BUCKLING OF THIN WALLED CYLINDERS

UNDER COMPRESSION OR TENSION AND TORSION

Thesis by

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Introduction

The loads received by a monocoque structure have led to this research, the original purpose of which was to investigate and, if possible, determine experimentally the relationship between axial compression and pure torsion on thin walled cylinders to cause buckling, as there is not much material of this type available for the designer. This was done in the first six series tested, and when these followed a definite relationship an additional series of cylinders was tested under axial compression and pure torsion and axial tension and pure torsion. It was found that this relationship still held, and it was then planned to investigate the connection between axial compression and pure bending in conjunction with axial tension and pure bending. Further it was planned to investigate the relationship between pure bending and pure torsion. One series of cylinders was tested in axial compression and pure bending in conjunction with axial tension and pure bending, and, although the results of these tests show promise that a definite relationship exists, the data obtained were for but one series with cylinders of but one diameter, thickness and length diameter ratio, and it is not felt that this relationship can be conclusively stated as existing for cylinders under axial compression-tension and pure bending or under pure torsion and pure bending.

Apparatus

The tests were made on small scale models of steel and brass "shim stock". The technique of manufacture and the special testing machines used were those developed by Dr. L. H. Donnell and described by him in N.A.C.A. Report 479. The figures mentioned in this paragraph only conform to the notation used in Report 479. As shown in this report, the results of small scale tests compare favorably with large scale tests. It is necessary to use great care in selection of stock and manufacture of specimens to secure uniformity of result, and despite this care it has so far been impossible to avoid scatter at high compression loads. The thickness of sheet was measured in the thickness tester, Fig. 14a. The modulus of elasticity E was obtained in the special testing machine and tensometer shown in Fig. 15a, and at the same time σ_p, the proportional limit stress, was determined for purposes of record. Sheets cut to 1/8 inch outside diameter greater than the diameter of the finished cylinder, were placed in a rolling machine around rods of such diameter as to obtain the proper curvature. These were then wrapped around an oiled wooden cylinder (to enable easy removal) and soldered. The rolled sheet was held tightly to the wooden cylinder by a special clamp and the edges soldered. (It has been found that buckling waves form freely over this soldered edge, and that the effect of the overlap, if small, is negligible). After removal the cylinders were soldered to end rings in turn soldered to butt plates to fit in the special testing machine shown in Plate 1. The decrease in effective length due to the rings has been allowed for in the following table. The testing machine, Plate 1, is capable of testing specimens in any combination of axial compression or tension and in pure bending and pure torsion at the same time.

The notation of figures, etc., from this point on applies to this paper.

Conclusions and Results

The relationship given by the empirical exponential formula:

$$1 - \sigma/\sigma_0 = (\tau/\tau_0)^n$$

where: σ = axial compression or axial tension in pounds

σ_0 = average of maximum axial compression values (with zero torsion) in pounds

τ = pure torsion in inch pounds

τ_0 = average of maximum pure torsion values (with zero compression or tension) in inch pounds. always positive.

$n (>1)$ = slope of the line determined by plotting experimental values of σ/σ_0 vs. τ/τ_0 on double log paper (see figures 3 and 5)

was determined conclusively to exist by experiment. Using cartesian coordinates with the compression-tension or σ/σ_0 axis as the ordinate axis, and torsion or τ/τ_0 as the abscissae axis, it will be noted that values range from zero to unity for axial compression values, from zero to unity for pure torsion (when neither compression nor tension is applied), and from zero to values greater than unity up to the elastic limit of the material in the cylinders. Figure 2 illustrates these conditions. In Figure 1 a characteristic curve on cartesian coordinates is shown, and the crossing of the compression axis in a perpendicular direction while the crossing of the torsion axis at a definite slope is shown. That the curves are symmetrical about the compression-tension axis is apparent as a change in the sign of shear for a symmetrical structure can make no difference. That the curve will pass through the torsional axis with a definite slope is physically apparent as the addition of tension obviously increases the ability of the structure to take a corresponding addition of torsion before buckling.

Figure 2 is a plot on cartesian coordinates of the points obtained from series G of Table I which in turn shows the physical characteristics of each series tested. In Figure 2 the solid line is a cubic parabola (n = 3). The dotted line represents that portion of the curve where the cylinders were stressed beyond their elastic limit. Figure 3 represents this Series G plotted on double log paper where the slope (or n) $\doteq$ 3.3.

Figure 4 represents a composite plot on cartesian coordinates of series A to G, while figure 5 illustrates the plots of series A to F inclusive, on double logarithmic paper. Figures 6-12 inclusive are self explanatory. In Fig. 11 it will be noted that the tension cage is attached. Table 2 tabulates the precise values attained on the individual cylinders of each series before failure by buckling.

