THE DERIVATION AND GRAPHICAL REPRESENTATION OF STRESS FACTORS IN AN ARCH DAM, INCLUDING THE EFFECT OF FOUNDATION YIELD.

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I. SYNOPSIS

A study of the variation in the factors leading to solution of arch dams, including the effect of abutment yield upon these factors, is herein presented.

Since a canvass of the entire range of variables as encountered in the actual practice of arch dam design is essential to the ultimate determination as to the range in which abutment yield cannot be ignored in determining arch dam stresses, a graphical exploration was found to be expedient.

Of necessity, a very definite type of arch was investigated under special end conditions. These conditions had been carefully enumerated under "Assumptions." However, an attempt was made to employ such particular end conditions that the actual mathematical values of the stress factors as charted can be utilized in designs in which the similarity of end conditions causes an error permissable ~~in degree~~ by the degree of uncertainty of the known variables. The computer should determine for himself the amount of error resulting from a change in the assumptions as, herein, specified before he uses the charts extensively.

Regardless of the range of validity of the actual mathematical values of the charts, the range in which the so-called "factor of ignorance" cannot assimilate the difference in stresses between fixed and yielding abutments, ~~are~~ is definitely traced through a comparison of the stress factor charts for fixed and yielding abutments. For this reason, a graph of the factors for the fixed abutment (which has appeared, in part, in a paper presented by

B. F. Jacobson.) has been included. It is felt that a reasonable change in the type of arch, waterload or the other assumptions, will, after a few isolated trials by the mathematical method, prove to the computer that the charts presented herein satisfactorally map for him, in general, the ranges in which he cannot ignore abutment yield.

II. INTRODUCTION

Some admirable work on the rather mathamatically complex phenomena of arch dam stresses which include the effect of abutment yield has been done by Dr. F. Vogt. Unfortunately, however, the computations involved in the stress calculations which include this bothersome abutment yield, as presented by Dr. Vogt, are so laborious that they preclude their inclusion in the ordinary dam design. However, B. F. Jacobson and Dr. F. Vogt have demonstrated by means of several isolated examples that abutment yield cannot be ignored in the case of relatively thick arches. A clear definition as to what constitutes a thick arch in the case of abutment yield investigation was not presented or could not possibly be presented with the meager canvass they had obviously taken. The necessary field of investigation was thus clearly outlined for a complete appreciation of the affect of abutment yield might be consumated. This investigation is undertaken herein, modified by the following basic conditions and assumptions. Again, these conditions and assumptions were chosen keeping in mind the desireability of extending the range of validity of the results obtained as far as possible.

Conditions and Assumptions

(1) A constant thickness arch is studied. This is done because of the present wide application of arches of this type.

(2) Uniform waterload is assumed. A small variation in the symmetry of p should not greatly effect the resulting arch stress, so that if the crown value of p is used, a fair degree of safety and approx-

imation is probably obtained.

(3) The modulus of elasticity of the foundation is taken as equal to that of concrete. This is supported by the facts that an arch dam should not be constructed in a locality in which this is not at least approximately true, and secondly, fairly large variations in the values of the elasticity moduli do not greatly affect these stresses.

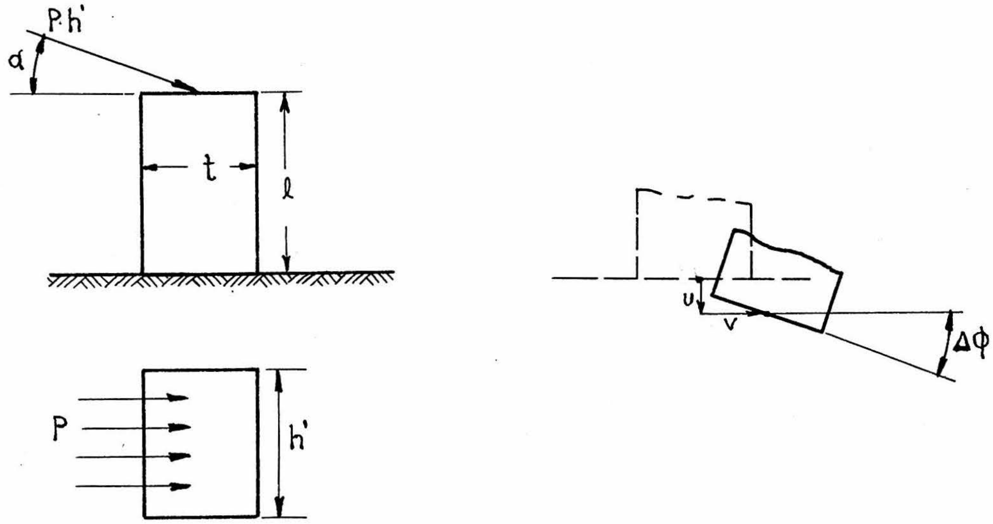
(4) The plane of the abutment is considered normal to the arch ring. This end condition is, of course, desireable where possible, in arch dam design; but in any case, a reasonable change from the normal, will not affect the stresses greatly.

(5) Poisson's ratio was taken as 1/8. Great accuracy is not required here because examination of several isolated pertinent examples revealed that variation in Poisson's ratio from 1/6 to 1/10 hardly affected the result.

(6) Two values of the ratio of loaded surface to arch thickness (which ratio is pertinent to the yield equations, as disclosed later) are examined and charted. Straight line interpolation is possible for values lying between the chosen two.

III. DEVELOPEMENT OF EQUATIONS

(1.) The following diagrams are as perDr. Vogt, except that the symbols have been altered to suit the convenience of the ensuing application.



