THE POWER FACTOR OF DIELECTRICS WHEN SUBJECTED TO FIFTY CYCLE VOLTAGES.

THESIS BY

RAYMOND WELLINGTON AGER.

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SUMMARY.

The work covered by this thesis consisted of the construction, calibration, and operation of an apparatus for the measurement of the power factors of small high voltage condensers. The apparatus is especially suited to the measurement of the power factors of small samples of liquid and solid dielectrics. The apparatus is simple in construction and in operation. Calculations, which are given in condensed form, indicate the accuracy which it is possible to obtain. The accuracy is satisfactory for most purposes.

The general shapes of the power factor vs temperature curves and the power factor vs voltage curves are indicated for porcelain, pyrex glass, paraffin, and transformer oil. These materials have power factors which rise as the temperature is increased. In the tests on transformer oil and on paraffin the power factor was found to be constant up to a critical value of the gradient. Above this critical value the power factor was found to increase. The critical gradient was found to be a function of the temperature. It is beleived that this bend indicates the first destructive effect in the process of puncture.

INTRODUCTION.

Some years ago, while the writer was in the employ of the Southern California Edison Company, a demand arose for some comparative information on different makes of porcelain insulators. Samples were obtained from the manufacturers, and the usual tests were performed. These tests consisted of a determination of the mechanical strength under both short and long time loadings, and a determination of the dielectric strength under definite voltage conditions. To suppliment these tests it was decided that a measurement of the phase angle or power factor should be made if suitable apparatus could be found. This measurement was to be made at a frequency of 50 cycles. It was thought that data of this kind might be related to the dielectric strength of the material and thus make it possible to determine the dielectric strength in this way and still have those insulators for other tests which would normally be destroyed in the puncture tests.

CIRCUIT.

The circuit usually used for such tests is the one commonly known as the Schering Bridge. However this circuit requires somewhat elaborate shielding, and it was decided to try a circuit suggested by Dr. J. S. Carrol of Leland Stanford University. That this circuit is essencially a bridge circuit can be seen from the simplified circuit diagram shown in Fig. 1. In this circuit the center tapped transformer, which is connected to ground at B, supplies voltage to the two condensers in series. If these two condensers have the same phase angle and the same impedance the connection between them will be at ground potential. If the two have different phase angles or different impedances there will be a voltage between point E and ground. This voltage will be a minimum when the two condensers have the same impedance, will not be zero as long as the two condensers have different phase angles. If the condensers are balanced to give this minimum reading and the different voltages recorded, the phase angle and power factor may be calculated as indicated in Appendix A. This calculation assumes that the power factor of the standard variable condenser is zero, and that the voltage indicating device shown in Fig. 1 as an electroscope does not effect the voltage of point E. The effect of the impedance of this meter upon the reading obtained is calculated in Appendix B. From these calculations it is seen that there is only one type of meter which can be used for this purpose and that type is the electrostatic. Of meters having the same value of impedance, one having pure resistance would be much better, but it is impossible to build meters of the resistance

