ANALYSIS OF HINGELESS ARCHES BY DEFLECTION MEASUREMENTS OF ELASTIC MODELS WITH MATHEMATICAL ANALYSES BASED ON THE THEORY OF ELASTICITY

A THESIS

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Presented in partial fulfillment of the requirements for the degree of Master of Science at the California Institute of Technology

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Introduction.

The engineer of today is often confronted with the task of finding the stresses in statically indeterminate structures. In the past, engineers have evaded the issue to a large extent. Most of them have dealt only with statically determinate structures or hafe used rough approximate methods of figuring stresses in statically indeterminate structures.

In recent years, the solution of involved structures by the use of deflection measurements on elastic models has been coming into favor. Perhaps the most ingenious of the means that have veen devised for making model analyses is the Beggs Deformeter. This apparatus was used to test a celluloid model of a fixed ended arch. The effect of a superstructure was studied by taking readings first with the model intact and then with the superstructure cut off. Two separate analytical solutions of the plain arch rib were made in addition to the study make with the Beggs Deformeter.

It is not deemed necessary to give a detailed explanation of the Beggs Deformator in this paper. The article, "The Use of Models in the Solution of Indeterminate Structures", by Professor Geo. E. Beggs may be found in the Journal of the Franklin Institute of March, 1927. If this paper is read carefully one can get as clear and complete a picture of the method as is possible without actually doing the experimental work.

The method explained by Professor Beggs for determining the sense of any reaction or stress component, namely, "If the image of the load-point in the microscope moves in the direction of the assumed load, the reaction component acts in the same direction as the corresponding displacement of the support", is perfectly general. In all cases, however, the relative sense can be determined merely by using the same order of insertion of the plugs. Since we usually know the sense for at least one position of the load, it is possible to obtain the sense without applying the above-mentioned rule.

Cardboard has been suggested as a material for making models. It has been found that very accurate results are not obtainable. The reason advanced is that the modulus of elasticity of cardboard is not the same in all directions. For this reason, the model was cut from celluloid. A thickness of 1/32" was used.

There were two main objects in mind when the work was started. One was to gain familiarity with methods of model analysis as typified by the Beggs' Method. The other was to become familiar with the characteristics of fixed ended arches. The actual experimental work was therfore planned with these two primary objects in view.

The model analysis was made first with the superstructure as shown in Plate I. The model was then cut at the quarter point point and a floating gage inserted. This made it possible not only to take readings of the thrust, moment, and shear at the quarter point, but also to take readings of the thrust, moment, and shear at the reaction, by inserting neutral plugs in the gages at the quarter point. This gave a means of determining the error introduced in the reactions by cutting the model.

The next step was to cut away the superstructure and test the plain arch rib. The plain arch rib was tested first with the floating gage at the quarter points and then with the model welded. This enabled several comparisons to be made. The relative errors introduced by the presence of the floating gage with neutral plugs at the quarter point, and by having the model welded at the quarter point, could be obtained. These values could both be compared with analytical values.

Extreme care was not taken in cutting out the model as it was thought that in actual practise the work would be done as quickly as possible and absolute accuracy would not be attained.

The following pages contain the theoretical bases for the mechanical and analytical solutions of a fixed ended arch used in this study.

A general proof of the principles involved in application of Beggs' Method is first presented. This proof applies to any isotropic body. In order to guard against possible arithmetical errors, two separate analytical methods were used. The theoretical basis for each of these methods is presented together with the actual numerical calculations in tabular form.

The first analysis was made by dividing the arch into sections such that $\frac{ds}{I}$ was constant. The method followed was the one outlined by William Cain in his discussion of C.S.Whitney's paper, "Design of Symmetrical Concrete Arches."[1]

The second analysis was made by dividing the arch into equal horizontal divisions. The method followed was the one proposed by F.J.Dulude in his article, "Arch Analysis by a Method Using Variable Elastic Weights."[2]

1. Transactions A.S.C.E. Vol. 88 Page 1030 (1925)

2. Eng. News Record Vol. 82 Page 471 (1919)

Theorectal Basis FOR Beggs' Method.

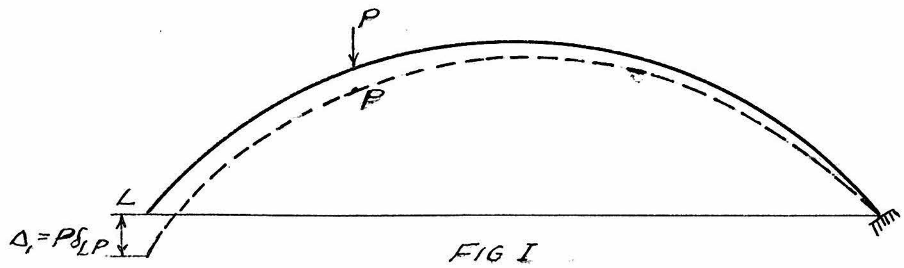


FIG I

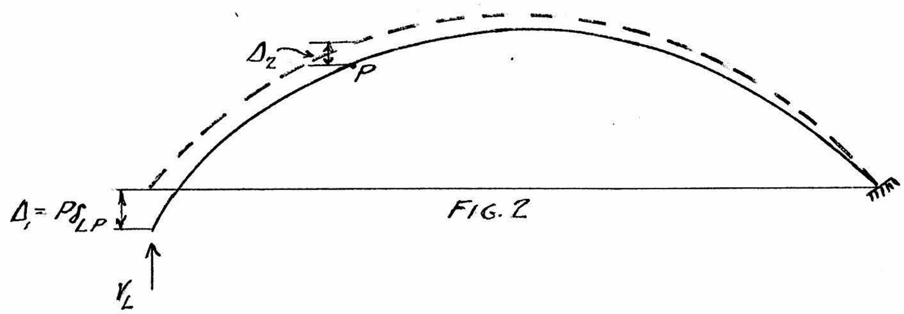


FIG. 2

Consider a fixed endedarch with load P at any point. When influence line values for V_L are wanted, plugs which allow vertical motion only are inserted in the gages at the left reaction. The amount of deformation caused by the gages we have previously calibated. In order to find value of V_L all that is necessary is to measure the vertical deflection at P. This statement may be proved as follows:

Let Fig.1 represent an arch cantilevered at right end with M_L and H_L sufficient to prevent rotation and horizontal translation. If, as is usual, δ_{LP} represents the deflection at L produced by unit load at P then the deflection at L, $\Delta_1 = P\delta_{LP}$

We know that V_L will have to be just large enough to move the arch back into its original position. This fact is brought out in Fig.2.

Again using the standard notation, let δ_{PL} represent the deflection at P produced by unit load at L.

Then $\Delta_2 = V_L \, \delta_{PL}$

And $\dfrac{\Delta_1}{\Delta_2} = \dfrac{P\delta_{LP}}{V\delta_{PL}}$

But by Maxwells Theorem

$$\delta_{LP} = \delta_{PL}$$

$$\therefore \frac{\Delta_1}{\Delta_2} = \frac{P}{V_L} \quad \text{OR IF} \quad P = 1 \; ; \; V_L = \frac{\Delta_2}{\Delta_1}$$

Expressions for obtaining influence line values for H and M_L may be obtained similarly.

That is $H = \dfrac{\Delta_2}{\Delta_1}$ and $M_L = \dfrac{\Delta_2}{\Delta_1}$ (scale of model).

The scale of the model enters into the expression for moment since the calibration of Δ_1 was made for a full size model.

Δ_1 was measured as tangential distance at unit radius from the center of rotation. When model drawn to scale is used, the distance must be divided by the scale in order to reduce it to unit rotation.

$$\text{Therefore } \frac{M_L}{P} = \frac{\dfrac{\Delta_2}{\Delta_1}}{\text{Scale of model}}$$

Or if P = 1, $M_L = \dfrac{\Delta_2}{\Delta_1}$ (Scale of model)

To obtain values of H, V_L and M_L all that is necessary therefore is to divide the deflection obtained experimentally by a previously determined constant.

In deriving the equations used as a basis for the application of the Begg's Method, Maxwells Theorem of reciprocal deflections was used.

A general proof of the applicability of this principle to any isotropic body was not available in any of the American text-books known to the authors of this paper.

A proof based on the conception of virtual work was found in a German text-book.[1] The principle of virtual work was then investigated in standard American books on theoretical mechanics.[2]

We will first develop the idea of virtual work.

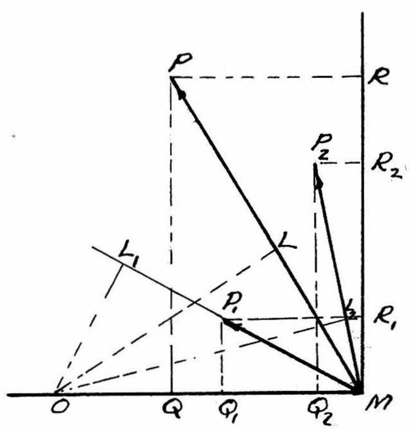


FIG. 1

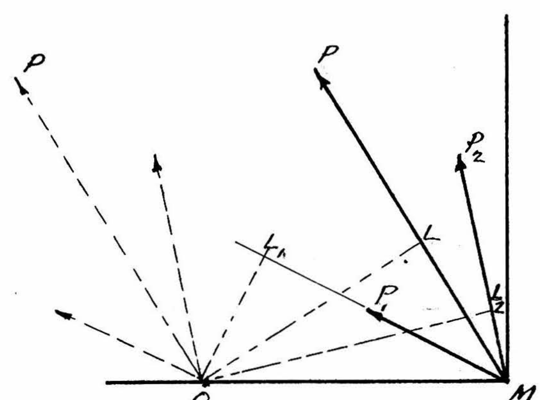


FIG. 2

In Fig.1, above, we have two forces P_1 and P_2 acting on the point M. The resultant of these two forces is P. P, P_1, P_2 have been broken down into components R, R_1 and R_2 in one direction and Q, Q_1 and Q_2 in a direction at right angles to each other. From any point O on either of the axes, perpendiculars have been drawn to the lines of action of the two forces P_1 and P_2 and their resultant P.

1. H. Muller-Breslau Die graphische Statik Der Baukonstruktionen Band 2 - 1. Abteilung Page 45.

2. Julius Weisbach - Mechanics of Engineering--Theoretical Mechanics.

We can now write the following relations

$\Delta MOL_1 \sim \Delta MP_1Q_1$

$\Delta MOL_2 \sim \Delta MP_2Q_2$

$\Delta MOL \sim \Delta MPQ$

(Sides are perpendicular and $\angle M = \angle M$, angle at M ppears in each triangle in every case.)

$\therefore \frac{Q_1}{P_1} = \frac{ML}{MO}$; $\frac{Q_2}{P_2} = \frac{ML_2}{MO}$, $\frac{Q}{P} = \frac{ML}{MO}$

But, from Fig.1, we see that $Q = Q_1 + Q_2$

Therefore $P\overline{ML} = P_1\overline{ML_1} + P_2\overline{ML_2}$ - - - - (1)

If we now think of the point M moving along the axis MO to point O, carrying with it the forces, unchanged in magnitude or in direction, the distances moved by the force P and its components will be ML, ML_1, and ML_2. Fig.2 brings out this fact very clearly. That is, these are the distances moved along their lines of action. The other component of their motion is at right angles and has no effect on work done by the force.

