THE STRESS DETERMINATION
BY THE PHOTO ELASTIC METHOD
OF THE SHRINKAGE STRESSES IN A GRAVITY DAM

Thesis by

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PURPOSE

The object of this experimental thesis was to determine the principal stresses in a theoretical gravity dam section on an elastic base due to a uniform shrinkage of the dam.

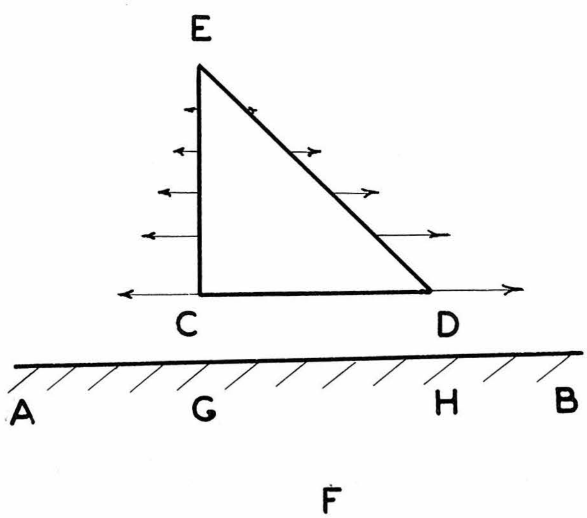
METHOD

The photo-elastic method is particularly adapted to this problem, only a two-dimensional stress analysis being required. A complete description of the theoretical background of this photo elastic method is given by Filon and Coker in their book, "Photo Elasticity", published by the Cambridge Press. A working summary of the theory together with some laboratory technique is given in a series of five articles published every two months in the General Electric Review by Filon starting with the November issue of 1920.

The only difference between the apparatus used here and that common throughout the other photo-elastic laboratories is that eight inch mirrors were used in obtaining parallel light instead of the more usual four or six inch lenses. A greater flexibility of apparatus and a larger working area on the model is permitted by this substitution of mirrors for lenses. Both mono-chromatic and white light were available but white light was used throughout this test by using a blue color filter in photographing the isochromatics.

To obtain the effect experimentally of having the dam shrink as a unit upon an elastic foundation, Dr. Biot suggested elongating the base by means of tension directly applied. Thus the base tends to elongate relative to the dam. Actually the reverse takes place; the dam shrinks relative to the base.

The most desirable procedure would be to stretch the dam CDE by heat or direct loading, then in this elongated condition fasten it securely along the base to the stress free elastic ground AB. This is a true representation of uniform shrinkage in a theoretical gravity dam section. A line of discontinuity in the stress pattern is encountered along AB, evident from the discontinuity in the elongation of either side of that line.



Roughly in this scheme the tip of the dam E is stress free; horizontal compression occurs in the ground in the region near the center of the dam base; tension gradually dissipates in the regions from G to A and from H to B.

To simulate this condition Dr. Biot's suggestion was acted upon. A homogeneous model of the dam and ground was made in one piece. The ground was stretched with a direct tensional load. This gave a high tension of say P lbs./sq.in. in the regions A and B, contrary to the above description of the problem. This must be corrected. Therefore, after the complete stress analysis had been determined a blanket horizontal compression can be arbitrarily laid down on the ground of

P lbs./sq.in. giving zero stress in the regions A and B; tension in the restricted regions G and H; and compression in the region F. This produces a most decided discontinuity along the base line CD. This is as it should be.

Thus it is seen that the analogy chosen holds for the dam proper, but not at all for the ground.

After the lines of constant shear and direction lines of principal stress had been determined it was found desirable to find the principal stress individually. Three methods were open:-

(a) Graphical integration.

(b) An electrical analogy.

(c) A membrane analogy.

Graphical integration is in rather common usage, but is long and tedious. No record was found of the electrical analogy having been used in this work. The Prandtl membrane analogy, also suggested by Dr. Biot, has been used in the solution of La Place's equation in problems of shafts carrying torque. Here soap membranes have been used.

A rubber membrane analogy was used on this problem. The analogy is based on the fact that both the sum of the principal stresses and the displacement of the membrane obey La Place's equation.

The model: Let P and Q be the principal stresses and   be Airy's stress function. Then:

$$\nabla^4\psi = 0 = \nabla^2 \cdot \nabla^2\psi$$

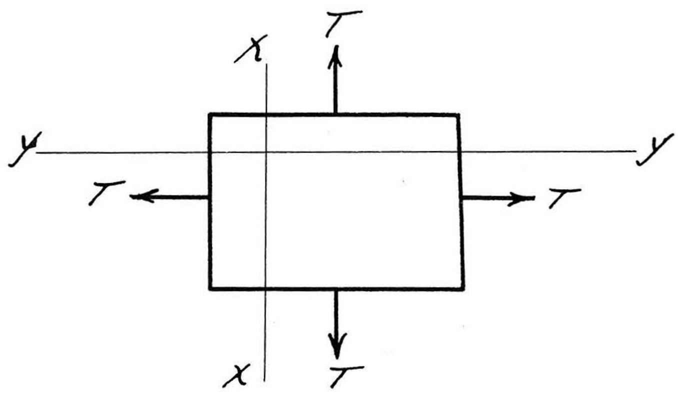
$$P_x = \frac{\partial^2\psi}{\partial y^2} \qquad Q_y = \frac{\partial^2\psi}{\partial x^2} \qquad P + Q = P_x + Q_y = \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial x^2}$$

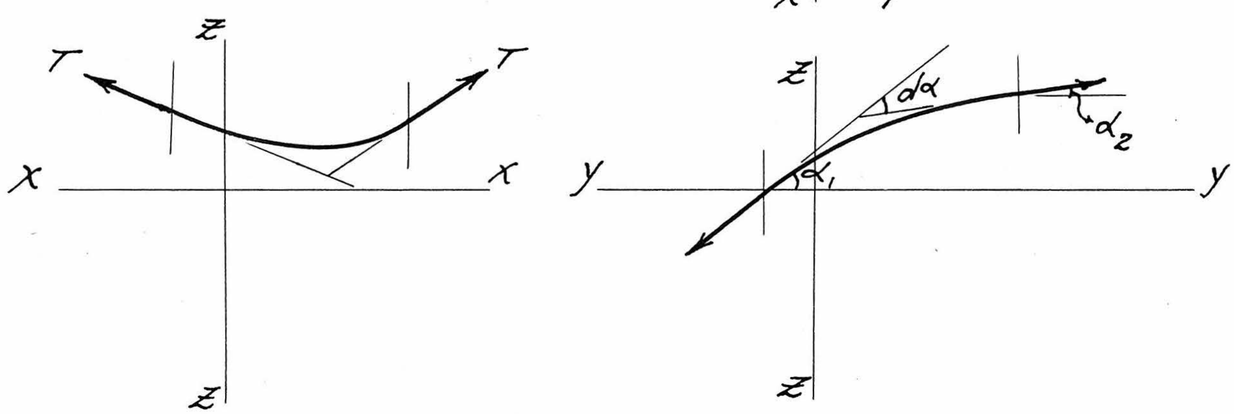
$$\nabla^2\psi = \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial x^2} = P + Q$$

$$\nabla^2(P+Q) = \nabla^2(\nabla^2\psi) = 0$$

Therefore

ELEMENT OF THE MEMBRANE





The membrane: Let the tension in the membrane be a constant T. Then it is in equilibrium as is evident from the view showing the xy plane.

