THE SCATTERING OF HARD X-RAYS

A Thesis by

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In Partial Fulfillment of the Requirements for the Degree

of Doctor of Philosophy

CALIFORNIA INSTITUTE OF TECHNOLOGY

Pasadena, California

1934

ABSTRACT.

An experimental investigation of the scattering of hard, monochromatic x-rays was undertaken to test the validity of the Compton, Dirac-Gordon, and Klein-Nishina formulas. X-rays from a tube excited by voltages up to 1,000 k.v. were monochromatized by means of a crystal spectrometer. A monochromatic beam of x-rays of wave-length 24 x-u. was passed through a C.T.R. Wilson expansion chamber. Stereoscopic pictures were taken of the Compton recoil electrons originating in the atoms of the gas in the expansion chamber. These photographs were analyzed, and the spatial distribution of the recoil electrons was studied.

The mathematical transformations necessary to reduce the scattering formulas to a form more easily tested by experiment, is presented.

It was concluded that the Klein-Nishina formula, which is based upon Dirac's relativistic interpretation of the quantum mechanics, was the one most nearly in accord with the experimental results. Small systematic differences were observed, however, which were thought to exceed the experimental error. The Dirac-Gordon formula, based on the Schroedinger wave-mechanics, was shown to be in bad disagreement.

THE SCATTERING OF HARD X-RAYS

Introduction

The phenomenon of the scattering of x-rays by matter was observed[1] very soon after the discovery of x-rays. That such scattering should exist may readily be explained on the basis of the electromagnetic theory of x-rays, for the electrons in matter will be set into vibration by the electromagnetic field. These electrons will then radiate x-rays of the same frequency as the incident radiation, according to the classical formula for the radiation from a vibrating charge.

It was upon this basis that Thompson[2] derived the well-known scattering formula

$$\frac{I_\varphi}{I_0} = \frac{e^4}{2m^2c^4}\left(1+\cos^2\varphi\right) \qquad (1.)$$

which gives the ratio of the intensity of x-rays scattered per unit solid angle at an angle φ with respect to the incident beam to the intensity of the incident beam. e and m are the charge and mass of the electron responsible for the scattering; c is the velocity of light. This formula was experimentally confirmed in the region of soft x-rays.[3]

In the region of hard x-rays and γ-rays, the experimental results were greatly at variance with the Thompson formula.[4,5,6] The total amount of scattering was found to be much less than that predicted, and the angular dependence of the scattered intensity was very different from the $(1+\cos^2\varphi)$ Law. In addition it was found that the radiation scattered at large angles was of much longer wavelength than that of the primary beam. These anamalous results received no explanation until 1922 when Compton[7] showed that the observed softening of scattered x-rays is in accord with the theory that x-rays are quanta of energy $h\nu$. He later showed[8] that due to the Doppler effect due to the recoil of the electrons responsible for the scattering, the intensity of scattered x-rays should be:

$$I_\varphi / I_o = \frac{e^4}{2m^2c^4} \frac{1+\cos^2\varphi + 2\alpha(1+\alpha)(1-\cos^2\varphi)}{[1+\alpha(1-\cos\varphi)]^5} \qquad (2.)$$

where $\alpha = h\nu / mc^2$, ν being the frequency of the primary beam.

Many other formulas were advanced at this time, but these were all superseded upon the development of wave mechanics. Using the Schroedinger wave equation as a basis, Dirac[9], and later Gordon[10], derived the following expression.

$$\frac{I_\varphi}{I_o} = \frac{e^4}{2m^2c^4} \frac{1+\cos^2\varphi}{\{1+\alpha(1-\cos\varphi)\}^3} \qquad (3.)$$

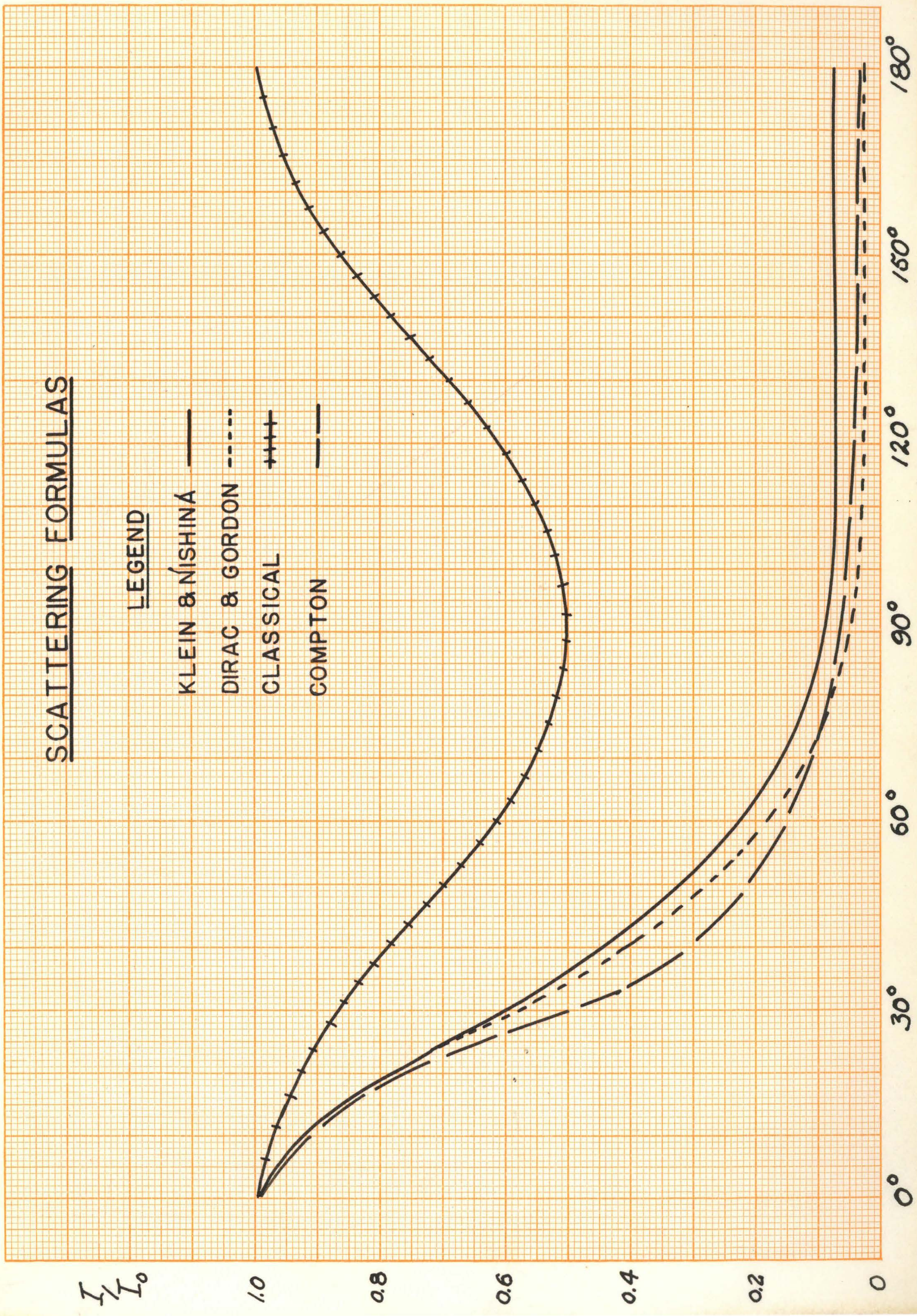
The most recent formula is that obtained by Klein and Nishina[11]. At present this formula is considered to have the soundest theoretical foundation, for it is derived from the relativistically correct equations of Dirac. The expression given by Klein and Nishina is