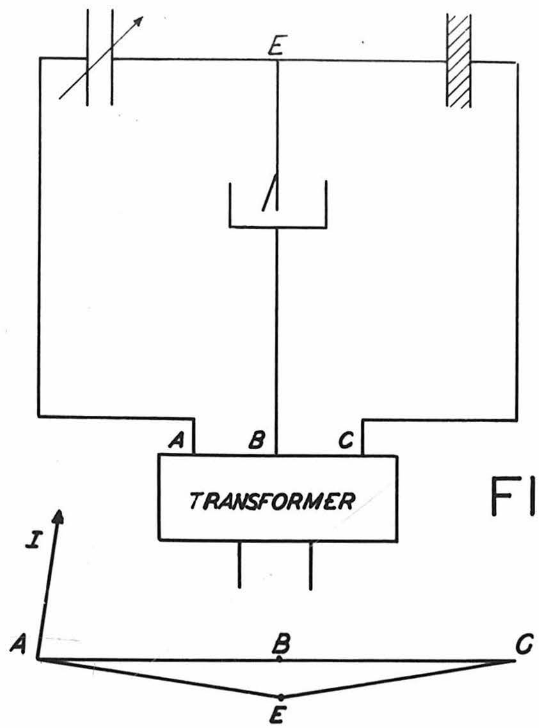

E
A
B
C
TRANSFORMER
I
A
B
C
E

FIG-1

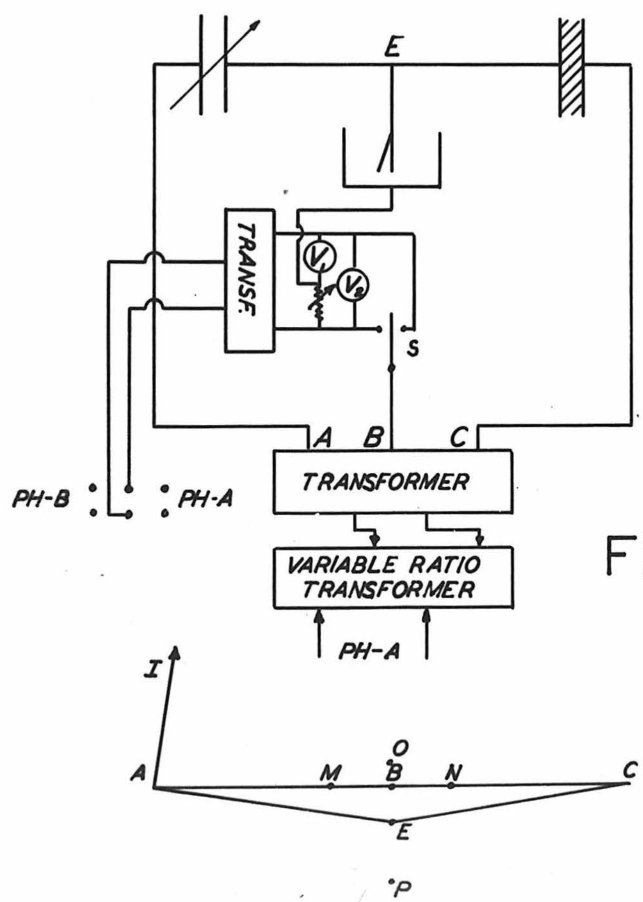

E
TRANSF.
V_1
V_2
S
A
B
C
TRANSFORMER
PH-B
PH-A
VARIABLE RATIO
TRANSFORMER
PH-A
I
O
A
M
B
N
C
E
P

FIG-2

type even approximating the impedance which can easily be obtained with electrostatic meters. The impedance of this instrument must be about one hundred times that of the test condensers in order to have an accuracy of one percent. For test condensers of the size used this would require an electroscope having a capacitance of 0.4 micro-microfarads. Such a capacitance is obviously impossible, hence we cannot expect to use direct voltage observation readings. To circumvent these difficulties a balancing and sensitizing transformer and potentiometer circuit is used as shown in the complete diagram in Fig. 2. When this transformer is connected to a source of voltage which is 90 degrees out of phase with that supplying the main transformer the potentiometer circuit may be so adjusted that the voltage on the electroscope will be independent of the position of the double throw switch S. This condition will be fulfilled when the case has the potential OB from ground with the switch in one position and the potential PB with the switch in the other position. The voltages across the electroscope will then be OE and PE respectively and these are observed as equal. When the potentiometer is so adjusted, the meters may be read, giving voltages OB and OP, and the power factor calculated. The calculation of the power factor in Appendix C with the biasing voltage included indicates that the impedance of the electroscope does not affect the final result. It would affect the magnitude of the indication of the electroscope and in that way it might effect the accuracy of the observation, but if the voltages on the electroscope can be made accurately equal to each other in all cases the indicated power factor will be always the same. In order to check experimentally the effect of the impedance of the electroscope the capacitance was increased by the addition of a sheet of metal which formed a condenser with the case. In this test no change in indicated power factor was found due to the presence of the sheet. The balancing and sensitizing transformer may also be used to definitely locate the minimum value where the two condensers have the same impedance. This is accomplished by connecting the transformer to the same phase as the main transformer, and setting the potentiometer so that the electroscope case has a potential midway between the two terminals of the transformer. Depending upon the position of the double throw switch the case of the electroscope will then have the potential BM or BN from ground and the voltage across the electroscope will be EM or EN.

APPARATUS.

No center tapped transformer of suitable voltage and capacity was available for this test so it was decided that we should use two potential transformers having as near as possible identical characteristics. Those selected were rated 16,500 to 110 volts, and had a capacity of

200 watts each. Tests performed in the Testing Laboratory of the Southern California Edison Company on these transformers showed them to have the same voltage ratio and the same phase angle on capacitive load to within the requirements of the test. The 110 volt windings were connected in parallel and supplied with voltage from an auto transformer which was controlled with dial switches. The 16,500 volt windings were connected in series and the center point grounded.

The potentiometer circuit was fed from a separate two winding transformer having a 400 volt secondary and a tapped primary so arranged that it could be connected to the same 110 volt source as the main transformers or to a 380 volt source on a three phase bank of transformers supplying power to the laboratory. This 380 volt source was displaced 90 degrees from the 110 volt source. To increase the range of the meters used in the potentiometer circuit, resistances were connected in series with them. V_2 is a 300 volt Weston model 155 voltmeter with a resistance connected in series to make its full scale reading 600 volts. V_1 is a 150 volt Weston model 155 voltmeter with a resistance connected in series to make its full scale reading 300 volts. The resistance in series with V_1 may be shorted out with a knife switch. The variable resistance in the potentiometer circuit is composed of a 1,000 ohm fixed resistor which may be shorted out, in series with a Bradley ohm carbon pile rheostat having a rated range of 1000 to 50,000 ohms.

A vacuum tube voltmeter was tried at one time but was discarded because the high grid capacitance and the low and variable grid to filament resistance made it unsuitable. A gold leaf electroscope having a sulphur bushing was put in place of the vacuum tube voltmeter and has given satisfactory service. Recently a cathetometer has been added which makes the reading of the gold leaf very much easier and more accurate. With this addition the electroscope is no longer the instrument which limits the accuracy obtainable. The present limit of accuracy is the reading of the voltmeters in the potentiometer circuit. These meters must be read with great accuracy as their readings enter the power factor equation as differences. If the line voltage fluctuates between the readings of the two meters the net result might be considerably in error. These meters might be read photographically or, by optical means, the images of the two might be superimposed so that simultaneous readings could be made.

Various arrangements of apparatus were tried with different designs and amounts of shielding. The arrangement now in use gives satisfactory results but could be changed to a considerable extent and possibly bettered. However, at the time that the engineers of the Southern

California Edison Company lost interest in the test, tho the apparatus looked very much as it does now, we were unable to check readings from day to day, and even readings taken in the morning just before noon could not be checked at one o'clock. These discrepancies were found to be caused by the presence of imperfect dielectrics in the field of the measuring circuit. A paraffined maple support, thought to be located in a position of practically zero field, was removed and grounded iron pipes substituted. A glass cylinder was replaced by three cast sulphur cones which now support the central electrode system. These two changes are the only important changes made to free the field of the central electrode system of all dielectric material which would change with time, temperature, or humidity. Sulphur is now the only solid dielectric in this field, and no changes have been noted in its behavior. Large cylinders, made of galvanized iron about 30 inches in diameter and 30 inches long, enclose the sample and the standard condenser. These cylinders are connected to the transformers so that the voltage between them is the sum of the transformer voltages. They are symetrically located with respect to the measuring circuit. With this arrangement of apparatus repeated checks have been made. These checks show that consistant results are obtained even tho made under widely varying conditions of temperature and humidity. A general view of the apparatus is

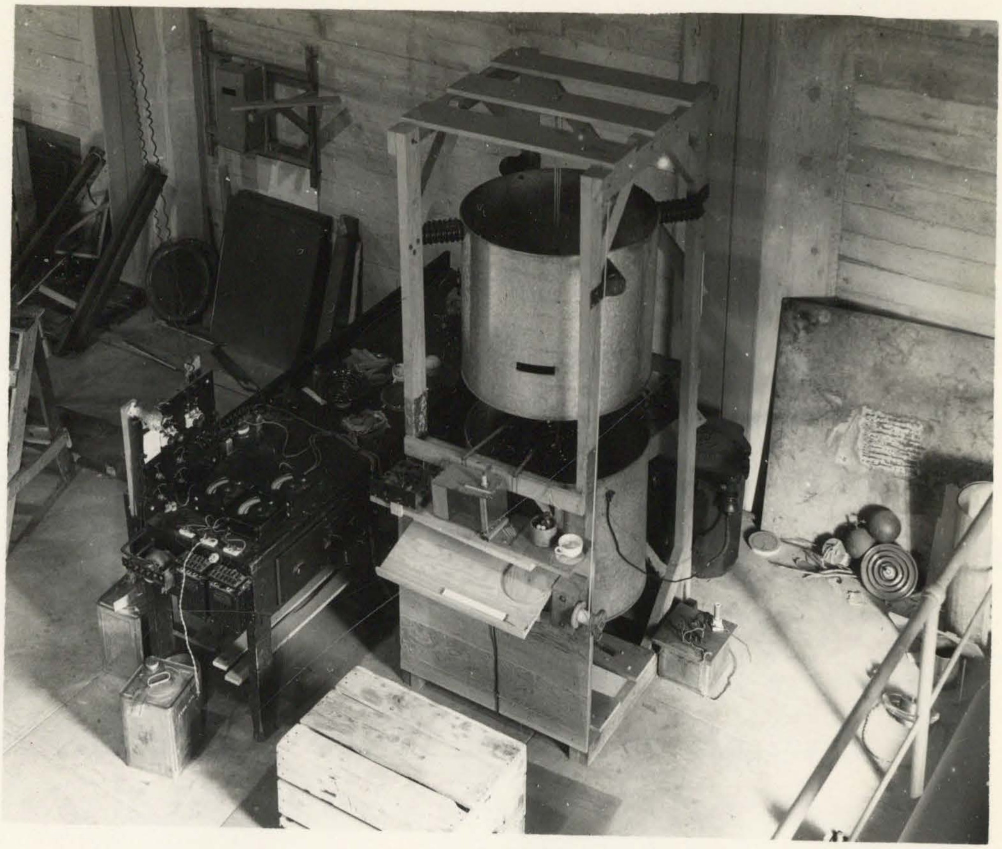


FIG. 3.