In the xz plane the angle that T makes with the xy plane is $\tan^{-1}\frac{\partial z}{\partial x}$.

If the angle be small the tangent may be taken equal to the angle. The component of each T acting in the z direction then is $T\frac{\partial z}{\partial x}$. Considering the element dx, the resultant component of force in the z direction then is:

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)T$$

Similar reasoning in the yz plane leads to

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)T$$

To have equilibrium the following must hold true:

$$\frac{\partial^2 z}{\partial x^2}T + \frac{\partial^2 z}{\partial y^2}T = 0 \qquad \text{or} \qquad \nabla^2 z = 0$$

It is seen that z can be used as P + Q. P - Q is determined from the isochromatic lines, and at the boundary of the model one of the principal stresses is either zero or a known load is being applied. Thus P and Q are known along the boundary giving immediately P + Q. Thus if the membrane be deformed in the z direction proportional to P + Q along the geometric shape of the model boundaries, z can be measured at every point within the model thus determining P + Q.

This procedure was followed so that a complete stress analysis resulted for the model.

The model was made up so that tension could be put into the ground by means of pins as shown in Figure 3. To be sure of axial loading the tension strap was made

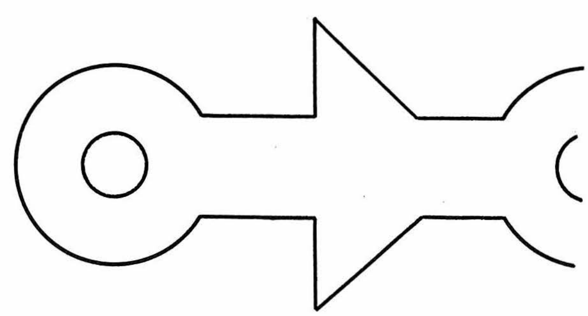


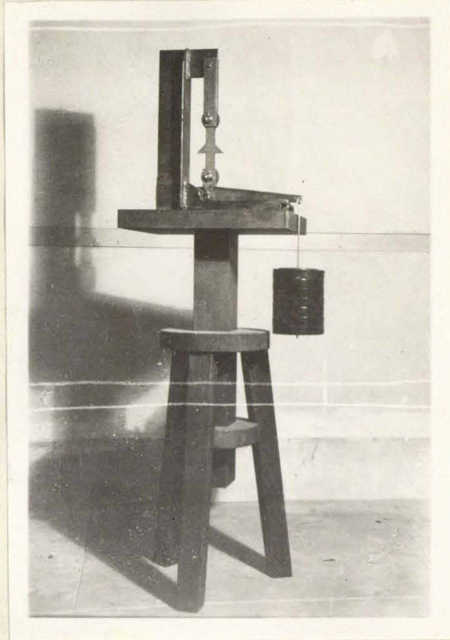
FIG. 3

symmetrical about its axis, thus the optical pattern was always a gauge of the symmetry of loading. The size of the model was limited to the eight inch working circle of light, the thickness being fixed by the standard 1/2" plates of bakelite available which were milled to 0.300 inches and then polished.

APPARATUS

It was necessary to construct two pieces of apparatus for the solution of this problem, a loading mechanism and the membrane mechanism.

Loading Machine

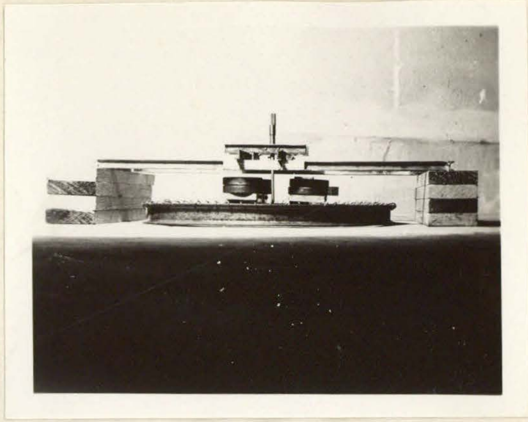


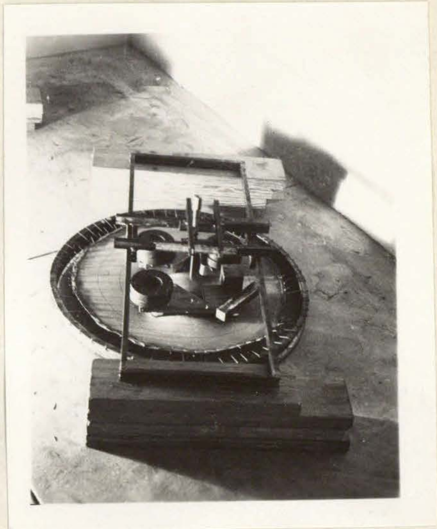
The loading frame is shown with the model in place in the accompanying photograph, Figure 4. Plans of its construction are provided in the appendix of original data. A load of 1220 lbs. per sq. in. was used in the strap of the model.

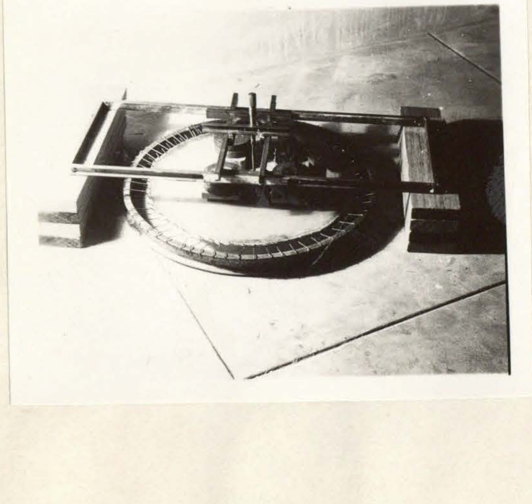
Membrane Apparatus

Several general views of the membrane with the deforming walls or boundaries are shown on the following page. The component parts, con-

THE MEMBRANE







sisting of a 1/64" rubber sheet stretched to twice its diameter on a hoop of channel iron may be seen. The concentric circles drawn before stretching remained circles on stretching. The deforming walls or boundaries are shown. Both a positive and a negative were made so as to be sure the rubber was at all times in contact with the boundary. The latter were made of 1/8" aluminum plate screwed to wood blocks to insure their retaining the correct shape. The negative and positive blocks were centered through the rubber by means of a ball and socket. The maximum difference in height in the boundary, (which was twice the model size) was 0.4 inches giving rise to a maximum slope in the membrane of approximately 45°. This steep slope was limited, however, to very restricted regions; namely, the corners, of the dam base. The elevation of the deformed membrane was obtained by means of a micrometer screw mounted upon a two way slide carriage provided with scales for purposes of orientation. Readings in elevation of the rubber sheet could be reproduced to within 0.003 inches, giving an accuracy of greater than one percent. Contact between the pointer and the rubber was noted by using a light focused to send light practically parallel to the rubber. Thus, when the pointer and its shadow met, contact was made.