Dr. Vogt has arrived at the following approximate formulae for the average displacement and rotation of the loaded part of the surfaces, for values of h' varying from t to 30t.

$$u = \frac{m^2-1}{m^2} \times \frac{N}{E_f \cdot t} \times \sqrt[3]{t^2 \cdot h'}$$

$$v = \frac{m^2-1}{m^2} \times \frac{S}{E_f \cdot t} \times \sqrt[3]{t^2 \cdot h'} + \frac{(m-2)(m+1)}{m^2} \times \frac{M}{E_f \cdot t} \times \frac{1}{1 + 1.1\frac{t}{h'}}$$

$$\Delta\phi = \frac{18}{\pi} \times \frac{m^2-1}{m^2} \times \frac{M}{E_f t^2} \times \frac{1}{1 + 0.25\frac{t}{h'}} + \frac{(m-2)(m+1)}{m^2} \times \frac{S}{E_f \cdot t} \times \frac{1}{1 + 1.1\frac{t}{h'}}$$

In which,

$N = P \cdot \sin\alpha =$ normal force per unit length,

$S = P \cdot \cos\alpha =$ shear " " " " " "

$M = P \cdot \ell \cdot \cos\alpha =$ moment " " "

1.(cont.)

or, in more simpler terms,

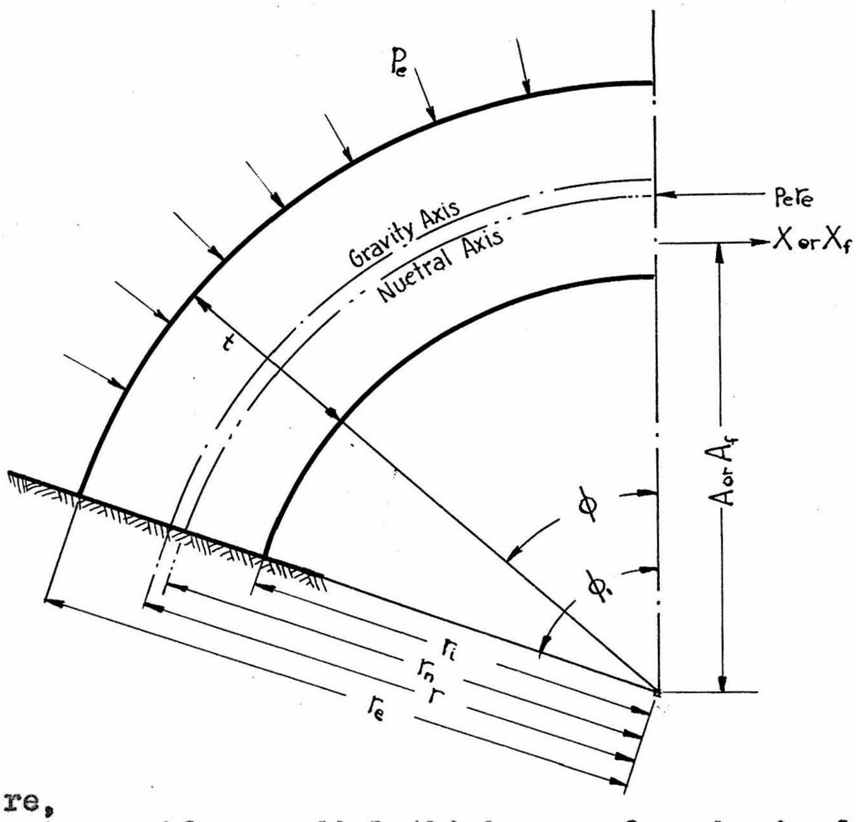
$$U = \xi \times \frac{N}{E_f}$$

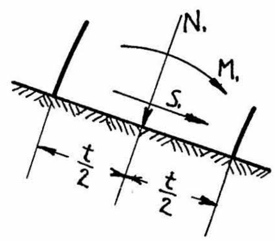
$$V = \xi \times \frac{S}{E_f} + \eta \times \frac{M}{E_f \cdot t}$$

$$\Delta\phi = \mu \times \frac{M}{E_f \cdot t^2} + \eta \times \frac{S}{E_f \cdot t}$$

where the values of the three variables is obvious after comparisom of these eguations with the previous ones.

2. The arch slice represented below, reveals the following relations;





Where,

t = uniform radial thickness of arch, in feet;

r = radius of the center line, in feet;

r_n = radius of the nuetral line, in feet;

r_e = radius of extrados, in feet;

r_i = radius of intrados, in feet;

p_e = normal radial pressure, in pounds per square foot, on the extrados

ϕ = angle with radius of crown

ϕ_1 = one halfe the central angle

E_A = modulus of elasticity of the arch

E_f = modulus of elasticity of the foundation

$c = r - r_n$

$X_{(f)}$ = additional crown force, applied normal to crown radius, necessary to bring crown section back to center

M = moment at any section

N = thrust at any section

S = shear at any section

$A_{(f)}$ = distance from arch center along crown radius at which it is necessary to locate $X_{(f)}$ in order to eliminate crown rotation

M_1 = moment at abutment

N_1 = thrust at abutment

S_1 = shear at abutment

then, referred to the center of gravity, the :

$$M_1 = X_f\left[A_f - r_n \cdot \cos\phi_1 - c \cdot \cos\phi_1\right] + p_e r_e \cdot c$$

$$N_1 = -X_f \cdot \cos\phi_1 + p_e r_e$$

$$S_1 = X_f \cdot \sin\phi_1$$

or, referred to the nuetral axis;

$$M = X_f\left[A_f - r_n \cdot \cos\phi_1\right]$$

3 Evaluation of the factors A and A_f

(a) For the arch with fixed abutments, the value of A is found by means of;

$$\int_{\varphi_1}^{0} \frac{M r_n}{E_A I} d\varphi = 0 \qquad \text{since} \qquad M = X(A - r_n \cos\phi_1)$$

which gives,

$$A = r_n \cdot \frac{\sin\phi_1}{\phi_1}$$

(b) For the arch with yielding abutments, the condition $\sum_{\phi_1}^{0} \alpha_A + \alpha_f = 0$ governs (from geometrical symmetry), where;

α_A = angular deflection of the arch = $\int_{\varphi_1}^{0} \frac{M r_n}{E_A \cdot I} \cdot d\varphi$

