

ECONOMIC SPACING OF COUNTERFORTS
IN REINFORCED CONCRETE RETAINING WALLS

Thesis by
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INTRODUCTION

The purpose of this paper is to determine the most economical spacing of counterforts, in reinforced concrete retaining walls of the counterforted type.

Previous attempts in this direction, as for instance by Mr. Paaswell in his " Retaining Walls ", have been based on mathematical formulae. This procedure necessitates the introduction of a great number of simplifying assumptions, such as neglecting the item of reinforcing steel, disregarding possible effects of changes in surcharge ratio, toe projection, and a few more.

It was suggested to the author that the inclusion of all significant variables in this problem, might give results considerably in divergence with current data on this subject. To make a positive check it was decided to make complete designs for a number of different, representative cases, applying only such simplifications as could be shown to be justifiable. The computations were carried out in tabular form, the final table being a compilation of the cost data - including only such items as will vary with changing counterfort spacing and on the basis of which the economic spacing was determined graphically.

The results of the investigation are shown graphically in four sets of diagrams, representing average conditions for two rates of toe projection, and two different shear conditions in the base slab. These four cases represent nearly limiting conditions, allowing the designer to interpolate for other values of toe projection and for intermediate conditions of shear reinforcement in base slab.

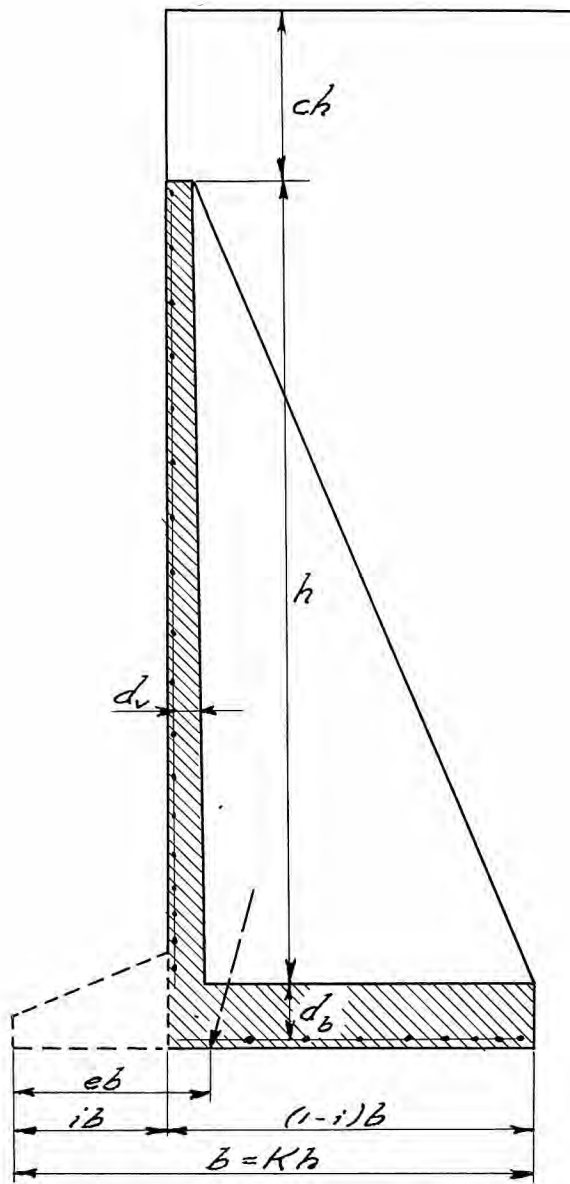
In a separate chapter the effects of changing the basic assumptions have been discussed, and certain rules developed which will permit the use of the curves for other than then used assumptions.

Several interesting discoveries were made, for instance that the rate of surcharge is nearly as important as the height in determining the economic spacing of counterforts, and that a number of other variables usually disregarded, must be included if the results shall be accurate.

It should also be noted that the total cost is not very sensitive to changes in spacing of counterforts, as long as this is reasonably close to the optimum condition. A tolerance of eighteen inches in either direction from the values given in the diagrams, is easily permissible.

The following independent variable factors were considered :

- | | |
|---|--------|
| 1. Height of wall to top of footing | " h " |
| 2. Rate of surcharge | " c " |
| 3. Ratio of toe projection to base width | " i " |
| 4. Thickness of counterforts | " t " |
| 5. Factor of Safety against overturning | " n " |
| 6. Base with or without shear reinforcement | |
| 7. Allowable soil pressure | " S, " |
| 8. Ratio of costs | |



TYPICAL WALL SECTION

NOTATIONS USED

(See sketch on page three)

- A_s Cross-sectional area of steel in square inches.
- b Width of base slab in feet.
- c Rate of surcharge.
- eb. Distance in feet from toe to point of application
of resultant soil pressure.
- f/c Ratio of cost of forms per square foot to cost of
concrete per cubic foot.
- h Height of wall in feet above top of footing.
- i Ratio of width of toe projection to total base width.
- K Ratio of width of base to height of wall above base
slab (b/h).
- M Moment in inch-pounds.
- m Spacing in feet from center to center of counterforts.
- n Factor of safety against overturning.
- R Resultant vertical pressure in pounds per linear foot.
- S Allowable soil pressure in pounds per square foot.
- S₁ Maximum soil pressure (at toe) in pounds per square
foot.
- S₂ Minimum soil pressure (at heel) in pounds per
square foot.
- s/c Ratio of cost of steel (in place) per pound to
cost of concrete (in place) per cubic foot.
- t Thickness of counterforts in inches.
- V Total resultant shear in pounds.
- v Unit shear in pounds per square inch.
- V_s Volume of steel in cubic inches.

COST ANALYSIS

The total cost of a counterforted retaining wall is composed of the following elements :

1. Cost of concrete ϕ in the vertical slab. This varies directly with the distance between counterforts. For practical reasons a minimum thickness of 10 inches was decided on.
2. Concrete in heel portion of the base slab, also varies directly with the spacing of counterforts. Minimum thickness chosen as 16 inches.
3. Concrete in the toe projection of the base slab. This item is entirely independent of the spacing of counterforts, and may be omitted from this consideration.
4. Concrete used as covering for reinforcing steel is independent of the spacing of counterforts. Consequently only the net thickness, d_v and d_b , of the two slabs has been included.
5. The volume of concrete in the counterforts is assumed evenly distributed along the full length of the wall, and is therefore inversely proportional to the spacing. To avoid irregularities in the final diagrams, it was decided to make the counterfort thickness a linear function of the height. The effect of using other thicknesses has been discussed in a separate chapter.
6. Cost of main, horizontal steel in the vertical slab. Increases directly with the distance between counterforts. The positive moment steel is considered continuous throughout

the full length of the wall, and allowance of 40 % in addition to this is made for the negative moment steel. The value used for the moment is $\frac{wm^2}{12}$ for both positive and negative moment, considering the slabs continuous across the counterforts.

7. Cost of main longitudinal steel in the heel portion of the base slab. The same remarks apply as above.
8. The amount of " spacer bars " in both slabs is practically independent of the spacing of counterforts, and is not included in the present investigation.
9. The steel used for reinforcing the toe cantilever of the base slab, is disregarded together with the concrete volume of this part.
10. The main tension steel in the counterforts, which are assumed to be designed as tension braces, when uniformly distributed over the full length of the wall, obviously remains of nearly constant total volume, irrespective of the spacing of counterforts. Omitted.
11. The cost of side forms for the counterforts should be distributed uniformly over the length of the wall, $\frac{1}{z}$ and consequently varies inversely with the spacing of counterforts.
12. All other form work may be considered entirely independent counterfortspacing, and is not included in the following cost analysis.

All these costs are to be taken as actual costs in place, with forms stripped and removed. A Pasadena building contractor was ~~ph~~ consulted on the question of prevailing costs as of today. Complete computations were worked out for other cost ratios. This revealed that variations in the ratio : cost of steel per pound to cost of concrete per cubic foot (s/c) has relatively small effect on the economic spacing, and a constant value corresponding approximately to the present price conditions~~s~~ was decided on.

As for the other cost ratio : form per square foot to concrete per cubic foot, a simple relation was found to exist to the economic spacing. The final diagrams were therefore worked out on the basis of prevailing costs, and an expression developed giving the correction~~t~~ to be applied for other ratios.

It should be noted that the costs compiled in tables 12 are not all inclusive, but that they give the differences in cost for various spacings of counterforts with reasonable accuracy. In order to obtain the actual total cost per linear foot of wall, a certain constant amount must be added to all figures under the same bracket in tables 12, this correction representing those cost elements which could be omitted from the present analysis.

GENERAL ASSUMPTIONS

1. For the purpose of this analysis a rigorously exact design can not well be carried out, and is believed to be unnecessary. The now customary procedure of neglecting the cantilever action in both slabs will be employed, considering the slabs supported at the counterforts only and carried continuously across these. For simplicity the moment, positive as well as negative, has been taken as $\frac{w m^2}{12}$.

The error committed in disregarding the cantilever action at the junction of both slabs, results in somewhat excessive thickness of the vertical slab at the base, and somewhat excessive use of steel in the base slab. Remembering that this analysis is concerned only with differences in units of volume and weight, caused by certain increments of counterfort spacing, one must realize that the error is not serious. On a much reduced scale it is undoubtedly comparable to the effect of employing shear reinforcement in the base slab, shown later to increase the economic spacing of counterforts.

2. The horizontal earth pressure is considered equivalent to the fluid pressure caused by a liquid one third as heavy as the soil retained, which is assumed to weigh 100 pounds per cubic foot.

3. The back of the face slab is taken as vertical.

4. All designs are carried out for an allowable soil pressure

S = 4 tons per square foot.

This value will be reached only at the toe of some of the higher walls, when the factor of safety of n = 2.5 is employed.

The effect of using a lower allowable soil pressure, for instance 3 tons per square foot, proved to be considerable for high walls with small toe projection. For walls 35 feet or less, in height this fact may be safely disregarded, unless the soil is of very much less bearing capacity than the assumed 4 tons per square foot. In such cases the values for economic spacing as given by the diagrams are too low.

This study will only be concerned with the following range of variables:

Height h from 15 ft. to 50 ft.

Surcharge c four values, 0, 0.25, 0.5, and 1.0

Cost ratio f/c ... from $f/c = 0.1$, to $f/c = 0.6$

Toe projection i .. two values, $i = 0.1$ and $i = 0.3$

The following reinforced concrete constants were used :

$f_c = 650$ pounds per square inch
 $f_s = 16,000$ " " "
 $v_c = 40$ " " "
 $n = 15$

COMPUTATION METHODS USED

An explanation of the methods employed in compiling the twelve different tables will now be presented, in each case stating the assumptions underlying, and showing cause for such approximations and simplifications as have been made.

TABLE 1, shows the thickness of the face slab for different values of h , c and m . This slab is assumed to be without shear reinforcement in all cases, and the thickness tabulated is always the higher of the two values given by shear and moment considerations respectively. For all but the very lowest walls, γ shear is found to be the determining factor, when the concrete constants given on page 9 were used.

Formulae will now be developed, giving wall thickness, d_v , as a function of the height, rate of surcharge and spacing of counterforts.

The intensity of horizontal pressure at the base of the vertical slab is :

$$w = 1/3 g.h (1 + c)$$

where " g " is the unit weight of the soil retained, and in all the following work made equal to 100 pounds per cubic foot. (The variations from this value are in all cases undoubtedly small enough to have negligible effect on the economic spacing of counterforts.)

