STRESS ANALYSIS IN A GRAVITY DAM THROUGH THE USE OF A MODEL.

A report upon experiments performed

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CONTENTS

A gravity dam is one which depends for its support upon its own weight, such that each section of the dam is supported entirely by the ground directly beneath it rather than by a foundation to the side, as is true in the case of the various types of arch and dome dams. The assumption is made in the knowledge that it is probably untrue in view of the fact that there will be friction between two adjacent parts of any structure, so that some resisting force will be set up to add to the stability of the individual section. Inasmuch, however, as this assumptive error is on the side of safety, one may proceed without danger on the theory that there is no lateral support.

For purposes of analysis, gravity dams are customarily divided into sections perpendicular to the horizontal cross-stream axis, it being assumed, as stated above, that each section is self-supporting and independent of any and all others. For analytical purposes the width of such a section is taken as unity, and its volume is therefore represented by the area of the cross section itself, and inasmuch as there are assumed to be no forces acting upon the section at the faces where it joins the adjacent section, either longitudinally or in shear any and all forceswhich act upon the unit section may be represented as acting upon its outer edges.

It may be shown that one can represent stresses in a large scale prototype of any structure by means of a model on a smaller scale, due to the fact that the deflection and loads upon substances made of different materials are proportional moduli of elasticity of the model and prototype. Therefore, if the modulus is constant for two given materials - that is, if it does not vary from point to point, and under different conditions of loading, results may be obtained using one as model, the other as prototype. This method of analysis is becoming more and more popular, both as an easy and rapid way of checking analytical calculations and as a means of original stress determination without mathematical analysis.

An example of this may be illustrated by the method of determining reactions of a continuous beam over a number of supports under a given loading, taking advantage of the fact, discussed later in this paper, that deflections at one point are to deflections at another inversely as the loads at the two points. A model beam is set up on a board on which the supports are represented by nails driven into a sheet of graph paper from which distances may be readily scaled. By deflecting the beam at any one reaction a unit amount, and measuring the ordinates at any given point of loading, in the direction of the load, the proportion of the load carried by that individual support is determined directly. This process is carried out for each support, accomplishing in half an hour what would be a normal day's work in analytical calculation. The percentage of error can be shown to be about two.

In the experiments carried on by the writer, a celluloid model of a triangular cross-section of a gravity dam having a base-to-height ratio of 0.70 was used, and known loads were applied to it at certain points in order to find the deflections at various other points under varying conditions of loading with an object of determining the internal work which would be performed by the dam.

From a study of the dam section, it is seen that the forces acting upon it, neglecting (as mentioned before) longitudinal and shear stresses transmitted from adjoining sections, are three in number, (a) the horizontal force due to the pressure of thewater behind the dam, (b) the gravitational force acting upon the section in a downward direction, and (c) the reaction of the foundation acting so as to oppose the other two, and hence upward and upstream. Inasmuch as the first two are determined easily, the weight of the section and the pressure of the water being known, it follows that the solution of the problem concerns chiefly the resultant forces opposing motion of the dam. These forces may be divided into components acting upward normal to the under surface of the dam, and those acting horizontally, causing shearing stress between the dam and its foundation.

The summation of the shearing stresses at the bottom of the dam equalsthe summation of the water pressures, or $\frac{wh^2}{2}$, in which h is the depth of water at the upstream faceoof the dam and w is the unit weight of water, that is, the weight of a unit cube, inasmuch as these two opposing forces are the only ones existing which have a horizontal component. It is the variation of this shearing stress along the base which is one of the ultimate purposes to which these experiments are preliminary.

Considering the differential cube at the base of the upstream face, it is seen that, inasmuch as there can be no appreciable shear as between water and a solid, the shear along the upstream vertical face is necessarily zero. Inasmuch as the shearing stresses on two adjacent faces of a cube must be equal in magnitude, there is no shear at the base of this small cube; in other words, the shearing stress at the upstream face of the dam is zero. It may then be considered as varying from zero up to a maximum value at the toe, along some curve whose shape is unknown.

Considering vertical reactions, if moments be taken at the base directly beneath the resultant of the weight, at which point the weight and the base shear have no moment, it becomes apparent that the resisting ground pressure has a resultant on the downstream side when the reservoir is full, which is the condition assumed. The ground pressures are therefore greatest near the toe of the dam. There can, of course, be no tension between the dam and its foundation, as this would tend to form cracks which would permit a water pressure under the dam tending to overturn it, but for economic reasons, the best dam is one in which the bearing stress at the upstream face approaches zero. For this reason, the bearing stress may also be assumed to follow some curve of unknown shape, from zero at the upstream face to a maximum value at the toe.

