

DESIGN OF AN OPTICAL OSCILLOGRAPH FOR
THE INVESTIGATION OF LIGHTNING PHENOMENA

Thesis by
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- # 5 - Motor Mounting

Calibration Curve for Mirror; Watts Input to Motor by Speed in r.p.m.

I. Lightning Problems

Although long recognized as an electrical phenomenon, Lightning continues to present many problems in the fields of Science and Engineering. Perhaps, of most immediate importance are those problems concerned with the protection of transmission lines and electrical apparatus, of explosives and oil storage reservoirs, against Lightning.

With these problems in mind it is then evident that first of all the phenomena of Lightning must, themselves, be clearly understood; some of these being, the generation and separation of cloud charges, the mechanism of breakdown, the factors determining the path which the stroke will follow, and the electrical characteristics of the stroke, i.e., the maximum current, steepness of wave front, quantity of electricity discharged and duration of current flow. A current-time oscillogram of the current in a stroke to ground, should give these latter undetermined electrical characteristics of the flash, and it is with these phenomena that this research problem is chiefly concerned.

Lightning Characteristics

The best authorities estimate the current in a Lightning flash to vary from 5,000 to 100,000 amperes, and the quantity of electricity to vary from a small fraction of a coulomb to 100 coulombs. C. T. R. Wilson¹ estimates the average quantity discharged in a flash to be 20 coulombs. It is generally believed that the current does not oscillate but is damped out at the end of the first half cycle; also that the wave is a flat-topped wave with a very steep wave front, being of the order of a micro-second.

II (a).

The Optical Oscillograph

While looking about for some optical scheme for measuring the current in a Lightning flash, for some scheme which would not be attended by the difficulties to be encountered with the Cathode-Ray Oscillograph, an article² was noted, describing an "Optical Oscillograph," which had been used for measuring condenser discharges. Further investigation indicated that this instrument would be most useful for Lightning investigation. Some of its advantages are as follows:-

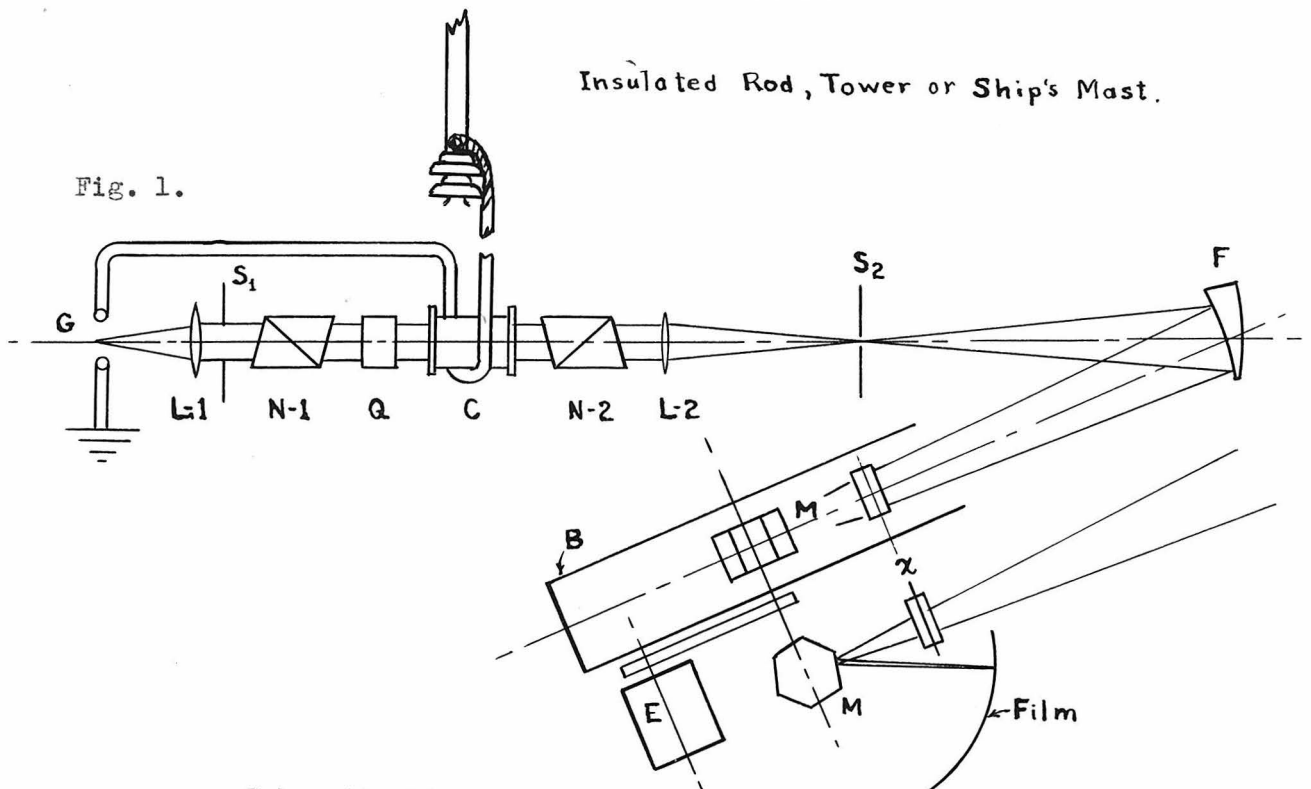
1. It has been used to record currents of 15,000 amperes, and within a range of frequencies up to 200,000 cycles. The upper limit with respect to frequency is determined by the light which can be gotten through the system and is for the most part a matter of cost and design.

2. As the impulse can be used to furnish the necessary light, the apparatus can be made operative at the right instant of time to record the transient or impulse wave. A small gap is placed in series with the electrical circuit, at the light source of the instrument, so that the impulse must form an arc here instantaneously with the flow of current.

3. Such an instrument is comparatively cheap, can be ruggedly designed for use in the field, requires little auxiliary apparatus and can be put in operation and left for hours at a time in anticipation of a lightning flash.

It is not, at least, in its present state of development, suitable for measurements of small currents, as a strong magnetic field is necessary to produce the magneto-optic effect; nor is it suitable for measurements of voltage.

II (b).

Principles of Operation²

- G = Gap across which impulse forms an arc.
- L-1 = Converging lens. 4-3/8" focal length, 2" diameter.
- S₁ = Stop to keep extraneous light out of the system.
- N-1 = 1" nicol prism.
- Q = Plate of Crystal Quartz - 3 cm. long, 5 cm. diameter.
- C = CS₂ cell - 2-1/2" long, 1" diameter.
- N-2 = 1" nicol prism.
- S₂ = Adjustable slit, adjustable both horizontally and vertically.
- L-2 = Converging lens. 2" diameter, 9-1/2" focal length.
- F = Fery Prism.
- x = 8" Focal length Camera Lens, located on mounting in Camera Box.

- M = Rotating Hexagonal Mirror.
 B = Camera Box.
 E = Electric motor for driving mirror.

A rod or tower would be insulated from ground with a set of compression insulators for approximately 100,000 V. A net of wires soldered to the rod above the insulators connects to a 0000 copper cable. This cable makes one turn about the CS₂ and then connects to one terminal of the small gap placed at the focus of converging lens (L-1). The other terminal of this gap connects solidly to ground.

When the tower is struck an arc forms across the small gap giving the necessary light at the correct instant of time. A very finely drawn wire of iron, lead or tungsten may be used to short the gap, which wire will explode, instantaneously with the arrival of the impulse, thus providing a good light source. Lens #1 renders the light coming from the gap plane parallel and nicol prism #1 in turn, produces "plane" polarized light. The Quartz plate, a piece of crystal quartz, cut perpendicularly to the Optic axis, produces rotary dispersion, i.e., rotates the plane of polarization of each wave length according to the equation,

$$\theta = A + \frac{B l}{\lambda^2} \quad \text{Form.(1).}$$

where l is length of plate, A and B are constants, λ the wave length of light and θ the angle of rotation.

The magnetic field produced by current flowing in the coil about the CS₂ cell rotates the planes of polarization of the light passing through the cell according to the same law as that for quartz.

Form. (2) $\theta' = A' + \frac{B'H}{\lambda^2}$, H being here the strength of the magnetic field. This rotation, small compared to that of the quartz plate, follows the variations of the electric current, practically instantaneously³, adding to or subtracting from that produced by the quartz plate.

Nicol prism #2 cuts out all those wave lengths, of which the planes of polarization lie in its crossed position. The light now appears approximately white to the eye, but certain colors are absent and a linear spectrum would show dark bands or lines as indicated in Fig. 2.

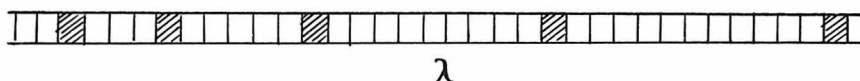


Fig. 2.

These dark lines move back and forth through the spectrum in unison with the current flowing through the coil about the CS₂ cell. They are not uniformly displaced in the spectrum, however, and the displacement of any one is not a linear function of the current.

In the equation

$$\theta' = A' + \frac{B'H}{\lambda^2} \quad \text{Form. (2).}$$

let $\theta = n\pi$, n being any whole number.

Solving

$$\lambda^2 = \frac{B'H}{n\pi - A'} \quad \text{Form. (3).}$$

which gives the wave length of the dark bands in a linear spectrum as in Fig. 2.

A Fery prism produces dispersion according to the equation

$$D = A'' + \frac{B''}{\lambda^2} \quad \text{Form. (4).}$$

Substituting for λ^2 , Form. (3),

$$D = A'' + B''(n\pi - A) \quad \text{Form. (5).}$$

which shows that the dark bands will be uniformly displaced in a spectrum formed by a Fery prism, and the displacement of these dark bands will be a linear function of the current.

A true current-time oscillogram of the current is now obtained by reflecting the beam of light from the Fery prism onto a rotating mirror from which it is again reflected coming to a focus on a film as indicated in Fig. 1.

III.

Design

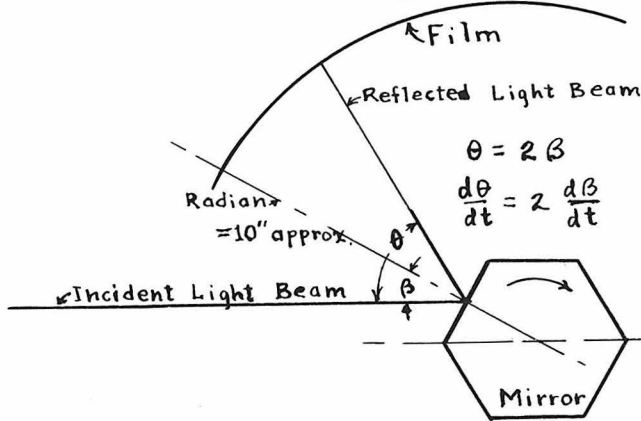
(a). General Considerations

On account of the losses in the Optical System and by reason of the high frequency or steep wave front currents to be measured there is need for a maximum of light through the system. The size of the nicol prisms fixes a limit to this maximum.

A very strong magnetic field is necessary to produce the magneto-optic effect. Lightning currents are thought to be immense, from 5,000 to 100,000 amperes, so a very few turns, perhaps even a fraction of one turn, may be sufficient for the coil about the CS₂ cell.

The wave front of a lightning discharge is of the order of a micro-second; therefore, if this wave front is to be measured accurately the spectrum must move over the film with a velocity of at least 1/16"

per micro-second or $\frac{1,000,000}{16} = 66,700''/\text{second}.$



Now as seen from Fig. 3, the light beam moves over the film with just twice the velocity of the radian drawn perpendicular to the mirror face, therefore, the necessary speed of the radian at the film is about 33,000"/second.

Fig. 3.

Assuming the perpendicular radian drawn from the face of the mirror to the film to be 10" in length,

$$\text{r.p.s.} = \frac{33,000}{2\pi \times 10} = 525 \qquad \therefore \text{r.p.m.} = 31,500,$$

the required speed of the mirror.

III. (b).

Rotating Mirror

There was first to be determined the necessary speed in r.p.m., the number of faces, the length and width of faces, the most suitable bearings and the best material for the mirror.

As shown above, the speed required of the mirror is approximately 30,000 r.p.m. By making the radian from mirror to film longer the r.p.m. could be reduced, but in this case the width of the faces, or what amounts to the same thing, the diameter of the mirror would need to be increased, as may be seen from Fig. 4.

