Precision Measurements

in the

Zeeman Effect

Thesis by

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Abstract

Calibration of a solenoid producing magnetic fields up to 7300 gauss has been carried out and a value of K, the field-current ratio determined. This value is K = 36.835 ± .011 gauss/ampere. Fields produced by four measured single layer standard solenoids were used as comparison standards. K was measured both at low currents and at the operating current used for spectroscopic measurements. A small decrease occurs in the value of K as the current increases. Expansion of the solenoid as it is heated by the passing current is largely responsible for this decrease.

Values of e/m have been obtained from the Zeeman separations produced in this magnetic field. Singlet lines of Cd λ6439, of Zn λ6362, of Ne λ5852 and λ6074, and of He λ5015 and λ4921 were used. The Zn and Cd lines required a careful theoretical examination of the g-values of the exciting levels. Ne required an application of the g-sum rule. No correction of g was required for He. Lines were excited in the positive column of a discharge. Liquid air cooling was applied to the He source. Zeeman patterns were photographed with a Fabry- Perot interferometer with the following results:

Cd λ6439: e/m = 1.7570 ± .0007
Zn λ6362: e/m = 1.7570 ± .0008
Ne λ5852 and λ6074: e/m = 1.7580 ± .0014
He λ5015 and λ4921: e/m = 1.7560 ± .0009

These results are stated as

$$e/m = (1.7570 \pm .0007) \times 10^7 \text{ abs. e.m.u. per gram.}$$

I Introduction

The primary purpose of these measurements has been an attempt to settle the controversy that existed regarding the true value of the specific charge of the electron. In 1929, as Birge[1)] pointed out, there existed two different and yet apparently equally acceptable values of e/m. From spectrscopic measurements Babcock [2)] and Houston [3)] obtained values which indicated e/m equals $(1.7606 \pm .0010) \times 10^7$ e.m.u./gm (units and factor 10^7 to be omitted hereafter) while Wolf [4)] by deflecting electrons in a magnetic field obtained $e/m = 1.7679 \pm .0018$. Such a large discrepancy might either be caused by an over estimation of the experimental accuracies or by an actual difference in e/m for bound and for free electrons.

Numerous more refined researches have been undertaken as attempts to decide between the two alternatives. In 1930 there appeared two values obtained by a method which measured electron velocities with oscillating electrostatic fields. When combined with the measured potential drop which the electrons had experienced, Perry and Chaffee [5)] obtained $e/m = 1.7610 \pm .0010$ and Kirchner [6)] obtained $e/m = 1.7602 \pm .0025$. In 1932 Campbell and Houston [7)] published the value $e/m = 1.7579 \pm .0025$ as obtained by Zeeman measurements on Zn and Cd singlet lines. Essentially the same method and apparatus were used as is to be described. The value being lower than any previous reliable value and yet not sufficiently accurate to decide whether the discrepancy was real or accidental, Professor Houston decided that it would be advise-

1) References are listed at the end.

able to repeat the problem with the aim of obtaining as accurate a value as was compatible with the inherent precision of the apparatus.

Recently various other methods have been applied and new values of e/m obtained. Dunnington [8] by a new oscillating electric fields and magnetic deflection metod obtained $e/m = 1.7571 \pm .0015$. Kirchner [9] has obtained an improved value of e/m by his method which equals $1.7585 \pm .0013$. From differences in $\nu(H_\alpha^1)$ and $\nu(H_\alpha^2)$ Gibbs and Williams [10] obtained $e/m = 1.757 \pm .001$. The results obtained from Zeeman measurements in Zn and Cd were recently published by Kinsler and Houston [11] and are $e/m = 1.7570 \pm .0010$.

A solenoid whose field is uniform, measureable, and capable of being controlled to the desired degree of precision is an essential piece of apparatus required for making precision measurements in the Zeeman effect. The solenoid used was the one described by Campbell and Houston [7&12]. It was built to produce the maximum field of the required uniformity from the available power supply. A Fabry-Perot interferometer was found to give sufficient resolution to obtain precise measurements of the longitudinal Zeeman splitting of the two σ components of the measured singlet lines.

The value which has been obtained is

$$e/m = (1.7570 \pm .0007) \times 10^7 \text{ e.m.u./gm}$$

II Calibration of the Magnetic Field

Solenoid

The details of construction of the air-core solenoid have been completely described else where 7,12). It will be sufficient to repeat only those facts which pertain to the understanding of its use and calibration. The coil is contained in and insulated from a brass container consisting of an inner brass tube of 6.3 cm inside diameter and an outer tube of 40 cm diameter. The tubes are 93 cm long and are closed at the ends with cast brass plates. The coil consists of 2449 turns of No.4 B.S. copper wire and has a resistance of 1.27 ohms at 20 ^{o} C.

The solenoid is cooled by pumping kerosene through it at a rate of some 200 liters per minute. Cooling of the kerosene in turn is obtained by passage through eight automobiles radiators which are continuously cooled with running water. The temperature of the solenoid is measured by a thermometer placed in thermal contact with the outer shell of the solenoid. The insulation resistance of the kerosene between the brass shell and the windings was measured from time to time and always found to be greater than 10^5 ohms. Comparing this resistance with that of the coil one sees that leakage of current is negligible.

Two compound generators connected in series, supplied the solenoid with an operating current of 190 amperes at 275 volts. The field circuit of one of the generators is controlled by a rheostat placed near the solenoid and thus the solenoid current can be regulated by varying the impressed voltage. Fifty-three kilowatts are required to produce and maintain the maximum operating field of 7000 gauss. This power is dissipated without the temperature of the outer shell exceeding 40°C.

The intensity of the magnetic field in absolute gauss is determined from the relation

1) $H = KI = KRP$

where K is an experimentally determined constant and I is the current in int.amperes as obtained from the potential P measured by a potentiometer across the terminals of a .001 ohm standard resistance R through which the current flows. The constant K has been determined from direct comparison of the magnets field with the fields produced by long single layer solenoids, whose magnetic constants can be computed from their turn densities and dimensions. Calibrations have been made at two separate periods more than a year apart. All of the spectroscopic measurements on Zn, Cd, and Ne were made between these calibrations so as to insure the detection of any change in K if it should take place during the actual use of the solenoid.

Standard Solenoids

Four different standard solenoids were used in the calibration. Two of these consisted of single layers of No.12 bare copper wire wound on linen bakelite tubes which had been threaded to take 10 turns / inch. Linen bakelite was used because it is known to be non-ferromagnetic. The other two were wound with No.20 enameled copper wire on tubes threaded for 28 turns/inch. One of these was wound on linen bakelite and the other on a brass tube upon which a layer of insulating varnish had been baked. All are of such a size that they can be slipped inside the inner tube of the large solenoid and were insulated from it throughout their entire length by a layer of empire cloth.

The number of turns per cm was determined by measurements with the glass scale, of cathetometer BL 8166, used in such a manner that coincidences were observed between the edges of turns and marks on the scale. Turn densities in the various intervals were then evaluated and averaged over all intervals. No appreciable deviation from uniformity was observed. However, as a precaution against a deviation existing in the value at the center of the solenoid as compared with the average over the entire length, the intervals were chosen so as to give progressively more weight to the center. The coherence of the result obtained upon comparison of the results of the four individual solenoids indicates the improbability of any one having a large error. Randomness in the turn density fluctuations also tends to reduce the size of any error that might arrise from non-uniformity. The glass scale was first calibrated by one person against a Gaertner type M1001 standard meter at Pomona College and later by another person against a glass decimeter scale belonging to the Mt. Wilson Observatory which in turn had been calibrated at the Bureau of Standards. By use of a traveling microscope with a micrometer eyepiece it was possible to investigate the scale throughout its entire length. Both calibrations led to the identical result that the scale is uniformly .032% too long at 21°C. This correction has been applied to all measurements made with the scale.

Bakelite solenoids have the advantage of ease in avoiding current leakage between the winding and the tube but they have the disadvantage of expanding and contracting with the season of the year. This is presumably caused by the change in humidity which results in a change in the moisture content of the bakelite. The truth of this explanation is supported by noticing in Table I that all three bakelite solenoids seem to expand and contract together at much the same rate. Each solenoid was measured before and after use so as to insure use of the appropriate turn density. The turn density of the brass solenoid showed no measurable change and after careful winding no leakage was detectable between the winding and the core.

The standard solenoids were usually used at a temperature slightly different from that at which they were measured. Consequently, it was necessary to actually measure the temperature and to maintain it fairly constant during a series of readings. A temperature coefficient of expansion of 2×10^{-5} was applied to all of the solenoids. This coefficient is near to those of brass, copper, and bakelite. Validity of this correction is apparent from the greater coherence of measurements, made over a temperature range of 30°C, when it is applied.

Table I gives the data on the solenoids. The constants K_s of these solenoids are determined from the relation

2) $$K_s = 0.4\pi n \cos\alpha \cos\varphi$$

where n is the number of turns per cm, α is the angle subtended at the center by the radius of the end winding, and φ is the angle of pitch. Errors given are the average deviation errors. Just the average error will be computed and carried over throughout this article. The average error has been computed instead of the mean deviation error which is obtained from it by division of it with the square root of the number of observations. This has been done in an attempt to allow for the unpredictable residual errors. Given errors represent uncertainties in the last place of the given values.