α_f = angular deflection of the abutments = $\mu \frac{M_1}{E_f t^2} + \eta \frac{S_1}{E_f \cdot t}$

$$\sum \int_0^{\varphi_1} \frac{M r_n}{E_A I} d\varphi + \left(\mu \frac{M_1}{E_f t^2} + \eta \frac{S_1}{E_f t}\right) = 0$$

or, putting in the values of M, M_1, S_1, and using the relations

$X = \frac{P_e r_e}{K}$ and $X_f = \frac{P_e r_e}{K_f}$

we get;

$$\sum \int_0^{\varphi_1} \frac{P_e r_e}{K_f} \frac{(A_f - r_n \cos\phi_1)}{E_A \cdot \frac{t^3}{12}} r_n \cdot d\varphi + \mu \frac{P_e r_e}{E_A t^2} \left(\frac{A_f - r_n \cos\phi_1}{K_f} + c \right) + \eta \frac{P_e r_e}{E_f K_f t} \sin\phi_1 = 0$$

or, since $E_A = E_f$ and integrateing;

$$\frac{12 r_n}{K_f t^3} (A_f \phi_1 - r_n \sin\phi_1) + \frac{\mu}{t^2} \left(\frac{A_f - r_n \cos\phi_1}{K_f} + c \right) + \frac{\eta}{t} \frac{\sin\phi_1}{K_f} = 0$$

factering out $\frac{A_f}{t}$, we obtain;

$$\frac{A_f}{t} = \frac{\left(12\left(\frac{r_n}{t}\right)^2 \sin\phi_1 - \eta\right) \sin\phi_1 - \mu\left(\frac{c}{t} K_f - \frac{r}{t} \cos\phi_1\right)}{12\left(\frac{r_n}{t}\right)\phi_1 + \mu}$$

(4) EVALUATION OF THE FACTORS K AND K_f

Letting ΔX denote the deformation of the fountdation and arch in a direction normal to the crown radius, and positive in value for deformations towards the center;

then: (a) For the arch fixed at the abutments:

$$\Delta X_A = \int_{\varphi}^{0} \frac{M}{E_A I} r_n^2 (\cos\phi - \cos\phi_1)\, d\varphi + \int_{\phi_1}^{0} \frac{1.2\, S_1}{G t} r_n \cdot \sin\phi \cdot d\varphi - \int_{\phi_1}^{0} \frac{N}{E t} \cdot r_n \cdot \cos\phi \cdot d\varphi$$

Placing $\Delta X = 0$; replacing M, S and N by their values in terms and X; and finally, using the value of $\frac{E}{G} = 2.4$ we may integrate and obtain,

$$\Delta X_A = 0 = -P_e r_e \sin\phi_1 \left(\frac{r_n}{t}\right) + X \left[\frac{r_n^2}{\frac{t^3}{12}} \left\{ A(\phi_1 - \sin\phi_1) - r_n \left(\sin\phi_1 - \frac{\phi_1}{2} - \frac{\sin 2\phi_1}{4}\right) \right\} + \frac{r_n}{t}\left(1.94\phi_1 - 0.47 \sin 2\phi_1\right) \right]$$

which, when solving for X , gives;

$$X = P_e r_e \cdot \frac{\left(\frac{r_n}{t}\right) \sin\phi_1}{12\left(\frac{r_n}{t}\right)^2 \left\{ A(\phi_1 - \sin\phi_1) - r_n\left(\sin\phi_1 - \frac{\phi_1}{2} - \frac{\sin 2\phi_1}{4}\right) \right\} + \frac{r_n}{t}\left(1.94\phi_1 - 0.47 \sin 2\phi_1\right)}$$

or, in shorter terms,

$X = \frac{P_e r_e}{K}$,where the value of K is obvious by comparisom

For use later, in the solution of K_f, the following relations are obtained, by putting in the value of $A = r_n \frac{\sin\phi_1}{\phi_1}$

then;

$$K = \left[12\left(\frac{r_n}{t}\right)^3 \frac{\sin\phi_1}{\phi_1}(\phi_1 - \sin\phi_1) + \psi\left(\frac{t}{r_n}, \phi_1\right) \right] \frac{t}{r_n} \cdot \frac{1}{\sin\phi_1}$$

where, $\psi\left(\frac{t}{r_n}, \phi_1\right)$ has grouped the following varibles;

$$-12\left(\frac{r_n}{t}\right)^3 \left(\sin\phi_1 - \frac{\phi_1}{2} - \frac{\sin 2\phi_1}{4}\right) + \frac{r_n}{t}\left(1.94\phi_1 - 0.47 \sin 2\phi_1\right)$$

(b) For the arch with yielding foundations, the identical geometrical end conditions(of no crown movement normal to the crown radius) exist. Also, it is to be noted that X becomes X_f and, A becomes A_f

(1) For the arch, with fixed abutments;

$$\Delta X_A = -p_e r_e\left(\frac{r_n}{t}\right)\sin\phi_1 + X\cdot 12\left(\frac{r_n}{t}\right)^2\cdot A(\phi_1 - \sin\phi_1) + X\cdot\psi\left(\frac{t}{r_n},\phi_1\right)$$

now, replacing A and X by A_f and X_f respectively, and also, putting in the value of $\psi\left(\frac{t}{r_n},\phi_1\right)$ as just found,

