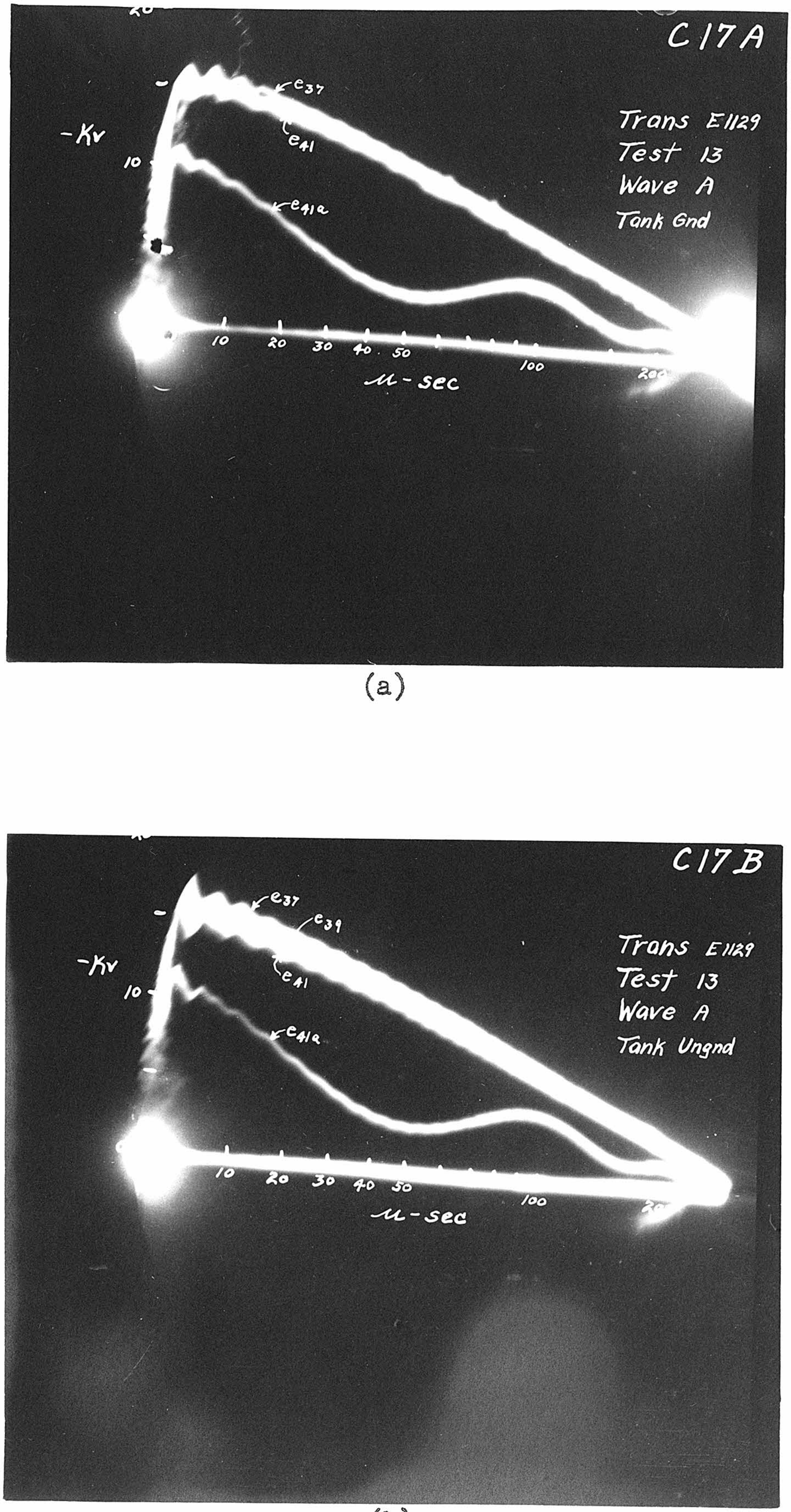
Fig. 147


C37D
Trans 473
Test 13
Wave H
Tank Gnd
-Kv
20
15
10
5
5
10
15
20
e37
e41a
e39
10
20
30
40
μ-sec
C14C
Trans 473
Test 15
Wave A
Tank Gnd
-Kv
20
10
10
20
e47
e43
e43a
e45
10
20
30
40
50
100
μ-sec

(a)

(b)

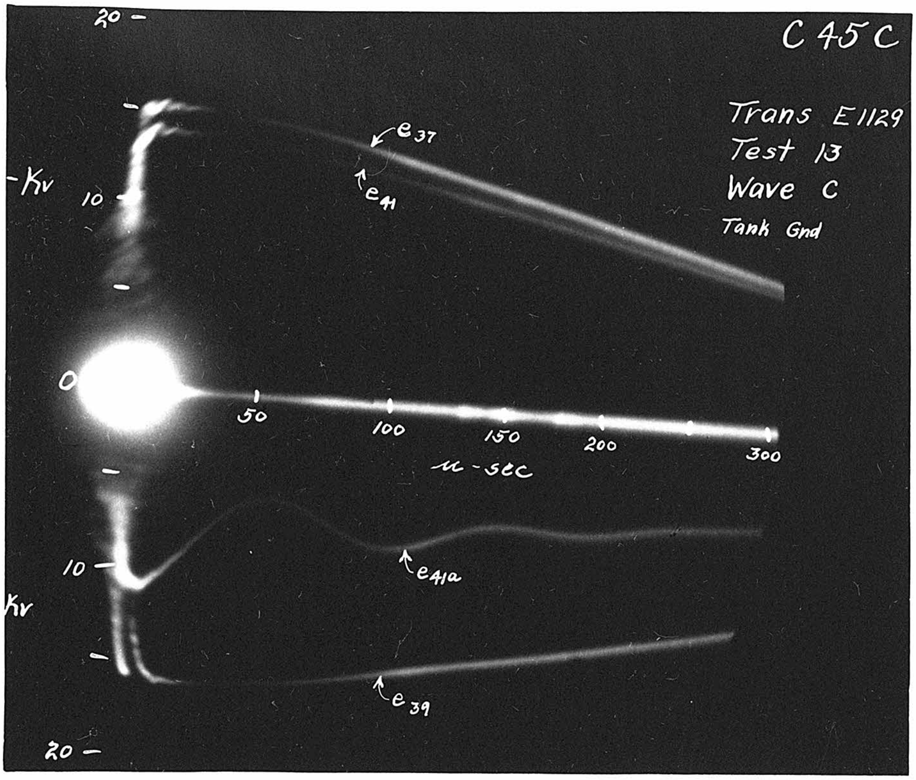
Fig. 148


C17A
Trans E1129
Test 13
Wave A
Tank Gnd
-Kv
10
e37
e41
e41a
10
20
30
40
50
100
200
µ-sec
C17B
Trans E1129
Test 13
Wave A
Tank Ungnd
-Kv
10
e37
e39
e41
e41a
10
20
30
40
50
100
200
µ-sec

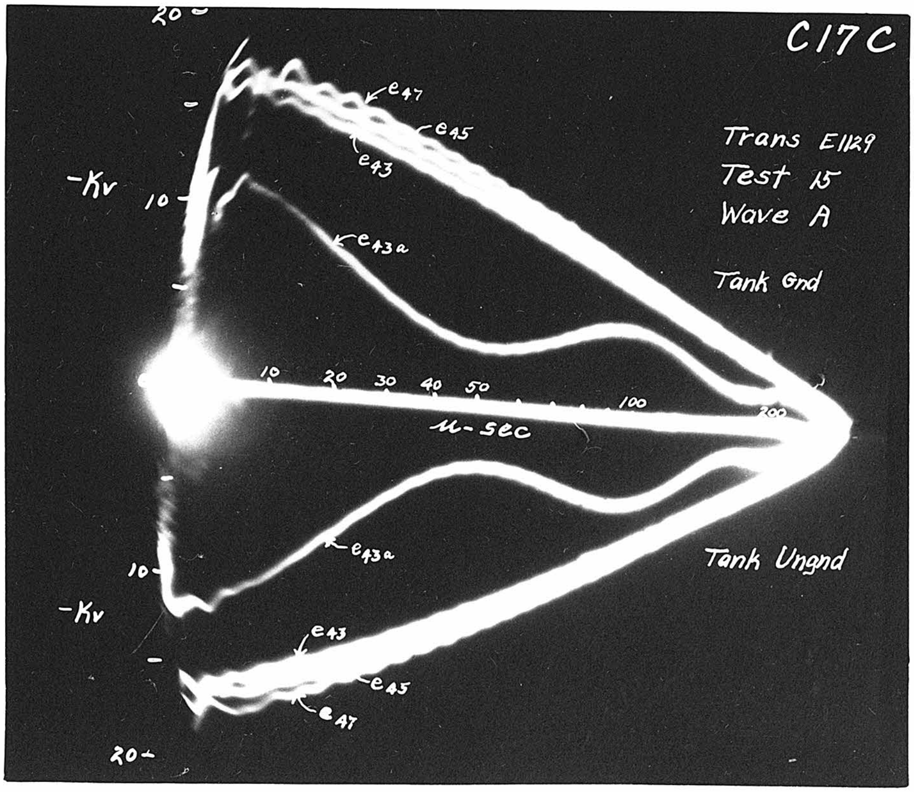
(a)

(b)

Fig. 149

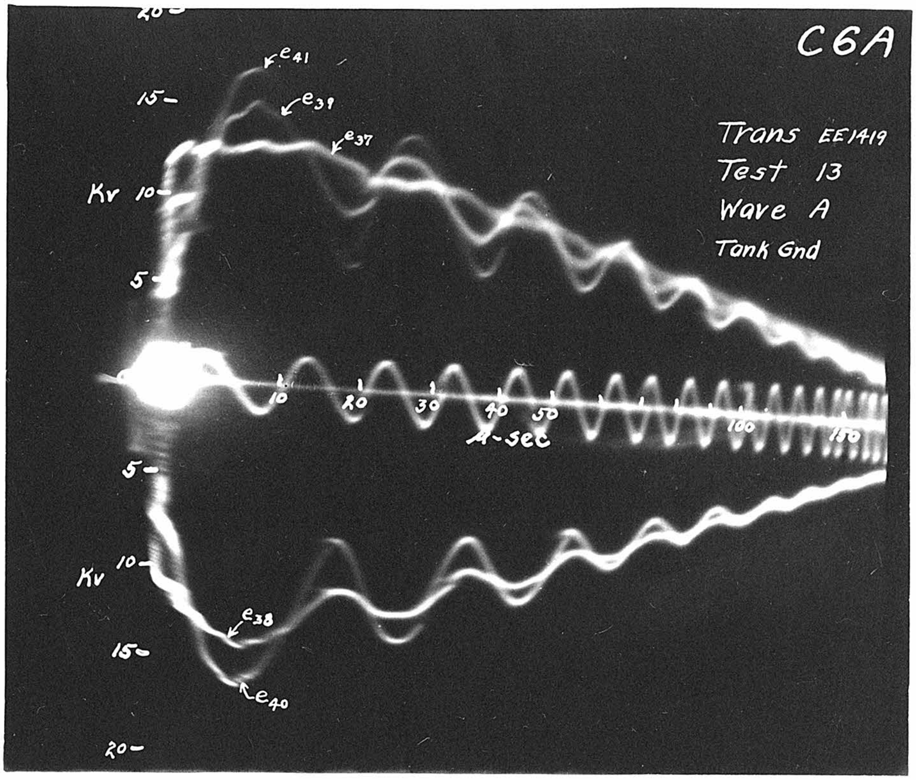

C 45 C
Trans E1129
Test 13
Wave C
Tank Gnd
20
10
-Kv
e37
e41
0
50
100
150
200
300
μ-sec
e41a
10
Kv
e39
20

(a)

