AN INVESTIGATION OF THE EFFECT OF BEARING AREA AND DEPTH UPON THE SETTLEMENTS OF FOOTINGS

Thesis by

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INTRODUCTION

Since the formulation, by Coulomb and Rankine, of their widely known theories of earth pressures, the science of soil mechanics has made little progress. These theories are now realized to be of classical interest rather than practical significance, for the conditions described by them are rarely, if ever, encountered in actual practise. Within recent years, widespread interest has been evinced in investigation of soil phenomena. However, due to the complexity of the factors entering into the action and reaction of soils, the advance of knowledge has been relatively slow. It is not to be expected that the theory of soil mechanics shall be put upon a thoroughly scientific basis for many years.

Experience and investigation have shown certain general characteristics of soil action under load. The most obvious, and of most practical interest, are; that the settlement of a bearing area with constant load per unit area varies as some function of the area; that the settlement varies inversely as the depth of the bearing plate beneath the surface of the ground; and that the settlement varies as the moisture content of the soil.

The investigations now underway, both experimental and theoretical, are pointing toward the ultimate solution of the exact effects of these characteristics. However, until the exact solutions have been determined it is desirable to have some means of analyzing the effect of bearing area, depth, and moisture content upon the settlement of foundations.

Since the effect of moisture content is so closely dependent upon the physical properties of the soil, such as, density, size of particles and shape of particles, beyond a general conclusion that an increase in moisture content means an increase in settlement, more exact determinations cannot be made until the completion of exhaustive experimentation.

This, fortunately is not the case with effect of bearing area and depth. It is the purpose of this paper to derive certain empirical relations which shall make possible the determination of the effect upon settlements of foundations of the bearing area and depth of footing. A knowledge of these factors is essential in the design of foundations as it is of prime importance that the footings be so designed as to secure a uniform settlement of the structure as a whole. Failure to do so will result in overstressing and perhaps cracking certain portions of the structure, with the resulting danger of ultimate failure.

CHAPTER I.

The results of this investigation are based upon a study of data furnished by the firm of Labarre and Converse, Consulting Engineers. This data was in the form of 97 load-settlement tests. The tests covered a wide range of soils, including decomposed rock, clay, sandy loam, loam, sandy silt, and silt. The tests were made with bearing areas of 1, 2, 3, 4, and 9 square feet, and at elevations varying from 3 to 20 feet below the surface of the ground. The load was applied by means of a jack and after the bearing plate had come to rest, the settlement was read to the nearest hundredth of an inch.

In analyzing the data, it was not considered suitable to derive a psuedo-theoretical equation and attempt to evaluate constants to give the desired results. The final relations were arrived at by pure induction. No attempt was made to embody any of the theory of soil mechanics, it being thought that strict adherence to inductive reasoning would result in a solution of the greatest simplicity.

In the investigation only that portion of each test up to the so-called "yield point" of the soil was considered: the "yield point" of the soil being that point at which small increase in load produces rapid increase in settlement; it being that bearing capacity which in practice is considered the ultimate safe capacity of the soil. The portions of the test loaded beyond this point, being of no practical significance, were, therefore, disregarded.

CHAPTER II.

The first step undertaken was the determination of the general form of the curves as described by the load-settlement data. A careful consideration disclosed that the data was best fitted by curves of the general form * $Y = aX^b$; where "Y" is the settlement and "X" is the load in pounds per square foot of bearing area.

Having determined this general relation the next step was to evaluate the constants "a" and "b". The procedure was as follows. First, the numerical values of "a" and "b" were determined for approximately 30 typical tests. Examples of these are:-

#69 Loam	#54 Sandy Loam	#41-B Silt
Bearing area = 2 sq.ft.	Bearing area = 4 sq.ft.	Bearing area = 2 sq.ft.
Elevation = - 7 ft.	Elevation = - 20 ft.	Elevation = - 17 ft.
$Y = \frac{11.6}{10^6} X^{1.213}$	$Y = \frac{16.6}{10^6} X^{1.10}$	$Y = \frac{3.55}{10^6} X^{1.25}$

In further discussions the slope of a curve will be taken to mean the slope of the tangent to the curve at the origin, and the curvature will mean the rate of curvature.

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* "Empirical Formulas", by Theodore Running, p. 42. John Wiley & Sons, 1917.