PROCEDURE

After much experimental work, it was found that for best results the bakelite available should be annealed before milling. This was done in an electrically operated oven, loaned by the Chemistry Department. Best results were obtained by placing the bakelite on a glass plate horizontally in the oven, allowing one hour for the temperature to rise to 80° F.. This temperature was held constant for one hour when the plate was turned. It was held then for another hour at this temperature, after which it was cooled at the rate of 8° per hour.

All curvature of the bakelite having been taken out, the plate was milled to 0.300 inches thickness in four cuts taken alternately on each side. Mr. Sandale then machined out the model. A small test beam was made out of the same piece for optical calibration. From this the difference between fringe orders was found to be 160 lbs. per sq. inch.

A stress-strain diagram was plotted to 1250 lbs. per sq. inch, giving a beautiful straight line relationship leading to a modulus of elasticity of 740,000 lbs. per sq. inch.

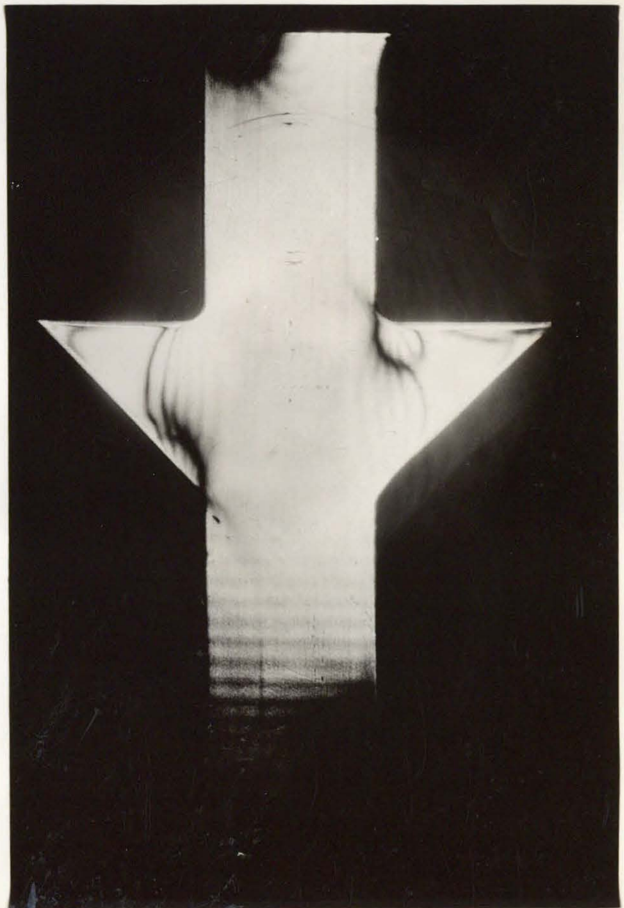
A complete set of isoclinics was taken on Eastman super-sensitive panchromatic film, a few samples of which are shown on the following pages. Isochromatics were taken on commerical ortho with the aid of a blue filter, (see Figure 5.) White light was used throughout.