For the determination of face slab thickness the following value of moment was used :

$$\underline{M_{\max.} = \frac{w m^2}{10} ,}$$

The resisting moment for the slab is :

$$\frac{1}{2} f_c \cdot k \cdot j \cdot d_v^2 \cdot l'$$

Equating this to the expression for the bending moment we get :

$$d_v = \sqrt{\frac{2 M}{f_c \cdot k \cdot j}} = \sqrt{\frac{2 \cdot 33.3 h (1 + c) m^2}{10 \cdot 650 \cdot 0.38 \cdot 7/8 \cdot 144}}$$

when we assume :

$$k = 0.38, \text{ and } j = 7/8 .$$

This can be reduced to :

$$d_v = 0.0146 m \sqrt{h (1 + c)} \dots \dots \dots (1)$$

Substituting the four values used for " c " :

- For c = 0 $d_v = 0.0146 m \sqrt{h}$
- " c = 0.25 $d_v = 0.0163 m \sqrt{h}$
- " c = 0.5 $d_v = 0.0179 m \sqrt{h}$
- " c = 1.0 $d_v = 0.0206 m \sqrt{h}$

The shear more frequently governs the thickness of the wall. In computing the shear it will be permissible to ^{deduct} the thickness " t " of the counterfort, from the effective span . As the increments in shear for equal increments in span ^{remain constant}, irrespective of the absolute value of this span, the correct value of the counterfort thickness is not required. This investigation is only concerned with differences in amount of material used.

In the case of the face slab, this deduction, representing thickness of counterforts, was chosen as one foot, while in the case of the base slab it was neglected altogether. The reason for this distinction being that the base thickness, when governed by shear is nearly always well above the practical $\frac{1}{8}$ minimum, while the face slab is not. A slight error is thus introduced in those cases where thickness is determined by minimum requirements.

Horizontal load at base of wall :

$$w = 1/3 \cdot 100 h (1 + c)$$

Shear :

$$V = \frac{w}{2} (m - 1') = \frac{m - 1}{6} 100 h (1 + c)$$

Allowing a unit shear of $v = 40$ pounds per square inch :

$$d_v' = \frac{V}{j \cdot l' \cdot v} = \frac{(m - 1) 100 h (1 + c)}{6 \cdot 7/8 \cdot 40 \cdot 144}$$

or :
$$\underline{d_v' = 0.0033 (m - 1) h (1 + c) \dots\dots\dots (2)}$$

For	c - 0	$d_v' - 0.0033 (m - 1) h$
"	c - 0.25	$d_v' - 0.00414 (m - 1) h$
"	c - 0.5	$d_v' - 0.00496 (m - 1) h$
"	c - 1.0	$d_v' - 0.0066 (m - 1) h$

A minimum thickness of 10" is chosen, or assuming 3" of protective cover outside the steel : $\underline{d_v \text{ min.} = 0.58'}$

Table 1 is compiled on the basis of equations (1) and (2), in all cases giving the highest of the two values.

Table 1 can also be used to determine the thickness at the top of the wall, by substituting the value c . h for h in that part of the table referring to c = 0. For instance :

h = 40, c = 0.5, m = 10 i.e. c . h = 20,

from Table I : d_v = 0.65'

Re TABLE II

The steel area for the top of the wall, considering a horizontal strip 1 foot wide, is found from the familiar reinforced concrete formulae :

$$A_s = \frac{M}{f_s \cdot j \cdot d_v}$$

For balanced reinforcement, j is nearly equal (for the chosen values of f_c and f_s) to 7/8, and the formula reduces to

$$A_s = \frac{M}{16,000 \cdot 7/8 \cdot d_v} = \frac{M}{14,000 d_v}$$

As d_v is given in feet and M in inch-pounds :

$$A_s = \frac{M}{16,000 \cdot 7/8 \cdot 12 d} = \frac{M}{168,000 d \text{ sq. in.}}$$

In the beginning an adjustment was applied to this value, using the correct value of " j ", but it was in every case found small enough to be neglected.

The minimum amount of steel to be used was chosen as :

1/2 inch round bars - 10 inches on centers,
or 0.24 square inches per vertical foot of slab.

Re TABLE III.

The first five columns are similar to table II, but represent the bottom one foot strip of the wall. A_s' is to be understood as the steel area in square inches of the bottom one foot strip, and A_s'' the same area for the top one foot strip, as copied from table II. As a reasonably accurate approximation, the total steel area is next taken as :

$$A_s = \frac{A_s' + A_s''}{2} h$$

Adding forty percent of this for negative moment steel, the total steel volume per linear foot of face slab is :

$$V_s = 1.4 A_s \cdot 12 = 16.8 A_s \text{ cu.in.},$$

and the weight :

$$\underline{0.283 V_s \text{ pounds}}$$

One sheet was worked out for each value of " h ", but only one of these - representing h = 25 ft. - was copied.

Re TABLE IV.

Consists of two sheets, one for toe projection $i = 0.1$, the other for $i = 0.3$. The ration K between the height and the base width, was found on the basis of a factor of safety against overturning of $n = 2.5$, and a maximum allowable soil pressure of $S = 4$ tons per square foot.

Paaswell, in his " Retaining Walls ", develops the following formula for K in terms of c , n and i :

$$K = \sqrt{\frac{0.33 (1 + 3c) n^2}{3 (1 + c) (1 - i^2)}} \dots \dots \dots (3)$$

$$\text{For } n = 2.5, \quad i = 0.3 : \quad K = 0.548 \sqrt{\frac{1+3c}{1+c}}$$

$$\text{For } n = 2.5, \quad i = 0.1 : \quad K = 0.530 \sqrt{\frac{1+3c}{1+c}},$$

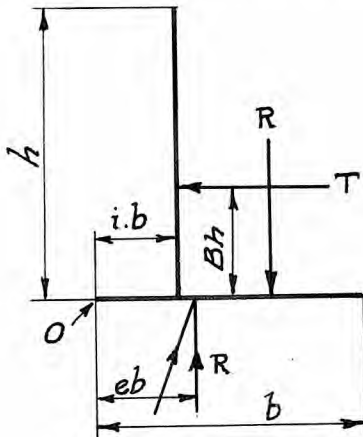
or K should be nt less than :

	c	0	0.25	0.50	1.00
i = 0;3 K	0.55	0.65	0.708	0.776	
i = 0.1 K	0.531	0.628	0.685	0.750	

The total weight of the soil resting on the heel slab is :

$$\frac{100 h (1+c) (1-i) b}{}$$

The distance " e " from the toe to the point of application of the resultant earth pressure may be found as follows :



$$R = g h (1+c) b$$

$$T = g h^2 J \frac{1+2c}{2} \quad (\text{Paaswell})$$

$$B = \frac{1}{3} \frac{1+3c}{1+2c} \quad (\text{Paaswell})$$

Moment about O :

$$R (ib + b \frac{1-i}{2}) - TBh - Re b = 0$$

or :

$$0 = g h (1+c) (1-i) b^2 \frac{1+i}{2} - g h^2 \frac{1}{3} \frac{1+2c}{2} \frac{1}{3} \frac{1+3c}{1+2c} h$$

$$- g h (1+c) (1-i) b^2 e$$

or :

$$0 = \frac{(1+c)(1-i^2)b^2}{2} - \frac{h^2}{18}(1+3c) - (1+c)(1-i^2)b^2 e$$

Or solving for "e" :

$$e = \frac{(1+c)(1-i^2)b^2/2 - h^2/18(1+3c)}{(1+c)(1-i)b^2} \dots\dots (4)$$

Introducing in this formula the value for $b = Kh$ as found from equation (3), it can be shown that :

for $i = 0.3, n = 2.5, \underline{e = 0.388}$

" $i = 0.1, n = 2.5, \underline{e = 0.33}$

The soil pressure intensity may then be found from the familiar formulae :

$$\begin{aligned} S_1 &= \frac{2R}{b} (2 - 3e) \\ S_2 &= \frac{2R}{b} (3e - 1) \end{aligned} \dots\dots (5)$$

When S_1 exceeds the maximum allowable pressure, the procedure is reversed. Inserting

$$\underline{S_1 = 8000, \quad R = b(1-i)(1+c)100h, \quad \text{in formula (5):}}$$

for $i = 0.3$:

$$e = 0.667 - \frac{S_1 b}{6 \cdot 0.7b(1+c)100h} = 0.667 - \frac{19.05}{h(1+c)}$$

For $i = 0.1$:

$$e = 0.667 - \frac{14.82}{h(1+c)}$$

Inserting these values in equation (4), "b" can be obtained. This is the procedure which was used in compiling tables 4.

Note that the maximum soil pressure is reached only when :

$$\begin{array}{llll} h (1 + c) & 70 \text{ ft.} & \text{for} & i = 0.3, \text{ and when :} \\ h (1 + c) & 50 \text{ ft.} & \text{for} & i = 0.1 . \end{array}$$

The effect of reducing the allowable soil pressure, or increasing the factor of safety will be discussed later.

Re TABLES V .

Hardly any comment is needed. The maximum moment in the heel portion of the base slab occurs at the heel, where the downward load is : $(w - S_2)$. The moment is :

$$M_{\text{max.}} = \frac{w - S_2}{12} \text{ m}^2 \text{ foot pounds} = \frac{(w - S_2) \text{ m}^2 \text{ inch lbs.}}{\underline{\hspace{10em}}}$$

Re TABLES VI a.

These give the thickness of the foot slab as based on the above expression for the moment , the slab being provided with shear reinforcement in the form of stirrups.

$$d_b = \sqrt{\frac{2 M}{f_c \cdot k \cdot j}} = \sqrt{\frac{2 M}{12 \cdot 650 \cdot 0.874 \cdot 0.379}} = \underline{\underline{0.0278 \sqrt{M} \text{ inches}}}$$

Re TABLES VI b.

These tables give the thickness of the base slab, when governed by an allowable unit shear of 40 pounds per square inch. To be exact, this shear should be taken at the face of the counterforts, reducing the effective span by the thickness of these.

As discussed on pp. 11 and 12 , under the heading of TABLE I, a negligible error will be committed by neglecting to make this deduction.

TABLES VIIa and VII b ,

simply give the volume of the heel portion of the base slab, for the two limiting conditions :

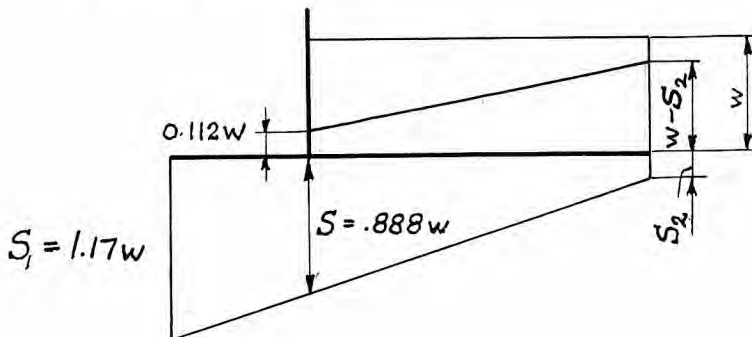
- a.) Base slab fully reinforced to take shear. Moment requirements govern the thickness.
- b.) No shear reinforcement. Allowable unit shear 40 pounds per square inch.

TABLE VIII ,

of which only the sheet for $h = 25$ feet has been copied, gives the amount of reinforcing steel in the base slab for the two conditions. It was ascertained that the shear stirrups total such small amount as to be entirely negligible.

