A STUDY OF THE DYNAMIC LONGITUDINAL STABILITY OF AIRPLANES WITH SPECIAL APPLICATION TO DESIGN

THESIS

by

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In partial Fulfillment of the Requirements for the

Degree of

MASTER OF SCIENCE

California Institute of Technology

Pasadena, California

1934

CONTENTS

SUMMARY

This thesis is a revision and extension of a previous work on longitudinal dynamic stability[1]. In the present work, an attempt is made to simplify the classical stability theory which is far too complicated and cumbersome to be of practical use in design.

By employing a non-dimensional set of units, and by making use of a graphical method, the range of dynamic stability for different values of fundamental parameters is defined graphically by means of diagrams, the coordinates of which are conveniently chosen as the position of the center of gravity and the size of the tail of the airplane. The coordinates, combined with two characteristic parameters, completely define the regions of dynamic longitudinal stability; while the diagrams themselves determine the period and the damping factor of the longitudinal oscillation, thus bringing the complicated theory into the range of applicability to design.

INTRODUCTION

Because of the inherent complexity of the classical stability theory, the airplane designer has not found it practical to apply the theory to actual design. Instead, he provides for a small amount of static stability, depending on the precedence of previous satisfactory designs from which certain empirical rules have been developed, for satisfactory dynamic characteristics. Such procedure, however, does not guarantee proper dynamic stability; in fact, undesirable properties in dynamic stability may develop in a plane which has the proper static stability. Thus, we see that in the process of a new design, it is desirable to know the dynamic longitudinal stability characteristics of an airplane in order to foresee and to avoid possible undesirable properties. Before this can be done, however, it is necessary to reduce as much as possible the complexity of the theory to the extent that it can be used practically.

In this paper, following an idea presented by Mr. S.B. Gates [1], an attempt is made to simplify the theory by reducing the number of variables that appear, and by applying a graphical method of representation of the stability characteristics. By reference to the charts that have been developed, it is possible to determine immediately the dynamic stability characteristics of an airplane, knowing its fundamental parameters. It is hoped that the development of these charts will contribute to design procedure by determining the longitudinal dynamic stability of an airplane.

THEORETICAL ANALYSIS

From classical aerodynamics[2], we know that in the most general case, the longitudinal stability of an airplane is determined by the roots of the quartic,

$$(1.0)\ \text{---}\ \begin{vmatrix} \lambda - X_u & -X_w & g\cos\theta_0 - X_q\lambda + \lambda w_0 \\ -Z_u & \lambda - Z_w & g\sin\theta_0 - Z_q\lambda - u_0\lambda \\ -M_u & -M_w & \lambda^2 - M_q\lambda \end{vmatrix} = 0\ ,$$

where:

X, Y, Z are the components of the resultant aerodynamic forces along the axes ÷ the mass of the airplane

L, M, N are the moments of the external forces about the axes ÷ the corresponding moments of inertia, A, B, C,

u_o, v_o, w_o are the components of the resultant velocity in the undisturbed motion. ($v_o = 0$)

θ_o = the upward inclination of the longitudinal axis to the horizontal.

For simplicity, wind axes are used in which the stability derivatives X_q and $Z_q = 0$, and $u_o = U$, $w_o = 0$. θ_o becomes the angle of climb in the undisturbed motion.

Since it is very desirable from an analytic standpoint to have the stability criteria for an airplane expressed in non-dimensional form, the dimensionless system devised by Mr. Glauert[3] is introduced. This system is based on the fact that an aerodynamic force can be expressed in the form $X = -C_D \frac{\rho}{2} S\bar{U}^2$; from which the force derivative is of the form,

$$\bar{X}_u = \frac{m\,dX}{du} = -C_D \rho S \bar{U}$$

or

$$m\bar{X}_u = -x_u \frac{S\rho\bar{U}}{m} = -A x_u\ .$$

Thus, in the new notation, the previous classical derivatives X_u, X_q, M_u, M_q, etc., are replaced by the dimensionless derivatives, x_u, x_q, m_u, m_q, etc., defined by the equations,

$$X_u = -Ax_u ,$$

$$X_q = -Ax_q ,$$

$$M_q = -Am_q ,$$

$$M_u = -\frac{A}{l} m_u , \qquad \text{where } A = \rho\frac{\bar{U}S}{m} .$$

By substituting these non-dimensional derivatives into the classical stability equation, it is found that all the parameters which occur are dimensionless. Apart from the derivatives themselves, the only parameter which affects the stability of the plane is

$$\mu = \frac{\bar{U}}{lA} = \frac{m}{\rho S l} = 13.08\frac{l_W}{l} \text{ (For sea level).}$$

In the dimensionless system, the unit of time is $\tau = \frac{m}{\rho S l}$ and the unit of length is l, taken as the distance from the center of gravity of the plane to the tail post. From this, the unit of velocity becomes $\frac{l}{\tau} = \frac{\bar{U}}{\mu}$, where the quantity μ may be called the " relative density" of the airplane since it is proportional to the mass and inversely proportional to the cube of the linear dimensions and the density. The quantity "μ", excepting the derivative coefficients, is the only parameter which effects the stability ; and it proves very useful in concisely summarizing several important parameters which occur in the theory. It is the only quantity in the dimensionless system in which the linear scale, the wing loading, and the density enter directly. Any variation in any one of these quantities is included in the one variable , μ.

In terms of the non-dimensional system, neglecting the derivatives z_q and x_q and using wind axes, the stability equation (1) can be expressed in the following manner: By making use of the equilibrium condition,

$$L = W\cos\theta_0 = \frac{1}{2}\rho\bar{U}^2 C_L S,$$

from which $g\cos\theta_0 = \rho\frac{S\bar{U}}{m}\frac{C_L}{2}\bar{U}\,;\; g\sin\theta_0 = \rho\frac{S\bar{U}}{m}\frac{C_L}{2}\bar{U}\tan\theta_0$,

we have

$$(2.0)\qquad \begin{vmatrix} \lambda'+x_u & x_w & \mu\frac{C_L}{2} \\ z_u & \lambda'+z_w & \mu\left(\frac{C_L}{2}\tan\theta_0-\lambda'\right) \\ m_u & m_w & \lambda'^2+m_q\lambda' \end{vmatrix} = 0.$$

Equation (2.0) can be reduced to the form, (3.0), i.e.,

$$(3.0)\qquad \lambda'^4+B_1\lambda'^3+C_1\lambda'^2+D_1\lambda'+E_1=0,$$ where the

coefficients B_1, C_1, D_1, and E_1 are functions of the stability derivatives; namely,

$$B_1 = z_w + m_q + x_u$$
$$C_1 = m_q(z_w+x_u)+\mu m_w + x_u z_w - x_w z_u$$
$$D_1 = m_q(x_u z_w - x_w z_u)+\mu m_w\left(x_u-\frac{C_L}{2}\tan\theta\right)-\mu m_u\left(\frac{C_L}{2}+x_w\right)$$
$$E_1 = \frac{C_L}{2}\mu m_w(z_u - x_u\tan\theta)-\frac{C_L}{2}\mu m_u(z_w - x_w\tan\theta)$$

It is noted that below the stalling speed of the airplane, all the non-dimensional derivatives except m_w, m_u, and x_w are positive. Longitudinal stability can thus be described completely in terms of these derivatives and "μ", which is always associated with either m_w or m_u.

Because the absolute values of two roots are large in comparison to the values of the other two, we use Bairstow's approximate method and express (3.0) in the form of a biquadratic,

$$(4.0) \quad \left[\lambda^2 + B_1\lambda + C_1\right]\left[\lambda^2 + \left(\frac{D_1}{C_1} - \frac{B_1E_1}{C_1^2}\right)\lambda + \frac{E_1}{C_1}\right] = 0.$$

The first quadratic represents the so-called " short oscillation" which has negligible influence on longitudinal stability below the stall since it is heavily damped and invariably stable. The second quadratic, however, represents the " Phugoid " motion, or a slightly damped periodic mode which directly determines the dynamic stability of an airplane.

GRAPHICAL REPRESENTATION OF STABILITY

From the classical theory, it is known that a system represented by

$$\lambda^4 + B_1\lambda^3 + C_1\lambda^2 + D_1\lambda + E_1 = 0$$

is stable if the following conditions are satisfied:

1. The coefficients $B_1, C_1, D_1,$ & E_1 must be positive;
2. Routh's discriminant, $R_1 = B_1C_1D_1 - D_1^2 - B_1^2E_1 > 0$.

If $B_1, C_1,$ or D_1 goes through zero, then the discriminant, R_1, reaches zero first. However, if E_1 goes through zero, R_1 increases. Hence, for stability, we must consider especially R_1 and E_1. If $R_1 = 0$, the condition of stability changes from a damped oscillation to a divergent one. If $E_1 = 0$, a subsidence changes into a divergence.* Hence we see that the regions of stability and instability can be separated from one another by working with the equations, $E_1 = R_1 = 0$. For a graphical representation, it is possible, by using the two equations $E_1 = 0$, $R_1 = 0$, to establish boundaries for the regions of stability.

* FOR ILLUSTRATION OF THE TYPES OF MOTION, SEE (5) PAGE 139.

If the variables "X" and "Y" are chosen as the coordinates, where "X" involves the C.G. postion and "Y" involves the size of the tail of the airplane, it is possible to establish on the "XY" plane boundaries separating the regions of instability from those of stability. Two fundamental parameters are varied, the other parameters remaining constant, thus giving a family of boundaries for stability.

It has been found convenient to take as the variables "x" and "Y", the following:

$$X = \frac{l^2}{k_B^2}\bar{h}$$

$$Y = \frac{1}{2}\frac{l^2}{k_B^2}\frac{S_t}{S}\frac{dC_{L_t}}{d\alpha_t}$$

where:

l = length from the C.G. to the tail post

k_B = radius of gyration about the Y axis

S_t = area of horizontal tail surfaces

S = area of wing

$\frac{dC_{L_t}}{d\alpha_t}$ = slope of lift curve of tail surface (horiz.)

$\bar{h}$ = distance of C.G. from the aerodynamic center of the wing section, EXPRESSED AS A FRACTION OF "t."

The "XY" plane is filled with two families of curves; namely,

1. $\bar{K} = \frac{D_1}{C_1} - \frac{B_1E_1}{C_1^2}$

2. $\frac{2\pi}{P} = \sqrt{\frac{E_1}{C_1} - \frac{1}{4}\left(\frac{D_1}{C_1} - \frac{B_1E_1}{C_1^2}\right)^2}$

which define the values of the roots of the quadratic,

$$\lambda^2 + \lambda\left(\frac{D_1}{C_1} - \frac{B_1E_1}{C_1^2}\right) + \frac{E_1}{C_1} = 0 \quad .$$

$\bar{K}$ and P in the non-dimensional system may be converted into the damping factor and the period, pound ft.-sec. system, by use of the appropriate factors:

$$P(\text{seconds}) = \tau P \quad ; \quad D.F. = \frac{\bar{K}}{2\tau}$$

Since $W = \rho \frac{\bar{U}^2}{2} S C_L$ from which $\bar{U} = \sqrt{\frac{2 l_w}{\rho C_L}}$,

$$\tau = \frac{m}{\rho S \bar{U}} = \frac{mg}{\rho g S \bar{U}} = \frac{l_w}{\rho g \sqrt{\frac{2 l_w}{\rho C_L}}} = \underbrace{\frac{1}{\sqrt{2}\, g \sqrt{\rho_0}}\sqrt{\frac{l_w C_L}{\sigma}}}_{=(0.451)}$$

C_L	$\tau\sqrt{\frac{\sigma}{l_w}}$	$\frac{1}{2\tau}\sqrt{\frac{l_w}{\sigma}}$
0.3	0.247	2.023
0.5	0.319	1.568
1.0	0.451	1.108
1.2	0.494	1.012

To get the damping factor and the period, we note the following relations:

$$D.F. = \bar{K}\left(\frac{1}{2\tau}\sqrt{\frac{l_w}{\sigma}}\right)\sqrt{\frac{\sigma}{l_w}}$$

$$P_{(\text{sec.})} = \left(\tau\sqrt{\frac{\sigma}{l_w}}\right)\sqrt{\frac{l_w}{\sigma}}\, P$$

To get the time to damp to one-half amplitude, we have the relation:

$$T_{\frac{1}{2}} = \frac{0.693}{D.F.} \, .$$

The family of period curves in the form,

$$\frac{2\pi}{P} = \sqrt{\frac{E_1}{C_1} - \frac{1}{4}\left(\frac{D_1}{C_1} - \frac{B_1 E_1}{C_1^2}\right)^2}$$

are very difficult to use for plotting. It has been found sufficienty accurate, except when E_1 becomes very small, to neglect the term, $\frac{1}{4}\left(\frac{D_1}{C_1} - \frac{B_1 E_1}{C_1^2}\right)^2$ in comparison with the term $\frac{E_1}{C_1}$. Thus, the period curves are reduced to the simpler form,

$$\frac{2\pi}{P} = \sqrt{\frac{E_1}{C_1}} \, .$$

Of the damping curves, $\bar{K} = 0$, is approximately the boundary defining the transition of the phugoid into an increasing oscillation. We note that

$$R_1 = B_1C_1D_1 - D_1^2 - B_1^2E_1 = 0.$$

D_1^2 is usually small in comparison to the other terms, hence we have as an approximation to $R_1 = 0$, the two curves, $B_1 = 0$, and $C_1D_1 - B_1E_1 = 0$, which is identically $\bar{K} = 0$, since $\bar{K} = \dfrac{D_1C_1 - B_1E_1}{C_1}$.

As the above approximation breaks down only where B_1 and C_1 are small, $\bar{K} = 0$ is a very good approximation to the $R_1 = 0$ boundary curve for all practical purposes since B_1 is large for any actual case.

DISCUSSION OF PARAMETERS

In keeping with the purpose of this work, it was found necessary to include two sets of parameters; those which correspond to the more modern type of airplane, and those which refer to the older type still being built. Thus, two groups of curves have been plotted; one group(Case I) using $\frac{dC_L}{d\alpha} = 4.8$ and $C_{D_p} = 0.02$, the other group (Case II) using $\frac{dC_L}{d\alpha} = 4.0$ and $C_{D_p} = 0.05$.

In order to represent conveniently the two families of curves, period and damping factor, it is necessary to limit the number of variables occuring, and to give constant values to those parameters which do not vary greatly or are not of great importance in stability. In order to arrive at satisfactory average parameters, it was necessary to collect flight test data, calculate the various parameters, and average those which seemed to lend themselves to such treatment. As a result, various simplifications were made as follows:

1. Lift

Case I

$$\frac{dC_L}{d\alpha} = 4.8$$

Case II

$$\frac{dC_L}{d\alpha} = 4.0$$

(These values are assumed to hold up to $C_L = 1.2$)

2. Drag

As is customary, the total drag is assumed to be of the form, $C_D = \frac{f}{S} + \frac{C_L^2}{e\pi AR}$. The values for C_D are:

Case I

$$C_D = 0.02 + 0.065\, C_L^2$$

Case II

$$C_D = 0.06 + 0.065\, C_L^2$$

3. Pitching Moment

Due to wings:

The pitching moment about the C.G. is given as;

$$C_M = C_{M_0} + \left(\frac{a}{t} - h_0\right) C_L ,$$

where h_0 is the aerodynamic center. ($0.22 \leqq h_0 \leqq 0.25$), and " a" is the horizontal distance from the leading edge of the wing to the C.G.

Due to tail:

The pitching moment due to the tail is assumed to be,

$$C_{M_t} = -\frac{l}{t}\frac{S_t}{S} C_{L_t} .$$

Due to fuselage:

If there are wind tunnel tests on the plane, then the effects of the fuselage can be accounted for. If tests are not available, then the following corrections may be applied to account for the fuselage effect on the moment curve:*

Type	Change in $\left(\frac{dC_M}{dC_L}\right)_{WING}$
Small, low-wing monoplanes (∾Northrop XFT-1)	$+0.025$
Large, low-wing monoplanes (∾Douglas DC-1)	$+0.035$

The above correction can be absorbed by using an " effective" a/t in the expression for the pitching moment for the wing alone. In the same manner, the vertical displacement of the C.G. must be corrected for; i.e., $\Delta \frac{a}{t} \doteq -\frac{1}{10}\frac{b}{t}$, where 'b' is the vertical distance from the C.G. to the chord line. Thus, the final "effective a/t" will be

$$\left(\frac{a}{t}\right)_{EFF.} = \left(\frac{a}{t}\right) + \left(\Delta \frac{a}{t}\right)_{FUSE.} \pm \left(\frac{1}{10}\frac{b}{t}\right)$$

and $\bar{h}$ will equal $\left[\left(\frac{a}{t}\right)_{EFF.} - h_0\right] = \frac{dC_M}{dC_L}$.

Downwash

$w = \frac{d\epsilon}{d\alpha}$ at the tail is taken as $= \dfrac{1+\frac{k}{\pi R_W}}{1+\frac{k}{\pi R_t}} = 0.5$.

Tail Length

A good average for l/t, or the ratio of the length from the C.G. to the tail post to the chord of the wing, was found to be 3.00.

Rotary Derivative

As in Gates' work, m_q for the total plane was taken as 5/4 of that due to the tail.

* Results from research, C.I.T. wind tunnel tests, by author.

Tail Efficiency

η_t , the ratio of the force on the horizontal surfaces when attached to the airplane to the force on the surfaces when at the same effective angle of attack in the free air stream, was found to be = 0.72 (Average value.)

LONGITUDINAL DYNAMIC STABILITY OF GLIDING FLIGHT

The dimensionless derivatives in the case of gliding flight become:

$$x_u = C_D \quad ; \quad z_u = C_L$$

$$x_w = -\frac{1}{2}\,\frac{1-\frac{k}{\pi R}}{1+\frac{k}{\pi R}}\,C_L \; ; \quad z_w = \frac{1}{2}\left(\frac{dC_L}{d\alpha} + C_D\right)$$

(C_D is neglected in the final analysis.)

$$m_w = -\frac{1}{\eta}\,\frac{1}{2}\,\frac{t}{l}\,\frac{k}{1+\frac{k}{\pi R}}\left[\bar{h} - \frac{l}{t}\,\frac{S_t}{S}\,\eta_t\,\frac{1-\frac{k}{\pi R_w}}{1+\frac{k}{\pi R_t}}\right]$$

WHERE: $\eta = \frac{k_B^2}{l^2}$; $\bar{h} = \left[\left(\frac{a}{t}\right)_{EFF.} - h_o\right]$

$$m_q = \frac{5}{8}\,\frac{1}{\eta}\,\frac{k}{1+k/\pi R_t}\,\frac{S_t}{S} \quad ; \quad m_u = 0 .$$

Substituting in X and Y, we have for the two cases:

Case I	Case II
$x_u = 0.02 + .065 C_L^2$	$x_u = 0.05 + .065 C_L^2$
$x_w = -0.3725 C_L$	$x_w = -0.2275 C_L$
$z_w = 2.4$	$z_w = 2.0$
$z_u = C_L$	$z_u = C_L$
$m_w = -0.8\bar{X} + 0.536\bar{Y}$	$m_w = -0.667\bar{X} + 0.327\bar{Y}$
$m_q = 5/4\,\bar{Y}$	$m_q = 5/4\,\bar{Y}$

The coefficients for the stability Quartic become:

CASE I

$$B_1 = m_q + z_w + x_u = \frac{5}{4}\bar{Y} + 0.065 C_L^2 + 2.42 \quad \text{(INDEPENDENT OF } \mu\text{)}$$

$$C_1 = m_q(z_w + x_u) + \mu m_w + x_u z_w - x_w z_u = \left(3.02\bar{Y} + .0812 C_L^2 \bar{Y} + .536\mu\bar{Y} - 0.8\mu\bar{X} + 0.529 C_L^2 + 0.048\right)$$

$$D_1 = m_q(X_u Z_w - X_w Z_u) + \mu m_w\left(X_u + \frac{C_D}{2}\right) =$$
$$\bar{Y}(0.06 + 0.661\, C_L^2 + 0.052\mu C_L^2 + 0.016\mu) - \bar{X}(0.024\mu + 0.078\mu C_L$$

$$E_1 = \mu m_w \frac{C_L}{2}\left(Z_u + X_u \frac{C_D}{C_L}\right) = \mu(0.268\,\bar{Y} - 0.4\bar{X})\left[C_L^2 + (0.02 + 0.065C_L^2)^2\right]$$

Case II

$$B_1 = \frac{5}{4}\bar{Y} + 0.065C_L^2 + 2.05$$

$$C_1 = (2.56 + .0812C_L^2 + .327\mu)\bar{Y} - .667\mu\bar{X} + .3575\,C_L^2 + 0.10$$

$$D_1 = (0.125 + 0.4466\,C_L^2 + 0.0318\mu C_L^2 + .0246\mu)\bar{Y} - (0.05\mu + 0.065\mu C_L^2)\bar{X}$$

$$E_1 = \mu(0.3270\,\bar{Y} - 0.667\bar{X})\left[C_L^2 + (0.05 + 0.065C_L^2)^2\right]$$

The family of non-dimensional damping curves can be written as
$$\bar{K} = \frac{D_1}{C_1} - \frac{B_1 E_1}{C_1^2} \quad \text{or} \quad \bar{K}C_1^2 = C_1 D_1 - B_1 E_1 .$$

Letting
$$B_1 = a\bar{Y} + b$$
$$C_1 = c\bar{Y} + d\bar{X} + e$$
$$D_1 = f\bar{Y} + g\bar{X}$$
$$E_1 = h\bar{Y} + j\bar{X}\,;$$

Then:
$$\bar{K}(c^2\bar{Y}^2 + d^2\bar{X}^2 + 2cd\bar{X}\bar{Y} + 2ce\bar{Y} + 2de\bar{X} + e^2)$$
$$= (cf - ah)\,\bar{Y}^2 + dg\bar{X}^2 + (cg + df - aj)\,\bar{X}\bar{Y} + (ef - bh)\bar{Y}$$
$$+ (eg - jb)\bar{X},$$

or,
$$\bar{K}(A\bar{Y}^2 + B\bar{X}^2 + C\bar{X}\bar{Y} + D\bar{Y} + E\bar{X} + F) = (\alpha\bar{Y}^2 + \beta\bar{X}^2 + \gamma\bar{X}\bar{Y} + \psi\bar{Y} + \theta\bar{X})$$

where:

$A = c^2$ $\qquad\qquad \alpha = (cf - ah)$

$B = d^2$ $\qquad\qquad \beta = dg$

$C = 2cd$ $\qquad\qquad \gamma = (cg + df - aj)$

$D = 2ce$ $\qquad\qquad \psi = (ef - bh)$

$E = 2de$ $\qquad\qquad \theta = (eg - jb)$

$F = e^2$

The family of period curves become:

$$\left(\frac{2\pi}{p}\right)^2 = \frac{E_1}{C_1} \quad \text{OR} \quad E_1 = \frac{4\pi^2}{p^2} C_1 .$$

Let $\frac{4\pi^2}{p^2} = \delta$, and substitute for E_1 and C_1;