If we let $ML = s$, $ML_1 = s_1$, and $ML_2 = s_2$, then, substituting in equation(1) we get the relation $Ps = P_1s_1 + P_2s_2$. The proof can easily ve extended to give us

$$Ps = P_1s_1 + P_2s_2 + P_3s_3 + \ldots\ldots\ldots\ldots (2)$$

If the forces are variable, either in direction or magnitude, then the formula (2) is correct only infinitely small spaces $\sigma_1, \sigma_2, \sigma_3$ etc. The law correspinging to the formula $P\sigma = P_1\sigma_1 + P_2\sigma_2 + P_3\sigma_3 \ldots$ is known as the law or principle of virtual velocities.

We have shown that the work done by the resultant is equal to the work of its components equation (2). If the forces at a point are in equilibrium, the resultant is zero. Therefore, the sum of the work values for the various forces acting on the point must be zero. That is $Ps = P_1s_1 + P_2s_2 + P_3s_3 \ldots\ldots\ldots = 0$

This equation, as stated, is true for all values of s, s_1, s_2, etc. so long as the forces remain in equilibrium and do not change in direction or magnitude ,

The formula $P\sigma = P_1\sigma_1 + P_2\sigma_2 + P_3\sigma_3 = 0$ is also true, provided the system is in equilibrium. In this case the values of P_1, P_2, P_3, etc may change both in magnitude and direction but values of $\sigma_1, \sigma_2, \sigma_3$ are taken so small that for that displacement may be considered as remaining constant.

When we apply the principle to structural design, the distortions due to applied loads are used as the values of $\sigma_1, \sigma_2, \sigma_3$ etc. This introduces an error since these elastic deflections are not infinitely small. In most structures they are so small that error introduced is negligible. If elastic deflections of a structure are very large, considerable error may be introduced.

T
This principle of virtual work has been developed for use in a general proof of Clapeyrons Law, $\Sigma \int Q\delta = \frac{\Sigma Q\delta}{2}$, for any isotropic body.

Clapeyrons Law is then used to demonstrate the applicibility of Maxwell's Law of Reciprocal Deflections to any isotropic body.

Let us consider the elementary cube dx dy dz in any isotropic solid. We will call the external loads ΣQ At any time between the application of the load and the final value of the load, expressions for the internal and external work during a short interval of time may be written. Let q represent the value of Q during this short intermediate interval. Expressions for the total internal and external works may be obtained by summing up the expressions for the short interval to extend over values of q from 0 to Q, and to extend over the whole body.

Let $\epsilon_x, \epsilon_y, \epsilon_z, \tau_x, \tau_y, \tau_z$ represent total deformations under the given loading. ϵ denotes normal deformations, τ denotes shearing deformations. Then $d\epsilon_x \cdots d\epsilon_z, d\tau_x \cdots d\tau_z$ will represent changes in deformation during the period when Q has its intermediate value q. That is $d\epsilon_x \cdots d\epsilon_z, d\tau_x \cdots d\tau_z$ will represent distances through which the stresses corresponding to the value q move. If we let $\sigma_x, \sigma_y, \sigma_z, \tau_x, \tau_y, \tau_z$ represent the value of these stresses, then the internal work may be expressed as $(\sigma_x d\epsilon_x + \sigma_y d\epsilon_y + \sigma_z d\epsilon_z + \tau_x d\tau_x + \tau_y d\tau_y + \tau_z d\tau_z)\,dV$

If we let δ be the final value of the deflections at the loads Q, then $d\delta$ is the distance traversed by the intermediate value q. Therefore, external work $= q \cdot d\delta$

Summing up over the whole body and from $q = 0$ to $q = Q$, also remembering that external work = internal work

$$\Sigma \int q\, d\delta = \iint (\sigma_x d\epsilon_x + \sigma_y d\epsilon_y + \sigma_z d\epsilon_z + \tau_x d\tau_x + \tau_y d\tau_y + \tau_z d\tau_z)\, dV$$

Applying Hooke's Law with the addition of terms containing Poisson's ratio to take care of the lateral contraction which accompanies every longitudinal pull, we can write the following equations:

$$d\varepsilon_x = \frac{1}{E}\left(d\sigma_x - \frac{1}{m}d\sigma_y - \frac{1}{m}d\sigma_z\right) \quad ; \quad d\gamma_x = \frac{1}{G}d\tau_x$$

$$d\varepsilon_y = \frac{1}{E}\left(d\sigma_y - \frac{1}{m}d\sigma_z - \frac{1}{m}d\sigma_x\right) \quad ; \quad d\gamma_y = \frac{1}{G}d\tau_y$$

$$d\varepsilon_z = \frac{1}{E}\left(d\sigma_z - \frac{1}{m}d\sigma_x - \frac{1}{m}d\sigma_y\right) \quad ; \quad d\gamma_z = \frac{1}{G}d\tau_z$$

$$\therefore \sigma_x d\varepsilon_x + \sigma_y d\varepsilon_y + \sigma_z d\varepsilon_z + \tau_x d\gamma_x + \tau_y d\gamma_y + \tau_z d\gamma_z =$$

$$\frac{1}{E}\left[\sigma_x d\sigma_x + \sigma_y d\sigma_y + \sigma_z d\sigma_z - \frac{1}{m}d(\sigma_y\sigma_z + \sigma_x\sigma_z + \sigma_y\sigma_x)\right] +$$

$$\frac{1}{G}\left[\tau_x d\tau_x + \tau_y d\tau_y + \tau_z d\tau_z\right]$$

By integrating this expression, we obtain

$$W_i = \frac{1}{2}\int\left[\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \frac{2}{m}(\sigma_y\sigma_z + \sigma_z\sigma_x + \sigma_y\sigma_x)\right]\frac{dV}{E} +$$

$$\frac{1}{2}\int\left[\tau_x^2 + \tau_y^2 + \tau_z^2\right]\frac{dV}{G}$$

Let u be any small but possible displacement of Q, let s_x, s_y, s_z t_x, t_y, t_z be simultaneous values of the internal displacements. We then know from the principle of virtual work that

$$\Sigma Q u = \int (\sigma_x s_x + \sigma_y s_y + \sigma_z s_z + \tau_x t_x + \tau_y t_y + \tau_z t_z)\, dV$$

However, these displacements may be any simultaneous values of deformations. Let us then use the final values of the deflections caused by the loads ΣQ themselves. Then we can write

$$\Sigma Q \delta = \int (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_x \gamma_x + \tau_y \gamma_y + \tau_z \gamma_z)\, dV$$

$$= \int \Big[\sigma_x \Big(\sigma_x - \frac{\sigma_y + \sigma_z}{m} \Big) + \sigma_y \Big(\sigma_y - \frac{\sigma_z + \sigma_x}{m} \Big)$$

$$+ \sigma_z \Big(\sigma_z - \frac{\sigma_x + \sigma_y}{m} \Big) \Big] \frac{dV}{E} + \int (\tau_x^2 + \tau_y^2 + \tau_z^2) \frac{dV}{G}$$

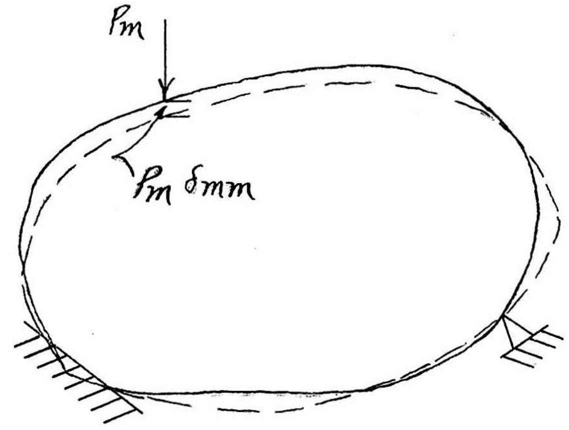
$$= \int \Big[\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \frac{2}{m} (\sigma_y \sigma_z + \sigma_z \sigma_x + \sigma_y \sigma_x) \Big] \frac{dV}{E} +$$

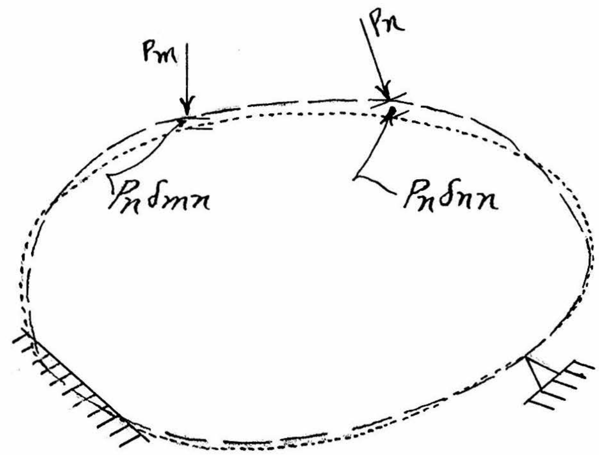
$$\int [\tau_x^2 + \tau_y^2 + \tau_z^2] \frac{dV}{G}$$

Comparing this value for $\Sigma Q \delta$ with obtained for W_i we see that

$$\Sigma Q \delta = 2 W_i \qquad \therefore \; W_i = \frac{\Sigma Q \delta}{2}$$

Therefore, regardless of the law of variation of q in increasing from 0 to Q, the work done will be equal to $\frac{\Sigma Q \delta}{2}$





Apply loads in order shown above.

Then External Work $= \frac{P_m}{2} \cdot P_m \delta_{mm} + P_m \cdot P_n \delta_{mn} + \frac{P_n}{2} \cdot P_n \delta_{nn}$

If loads are applied in reverse order , it is evident that

External Work $= \frac{P_n}{2} \cdot P_n \delta_{nn} + P_n \cdot P_m \cdot \delta_{nm} + \frac{P_m}{2} \cdot P_m \delta_{mm}$

Regardless of the order of application of the loads the amount of work done must be identical. Since first and last terms in each case are identical, the middle terms must be equal.