Null Method Calibration

The first method applied for comparing the constants of the standard solenoids with that of the large solenoid was a null balancing method. Because of the limitation in the strength of fields producible by the standard solenoids this method is only applicable to measurements of K when the current I is less than one ampere. It is not desireable to place too much confidence in the constancy of K over a wide range of currents because the change in temperature, the effect of the external surroundings, and the internal electro-magnetic force distortions will probably affect its value. Consequently, an additional method was used which permitted an evaluation of K at all currents.

Table I

a. Data on the standard solenoids

Solenoid	Effective diam. in cm.	Length	$0.4\pi\cos\alpha\cos\varphi$
Bakelite No. 1	6.00	89.7	1.25371 ± 3
Bakelite No. 2	5.97	89.5	1.25374 ± 1.5
Bakelite No.3	5.92	90.4	1.25395 ± 2
Brass	5.86	90.0	1.25397 ± 2

b. Measurement of Turn Density

Solenoid	Date	Readings	Turns per cm.	K_s
Bakelite No. 1	12/21/31	40	3.9326 ±4	4.9303 ±5
	1/29/32	30	3.9340 ± 3	4.9321 ±4
Bakelite No.2	12/21/31	50	3.9389 ± 2	4.9384 ± 3
	1/10/32	44	3.9403 ±7	4.9401 ±9
	6/20/32	60	3.9353 ±2	4.9338 ±3
	3/3/33	18	3.9406 ±3	4.9405 ±4
Bakelite No. 3	12/23/31	30	11.0295 ±15	13.8304 ±19
	2/4/32	34	11.0322 ±8	13.8338 ±11
Brass	12/23/31	30	11.0220 ±10	
	2/4/32	30	11.0232 ±5	13.8224 ±10
	2/1/33	30	11.0235 ±10	

The arrangement of the apparatus used for determining the ratio between the constant of the large solenoid S and that of a standard solenoid is shown in Figure 1.

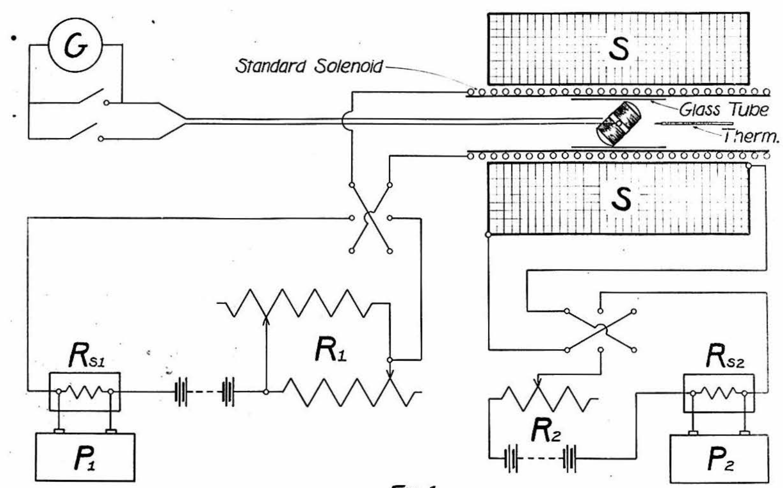


FIG.1

The ratio of the currents in the two solenoids with their fields in opposition was varied until they balanced as was indicated by the wall-type ballistic galvanometer G experiencing no deflection when the flip coil is rotated through 180 degrees. Then

3) $K = K_s(I_s/ I)$

where the subscript s refers to the standard solenoid and I_s/I is the ratio of currents for which the opposing magnetic fields balance. In actual practice the method consisted of reading a series of small deflections as the current in the standard solenoid was varied by changing R_1 and then another series upon reversal of both currents. When the two sets of current ratio - deflection curves are plotted they intersect at the true balance point and eliminate the effect of the earth's magnetic field.

Since the currents were measured by means of the potentiometers P_1 and P_2 across the terminals of the standard resistances R_{s1} and R_{s2}, the actual observed ratio of balance was that of P_s/P where P refers to potential in volts and the subscript s, to the standard solenoid. Since

4) $$I_s/I = (P_s/P)x(R_s/R_s)$$

it was necessary to know accurately the ratio of the various standard resistances. Table II contains the adopted values of standard resistances as used in these calculations. Determinations of their values will be discussed later.

Table II

Values of standard resistances

Nominal value	Current (amp)	Bur. Stds. value	Value rel. 10 ohm	Adopted value
10.0		10.000	10.000	10.000
1.0		1.0000	1.00005	1.0000
0.1	1.5	.10004	.10005	.100045
	15	.10003		.10003
.01	1	[illegible]	.010002	.010002
	15		.010000	.010000
.001	60	.0010003		.0010003
	190		.00099996	.00099996
	300	.0009994		.0009994

Three flip coils were used. One contained 50,000 turns of No.40 B.and S. enameled copper wire while the others were wound with 4000 and 7000 turns respectively. Table III contains the original measurements, corrections, and corrected results of each of six sets of observations made. The column under P_s/P consists of the observed potential ratio at balance as obtained from the graph. Column T_s contains the observed temperature of the standard solenoid. R/R_s contains the appropriate ratio of shunts as obtained from Table II. Column I_s/I contains the corrected current ratio after temperature and shunt corrections have been applied.

Table IV contains the final results of all the null method measurements. The errors given are in each case average deviations. In cases where both the external deviation (R_e) and the internal deviations (R_i) are calculable the larger of the two is always used. Because of an uncertainty in the value of the .01 ohm standard shunt in some of the measurements, the average deviation error has been increased by .005% in such cases. The results indicate that the four different standard solenoids are equally reliable since they lead to coherent results and that the large solenoid did not change with time. An estimate of the general reliability of the equipment is also obtained. However, the chief purpose of this method is to compare the obtained value of $K = 36.872 \pm 5$, with the value obtained by the second method at corresponding currents as a check on the reliability of the latter method.

Table III

Null method calibration

Bakelite Solenoid No.1
50000 turn flip coil
12/23/31

P_s/P	T_s	R/R_s	I_s/I
.074796	30°C		7.4770
797		1.0000	771
790	28	.0100015	767
793			770
787	26		767
795		"	775
796	31		768
794			766
793	32		763
791			761
			7.4768±3

Bakelite Solenoid No. 2
50000 turn flip coil
12/23/31

P_s/P	T_s	R/R_s	I_s/I
.074671	26°C		7.4653
685		1.0000	667
663	24	.0100015	649
677			663
694	28		674
695		"	675
693	30		668
694			669
688	30		663
691			666
			7.4665±5

Bakelite Solenoid No.3
50000 turn flip coil
12/24/31

P_s/P	T_s	R/R_s	I_s/I
.026674	38°C		2.6660
674		1.0000	660
668	33	.0100015	658
668			658
0.26674	30	1.0000	659
674		.10004	659
672	30		657
672		"	657
675	30		660
674			659
			2.6659±2

Brass Solenoid
7000 turn flip coil
12/25/31

P_s/P	T_s	R/R_s	I_s/I
0.26692	26°C		2.6680
695		1.0000	683
684	25	.10004	672
688			676
.026691	31	1.0000	682
693		.0100015	684
683	35		673
687			677
			2.6679±3

Table III (cont)

Bakelite Solenoid No.2 7000 turn flip coil 6/28/32				Brass Solenoid 4000 turn flip coil 12/5/32			
P_s/P	T_s	R/R_s	I_s/I	P_s/P	T_s	R/R_s	I_s/I
0.74765	28°C		7.4726	0.26692	24°C		2.6677
767		1.0000	728	698		1.0000	683
769	27	.100045	731	694	25	.10005	679
771			733	695			680
.074754	26	1.0000	739	692	24		679
751		.0100015	735	691			678
751	25		736	697	33		679
756			741	697			679
			7.4733±4				2.6679±1

Table IV

Results of null method calibration

Solenoid	Date	Mean I_s/I	K_s	K
Bakelite No.1	12/23/31	7.4768 ±7	4.9303	36.863 ±4
Bakelite No.2	12/23/31	7.4665 ±9	4.9384	36.873 ±5
	6 /28/32	7.4733 ±8	4.9338	36.871 ±4.5
Bakelite No.3	12/24/31	2.6659 ±3	13.8304	36.870 ±6
Brass	12/25/31	2.6679 ±4	13.8224	36.877 ±5
	12/3 /32	2.6679 ±2	13.8224	36.877 ±4
			Mean	36.872 ± 5
				$(R_e/R_i = .80)$

Calibration with a Mutual Inductance

The arrangement used to determine the solenoid constant K under the actual conditions prevailing during a spectroscopic exposure is shown in Figure 2.