Then

$$(h - c\delta)\bar{Y} + (j - d\delta)\bar{X} - \delta e = 0$$

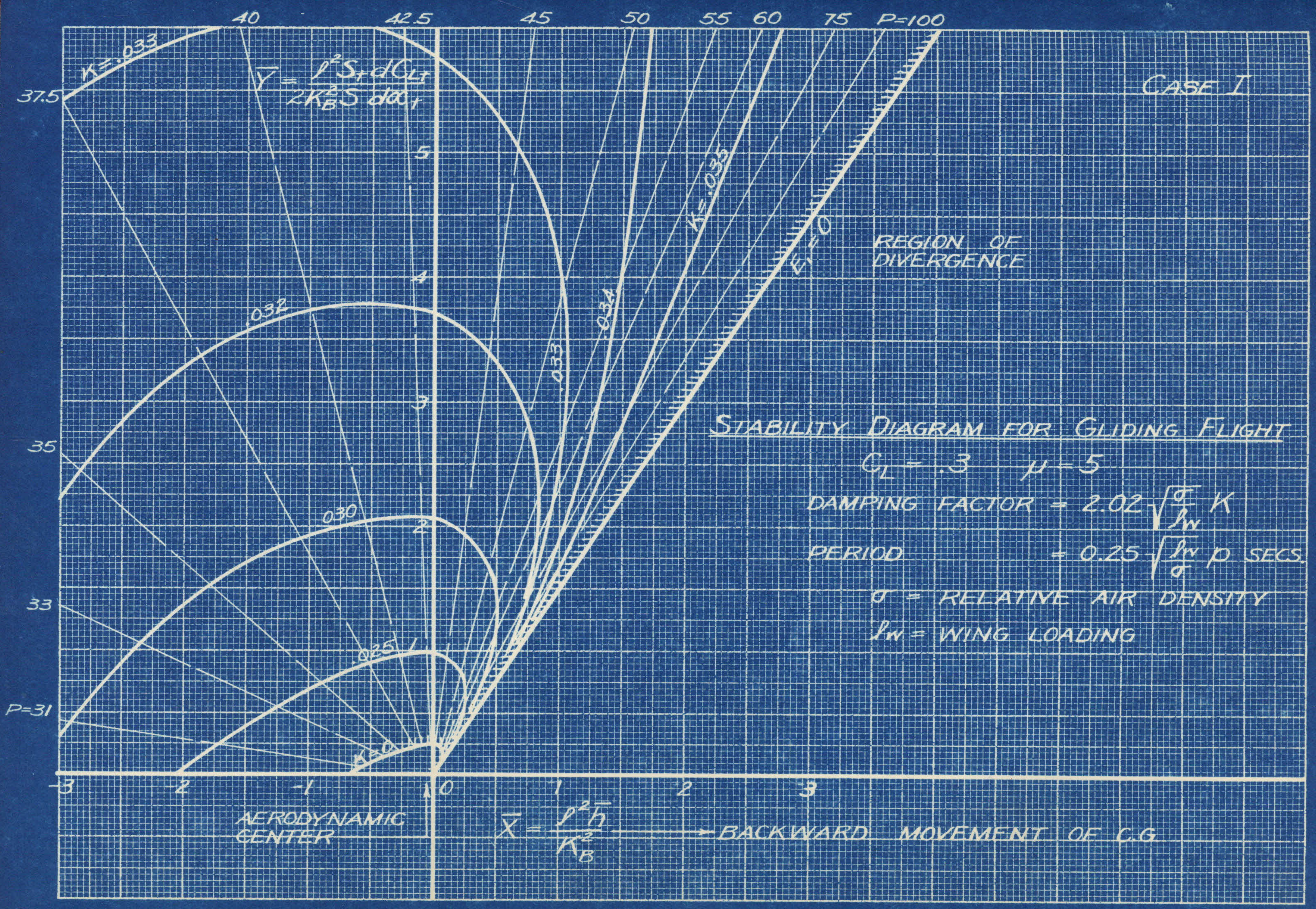
or,

$$\bar{Y} = \left[\frac{d\delta - j}{h - c\delta}\right]\bar{X} + \left[\frac{\delta e}{h - c\delta}\right] .$$

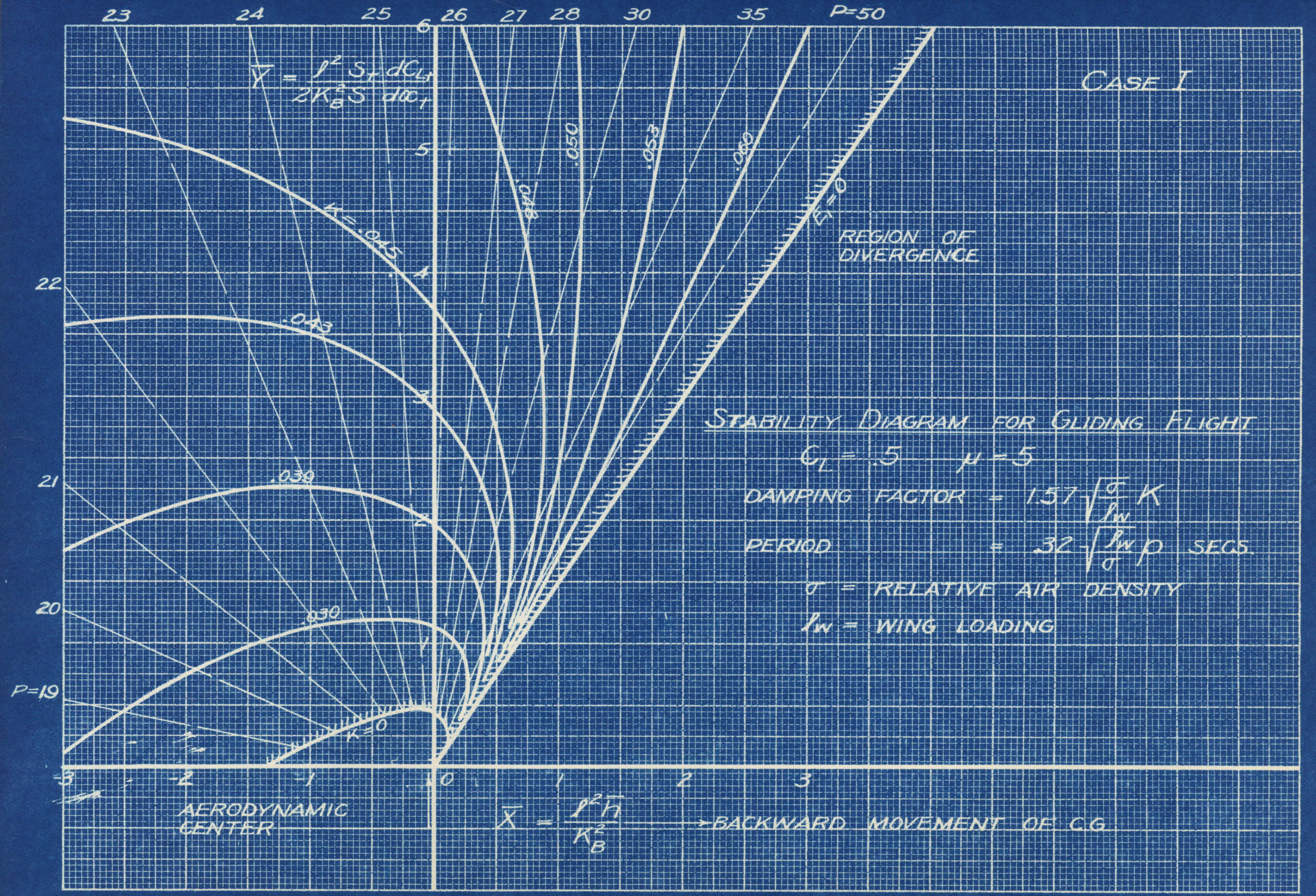
It is now possible to represent these two families of curves on the " XY " plane by assigning various values to the parameters, μ and C_L, at the same time varying the constant values assigned to $\bar{K}$ and p. This has been done for the two groups of curves; i.e., for Cases I & II. There are sixteen curves for each case, consisting of four groups, each group for a constant value of "μ". The $E_1 = 0$ and $\bar{K} = 0$ curves on each diagram represent the boundaries separating the stable from the unstable regions. Each diagram is covered with a series of curves of constant period and of constant damping factor. (p and $\bar{K}$)

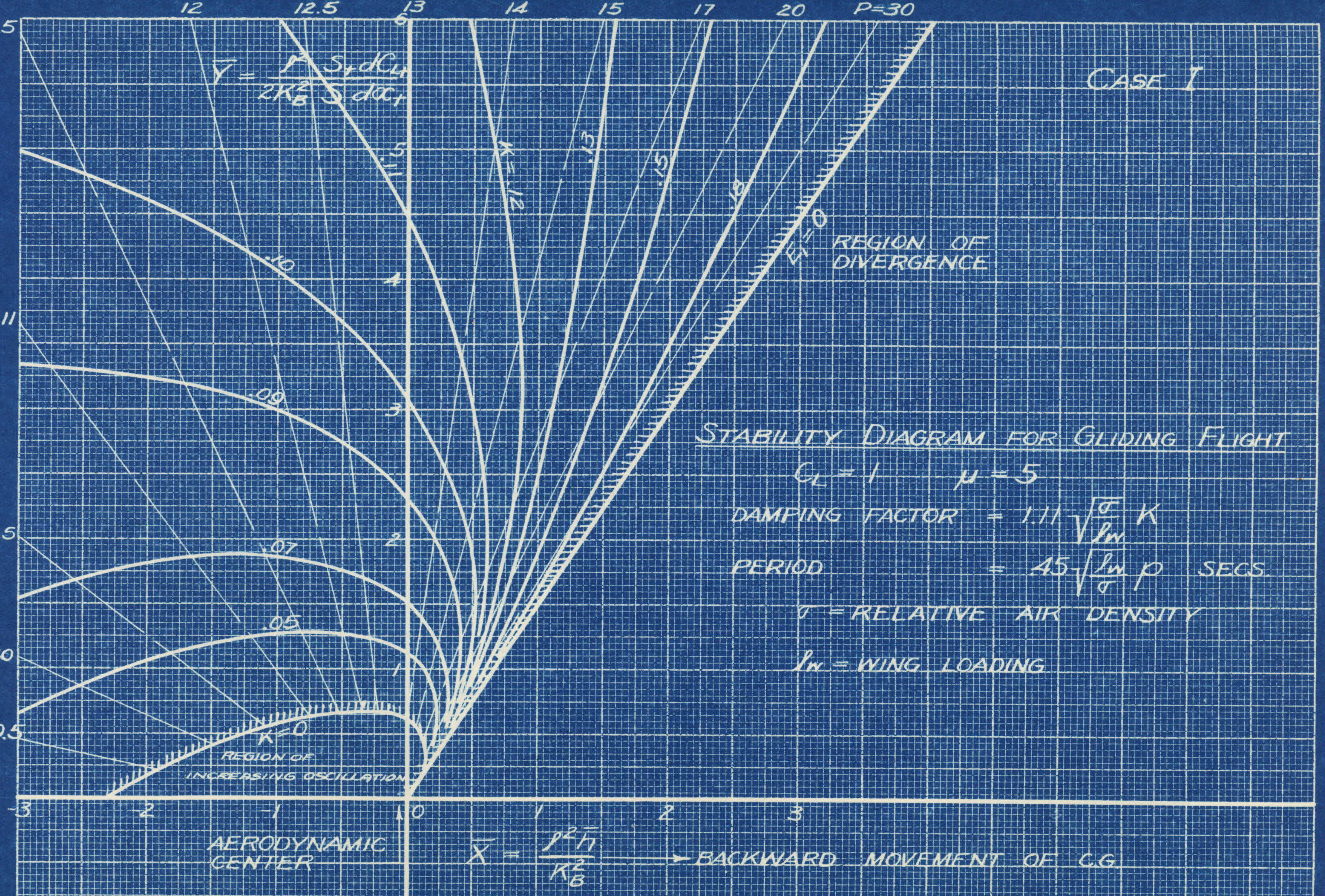
DYNAMIC STABILITY DIAGRAMS

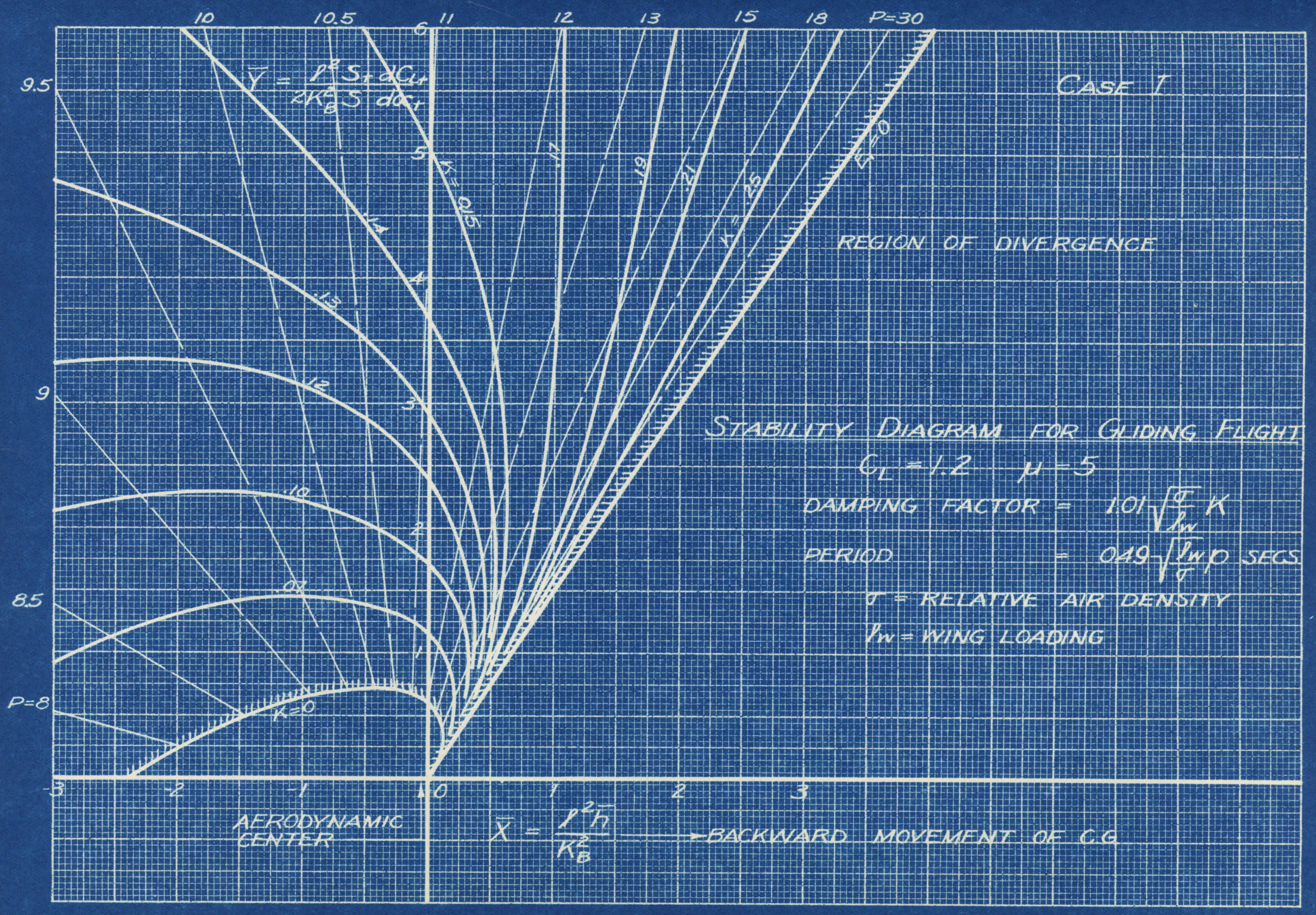
CASE I



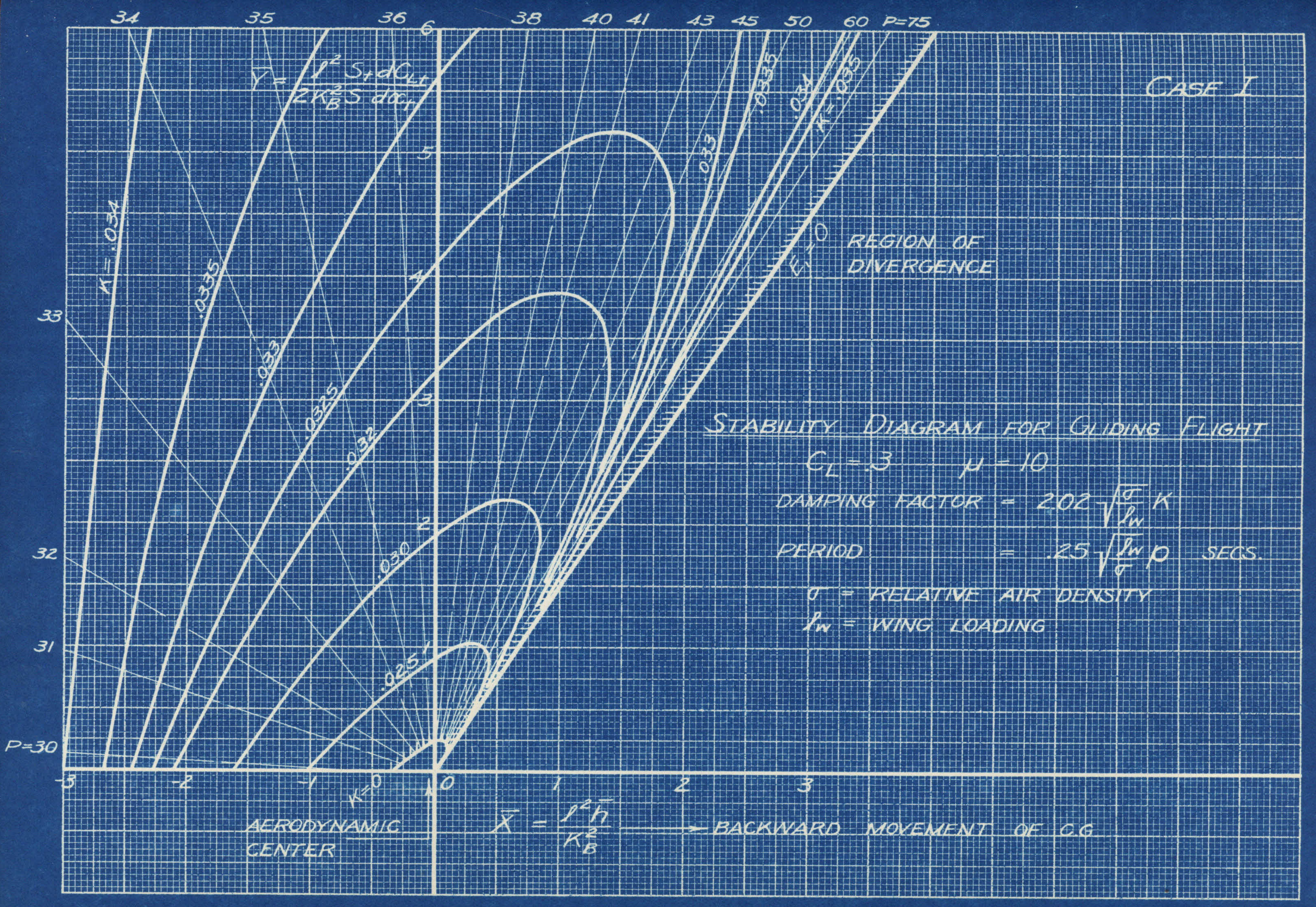
CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .3 \quad \mu = 5$
DAMPING FACTOR $= 2.02 \sqrt{\frac{\sigma}{l_W}}\, K$
PERIOD $= 0.25 \sqrt{\frac{l_W}{\sigma}}\, p$ SECS.
σ = RELATIVE AIR DENSITY
l_W = WING LOADING
REGION OF DIVERGENCE
$\bar{Y} = \frac{l^2 S_T\, dC_{LT}}{2K_B^2 S\, d\alpha_T}$
$\bar{X} = \frac{l^2 \bar{h}}{K_B^2}$ → BACKWARD MOVEMENT OF C.G
AERODYNAMIC CENTER
K = .033
K = .035
K = 0
.032
.030
.025
.034
.033
40
42.5
45
50
55
60
75
P=100
37.5
35
33
P=31
-3
-2
-1
0
1
2
3
4
5

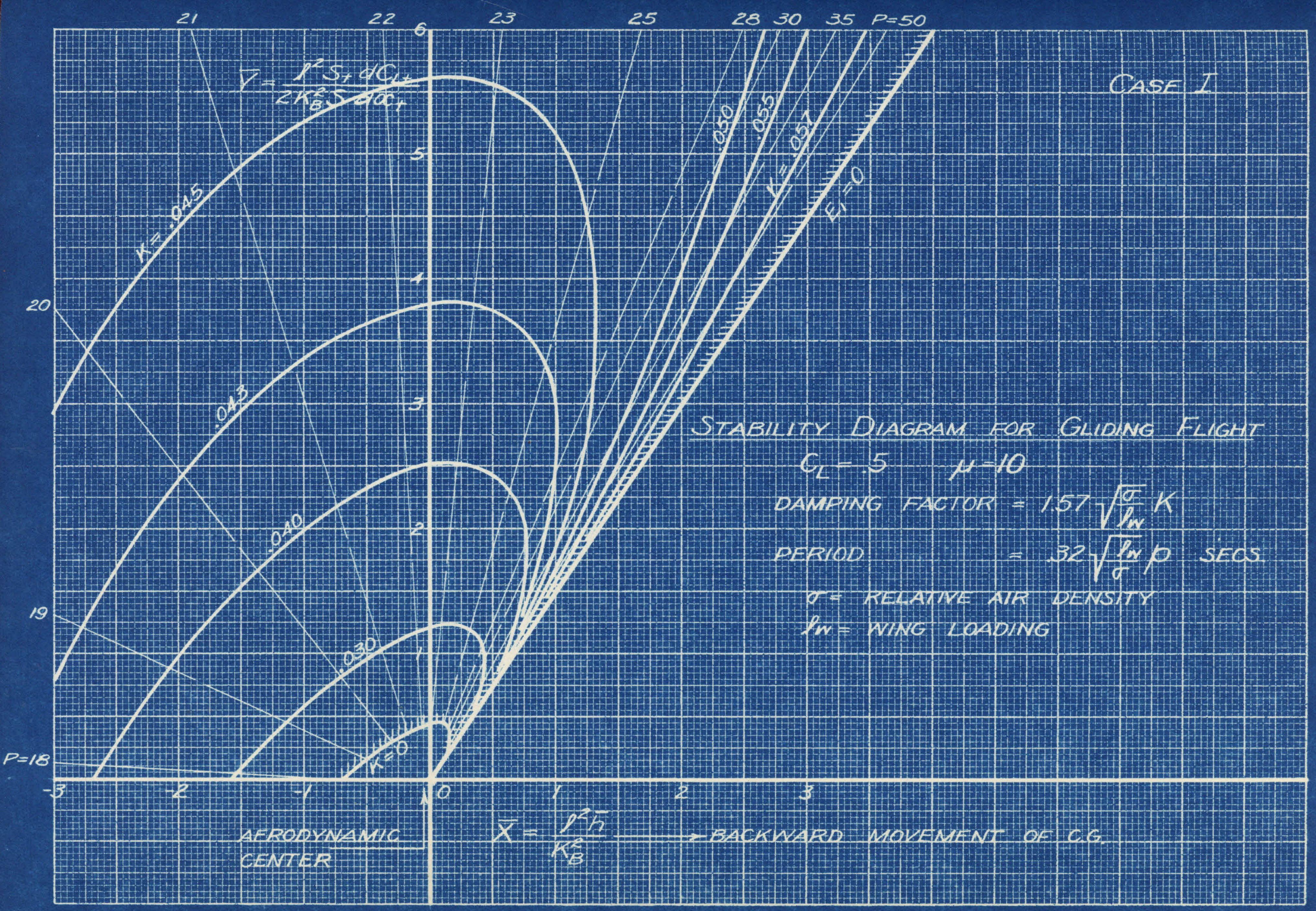

CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .5 \quad \mu = 5$
DAMPING FACTOR $= 1.57\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= .32\sqrt{\frac{l_w}{\sigma}}\,P$ SECS
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$E_1 = 0$
$\bar{V} = \frac{l^2 S_t}{2K_B^2 S}\frac{dC_{L_t}}{d\alpha_t}$
$\bar{X} = \frac{l^2\bar{h}}{K_B^2}$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
K=.045
.043
.039
.030
K=0
.048
.050
.053
.060
23
24
25
26
27
28
30
35
P=50
22
21
20
P=19
-3
-2
-1
0
1
2
3
4
5
6