$\therefore P_m \cdot P_n \cdot \delta_{mn} = P_n \cdot P_m \cdot \delta_{nm} \quad \text{or} \quad \delta_{mn} = \delta_{nm}$

Theoretical Basis for Cain's Method

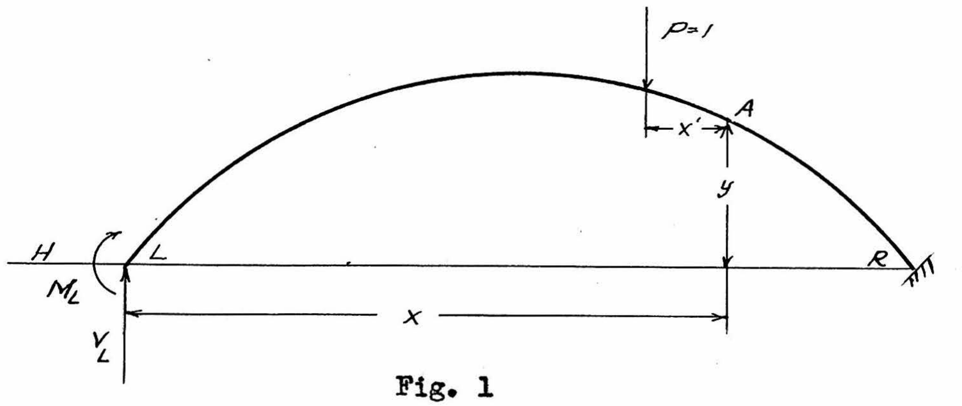


Fig. 1

Notation:- $\frac{ds}{EI} = dw$

M_r = moment of reactions at any section

M_p = moment of loads at any section

$\Delta_r x$ = horizontal displacement of the left end due to the reactions

$\Delta_r y$ = vertical displacement of the left end due to the reactions

$\Delta_r \phi$ = angular displacement of the left end due to the reactions

$\Delta_p x$ = horizontal displacement of the left end due to the loads

$\Delta_p y$ = vertical displacement of the left end due to the loads

$\Delta_p \phi$ = angular displacement of the left end due to the loads

If rib is rigidly fixed at the abutments

Then

$$\Delta_r x + \Delta_p x = 0$$

$$\Delta_r y + \Delta_p y = 0$$

$$\Delta_r \phi + \Delta_p \phi = 0$$

From Fig. 1 we see that

$$M_r = M_L + V_L x - Hy$$

Then $$\Delta_r x = \int_L^R \frac{M_r y\,ds}{EI} = \int_L^R (M_L y + V_L xy - Hy^2)\,dw$$

$$\Delta_r y = \int_L^R \frac{M_r x\,ds}{EI} = \int_L^R (M_L x + V_L x^2 - Hxy)\,dw$$

$$\Delta_r \phi = \int_L^R \frac{M_r\,ds}{EI} = \int_L^R (M_L + V_L x - Hy)\,dw$$

These equations are somewhat complex. If we shift the origin of co-ordinates to the so-called elastic center, the center of gravity of the dw terms, some of the terms will disappear.

$\int_L^R y\,dw$; $\int_L^R x\,dw$; and $\int_L^R xy\,dw$ will equal zero.

Let ($\frac{l}{2}$, b) be the new origin of co-ordinates.

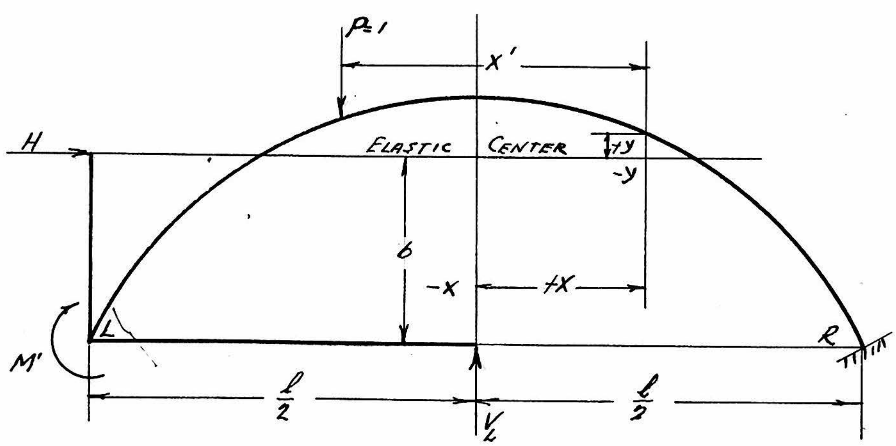


Fig. 2

Then, shifting axes

$$M_r = M_L + V_L\left(x - \frac{l}{2}\right) - H(y - b)$$

or $$M_r = M_L + V_L x - V_L \frac{l}{2} - Hy + Hb$$

Let $$M' = M_L - V_L \frac{l}{2} + Hb$$

Then $$M_r = M' + V_L x - Hy$$

We then have

$$\Delta_r x = \int_L^R (M'y + V_L xy - Hy^2)\,dw$$

$$\Delta_r y = \int_L^R (M'x + V_L x^2 - Hxy)\,dw$$

$$\Delta_r \phi = \int_L^R (M' + V_L x - Hy)\,dw$$

But we have chosen our axes in such a manner that

$$\int_L^R x\,dw = \int_L^R y\,dw = \int_L^R xy\,dw = 0$$

There remains then

$$\Delta_r x = -\int_L^R Hy^2\,dw$$

$$\Delta_r y = \int_L^R V_L x^2\,dw$$

$$\Delta_r \phi = \int_L^R M'\,dw$$

FOR $P = 1$

$$\Delta_p x = \int_p^R M_p y \, d\omega = -\int_p^R x' y \, d\omega$$

$$\Delta_p y = \int_p^R M_p x \, d\omega = -\int_p^R x' x \, d\omega$$

$$\Delta_p \phi = \int_p^R M_p \, d\omega = -\int_p^R x' \, d\omega$$

But $\Delta_r + \Delta_p = 0$, since abutments are rigidly fixed.

$$\therefore -\int_L^R H y^2 d\omega = -\int_p^R x' y \, d\omega ; \text{ OR } H = -\frac{\int_p^R x' y \, d\omega}{\int_L^R y^2 d\omega}$$

Similarly

$$V_L = \frac{\int_p^R x' x \, d\omega}{\int_L^R x^2 d\omega} \quad \text{AND} \quad M' = \frac{\int_p^R x' d\omega}{\int_L^R d\omega}$$

To further simplify the equations we will dividethe arch ring into N divisions such that Δw is constant.

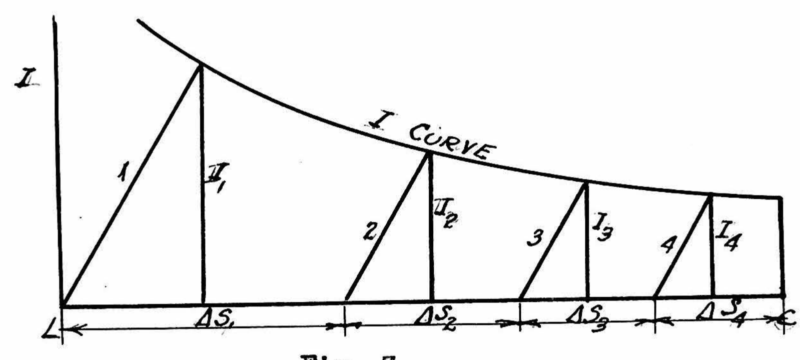


Fig. 3

Method: Assume length of Δs_1, measure I_1 at its mid-point, compute $\Delta s_1/I_1$. Atend of Δs_1 draw line 2 parallel to line 1. Measure I_2. Compute $\Delta s_2 = \Delta s_1/I_1 \times I_2$ and lay it off. Continue until center is reached. Distribute the error proportionally and begin again. The third trial should always give accurate results.

Then

$$H = -\frac{\sum_P^R yx'}{\sum_L^R y^2} \qquad V_L = \frac{\sum_P^R xx'}{\sum_L^R x^2} \qquad M' = \frac{\sum_P^R x'\Delta\omega}{2\sum_C^R \Delta\omega}$$

It is our purpose to get influence line values. Since we can get values for loads on left half from those on right half, formulas will be derived which apply only to loads on the right half of the arch.

Let n = number of points such as 1',2'.... between R and the load.

d = horizontal distance from crown to P.

x' = horizontal distance from P to points such as 1',2'... to its right.

$$\therefore 2\sum_B^C \Delta\omega = N.\Delta\omega \; ; \; x' = x - d$$

$$M' = \frac{1}{N}\sum_R^P x' = \frac{1}{N}\sum_R^P (x-d)$$

If co-ordinates of 1',2'.... are (x_1, y_1) , (x_2, y_2) ....

$$\sum_R^P (x-d) = x_1 - d + x_2 - d + \ldots + x_n - d$$

$$\therefore M' = \frac{(x_1 + x_2 + \ldots + x_n) - nd}{N}$$

If a unit load is placed successively at the points 1', 2' .... 10', d takes the values x_1, x_2, ...x_{10} and n takes the soccessive values 0,1, 2, ...9. In writing values of n, a diagram such as Fig. 4 should be used. The value of n does not include the point where the unit load is placed.

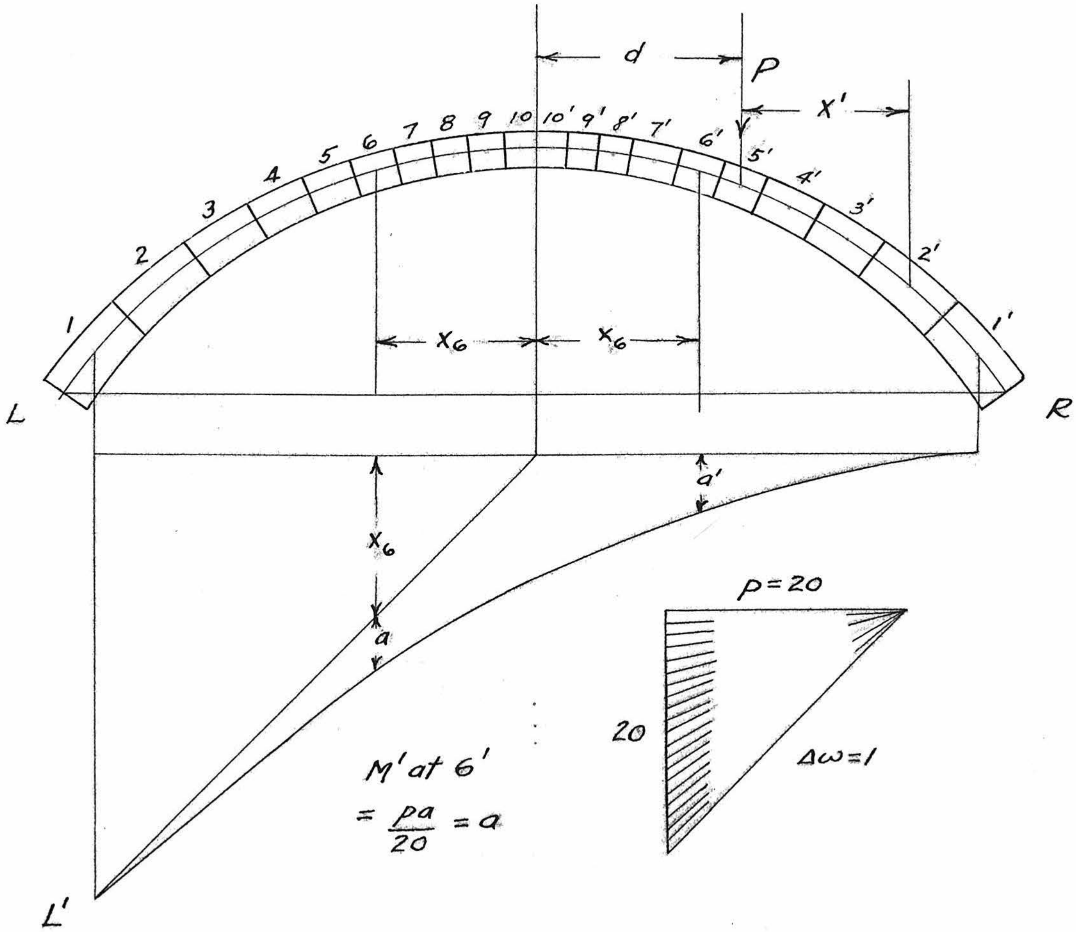


Fig. 4

We have made the statement that the influence line values for the left half of the arch can be obtained from those for the right half. It is evident that H_L values will be identical for symmetrically placed points. Values of V_L can be obtained by subtracting the value for the corresponding point on the right half from unity.