$$\frac{I_\varphi}{I_o} = \frac{e^4}{2m^2c^4} \frac{1+\cos^2\varphi}{[1+\alpha(1-\cos\varphi)]^3} \left\{ 1 + \frac{\alpha^2(1-\cos\varphi)^2}{(1+\cos^2\varphi)[1+\alpha(1-\cos\varphi)]} \right\} \qquad (4.)$$

A plot of these formulas is given in figure 1. It is to be noted that equations (2.), (3.), and (4.) approach the value given by the classical formula as α approaches zero; i.e., in the region of long wavelengths. This is in agreement with the correspondence principle.

The remainder of this paper will be concerned with the experimental methods of measuring the scattered intensity of x-radiation and the correlation of the results with the several formulas.

FIGURE I.



SCATTERING FORMULAS
LEGEND
KLEIN & NISHINA
DIRAC & GORDON
CLASSICAL
COMPTON
0°
30°
60°
90°
120°
150°
180°
I/I₀
1.0
0.8
0.6
0.4
0.2
0

Methods of Investigation of the Scattering Law

There are three general methods of testing the validity of the scattering formulas, as follows:

1. Measurement of scattered intensity at various angles
2. Absorption measurements
3. Measurements of the spatial distribution of the Compton recoil electrons.

In all of these methods it is important to use monochromatic x-rays of short wavelength, for it is in the region of hard-radiation that the differences among the scattering formulas become most pronounced. If the second or the third method be used, it is imperative that the photoelectric effect be negligible, as it will be only if hard x-rays and scatterers of small atomic number be employed.

The first method is the most direct and is the one which has been used by Kohlrausch , Compton , and others for showing the incorrectness of the Thomson formula (1.) in the region of short wavelength x-rays. The experimental procedure is briefly as follows. A beam of x-rays is allowed to fall on a scatterer, such as a block of carbon. The scattered x-rays are detected by a small ionization chamber which is fastened to an arm free to move about an axis through the scatterer. This arrangement permits of varying the angle φ in the scattering formulas from zero to 180 degrees. This method is open to four objections. First, it is difficult to get a monochromatic beam of sufficiently high intensity. Second, inasmuch as the wavelength of the scattered beam is a function of the angle, it is necessary to know the ionization function of the ionization chamber. As a rule this can be only approximately determined. Third, part of the scattered radiation will be absorbed in the scatterer. This is particularly true at large angles. Fourth, the fact that the measurements at large angles are particularly inaccurate is unfortunate, for, as can be seen from Figure 1., the region of large angles is the one where the discrepancies among the various formulas are most prominent.

The second method, that of measuring absorption coefficients, is the one most generally used because it is capable of high accuracy without too elaborate an experimental arrangement. This method is only capable of measuring the integrated value of the scattering formulas and hence it tells nothing at all about the angular distribution of the scattered x-rays. It is, however, possible to distinguish between the various formulas by this method for wavelengths shorter than 100 x-units. Absorption measurements in this range are in definite agreement with the Klein and Nishina formula.[12]

The third method, that of determining the spatial distribution of the scattered electrons produced by Compton recoil, is the one used in the investigation here reported. No information as to the absolute values of the intensities has been obtained by this method, nor is this necessary since the absorption method gives such excellent results in this connection. One can, however, determine the ratios of intensities of x-rays scattered in the various angular intervals. The theory of this method is, briefly, as follows. The scattering formulas predict the intensity, and, what is the same thing, the number of quanta scattered at any angle φ. For every quantum scattered at an angle φ there is an electron scattered at an angle ϑ, where φ and ϑ are connected by the Compton relation:

$$\cot \frac{\varphi}{2} = -(1+\alpha)\tan\vartheta \qquad (5.)$$

Hence, one can calculate from the scattering formula the number of electrons to be expected in the interval from ϑ to $\vartheta + d\vartheta$.

This method was first used by Skobelzyn . A filtered beam of γ-rays from Radium (B + C), after collimation by lead slits, was allowed to pass through a Wilson cloud chamber. The recoil electron tracks were bent in a magnetic field. Both the angle made with the incident γ-ray beam and the radius of curvature of each track were measured, so that it was possible to calculate the wavelength of the incident quantum by means of the relation

$$\frac{E}{h\nu} = \frac{2\alpha}{1+2\alpha+(1+\alpha)^2\tan^2\vartheta} \qquad (6.)$$

where, as before, $\alpha = \frac{h\nu}{mc^2}$ and E is the energy of the recoil electron as measured by its curvature in the known magnetic field. Skobelzyn obtained about 1300 tracks in all, of which 900 were obtained with radiation filtered through 3 mm of lead and 400 with radiation which had been filtered through 11.3 mm of lead. He concluded that the Klein and Nishina formula more nearly fitted the observed results than either the Dirac-Gordon or the Compton formula. He also found, however, large deviations in certain angular intervals from the number predicted by the Klein and Nishina formula, these deviations being too large to be explained as experimental errors or statistical fluctuations. He points out in particular that the experimental distribution has a minimum not predicted, he says, by any of the scattering formulas. This may be explained by the fact that he did not plot the Klein-Nishina distribution carefully enough, for as shown in Figure 6 , a minimum actually exists.

