AN ANALYSIS OF THE STRESSES IN THE WALL SLAB OF A COUNTERFORTED RETAINING WALL.

Thesis by

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GENERAL STATEMENT OF PROBLEM

The purpose of this paper is to analyze the stresses in the wall slab of a counterforted retaining wall and to determine from this analysis, if possible, whether or not the conventional method of design for the slab is economical.

The conventional method of designing the wall slab of a counterforted retaining wall is to consider the wall as a series of independent horizontal strips between the counterforts. These strips are then designed either as continuous beams or as simple beams extending from one counterfort to the next. This method completely neglects the effect of cantilever action from the horizontal base slab where it joins the vertical wall. In the case of large counterfort spacing this effect may be considerable and it is possible that some reduction in material could be made if this effect were accounted for in the design calculations.

In order to throw some light on the effect of neglecting the cantilever action, the following analysis has been made on a wall slab designed by the conventional method. The dimensions of the slab are as given in the drawing on page 5.

ASSUMPTIONS.

Throughout the deflection calculations the moment of inertia, "I", of the horizontal beam strips and the vertical cantilever strips was assumed as that of solid concrete of the full depth of the beam or cantilever. Under this assumption no allowance was made for the effect of reinforcing. This would not have been possible anyway without assuming some sustem of reinforcing in the cantilevers.

In the determination of the flexural and shear stresses, the assumption was made that sufficient reinforcing steel was in place to take all tensile stresses due to bending, both in the cantilevers and in the beam strips. It was further assumed that this steel was imbedded in concrete three inches measured from the face of the concrete to the center of the steel bars.

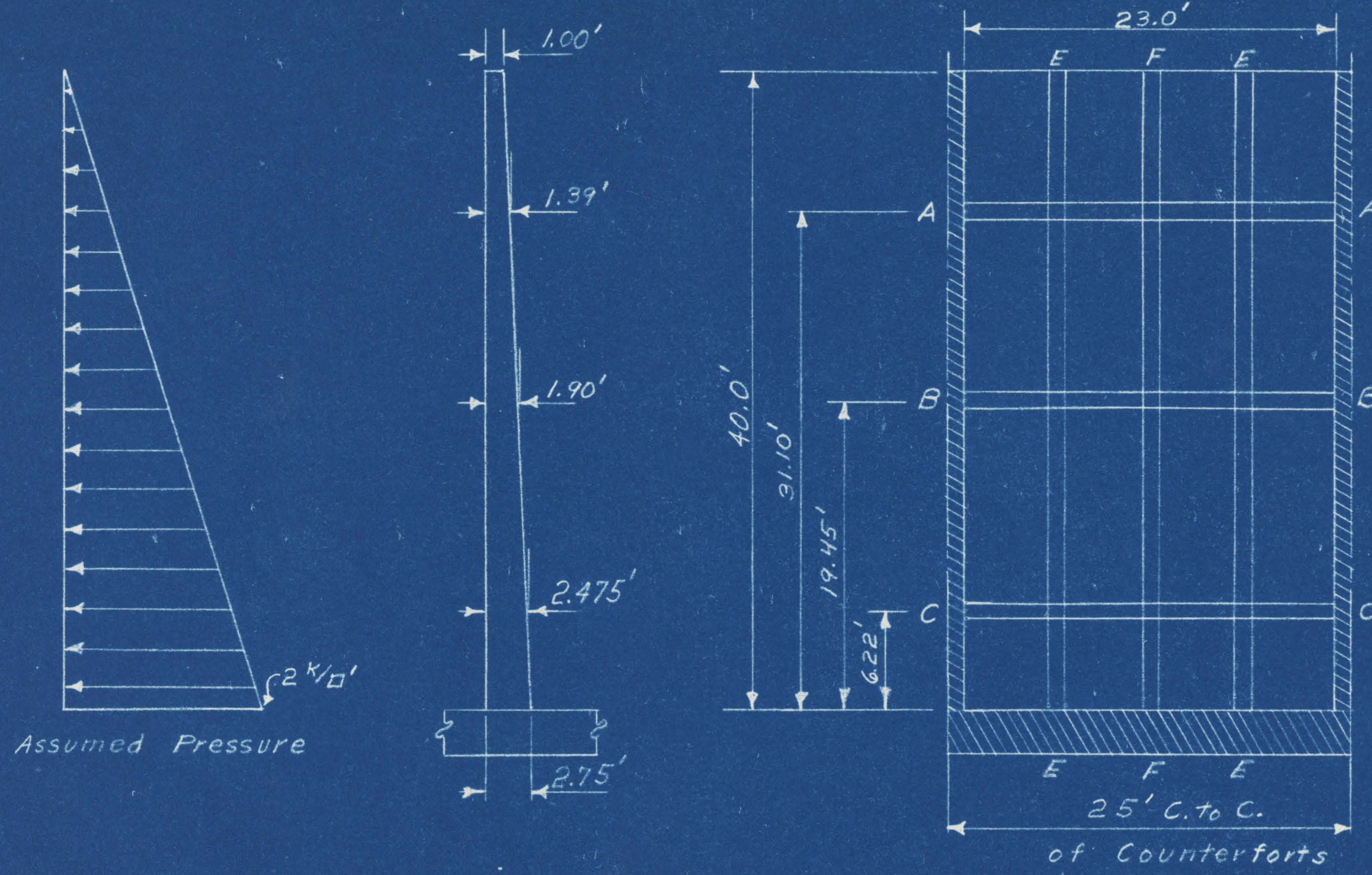
In the calculation of the cantilever and beam strip deflections, no allowance was made for the twist of either cantilever or beam strips in order to afford continuity of the wall. This effect was assumed negligible.