ISOCLINIC PICTURES

0°

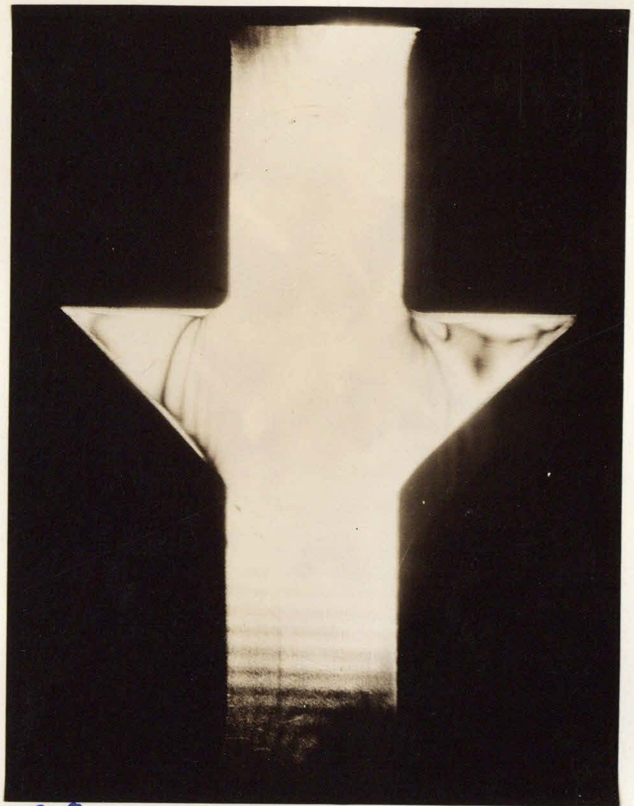
3°

5°

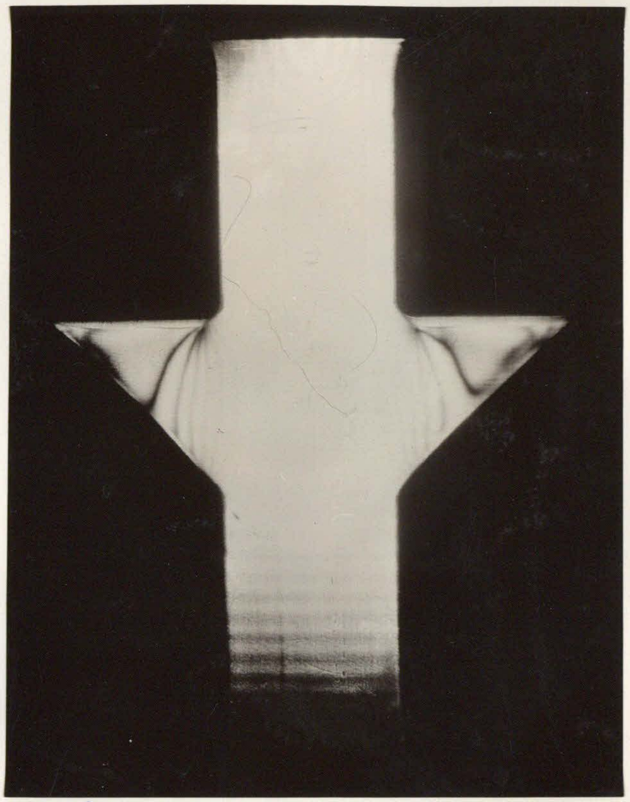


15°

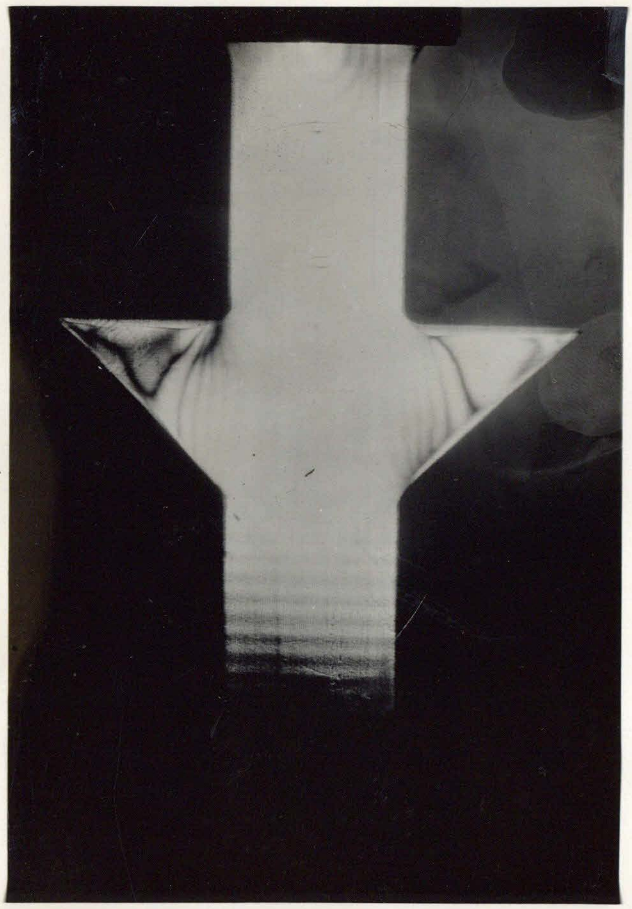
ISOCLINIC PICTURES



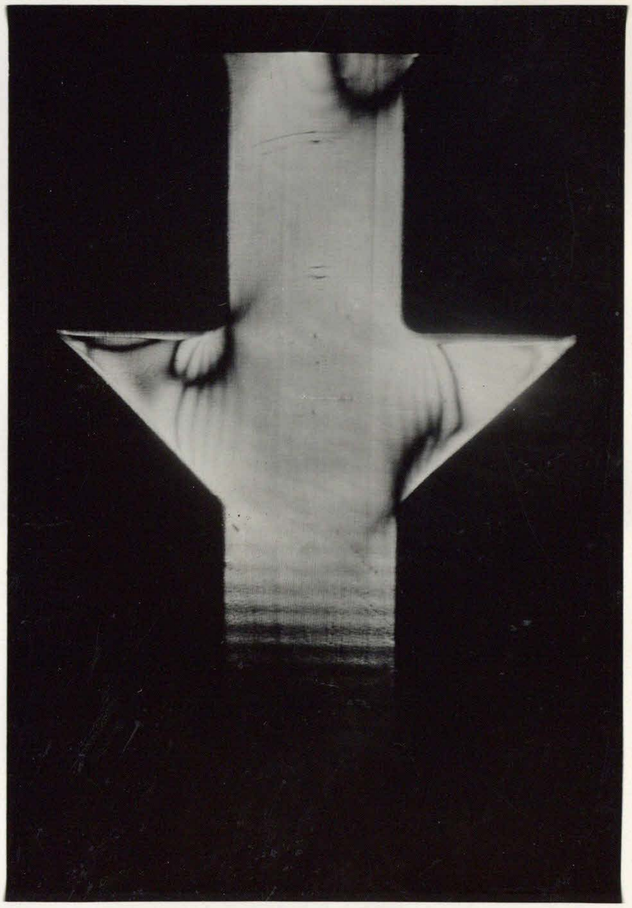
30°



45°



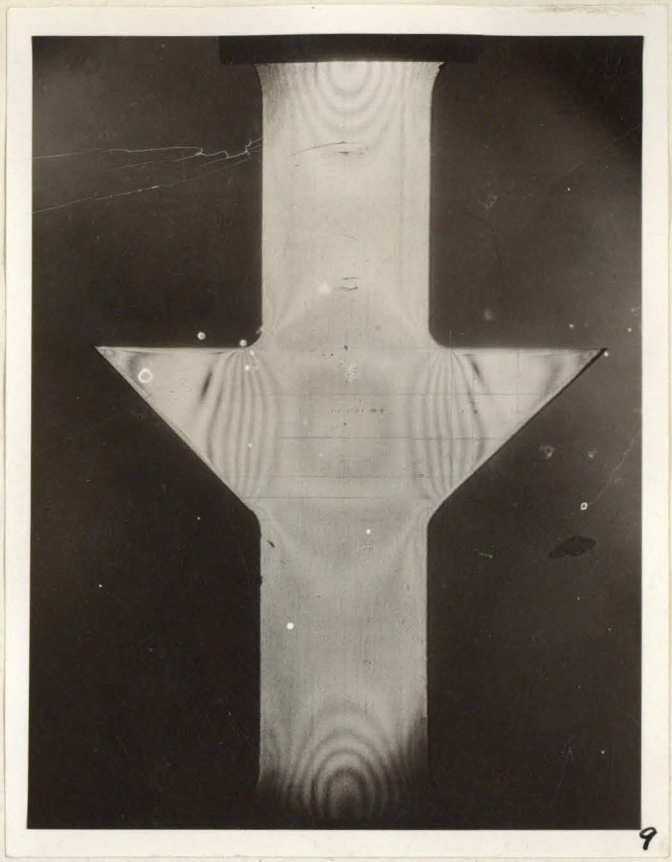
60°



75°

ISOCHROMATIC
PICTURE

FIG. 5



The sum of the principal stresses was found for all points on the boundary of the model from which the boundary walls were made. The deformed membrane was then surveyed, the data thus obtained being used for the computation of the principal tensions and compressions.