The ultimate purpose of these experiments, but one which is beyond the scope of this paper, is to find the curves which these two stresses follow. The stress is temporarily assumed to follow a straight line whose end points are determined such that the area under the line represents the force which is resisted. To this straight line is to be fitted a parabola such that an equal area will be above and below the line, and hence the area under the curve as fitted will be the same as that under the line. The parabola will have the form

$$y = \mu(x^2 - a^2)$$

in which y and x are the ordinate and abscissa, respectively, along the force line assumed, and may then be referred to the same co-ordinates as is the equation of that line.

In the analysis it is necessary to consider the principle of least work, which may be used frequently in the solution of problems that may not be solved solely by means of the three fundamental conditions of static equilibrium, as in the simple case of a table or chair with four legs, which having one more unknown than there are equations, is therefore redundant and must be solved by some means involving indeterminate structures. The principle of least work states that the stresses in a redundant structure have values such that the internal energy of all the stresses is at a minimum. The proof of this is given by Merriman as follows:

Let $P_1, P_2, P_3 \cdots\cdots P_n$ be n forces in equilibrium. Then let a small internal displacement be made without performing work on the system as a whole such that the point of application of each of the forces move through small distances δe_1 δe_2 $\delta e_3 \cdots\cdots \delta e_n$. As no outside work was done the total work is equal to zero, hence

$$P_1 \delta e_1 + P_2 \delta e_2 + P_3 \delta e_3 \ldots\ldots + P_n \delta e_n = 0$$

A law of mechanics states that for cases of stable equilibrium the integral of this expression is at a minimum, so that $P_1 e_1 + P_2 e_2 + P_3 e_3 + \ldots\ldots P_n e_n$ = a minimum in which $e_1 \; e_2 \; e_3 \ldots\ldots e_n$ are the total distances through which the various forces have moved in reaching a state of equilibrium. The work done by each of the forces P in increasing from O to P is therefore $\frac{Pe}{2}$, and equals the energy stored by the internal stresses which balance the forces. Therefore the internal energy of the entire system is at a minimum.

It may be shown that the internal energy which is stored in a system is exactly equal to the external work to which the system is subjected. By the use of this fact, it is possible to determine the internal energy of the system through an analysis of the external work done upon it. In the case of the dam, this work consists of half the summation of the loadings times the distance through which each acts.

Inasmuch as it is impossible to determine the movement of each point when the entire section is under stress, an approximation must be made, even using the model. In this case, points were selected along the two faces of the dam upon which stresses act, for determination of the movement of each of the points under various conditions of loading. The deflections under different loads are additive; that is, if the model has a deflection a at a given point under a load at one point, and a deflection b under a loading at another, it will have a deflection a + b under the combined loadings at the same time, and this may obviously be extended to include loading at all points.

The law of reciprocal deflection, which was cited briefly in an example given, states that if a given amount of work is done at one place, the summation of the work done at a number of other places is the same if the body remains in equilibrium. Hence if a weight is applied at one point, causing a certain deflection, if there are a number of other forces acting upon the body the applied work will be equalled by the work done on the other forces. This fact may also be reversed, stating that if a number of different forces do work on a body, work may be done on an applied force at another point.

The application of this experimental data is an extension of this latter principle. A load is actually applied at a number of given points, noting in each case the deflection at other points, as well as the actual work performed by measuring the deflection under the applied load.

The reversal of this process is carried out in the analysis. By studies involving the resultants of the water pressure and of the gravitational action at any of the other points, it is possible to arrive at the work which would actually be done upon the model, using the deflection obtained for the horizontal and vertical loadings at the base. A simple proportion then exists between the work actually done upon the model, and the theoretical work which would be done upon it under full load, as by summing the amounts of the theoretical work performed a result is obtained which is to the actual work done as the reaction to the theoretical applied loads is to the load actually applied to the model.

In this way, it is possible to obtain the resultant thrusts and shears at each of the control points for any given dam. By plotting these, a curve may be drawn through them, and then related, as previously suggested, to a parabola diverging from a straight line.

Experimental Work.

The model set up was a celluloid triangle 15 inches long, $10\frac{1}{2}$ inches along the base, and about 0.075 inches in thickness. This represented a cross-sectional area through the dam , the vertical face being considered as the upstream side. The base, and the vertical edge were each divided into ten parts, the nine divisions being marked by lines ruled on the face of the triangle.

Small bridges, a part of the Beggs deformeter apparatus, were used to support the pins which held the two corners of the base. The pin holes were drilled exactly at the intersection of the edges prolonged, at the centers of small circles of 1/5" radius. Further supports for the rest of the triangle were provided in the form of small glass blocks which prevented it from excessive sagging.