The wall slab was assumed to be a slab rigidly fixed on three sides and free on the fourth side. Although this may not be strictly true, it is believed to be sufficiently near the truth for purposes of this analysis.

CALCULATIONS.

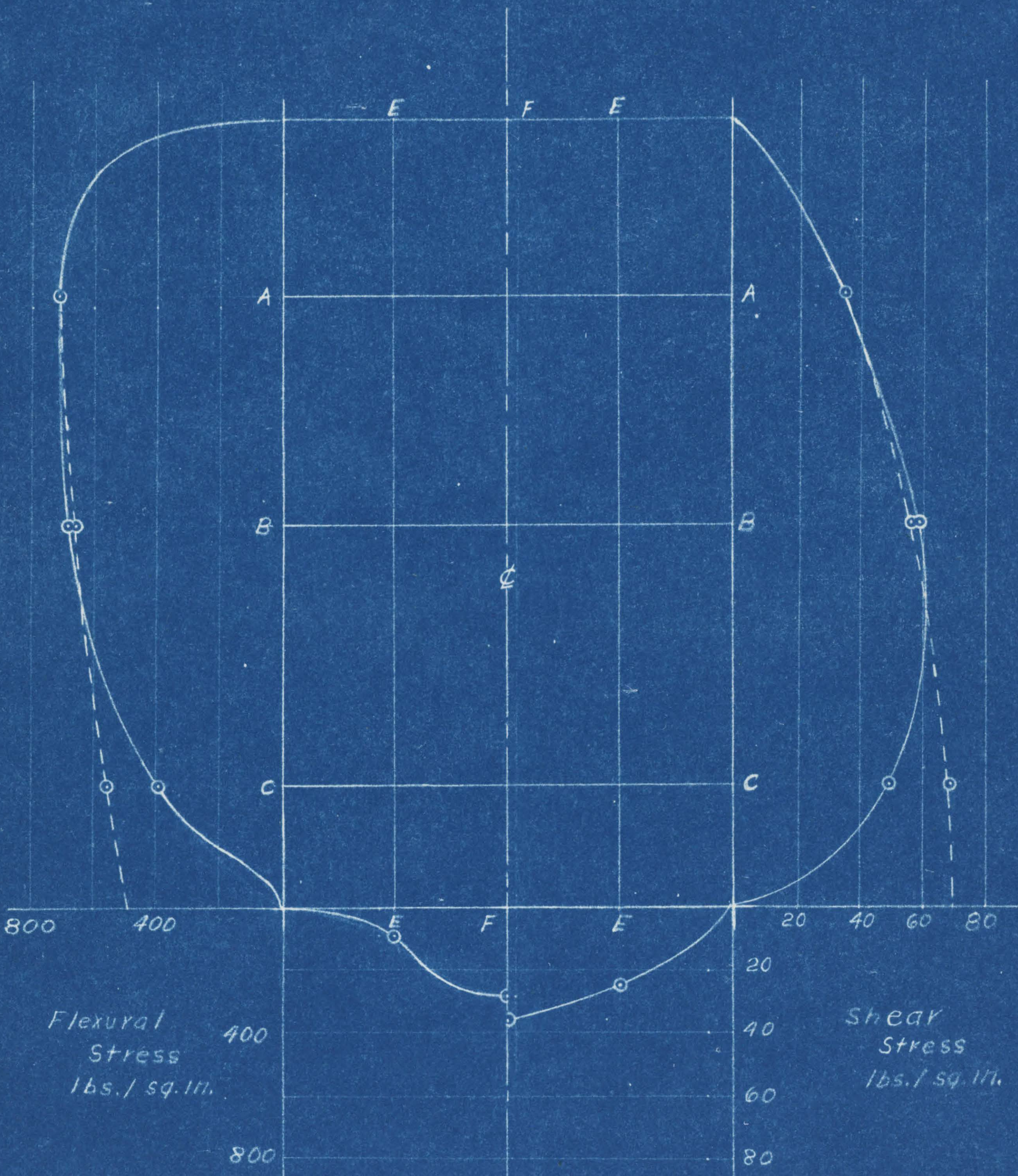
The method of calculation used is based on the principle of dividing the slab into vertical cantilever strips and horizontal beam strips. Then the load is distributed between these two systems of strips so that the corresponding points in each system will have an equal deflection. This load distribution is accomplished by a trial and error process until the deflections are approximately equal.