In his paper[13] Skobelzyn himself regrets that no source of hard monochromatic radiation is available, and that for this reason it was necessary to bend the tracks in a magnetic field. This magnetic deflection of the electrons introduces not only the errors of measurement of the curvature of the tracks but also gives rise to the more important difficulty that it makes extremely uncertain the measurement of the angle that the tangent to the path of the electron makes at the point of ejection with the direction of the beam of incident radiation. Sometimes the exact point of origin of the tracks is not clearly shown in the photographs, while the scattering of the slow electrons ejected at large angles with the direction of the beam complicates both the measurement of curvature and the measurement of angles.

The inhomogeneity of the incident radiation also greatly complicates the calculations. Skobelzyn resorted to the scheme of dividing the radiation spectrum into frequency intervals and determining the realtive intensity and effective wavelength in each interval.

As a consequence of these objections, it was decided to repeat the experiment, using a monochromatic beam of hard x-rays. Under these conditions the necessity of using a magnetic field and the resulting difficulty of measuring radii of curvature and determining direction of tangents is avoided, and also one need not deal with "effective wavelengths", as did Skobelzyn.

Experimental Arrangement

The arrangement of the apparatus is shown in Figure 2.

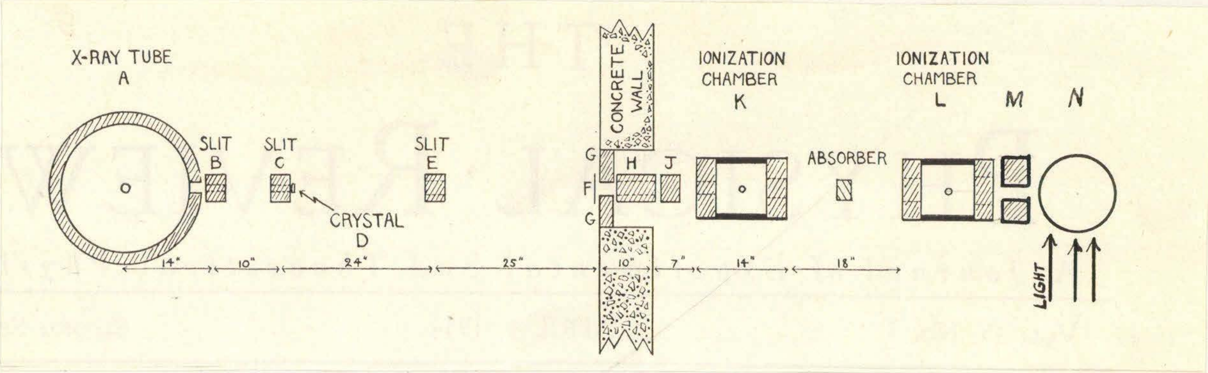


Figure 2.

The source of radiation was the x-ray tube A, which could be excited by voltages up to 1, 000 kilovolts. At this voltage, the tube current could be raised to 8 milliamperes without overheating the gold target. This furnished a very intense source of hard x-rays. Slits in the lead blocks B and C defined a plane horizontal x-ray beam, which was incident on the rock salt crystal D, set at such an angle that radiation of the desired wavelength was reflected from the internal atom planes. The slit in the lead block E was so placed that the monochromatic reflected beam was transmitted, while the unreflected beam was absorbed in the block. The monochromatic radiation then passed through the slits H, K, and L into the cloud chamber N. M was a vertical slit of adjustable width for limiting the breadth of the beam, the thickness in the vertical direction having been defined by the previous slits. The shutter J was operated by an electromagnet so arranged that the shutter opened just at the time of expansion of the chamber.

The C. T. R. Wilson expansion chamber employed in this investigation was essentially that described by Simon and Loughridge[14]. The expansion was made by removing the air under the piston, the air flowing out through a magnetically operated valve. The tracks were illuminated with parallel light from a carbon arc, the arc being flashed at the time of expansion by passing a current of 400 amperes through it. A magnetically operated shutter, placed in the path of the light beam, was opened only at the time of expansion, thus preventing undue heating of the gas in the expansion chamber. Thirty volts, applied between the floor and the roof of the expansion chamber, served to sweep out stray ions. This voltage was not cut off at the time of the expansion. The cycle of operations was carried out by means of electrical contacts on rotating disks.

Two plane face-silvered mirrors were placed perpendicular to the roof of the expansion chamber, as shown in figure 2. It was found that an evaporated aluminum coating[15] on the mirror was much superior to a silver coating, the latter tending to tarnish very rapidly. The camera was placed midway between the two mirrors at a distance of 57 cm from the chamber, and stereoscopic pictures were obtained by photographing the chamber and the images of the chamber in the two mirrors. The pictures so obtained are in triplicate, as isshown in figure 3. The line down the center of each picture indicates the position and direction of the x-ray beam and is of great aid in the analysis of the photographs.

A "Leica" camera, having a lens of 50mm focal length and a focal ratio of 3.5 was used for photographing the tracks. It was loaded with 36mm super-sensitive panchromatic motion picture film.