A Bausch and Lomb microscope, no. 1412, was used to measure the deflections. A small dot of ink was placed on the triangle adjacent to the point whose deflection was desired, the cross-hair being brought tangent to this dot of ink for each reading.

Five one-pound weights, wired together and connected by means of a flexible trout cord to a wire stirrup which passed around the triangle ~~and~~ could be applied at any of the nine points on the base of the triangle, at each of which there was a notch to hold it in place. The line passed from the triangle over a moveable glass rod at the edge of the table, so that the weights were free to hang vertically. A drawing stool was used to rest the weights upon during 'no load' conditions.

In operation, the microscope was kept at one place throughout each series, the weight being moved from point to point. This was obviously because of the greater ease in moving the weights, although perhaps for experimental purposes the reverse would have been slightly superior because of the advisability of obtaining all results at any one point under approximately the same conditions. The procedure in taking any set of readings was as follows:

1) With no load, the cross-hair was brought tangent to the point.

2) The load was applied by releasing the weights.

3) The reading of the scale was taken and recorded to tenths of the least reading (To nearest ten-thousandth of one inch.

4) The cross-hair was set on the new position.

5) The load was removed, by placing the weights upon a

rest so that there was no tension in the cord.

6) The scale was again read and recorded for load conditions.

7) Same as (1) etc., etc.

This procedure continued until five readings had been taken under load and under no load conditions. The results were inspected, and if no wide discrepancy appeared in the readings, an average was taken for each condition. These averages were then compared to determine the deflection to be recorded.

Deflections were first obtained for the normal reactions, measurements being taken for points along the base. Due to the impossibility of making direct readings to give the shear on the base, it was found necessary to apply the loads at an angle and then to correct for the discrepancy, finding the horizontal component of the angle. Loads were applied at 45°, deflections being taken along the base both horizontally and vertically, and horizontally along the water face, for all positions of the loads.

An inspection of the set up will show that multiplication of the deflections by $\sqrt{2}$ gives the same result as would a corresponding increase of the weight, which, in turn, is equivalent to two forces of the original weight acting, one tangentially, the other normal. By subtracting the deflections already found due to the normal load, it is theoretically possible to obtain the effect of the pure tangential load. However, it was found that due to absolute bending of the model due to the applied load, results obtained in this way as due to the pure tangential load were so much in error as compared with the probable true deflection that this method was abandoned, and the horizontal components used along, without correction.

The data recorded includes a tabulation of the deflections at each of the control points, in ten-thousandths of an inch, resulting in a five pound load applied horizontally and vertically at each of the nine points along the base. This is now in a condition to be used in further calculations for any particular dam, according to the methods as outlined before.

It is felt that the results obtained would have been much better had it been possible to obtain them from a set up which did not involve bending in the triangle to so great an extent as did the conditions used, in which it acted largely as a beam supported by the two pins. Some bending is inevitable, and indeed the success of the problem depends upon it, but the writer feels that it was largely due to this fact that it was impossible to obtain more perfect figures for the tangentially loaded deflections, as in a beam of varying moments of inertia, the direction of application of the load becomes more involved than was assumed to be the case.

In conclusion the writer wishes to express his appreciation for the help of the staff of the California Institute, and particularly that of Prof. Eugene Kalman, under whose direction the entire series of experiments were carried on.

DEFLECTIONS DUE TO NORMAL LOAD (5#)

LOAD AT	DEFLECTION AT	DEFLECTION ($\frac{1}{10000}$ of 1 inch
1	1	59
	2	48
	3	57
	4	41
	5	43
	6	40
	7	41
	8	33
	9	25
2	1	44
	2	58
	3	56
	4	72
	5	58
	6	60
	7	60
	8	40
	9	43
3	1	52
	2	51
	3	84
	4	74
	5	70
	6	81
	7	76
	8	66
	9	61

DEFLECTIONS DUE TO NORMAL LOAD (Cont.)

LOAD AT	DEFLECTION AT	DEFLECTION ($\frac{1}{10000}$ of inch)
4	1	43
	2	54
	3	77
	4	95
	5	81
	6	81
	7	81
	8	73
	9	88
5	1	46
	2	57
	3	72
	4	95
	5	111
	6	89
	7	94
	8	92
	9	101
6	1	41
	2	64
	3	78
	4	94
	5	97
	6	108
	7	123
	8	91
	9	97

DEFLECTIONS DUE TO NORMAL LOAD (Cont.)