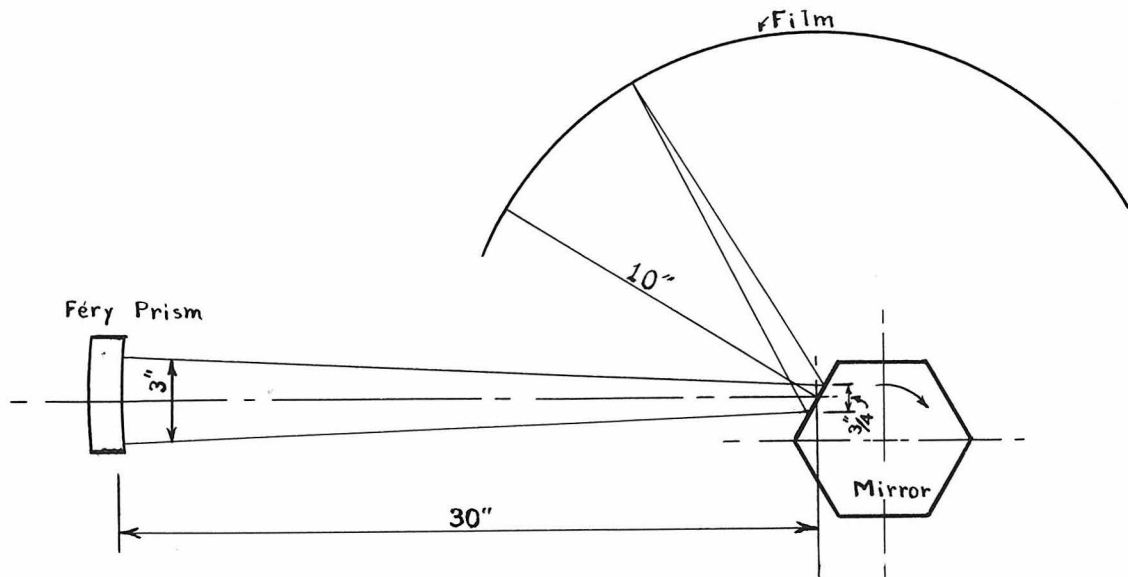


Fig. 4.

Difficulties in polishing and balancing, as well as cost, mount rapidly with the mirror diameter. As a further limitation, it was both difficult and expensive to make a Fery prism with focal length longer than 40". If at the prism the beam of light is 3" in diameter and from there focused at a distance of 40" and if the mirror is placed 10" from the focus (as shown in Fig. 4), then the beam will be 3/4" in diameter at the mirror.

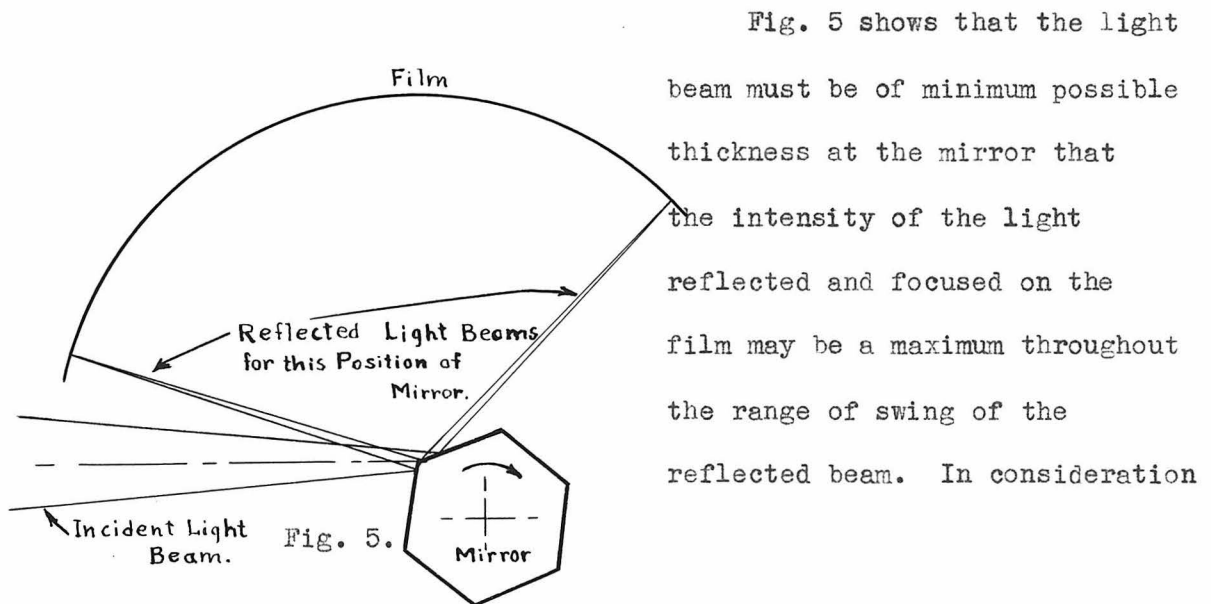


Fig. 5 shows that the light beam must be of minimum possible thickness at the mirror that the intensity of the light reflected and focused on the film may be a maximum throughout the range of swing of the reflected beam. In consideration

of these limitations a general compromise was made using the following dimensions:- See Drawing #1.

Length of radian from mirror face to film	=	10" approx.
Mirror speed in r.p.m.	=	30,000
No. of faces	=	6
Dia. of circumscribed circle for Hex.mirror	=	2-1/4"
Width of face	=	1-1/8"
Length of mirror	=	1-1/2"

A narrow beam of light incident to a face of a hexagonal mirror is reflected within a 120° arc as the mirror rotates, a 90° arc for an octagonal mirror and 180° arc for a square mirror. The 180° swing requires an excessively long film, the octagonal mirror is quite difficult to make and its faces are narrow, hence the hexagonal mirror was chosen as the more desirable.

In order to raise the "critical speed" of the mirror and shaft well above 30,000 r.p.m. the distance between bearings (see Drawing #1) was made as short as possible. Besides, it is best that the spectrum be not too long, as the intensity of the light in the spectrum diminishes directly as its length is increased. One inch was taken as a maximum length for the spectrum and the mirror was made 1-1/2" in length.

Material for Mirror

- Requirements:-
- (a). High reflecting power - preferably not less than 70%.
 - (b). A non-rusting, non-corrosive material.
 - (c). Of a high and definite tensile strength.
 - (d). Not too hard or brittle to work by ordinary means.

A mirror made of glass and silver-plated can be made considerably cheaper than one made of steel or other metal. The tensile strength of glass is, however, an uncertain quantity varying from 2,000 to 10,000 lbs. per sq. in.. Severe stresses may be set up by temperature changes and it is known that high speed rotating glass mirrors may fly to pieces very unexpectedly.

Silver has a high reflecting power in the visible range but poor in the violet and ultra-violet, - besides tarnishes to some extent. The magnalium alloys have good reflecting power for a short time but are entirely unsuited on account of their tendency to fracture and crumble. Stellite is too hard and brittle to turn on a lathe.

Nitralloy, a steel alloy which can be case hardened to a very high degree of hardness, was selected as the material for the mirror. It is practically non-corrosive and has a very high tensile strength; its maximum reflecting power is unknown. It was planned to polish the faces as well as possible, after case-hardening. In case the reflecting power was then found to be insufficient the faces could be plated with a thin layer of chromium by evaporation, (.0002" to .0004" in thickness). Chromium is known to have a reflecting power of 70% (approx.) and to be

non-corrosive. The work spent in polishing the steel would not be wasted, for, to polish chromium plating satisfactorily, it is necessary that the original surface be first brought to an optical finish and that the chromium be deposited in a very thin layer.

Centrifugal Forces Acting on Mirror

Material - Nitralloy Steel - Tensile Strength 120,000 lbs./sq. in.

Weight = 600 lbs./cu.ft. = .348 lbs./cu.in.

Diameter = 2-1/4". R.p.m. = 30,000 r.p.s. = 500.

Velocity at surface = 3540"/second.

Centrifugal force = $\frac{mV^2}{r} = \frac{.348 \times 3540^2}{32.2 \times 12 \times 1.125} = 10,000 \text{ lbs./sq.in.}$

Chromium plating is not affected by this stress.

Critical Speed of Rotating Mirror⁴

In designing a device such as this high speed rotating mirror, it is most essential that the "critical speed" be far above or far below the operating speed. The critical speed, in its most simple form, is that speed at which the bending moment produced by the centrifugal forces, which arise in turn, from the deflection caused by the weight of the rotating member, is just balanced by the restraining forces of the rotating member, this being with perfect dynamic balance in other respects. It corresponds to the natural period of vibration of the rotating member. Unbalancing, aside from that due to deflection under load, lowers the critical speed.

The calculation of critical speed is at best only a fair estimate, as is evident from the following assumptions.

$$\text{Lowest critical speed} = \omega_{cr} =$$

$$\sqrt{\frac{g \sum_1^n m_n y_n}{\sum_1^n m_n y_n^2}} \quad g = \text{acceleration of gravity.}$$

m_1, m_2, m_n = rotor masses.

y_1, y_2, y_n = corresponding deflections.

For a single concentrated load at the center of a beam supported at both ends,

$$\omega_{cr} = \sqrt{\frac{g}{\gamma_{max}}} \quad \text{Form. (6).}$$

Fig. 6 shows the rotating member and Fig. 7 an equivalent shaft and load, assuming,

entire weight concentrated at center, shaft supported at bearings (not fixed) and diameter of shaft as 9/16".

These assumptions may be considered safe as the bearings are, in fact, partially fixed and the

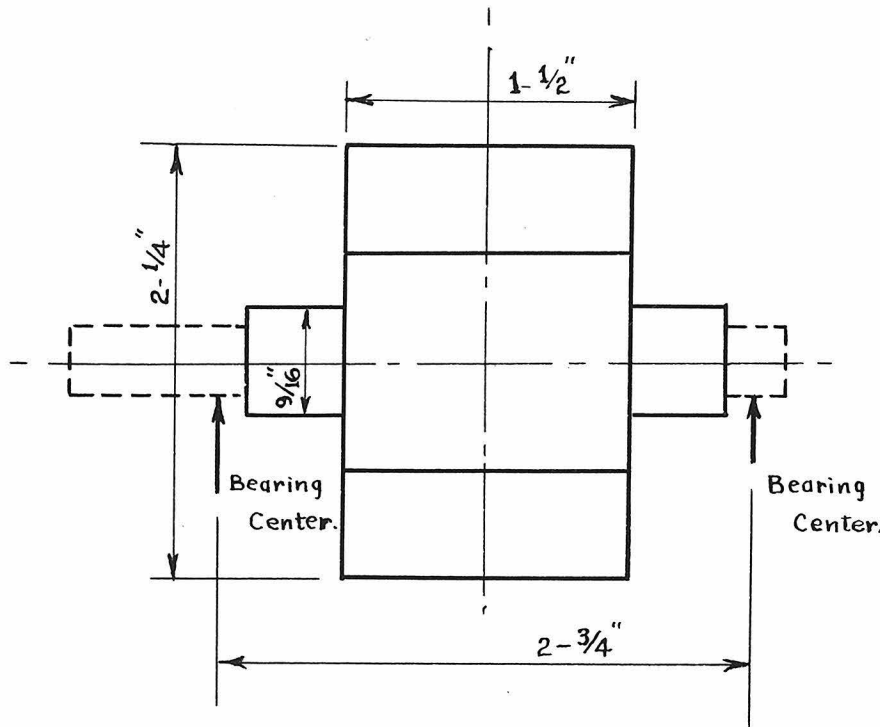


Fig. 6.

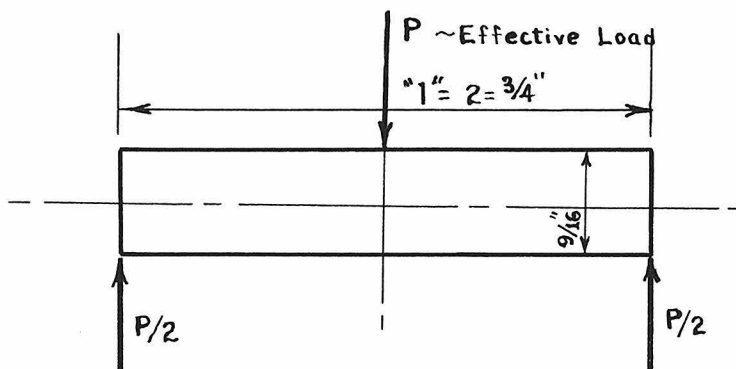


Fig. 7.

shaft diameter is much greater than 9/16" throughout a large portion of its length; both factors tend to increase the stiffness and to raise the critical speed.

$$\text{Wt./cu.in.} = .35\#.$$

Wt. of rotating member = P

$$P = .35\pi(1^2 \times \frac{1}{2} + (\frac{9}{32})^2 \times \frac{1}{4}) = 1.76\#$$

I = moment of inertia of shaft cross-section, about a center axis

$$= \frac{\pi d^4}{64} = .0491 \times .563^4 = .00495 \text{ in.}^4.$$

$y_{\text{max.}}$ = max. deflection of shaft under load P

$$y_{\text{max.}} = \frac{Pl^3}{48 EI} = \frac{1.76 \times 2.75^3}{48 \times 30 \times 10^6 \times .00495} = 5.16 \times 10^{-6} \text{''}$$

$$\therefore \omega_{\text{cr}} = \sqrt{\frac{g}{y_{\text{max.}}}} = \sqrt{\frac{32.2 \times 12}{5.16 \times 10^{-6}}} = 8.64 \times 10^3 = 8640 \text{ rad./sec.}$$

$$\therefore \underline{\text{r.p.m.}} = \frac{8640}{2\pi} \times 60 = 82,500 = \text{lowest critical speed.}$$

A previous design gave a critical speed of 30,000 r.p.m. In the final design the length "l" of shaft was made as short as possible and the diameter increased considerably with the result that the critical speed was raised to 82,500 r.p.m..