(For $h = 15$, $c = 1.0$, the weight of stirrups, when distributed uniformly over the full length of wall, equals approximately 1.0 pounds per foot. For $h = 40$, $c = 0.5 : 10$ lbs./ft. or in both cases less than 9 % of the weight of the main steel.)

In the case of $i = 0.3$, $e = 0.388$, the following development was used :



Cross hatched area represents resultant, downward load on heel slab.

Average load intensity (see sketch) :

$$\frac{1}{2} (w - S_2 + w - S) , \text{ where } S_2 = 0.2295 w$$

i.e. 0.4412 w

Maximum load intensity = (w + S₂) = 0.7705 w

Or :

$$\text{Average moment} = M_{av.} = \frac{.4412}{.7705} \cdot M_{max.} = \underline{0.572 M_{max.}}$$

Steel area for each one foot strip of slab (adding 40 % for negative moment steel) :

$$A_s = 1.4 \frac{M_{av.}}{16,000 \cdot 7/8 \cdot d_b} = \underline{\frac{M_{av.}}{10,000 d_b}}$$

Total area for entire heel portion of base slab = (1 - i) b \cdot A_s

And volume per linear foot :

$$V_s = 12 (1 - i) b \cdot A_s$$

Weight per linear foot :

$$\underline{\text{Weight} = 3.4 (1 - i) b \cdot A_s}$$

Similar expressions were worked out for other values of i and e .

TABLES IX AND X .

The economic spacing of counterforts was first worked out assuming certain arbitrary values for counterfort thickness, one for each height in accordance with practical considerations. It was found that the curves developed on this basis were quite irregular, and to remedy this trouble the thickness was made a linear function of the

height. Afterwards a rule was evolved giving the correction to be applied for other counterfort thicknesses. (See about this in a special chapter later.)

In TABLE IX the minimum thickness (for $h = 15$ ft.) was chosen as ten inches, and the maximum (for $h = 50$ ft.), thirty inches. This gives a nearly constant " slenderness ratio " for all counterforts.

The approximation was made of not deducting the thickness of the face slab from the volume and the side areas of the counterforts. The error in actual volume and area is from five to ten percent, and will result in slightly too high values ϕ for the economic spacing.

Tables IX and X were copied for only one value (0.1) of toe projection ratio " i " .

TABLE XI,

which was copied for the two cases $h = 25$, $i = 0.1$

and $h = 25$, $i = 0.3$,

gives a summary of the volume of concrete, weight of steel, and area of forms involved, in the two limiting cases :

- a.) Base slab thickness determined by moment
- b.) " " " " " shear.

Only the variable factors were included, omitting all form work except counterfort side forms, all spacer bars, entire volume of toe projection, and protective covering outside the reinforcement.

TABLES XII a, and XII b,

give the final compilation of the variable costs, based on prevailing prices as obtained from a Pasadena contractor, Mr. South, and also the cost for two other values of the ratio " f/c ". The unit steel cost was assumed to be a constant fraction of the unit concrete cost :

$$\text{Cost of steel per pound} = \frac{1}{74} \text{ times cost of concrete per cu. ft.}$$

The prevailing cost ratios are represented in the next to the last column, and ~~f/c~~ ^{are} assumed to be :

Concrete in place	\$ 0.259 per cubic foot
Formwork erected and removed	\$ 0.07 per square "
Steel in place	\$ 0.035 per pound

The cost figures are naturally correct only for these unit prices, but the indicated economic spacing of counterforts is correct provided the two ratios, "f/c " and " s/c ", are as shown :

$$f/c = \frac{\text{cost of forms per square foot}}{\text{cost of concrete per cu. foot}} = 0.27$$

$$s/c = \frac{\text{cost of steel per pound}}{\text{cost of concrete per cu. foot}} = \frac{1}{74} .$$

The effect of changing these ratios will be discussed in the following chapter.

In each case the values in the last three columns of TABLE XII was plotted as ordinates with the spacing as abscissus, and the spacing for the most economical condition fixed by judgment of the curve passed through the points.

On the following pages are shown a few examples of the cost curves thus obtained, illustrating the saving possible by a correct choice of counterfort spacing. It must ~~h~~ be remembered that the total cost values are not all inclusive, and that the apparent percentage saving is somewhat exaggerated. Notice that all curves are relatively flat near the "bottom", substantiating a previous statement that a tolerance in the spacing of eighteen inches each way from the optimum condition, is easily permissible.

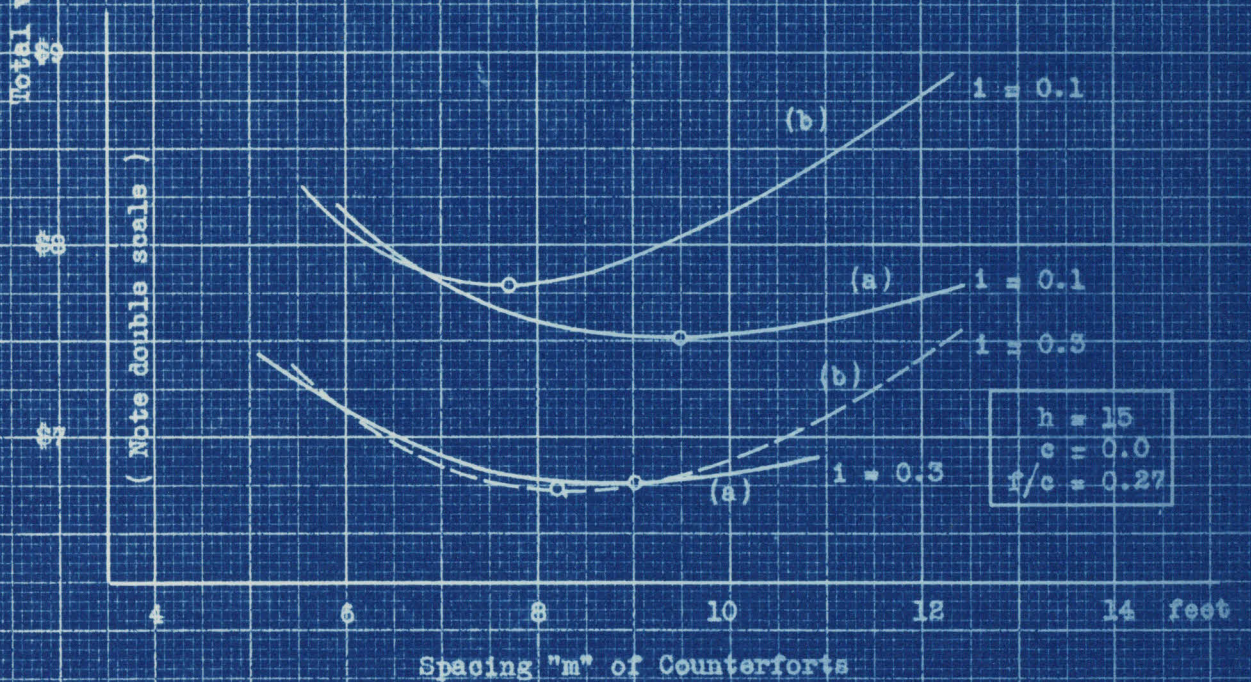
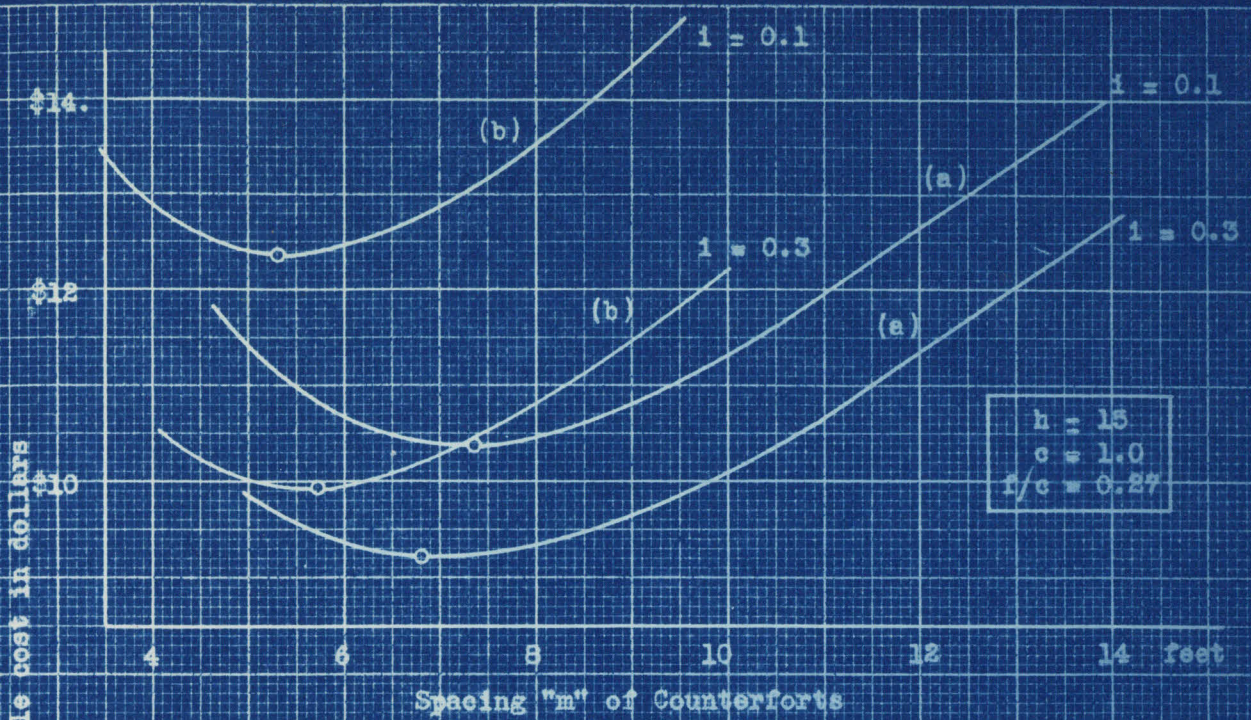
Notice also the considerable saving obtained by using shear reinforcement in the base slab, and that a toe projection ratio of 0.3 in all cases is more economical than a toe projection ratio of 0.1.

The curves shown on pp. 26 and 27 were obtained by plotting the economic spacing as found from TABLES XII.

The diagrams on page 26 represent the case when the toe projection ratio "i" equals 0.3. It was not considered necessary to extend the curves beyond the point where the factor of safety against overturning just equals 2.5 for a maximum soil pressure of 8000 pounds per square foot. Simple straight lines were drawn, never missing the actual points by more than a few tenths of a foot, as indicated by the small circles.

The diagrams on p. 27 illustrate the case of "i" = 0.1. Notice that the full lines cover practically the entire range of heights and rates of surcharge as encountered in practical design, and that within this range the curves very much resemble those given on page ~~27~~ 26.

In the concluding chapter a few examples are given of how to use the diagrams in determining the economic spacing for any given case.