LOAD AT	DEFLECTION AT	DEFLECTION ($\frac{1}{10000}$ inch)
7	1	33
	2	47
	3	81
	4	98
	5	93
	6	104
	7	117
	8	103
	9	115
8	1	31
	2	48
	3	71
	4	87
	5	95
	6	110
	7	111
	8	115
	9	122
9	1	20
	2	39
	3	56
	4	77
	5	74
	6	93
	7	95
	8	114
	9	136

DEFLECTIONS DUE TO TANGENTIAL LOAD

LOAD AT		NORMAL DEFLECTION At Slant	Corr for Wt	Corr. for Norm.	TANGENTIAL DEFLECTION Slant	Tangent.		DEFL. VERT. FACE At Slant	Tang.
1	1	40	57	-2	42	59	a	19	27
	2	27	38	-10	36	57	b	11	16
	3	17	24	-27	25	35	c	1	1
	4	15	21	-20	34	48	d	-9	-13
	5	7	10	-33	31	44	e	-24	-34
	6	6	8	-32	25	35	f	-35	-50
	7	6	8	-33	23	33	g	0	0
	8	3	4	-27	23	33	h	-30	-42
	9	0	0	-25	11	16	i	-41	-58
2	1	30	42	-2	26	37	a	40	+57
	2	38	54	-4	32	45	b	35	+50
	3	29	41	-15	29	41	c	5	+7
	4	18	25	-47	30	42	d	-3	-3
	5	13	18	-40	27	38	e	-10	-14
	6	11	16	-44	22	31	f	-21	-30
	7	24	34	-26	21	30	g	5	7
	8	12	17	-23	19	27	h	-23	-33
	9	5	7	-36	12	17	i	-34	-48
3	1	29	41	-9	22	31	a	32	
	2	32	45	-6	25	35	b	40	
	3	36	51	-33	27	38	c	20	
	4	22	31	-43	28	40	d	8	
	5	17	24	-46	25	35	e	-2	
	6	15	21	-60	23	33	f	-13	
	7	31	44	-32	19	27	g	+6	
	8	30	42	-24	19	27	h	-15	
	9	6	8	-53	12	17	i	-25	

DEFLECTIONS DUE TO TANGENTIAL LOAD (con't)

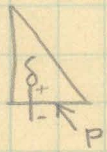
LOAD AT	NORMAL DEFLECTION At	Slant	Normal	Corr. for Norm	TANGENTIAL DEFLECTION Slant	Tangent	DEFL. VERT. FACE At	Defl.	Tang
4	1	28	40	-3	17	24	a	28	40
	2	31	44	-10	23	33	b	41	58
	3	31	44	-33	19	27	c	20	28
	4	28	40	-55	25	35	d	12	17
	5	25	35	-46	25	35	e	8	11
	6	23	33	-48	20	28	f	6	8
	7	43	61	-20	20	28	g	11	16
	8	40	57	-16	20	28	h	-1	-1
	9	20	28	-60	13	18	i	-10	-14
5	1	21	30	-16	13	18	a	13	18
	2	26	37	-14	20	28	b	37	52
	3	30	42	-30	15	21	c	19	27
	4	25	35	-60	22	31	d	20	28
	5	40	57	-54	25	35	e	15	21
	6	30	42	-47	16	23	f	10	14
	7	45	64	-30	16	23	g	6	8
	8	50	71	-21	18	25	h	+9	13
	9	26	37	-75	11	16	i	8	11
6	1	19	27	-14	10	14	a	5	7
	2	22	31	-33	13	18	b	30	42
	3	29	41	-37	11	16	c	18	25
	4	26	37	-57	18	25	d	20	28
	5	38	54	-43	18	25	e	23	33
	6	46	65	-43	18	25	f	26	37
	7	61	86	-37	13	18	g	0	0
	8	67	95	+4	15	21	h	21	30
	9	39	55	-42	11	16	i	20	28

DEFLECTIONS DUE TO TANGENTIAL LOAD (Cont.)

LOAD AT	NORMAL DEFLECTION				TANGENTIAL DEFLECTION		DEFL. VERT FACE		
	At	Slant	Norm.	Corr. for Norm.	Slant	Tangent	At.	Defl.	Tang.
7	1	14	20	−13	5	7	a	5	7
	2	19	27	−20	9	13	b	26	37
	3	25	35	−46	5	7	c	23	33
	4	25	35	−63	13	18	d	20	28
	5	41	58	−35	15	21	e	29	41
	6	41	58	−46	12	17	f	27	38
	7	83	117	0	14	20	g	2	3
	8	84	119	+16	15	21	h	28	40
	9	47	67	−48	9	13	i	35	50
8	1	9	13	−18	3	4	a	7	10
	2	15	21	−27	3	4	b	22	31
	3	21	30	41	3	4	c	22	31
	4	25	35	52	9	13	d	24	34
	5	40	57	38	6	8	e	33	47
	6	38	54	56	13	18	f	42	59
	7	92	130	+19	8	11	g	+4	6
	8	98	138	+23	10	14	h	38	54
	9	63	89	−33	9	13	i	48	68
9	1	8	11	−9	0	0	a	6	8
	2	9	13	−26	0	0	b	23	33
	3	19	27	−23	0	0	c	20	28
	4	24	34	−43	2	3	d	28	40
	5	40	57	−17	0	0	e	41	58
	6	39	55	−38	8	11	f	47	66
	7	86	121	+26	6	8	g	−4	−6
	8	99	140	+26	2	3	h	41	58
	9	82	116	−20	9	13	i	39	55