The reader is referred to Appendix A for a copy of sample calculations.



Wall Slab of a Counterforted Retaining Wall
which was designed neglecting cantilever action.

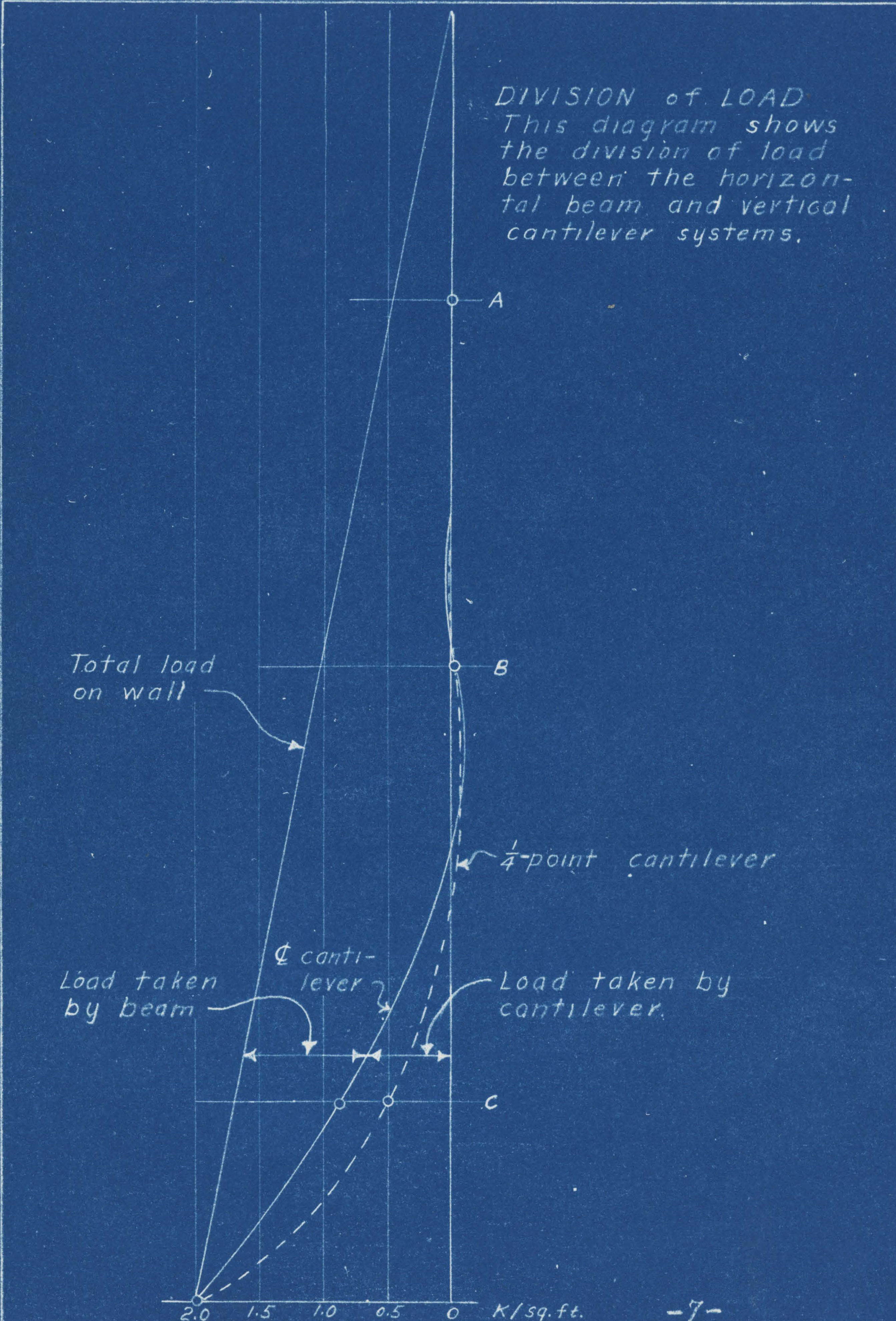
STRESS DISTRIBUTION — Flexure & Shear



Symmetrical about ℄

Considering cantilever action ————

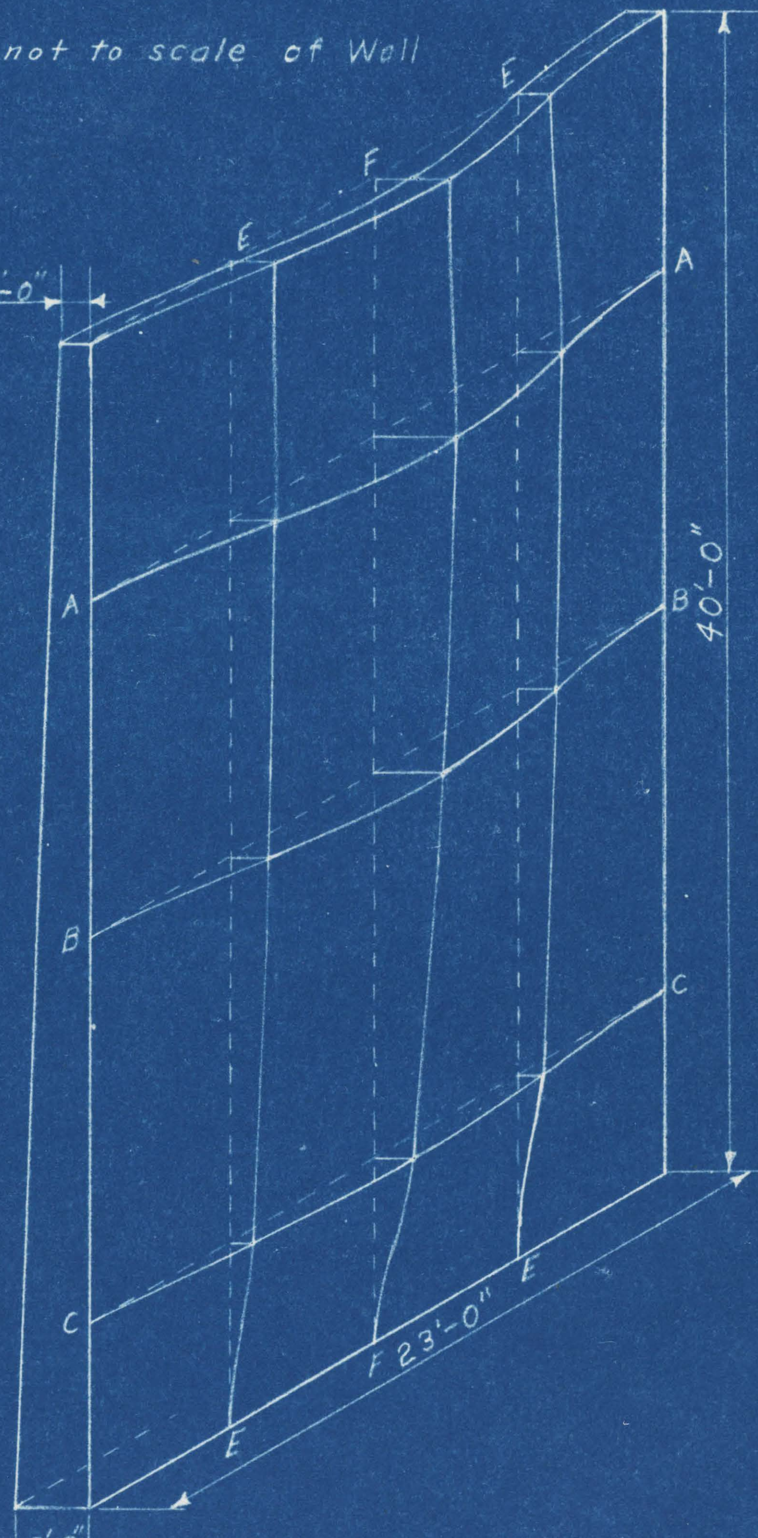
Neglecting cantilever action - - - -


DIVISION of LOAD
This diagram shows the division of load between the horizontal beam and vertical cantilever systems.
A
B
C
Total load on wall
1/4-point cantilever
℄ cantilever
Load taken by beam
Load taken by cantilever.
2.0
1.5
1.0
0.5
0
K/sq.ft.

ISOMETRIC VIEW of WALL

Showing bulging effect

Deflections not to scale of Wall



CONCLUSIONS.

A study of the calculations and of the stress diagrams, as shown on the previous pages, leads the author to believe that the conventional method of design of the wall slab is not too uneconomical. From the detail of the calculations and to some extent visible from the resulting curves it becomes apparent that any decrease in the thickness of the wall near its base would greatly reduce the proportion of the load taken by the cantilevers. This decrease in cantilever load must be absorbed as an additional load on the beam strips. Consequently, the beam strips having greater load will require nearly as great a depth as if designed neglecting cantilever effect. Thus very little saving in concrete appears possible from consideration of the cantilever effect in the design.