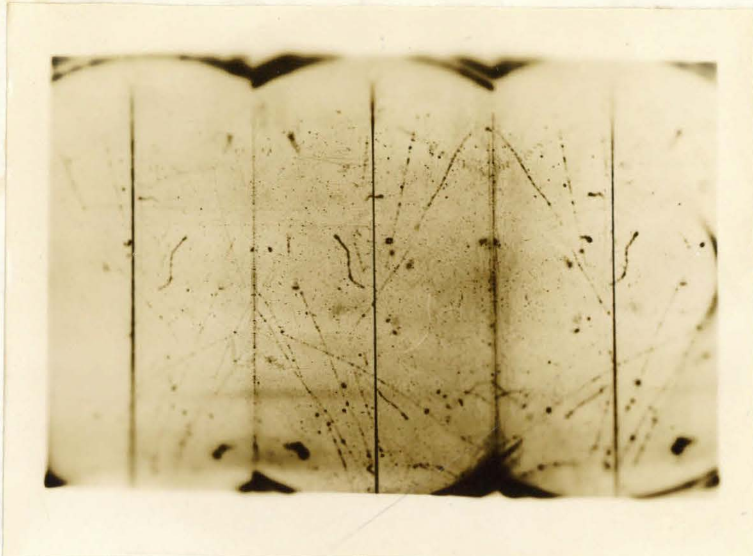


Figure 3.

Experimental Procedure

Due to the fact that the x-ray tube was being used for medical work it was necessary to replace the apparatus in position each time a set of measurements were made. This was facilitated by mounting the spectrograph and the cloud chamber on frames equipped with casters. The spectrograph was first rolled into position and adjusted for maximum intensity of x-rays through the slits A and B. The crystal was then adjusted to the proper angle for obtaining the desired wavelength (24 x u for this work) by taking photographs of the direct and reflected beams at J with the slits E and H removed. The separation of the direct and the reflected beams at J (169 cm from the crystal) is about 14mm, and can be clearly seen from figure 4. The crystal could be approximately adjusted by observing the direct and reflected beams with a fluoroscope; reflected beams as short as 50 x u could be observed by this means. Inasmuch as the radiation was taken at a very small angle with the target, it came effectively from a line source. The geometry of the slits was such that the arrangement used gave a reflected beam of about 5 x u width.[12] This was borne out by the spectrograms taken at J (cf. figure 4.).

PHOTOGRAPH OF DIRECT AND REFLECTED X-RAY BEAMS AT "J"

Figure 4.

The slits K and L were next adjusted, first by geometrical means and finally by plotting the position of the slits against the rate of discharge of the ionization chambers and then setting the slits at the point at which the maximum ionization had been observed. The cloud chamber was then rolled into position. It was adjusted by removing the ionization chambers and sending a beam of visible light through the slits and the cloud chamber, making sure that it passed through the center of the latter. It was important that this be properly done, for if the x-ray beam was too near one side of the chamber, either top or bottom, the electron tracks going either up or down would be so short as to make measurements uncertain. This unfortunately happened on two occasions in the present work and both sets of data had to be discarded. Two other sets of data were never analyzed for the reason that the intensity of the reflected beam was small.

After the cloud chamber had been placed in position, the sheet of light from the arc was directed so as to fill the cloud chamber as completely as possible without allowing any light to hit the floor or the roof of the cloud chamber, as this would fog the photograph.

The operations outlined above usually required about 12 hours. The remainder of the time during which work could be carried on was employed in taking photographs, about a hundred being taken before removing the apparatus.

Expansions were made at 1½ minute intervals, this much time being allowed for the chamber to reach temperature equilibrium. The fact that this time is large compared to that allowed by other cloud chamber workers is due to the fact that the cloud chamber used in the present investigation is large in comparison with other chambers.

Analysis of Photographs

The photographs having been developed, the film was placed behind the same lens that was used in the photography. A strong light source was placed behind the film, and the light projected on to two mirrors, the geometry of the film, lens, and mirrors being the same as that used in originally photographing the tracks. The tracks were projected on a ground glass screen held near the bottom end of the mirrors. In general, the image of any track was seen to be triple on the ground glass, but by holding the screen at the proper angle and in the proper position the three images could be made to coincide. The position of this single image is then the same as that of the track in the cloud chamber at the time it was photographed. It was upon this principle that the measurements of angle were based.

The lens and mirror system was placed in a horizontal position so that the image of a line parallel to the x-ray beam (the dark line along the middle of each photograph) was vertical. It is then merely necessary to measure the angle that the projected image of each track makes with the vertical. This was done by the use of a rectangular piece of ground glass with a line drawn on it parallel to one edge. A protractor was fastened by means of hinges to this edge and a plumb bob swung from the center of the protractor. Then, to measure the angle of a track it was necessary simply to bring the line on the ground glass plate into coincidence with the position of the track in space, the angle being read directly by observing the position of the plumb bob along the circular scale of the protractor.

The best tracks could be measured with an accuracy of 1 or 2 degrees. The tracks making large angles with the floor of the chamber were the most difficult to measure, and the error on these tracks might occasionally be as large as 8 degrees. The number of tracks so unfavorably placed is small, however, and the errors tend to average out. The electrons ejected at large angles are slow, and hence badly scattered and difficult to measure; it was therefore decided to include in the results no tracks making an angle greater than 70 degrees.

In spite of the fact that the x-ray tube was shielded by 2 inches of lead and the cloud chamber was placed in a room shielded from the tube room by 10 inches of cement, there still existed an appreciable amount of scattered radiation in the vicinity of the cloud chamber. This resulted in quite a number of spurious tracks in the photographs; most of which, however, originated in the walls of the chamber, so that by counting only those tracks which originated in the well-defined x-ray beam, the number ofspurious tracks included would be negligibly small. The variation of energy and range with the angle of ejection also provides a criterion for selecting tracks, but it is almost never necessary to use this criterion.

The method of projecting the tracks which was employed is particularly suitable for stereoscopic viewing. On placing a large ground glass plate flat against the end of the mirrors, three images of each track can be seen. Due to the fact that ground glass scatters most of the light in the forward direction it is possible to place one's head in such a position that one of the images completely vanishes. Of the remaining two, one appears very intense to the right eye, while the other appears very intense to the left eye. This is just the condition necessary for good stereoscopic vision, and with a little practice the single tracks can be made to stand out in space. This procedure is sometimes very useful when several tracks are close together , and frequently serves as an aid to measuring.

Before the data can be compared with the scattering formulas it is necessary to transform the formulas.

Electron Distribution Predicted by Scattering Formulas.