FIG. 4.

shown in Fig. 3. The upper condenser and the sulphur cones are shown in Fig. 4. Readings tho consistent among themselves may be considerably in error. In order to get accurate readings with this apparatus it is necessary that there be no convergence of flux lines from the lower cylinder to the central electrode system which are not wholly in the dielectric under test, and no convergence of flux lines from the upper cylinder which do pass thru the sample. If there is in either case convergence of lines as stated the power factor indicated will be less than it should be. The actual location of the ground plane around the central electrode system may be found by placing grounded conductors in the field and noting the effect they have upon the capacity required in the standard condenser for balance. If the capacity required is unchanged by the insertion of the grounded electrode this electrode must, in general, conform with the ground plane. Having found the location of this ground plane one can form a quite accurate idea as to the probable error with any particular arrangement. One might say in general that the errors will be very small when the ground plane comes down close to the sample under test, the upper electrode of the sample is nearly enclosed in the sample, and the leads to the electroscope and the two condensers are small.

CALIBRATION.

The apparatus was calibrated in the following manner. A 12 inch sphere was suspended in the lower cylinder. The condenser formed by the sphere and the enclosing cylinder was considered as a perfect condenser. A sketch made to scale is shown in Fig. 5. If the apparatus were perfect the indicated power factor would always be zero for this arrangement. The "zero power factor curve" which was obtained is shown in Fig. 6. Considering that the power factor scale has been somewhat elongated the points fall fairly well on the curve. Another calibration was made in which resistance was inserted in the lead connecting the two condensers. By having this resistance below the point where the electroscope lead was taken off, the condenser in the lower cylinder was made to have a power loss. Measurements were made of the resistance added using a "B" battery and a Rawson multimeter. Knowing the resistance and the power factor the impedance of the condenser was calculated. This should remain constant. In a series of tests made on three different days and covering a voltage range of from 5,000 to 10,000 volts and power factors of from 0.017 to 0,102 the average variation of the readings from the mean was about 2.0%. Using the mean value of the impedance the capacitance of the condenser was calculated and found to be 19.2 centimeters. As the sphere was hanging centrally in the cylinder and was arranged to be nearly

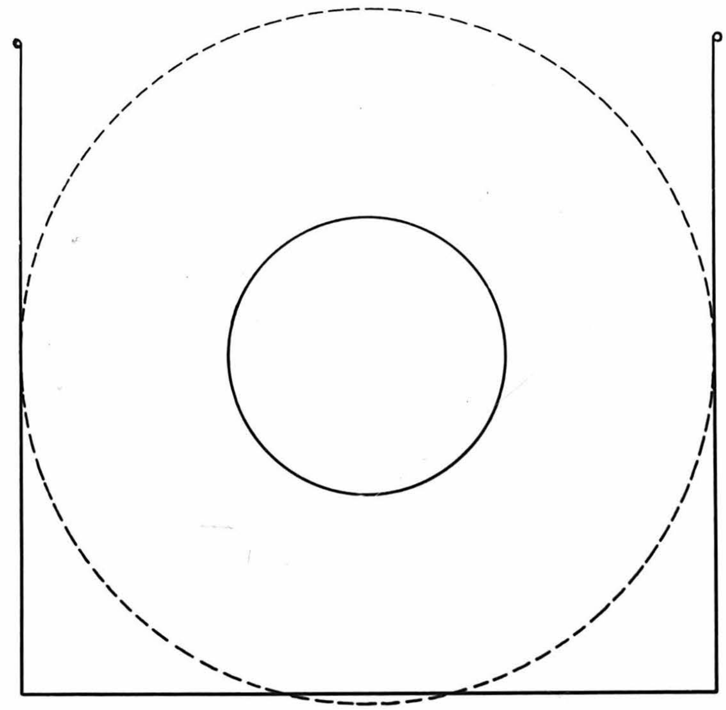


FIG 5

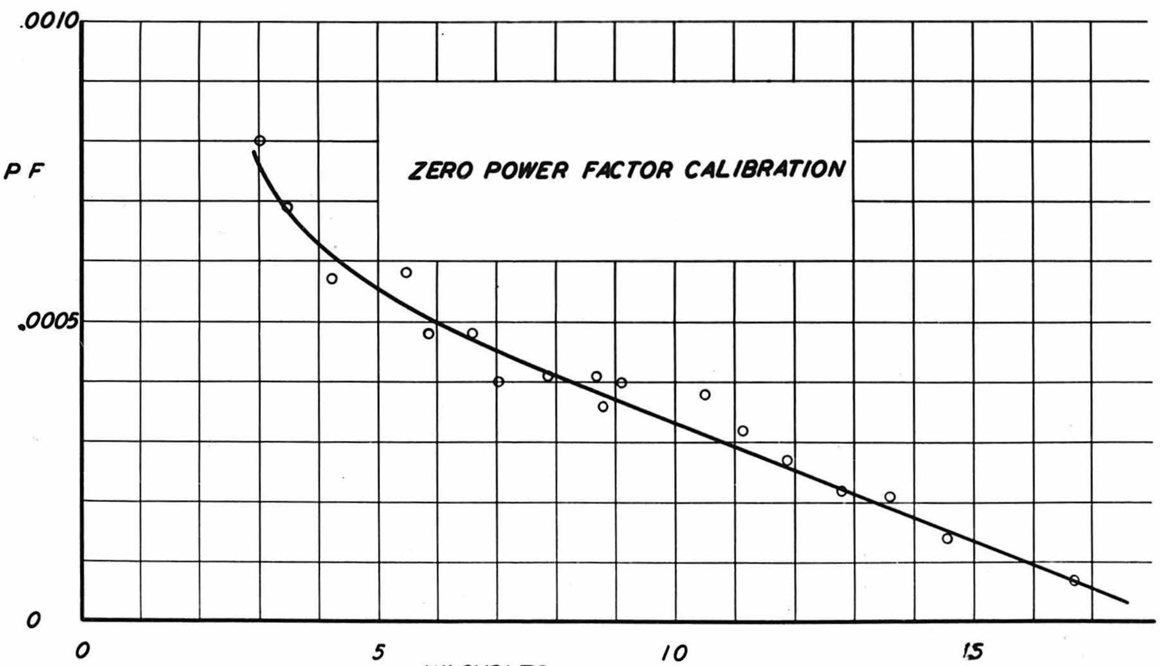

ZERO POWER FACTOR CALIBRATION
PF
.0010
.0005
0
0
5
10
15
KILOVOLTS

FIG 6

the same distance from the bottom as from the sides a calculation of the capacitance of two concentric spheres having the same spacing as the spacing to the sides of the above arrangement was made. This gave a value of 24.6 centimeters. The dotted circle in Fig. 5 shows the dimensions of this mythical sphere. Thus the actual arrangement gave a capacitance which was 78 percent that of two concentric spheres having about the same spacing. This seem like a reasonable value. However too great significance should not be given to this last test, as it is not beleived that the Rawson multimeter used in measuring the resistances was highly accurate on its most sensitive scale. Also the resistances used had slight tendencies to change with time and to polarize. These resistances were liquid resistors in small glass tubes having copper electrodes. The liquids used were mixtures of acetone, alcohol, and xylene.