TABLE I

Properties of cylinders in series tested

Series	Mat.	Length	Diam.	Thick.	$E \times 10^{-6}$	$\sigma_p \times 10^{-3}$	n
A	steel	5.315	1.88	.00204	31.4	57.7	1.992
B	brass	5.315	1.88	.0032	16.5	27.0	4.25
C	steel	5.315	3.75	.00295	30.6	48.6	2.625
D	steel	11.315	1.88	.00204	27.06	53.3	2.156
E	steel	5.315	1.88	.00295	30.6	48.6	1.0
F	steel	1.32	3.75	.00204	31.4	57.7	2.781
G	steel	5.315	1.88	.00395	29.6	36.0	3.33

TABLE II

Experimental Results Used in Plotting Curves

Series A

lbs.	0	20	40	60	80	100	120	170	172	230	192	218	231
lb.in.	55	52	49	47	40	39	36	30	20	10	0	0	0

Series B

lbs.	20	45	90	100	120	150	175	190	230	250
lb.in.	70	64	50	64	48	60	56	54	30	0

Series C

lbs.	0	0	50	100	150	155	170	200	200	220	230	250	279	287	287
lb.in.	217	240	226	178	180	174	210	210	126	96	70	100	72	0	0

Series D

lbs.	0	40	97	100	100	120	125	130	140	150	151
lb.in.	36	34	23	12	16	24	0	0	0	11	10

Series E

lbs.	0	0	100	100	200	200	200	300	300	420	430	455	470	490	520
lb.in.	106	110	96	100	68	78	90	64	68	20	20	10	10	0	0

Series F

lbs.	0	25	50	75	100	132
lb.in.	160	170	128	130	96	0

Series G

lbs.	-980	-800	-800	-700	-600	-600	-500	-500	-400	-400	-300	-300	-200	-200
lb.in.	0	166	200	220	248	224	220	234	215	218	222	210	204	210

lbs.	-100	-100	0	0	0	0	100	100	200	200	300	380	400	500	500
lb. in.	206	216	178	196	186	184	172	180	166	174	144	150	124	0	75

lbs.	500	500	540	595	613	610	660
lb.in.	132	140	100	40	0	0	0

Discussion

The manufacture, test and choice of material for these experiments must be carefully done or results will not be uniform or accurate.

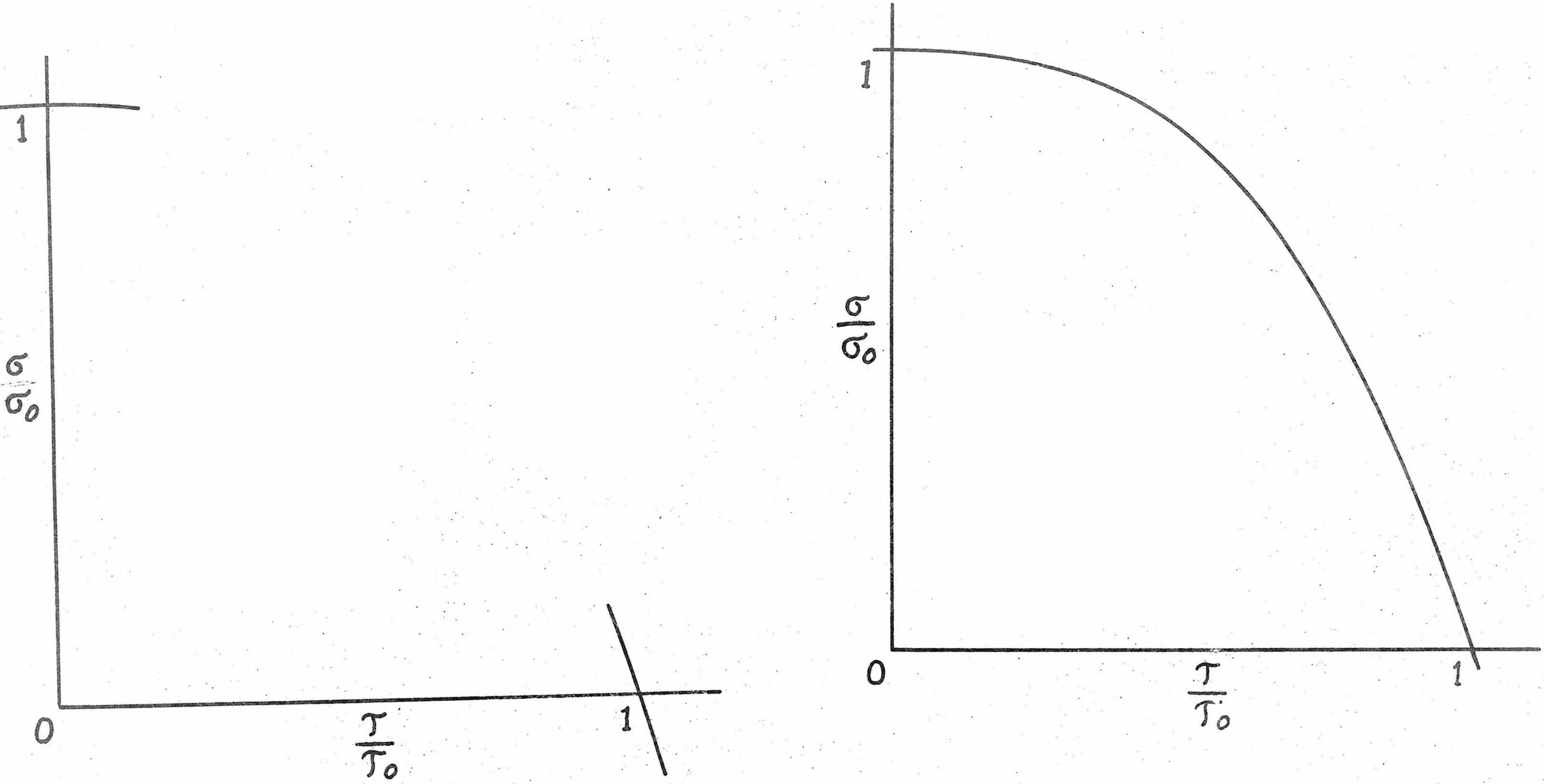
As a general rule the rolled metal stock is not isotropic and is definitely anisotropic as shown by the different moduli of elasticity in series D cut longitudinally from the sheet stock and F cut across the sheet. Any attempt to condense the physical characteristics of the cylinders by forming parameters of the several variables to be accurate would of necessity consider this fact.