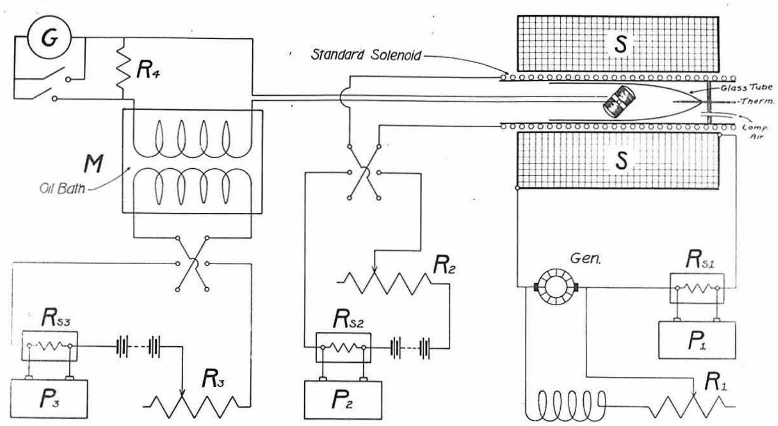


FIG. 2

Resistance R_4 was adjusted so that nearly full scale deflection of the galvanometer G was obtained when the flip coil was operated in the field of the standard solenoid. The current in the primary of mutual inductance M was then varied until its reversal by means of a switch gave a deflection equal to that produced by the flip coil. Assuming the resistance of the galvanometer circuit and the constant of the galvanometer to remain fixed, this gave calibration of the mutual inductance and the flip coil in terms of the standard solenoid constant and the ratio of the two currents.

Using the notation

K_s - constant of the standard solenoid (gauss/ampere)

I_s' - current in the standard solenoid (amperes)

F - magnetic area of flip coil

M - value of mutual inductance

I_m' - current in inductance primary which gave the same deflection as I_s' in standard solenoid.

we have

5) $2\ F\ H_s = 2\ M\ I_m'$

but $H_s = K_s\ I_s$ and therefore

6) $F\ K_s\ I_s' = M\ I_m'$ or $M/F = K_s\ (I_s'/\ I_m')$

Having calibrated the mutual inductance in this manner it could then be used to calibrate the large solenoid. The required data was obtained by decreasing the galvanometer shunt R_4 until the galvanometer gave almost full scale deflection when the coil was turned over in the field of the large solenoid and then increasing the current in the primary of the mutual until upon its reversal one obtained the same deflection. From considerations similar to those leading to equation (6) one obtains

7) $K = M/F\ (I_m'\ /\ I)$

where I_m and I are respectively the currents in the inductance primary and the large solenoid leading to the same galvanometer deflection. Combining with equation (6) one obtains

8) $K = K_s\ (I_s'\ /\ I_m')(I_m/\ I)$

Equations connecting change in magnetic flux $\Delta\varphi$ in the galvanometer circuit with the galvanometer deflection θ are

$$9)\qquad Q = \frac{\Delta\varphi}{10^8\left(R_c + R_m + \frac{R_4 R_g}{R_4 + R_g}\right)} \cdot \frac{R_4}{R_4 + R_g}$$

and

$$10)\qquad Q = C\theta$$

where Q is the charge passing through the galvanometer, R_g is the resistance of the galvanometer, R_c is the resistance of the flip coil, R_m is the resistance of the mutual inductance secondary, R_4 is the resistance of the galvanometer shunt, and C is the galvanometer constant in coulombs/radian. In addition to the resistances and C the constancy of F and M must be considered inorder to estimate the validity and accuracy of the method.

(a) <u>Galvanometer constant C</u>

C must not change duming the calibration of the inductance with any of the solenoids. Although C is not strictly a constant throughout the entire range of deflections it is nearly so for any narrow range of deflection values. In the actual measurements, flip coil deflections were interspersed between inductance deflections over a narrow range of values and the current ratio of corresponding deflections, (I_m/I) or (I_s'/I_m'), obtained by graphical interpolation. This serves to eliminate both the error of changes in C and that arrising from the non-linearity of the gharge-deflection graph. If the deflections were of the same direction and magnitude and if the galvanometer circuit was well insulated so that the zero did not drift, the deflections could be repeated to within the measurable accuracy of .02%. The galvanometer used was a Leeds and Northup Type P ballistic galvanometer placed 40 feet from the large solenoid. The magnetic field of the latter was found to affect the galvanometer constant by disturbing the permanent magnets of the galvanometer when it was closer than 15 feet to the magnet.

(b) Resistances in the galvanometer circuit.

These resistances must remain constant in any one calibration run to within .03% for otherwise the individual galvanometer readings will neither be reproducible nor will the deflection-current graph line have a repeatable location and slope. It will be shifted and rotated by relatively the same amount that the resistances change. As can be seen from equation (9) the deflection changes approximately inversely with the sum of $(R_c + R_m + R_4)$ and directly with R_4/R_g. Other terms are neglected because R_4 is small. Consequently, the temperature of the resistances must be controlled. The values of the various resistances were usually R_c = 1319 ohms, R_g = 2046 ohms, R_m = 9475 ohms, and R_4 = 50 ohms when calibrating the large solenoid at high currents.

Although the resistance of the flip coil was relatively small as compared with the total resistance, it needed to be controlled because its temperature changed by as much as 25°C.

This resulted in a change in its resistance of about 125 ohms which corresponds to a change of about 1% in the resistamce of the entire circuit. The flip coil absorbed heat from the surrounding solenoids. Finding it impossible to eliminate this absorption by surrounding the coil with a double-walled evacuated silvered glass tube, it consequently became necessary to bring the coil to some equilibrium temperature and hold it there. Such a state was attained by surrounding the coil with a glass tube and controlling the temperature by blowing a current of air along the outside of this tube. With this arrangement, temperature equilibrium was attained at 30°C in about 10 minutes and was maintained indefinitely.

Control of the secondary resistance was important both because it was the predominate resistance and because it absorbed heat from the primary of the mutual inductance. A change of 1°C in its temperature changed the resistance of the circuit by 37 ohms which corresponds to .25% of the entire resistance. This was reduced to a negligible amount by immersing the whole mutual inductance in a bath of transformer oil.

Changes in R_4 and R_g had no apparent effect since they change together while remaining at room temperature and thus R_4/R_g remained constant. The above precautions reduced the change of resistance during the time of a series of measurements to less than .03%.

(c) Magnetic area F of flip coil.

This must remain the same throughout all measurements. Two small flip coils were used and both led to the same result which indicates the probability that neither had a constant error of any considerable magnitude. Both were strongly built and operated by springs so as to insure a constant rotation through 180 degrees. Each layer of the coil wire was shellaced to prevent leakage between turns and the frame holding the coil was insulated from the surrounding solenoid with a glass tube. Both coils were wound on redmanol spools and all parts of each coil and its supporting frame were made from materials of non-ferromagnetic nature. Temperature effects on the value of F were negligible because of the small expansion coefficient of redmanol.

(d) Constancy of the Mutual Inductance M.

The value of M must remain constant over a wide range of primary currents. It was necessary to change this current by a factor of 200. Two inductances were used in two different locations. Each was located midway between the floor and the ceiling of a nearby room and as far as possible from all visible ferromagnetic material. One consisted of a low resistance (2 ohms) primary of No.12 S.C.C. copper wire wound on a micarta tube 8 inches in diameter and 15 inches long. The secondary consisted of 12 lbs. of No.32 S.C.C. enameled copper wire wound in three coils and rigidly supported within the primary. The second was made from the first by enlargement of both the primary and secondary.

As a check upon the constancy of M, the deflections resulting from the reversal in the primary of currents differing by a factor of over 100 and in the same range as used in the solenoid calibration, was compared with the steady deflections due to constant currents through the galvanometer. The arrangement used is that shown in Figure 3.

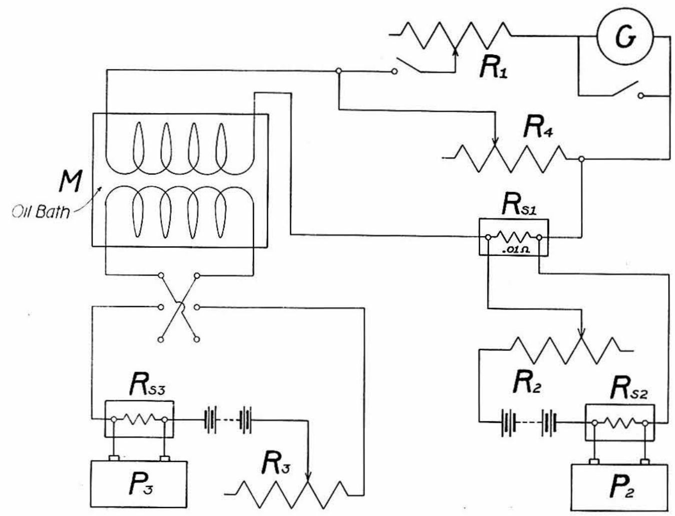


FIG. 3

The most general expression for the current in the galvanometer G is

$$11) \qquad i = \frac{E - L\frac{di}{dt} - \frac{C}{10^8}\frac{d\theta}{dt}}{r}$$

where

E - applied voltage

L - self inductance of the circuit

C - torque in(dyne cm/ amp) produced by the reaction of the current in the galv. coil with the field of its permanent magnets.