In regard to the economical use of reinforcing steel, it may be possible to effect a saving by reinforcing the base of the wall for cantilever action, thus reducing the amount of horizontal reinforcing necessary in this region. However, the answer to this question is beyond the scope of this paper and the idea is merely suggested as a possibility.

The most serious difficulty with the design of a wall slab by considering the effect of cantilever action is that the only method so far developed is to assume a design and then check the stresses. With an analysis of stress as tedious as this one, unless the wall were very large, the amount of extra labor necessary to effect such a design would be an unwarranted expense.

In any further investigation of this subject by a similiar method of analysis, the author would suggest that the investigator use a greater number of horizontal strips near the base of the wall in his calculations as the cantilever effect beyond a certain height becomes relatively insignificant.

APPENDIX A

C A L C U L A T I O N S

OUTLINE of CALCULATIONS:

All deflection coefficients for the beam strips in bending were taken from the A.I.S.C. Steel Construction Manual. All other deflection coefficients were calculated using the formulas:

Bending $\delta = \int \frac{M m \, dx}{I}$

Shear $\delta = \int \frac{V v \, dx}{\frac{2}{5} I}$

E being constant throughout the problem, it was assumed as unity to simplify computations.

All units of length are in feet.

Coefficients of deflection for cantilevers "J_{A-B}" means the deflection at strip A-A of the cantilever in question due to a unit load on the cantilever at strip B-B, etc.

Coefficients of deflection for beams "$K_{BE_{L-R}}$" means the deflection at point BE (left) due to a unit load on the beam strip at BE (right), etc.

The beam strips are 1 ft wide.

The cantilever strips are $\frac{23}{40}$ ft. wide.

In many instances the use of Maxwell's reciprocal deflections served to shorten the work considerably.

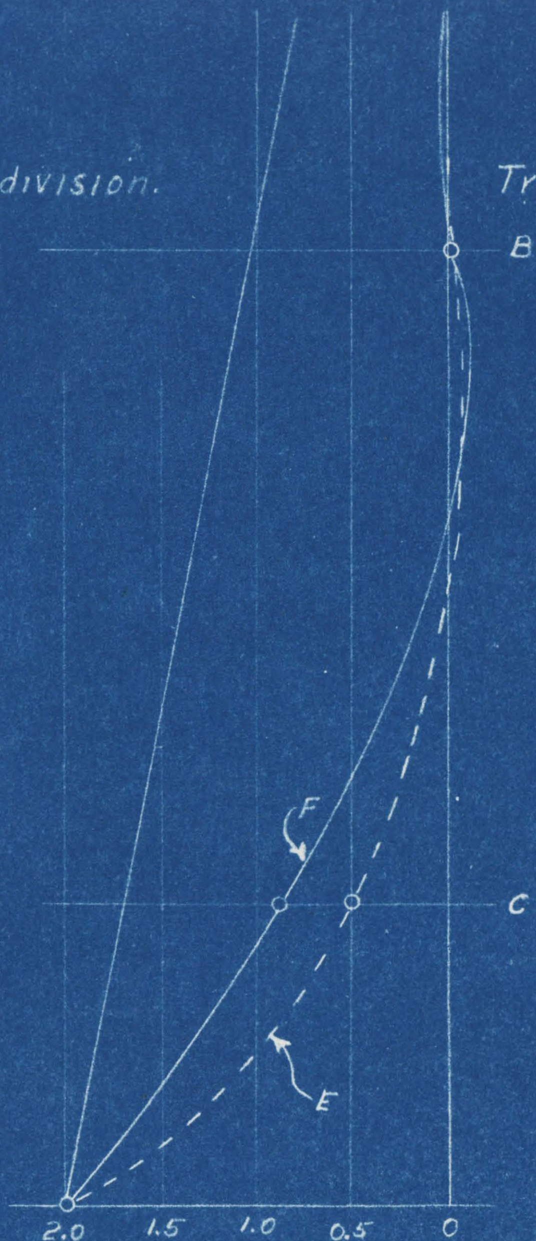
Coefficients of Deflection "K" for Beam Strips.