Bearings

For many applications of light loading and high speeds, ball bearings are often more desirable than bronze bearings in that they do not wear so rapidly and are freer from frictional losses and lubrication difficulties.

Bearings specially selected at the factory for small tolerances, and of a separable type were employed; normal speed being 10,000 r.p.m. with a maximum load of 48#. The separable type is most satisfactory as the end pressure may be adjusted for most quiet operation while running (see Drawing #1.) A wringing fit was made for the ball race on the shaft, it being .0002" tight while the housing was made .0002" large for the stationary ring. A shrink fit of any kind would deform the ball race and ring and cause chattering so the ball races were fastened tightly to the shaft by nuts (see Drawing #1.)

The base was made extremely heavy to help eliminate vibrations and deformations of any sort.

III (c). Fery Prism⁵.

The Fery prism produces the spectrum and focuses it onto the film. Further, as explained under "Principles of Operation", it produces dispersion according^{to} the law:

$$D = K_1 + \frac{K_2}{\lambda^2}$$

making the displacement of the dark bands in the spectrum a linear function of the current in the surge.

As stated above the focal distance (see Fig. 4.) should not be greater than 40".

The spectrum should not exceed 1" in length; the shorter the spectrum the greater the intensity of light at the film while on the other hand it is desirable to make the spectrum long so as to give a larger and more sensitive scale of current.

The dimensions of the rotating mirror being now fixed, in no case will the light beam leaving the Fery prism be greater than 3" in diameter. The diameter of the Fery prism was accordingly taken as 4".

The following calculation illustrates the method of design:-

Size of glass block - 4-1/2" x 4-1/2" x 1-1/4".

Kind of glass - Boro-silicate - BSC-7.

Indices of Refraction

N_d	$\lambda =$	b	C	d	e	F	g	h	
1.5151		7065	6563	5875	5461	4861	4359	4047	3700
		b-C	C-d	d-e	e-F	F-g	g-h		
		.00143	.00246	.00191	.00365	.00433	.00359		

$\therefore n_C = 1.5151 - .00246 = 1.51264$
 $n_g = \text{---} = 1.52499$
 $n_h = \text{---} = 1.52858.$

The wave lengths which may here be considered useful for photographic purposes range from $\lambda = 6563$ to $\lambda = 3700$ (approx.).

A prism of maximum thickness and maximum angle r is cut from the glass block. Dimension "A" must not be cut down to a fine edge (Fig. 8) else the prism would be easily broken.

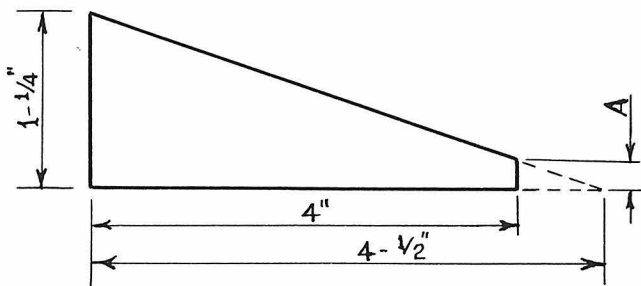


Fig. 8.

$r_1 = 15^\circ - 31' - 37.55''.$

The faces of the prism are, in fact, spherical, but a plane triangle may be used for calculating the length of spectrum, without error.

First solve for the angle (i_1), Fig. 9, of minimum dispersion, using an index of refraction n_g , (for a wave length midway in the spectrum). Then angle $\alpha = 90^\circ$.

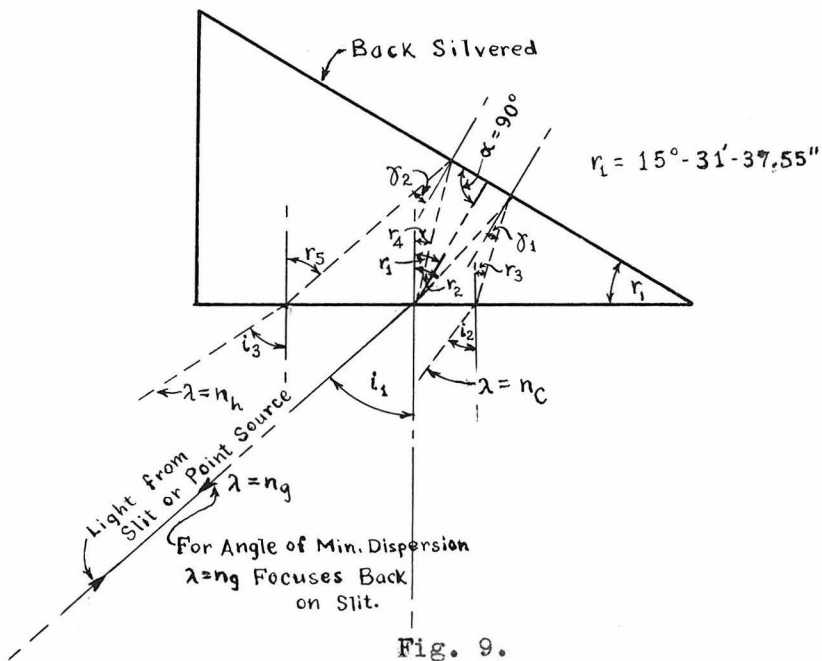


Fig. 9.

$$\delta_1 = r_2 - r_1 = 7' 37.8''$$

$$r_3 = 90 - [180 - r_1 - (90 - \delta_1)] = r_1 - \delta_1 = 15^\circ 24'$$

$$\sin i_2 = n_c \sin r_3 = .40168$$

$$i_2 = 23^\circ 41' 2''.$$

For n_h .

$$\sin i_1 = n_h \sin r_4 = .40815$$

$$\therefore r_4 = 15^\circ 29' 11.9''$$

$$\delta_2 = r_1 - r_4 = 2' 25.7''$$

$$r_5 = 180 - [(90 - \delta_2) + (90 - r_1)] = \delta_2 + r_1$$

$$r_5 = 15^\circ 34' 3.25''.$$

For n_g .

$$\begin{aligned} \sin i_1 &= n_g \sin r_1 \\ &= 1.52499 \times .26764 \\ &= .40815 \end{aligned}$$

$$\therefore i_1 = 24^\circ 5' 49.4''.$$

For n_c .

$$\begin{aligned} \sin i_1 &= n_c \sin r_2 \\ &= 1.51264 \sin r_2 \\ &= .40815 \end{aligned}$$

$$\therefore \sin r_2 = .26983$$

$$r_2 = 15^\circ 39' 15.4''.$$

$$\sin i_3 = n_h \sin r_5 = .41024$$

$$i_3 = 24^\circ 13' 10.75''.$$

$$\delta = i_3 - i_2 = 32' 8.75'' = .5357^\circ$$

$$\mu = \text{length of spectrum } (\lambda = c \text{ to } \lambda = h) = \frac{.5357 \times \pi}{180} \times 36'' = .337''$$

allowing 50% for $n_h - n_{3700}$

$$\mu = .505''$$

The length⁵ of the spectrum is a constant for any position of the slit on the circle "B" Fig. 11.

In the above calculation μ was too short. A block of Light Flint glass, 6" x 6" x 1-1/2" in the rough, was procured and having an index of refraction, $n_g = 1.61$ approx. One quarter inch in thickness was allowed for roughing out the

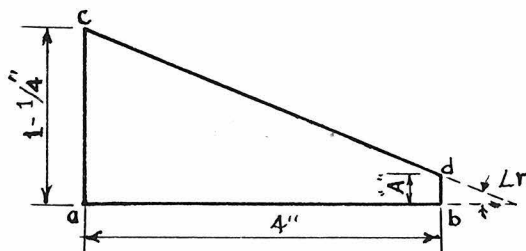


Fig. 10.

blank and dimension "A" Fig. 10 was made 3/16".

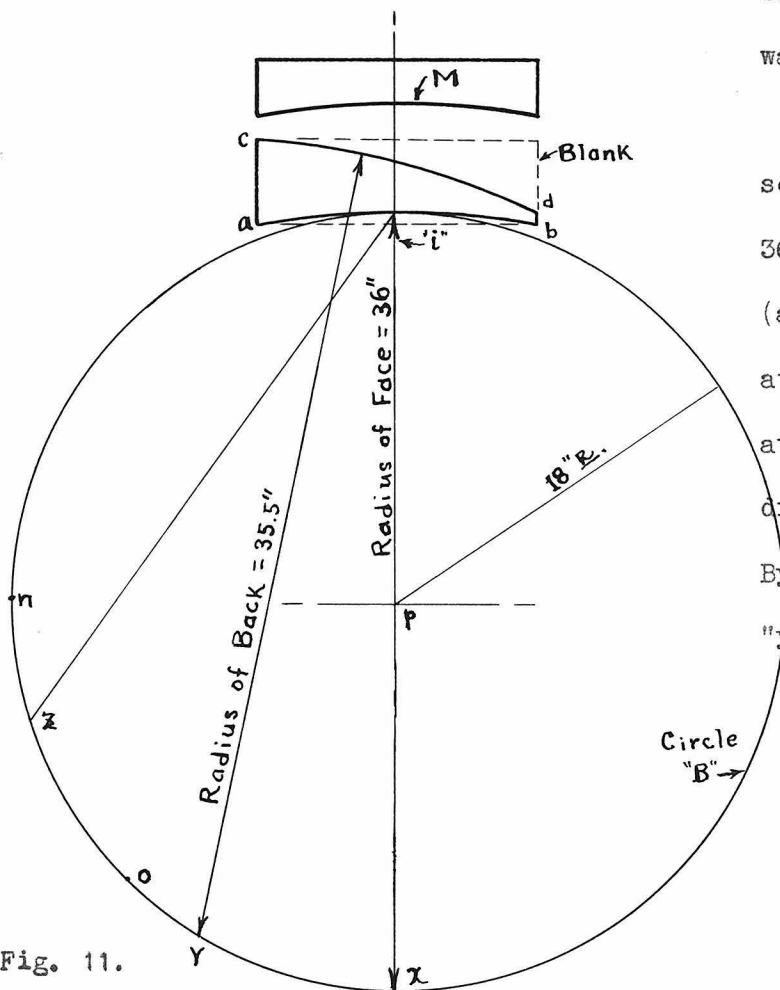


Fig. 11.

This blank was laid out to scale and an arc of a circle 36" rad. drawn through points (a) and (b) Fig. 11, with center at "x". Circle "B" with center at "p", and radius of 18" was drawn tangent to the prism face. By trial another arc, center at "y" was drawn through points

(c) and (d). Angle "r" was measured and the angle "i" of minimum dispersion calculated. The length of spectrum (- 6500 to - 3700) was about $7/8$ ", which was satisfactory.

It is a property of the Fery prism that, if a diverging beam, from a slit or other source located on the tangent circle "B", is projected into the prism, it will produce a spectrum and focus it at some point on this circle "B". If the slit is placed at "z", giving the angle "i" of minimum dispersion, the spectrum will be focused in the vicinity of "z". In this device it is necessary to separate the position of slit and spectrum, on the circle "B", to some such positions as "n" and "o" respectively.

A Fery prism is subject to astigmatism. To correct for this the back of the prism was not silvered, a mirror "M" with the same radius of curvature as the back of the prism, being mounted behind the prism with adjusting screws for three positions (Drawing #2.) One position of this mirror may always be found which will correct the astigmatism present in the spectrum.

III (d).