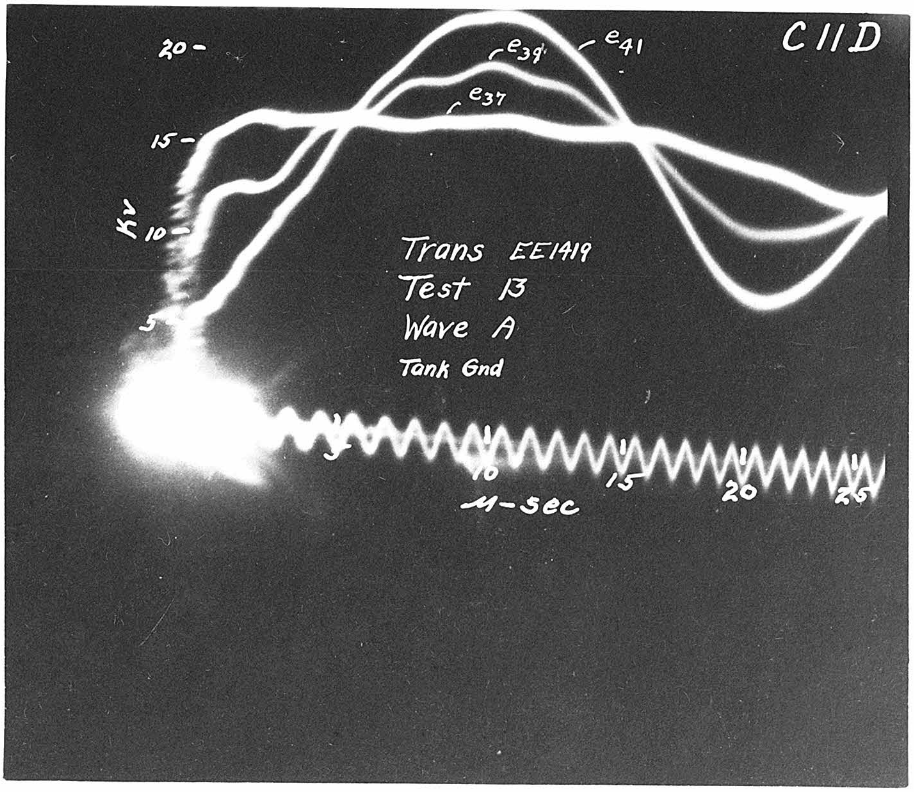

C17C
Trans E1129
Test 15
Wave A
Tank Gnd
20
10
-Kv
e47
e45
e43
e43a
10
20
30
40
50
100
200
μ-sec
e43a
Tank Ungnd
10
-Kv
e43
e45
e47
20

(b)

Fig. 150

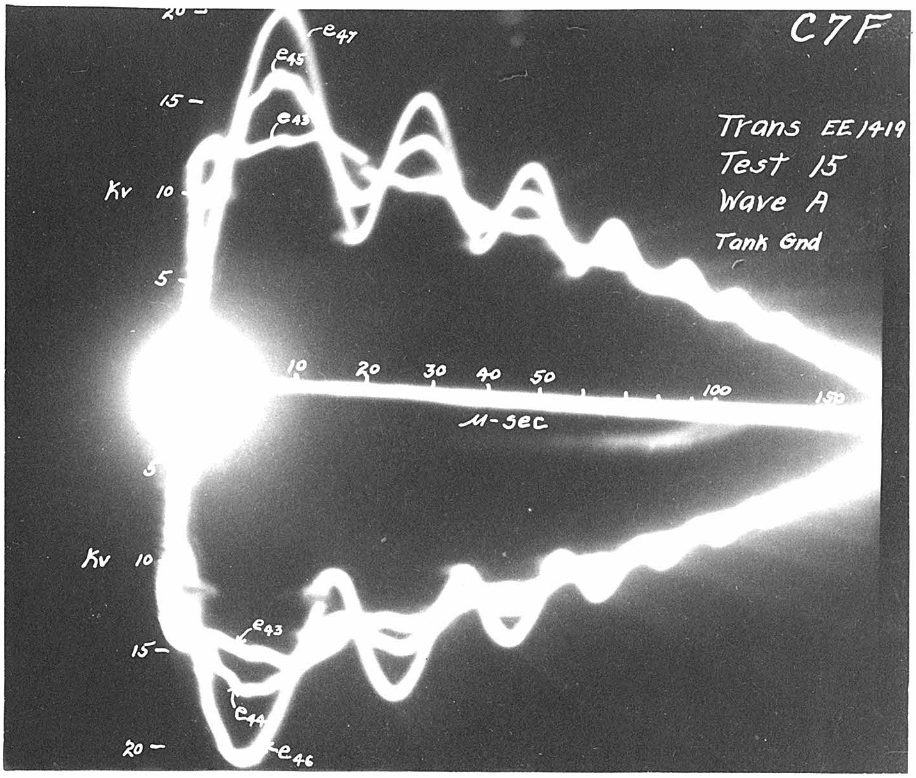

C6A
Trans EE1419
Test 13
Wave A
Tank Gnd
Kv
20
15
10
5
5
10
15
20
e_{41}
e_{39}
e_{37}
e_{38}
e_{40}
10
20
30
40
50
100
150
μ-sec

(a)

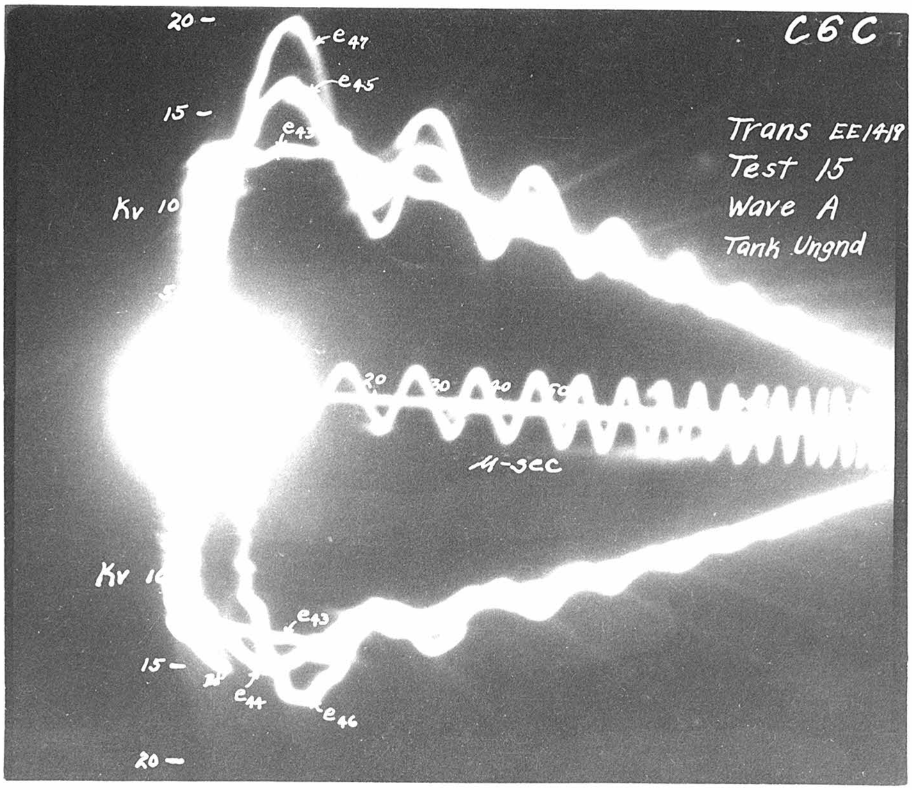

C11D
Trans EE1419
Test 13
Wave A
Tank Gnd
Kv
20
15
10
5
e_{39}
e_{41}
e_{37}
5
10
15
20
25
μ-sec

(b)

Fig. 151

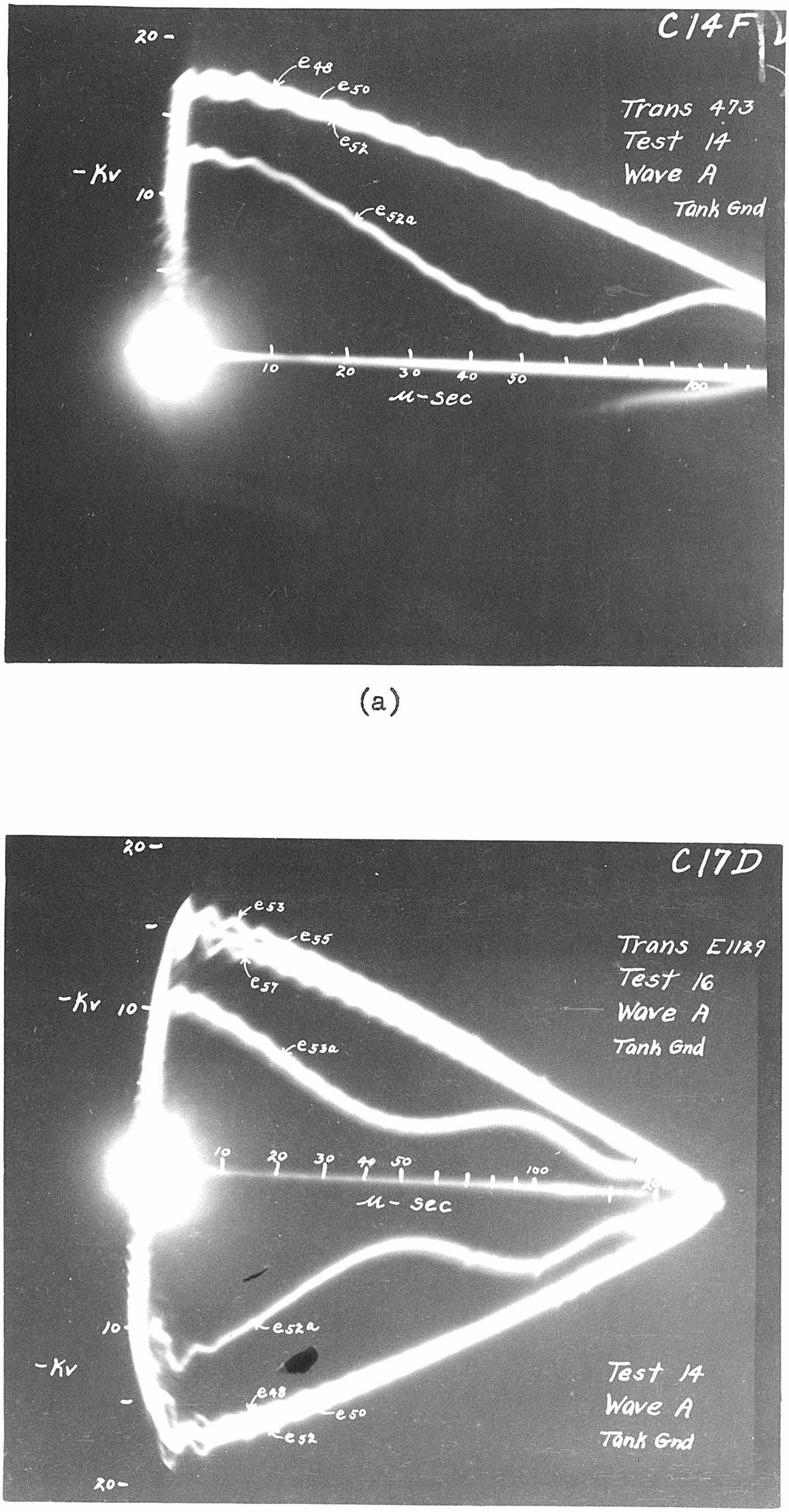

C7F
Trans EE1419
Test 15
Wave A
Tank Gnd
20
15
Kv 10
5
5
Kv 10
15
20
e47
e45
e43
e43
e44
e46
10
20
30
40
50
100
150
μ-sec

(a)


C6C
Trans EE1419
Test 15
Wave A
Tank Ungnd
20
15
Kv 10
Kv 10
15
20
e47
e45
e43
e43
e44
e46
20
30
40
50
μ-sec

(b)