(which equals $\left(\frac{r_n}{t}\right)\cdot K\cdot\sin\phi_1 - 12\left(\frac{r_n}{t}\right)^3\frac{\sin\phi_1}{\phi_1}(\phi_1 - \sin\phi_1)$)

we get;

$$\Delta X_A = -p_e r_e\left(\frac{r_n}{t}\right)\sin\phi_1 + \frac{p_e r_e}{K_f}\cdot 12\left(\frac{r_n}{t}\right)^2\cdot\frac{A_f}{t}(\phi_1 - \sin\phi_1) + p_e r_e\left(\frac{r_n}{t}\right)\frac{K}{K_f}\sin\phi_1$$
$$-\frac{p_e r_e}{K_f}\cdot 12\cdot\left(\frac{r_n}{t}\right)^3\frac{\sin\phi_1}{\phi_1}(\phi_1 - \sin\phi_1)$$

in which we have placed $X_f = \frac{p_e r_e}{K_f}$

(2) For the foundation,

From an algebraic summation of the components of the deflections in the foundations(components to be in the direction normal to the crown radius)

we get;

$$\Delta X_f = \xi\left[S_1\cdot\sin\phi_1 - N_1\cdot\cos\phi_1\right] + \eta\frac{M_1}{t}\sin\phi_1$$
$$+\left[\mu\frac{M_1}{t^2} + \eta\frac{S_1}{t}\right]\left[r_n(1-\cos\phi_1) - c\cdot\cos\phi_1\right]$$

or, putting in the values of S_1, M_1, and N_1 and reducing as far as possible,

$$\Delta X_f = \xi\cdot p_e r_e\left[\frac{1}{K_f} - \cos\phi_1\right] + \eta\left[\frac{1}{K_f}\left(\frac{A_f}{t} - \frac{r}{t}\cos\phi_1\right)\sin\phi_1 + \frac{c}{t}\sin\phi_1\right]p_e r_e$$
$$+\mu\frac{p_e r_e}{K_f}\left(\frac{A_f}{t} - \frac{r}{t}\cos\phi_1\right)\left(\frac{r_n}{t} - \frac{r}{t}\cos\phi_1\right) + \mu\cdot\frac{c}{t}\cdot p_e r_e\left(\frac{r_n}{t} - \frac{r}{t}\cos\phi_1\right)$$
$$+\eta\frac{p_e r_e}{K_f}\sin\phi_1\left(\frac{r_n}{t} - \frac{r}{t}\cos\phi_1\right)$$

Combining the deflections of the arch and the foundations, and imposing the geometrical end condition as heretofore mentioned,

$$\sum \Delta X_A + \Delta X_f = 0$$

which gives;

$$-\frac{r_n}{t}\sin\phi_1 + \frac{1}{K_f}\cdot 12\left(\frac{r_n}{t}\right)^2\cdot\frac{A_f}{t}(\phi_1-\sin\phi_1) - \frac{1}{K_f}\cdot 12\left(\frac{r_n}{t}\right)^3\frac{\sin\phi_1}{\phi_1}(\phi_1-\sin\phi_1)$$
$$+\frac{1}{K_f}\left(\frac{r_n}{t}\right)\sin\phi_1\cdot K + \xi\cdot\frac{1}{K_f} - \xi\cos\phi_1 + \frac{1}{K_f}\eta\cdot\sin\phi_1\left(\frac{A_f}{t}-\frac{r}{t}\cos\phi_1\right)$$
$$+\eta\sin\phi_1\cdot\frac{c}{t} + \frac{1}{K_f}\mu\left(\frac{A_f}{t}-\frac{r}{t}\cos\phi_1\right)\left(\frac{r_n}{t}-\frac{r}{t}\cos\phi_1\right) + \mu\cdot\frac{c}{t}\cdot\left(\frac{r_n}{t}-\frac{r}{t}\cos\phi_1\right)$$
$$+\frac{1}{K_f}\cdot\eta\cdot\sin\phi_1\left(\frac{r_n}{t}-\frac{r}{t}\cos\phi_1\right) = 0$$

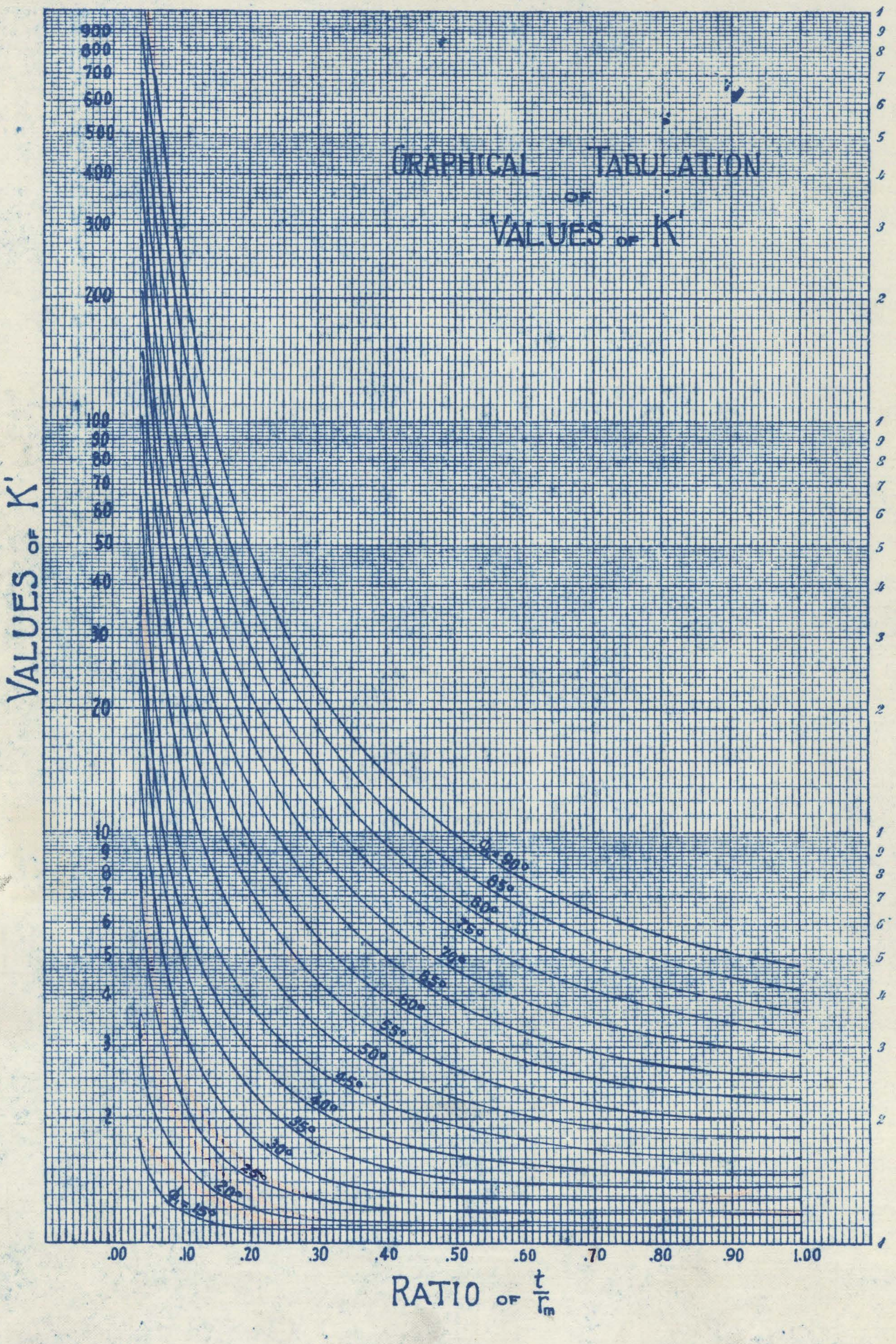
or, upon solving for and reducing as far as possible, the ultimate stress factor, involving foundation yield, is obtained.