SAMPLE TRANSCRIPTION OF ORIGINAL NOTES

NORMAL DEFLECTION OF BASE, LOAD AT 45°



	$W_1 I_1$		$W_2 I_1$		$W_3 I_1$		$W_4 I_1$		$W_5 I_1$	
	No Ld	Ld.	No Ld	Ld.	No Ld	Ld	No Ld	Ld	No Ld	Ld
	233	270	228	252	231	268	217	254	163	189
	229	270	230	264	230	266	224	247	158	181
	229	269	228	258	227	250	221	250	162	178
	229	269	229	261	230	258	217	244	154	179
	231	271	227	256	231	254	220	250	156	178
Av.	230	270	228+	258	230	259	221-	249	159-	181
Net Defl.	+40		+30		+29		+28		22	

	$W_6 I_1$		$W_7 I_1$		$W_8 I_1$		$W_9 I_1$	
	No Ld	Ld	No Ld	Ld	No Ld	Ld	No Ld	Ld
	157	170	144	156	144	154	139	150
	152	179	142	157	138	148	141	145
	156	176	141	155	141	149	138	148
	158	173	145	157	142	152	138	147
	154	175	143	159	140	149	140	146
Av.	155+	175-	143	157	141	150+	139	147
Net Defl.	+19		+14		+9		+8	

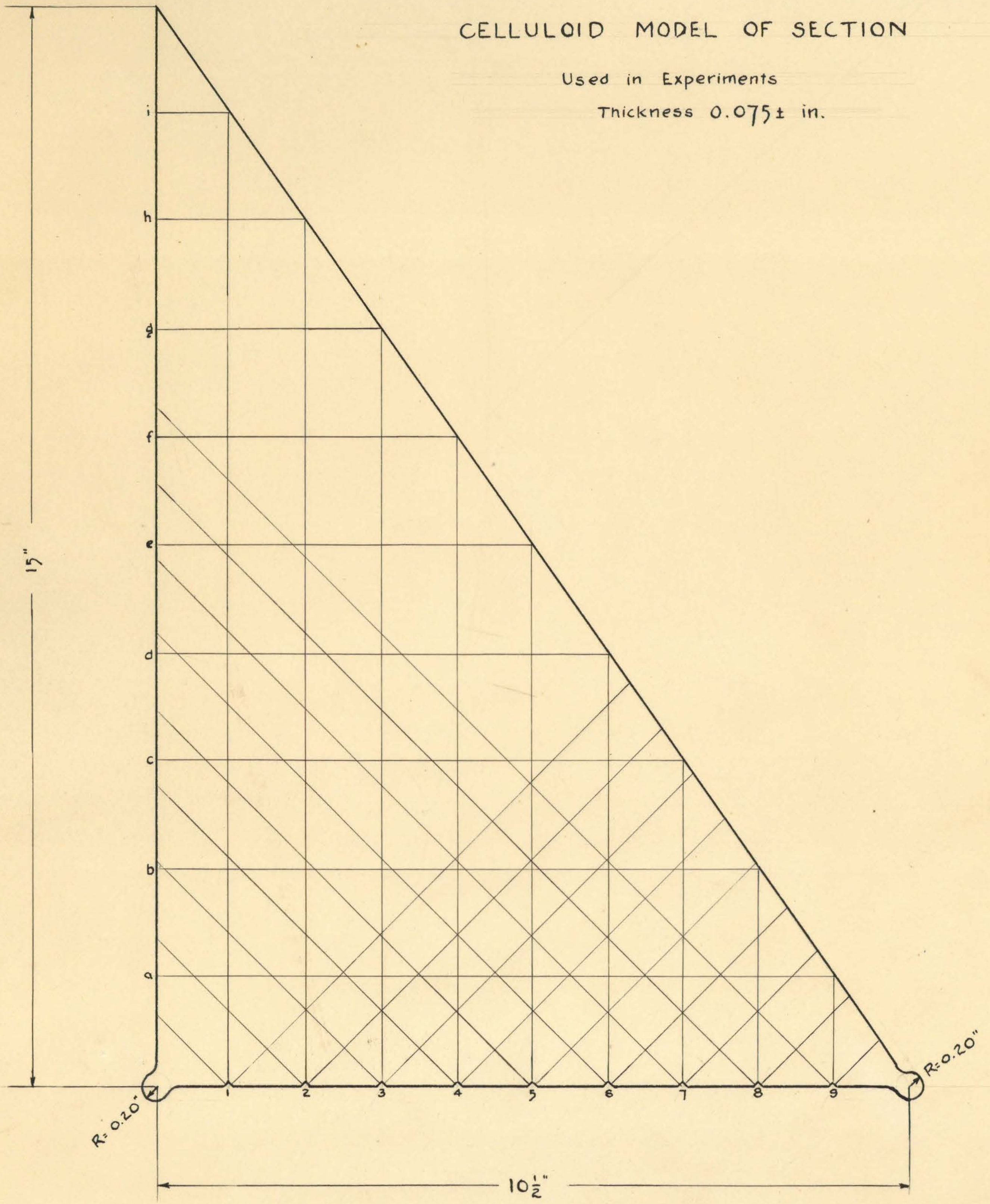
	$W_1 I_2$		$W_2 I_2$		$W_3 I_2$		$W_4 I_2$		$W_5 I_2$	
	No Ld	Ld	No Ld	Ld	No Ld	Ld	No Ld	Ld	No Ld	Ld
	342	371	356	391	359	386	357	396	353	385
	347	372	356	390	357	390	363	390	362	382
	347	376	353	389	360	391	363	392	361	384
	348	370	352	392	359	392	363	391	360	387
	346	375	359	401	353	391	361	393	360	386
Av.	346	373	355-	393	358	390	361+	392+	359	385
Net Defl	+27		+38		+32		+31		+26	