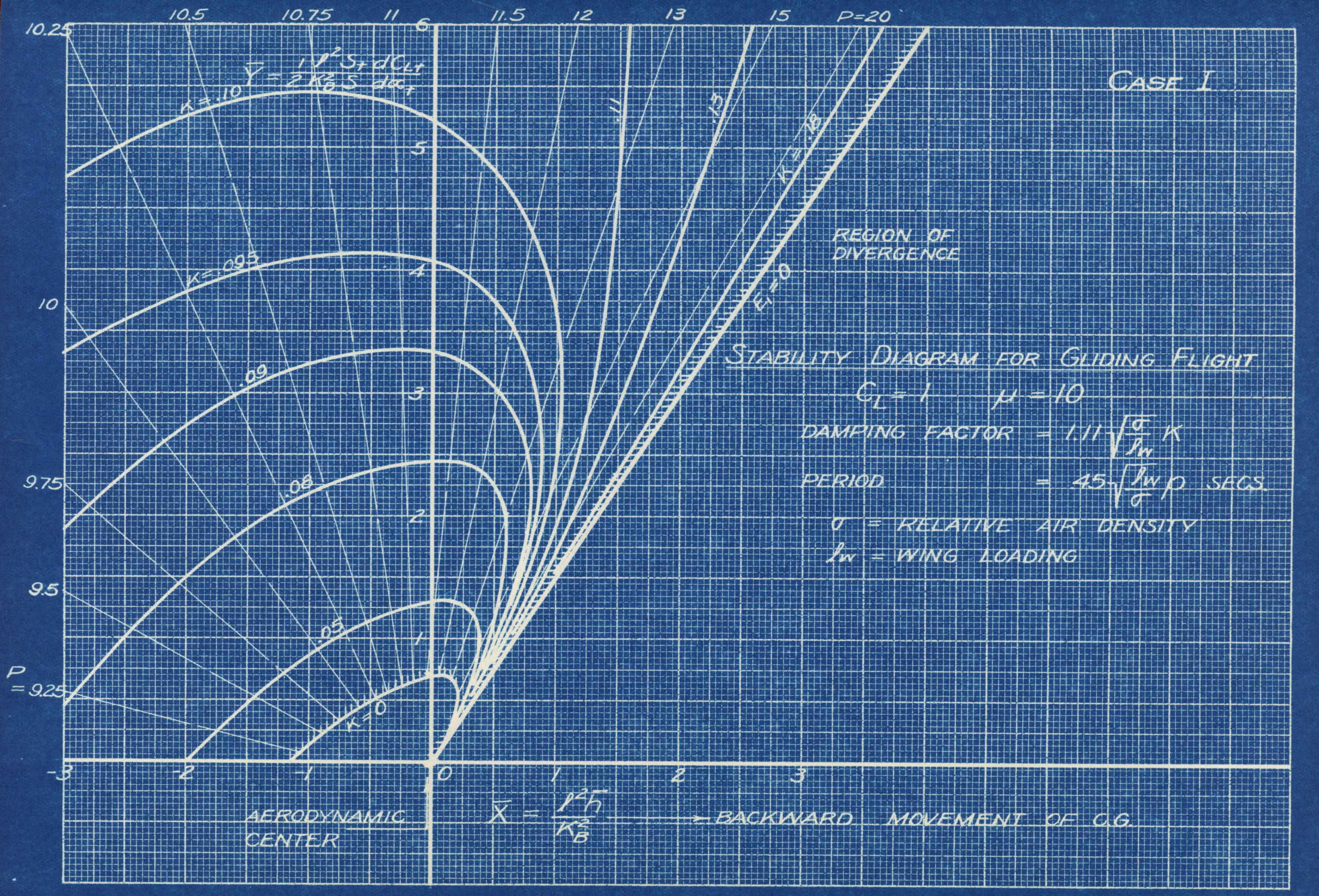

CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1 \qquad \mu = 5$
DAMPING FACTOR $= 1.11\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= .45\sqrt{\frac{l_w}{\sigma}}\,p$ SECS.
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
REGION OF INCREASING OSCILLATION
$\overline{Y} = \frac{\mu}{2K_B^2}\frac{S_t}{S}\frac{dC_{Lt}}{d\alpha_t}$
$\overline{X} = \frac{\mu^2 \overline{h}}{K_B^2}$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
K = .12
K = 0
P = 30
P = 9.5
.05
.07
.09
.10
.11
.13
.15
.18

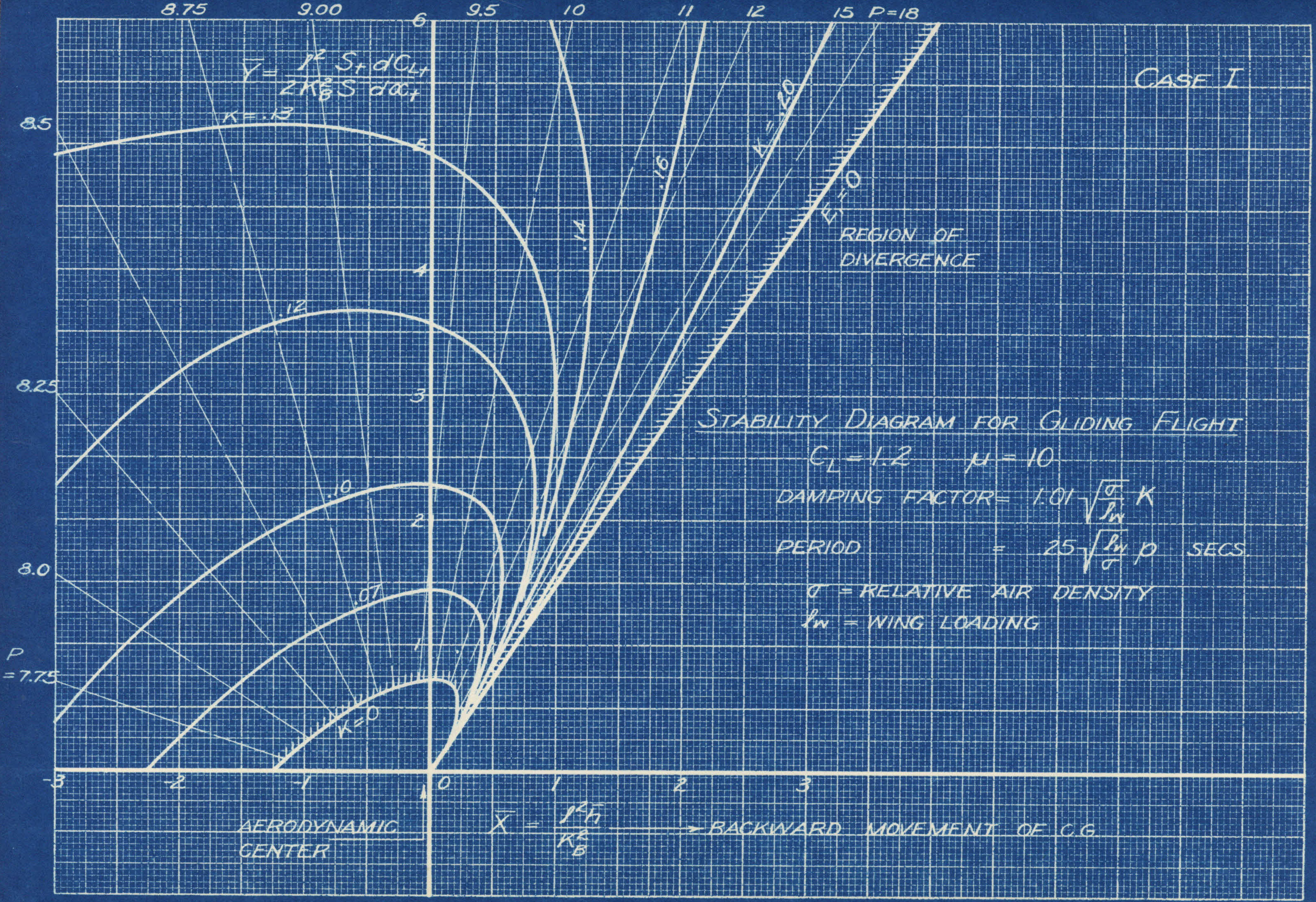


CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1.2 \quad \mu = 5$
DAMPING FACTOR = $1.01\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD = $.049\sqrt{\frac{l_w}{\sigma}}\,p$ SECS
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$\overline{Y} = \frac{l^2 S_t\, dC_{Lt}}{2K_B^2 S\, d\alpha_t}$
$\overline{X} = \frac{l^2 \overline{h}}{K_B^2}$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
P=8
8.5
9
9.5
10
10.5
11
12
13
15
18
P=30
K=0
.07
.10
.12
.13
.14
K=.015
.17
.19
.21
K=.25
-3
-2
-1
0
1
2
3
4
5
6

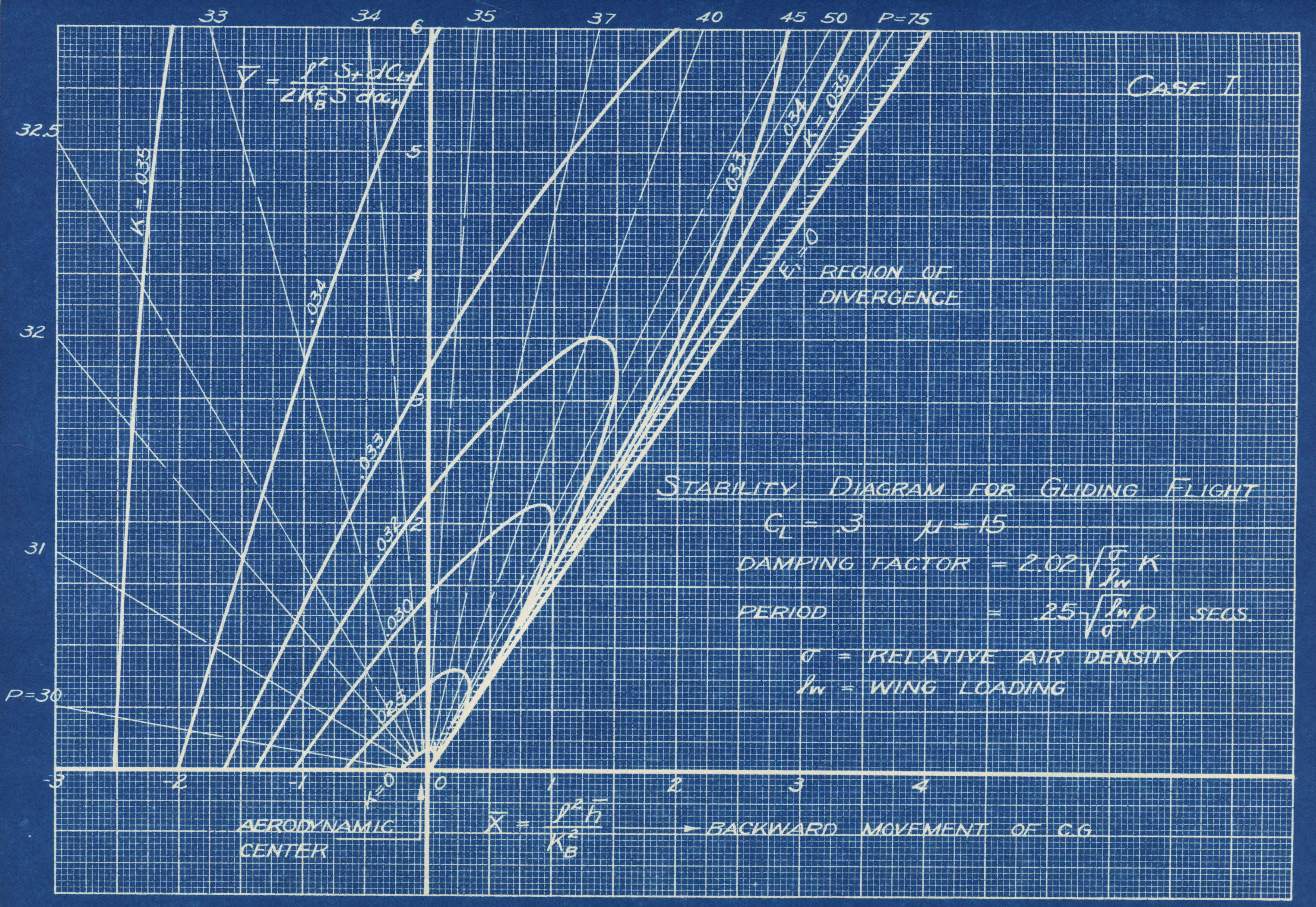

CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .3 \quad \mu = 10$
DAMPING FACTOR $= 2.02 \sqrt{\frac{\sigma}{l_w}}\, K$
PERIOD $= .25 \sqrt{\frac{l_w}{\sigma}}\, p$ SECS.
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$E_1 = 0$
$\bar{Y} = \frac{l^2 S_t\, dC_{Lt}}{2K_B^2 S\, d\alpha_t}$
$\bar{X} = \frac{l^2 \bar{h}}{K_B^2}$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
K = .034
.0335
.033
.0325
.032
.030
.025
K = 0
K = .035
.034
P = 30
P = 75
34 35 36 38 40 41 43 45 50 60
31 32 33
-3 -2 -1 0 1 2 3
1 2 3 4 5 6


CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .5$ $\mu = 10$
DAMPING FACTOR = $1.57\sqrt{\frac{\sigma}{l_W}}\,K$
PERIOD = $.32\sqrt{\frac{l_W}{\sigma}}\,P$ SECS.
σ = RELATIVE AIR DENSITY
l_W = WING LOADING
$\overline{V} = \frac{l^2 S_t}{2K_B^2 S}\frac{dC_{L_t}}{d\alpha_t}$
$\overline{X} = \frac{l^2 \overline{h}}{K_B^2}$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
K = .045
.043
.040
.030
K = 0
.050
.055
K = .057
$E_1 = 0$
P=18
19
20
21
22
23
25
28
30
35
P=50
-3
-2
-1
0
1
2
3
4
5
6

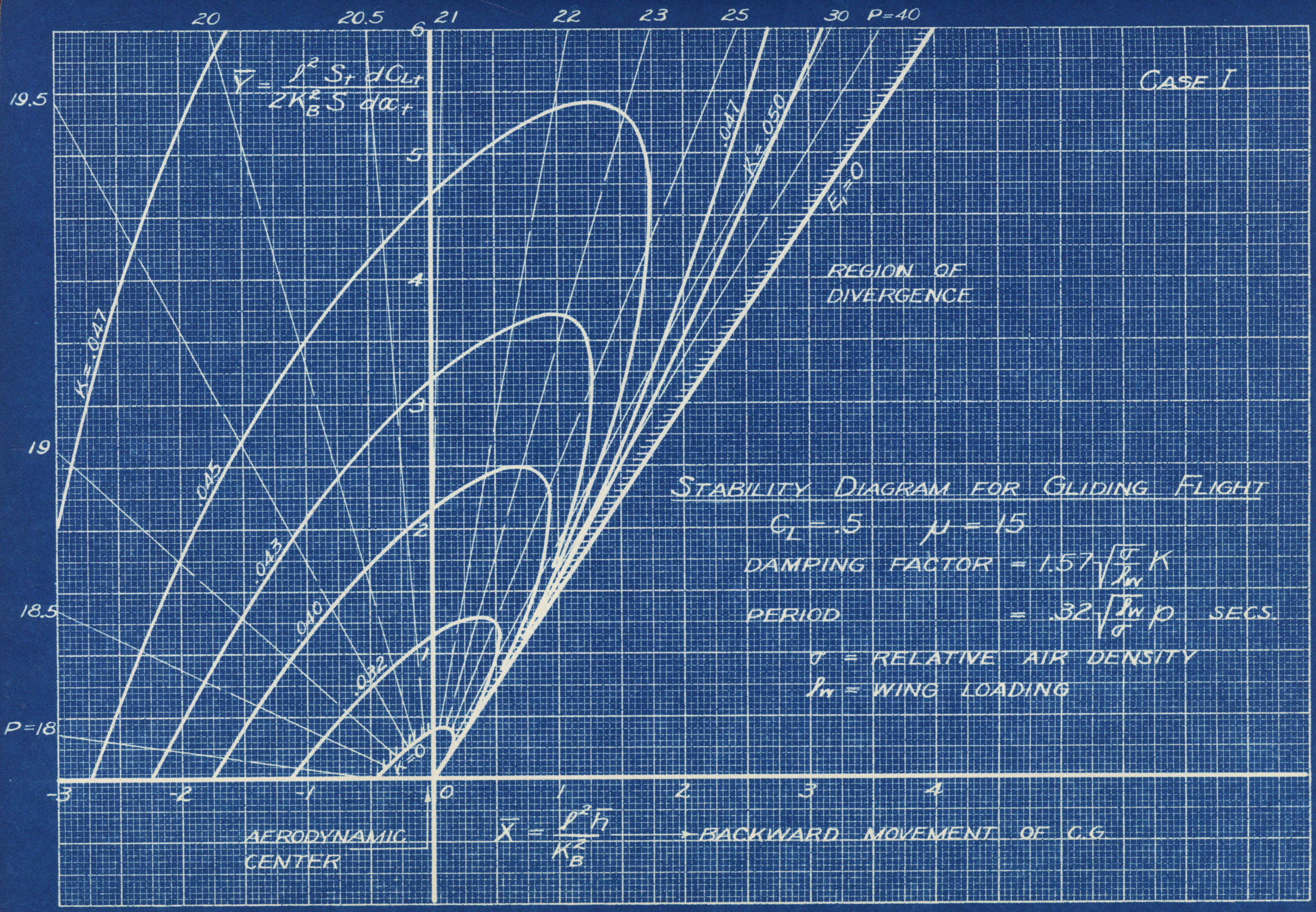

CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1 \quad \mu = 10$
DAMPING FACTOR $= 1.11\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= 45\sqrt{\frac{l_w}{\sigma}}\,P$ SECS.
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$E_1 = 0$
$\bar{Y} = \frac{1}{2}\frac{l^2 S_T}{K_B^2 S}\frac{dC_{LT}}{d\alpha_T}$
$\bar{X} = \frac{l^2 h}{K_B^2}$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
K = .10
K = .095
.09
.08
.05
K = 0
.11
.13
K = .18
P = 9.25
9.5
9.75
10
10.25
10.5
10.75
11
11.5
12
13
15
P = 20



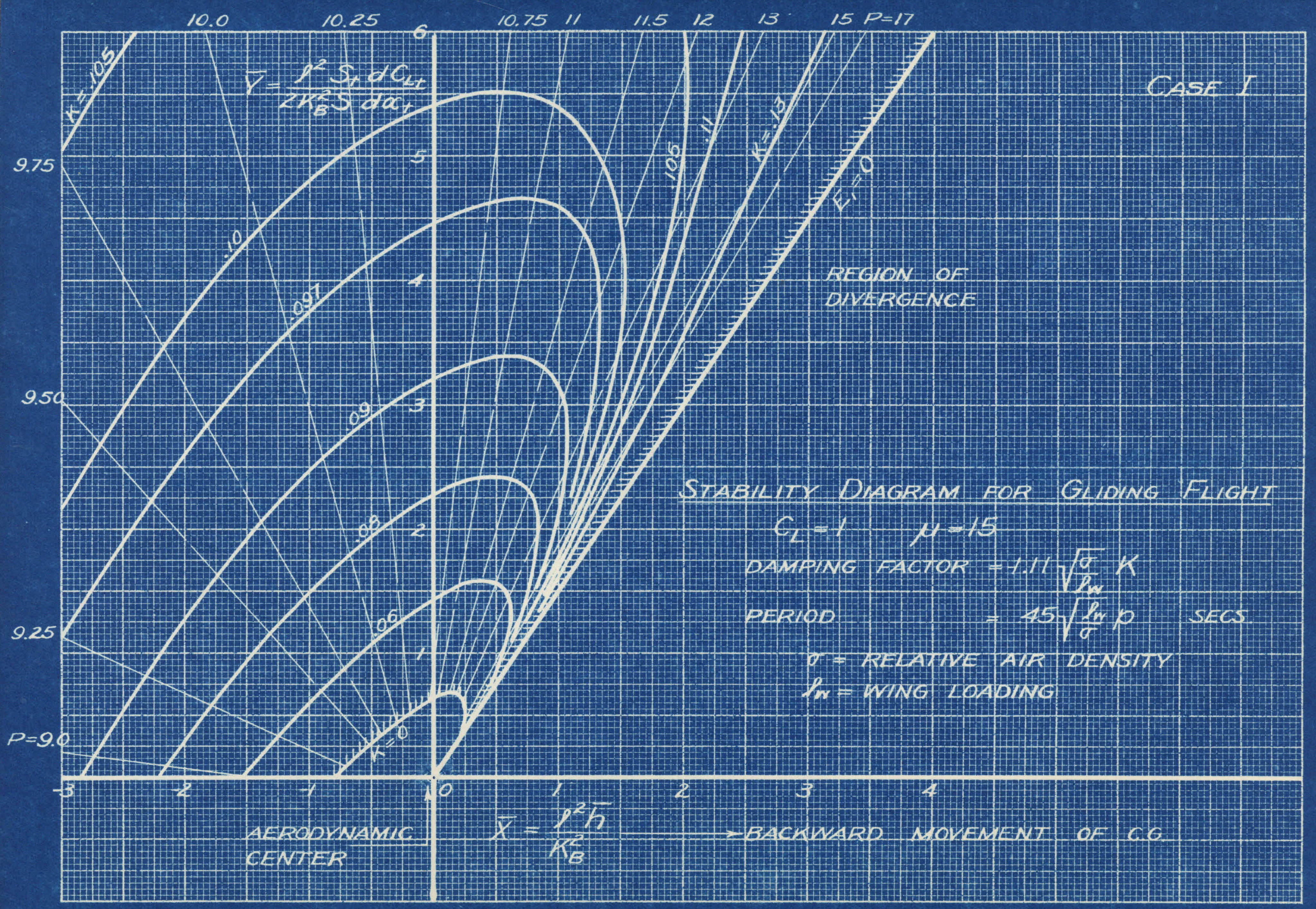
Case I
Stability Diagram for Gliding Flight
$C_L = 1.2 \quad \mu = 10$
Damping factor $= 1.01\sqrt{\frac{\sigma}{l_w}}\,K$
Period $= 25\sqrt{\frac{l_w}{\sigma}}\,p$ secs
σ = Relative air density
l_w = Wing loading
$\bar{Y} = \frac{l^2 S_t\, dC_{L_t}}{2K_B^2 S\, d\alpha_t}$
$\bar{X} = \frac{l^2 \bar{h}}{K_B^2}$ → Backward movement of C.G.
Aerodynamic center
Region of divergence
$E = 0$
K = .13
.12
.10
.07
K = 0
.14
.16
K = .20
P = 18
15
12
11
10
9.5
9.00
8.75
8.5
8.25
8.0
P = 7.75

