STUDY OF

THE BREADTH AND STRUCTURE OF THE COMPTON MODIFIED LINE

WITH THE

MULTI-CRYSTAL SPECTROGRAPH

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Thesis by

HARRY ALLISTER KIRKPATRICK

In Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy

California Institute of Technology

Pasadena, California

1931

OUTLINE

I. <u>History of the Compton Effect</u>

    a. Discovery of scattered radiation   (1895)

    b. Softening, and attempts to explain it
        1. Work of Sagnac, Villard, Barkla, Sadler, Mesham
        2. Thomson-Stokes pulse theory
        3. Laue's discovery, Bragg's crystal spectrometer
        4. "J"-radiation
        5. Later work by Barkla, Gray, Plimpton, Compton

    c. Compton-Debye quantum theory of scattering (1923)
        1. Compton's experimental work
        2. Opposition from Duane, Clark et al
        3. Confirmation of Compton's predicted shift
        4. Intensity relations, primary and scattered rays
        5. Intensity relations, modified and unmodified

    d. Work in this laboratory on Compton Effect by Dumond
                              (1925-28)

II. <u>Multi-crystal Spectrograph</u>

    a. Design of instrument
    b. Construction details
    c. Testing crystals for planeness
    d. Orientation of crystals
    e. Non-selective scattering at the crystals and wedges

III. <u>Theory Relating Breadth of Line to Electron Momenta</u>

    a. Introductory outline
    b. Analytical solution of the line breadth

IV. <u>Experimental Work</u>

    a. Geometrical dispositions of tube and scatterer
       and exposure times
    b. Results
    c. Discussion of results
    d. Absolute velocities of electrons

SUMMARY

Purpose

Dumond has treated theoretically the breadth of the Compton modified line in x-ray scattering as a function of the initial velocities, or momenta, of the electrons in the scattering body, and has shown that the modified line given by a class of electrons whose velocities range between βc and $(\beta + d\beta)c$ should consist of a uniform distribution of intensity over a wave-length range approximately equal to $4\beta\lambda_1\sin\frac{1}{2}\Theta$, centered approximately at the position of simple Compton scattering by stationary electrons.

The purpose of this research has been to test this prediction, particularly with respect to Θ, the scattering angle. (λ_1 is the primary wave-length.)

Experimental Method

In previous work on scattered x-rays homogeneity of scattering angle has been greatly sacrificed to obtain intensity. Since the position of the Compton line depends upon the scattering angle, homogeneity is important to avoid spurious breadth of the spectral line.

A spectrograph has been constructed which consists of fifty small calcite Seeman spectrographs so oriented as to focus the spectra from all of them at the same position on a photographic film. The geometry of the instrument and of the arrangement of the scatterer and x-ray tube furnishing the primary radiation is such that an extremely homogeneous angle of scattering is secured over the whole of an extended scattering body.

Results

Very good spectrograms of molybdenum K-radiation scattered by graphite at angles of 63 °, 90°, 156° have been secured.

The excess breadth of the modified line over that of the unmodified has been shown to be real.

The relative breadths of the modified lines at half their maximum height were determined from microphotometer curves made from each of the three negatives, and proved to be in very close agreement with the ratios of the sines of half the scattering angles as predicted.

The possibility that our results might have been affected by multiple scattering was tested by the use of a specially constructed scatterer, and no observable difference was found.

Scattering of silver K-radiation at 156° gave a narrower line than molybdenum, in at least qualitative agreement with the theoretical dependence of breadth on primary wave-length.

The theory seems to be completely supported by the experimental results thus far obtained.

I. HISTORY OF THE COMPTON EFFECT

It is the purpose of this thesis to present a brief history of the study of the Compton shifted or modified line in the x-ray spectrum, and to describe in detail the work done on it here at the Norman Bridge Laboratory of physics by the author in collaboration with Dr. Jesse W. M. Dumond who preceded him in the work and who has continued to be the guiding spirit in this research.

a. Discovery of Scattered X-radiation

The existence of secondary x-radiation from matter irradiated by primary x-rays was observed almost immediately after Röntgen's momentous discovery in 1895 of the primary x-rays themselves. Optical spectroscopic methods were not immediately adaptable to the new radiation and the first investigations of its character were necessarily made by absorption measurements. It was soon commonly believed that primary x-rays were electromagnetic radiation identical with light except for shorter wave-length, and this belief was strengthened by subsequent investigations culminating in 1912 with the discovery by Laue[1]* and Friedrich and Knipping[2] of interference in their reflection from crystal planes.

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* See last page for all references.

Among the first questions to be answered in regard to the secondary radiation were these: (1) Is it of the same electromagnetic nature as the primary radiation, or is it corpuscular, or both? (2) What rules govern its intensity relative to that of the primary radiation? (3) How does it compare in "hardness" or penetrability with the primary radiation? The first question did not long remain unanswered. All experiments led to the conclusion that this secondary radiation was indeed x-radiation. The second question was answered, and quite satisfactorily so at the time, by the electromagnetic pulse theory proposed by Stokes[3] and J. J. Thomson[4] which, although now no longer tenable, was the first satisfactory hypothesis regarding the nature of x-rays. It was so satisfactory, in fact, that it was abandoned only with great reluctance upon the discovery of interference phenomena* and the necessary advent of the quantum radiation theory.

In view of the very definite answer to the first of the above questions, the third resolved itself into the question as to whether or not the scattered radiation had

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* The pulse theory was retained at first in spite of interference phenomena, the supposition being that the wave-train itself was a product of the crystal reflection, and not the original form of the radiation. See discussion by J. A. Gray, Jour. Fr. Inst., 190, 633 (1920) and A. H. Compton, Nature 108, (1921).

suffered a change in wavelength, and it is this whole question of change of wave-length during scattering which has been the subject of this research.

b. Softening of Scattered Radiation, and Attempts to Explain It.

Sagnac, who probably did more than anyone else toward establishing the existence of secondary x-rays, decided that scattered x-rays were softer (less penetrating) than their primary rays. He studied the secondary radiation from metals and also discovered that there was secondary radiation given off by air. His reports are largely qualitative but he was led to positive conslusions, as may be seen from the following quotation;* "But one cannot say that these S-rays (scattered rays) emitted by M (the scattering material) are simply chosen from the incident bundle as though it were simple selective diffusion; they are transformations of the x-rays. If M is a material such as zinc or lead the very slightly penetrating S-rays which it emits do not appear to exist in notable quantities in the radiation from any of the vacuum tubes actually employed."[5] He also reported that the secondary rays from air were less penetrating than the primary x-rays[6] and was confirmed in this opinion by Villard[7].

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* The translation and explanatory parentheses are mine.

Later experimenters however were not so certain
that this was actually true. Barkla, who was responsible
for much of the early work on the subject, at first con-
sidered that the difference in penetration lay within the
experimental error and says; "The absorption-coefficients
for the primary and secondary radiations cannot differ by
more than ten percent of their value, for the radiations
experimented on. I conclude therefore that the penetra-
bility of the primary and secondary rays is practically
the same."[8] This was indeed to be expected according to
the J. J. Thomson theory which regarded scattering as the
result of the accelerations of electrons traversed by the
primary electromagnetic pulse. If the electrons were free
to respond to this changing electric field, the energy
radiated in all directions as a result of their accelera-
tions should consist of pulses identical with those of the
primary, and while the intensity would naturally be dimin-
ished, the penetrability should be the same. About a
year later however Barkla was forced to agree with Sagnac
that the secondary radiation from metals was actually
softer than the primary[9] and set himself the task of de-
termining the reason. In 1908 after an extended study of
secondary radiation from numerous elements, Barkla and
Sadler[10] published the important conclusion that second-
ary radiation was of two types. The first type, they still

maintained, was in accord with the Thomson theory and was unchanged by the scattering process. It was found to predominate in, or comprise the whole of the secondary radiation from the lighter elements, and formed a much smaller percent of that from the heavier. They ascribed this radiation to electrons controlled by the electric forces in the primary pulse, and stated that the experimental evidence in support of Thomson's theory was "overwhelming" in this type. The second type was an extremely homogeneous radiation characteristic of the scattering element and independent of the primary radiation except that it did not appear unless the primary radiation included radiation at least as penetrating. They concluded that this must also be x-radiation since it was too penetrating to be of corpuscular nature, was not deflected in electric or magnetic fields, had the same absorption coefficients as similar primary radiation, and was scattered by air in the same proportion. They rightly attributed its origin to forces in the atom itself, brought into play but not controlled by the electric field of the primary radiation.

Their explanation assumed that the electron had a natural period of vibration in the atom. Consequently if it were traversed by a thin pulse whose time of passing was long in comparison with half the natural period of the electron, it would be displaced but the restoring

forces would be gone before it could respond to them and it would be left to vibrate with its own frequency. If a thick pulse passed over it, it would be more nearly, or completely, restored to its equilibrium position and left with less, or no, energy to radiate. They pointed out that if this were true a wave train might be expected instead of a pulse, but thought that probably the energy of vibration was transmitted from electron to electron in the atom and that the result was more like a number of isolated pulses. This accounted satisfactorily for the requirement of radiation at least as penetrating in the primary. If the pulses were all of long duration, the electrons were gently restored to their original positions and there was no radiation of this type as a result. When it did occur its intensity was a considerable part of the total intensity and its presence accounted for the observed increase in absorption of the secondary over that of the primary radiation which necessarily contained still harder components. They also considered the possibility that the process might involve ionization, but thought that it was highly improbable, partly because they did not get as great an increase with increasing primary intensity   in this form of radiation as they did in the ionization of air, and partly because ionization occurred in the lighter elements without the appearance of characteristic x-radiation.

Further investigations using the lighter elements
as scatterers showed however that there was still something
lacking in the theory. The secondary radiation was defi-
nitely softer even when the characteristic radiation was
not present. Sadler and Mesham[11] used scatterers of high
atomic weights to secure homogeneous rays, and studied the
tertiary scattering from carbon. They found that it was
neither as hard nor as homogeneous as the radiation pro-
ducing it, but they inclined "more to the belief that an
actual modification of the x-ray occurs in its passage
through matter, a general softening taking place * * *"
rather than that the modification occurred in the scatter-
ing process itself.

The introduction of the crystal spectrograph by
W. H. Bragg in 1912 following the Laue-Friedrich and
Knipping discovery gave a new impetus to the study of
primary x-rays, but the limitation of total intensity
imposed by the necessary system of slits at first pre-
vented it from being successfully used for the study of
secondary radiation, in which the intensity is extremely
small. As a result, the whole of that subject remained
in a rather unsatisfactory and uncertain state for another
decade. It was thought by some[12] that the difference in
penetration in the case of lighter elements was due to the

existence of a characteristic "J" radiation, shorter in wave-length than the K-series, but this idea was gradually abandoned as it was never detected directly, did not agree with the accepted Bohr atom model, and no critical absorption limits were ever found for it.

Another explanation of the softening effect of scattering was that there might be a greater scattering of the softer components than of the hard, but this also had to be given up when experimental evidence came to light showing the reverse to be true.

In 1920 Gray[13] showed that an electromagnetic pulse would be lengthened if it were scattered by an electron or atom whose dimensions were comparable with the thickness of the pulse. If it were a train of waves this would not be true. At this time it was thought that the primary radiation came in pulses and that the periodicity characteristic of a wave-train, which appeared to be present in crystal reflection, was manufactured by the crystal. Consequently it should be possible to test this hypothesis by merely comparing the radiation before and after reflection from a crystal. Gray tried this but did not succeed in getting definite results. Plimpton[14] repeated the attempt and could detect no difference. A. H. Compton, who was at that time supporting the idea that the softening was due entirely to fluorescence in the scatterer,

repeated the experiment[15] with shorter wave-lengths and obtained more conclusive evidence that there was no difference. However, it soon became evident that the true answer was not contained in either the classical scattering theory or a purely fluorescence theory.

In a comprehensive survey of the whole question of secondary x-radiation Compton published[16] in October 1922 the first curves obtained by spectroscopic methods which showed definitely that secondary radiation from graphite consisted, principally, at least, of longer wave-lengths, and suggested that the explanation could be gotten only by treatment on a quantum basis. The classical theory failed to account for the fact of "excess scattering" (greater scattering in the forward direction than backward) and for the change in wave-length. The fluorescence theory had to be given up because the change in wave-length did not seem to depend upon the nature of the radiator and because the secondary radiation was found to be very much polarized. The work of Sadler and Mesham, and of Gray in gamma-rays, had disposed of the idea that the softening was due to greater scattering of the softer components, and the Doppler theory on the classical basis required extremely unlikely assumptions as to the motion of the scattering particles.