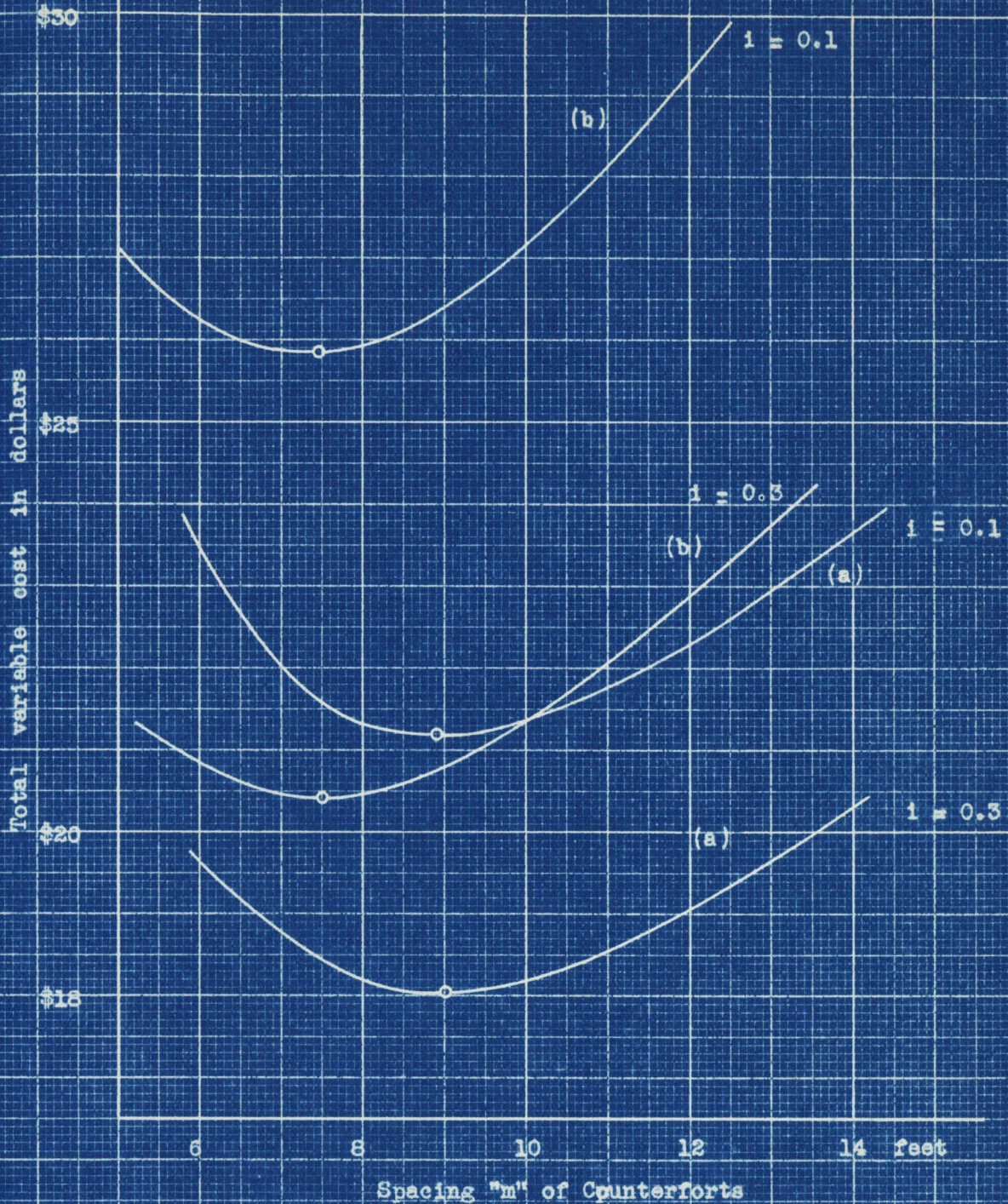


Note :

(a) represents condition when base slab is reinforced against shear

(b) represents condition when no shear reinforcement is used.

VARIABLE COST CURVES



$h = 25, \quad c = 0.25, \quad f/c = 0.27$

Note :

- (a) represents condition when base slab is reinforced against shear.
- (b) represents condition when no shear reinforcement is used.

VARIABLE COST CURVES

$h = 40$
$c = 0.25$
$l = 0.1$
$f/c = 0.27$

Total variable cost in dollars

Spacing "m" of Counterforts

Note :

- (a) represents condition when base slab is reinforced against shear
- (b) represents condition when no shear reinforcement is used.

\$75

\$70

\$65

\$60

$S_1 = 8000$

$S_1 = 6000$

(b)

(b)

$S_1 = 6000$

(a)

Effect of changing allowable soil pressure

$S_1 = 8000$

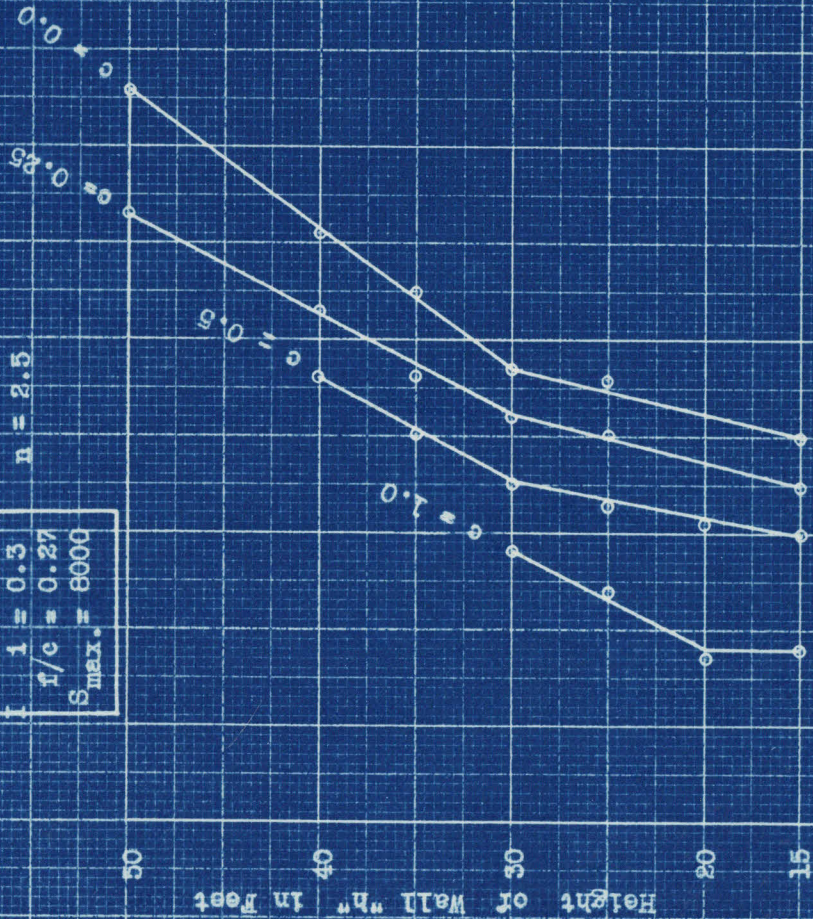
(a)

VARIABLE COST CURVES

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50 X 20 to our Dept.

ECONOMIC SPACING OF COUNTERFORTS

$n = 2.5$
 $l = 0.3$
 $f/c = 0.27$
 $S_{max.} = 8000$

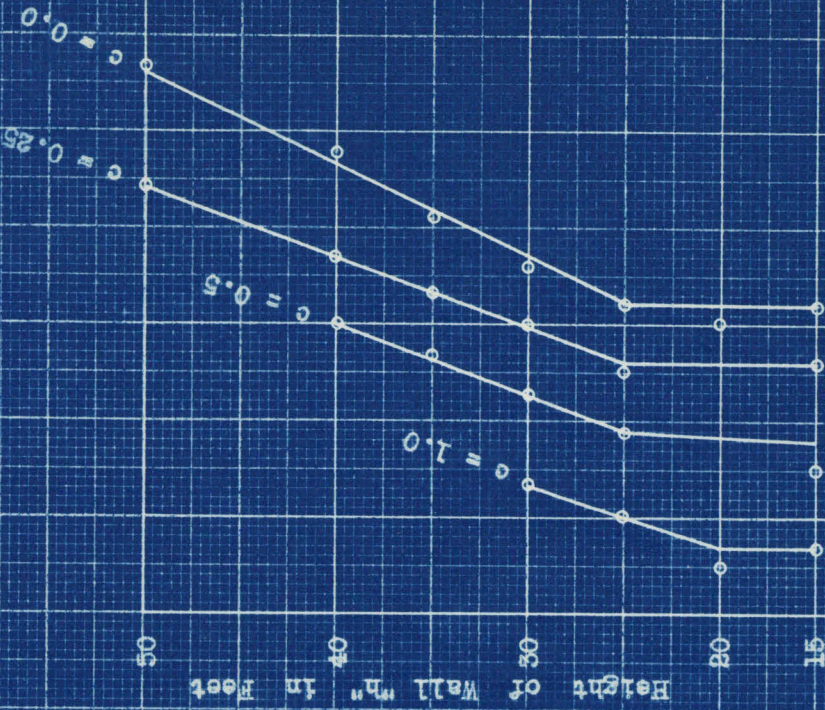


(a)

Base Slab fully reinforced for Shear

Economic Spacing "m" in Feet

ECONOMIC SPACING OF COUNTERFORTS



(b)

No Shear Reinforcement used

$v = 40 \text{ lbs./sq.in.}$

Economic Spacing "m" in Feet

ECONOMIC SPACING OF COUNTERFORTS

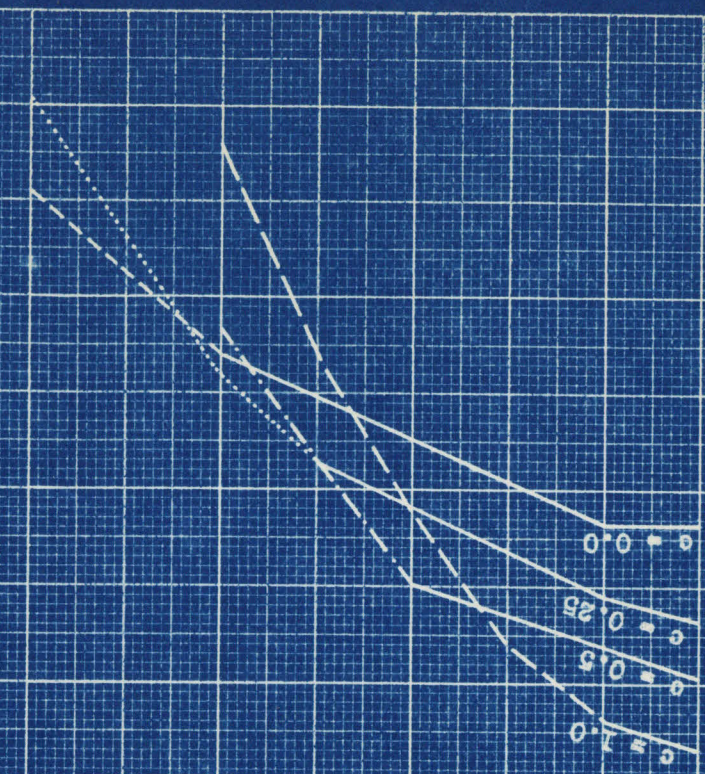
$l = 0.1$
 $f/c = 0.27$
 $S_{max.} = 8000$

Legend:

Full Lines : Factor of safety " n " = 2.5
Dotted " : " n " greater than 2.5

Height of Wall in Feet

Height of Wall in Feet



(a)

(b)

Base slab fully reinforced for shear

No Shear Reinforcement used

Economic Spacing "m" in Feet

Economic Spacing "m" in Feet

8

10

12

14

6

8

10

12

EFFECT OF CHANGING ASSUMPTIONS.

In the following section an analysis will be made of the effect of changing the assumptions upon which this whole work is based.

COST RATIOS.

a) It is quite conspicuous from TABLES XII that a simple relation exists between the economic spacing, and the ratio of cost of forms to cost of concrete. By simple graphical interpolation, using semilogarithmic paper, it was found that each increase of 0.1, one tenth, in the "f/c" ratio, corresponds to an average increase of 4.3 per cent in the economic spacing of counterforts.