MEASUREMENTS ON PORCELAIN AND PYREX PLATES.

Some tests were made on porcelain plates or discs which were about six inches in diameter and three-fourths inch thick, and also on pyrex glass plates which were made in the normal pie plate molds except that the molds had been separated so that the thickness was increased to about five-eighths inch. The surfaces of these samples were very nearly plane and cast iron electrodes were cemented to them with sealing wax. The volume of the wax in the field was only a very small percentage of the volume of the sample. The tests made on these samples consisted of readings taken of power factor against time and temperature against time as the sample cooled from an elevated temperature. Fig. 7 shows typical curves of this type. A curve which was derived from these curves giving power factor as a function of temperature is shown in Fig.8. Other tests were made in which the temperature of the sample was held constant and readings made of power factor at different voltages. In these tests the power factor was constant. Considerable care had to be taken to keep the surface of the insulator clean so that the leakage over the surface would be negligible. A very satisfactory way was found. The surfaces were first scoured with lava soap and water, and then polished with paper towels. When the sample had been properly cleaned a charge could be built up on the central electrode system using a rubber fountain pen which had been charged by rubbing on wool. The charge so built up would leak off very slowly. Calculations of the losses due to this leakage as estimated from the time constant and the known capacitances show that this loss is negligible in comparison with those observed on test. During these tests it was observed that the thermometers used to measure the temperatures of the electrodes had a considerable effect on the indicated power factor. No precise tests were made to evaluate this but it appears

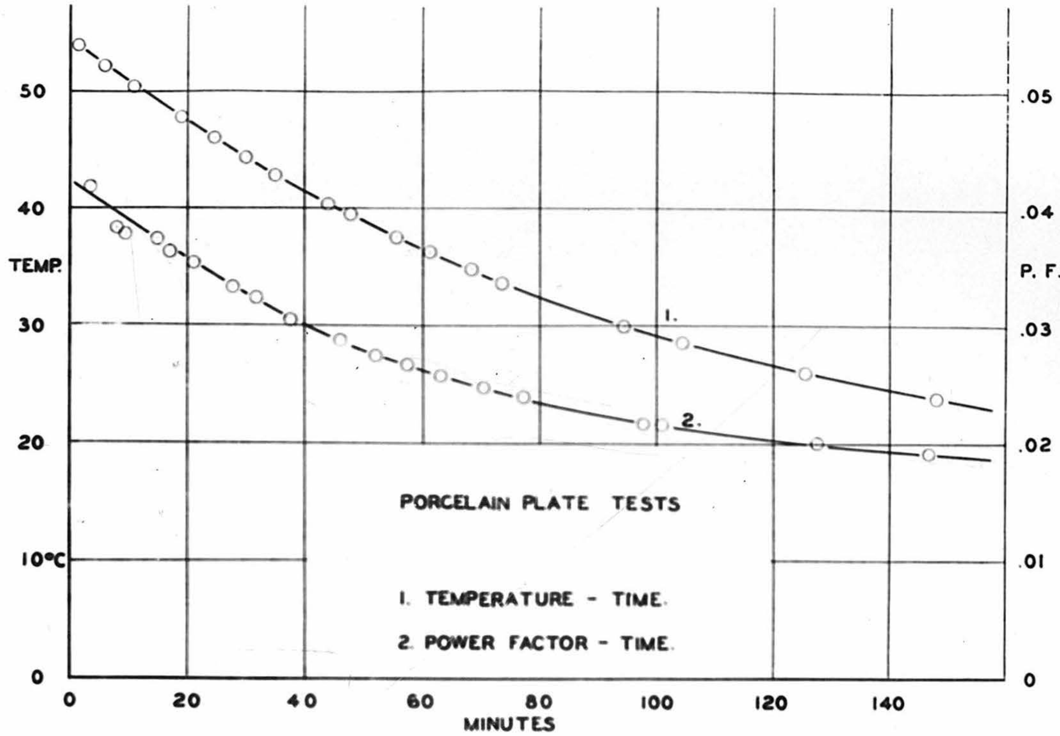

50
40
TEMP.
30
20
10°C
0
.05
.04
P. F.
.03
.02
.01
0
0
20
40
60
80
100
120
140
MINUTES
1.
2.
PORCELAIN PLATE TESTS
1. TEMPERATURE - TIME.
2. POWER FACTOR - TIME.

FIG 7

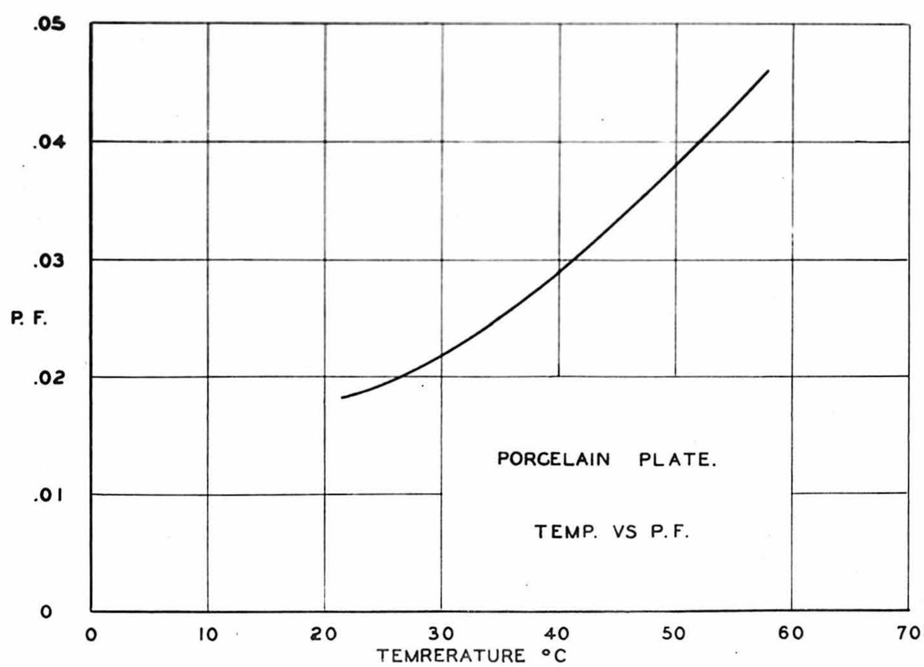

.05
.04
.03
P. F.
.02
.01
0
0
10
20
30
40
50
60
70
TEMRERATURE °C
PORCELAIN PLATE.
TEMP. VS P.F.

FIG 8

that the thermometers may effect the power factor by at least 0.0005.

MEASUREMENTS ON PORCELAIN SHELLS.