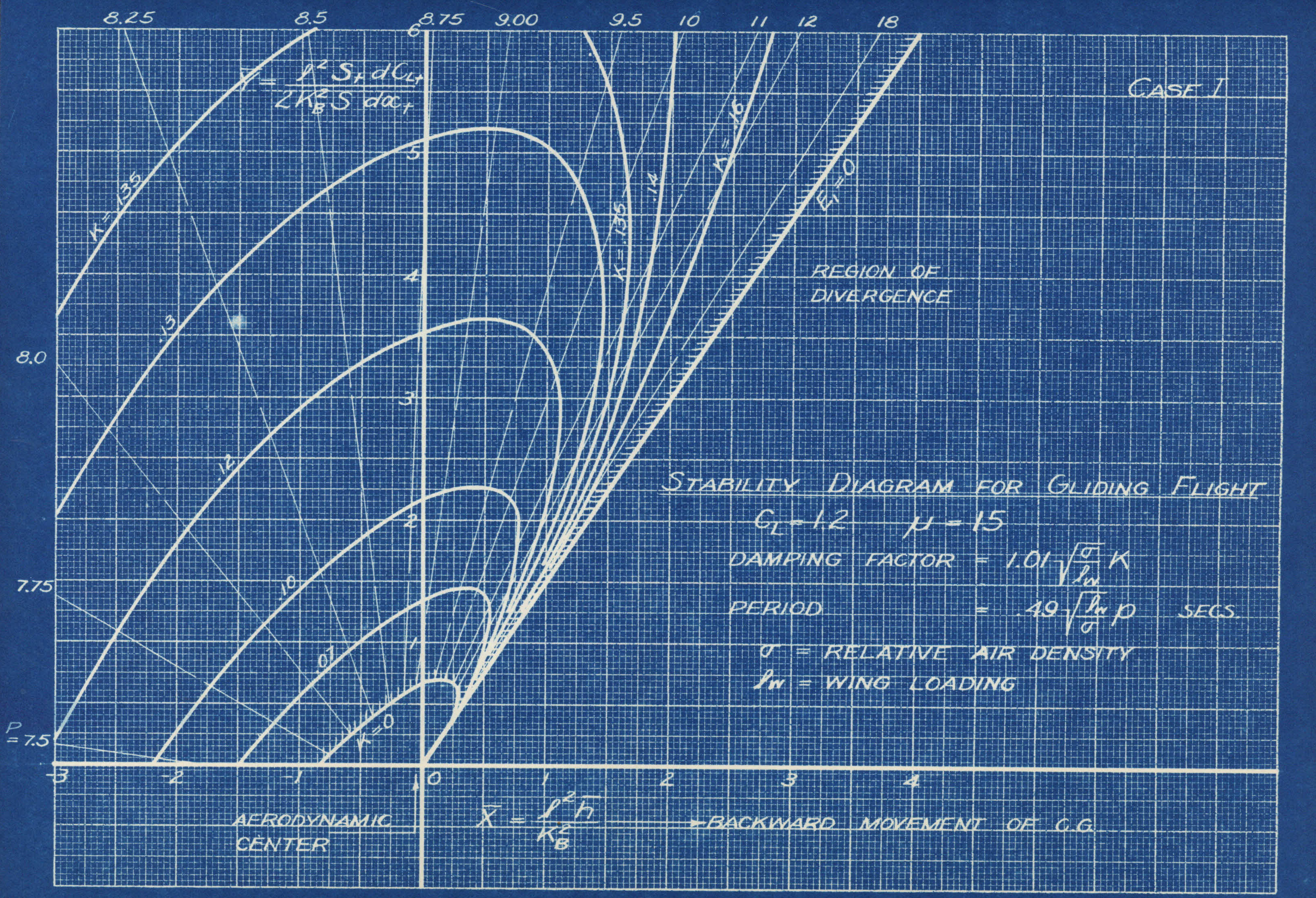


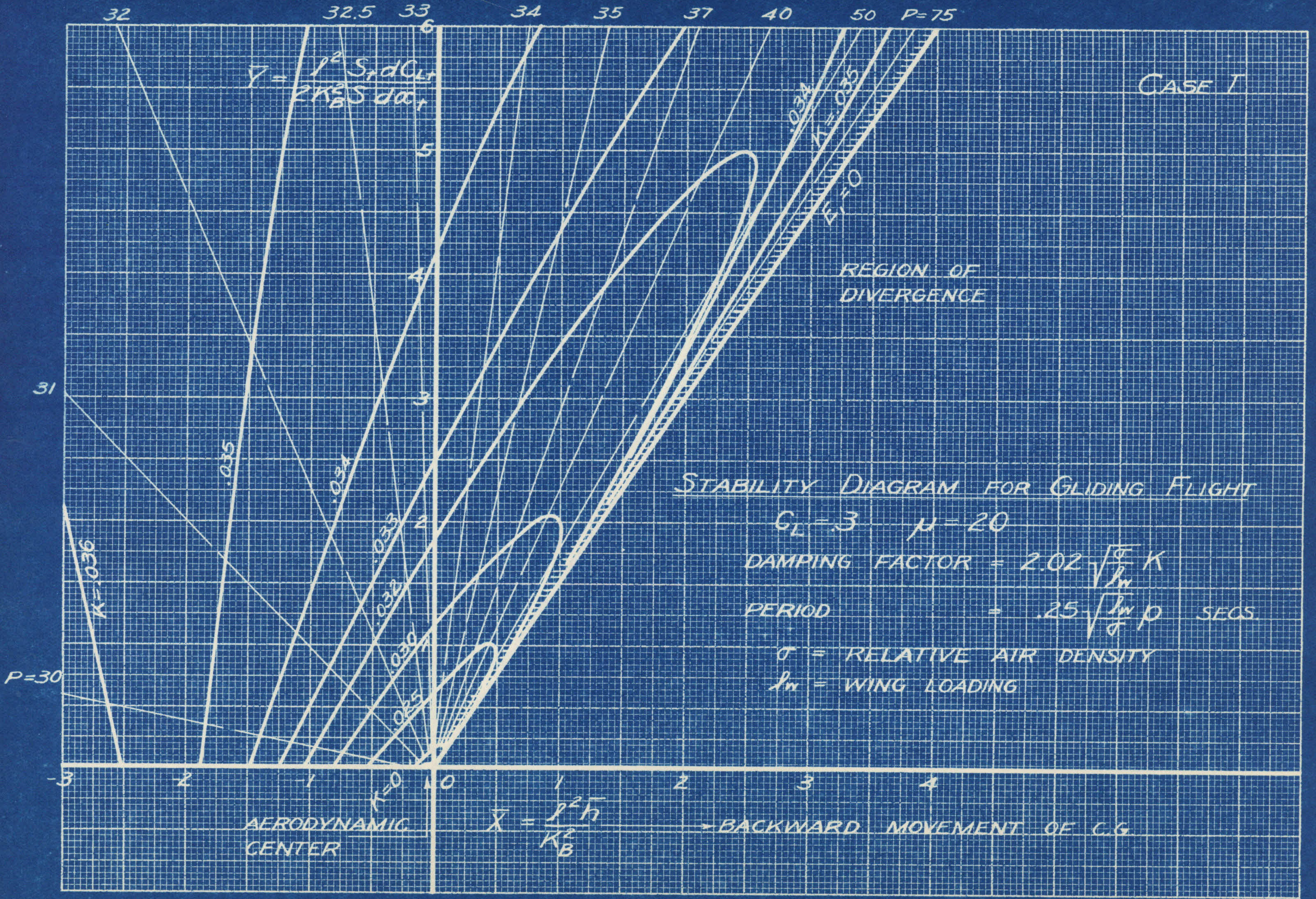
CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .3 \quad \mu = 15$
DAMPING FACTOR $= 2.02\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= 25\sqrt{\frac{l_w}{\sigma}}\,P$ SECS.
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
$\bar{Y} = \frac{l^2 S_t \, dC_{Lt}}{2K_B^2 S \, d\alpha_t}$
$\bar{X} = \frac{l^2 \bar{h}}{K_B^2}$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
REGION OF DIVERGENCE
K = .035
.034
.033
.032
.030
.025
K=0
33
34
35
37
40
45
50
P=75
32.5
32
31
P=30
-3
-2
-1
0
1
2
3
4
5
6



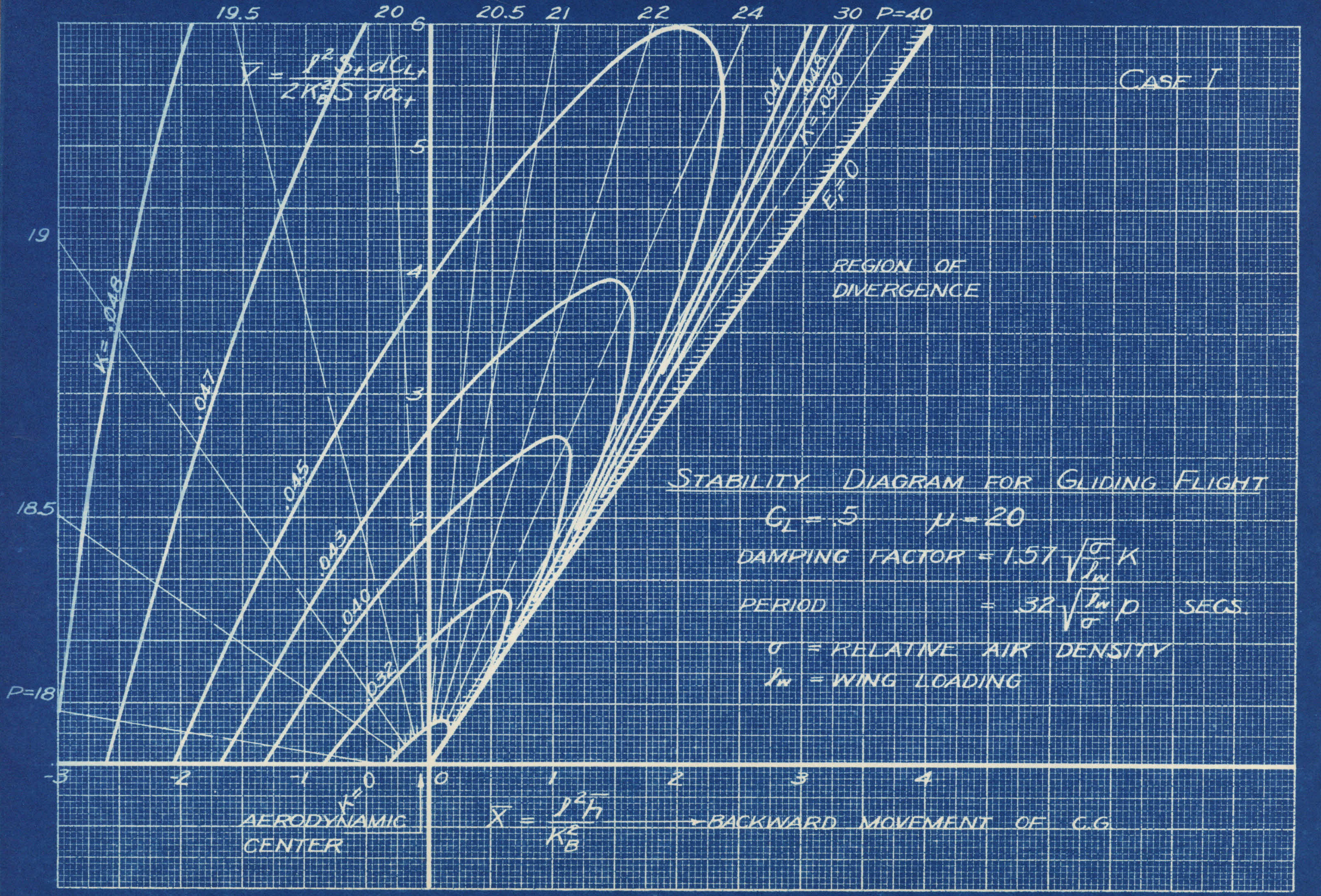
CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .5 \quad \mu = 15$
DAMPING FACTOR $= 1.57\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= .32\sqrt{\frac{l_w}{\sigma}}\,p$ SECS.
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$\bar{Y} = \frac{l^2 S_t\, dC_{Lt}}{2K_B^2 S\, d\alpha_t}$
$\bar{X} = \frac{l^2 h}{K_B^2}$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
K=.047
.045
.043
.040
.032
K=0
.047
K=.050
E=0
20
20.5
21
22
23
25
30
P=40
19.5
19
18.5
P=18
-3
-2
-1
0
1
2
3
4
5
6

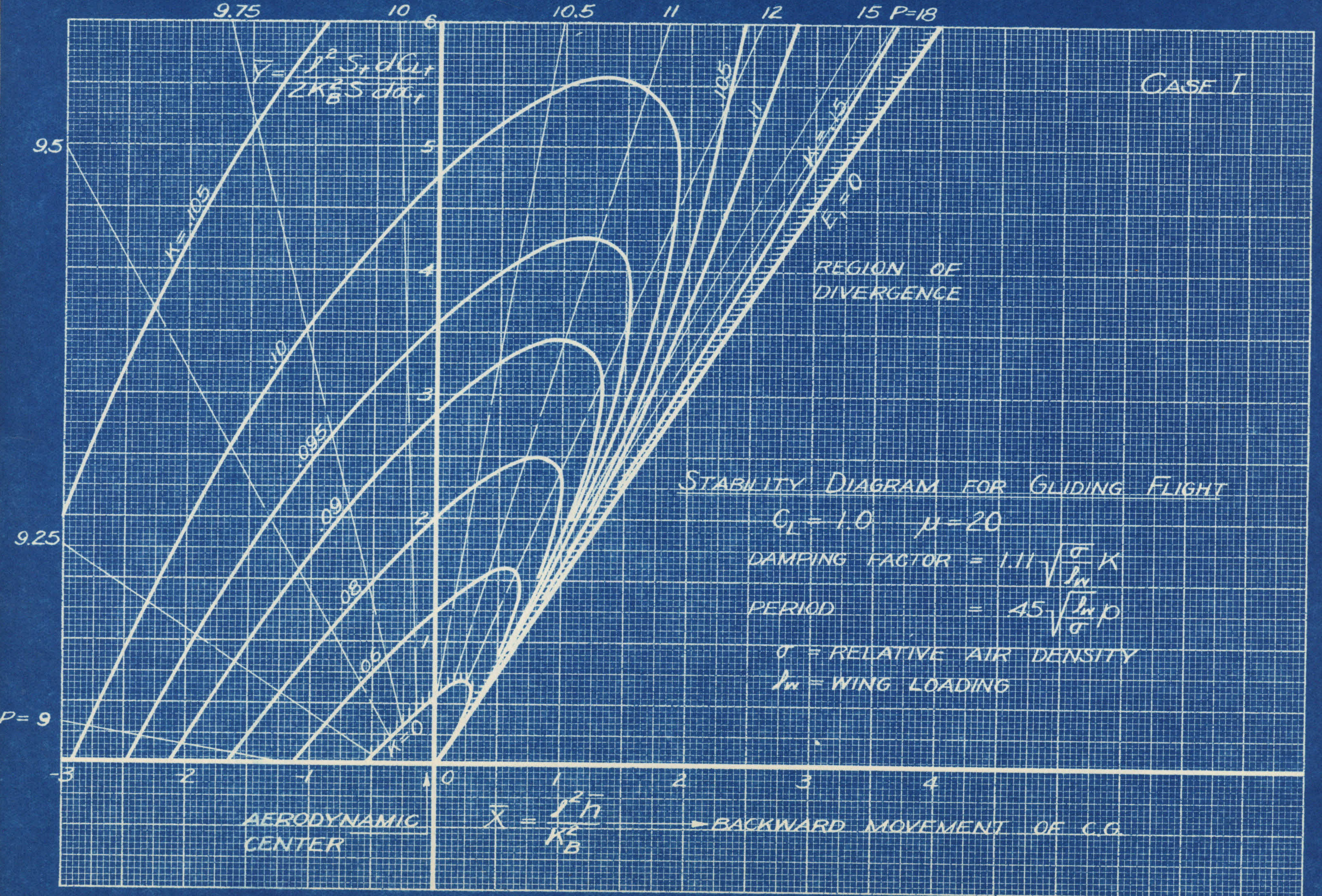

CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1$ $\mu = 15$
DAMPING FACTOR $= 1.11 \sqrt{\frac{\sigma}{l_w}}\, K$
PERIOD $= 45 \sqrt{\frac{l_w}{\sigma}}\, p$ SECS
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$\bar{Y} = \frac{l^2 S_t \, dC_{Lt}}{2K_B^2 S \, d\alpha_t}$
$\bar{X} = \frac{l^2 \bar{h}}{K_B^2}$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
$E_1 = 0$
K = .105
K = .13
.11
.105
.10
.097
.09
.08
.06
K = 0
10.0
10.25
10.75
11
11.5
12
13
15
P=17
9.75
9.50
9.25
P=9.0
-3
-2
-1
0
1
2
3
4
5
6


CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1.2 \quad \mu = 15$
DAMPING FACTOR $= 1.01\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= .49\sqrt{\frac{l_w}{\sigma}}\,p$ SECS.
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$E_1 = 0$
$\bar{Y} = \frac{l^2 S_t \, dC_{L_t}}{2K_B^2 S \, d\alpha_t}$
$\bar{X} = \frac{l^2 h}{K_B^2}$
→BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
K = .135
.13
.12
.10
.07
K = 0
K = .135
.14
K = .16
8.25
8.5
8.75
9.00
9.5
10
11
12
18
8.0
7.75
P = 7.5
-3
-2
-1
0
1
2
3
4
5
6

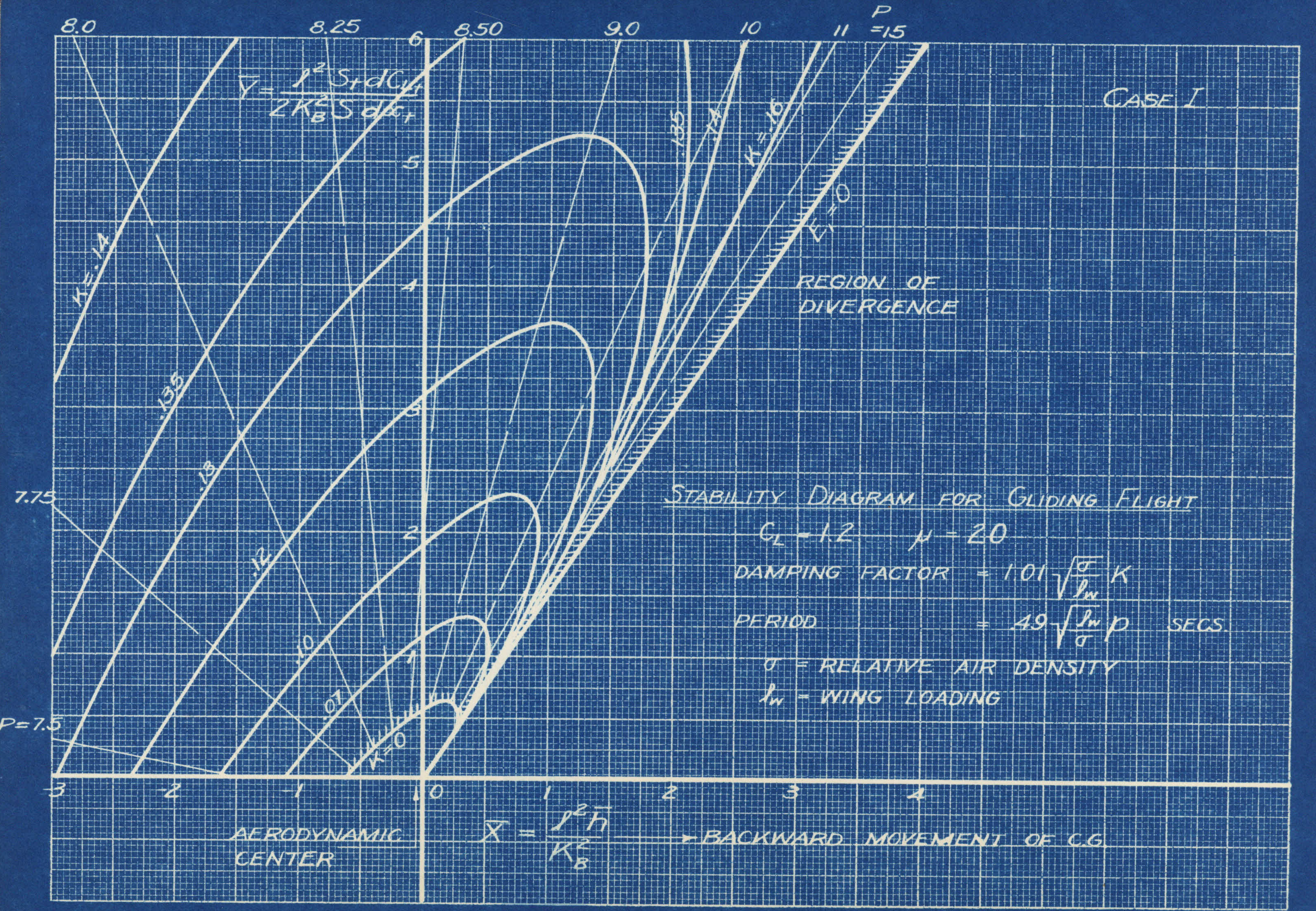


CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .3 \quad \mu = 20$
DAMPING FACTOR $= 2.02\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= .25\sqrt{\frac{l_w}{\sigma}}\,P$ SECS.
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$\bar{Y} = \frac{l^2 S_t \, dC_{L_t}}{2K_B^2 S \, d\alpha_t}$
$\bar{X} = \frac{l^2 \bar{h}}{K_B^2}$
→BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
32
32.5
33
34
35
37
40
50
P=75
31
P=30
K=.036
.035
.034
.033
.032
.030
.025
K=0
.034
K=.035
$E_1=0$
-3
-2
-1
0
1
2
3
4
5
6


CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .5 \quad \mu = 20$
DAMPING FACTOR $= 1.57\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= .32\sqrt{\frac{l_w}{\sigma}}\,p$ SECS.
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$\overline{Y} = \frac{l^2 S_t\, dC_{L_t}}{2K_B^2 S\, d\alpha_t}$
$\overline{X} = \frac{l^2 h}{K_B^2}$
→ BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
$K=.048$
.047
.045
.043
.040
.032
$K=.050$
.048
.047
$E_1=0$
$K=0$
P=18
18.5
19
19.5
20
20.5
21
22
24
30
P=40
-3
-2
-1
0
1
2
3
4
5
6