<u>c</u>. <u>Compton-Debye Quantum Theory of X-ray Scattering</u>

In 1923 Compton[17] and Debye[18] published independently
and almost simultaneously, nearly identical theories of
x-ray scattering on a purely quantum basis. Assuming that
a quantum of radiation of energy $h\nu_0$ and momentum $h\nu/c$ is
scattered in a definite direction by a single free electron
in a collision in which the laws of conservation of energy
and of momentum hold, they arrived at the following expres-
sion for the change in wave-length suffered by the quantum;

$$\delta\lambda = (2h/mc)\sin^2\tfrac{1}{2}\theta \qquad)$$
$$= (h/mc)(1-\cos\theta), \quad) \qquad ----- (1)$$

where h is Planck's constant, m is the mass of the electron,
c is the velocity of light, θ is the scattering angle or
the angle between the initial velocity of the quantum and
its velocity after scattering, and h/mc = .0242 Angstrom
units. It is to be noted that this shift is independent
of the nature of the scattering body, which follows of
course from the assumption that the free electrons are the
scattering agents, but, more than that, it is independent
of the initial wave-length of the radiation. In the same
paper Compton published ionization curves comparing the
primary K-spectrum of molybdenum with the same radiation
scattered by graphite, and showing a shift in approximate
agreement with Eq. (1). In November of the same year he
published more complete experimental data[19] including

scattering at angles of 45°, 90°, and 135°, showing in-
creases in wave-length which varied with the scattering
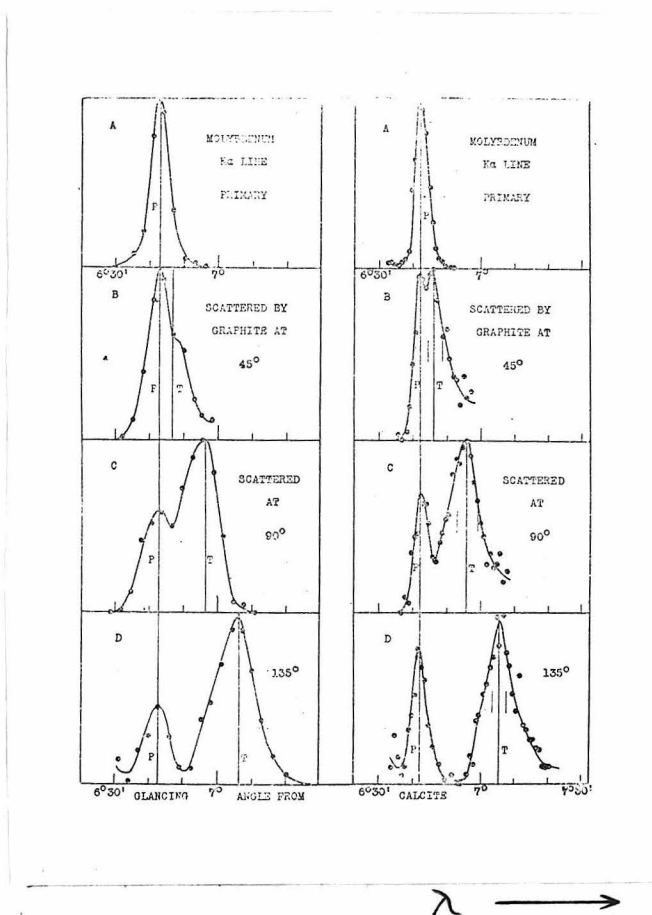angle as predicted. They also showed unmistakably the

Fig. 1. Compton's data on shift.

presence of an unmodified component which was greater in
intensity at small angles and became less at large angles.
See Fig. 1. He noted that the modified line was broader
than the unmodified, and suspected that some of the ad-
ditional breadth was real, although much of it could be
attributed to the fact that his scattering occurred through-

out a considerable range of angles and therefore his scattered radiation was less homogeneous than the primary. It would be difficult to exaggerate the importance of these last three papers, particularly those by Compton inasmuch as he supported his theoretical predictions with experimental facts. It is probably not too much to say that they contributed more to the subject of x-ray scattering than the sum total of all the work that had been done up to that time. The quantum theory of scattering was provided as a working basis for further research, the spectroscopic method was shown to be a tool applicable to the problem, and a considerable increment of new and exceedingly important knowledge was added to a rather incomplete and poorly correlated set of data.

The existence of the "Compton effect" was almost immediately confirmed by Ross[20], but an unfortunate series of experiments at Harvard[21] kept the issue in doubt for about a year and brought a considerable number of investigators into the debate. Clark and Duane consistently failed to get modified radiation according to Compton's prediction but did get indications of additional radiation elsewhere which they ascribed to the collision of photoelectrons with neighboring atoms in the scatterer, and called "tertiary" radiation. Similar results were gotten

by others working in the same laboratory but were finally found to be due to experimental conditions. After these conditions were corrected the tertiary radiation disappeared and some of the best measurements of the Compton shift were obtained there.

The capitulation of the Harvard group did not however end all opposition to the quantum theory of x-ray scattering. Barkla had, at the time of Compton's discovery, abandoned the idea of a characteristic "J"-radiation[12] in favor of a "J-transformation" which was a transformation toward longer wave-length which occurred, he thought, _after scattering_ and during transmission through scattering substances. He suggested poor technique on the part of Compton in his experimental work and attacked his theory rather strenuously at the outset[22], but was later compelled to give ground, and in 1925 sponsored a paper by R. T. Dunbar[23] in which the author admitted that the accumulated experimental data furnished "a good deal of evidence in favour of the quantum theory of scattering", and suggested that classical scattering might occur sometimes and Compton scattering at other times, depending upon unknown conditions. By this time practically all doubters had been convinced.

At this point the study of the Compton effect may be conveniently divided into three parts under the follow-

ing heads: (1) The wave-length shift of the spectral line,
(2) The intensity relations of primary and scattered radi-
ation, and the relative intensity of modified and unmodified
scattering. (3) The breadth and structure of the modified
line.

The first two will be treated rather briefly here, as
it is the third which has been the particular subject of
the research to be reported in this paper.

The shift of the spectral line has been measured by
a number of experimenters. A quite complete bibliography
is given by Professor Compton in his book, "X-rays and
Electrons"[21]. Since the publication of that volume his
equation $\delta\lambda = (h/mc)(1 - \cos\theta)$ has been checked by many
others[24] and has now been universally accepted. As has
been stated above, the first confirmation was by Ross who
was also the first to obtain the Compton effect photo-
graphically. He secured good spectrograms showing the
modified Mo K_α doublet scattered at 90° from paraffin.
This scattered radiation was in a broad, unresolved line,
while the primary lines were narrow and clearly resolved.
This point will be discussed later. The shift was in
excellent agreement with Compton. He also attempted to
detect the effect in visible light, using a Lummer-
Guercke interferometer and the green Hg line scattered
at 180° from paraffin, but was not successful, as the

intensity of scattered radiation varies, at least approximately, as the inverse of the wave-length cubed[25].

Among the first and most conclusive substantiations of the new effect was that by Becker, Watson, Smythe, Brode and Mott-Smith[24] in this laboratory. They obtained negatives showing unmistakably the scattered α and β lines of molybdenum, both unmodified and shifted by the predicted amount.

These were the forerunners of others and in this respect the Compton-Debye theory has met complete success.

4.   In the realm of intensity however it has not been so fortunate. Both Compton and Debye treated the ratio of intensity of scattered radiation to that of the primary, as a function of the scattering angle. Later Jauncey[26] and Breit[27] derived somewhat similar relations, but none have achieved more than qualitative agreement with the facts. Thomson's classical theory gave as the intensity of scattering of an unpolarized beam by a single electron,

$$I_e = I \frac{e^4}{2m^2 r^2 c^4} (1 + \cos^2\theta) \;-\;-\;-\;-\;-\;-\;-\;-\; (2)$$

where r is the distance from the electron, and the other symbols have their usual meaning. This gives symmetry in the forward and backward directions, and is shown by the curve marked 0 in Fig. 2. If the scattering electron recoils appreciably from both the incident and the scattered ray, as it must when struck by a quantum of higher energy

(shorter wave-length), the scattering should on any theory be less than is given by Eq. (2). Breit has shown[28] that the minimum value which scattering can have is given by this classical value divided by $\left[1 + \frac{h\nu}{mc^2}(1 - \cos\theta)\right]^4$. This curve and others of the above mentioned theoretical curves are shown in Fig. 2. Experimentally, scattered
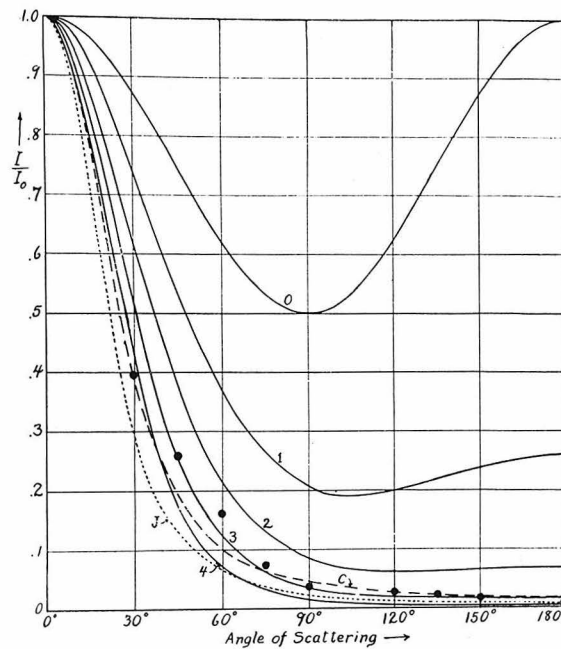

Fig. 2. Intensity of Scattering at
Different Angles.

(From Compton's "X-rays and Electrons")

O represents the Thomson classical theory
4 is Breit's minimum curve
C is Compton's theoretical curve
J is Jauncey's   "   "   .

intensity lies between the classical curve and Breit's
minimum curve, but conforms more nearly to the latter.
No really satisfactory theoretical treatment has yet
been evolved, although all the quantum theory formulas
have the advantage of accounting at least qualitatively
for the excess scattering which is found in the forward
direction.

5.   Since the intensity in the modified line is quite
intimately connected with its breadth and structure which
will be discussed in detail later, it will suffice here
to mention the first work done on the subject, by Ross[29]
and Woo[30]. Ross made a microphotometric study of his own
photographic spectra and those obtained in collaboration
with Webster, and also measured the relative intensities
given by Compton's curves. This gave him data on scatter-
ing at 90° from carbon (6), aluminum (13), sulphur (16),
copper (29), silver (47), and lead (82). Plotting the ratio
of unmodified to modified intensity against atomic number
gave him roughly a straight line, leaving out the point
for lead which did not fall anywhere near it. Varying the
scattering angle while using the same scatterer gave the
following results in the case of graphite;

Angle	Ratio
30°	5.0
60°	1.05
90°	.59

These results, which Ross considered as only very approxi-
mate, do not agree very well with the later and more care-
ful work of Woo. He used an ionization spectrometer with
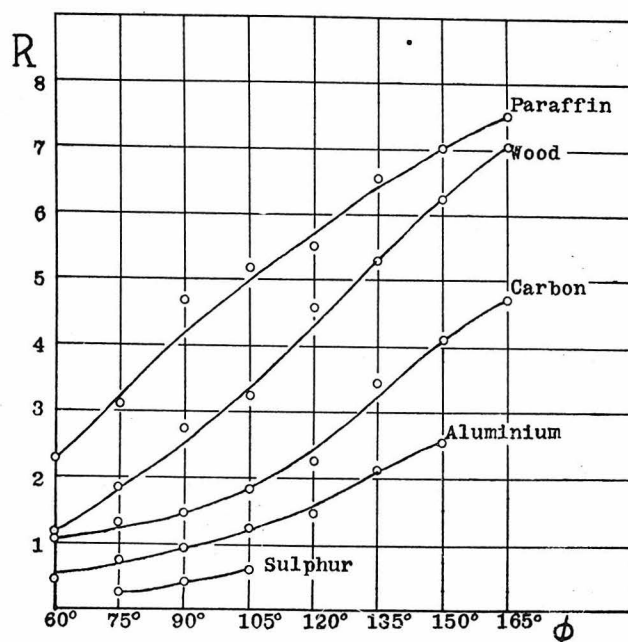a methyl bromide chamber and studied scattering at inter-

Fig. 3. The ratio, R, of modified to unmodified intensity
plotted against scattering angle (by Woo).