It is not so evident, however how to get influence line values for M' for the left half of the arch.

Let us proceed as follows:

We know that $$M' = \frac{\int_P^R x'dw}{\int_L^R dw}$$

If a unit load is placed at any point P. the moment at any other point of the cantilever fixed at R = $-x'$.

The angular deflection of the tangent at L with respect to tangent at R $= \Delta\phi = \int_P^R M\,dw = -\int_P^R x'\,dw$

We see that this is the numerator in the expression for M'. From Maxwell's Theorem of Reciprocal Deflections, the angular displacement at L produced by a unit vertical load at P is equal to the vertical displacement at P produced by a unit moment at L. Let us denote this vertical displacement at P produced by unit moment at L by δ_{MP}. Expressed in mathematical terminology we have just stated that $\delta_{MP} = \delta_{PM}$

Let us now apply a unit moment at L. We know that the vertical deflection at any point is equal to the bending moment at that point when the arch is loaded with the M/EI diagram. δ_{MP} can thus be obtained graphically by loading each division with its $M\,\Delta x/EI$ value or, since $M=1$, with its Δw value.

Then $$M' = \frac{\delta_{MP}}{\sum_L^R \Delta w}$$

In Fig. 4, scales have been so chosen that ordinates represent $\frac{\delta_{MP}}{\sum_L^R \Delta w}$ or M' instead of δ_{MP}. This was done by making the pole distance equal to the sum of the elastic loads. This causes the end tangents to make an angle of 45° with each other. Also, since pole distance x a' must equal the bending moment in a simple beam, $20a' = \delta_{MP}$. But $M' = \frac{\delta_{MP}}{20}$ $\therefore$ $M' = \frac{20a'}{20} = a'$

Since vertical distance $= M'$, then for any symmetrical points, such as 6, 6'

M' at $6 = x_6 + a$; M' at $6' = a'$

but, by symmetry, $a = a'$ $\therefore$ M' at $6 = x_6 + a'$

Similarly, M' at $5 = x_5 + a_5'$ etc.

We have thus shown that influence line values for M', V, and H can be obtained for entire arch by moving unit load across the half arch only.

We have found that $$V_L = \frac{\Sigma_R^P x x'}{\Sigma_R^L x^2}$$

From Fig. 4 we see that

$$V_L = \frac{x_1^2 + x_2^2 ---- x_n^2 \quad - d(x_1 + x_2 ---- x_n)}{\Sigma_R^L x^2}$$

Since $x' = x - d$ and $x x' = x^2 - d x$

We have found that $$H = -\frac{\Sigma_R^P y x'}{\Sigma_R^L y^2}$$

From Fig. 4 we see that

$$\Sigma_R^P y x' = [y_1(x_1 - d) --- y_n(x_n - d)] = (x_1 y_1 ---- x_n y_n) - d(y_1 + ----- + y_n)$$

Since $x' = x - d$ and $y x' = x y - d y$

In the above formulas x is always positive, but y may be either positive or negative. Care must be taken to give it its proper sign.

Then $$H = \frac{-(+ x_1 y_1 + ---- x_n y_n) + d(y_1 + ----- y_n)}{\Sigma_R^L y^2}$$

Once the values of M', V_L, and H are obtained, the arch becomes statically determinate and the values of the thrust moment and shear at any point may be obtained from the equations of statics.

Point	1	2	3	4	5
	y	y^2	x	x^2	xy
1'	-36.98	1367.52	99.75	9950	-3688.8
2'	-17.28	298.60	80.20	6432	-1385.9
3'	- 7.53	48.34	67.35	4536	- 507.1
4'	- 0.48	0.18	56,05	3142	- 26.9
5'	+ 4.42	19.54	46.15	2132	+ 204.0
6'	+ 7.92	53.94	37.00	1369	+ 293.0
7'	+10.42	108.58	28.40	807	+ 295.9
8'	+12.32	151.78	20.00	400	+ 246.4
9'	+13.32	177.42	11.95	143	+ 159.2
10'	+13.92	193.76	3.85	15	+ 53.6
	$\sum_B^C y = +.05$	2419.66	450.70	28924	-5608.7
		× 2		× 2	+1252.1
		4839.32		57848	$\sum_B^C xy =$ -4356.6

Point	6	7	8	9	10	11
	n	d	nd	$X_1 + \dots X_n$	$X_1 + \dots + X_n - nd$	$M' = \frac{\text{col } 10}{20}$
1'	0	99.75	0.00	0.00	0.00	0.000
2'	1	80.20	80.20	99.75	19.55	0.977
3'	2	67.35	134.70	179.95	45.25	2.263
4'	3	56.05	168.15	247.30	79.15	3.957
5'	4	46.15	184.60	303,35	118.75	5.983
6'	5	37.00	185.00	349.50	164.50	8.225
7'	6	28.40	170.40	386.50	216.10	10.805
8'	7	20.00	140.00	414.90	274.90	13.745
9'	8	11,95	95.60	434.90	339.30	16.965
10'	9	3.85	34.65	446.85	412.20	20.610
10	X_{10} +		20.610 =			24.460
9	X_9 +		16.965 =			28.915
8	X_8 +		13.745 =			33.745
7	X_7 +		10.805 =			39.205
6	X_6 +		8.225 =			45.225
5	X_5 +		5.938 =			52.088
4	X_4 +		3.957 =			60.007
3	X_3 +		2.263 =			69.613
2	X_2 +		0.977 =			81.77
1	X_1 +		0.000 =			99.750

Point	12	13	14	15	16	17	18
	n	$x_1^2+x_2^2+\dots x_n^2$	$x_1+x_2+\dots x_n$	$d-x$	$d(x_1+\dots x_n)$	col 13 - col 19	$V_L = \frac{\text{col 17}}{57848}$
1'	0	0	0.00	99.75	0	0	.000
2'	1	9950	99.75	80.20	8000	1950	.034
3'	2	16382	179.95	67.35	12120	4262	.074
4'	3	20918	247 30	56.05	13861	7057	.122
5'	4	24060	303 35	46.15	14000	10060	.174
6'	5	26190	349 50	37.00	12932	13258	.229
7'	6	27559	386.50	28.40	10977	16582	.287
8'	7	28366	414.90	20.00	8298	20068	.347
9'	8	28766	434.90	11.95	5197	23569	.407
10'	9	28909	446.85	3.85	1720	27189	.470
10		1-	.470 =				.530
9		1-	.407 =				.593
8		1-	.347 =				.653
7		1-	.287 =				.713
6		1-	.229 =				.771
5		1-	.174 =				.826
4		1-	.122 =				.878
3		1-	.074 =				.926
2		1-	.034 =				.966
1		1-	.000 =				1.000

Point	19	20	21	22	23	24	25
	n	$x_1y_1+\dots+x_my_m$	d	$y_1+\dots+y_m$	$d(y_1+\dots y_m)$	$-col20+col23$	$H=\frac{col\,24}{4839.32}$
1'	0	0.0	99.75	- 0.00	0.0	0.0	.000
2'	1	-3688.8	80.20	-36.98	-2965.8	723.0	.149
3'	2	-5074.7	67.35	-54.26	-3654.4	1420.3	.293
4'	3	-5581.8	56.05	-61.79	-3463.3	2118.5	.438
5'	4	-5608.7	46.15	-62.27	-2873.8	2734.9	.565
6'	5	-5404.7	37.00	-57.85	-2140.5	3264.2	.675
7'	6	-5111.7	28.40	-49.93	-1418.0	3693.7	.763
8'	7	-4815.8	20.00	-39.51	- 790.2	4025.6	.832
9'	8	-4569.4	11.95	-27.19	- 324.9	4244.5	.877
10'	9	-4410.2	3.85	-13.87	- 53.4	4356.8	.900

Point	26 M'	27 H_L	28 b	29 $H_L b$	30 $\frac{l}{2}$	31 V_L	32 $V_L \cdot \frac{l}{2}$	33 M_2
1'	0.000	.000	52.58	0.00	111.4	.0000	0.00	0.00
2'	0.977	.149	"	7.83	"	.034	3.79	+ 5.02
3'	2.263	.293	"	15.41	"	.074	8.24	+ 9.43
4'	3.957	.438	"	23.03	"	.122	13.59	+13.40
5'	5.938	.565	"	29.71	"	.174	19.38	+16.27
6'	8.225	.675	"	35.49	"	.229	25.51	+18.21
7'	10.805	.763	"	40.12	"	.287	31.97	+18.96
8'	13.745	.832	"	43.75	"	.347	38.66	+18.84
9'	16.965	.877	"	46.11	"	.407	45.34	+17.74
10'	20.610	.900	"	47.32	"	.470	52.36	+15.57
10	24.460	.900	"	47.32	"	.530	59.04	+12.74
9	28.915	.877	"	46.11	"	.593	66.06	+ 8.97
8	33.745	.832	"	43,75	"	.653	72.74	+ 4.76
7	39.205	.763	"	40,12	"	.713	79.43	- 0.10
6	45.225	.675	"	35.49	"	.771	85.89	- 5.17
5	52.088	.565	"	29,71	"	.826	92.02	-10.22
4	60.007	.438	"	23.03	"	.878	97.81	-14.77
3	69.613	.293	"	15.41	"	.926	103.16	-18.14
2	81.177	.149	"	7.83	"	.966	107.61	-18.60
1	99.750	.000	"	0.00	"	1.000	111.40	-11.65

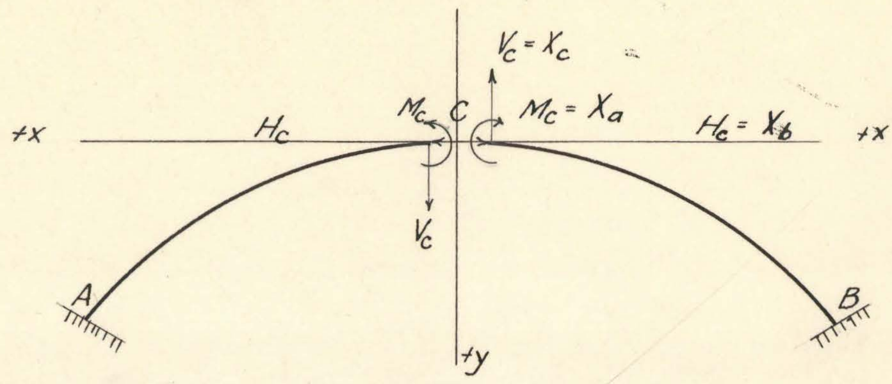
$$M_2 = M' + H_L b - V_L \cdot \frac{l}{2}$$