Camera Box

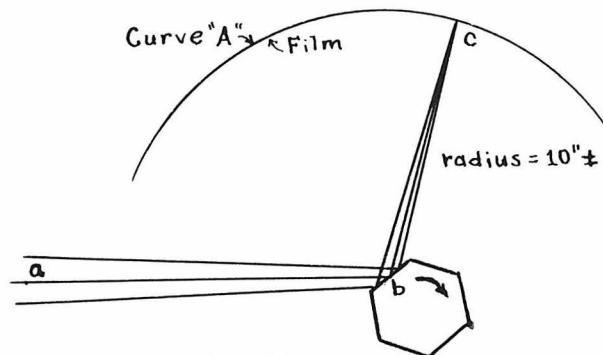


Fig. 12.

Drawing #3 shows the main features in the design of the camera box. The distance from mirror to film was taken as $10" \pm$. It is to be noted that the curve "A", Fig. 12, is not a circle.

The light beam must focus at the film throughout the length of the film, which means that the distance from "a" to "b" plus that from "b" to "c" equals a constant,

$$\overline{ab} + \overline{bc} = k,$$

for all positions of the rotating mirror. The curve "A" was accordingly plotted and a templet made to locate the position of the film.

Both the Camera Box and Optical Bench were made light-tight throughout. A camera bellows directs the light from the Fery prism onto the rotating mirror (see Drawings #3 and #4.) By this means the camera box may be moved in or out or rotated with respect to the Fery prism, as may be required. (See Fig. 11.)

III (e).

Coil for CS₂ Cell

From that which is known of the nature of lightning it is plain that the inductance of the coil about the CS₂ cell must be a minimum, else the stroke may side flash to ground, not passing through the coil. An opposing limitation is that a very strong magnetic field is necessary to produce the magneto-optic effect. From an inspection of an oscillogram of a condenser discharge, taken at the Mount Wilson Observatory, it was noted that approximately 3000 amperes flowing in a coil of one turn and 1-1/2" in diameter would be needed to produce a perceptible shift in the dark bands of the spectrum. Using one turn, the current scale as formed by the spectrum would range from 3000 ± to 150,000 ± amperes.

The current scale may be widened, in the design of the Fery prism, to give a greater deflection for a given current. There is, however, a limit to this, for as has been shown, one of the chief problems in recording high frequency oscillations is that of getting sufficient light through the

system. The intensity of the light varies inversely with the length of the spectrum.

The use of one turn for the coil is based on the estimated values of lightning currents, which may be far from correct.

Inductance of Coil

Wire - 0000 copper cable

$$L = \pi a \mu' + 4\pi a \left(\log \frac{8a}{r} - 2 \right)$$

dia. = .46"

El. & Mag. - Jeans - p.445.

$$r = .23" = .575 \text{ cm.}$$

$$a = \text{rad. of coil} = 3/4" = 1.91 \text{ cm.}$$

$$\therefore L = \pi a \left(-7 + 4 \log \frac{8 \times 1.91}{.575} \right) = 36.6 \text{ abhenries.}$$

$$\underline{L = 36.6 \times 10^{-9} \text{ henries.}}$$

It is not believed that the inductance of a coil of one turn is sufficient to alter the wave shape of the current appreciably, for assume $i = 10^5$ amps.,

$$\text{Energy} = \frac{1}{2} Li^2 = \frac{1}{2} \times 36.6 \times 10^{-9} \times 10^{10} = 183 \text{ watt sec.,}$$

an amount, insignificant compared to that of any

lightning stroke to ground.

The voltage appearing across the top of the coil for a current of 50,000 amperes, and wave front equal to that of a frequency of 10^6 cycles/sec.

$$= 50,000 \times 2\pi \times 10^6 \times 36.6 \times 10^{-9} = 11,500 \text{ volts.}$$

The coil should be insulated on the top, where the two ends cross, with a piece of 1/4" micarta.

IV.

Time Lag in the System

(a). The time required for the magneto-optic effect (Faraday effect) to take place in the CS_2 cell is entirely negligible compared to time

intervals of the order of a micro-second. The action of this cell involves the partial rotation or polarization of the molecules of the carbon-bisulphide, which are under the influence of the magnetic field. A test by Bichat and Blondlot³ showed that this effect is practically instantaneous.

(b). In measuring real lightning currents the light developed at the light source should be sufficient without the use of a fine wire. No appreciable time delay could then enter in, as the time for the breakdown of a short gap is of the order of 10^{-8} seconds or less as shown by measurements of Dunnington⁶ at Berkeley, California.

A wire of some metal such as tungsten or platinum, when exploded gives a light far more intense than that produced by the same current in a discharge through air. For measuring condenser discharges in the laboratory, it is convenient, therefore, to place a fine (1 or 2 mils) wire of tungsten at the light source. If this wire is too large or the gap too short, considerable time lag may be introduced into the system. Experience shows that a 2 mil tungsten wire, $3/4$ of an inch long will cause no measurable time lag when the time scale is $1/16$ inch per micro-second. A metal of higher resistance and lower melting point will, of course, explode in less time than one of tungsten. A further improvement in focusing of the optical system might make the use of a fine wire at the light source unnecessary.

A fair estimate of the time required for the wire to explode may be had by plotting a temperature-time curve for the point at the center of the wire from the equation derived in the addenda to this report. The lack of accurate data regarding coefficients of heat conduction, resistance, etc., at high temperatures, limits the use of such an equation.

V.

THE COMPLETED INSTRUMENT

Following the design outlined above the Optical Oscillograph was built and tested. Figure 13 below shows the instrument completed and assembled.

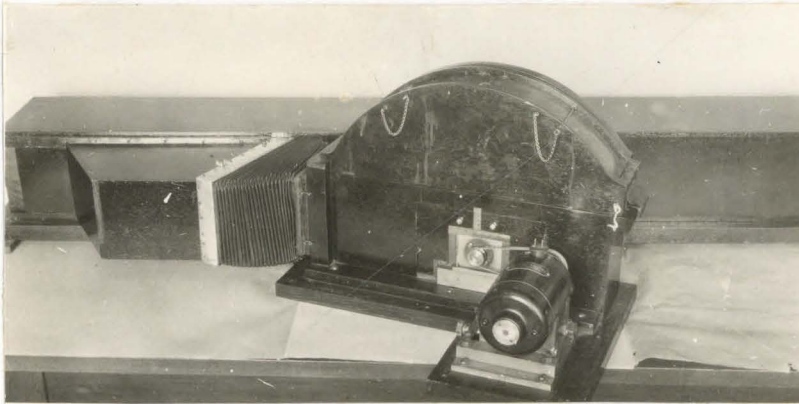
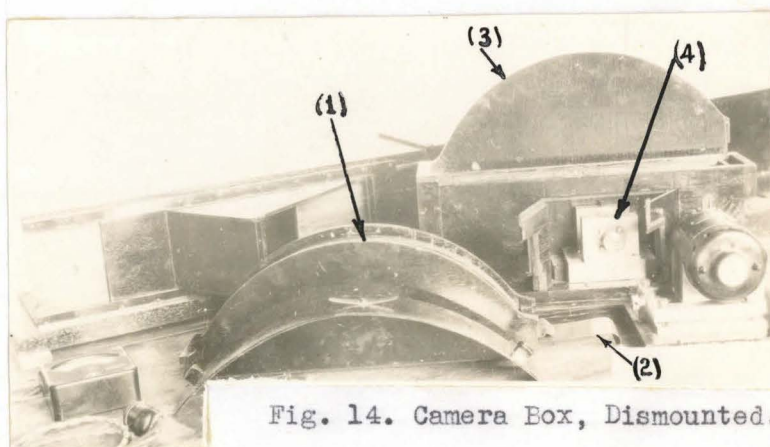


Fig. 13. The Assembled Instrument

Figure 14 shows the Camera Box and Rotating Mirror in more detail. The top part of the Camera Box is light, tight and removable; it holds the film which must be loaded in a dark room.



- (1) Film holder.
- (2) Shutter.
- (3) Camera Box.
- (4) Rotating Mirror.

Fig. 14. Camera Box, Dismounted.

Figure 15 shows the Optical Bench and various parts placed in operating positions.



Fig. 15. Optical Bench.

G , is the gap in which is placed the fine tungsten wire.

M , is a plane mirror placed close behind the gap.

L₁, is a 2" diameter, 4-3/8" focal length lens.

S₁, is a stop which limits the light beam to a size which the Nicol prisms can handle and keeps extraneous light out of the system.

N₁, is a one inch Nicol prism.

Q , is a crystal quartz plate, 2" in dia. Either of the two plates provided, one 2 cm. the other 5 cm. long may be used here.

C., is the carbon bisulphide cell with coil in place.

N₂, is the second Nicol prism.

L₂, is a 2" diameter, 9-1/2" focal length lens.

S₂, is a double slit on which lens L₂ is focused.

F , is the Fery prism.

A camera bellows connects the camera box to the housing over the Optical Bench; this provides much flexibility in focusing the Fery prism. However, it is necessary that both camera box and optical bench be placed on a solid plane table so that the light beam from the Fery prism enters the camera box at exactly the correct elevation.

After testing, it was found advisable to make several changes in the instrument. The camera box and bellows were set at a different angle with respect to the optical bench to reduce distortion in the focusing of the Fery prism. An 8" focal length camera lens* was placed in the front end, position "x", Fig. 1., of the camera box and directly in the path of the light beam from the Fery prism. Here also was placed a 2" diameter 5" focal length double concave lens, the effect of which was to increase the effective focal length of the camera lens to about 15". The advantage of this system is that the magnification of the Fery prism, which was large was much reduced, making possible much sharper focusing of the spectrum on the film.

*The camera lens was borrowed.

There would be no advantage in using this additional camera lens in the measuring of very large currents, such as lightning; as there with the large amount of light available even a large broad spectrum would most likely come out clean and sharp.

VI.

RESULTS OF LABORATORY TESTS

The circuit used for testing in the laboratory is that given in Fig. 16.

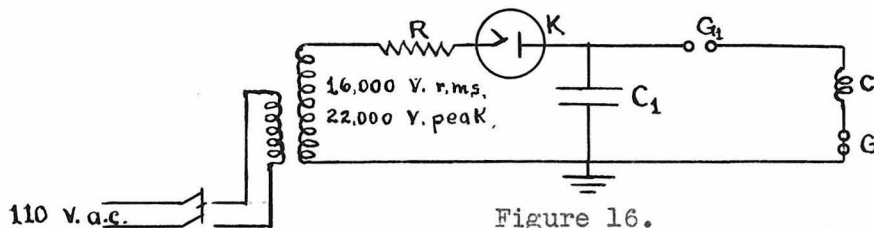


Figure 16.

G is the gap at the light source of the instrument across which the fine tungsten wire is stretched.

C is the coil wound about the carbon bisulphide cell.

C₁ is a 1.75 micro-farad condenser which can be charged to about 25,000 volts.

G₁ is a gap or switch which breaks down when the condenser becomes fully charged.

R is a resistance of about 100,000 ohms (a water tube) placed in the circuit to limit the charging current of the condenser to prevent burning out the Kenotron rectifier tube K.

The circuit C₁G₁CG must be made of a large low resistance conductor in order that a large current may flow.

Following are a number of current-time oscillograms taken under the conditions specified. It is to be noted that in these oscillograms any one single light or dark band of the group may be used. The time axis is a horizontal line drawn through the center of the single light or dark band under consideration, and the current axis, a vertical line.

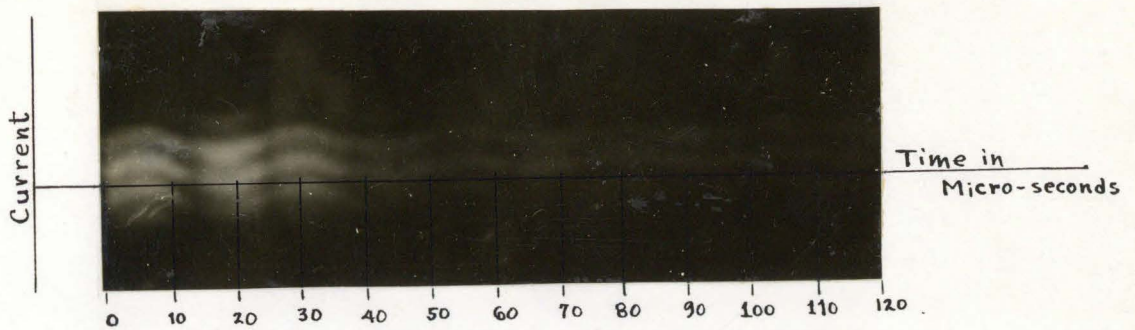


Figure 17.