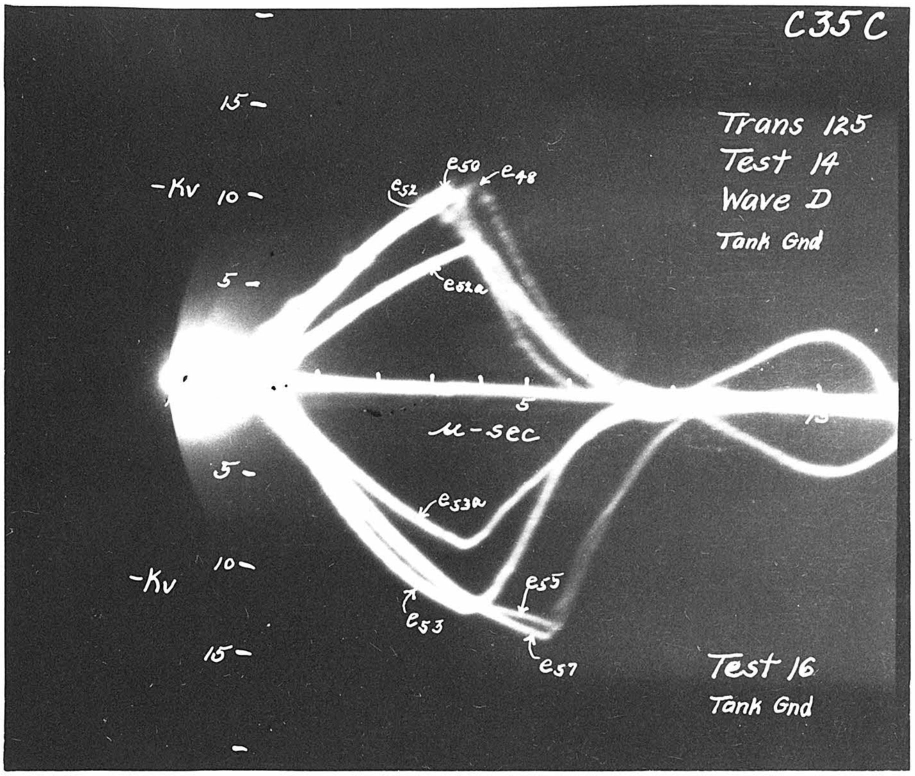
Fig. 152


C14F
20
e48
e50
e52
-Kv
10
e52a
Trans 473
Test 14
Wave A
Tank Gnd
10
20
30
40
50
100
μ-sec
C17D
20
e53
e55
e57
-Kv
10
e53a
Trans E1129
Test 16
Wave A
Tank Gnd
10
20
30
40
50
100
μ- sec
10
e52a
-Kv
e48
e50
e52
20
Test 14
Wave A
Tank Gnd

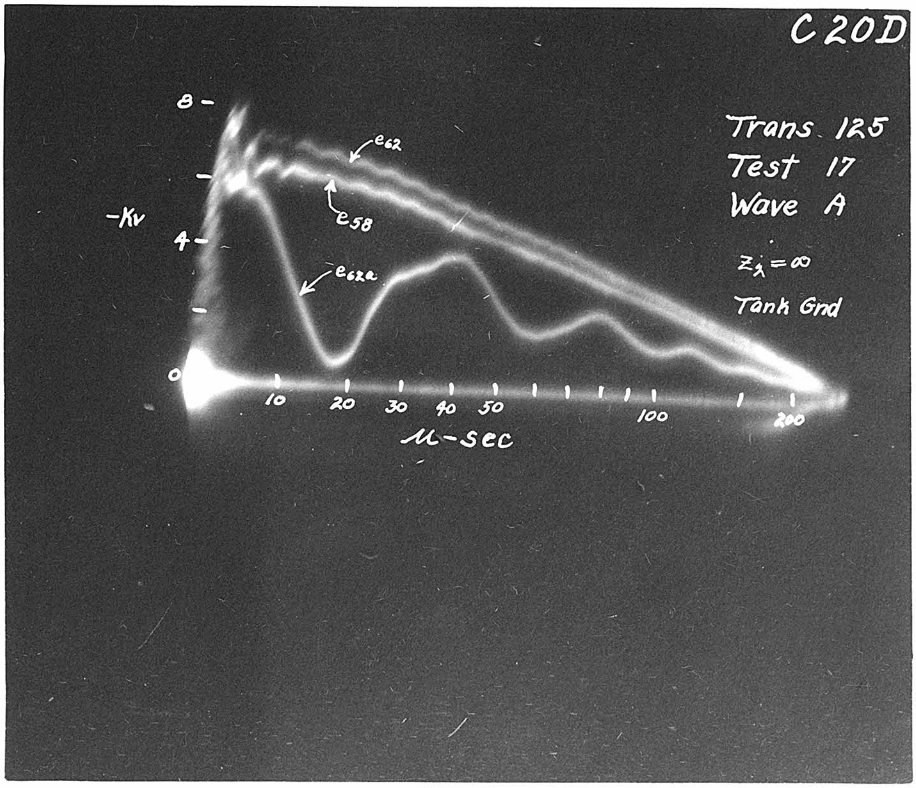
(a)

(b)

Fig. 153

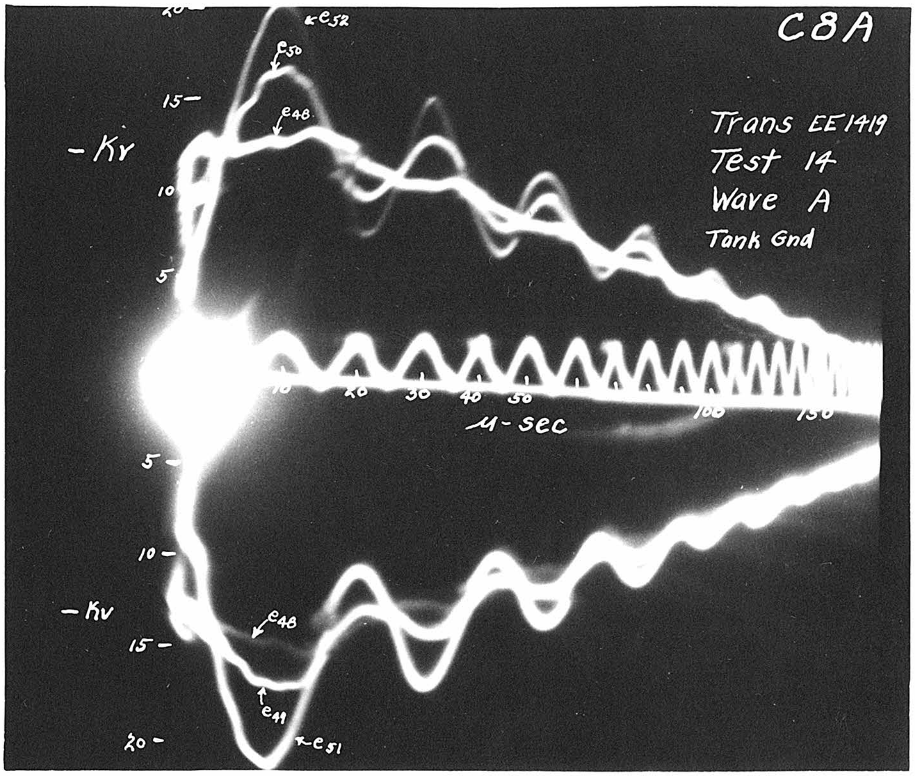

C35C
15
-Kv 10
5
5
-Kv 10
15
e52
e50
e48
e52a
e53a
e53
e55
e57
5
15
μ-sec
Trans 125
Test 14
Wave D
Tank Gnd
Test 16
Tank Gnd

(a)

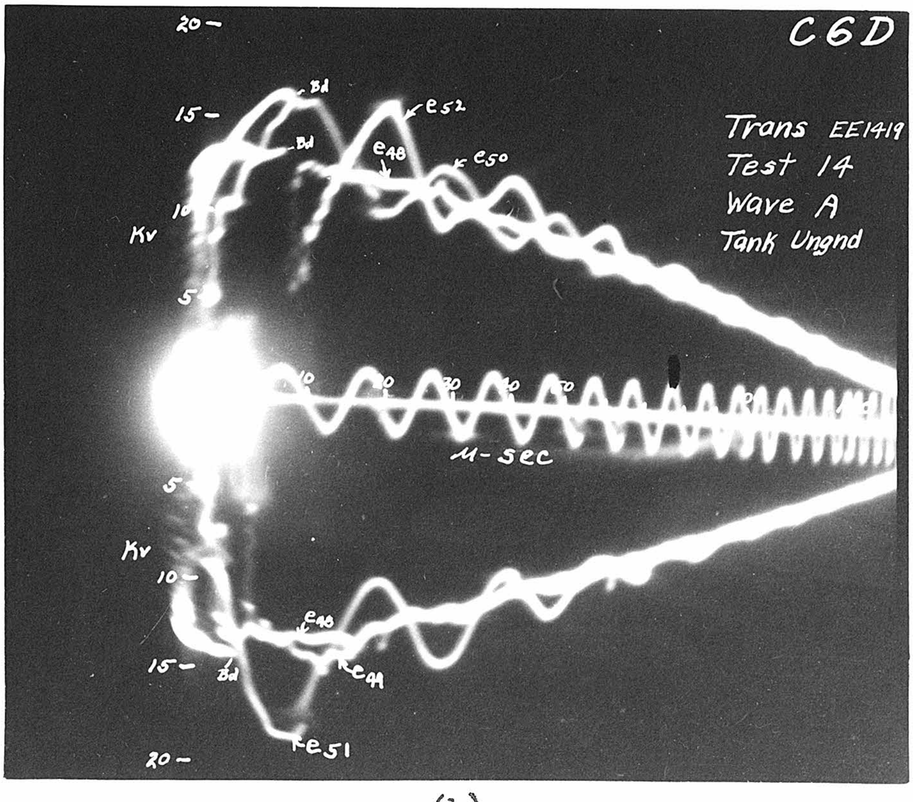

C20D
8
-Kv
4
0
e62
e58
e62a
10
20
30
40
50
100
200
μ-sec
Trans 125
Test 17
Wave A
$Z_\lambda = \infty$
Tank Gnd

(b)

Fig. 154

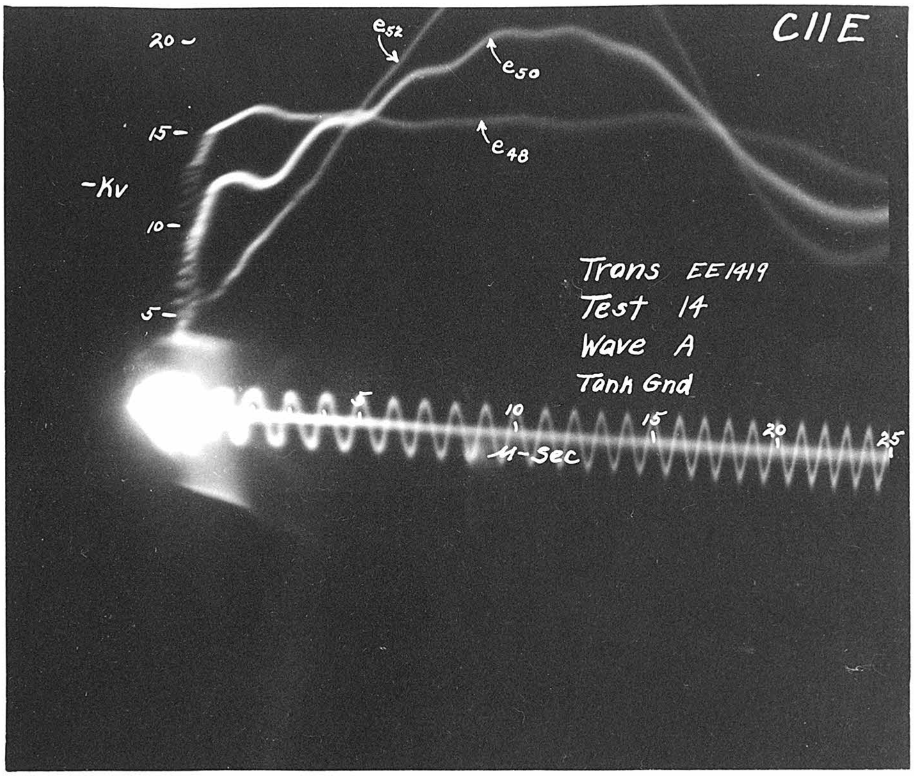

C8A
Trans EE1419
Test 14
Wave A
Tank Gnd
-Kv
-Kv
20
15
10
5
5
10
15
20
e52
e50
e48
e48
e49
e51
10
20
30
40
50
100
150
μ-sec