Point	34	35	36	37	38	39
	$54.3 H_L$	M_L	$59.2 V_L$	X	$1(X-52.2) = M_1$	$\frac{M_L}{4}$
1'	0.00	0.00	0.00			0.00
2'	8.09	+ 5.02	2.01			+1.06
3'	15.91	+ 9.43	4.38			+2.10
4'	23.78	+13.40	7.22			+3.16
5'	30.68	+16.27	10.30			+4.11
6'	36.65	+18.21	13.56			+4.88
7'	41.43	+18.96	16.99			+5.48
8'	45.18	+18.84	20.54			+5.80
9'	47.62	+17.74	24.09			+5.79
10'	48.87	+15.57	27.82			+5.48
10	48.87	+12.74	31.38			+4.75
9	47.62	+ 8.97	35.11			+3.54
8	45.18	+ 4.76	38.66			+1.76
7	41.43	- 0.10	42.21			-0.68
6	36.65	- 5.17	45.64			-3.82
5	30.68	-10.22	48.90			-8.00
4	23.78	-14.77	51.98	56.05	3.85	-9.58
3	15.91	-18.14	54.82	67.35	15.15	-5.62
2	8.09	-18.60	57.19	80.20	28.00	-2.50
1	0.00	-11.65	59.20	99.75	47.55	0.00

Point	41	42
	Thrust $\frac{1}{4}$	Shear $\frac{1}{4}$
1'	+0.000	.000
2'	+0.150	-.037
3'	+0.294	-.065
4'	+0.448	-.087
5'	+0.582	-.098
6'	+0.705	-.100
7'	+0.810	-.086
8'	+0.900	-.062
9'	+0.965	-.030
10'	+1.015	+.020
10	+1.040	+.069
9	+1.050	+.137
8	+1.035	+.211
7	+1.000	+.296
6	+0.948	+.390
5	+0.872	+.487
4	+0.338	-.307
3	+0.229	-.200
2	+0.120	-.100
1	+0.000	.000

Note- The above values were obtained graphically.

Theoretical Basis For Deludes Method



The deflections are the deflections of the cut faces at the crown.

δ'_a = angular deflection of cut face

δ'_b = horizontal deflection of cut face

δ'_c = vertical deflection of cut face

M' is the cantilever moment of applied loads.

m_a is the moment at any point of the cantilever with unit moment applied at the crown and no other forces acting

m_b is the moment at any point of the cantilever with unit horizontal forces applied at crown, no other forces acting

m_c is the moment at any point of cantilever with unit vertical forces applied at crown no other forces acting

$$m_c = \begin{cases} -x & \text{for left half} \\ +x & \text{for right half} \end{cases}$$

Values of moment taken as positive when causing tension in the lower fibres.

Applying the well known formula $\delta = \int \frac{Mm\,ds}{EI}$

We have

$$\delta_a' = \int_C^A \frac{M_L'\,ds}{EI} + \int_C^B \frac{M_R'\,ds}{EI} = \int_A^B \frac{M'\,ds}{EI}$$

$$\delta_b' = \int_C^A \frac{M_L'\,y\,ds}{EI} + \int_C^B \frac{M_R'\,y\,ds}{EI} = \int_A^B \frac{M'\,y\,ds}{EI}$$

$$\delta_c' = -\int_C^A \frac{M_L'\,x\,ds}{EI} + \int_C^B \frac{M_R'\,x\,ds}{EI}$$

Let δ_{mn} = deflection at m due to a unit loading at n in the base system

Then

$$\delta_{aa} = \int_A^B \frac{ds}{EI} = 2\int_A^C \frac{ds}{EI}$$

$$\delta_{bb} = \int_A^B \frac{y^2\,ds}{EI} = 2\int_A^C \frac{y^2\,ds}{EI}$$

$$\delta_{cc} = \int_A^B \frac{x^2\,ds}{EI} = 2\int_A^C \frac{x^2\,ds}{EI}$$

$$\delta_{ab} = \delta_{ba} = 2\int_A^C \frac{y\,ds}{EI}$$

$$\delta_{ac} = \delta_{ca} = -\int_A^C \frac{x\,ds}{EI} + \int_B^C \frac{x\,ds}{EI} = 0$$

$$\delta_{bc} = \delta_{cb} = -\int_A^C \frac{xy\,ds}{EI} + \int_B^C \frac{xy\,ds}{EI} = 0$$

THE GENERALLEQUATIONS FOR A TRIPLY REDUNDANT STRUCTURE ARE AS FOLLOWS:

$$\delta_a = 0 = \delta_a' + X_a \delta_{aa} + X_b \delta_{ab} + X_c \delta_{ac}$$

$$\delta_b = 0 = \delta_b' + X_a \delta_{ba} + X_b \delta_{bb} + X_c \delta_{bc}$$

$$\delta_c = 0 = \delta_c' + X_a \delta_{ca} + X_b \delta_{cb} + X_c \delta_{cc}$$

THESE REDUCE TO :

$$\delta_a = 0 = \delta_a' + X_a \delta_{aa} + X_b \delta_{ba} \text{ ----- } (1)$$

$$\delta_b = 0 = \delta_b' + X_a \delta_{ab} + X_b \delta_{bb}$$

$$\delta_c = 0 = \delta_c' + X_c \delta_{cc}$$

FROM WHICH

$$X_a = - \frac{(\delta_a' \delta_{bb} - \delta_b' \delta_{ab})}{(\delta_{aa} \delta_{bb} - \delta^2_{ab})}$$

$$X_b = - \frac{(\delta_b' \delta_{aa} - \delta_a' \delta_{ab})}{(\delta_{aa} \delta_{bb} - \delta^2_{ab})}$$

$$X_c = - \frac{\delta_c'}{\delta_{cc}}$$

ALSO FROM (1)

$$X_a = - \frac{(X_b \delta_{ba} + \delta_a')}{\delta_{aa}}$$

THEREFORE WE HAVE FINALLY (E CONSTANT)

$$X_a = M_c = -\frac{H_c\int_A^B \frac{y\,ds}{I} + \int_A^B \frac{M'ds}{I}}{\int_A^B \frac{ds}{I}}$$

$$X_b = H_c = -\frac{\int_A^B \frac{M'y\,ds}{I}\int_A^B \frac{ds}{I} - \int_A^B \frac{M'ds}{I}\int_A^B \frac{y\,ds}{I}}{\int_A^B \frac{ds}{I}\int_A^B \frac{y^2ds}{I} - \left(\int_A^B \frac{y\,ds}{I}\right)^2}$$

$$X_c = V_c = -\frac{\int_C^B \frac{M_R'x\,ds}{I} - \int_C^A \frac{M_L'x\,ds}{I}}{\int_A^B \frac{x^2ds}{I}}$$

These formulas are the basis from which Mr. Deludés formulas are derived. In order to conform to his notation we will let $m =$ cantilever moment

m_L for left half

m_R for right half

The arch will be divided so that the horizontal projections of the divisions will be equal.

We will let S the length of adivision

and $\frac{S}{I} = \Delta$

Since the arch is symmetrical it would seem at first sight that it would be possible to make the summations for one half the arch and then multiply this by two. This may indeed be done for all terms which do not involve m, for those in which m occurs the summation must be made directly over the full length of the arch.

Therfore the equations can be rewritten as follows:-

$$M_c = \frac{\sum_A^B m\Delta - H_c \sum_A^B y\Delta}{\sum_A^B \Delta} = \frac{\sum_A^B m\Delta - 2H_c \sum_A^C y\Delta}{2\sum_A^C \Delta}$$

$$H_c = -\frac{+\sum_A^B my\Delta \sum_A^B \Delta - \sum_A^B m\Delta \sum_A^B y\Delta}{\sum_A^B \Delta \sum_A^B y^2\Delta - \left(\sum_A^B y\Delta\right)^2}$$

$$= -\frac{+\sum_A^B my\Delta \cdot 2\sum_A^C \Delta - \sum_A^B m\Delta \cdot 2\sum_A^C y\Delta}{2\left(\sum_A^C \Delta \sum_A^C y^2\Delta - \left\{\sum_A^C y\Delta\right\}^2\right)}$$

$$V_c = \frac{+\sum_C^A m_L x\Delta - \sum_C^B m_R x\Delta}{\sum_A^B x^2\Delta} = \frac{\sum_C^A (m_L - m_R) x\Delta}{2\sum_A^C x^2\Delta}$$

All of the terms in the above equations can be directly evalulated easily except $\sum m\Delta$, $\sum my\Delta$, AND $\sum (m_L - m_R) x\Delta$ It so happens that these terms may also be evaluated easily if the following method is used.

Take a load of unity at any point say point 6

Then $\Sigma m\Delta = (m_0\Delta_0 + m_1\Delta_1 + m_2\Delta_2 + \dots + m_5\Delta_5)$

But $m_0 = 6S$; $m_1 = 5S$; $m_2 = 4S$; etc.

$\therefore \Sigma m\Delta = S(6\Delta_0 + 5\Delta_1 + 4\Delta_2 + \dots + \Delta_5)$ for load at point 6

Similarly $\Sigma m\Delta = S(7\Delta_0 + 6\Delta_1 + 5\Delta_2 + \dots + 2\Delta_5 + \Delta_6)$ for load at point 7

Therefore increase in $\Sigma m\Delta = S\Sigma_0^{m-1}\Delta$

Similarly for $\Sigma my\Delta$

$\Sigma my\Delta = S(6\Delta_0 y_0 + 5\Delta_1 y_1 + \dots + \Delta_5 y_5)$ for load at point 6

$\Sigma my\Delta = S(7\Delta_0 y_0 + 6\Delta_1 y_1 + \dots + 2\Delta_5 y_5 + \Delta_6 y_6)$ for load at point 7

Therefore increase in $\Sigma my\Delta = S\Sigma_0^{m-1}\Delta y$

In the same manner we can show that

increase in $\Sigma mx\Delta = S\Sigma_0^{m-1} x\Delta$

If we let the value in parenthesissuch as $(6\Delta_0 y_0 + 5\Delta_1 y_1 + \dots + \Delta_5 y_5)$ for point 6, $= [\Delta y]_6$ then we may express the relation shown above as

$$S[\Delta y]_7 = S[\Delta y]_6 + S\Sigma_0^{m-1}\Delta y$$

or in general $[\Delta y]_m = [\Delta y]_{m-1} + \Sigma_0^{m-1}\Delta y$

similarly $[\Delta x]_m = [\Delta x]_{m-1} + \Sigma_0^{m-1}\Delta x$

and $[\Delta]_m = [\Delta]_{m-1} + \Sigma_0^{m-1}\Delta$

From these equations it is easy to see that these values may be obtained by successive addition beginning at point (I)

at point (I) $[\Delta] = \Delta_0$; $[y\Delta] = y_0\Delta_0$; $[x\Delta] = x_0\Delta_0$

Incorporating these values in the formulas already obtained we have for a final result

$$H_c = -\frac{S}{2}\frac{([y\Delta]\Sigma\Delta - [\Delta]\Sigma y\Delta)}{[\Sigma y^2\Delta\Sigma\Delta - (\Sigma y\Delta)^2]}$$

$$V_c = \frac{\pm S[x\Delta]}{2\Sigma x^2\Delta}$$

$$M_c = \frac{[\Delta] - 2H_c\Sigma y\Delta}{2\Sigma\Delta}$$

Since we are after influence line values only;at any time only one cantilever will be loaded. All terms containing m in the unloaded cantilever will become equal to zero. Therefore all the summations indicated above can be done for one half the arch.