vals of 15° from 60° to 165° inclusive, using paraffin,
wood, carbon, aluminum and sulphur as scatterers. His
curves are shown in Fig. 3. The ratio of modified to un-
modified increases with angle in all cases and decreases
with atomic number. This latter fact supports the belief
that the modified scattering comes from the loosely bound
electrons. He calls attention to the fact that paraffin,

which has the highest curve, is composed of carbon and
hydrogen while wood, which has the next highest, has oxy-
gen in addition, and both lie above the curve for carbon
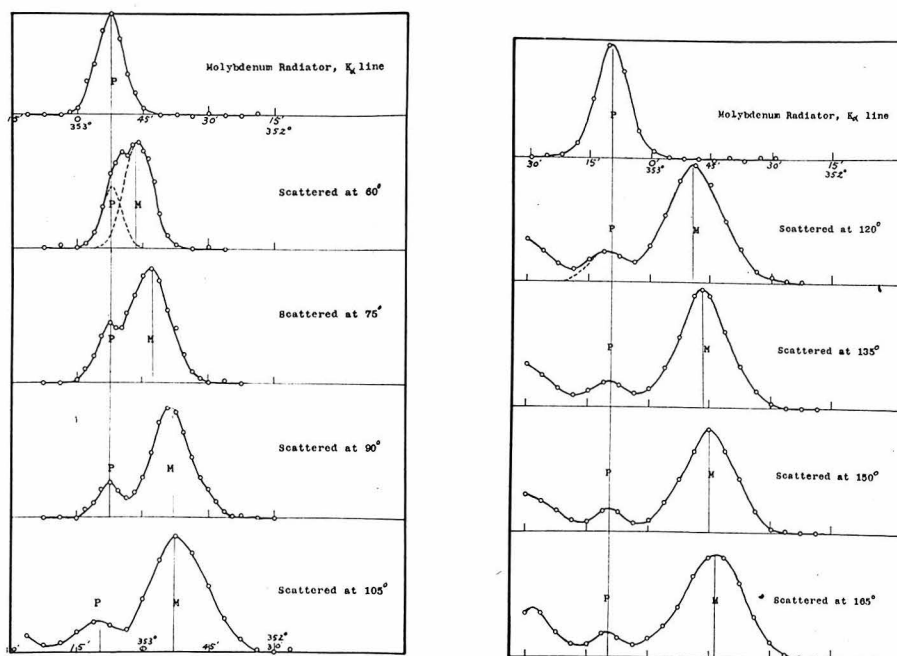
Fig. 4. Intensity ditribution of x-rays scattered from
paraffin. P marks the position of the primary K_α line,
and M that of the modified line according to Eq. (1). (Woo).

itself. Evidently hydrogen is the most effective in pro-
ducing the modified line. He also tried a lithium radiator
cleaned and kept in an atmosphere of hydrogen and found that
practically all the scattering was modified, the presence of

the unmodified line at all being questionable. Fig. 4 shows the distribution of intensity he secured from paraffin at various angles. Jauncey, in a paper on the quantum theory of the unmodified line[31], had derived a formula for the above ratio. Woo's values fell consistently below Jauncey's estimate. Woo also observed that the ratio was independent of the intensity of the initial radiation, and agreed with the earlier suspicions that the modified line was broader than the primary or the unmodified.

d. <u>Work in this Laboratory on the Compton Effect by Dumond.</u>

Work on the Compton effect was begun here at this laboratory in 1925 by Dr. Jesse W. M. Dumond with the purpose of investigating the cause of the apparent increased breadth of the modified line in the x-ray spectrum, and the question as to whether scattering was to be attributed to free electrons or not. His first experimental work[32,33,34] was done with a specially constructed x-ray tube containing both the scatterer and a small Seeman type spectrograph embodied in the anticathode. The advantage of this type of apparatus is evident. The distance from the focal spot of the target to the scatterer was only about half an inch and the distance from the scatterer to the spectrograph was about an inch. The gain in intensity over anything that had been previously attained promised to be quite consider-

able, but unfortunately he found that the primary intensity had to be limited because the least overheating of the anti-cathode caused vaporization and consequent coating of the scatterer with molybdenum. Although the expected reduction of exposure time was not fully realized, the results were quite satisfactory with both aluminum and beryllium as scatterers. In the last mentioned paper[34] he gave a theoretical interpretation of the breadth of the line as a type of Doppler broadening, and derived a relation between the line structure and the velocity distribution of electrons in the atom. His observed line structure was compared with the line structures to be expected on this basis when four different assumptions were made as to the velocity distribution.

These assumptions were; " 1. That electrons in the solid scattering substance have the velocity distribution required by a wave-mechanical model for a free atom of that substance far removed from neighbors. 2. That electrons may be divided into two classes, one class the metallic or conductive electrons in the state of a degenerate electron gas subject to the Pauli Exclusion Principle and having the velocity distribution derived by Sommerfeld, and the other class as in the 1st assumption unperturbed by the neighboring atoms. 3. That electrons may be divided into two classes as before but that those forming an electron gas have the classical velocity dis-

tribution required by the Maxwell-Boltzmann equipartition
law. 4. That electrons have the velocity distribution re-
quired by the older Bohr-Sommerfeld atom model with point
electrons executing Kepler orbits and as in the first as-
sumption unperturbed by neighboring atoms."

His experimental results gave a broad, diffuse
shifted line, and comparison of microphotometer curves
with curves constructed on the basis of each of the above
assumptions showed confirmatory evidence for the wave-
mechanics model of the atom and for the Sommerfeld theory
of a degenerate electron gas for the conduction electrons,
on the basis of Fermi statistics.

II.   THE MULTI-CRYSTAL SPECTROGRAPH.

As yet no satisfactory method had been devised to
study scattering at angles other than in the neighborhood
of 180° where inhomogeneity of angle causes the least
spuriou breadth of the modified line. It was evident
that if the breadth of the Compton line were due to the
velocity distribution of the electrons doing the scatter-
ing, as seemed most probable, then it should be possible
to gain a good deal of information from a study of scat-
tering at various angles. For this purpose Dr. Dumond
devised the fifty-crystal spectrograph[35] which was built
in the Institute shops by Mr. Bressler under the super-

vision of Mr. Julius Pearson, and has been successfully adjusted and used in this research by Dr. Dumond and the author.

<u>a</u>. <u>Design of the Instrument.</u>

In effect this instrument is an application of the principle of the Rowland concave grating, in two dimensions,
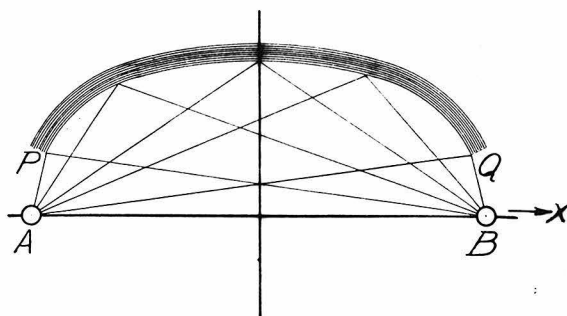

Fig. 5. Flexible crystal Rowland grating.

to x-ray crystal spectroscopy. However a considerable modification is necessary. The conditions for Bragg reflection impose two requirements which cannot be satisfied by a single surface of a crystal having its atomic planes parallel to that surface as in Fig. 5, if radiation from a point source is to be focussed at a point. The first of these requirements is that at any given point the angles of incidence and reflection must be equal,

and the second is that these angles must be the same at
all points and yet the rays must converge to some single
point B on the photographic film. However since it is
not necessary to have the atomic planes parallel to the
surface of the reflector, and since the study of scatter-
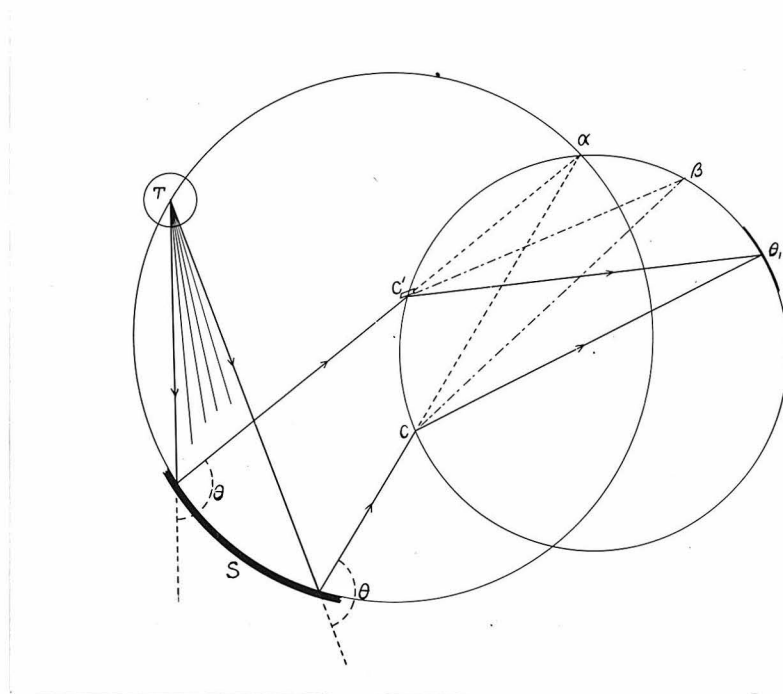ed radiation permits the use of a large scatterer as an

Fig. 6. Geometrical arrangement of x-ray tube, scatterer
and multi-crystal spectrograph.

extended source, the following very simple solution was
hit upon by Dr. Dumond. If, as in Fig. 6, we place the
target of an x-ray tube, T, and an extended scattering
body, S, on the circumference of a circle, all radiation
scattered at a given angle θ will converge to a point α

also on the circumference of the same circle. If we now
construct another circle passing through the point α with
an arc CC' intercepting the wedge of converging scattered
rays, and if we place along this arc small crystal spec-
trographs with their reflecting planes perpendicular to
the figure and all meeting in a line through the point β
such that the angle $\alpha C\beta$ is the Bragg reflecting angle for
a particular wave-length λ of radiation, by the geometry
of the figure all radiation of that particular wave-length
will be reflected to a point θ_1. Thus a film placed on the
circumference of the smaller circle at θ_1 will have record-
ed on it the spectrum of the radiation in the neighborhood
of the wave-length λ. This arrangement has the double
advantage of permitting the use of a large scattering body
with the accompanying gain in total intensity of scattered
radiation, while securing also extremely good homogeneity
of scattering angle which has been secured in no other
way. The latter is by far the most important feature of
the instrument, since it must be admitted that much of the
first named advantage is lost because the distance from the
x-ray tube to parts of the scatterer must be rather large,
especially when the set-up is for large scattering angles.

b. Construction Details

Fig. 7 shows the plan and elevation of the spectro-
graph and Fig. 8 shows views of the completed instrument.

Fifty small cylindrical Seemann-type spectrographs, shown in Fig. 9 were used. They are 1/4 inch in diameter and the calcite crystal in each has a cleavage face about 3/16 inches wide and an inch long. The crystals were ground by Mr. Pearson to fit accurately in a triangular
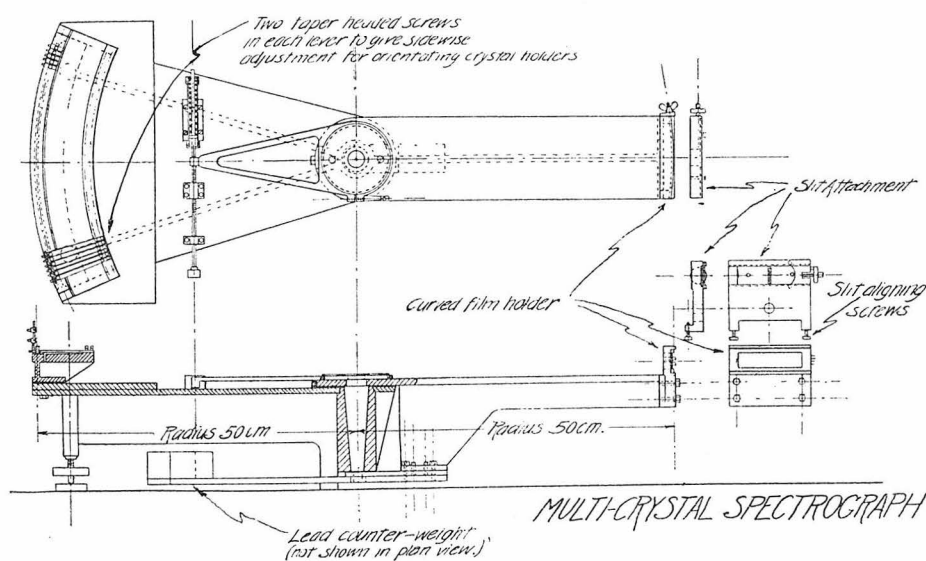

Fig. 7. Plan and elevation of multicrystal spectrograph.