Some sample porcelain suspension insulator shells obtained from the Westinghouse Company were used in these tests. The first electrodes used were mercury. The iron cap used with the shell was held inverted and a quantity of mercury poured into it. The shell was then inverted and the head placed in the cap. Mercury was then poured into the pin hole and connection to this was obtained by hanging a brass rod so as to dip into it. This arrangement gave very nice curves as long as the voltage on the sample was kept down to fairly low values. When test runs were made between power factor and voltage, however, the power factor obtained was found to be approximately constant up to a certain voltage, and then the value would rise. Accompanying this rise the readings would usually become unsteady. The reason for this was first thought to be corona in the small air spaces in the rough sanded surfaces into which the mercury did not penetrate. To fill up these air spaces transformer oil was used. With the oil the voltage vs power factor curves became straight lines, but the power factor was higher than expected. Further tests showed that although the sanded surfaces were ground smooth the above noted effects were still present. Also it was found that the oil was effective only on the outside of the head of the shell. In Fig. 9 curve 1. was obtained with oil filling these small spaces and also the space between the cap and the porcelain above the mercury. Curve 2. is drawn to fit fairly well the other three runs which were made on the shell as received, the shell as received but with oil in the pin hole only, and the shell having all sanded surfaces smoothed by grinding no oil being used. There were slight differences between the temperatures when the curves were taken so that a close agreement between the different curves cannot be expected. When the surfaces which were supposed to be conducting were coated with a conducting paint the power factor was further raised and was independent of the voltage. From this it was concluded that the reason for the variations observed lay in the behavior of the air space between the cap and the shell above the mercury. If this air space were considered as a perfect condenser in series with a part of the porcelain condenser we would expect that the power factor would be lower than that of the porcelain condenser alone. If this air space were to go into corona on increasing the voltage we would expect that the power factor would increase and conditions would become somewhat unsteady. If the above were true and oil was added to fill this space the power factor would be increased slightly due to the losses in the oil and also due to its higher dielectric constant. Also since oil has a higher dielectric strength than air

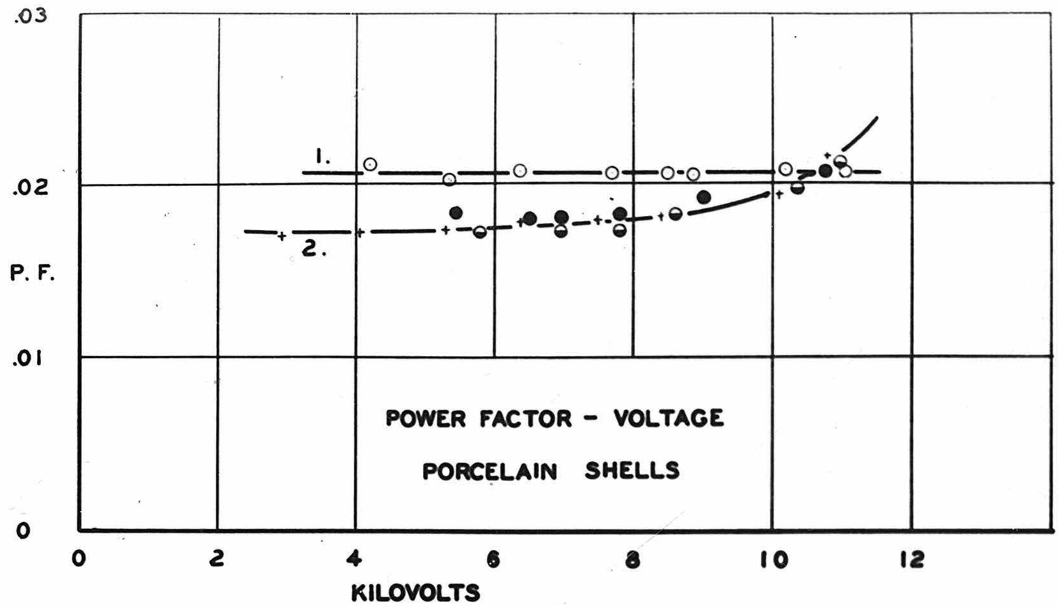

.03
.02
P. F.
.01
0
1.
2.
POWER FACTOR - VOLTAGE
PORCELAIN SHELLS
0
2
4
6
8
10
12
KILOVOLTS

FIG 9

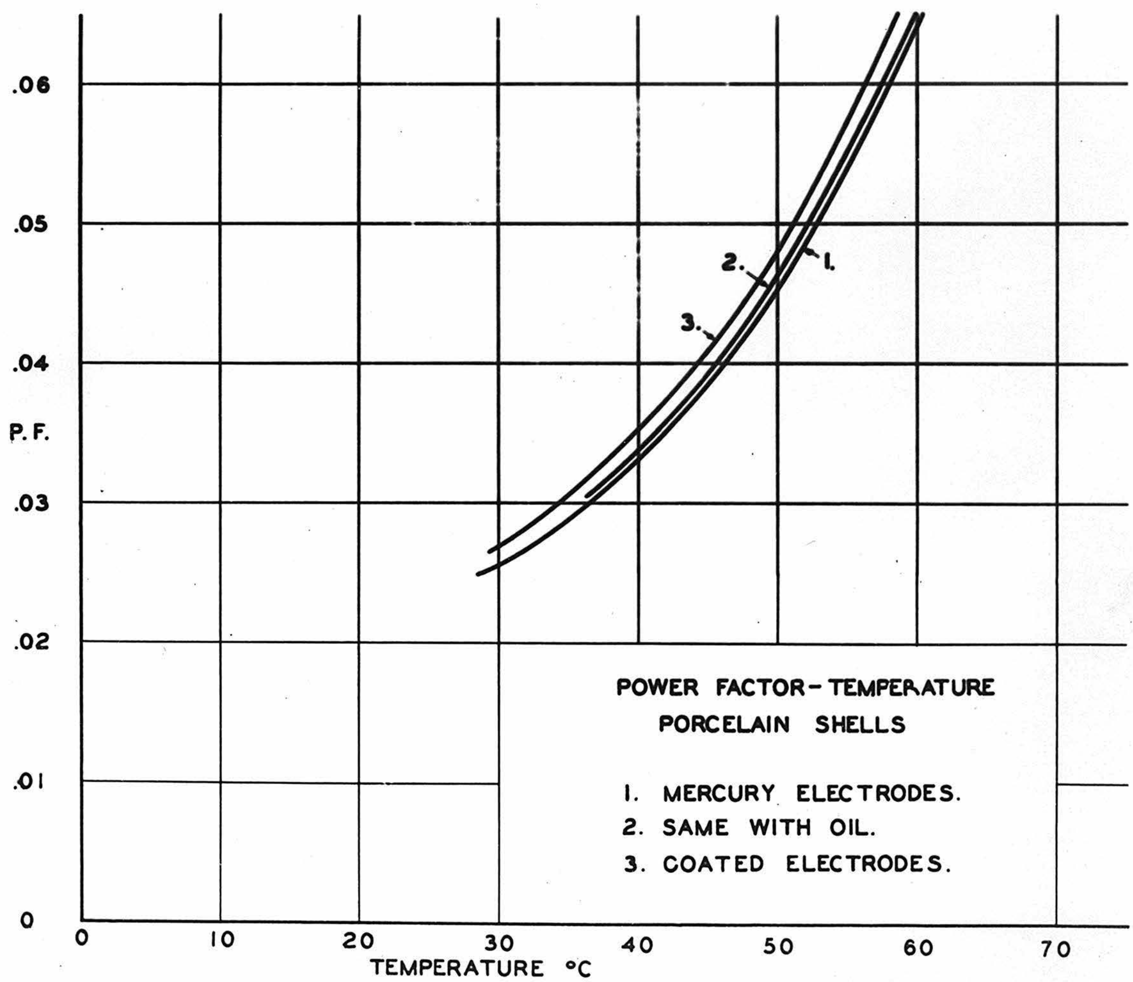

.06
.05
.04
P.F.
.03
.02
.01
0
2.
1.
3.
POWER FACTOR-TEMPERATURE
PORCELAIN SHELLS
1. MERCURY ELECTRODES.
2. SAME WITH OIL.
3. COATED ELECTRODES.
0
10
20
30
40
50
60
70
TEMPERATURE °C