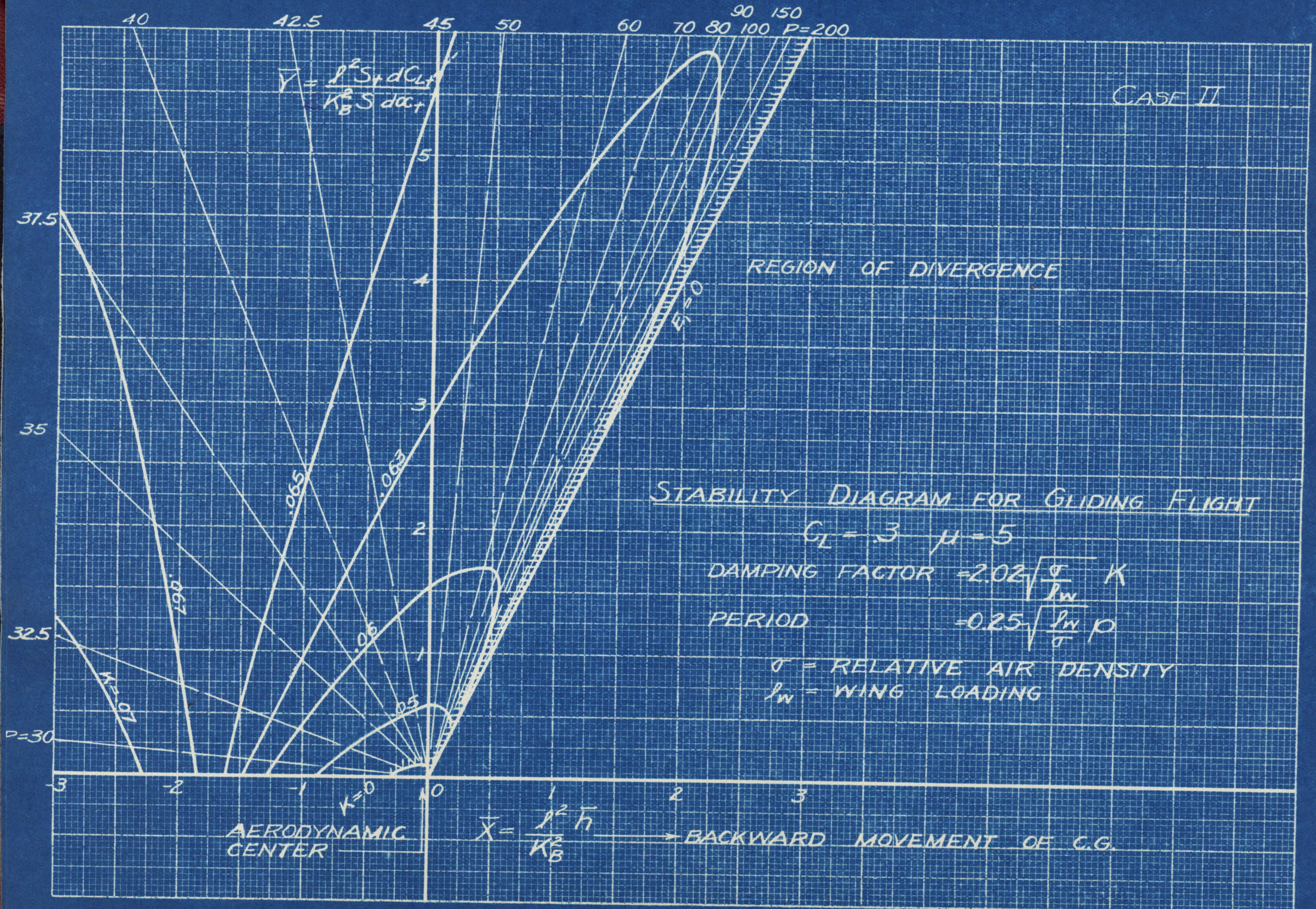
CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1.0 \quad \mu = 20$
DAMPING FACTOR $= 1.11\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= 45\sqrt{\frac{l_w}{\sigma}}\,P$
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$\bar{Y} = \frac{l^2 S_t}{2K_B^2 S}\frac{dC_{Lt}}{d\alpha_t}$
$\bar{X} = \frac{l^2 h}{K_B^2}$
BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
$K = .105$
.10
.095
.09
.08
.06
$K = 0$
$E_1 = 0$
$P = 9$
9.25
9.5
9.75
10
10.5
11
12
15
$P = 18$

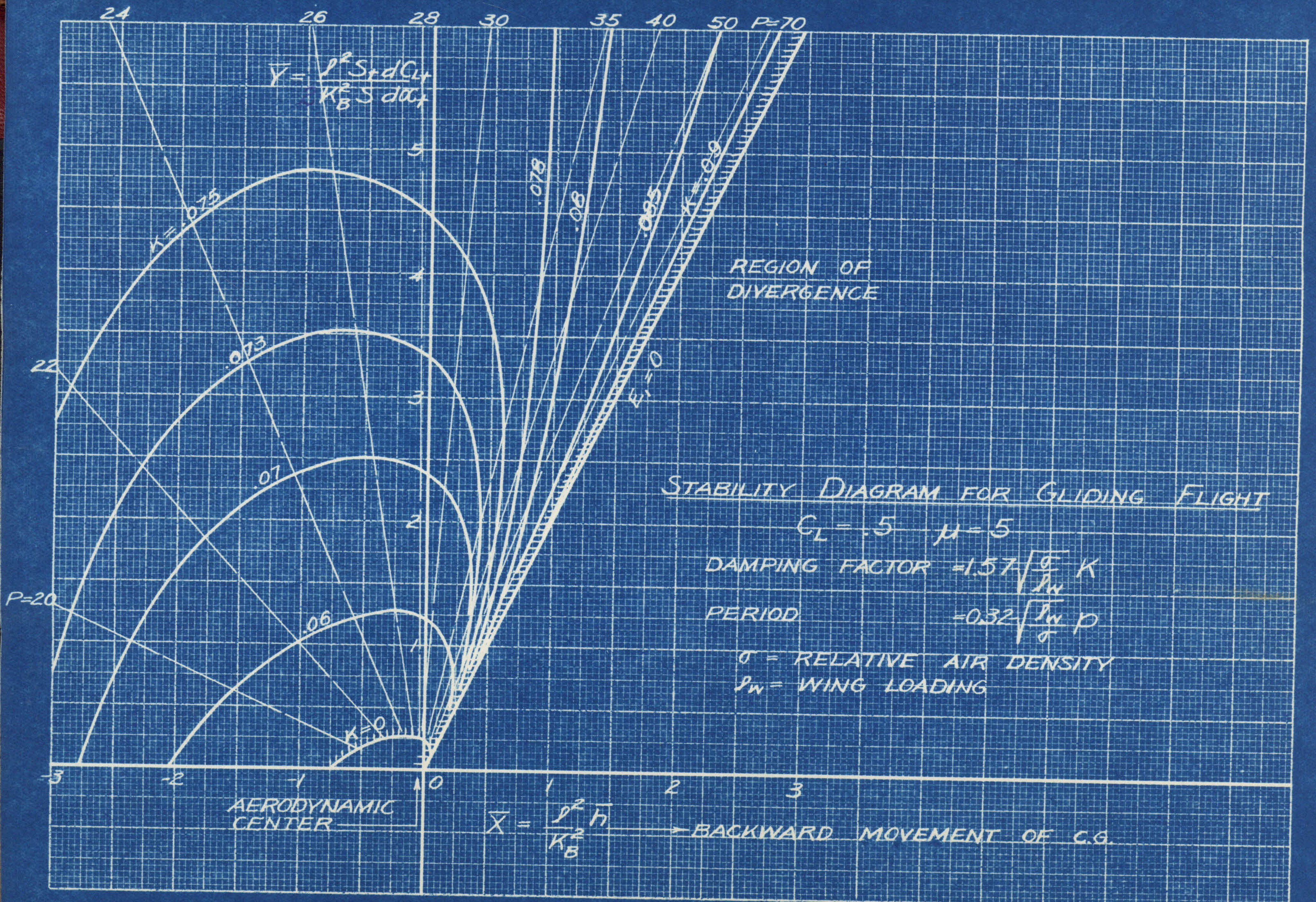


CASE I
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1.2 \quad \mu = 20$
DAMPING FACTOR $= 1.01\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= 49\sqrt{\frac{l_w}{\sigma}}\,p$ SECS.
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$\overline{Y} = \frac{l^2 S_t\, dC_{Lt}}{2K_B^2 S\, d\alpha_t}$
$\overline{X} = \frac{l^2 \overline{h}}{K_B^2}$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
8.0
8.25
8.50
9.0
10
11
P =15
7.75
P=7.5
K=.14
.135
.13
.12
.10
.07
K=0
.135
.14
K=.16
E=0

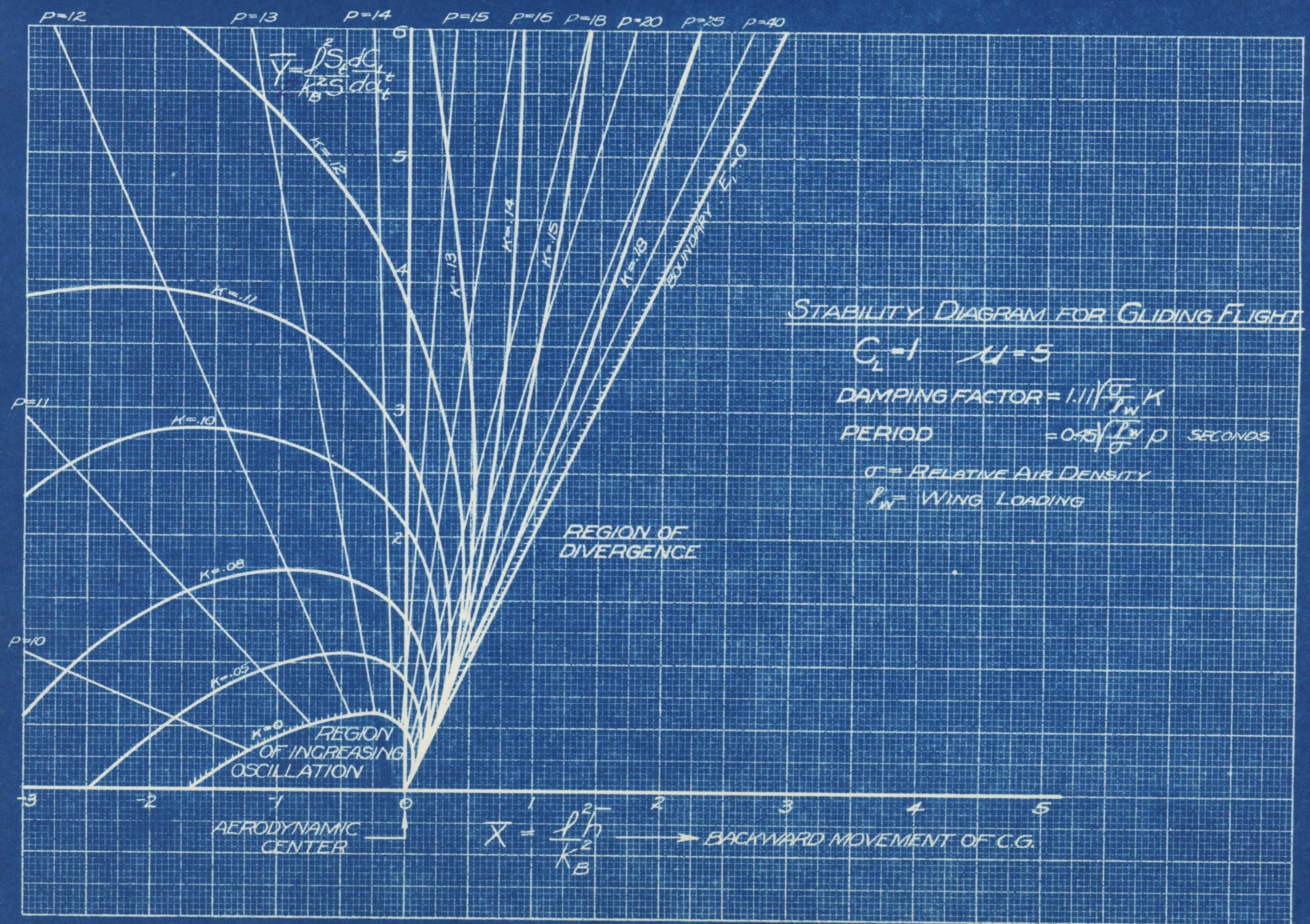
DYNAMIC STABILITY DIAGRAMS

CASE II

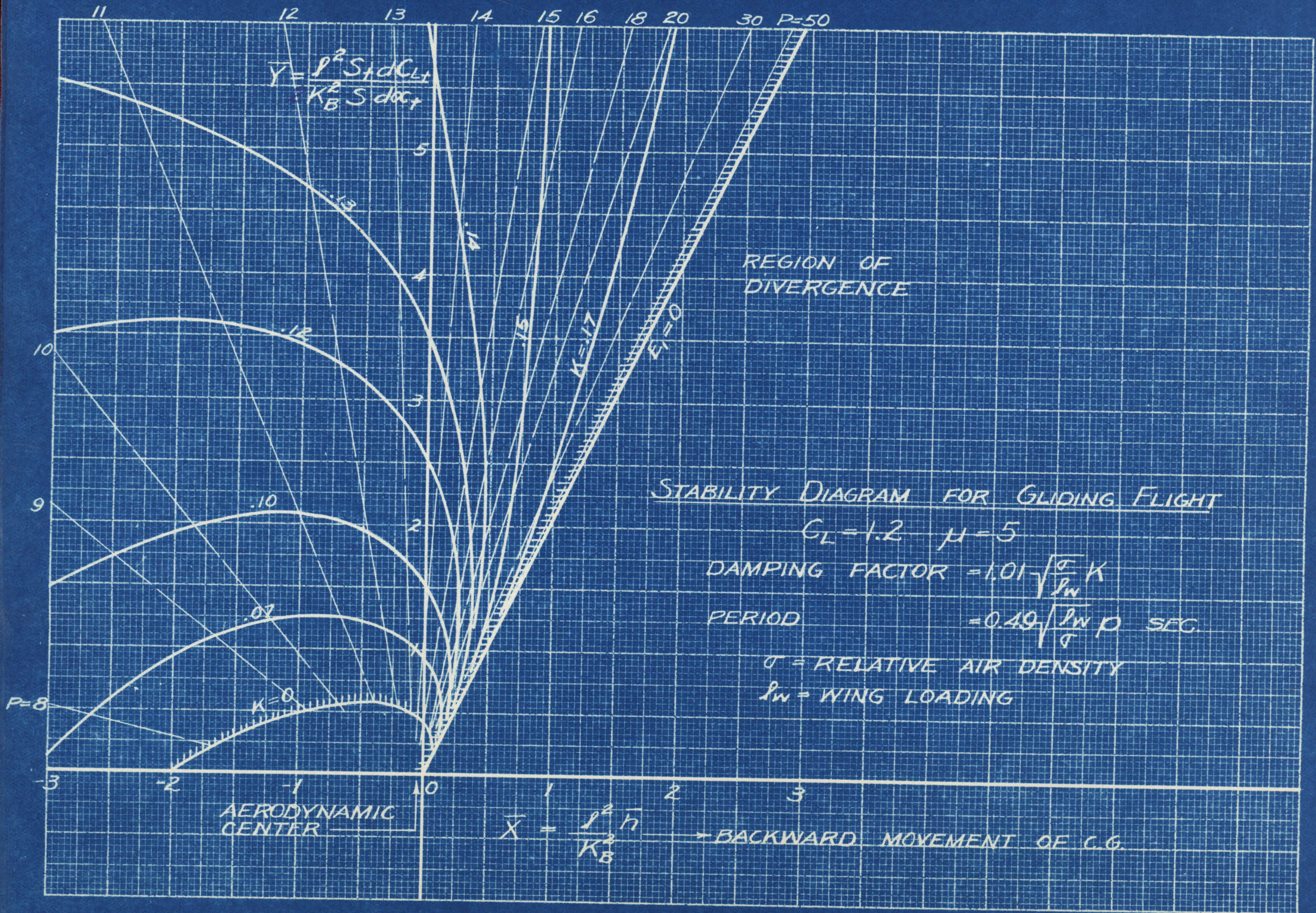

CASE II
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .3 \quad \mu = 5$
DAMPING FACTOR $= 2.02\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= 0.25\sqrt{\frac{l_w}{\sigma}}\,P$
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$E_1 = 0$
$\bar{Y} = \frac{l^2 S_t\, dC_{Lt}}{K_B^2 S\, d\alpha_t}$
$\bar{X} = \frac{l^2 \bar{h}}{K_B^2}$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
K=0
.05
.06
.063
.065
.067
K=.07
P=30
32.5
35
37.5
40
42.5
45
50
60
70
80
90
100
150
P=200
-3
-2
-1
0
1
2
3
4
5



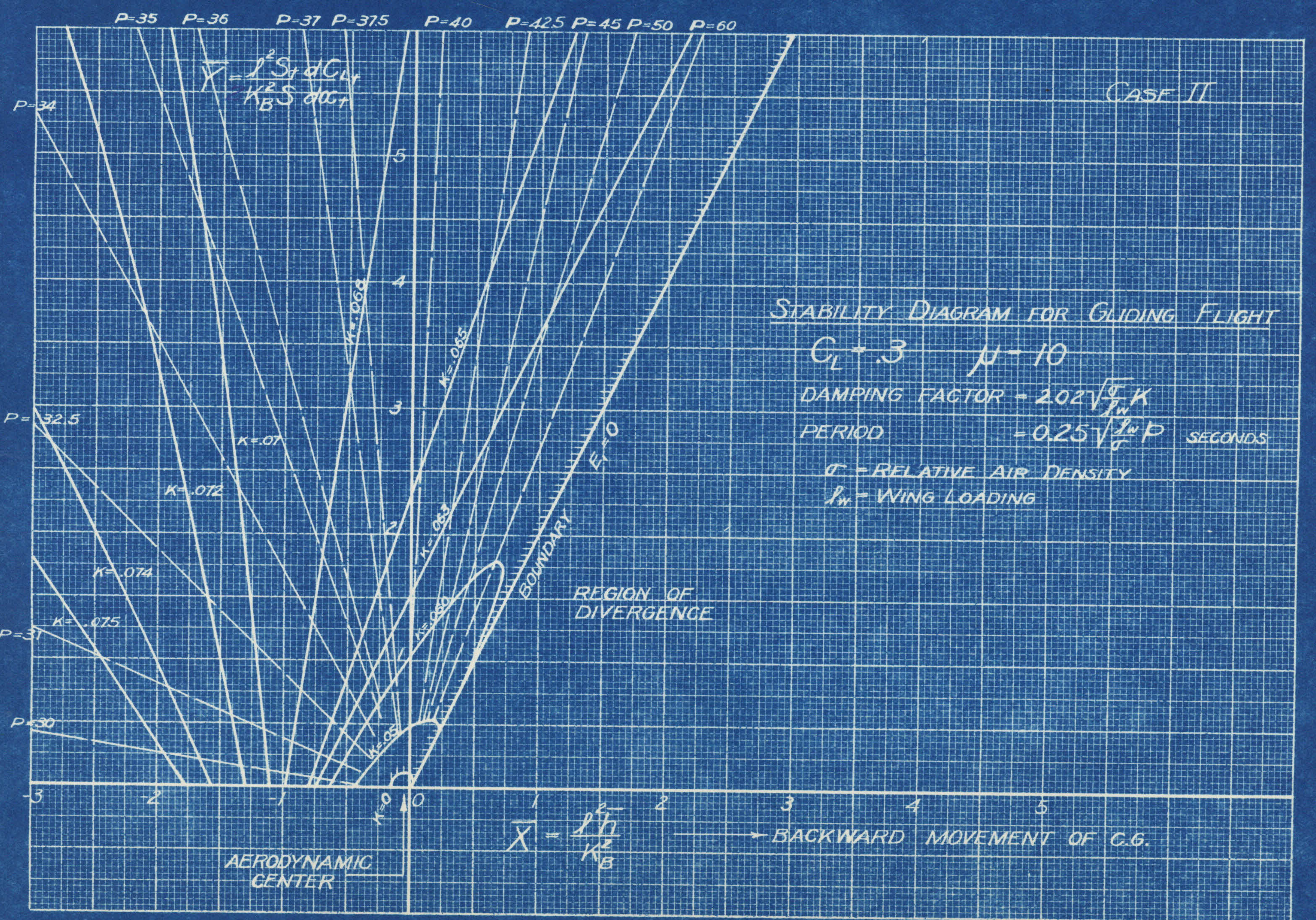
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .5 \quad \mu = 5$
DAMPING FACTOR $= 1.57\sqrt{\frac{\sigma}{\rho_w}}\,K$
PERIOD $= 0.32\sqrt{\frac{\rho_w}{\sigma}}\,p$
σ = RELATIVE AIR DENSITY
ρ_w = WING LOADING
REGION OF DIVERGENCE
$\bar{Y} = \frac{\rho^2 S_t\, dC_{L_t}}{K_B^2 S\, d\alpha_t}$
$\bar{X} = \frac{\rho^2 \bar{h}}{K_B^2}$
BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
$E_1 = 0$
K=0
.06
.07
.073
K=.075
.078
.08
.085
K=.09
P=20
22
24
26
28
30
35
40
50
P=70
-3
-2
-1
0
1
2
3
4
5