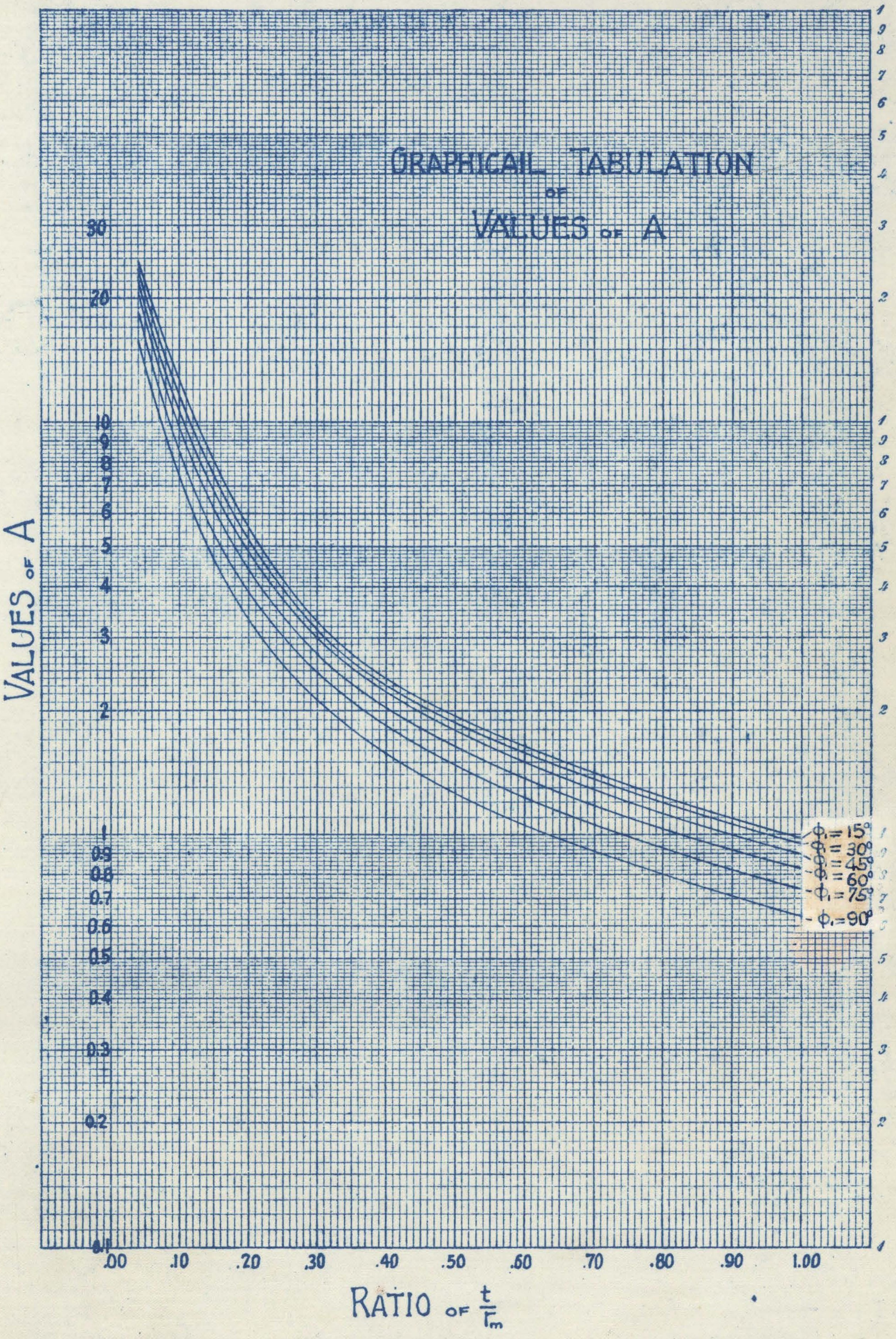
$$K_f = \frac{12\left(\frac{r_n}{t}\right)^2(\phi_1-\sin\phi_1)\left(\frac{A_f}{t}-\frac{r_n}{t}\frac{\sin\phi_1}{\phi_1}\right) + \frac{r_n}{t}\sin\phi_1\cdot K + \xi + \eta\sin\phi_1\left(\frac{A_f}{t}-\frac{r}{t}\cos\phi_1\right) + \mu\left(\frac{A_f}{t}-\frac{r}{t}\cos\phi_1\right)\left(\frac{r_n}{t}-\frac{r}{t}\cos\phi_1\right) + \eta\sin\phi_1\left(\frac{r_n}{t}-\frac{r}{t}\cos\phi_1\right)}{\left(\frac{r_n}{t}\right)\sin\phi_1 + \xi\cos\phi_1 - \eta\cdot\sin\phi_1\cdot\frac{c}{t} - \mu\cdot\frac{c}{t}\cdot\left(\frac{r_n}{t}-\frac{r}{t}\cos\phi_1\right)}$$

To reiterate, $X_f = \frac{P_e r_e}{K_f}$. M, T, and S are found directly from X_f as shown before.