In the construction of the curves for this paper the values of maximum compression and maximum torsion have been chosen as the mean of the maximum values secured experimentally. The use of the maximum torsion (τ_0) as calculated from Technical Report No. 479, N.A.C.A. 1933, by Dr. L. H. Donnell, and the use of maximum compression (σ_0) as calculated from Technical Report No. 473, N.A.C.A. 1933, by E. E. Lundquist, would change the value of n but not the nature of the relationship.

In Figure 2 it will be noted that compression decreases the ability of the cylinder to resist torsion while tension increases it. The empirical equation represents this relationship definitely. In Fig. 2, the value of the point at which this relationship breaks down, namely maximum tension and torsion values at the elastic limit, about -.8 is interesting in comparison with the maximum compression and minimum torsion value of 1.0 at the stability limit. Further experimental work might develop a bona fide relationship between these points which would be of value.

The author wishes to thank Dr. Th. von Kármán, Director of the Guggenheim Aeronautics Laboratory at the California Institute of Technology, for the opportunity for making these researches. The researches were suggested by and carried out under the direction of Dr. L. H. Donnell.

Acknowledgment is also due to L. Secretan, who cooperated in the first research, and to Dr. A. L. Klein and E. E. Sechler (all of the staff of the Institute) for numerous helpful suggestions, and to Dr. E. E. Lundquist of the N.A.C.A. for his suggestion as to the choice of maximum torsion and compression values.



$\frac{\sigma}{\sigma_0}$
1
0
$\frac{T}{T_0}$
1
$\frac{\sigma}{\sigma_0}$
1
0
$\frac{T}{T_0}$
1

(A) (B)