Oscillogram taken on ordinary roll film.

Speed of rotating mirror - - - 16,000 r.p.m.

Scale of time axis - - - $\left(\frac{16,000}{60} \right) \times 2 \times (2 \times 10'') \times \pi = 33,500''/\text{second}$
 $= .0335''/\text{micro-second} = 1/32''/\text{micro-second, approx.}$

Radius of light beam is 10" in all cases.

Number of turns on CS₂ cell - - - 8.

The camera lens mentioned on page 23 was not used.

Frequency of damped oscillation - - - - 38,500 cycles approx.

A method for calibrating the instrument for current is included in Part VII of this report.



Figure 18.

Figure 18 is an oscillogram taken under the same conditions as that in Figure 17, except that it was taken on super-sensitive panchromatic film, which gives a wider spectrum.



Figure 19.

Oscillogram taken on ordinary roll film.

Speed of rotating mirror - - - 20,000 r.p.m.

Scale of time axis - - - - - .0418"/micro-second.

Number of turns on CS₂ cell ▸ - - 4.

Frequency of damped oscillations ▸ - - 89,000 cycles.

The camera lens was not used. An adjustment of the Fery prism and of the camera box, different from that used for Figures 17 and 18, was employed. This would necessitate a separate calibration for the current scale.

Length of quartz plate - - - 5 cm.



Figure 20-a.



Figure 20-b.

Figure 20-a

Figure 20b.

Speed of mirror - - 24,000 r.p.m.

20,000 r.p.m.

Scale of time axis .05"/micro.sec.

.0418"/micro-sec.

No. of turns on CS₂ cell 4

12

Length of quartz plate 5 cm. in both cases.

All other data for Figures 20-a and 20-b same as for Figure 19.

As seen from Figure 20-b, the scale of the current axis depends on the number of turns in the coil about the CS₂ cell.

All the above oscillograms are the result of early attempts to operate the instrument and are not the best that can be obtained with it.

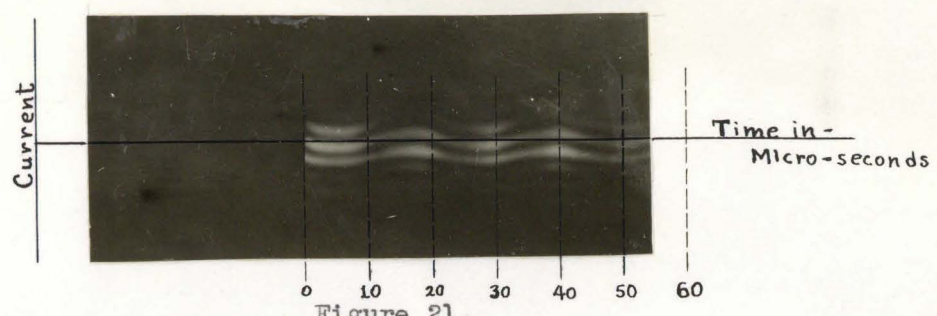


Figure 21.

Oscillogram taken on super-sensitive panchromatic film.

Speed of rotating mirror - - - 16,000 r.p.m.

Scale of time axis - - - - - .0335"/micro-second.

Number of turns on CS₂ cell 8.

Length of quartz plate - - - - 2 cm.

Frequency of damped oscillations 45,000 cycles.

The camera lens mentioned on page 23 was used, being placed in the location marked "x" of Figure 1.

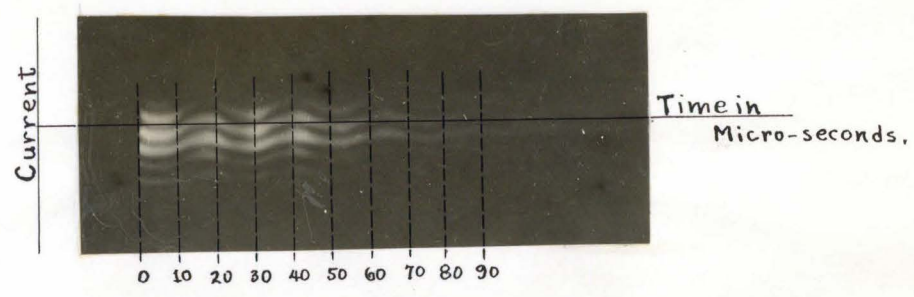


Figure 22.

Oscillogram taken on super-sensitive panchromatic film.

Speed of rotating mirror - - - - 10,000 r.p.m.

Scale of time axis - - - - - .021"/micro-second.

Other data same as for Figure 21.

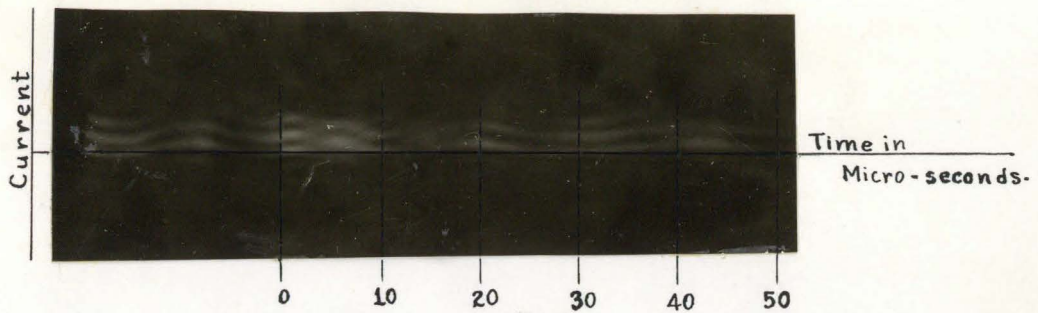


Figure 23.

For the sake of economy four discharges were recorded on each loading of film in the camera box. Thus occasionally the oscillograms overlapped as in Figure 23, above. The surge recorded on the left in Figure 23 is the same in all details as that one in Figure 21. The characteristics of the one on the right are listed below.

Oscillogram taken on super-sensitive panchromatic film.

Speed of rotating mirror - - - 25,000 r.p.m.

Scale of time axis - - - - - .053" / micro-second.

Other data are the same as for Figure 21.

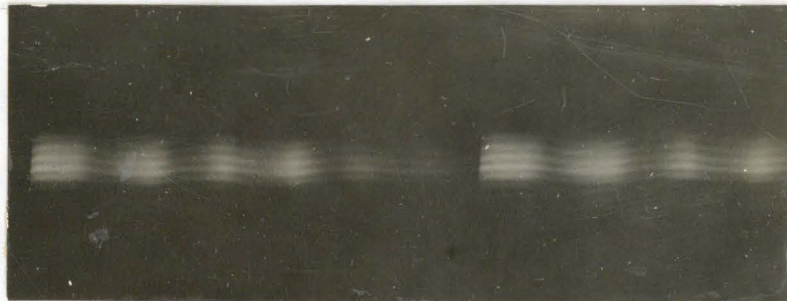


Figure 24.

In the oscillogram of Figure 24, the quartz plated used was 5 cm. long.

All other data are the same as those under Figure 21.

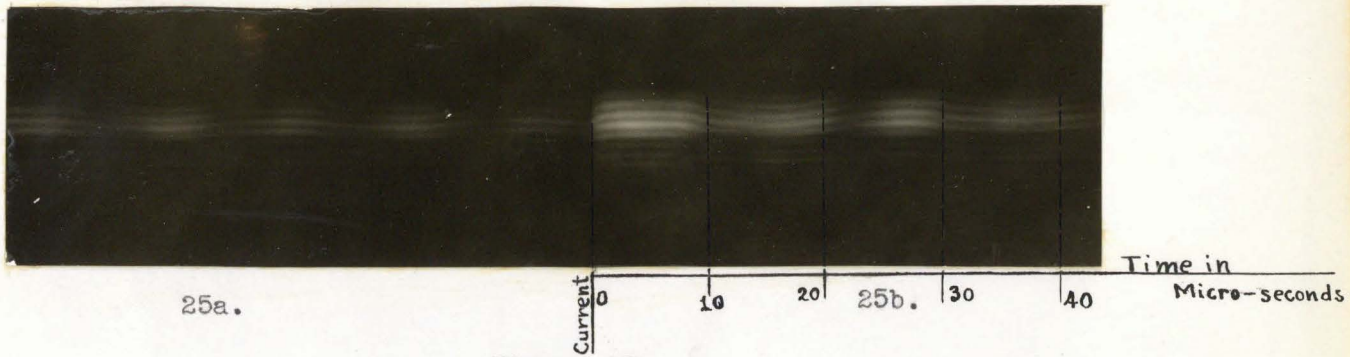


Figure 25.

Oscillogram taken on super-sensitive panchromatic film.

Speed of rotating mirror - - - 29,000 r.p.m.

Scale of time axis $\tau \tau \tau$ - - - .061" / micro-second.

Number of turns on CS_2 cell , 8.

Length of quartz plate 5 cm.

Frequency of damped oscillations 45,000 cycles.

The camera lens was used.

The oscillogram in the left of Figure 25 fell on the end of the film in the camera box, as explained in the Figure at the bottom of page 8. The light beam from the mirror was divided into two sections, recording an oscillogram at each end of the film. Hence, note the dimness of the record, Figure 25a.

In this test the slit S_2 was opened to $1/32$ " square, about four times the size used in previous tests. This explains the brightness of the record 25b even though the time scale was quite large.

The data for Figure 26 are just the same as for Figure 25. The



Figure 26.

details of this oscillogram are spoiled by the overlapping of a second surge; however, it is a good record of a surge virtually damped out. The camera lens used in these later tests was too small to accommodate the entire beam converging from the Fery prism. Part of the beam hit the holder of the lens, which explains the dimness of the lower half of the spectrum (oscillogram) of Figure 26.

To measure the current accurately in these oscillograms it is first necessary to calibrate the instrument for the particular combination used, the combination of quartz plate, coil for the CS_2 cell, and position of the camera box with respect to the optical bench. A fine line is then carefully drawn for the time axis and the oscillogram observed under a microscope having finely graduated rulings in the field of vision.

The rotating mirror was driven at 33,000 r.p.m. for 30 minutes to test its reliability at this speed.

VII.

METHOD OF OPERATION

(a) For Laboratory Work.

Place lens L_1 at exactly $4\text{-}\frac{3}{8}$ " from the tungsten wire placed in the gap G and lens L_2 at exactly $9\text{-}\frac{1}{2}$ " from the slit S_2 . Remove the mirror block M (Fig. 15), place a strong point source of light such as an electric arc in front of the system (if an electric arc is used, it is necessary to keep the electrodes adjusted to a single point source as near as possible); remove the cover to the mirror of the Fery prism, also the shutter and lid to the camera box; open the vertical slit of S_2 wide and place an old piece of strip film in the film holder of the camera box. Now adjust the position of the Fery prism and of the camera box until the spectrum reflected from the rotating mirror falls upon the strip film; turn Nicol prism N_2 until the lines in the spectrum are clear and easily discernible. If the lines in the spectrum are not parallel to the camera box, turn the quartz plate Q either

backward or forward a little. Rotate the rotating mirror slightly by hand and observe the spectrum as it travels from one end of the film holder to the other. If the spectrum fades out toward one end of the film holder it is owing to the fact that the beam from the Fery prism is not falling upon the rotating mirror at the right angle; this can be corrected by adjusting the screws on the back of the Fery prism mirror and the lever on the top of the Fery prism itself. Stop down the slit S_2 to $1/32''$ to $1/64''$ in both directions. By moving the camera box, adjust its position relative to the optical bench so that the spectrum focuses exactly on the position of the film in the film holder.

Locate the point source of light as nearly as possible on the center line of the optical system and directly behind the fine wire in the gap G. Adjust the holder of the gap G until the image of the fine wire falls exactly and undistorted on the slit S_2 . If a piece of white paper is placed about 6" beyond the slit S_2 the image of the fine wire may be easily seen when properly lined up. Set all adjustments tightly, replace the mirror block M behind the gap G, and replace the covers to the optical bench. Load the film holder in a dark room and place in the camera box. Before setting off the condenser discharge, bring the mirror up to the desired speed and remove the shutter from the film holder.

A camera lens located as explained on page 23 will greatly increase the sharpness of focus of the spectrum. The diameter of such a lens must be sufficient to handle the light beam from the Fery prism, about $1-1/2''$.

(b). It is not expected that the leads provided on the optical bench would be used when measuring lightning currents. The instrument would be placed in a nearly vertical position and all conductors made as short and direct as possible.

The motor used at present will not drive the mirror above 20,000 r.p.m. continuously without overheating.

(c). For Measurement of Mirror Speed.