groove with their cleavage faces lying accurately in the diametrical planes of the cylinders. A few thousandths of an inch from the crystal face and practically on the axis of the cylinder is the edge of the brass wedge which defines the reflected beam. These fifty units are fitted accurately into fifty holes bored as close together as was consistent with mechanical strength in a single bronze casting. The holes are carefully located on a circular arc which forms part of the major circle of the instrument
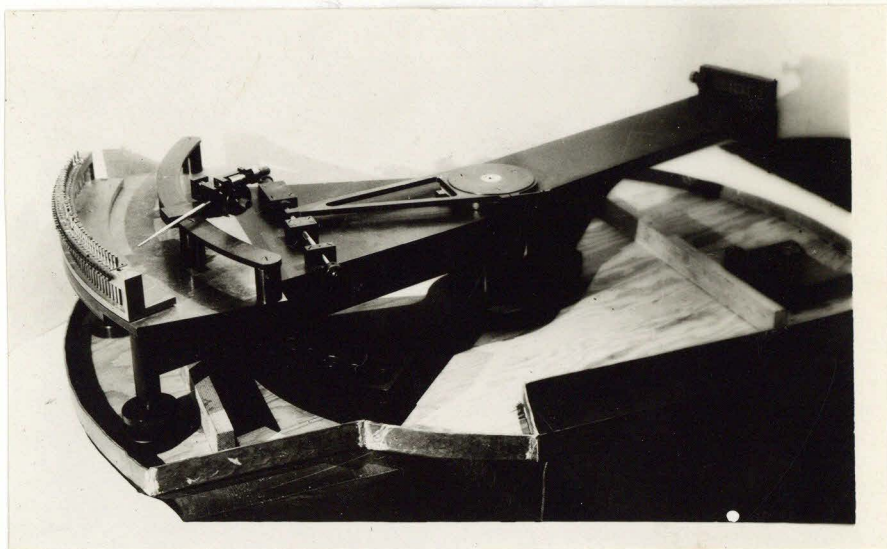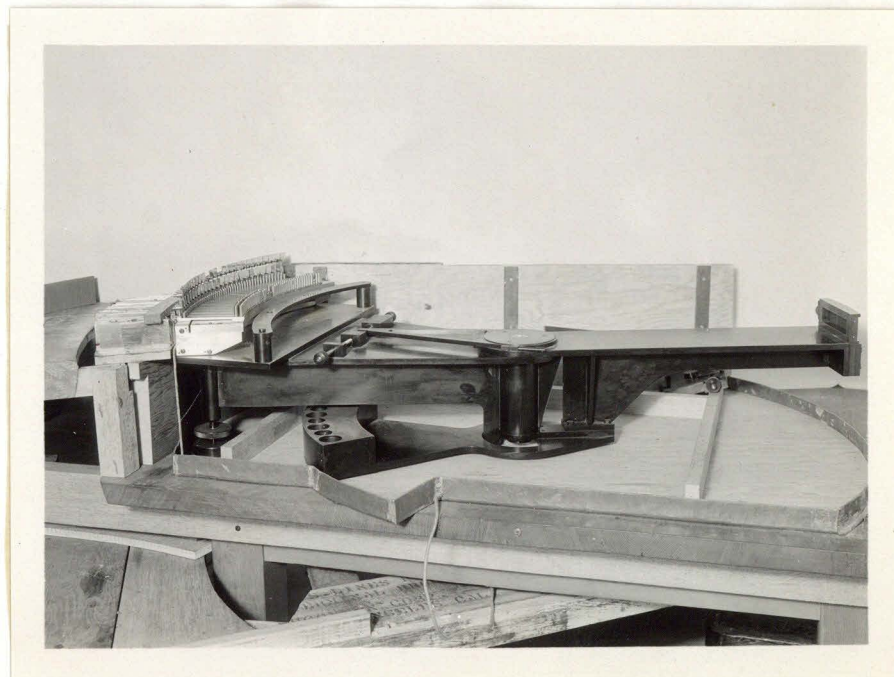
Fig. 8a.

Fig. 8b.

Fig. 8c.

and has a diameter of about one meter. The curved cast-
ing has two channels milled out on either side to such a
depth that the holes open into the channels so as to form
two elongated rectangular windows on diametrically opposite
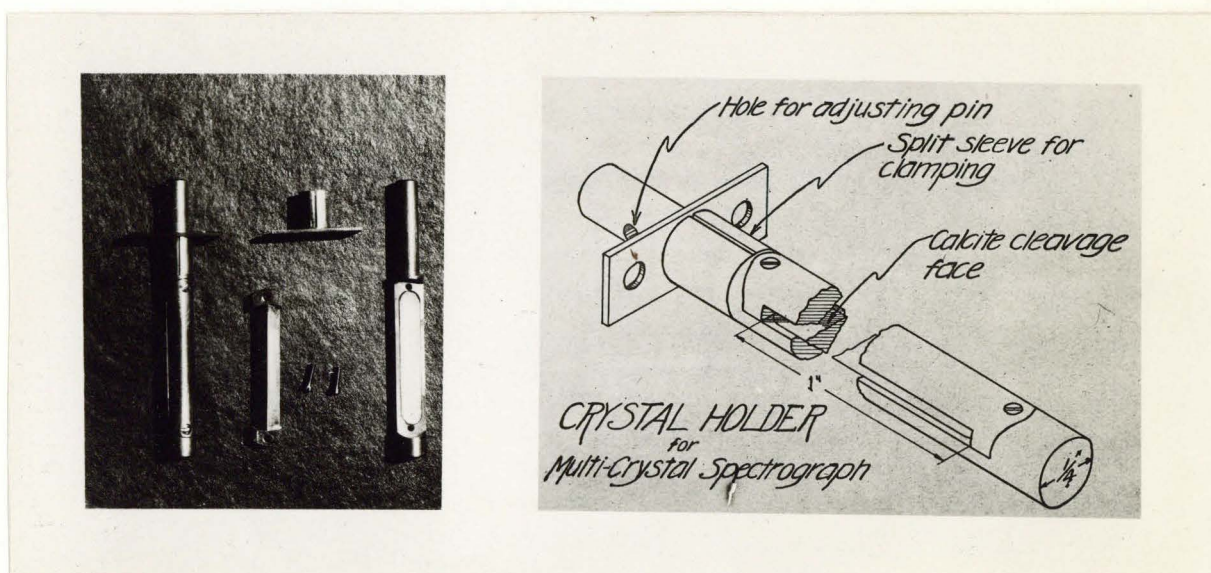sides of each hole. The x-rays pass through these windows.

Fig. 9. Crystal holders, before and after assembling.

The casting which remains between two adjacent holes pro-
vides an arc of contact of nearly 90° on two opposite
sides of each crystal holder. This is sufficient to pre-
vent x-radiation from leaking between the crystal holders.
The film holder or cassette is mounted on a counterbalanced
arm pivoted at the center of the major circle and can be
be swung through an angle sufficient to explore the spec-

trum through three orders. The film cassette can be removed and replaced by the slit shown, to be used with an ionization chamber and electrometer, but up to the present time the instrument has been used only photographically.

As originally built, the rotation of all the crystals to their proper orientations was accomplished by means of a single rotating mechanism which could be clamped at any position on an arc parallel to that of the crystals, and consisted of a slow motion screw to move one end of a steel arm about 15 cm long, the other end of which was inserted in the hole in the upper end of the holder of the crystal to be adjusted. This is shown in Figs. 8a and 8b. When properly orientated the crystals were held in place by set screws in the bronze casting. This arrangement was found to be unsatisfactory. It was impossible to insert or withdraw the steel arm without disturbing the crystal, and although the set screw exerted its pressure on the split sleeve shown in Fig. 9 and not directly on the holder itself, it usually rotated the crystal through an angle larger than the correction that had just been made in the position of the crystal. Consequently it was found advisable to change the design of this part of the instrument, and the device shown in Figs. 7 and 8c was adopted and found to be quite satisfactory. Each crystal holder is equipped with a steel arm

10 cm long. One end can be clamped rigidly to the upper end of the crystal holder by means of a small set screw. The other end extends along a radius toward the center of the major circle and rests on a bronze plate attached to the original casting. There are two tapered holes in each arm near this end and through them two taper headed screws are threaded into two holes in the bronze plate which are not both on the same radial line. When screwed down, the tapered heads bear on opposite sides of their respective holes, and thus the end of the lever can be moved slightly either way by turning down on one screw and up on the other. This has the advantage also that no auxiliary locking device is needed and there is no necessity of disturbing the adjustment in any way after the crystal has been properly orientated.

a. Testing Crystals for Planeness.

In the study of scattered radiation it is important that each crystal have atomic "planes" which are accurately plane throughout the length of that part of the crystal which is in use. This is because the radiation comes from an extended source and each point on the photographic film receives radiation from all points along this length of the crystal. Thus a twisted crystal will give a broadened, more diffuse line whereas with a point source the line would be merely inclined or curved. For this reason it

was necessary to test each crystal for planeness, and
since this could not be done until it was split, ground
and mounted in its holder, and since about thirty percent
of the crystals failed to pass, the work of testing and
of replacing the ones that had to be discarded proved to
be one of the principle tasks involved in constructing
the instrument. The testing was done with the crystals
in place in the instrument. An x-ray tube was mounted in
front, close to the crystal to be tested so that only a
small fraction of the length of the crystal contributed
to the spectral lines on the film. Thus it was possible
to explore it throughout its length by simply raising and
lowering the x-ray tube and determining whether the lines
reflected selectively from the various parts coincided on
the film. Usually four different parts were checked a-
gainst the middle. The tube was first placed so that the
focal spot was at the level of the middle of the crystal,
and a central horizontal zone on the film was shielded by
a brass strip while a one-minute exposure was made. Then
the tube was raised until in line with the upper part of
the surface of the crystal, and an exposure made on the
central zone only while the part previously exposed was
shielded. The film was then moved a few millimeters lat-
erally in the cassette and a similar comparison was made
between the surface at the middle and at a point one

fourth of the way down, and so on to the bottom. Lastly, as a check, the top was compared with the bottom.

Fig. 10a shows a sample negative revealing one of the worst crystals found. The last exposure, on the right, shows a difference between the top and bottom
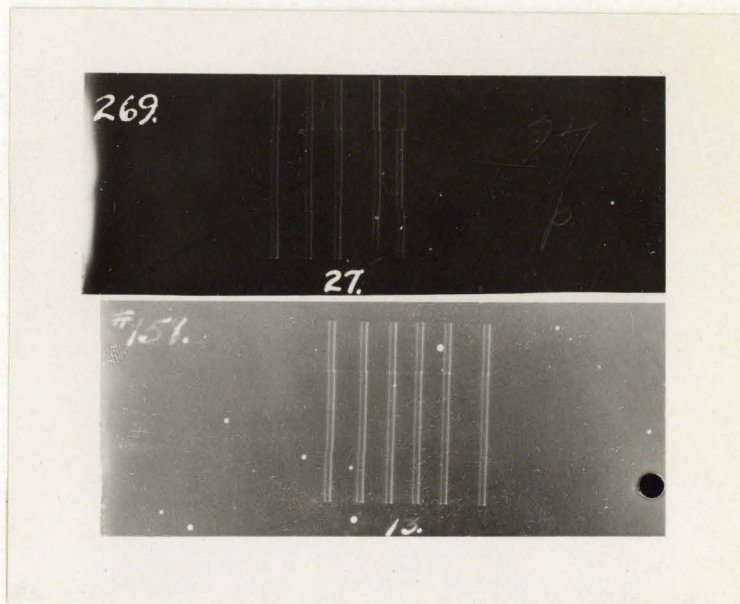

Fig. 10a and 10b. Tests of planeness of crystals.

nearly equal to half the separation of the K doublet. Fig. 10b shows an investigation of another crystal at intervals of one sixth of its length but with no comparison directly between the top and bottom. In both there is apparently a nearly uniformly helicoidal surface throughout the length, although this was not true of all crystals. On both negatives the central segments

of the lines are from the central part of the crystal,
with the exception mentioned above of the right hand
pair in Fig. 10a.

In some crystals this distortion was found to be
a mechanical strain, probably torsional, imposed by the
holder and was corrected by loosening slightly the screws
which held the wedge and crystal in place. At first a
small touch of Duco cement was applied to the back of
each crystal, at one end, before placing it in its holder,
as an added precaution against subsequent change of pos-
ition of the crystal. Apparently the cement did not
shrink evenly while drying and there occurred a slight
rotation of that end of the crystal about some axis. If
this axis had an appreciable component in the direction
of the length of the crystal, the result was a twist.
If a slight loosening of the wedge screws permitted the
rest of the crystal to take the same orientation as the
cemented end and straightened the crystal out, it could
be used. Otherwise it had to be replaced.