(a)

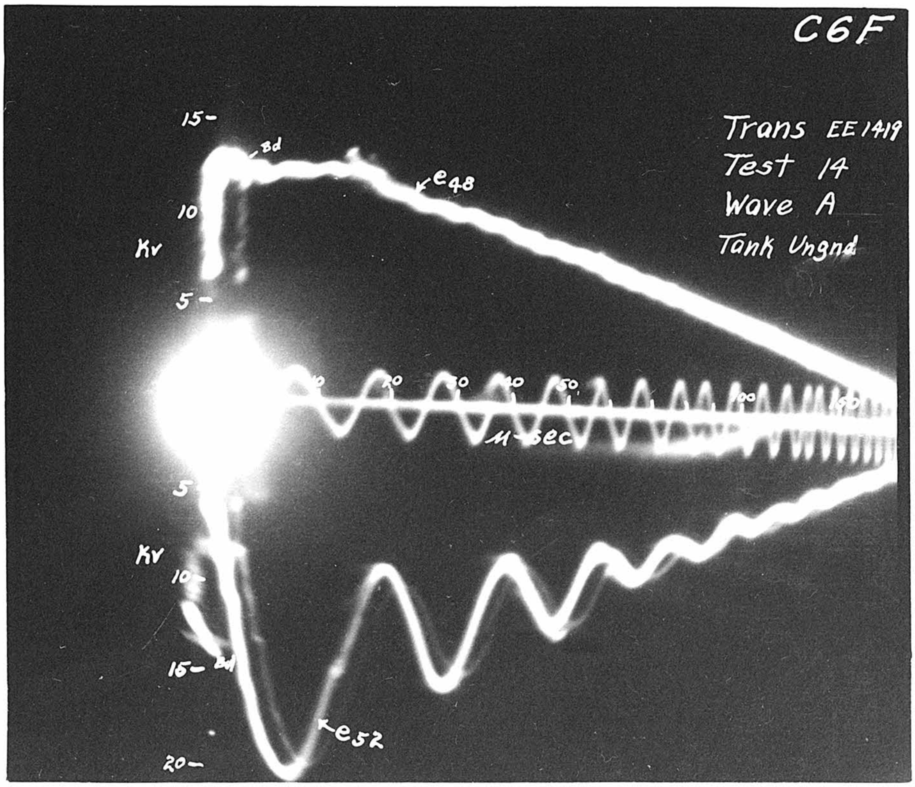

C6D
Trans EE1419
Test 14
Wave A
Tank Ungnd
Kv
Kv
20
15
10
5
5
10
15
20
Bd
Bd
Bd
e52
e48
e50
e48
e49
e51
10
20
30
40
50
μ-sec

(b)

Fig. 155

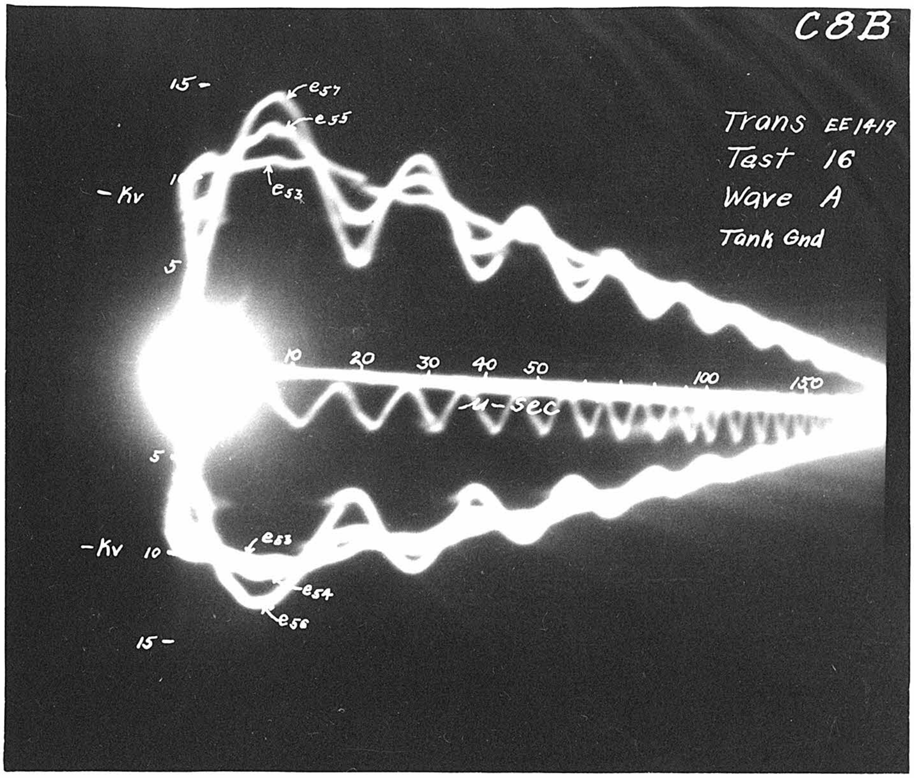

C11E
20
15
10
5
-Kv
e52
e50
e48
Trans EE1419
Test 14
Wave A
Tank Gnd
5
10
15
20
25
μ-sec

(a)

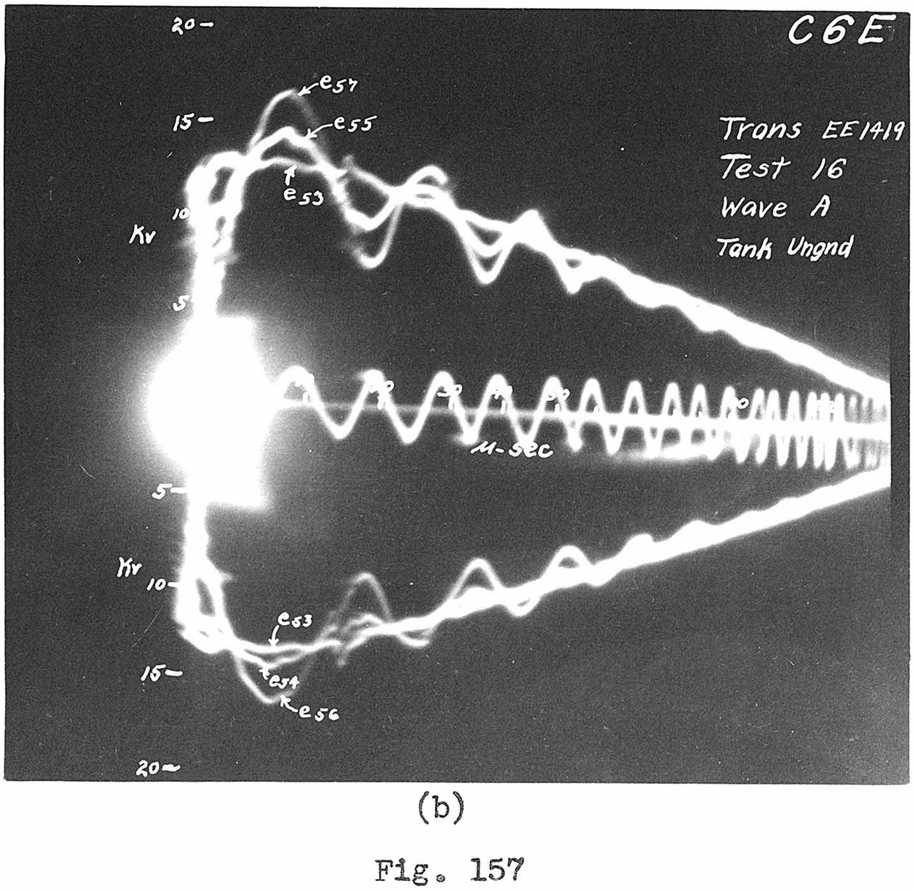

C6F
Trans EE1419
Test 14
Wave A
Tank Ungnd
15
Bd
e48
10
Kv
5
10
20
30
40
50
100
150
μ-sec
5
Kv
10
15
Bd
e52
20

(b)

Fig. 156

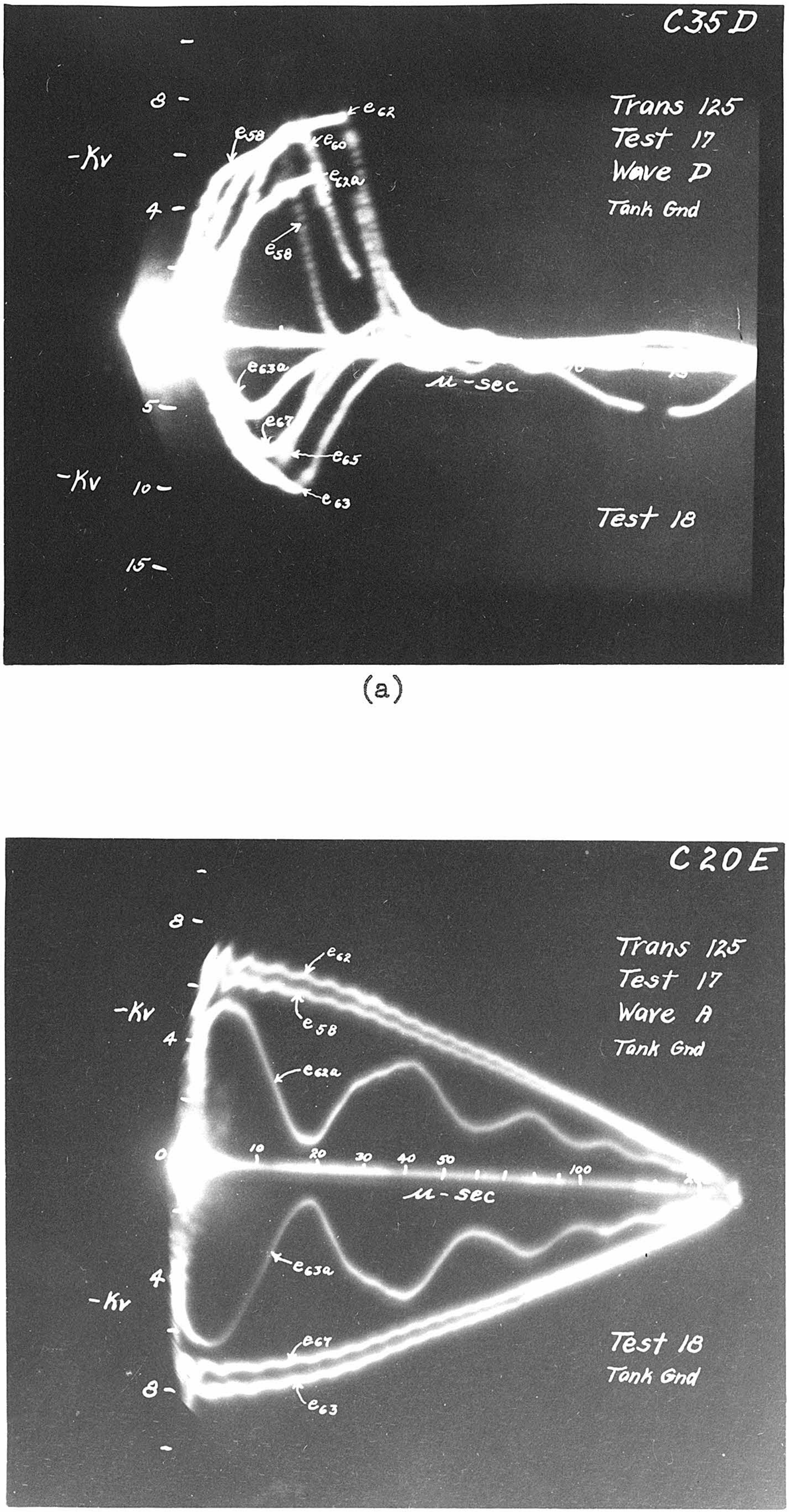

C8B
Trans EE1419
Test 16
Wave A
Tank Gnd
15
e57
e55
e53
-Kv
10
5
10
20
30
40
50
100
150
μ-sec
5
-Kv
10
e53
e54
e56
15

(a)


C6E
Trans EE1419
Test 16
Wave A
Tank Ungnd
20
15
e57
e55
e53
Kv
10
5
μ-sec
5
Kv
10
e53
15
e54
e56
20