The diagram of Figure 27 illustrates the method by which the speed of

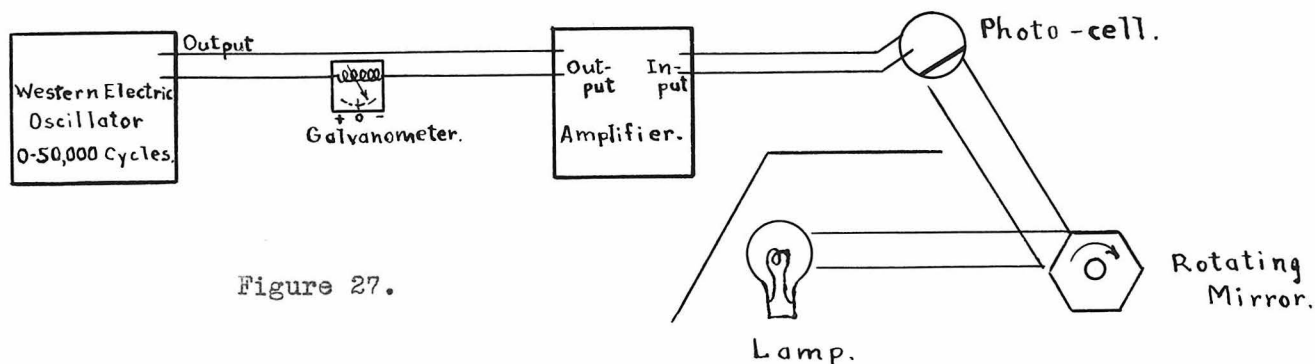


Figure 27.

the mirror was measured and a calibration curve obtained.

By coupling the amplified impulses of the photo-cell to the output of the Western Electric oscillator, as shown above, a beat frequency equal to the difference of the frequencies of the two circuits was produced. The natural mechanical frequency of the needle of the galvanometer was about 30 cycles. When the beat frequency came in the neighborhood of this frequency, the needle would flutter, and as more careful adjustment of the W.E. oscillator brought the beat frequency lower and lower, the needle would oscillate slowly to and fro, and when the needle ceased to move, the frequency of the impulses to the photo-cell must of course have been the same as that of the oscillator. As each face of the mirror gave rise to a single impulse in the photo-cell and each impulse corresponded to a single cycle in the output of the amplifier, (as the impulses were uni-directional), the speed of the mirror in r.p.m. was equal to,

$$= 60 \times \frac{\text{frequency of oscillator}}{6} = 10 \times \text{frequency of oscillator}$$

A calibration curve was plotted between r.p.m. and watts input to the motor, driving the rotating mirror, which is included in this report.

(d). To Calibrate the Instrument for Current.

A suggestion for doing this is to connect the coil of the CS₂ cell in the low side of a spot-welding transformer of large current rating, as for example, 4000 or 5000 amperes for a small time interval. Use an ordinary

electric lamp for the light source and rotate the mirror by hand. The r.m.s. value of the current may be determined by measuring the current in the primary if one also knows the ordinary design data of the transformer. An oscillogram of a few cycles taken in this manner may be used to determine the current scale of oscillograms of condenser discharges.

(e). Care of the Instrument

The several parts of this instrument are mostly high quality optical goods and require much care in handling. Soft cotton may be used for cleaning the lens and prisms. The Nicol prisms are soft and brittle, thus easily scratched or chipped. The rotating mirror, which is made of a steel alloy, should not be exposed to dampness or fumes. The cover to the mirror of the Fery prism should be kept in place when the prism is not in use. All optical surfaces should be cleaned before taking an oscillogram. The ball bearings to the rotating mirror should be oiled with about two drops of a very light oil, such as typewriter oil, for every half hour of operation. Do not put so much oil in the bearings that it is thrown out onto the mirror. It will be found necessary occasionally to clean the commutator of the motor.

VIII. SUGGESTED WORK FOR COMPLETING AND PERFECTING THIS INSTRUMENT

It is believed that an optical system having two slits would make possible much sharper focusing of the spectrum and give a current scale which could be more accurately read. Slit S_2 would be a vertical slit only; the other slit would be placed at the focus of the Fery prism and be a horizontal one. Between this slit and the rotating mirror would be placed the camera lens as mentioned above.

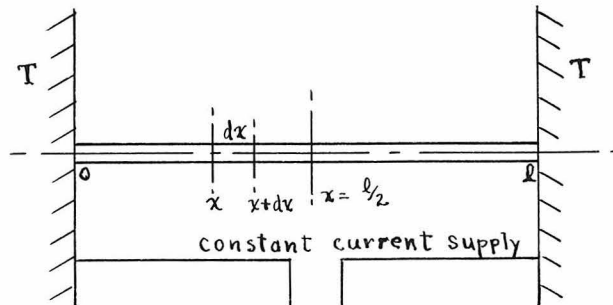
The instrument should be calibrated for current either by the method outlined above, or by some other suitable method.

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Addenda. Temperature Rise in a Wire Owing to I^2R Loss

A metal wire connects two metal plates as shown in the figure and forms part of an electrical circuit. The problem is to find the equation for temperature of the wire; this equation to hold for the following assumptions and conditions.



We will assume, 1. that the plates are kept at a constant temperature T .

2. That ~~the voltage E of the battery~~ ^{current in the Electrical circuit.} remains constant.
3. That the expression for resistance per unit length of the wire is $r = r_0(1 + \alpha V)$, where α is the coefficient of increase in resistance with unit increase in temperature.
4. That the heat loss by radiation and conduction to the medium surrounding the wire is proportional to the 1st power of the temperature rise of the wire above the surrounding medium.

Let the following symbols represent the quantities designated:

V = temperature at a point x of the wire and at a time t ,

r = resistance per unit length of wire in ohms per cm.

h = coefficient of heat conduction for the wire in $\text{cal/cm}^2/\text{C}/\text{sec}$.

$$= \frac{\text{cal cm.}}{\text{°C. cm}^2 \cdot \text{sec.}}$$

ρ = coefficient of heat for the wire = $\text{cal/gm.}/\text{°C} = \frac{\text{cal}}{\text{gm.} \cdot \text{°C.}}$

p = density of the wire = $\frac{\text{gm}}{\text{cm}^3}$

l = length of wire in cm.

a = cross-sectional area of the wire in cm^2 .

T = temperature of end plates and of surrounding medium and is constant

$e = \text{constant of emissivity} = \frac{\text{calories}}{\text{cm}^2 \times \text{degrees C rise in temp. above surrounding med.} \times \text{sec}}$

$\sigma = \text{perimeter of wire in cm.}$

$r_0 = \text{resistance of wire at temperature T.}$

$A = \frac{e\sigma - \alpha f}{ha}$

$B = \frac{\gamma + e T \sigma}{ha}$

$k = \frac{p \rho}{h}$

$\gamma = .239 i^2 r_0$

$q = \text{heat generated per unit length of wire}$

$\text{per second } [=] \frac{\text{cal}}{\text{cm.} \times \text{sec}} = \frac{\text{joules}}{4.185 \times \text{cm}} = \frac{i^2 r}{4.185 \text{ cm.}}$

$q = .239 i^2 r, \frac{\text{cal}}{\text{cm. sec.}}$

Boundary Conditions

From symmetry it is evident that the center of the wire will always be a point of maximum temperature; this gives condition 1. Condition 2 follows from the assumptions made. The switch is thrown at the time $t = 0$, which gives condition 3. After an infinite time the temperature of the wire will have arrived at a steady state; this means then that $\frac{\partial V}{\partial t} = 0$ which is condition 4a. Condition 4b follows from the solution for the steady state, given below in equation (8).

1. $\frac{\partial V}{\partial x} = 0$ at $x = l/2$ at all times.
2. $V = T$ at $x = 0$ and at $x = l$ at all times.
3. $V = T$ at $t = 0$, for all values of x .
- 4a. $\frac{\partial V}{\partial t} = 0$ when $t = \infty$ for all values of x .
- 4b. $\frac{\partial V}{\partial x} = \text{a value to be determined from the equation for the steady state heat flow; see equation 8 below.}$

The heat generated in a section of the wire between x and $x + dx$ is

$$= q dx \quad \text{cal /sec.}$$

The total heat flow at x is in the negative direction and $\frac{\partial V}{\partial x} = -ha \frac{\partial V}{\partial x}$ cal/sec. \rightarrow $\frac{\partial V}{\partial x}$ = heat flow out of section

The total heat flow at $x+dx$ is in the negative direction and

$$= -ha \left(\frac{\partial V}{\partial x} - \frac{\partial}{\partial x} \frac{\partial V}{\partial x} dx \right) \quad \text{cal/sec.} = \text{heat flow into section } dx.$$

As heat is being generated in the wire it is clear that more heat must flow out at x than flows in at $x+dx$.

$$\text{Heat storage in section } dx = \rho a p \frac{\partial V}{\partial t} dx = Q. \quad \text{cal/sec.}$$

Heat lost by radiation and convection in section $dx = e(V-T)\sigma dx \quad \text{cal/sec.}$

Now, heat storage in section dx equals the heat flow into the section plus the heat generated in the section, minus the heat lost by radiation and convection and minus the heat flow out of the section per second. This is expressed in the following equation.

$$5. \quad \rho a p \frac{\partial V}{\partial t} dx = -ha \left(\frac{\partial V}{\partial x} - \frac{\partial}{\partial x} \frac{\partial V}{\partial x} dx \right) + ha \frac{\partial V}{\partial x} + q dx - e(V-T)\sigma dx.$$

$$\rho a p \frac{\partial V}{\partial t} = ha \frac{\partial^2 V}{\partial x^2} + q - e(V-T)\sigma.$$

To be effective the small tungsten wire at the light source must explode in a small fraction of a micro-second or less. Actually, we know that during this time the current of the condenser discharge is increasing very rapidly. The simplifying assumption will be made that the current remains constant.

$$\text{Then} \quad q = .239 i^2 r = \gamma(1 + \alpha V)$$

$$6. \quad \rho a p \frac{\partial V}{\partial t} = ha \frac{\partial^2 V}{\partial x^2} + \gamma(1 + \alpha V) - e\sigma(V-T).$$

$$\frac{\rho p}{h} \frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2} - \gamma \frac{e\sigma - \alpha \gamma}{ha} + \frac{\gamma + e\sigma T}{ha}$$

$$k \frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2} - AV + B.$$

The equation for the steady state is,

$$7. \quad \frac{d^2 V'}{dx^2} - AV' = -B. \quad , \text{ the complementary solution of which is}$$

$$V_1 = J \sinh \sqrt{A} \cdot x + P \cosh \sqrt{A} \cdot x, \text{ and the particular solution is}$$

$$V_2 = \frac{B}{A}.$$

The complete solution for the steady state is, therefore,

$$8. \quad V' = \frac{B}{A} + J \sinh \sqrt{A} \cdot x + P \cosh \sqrt{A} \cdot x, \text{ where } J \text{ and } P \text{ are constants.}$$

When t equals to infinity the equation for V , the solution of equation 6 must reduce to the steady state solution, 8, above.

If we make the assumption that,

$$9. \quad V = u e^{-\left(\frac{A}{K}t\right)} + V' \quad , \text{ where } u \text{ may be a function of both } t \text{ and } x, \text{ also, and substitute this into equation 6, an equation in } u \text{ results, the}$$

solution of which is well known. The equation in u is,

$$10. \quad \frac{\partial^2 u}{\partial x^2} = k \frac{\partial u}{\partial t}.$$

Let $u = XT$, where $X = f(x)$ only and $T = f(t)$ only. This substituted into 10 gives the two equations below, 11 and 12.

$$10a. \quad T \frac{\partial^2 X}{\partial x^2} = k X \cdot \frac{\partial T}{\partial t}. \quad \text{If we divide this through by } XT \text{ the variables will be completely separated.}$$

$$10a. \quad \frac{1}{X} \frac{\partial^2 X}{\partial x^2} = \frac{k}{T} \frac{\partial T}{\partial t}. \quad \text{Two such functions can only be equal if each is equal to the same constant. Let us put,}$$

$$11. \quad \frac{1}{X} \frac{d^2 X}{dx^2} = - \frac{m^2}{k} \quad \text{and}$$

$$12. \quad \frac{1}{T} \frac{d T}{dt} = - \frac{m^2}{k}.$$

The solutions of 11 and 12 are well known and are respectively,

$$X = E_m \cos mx + F_m \sin mx,$$

$$T = C_m e^{-\frac{m^2 t}{k}}$$

The most general form in which to write the solution for u is then,

$$13. \quad u = \sum_{m=0}^{\infty} e^{-\frac{m^2 t}{k}} (E_m \cos mx + E_m \sin mx)$$

Substituting this value of u in equation 9. we obtain the general

solution for V ,

$$14. \quad V = e^{-\frac{At}{R}} \left\{ \sum_{m=0}^{\infty} e^{-\frac{m^2 t}{k}} \cdot (E_m \sin mx + F_m \cos mx) \right\} + \frac{B}{A} + J \sinh \sqrt{A} \cdot x + P \cosh \sqrt{A} \cdot x$$

We will now apply the boundary conditions to the above equation and determine the constants E_m , F_m , m , P , and J .