In studying these 30 typical equations, considering a group of tests on one type of soil such as loam, certain characteristics were noted. It was seen that as the bearing area was increased the slope of the curve (constant "a") was increased. Also, as the bearing area was increased the curvature (constant "b") was increased. These facts were evident both from the curves and from the equations. This then made the general equation of the form:-

$$Y = m \cdot f(A) \cdot (X)^{kf'(A)}$$

A = bearing area

k and m = constants

In the same manner, considering the effect of change of elevation, it was found that both slope and curvature were decreased as the depth was increased. With "t" representing the depth the general form of the equation then was:-

$$Y = m \frac{f(A)}{f(t)} (X)^{k \frac{f'(A)}{f'(t)}}$$

It was then necessary to determine more exact values of f(A), f(t), f'(A), and f'(t). Considering first the exponent of "X", it was noted that in every case it ranged within the limits 1 and 2. This meant that the effect of change of bearing area or depth was small and the relationship, therefore, was something other than $k\frac{f'(A)}{f'(t)}$. An examination of the values suggested that the variation was caused by a relationship of the form $k\sqrt[t]{A}$. A study of data wherein two tests were made at the same location, for example:-

#29-A	#29-B
Bearing area = 2 sq.ft.	Bearing area = 2 sq.ft.
Elevation = -15.5 ft.	Elevation = -10 ft.
$Y = \frac{1.24}{10^6} X^{1.35}$	$Y = \frac{2.24}{10^6} X^{1.387}$

showed that this relationship was probable. The general equation then was of the form:-

$$Y = m\frac{f(A)}{f(t)} X^{k\sqrt[t]{A}}$$

The next step then was to proceed in the same manner for $m\frac{f(A)}{f(t)}$. Examination of the data showed that the value of this factor varied markedly as the bearing area and depth varied. This suggested that the relationship could be expressed as $k\frac{A}{t}$. Testing this on data, such as #29-A and #29-B, led to the conclusion that it was best expressed as $m\frac{\sqrt{A}}{t}$. The general equation then was in the form:-

$$Y = m\frac{\sqrt{A}}{t} X^{k\sqrt[t]{A}}$$

A further study, however, showed that the exponent $k\sqrt[t]{A}$ was not correct. For, while with small bearing areas and large depths the results seemed correct, it was seen that large bearing areas and small depths would give impossible values. It was evident that the effect of depth "t" was too marked. This effect could be reduced to the proper amount by the inclusion of a constant "c", thus $k\sqrt[ct]{A}$. Testing this upon the data showed that the assumption of this constant was justified. The general equation then was of the form:-

$$Y = m\frac{\sqrt{A}}{t} \quad X^{k\sqrt[ct]{A}}$$

Considering then the coefficient $m\frac{\sqrt{A}}{t}$, and testing it thoroughly upon the data, it was discovered that the function was not $A^{1/2}$, but was something slightly different, depending upon the soil considered. The coefficient was therefore written as $m\frac{A^n}{t}$ where "n" was a constant to be determined. The equation then was:-

$$Y = m\frac{A^n}{t} \quad (X)^{k\sqrt[ct]{A}}$$

with Y being the settlement in inches, X being pounds per square foot of bearing area, and m, n, k, and c being constants that must be determined.

It should be understood that the constants must be determined each time the equation is used. The values of the constants determined for sandy loam at one location are not necessarily the same for sandy loam at another location. This fact is important and should be noted, for failure to do so will lead to erroneous results.

To determine the constants m, n, k and c, two tests are required. Having the constants determined the equation then holds true for different values of bearing area (A) and depth (t). It should further be noted that the relationship of the depth does not hold for extreme projections. That is, if the constants are determined for a depth of 2', considerable error may be expected if the equation is applied to a depth of 30'. However, for changes of depth from 5' to 10' or from 15' to 25' the error introduced will be relatively small.

To make the equation easier to handle and less sensitive to inaccuracies the final form was written:-

$$Y = m\frac{A^n}{t}\left(\frac{x}{1000}\right)^{\sqrt[k]{\frac{ct}{A}}}$$

Y = settlement in inches.

x = pounds per square ft. of bearing area.

A = bearing area. sq ft.

t = depth of bearing plate. ft.

The following is an example illustrating the use and accuracy of the equation.