The tabular solution of the arch under consideration is shown in the body of this report.

Point	1	2	3	4	5	6	7	8	9
	x	y	Δ	$\sum_0^m \Delta$	$[\Delta]$	$y\Delta$	$\sum_0^m y\Delta$	$[y\Delta]$	$x\Delta$
0	105.83	58.50	1.96	1.96	0.0	114.66	114.66	0.0	207.43
1	94.69	45.20	3.21	5.17	1.96	145.09	259.75	114.66	303.95
2	83.55	34.15	4.30	9.47	7.13	146.85	406.60	374.41	359.27
3	72.41	25.00	5.14	14.61	16.60	128.40	535.10	781.01	372.19
4	61.27	17.40	5.89	20.50	31.21	102.49	637.59	1316.11	360.88
5	50.13	11.30	6.56	27.06	51.71	74.13	711.52	1953.70	328.85
6	38.99	6.80	7.17	34.23	78.77	48.76	760.48	2665.42	279.56
7	27.85	3.25	7.64	41.87	113.00	24.83	785.31	3425.90	212.77
8	16.71	1.10	8.06	49.93	154.87	8. 87	794.18	4211.21	134.68
9	5.57	.05	8.38	58.31	204.80	0.42	794.60	5005.39	46.68
C					233.95			5402.69	
S			58.31			794.60			2606.26

Point	10	11	12	13	8	14	5	15
	$\sum_0^m x\Delta$	$[x\Delta]$	$x^2\Delta$	$y^2\Delta$	$[y\Delta]$	$[y\Delta] \leq \Delta$	$[\Delta]$	$[\Delta] \leq y\Delta$
0	207.43	0.0	21952	6708	0.0	0	0.0	0
1	511.38	207.43	28781	6558	114.66	6686	1.96	1557
2	870.65	718.81	30017	5015	374.41	22009	7.13	5665
3	1242.84	1589.46	26950	3213	781.01	45541	16.60	13190
4	1603.72	2832.30	22111	1783	1316.11	76742	31.21	24799
5	1932.57	4436.02	16485.	838	1953.70	113920	51.71	41089
6	2212.13	6368.59	10900	332	2665.42	155420	78.77	62591
7	2424.90	8580.72	5926	81	3425.90	199764	113.00	89790
8	2559.58	11005.62	2251	8	4211.21	245556	154.87	123060
9	2606.26	13565.20	260	0	5005.39	291864	204.80	162734
C		14868.33			5402.69	315030	233.95	185897
S			165633	24536				

Point	16	17	18	19	20	21	11	22
	$Q = 14 - 15$	$H_c = \frac{SQ}{K}$	$s[\Delta]$	$2 H_c \Sigma y \Delta$	$R = 18 - 19$	$M_c = \frac{R}{2\Sigma\Delta}$	$[x\Delta]$	$\frac{\pm s(x\Delta)}{2\Sigma x^2 \Delta}$
0	0000	.000	0.00	0.00	0.00	0.000	0.00	.000
1	5129	.043	21.83	68.34	-46.51	-0.399	207.43	.007
2	16344	.114	79.43	181.17	-101.74	-0.872	718.81	.024
3	32351	.225	184.92	357.57	-172.56	-1.480	1589.46	.053
4	51943	.362	347.68	575.29	-227.61	-1.952	2832.30	.095
5	72831	.508	576.05	807.31	-231.26	-1.983	4436.02	.149
6	92829	.647	877.50	1028.21	-150.71	-1.292	6368.59	.214
7	109974	.766	1258.82	1217.33	+ 41.49	+0.356	8580.72	.289
8	122496	.854	1725.25	1357.18	+368.07	+3.156	11005.62	.370
9	149130	.900	2281.47	1430.28	+851.19	+7.300	13565.20	.456
C	159134	.900	2606.20	1430.28	+1175.92	+10.080	14868.33	.500

$$K = 2\left[\Sigma y^2 \Delta \, \Sigma \Delta - (\Sigma y \Delta)^2\right] = 2\left[24{,}536 \times 58.31 - (794.60)^2\right] = 1{,}598{,}610$$

Point	22	21	23	24	25	26
	V_L	M_c	$(111.4)V_c$	$66.5 H_c$	$111.4-x$	M_L
0'	0.000	.000	0.00	0.00	0.00	0.00
1'	.007	.399	.78	2.86		+1.68
2'	.024	.872	-2.67	7.58		+4.04
3'	.053	-1.480	-5.90	14.96		+7.58
4'	.095	-1.952	-10.58	24.07		+11.54
5'	.149	-1.983	-16.60	33.78		+15.20
6'	.214	-1.292	-23.84	43.03		+17.90
7'	.289	+0.356	-32.19	50.94		+19.11
8'	.370	+3.156	-41.22	56.79		+18.73
9'	.456	+7.300	-50.80	59.85		+1635
C	.500	+10.08	+55.70	59.85	-111.40	+14.23
9	.544	+7.30	+50.80	59.85	-105.83	+12.12
8	.630	+3.156	+41.22	56.79	-94.69	+ 6.48
7	.711	+0.356	+32.19	50.94	-83.55	- 0.06
6	.786	-1.292	+23.84	43.03	-72.41	- 6.83
5	.851	-1.983	+16.60	33.78	-61.27	-12.87
4	.905	-1.952	+10.58 m	24.07	-50.13	-17.43
3	.947	-1.480	+ 5.90	14 96	-38.99	-19.61
2	.976	- .872	+ 2.67	7.58	-27.85	-18.47
1	.993	- .399	+0 . 78	2.86	-16.71	-13.47
0	1.000				- 5.57	- 5.57

Point	27	21	28	29	30
	M_c	52.2 V_c	12.2 H_c	52.2-x	$M_{1/4}$
0'	0.00	0.00	0.00		0.00
1'	-.37	-.40	.52		-.25
2'	-1.25	-.87	1.39		-.73
3'	-2.77	-1.48	2.74		-1.51
4'	-4.96	-1.95	4.42		-2.49
5'	-7.79	-1.98	6.19		-3.58
6'6'	-11.18	-1.29	7.89		-4.58
7'	-15.10	+ .36	9.34		-5.40
8'	-19.32	+3.16	10.41		-5.75
9'	-23.80	+7.30	10.98		-5.52
C'	+26.10	+10.08	10.98	-52.20	-5.04
9	+23.80	+7.30	10.98	-46.63	-4.55
8	+19.32	+3.16	10.41	-35.49	-2.60
7	+15.10	+ .36	9.34	-24.35	+ .45
6	+11.18	-1.29	7.89	-13.21	+4.57
5	+7.79	-1.98	6.19	- 2.07	+9.93
4	+4.96	-1.95	4.42		+7.43
3	+2.77	-1.48	2.74		+4.03
2	+1.25	-.87	1.39		+1.77
1	+ .37	-.40	.52		+ .49
	0	0	0		0

Discussion of Results Obtained

The results obtained are best shown by means of examination and comparison of plotted curves.

Plates 1 and 1A give the essential dimensions of the arch tested. The values of moment of inertia shown are merely relative values. The marked variation of the I curve from that corresponding to $I = I_c \sec\alpha$ should be noted.

The parabolic arch rib with same rise and span does not depart radically from the actual arch. The value of the horizontal reaction for the parabolic arch with $I = I_c \sec\alpha$ will be developed for purpose of comparison with the value obtainedby measuring the area under the curves obtained experimentally.

We know that for a load P, at any point,

$$H = \frac{15PL}{64h}(1-K^2)^2$$

In order to find H for a uniform load, it will be necessary to express P as $w\,dx$ and integrate.

That is $$H = \frac{15 \times 222.8 \times 2w}{64 \times 66.5}\int_0^{L/2}\left(1-\frac{x^2}{L^2}\right)^2 dx$$

$$= 1.57w\left[x - \frac{2x^3}{3L^2} + \frac{x^5}{L^4}\right]_0^{L/2}$$

$$= 1.57wL\left[\frac{1}{2} - \frac{1}{12} + \frac{1}{160}\right]$$

$$= 151\,w$$

By mesuring the area under the curve we find that $H = 94.5\omega$.

This value is practically the same as the area under the curve obtained analytically. This shows that the extreme variation of I form Ic sec influences the results considerably. The cahnge in the position of the reaction locus also shows the effect of this large increase in I toward the springing line.

The results obtained from the two analytical methods agreed very closely. Plates 2, 3, and 4, show this agreement. The value for moment differed somewhat in the two methods. The average of these two values was determined for use in comparison with results from Beggs Deformeter method.

Plate 5 shows the characteristic curves of moment at support, quarter point, and crown. From this plate the effect of a load ar any point on each of these three points may be seen.

The Beggs aparatus apparently gives almost perfect accuracy for thrust and shear. Curves 2 and 4 of Plate 6 are examples of this fact. Curves 1 and 3 show the relieving effect of the superstructure. By measuring the area under the curves 2 and 3, it was ascertained that the superstructure reduced the horizontal reaction from 94.5ω to 89.1ω. The fact that there is some difference between curves 1 and 3 shows that there is some experimental error introduced as a result of cutting the model. The two curves represent two different methods of simulating actual conditions after the model has been cut. The curves will be studies in an attempt to se which method is the better.

In Plate 7 there is another example of the remarkable accuracy obtained with the Beggs Deformeter. The slight departure of curve 2 indicates the possibility that the welded model more truly represents the actual structure than the gage with neutral plugs.

Plates 8 and 9 show that the values for moment at points close to the cut are best obtained when the arch is welded. Conversely, points at considerable distance from the cut are best obtained when the gage with neutral plugs is at the cut.

There are several reasons for the fact that the experimental values for moment are somewhat in error, while the values for thrust and shear are almost exact. Any error in setting up the model, so that slight eccentricity is introduced, would materially affect the moment values. The vaules of thrust and shear would be changed very little. Also, the readings obtained for moment were considerably smaller than those obtained for thrust and shear. The percent error introduced, in case of moments, was therefore much larger.

In Plates 11, 12, and 13 the relieving effect of the superstructure is again illustrated. For a uniform load over the whole structure the moment at the quarter point is reduced form 72 ω to 49 ω.

In the design of arches with superstructures this effect is not usually taken advantage of. In other words the safety factor is simply increased.

The results obtained were all sufficiently accurate for practical purposes, regardless of whether the arch was welded or the gage with neutral plugs was at the quarter point. The real advantage of the Beggs method shows up when it is desired to analyze a structure as multiply-indeterminate as the arch with superstructure. A mathematical solution of such a problem would require a great deal more time than a solution by means of the Beggs Deformeter.

The rigid proof of the applicability of the Beggs Method to any isotropic body should be of considerable interest.