θ - angle of galv. coil relative to zero position.

$$r = (R_g + R_1) + \frac{R_m \cdot R_4}{R_m + R_4}$$

The equation of motion of the galvanometer coil is

$$\text{12)} \quad I \frac{d^2\theta}{dt^2} + k \frac{d\theta}{dt} + \tau\theta = C\,i$$

where

k - damping constant on open circuit

I - moment of inertia of coil

τ - galv. suspension constant in (dyne cm/radian)

Substituting i from (11) into (12) and simplifying one obtains

$$\text{13)} \quad \left(I - \frac{C^2 L}{10^8 r^2}\right) \frac{d^2\theta}{dt^2} + \left(\frac{C^2}{10^8 r} + k\right) \frac{d\theta}{dt} + \tau\theta = \frac{EC}{r}$$

Since the galvanometer was used in a critically damped state, (i.e.) just becoming non-oscillatory, one obtains from the "auxiliary equation" of the above second order differential equation

$$\text{14)} \quad \left(\frac{C^2}{10^8 r} + k\right) = 4\tau I$$

as the condition for critical damping. The term containing L has been discarded for simplicity since subsequent calculations and observations showed it to be negligible when compared with I. Measurement showed the critical damping resistance r = 12,200 ohms.

The relation between the final steady current i_f through the galvanometer and its final deflection θ_f is

15) $i_f = \gamma \theta_f$

From Kirchoff's Laws of branched currents it is possible to show that a current I_r in the circuit $(R_2 - R_{s2} - R_{s1})$ will produce a current

16) $$i_f = \frac{R_{s1}}{R} \cdot \frac{R_4}{(R_1 + R_g) + R_4} \cdot I_r$$

where

$$R = R_m + \frac{(R_1 + R_g) R_4}{(R_1 + R_g) + R_4}$$

and

R_{s1} = .01 ohm standard resistance.

Combining (15) and (16) there results

17) $$I_r = \frac{\{(R_1 + R_G) + R_4\}}{R_4 \cdot R_{s1}} \cdot \frac{\gamma}{C} \theta_f$$

as the relation between the current measured by the potentiometer P_2 and the deflection of the galvanometer G. By measurement of the current sensitivity and of the logarithmic decrement and period on open circuit it was possible to evaluate the galvanometer constants C, k, γ and I. From these constants it was calculated that for a steady current and with the galvanometer critically damped the deflection should be within .01% of its final value 52 seconds after the current I_r was made. Observation gave a value of 55 seconds. Readings were always taken 100 seconds after closing the circuit which made it a certainty that the deflection had reached its maximum value.

The other required measurement is that of the deflection caused by the instantaneous current that passes through the galvanometer upon reversing the current in the primary of the mutual inductance M. When a current passes instantaneously through a ballistic galvanometer one has the equation

18) $$C\int i\ dt = I\int \frac{d^2\theta}{dt^2}\cdot dt$$

Integrating each side

19) $$C\ Q = I(d\theta/dt)_{t=o}$$

where Q is the charge passing, is obtained. The general solution of the motion of a critically damped galvanometer when E = 0 in equation (13) is

20) $$\theta = (A + Bt)\ e^{-\frac{k't}{2I}}$$

where $$k' = \left\{(C^2/\ 10^8 r)+k\right\}$$

In this particular case the initial conditions at t = 0 are $\theta = 0$ and from (19) $d\theta/dt = CQ/I$. Applying these , equation (20) becomes

21) $$\theta = \frac{C\ Q}{I}\ t\ e^{-\frac{k't}{2I}}$$

The time of the first elongation θ_1 is obtained from

22) $$d\theta/dt = 0=(1 - k't/2I)$$

and substituting this value in (21) one obtains

23) $$\theta_1 = 2CQ/k'e$$

But the charge passing through the galvanometer when the inductance current Im is reversed is

$$24)\qquad Q = \frac{2\, I_m M}{R} \; \frac{R_4}{(R_g + R_1) + R_4}$$

Eliminating Q between (23) and (24)

$$25)\qquad I_m = \frac{\{(R_1 + R_G) + R_4\}\, R}{R_4 \cdot 2M} \cdot \frac{k' e}{2C}\, \theta_1$$

is obtained as the relation between the current I_m and its corresponding deflection θ_1. Dividing (25) by (17) one obtains

$$26)\qquad \frac{I_m}{I_r} = \frac{k' e}{4 T M R_{s1}} \; \frac{\theta_1}{\theta_f}$$

If the value of k' is substituted and I'_m/I'_r is the ratio of currents making $\theta_1 = \theta_f$, the final result is

$$27)\qquad M = \frac{e}{4 T R_{s1}} \left(\frac{C^2}{r} + k \right) \left(\frac{I'_r}{I'_m} \right)$$

as the equation for investigating changes of M with the size of the primary current.

The experimental procedure consisted of taking a series of observations of inductance deflections intermingled with steady current deflections at low currents in the mutual, then at high currents, and then again at low currents. By graphical interpolation the ratio (I'_r/I'_m) could then be obtained at both high and low currents. If M does not change these ratios should be the same and thus they are a measure of the constancy of M. The accuracy depends largely upon the constancy of the constant factor in the above equation (27). It was possible to adjust R_1 and R_4 so that both r = 12,200 ohms and a full scale deflection was obtained. This was possible since the two conditions could always be satisfied by the two variables.

Table V contains the results of three sets of measurements.

Table V

Direct current - mutual inductance comparison.

I_m/I_r low cur.	Im/I_r high cur.	Ratio $\frac{low}{high}$
.27726	.27703	1.0008
711	699	004
698	698	000
433	448	.9996
434	449	.9996
435	444	.9997
474	456	1.0009
465	454	004
457	449	003
		1.0002 ±4

These results are not as accurate as they could have been made with a better galvanometer since it was found to be difficult to work rapidly enough so that neither of the galvanometer constants C or τ had time to change. However, they indicate that the value of M is constant to within .03%.

Results of Mutual Inductance Calibration of Large Solenoid.

Tables VI and VII contain the original measurements by which the two mutual inductances were calibrated. The accuracy of these observations is seen to be of the same order as obtained in the null method work. Tables VIII and IX contain the observed values in the calibration of the

Table VI

Calibration of Mutual Inductance No.1   7000 turn flip coil

Solenoid	P_s/P_m	T_s	R_m/R_s	I_s/I_m	$K_s(I_s/I_m)$
Bak.No.1	9.5541	25°C	10.000	955.00	
	572	30	.100035	25	
1/29/32	563	28		18	4710.9 ± 8
	570	40	"	03	
	575	23		35	
	K_s = 4.9321			955.16 ±12	
Bak.No.2	9.5387	29		953.39	
	416	29		69	
1/10/32	383	50	"	18	4709.9 ± 11
	383	27		39	
	395	28		49	
	K_s = 4.9401			953.42 ±13	
Bak.No.3	3.4065	38	10.000	340.38	
	072	38	.100045	45	
2/4/32	079	53		49	4710.1 ±11
	091	53	"	61	
	K_s = 13.8338			340.48 ± 6	
Brass	3.4079	31		340.59	
	106	35		92	
2/4/32	077	34		54	4708.3 ±13
	073	42	"	47	
	081	34		57	
	093	33		71	
	K_s = 13.8224			340.62 ± 10	
				Mean	4709.8 ± 11

(R_e/R_i = .67)

Table VII

Calibration of Mutual Inductance No.2 4000 turn flip coil

Solenoid	P_s/P_m	Temp.	R_m/R_s	I_s/I_m	$K_s(I_m/I_m)$
Brass	9.6896	26 C	10.000	968.38	
	891		.10005	33	
2/10/33	927			69	
	879	26		21	
	860		"	02	
	893	25.5		35	
	921			63	13385.6 ± 25
	904		"	46	
	924	26		66	
	881			23	
	902		"	44	
	903	26		45	
	934			76	
	875		"	17	
	874	26		16	
	K_s = 13.8224			968.40 ± 17	
Bak. No.2	2.7102	22	"	270.87	
	103			88	
3/3/33	1o1	22		86	
	107		"	92	
	104	21.5		89	13383.3 ± 18
	101			86	
	113	21.5	"	98	
	100			85	
	K_s = 4.9405			270.89 ± 3	
				Mean	13384.3 ± 21

$(R_e/R_i = .58)$

Table VIII

Calibration of Large Solenoid with Mutual No. 1 Jan. 1932

Current	P_m/P	Temp.	R/R_m	I_m/I
1 amp.	.078243	22°C	.99999	.0078243
	270	48	10.000	290
	258	22	"	258
	0.78230	49	.100045	290
	240	24	10.000	274
				.0078266 ± 16
15 amp.	.078190	23	.10003	.0078219
	274	23	.99999	301
	255	24.5		285
	266	28	"	302
	192	25.5	"	225
				.0078266 ± 32
150 amp.	.78240	25	.0010001	.0078214
	285	40	.100045	279
	282	40		276
	273	28	"	247
	277	36		266
	252	39		249
	241	34	"	229
	252	36		244
				.0078251 ± 17
200 amp.	.78284	49	.00099996	.0078288
	257	41	.100045	246
	210	37		186
	213	41		203
	262	44		256
				.0078236 ± 33

Table IX

Calibration of Large Solenoid with Mutual No.2 March 1933

Current	P_m/P	Temp	R/R_m	I_m/I
1 amp.	.27540	22°C	.10005	.0027555
	541		10.000	556
	528			543
	531		"	546
	528			543
	525			540
	.027551	"	1.0000	551
	552		10.000	552
	530			530
	526			526
	528			528
	543	"		543
	558		"	558
	549			549
	560			560
	554		"	554
	559	"		559
	550			550
	549		"	549
	536			536
				.0027546 ± 8
5 amp	.27534	22°C	.10004	.0027545
	542		10.000	553
	532			543
	547	"	"	558
	543			554
	536			547
	536	"		547
	540		"	551
	538			549
	546			557
	539	"		550
	540			551
	534			545
	531		"	542
				.0027549 ± 4

Table IX (cont.)