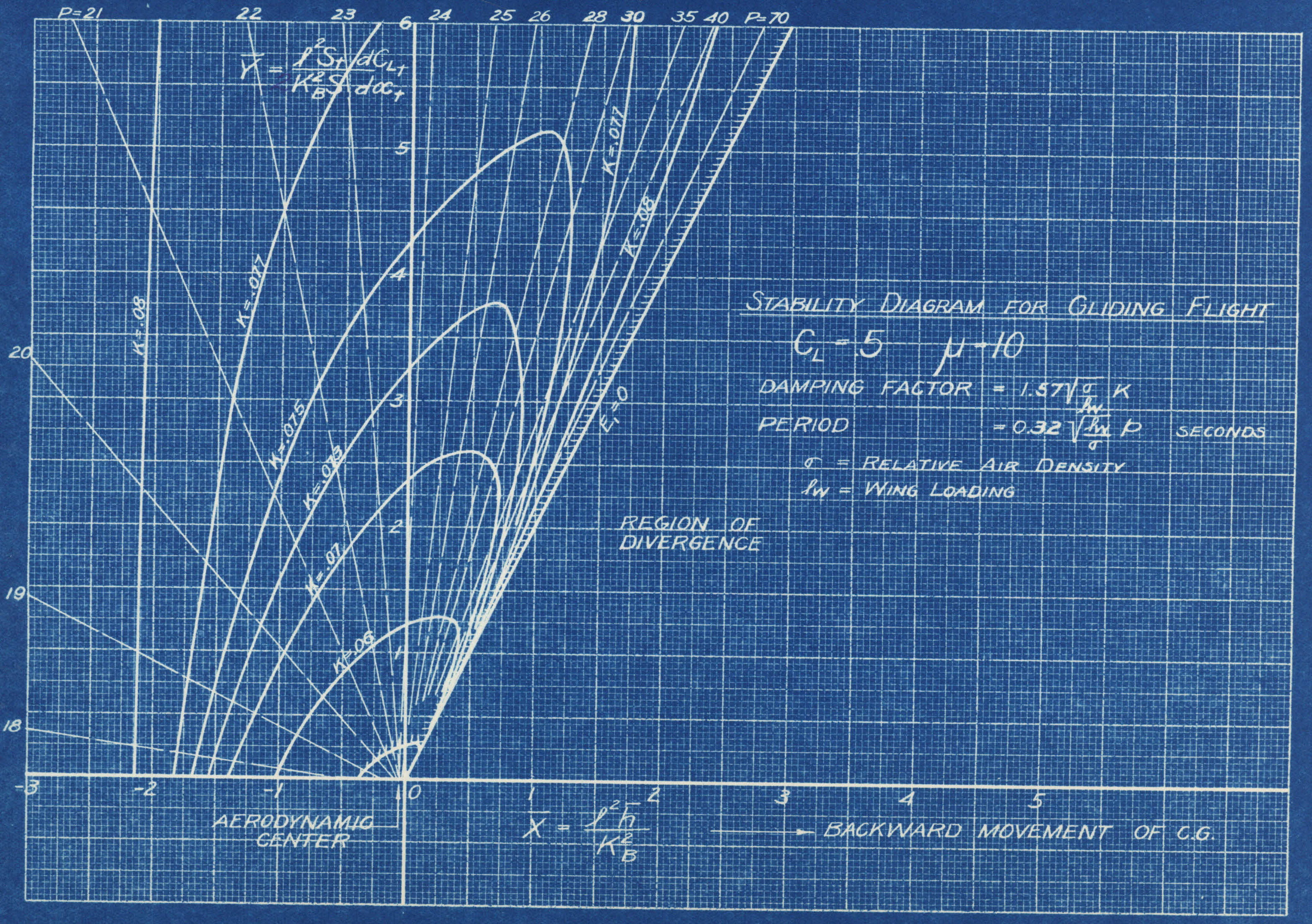
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1 \quad \mu = 5$
DAMPING FACTOR $= 1.11\sqrt{\frac{\sigma}{\rho_w}}\, K$
PERIOD $= 0.45\sqrt{\frac{\rho_w}{\sigma}}\, p$ SECONDS
σ = RELATIVE AIR DENSITY
ρ_w = WING LOADING
$\bar{Y} = \frac{l^2 S_t}{k_B^2 S} \frac{dC_{L_t}}{d\alpha_t}$
REGION OF DIVERGENCE
REGION OF INCREASING OSCILLATION
BOUNDARY $E_1 = 0$
AERODYNAMIC CENTER
$\bar{X} = \frac{l^2 h}{k_B^2}$ → BACKWARD MOVEMENT OF C.G.
p=10
p=11
p=12
p=13
p=14
p=15
p=16
p=18
p=20
p=25
p=40
K=0
K=.05
K=.08
K=.10
K=.11
K=.12
K=.13
K=.14
K=.15
K=.18
-3
-2
-1
0
1
2
3
4
5
6

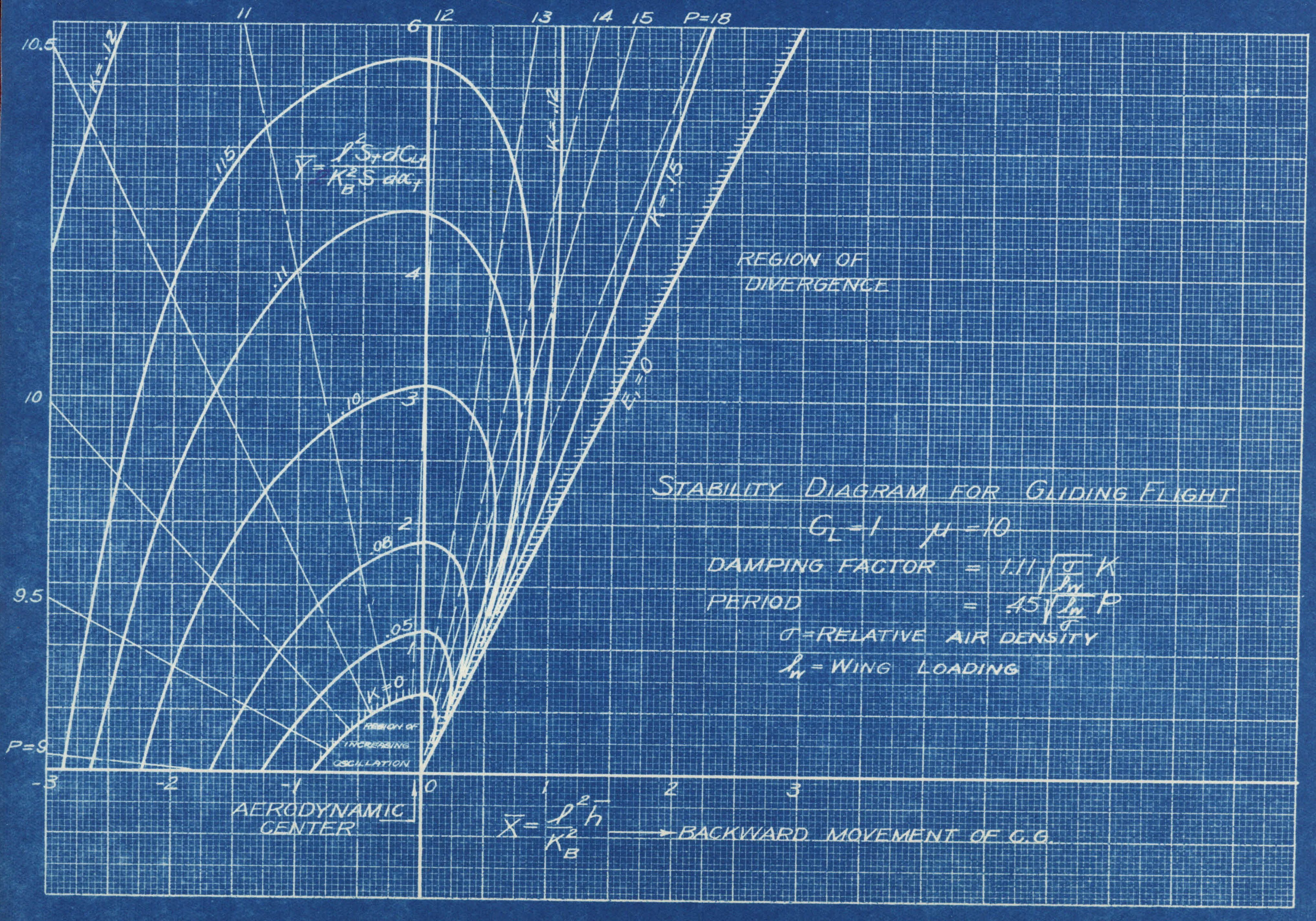


STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1.2 \quad \mu = 5$
DAMPING FACTOR $= 1.01\sqrt{\frac{\sigma}{\rho_W}}\,K$
PERIOD $= 0.49\sqrt{\frac{\rho_W}{\sigma}}\,P$ SEC.
σ = RELATIVE AIR DENSITY
ρ_W = WING LOADING
REGION OF DIVERGENCE
$\overline{Y} = \frac{\ell^2 S_t\, dC_{L_t}}{K_B^2 S\, d\alpha_t}$
$\overline{X} = \frac{\ell^2}{K_B^2}\overline{h}$
→ BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
$E_1 = 0$
K = .17
K = 0
P = 8
P = 50

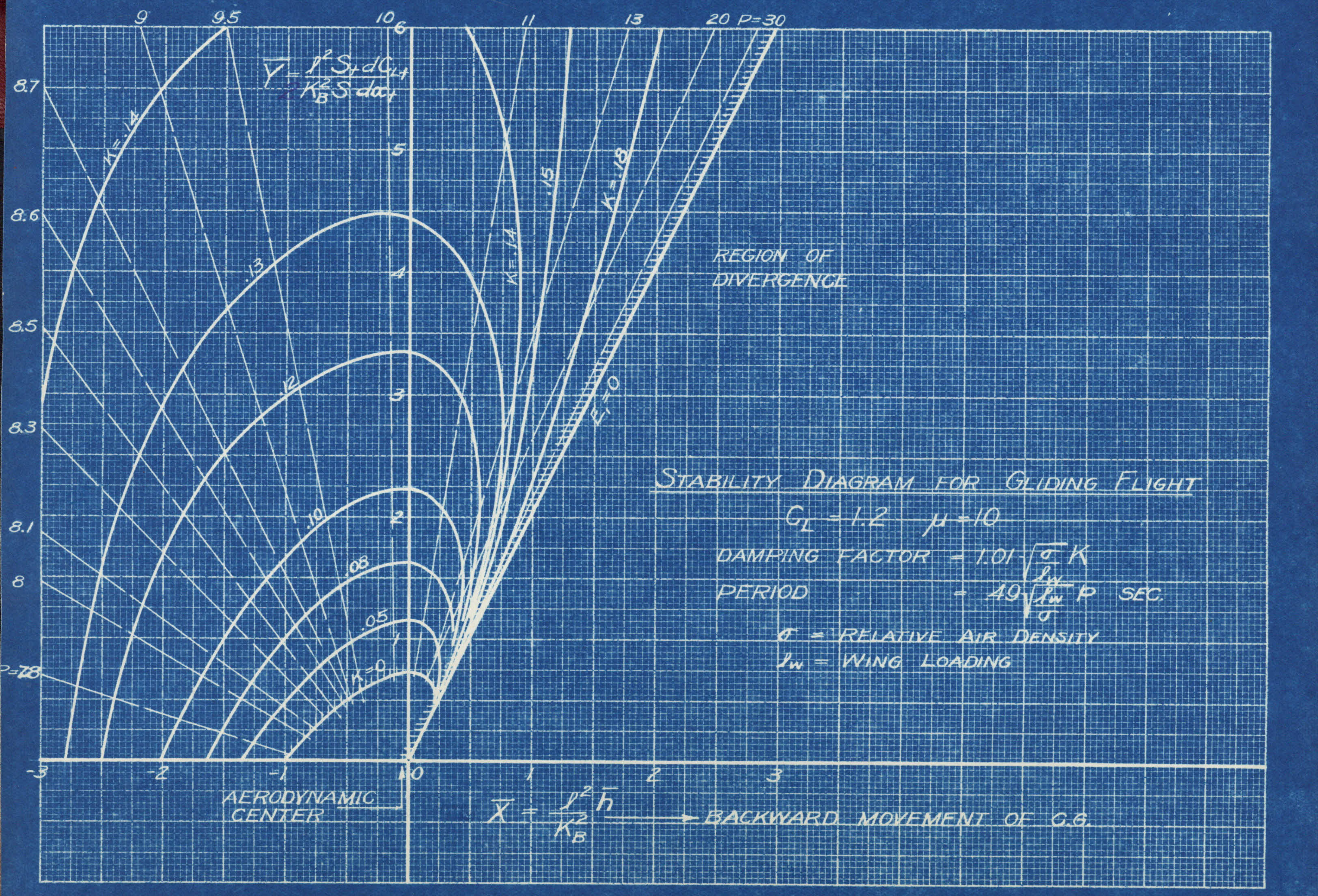


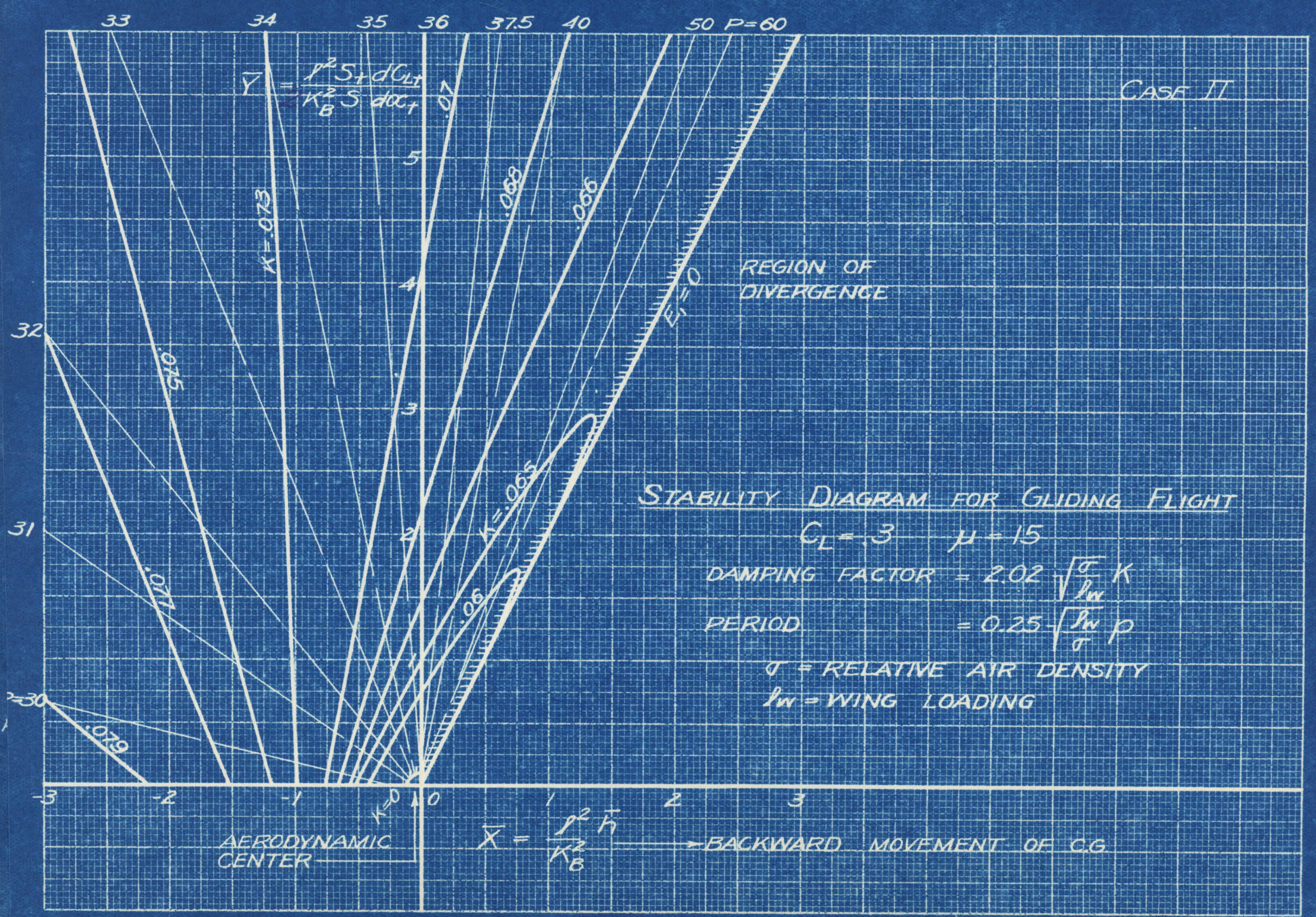
CASE II
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .3 \quad \mu = 10$
DAMPING FACTOR $= 2.02\sqrt{\frac{\sigma}{l_w}}K$
PERIOD $= 0.25\sqrt{\frac{l_w}{\sigma}}P$ SECONDS
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
$\overline{Y} = \frac{l^2 S_t}{K_B^2 S} \frac{dC_{L_t}}{d\alpha_t}$
$\overline{X} = \frac{l h}{K_B^2}$
→ BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
REGION OF DIVERGENCE
BOUNDARY $E_1 = 0$
P=35
P=36
P=37
P=37.5
P=40
P=42.5
P=45
P=50
P=60
P=34
P=32.5
P=31
P=30
K=.07
K=.072
K=.074
K=.075
K=.066
K=.065
K=.063
K=.060
K=.05
K=0


STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .5 \quad \mu = 10$
DAMPING FACTOR $= 1.57\sqrt{\frac{\sigma}{l_W}}\,K$
PERIOD $= 0.32\sqrt{\frac{l_W}{\sigma}}\,P$ SECONDS
σ = RELATIVE AIR DENSITY
l_W = WING LOADING
REGION OF DIVERGENCE
$\bar{Y} = \frac{l^2 S_T}{K_B^2 S}\frac{dC_{L_T}}{d\alpha_T}$
$\bar{X} = \frac{l^2 h}{K_B^2}$
BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
P=21
22
23
24
25
26
28
30
35
40
P=70
K=.08
K=.077
K=.075
K=.073
K=.07
K=.06
$E_1=0$
20
19
18
-3
-2
-1
0
1
2
3
4
5
6

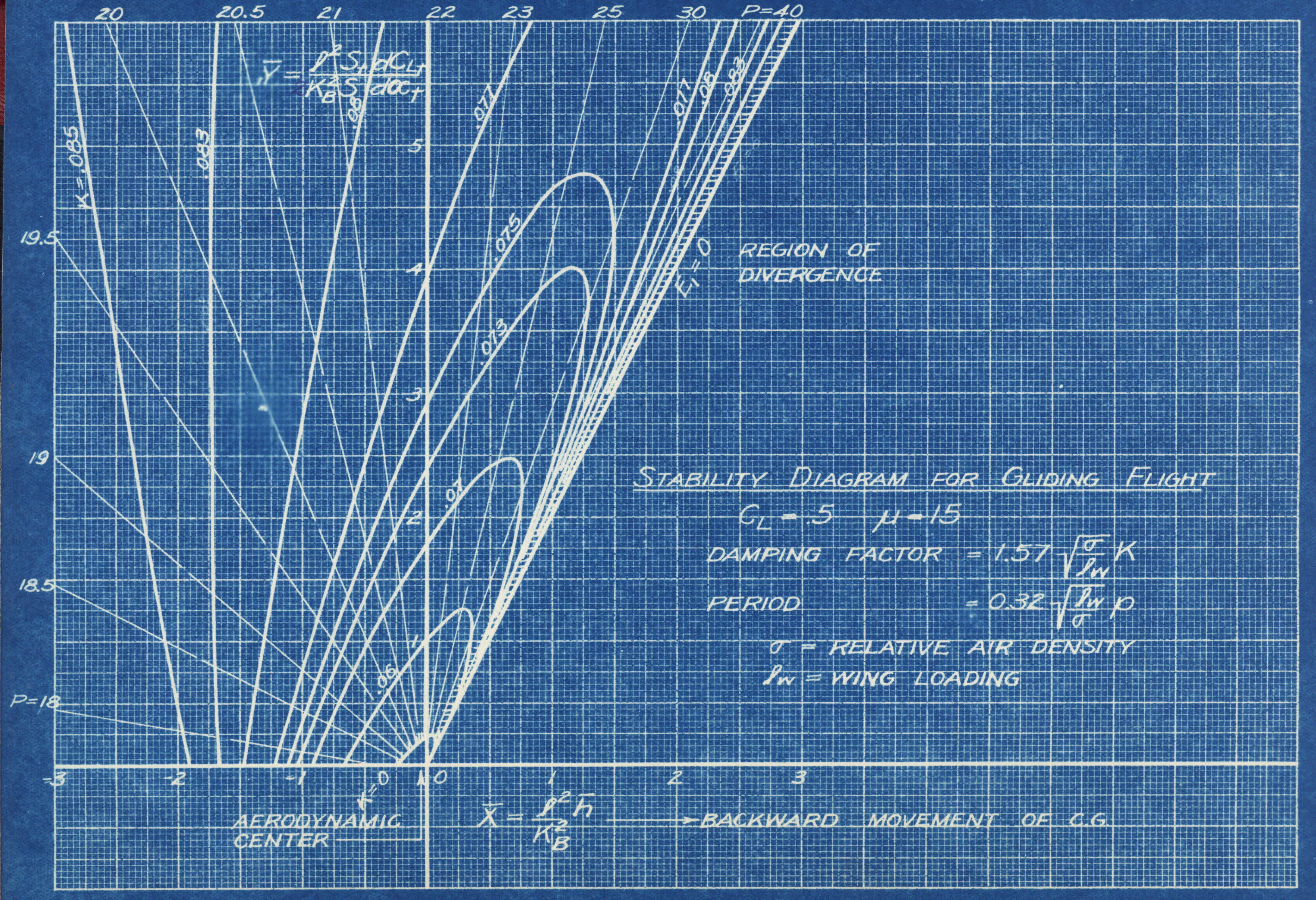


STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1 \quad \mu = 10$
DAMPING FACTOR $= 1.11\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= .45\sqrt{\frac{l_w}{\sigma}}\,P$
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
REGION OF INCREASING OSCILLATION
$\bar{Y} = \frac{l^2 S_t\, dC_{Lt}}{K_B^2 S\, d\alpha_t}$
$\bar{X} = \frac{l^2}{K_B^2} h$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
$E_1 = 0$
K = .15
K = .12
K = 0
P = 9
P = 18
11
12
13
14
15
.05
.08
.10
.11
.115
-3
-2
-1
0
1
2
3
4
6
9.5
10
10.5

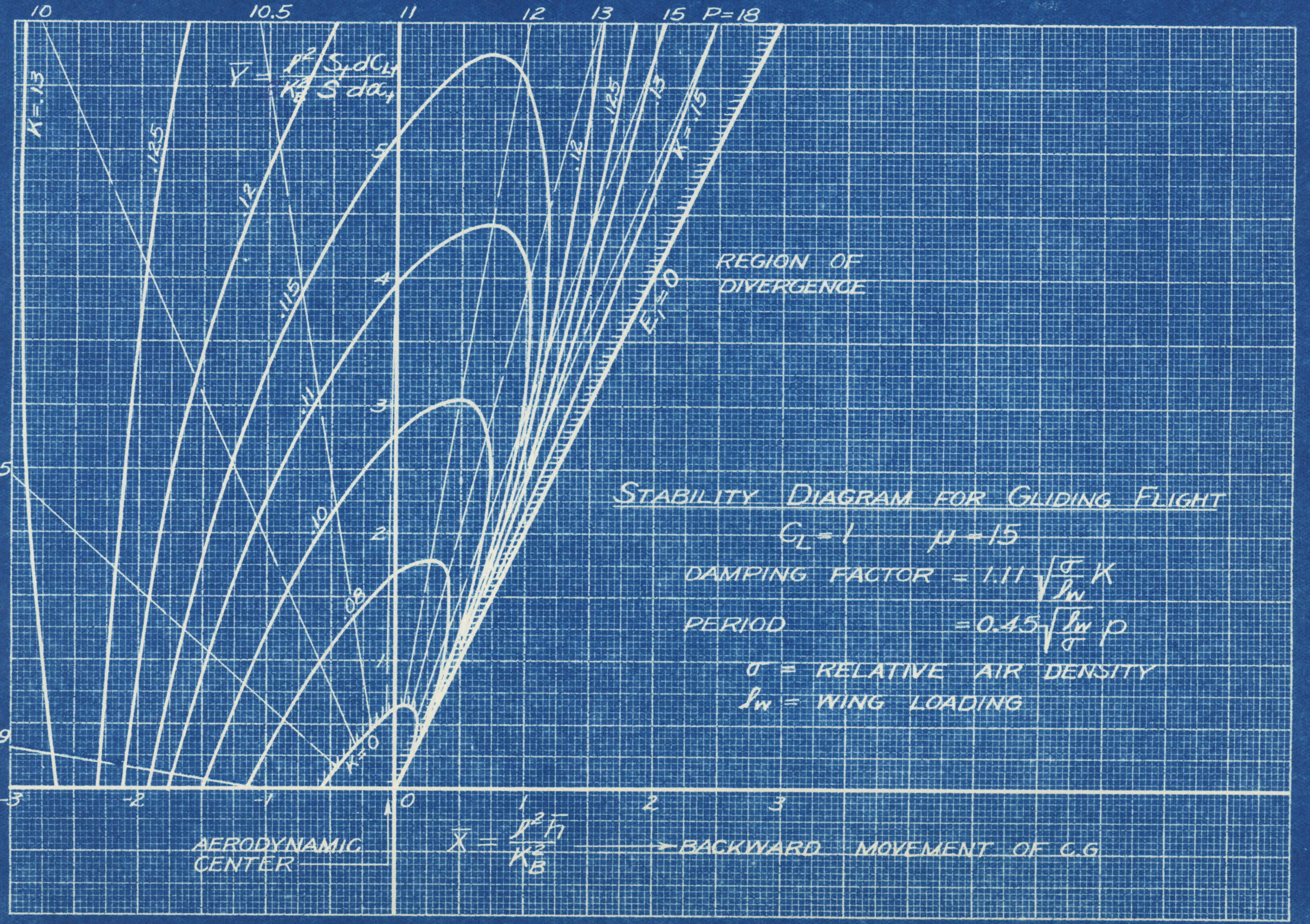

STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1.2 \quad \mu = 10$
DAMPING FACTOR $= 1.01\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= 49\sqrt{\frac{l_w}{\sigma}}\,P$ SEC.
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$\overline{Y} = \frac{l^2 S_t\, dC_{Lt}}{K_B^2 S\, d\alpha_t}$
$\overline{X} = \frac{l^2}{K_B^2}\overline{h}$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
K=.14
K=.18
.15
K=0
.05
.08
.10
.12
.13
$E_1=0$
9
9.5
10
11
13
20
P=30
8.7
8.6
8.5
8.3
8.1
8
-3
-2
-1
0
1
2
3
4
5
6