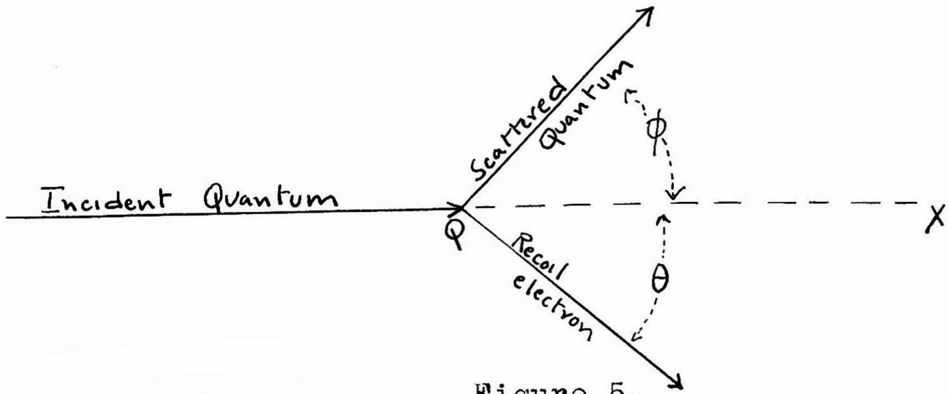


Figure 5.

Consider a quantum of energy $h\nu_0$ progressing in the direction of the x axis. What is the probability that an electron at Q will scatter the quantum inside the conical shell ϕ to $\phi+d\phi$? According to the Klein-Nishina formula the energy scattered at a direction ϕ is

$$\frac{I_\phi}{I_0} = \frac{e^4}{2m^2c^4}\,\frac{1+\cos^2\phi}{\left[1+\alpha(1-\cos\phi)\right]^3}\left\{1+\frac{\alpha^2(1-\cos\phi)^2}{(1+\cos^2\phi)(1+\alpha[1-\cos\phi])}\right\} \qquad (3)$$

To find the number of quanta N_ϕ scattered in the direction ϕ we must divide this energy by the energy of the quanta, E_ϕ, scattered at this angle.
That is, $N_\phi = I_\phi/E_\phi$.
Similarly the number of incident quanta, N_0, is $N_0 = I_0/E_0$.
The probability of a quantum being scattered in unit solid angle between the directions ϕ and $\phi+d\phi$ will then be

$$P_\phi\, d\phi = \frac{N_\phi}{N_0} = \frac{I_\phi}{I_0}\cdot\frac{E_0}{E_\phi} \qquad (7)$$

The quantity I_ϕ/I_0 is to be taken from the scattering formula, while E_0/E_ϕ can be obtained from the Compton scattering relation, $\frac{1}{\nu_\phi} - \frac{1}{\nu_0} = \frac{h}{mc^2}(1-\cos\phi)$.

Multiplying by ν_0 ,

$$\frac{\nu_0}{\nu_\phi} - 1 = \frac{h\nu_0}{mc^2}(1-\cos\phi)$$

or (8)

$$\frac{E_0}{E_\phi} = 1 + \alpha(1-\cos\phi)$$

where $\alpha = \frac{h\nu_0}{mc^2}$

Substituting in the probability equation,

$$P_\phi\, d\phi = -\frac{\pi e^4}{m^2c^4}\cdot\frac{1+\cos^2\phi}{[1+\alpha(1-\cos\phi)]^2}\left\{1+\frac{\alpha^2(1-\cos\phi)^2}{(1+\cos^2\phi)(1+\alpha[1-\cos\phi])}\right\}\sin\phi\, d\phi \qquad (9)$$

$d\Omega$ being replaced by $-2\pi\sin\phi\, d\phi$ (ϕ is taken as essentially negative, since θ is positive.) Now for every photon scattered at an angle ϕ there is an electron scattered at an angle θ where θ and ϕ are related by the equation

$$\cos\phi = \frac{a-1}{a+1} \qquad (10)$$

where $a = (1+\alpha)^2\tan^2\theta$

This equation is derived from the energy and momentum relations. Differentiating equation (10)

$$\sin\phi\, d\phi = -\frac{2\,da}{(a+1)^2}\ .$$

Substituting these values into equation (9)

$$P_\phi\, d\phi = P_\theta\, d\theta = \frac{2\pi e^4}{m^2c^4}\cdot\frac{1}{(a+1+2\alpha)^2}\left\{1+\frac{(a-1)^2}{(a+1)^2}+\frac{4\alpha^2}{(a+1)(a+1+2\alpha)}\right\}da \qquad (11)$$

$P_\theta\, d\theta$ is the probability that a quantum will project the electron at an angle θ . $P_\theta\, d\theta$ is set equal to $P_\phi\, d\phi$ since for every quantum scattered at an angle ϕ there is an electron projected at an angle θ . To find the number of electrons per quantum scattered between θ_1 and θ_2 we need only integrate the above formula between the corresponding limits a_1 and a_2 . This is done by splitting the equation into partial fractions. The details of the integration will be omitted. The integrated result is

$$N_{\theta_1}^{\theta_2}(K.N.) = \frac{4\pi e^4}{m^2c^4}\Bigg| -\left[\frac{1}{2\alpha^2}+\frac{1}{2\alpha^3}-\frac{1}{4\alpha}\right]\log(a+1)$$

$$+\left[\frac{1}{2\alpha^2}+\frac{1}{2\alpha^3}-\frac{1}{4\alpha}\right]\log(a+1+2\alpha) - \left[\frac{1}{2}+\frac{1}{\alpha}+\frac{1}{2\alpha^2}\right]\frac{1}{(a+1+2\alpha)}$$

$$-\frac{1}{2\alpha^2}\cdot\frac{1}{(a+1)}+\frac{\alpha}{2(a+1+2\alpha)}\Bigg|_{a_1}^{a_2} \qquad (12)$$

This is the Klein-Nishina expression for the number of electrons between a_1 and a_2. The accuracy of this result can be tested by integrating between 0 and 90 degrees, since for every electron that is propelled there is a quantum scattered; hence the total number of electrons scattered per incident quantum should give the total absorption coefficient. For $\theta = 0$, $a = 0$; for $\theta = 90$, $a = \infty$. Substituting these limits into equation (12) we obtain

$$N_0^{90}(K.N.) = \frac{\pi e^4}{m^2c^4}\left\{\frac{1+\alpha}{\alpha^2}\left[\frac{2(1+\alpha)}{1+2\alpha}-\frac{1}{\alpha}\log_e(1+2\alpha)\right]\right. \qquad (13)$$

$$\left.+\frac{1}{2\alpha}\log_e(1+2\alpha)-\frac{1+3\alpha}{(1+2\alpha)^2}\right\}$$

which is the usual expression for the total absorption For long wave-lengths ($\alpha = 0$) this converges to the classical expression $\frac{8}{3}\cdot\frac{\pi e^4}{m^2c^4}$.