TEST DATA

at

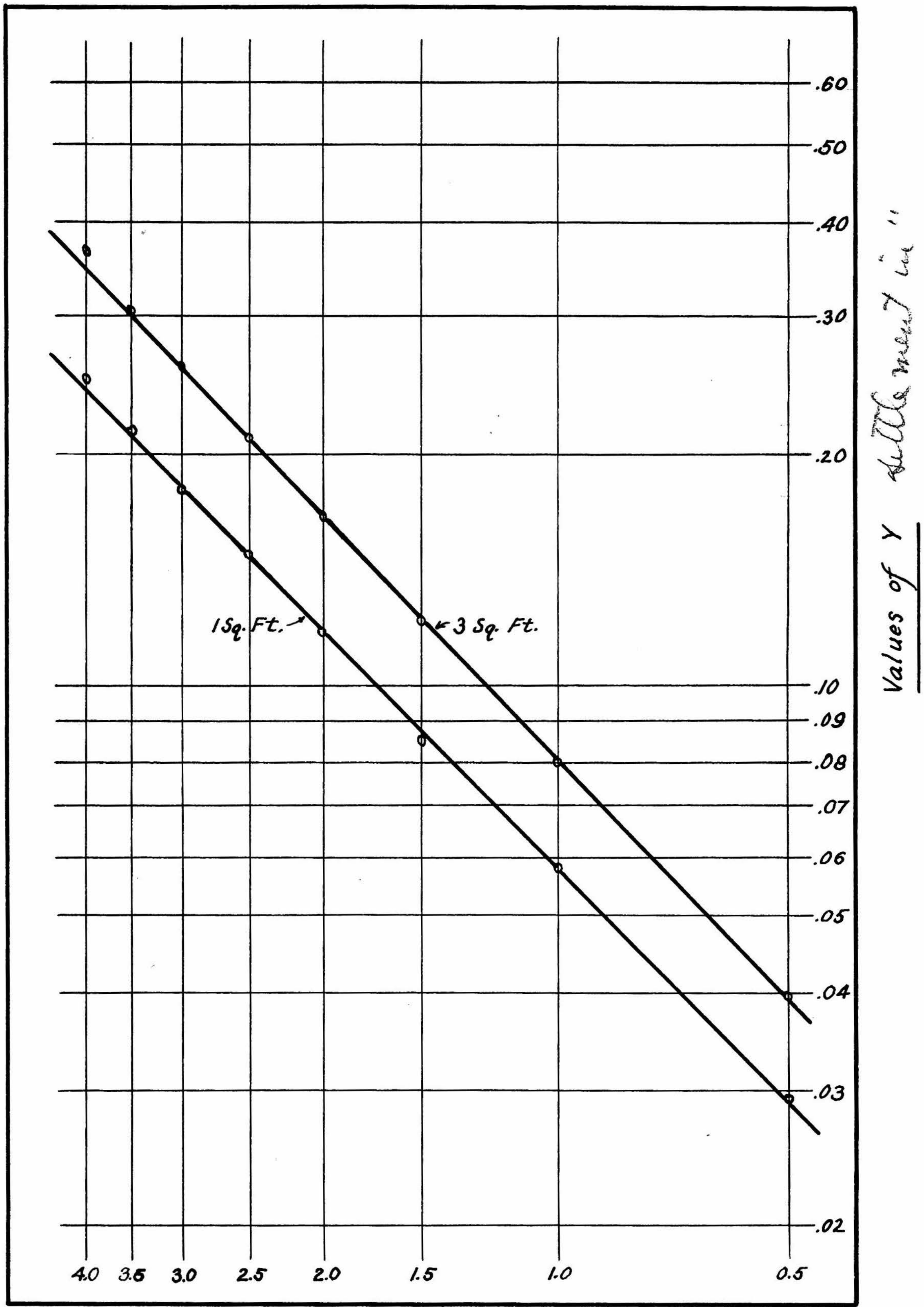
Willowbrook Junior High School, Compton District

Elevation = -5'

Load	Settlement in Inches		
Lb./sq.ft.	1 sq.ft.	3 sq.ft.	9 sq.ft.
500	0.030	0.040	0.058
1000	0.060	0.081	0.118
1500	0.087	0.125	0.175
2000	0.120	0.168	0.236
2500	0.150	0.217	0.292
3000	0.180	0.267	0.357
3500	0.223	0.335	0.435
4000	0.265	0.412	0.540