Given m_1 for a price ratio $f/c = 0.1$,
wanted m_2 " " " $f/c = n \cdot 0.1$:

$$\underline{m_2 = (1.043)^n \cdot m_1}$$

Exemple :

$$\begin{aligned} f/c = 0.27 & \dots \dots \dots m_1 = 6.4 \\ f/c = 0.60, & \quad m_2 = (1.043)^{0.33} \cdot 6.4 = 1.149 \cdot 6.4 = 7.4 \end{aligned}$$

In no case was the deviation from this rule found to be appreciable.

b). For a few representative cases the effect of reducing by 25 % the ratio "s/c" of steel cost to concrete cost, was found to be negligible. A still greater reduction will tend to increase slightly

the economic spacing of the counterforts for the lower walls.

2. Toe Projection.

Occasionally local conditions makes it necessary entirely to omit any toe projection. It is reasonable to assume that the curves for "i" = 0.1, cover this case close enough, but if desired, one could extrapolate β from the two diagrams, considering a straight line relation to exist between economic spacing and toe projection.

If possible, the designer will choose the most economical value for the toe projection, which ordinarily corresponds very closely to "i" = 0.3; the diagrams on page 26 will therefore be directly applicable in most cases.

If, for any reason an intermediate value of "i" is selected, interpolation between the two diagrams will with sufficient accuracy give the desired economic spacing.

3. Partial Use of Shear Reinforcement.

In practice the base slab may be made somewhat thicker than required by moment considerations, but not quite thick enough to produce a maximum shear intensity of $v = 40$ pounds per square in.^{only} The designer may then with sufficient accuracy select an intermediate value between the two indicated for these extreme conditions, using his judgment as to which of the two values he should "favor".

Attention is at this point called to the very substantial saving effected by full use of shear reinforcement in the base slab. In itself this is a much more important economy, than the saving made possible by changing the counterfort spacing within a wide range.

Example :

$$h = 35 \text{ ft. } , \quad c = 0.25 , \quad i = 0.1$$

Total saving per linear foot by using shear reinforcement. . . . \$11.70

" " by changing "m" from 12ft. to 8 ft. \$ 4.60

The effect of using shear reinforcement in the face slab, would be comparable to the effect of doing the same thing with the base, but to a much smaller extent. Even the very highest walls (40 - 50 ft) do not assume unreasonable face slab thickness, while the footing must be very thick to resist shear without any stirrups or bent-up bars. For this reason the effect of using shear reinforcement in the face slab was not investigated in detail.

4. Counterfort Thickness.

The economic spacing of counterforts was determined for two different values of "t" , in each of the following cases : $i = 0.3$, $h = 30, 35, 40, 45$ and 50 feet, for all four values of c . Plotting the results on cross-section paper, the effect seemed to be fairly consistent, and justifying the formulation of this approximate rule :

For each one inch added counterfort thickness, add one and one half inch to the economic spacing as found from the curves.

These are based on the following values :

Height	15'	20'	25'	30'	35'	40'	50'
Thickness	10"	12.9"	15.7"	18.6"	21.4"	24.3"	30"

5. Changing Main Design Assumptions.

The probable results of substituting a rigorously exact design for the simplified method used here, was discussed under "General Assumptions."

6. Variation in Allowable Soil Pressure S.

All tables were compiled under the assumption that the soil will safely carry a load of 4 tons per square foot. This may seem to be somewhat higher than a strictly average condition, especially since the actual soil pressure is somewhat higher than S_1 as given in TABLES IV, due to the fact that the weight of the base slab was disregarded. (A four foot slab would impose an additional 600 pounds per square foot.)

It should be noted, however, that a factor of safety of $n = 2.5$, calls for base widths in excess of the requirements for a safe load on the soil, except in the cases of the very highest walls with high rates of surcharge.

Substituting $S = 3$ tons per square foot in the place of 4 tons per square foot, the following high wall was investigated :

(See sheet 25.)

$$h = 40 \text{ ft. } , c = 0.25 , i = 0.1 , f/c = 0.27$$

a.) Base slab reinforced for shear

(b) No Shear reinforcement

S = 6000

8000

S * = 6000

8000

$m_{ec.} = 12.1$

11.3

$m_{ec.} = 11.8$

9.4

In the first case, nearly \$2.00 would be wasted per linear foot by using the value given by the curves, instead of the proper minimum cost value. In the second case the difference would be almost negligible.

As all the cost curves are fairly flat near the minimum point, a deviation of not more than one foot in either direction from the optimum value of spacing, in no case makes appreciable difference in the total cost. It should be noted that the cost curves invariably are steeper on the left hand side. In case of doubt, an adjustment should therefore preferably be made upwards.

Referring to the trend indicated by the example on the preceding page, and carrying in mind that the cost ϕ is not very sensitive to changes in counterfort spacing as long as this is reasonably close to the most economical value, it is suggested to make a qualitative correction ~~#####~~ of "m", upward if the soil is of much lower bearing capacity, downward if it is materially higher. Before this correction is made, TABLE IV should be consulted to see whether S_1 is actually 8000 pounds per square foot, in the case considered.

The case quoted may serve as an indication of γ the order of magnitude of the correction. It should naturally be smaller in the case of the low walls, somewhat greater for higher walls with higher rates of surcharge.

7. Variation in Safety Factor "n".

The base width of the higher walls is not governed by this consideration, as much as by the allowable soil pressure. A low wall with a high surcharge will then ~~be~~ nearly represent the most unfavorable condition, as far as this effect is concerned. Taking a 15 foot wall with a 100 % surcharge, the effect on the economic spacing was found to be negligible for an increase of n from 2.5 to 3.0. For a higher wall the difference would be proportionately smaller, and it is confidently stated that this consideration may be disregarded in all practical cases.

CONCLUSION.

The diagrams on pp. 26 and 27 give the results of this investigation in concentrated form, and used in conjunction with the remarks under the heading "Effect of Changing Assumptions", should prove of value to designers.

The investigation brings out the following facts :

1. The saving obtained by changing the spacing of counterforts within a range of two or three feet, in no case amounts to more than two per cent of the total cost of the wall.
2. All cost curves are steeper on the left hand side (low values of "m") than on the other, proving that an increase in the value of m , is less apt to be costly than a decrease.
3. The investigation plainly shows the economy of using shear reinforcement, the saving obtained by this measure being of a much higher order than that possible through reasonable variations of counterfort spacing.
4. TABLES XII illustrate that the steel item should not be left out of a study of this type. Although the steel cost is usually less than 10 per cent of the total cost, the changes are comparable in magnitude to the changes in both concrete and form cost, at least near the point of most economical spacing.
5. The cost data also illustrate the well known fact that the vertical wall should be placed near the point of application of the resultant earth pressure, for economic design. The total cost for corresponding cases is invariably higher ~~for~~ for a toe projection ratio $i = 0.1$, than for $i = 0.3$.

APPLICATION OF DIAGRAMS
ON SOME ACTUAL DESIGNS.

The following examples represent actual designs of walls,
as taken from :

George Paaswell : Retaining Walls ,

and Ketchum : Walls, Bins And Elevators ,

and illustrate the use of the tables and of the curves on pp. 26 and 27.

1. $h = 25$, $c = 0.25$, $i = 0.0$, $S = 8000$, $n = 2.0$, $v = 40$

Spacing used : $m = 10$ feet

Sheet 27 covers the case of $i = 0.1$, and $n = 2.5$

and gives $m_{ec.} = 7.4$ feet

Sheet 26 covers the case of $i = 0.3$, and $n = 2.5$

and gives $m_{ec.} = 7.6$ feet,

which indicates that a smaller value (0.0) of "i" would certainly not correspond to a counterfort spacing greater than 7.4 feet, and probably somewhat less. The saving obtained by changing the spacing from the 10 feet which was used, to the economic spacing, according to the top curve on page 24 (which nearly covers the identically same condition) would be

\$ 1.40 (or almost 5 % of the total cost) per foot.

2. $h = 24$, $c = 0.25$, $i = e = 0.28$, $n = 2.0$ ⁺ , No shear reinforce.

Spacing ~~not~~ indicated.

From sheet 26, for $h = 25$, $c = 0.25$, Case (b) :

$m_{ec.} = 7.6$ feet.

3. Bureau of Construction, Pittsburgh, Pa. Dilworth Street Wall :

$h = 28$, $c = 0.0$, $i = 0$, $n = \text{approx. } 2.0$, $v = 40$

Spacing used : $m = 9.0$ feet. (Design also carried out for

$m = 7.5$ Feet, which was found to be less economical.)

The most nearly corresponding case is found on sheet 27,
and is :

$$h = 30 , c = 0 , i = 0.1 , n = 2.5$$

$$\text{which gives } \underline{m_{ec} = 8.5 ,}$$

checking the statement that a 9 foot spacing is more economical
than a 7.5 feet spacing.

4. U.S. Reclamation Service Walls :

$$a) h = 15 , c = 0 , i = 0.38 , n = 2.5 , v = 60$$

$$\text{Spacing used : } \underline{m = 8.0 \text{ feet}}$$

Sheet 26, diagrams (b) covers the case :

$$h = 15 , c = 0 , i = 0.3 , n = 2.5 , v = 40 ,$$

$$\text{and indicates } \underline{m_{ec} = 8.2 \text{ feet.}}$$

$$b) h = 15 , c = \text{approx. } 0.2 , i = 0.37 , v = 60$$

$$\text{Spacing used : } \underline{m^* = 8.0 \text{ feet}}$$

Sheet 26, diagrams (b) is applicable :

$$h = 15 , c = 0.2 \text{ (by interpolation) , } i = 0.3 , v = 40$$

$$\text{Economic spacing: } \underline{m_{ec} = 7.8 \text{ feet.}}$$

5. Illinois Central R.R. Walls :

$$h = 20 , c = 0.5 , i = 0.23 , n = \text{approx. } 2.2 , v = 80 (?)$$

$$\text{Spacing used : } \underline{m = 10 \text{ feet}}$$

Sheet 26 :

$$h = 20 , c = 0.5 , i = 0.3 , n = 2.5 , \text{ interpolating between}$$

diagram (a) and (b) : $m_{ec} = 7.5$

Sheet 27 :

$$h = 20, c = 0.5, i = 0.3, n = 2.5 : m_{ec.} = 7.4$$

For $i = 0.23$ an intermediate value of $m = 7.5$, would be the most economical spacing, but of course the difference in cost for variations in spacing of "m" amounting to only a few tenths, is almost imperceptible. The increase in cost by using $m = 10$ feet instead of the economic spacing, is approximately \$ 0.80 per linear foot of the retaining wall, or between 3 and four per cent of the total cost.

6. Corrugated Bar Company :

$$h = 23, c = 0, i = 0.29, n = 2.5, \text{ No shear reinforcement.}$$

Spacing used : $m = 10$ feet

Sheet 26, case (b) :

$$h^* = 23, c = 0, i = 0.3, n = 2.5 : m_{ec.} = 8.2 \text{ feet}$$

Difference in cost between the two spacings :

$$\underline{\$ 0.50 \text{ per linear foot, or about } 2\% \text{ of total.}}$$

In conclusion The nice agreement between Table 27 in Paaswell : Retaining Walls and the right hand curve of diagram (b) p. 26 of this paper is pointed out. The top line in Paaswell's table covers practically the identically same case, and is based on nearly identical assumptions, without - however - considering certain minimum requirements, and without including the cost of steel.

h	15	20	25	30	35	40	50
$m_{Paaswell}$	7;5	8.1	8.6	8.9	9.3	9.6	10.2
m_{Omsted}	8.2	8.2	8.2	8.7	9.2	9.7	10.6

The same table in Passwell's book, also gives the effect of using other values of the ratio cost of forms to cost of concrete. By applying the formula worked out on p. 28 of this paper, it is seen that the agreement is nearly perfect.