Fig. 1

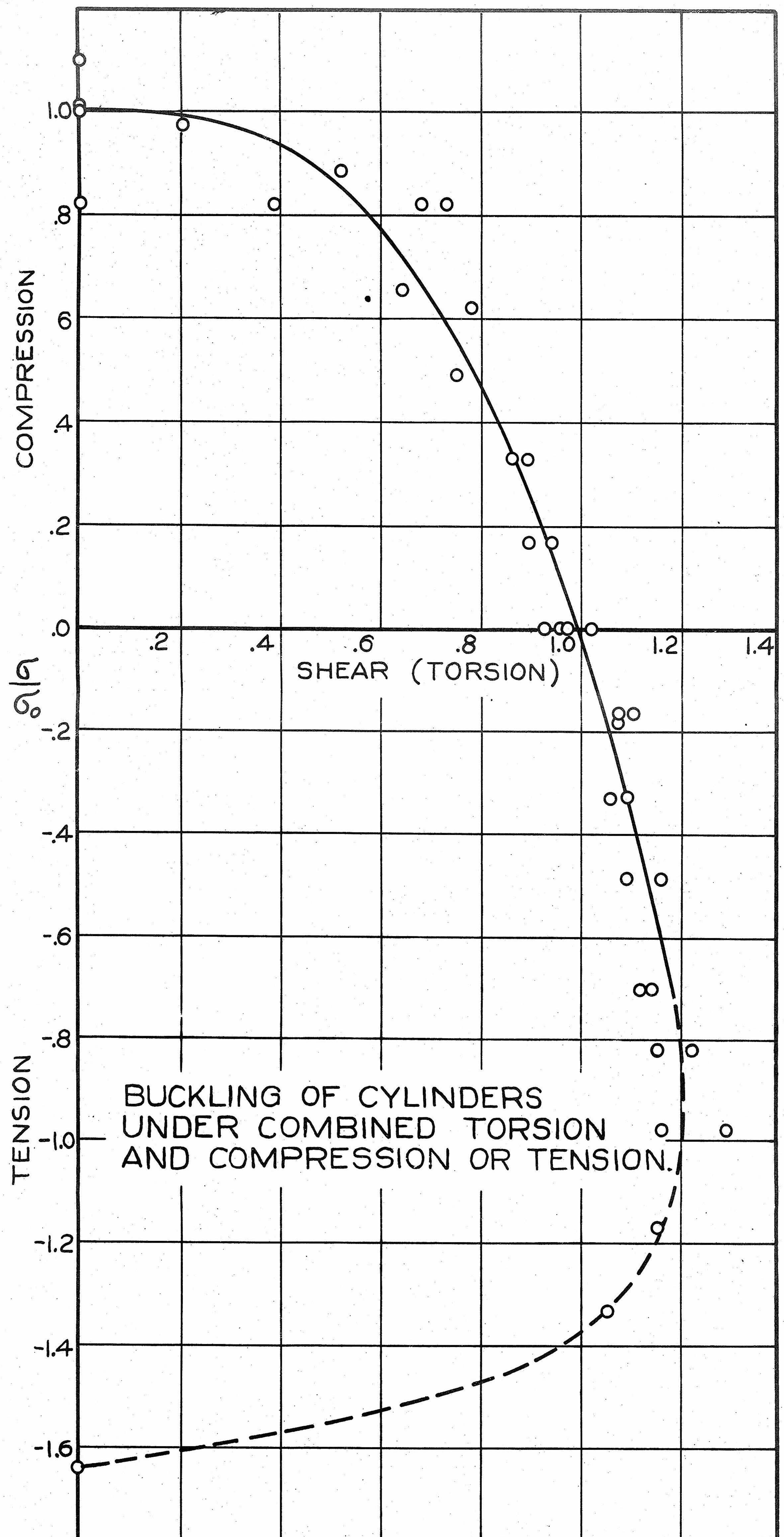

COMPRESSION
TENSION
σ/σ_0
SHEAR (TORSION)
1.0
.8
6
.4
.2
.0
-.2
-.4
-.6
-.8
-1.0
-1.2
-1.4
-1.6
.2
.4
.6
.8
1.0
1.2
1.4
BUCKLING OF CYLINDERS
UNDER COMBINED TORSION
AND COMPRESSION OR TENSION.