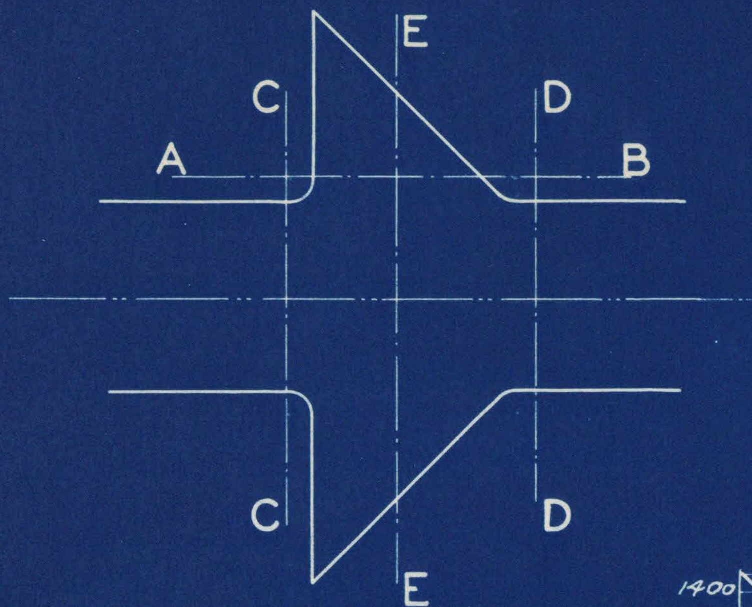
Checks against equilibrium were made by obtaining the forces perpendicular to and the shears along various sections through the model to check on the experiments' accuracy. The results are shown on the following page.

They show:-

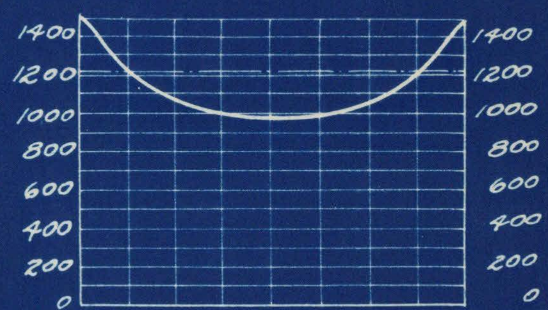
(a) A 15% error in the shear along AB.

(b) An 8% error in the tension along AB.

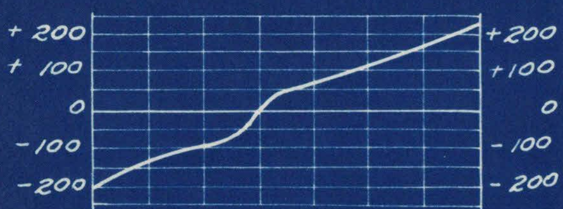
CHECKS AGAINST EQUILIBRIUM



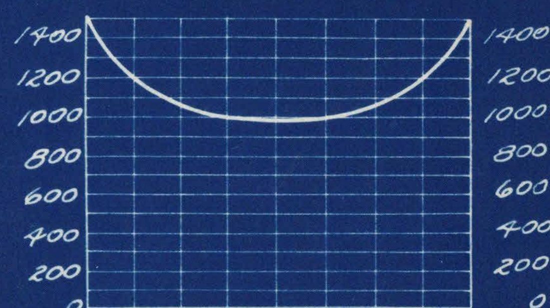
LINES ALONG WHICH CHECKS WERE MADE



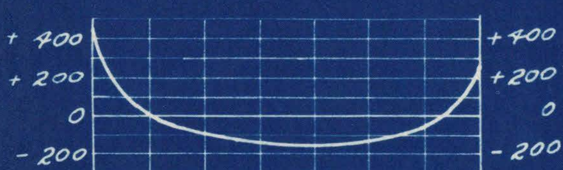
TENSION ALONG CC



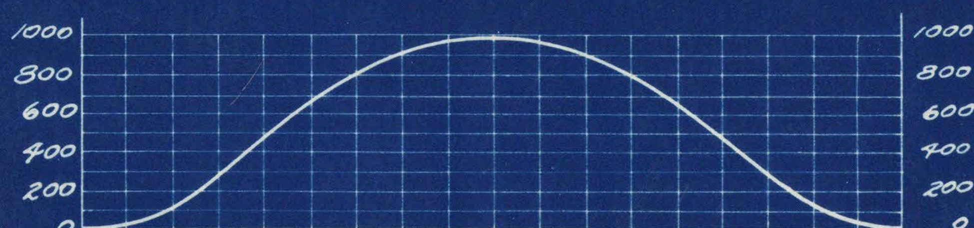
SHEAR ALONG AB



TENSION ALONG DD



TENSION ALONG AB



TENSION ALONG EE

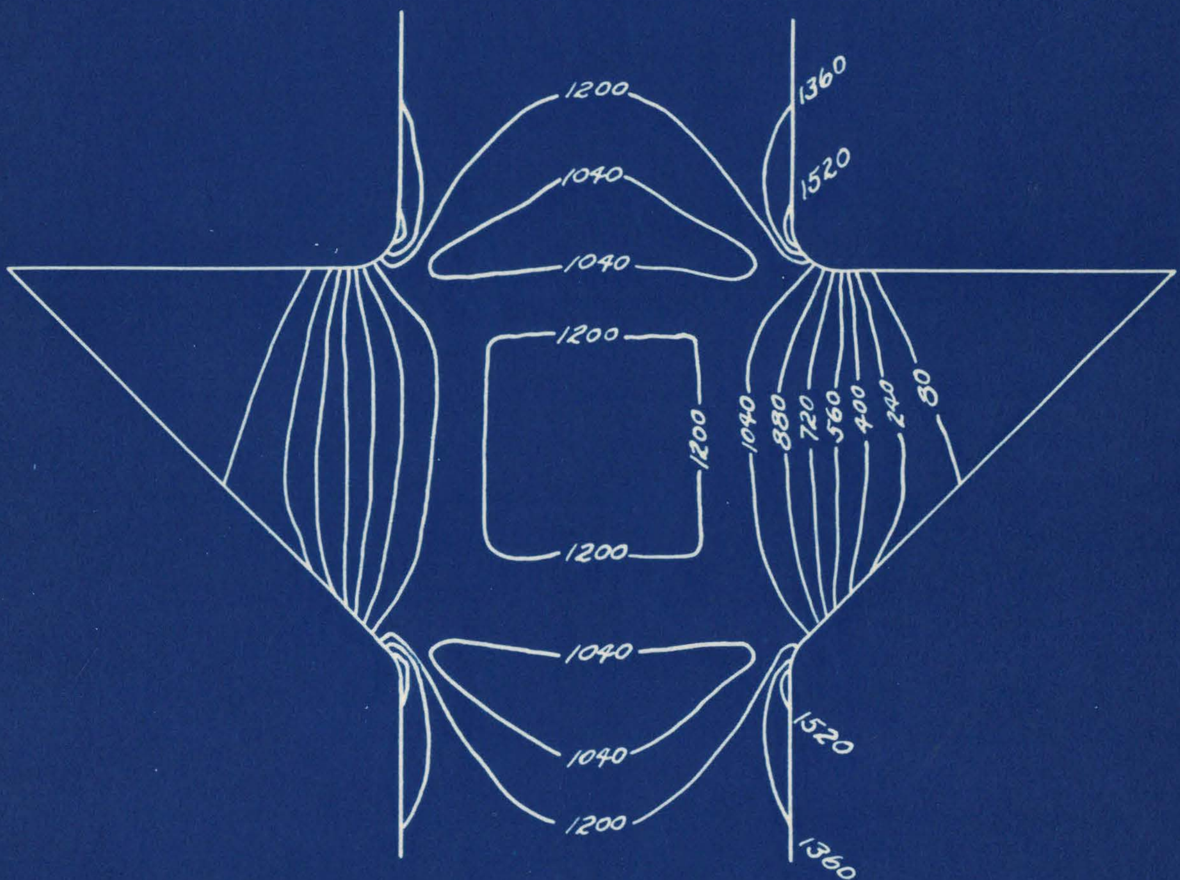
(c) A 5% error in the tension along CC.