Several substances other than Duco cement were
also tried, but nothing was found to be entirely satis-
factory. The later replacements were not cemented in at
all, entire reliance being placed on a good mechanical
fit, and protection of the instrument from jarring by
mounting its table on rubber blocks one inch thick.

d, Orientation of the Crystals

The orientation of the crystals was accomplished photographically using the $K\alpha$ lines from a molybdenum target tube. The tube was mounted as shown in Fig. 11 so that it could be rotated about the point α_1 as center,
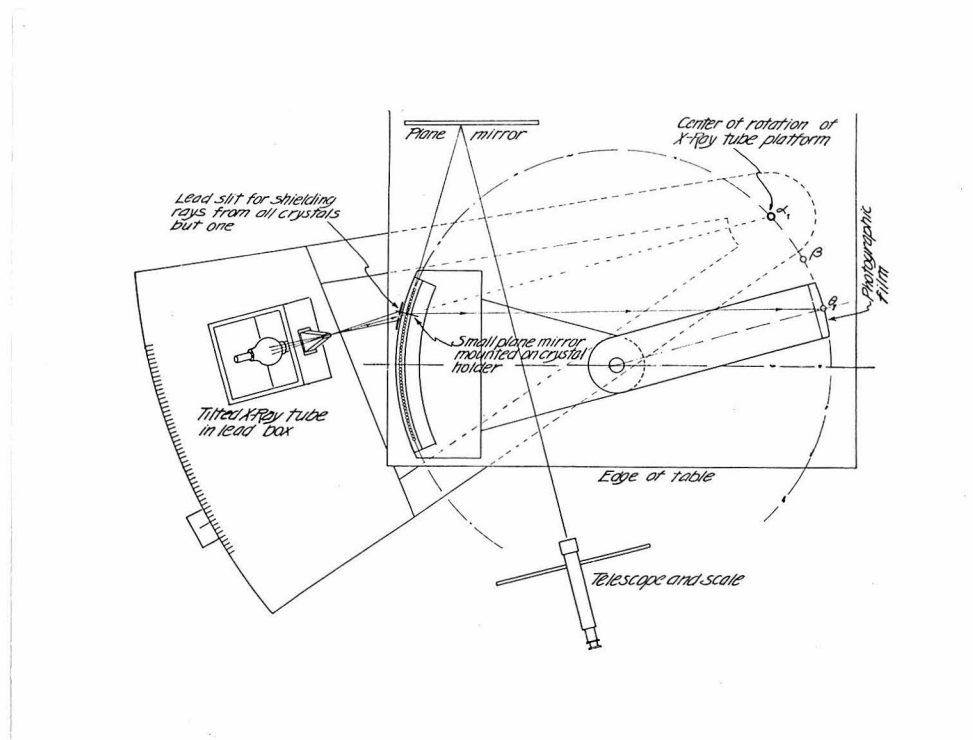

Fig. 11. Tube mounted for orientation of crystals.

and positions were marked on an arc on the moving frame supporting it so that the focal spot could be set at will on the extension of the radius through any desired crystal. One of the middle crystals was selected as a reference crystal, rotated by hand with a steel rod inserted in the

hole through the top of its holder, until the K_α lines
in the first order appeared in a fluoroscope at about the
center of the film holder at θ_1. The crystal holder was
then locked to its adjusting arm by the small set screw,
the two taper-headed screws at the end of the adjusting
arm were turned down snugly and permanently, and the steel
rod was withdrawn. Then each of the other crystals in
turn was orientated so that the spectral lines reflected
from it coincided on the film with those from the refer-
ence crystal.

The process was as follows. The first rough set-
ting was made with the fluoroscope as in the case of the
reference crystal. This usually put the spectral lines
within a few millimeters of their proper positions at
the film holder. The crystal holders were all clamped
to their adjusting arms after this operation and, in
general, all subsequent adjustments were made by means
of the taper-headed screws. The positions of the crystals
were then tested photographically. An Eastman duplitized
x-ray film in its black paper envelope was put into the
film holder and exposures made in the following way. The
x-ray tube was swung into position in front of the refer-
ence crystal. A small moveable lead plate to shield the
neighboring crystals but with a slit to permit the radia-
tion to reach the reference crystal, was put in position

as shown in Fig. 11. A central longitudinal zone of the
film, about one third of the width of the 1x4-inch open-
ing in the front of the film holder, was shielded from
the radiation by a strip of 1/8-inch brass, and a one-
minute exposure was taken. Thus only the extremities of
the refernce spectral lines show on the film, the central
third of their length being shielded off. The tube was
then moved into position in front of crystal No. 1, a
similar lead slit was placed in front of it, the film
was shielded this time in such a way as to expose only
the central third of the spectral lines, and a second
exposure was made. The film envelope was then slipped
a short distance in the film holder and the process re-
peated for the second crystal, and so on until the set-
ting of each crystal had been photographically recorded
with that of the reference crystal. Usually five of
these double exposures were made on each film and then
the ten films were developed, fixed, washed and dried in
a specially constructed holder, at the same time. Fig. 12
shows one of the films with four double exposures on it.
The segments of the lines at the top and bottom are in
each case those from the reference crystal. On this
film crystal No. 6 is shown to be satisfactorily oriented
while the others are not.

As a basis for calculating the angle through which

it was then necessary to rotate crystal No. 19 to bring
the lines into coincidence, the distance d was measured
on the negative with a parallel plate comparator built
for the purpose[36].

After measurements had been made on all the films,
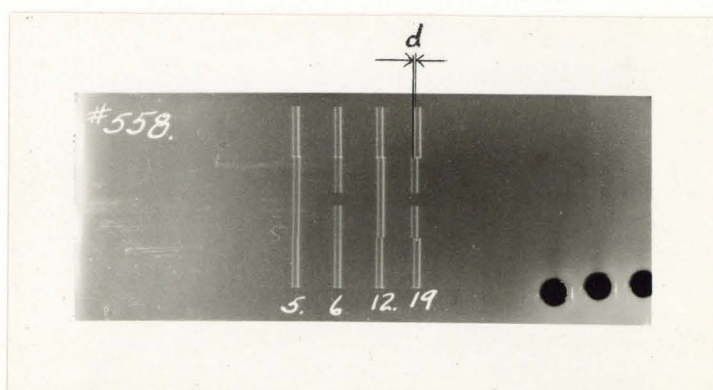the next step was to make the indicated corrections to

Fig. 12. Test of orientation of four different crystals.

the crystal settings. On the end of each adjusting arm
near the crystal holder, had been mounted an excellent
small mirror one half inch square cut from an Eastman
Kodak Company filter to secure a good plane surface, and
gilded on the front surface through the kindness of Dr.
C. Hawley Cartwright to whom we are indebted for volun-
teering his experience and equipment for the task. At
the center of the instrument was placed a larger plane
mirror which could be rotated about a vertical axis. A
well lighted scale was placed about three meters from

this mirror. Light from the scale was reflected from the large mirror to the desired small mirror, back to the large mirror and thence to a telescope to form an image of the scale (see Fig. 11),[*] and by this means a very small rotation of the crystal could be detected and measured. Since the x-ray beam moves through the same angle as the crystal, while the reflected light ray from the moving mirror moves through twice that angle, we have for s, the change of apparent position of the scale in the telescope required to bring the crystal into the proper orientation, the relation

$$s = 2dr_2/r_1 \qquad - - - - - - - - - (3)$$

where r_1 is the distance from the crystal to the film and r_2 is the length of the optical path from the scale to the small rotating mirror. This correction was made as carefully as possible for each crystal and then another photographic check was made just as before. This cycle was repeated, eliminating each time a few that were found to be in satisfactory adjustment, until all coincided with the reference crystal to within at least one fifth of the width of the K_{α_1} line, and the majority were within one tenth of that distance.

The accuracy of these orientations is attested by the sharpness of the narrower spectral lines shown in Fig. 22, of negatives obtained with scattered radiation,

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[*] See also Fig. 12a. Fig. 11 shows earlier arrangement, before the addition of permanently mounted mirrors, etc.

Fig. 12a. Showing small mirrors mounted on crystal holders, larger central, rotating mirror, and telescope. The scale was located behind behind the camera.
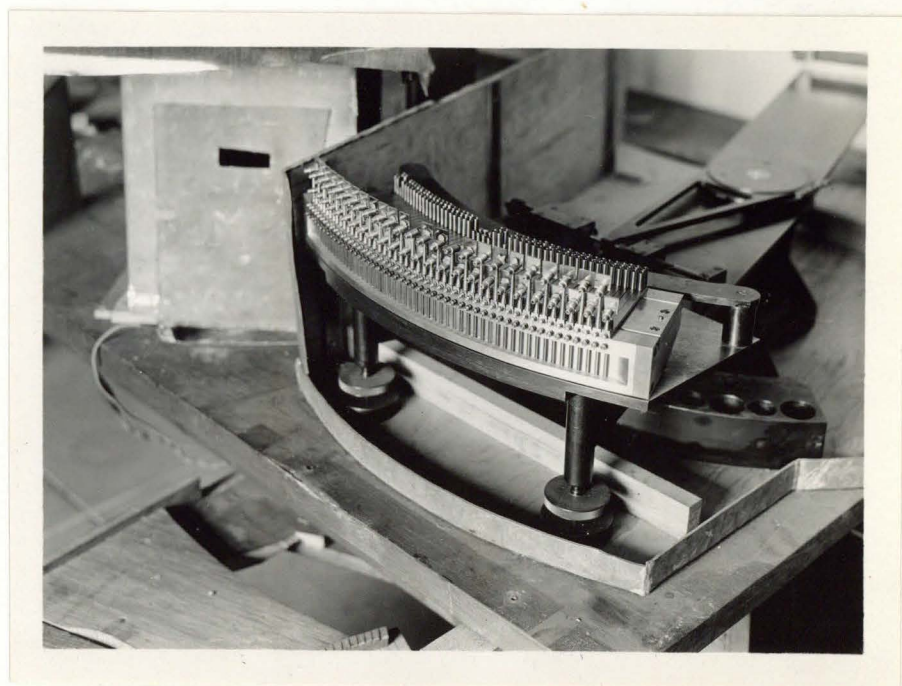

Fig. 12b. Closer view showing crystal apertures, mirrors and adjusting levers. Most of the taper-headed screws are covered by protecting brass cylinders.

using all fifty crystals. At the time this is written
the instrument has been in use about nine months and
while no photographic check up on the individual crys-
tals has been made in that time, no diminishing of the
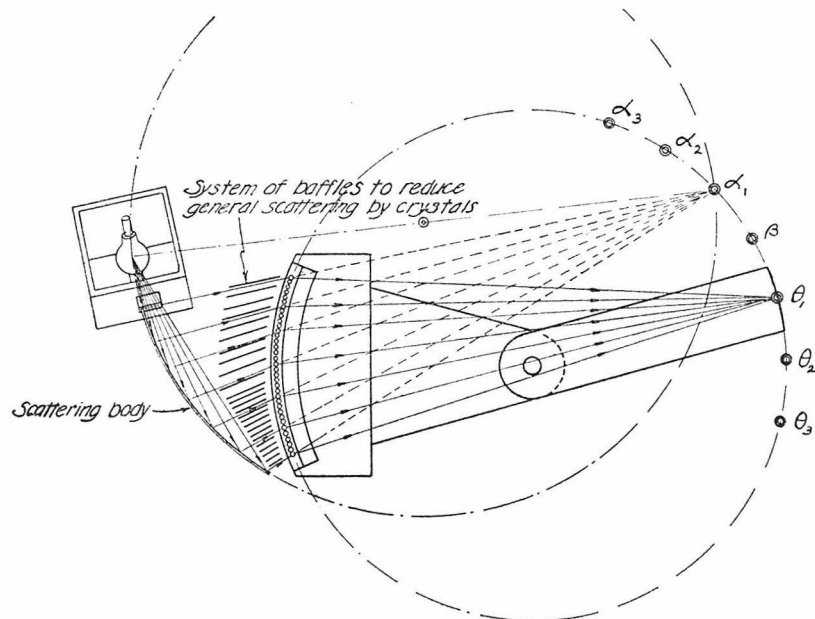sharpness of spectral lines can be noted and evidently

Fig. 13. Showing arrangement for homogeneous scattering
at 90° in the first order, and also positions of images
and virtual sources for the first three orders.

most of them have remained in their positions exceeding-
ly satisfactorily.

It should be noted that when the crystals are ori-
ented for one wave-length and one order, they are also
oriented properly for all other wave-lengths and orders,
as may easily be seen from the geometry of the instrument.
In Fig. 13 the positions chosen on the major circle for

the spectral images and sources in three different orders
are shown. In the figure θ refers to the images, α to
the virtual sources, β is the point where the reflecting
crystal planes would converge if produced. The geometry
of the instrument permits a certain freedom in the choice
of these positions. A number of practical considerations
entered into the final choice made. These are: (1) the
limits imposed by the construction of the instrument.
The opening of the rectangular windows in the bronze cast-
ing requires that

$$- 90° < \alpha < +90°.$$

(All angles are measured positively to the right from a
point 0 diametrically opposite the center of the arc oc-
cupied by the fifty crystals.) The design of the lead
box inclosing the instrument limits the swing of the arm
carrying the film cassette so that

$$- 12° < \theta < +45°.$$

The glancing crystal angle cannot exceed $30°$ because of
the angle of the wedges used. This limits the instrument
to the fourth and lower orders. (2) α must be as large as
possible to permit working advantageously at large scat-
tering angles. (3) The rays incident on the film must
be kept as nearly normal as possible, especially as dupli-
tized film (sensitized on both sides) is used.