Applying condition 2, that $V = T$ at all times at $x = 0$ and $x = l$, we see that the coefficients of $e^{-\frac{m^2 t}{k}}$ must vanish for both cases.

Therefore, $F_m = 0$ and if we put

$$m = \frac{n \pi}{l}, \quad \sin mx \text{ will equal } 0 \text{ when } x = l.$$

$$x = 0 \quad T = \frac{B}{A} + P \quad \text{i.e.} \quad P = T - \frac{B}{A}. \quad 15.$$

$$x = l, \quad T = \frac{B}{A} + J \sinh \sqrt{A} \cdot l + P \cosh \sqrt{A} \cdot l$$

$$\text{Therefore, } 16. \quad J = \left(T - \frac{B}{A} \right) \frac{1 - \cosh \sqrt{A} \cdot l}{\sinh \sqrt{A} \cdot l}$$

Applying condition 3 that at $t = 0$, $V = T$ for all x , to 14, we obtain,

$$17. \quad T = \sum_{n=1}^{\infty} E_m \sin nx + \frac{B}{A} + J \sinh \sqrt{A} \cdot x + P \cosh \sqrt{A} \cdot x.$$

Substituting for J and P in the above, we arrive at the result,

$$18. \quad \sum_{n=1}^{\infty} E_m \sin mx = \left(T - \frac{B}{A} \right) \cdot \left[1 - \cosh \sqrt{A} \cdot x - \frac{1 - \cosh \sqrt{A} \cdot l}{\sinh \sqrt{A} \cdot l} \cdot \sinh \sqrt{A} \cdot x \right]$$

note, $m = \frac{n \pi}{l}$.

Treating the left hand member as a Fourier series, we obtain the following value of E_m ,

$$19. \quad E_m = \frac{2}{l} \left(T - \frac{B}{A} \right) \frac{(1 - \cos n \pi) A}{m(m^2 + A)}, \quad \text{where } m = \frac{n \pi}{l}.$$

The complete solution for V now becomes,

$$20. \quad V = \left(T - \frac{B}{A}\right) \left\{ e^{-\frac{A}{K} t} \frac{2}{A} \frac{l^2}{l^2} \sum_{n=1}^{\infty} e^{-\frac{n^2 \pi^2}{k l^2} t} \frac{(1 - \cos n \pi)}{n^3 \pi^3 + n \frac{l^2}{A} n} \sin \frac{n \pi}{l} x + \right. \\ \left. + \left[\cosh \sqrt{A} \cdot x + \frac{1 - \cosh \sqrt{A} \cdot l}{\sinh \sqrt{A} \cdot l} \sinh \sqrt{A} \cdot x \right] \right\} + \frac{B}{A} .$$

Conditions 1 and 4 were used to check the above result.

SUMMARY OF RESULTS OF RESEARCH WORK

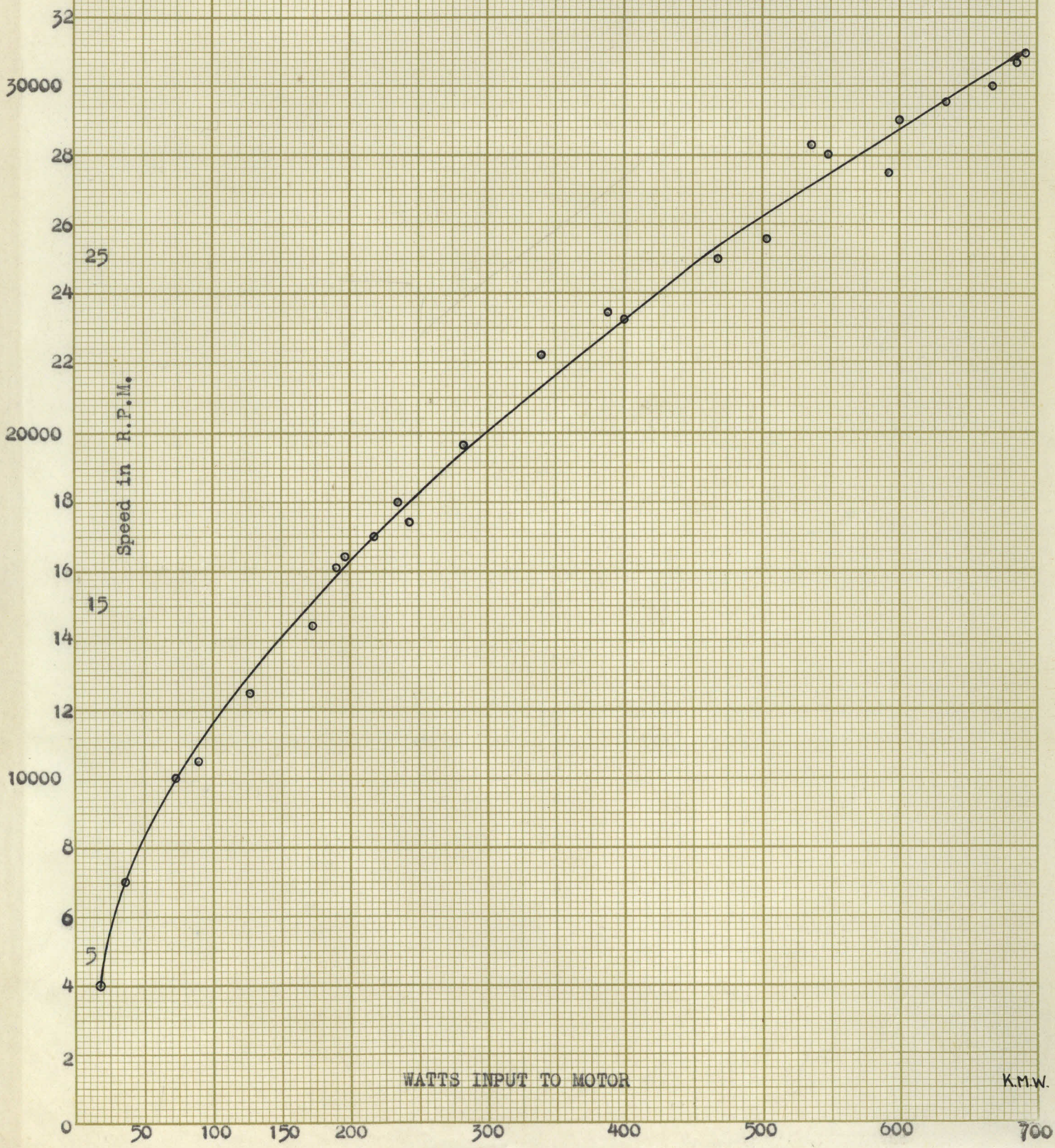
The Optical Oscillograph was designed and built for the particular purpose of measuring lightning currents, but so far there has not been an opportunity to take it into the field and try it on actual lightning discharges. It was, however tested in the laboratory, obtaining oscillograms such as those included in this report.