Fig. 2

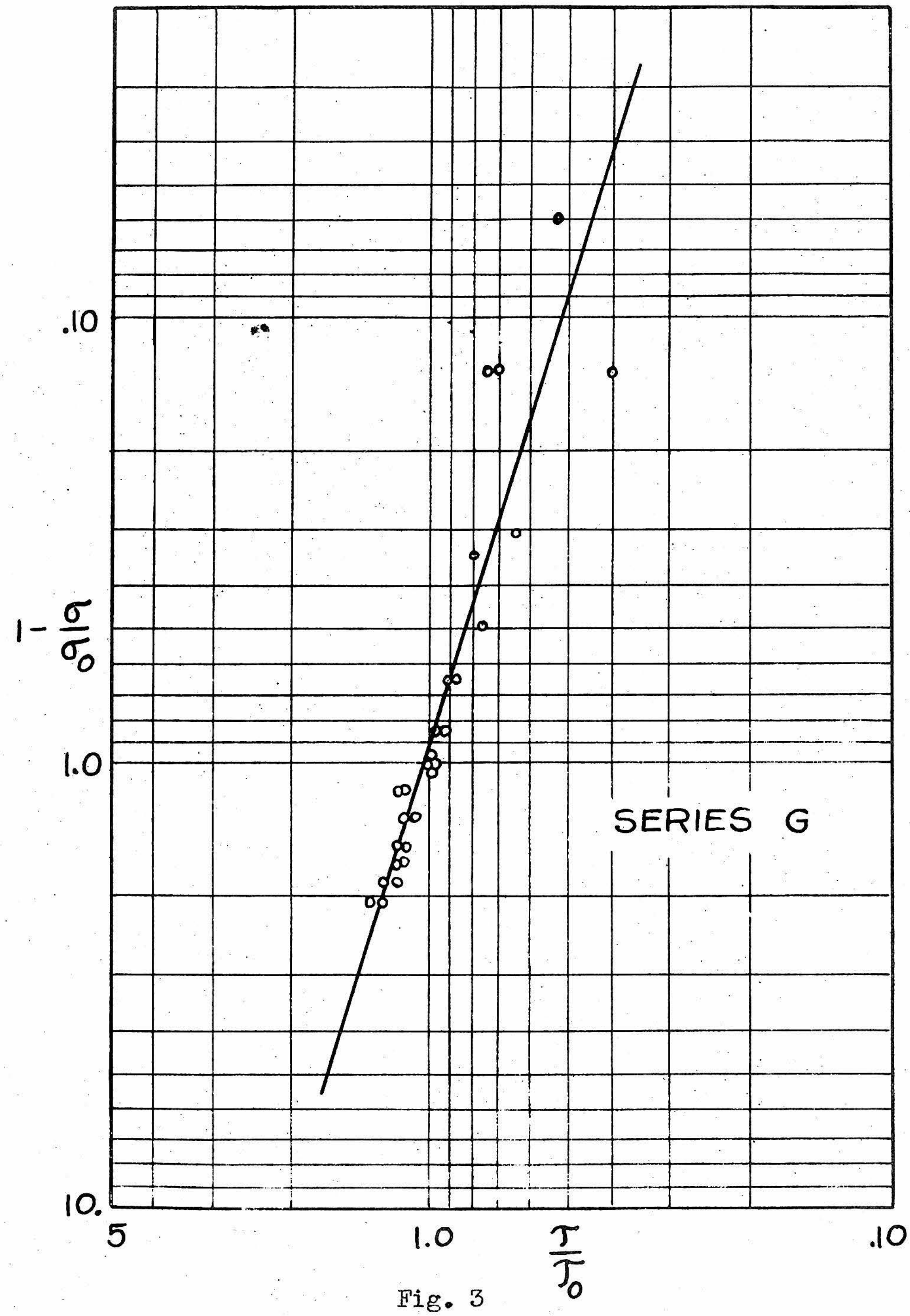


Fig. 3

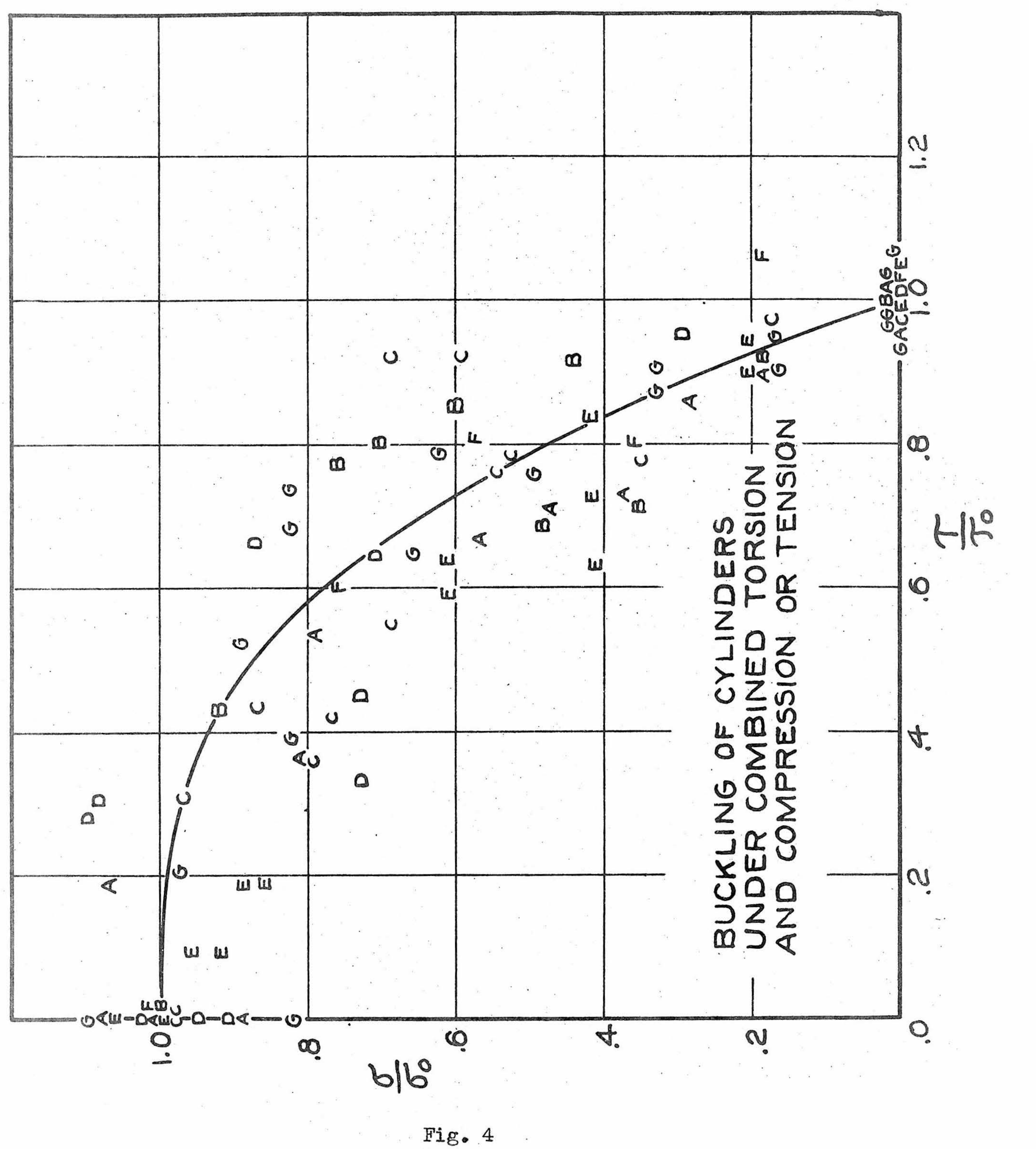

BUCKLING OF CYLINDERS
UNDER COMBINED TORSION
AND COMPRESSION OR TENSION