190 amp.	2.7518	30° C	.00099996	.0027525
	521		1.0000	528
	530	38		537
	532			539
	531		"	538
	513	38		520
	522			529
	528		"	535
	516	36.5		522
	518			524
	514		"	520
	511	38		518
	513			520
	522			529
	526	39		533
	518		"	526
	516	39		524
	533			541
	527			535
				.0027529 ± 6

Table X
Results of Mutual Inductance Method

	Mutual No.I $K_s(I_s/I_m) = 4709.8 \pm 11$		Mutual No.2 $K_s(I_s/I_m) = 13384.3 \pm 21$	
Current	I_m/I	K	I_m/I	K
1 amp.	.0078266 ± 16	36.862 ± 9	.0027546±8	36.868±11
5 amp.			.0027549±4	36.872±7
15 amp.	.0078266±32	36.862±15		
150 amp.	.0078251±17	36.855±9		
190 amp.	.0078236±33	36.848±15	.0027529±6	36.846±9
Adopted value of K for operating conditions				36.835±11

large solenoid at various currents in terms of the two inductances. Table X gives the resulting values of K as obtained from the two sets of measurements. The general agreement between the two inductances is a further support of their individual validity, The average value of K at low currents is 36.866 ± 9 which is in satisfactory agreement with that of 36.872 ± 5 obtained by the null method. The average deviation is nearly twice that of the earlier method which is to be expected since each of its two parts has approximately the same error as the former. Because of greater consistency and improved apparatus the results with the second inductance are given twice the weight of those obtained with inductance No.1.

The constant K apparently decreases with increased currents. All the values in Table X are reduced to 22° C. Since it was probable that the temperature of the solenoid was higher than that measured for the brass case, this correction may be larger and consequently would increase the value of K. Recently, support of this hypothesis was obtained by measurement of the resistance of the solenoid coil when at operating temperature and then at room temperature. The respective values of the resistances were 1.274 ohms at room temperature of 22.5° C and 1.42 ohms at an observed operating temperature of 40° C. The change in resistance indicates an internal temperature of 50° C instead of 40° C. However, the Zeeman effect data and the calibration were made under identical conditions and consequently no correction is applied to the value of K used for spectroscopic observations. The adopted value of K given in the table differs from that tabulated for 190 amperes by the amount of this correction and by .005% to change from international to absolute gauss.

Standards and Equipment Used.

All currents were measured with standard resistors and potentiometers. Potentiometers used were one "Queens" BL8922, one "Type K" BL3286,+two "Brooks" BL3251 and EE1502 deflection potentiometers. The "Queens" had been checked against a "Wolf" potentiometer and was used to standarize the others. The slide wire of the "TypeK" was found to read low by .08% at all readings. When it was necessary to use the deflection potentiometers they were used in so far as possible with small deflections.

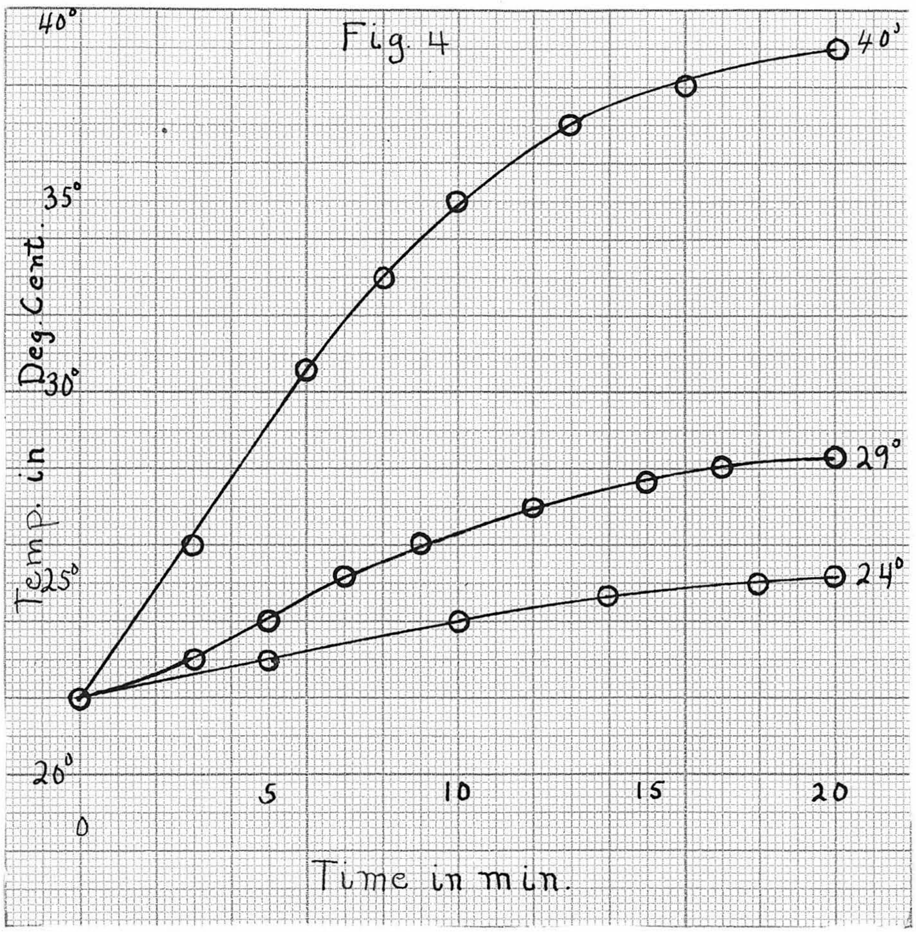
The 1 ohm BL398A and 10 ohm BL397A resistances, of the hermetically sealed in oil Bur.of Stds. type, were newly purchased and have Bur. of Stds. certificates guaranteeing the values given in Table II to within .005%. The respective mean temperature coefficients of resistance are +.000009 and +.000008 per degree centigrade. The ~~point~~ .01 ohm BL395 resistor was of the Reichsanstalt type. The .1 ohm BL390 and the .001 ohm BL389 shunts were Leeds and Northup standard resistors for large currents. They were recently calibrated at the Bureau of Standards for various currents. Being solely interested in the ratios of the resistances, they have been intercompared from time to time. One method consisted in comparing their ratios with a known 10/1 ratio at various currents, and the other a direct comparison by passing the same current through two of them and measuring the potentials across each. The results are tabulated in Table II and have been applied to obtain correct current ratios from observed potential ratios.

The same Weston Standard Cell BL 3285 was used for all readings and thus any variations affected all observed currents to the same extent at any one time and thus had no affect on the ratios. For the past three years it has been compared with newly purchased cells with Bur. of Stds. certificates and has been found to gradually change from 1.01866 to 1.01857 volts. As fluctuations in its value would affect spectroscopic measurements it was compared while subject to room temperature conditions with two thermo-stated cells. Over a three week period the maximum deviation from the mean was .008% and the mean value was 1.01857 $\pm$.00004 int. volts.

Measurement and Control of the Solenoid Current.

In addition to knowing K it is essential to be able to measure and control the current I in the large solenoid inorder to obtain any desired magnetic field strength. Currents were measured by a null balance potentiometer conn-ected across the terminals of the .001 ohm standard shunt.

As may be seen in Table II this shunt's resistance decreases 9 parts in 10000 when the current changes from 60 amperes to 300 amperes. As this change is obviously caused by heating it was necessary to allow the resistance to always operate at the same temperature. Figure 4 gives a graph of its temperature as plotted against the elapsed time since the various currents were started.



If one allows the current to pass for ten minutes before any measurements are made, the maximum remaining possible change is .01% and consequently this procedure has been followed. An estimate of the actual value of the resistance at 190 amperes has been obtained from energy dissipation considerations.

Assume the energy loss equals $L\,(T_s - T_r)$ where T_r is the room temperature and T_s is the resistance temperature. Then equating energy input to output one obtains

28) $$I^2 R = L(T_s - T_r)$$

but

29) $$\Delta R = \alpha \Delta T$$

where α is the resistance temperature coefficient. The result of sustituting ΔT into (28) is

30) $$\alpha I^2 R = L \Delta R$$

From (30) and the values of R at 60 and 300 amperes one may evaluate α and L and then deduce the equation

31) $$\frac{R_I - R_{60}}{R_{300} - R_{60}} = \frac{I^2 - 3600}{90000 - 3600}$$

This equation gives R = .00099996 ohms at 190 amperes. This value of the resistance was applied in obtaining the solenoid constant K. Never-the-less, should there exist an error in the assumed value of the shunt, it would have no influence upon the accuracy of the spectroscopic measurements. The same shunt was used under like conditions and hence its value and likewise any error cancels out in the final result.

Constancy of the current was maintained by manual control of the current in the field coils of the motor-generator set. It was found possible to hold the current steady to within .015%. This error combined with that in K makes the average error in the magnetic field less than .035%.