24 pages similar to this

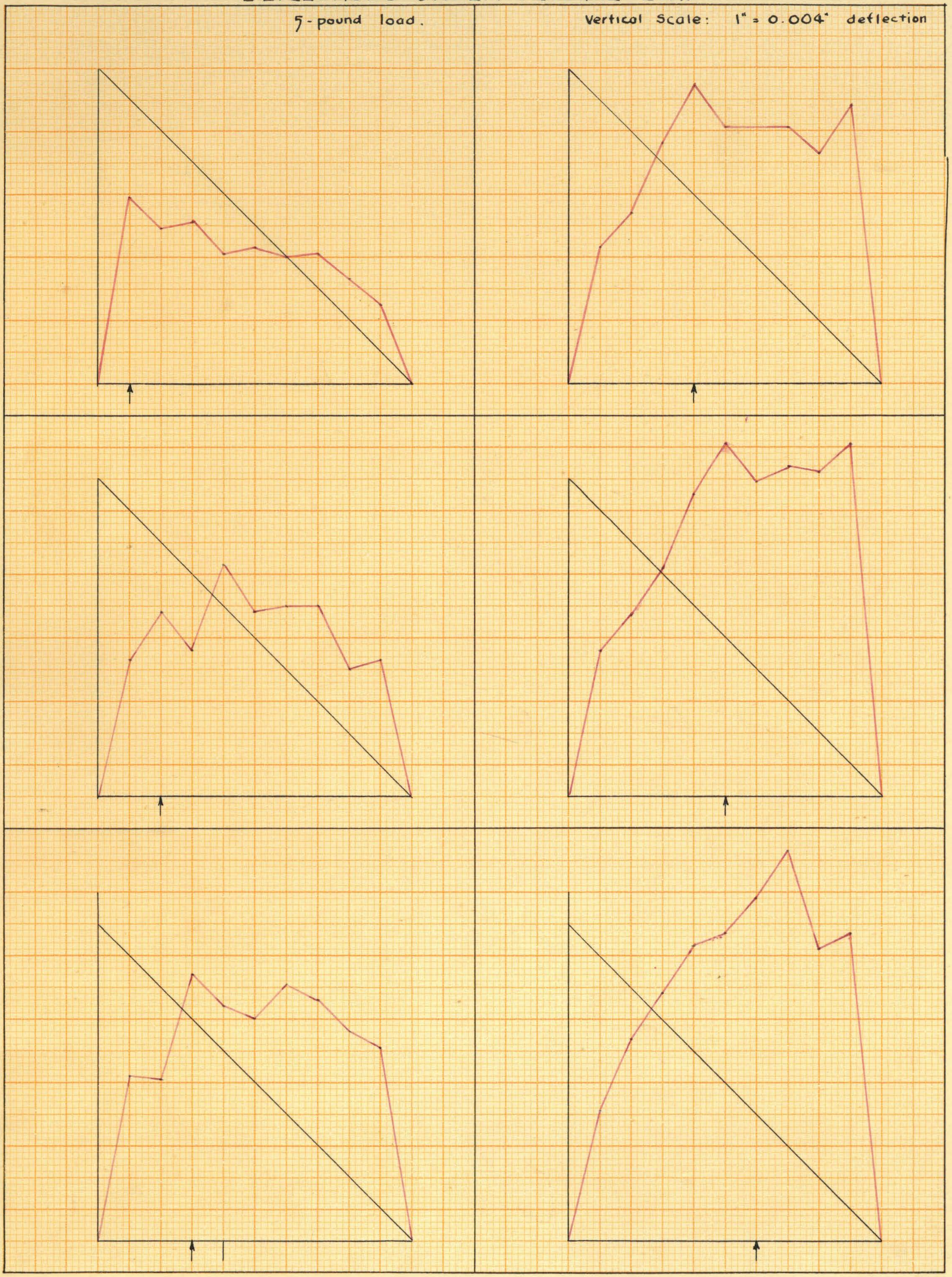
CELLULOID MODEL OF SECTION

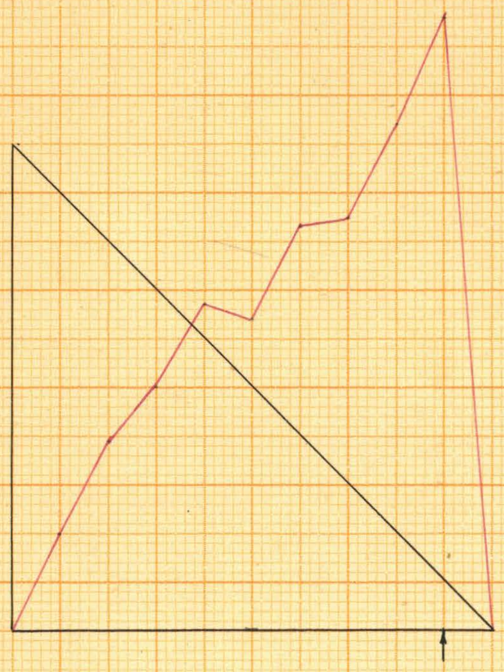
Used in Experiments

Thickness 0.075± in.