τ/τ_o
σ/σ_o
0
.2
.4
.6
.8
1.0
1.2

Fig. 4

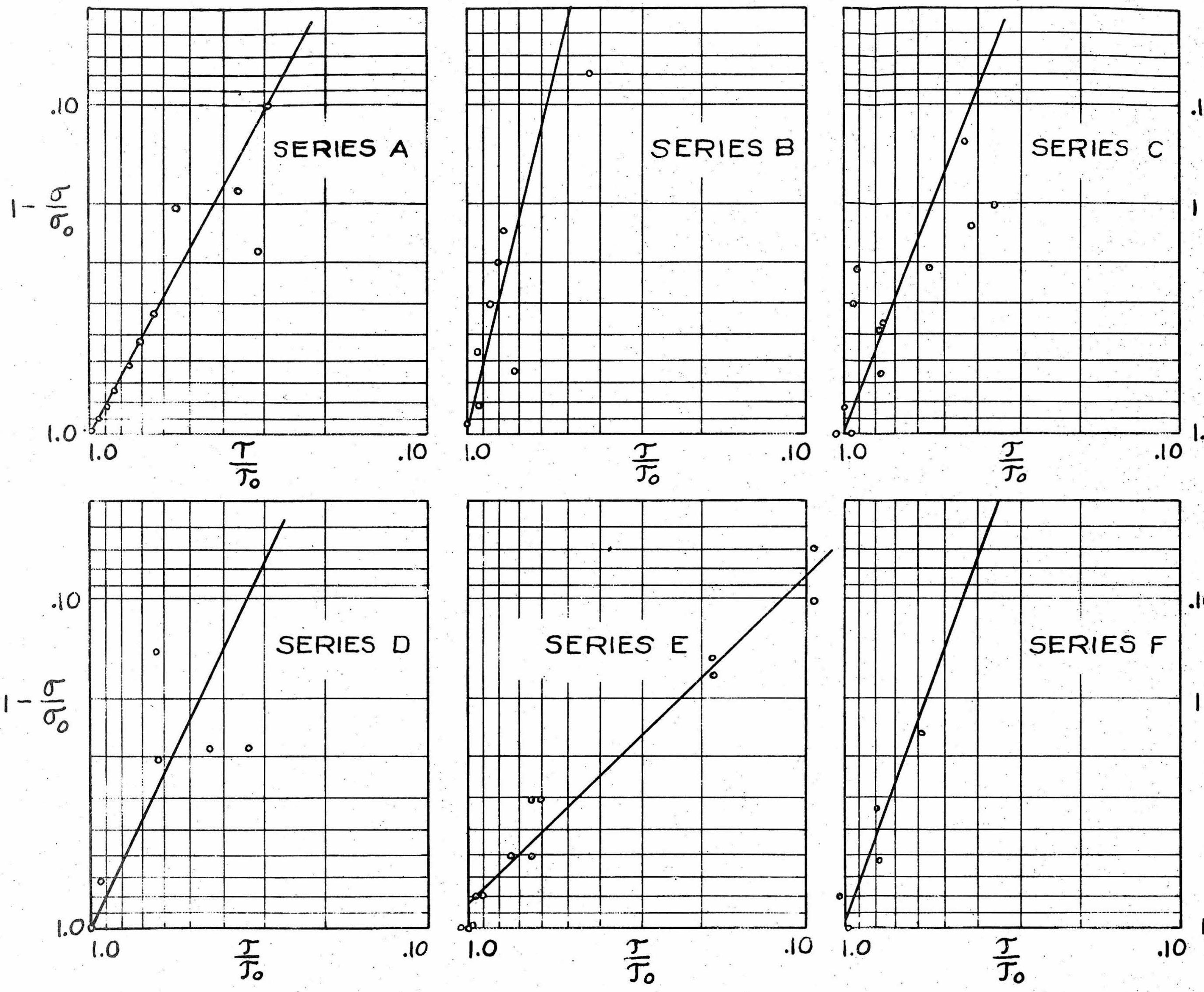

SERIES A
SERIES B
SERIES C
SERIES D
SERIES E
SERIES F
.10
1.0
1 - σ/σo
T/To

Fig. 5