Point & load	Flexure	Shear	"K" Total
AE_L-C	141.0	5.2	146.2
AE_L-L	119.0	8.1	127.1
AE_L-R	57.4	2.7	60.1
AF-E	141.0	5.2	146.2
AF-C	283.0	10.4	293.4
BE_L-C	46.2	3.8	50.0
BE_L-L	39.0	5.9	44.9
BE_L-R	18.7	2.0	20.7
BF-E	46.2	3.8	50.0
BF-C	92.3	7.6	99.9
CE_L-C	25.1	2.9	28.0
CE_L-L	21.2	4.5	25.7
CE_L-R	10.2	1.5	11.7
CF-E	25.1	2.9	28.0
CF-C	50.2	5.8	56.0

Coefficients of Deflection "J" for Cantilevers

Point & load	Flexure	Shear	Total	"J" $\times \frac{40}{23}$
A-A	9,460	39	9,499	16,500
A-B	3,765	21	3,786	6,580
A-C	399	6	405	703
B-A	3,765	21	3,786	6,580
B-B	1,860	21	1,881	3,270
B-C	252	6	258	448
C-A	399	6	405	703
C-B	252	6	258	448
C-C	86	6	92	160

Trial load division. Trial #14



Point	Total load	Beam load	Cantilever load
AE	0.45	0.45	0.0
BE	1.03	1.0595	-0.0295
CE	1.69	1.20	0.49
AF	.45	0.45	0.0
BF	1.03	1.085	-0.055
CF	1.69	0.81	0.88

Trial loading deflections: Trial #14

BEAM SYSTEM

Point AE_L

$\delta_{A\text{-}E_L} = K_{AE_L\text{-}L} \times \text{Load } AE$

127.1 × .45 = 57.2

$+ K_{AE_L\text{-}C} \times \text{Load } AF$

146.2 × .45 = 65.8

$+ K_{AE_L\text{-}R} \times \text{Load } AE$

60.1 × .45 = 27.1

Total δ_{AE} ⟶ +150.1

Point AF

$\delta_{AF} = K_{AF\text{-}E} \times \text{Load } AE$

146.2 × .45 = 65.8

× 2 = 131.6

$+ K_{AF\text{-}C} \times \text{Load } AF$

293.4 × .45 = 131.8

Total δ_{AF} ⟶ 263.4

Point BE_L

$\delta_{BE_L} = K_{BE_L\text{-}L} \times \text{Load } BE$

44.9 × 1.0595 = 47.6

$+ K_{BE_L\text{-}C} \times \text{Load } BF$

50.0 × 1.085 = 54.2

$+ K_{BE_L\text{-}R} \times \text{Load } BE$

20.7 × 1.0595 = 21.9

Total δ_{BE} ⟶ 123.7

Point BF

$\delta_{BF} = K_{BF\text{-}E} \times \text{Load } BE$

50.0 × 1.0595 = 52.95

× 2 = 105.9

$+ K_{BF\text{-}C} \times \text{Load } BF$

99.9 × 1.085 = 108.3

Total δ_{BF} ⟶ 214.2

Point CE_L

$\delta_{CE_L} = K_{CE_L\text{-}L} \times \text{Load } CE$

25.7 × 1.20 = 30.8

$+ K_{CE_L\text{-}C} \times \text{Load } CF$

28.0 × .81 = 22.7

$+ K_{CE_L\text{-}R} \times \text{Load } CE$

11.7 × 1.20 = 14.0

Total δ_{CE} ⟶ 67.5

Point CF

$\delta_{CF} = K_{CF\text{-}E} \times \text{Load } CE$

28.0 × 1.20 = 33.6

× 2 = 67.2

$+ K_{CF\text{-}C} \times \text{Load } CF$

56.0 × .81 = 45.3

Total δ_{CF} ⟶ 112.5

Trial loading deflections: Trial #14

CANTILEVER SYSTEM

Point AE

$\delta_{AE} = J_{A-A} \times$ Load AE
16,500 x 0 = 0.
+ $J_{A-B} \times$ Load BE
6,580 x -.0295 = -194.0
+ $J_{A-C} \times$ Load CE
703 x .49 = 345.0
Total δ_{AE} → 151.0

Point BE

$\delta_{BE} = J_{B-A} \times$ Load AE
6,580 x 0 = 0
+ $J_{B-B} \times$ Load BE
3,270 x -.0295 = -96.5
+ $J_{B-C} \times$ Load CE
448 x .49 = 219.5
Total δ_{BE} → 123.0

Point CE

$\delta_{CE} = J_{C-A} \times$ Load AE
703 x 0 = 0
+ $J_{C-B} \times$ Load BE
448 x -.0295 = -13.2
+ $J_{C-C} \times$ Load CE
160 x .49 = 78.3
Total δ_{CE} → 65.1