III Spectroscopic Measurements

Interferometer

Zeeman splitting was observed and measured with use of a Fabry-Perot interferometer. The instrument used was the one previously described in connection with the measurement of hydrogen fine structure,[13] and was placed between the collimator and prism of the spectrograph. The entire spectrograph was mounted on a concrete slab supported by 12 tennis balls and placed two meters from the solenoid. This arrangement was stable. Vibrations in the room resulting from the operation of other equipment did not disturb the interferometer.

The plate surfaces were obtained from evaporation of silver in a vacuum until a density was obtained which gave a reflection of 90 to 95 percent. The process and apparatus used was that developed by Strong [14]. Following this procedure it was relatively easy to obtain plates which showed 35 visible reflections of a 25 watt lamp and yet were sufficiently transparent so that excessive exposure times were not required. An estimation of the resolving power was obtained as half the number of reflections multiplied by the order of interference. By order of interference is meant, the number of wave lengths in twice the distance between the plates. Maximum resolving powers were 2000000 for Zn and Cd, 1500000 for Ne, and 660000 for He. Aluminum and gold surfaces were also tried, howevar, silver was found to produce sharper lines in the region 4900 - 7300 Angstroms. The central 1.5 cm of 2.5 cm diameter mirrors was used.

The interferometer was enclosed in a tight wooden box from which the collimator slit and the camera projected through felt gaskets. The box rested on a felt pad and it was thus possible to remove the box without disturbing the adjustment of the interferometer. Collimator, prism, and interferometer were separately supported and could be independently brought into alignment. Exposures were short, being of from 2 to 10 minutes, and thus thermo-static control of the interferometer temperature was unnecessary.

The Fabry-Perot interferometer is particularly adapted to accurate measurement of the longitudinal Zeeman separation of singlet lines. In the ideal case, only the two displaced σ components are present while the undisplaced π component is missing if one observes parallel to the field. Therefore the dispersion of the instrument may be so adjusted, by altering the distance between the mirrors, so that fringes due to one component lie midway between the adjacent orders of the other. The distance required for even spacing ranged from .39 cm at $\frac{1}{2}$ order of separation, with no overlapping taking place, to 3.5 cm at 4.5 orders separation, when four overlappings of the other component have taken place.

The actual measurement of the splitting $2\,\Delta\nu$, in cm^{-1}, of the two σ components was obtained from the difference in order of their fringe systems at the center of the interferometer. The relation for the fractional order at the center in terms of the diameters of the fringes is

32) $$P = \frac{D_i^2}{D_i^2 - D_{i-1}^2} - i$$

where D_i is the linear diameter of the i-th ring from the center of the pattern 7,12). The diameters of roughly ten fringes of each component were measured on a comparator. From the table of values of D_i^2 an average denominator could then be found. Dividing this into the D_i^2 terms, values of p the fractional part of the order could then be determined and averaged. From the difference in order, $(i+p) - (p'+i')$, between the fringes corresponding to $+\Delta\nu$ and $-\Delta\nu$ in the Zeeman pattern, the separation expressed in cm^{-1} is

33) $$2\,\Delta\nu = \frac{(p+i) - (p'+i')}{2nd}$$

The index of refraction of air n is required because the distance d between the plates was measured in terms of the wave lengths in air of the Ne secondary standard lines as expressed in int. ang. units. Division by n thus reduces $\Delta\nu$ to cm^{-1} in a vacuum as is required. The method of Lord Rayleigh 15) was followed in evaluating d, resulting in an accuracy to within 2 parts in 1000000.

Sources of Radiation

The source of radiation used for Zn, Cd, and Ne was almost of identical design with that described in previous publications.[7,12] Fig.1) The tube was made of Pyrex glass and was operated at a lower temperature than the previous one. This served to reduce the Doppler broadenimg. The discharge was limited to a 6 cm constriction by means of the re-entrant window passed through the copper anode and by an opaque glass stop placed in front of the cathode. The constriction was viewed end on. This design was used so as to make certain that only light reaches the slit of the spectrograph, which has originated in the calibrated section of the magnetic field.

Both the constriction from which the radiation originates and the calibrating flip coils occupied essentially the same space when placed in the solenoid. Therefore, it is unnecessary to apply any correction for the falling off of the magnetic field either radially or longitudinally, since the field measured is that used. At most this correction cannot exceed .005%.

Four 500 volt D.C. generators connected in series produced the current for operating the discharge tube. Control was maintained with a series resistance. At 350 m.a. the Zn and Cd lines were of sufficient intensity to be photographed in three minutes on Wratten and Wainwright Hypersesitive Panchromatic plates. Ne pictures were taken at an operating current of 80 m.a. At this low current the source remained cold. For the Zn and Cd measurements He was continuously circulated through the tube by a two stage diffusion pump. Three liquid air charcoal traps were used for purification of the gas, Ne was circulated alone.

Figure 5 shows the discharge tube used fot excitation of the He spectra.

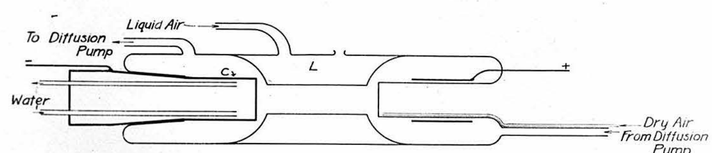


FIG. 5

A water cooled aluminum catode C is fitted into the tube through a ground glass joint. This joint maintains a vacuum indefinitely and requires only a very small amount of stopcock grease. Not enough organic vapors are emitted to introduce unwanted impurities into the discharge. The constriction is cooled by a continuous flow of liquid air into the surrounding jacket L.

Calculations show the effect of the relatively large paramagnetic suseptibility of the liquid oxygen in liquid air is negligible. Even if it were pure oxygen with χ equal to $260 \cdot 10^{-6}$/gm the maximum effect is .02%. Liquid nitrogen, which is diamagnetic, was also used. No measurable difference existed in the results obtained with it as compared with those obtained using liquid air. The suseeptibility of the aluminum used was found to be paramagnetic with $K = 2.2 \cdot 10^{-6}$/cc.

The re-entrant window inside the aluminum anode tubing became cold through radiation and conduction from the liquid air. Moisture was then collected which clouded the window. Blowing dry air upon the window removed this difficulty. The anode end of the tube was painted black with the single exception of the window. Consequently, just light from the constriction reached the spectrograph.

This source was operated at a current of 250 m.a. During a 5 minute exposure less than one lb. of liquid air was required to keep the cooling jacket filled. Liqiud air was blown into the jacket with compressed air, through a double walled siphon. The widths of the He lines were about $\frac{3}{4}$ as great when liquid air cooling was used as compared with the widths when just water cooling was applied. These observed widths indicate about 50% efficiency of cooling. A pressure of approximately 5 mm of Hg was used. Although lower pressures of gas would have led to sharper lines, it was found impossible to operate the source at lower pressures. The magnetic field tended to spiral the electrons and ions into the walls and put out the source.

Equation for e/m

The primary purpose of these measurements has been to determine e/m. The relation between e/m and the Zeeman splitting of the two σ components of a singlet line is

34) $$e/m = 4\pi c/\underline{a}H \cdot \Delta\nu$$

where c is the velocity of light in a vacuum, H is the magnetic field in abs. gauss, $\Delta\nu$ is the displacement of one component in cm^{-1} in a vacuum, and a is a constant approximately equal to one arrising from theoretical considerations of the g- values of the excited levels. For the normal case a is one. In general it varies from element to element and from line to line. For discussion of results H is replaced by its equal KI and the equation written

35) $$e/m = (2\pi c/K) \times (2\,\Delta\nu/I) \times (1/\underline{a})$$

where $2\pi c/K$ is the same in all cases and $2\,\Delta\nu/I$ is the spectroscopically measured ratio of Zeeman splitting to current in the solenoid.

a depends upon the perturbing effect of neighboring configurations upon the two levels from which the observed line arrises. It is directly connected with their g-values. Its definition and method of calculation has been described before.[7,16] Consequently just its value will be calculated and used.

Measurements in Zn and Cd.

Lines employed were $^1P - ^1D$ lines Cd λ6439 and Zn λ6362. Both lines are sharp and are free from hyper-fine structure. The interferometer mirrors were placed at distances such that the two corresponding σ components were separated by from 1.5 to 4.5 orders of interference at maximum field strength. For each interferometer separation a number of exposures were taken. Currents ranged on both sides of even spacing of the fringes. This serves to cancel the effect caused by the apparent shifting of maximum when two broad lines are close together. In no case was a result used if the lines were more than 2% off from even spacing.

Table XI contains the collected data of the measurements of $2\Delta\nu/I$ for the two lines. The first column contains approximate difference in order for each distinct adjustment of distance between the plates. Column 2 contains the number of plates measured at each such separation. The column under $2\Delta\nu/I$ consists of its average value at each separation as obtained from measurement of the plates. The given error is the average error of these measurements. The mean $2\Delta\nu/I$ is obtained by weighting these values according to the number of plates measured. Both the external and internal average deviation error is given for the mean $2\Delta\nu/I$. The small value of R_e/R_i in each case indicates that the largest error is the accidental error arrising from the comparator measurement of the photographic plate. Thus, the assumed average deviation error in $2\,\Delta V/I$ has been chosen nearer to that of the external error.