FIG 10

the corona would be eliminated so that the power factor should be constant. If this air space were shorted out by conducting paint the power factor should be further raised to that of the porcelain itself. The calculations indicated in Appendix D were made with the above in mind. These calculations indicate that although the power factor of the porcelain was 0.020 as would have been indicated by a test made with the air space short circuited, the value obtained with the oil would be 0.01925, while if the test were made without oil or conducting paint the value would be 0.01865. It is remarkable that the ratios of the power factors obtained on a shell tested under these conditions should give this ratio almost exactly. Fig. 10 shows curves for these tests. There are two reasons which may be given for the differences between the power factors obtained on the porcelain shells and those on the porcelain plates. The most important is probably a difference in the composition of the porcelain. The porcelain plates were furnished be the Lapp Insulator Co. while the shells were furnished by the Westinghouse Co. The other reason is that there was probably considerable dielectric flux set up between the lower cylinder and the large electrode which was used as the top plate of the condenser made with the porcelain plate.

MEASUREMENTS ON TRANSFORMER OIL.

It was desired to have the class in course E.E. 223, which was studying dielectrics, run some tests on plastic insulation. A set of electrodes were hurriedly turned out and the insulating material to be tested poured around them. After their tests were completed the apparatus was cleaned up and a run started on transformer oil. The oil was heated in an open vessel for some time, and then readings were made of power factor against voltage for constant temperatures. Upon plotting the curves, which are shown in Fig. 11, it was found that each had a bend in it. This bend occurred at progressively lower voltages as the temperature was increased. Since the electrodes had not been made with a calculation of the gradient in mind and the exact spacing was unknown, only an estimate of the maximum gradient existing in the oil during the times of the tests is possible. In the neighborhood of the bends this is estimated to have a peak value of about 75,000 volts per centimeter. In order to get data from which one could compute the gradient on the sample accurately, a new set of electrodes was made up in the form of a sphere against a plane and having a micrometer screw for the determination of the spacing. At first the upper electrode was made of brass and the lower of iron. On trying to operate at temperatures above atmospheric the oil sludged badly. As

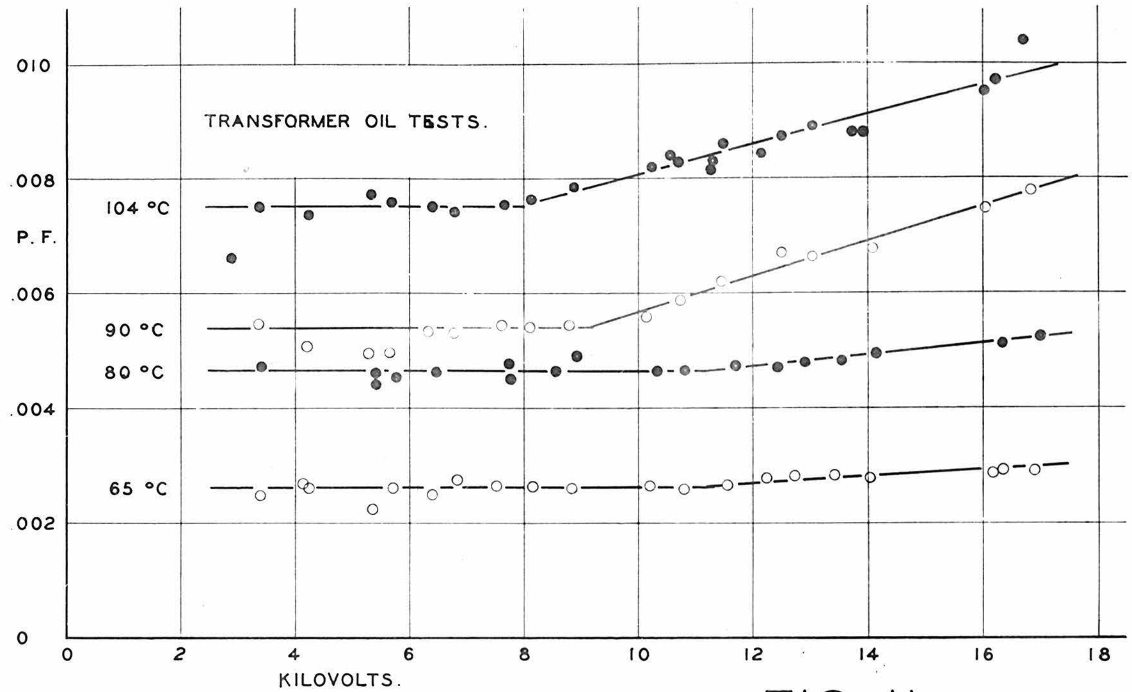

TRANSFORMER OIL TESTS.
010
.008
.006
.004
.002
0
P. F.
104 °C
90 °C
80 °C
65 °C
0
2
4
6
8
10
12
14
16
18
KILOVOLTS.

FIG 11

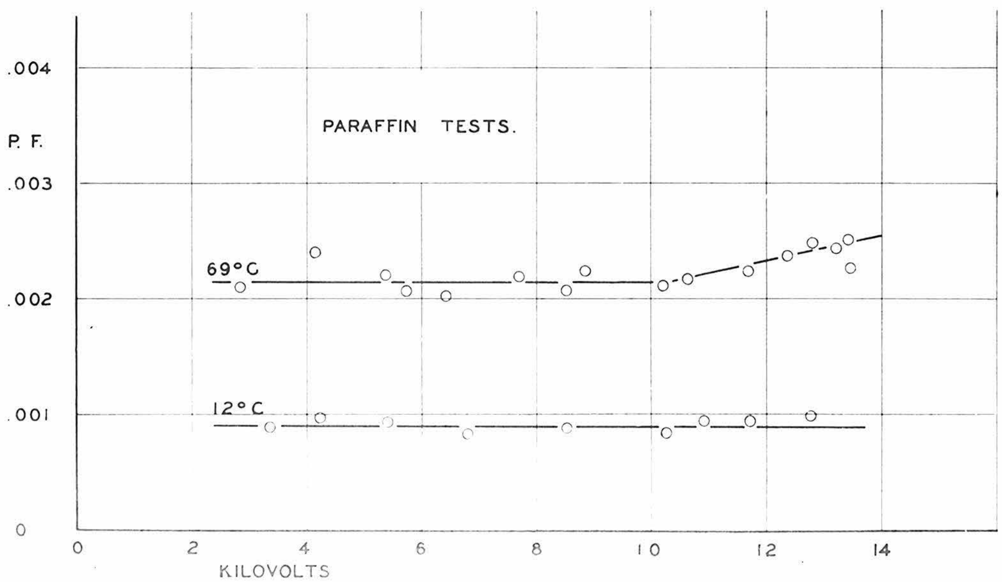

PARAFFIN TESTS.
.004
.003
.002
.001
0
P. F.
69°C
12°C
0
2
4
6
8
10
12
14
KILOVOLTS

FIG 12

copper has a tendency to accelerate this process the upper electrode was changed to iron. Even with the iron electrode sludging proceeded at such a rate that no satisfactory results were obtained. To prevent this sludging of the oil one would have to construct apparatus in which the oil could be kept out of contact with the air. Either the space above the oil should be evacuated or should contain an inert gas. Further work in this field might lead to valuable information pertaining to dielectric breakdown.