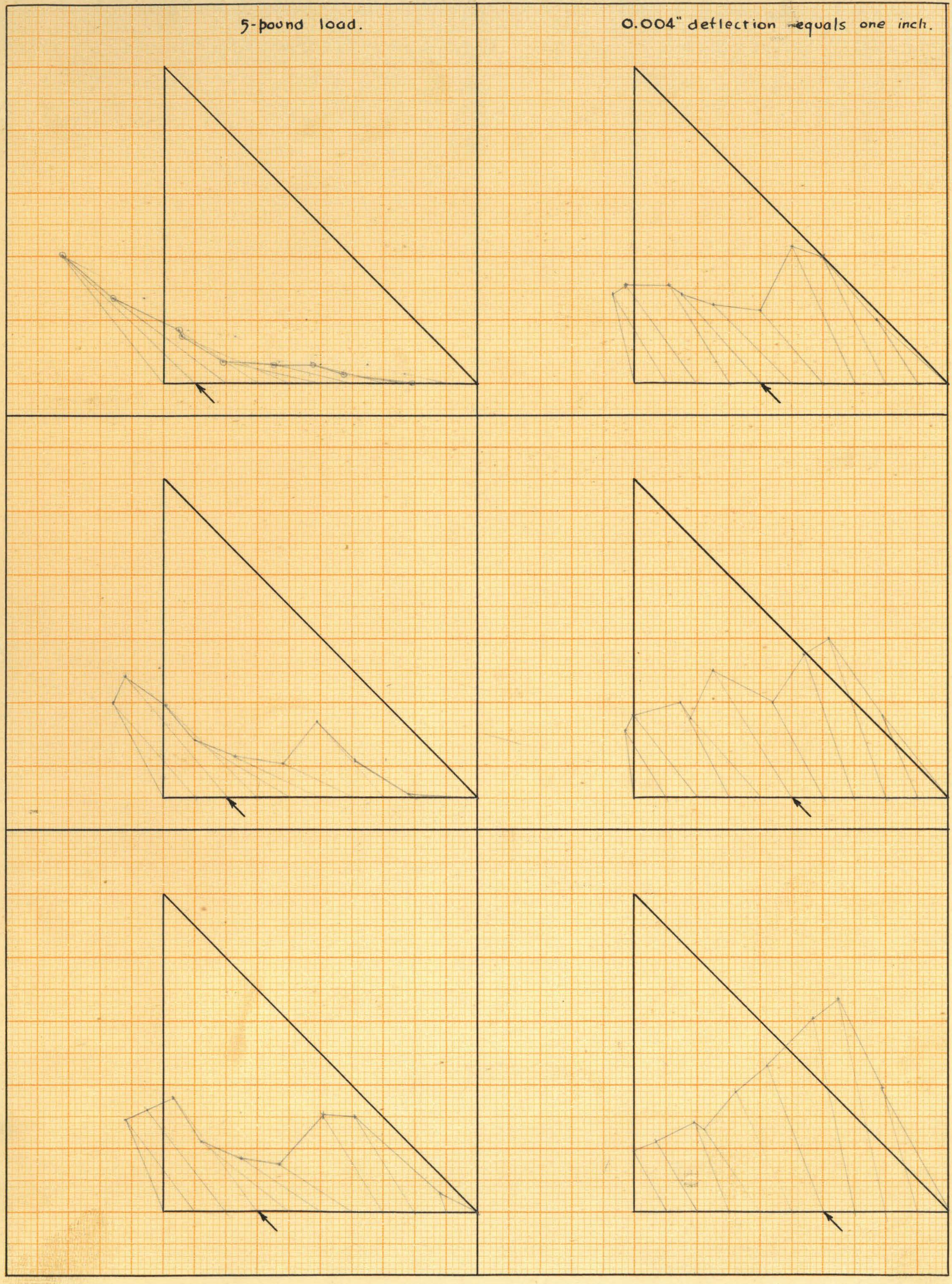


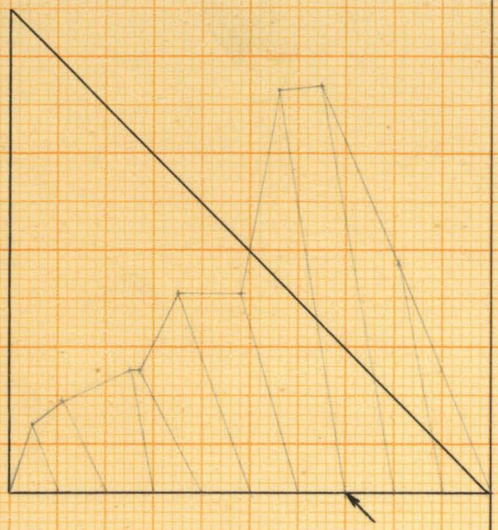
DEFLECTIONS UNDER NORMAL LOAD

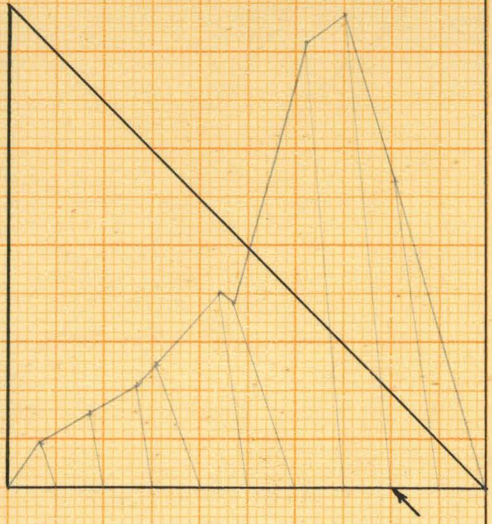


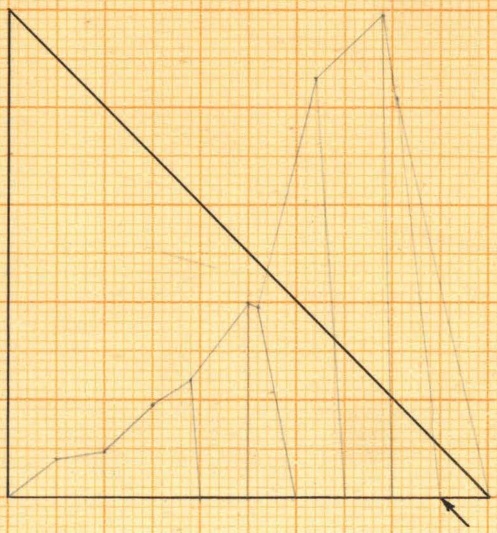


BASE DEFLECTIONS UNDER INCLINED LOAD









DEFLECTIONS OF UPSTREAM FACE UNDER INCLINED LOAD