The expression for the Dirac-Gordon formula is simply obtained by omitting the last term in equation (9), the integrated result being

$$N_{\theta_1}^{\theta_2}(D.G.) = \frac{4\pi e^4}{m^2c^4}\left|\left(\frac{1}{2\alpha^3}+\frac{1}{2\alpha^2}\right)\log_e\frac{a+1+2\alpha}{a+1} - \left(1+\frac{1}{\alpha}+\frac{1}{2\alpha^2}\right)\cdot\frac{1}{a+1+2\alpha} - \frac{1}{2\alpha^2(a+1)}\right|_{a_1}^{a_2} \qquad (14)$$

The corresponding result for the Compton formula is obtained by substituting into equation (7) the value of I/I_0 given by equation (2). Again transforming from ϕ to a by equation (10) we find the Compton expression for the probability of ejection of a recoil electron in an interval da is given by

$$P_\theta d\theta = \frac{2\pi e^4}{m^2c^4}\left\{\frac{2a^2}{(a+1+2\alpha)^4}+\frac{2+8\alpha(1+\alpha)}{(a+1+2\alpha)^4}\right\}da \qquad (15)$$

Upon integrating between the limits a_1 and a_2 this expression becomes

$$N_{\theta_1}^{\theta_2} = \frac{4\pi e^4}{m^2c^4}\left|-\frac{1}{a+1+2\alpha}+\frac{1+2\alpha}{(a+1+2\alpha)^2} - \frac{2}{3}\cdot\frac{(1+2\alpha)^2}{(a+1+2\alpha)^3}\right|_{a_1}^{a_2} \qquad (16)$$

Equations (11) and (15) express the number of recoil electrons per unit interval da. In order to obtain the expression for the number of electron tracks per degree da must be replaced by its equivalent expression in $d\theta$.

$$a = (1+\alpha)^2\tan^2\theta$$

$$da = (1+\alpha)^2\tan\theta\sec^2\theta\, d\theta = \frac{\sqrt{a}}{1+\alpha}\left[(1+\alpha)^2+a\right]d\theta$$

The substitution of the above expression for da into equation (11) gives the Klein-Nishina expression for the probability that a given quantum will scatter a given electron inside a conical shell, the elements of which make an angle a and $a+d\theta$ with the direction of the quantum. It is

$$P_\theta\, d\theta = \frac{2\pi e^4}{m^2 c^4} \cdot \frac{\sqrt{a}\,\left[a + (1+\alpha)^2\right]}{(1+\alpha)(a+1+2\alpha)^2} \left\{ 1 + \frac{(a-1)^2}{(a+1)^2} + \frac{4\alpha^2}{(a+1)(a+1+2\alpha} \right\} d\theta \qquad (17)$$

The corresponding expression for the Dirac-Gordon formula is obtained merely by omitting the last term in the above expression.

$$P_\theta\, d\theta = \frac{2\pi e^4}{m^2 c^4} \cdot \frac{\sqrt{a}\,\left[a + (1+\alpha)^2\right]}{(1+\alpha)(a+1+2\alpha)^2} \left\{ 1 + \frac{(a-1)^2}{(a+1)^2} \right\} d\theta \qquad (18)$$

From equation (15) we find that the Compton formula predicts the following distribution.

$$P_\theta\, d\theta = \frac{2\pi e^4}{m^2 c^4} \cdot \frac{\sqrt{a}\,\left[a + (1+\alpha)^2\right]}{(1+\alpha)(a+1+2\alpha)^2} \left\{ 1 + \frac{(a-1)^2}{(a+1)^2} + \frac{8\alpha(1+\alpha)}{(a+1)^2} \right\} \times \frac{(a+1)^2}{(a+1+2\alpha)^2} \cdot d\theta \qquad (19)$$

Equations (17), (18), and (19) have been plotted in figure 6, where α has been given the value 1. This corresponds to a wave-length of about 24 x-u., or 510 k.v. x-rays. The ordinates of each curve have been adjusted so as to make the total area between 0 and 70 degrees the same. The Klein-Nishina formula predicts a great deal more recoil electrons at small angles to the beam than either of the other two formulas. This will be more clearly seen from the integrated values.

Results.

As was previously stated, two sets of photographs were taken, each set containing about 100 pictures. In each case x-rays of about 24 x-u. were used. In the first set a beam of x-rays $1\frac{1}{2}$ cm in width was used, which afforded a total of 265 tracks between 0 and 70 degrees. In the second set a beam width of $3\frac{1}{2}$ cm was used, and 430 tracks were obtained. Hereafter these will be referred to as set 1 and set 2.

It will first be shown that the two sets are consistent with each other. Thenceforward it will be permissible to deal with the total 695 tracks. The first row in table 1 shows the observed distribution of 265 tracks. The second row shows the distribution of the 430 tracks of set 2, reduced in the ratio 265:430. The agreement is surprisingly good.

Table 1.

Angular interval	0 - 20	20 - 40	40 - 60	60 - 70
Set 1	57	87	95	26
Set 2	55	92	95	23

Table 2 is the same as table 1, except that the interval from 0 to 70 degrees is divided in a different manner.

Table 2.

Angular interval	0 - 15	15 - 30	30 - 50	50 - 70
Set 1	40	63	92	69
Set 2	33	67	94	71

The agreement is again good, except in the first column, and here the difference is just equal to the probable statistical error.