(d) A 5% error in the tension along DD.

(e) A 2% error in the tension along EE.

All results were drawn up and are presented in the following prints.

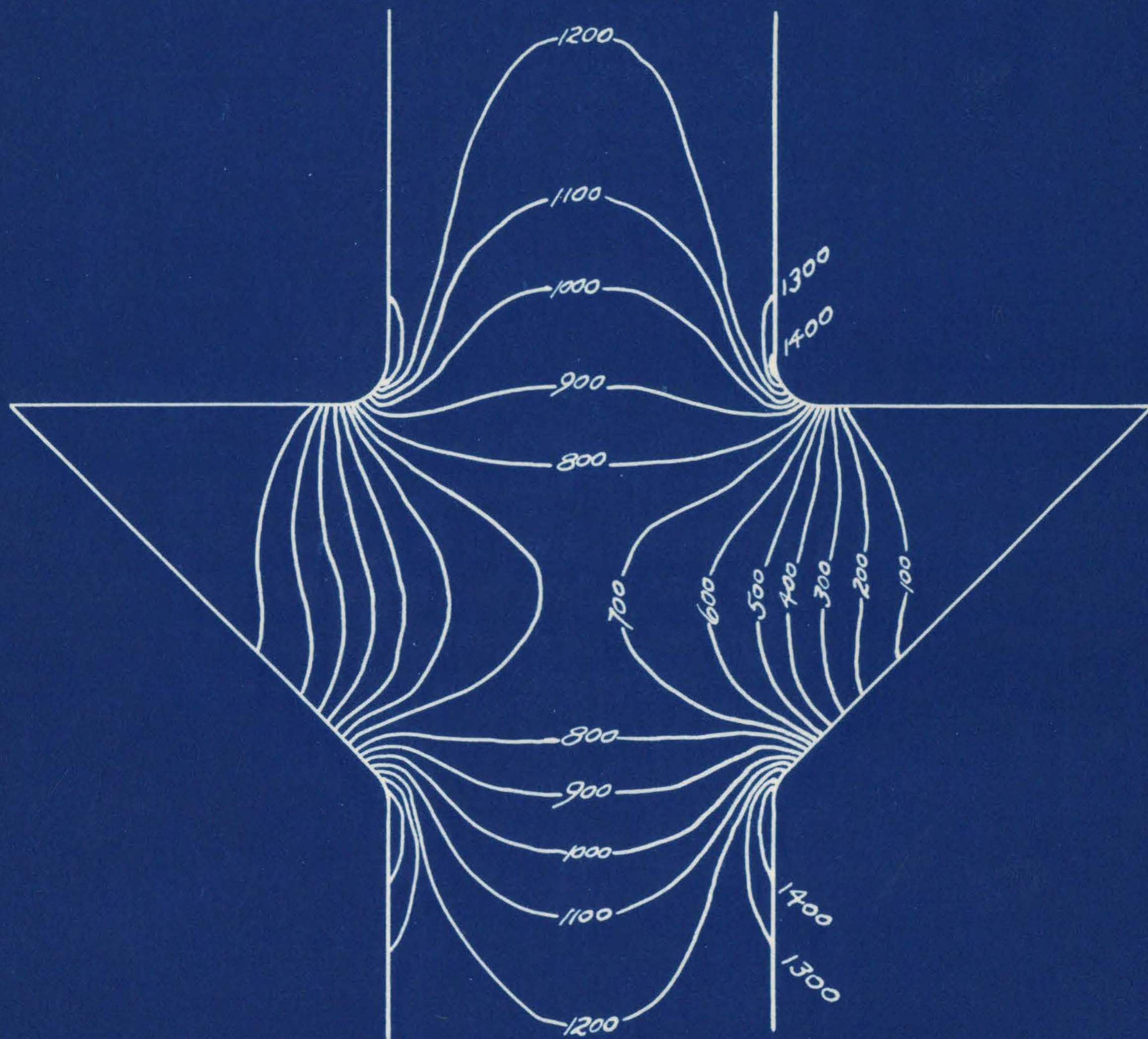
ISOCHROMATIC