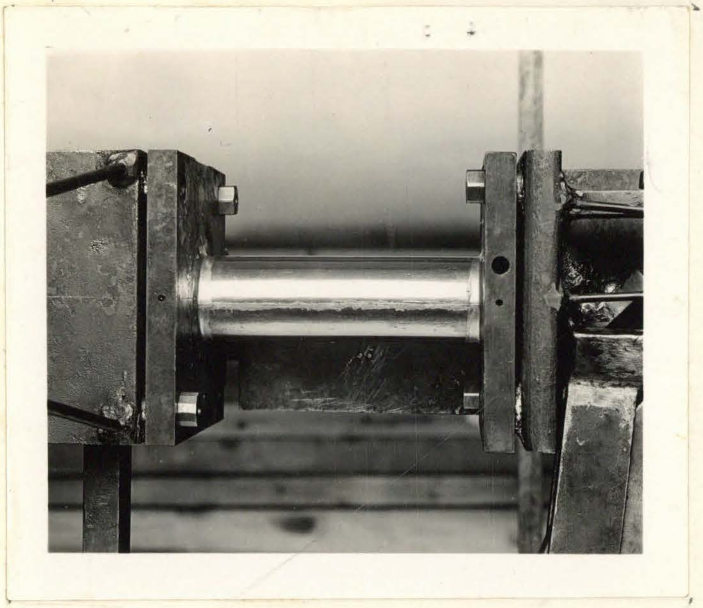


Fig. 6
A Typical Specimen "G" Series

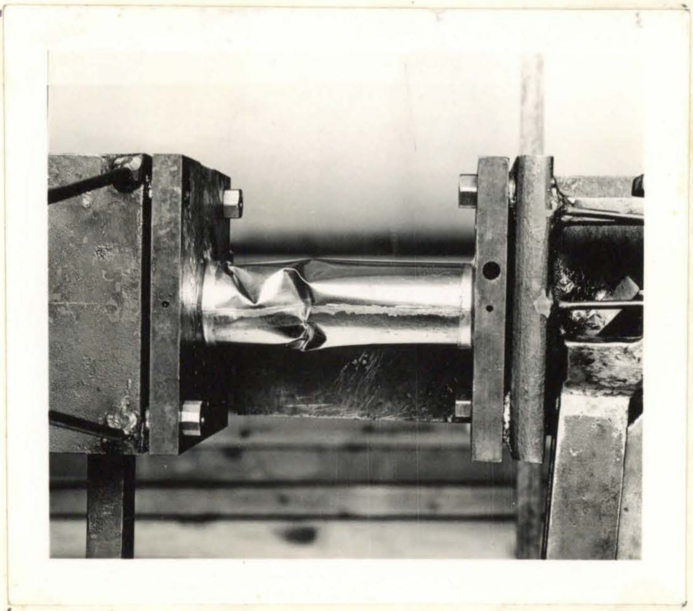


Fig. 7
Compression and Torsion Buckling
Series "A"