Equations (12), (14), and (16) each express the probability that an electron will be scattered between the angular limits ϕ_1 and ϕ_2. Hence for a given value of α, if 695 tracks are observed between 0 and 70 degrees, these equations predict how the tracks should be divided among the various subintervals. This calculation has been made, and the results are compared with experiment in table 3.

Table 3.

Angular interval	0 - 15	15 - 30	30 - 50	50 - 70
Observed	93	172	242	185
Klein-Nishina	108	167	223	196
Dirac-Gordon	79	133	241	242
Compton	96	181	214	204

The observed values are given in the first row, the values calculated from the Klein-Nishina, Dirac-Gordon, and Compton formulas in rows 2, 3, and 4 respectively. The agreement with any one of the formulas is not very good. The probable statistical error is exceeded in the case of the Klein-Nishina formula in two columns, in the case of the Dirac-Gordon in three. On the whole the Klein-Nishina formula is, perhaps, in closest agreement.

Division of the range 0 to 70 degrees into the intervals shown in table 4 does not change matters appreciably.

Table 4.

Angular interval	0 - 20	20 - 40	40 - 60	60 - 70
Observed	146	236	249	64
Klein-Nishina	163	220	220	94
Dirac-Gordon	121	200	262	111
Compton	154	234	210	98

In this table there is a startling disagreement in the interval 60 to 70 degrees. Can this be accounted for? It has already been stated that this is the region in which the measurements were most unreliable, first because many of the tracks were pointed nearly towards the camera, second because tracks nearly vertical soon leave the light beam and are therefore short, and third because the recoil electrons have low energy and are therefore badly scattered. Electrons emitted at an angle of 70 degrees have but $1\frac{1}{2}$ cm range. In addition it should be pointed out that errors in the other angular intervals are compensated for by errors in the neighbouring intervals. Since, however, tracks making an angle greater than 70 degrees with the x-ray beam were not measured, there is less compensating effect in the 60 to 70 degree interval. These conclusions are further substantiated by figure 6, wherein the number of tracks per degree is plotted against the angle of the tracks. If the right limb of the curve through the experimental points is extended downwards it will cut the axis at

75 degrees. This could be accounted for by assuming that the curve would have a foot on it, but this would be in entire disagreement with all of the scattering formulas.

As a consequence of these considerations it was decided to leave out of the data all tracks at angles greater than 60 degrees. The remaining 632 tracks were then divided as shown in table 5.

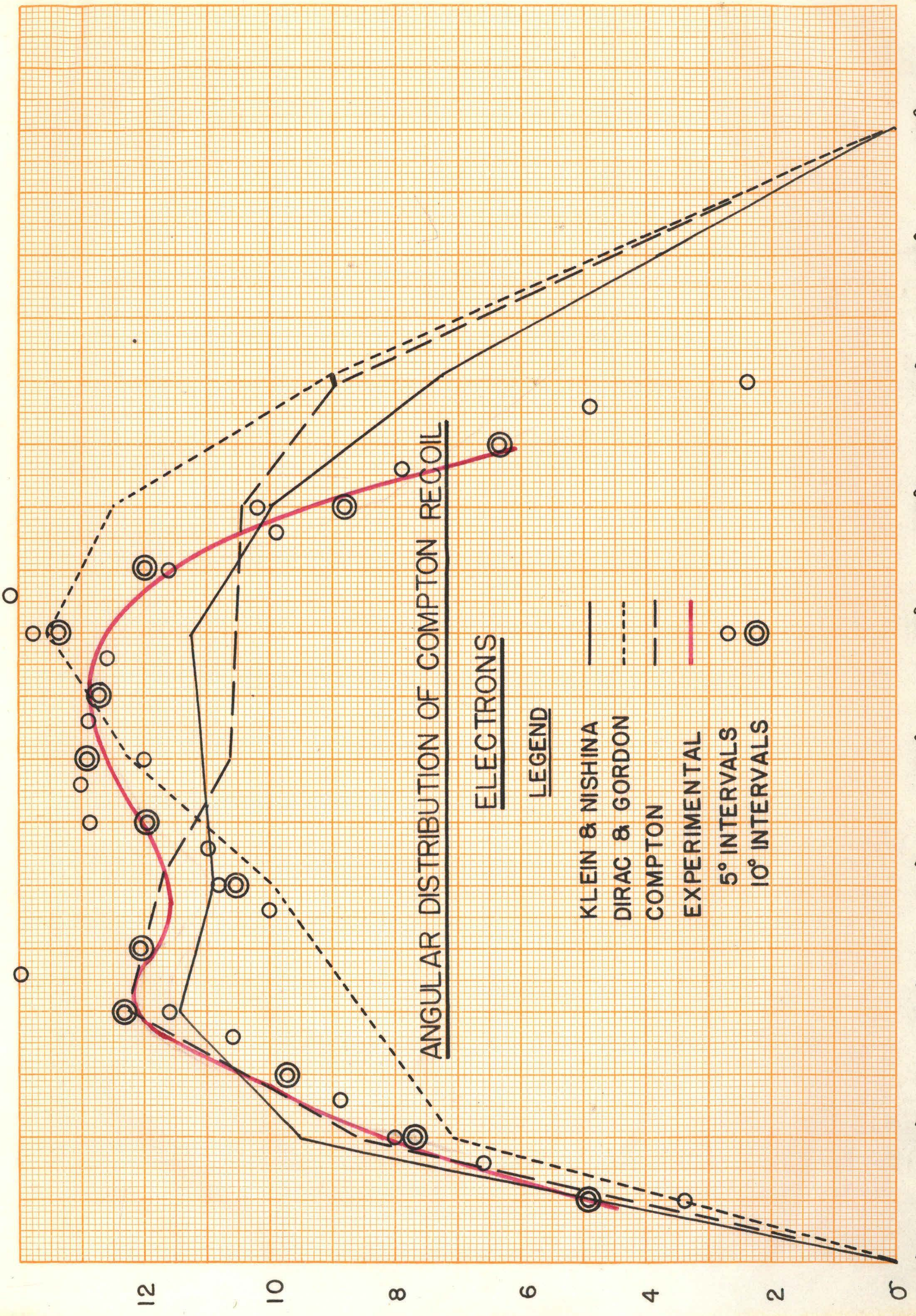
Table 5.