LINES OF CONSTANT
P - Q



TAKEN FROM PHOTOGRAPH

LINES OF CONSTANT

P + Q



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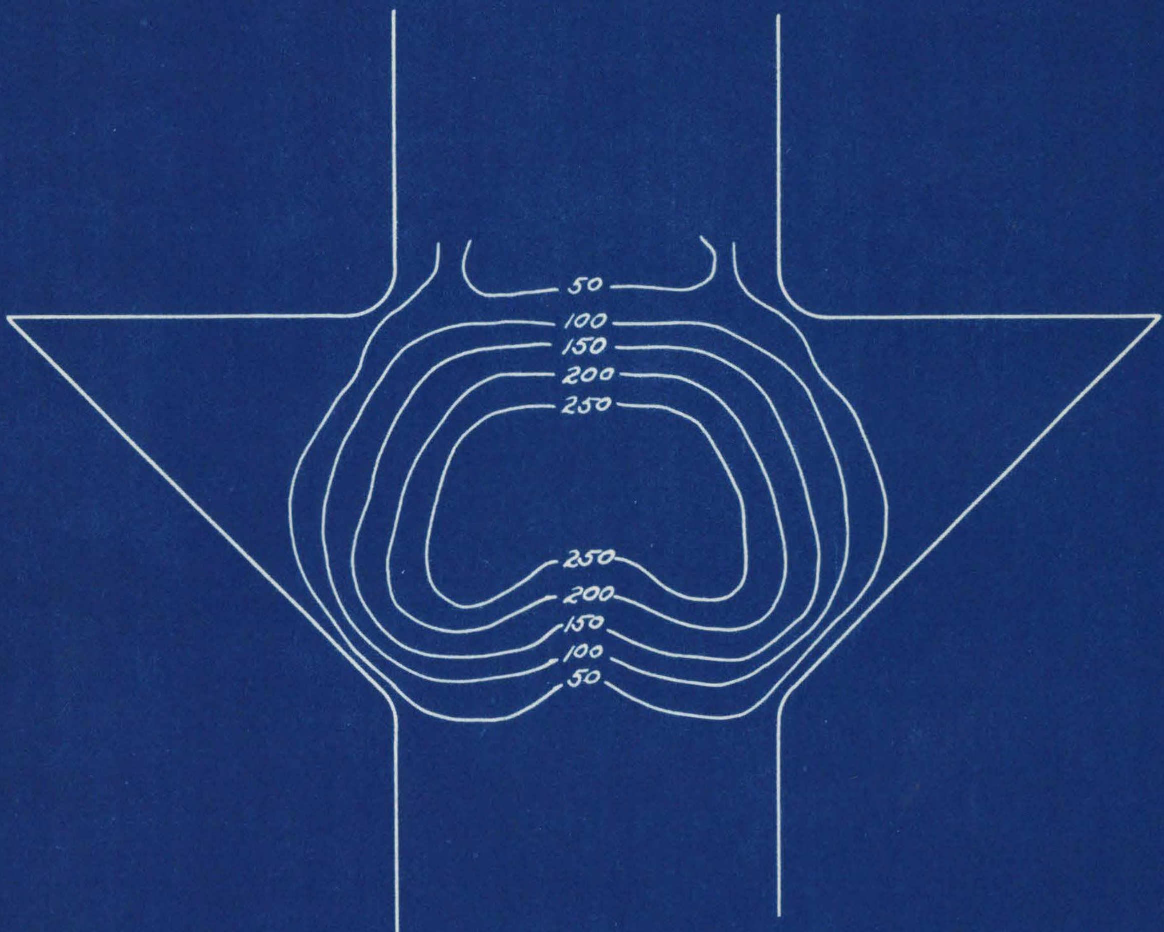
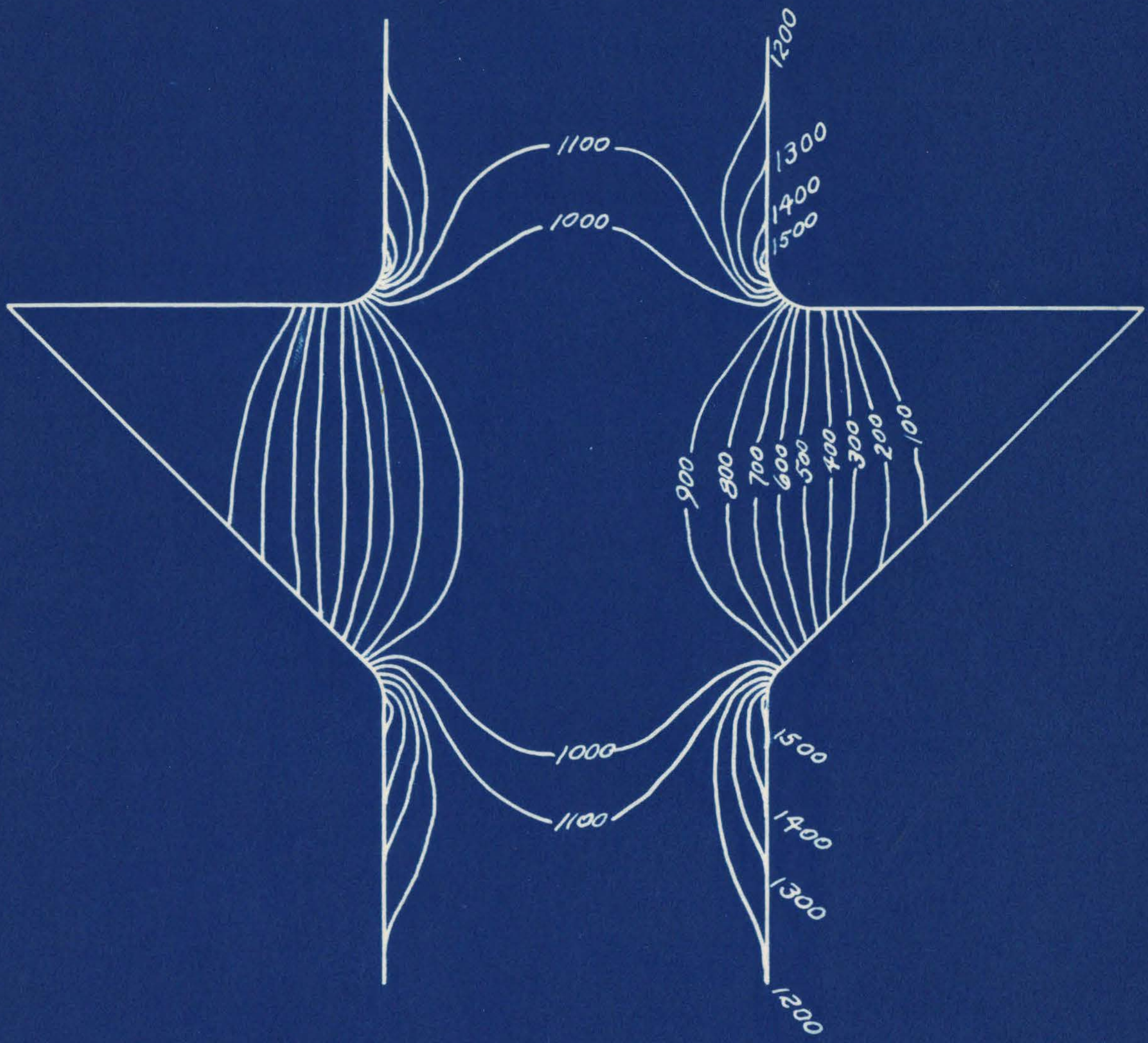
MEMBRANE