I

LogLog Scale



Values of $\frac{X}{1000}$

First plot the values of settlement against load of the tests on 1 sq. ft. and 3 sq. ft. on a logarithmic scale, $\log Y = \log (m \frac{A^n}{t}) + (k \sqrt[ct]{A}) \log \frac{x}{1000}$. Then the slope of the line, Figure No. 1, equals $k\sqrt[ct]{A}$ and the intercept of the line with the Y axis (where $\frac{x}{1000} = 1$) determines the value of $\log (m \frac{A^n}{t})$. The equations as determined from this procedure are:-

<u>1 sq.ft. area</u>

$$Y = \frac{5.95}{100} \left(\frac{x}{1000}\right)^{1.022}$$

<u>3 sq.ft. area</u>

$$Y = \frac{8.35}{100} \left(\frac{x}{1000}\right)^{1.050}$$

Since ~~Therefore~~ $k\sqrt[5c]{1} = 1.022$

$$k\sqrt[5c]{3} = 1.050$$

Therefore $k = 1.022$

and

$$\frac{1}{5c} \log \frac{3}{1} = \log \frac{1.05}{1.022}$$

$$c = \frac{1}{5} \frac{\log 3}{\log 1.025} = 8.67$$

Also

$$\frac{m \cdot 1^n}{5} = \frac{5.95}{100}$$

$$\frac{m \cdot 3^n}{5} = \frac{8.35}{100}$$

Therefore $m = \frac{5 \cdot 5.95}{100} = \frac{29.78}{100}$

and

$$\log m + n \log 3 - \log 5 = \log 8.35 - \log 100$$

$$\underline{\log m + n \log 1 - \log 5 = \log 5.95 - \log 100}$$

$$n \log 3 \qquad = \log 8.35 - \log 5.95$$

$$n = \log \left(\frac{8.35}{5.95}\right) \div \log 3 = 0.31$$

Therefore,

$$m = \frac{29.78}{100}$$

$$n = 0.31$$

$$k = 1.022$$

$$c = 8.67$$

The equation then is of the form

$$Y = \frac{29.78}{100} \cdot \frac{A^{(0.31)}}{t} \left(\frac{x}{1000}\right)^{1.022 \sqrt{\frac{8.67t}{A}}}$$

Then substituing values of

$$A = 9 \text{ sq.ft.}$$

$$t = 5 \text{ ft.}$$

$$Y = \frac{11.74}{100} \left(\frac{x}{1000}\right)^{1.078}$$

The values given by this equation are:-

9 sq.ft. bearing area.

Elevation = -5'