For molybdenum K-radiation and calcite the glancing

angle of about 7° may be assumed to represent roughly the center of the small spectral region to be studied. We therefore decided to use the following values;

$$\theta_1 = 16^o \qquad \theta_2 = 2^o \qquad \theta_3 = -12^o$$
$$\alpha_1 = 44^o \qquad \alpha_2 = 58^o \qquad \alpha_3 = 72^o$$
$$\psi_1 = 72^o \qquad \psi_2 = 79^o \qquad \psi_3 = 80^o$$

where ψ represents the angle between the tangent to the major circle at the film and the ray least nearly normal.

Considerable care has been taken to insure that the position of the film is always the same. The containing envelopes are of uniform thickness of black paper, folded only once so that there is nowhere more than one layer in front of or behind the film, and the ends are closed by a thin layer of cement between the two thicknesses of paper. The upper edge is left open and the loading is done in the dark. The film is held in place against the circular arc of the frame of the cassette by the tension of a flexible brass strip, and this has vertical reinforcing ribs on the back to prevent bowing in and forcing the film too far forward in the middle. The importance of these precautions is evident from the consideration that the beams from the crystals located at the extreme ends of their arc are converging at an angle of about 23°.

e. <u>Non-selective Scattering at the Crystals and Wedges</u>.

The first negatives of scattered radiation obtained

were featured by a background much too intense to permit satisfactory measurements on the fainter lines. Since the film was completely protected by the lead case from stray radiation, it seemed probable that the cause of this undesirable blackening was non-selective scattering by the crystals and wedges. This non-selective scattering on the part of the crystal could occur by Compton modified scattering inasmuch as that is non-coherent and therefore not subject to the Bragg restriction of equality of angles of incidence and reflection.

With an extended scattering body, each crystal unit was exposed to radiation from a large part of the scatterer and a certain part of this would be scattered again by the unit in the direction of the film. While the intensity of this doubly scattered radiation would certainly be quite small in comparison with the radiation received by the unit, it must be remembered that the area of the scatterer from which it was permitted to come was very great as compared with the very small area which gave rise to the radiation selectively scattered to form the spectrum from that particular crystal. Hence the total intensity might be quite comparable to that in the lines of the spectrum.

Accordingly a set of baffle plates were introduced to limit the area of the scatterer which could supply radiation to each crystal. These are shwon in Fig. 13.

The result was satisfactory beyond expectations. The
background disappeared almost completely.

III. THEORY RELATING BREADTH OF THE COMPTON LINE TO
ELECTRON MOMENTA.

a. Introductory Outline

The first suggestion that the motion of the electrons
might be the cause of the breadth of the Compton line seems
to have come from Ross[20]. In the first report of his exper-
imental confirmation of Compton's shifted line he noted the
fact that it was broader than the unshifted line and sug-
gested that this width might yield some important inform-
ation about the motion of the scattering electron.

Jauncey[31,38], deBroglie[39] Wentzel[40] Dumond[34] and
Chandrasekhar[41] have treated the subject theoretically.

The present treatment, which is due to Dumond[37]
makes use of the following assumptions: (1) the conserv-
ation of momentum and energy in the scattering process,
(2) that the electron binding energy is negligible in
comparison with the energy transferred to the electron
from the quantum, (3) that the initial velocity of the
electron is small compared with that of light, and (4)
that the probability of scattering by electrons in a
given velocity class is proportional to the number in
that class. Proceeding from these assumptions it is shown
that the breadth of the modified line at any given frac-

tional part of its height is approximately proportional to the velocity of the electrons in any given class in the scatterer, to the initial wave-length, and to the sine of half the scattering angle. Expressed mathematically,

$$\Delta\lambda = 4\beta_1\lambda_1\sin\tfrac{1}{2}\theta \quad \text{(approximately)}. - - (4)$$

As will be shown later, this equation is supported by our experimental results.

It is now well known that the change of wave-length with scattering angle can be derived from a purely wave hypothesis as well as on the quantum theory, by considering it as a Doppler effect from a moving scattering body. Regardless of the point of view, it should cause no confusion to speak of it as a Doppler effect, and this we do without wishing to commit ourselves to any explanation of the mechanical process involved in the scattering, but merely referring to the analogous result. The Compton effect may then be thought of as the superposition of two Doppler effects. The first is that due to the _velocity given to the scattering electron by the radiation._ This is always in such a direction with respect to the propagation of the radiation that it results in an increase in the wave-length and causes the well-known Compton shift. The second is due to the _initial velocity of the electron_ which may be in any direction and consequently may either add to or subtract from the effect of the first.

This second velocity, on the whole, is less than the first and causes the line breadth.

The result of the first is the Compton-Debye equation

$$\partial\lambda = (2h/mc)\sin^2\tfrac{1}{2}\theta \quad ------- \quad (1)$$

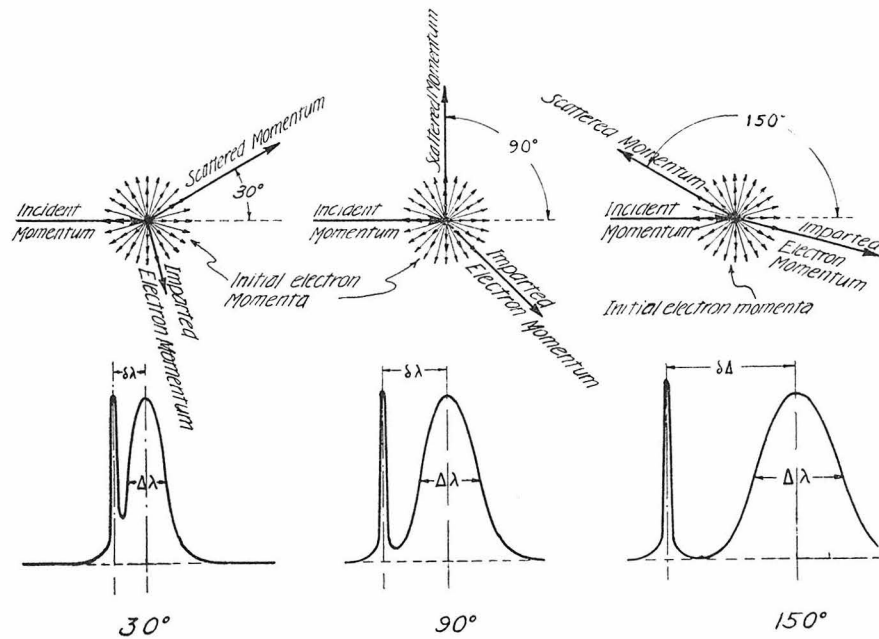The result of the second has been expressed in Eq. 4.

Fig. 14. Schematic illustration of Compton scattering by randomly directed moving electrons for three scattering angles, together with idealized spectra of the resulting scattered radiation.

In Fig. 14 is shown diagrammatically the scattering process for three different angles, and below, the resulting shifts and line breadths. Fig. 15 shows relative breadth and relative shift curves for angles up to 180° plotted on a percentage scale in which the value of each at 180° is taken as 100%. Relative breadth is here computed from

a formula derived below, which is more accurate than Eq. (4).

It is to be noted that the shift is proportional to the sine squared of half the scattering angle while the breadth is proportional simply to the sine itself. This has a simple qualitative explanation in the fact that the
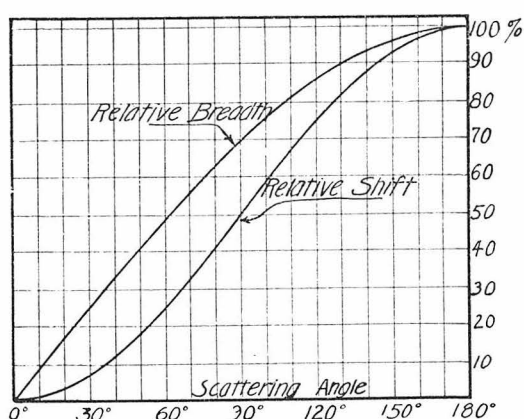

Fig. 15. Relative breadth and relative shift as a function of scattering angle.

shift increases with angle for two reasons, while the breadth increases because of only one. The Doppler effect from any moving scatterer increases with scattering angle, and in the Compton effect the velocity of the moving scatterer itself increases with scattering angle, while the breadth is due to the initial velocity alone.

b. Analytical Solution of the Line Breadth.

Case of one free scattering electron only. Following de Broglie[39] we write the equation expressing the

conservation of energy in a collision between a quantum of radiation and a moving electron, and the three equations expressing the conservation of momentum along three rectangular coordinate axes, letting the direction of motion of the primary quantum be the positive x-axis. See Fig. 16.
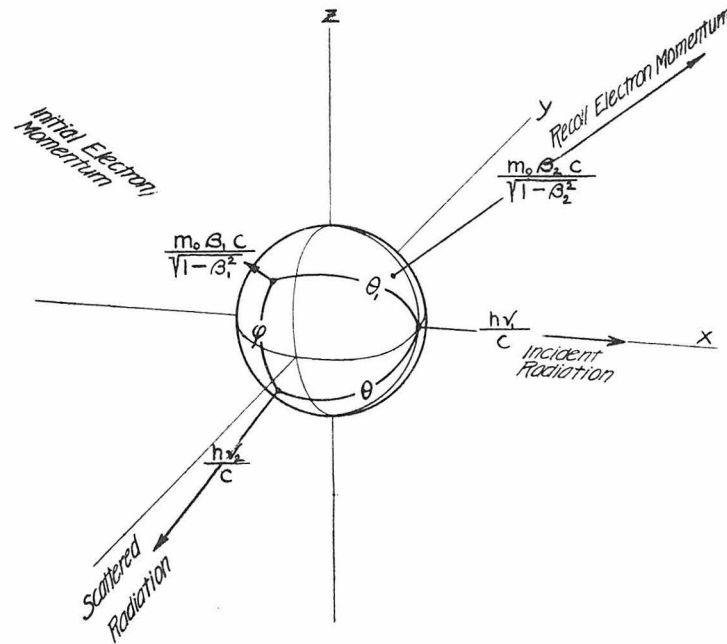

Fig. 16. Various angles and vectors involved in Compton scattering by an electron having initial momentum whose vector is here exaggerated.

$$
\begin{aligned}
h\nu_1 + m_0 c^2/(1 - \beta_1^2)^{1/2} &= h\nu_2 + m_0 c^2/(1 - \beta_2^2)^{1/2} \\
h\nu_1/c + (m_0\beta_1 c/(1 - \beta_1^2)^{1/2})a_1 &= (h\nu_2/c)p + (m_0\beta_2 c/(1 - \beta_2^2)^{1/2})a_2 \\
(m_0\beta_1 c/(1 - \beta_1^2)^{1/2})b_1 &= (h\nu_2/c)q + (m_0\beta_2 c/(1 - \beta_2^2)^{1/2})b_2 \\
(m_0\beta_1 c/(1 - \beta_1^2)^{1/2})c_1 &= (h\nu_2/c)r + (m_0\beta_2 c/(1 - \beta_2^2)^{1/2})c_2
\end{aligned} \right\} - (5)
$$

Here ν_1 is the frequency of the primary quantum. The velocity of the electron before scattering is $\beta_1 c$ and has direction cosines a_1, b_1, c_1. The scattered quantum has the

frequency ν_2' and direction cosines p,q,r. The recoiling electron has a velocity $\beta_2 c$ and direction cosines a_2, b_2, c_2. The angles are as shown in the figure. The sphere in this figure and in Fig. 17 has no special significance, being merely to aid in visualizing the figure in three dimensions.

Eliminating a_2, b_2, c_2 and β_2 from Eqs. (5) we obtain for the change in wave-length

$$\lambda_2 - \lambda_1 = \frac{\beta_1(\cos\theta_1 - \cos\phi)}{1 - \beta_1\cos\theta_1}\lambda_1 + \frac{2\alpha\lambda_1\sin^2\frac{1}{2}\theta}{1 - \beta_1\cos\theta_1} \quad - - - (6)$$

where $\alpha = h\nu_1/m_0 c^2$. If β_1 is small so that we can consider the denominator as equal to one, the second term on the right is exactly the Compton shift in the first form of Eq. (1). The first term then represents the modification caused by the initial speed of the electron, β_1.