TABLE I

Thickness of Face Slab.

d_v . . . thickness required by moment. Minimum : 0.58'
 d_v' . . . " " " " shear

h	m >	6	8	10	12	14	16
15	d_v	0.34	0.45	0.56	0.68	0.79	0.90
20	"	0.39	0.52	0.65	0.78	0.92	1.05
25	d_v'	0.41	0.58	0.74	0.91	1.07	1.24
30	"	0.49	0.69	0.89	1.09	1.28	1.48
35	"	0.58	0.81	1.04	1.27	1.50	1.73
40	"	0.66	0.93	1.19	1.45	1.72	1.98
50	"	0.83	1.16	1.49	1.82	2.15	2.48
15	d_v	0.38	0.51	0.63	0.76	0.88	1.01
20	d_v'	0.42	0.58	0.75	0.91	1.08	1.24
25	"	0.52	0.73	0.93	1.14	1.35	1.55
30	"	0.62	0.87	1.12	1.37	1.61	1.86
35	"	0.73	1.02	1.31	1.60	1.89	2.18
40	"	0.83	1.16	1.49	1.82	2.15	2.48
50	"	1.04	1.45	1.86	2.28	2.69	3.11
15	d_v'	0.42	0.55	0.70	0.83	0.98	1.13
20	"	0.50	0.70	0.89	1.09	1.29	1.49
25	"	0.62	0.87	1.12	1.37	1.61	1.86
30	"	0.75	1.05	1.35	1.64	1.94	2.24
35	"	0.87	1.22	1.57	1.92	2.26	2.61
40	"	0.99	1.39	1.78	2.18	2.58	2.97
50	"	1.24	1.74	2.24	2.73	3.22	3.72
15	d_v'	0.50	0.70	0.89	1.09	1.29	1.49
20	"	0.66	0.92	1.19	1.45	1.72	1.89
25	"	0.83	1.16	1.49	1.82	2.15	2.48
30	"	0.99	1.39	1.78	2.18	2.58	2.97
35	"	1.16	1.62	2.08	2.54	3.01	3.74
40	"	1.32	1.85	2.38	2.90	3.43	3.96

c = 0

c = 0.25

c = 0.50

c = 1.0

TABLE II

Steel at Top of Vertical Wall

Based on $M = w m^2/12$

	m	M	d	A _s		m	M	d	A _s
	ft.	in.lb.	ft.	sq.in.		ft.	in.lb.	ft.	sq.in.
ch = 5' w' = 167	5	4,000	.58	.240	ch = 20 w' = 667	5	17,000	.58	.240
	6	6,000	"	"		6	24,000	"	.244
	7	8,200	"	"		8	42,600	"	.433
	8	10,700	"	"		10	66,700	.65	.608
	10	16,700	"	"		12	96,000	.78	.725
ch = 7.5 w' = 250	5	6,300	.58	.240	ch = 25 w' = 833	5	20,800	.58	.240
	6	9,000	"	"		6	30,000	"	.304
	7	12,300	"	"		7	40,800	"	.413
	8	16,000	"	"		8	53,300	"	.540
	10	25,000	"	.254		10	83,300	.74	.662
ch = 10 w' = 333	5	8,300	.58	.240	ch = 30 w' = 1000	5	25,000	.58	.253
	6	12,000	"	"		6	36,000	"	.364
	7	16,300	"	.248		7	49,000	"	.496
	8	21,300	"	.325		8	64,000	.69	.546
	10	33,300	"	.507		10	100,000	.89	.661
ch = 15' w' = 500	4	8,000	.58		ch = 35 w' = 1170	6	42,100	.58	.428
	6	18,000	"			7	57,300	.70	.481
	7	24,500	"			8	74,800	.81	.543
	8	32,000	"			10	117,000	1.04	.662
	10	50,000	"			12	168,300	1.27	.780

TABLE III

Steel per Linear Foot of Vertical Wall, Based on $M = \frac{w m^2}{12}$,
and 40 per cent allowance for negative moment steel.

	m	Base of Wall		Top of Wall			A _s	V _s	Weight
		M	d'	A _s '	d''	A _s ''			
h = 25 c = 0 w = 833	6	30,000	.58	.304	.58	.240	6.80	114.2	32.4
	7	40,800	"	.414	"	"	8.17	137.3	38.9
	8	53,300	"	.540	"	"	9.75	164.0	46.4
	10	83,300	.74	.662	"	"	11.30	190.0	53.8
	12	120,000	.91	.776	"	"	12.70	213.5	60.5
h = 25 c = 0.25 w = 1042	5	26,000	.58	.264	.58	.240	6.30	105.8	30.0
	6	37,500	"	.380	"	"	7.75	130.2	36.9
	7	51,100	.63	.476	"	"	8.95	150.3	42.6
	8	66,800	.73	.538	"	"	9.72	163.3	46.3
	10	104,200	.93	.660	"	"	11.25	189.0	53.5
	12	150,000	1.14	.774	"	.300	13.43	226.0	64.0
h = 25 c = 0.5 w = 1250	5	31,300	.58	.317	.58	.240	6.96	117.0	33.1
	6	45,000	.62	.426	"	"	8.32	140.0	39.6
	7	61,300	.75	.481	"	"	9.01	151.5	42.9
	8	80,000	.87	.540	"	.280	10.25	172.3	48.8
	10	125,000	1.12	.656	"	.423	13.48	226.5	64.1
h = 25 c = 1.0 w = 1670	5	41,700	.67	.366	.58	.240	7.58	127.5	36.0
	6	60,000	.83	.425	"	.304	9.11	153.2	43.3
	7	81,700	1.00	.481	"	.414	11.18	188.0	53.2
	8	106,700	1.16	.541	"	.540	13.53	227.5	64.5
	10	166,700	1.49	.658	.74	.662	16.50	277.5	78.5

TABLE IV a.

Width of Base Slab, Soil Pressure Intensity.

$$i = 0.3, n = 2.5 : K = 0.548 \sqrt{\frac{1 + 3c}{1 + c}} ; S_{max.} = 8000$$

$$S_1 = \frac{2R}{b} (2 - 3e) ; S_2 = \frac{2R}{b} (3e - 1)$$

h	c	K	b = K.h	(1-i)b = 0.7b	"R" per foot of wall	e	S ₁ lb/sq.ft.	S ₂ lb/sq.ft.
15	0	.55	8.25	5.78	8,650	.388	1750	346
	.25	.65	9.75	6.82	12,800	"	2220	416
	.50	.708	10.62	7.44	16,700	"	2620	515
	1.00	.776	11.62	8.15	24,400	"	3540	663
20	0	.55	11.00	7.70	15,400	.388	2340	460
	.25	.65	13.00	9.10	22,750	"	2930	575
	.50	.708	14.18	9.93	29,800	"	3500	690
	1.00	.776	15.32	10.72	42,900	"	4690	920
25	0	.55	13.75	9.62	24,000	.388	2920	570
	.25	.65	16.25	11.38	35,600	"	3660	720
	.50	.708	17.70	12.38	46,400	"	4380	860
	1.00	.776	19.40	13.58	67,900	"	5850	1150
30	0	.55	16.50	11.55	34,700	.388	3520	690
	.25	.65	19.50	13.65	51,200	"	4400	860
	.50	.708	21.20	14.84	66,800	"	5270	1030
	1.00	.776	23.30	16.30	97,800	"	7000	1375
35	0	.55	19.25	13.50	47,300	.388	4110	805
	.25	.65	22.75	15.93	69,800	"	5120	1005
	.50	.708	24.80	17.37	91,200	"	6150	1210
	1.00	.776	27.15	19.00	133,000	"	82000	1610
40	0	.55	22.00	15.40	61,600	.388	4680	920
	.25	.65	26.00	18.20	91,000	"	5850	1150
	.50	.708	28.30	19.80	119,000	"	7040	1380
	1.00	.85	34.00	23.80	190,500	.432	7880	3320
50	0	.55	27.50	19.23	96,200	.388	5840	1145
	.25	.65	32.50	22.75	142,000	"	7310	1435
	.50	.75	37.50	26.25	197,000	.415	7950	2580
	1.00	.96	48.00	33.60	336,000	.479	7900	6100

TABLE IV b.

Width of Base Slab, Soil Pressure Intensity.

$$i = 0.1, \quad n = 2.5 : K = 0.53 \sqrt{\frac{1 + 3c}{1 + c}} \quad (\text{or greater})$$

$$S_1 = \frac{2R}{b} (2 - 3e) ; \quad S_2 = \frac{2R}{b} (3e - 1).$$

h	c	K	b = K.h	(1-i)b = 0.9b	"R" per foot of wall	e	S ₁ lb/sq.ft.	S ₂ lb/sq.ft.
15	0	.531	7.98	7.18	10,780	.33	2700	0
	.25	.628	9.42	8.48	15,900	"	3380	0
	.50	.685	10.28	9.25	20,800	"	4050	0
	1.00	.750	11.23	10.11	30,400	"	5410	0
20	0	.531	10.62	9.56	19,120	.33	3600	0
	.25	.628	12.56	11.30	28,250	"	4500	0
	.50	.685	13.70	12.32	37,000	"	5400	0
	1.00	.750	15.00	13.50	54,000	"	7200	0
25	0	.531	13.29	11.96	29,900	.33	4500	0
	.25	.628	15.70	14.13	44,200	"	5640	0
	.50	.685	17.12	15.42	57,800	"	6750	0
	1.00	.830	20.75	18.67	93,300	.371	7970	1020
30	0	.531	15.92	14.32	43,000	.33	5400	0
	.25	.628	18.83	16.93	63,500	"	6740	0
	.50	.685	20.56	18.47	83,000	"	8090	0
	1.00	.973	29.20	26.30	157,900	.420	8000	2810
35	0	.531	18.60	16.73	58,600	.33	6300	0
	.25	.628	22.00	19.80	86,700	"	7880	0
	.50	.791	27.67	24.90	131,000	.385	8000	1470
	1.00	1.144	40.10	36.10	252,700	.455	8000	4600
40	0	.531	22.24	20.02	80,200	.33	7200	0
	.25	.695	27.80	25.00	125,000	.371	7990	1020
	.50	.888	35.52	31.95	192,000	.420	8000	2810
	1.00	1.350	54.00	48.60	388,500	.482	7850	6430
50	0	.587	29.30	26.37	159,300	.371	8000	1020
	.25	.848	42.40	38.20	238,200	.430	8000	3260

TABLE V b.