(b)

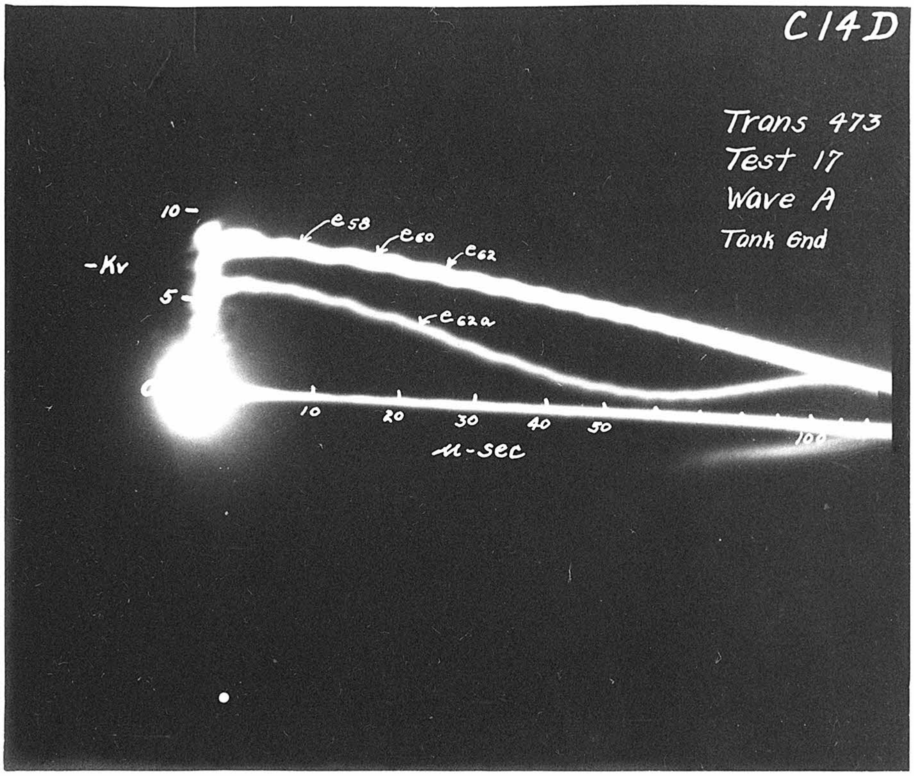
Fig. 157


C35 D
8
-Kv
4
5
-Kv
10
15
e58
e62
e60
e62a
e58
e63a
e67
e65
e63
u-sec
Trans 125
Test 17
Wave D
Tank Gnd
Test 18
C 20 E
8
-Kv
4
0
4
-Kv
8
e62
e58
e62a
10
20
30
40
50
100
u-sec
e63a
e67
e63
Trans 125
Test 17
Wave A
Tank Gnd
Test 18
Tank Gnd

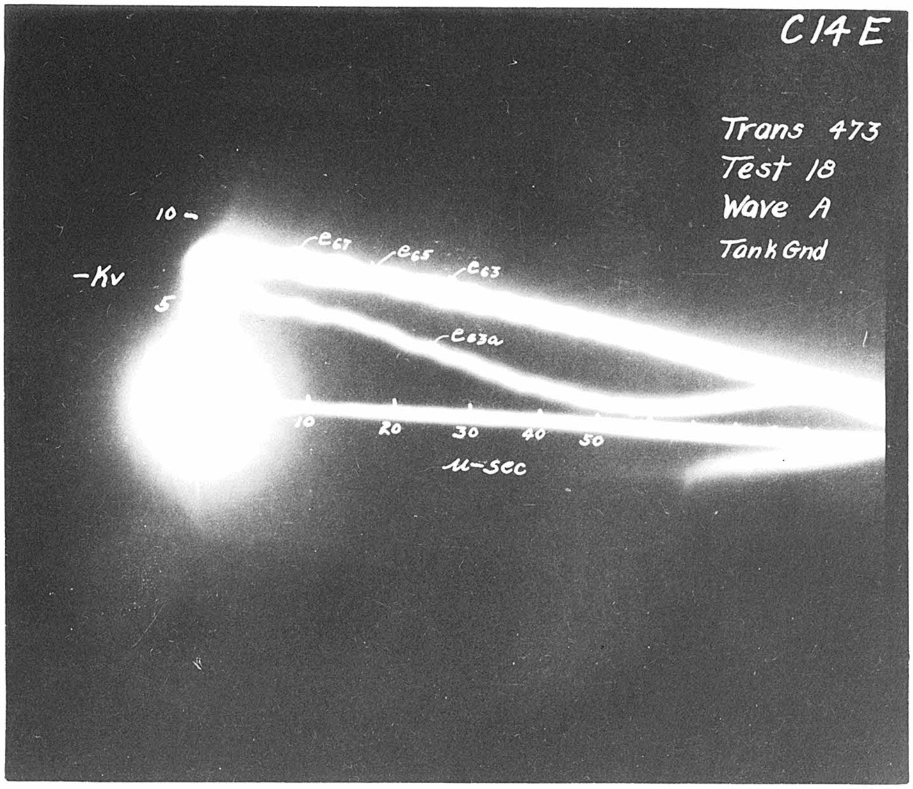
(a)

(b)

Fig. 158

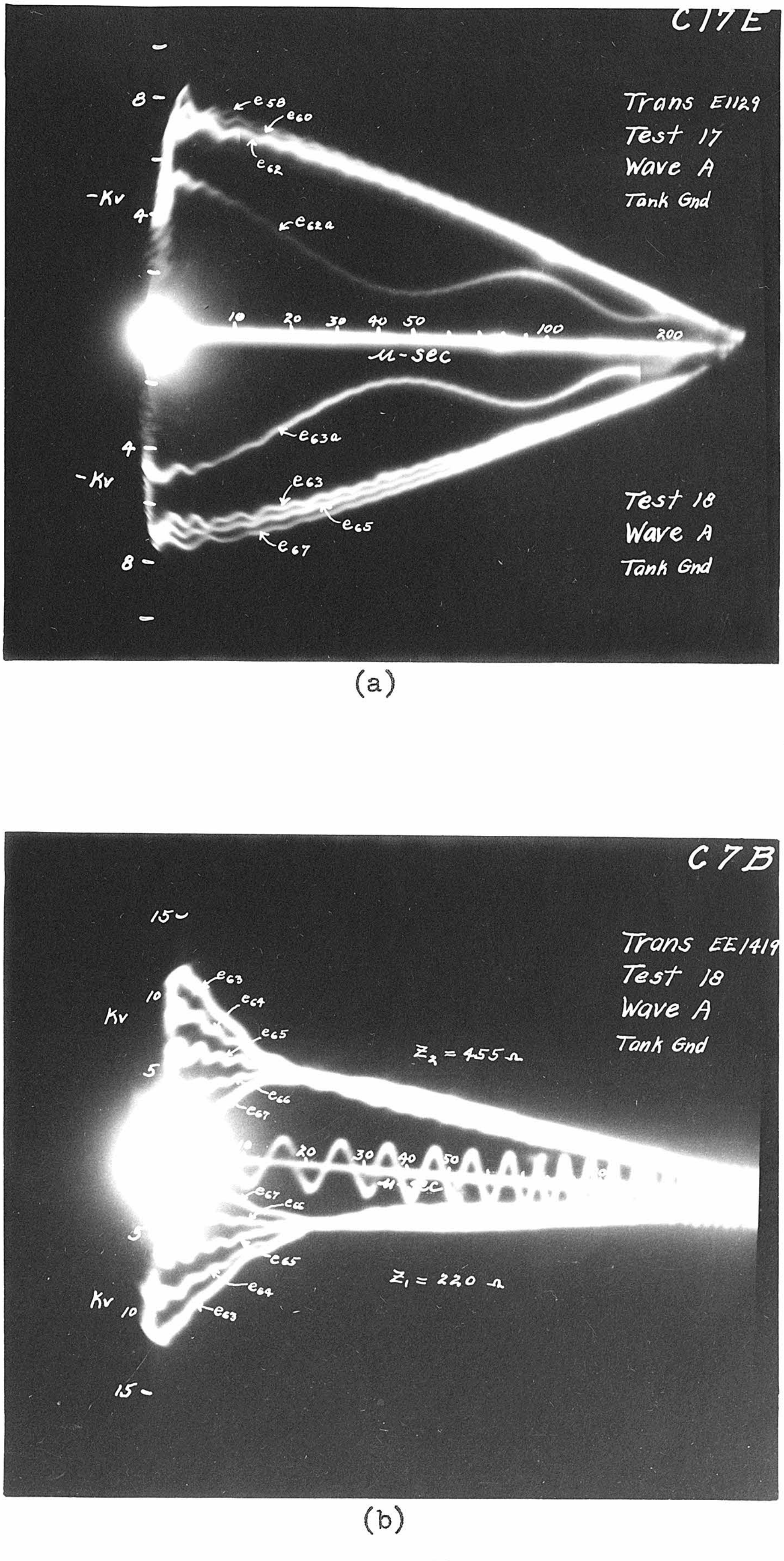

C14D
Trans 473
Test 17
Wave A
Tank Gnd
10
-Kv
5
0
e_{58}
e_{60}
e_{62}
e_{62a}
10
20
30
40
50
100
μ-sec

(a)


C14E
Trans 473
Test 18
Wave A
Tank Gnd
10
-Kv
5
e_{67}
e_{65}
e_{63}
e_{63a}
10
20
30
40
50
μ-sec

(b)

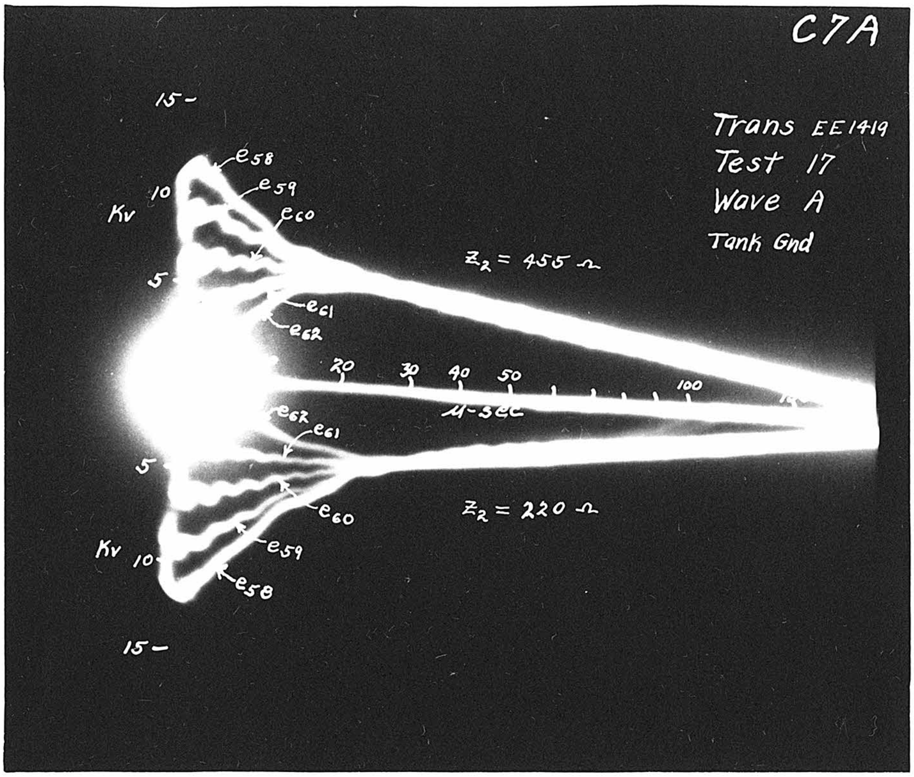
Fig. 159


C17E
Trans E1129
Test 17
Wave A
Tank Gnd
8
-Kv
4
e58
e60
e62
e62a
10
20
30
40
50
100
200
μ-sec
e63a
4
-Kv
e63
e65
e67
8
Test 18
Wave A
Tank Gnd
C7B
15
Kv
10
5
e63
e64
e65
e66
e67
Trans EE1419
Test 18
Wave A
Tank Gnd
Z2 = 455 Ω
20
30
40
50
μ-sec
e67
e66
e65
e64
e63
5
Kv
10
15
Z1 = 220 Ω