The value of K_f was laboriously computed for two ratios of $\frac{h'}{t}$ the results of which are presented graphically in order to facilitate interpolation.

IV. GRAPHICAL TABULATION


GRAPHICAL TABULATION OF VALUES OF K'
VALUES OF K'
RATIO OF $\frac{t}{r_m}$
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
.00
.10
.20
.30
.40
.50
.60
.70
.80
.90
1.00
θ_1 = 90°
85°
80°
75°
70°
65°
60°
55°
50°
45°
40°
35°
30°
25°
20°
θ_1 = 15°


GRAPHICAL TABULATION
OF
VALUES OF A
VALUES OF A
RATIO OF $\frac{t}{r_m}$
$\phi_1 = 15°$
$\phi_1 = 30°$
$\phi_1 = 45°$
$\phi_1 = 60°$
$\phi_1 = 75°$
$\phi_1 = 90°$
.00
.10
.20
.30
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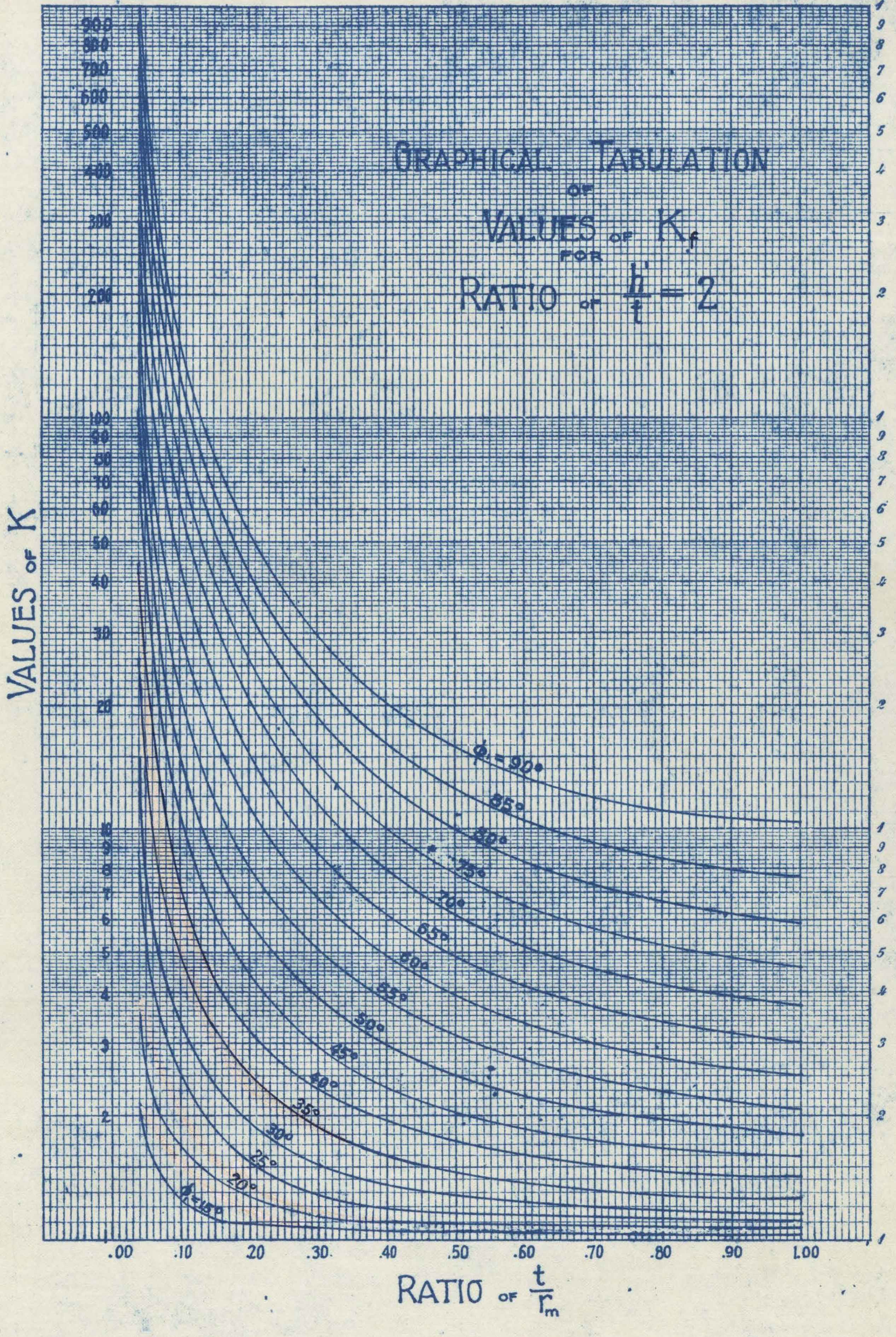
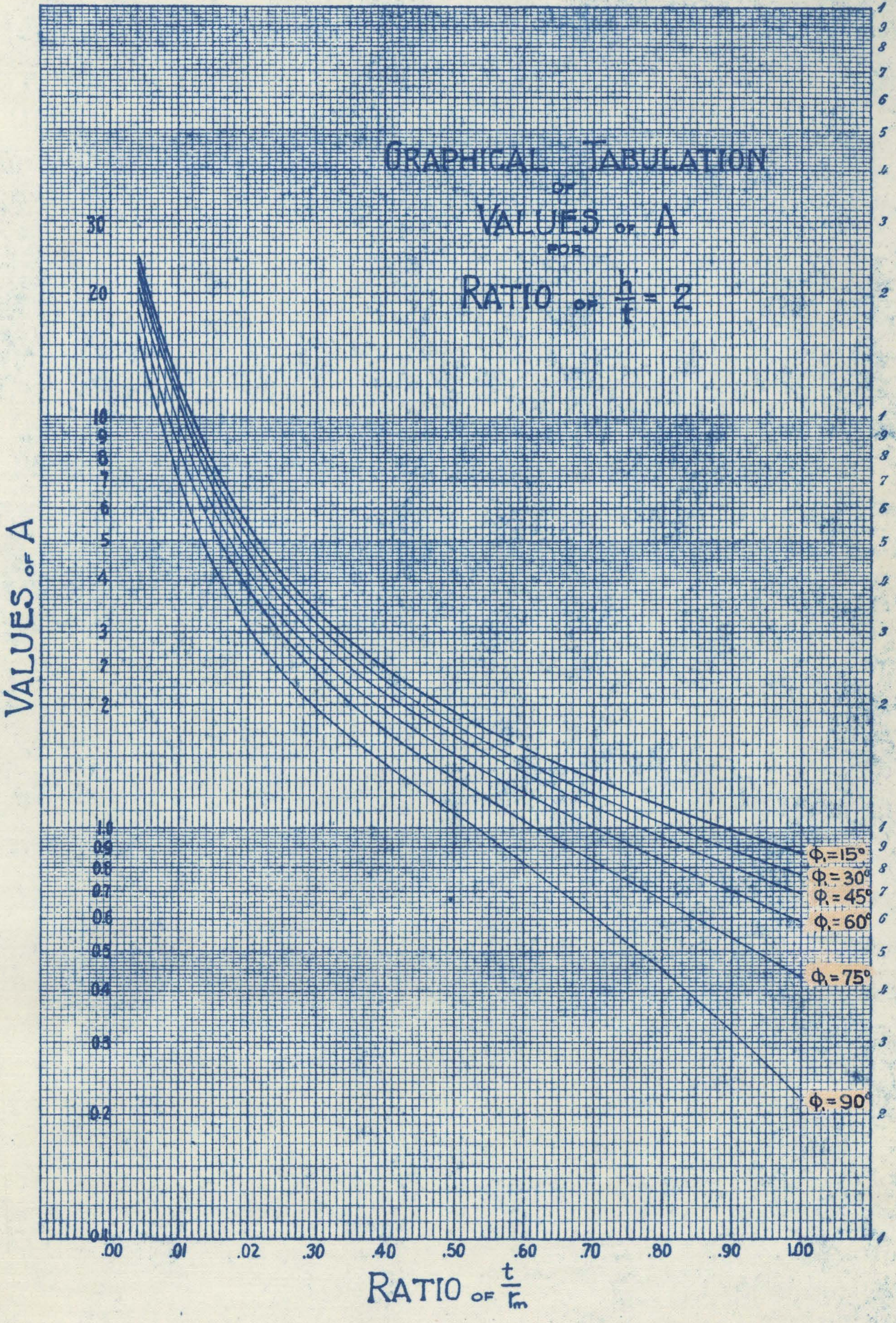
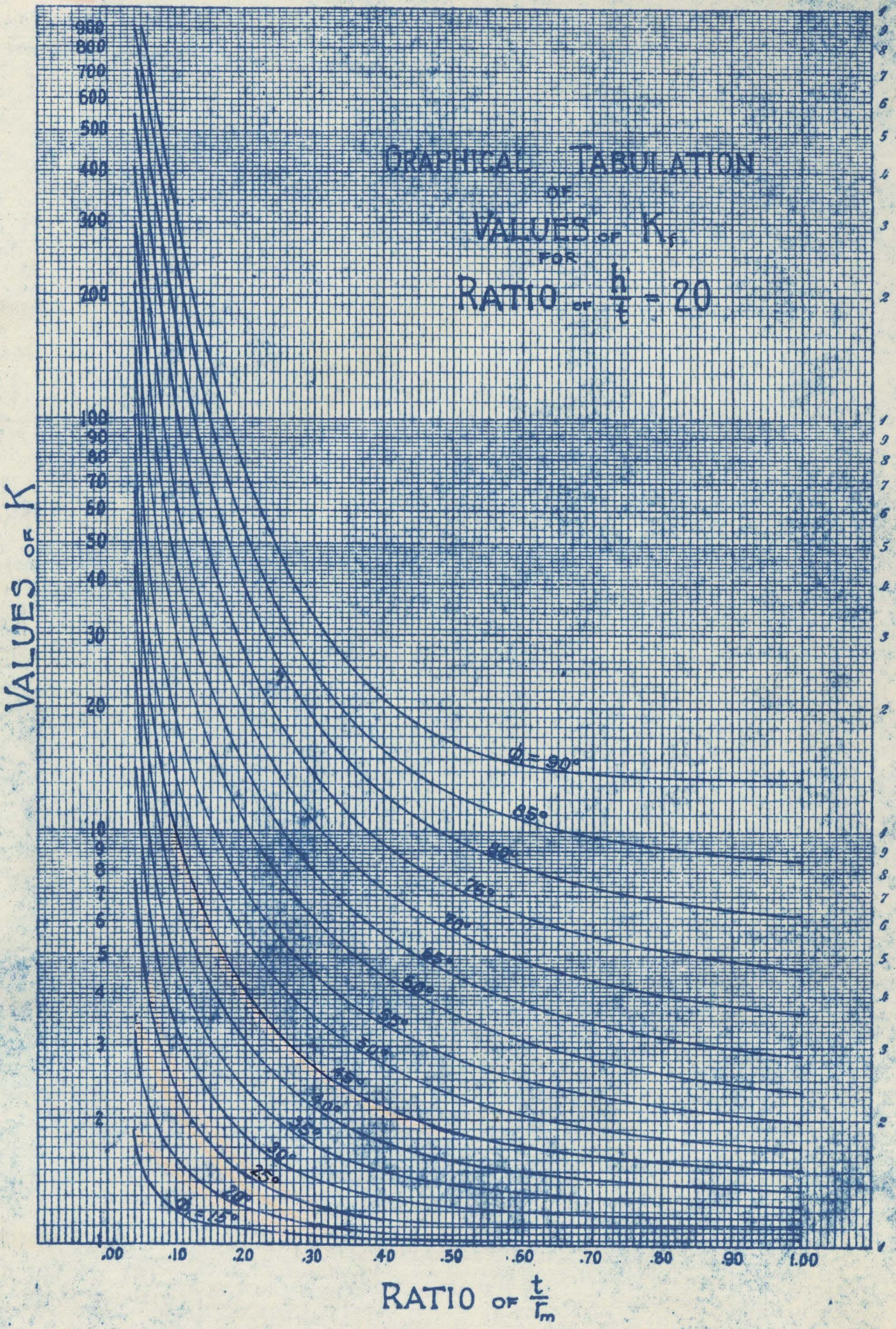
.50
.60
.70
.80
.90
1.00