Calibration of Microscopes for Deformeter Plugs

Inst. No.	Trial No.	Thrust	Shear	Moment x5	Moment
1401	1	1.887	1.863	1.820	
	2	1.884	1.856	1.826	
	3	1.887	1.856	1.829	
	Ave.	1.886	1.858	1.825	0.3650
1412	1	1.903	1.859	1.845	
	2	1.904	1.866	1.837	
	3	1.905	1.862	1.841	
	Ave.	1.904	1.862	1.841	0.3682
1451	1	1.894	1.860	1.842	
	2	1.907	1.858	1.848	
	3	1.903	1.869	1.849	
	Ave.	1.901	1.862	1.846	0.3692
Average for all instruments		1.897	1.861		0.3675

Average for all instruments 2.672×10^{-5} inches per division.

Thrust, Shear and Moment at Right Reaction

Before Model Was Cut At Quarter Point.

Target Points On Roadway

Point	Initial Reading	Final Reading	Diff.	Thrust	Shear	Moment
0	2.387	2.366	-0.021	-0.011		
1	2.705	2.721	+0.016	+0.008		
2	3.384	3.200	+0.184	+0.097		
3	3.900	4.360	+0.460	+0.242		
4	4.305	3.492	+0.813	+0.428		
5	3.444	4.615	+1.171	+0.618		
6	4.020	2.538	+1.482	+0.782		
7	4.470	6.260	+1.790	+0.945		
8	4.025	5.985	+1.880	+0.992		
9	2.295	4.215	+1.920	+1.011		
0	4.112	4.072	-0.040	-	-0.021	
1	4.295	4.440	+0.145		+0.078	
2	2.975	3.620	+0.645		+0.346	
3	5.675	6.877	+1.202		+0.646	
4	3.592	5.070	+1.478		+0.794	
5	4.784	3.264	+1.520		+0.817	
6	2.760	3.912	+1.152		+0.620	
7	5.690	4.982	+0.708		+0.386	
8	3.390	3.583	+0.193		+0.104	
9	3.256	3.256	+0.000		+0.000	
0	3.108	3.082	-0.026			- 1.41
1	4.355	4.388	+0.033			+ 1.80
2	3.210	3.375	+0.165			+ 8.98
3	6.160	6.412	+0.252			+13.70
4	4.278	4.495	+0.217			+11.80
5	3.992	4.052	+0.060			+ 3.26
6	3.333	3.237	-0.096			- 5.22
7	5.488	5.209	-0.279			-15.20
8	3.633	3.383	-0.250			-13.70
9	3.244	3.292	+0.048			+ 2.64

Calibration Constant x 20 gives Moment in Ft.-lbs.

RIGHT REACTION-MODEL WITH SUPERSTRUCTURE-FLOATING GAGE AT $\frac{1}{4}$ POINT

Point		I.R.	F.R.	Diff.	V.R.	I.R.	F.R.	Diff.	H.R.	I.R.	F.R.	Diff	Mom.
0	1	3.884	3.891	.007		3.915	3.896	.019	.017	3.903	3.900	.003	
	2	4.603	4.610	.007		4.688	4.640	.048		4.670	4.670	.000	
1	Av			- .007	-.004			.032	.017			.002	+0.08
1	1	3.198	3.810	.018		3.172	3.303	.131		3.223	3.283	.041	
	2	4.335	4.315	.020		4.260	4.391	.131		4.308	4.341	.033	
	Av			.019	.010			.131	.07			.037	-2.02
2	1	4.128	3.953	.175		3.582	4.237	.655		3.838	3.998	.150	
	2	4.307	4.130	.177		3.840	4.470	.630		3.992	4.127	.135	
	Av			.176	.093			.642	.35		4.616	.143	-7.80
3	1	4.710	4.283	.427		3.626	4.800	1.174		4.392	4.616	.224	
	2	4.105	3.687	.418		3.315	4.473	1.158		3.793	4.030	.237	
	Av			.422	.220			1.166	.63			.230	-12.54
4	1	4.601	3.862	.739		3.484	4.952	1.468		4.147	4.332	.185	
	2	4.169	3.421	.748		3.840	4.536	1.497		3.683	3.868	.185	
	Av			.744	.390			1.482	.80			.185	-10.10
5	1	4.358	3.268	1.090		3.605	4.555	1.490		3.780	3.835	.055	
	2	4.716	3.622	1.094		3.425	4.905	1.480		4.152	4.202	.050	
	Av			1.092	.580			1.485	.80			.053	-2.96
6	1	4.672	3.247	1.425		3.587	4.785	1.198		4.170	4.079	.091	
	2	4.765	3.354	1.411		3.448	4.646	1.198		4.130	4.032	.098	
	Av			1.418	.750			1.198	.64			.094	+5.12
7	1	5.370	3.696	1.674		3.239	3.914	.675		3.726	3.457	.269	
	2	4.360	2.707	1.653		3.174	3.842	.668		3.660	3.400	.260	
	Av			1.663	.880			.671	.36			.264	+14.40
8	1	4.515	2.690	1.825		4.388	4.565	.177		4.852	4.592	.260	
	2	4.737	2.920	1.817		3.518	3.687	.169		3.740	3.483	.257	
	Av			1.821	.960			.173	.09			.259	+14.10
9	1	5.610	3.720	1.890		4.641	4.658	.017		4.618	4.697	.079	
	2	4.370	2.467	1.903		3.500	3.537	.037		3.473	3.552	.079	
	Av			1.897	1.000			.027	.015			.079	-4.31

LEFT REACTION-MODEL WITH SUPERSTRUCTURE-FLOATING GAGE AT $\frac{1}{4}$ POINT

Point		I.R.	F.R.	Diff.	V.R.	I.R.	F.R.	Diff.	Mom.	I.R.	F.R.	Diff.	H.R.
0	1	4.845	2.942	1.903		3.928	3.870	.058		3.880	3.880	.000	
	2	5.506	3583	1.923		4.567	4.512	.055		4.518	4.542	.024	
	Av			1.913	1.008			.057	+ 3.08			.012	.006
1	1	3.117	1.263	1.854		2.047	2.339	.292		2.264	2.100	.164	
	2	5.260	3.392	1.868		4.173	4.470	.297		4.392	4.236	.156	
	Av			1.861	.98			.295	−16.08			.160	.086
2	1	4.762	3.057	1.705		3.764	4.072	.308		4.232	3.587	.645	
	2	5.253	3.548	1.705		4.257	4.553	.296		4.718	4.070	.648	
	Av			1.705	.90			.302	−16.47			.646	.35
3	1	5.222	3.773	1.449		4.428	4.570	.142		4.807	3.644	1.163	
	2	4.660	3.211	1.449		3,852	4.000	.148		4.507	3.357	1.150	
	Av			1.449	.76			.145	−7.920			1.156	
4	1	4.800	3.677	1.123		3.819	3.794	.025		4.523	3.100	1.423	.62
	2	4.392	3.245	1.147		3.812	3.790	.022		3.315	1.885	1.430	
	Av			1.134	.60			.024	+ 1.28			1.426	.77
5	1	4.213	3.450	.763		3.916	3.725	.191		4.522	3.106	1.416	
	2	4.555	3.784	.771		4.260	4.078	.182		4.878	3.461	1.417	
	Av			.767	.40			.187	+10.18			1.417	.76
6	1	4.191	3.740	.451		4.080	3.860	.220		4.825	3.663	1.162	
	2	4.315	3.853	.462		4.188	3.971	.217		4.655	3.504	1.151	
	Av			.456	.24			.219	+11.92			1.156	.62
7	1	4.636	4.442	.194		4.622	4.464	.158		4.865	4.224	.6410	
	2	3.647	3.446	.201		3.611	3.470	.141		3.847	3.205	.642	
	Av			.198	.104			.150	+ 8.19			.642	.34
8	1	4.547	4.495	.052		4.553	4.498	.055		4.610	4.432	.178	
	2	3.855	3.802	.053		3.847	3.812	.035		3.917	3.740	.177	
	Av			.052	.027			.045	+ 2.45			.177	.095
9	1	4.659	4.659	.000		4.658	4.658	.000		4.658	4.658	.000	
	2	3.477	3.477	0000		3.478	3.478	.000		3.481	3.483	.002	
	Av			.000	.000			.000	0.00			.001	.000

QUARTER POINT-MODEL WITH SUPERSTRUCTURE- FLOATING GAGE AT $\frac{1}{4}$ POINT

Point		I.R.	F.R.	Diff.	Thrust	I.R.	F.R.	Diff.	Shear	I.R.	F.R.	Diff.	Mom.
0	1	2.918	2.950	.032	.01	2.921	2.920	.001		2.993	3.000	.007	
	2	4.552	4.560	.008		4.555	4.555	.000		4.562	4.580	.018	
	Av			.020	.01			.000	.000			.011	−0.60
1	1	3.483	3.590	.107		4.287	4.368	.075		3.490	3.492	.002	
	2	4.287	4.369	.082		3.462	3.528	.066		4.331	4.331	.000	
	Av			.094	.05			.070	−.04			.001	−0.06
2	1	4.787	5.025	.238		4.770	5.083	.313		4.893	4.961	.068	
	2	3.917	4.170	.253		4.140	4.472	.332		4.037	4.112	.075	
	Av			.245	.13			.322	−.17			.071	−3.87
3	1	3.337	4.642	1.305		4.368	3.680	.688		4.009	4.067	.058	
	2	3.220	4.523	1.303		4,241	3.588	.653		3.886	3.948	.062	
	Av			1.304	.69			.670	+.36			.060	−3.27
4	1	3.907	5.660	1.753		4.502	4.213	.289		4.350	4.319	.031	
	2	2.953	4.700	1.747		3.978	3.690	.288		3.823	3.780	.043	
	Av			1.750	.92			.288	+.155			.037	+2.02
5	1	4.393	6.192	1.799		5.299	5.247	.052		5.289	5.237	.052	
	2	3.267	5.057	1.790		4.212	4.157	.055		4.193	4.138	.055	
	Av			1.795	.945			.054	+.029			.054	+2.92
6	1	3.618	5.087	1.469		4.337	4.406	.070		4.383	4.328	.055	
	2	3.341	4.794	1.453		4.043	4.097	.054		4.093	4.041	.052	
	Av			1.461	.77			.062	−.033			.054	+2.92
7	1	3.733	4.552	.819		3.519	3.560	.041		4.172	4.137	.035	
	2	3.129	3.921	.792		3.500	3.562	.062		3.563	3.525	.038	
	Av			.805	.425			.051	−.027			.037	+1.99
8	1	5.175	5.392	.217		5,270	5.290	.020		5.284	5.279	.005	
	2	3.717	3.938	.221		3.570	3.592	.022		3.837	3.837	.000	
	Av			.219	.115			.021	−.011			.002	+0.14
9	1	4.412	4.412	.000		4.385	4.387	.002		4.382	4.383	.001	
	2	3.473	3.460	.013		3.466	3.466	.000		3.470	3.470	.000	
	Av			.006	.003			.001	.000			.000	0.00