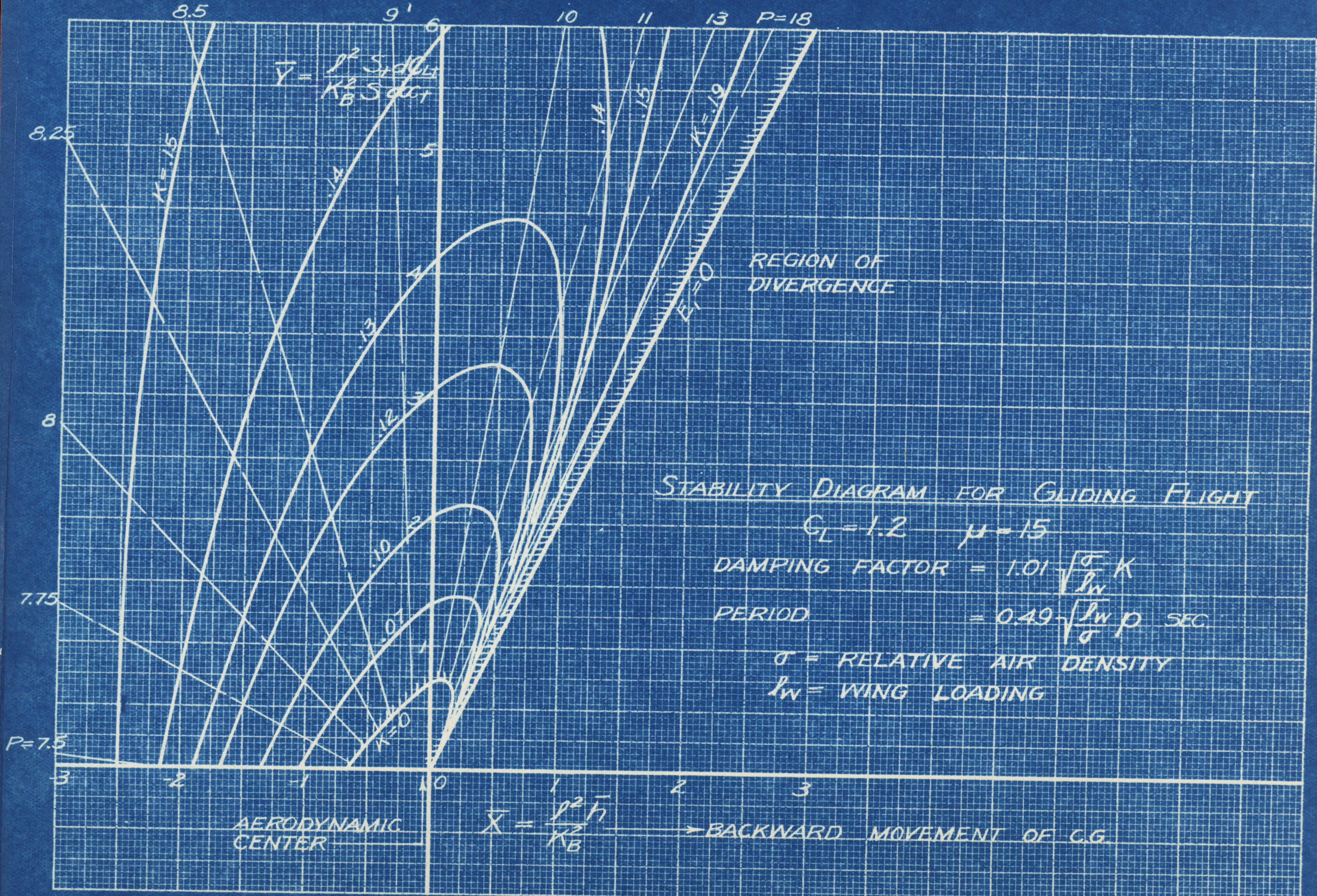
CASE II
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .3 \quad \mu = 15$
DAMPING FACTOR $= 2.02 \sqrt{\frac{\sigma}{l_w}}\, K$
PERIOD $= 0.25 \sqrt{\frac{l_w}{\sigma}}\, P$
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$E_1 = 0$
$\bar{Y} = \frac{r^2 S_t\, dC_{Lt}}{2K_B^2 S\, d\alpha_t}$
$\bar{X} = \frac{r^2 \bar{h}}{K_B^2}$
BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
33
34
35
36
37.5
40
50
P=60
32
31
P=30
K=.073
.075
.077
.079
.068
.066
K=.065
.06
K=0
-3
-2
-1
0
1
2
3
4
5

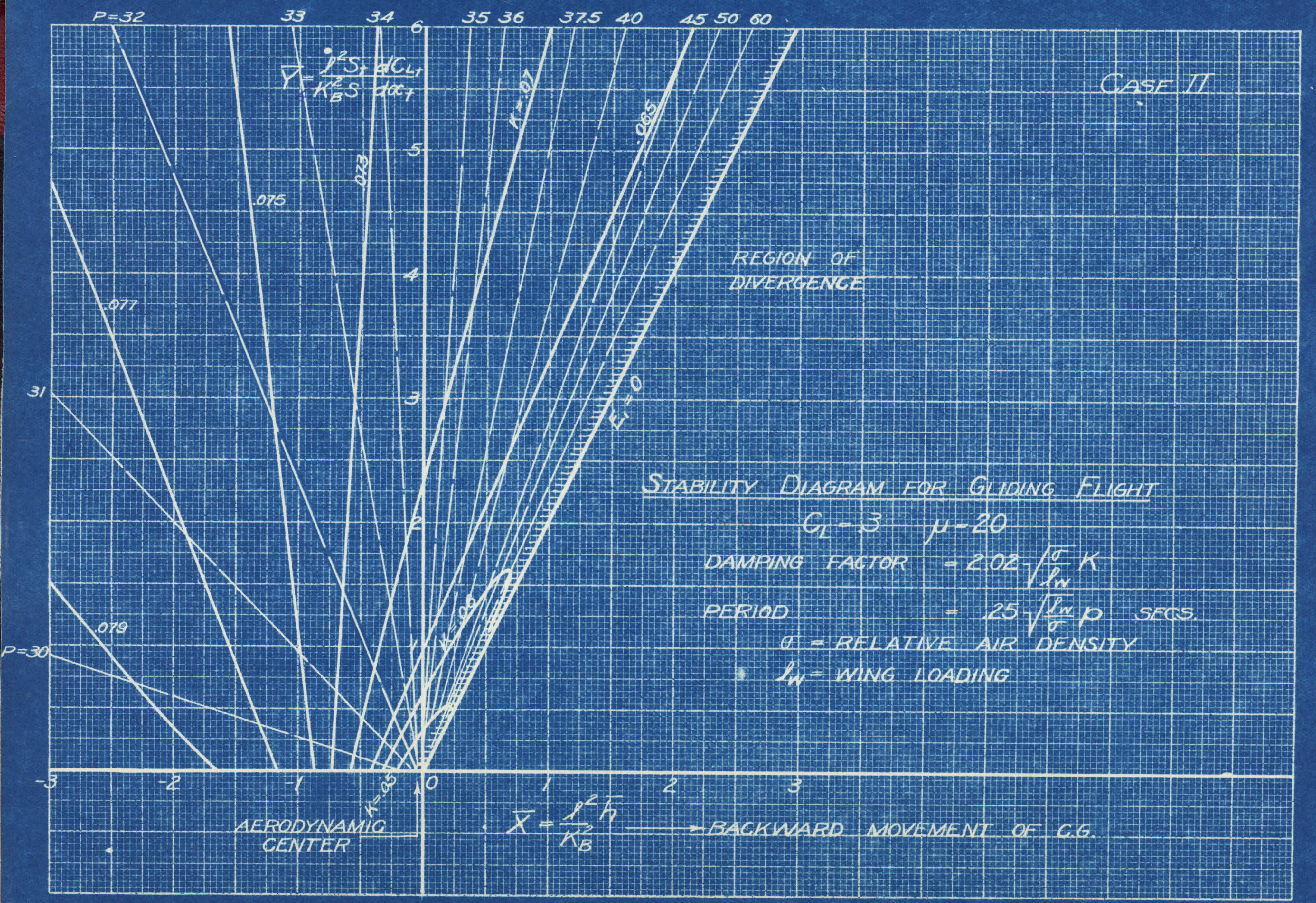


STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .5 \quad \mu = 15$
DAMPING FACTOR $= 1.57\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= 0.32\sqrt{\frac{l_w}{\sigma}}\,P$
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$E_1 = 0$
$\bar{Y} = \frac{l^2 S_t \, dC_{Lt}}{K_B^2 S \, d\alpha_t}$
$\bar{X} = \frac{l^2}{K_B^2}\bar{h}$
BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
K=.085
.083
.077
.075
.073
.07
.06
P=18
18.5
19
19.5
20
20.5
21
22
23
25
30
P=40
-3
-2
-1
0
1
2
3
4
5

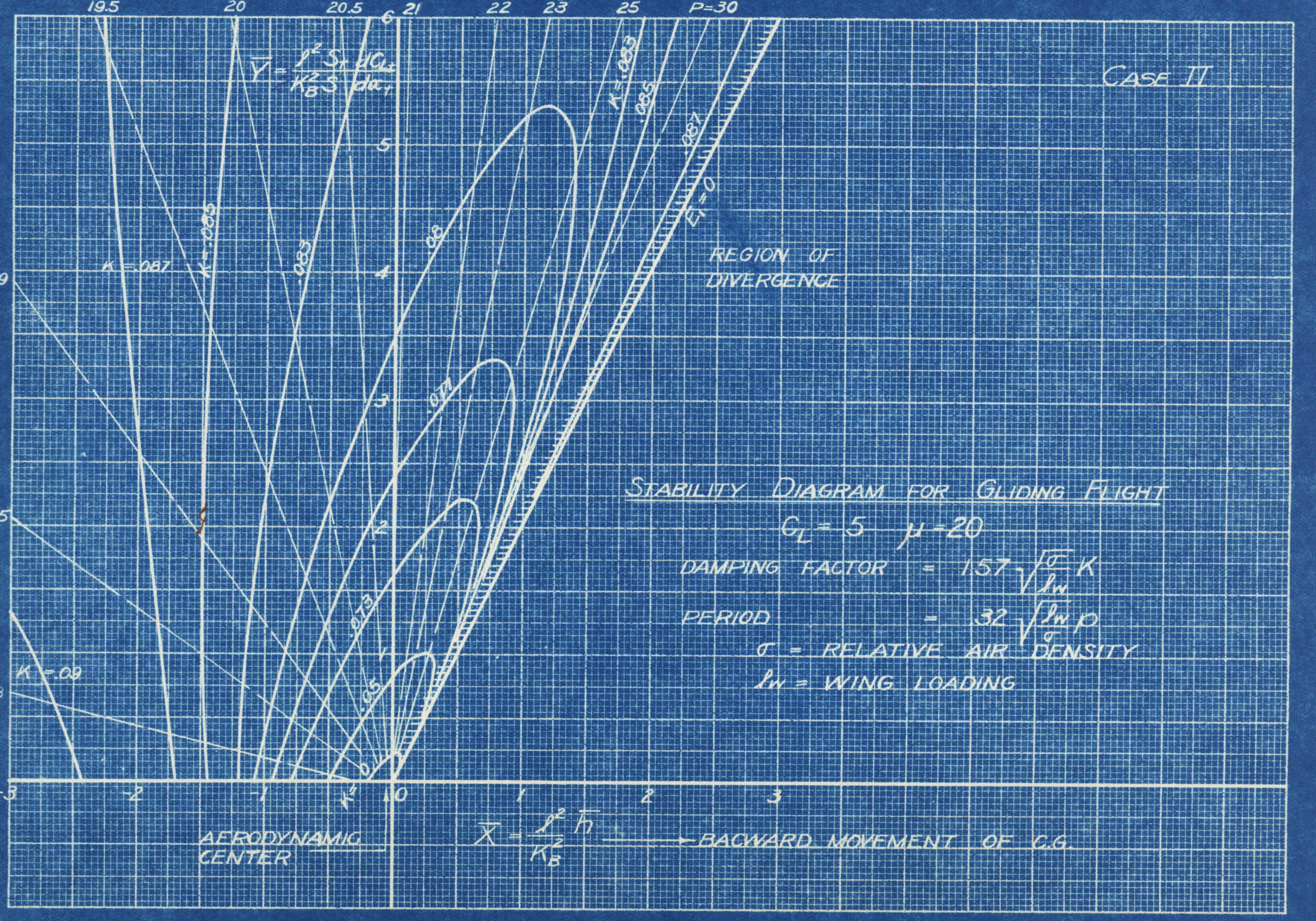


STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1 \qquad \mu = 15$
DAMPING FACTOR $= 1.11 \sqrt{\frac{\sigma}{l_w}}\, K$
PERIOD $= 0.45 \sqrt{\frac{l_w}{\sigma}}\, P$
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$E_1 = 0$
$K = .15$
$K = .13$
$K = 0$
$\bar{Y} = \frac{r^2}{K_B^2} \frac{S_1}{S} \frac{dC_{L_1}}{d\alpha_1}$
$\bar{X} = \frac{r^2 \bar{h}}{K_B^2}$
→BACKWARD MOVEMENT OF C.G
AERODYNAMIC CENTER
P=9
9.5
10
10.5
11
12
13
15
P=18

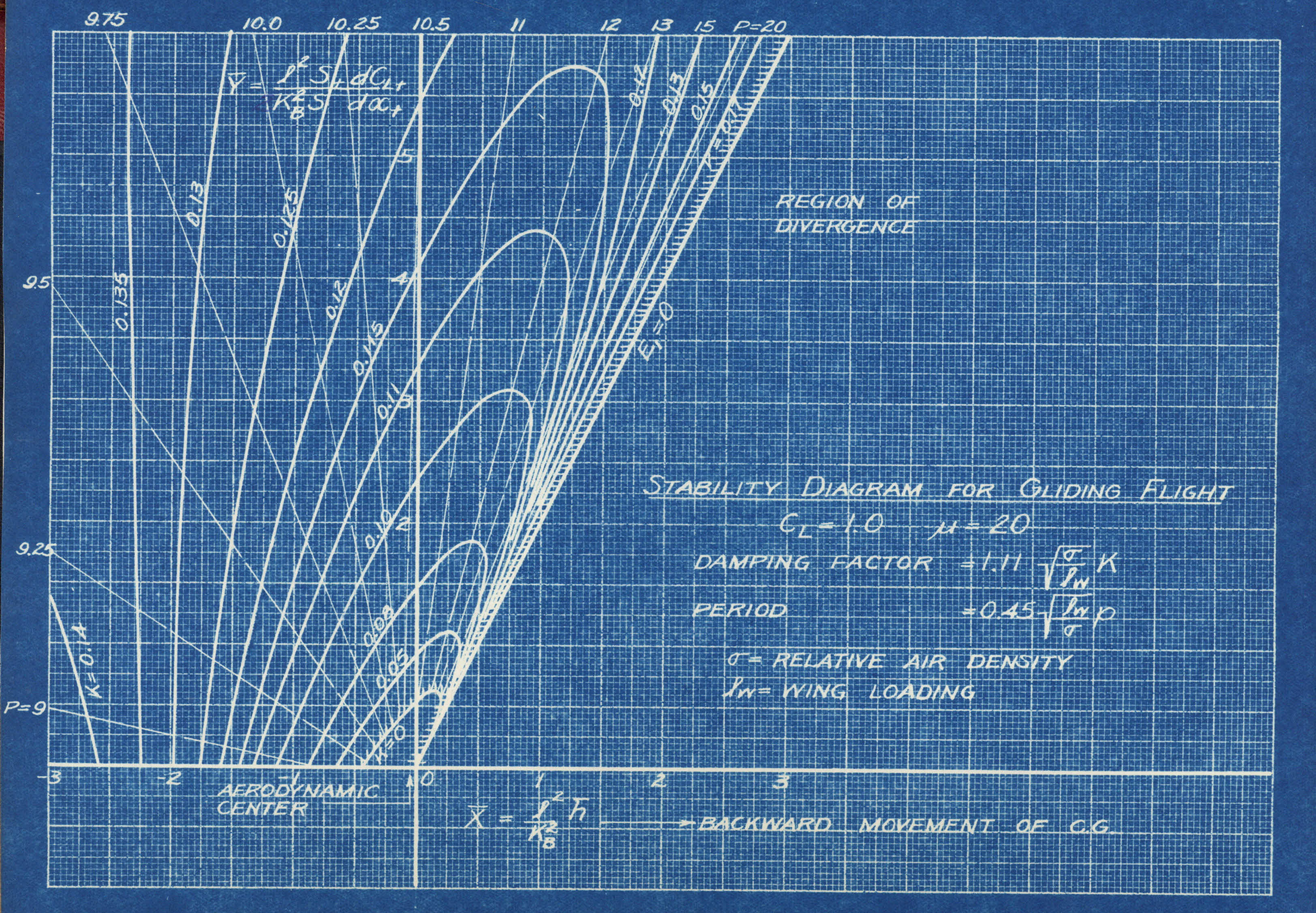

STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1.2 \quad \mu = 15$
DAMPING FACTOR $= 1.01\sqrt{\frac{\sigma}{l_W}}\,K$
PERIOD $= 0.49\sqrt{\frac{l_W}{\sigma}}\,P$ SEC.
σ = RELATIVE AIR DENSITY
l_W = WING LOADING
REGION OF DIVERGENCE
$E = 0$
$\bar{Y} = \frac{\rho^2 S_t\, dC_{Lt}}{K_B^2 S\, d\alpha_t}$
$\bar{X} = \frac{\rho^2 \bar{h}}{K_B^2}$
AERODYNAMIC CENTER
→BACKWARD MOVEMENT OF C.G.
P=7.5
7.75
8
8.25
8.5
9
10
11
13
P=18
K=.15
K=.19
K=0
-3
-2
-1
0
1
2
3



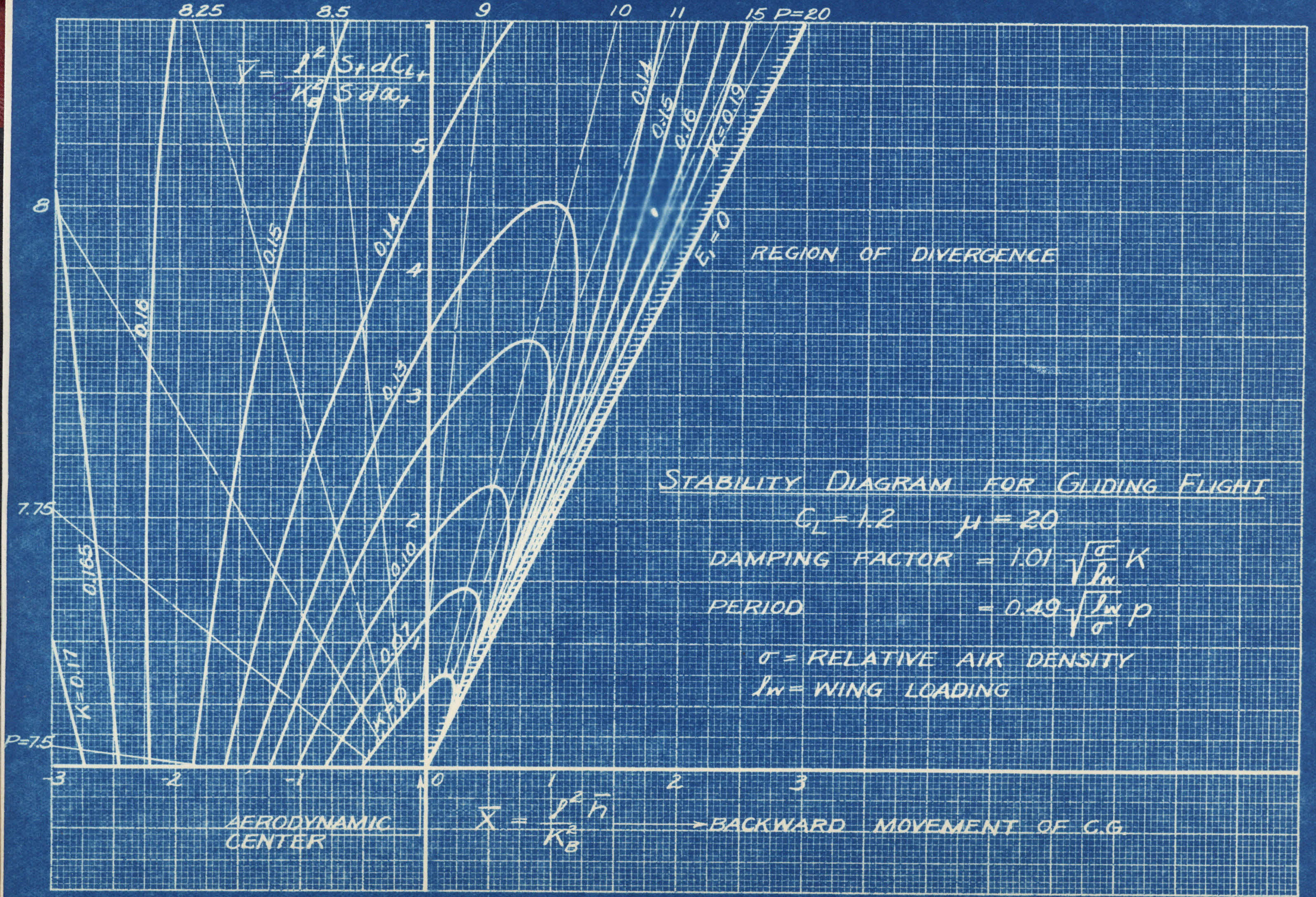
CASE II
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .3 \quad \mu = 20$
DAMPING FACTOR $= 2.02\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= .25\sqrt{\frac{l_w}{\sigma}}\,p$ SECS.
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$\overline{Y} = \frac{l^2 S_t}{K_B^2 S}\frac{dC_{L_t}}{d\alpha_t}$
$\overline{X} = \frac{l^2}{K_B^2}\overline{h}$
BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
P=32
33
34
35
36
37.5
40
45
50
60
31
P=30
.075
.077
.079
.073
.065
K=.07
K=.09
K=.05
$K_1=0$
-3
-2
-1
0
1
2
3
4
5
6