MEASUREMENTS ON PARAFFIN.

The difficulties encountered in working with oil suggested a similar test for paraffin. Two different kinds were tried but in both cases sludging was present in sufficient extent to make the results of little value. However in one test a curve was obtained which showed that a bend in the power factor vs voltage curve was present. This curve is shown in Fig. 12. It would be interesting to find out what would happen to the bend in the curve at temperatures around the melting point.

RECENT WORK ON SPECIALLY SHAPED PORCELAIN PLATES.

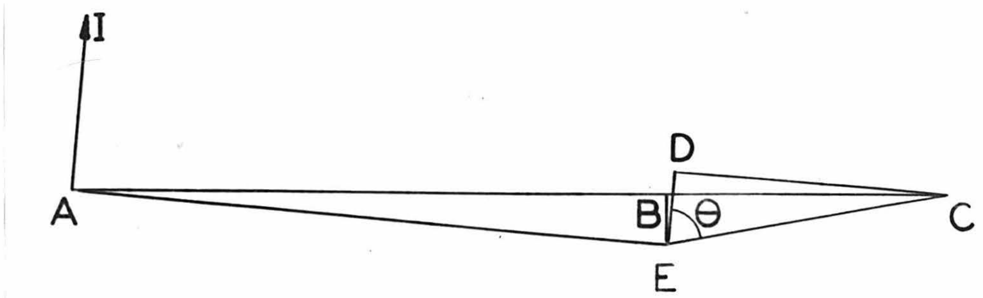
To get a material which would not change with time it was decided to try porcelain again. The previous tests made on solid dielectrics were made under conditions which precluded any possibility of high gradients. To obtain these higher gradients the discs were ground with metal tools and carborundum powder so as to have a spherical depression in one side. The blanks used had thicknesses of about three-fourths inch. Two of these discs have been finished at present but trouble is being experienced in getting away from edge discharge. On one sample having gold electrodes plated on it by the usual ceramic process, edge discharge was found at a quite low voltage. No method of shielding this edge was found that was satisfactory. This sample has a thickness of 0.015 inches at its thinest section and has been operated at voltages up to 10,000 volts (effective) at a temperature of 20 degrees Centigrade with no evidence of a bend. When the edge discharge has been eliminated it is to be tested up to 17,000 volts. Another sample having a minimum thickness of 0.008 inches was coated with a paint made of a mixture of collodion and graphite gave erratic results which are thought to be caused by lack of proper conductivity in the paint. A pyrex glass sample is also in the process of preparation.

CONCLUSIONS.

The apparatus is satisfactory for the determination of the power factor of small high voltage condensers. The accuracy which can be obtained is satisfactory for most purposes. The simplicity of operation and calculation make the apparatus one which can be used by any one having moderate skill in the handling of scientific instruments. The absence of complicated shielding should make this apparatus stable over long periods of time.

The general shapes of the power factor vs temperature curves and the power factor vs voltage curves have been obtained for porcelain, pyrex glass, paraffin, and transformer oil. A bend in the power factor vs voltage curve has been detected in paraffin and transformer oil. This is believed to be the first destructive effect in the process of puncture. With further work done along this line the purpose of the experiment will be greatly advanced, as it may give a way of determining the maximum gradient which may be used in electrical apparatus if all deterioration of the insulation, from electrical causes, is to be avoided.

APPENDIX A.



In the above diagram the point B is the tap on the transformer winding which is connected to ground. The line AE represents the voltage across the standard variable condenser which is assumed to have no losses so that it lags the current I by 90 degrees. EC is the voltage across the sample and it lags the current by an angle θ. It is assumed that the adjustment has been made so that BE is perpendicular to AB. BE is the voltage across the electroscope and the voltages AB and BC are known from the transformer ratios. The power factor is cosine θ.

$$\cos\theta = \frac{DE}{CE} \qquad \frac{DE}{AB + BC} = \frac{BE}{AE}$$

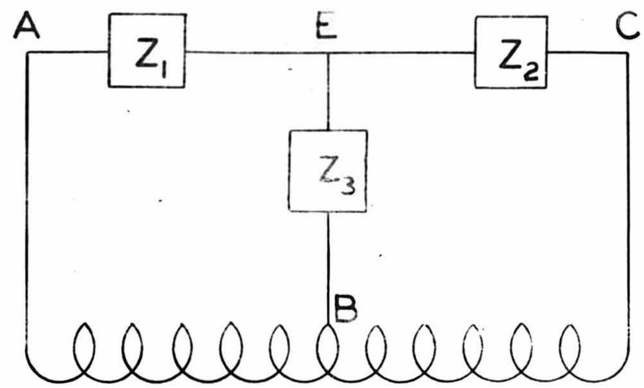
$$\cos\theta = \frac{BE\ (AB + BC)}{CE \times AE} = \frac{BE\ (AB + BC)}{\sqrt{(AB^2 + BE^2)(BC^2 + BE^2)}}$$

if AB = BC, then $\cos\theta = \frac{2\ BE \times AB}{AB^2 + BE^2}$

If we can neglect BE^2 in comparison with AB^2

$$\cos\theta = \frac{2\ BE}{AB}$$

APPENDIX B.



The following calculation was made to determine the effect of the shunt impedance Z_3. In it the currents and voltages are vectors expressed in complex notation and the impedances are complex operators.

E = the voltage from A to B, and also from B to C.

$$E - I_1 Z_1 - I_3 Z_3 = 0$$

$$E + I_3 Z_3 - I_2 Z_2 = 0$$

$$I_1 = I_2 + I_3$$

From these equations we get

$$I_3 Z_3 = \overline{BE} = E Z_3 \frac{Z_2 - Z_1}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

For Z_3 very large in comparison with the other impedances we get

$$I_3 Z_3 = E \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

Now if we let the actual reading divided by the reading when Z_3 approaches infinity be equal to M, we get

$$M = \frac{Z_3 (Z_1 + Z_2)}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

We are interested in the scalar magnitude of M. However we must still use the complex values for the impedance operators.

We may now consider the possible variations in the impedances.