GRAPHICAL TABULATION
OF
VALUES OF K_f
FOR
RATIO OF $\frac{h'}{t} = 2$
VALUES OF K
RATIO OF $\frac{t}{r_m}$
.00
.10
.20
.30
.40
.50
.60
.70
.80
.90
1.00
φ = 90°
85°
80°
75°
70°
65°
60°
55°
50°
45°
40°
35°
30°
25°
20°
φ = 15°


GRAPHICAL TABULATION
OF
VALUES OF A
FOR
RATIO OF $\frac{h}{t}$ = 2
VALUES OF A
RATIO OF $\frac{t}{r_m}$
ϕ_1 = 15°
ϕ_1 = 30°
ϕ_1 = 45°
ϕ_1 = 60°
ϕ_1 = 75°
ϕ_1 = 90°
.00
.01
.02
.30
.40
.50
.60
.70
.80
.90
1.00
30
20
10
0.1


GRAPHICAL TABULATION OF VALUES OF K_f FOR RATIO OF $\frac{h}{t}$ = 20
VALUES OF K
RATIO OF $\frac{t}{r_m}$
ϕ = 90°
85°
80°
75°
70°
65°
60°
55°
50°
45°
40°
35°
30°
25°
20°
ϕ = 15°
.00
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.50
.60
.70
.80
.90
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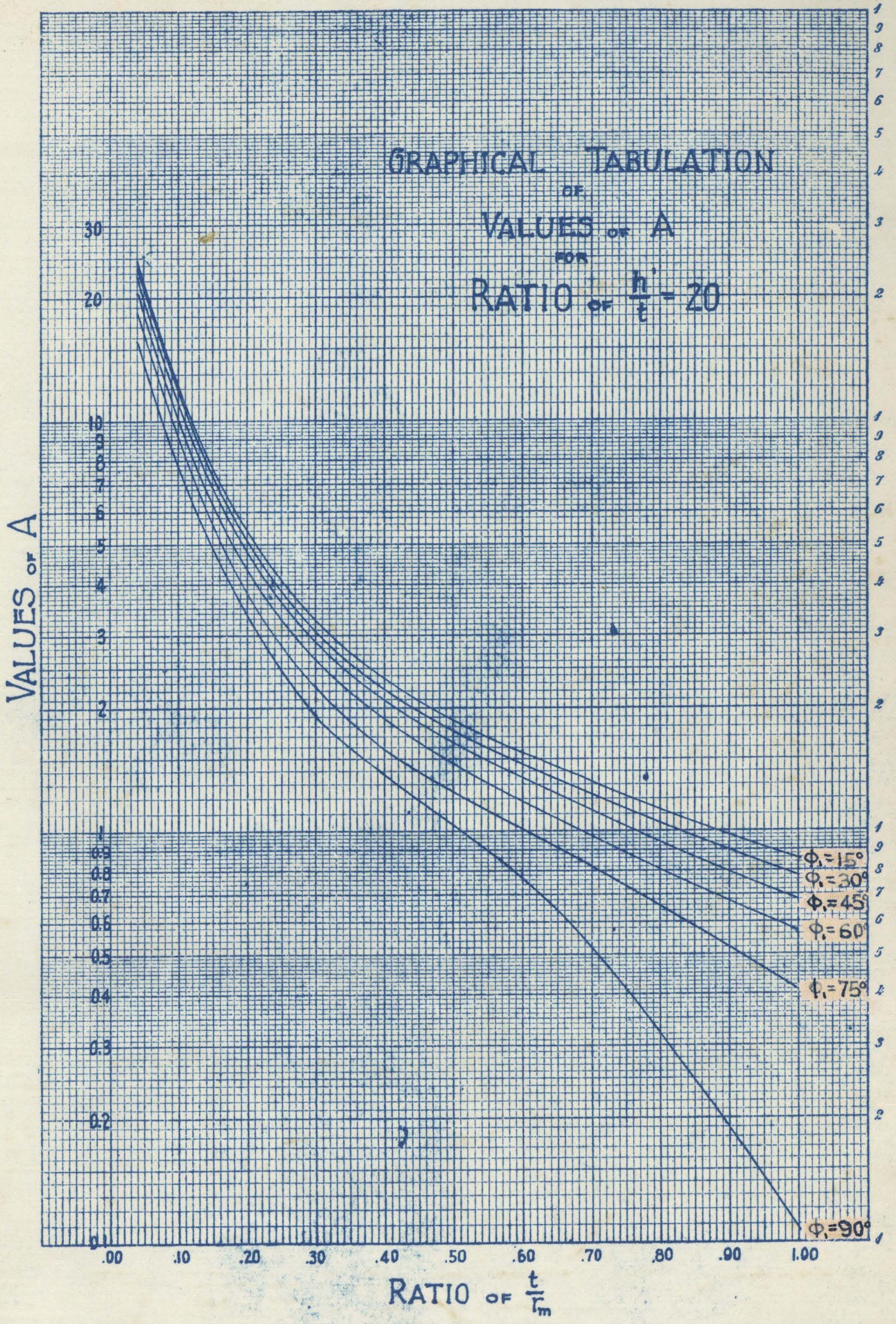

GRAPHICAL TABULATION OF VALUES OF A FOR RATIO OF h'/t = 20
VALUES OF A
RATIO OF t/Tm
30
20
10
9
8
7
6
5
4
3
2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
.00
.10
.20
.30
.40
.50
.60
.70
.80
.90
1.00
φ1=15°
φ1=30°
φ1=45°
φ1=60°
φ1=75°
φ1=90°