Since we are interested particularly in the breadth of the shifted line, let us choose our origin at the "center" of the shifted line (the shifted position for scattering by free, initially stationary electrons) and a coordinate w measured positively in the direction of increasing wave-length. Let

$$\lambda_c = \lambda_1 + 2\alpha\lambda_1\sin^2\frac{1}{2}\theta \quad - - - - - - (7)$$

$$w = \lambda_2 - \lambda_c$$

$$= \lambda_2 - \lambda_1 - 2\alpha\lambda_1\sin^2\frac{1}{2}\theta \quad - - - (8)$$

Rearranging Eq. (6),

$$(\lambda_2 - \lambda_1)(1 - \beta_1\cos\theta_1) = \lambda_1\beta_1(\cos\theta - \cos\phi) + 2\alpha\lambda_1\sin^2\frac{1}{2}\theta$$

$$w(1 - \beta_1\cos\theta_1) = \lambda_1\beta_1(\cos\theta_1 - \cos\phi) - 2\alpha\lambda_1\beta_1\cos\theta_1\sin^2\tfrac{1}{2}\theta$$

$$= \beta_1\cos\theta_1(\lambda_1 - 2\alpha\lambda_1\sin^2\tfrac{1}{2}\theta)$$

$$w = \frac{(\lambda_c\cos\theta_1 - \lambda_1\cos\phi)\beta_1}{1 - \beta_1\cos\theta_1} \quad - - - - (9)$$

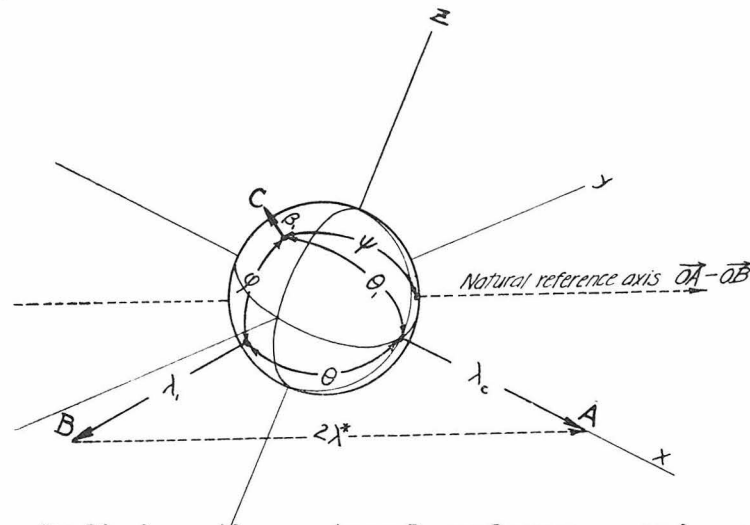This equation can be much simplified by shifting our reference axis from the direction of the incident quantum

Fig. 17. Defining the natural reference axis and the angle ψ, and showing graphically the quantity λ^*.

to the direction of the change in momentum suffered by a quantum in the simple Compton case of scattering by a single stationary electron at an angle θ. This is illustrated in Fig. 17.

Now let the vector OA be equal in length to λ_c but in the direction of the incident quantum, while OB is equal in length to λ_1 in the direction of the scattered quantum. This inversion is due to the fact that the momenta are inversely proportional to the wave-lengths.

Let the vector OC have the length β_1 and lie in the direction of the initial velocity of the electron. We now define a new wave-length λ^* such that the vector BA has a length $2\lambda^*$. If we write the numerator of Eq (9) as the difference of two scalar products,

$$OC \cdot OA - OC \cdot OB = OC \cdot (OA - OB)$$
$$= OC \cdot BA$$
$$= 2\beta_1 \lambda^* \cos\psi.$$

Thus we may write

$$w = \frac{\cos\psi}{1 - \beta_1 \cos\theta_1} \, 2\beta_1 \lambda^* \quad - - - - - (10)$$

Now β_1 is very small in most practical cases and if we neglect it we have at once that

$$- 2\beta_1 \lambda^* \leq w \leq 2\beta_1 \lambda^* \quad - - - - - - - - - - (11)$$

since the new natural reference axis is fixed by the directions of the incident and scattered radiation, and the angle ψ is determined solely by the direction of motion of the scattering electron and may have any value from 0° to 180°.

If we retain β_1 but consider β_1^2 as negligible with respect to one, and put in the value of θ_1 in terms of ψ, θ, and ϕ we get the inequality (11) with the term $2\beta_1^2 \lambda^* \cos\frac{1}{2}(\pi - \theta)$ added to each side. This gives the same range of values for w, but displaced slightly toward longer wave-lengths.

<u>Case of ensemble of electrons of speed βc and random direction.</u> It is seen from the above that the breadth of the shifted line due to an ensemble of electrons of equal, randomly directed velocities would be

$$\Delta \lambda = 4\beta_1 \lambda^* \quad - - - - - - - - - \quad (12)$$

It remains now to be seen what the structure of this line will be. We shall get a sufficiently good approximation if we neglect β_1 with respect to unity and put the denominator of (10) equal to unity at the outset. Since the initial electron velocities are equally distributed as to direction, it is easily shown that the probability of scattering by an electron whose velocity makes an angle between ψ and $\psi + d\psi$ with the above defined reference axis is given by the equation

$$P(\psi) \; d\psi = \tfrac{1}{2} \sin\psi \, d\psi. \quad - - - - - - - \quad (13)$$

Differentiating Eq. (10) we have that

$$dw = -2\beta_1 \lambda^* \sin\psi \, d\psi, \quad - - - - - \quad (14)$$

(neglecting β_1) and from these two equations we get that

$$P(w) \; dw = (4\beta_1 \lambda^*)^{-1} dw \quad - - - - - \quad (15)$$

Thus we see that the probability of a given deviation from the Compton shift is independent of the deviation w, and the contribution of this particular class of electrons to the line structure may be represented by a rectangle of width $4\beta_1 \lambda^*$ and area proportional to the number of electrons in that particular speed class. See Fig. 18.
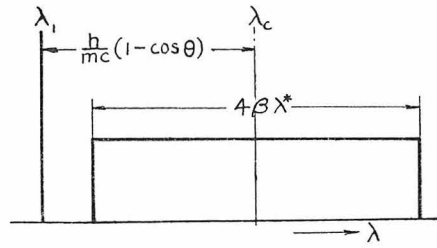
Fig. 18. Spectral distribution curve of originally monochromatic radiation after scattering through angle θ by ensemble of electrons of initial speeds βc and random directions.

As stated above this spectral distribution is not quite centered on the Compton shifted position but is displaced to the right by the amount $2\beta_1^2 \lambda^* \cos\frac{1}{2}(\pi - \theta)$. The error in Eq. (15) is about 3 percent in the widest part of the line structure and much less in the narrower parts.

$\underline{\text{Case of an ensemble of electrons with random direc-}}$ $\underline{\text{tions and any speed distribution, } \Phi(\beta).}$ If we have a given distribution of electron velocities in the scatter-
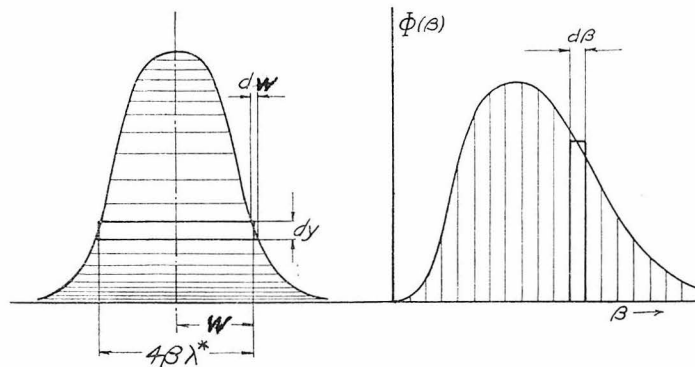

Fig. 19. Relation between spectral intensity distribution (left) and population of electron speed states (right).

ing body, for example such as shown by the right hand curve $\Phi(\beta)$ of Fig. 19, we can construct the spectral line by building it up of rectangles of decreasing lengths, whose dimensions are determined in the above manner. That is, for each element of area $\Phi(\beta)d\beta$ we construct a rectangle $4\beta\lambda^*dy$ whose area is proportional to that of the former. If k is the proportionality constant,

$$2w \, dy = k\Phi(\beta) \, d\beta$$
$$= k\Phi(w/2\lambda^*) \, dw/2\lambda^* \quad - - - - - (16)$$

since $w = 2\beta\lambda^*$. From this the equation of the line structure can be gotten by integrating from $y = 0$, $w = \infty$ to $y = y$, $w = w$.

$$y = - k' \int_{w=\infty}^{w=w} w^{-1}\Phi(w/2\lambda^*)dw \quad - - - (17)$$

Dependence of line breadth on scattering angle. When the primary wavelength and the scattering material are kept the same while varying the scattering angle, the only variable affecting the lengths of the elementary rectangles going to make up the scattered line, is λ^*. Therefore the effect on the spectral line at any fractional part of its height is simply to increase or diminish its breadth in the same ratio as the change in λ^*. What this change will be can be most readily seen by referring to Fig. 20. Since λ_c and λ_1 are very nearly equal,

$$\lambda^* = \lambda_1 \sin\tfrac{1}{2}\theta \quad - - - - - - - (18)$$

approximately, and the breadth of the line at half its

maximum height, for instance, is proportional to the sine of half the scattering angle as has been asserted above in Eq. (4). This gives a very satisfactory opportunity for a simple experimental test of the theory.
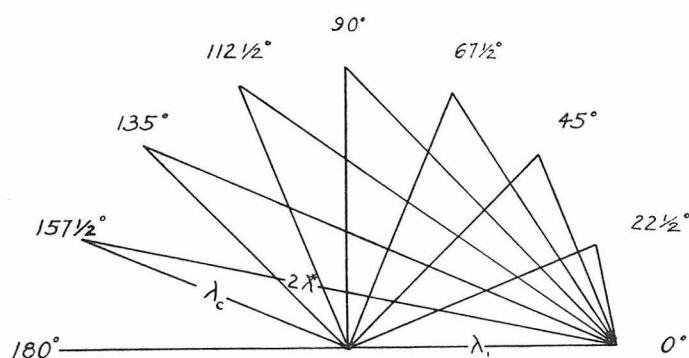

Fig. 20. Graphical construction for $\lambda*$.

IV. EXPERIMENTAL WORK TO TEST EQUATION (4).

a. Geometrical Dispositions of Tube and Scatterer, and the Exposure Times Required.

To test the change of breadth with scattering angle, three exposures were made at angles of $63°30'$, $90°$, and $156°27'$, using molybdenum K-radiation, and carbon in the form of graphite from the Acheson Graphite Company as the scatterer.

The geometrical arrangements of the tube and scatterer with respect to the spectrograph are shown in Fig. 21.

The spectrum in the first order was used. The necessary
exposure time depends principally on the distances from
tube to scatterer since the inverse square law is oper-
ative over that part of the path of the radiation, while
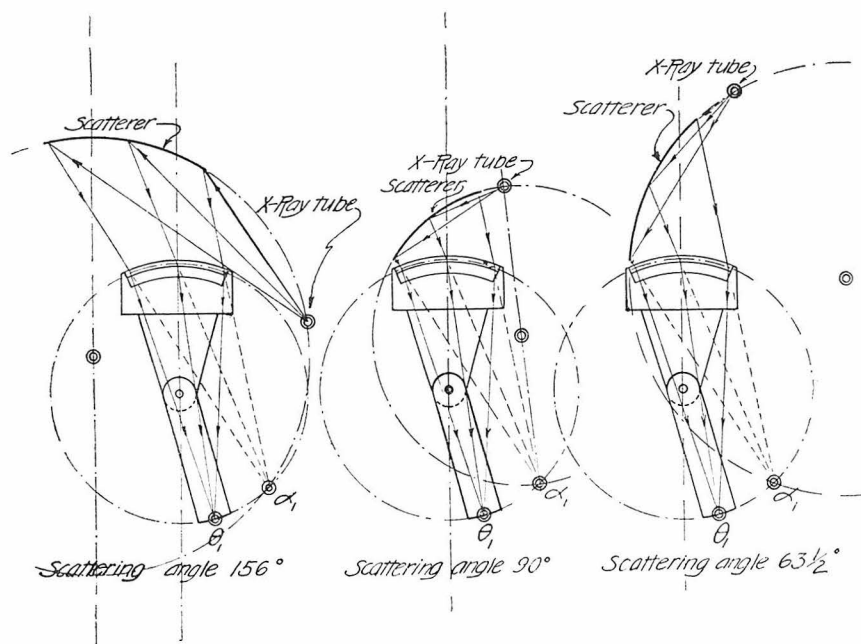the intensity after scattering is roughly proportional to

Fig. 21. Geometrical dispositions of x-ray tube, scatterer
and spectrograph for the three scattering angles studied.

the inverse of the first power only, of the distance.
Having experimentally determined the time required for
one scattering angle, it is possible to get a very good
estimate of the time required with any other arrangement
by calculating in each case the mean value of the inverse
squares of the distances from the tube to the positions
on the scatterer from which each crystal receives its ra-
diation. The x-ray tube, an ordinary water cooled Coolidge

tube, was run continuously twenty-four hours a day at about 50 kv and 20 ma.