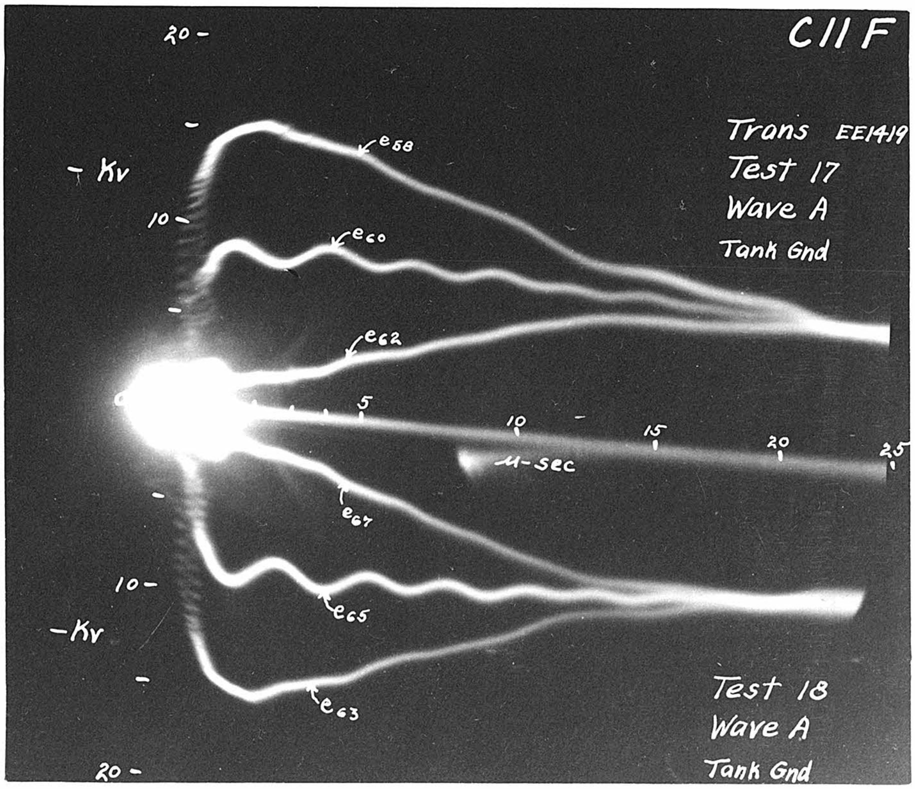
(a)

(b)

Fig. 160

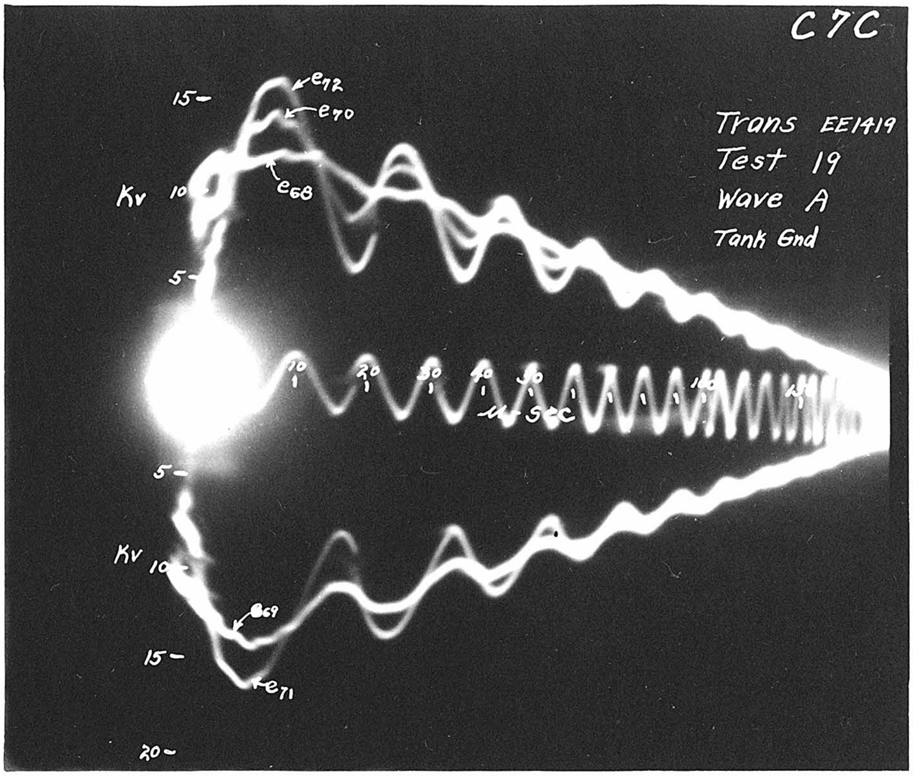

C7A
Trans EE1419
Test 17
Wave A
Tank Gnd
15 -
10
Kv
5
e58
e59
e60
e61
e62
Z2 = 455 Ω
20
30
40
50
100
µ-sec
e62
e61
5
e60
Z2 = 220 Ω
Kv
10
e59
e58
15 -

(a)

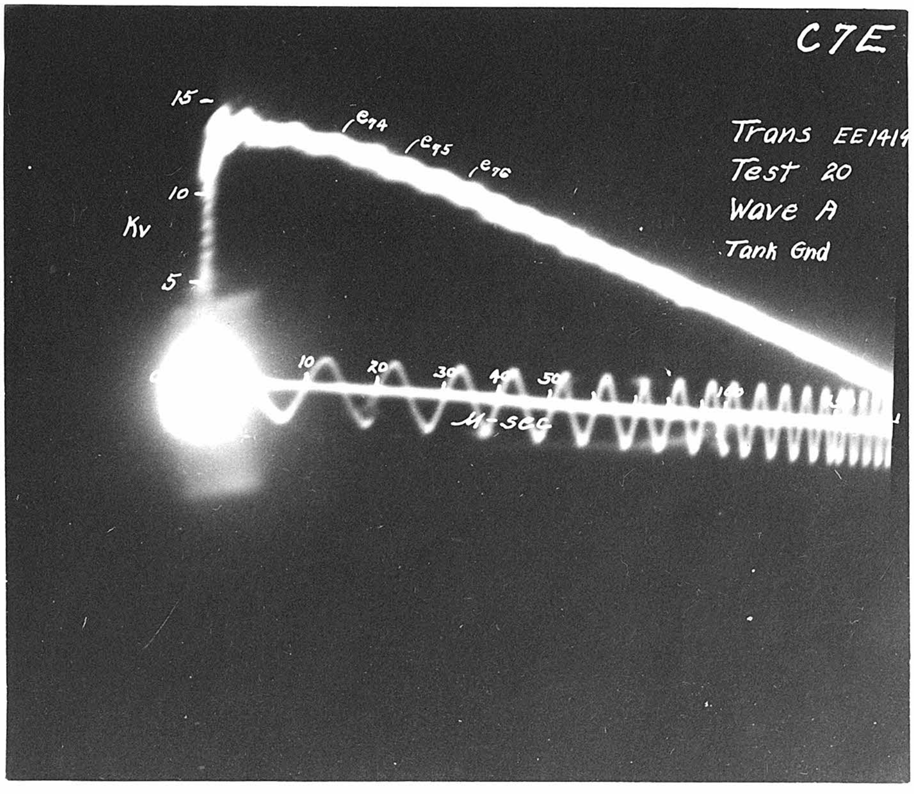

C11F
20 -
Trans EE1419
Test 17
Wave A
Tank Gnd
- Kv
e58
10 -
e60
e62
5
10
15
20
25
µ-sec
e67
10 -
e65
- Kv
e63
Test 18
Wave A
Tank Gnd
20 -

(b)

Fig. 161

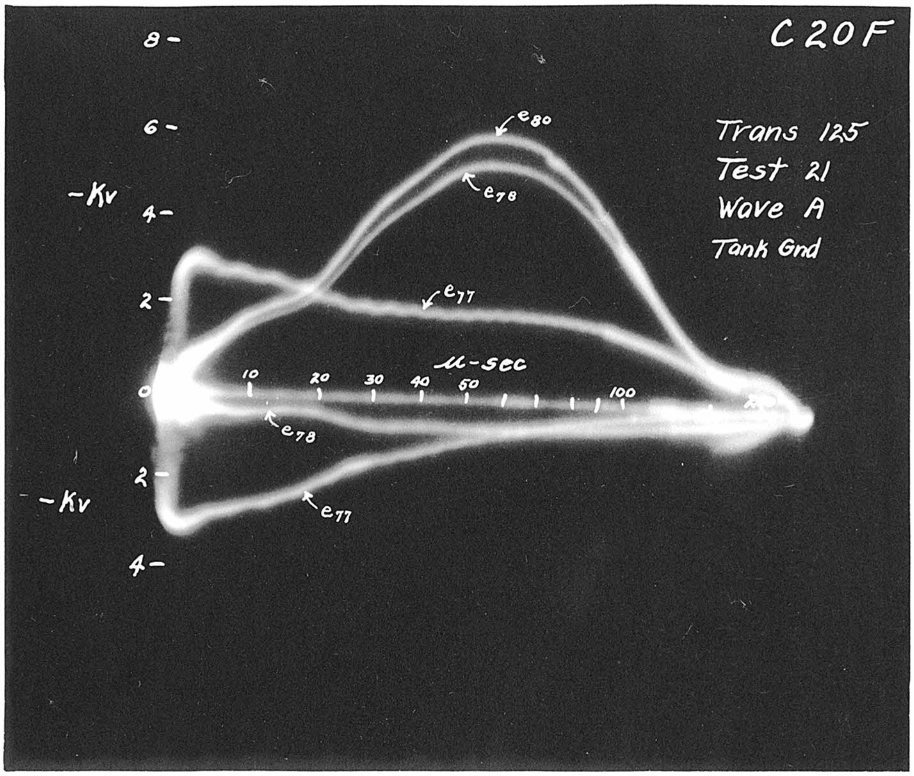

C7C
15
e72
e70
e68
Kv
10
5
10
20
30
40
50
100
μ-sec
Trans EE1419
Test 19
Wave A
Tank Gnd
5
Kv
10
e69
15
e71
20

(a)

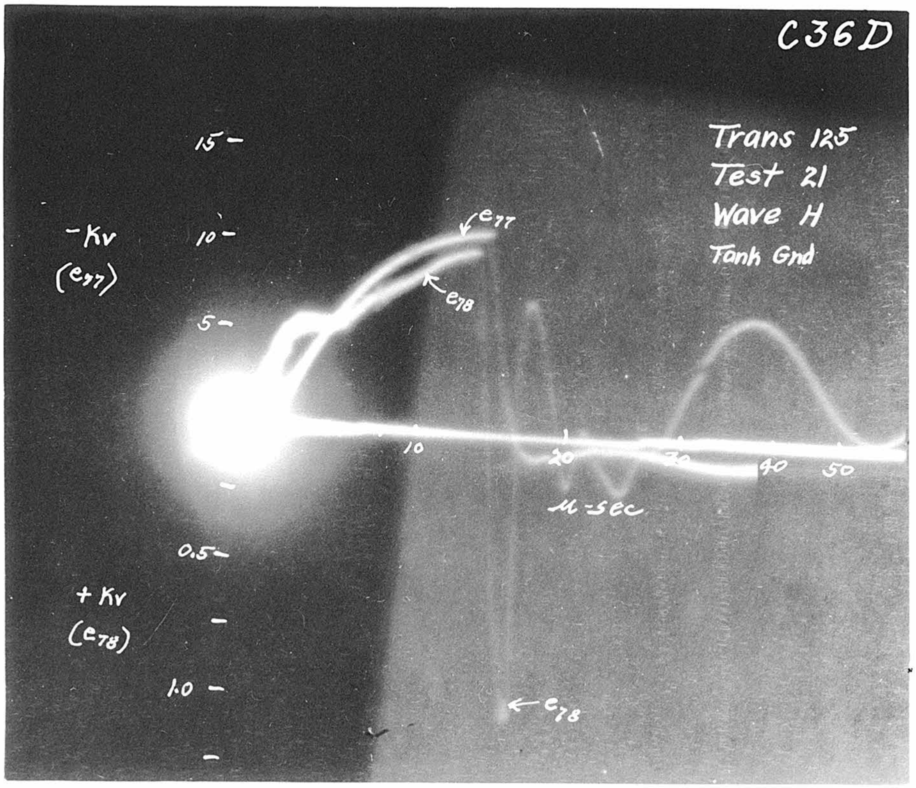

C7E
15
e74
e75
e76
10
Kv
5
10
20
30
40
50
100
μ-sec
Trans EE1419
Test 20
Wave A
Tank Gnd