Moments in Base Slab.

$$M_{\max} = (w - S_2)m^2 \text{ in.lbs.}$$

$$i = 0.1$$

h	m	6	8	10	12	14	16	
15		54,000	96,000	150,000	216,000	294,000	384,000	c = 0.0
20		72,000	128,000	200,000	288,000	392,000	512,000	
25		90,000	160,000	250,000	360,000	490,000	640,000	
30		108,000	192,000	300,000	432,000	588,000	768,000	
35		126,000	224,000	350,000	504,000	685,000	896,000	
40		144,000	256,000	400,000	576,000	784,000	1024,000	
50		"	"	"	"	"	"	
15		67,500	120,000	187,500	270,000	367,000	480,000	c = 0.25
20		90,000	160,000	250,000	360,000	490,000	640,000	
25		112,500	200,000	312,000	450,000	612,000	800,000	
30		135,000	240,000	375,000	540,000	735,000	960,000	
35		157,500	280,000	437,000	630,000	858,000	1120,000	
40		144,000	256,000	400,000	576,000	784,000	1024,000	
50		108,000	192,000	300,000	432,000	588,000	768,000	
15		81,000	144,000	225,000	324,000	441,000	576,000	c = 0.50
20,		108,000	192,000	300,000	432,000	588,000	768,000	
25		135,000	240,000	375,000	540,000	735,000	960,000	
30		162,000	288,000	450,000	648,000	882,000	1152,000	
35		137,000	244,000	380,000	550,000	745,000	973,000	
40		115,000	205,000	320,000	461,000	627,000	820,000	
15		108,000	192,000	300,000	432,000	588,000	768,000	
20		144,000	256,000	400,000	576,000	784,000	1024,000	
25		"	"	"	"	"	"	
30		115,000	205,000	320,000	461,000	627,000	820,000	
35		87,000	154,000	240,000	346,000	470,000	615,000	
40		,58,000	102,000	160,000	230,000	314,000	410,000	

TABLE VI b

Thickness of Base Slab.
 $i = 0.1$
 Based on Shear Requirements

h	m	6	8	10	12	14	16
15		12.50	14.30	17.87	21.45	25.00	28.60
20		14.30	19.07	23.82	28.60	33.35	38.10
25		17.88	23.85	29.80	35.75	41.70	47.70
30		21.47	28.65	35.80	43.00	50.10	57.30
35		25.05	33.40	41.75	50.10	58.45	66.80
40		28.60	38.10	47.60	57.15	66.70	76.20
50		"	"	"	"	"	"
15		13.42	17.90	22.35	26.85	31.30	35.80
20		17.88	23.85	29.80	35.75	41.70	47.70
25		22.35	29.80	37.25	44.70	52.15	59.60
30		26.80	35.75	44.70	53.65	62.60	71.50
35		31.30	41.70	52.15	62.60	73.00	83.50
40		28.60	38.10	47.60	57.15	66.70	76.20
50		21.47	28.65	35.80	43.00	50.10	57.30
15		16.10	21.47	26.82	32.20	37.45	42.90
20		21.45	28.62	35.75	42.90	50.10	57.20
25		26.80	35.75	44.70	53.65	62.60	71.50
30		32.20	42.90	53.60	64.40	75.10	85.80
35		27.20	36.20	45.30	54.40	63.40	72.50
40		22.85	30.50	38.10	45.75	53.40	61.00
15		21.45	28.62	35.75	42.90	50.10	57.25
20		28.60	38.10	47.60	57.15	66.70	76.20
25		35.75	47.60	59.60	71.50	83.40	95.30
30		22.85	30.50	38.10	45.75	53.40	61.00
35		17.17	22.85	28.60	34.30	40.05	45.70
40		12.50	15.25	19.05	22.85	26.70	30.50

c = 0.0

c = 0.25

c = 0.5

c = 1.0

TABLE VII a

Volume of Heel Slab per Linear Foot
 $i = 0.1$
 Thickness Based on Moment Requirements
 (Slab reinforced for Shear)

h	0.9b	m						
		6	8	10	12	14	16	
15	7.18	7.5	7.5	7.5	7.7	9.0	10.3	c = 0.0
20	9.56	10.0	10.0	10.0	11.9	13.9	15.9	
25	11.96	12.5	12.5	13.8	16.7	19.4	22.2	
30	14.32	14.9	14.9	18.2	21.8	25.4	29.1	
35	16.73	17.4	18.4	23.0	27.5	32.0	36.6	
40	20.02	20.8	23.4	29.4	35.2	41.1	47.0	
50	26.37	27.4	30.8	38.6	46.3	54.1	61.8	
15	8.48	8.8	8.8	8.8	10.2	11.9	13.6	c = 0.25
20	11.30	11.8	11.8	13.1	15.7	18.3	21.0	
25	14.13	14.7	14.7	18.3	22.0	25.6	29.3	
30	16.93	17.6	19.2	24.0	28.9	33.6	38.4	
35	19.80	20.6	24.3	30.4	36.5	42.6	48.6	
40	25.00	26.0	29.3	36.7	44.0	51.4	58.6	
50	38.20	39.8	39.8	48.5	58.1	67.6	77.5	
15	9.25	9.6	9.6	10.2	12.2	14.2	16.2	c = 0.50
20	12.32	12.8	12.8	15.6	18.8	21.9	25.0	
25	15.42	16.0	17.5	21.9	26.2	30.6	35.0	
30	18.47	19.2	23.0	28.7	34.5	40.2	46.0	
35	24.90	25.9	28.6	35.7	42.9	50.0	57.0	
40	31.95	33.3	33.6	42.0	50.4	58.8	67.2	
15	10.11	10.5	10.5	12.8	15.4	18.0	20.6	c = 1.0
20	13.50	14.1	15.8	19.8	23.8	27.8	31.6	
25	18.67	19.5	24.5	30.6	36.7	42.9	49.0	
30	26.30	27.4	27.7	34.5	41.5	48.4	55.4	
35	36.10	37.6	37.6	41.1	49.4	57.5	65.9	
40	48.60	50.6	50.6	50.6	54.2	63.3	72.4	

TABLE VII b

Volume of Heel Slab per Linear Foot
 $i = 0.1$
 Thickness Based on Shear Requirements.
 No Shear Reinforcement.

h	0.9b	m						
		6	8	10	12	14	16	
15	7.81	7.5	8.6	10.7	12.8	15.0	17.1	c = 0.0
20	9.56	11.4	15.2	19.0	23.7	26.6	30.3	
25	11.96	17.8	23.8	29.7	35.6	41.5	47.5	
30	14.32	25.6	34.2	42.7	51.3	59.8	68.4	
35	16.73	35.0	46.6	58.2	69.9	81.5	93.2	
40	20.02	47.8	63.6	79.5	95.4	111.3	127.2	
50	26.37	62.8	83.6	104.7	125.5	148.8	167.5	
15	8.48	9.5	12.7	15.8	19.0	22.1	25.3	c = 0.25
20	11.30	16.8	22.4	28.0	33.6	39.2	44.9	
25	14.13	26.3	35.1	43.9	52.6	61.4	70.2	
30	16.93	37.8	50.4	63.1	75.6	88.4	101.0	
35	19.80	51.60	68.9	86.0	103.3	120.5	137.9	
40	25.00	59.6	79.4	99.2	119.1	139.0	158.9	
50	38.20	68.3	91.2	114.0	136.8	159.3	182.3	
15	9.25	12.4	16.5	20.7	24.8	28.9	33.1	c = 0.50
20	12.32	22.0	29.4	36.7	44.0	51.5	58.7	
25	15.42	34.4	45.9	57.5	69.0	80.5	91.9	
30	18.47	49.6	66.1	82.5	99.1	115.7	132.0	
35	24.90	56.5	75.2	94.0	113.0	131.7	150.5	
40	31.95	60.9	81.2	101.3	121.8	142.2	162.5	
15	10.11	18.1	24.1	30.1	36.2	42.2	48.2	c = 1.0
20	13.50	32.2	42.9	53.5	64.2	75.0	85.7	
25	18.67	55.6	74.1	92.8	111.3	129.8	148.3	
30	26.30	50.1	66.8	83.5	100.3	117.0	133.8	
35	36.10	51.6	68.7	86.1	103.2	120.5	137.5	
40	48.60	50.6	61.8	77.2	92.5	108.1	123.5	

TABLE VIII

Steel in Base Slab

i = 0.3 e = 0.388	m	By Mom. d inch.	M _{mean} = 0.572 M _{max.} in.lb.	d determined by Moment		d' determined by Shear		
				A _s = $\frac{M}{10,000d}$	Total Steel per Lin.Ft.	d'	d/d'	Total Steel per Lin.Ft.
	6	12.5	39,700	0.317	10.4	13.78	.908	9.4
h = 25	8	"	70,600	0.565	18.5	18.40	.680	12.6
c = 0	10	"	110,200	0.882	28.8	23.00	.544	15.7
.7b = 9.62	12	14.64	159,000	1.086	35.6	27.60	.530	18.9
	14	17.10	216,000	1.263	41.4	32.20	"	22.0
	16	19.53	282,000	1.442	47.2	36.80	"	26.1
	6	12.50	51,600	0.413	16.0	17.90	.698	11.2
h = 25	8	"	91,800	0.734	28.4	23.90	.523	14.8
c = 0.25	10	13.92	143,000	1.030	39.8	29.85	.466	18.5
.7b = 11.38	12	16.70	206,300	1.236	47.8	35.80	"	22.3
	14	19.48	281,000	1.442	55.8	41.75	"	26.0
	16	22.27	366,500	1.648	63.8	47.75	"	29.8
	6	12.50	59,500	0.476	20.1	20.65	.605	12.2
h = 25	8	"	105,800	0.846	35.6	27.55	.454	16.1
c = 0.5	10	14.95	165,300	1.105	46.5	34.40	.435	20.2
.7b = 12.38	12	17.95	238,000	1.326	55.8	41.30	"	24.2
	14	20.95	324,000	1.547	65.2	48.20	"	28.3
	16	23.93	423,000	1.766	74.2	55.10	"	32.2
	6	12.50	79,200	0.633	29.2	27.45	.455	13.3
h = 25	8	13.80	141,000	1.022	47.2	36.60	.377	13.3
c = 1.0	10	17.25	220,000	1.276	58.9	45.80	"	22.2
.7b = 13.58	12	20.70	317,000	1.532	70.8	55.00	"	26.7
	14	24.17	432,000	1.790	82.8	64.10	"	31.2
	16	27.63	564,000	2.040	94.0	73.25	"	35.4

TABLE IX

Volume of Counterforts

i = 0.1

h	15	20	25	30	35	40	50	feet
Thickness	10	12.86	15.71	18.57	21.43	24.30	30	inches
h	Volume per Coun.ft.	Volume per Lin. Ft. for different values of "m"						
		6	8	10	12	14	16	
15	45	7.5	5.6	4.5	3.7	3.2	2.8	c = 0.0
20	102	17.1	12.8	10.2	8.5	7.3	6.4	
25	196	32.6	24.4	19.6	16.3	14.0	12.2	
30	333	55.5	41.6	33.3	27.7	23.8	20.8	
35	523	87.2	65.5	52.3	43.6	37.3	32.7	
40	811	135.1	101.2	81.1	67.6	57.9	50.7	
50	1648	274.5	206.0	164.8	137.3	117.7	103.0	
15	53	8.8	6.6	5.3	4.4	3.8	3.3	c = 0.25
20	121.	20.2	15.1	12.1	10.1	8.6	7.6	
25	231	38.5	28.9	23.1	19.3	16.5	14.4	
30	393	65.4	49.1	39.3	32.7	28.1	24.6	
35	620	103.3	77.5	62.0	51.6	44.3	38.7	
40	1012	168.7	126.5	101.2	84.3	72.3	63.2	
50	2390	398.0	298.5	239.0	199.0	170.8	149.3	
15	58	9.6	7.2	5.8	4.8	4.1	3.6	c = 0.5
20	132	22.0	16.5	13.2	11.0	9.4	8.2	
25	253	42.2	31.6	25.3	21.1	18.1	15.8	
30	429	71.5	53.6	42.9	35.7	30.6	26.8	
35	779	129.8	97.3	77.9	64.9	55.6	48.6	
40	1292	215.3	161.4	129.2	107.7	92.3	80.8	
15	63	10.6	7.9	6.3	5.3	4.5	4.0	
20	145	24.1	18.1	14.5	12.0	11.3	9.0	
25	305	50.9	38.1	30.5	25.4	21.8	19.1	
30	610	101.8	76.3	61.0	50.8	43.6	38.1	
35	1129	188.0	141.0	112.9	94.1	80.6	70.5	