Considerable care was exercised to insure that the scatterer and tube were accurately placed. A radius arm was erected which carried at its outer end a plumb-bob which could be raised and lowered into close proximity with the curved scatterer and the point α_1. Two auxiliary plumb-lines were used to place the x-ray tube in position by sighting from two directions on the focal spot and the principal plumb-line above it. The scattering angle was determined by measuring carefully the angle through which the radius arm was turned when moved from the position above the focal spot to that above α_1.

Inhomogeneity of scattering angle is caused principally by the width of the focal spot and the thickness of the scatterer, but in no case could have been greater than one degree. The corresponding excess breadth could be only of the order of magnitude of the width of the narrower unshifted lines.

b. Results.

The spectra of scattered radiation secured at these three angles are shown in Fig. 22. It will be noted that the unshifted lines are sharp and clearly resolved. This fact furnishes unambiguous proof that the breadths of the shifted lines are not due to poor resolution or poor ori-
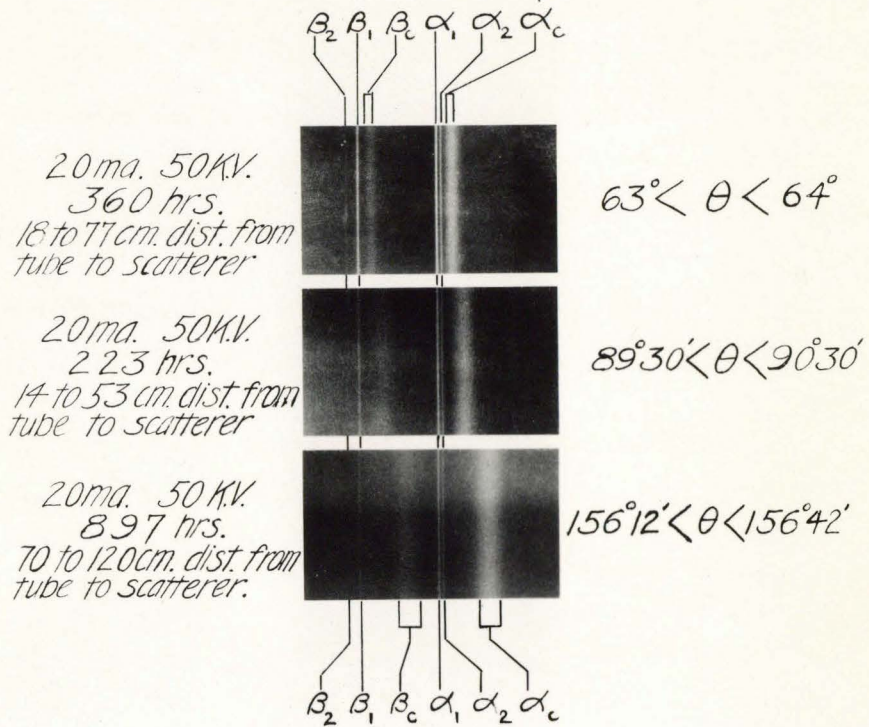
Fig. 22.

entation of the crystals.

Fig. 23 is a reproduction of sample microphotometer curves secured from the three negatives shown in Fig. 22.
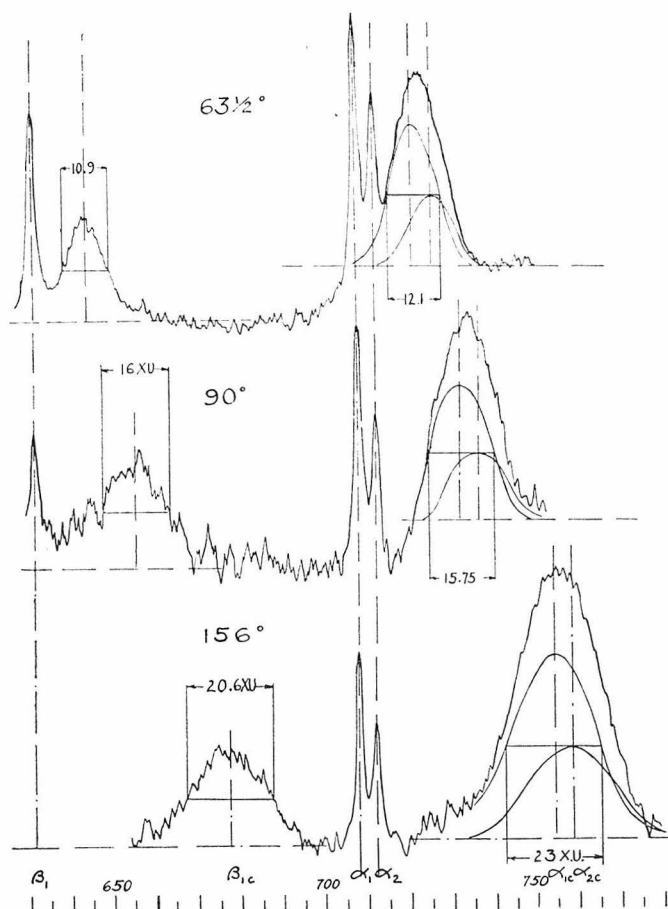
Fig, 23. Typical microphotometer curves of the spectra shown in Fig. 22.

The modified α-doublet has been resolved into its two components by an analytical-graphical method devised by Dr. Dumond[37] which is quite simple of application in this case since we know the relative intensities of the two components to be in the ratio 2:1 and that their separation is 4 X.U., exactly as with the primary lines.

The β line is also a doublet but the separation is only
0.6 X.U. and the ratio of the intensities of the two com-
ponents is much greater than in the case of the α-doublet,
so the effect of the smaller one is considered negligible.

Five different microphotometer curves from each of
the three films shown were measured to obtain the data
given in Table I.

Table I. Breadths in x-units at half maximums.

63°			90°			156°			
α_{12c}	α_{1c}	β_{1c}	α_{12c}	α_{1c}	β_{1c}	α_{12c}	α_{1c}	β_{1c}	
13.0	12.1	12.0	16.2	15.6	13.3	22.4	22.8	22.4	
12.4	11.5	10.9	16.4	15.75	16.0	21.8	21.2	19.4	
13.0	12.1	11.5	15.75	15.15	15.75	23.0	23.0	20.6	
13.0	12.1	12.1	16.4	15.75	16.0	23.0	23.0	20.6	
13.0	12.1	10.9	16.4	15.75	15.2	22.2	21.9	21.5	
12.9	12.0	11.5	16.2	15.6	15.2	22.5	22.4	20.9	Avg.
11.8			15.4				21.6		Grand average α_{1c} and β_{1c}
51%			69%				97%		Theor. relative breadths

These runs were taken at different heights on the films in
order to eliminate irregularities due to the grain of the
film and other spurious effects. Also the reliability of
the individual runs was increased by using the image of a
slit parallel to the spectral lines, instead of the usual
small spot of light, to scan the films. The microphotometer
itself is very reliable. Successive runs across the same

part of any film were found to be practically identical, and the small irregularities in the curves can be said to be due entirely to the films.

The breadths of α_{12_c} and β_{1_c} at half maximum were measured directly from the curves. Those of α_{1_c} were measured from the graphical decompositions, as shown in Fig. 23.
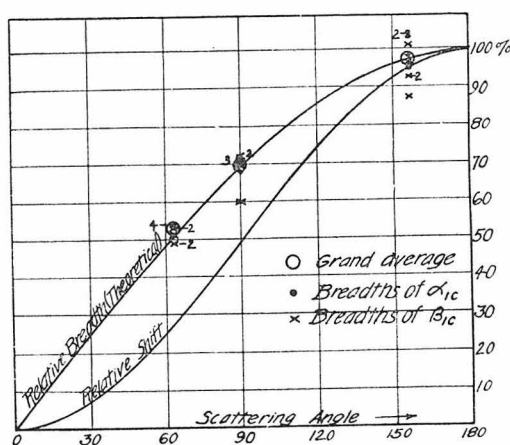

Fig. 24. Measured relative breadths of shifted lines compared with theoretical curve.

A comparison of the experimental results with theory is shown in Fig. 24. The full line curves are the same ones shown in Fig. 2, the relative breadth curve being a $\sin\frac{1}{2}\theta$ curve while the relative shift curve is a $\sin^2\frac{1}{2}\theta$ curve. The plotted points are the averages from Table I. Since the curves represent only relative values it was necessary to fit one point of each set arbitrarily, and this was done with the upper points. The others are seen to fall quite close to their predicted positions, and

indicate a very good agreement with the theoretical result that the relative breadths are proportional to $\sin\frac{1}{2}\theta$.

It should also be noted that the results are apparently in agreement with another prediction of the theory. If we combine Eqs. (12) and (18), we have,

$$\Delta\lambda = 4\beta_1\lambda_1\sin\tfrac{1}{2}\theta \quad \text{(approximately)} - - (19)$$

which tells us that the breadth is also proportional to the primary wave-length. This seems to be borne out by the figures in Table I, since the average breadths of α_{1c} are systemmatically greater than the average breadths of β_{1c}

However, the precision of the breadth measurements hardly warrants more than a mention of this apparent agreement, and a claim to confirmation on this point must await more data.

Confirmation of Compton's equation. The positions of the shifted lines according to the Compton formula (Eq. (1)) are shown on Fig. 23 as vertical broken lines, and the actual lines appear to occur precisely at those positions.

c. Discussion of Results.

After these results had been obtained, but before their publication, two other experimenters published results which up to the present time have not been reconciled with ours. Bearden[42] and Gingrich[43] both reported the partial resolution of the modified K -doublet into

two peaks, using double crystal spectrometers.

Both had much less homogeneity of scattering angle than ours and for that reason should have gotten broader lines for a given scattering angle, although that was largely offset by the fact that they worked at large angles only. Bearden used a silver target tube so his primary radiation was some what shorter in wave-length than ours. Gingrich worked with molybdenum radiation but with much greater primary intensities. Bearden's lines would be narrower because of the shorter wave-length, but Gingrich's should be broader than ours because of his in-homogeneity of angle.

At first it was thought that because of our much more extended scatterer there was a possibility that multiple scattering might be present to a greater extent in our ex-periment. This would broaden the modified line, first be-cause multiply modified radiation would have a greater in-crease in wave-length, and second because radiation unmod-ified at the first scattering and modified by scattering at a smaller angle the second time would be less changed in wave-length than any of the singly scattered radiation. Accordingly a scatterer was constructed of graphite divided into units screened from each other by lead strips in such a way that there was practically no interference with single scattering, but preventing the exchange of scattered

radiation between the units. This confined the multiple scattering to volumes less than that of the scatterer used by Gingrich and not much larger than that of Bearden's. The result of this test was a negative which could not be distinguished from the corresponding one obtained without this precaution. It is now certain that our modified lines are not appreciably broadened by multiple scattering.

We have also used a silver target tube to see if the resolution appeared with shorter wave-lengths, but while the line is narrower, as it should be theoretically, there is no evidence of any resolution into the two components.

Furthermore it is difficult to understand how there could be any such resolution of peaks so close together and as broad as we find them to be, unless the shape of the peaks were greatly different from the inverted parabola which appears best to describe the ones we get.

At present we have no explanation to offer for the discrepancy between their results and ours. The double crystal spectrometer is capable of higher resolution than we can claim, but on the other hand, our resolution is sufficiently good as is attested by the separation and sharpness of the unmodified lines. The ionization chamber method which must be used with the double crystal spectrometer is subject to many difficulties. The background intensity is great, and accidental fluctuations in

the ion current occur which are of the order of magnitude of the peaks sought for, so reliable results are obtained with scattered radiation only by exercising great care and taking a great deal of data. Our photographic results are entirely reproducible.

At the present time it seems to us most probable that the broad line which we get represents the true shape of the Compton modified line, and that the modified K_α-doublet cannot be resolved by the spectrometer into its two components. Previous researches by Sharp,[44] Dumond,[33,34] and Nutting[45] have yielded lines quite similar to ours, but the most satisfactory confirmation of our results has recently come from the work of Mr. Archer Hoyt here in this laboratory. Using a double crystal spectrometer and an unusually sensitive and dependable methyl bromide ionization chamber, both designed and constructed by Dr. Dumond and himself, he has obtained ionization curves of scattering by graphite showing in each case a broad shifted line, and the curve indicated by plotting a number of his runs on the same graph shows no indication of resolution into the two components.

d. Absolute Velocities of Electrons.

As yet no careful study of the absolute electron velocities to be expected from the structure of our modified lines has been made. It can be said however that the

breadth of the lines at half maximum corresponds to a class of electrons for which $\beta = 0.0076$, which is equivalent to about 15 volts. This seems a reasonable value for electrons in the carbon atom.

It is our intention to make an analysis of the above and additional data in the near future, to determine as nearly as possible the complete distribution of electron velocities in the atom on the assumptions and theoretical conclusions discussed herein.

At the present time we are seeking confirmation of the predicted variation of breadth with primary wave-length, by using radiation from other targets than molybdenum.

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