CASE II
STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = .5 \quad \mu = 20$
DAMPING FACTOR $= .157 \sqrt{\frac{\sigma}{l_W}}\, K$
PERIOD $= .32 \sqrt{\frac{l_W}{\sigma}}\, P$
σ = RELATIVE AIR DENSITY
l_W = WING LOADING
REGION OF DIVERGENCE
$E_1 = 0$
$\overline{Y} = \frac{l^2 S_t}{K_B^2 S} \frac{dC_{L_t}}{d\alpha_t}$
$\overline{X} = \frac{l^2}{K_B^2} \overline{h}$ → BACWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
P=18
P=30
19.5
20
20.5
21
22
23
25
19
18.5
K=.087
K=.085
.083
.08
.077
.073
.065
K=.083
.085
.087
K=.09
-3
-2
-1
0
1
2
3



STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1.0 \quad \mu = 20$
DAMPING FACTOR $= 1.11 \sqrt{\frac{\sigma}{l_W}}\, K$
PERIOD $= 0.45 \sqrt{\frac{l_W}{\sigma}}\, P$
σ = RELATIVE AIR DENSITY
l_W = WING LOADING
REGION OF DIVERGENCE
$\bar{Y} = \frac{l^2 S_t}{K_B^2 S} \frac{dC_{L_t}}{d\alpha_t}$
$\bar{X} = \frac{l^2}{K_B^2} h$ → BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
$E_1 = 0$
K = 0.4
K = 0.17
P = 9
P = 20
9.75
10.0
10.25
10.5
11
12
13
15
9.5
9.25
0.135
0.13
0.125
0.12
0.115
0.11
0.10
0.08
0.05
0.15
-3
-2
-1
0
1
2
3


STABILITY DIAGRAM FOR GLIDING FLIGHT
$C_L = 1.2$ $\mu = 20$
DAMPING FACTOR $= 1.01\sqrt{\frac{\sigma}{l_w}}\,K$
PERIOD $= 0.49\sqrt{\frac{l_w}{\sigma}}\,P$
σ = RELATIVE AIR DENSITY
l_w = WING LOADING
REGION OF DIVERGENCE
$\bar{Y} = \frac{l^2}{K_B^2}\frac{S_t}{S}\frac{dC_{L_t}}{d\alpha_t}$
$\bar{X} = \frac{l^2}{K_B^2}\bar{h}$
→BACKWARD MOVEMENT OF C.G.
AERODYNAMIC CENTER
8.25
8.5
9
10
11
15
P=20
8
7.75
P=7.5
K=0.17
0.165
0.16
0.15
0.14
0.13
0.10
0.07
K=0
0.14
0.15
0.16
K=0.19
$E_1 = 0$
-3
-2
-1
0
1
2
3
5
4
3
2

CONCLUSIONS FROM STUDY OF DIAGRAMS

From study of the dynamic stability diagrams, the following conclusions have been derived:

(1) The damping factor and the period, $\overline{K}$ and P, should be as large as possible in order to avoid the region of increasing oscillation.

(2) The damping factor decreases and the period increases with an increase in altitude and wing loading, other quantities remaining constant.

(3) The period is increased by a backward movement of the center of gravity of the airplane and by an increase in the horizontal tail surface area, if the C.G. is behind the aerodynamic center.

(4) Instability may arise through an increasing oscillation or a divergence. As the speed is decreased, i.e., C_L increased, the danger of instability due to increasing oscillation increases.

(5) A large moment of inertia about the lateral axis of the airplane increases the danger of instability; hence the distribution of the weight along the longitudinal axis should be as compact as possible.

(6) When C_L is small, the damping is roughly independent of the coordinates, i.e., the C.G. position and the tail area. As C_L increases, the dependence of $\overline{K}$ on $\overline{X}$ and $\overline{Y}$ increases.

(7) The diagrams show that it is desirable to arrive at static stability by means of a fairly large horizontal tail surface rather than by having the C.G. far forward and using a smaller tail surface.

(8) Reasonable static stability with a relatively large horizontal tail surface insures dynamic gliding stability. Large static stability, however, makes the period short and the damping factor small. ($\bar{X}$ large negatively and $\bar{Y}$ small.)

(9) An increase in the slope of the lift curve increases the region of dynamic stability. (Compare Cases I & II.)

Conclusions Important for Design

(1) The plane should have a slight static stability obtained with a relatively large horizontal tail surface, C.G. to the rear.

(2) The moment of inertia about the lateral axis should be as small as possible.

(3) For assurance that the plane will be stable with power-on, the boundaries of gliding stability should not be approached too closely.

PROCEDURE FOR USE OF CHARTS IN DESIGN

For practical use of the stability diagrams, the following properties of the airplane must be known:

W = weight of plane

S = wing area (From this and the span, find $\overline{AR}_w$)

S_t = area of horizontal tail surface

a = distance, horizontally, from C.G. to leading edge of wing.

b = vertical distance of C.G. from chord line

t = mean aerodynamic chord

h_o = aerodynamic center of wing

$\bar{h}$ = $\left[\left(\frac{a}{t}\right)_{EFF} - h_o\right]$

$$(a/t)_{eff.} = \left[\left(\frac{a}{t}\right) + \left(\Delta \frac{a}{t}\right)_{FUSE.} \pm \left(\frac{1}{10}\frac{b}{t}\right)\right]$$

l = length from the C.G. to the tail post.

k_B = radius of gyration about the lateral axis.

AR_t = aspect ratio of horizontal tail surfaces.

From this $\frac{dC_{L_t}}{d\alpha_t} = \frac{k}{1 + k/\pi AR_t} \cdot (k \doteq 5.5)$.

Outline of Procedure

(1) Determine which set of curves to use by referring to wind tunnel test or by calculating $\frac{dC_L}{d\alpha}$ and C_{D_p}.

If $\frac{dC_L}{d\alpha} \doteq 4.8$ and $C_{D_p} \doteq 0.02$, use Case I.

If $\frac{dC_L}{d\alpha} \doteq 4.0$ and $C_{D_p} \doteq 0.05$, use Case II.

(2) From the given properties of the plane, calculate "$\bar{X}$" and "$\bar{Y}$", where

$$\bar{X} = \bar{h}/\eta \quad , \quad \frac{1}{\eta} = \frac{l^2}{k_B^2}.$$

and

$$\bar{Y} = \frac{1}{2}\frac{1}{\eta}\frac{S_t}{S}\frac{k}{1 + k/\pi AR_t}$$

(3) Determine "μ", the " relative density" of the plane,

$$\mu = \frac{m}{\rho S l} = 13.08 \frac{l_w}{l} \quad \text{(For sea level)}$$

where l_w = W/S = wing loading.

(4) Find K and p (non-dimensional units) for the series of lift coefficients, 0.3, 0.5, 1.0, 1.2, using the factor "μ" calculated in step (3). Interpolate for the proper μ if necessary.

(5) By the use of the factors appearing on each diagram, calculate the Damping Factor and the Period in the proper units. The time to damp to half amplitude is given by the formula,

$$T_{\frac{1}{2}} = \frac{0.693}{D.F.} \text{ seconds.}$$

The shaded lines on each curve sheet represent the boundaries of dynamic longitudinal stability in gliding flight. The plane is dynamically stable if the point (X, Y) falls within the boundaries for all the lift coefficients. Any inherent instability should appear immediately by reference to the diagrams.

POSSIBLE RANGE OF PARAMETER VALUES

Quantity		Range
l^2/k_B^2	----------	5 - 25
S_t/S	----------	0.10 - 0.20
h_o	----------	0.20 - 0.25
$\bar{h}$	----------	-0.05 - 0.20
$\frac{dC_{L_t}}{d\alpha_t}$	----------	2.00 - 4.00
$\bar{X}$	----------	-1.00 - 4.00
$\bar{Y}$	----------	0.50 - 8.00
μ	----------	5.0 - 20.0

ACCURACY OF DIAGRAMS

It has been found that the theoretical and measured longitudinal stability characteristics for gliding flight agree reasonably well if neutral stability is not approached too closely. A reasonably stable plane in gliding flight will MOST LIKELY be stable with power on; but if gliding flight characteristics approach the boundaries of instability too closely, then dynamic stability with power-on is not assured. As far as the accuracy of the charts, themselves are concerned, two practical examples of their use will serve to illustrate.

EXAMPLES OF THE USE OF THE DIAGRAMS

(1) For the first example illustrating the use of the charts, consider the Doyle O-2 biplane. Its properties are as follows: (4)

$W = 1315$ #

$S = 159.5$ sq. ft.

$b = 30.0$ ft.

$S_t = 18.7$ sq. ft.

$a/t = .34$

$b/t = -.36$

$h_o = .25$ (Estimated)

$\bar{h} = (.34 - .036) - (.25) = 0.054$ (Effect of fuselage neglected

$l = 11.8'$

$k_B = \sqrt{16.4}$

$AR_t = 4.33$; $AR_W = 5.65$

$$\frac{dC_{L_t}}{d\alpha_t} = \frac{5.5}{1 + \frac{5.5}{\pi AR_t}} = 3.93$$

Procedure:

1. $$\frac{dC_L}{d\alpha} = \frac{k}{1+\frac{k}{\pi AR_w}} = \frac{5.5}{1+\frac{5.5}{\pi 5.65}} = 4.22$$

C_{D_p} = .04 -- .05 (Estimated)

Use Charts for Case II.

2. $$\bar{X} = \frac{\bar{h}}{\eta} \; ; \quad \frac{1}{\eta} = \frac{l^2}{k_B^2} = \frac{(11.8)^2}{16.4} = 8.5 \; ; \quad \bar{X} = .46$$

$$\bar{Y} = \frac{1}{2}(8.5)\frac{18.7}{159.5}(3.93) = 1.96$$

3.

$$\mu = 13.08\frac{l_w}{l} = \frac{13.08}{11.8}\,\frac{1315}{159.5} = 9.15.$$

4. It will be sufficiently accurate to use the charts for μ = 10.00. Referring to the charts, we have:

$$\sqrt{\frac{l_w}{\sigma}} = \sqrt{8.28} = 2.88$$

$$\sqrt{\frac{\sigma}{l_w}} = 0.347$$

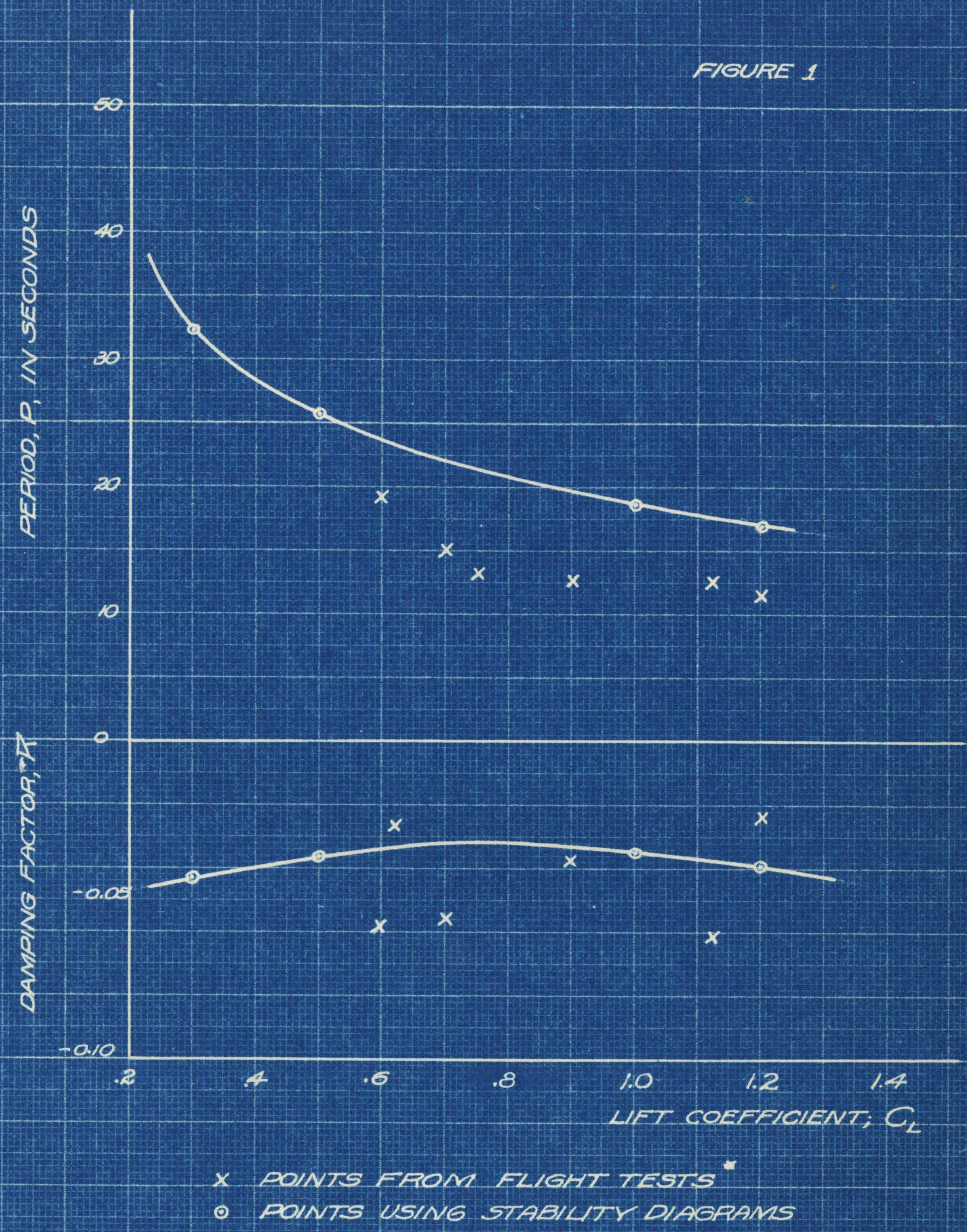
$(\sigma = 1)$

C_L	$\bar{K}$	p
0.3	0.043	32.3
0.5	0.036	25.6
1.0	0.035	18.7
1.2	0.039	16.9

where $\bar{K}$ and p are in English units.

Discussion

The above values for $\bar{K}$ and p are plotted in Fig. 1. The agreement with experimental data on the Doyle 0-2 [4] is very good for the damping factors. For the period, the agreement is good for high speed, i.e., low C_L's; but gives a high value at the higher lift coefficients. In the respect that the designer is interested fundamentally IN THE PERIOD AND



VALUES OF DAMPING FACTOR AND PERIOD OF THE PHUGOID FOR THE DOYLE O-2 AIRPLANE, GLIDING FLIGHT.

* (See N.A.C.A. T.R. 442, page 13, figures & and 15)

damping factor in the range of high speed, the charts give good results for this particular case.

(2)

As a second problem for illustration, consider the new Douglas Transport, the properties for which are the following:

$W = 17{,}500$ #

$S = 939$ sq. ft.

$S_t = 145.6$ sq/ ft.

$a/t = .25$

$b/t = + .13$

$h_o = .23$

$\bar{h} = (.25 + .013 + .025) - .23 = 0.058$

$l = 36.62$ FT.

$k_B = \sqrt{45.00}$

$AR_t = 4.75$

$\frac{dC_L}{d\alpha} = 4.7$ (From wind tunnel test)

__Procedure:__

1. $\frac{dC_L}{d\alpha} = 4.70$

   $C_{D_p} = 0.02$

   Use charts for Case I.

2. $$\bar{X} = \frac{\bar{h}}{\eta} = \frac{.058(36.62)^2}{45} = 1.74 \quad \text{WHERE } \frac{1}{\eta} = \frac{l^2}{k_B^2} = 29.8$$

   $$\bar{Y} = \frac{1}{2}\frac{1}{\eta}\frac{S_t}{S}\frac{dC_{L_t}}{d\alpha_t} = \frac{29.8}{2}\frac{5.5}{1+\frac{5.5}{\pi 4.75}}\frac{145.6}{939.0} = 9.30$$

3. $$\mu = 13.08\frac{h_w}{l} = \frac{13.08(18.65)}{36.62} = 6.65$$

4. Referring to the charts for Case I, using the coordinates $\bar{X}$ and $\bar{Y}$ from (2), we see that the plane is well within the stable range. This is due to the fact that $\bar{Y}$ is very large, i.e., the tail area is large. The values for $\bar{K}$ and $\bar{p}$ are :

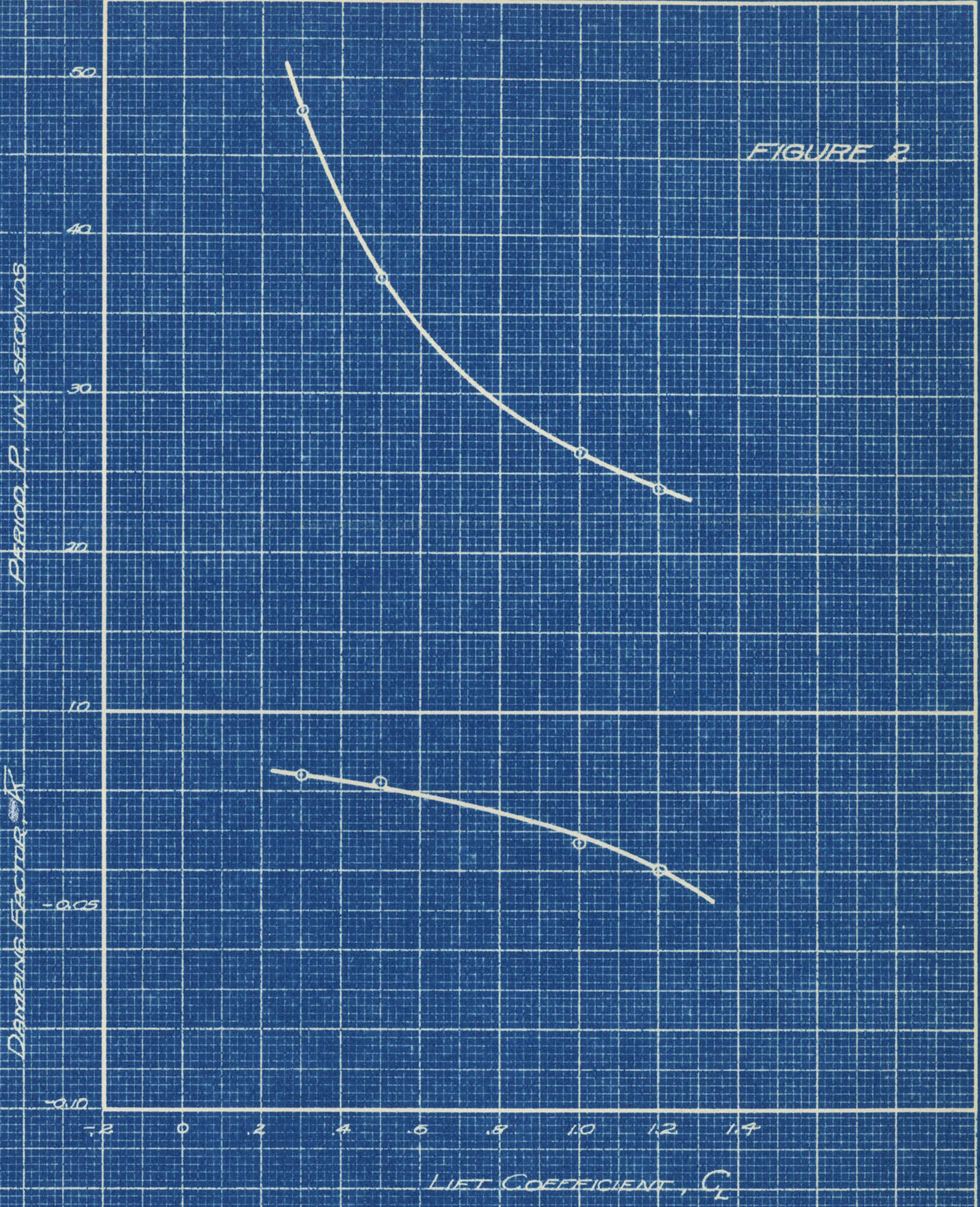
$\mu = 5$			$\mu = 10$		$\mu = 6.65$	
C_L	$\bar{K}$	p	$\bar{K}$	p	$\bar{K}$	p
0.3	.034	47.0	.0335	39.5	.034	44.5
0.5	.052	28.0	.047	24.0	.050	26.7
1.0	.130	14.3	.120	12.0	.126	13.5
1.2	.180	12.0	.160	10.0	.173	11.3

Final Values

C_L	$\bar{K}$	p
0.3	.016	48.0
0.5	.018	37.3
1.0	.033	26.3
1.2	.040	24.0

5. <u>Discussion:</u> The above values of $\bar{K}$ and p are plotted in Figure 2. From actual flight tests, the Douglas Transport proved to be dynamically stable. The data from the Charts indicate a similar result. From an observation at high speed, gliding flight, the period of the phugoid was found to be about fifty(50) seconds. For a $C_L = 0.3$, corresponding to the high speed range, the value for the period is 48.0 sec. which agrees very well with the observed value.

FIGURE 2



VALUES OF DAMPING FACTOR AND PERIOD FOR THE DOUGLAS TRANSPORT, GLIDING FLIGHT

ACKNOWLEDGEMENTS

The author wishes to express sincere appreciation to Dr. C.B. Millikan, Dr. T. von Kårmån, and Dr. W.B. Oswald for assistance and helpful suggestions rendered during the progress of this thesis. He also wishes to express appreciation to Mr. J.E. Lipp for assistance in the construction of the stability diagrams.

SUGGESTIONS FOR FUTURE RESEARCH

The same type of analysis as that appearing in this thesis should be attempted for power-on flight, thus completely covering the range which should be investigated for stability characteristics. Before any degree of success in this further application can be attained, however, there must be more research done on the subject of the effect of the propeller slipstream. An analysis for power-on flight has been made by Gates[(1)], but for the same reason that the present work was done on gliding flight , that of making available modern data, his analysis is a little obsolete for use in modern design.

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(1) Gates, S.B. , A Survey of Longitudinal Stability Below the Stall, with an Abstract for Designers' Use, R. & M. 1118.

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(3) Glauert, H., A Non-dimensional Form of the Stability Equations of an Airplane, R. & M. 1093.

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(5) Hall, S.S. & England, T.H., Aircraft Performance Testing, page 138.