TABLE X

Side Areas of Counterforts
i = 0.1

h	Areas each Coun.ft.	Area per Lin.Ft. for different Values of "m"						
		6	8	10	12	14	16	
15	108	17.9	13.5	10.8	9.0	7.7	6.7	c = 0.0
20	191	31.8	23.9	19.1	16.0	13.7	12.0	
25	299	49.7	37.3	29.8	24.9	21.3	18.6	
30	430	71.6	53.8	43.0	35.8	30.7	26.9	
35	586	97.6	73.2	58.6	48.8	41.9	36.6	
40	801	133.5	100.0	80.1	66.7	57.2	50.0	
50	1318	219.5	164.8	131.8	109.8	94.2	82.4	
15	127	21.2	15.9	12.7	10.6	9.1	8.0	c = 0.25
20	226	37.7	28.2	22.6	18.8	16.2	14.1	
25	353	56.8	44.1	35.3	29.4	25.2	22.0	
30	508	84.6	63.5	50.8	42.3	36.3	31.8	
35	694	115.7	86.7	69.4	57.8	49.5	43.4	
40	1000	166.7	125.0	100.0	83.3	71.5	62.5	
50	1910	318.5	238.7	191.0	159.7	136.5	119.3	
15	138.8	23.1	17.4	13.9	11.6	9.9	8.7	c = 0.5
20	246	41.1	30.8	24.6	20.6	17.6	15.4	
25	386	64.4	48.3	38.6	32.2	27.6	24.1	
30	554	92.4	69.3	55.4	46.3	39.6	34.6	
35	872	145.3	109.0	87.2	72.7	62.2	54.4	
40	1278	212.7	159.7	127.8	106.4	91.3	79.8	
15	152	25.3	19.0	15.2	12.6	10.8	9.5	c = 1.0
20	270	45.0	33.7	27.0	22.5	19.3	16.9	
25	466	77.7	58.3	46.6	38.8	33.3	29.1	
30	789	131.6	98.6	78.9	65.8	56.3	49.3	
35	1263	210.5	158.0	126.3	105.5	90.3	79.0	
40	1943	324.0	242.7	194.3	162.0	138.8	121.5	

TABLE XII a (Base Slab Thickness from Moment)

Variable Cost of Retaining Wall per Linear Foot

i = 0.3 m	m	Total Concrete Cost	Total Steel Cost	Cost of Forms			Total (a)	Total (b)	Total (c)
				$\frac{f}{c}=.2$	$\frac{f}{c}=.27$	$\frac{f}{c}=.5$			
				(a)	(b)	(c)			
h = 25 c = 0 t = 15	6	12.81	1.50	2.07	2.80	5.18	16.38	17.11	19.49
	8	11.20	2.27	1.56	2.11	3.91	15.03	15.58	17.38
	10	10.74	2.89	1.25	1.69	3.13	14.88	15.32	16.76
	12	11.09	3.36	1.05	1.41	2.61	15.50	15.86	17.06
	14	11.65		0.89	1.20	2.22			
	16						(9.5)	(9.6)	(10.3)
h = 25 c = 0.25 t = 15	6	14.51	1.85	2.46	3.32	6.15	18.82	19.68	22.51
	8	13.08	2.61	1.85	2.49	4.61	17.54	18.18	20.30
	10	12.93	3.26	1.48	2.00	3.70	17.67	18.19	19.26
	12	13.50	3.91	1.23	1.66	3.07	18.64	19.07	20.48
	14	14.32	4.56	1.05	1.42	2.63			
	16						(8.8)	(9.0)	(9.9)
h = 25 c = 0.5 t = 15	6	15.57	2.09	2.68	3.61	6.68	20.34	21.27	24.34
	8	14.30	2.95	2.01	2.71	5.02	19.26	19.96	22.27
	10	14.49	3.87	1.61	2.17	4.02	19.97	20.53	22.38
	12	15.43		1.34	1.81	3.35			
	14	16.73							
	16						(8.2)	(8.3)	(8.9)
h = 25 c = 1.0 t = 15	6	17.30	2.54	2.94	3.96	7.33	22.78	23.80	27.17
	8	16.57	3.91	2.21	2.98	5.52	22.69	23.46	26.00
	10	17.77	4.81	1.76	2.38	4.41	24.34	24.96	26.99
							(7.2)	(7.4)	(8.1)

TABLE XII b (Base Slab Thickness from Shear)

Variable Cost of Wall per Linear Foot

i = 0.3	m	Total Concrete Cost \$	Total Steel Cost \$	Cost of Forms			Total (a)	Total (b)	Total (c)
				$\frac{f}{c} = .2$ (a)	$\frac{f}{c} = .27$ (b)	$\frac{f}{c} = .5$ (c)			
h = 25 c = 0 t = 15	6	13.08	1.47	2.07	2.80	5.18	16.62	17.35	19.73
	8	12.43	2.07	1.56	2.11	3.91	16.06	16.61	18.41
	10	12.92	2.43	1.25	1.69	3.13	16.60	17.04	18.48
	12	13.80	2.78	1.05	1.41	2.61	17.63	17.99	19.19
	14	14.80							
	16						(8.0)	(8.2)	(8.9)
h = 25 c = 0.25 t = 15	6	15.80	1.68	2.46	2.46	6.15	19.94	20.80	23.63
	8	15.86	2.14	1.85	1.85	4.61	19.84	20.48	22.60
	10	16.85	2.52	1.48	1.48	3.70	20.85	21.37	23.07
	12	18.20	3.02	1.23	1.23	3.07	22.45	22.88	24.29
	14	19.77							
	16						(7.2)	(7.5)	(8.3)
h = 25 c = 0.5 t = 15	6	17.76	1.81	2.68	3.61	6.68	22.25	23.18	26.25
	8	18.32	2.27	2.01	2.71	5.02	22.60	23.130	25.61
	10	19.70	2.95	1.61	2.17	4.02	24.26	24.82	26.67
	12	21.70							
	14								
	16						(6.4)	(6.9)	(7.8)
h = 25 c = 1.0 t = 15	6.5	21.60	1.70	3.53	4.76	8.82	26.83	28.06n	32.12
	6	21.65	1.98	2.94	3.96	7.33	26.57	27.59	30.96
	7	22.22	2.44	2.52	3.40	6.30	27.18	28.06	30.96
	8	23.25	2.88	2.21	2.98	5.52			31.65
	10	26.15	3.52	1.76	2.38	4.41			
							(5.8)	(6.0)	(6.5)

TABLE XII a (Base Slab Thickness from Moment)

Variable Cost of Retaining Wall per Linear Foot

c = cost of concr. per cu. foot = \$ 0.259

f = " " forms " sq. " = 0.1c , 0.27c & 0.5c

s = " " steel " pound = 1/74.c = \$ 0.035

i = 0.1	m	Total Concrete Cost	Total Steel Cost	Cost of Counterft. fo.			Total (a)	Total (b)	Total (c)
				$\frac{f}{c}=0.1$	$\frac{f}{c}=0.27$	$\frac{f}{c}=0.6$			
				(a)	(b)	(c)			
h = 25 c = 0 t = 15.7	6	15.44	1.46	1.29	3.48	7.73	18.19	20.38	24.63
	8	13.32	2.32	.97	2.61	5.80	16.61	18.25	21.44
	10	12.93	2.88	.77	2.08	4.62	16.58	17.89	20.43
	12	13.37	3.34	.65	1.74	3.86	17.36	18.45	20.57
	14	14.00	3.81	.55	1.49	3.31			
	16						(9.0)	(9.6)	(10.8)
h = 25 c = 0.25 t = 15.7	6	17.53	1.85	1.52	4.11	9.12	20.90	23.49	28.50
	8	15.55	2.70	1.14	3.08	6.85	19.39	21.33	25.10
	10	15.62	3.27	.91	2.47	5.49	19.80	21.36	24.38
	12	16.27	3.98	.76	2.06	4.58	21.01	22.31	24.83
	14	17.16	4.75	.65	1.76	3.91			
	16						(8.5)	(8.9)	(10.0)
h = 25 c = 0.5 t = 15.7	6	18.97	2.15	1.67	4.51	10.02	22.79	25.63	31.14
	8	17.42	3.07	1.25	3.38	7.51	21.74	23.87	28.00
	10	17.73	3.96	1.00	2.70	6.00	22.69	24.39	27.69
	12	18.73	4.75	.83	2.25	5.00			28.48
	14			.71					
	16						(8.0)	(8.5)	(9.7)
h = 25 c = 1.0 t = 15.7	6	22.67	2.60	2.02	5.44	12.10	27.29	30.71	37.37
	8	21.85	3.97	1.51	4.08	9.07	27.33	29.90	34.89
	10	23.07	4.89	1.21	3.26	7.24	29.17	31.22	35.20
	12	27.55		1.01	2.72	6.04			
	14						(7.0)	(7.6)	(8.6)

TABLE XIII b (Base Slab Thickness from Shear)

Variable Cost of Retaining Wall per Linear Foot

i=0.1	m	Total Concrete Cost \$	Total Steel Cost \$	Cost of Forms			Total (a)	Total (b)	Total (c)
				$\frac{f}{c}=0.1$	$\frac{f}{c}=0.27$	$\frac{f}{c}=0.6$			
				(a)	(b)	(c)			
h = 25 c = 0 t = 15.7	6	16.80	1.33	1.29	3.48	7.73	19.42	21.61	25.86
	8	16.23	1.95	.97	2.61	5.80	19.15	20.79	23.98
	10	17.03	2.29	.77	2.08	4.62	20.09	21.40	23.94
	12	18.27	2.65	.65	1.74	3.86	21.57	22.66	24.78
	14	19.70	3.00	.55	1.49	3.31			
	16						(7.6)	(8.1)	(9.0)
h = 25 c = 0.25 t = 15.7	6	20.55	1.57	1.52	4.11	9.12	23.64	26.23	31.24
	7								
	8	20.82	2.04	1.14	3.08	6.85	24.00	25.94	29.71
	10	22.25	2.43	.91	2.47	5.49	25.59	27.15	30.17
	12	24.20	2.97	.76	2.06	4.58		29.23	
	14	26.40	3.56	.65	1.76	3.91		31.72	
	16						(7.0)	(7.5)	(8.5)
h = 25 c = 0.5 t = 15.7	6	23.75	1.78	1.67	4.51	10.02	27.20	30.04	35.55
	7	24.05	1.96	1.43	3.86	8.60	27.44	29.87	34.61
	8	24.80	2.23	1.25	3.38	7.51	28.28	30.41	34.54
	10	26.95	2.90	1.00	2.70	6.00	30.85	32.55	35.85
	12	29.80	3.59	.83	2.25	5.00			
	14								
	16						(6.2)	(6.7)	(7.6)
h = 25 c = 1.0 t = 15.7	6	32.00	1.89	2.02	5.44	12.10	35.91	39.33	45.99
	8	34.70	2.76	1.51	4.08	9.07	38.97	41.54	46.53
	10	39.10	3.38	1.21	3.26	7.24	43.69	45.74	49.72