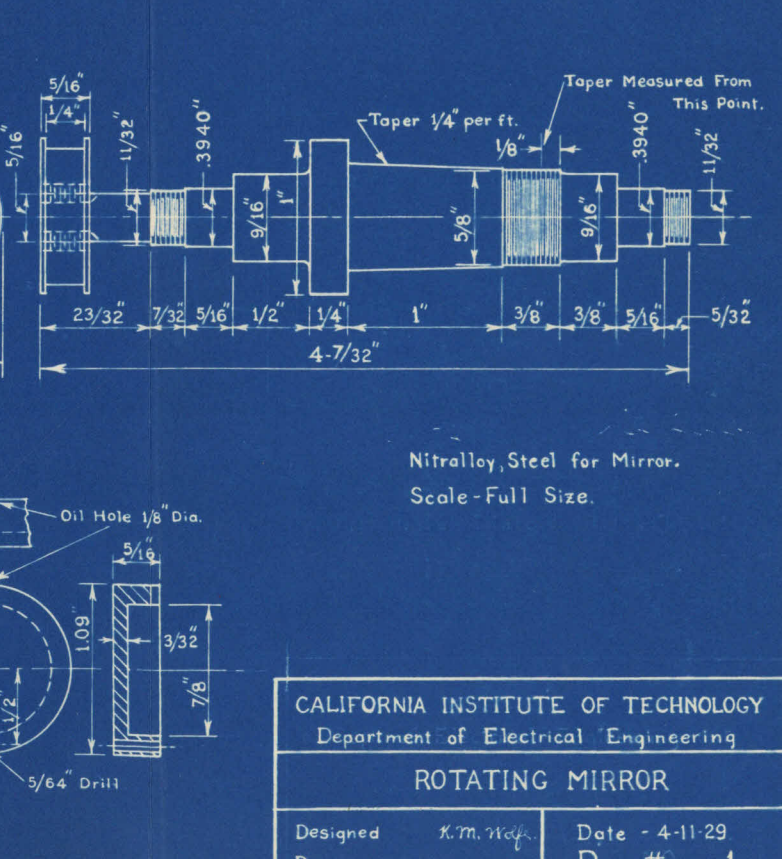
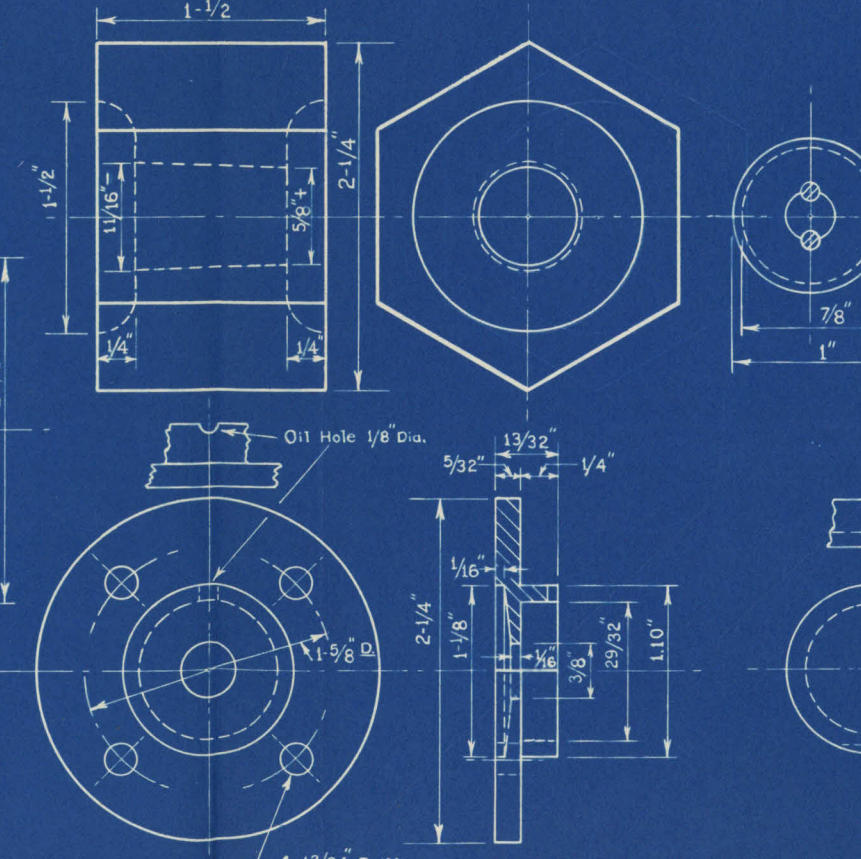
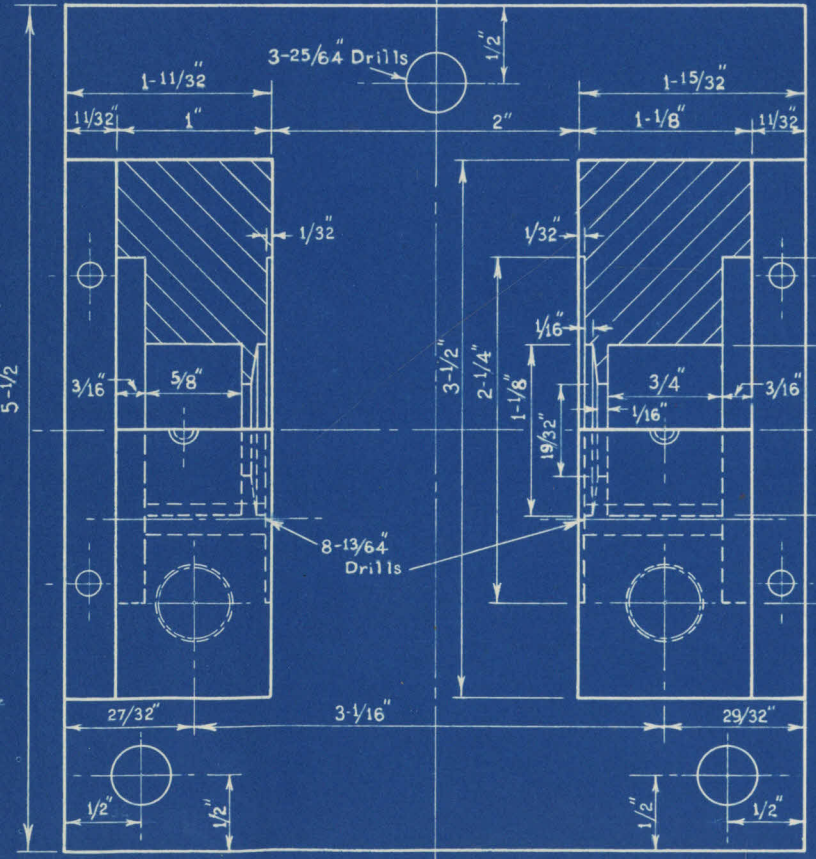
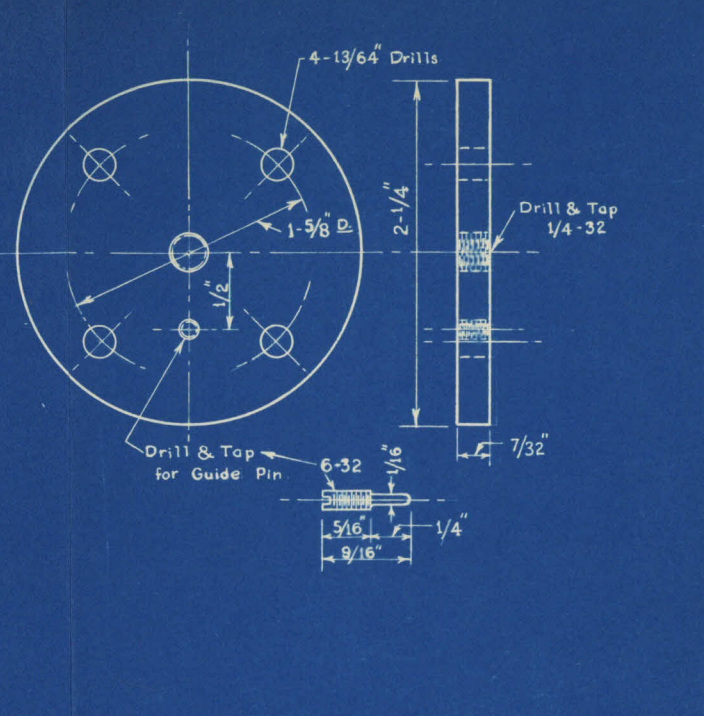
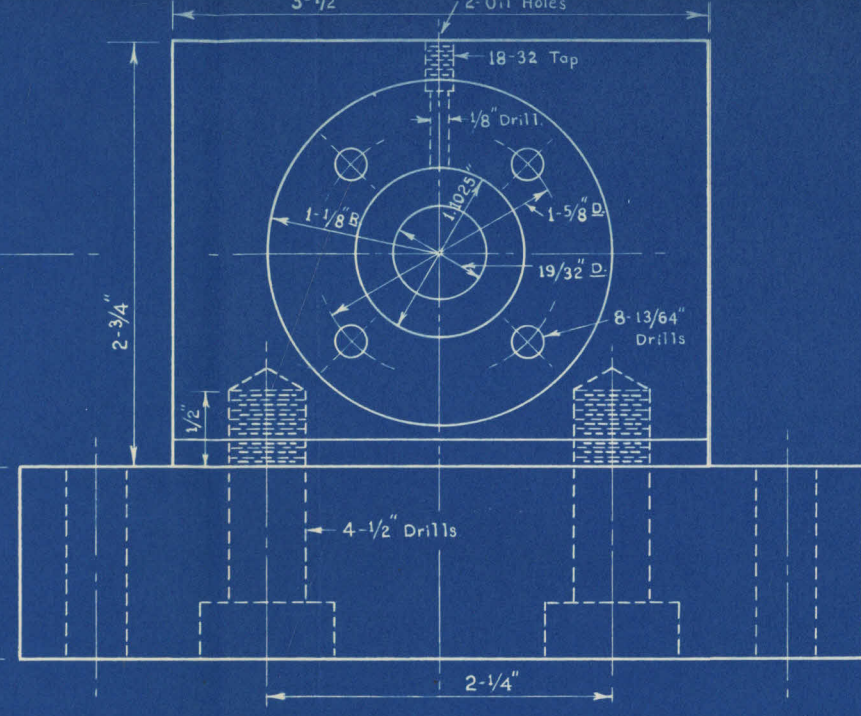
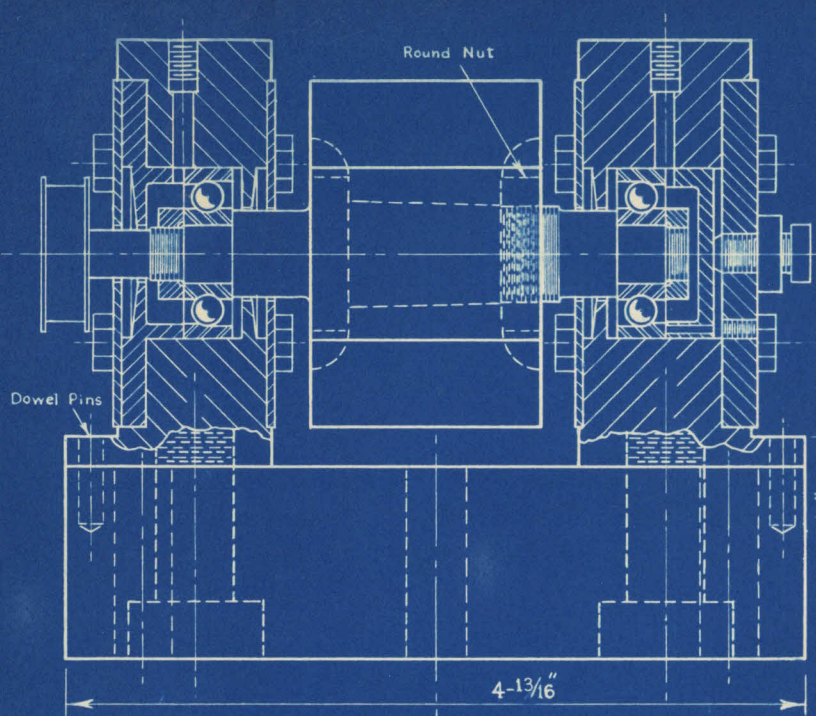
In the oscillograms a time scale of $1/16''$ per micro-second was easily secured; with a $1/2$ h.p. motor to drive the mirror at a higher speed it would not be at all difficult to get a time scale of $1/8''$ per micro-second. There is evidence concerning lightning to indicate that this time scale is sufficient to give good definition of any ordinary lightning surge oscillogram.

The current scale is not entirely satisfactory for measuring condenser discharges because the amplitude of the curves is so small. Knowing, though, that the charge of an average lightning discharge may be from 50 to 500 times that of the condenser used in the laboratory tests, one would expect that oscillograms of lightning currents would show a much wider deflection along the current axis. (The average charge of a lightning discharge is about 20 coulombs while the charge of the condenser used in the laboratory tests never exceeded .1 coulomb).

In measuring lightning currents the difficult problem will be to get the lightning to strike a particular rod or tower. This means that to get results within a reasonable length of time the instrument used must be set up on some mountain peak infested with numerous thunderstorms, and most likely in some isolated region. This oscillograph is admirably adapted for such a task due to its portability, its ease of operation and the certainty with which a surge is recorded.

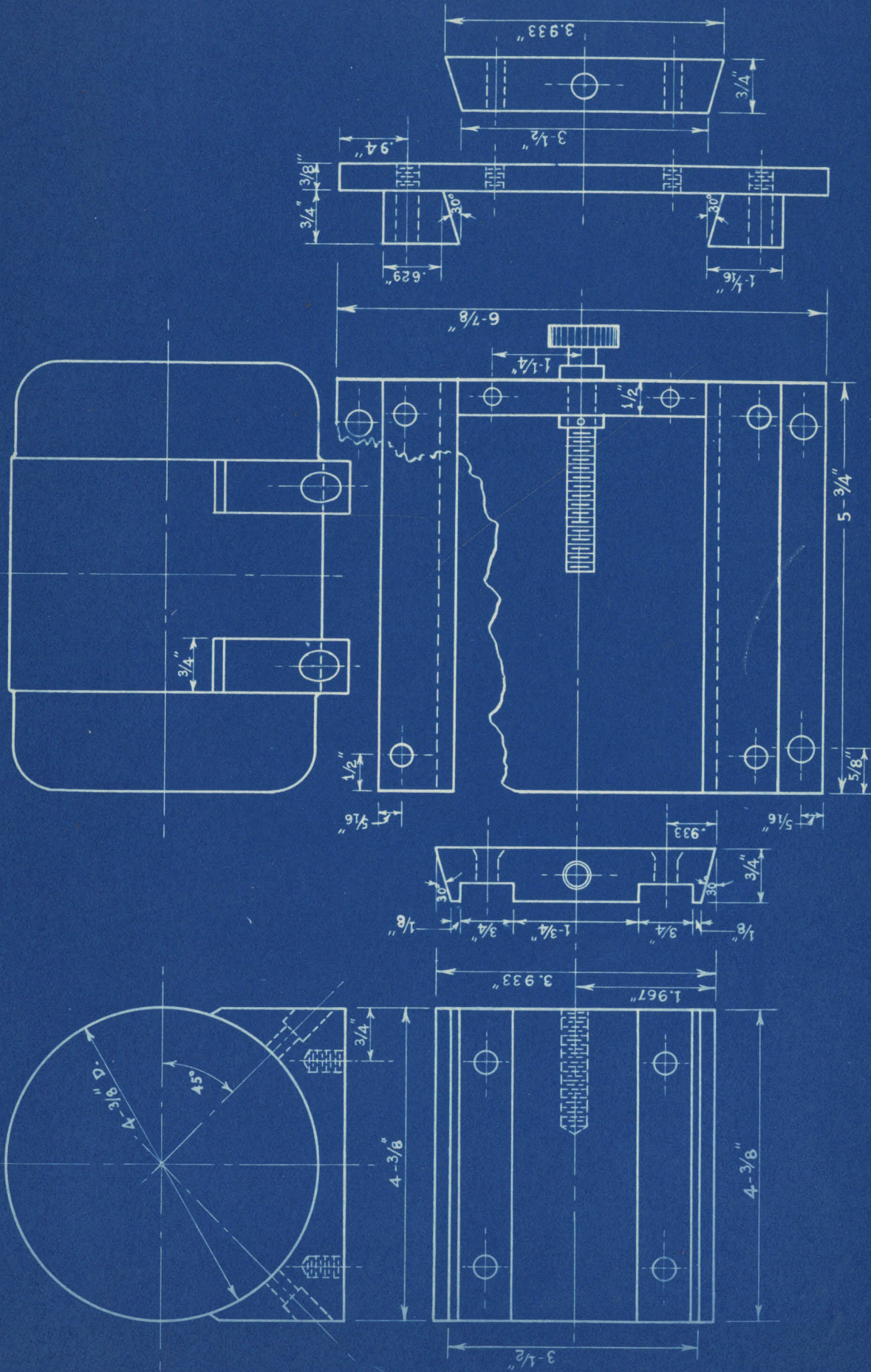
CALIBRATION CURVE OF MIRROR SPEED.





Niralloy Steel for Mirror.
Scale - Full Size.

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Department of Electrical Engineering	
ROTATING MIRROR	
Designed	K. M. McPh.
Drawn	"
Date	4-11-29
Dwg. #	--- 1



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 Department of Electrical Engineering

MOTOR MOUNTING

Designed K. M. Wofford
 Drawn ~10-10-30 ~"
 Scale ~ 6" = 1'
 Dwg. NO. 5