(b)

Fig. 162

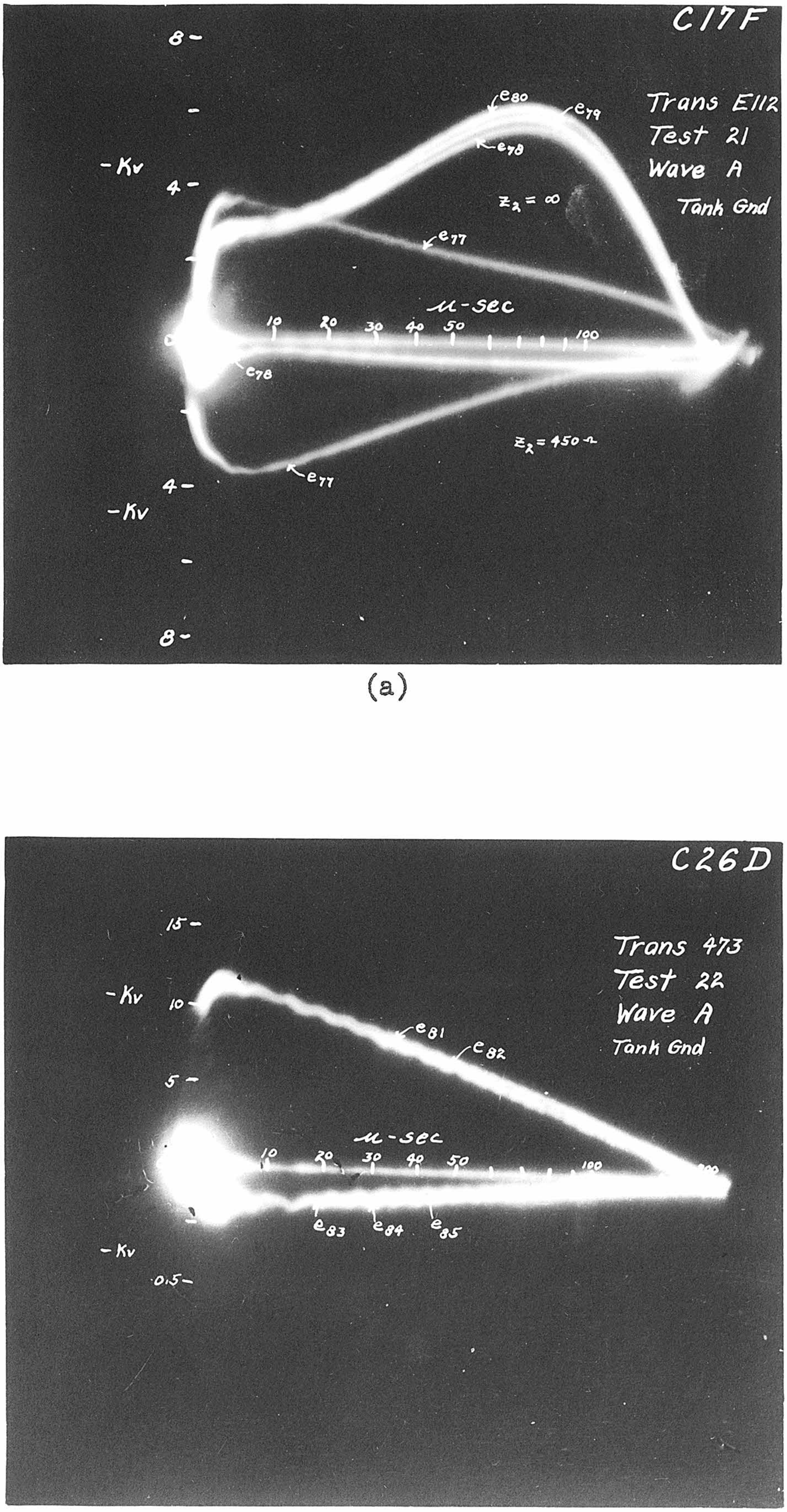

C 20 F
Trans 125
Test 21
Wave A
Tank Gnd
8
6
4
2
0
2
4
-Kv
-Kv
e_{80}
e_{78}
e_{77}
μ-sec
10
20
30
40
50
100
e_{78}
e_{77}

(a)


C36D
Trans 125
Test 21
Wave H
Tank Gnd
15
10
5
0.5
1.0
-Kv (e_{77})
+Kv (e_{78})
e_{77}
e_{78}
10
20
30
40
50
μ-sec
e_{78}

(b)

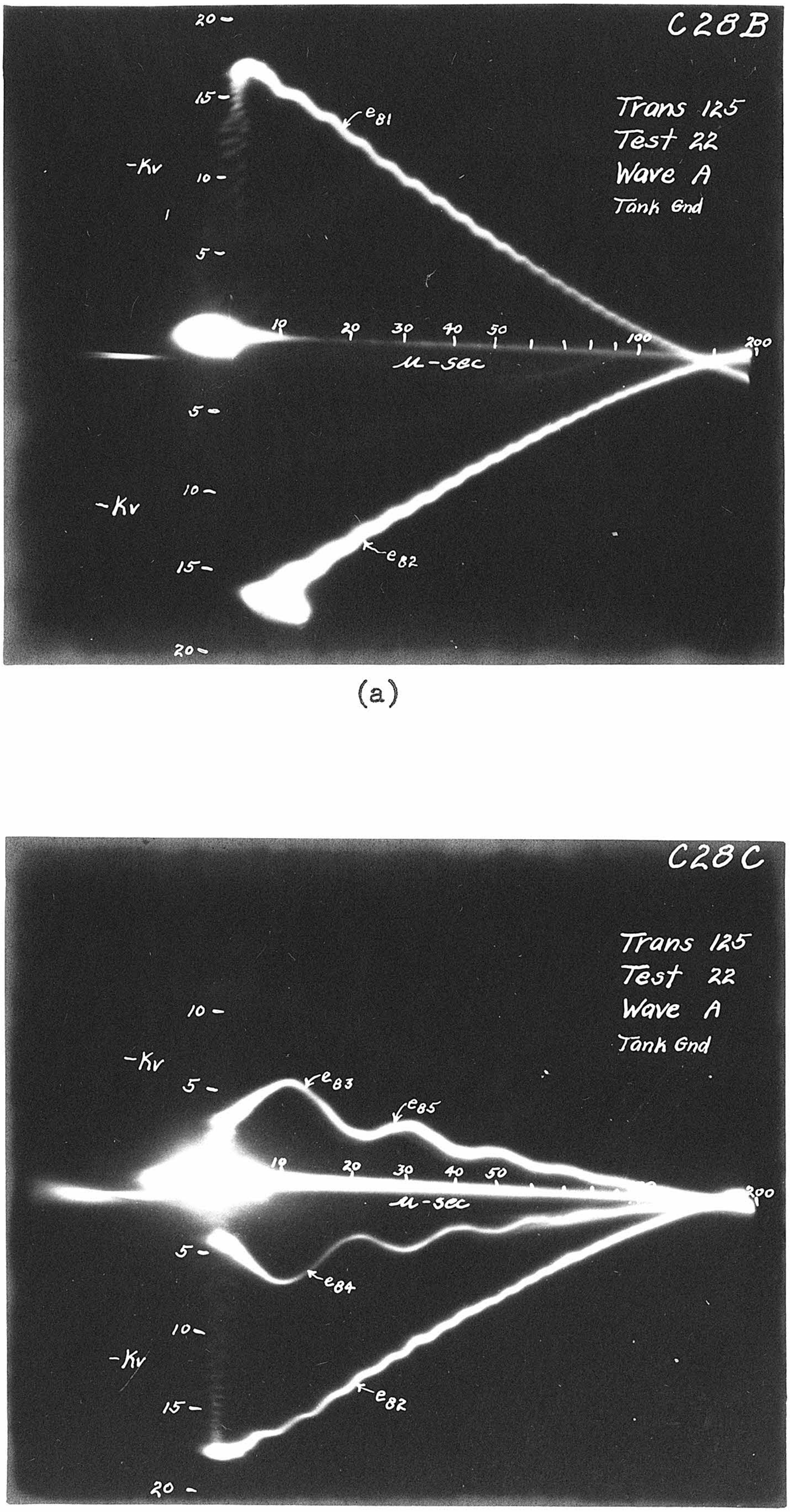
Fig. 163


C17F
8
-Kv
4
0
4
-Kv
8
Trans E112
Test 21
Wave A
Tank Gnd
e80
e79
e78
e77
$z_2 = \infty$
μ-sec
10
20
30
40
50
100
e78
$z_2 = 450 \Omega$
e77
C26D
15
-Kv
10
5
-Kv
0.5
Trans 473
Test 22
Wave A
Tank Gnd
e81
e82
μ-sec
10
20
30
40
50
100
200
e83
e84
e85

(a)

(b)

Fig. 164


C28B
Trans 125
Test 22
Wave A
Tank Gnd
-Kv
20
15
10
5
5
10
15
20
-Kv
e81
e82
10
20
30
40
50
100
200
u-sec
C28C
Trans 125
Test 22
Wave A
Tank Gnd
-Kv
10
5
5
10
15
20
-Kv
e83
e85
e84
e82
10
20
30
40
50
200
u-sec

(a)

(b)

Fig. 165

V. CONCLUSION

In conclusion, it is felt that, in the main, the eight-fold objective of the investigation has been attained.

Perhaps the most important conclusion to be drawn is that the complex phenomena of transient voltages in transformer windings can be completely explained, measured, and, to a large degree, calculated.

APPENDIX I

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APPENDIX II

Impulse Current Measurements

Impulse current measurements reduce to the measurement of an impulse voltage since current must be translated into voltage in order to be used in the electrostatic deflecting system of the cathode ray oscillograph. The only new feature in the problem of impulse current measurement is the production of voltage proportional to the current. The obvious way of doing this is by using the voltage drop across a resistance.

The characteristics of the resistor used for current measurement are of prime importance. It must maintain a constant resistance with time and with magnitude of current passing through it. The volt-ampere characteristic must be a straight line as shown in Fig. 168-b. Many composition resistance materials show volt-ampere curves which bend over as the one in Fig. 168-a. Carborundum resistance rods usually show this effect in some degree.

Resistors must also be non-inductive to be suited to current measurement. The volt-ampere characteristic of a pure inductance gives the loop shown in Fig. 168-c. The effect of inductance in a resistor is to give the volt-ampere curve of Fig. 168-d, which is that of a two ohm Vitrohm resistor.

A suitable current measuring resistor is shown in Fig. 3-b. The resistance rods are zirconium oxide composition rods which have a very good volt-ampere characteristic and little inductance. A water tube may be considered as a standard of pure resistance but is somewhat inconvenient for practical use. The resistance of a water tube is hard to measure accurately; it is subject to considerable variation with temperature change; and it is affected by evaporation and solution of electrode materials.

Figs. 7 and 8 show the oscillograph circuit set up for taking volt-ampere curves. Care must be taken to avoid error due to voltage divider current. Error in voltage measurements may be appreciable if the voltage drop in the current resistor is included.

Current has been measured successfully by using the magnetic field of a coil carrying the current. Impulse currents from 200 to 1500 amperes can be measured by using rectangular Helmholtz coils of a single turn. The use of a coil with several turns introduces spurious effects due to the inductance of the coil. Coil measurements of current are in general unsatisfactory.

APPENDIX III

The Determination of Cable Surge Impedance

The use of a resistance-cable voltage divider makes the precise determination of the cable surge impedance essential. A simple method of doing this is to use a variable cable terminal resistance, (R_1 of Fig. 17), and make records of the voltage appearing across this terminal resistor when a steep front wave is sent through the cable. The proper value of terminal resistance is the one which gives no reflections.