The constants a are slightly different from those previously published because of a small error in previous calculations. In evaluating $2\pi c/K$, c has been taken as $2.99796 \cdot 10^{10}$ cm/sec. and K is the value for operating conditions as given in Table X. The uncertainties given in the resulting values of e/m are obtained from combination of those of K and $2\,\Delta V/I$. The only remaining possible error is in the value of a. It has been calculated from theoretical considerations and is as accurate, as the applied theory is correct. The increased coherence of Zn and Cd values after a is applied is a support of its validity. The measurements with Ne and He, whose results follow, were undertaken so as to eliminate this uncertainty as a is either 1.000000 or does not enter into consideration.

Table XI

Spectrscopic measurements in Zn and Cd.

Cd			Zn		
Order diff.	No. of plates	$2\,\Delta\nu/I$	Order diff.	No. of plates	$2\,\Delta\nu/I$
3.5	8	.0034346±16	4.5	9	.0034356±15
4.5	6	350±16	3.5	4	352±12
4.5	5	352±24	3.5	3	340±24
4.5	4	346±10	3.5	2	371±40
3.5	1	353±5	4.5	3	352±29
1.5	2	356±5		Mean	.0034354±20 ±8
4.5	3	331±8			(R_e/R_i = .40)
3.5	4	360±11			
	Mean	.0034348±15 ±6			
		(R_e/R_i = .40)			
<u>a</u>	.99978				.999955
$2\,\Delta\nu/I\underline{a}$	.0034356 ± 10				.0034357 ± 12
$2\pi c/K$	51141 ± 15				51141 ± 15
e/m	1.7570 ± 7				1.7570 ± 8

Measurements in Ne

These measurements were made with two purposes in view. One was to obtain another value of e/m; and the other was to investigate the validity of the g-sum rule in a particular case when subjected to measurements of high precision. A determination of e/m from Ne is desirable since no correction factor must be applied and since its spectral type is different from that of Zn and Cd.

The g-sum rule states that the sum of the Lande g-values for all the levels in a specific electronic configuration having the same J-value is the same for all types of coupling and for all magnetic field strengths. Ne lines λ5852 and λ6074 were found to be sufficient to make a precise check of this rule. Table XII contains information regarding these lines.

Table XII

λ5852			λ6074		
$2p^5 3s\ {}^1P_1$	-	$2p^5 3p\ (2p_1)$	$2p^5 3s\ {}^3P_1$	-	$2p^5 3p\ (2p_3)$
g = 1.000		g = 0/0	g = 1.500		g = 0/0
J = 1		J = 0	J = 1		J = 0

The g-values given are those calculated assuming Russell-Saunders coupling. As may be noticed in any table of spectroscopic levels [17] these levels are the only ones in the above electronic configurations with the respective given J-values. Therefore, the g-sum rule may be applied. Assuming no splitting takes place in a magnetic field of the levels with J = 0, the equation for the separation in cm^{-1} of the two σ components if one level has J = 0 and the other J = 1 and g = g is

36) $\Delta\nu = (gH/4\pi c)(e/m)$

Replacing H by its equal KI one obtains

37) $g_1(e/m) = (2\pi c/K)(2\Delta\nu_1/I_1)$

for λ5852 where g_1 refers to the value of g for this line and $2\Delta\nu_1/I_1$ the spectroscopically measured ratio of splitting to current. Similarly

38) $g_2(e/m) = (2\pi c/K)(2\Delta\nu_2/I_2)$

is obtained for λ6074. Applying the g-sum rule

39) $g_1 + g_2 = 1.000 + 1.500 = 2.500$

these equations give

40) $e/m = (2\pi c/2.5K)\left(\frac{2\Delta\nu_1}{I_1} + \frac{2\Delta\nu_2}{I_2}\right)$

as an equation for determining e/m.

Table XIII contains the results of the Ne measurements. The values of g_1 and g_2 have been separately computed by use of equations (37) and (38) and with an assumed value of e/m=1.7570, the generally accepted value at present. Their sum is seen to be 2.5000 to within .07%. Therefore it seems that at least this case the g-sum rule is valid to better than .1%.

The value of e/m has been obtained, after assumption of the validity of the g-sum rule, by means of equation (40).

Table XIII

Measurements in Neon

Order diff.	λ5852 $2\Delta\nu_1/I_1$	Order diff.	λ6074 $2\Delta\nu_2/I_2$
3.5	.0035520	4.5	.0050345
	581		362
	603		405
			336
2.5	556		
	549	3.5	430
	537		415
	570		375
	556		360
	576		372
	583		431
	559		388
	576		405
	528		349
			405
	.0035560 ± 20		336
			.0050381 ± 29

g (e/m = 1.7570)

λ5852 1.0350 ± 7

λ6074 1.4667 ± 9

Σg = 2.5017 ± 16

e/m (eq. 40) = 1.7580 ± 14

Measurements in He

The investigation of He singlet lines was next pursued in an attempt to determine e/m. The theory is completely accurate for He and there are no theoretical correction factors to be applied. He lines $^1S - {^1P}$ λ5015 and λ7281 and $^1D - {^1P}$ λ4921 and λ6678 were investigated.

It was found impossible to measure the original plates with sufficient accuracy. Microphotometer curves were made of the fringe pattern at a (6 to 1) magnification and then the fringe diameters measured from the microphotometer plates. Initial measurements at 1.5 orders separation led to four widely differing results for the four lines investigated. Upon placing the interferometer mirrors at such a distance that no overlapping took place a third component appeared between each pair of corresponding σ fringes.

This component appeared at a position corresponding to that expected for the undisplaced π component. Calculations made from the size of the solid angle subtended at the source, by the condensing lens indicated that its intensity should be .08% of that of either σ component. Intensity marks were placed on a plate by means of the step weakener described by Houston and Hsieh.[18] From the microphotometer of the He lines and the intensity marks it was then possible to determine the relative intensities of the π and σ components. The π component was found to be 3.3% as intense as a σ component. At 1.5

orders separation of corresponding fringes the weak third component is not resolvable. It does ,however, displace the apparent positions of the centers of gravity of the two strong components toward it. More careful alignment of the solenoid axis along the optical path, reduction of the size of the condensing lens, and reducing the amount of reflected light were all found to be ineffectual towards eliminating this spurious component.

Figure 6 is a He photograph at $\frac{1}{2}$ order separation made on a W.and W. Hyper. Panchr. plate. λ6678, on the right, shows faintly the third component. Zeeman components belonging to the same order are bracketed in all figures. Fig. 7 is a He photograph at 1.5 orders separation made on an Eastman #33 plate. λ5015 is shown on the right. Fig. 8 shows a microphotometer curve of λ6678 at $\frac{1}{2}$ order separation. The π component is very noticeable. Fig. 9 is a microphotometer curve of λ5015 at 1.5 orders. Although the π component is not resolved its presence is made apparent by the alternation in intensities of the minima. All these pictures were taken at currents which corresponded to even spacing of the σ components.

Being unable to eliminate the π component it thus became necessary to correct for its influence upon the positions of the true centers of gravity of the adjacent σ components. The apparent distance between two fringes was taken as that