Angular interval.	Observed.	Klein-Nishina.	Dirac-Gordon.	Compton.
0 - 10	50	56	42	47
10 - 20	97	115	87	112
20 - 30	120	116	98	127
30 - 40	120	115	120	114
40 - 50	127	128	141	107
50 - 60	120	112	143	109
Total difference		42	77	62

The agreement with the Klein-Nishina formula can be seen to be very good in all intervals except the second. However it must be stated that set 1 and set 2 agree in giving lower values than predicted by the Klein-Nishina formula in both the first two intervals.

In figure 6 is plotted the recoil electron distribution as predicted by the three scattering formulas. The total area under each curve, between 0 and 70 degrees has been made equal to 695. The double circles represent the number of tracks per degree, using

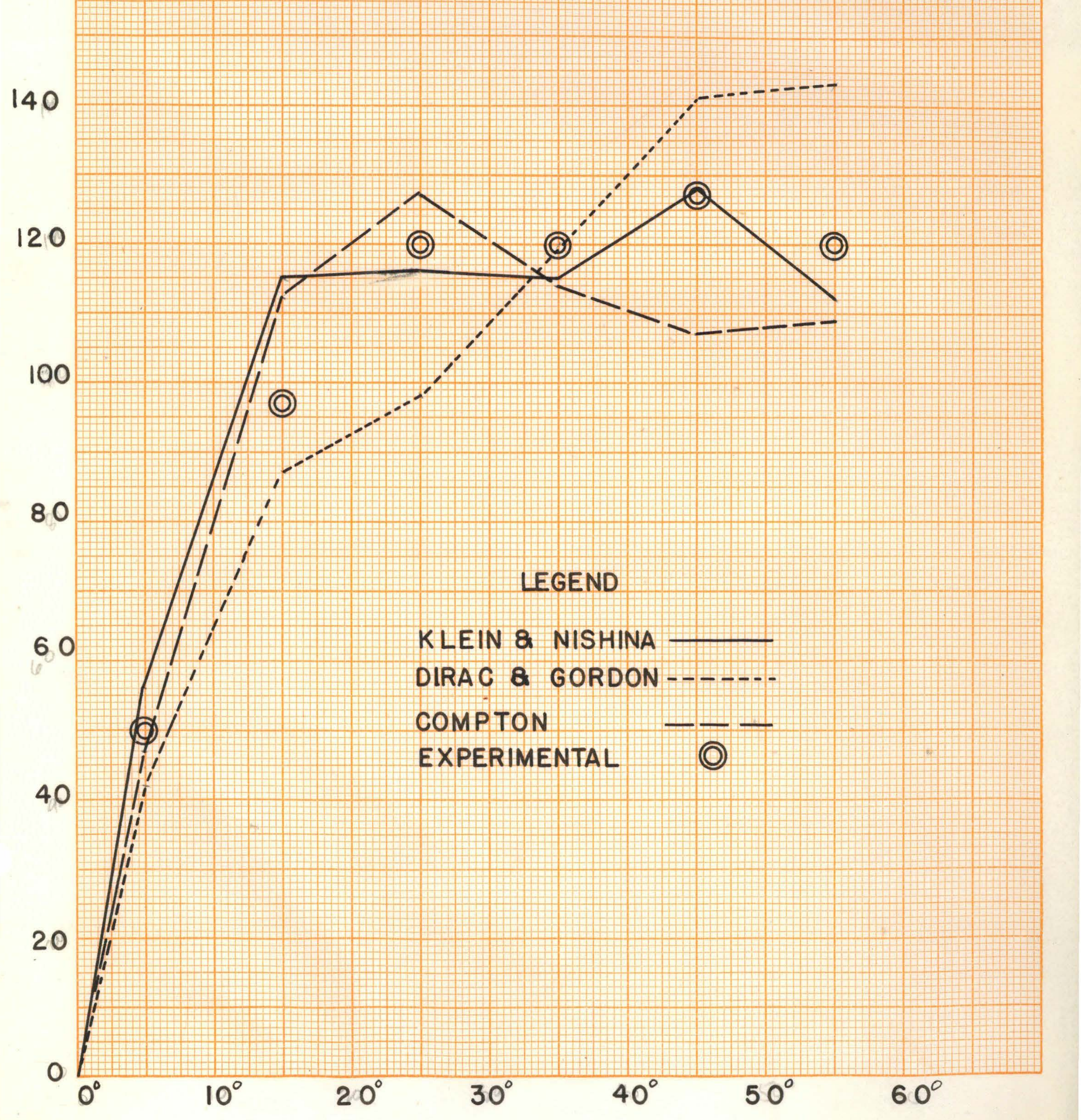
FIGURE 6



ANGULAR DISTRIBUTION OF COMPTON RECOIL
ELECTRONS
LEGEND
KLEIN & NISHINA
DIRAC & GORDON
COMPTON
EXPERIMENTAL
5° INTERVALS
10° INTERVALS
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
0
2
4
6
8
10
12

FIGURE 7

PLOT OF TABLE 5



overlapping intervals of 10 degrees. The single circles represent overlapping intervals of five degrees; hence the statistical fluctuations are much larger than in the previous case. The red line is a smooth curve through the experimental points. Two facts should be noted. First, the experimental curve has a minimum at exactly the same point as the Klein-Nishina curve. Second, the experimental curve has the same general outline as the Klein-Nishina curve. The agreement would be still better if the experimental curve were plotted on the basis of 632 tracks between 0 and 60 degrees, for this would shift all of the theoretical curves upward, leaving the experimental curve unchanged.

It is concluded from this investigation that the Klein-Nishina formula is in closer agreement with the experimental results than either of the other two formulas. However certain discrepancies appear which seem to be outside of experimental error. The Dirac-Gordon formula is in complete disagreement, the older Compton formula fitting better than it does.

In conclusion I wish to express my appreciation to Dr. C.C. Lauritsen for suggesting this problem, and for valuable suggestions which were used in analyzing the data.

I am also indebted to Mr. Louis Ridenour and to Mr. John Read for their assistance in the preparation of this thesis.

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