An example of such a test is shown in Fig. 166. The oscillogram (a) shows the effect of terminal resistance less than cable surge impedance. Wave front A suffers partial reflection with reversed polarity and returns through the cable to the test end where it reflects at the high divider resistance, (R_2 of Fig. 17 omitted), without reversal of polarity. The reflected wave then returns to the oscillograph end of the cable and produces the depression in the voltage wave at B. Reflections again occur with reversal of polarity, the second reflected wave returning at C.

Similar effects are evident when the terminal resistance is higher than the cable surge impedance except that no reversal of polarity is experienced. The oscillogram of Fig. 166-b shows this condition. Correct value of terminal resistance should result in a smooth curve which is approached in Fig. 167-a. The small break in this curve at B was due to the inductance of the terminal resistor which in this case was a fine wire about six inches long.

The cable in the example just cited had a length of 500 feet. The time required for the wave to traverse the cable twice may be seen to be two microseconds. The velocity of propagation was then 500 feet per microsecond or half the speed of light.

Several other methods of determining cable surge impedance have also been described.[1]

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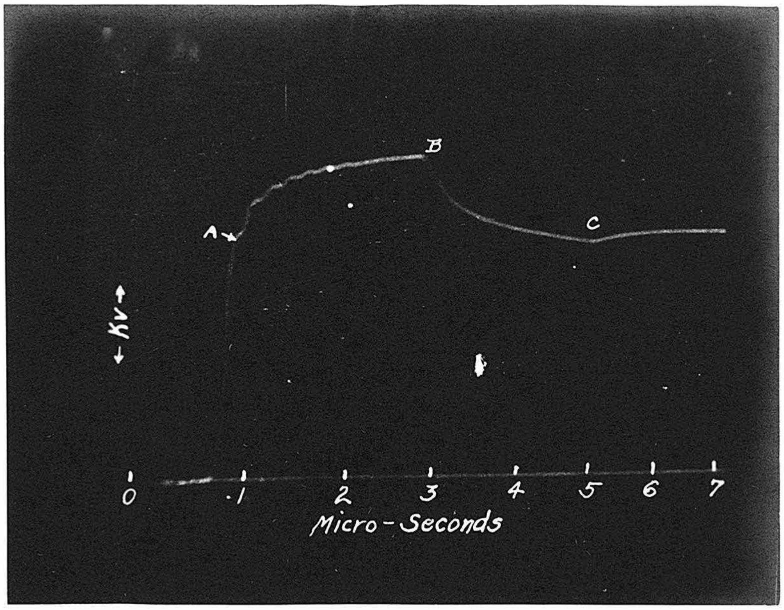
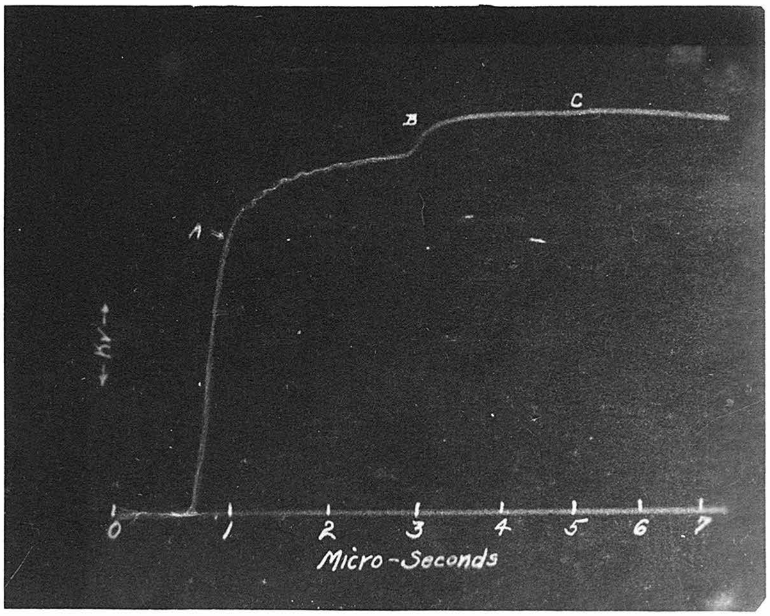
[1] "Study of Effects of Short Lengths of Cable on Traveling Waves," K. B. McEachron, J. G. Hemstreet, and H. P. Seelye. A.I.E.E. Trans., Vol. 49, October 1930. p. 1432.

Also "Traveling Waves on Transmission Lines with Artificial Lightning Surges," K. B. McEachron, J. G. Hemstreet, and W. J. Rudge. A.I.E.E. Trans., Vol. 49, July 1930. p. 885.

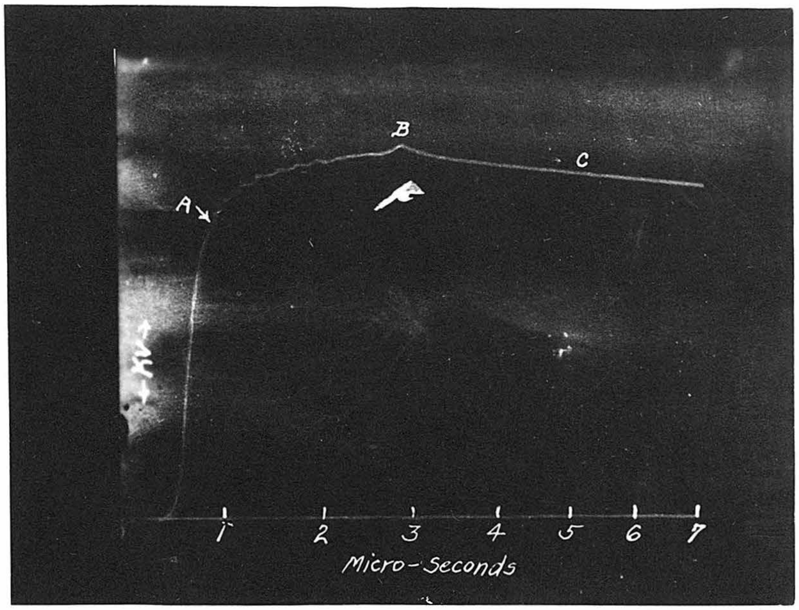

A
B
C
Kv
0
.1
2
3
4
5
6
7
Micro-Seconds

(a)

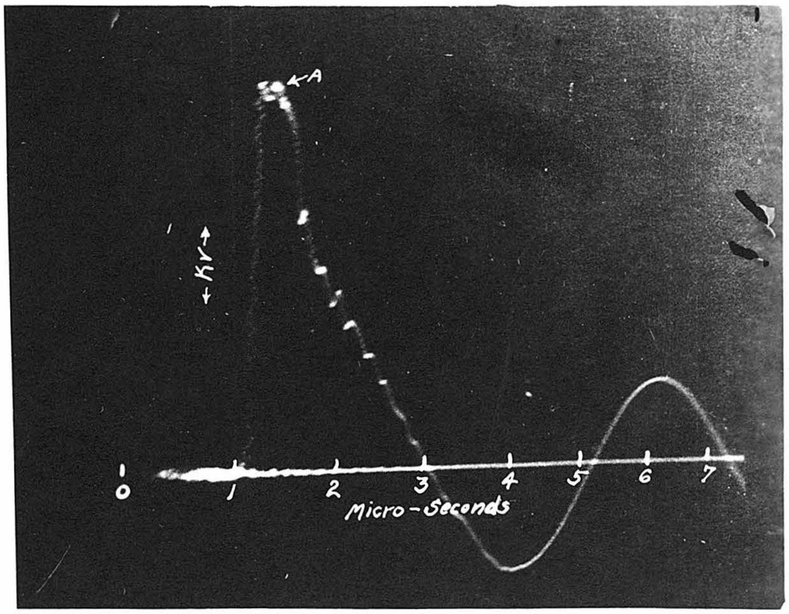

A
B
C
Kv
0
1
2
3
4
5
6
7
Micro-Seconds

(b)

Fig. 166

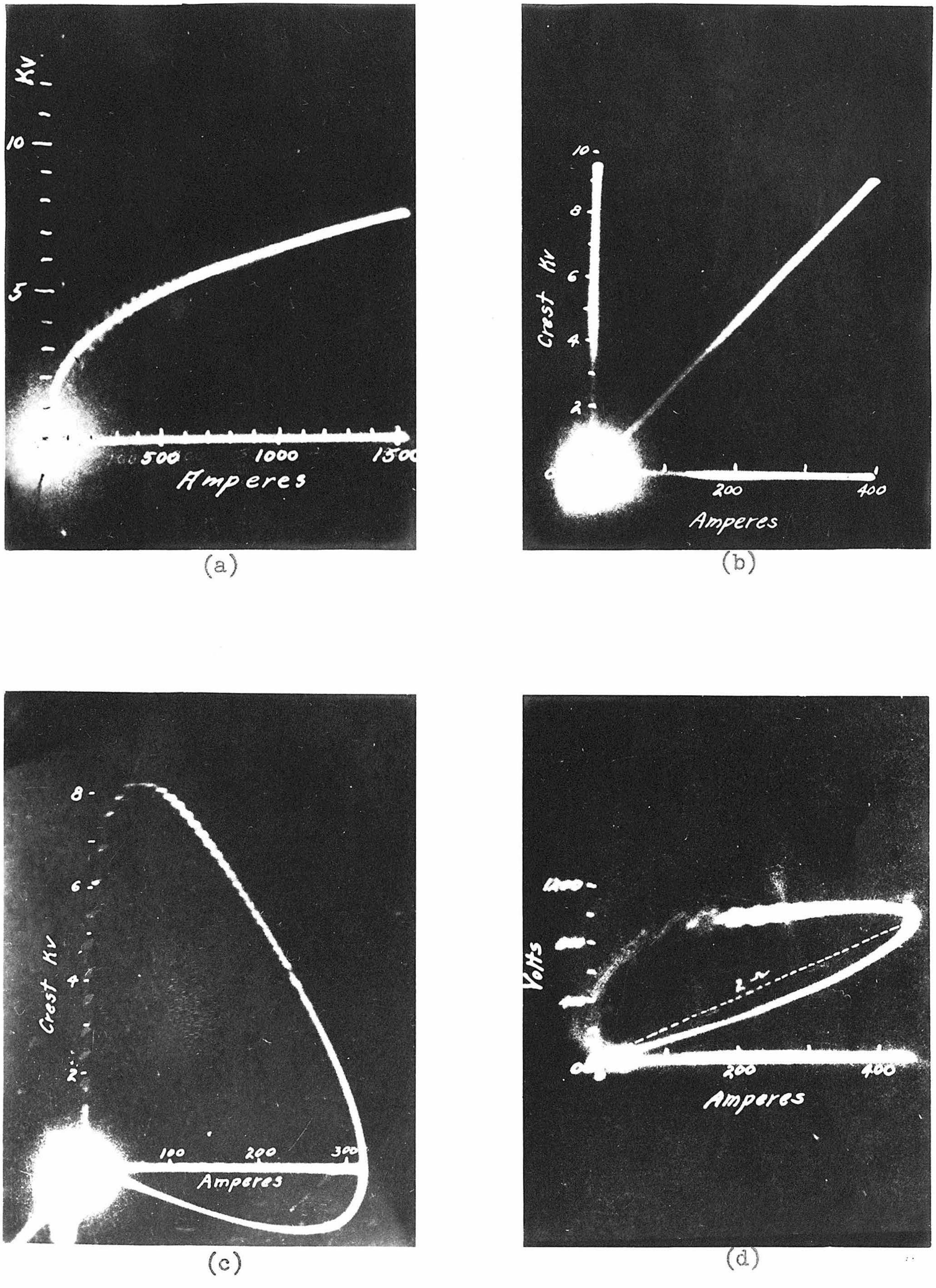


(a)



(b)

Fig. 167


Kv
10
5
500
1000
1500
Amperes
10
8
6
4
2
0
Crest Kv
200
400
Amperes
8
6
4
2
Crest Kv
100
200
300
Amperes
Volts
0
200
400
Amperes

(a) (b)

(c) (d)

Fig. 168