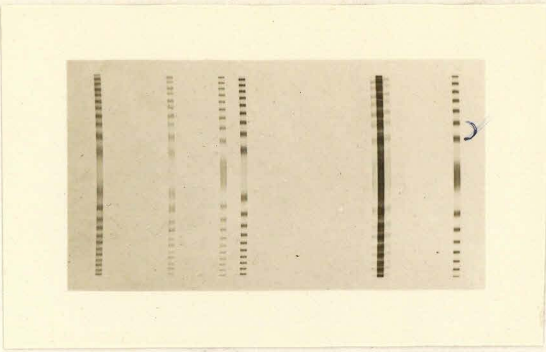


Fig. 6

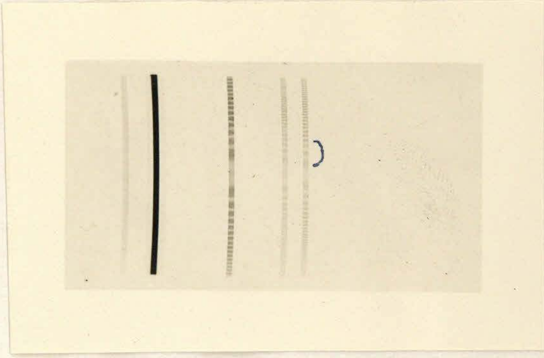


Fig. 7

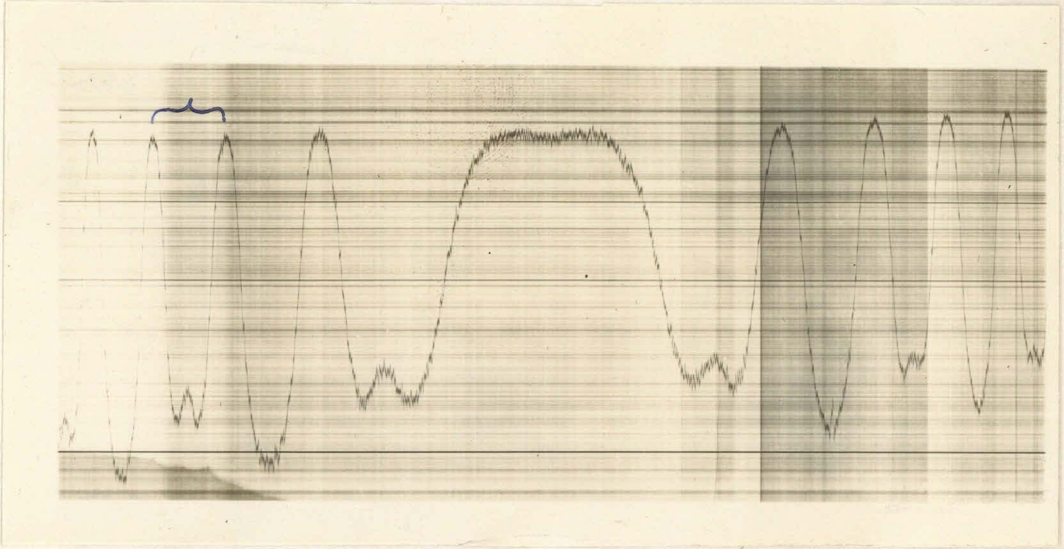


Fig. 8

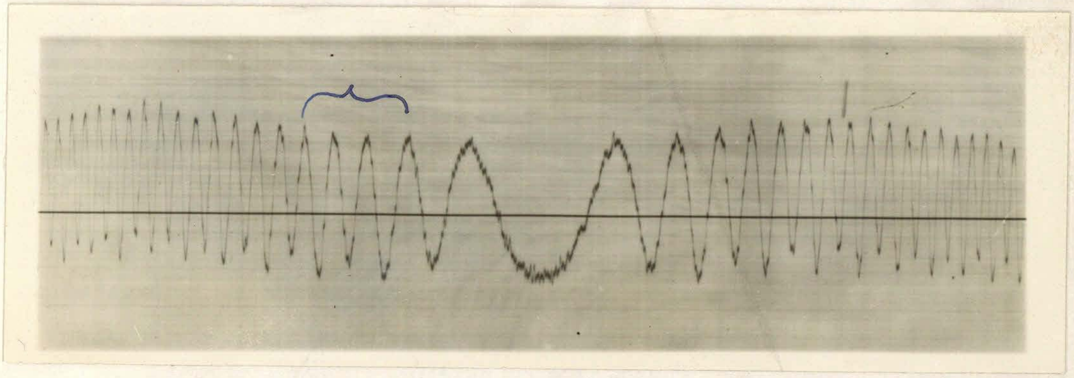


Fig. 9

between the points obtained by estimating the positions of their respective centers of gravity, giving weight down to half maximum intensity only.

In deriving the displacement of the center of gravity the following notationmwas used:

2D - distance between fringes

2d - width of a fringe at $\frac{1}{2}$ makimum intensity

A - intensity of a σ component

B - intensity of a π component

x - distance from origin (origin is at the true center of a σ fringe having a π component to the right)

x_c - apparent displacement of σ component

$$I = I_0 e^{-\alpha x^2}$$

The intensity about x = 0 is then approximately

41) $$I = A\left(e^{-\alpha(x+2D)^2} + e^{-\alpha x^2} + e^{-\alpha(x-2D)^2}\right) + B e^{-\alpha(x-D)^2}$$

The equation for the center of gravity under a curve y = f(x) is

42) $$x_c = \frac{\int x\, y\, dx}{\int y\, dx}$$

Thus if the deflection of the microphotometer is assumed proportional to intensity as was nearly the case for the intensities used the displacement of the fringe as reproduced by the microphotometer is

44) $$x_c = \frac{\int I\, x\, dx}{\int I\, dx}$$

Substituting in (44) the value of I from (41) and the limits -d and +d one obtains

$$45)\quad x_c = \frac{B}{A}\,\frac{\int_{-d}^{d} x\, e^{-\alpha(x-D)^2}\,dx}{\int_{-d}^{d} e^{-\alpha x^2}\,dx}$$

Now from the definition of d, $I = \frac{1}{2} I_o$ for $x = d$, and thus we may write

$$46)\quad \tfrac{1}{2} I_o = I_o e^{-\alpha d^2}$$

which results in $\alpha = 0.7/d^2$. Using this and working out the integrals in equation (45) the result is

$$48)\quad x_c = \frac{B}{A}\cdot 2D\,\frac{\left[f\left(\frac{5}{6}\left(1+\frac{D}{d}\right)\right) - f\left(\frac{5}{6}\left(\frac{D}{d}-1\right)\right) - \frac{3}{5}\frac{d}{D}\left(e^{-.7\left(\frac{D}{d}-1\right)^2} - e^{-.7\left(\frac{D}{d}+1\right)^2}\right)\right]}{4 f(.833)}$$

where

$$49)\quad f(x) = \int_0^x e^{-x^2}\,dx$$

f (x) may be looked up in tables. Thus it is possible to obtain the displacement of a σ component x_c , in terms of the distance between fringes 2D, if the ratios B/A and D/d are known.

Table XIV contains data on the derived corrections. The column under D/d contains values as obtained from actual measurement on microphotometer plates of the widths d at $\frac{1}{2}$ maximum intensity. Column 4 contains the line widths $\Delta\nu$ in cm^{-1} at $\frac{1}{2}$ maximum intensity as calculated from the observed D/d. The next column is calculated from equation(48). The percent correction for the fractional order of the displaced fringe is then obtained by assuming A/B equals the observed value of 30. Column 7 contains the percent effect on $2\,\Delta\nu/I$. For 1.5 orders this value is equal to 2/3 that in column 6 and for 1/2 order it is twice, as may easily be seen from considerations of the effect of the change in each d upon the value of $(i_1 + d_1) - (i_2 + d_2)$.

****

Table XIV

Correction factors for He

Order diff.	λ	D/d	$\Delta\nu$ in cm^{-1}	$\frac{x_c}{2D \cdot B/A}$	% cor. to d_i	% cor. $2\Delta\nu/I$
1.5	4921	1.1	.20	.183	-.61	-.41
	5015	1.35	.16	.143	-.48	-.32
	6678	1.6	.135	.098	-.33	-.22
	7281	2.15	.10	.036	-.115	-.077
.5	4921	2.95	.22	.004	+.013	+.026
	5015	3.45	.19	.001	+.003	+.007

Table XV contains the results at 1.5 orders separation for the various lines. Although the apparent accuracy of measurement of $2\Delta\nu/I$ is considerable, too much dependence cannot be placed in these results since an error of 10% in the value of B/A would have a 10% effect on the correction and an error of 10% in the value of D/d would have an even greater effect. Such an error is entirely probable. Results are all high as compared with others obtained. This may be caused by the application of too small a correction factor.

Table XV

He measurements at 1.5 orders

$2\Delta V/I$	λ4921	λ5015	λ6678	λ7281
	.0034534	.0034536	.0034484	.0034425
	513	539	505	412
	534	545	465	415
	506	550		382
	473	517	.0034485±14	450
	440	510		431
	446	485		
		523		.0034419±16
	.0034492±33	536		
		490		
		.0034523±19		
Cor. from Table XIV	-.41%	-.32%	-.22%	-.077%
Cor. $2\Delta V/I$	34355	34412	34409	34392
$2\pi c/K$		51141±15		
e/m	1.7569±18	1.7600±12	1.7597±10	1.7588±11

The line λ7281 was measured on Eastman Spec. Type I-K plates. It required a 20 minute exposure upon being sensitized with an ammonia solution. This line is the most suitable one to be used in obtaining more satisfactory results, since if it could somewhat sharpened the correction factor would be negligible. All these results are too uncertain to be used in the general average of values of e/m.

Measurements of λ5015 and λ4921 at 1/2 order of interference of corresponding fringes are contained in Table XVI. The correction factor is very small and the values of e/m are thus reliable. As only four measurable fringes of each component occur on a plate, it has been necessary to remeasure the plates two or three times so as to obtain an adequate number of individual values of p, the fractional order of interference, inorder that the average will be close to the true value. Thus accidental errors of measurement are largely responsible for any uncertainty in these values of $2\Delta\nu/I$

Table XVI

Measurements in He at 1/2 order separation.

$2\Delta\nu/I$	λ4921	λ5015
	.0034331	.0034311
	396	338
	341	290
	343	324
	317	309
	339	336
		311
	.0034344 ± 17	.0034319 ± 14
Cor. Tb.XIV	+ .026%	+ .007%
Cor. $2\Delta\nu/I$	.0034353 ± 17	.0034322 ±14
$2\pi c/K$	51141 ± 15	51141 ± 15
e/m	1.7568±10	1.7552±9
	Mean e/m =	1.7560 ± 9

The values of e/m as obtained from each of the four elements are

Cd - 1.7570 ± 7

Zn - 1.7570 ± 8

Ne - 1.7580 ± 14

He - 1.7560 ± 9

Av. e/m - 1.7570 ± 7 $(R_e/R_i = .50)$

An average between the external and internal average deviation error has been taken as the final error.

The results of these measurements may be stated as

$$e/m = (1.7570 \pm .0007) \times 10^7 \text{ e.m.u./gm.}$$

This value of e/m is one of the most accurate values that has been obtained and is in excellent agreement with recent other determinations.

The writer wishes to express his gratitude to Professor Houston who originally conceived of this work and whose advice and collaboration greatly facilitated its completion. Professor Bowen and other members of the staff are also thanked for their advice and help. The aid of Dr. Hsieh, Mr. Strong, and Kathleen Kinsler in making experimental measurements is greatly appreciated.

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