Point AF

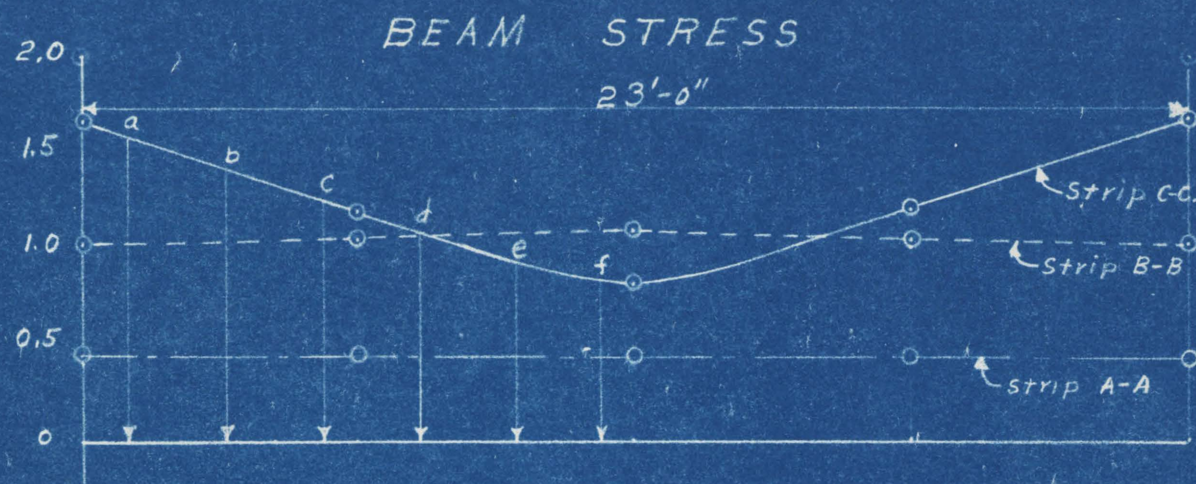
$\delta_{AF} = J_{A-A} \times$ Load AF
16,500 x 0 = 0
+ $J_{A-B} \times$ Load BF
6,580 x -.055 = -362.0
+ $J_{A-C} \times$ Load CF
703 x .88 = 619.0
Total δ_{AF} → 257.0

Point BF

$\delta_{BF} = J_{B-A} \times$ Load AF
6,580 x 0 = 0
+ $J_{B-B} \times$ Load BF
3,270 x -.055 = -179.7
+ $J_{B-C} \times$ Load CF
448 x .88 = 395.0
Total δ_{BF} → 215.3

Point CF

$\delta_{CF} = J_{C-A} \times$ Load AF
703 x 0 = 0
+ $J_{C-B} \times$ Load BF
448 x -.055 = -24.6
+ $J_{C-C} \times$ Load CF
160 x .88 = 140.6
Total δ_{CF} → 116.0



Calculation of Beam End Moments.

Strip C-C.

Element	Load-P.	ℓ^2	a	a^2	b	b^2	$M_L = \frac{Pab^2}{\ell^2}$	$M_R = \frac{Pa^2b}{\ell^2}$
a	3.16	529	1	1	22	484	2.89	.13
b	2.91	↓	3	9	20	400	6.60	.99
c	2.52		5	25	18	324	7.82	2.14
d	2.16		7	49	16	256	7.32	3.20
e	1.85		9	81	14	196	6.17	3.97
f	1.46		10.75	115.5	12.25	150	4.46	3.91
	13.86						35.26	14.34

End Mom. C-C = 35.26 + 14.34 = 49.60 ft-k

End Shear C-C = 13.86 k

End Mom. C-C (Neglecting cantilever action) = $\frac{1.69 \times 23^2}{12} = 74.6$ ft.-k.

Strip B-B.

End Mom. B-B = $\frac{1.03 \times 23^2}{12} + \frac{5 \times .028 \times 23^2}{48} = 46.9$ ft.-k

End Shear B-B = 12.17 k

End Mom. B-B (Neglecting cantilever action) = $\frac{1.03 \times 23^2}{12} = 45.4$ ft.-k

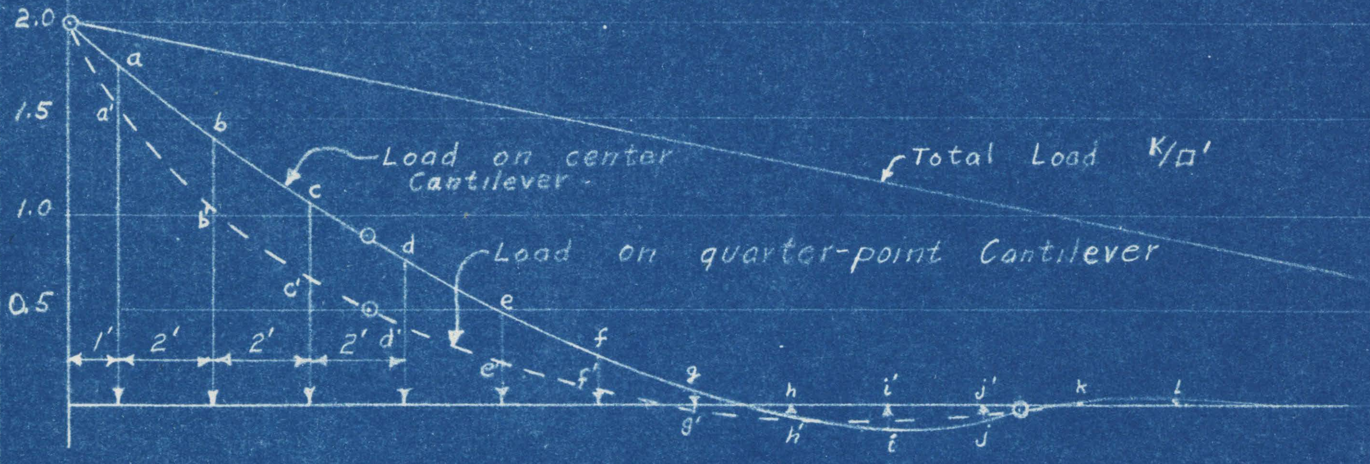
Strip A-A.