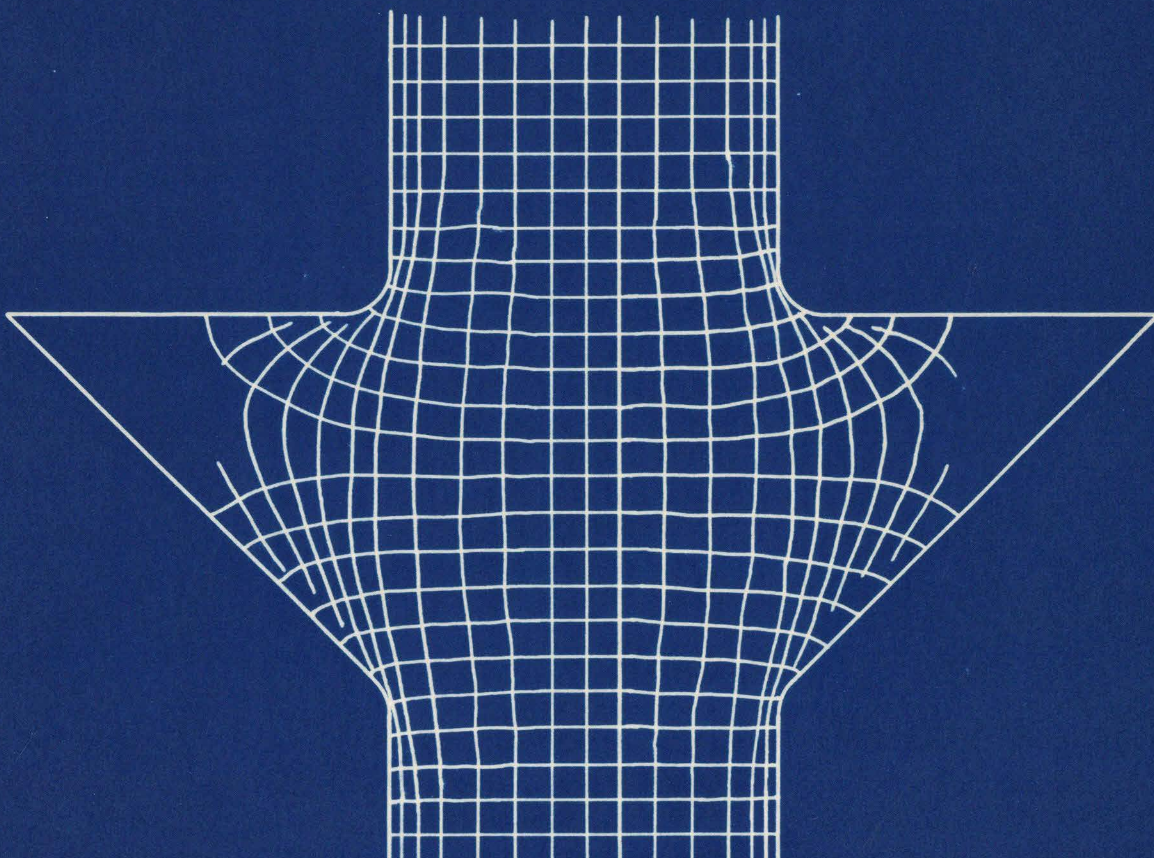
LINES OF CONSTANT
Q - COMPRESSION



LINES OF CONSTANT P - TENSION



DIRECTION LINES OF PRINCIPAL STRESS



TAKEN FROM THE ISOCLINICS

CONCLUSIONS

The model test was of a theoretical condition that never exists in the field, and for that reason the results shown in the following page are only of philosophical interest. Points of high stress are indicated, however, and a general feeling for the behavior of dams under shrinkage stresses can be obtained from the results.

The accuracy of the method is remarkably good, especially as the membrane boundaries were in this case rather crude. It can be said from this experience that if bad stress concentrations do not exist, the membrane method is accurate, easily applied, and convenient; but it has the disadvantage that each set of boundary walls is unique and in general cannot be used to fit any succeeding problem.

Results shown hold only in the dam proper, not in the foundation material. If an insight is desired into this stress condition, a rough picture may be had by superimposing a uniform horizontal compression on the ground of 1200 lbs. per sq. inch. This gives a line of discontinuity along the union of the dam with the ground. A discussion of this condition is given on page 2 of this paper.

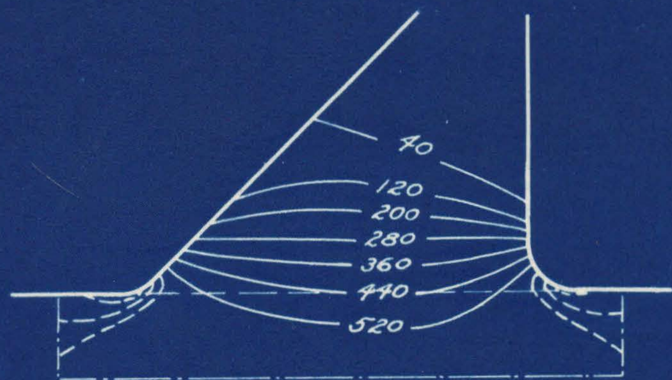
It is my opinion that these results are significant; that a philosophical approach to shrinkage stresses in gravity dams is to be had from them; and that the membrane method is applicable to design offices where the photo-elastic method of stress determination is used.

STRESSES IN A GRAVITY DAM

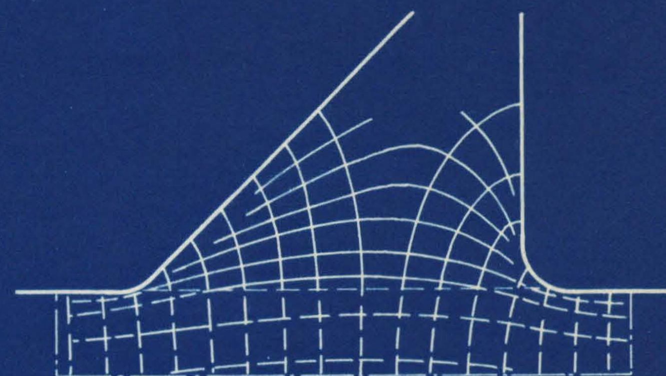
1.65×10^{-3} IN. PER IN.

SHRINKAGE

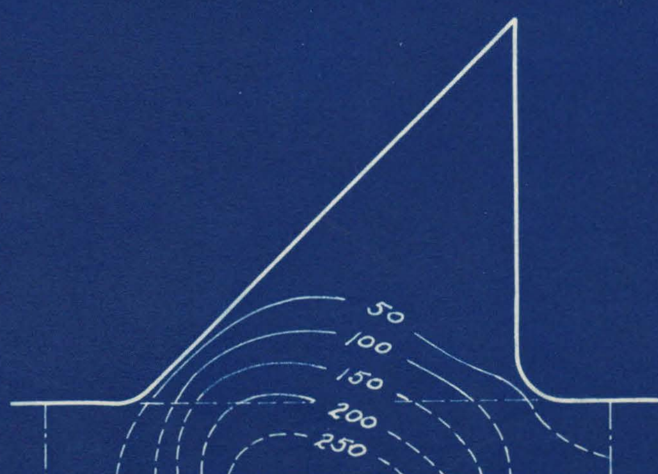
#/□



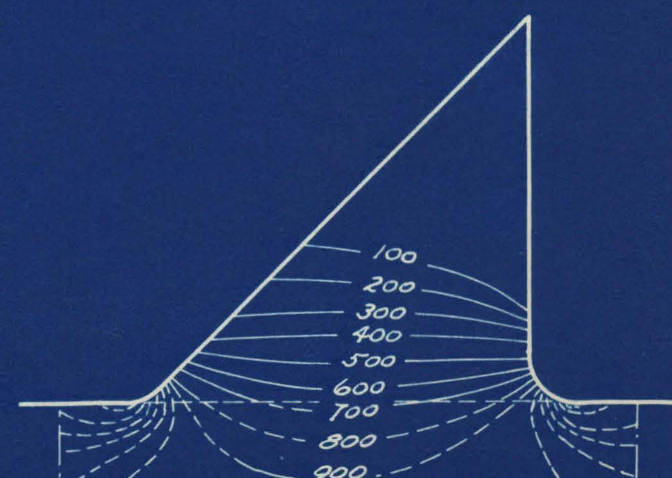
SHEAR



DIRECTION OF MAX. STRESSES



COMPRESSION



TENSION