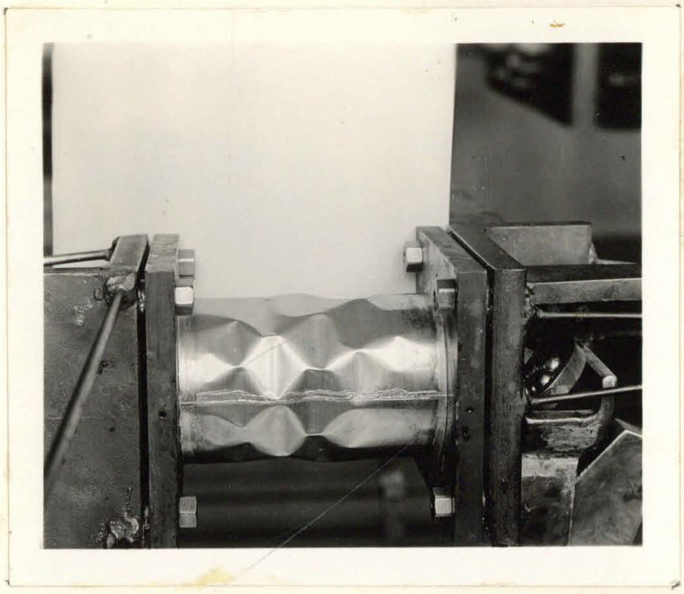


Fig. 8
Axial Compression Buckling
Series "C"

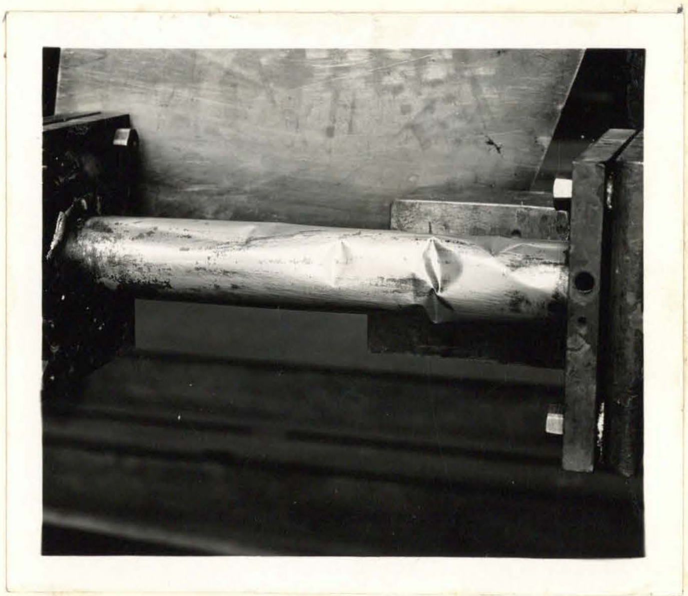


Fig. 9
Compression and Torsion Buckling
Series "D"

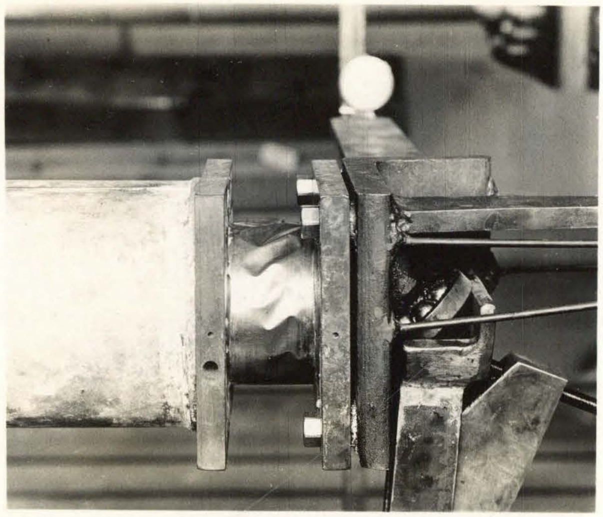


Fig. 10
Compression and Torsion Buckling
Series "F"

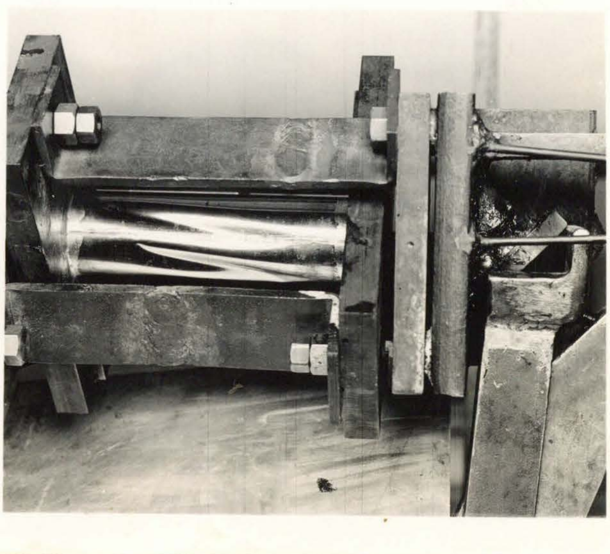


Fig. 11
Tension and Torsion Buckling
Series "G"

Fig. 12
"G" Series Cylinders After
Test