No cantilever effect.

End Mom. A-A = $\frac{.45 \times 23^2}{12} = 19.8$ ft-k

End Shear A-A = .45 x 11.5 = 5.18 k.

CANTILEVER STRESS



Calculation of Cantilever Base Moments:

Center

Element	Load	Mom-Arm.	Moment
a	3.56	1.0	3.56
b	2.76	3.0	8.28
c	2.12	5.0	10.06
d	1.52	7.0	10.63
e	0.96	9.0	8.63
f	0.54	11.0	5.94
g	0.16	13.0	2.08
h	-0.11	15.0	-1.65
i	-0.27	17.0	-4.58
j	-0.18	19.0	-3.42
k	0.07	21.0	1.47
l	0.09	23.0	2.07

Σ Mom. ⟶ 43.61 ft-K/ft

Σ Shear 11.22 K/ft.

$$f_c = \frac{43.61 \times 12{,}000}{6 \times .866 \times .403 \times 30^2} = 277 \text{ psi}$$

$$v = \frac{11.22 \times 1000}{12 \times .866 \times 30} = 36 \text{ psi}$$

Quarter-point

Element	Load	Mom-Arm	Moment
a'	3.10	1.0	3.10
b'	2.08	3.0	6.23
c'	1.32	5.0	6.60
d'	0.80	7.0	5.60
e'	0.44	9.0	3.96
f'	0.17	11.0	1.87
g'	-0.07	13.0	-.91
h'	-0.11	15.0	-1.65
i'	-0.18	17.0	-3.68
j'	-0.09	19.0	-1.71
k'	0	21.0	0
l'	0	23.0	0

Σ Mom ⟶ 13.78 ft-K/ft

Σ Shear 7.46 K/ft

$$f_c = \frac{13.78 \times 12{,}000}{6 \times .866 \times .403 \times 30^2} = 87.5 \text{ psi.}$$

$$v = \frac{7.46 \times 1000}{12 \times .866 \times 30} = 24 \text{ psi.}$$

The above calculations are assuming the cantilevers were adequately reinforced to take these stresses.

Calculation of Beam Stress:

Strip C-C

With cantilever action:

$$f_c = \frac{49.6 \times 12{,}000}{6 \times .866 \times .403 \times (26.74)^2} = 397 \text{ psi.}$$

$$v = \frac{13.86 \times 1000}{12 \times .866 \times 26.74} = 50 \text{ psi.}$$

Without cantilever action:

$$f_c = \frac{74.6 \times 12{,}000}{6 \times .866 \times .403 \times (26.74)^2} = 594 \text{ psi.}$$

$$v = \frac{1.69 \times 11.5 \times 1000}{12 \times .866 \times 26.74} = 69.8 \text{ psi.}$$

Strip B-B

With cantilever action:

$$f_c = \frac{46.9 \times 12\,000}{6 \times .866 \times .403 \times (19.8)^2} = 685 \text{ psi.}$$

$$v = \frac{12.17 \times 1000}{12 \times .866 \times 19.8} = 59.1 \text{ psi.}$$

Without cantilever action:

$$f_c = \frac{45.4 \times 12\,000}{6 \times .866 \times .403\,(19.8)^2} = 664 \text{ psi.}$$

$$v = \frac{1.03 \times 11.5 \times 1000}{12 \times .866 \times 19.8} = 57.6 \text{ psi.}$$

Strip A-A

With or Without cantilever action:

$$f_c = \frac{19.8 \times 12\,000}{6 \times .866 \times .403\,(13.7)^2} = 713 \text{ psi.}$$

$$v = \frac{5.18 \times 1000}{12 \times .866 \times 13.7} = 36.4 \text{ psi.}$$

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