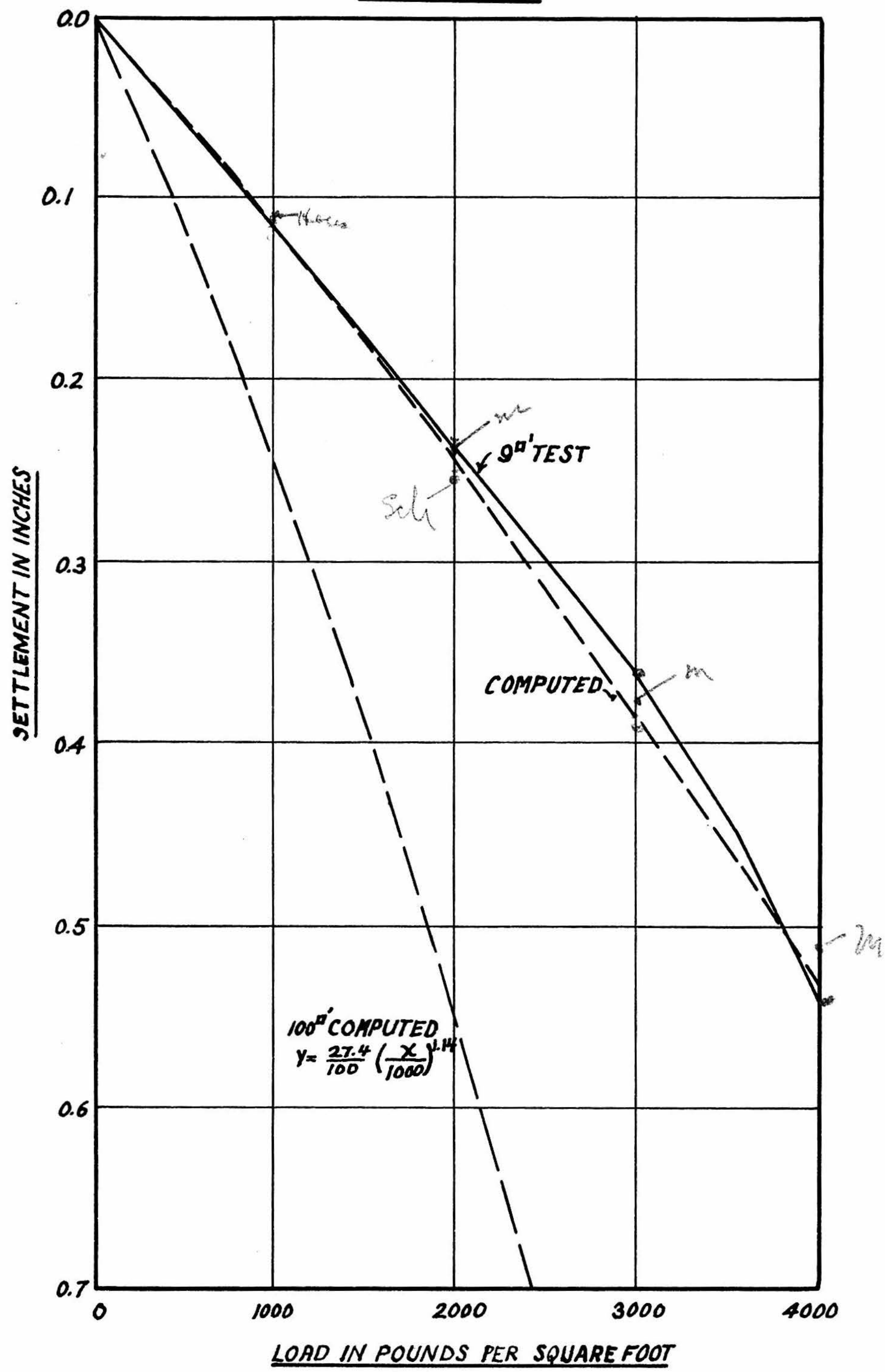
$$Y = \frac{11.74}{100} \left(\frac{x}{1000}\right)^{1.078}$$

Load	Settlement in Inches	
Lbs./sq.ft.	Test	Computed
500	0.058	0.057
1000	0.118	0.118
1500	0.175	0.182
2000	0.236	0.248
2500	0.292	0.315
3000	0.357	0.383
3500	0.435	0.452
4000	0.540	0.522

This data is plotted in Figure 2.

II

COMPARISON OF COMPUTED WITH ACTUAL SETTLEMENTS



CONCLUSION

While the equation developed in this paper gives a close correlation with test data, its results for practical applications are not final. This limitation is due to the neglect of certain factors which cannot well be embodied in such an equation. The difficulty entailed in an attempt to formulate an expression embracing such variable conditions as moisture content, effect of proximity of footings and non-homogeneity of soil, is obvious. However, the effects of these conditions do not make the results any less valid. In every instance, (except in the case of pure sand) under uniform load an increase in bearing area will result in an increase in settlement and an increase in depth results in a decrease of settlement. Therefore, the use of the equation is a step closer to the exact solution.

In an application of the equation it should be noted that the accuracy of the results depends for the most part upon the accuracy of the test data. This does not mean accuracy in reading the settlement to the thousandth of an inch, for slight inaccuracies of this sort can be eliminated by drawing a smooth curve through the plotted points of the test data. However, an error of the sort in which one tneth of an inch is added to each measured settlement will lead to entirely erroneous results.

If the test data does not contain serious errors the results obtained by the use of the equation will closely approximate the actual settlements. The value of the equation is demonstrated in the design of footings for a building. In the usual case a typical interior footing is subjected to a load approximately twice the load on a typical exterior footing. If the interior footing then is designed with twice the bearing area, it is obvious that the settlement will be much larger than that of the exterior footing, thus causing undesirable secondary stresses. This condition of unequal settlements would be to a great extent eliminated by the use of the equation developed in the body of this paper.

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