If Z_1 Z_2 Z_3 are equal in absolute value and all have the same character (that is capacitive with low phase angle) then $M = 0.667$

If $Z_3 = 10Z_1 = 10Z_2$ in absolute magnitude and they all have the same character as above. Then $M = 0.95$

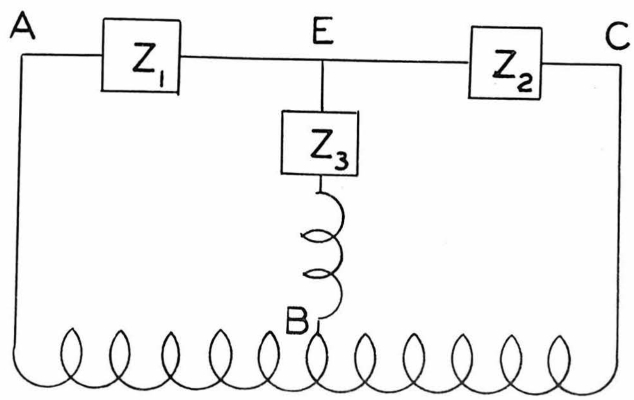
If $Z_3 = 20Z_1 = 20Z_2$ in absolute magnitude and they all have the same character. Then $M = 0.975$

If, however, $Z_1 = Z_2 = Z_3$ in absolute value and Z_3 has a phase angle which is different from those of the other two by 90 degrees. Then $M = 0.895$

If $Z_3 = 10Z_1 = 10Z_2$ in absolute value and Z_3 has a phase angle about 90 degrees from those of the other two Then $M = 1.000$ Practically.

From these considerations it is seem that it would be desirable to have Z_3 about ten times as large as the other two and have a phase angle difference of about 90 degrees. As Z_1 and Z_2 in these tests are condensers having low phase angles and capacitances of the order of 30 centimeters, and therefore impedances of the order of 10^8 ohms at 50 cycles, we would require a resistance type meter having an impedance of the order of 10^9 ohms.

APPENDIX C.



The following calculation was made to determine the effect which the biasing voltage induced into the transformer winding between point B and impedance Z_3 has upon the voltage across the impedance Z_3. If we let the voltages induced in the two halves of the main transformer be each equal to E, and the voltage induced in the biasing transformer be e_1 we may solve for I_3 which is the current flowing in Z_3. The equations used in this solution are:-

$$E + je_1 - I_3Z_3 - I_1Z_1 = 0$$

$$E - je_1 + I_3Z_3 - I_2Z_2 = 0$$

$$I_1 = I_2 + I_3$$

Solving these we obtain:-

$$I_3 = \frac{E(Z_2 - Z_1) + je_1(Z_1 + Z_2)}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

Reversing e_1 and calling it e_2 and reversing I_3 calling it I_3' we get from a similar calculation:-

$$I_3' = \frac{-E(Z_2 - Z_1) + je_2(Z_1 + Z_2)}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

Now if we make $I_3 = I_3'$ we have the condition that the voltage across the electroscope is the same for either position of the double throw switch on the sensitizing apparatus. This gives

$$j(e_2 - e_1)(Z_1 + Z_2) = 2E(Z_1 - Z_2)$$

but $Z_1 = -jX$ and $Z_2 = R - jX$ when R is small in comparison with X.

Substituting these values we obtain:-

$$e_2 - e_1 = \frac{-ER}{Z_1}$$

But we read on the meters

$$V_2 = e_1 + e_2 \qquad \text{and } V_1 = e_1$$

Therefore

$$e_2 - e_1 = V_2 - 2V_1$$

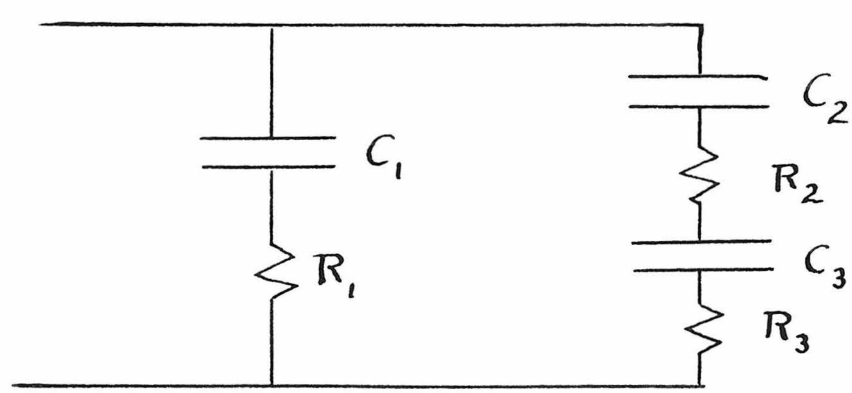
And

$$\frac{V_2 - 2V_1}{E} = \frac{R}{Z_1} = \text{the power factor.}$$

It is to be noticed that this is independent of the impedance of the voltage indicating device, and is correct as long as R is small enough in comparison with Z_1 so that R^2 can be neglected in comparison with Z_1^2.

APPENDIX D.

Calculations were made to determine the effect of the air space between the cap and the porcelain above the mercury in the tests on porcelain shells. For simplicity in these calculations all fringing of the flux is neglected. The porcelain was assumed to float on the mercury, and the condition below the level of the mercury on the outside of the shell was taken as that of a hemispherical condenser in parallel with a short section of a cylindrical condenser. These two condensers took care of the closed end of the pin hole and the short section of the straight portion of the head which was below the mercury level. As the mercury level in the pin hole was higher than that on the outside and was practically level with the upper edge of the cap, it was assumed that the cylindrical porcelain condenser was surrounded by a coaxial air or oil condenser. The equivalent diagram is shown below.



From calculations based on the weight of the shell and the weight of the mercury in the pinhole the depth which the head of the shell sinks into the mercury was found to be 2.47 inches. The capacitances of the portion submerged total 2.93 K centimeters. Where K is the dielectric constant. The capacity of the portion of the head above the level of the mercury was 1.00 K centimeters. The capacitance of the air or oil condenser on the outside of this latter condenser is 8.6 k centimeters.

The general equation for the power factor of the network shown is given in the following formula:-

$$P.F. \simeq \frac{R_1(R_2+R_3)^2 + R_1\left(\frac{1}{\omega C_2}+\frac{1}{\omega C_3}\right)^2 + R_1^2 R_2 + R_1^2 R_3 + R_2\left(\frac{1}{\omega C_1}\right)^2 + R_3\left(\frac{1}{\omega C_1}\right)^2}{\frac{1}{\omega C_1}(R_2+R_3)^2 + \frac{1}{\omega C_1}\left(\frac{1}{\omega C_2}+\frac{1}{\omega C_3}\right)^2 + R_1^2\frac{1}{\omega C_2} + R_1^2\frac{1}{\omega C_3} + \left(\frac{1}{\omega C_1}\right)^2\left(\frac{1}{\omega C_2}+\frac{1}{\omega C_3}\right)}$$

If we now let K = 6 for porcelain

k = 1 for air

k = 2.5 for oil

P.F. = 0.020 for porcelain

= 0.000 for air

= 0.0025 for oil

then $1/\omega C_1 = 50\ R_1$ $\quad 1/\omega C_2 = 150\ R_1$

$1/\omega C_3 = 100\ R_1$ $\quad R_2 = 3\ R_1$ $\quad R_3 = 0$

for the condition with air above the mercury.

Substituting these values we get P.F. = 0.01865

For the condition with oil above the mercury we must use these new values:-

$1/\omega C_3 = 40\ R_1$ and $R_3 = 0.1\ R_1$

When we substitute these values we get:-

P.F. = 0.01925.