RIGHT REACTION PLAIN ARCH RIB FLOATING GAGE AT 1/4 POINT

Point		I.R.	E.R	Diff.	V.R.	L.R.	F.R.	Diff.	H.R.	I.R.	F.R.	Diff.	Moment
1	1	3.687	3.639	.048	.025	3.553	3.732	.179	.096	3.674	3.630	.044	-2.34
	2												
	Av												
2	1	4.759	4.582	.177		4.430	5.029	.689		4.773	4.589	.184	
	2	4.052	3.858	.194		3.606	4.036	.700		4.050	3.871	.179	
	Av			.186	.098			.695	.373			.182	-9.88
3	1	4.003	3.560	.443		3.571	4.793	1.222		4.332	4.072	.260	
	2	3.737	3.288	.449		2.867	4.113	1.246		3.924	3.640	.284	
	Av			.446	.235			1.234	.663			.272	-14.80
4	1	4.730	4.038	.692		3.580	5.222	1.642		4.545	4.234	.311	
	2	5.330	4.264	.706		4.152	5.800	1.648		5.125	4.842	.283	
	Av			.699	.368			1.645	.883			.297	-16.15
5	1	5.417	4.338	1.079		4.066	5.685	1.621		4.945	4.826	.119	
	2	5.770	4.666	1.104		4.422	6.032	1.610		5.267	5.162	.105	
	Av			1.091	.575			1.616	.867			.112	-6.10
6	1	5.122	3.664	1.458		3.807	4.963	1.156		4.303	4.503	.200	
	2	4.959	3.478	1.481		3.609	4.781	1.172		4.121	4.303	.179	
	Av			1.470	.775			1.164	.625			.190	+10.35
7	1	4.770	3.040	1.730		3.602	4.142	.540		3.717	4.080	.363	
	2	5.168	3.433	1.728		4.012	4.552	.540		4.108	4.474	.366	
	Av			1.729	.912			.540	.290			.365	+19.85
8	1	5.245	3.400	1.845	.973	4.248	4.372	.124	.066	4.190	4.442	.252	+13.70
	2												
	Av												

LEFT REACTION-PLAIN ARCH RIB-FLOATING GAGE AT $\frac{1}{4}$ POINT

Point		I.R.	F.R.	Diff.	V.R.	I.R.	F.R.	Diff.	H.R.	I.R.	F.R.	Diff.	Moment
1		4.578	2.713	1.865	.983	3.728	3.548	.180	.096	3.471	3.810	.339	18.45
2	1	5.523	3.820	1.703		5.006	4.322	.684	.373	4.508	4.832	.324	
	2	4.837	3.148	1.689		4.325	3.615	.710		3.839	4.147	.308	
	Av			1.696	.894			.697	.373			.316	17.20
3	1	4.898	3.452	1.446		4.747	3.570	1.177		4.110	4.231	.121	
	2	3.504	2.047	1.457 1.452		3.407	2.126	1.281		2.720	2.842	.122	
	Av			1.452	.766			1.229	.657	5.050		.122	6.62
4	1	4.940	3.797	1.143		5.227	3.558	1.669		5.050	4.940	.110	
	2	5.577	4.419	1.158		5.824	4.150	1.674		5.050	4.939	.111	
	Av			1.150	.606			1.672	.895			.110	6.02
5	1	4.905	4.143	.762		5.707	4.062	1.645		5.002	4.750	.252	
	2	5.609	4.867	.742		6.053	4.413	1.640		5.368	5.113	.255	
	Av			.752	.396			1.642	.878			.254	13.80
6	1	4.588	4.185	.403		4.958	3.836	1.122		4.527	4.273	.254	
	2	4.398	4.017	.381		4.768	3.645	1.123		4.335	4.090	.245	
	Av			.392	.206			1.122	.602			.250	13.60
7	1	4.196	4.053	.143		4.164	3.640	.524		3.972	3.847	.125	
	2	4.379	4.238	.141		4.552	4.052	.500		4.367	4.238	.129	
	Av			.142	.075			.512	.274			.127	6.90
8	1	4.310	4.290	.020	.011	4.348	4.270	.078	.042	4.321	4.278	.043	2.34

WITHOUT SUPERSTRUCTURE DATA FOR LEFT 1/4 POINT

Point		I.R	F.R.	DIFF.	THR.	I.R.	F.R.	DIFF.	SHEAR	I.R.	F.R.	DIFF.	MOM.
1	1	3.573	3.727	.154	.081	3.598	3.703	.105	-.056	3.632	3.659	.027	-1.47
	2												
	A												
								.481					
2	1	4.407	4.943	.536		4.450	4.931	.481		4.602	4.760	.158	
	2	3.720	4.273	.553		3.693	4.180	.487		3.843	4.011	.168	
	A			.544	.286			.484	-.260			.163	-8.86
3	1	3.303	5.061	1.758		4.577	3.820	.757		4.122	4.232	.110	
	2	1.672	3.442	1.770		2.947	2.197	.750		3.477	3.562	.085	
	A			1.764	.930			.753	+.404			.097	-5.27
4	1	3.387	5.405	2.018		4.560	4.248	.312		4.436	4.380	.056	
	2	3.998	6.005	2.007		5.167	4.852	.315		5.032	4.965	.067	
	A			2.013	1.060			.313	+.168			.062	+3.37
5	1	3.979	5.803	1.824		4.899	4.897	.002		4.952	4.837	.115	
	2	4.348	6.157	1.809		5.250	5.262	.012		5.293	5.183	.110	
	A			1.816	.957			1005	-.003			.112	+6.12
6	1	3.804	5.013	1.209		4.348	4.480	.132		4.457	4.367	.090	
	2	3.618	4.803	1.185		4.163	4.282	.119		4.269	4.172	.097	
	A			1.197	.630			.126	-.068			.093	+5.08
7	1	3.638	4.178	.540		3.858	3.962	.104		3.937	3.889	.048	
	2	4.048	4.575	.527		4.262	4.357	.095		4.325	4.287	.038	
	A			.534	.282		[illegible]	.100	-.054			.043	+2.34
8	1	4.264	4.344		.042	4.291	4.291	.000	.000	4.290	4.292	.002	

Point	I.R.	F.R.	DIFF.	THR.	I.R.	F.R.	DIFF.	SHEAR	I.R.	F.R.	DIFF.	MOM.
1	4.343	4.332	0.011	.006	4.272	4.403	0.131	.070	4.317	4.352	-.036	-1.91
22	4.303	4.132	0.171	.090	3.912	4.525	0.613	.330	4.129	4.297	-.168	-9.16
3	6.419	6.010	0.409	.216	5.619	6.827	1.208	.649	6.082	6.351	-.269	-14.12
4	6.822	6.091	0.731	.385	5.660	7.275	1.615	.867	6.342	6.583	-.241	-13.13
5	4.088	2.948	1.140	.601	2.704	4.327	1.623	.872	3.491	3.561	-.070	-3.82
6	5.983	4.523	1.460	.769	4.650	5.827	1.177	.632	5.333	5.172	+.161	+8.77
7	4.638	2.907	1.731	.912	3.464	4.060	0.596	.320	3.953	3.596	+.357	+19.46
8	4.572	2.640	1.932	1.000	3.554	3.680	0.126	.077	3.724	4.333	-.309	-16.84

RIGHT-REACTION PLAIN ARCH RIB- MODEL WELDED AT QUARTER POINT

RIGHT REACTION PLAIN ARCH RIB NEUTRAL PLUGS AT $\frac{1}{4}$ POINT

Point		I.R	F.R	DIFF	V.R.	I.R.	F.R.	DIFF	H.R.	I.R.	F.R.	DIFF	MOM
1	1	3.687	3.639	.048	.025	3.553	3.732	.179	-.096	3.630	3.674	.044	-2.34
	2												
	Av												
2	1	4.759	4.582	.177		4.340	5.029	.689		4.589	4.773	.184	
	2	4.052	3.858	.194		3.606	4.306	.700		3.871	4.050	.179	
	Av			.1855	.098			.695	-.373			.1815	-9.88
3	1	4.003	3.560	.443		3.571	4.793	1.222		4.072	4.332	.260	
	2	3.737	3.288	.449		2.867	4.113	1.246		3.640	3.924	.284	-14.80
	Av			4446	.235			1.234	-.663			.272	
4	1	4.730	4.038	.692		3.580	5.222	1.642		4.234	4.545	.311	
	2	5.330	4.624	.706		4.152	5.800	1.648		4.842	5.125	.283	
	Av			.699	.368			1.645	-.883			.297	-16.15
5	1	5.417	4.338	1.079		4.066	5.685	1.621		4.826	4.945	.119	
	2	5.770	4.666	1.104		4.422	6.032	1.610		5.162	5.267	.105	
	Av			1.091	.575			1.616	-.867			.112	-6.10
6	1	5.122	3.664	1.458		3.807	4.963	1.156		4.503	4.303	.200	
	2	4.959	3.478	1.481		3.609	4.781	1.172		4.303	4.121	.179	
	Av			1.470	.775			1.164	-.625			.190	10.35
7	1	4.770	3.040	1.730		3.602	4.142	.540		4.080	3.717	.363	
	2	5.168	3.433	1.728		4.012	4.552	.540		4.474	4.108	.366	
	Av			1.729	.912			.540	-.290			.3645	19.85
8	1	5.245	3.400	1.845	.973	4.248	4.372	.124	-.066	4.442	4.190	.252	13.70
	2												
	Av												

	LEFT REACTION			PLAIN ARCH RIB				MODEL WELDED				
	I.R.	F.R.	DIFF.	THR.	I.R.	F.R.	DIFF.	SHEAR	I.R.	R.R.	DIFF.	MOM.
1	5.252	3.407	1.845	.972	4.394	4.255	.139	.075	4.171	4.497	-.326	-17.77
2	5.103	3.358	1.745	.920	4.497	3.883	.614	.330	4.040	4.407	-.367	-20.00
3	6.937	5.488	1.449	.764	6.803	5.603	1.200	.644	6.142	6.291	-.149	- 8.12
4	6.994	5.844	1.150	.606	7.203	5.615	1.588	.853	6.359	6.249	.110	6.00
5	3.877	3.148	.729	.384	4.313	2.734	1.579	.848	3.670	3.377	.293	15.97
6	5.547	5.062	.485	.256	5.841	4.678	1.163	.625	5.395	5.112	.283	15.42
7	3.848	3.692	.156	.082	4.041	3.483	.558	.300	3.848	3.691	.157	8.56
8	3.647	3.620	.027	.014	3.687	3.582	.105	.056	3.648	3.613	.035	1.91

PLAIN ARCH RIB　　LEFT REACTION　　NEUTRAL PLUGS AT $\frac{1}{4}$ PT

Point		I.R.	F.R.	DIFF	V.R.	I.R.	F.R.	DIFF	H.R.	I.R.	F.R.	DIFF.	MOM.
1	1	4.578	2.713	1.865	.983	3.728	3.548	.180	.096	3.471	3.810	.339	-18.45
2	1	5.523	3.820	1.703		5.006	4.322	.784		4.508	4.832	.324	
	2	4.837	3.148	1.689		4.325	3.615	.710		3.839	4.147	.308	
	Av			1.696	.894			.697	.373			.316	-17.20
3	1	4.898	3.452	1.446		4.747	3.570	1.177		4.110	4.231	.121	
	2	3.504	3.452	1.457		3.407	2.126	1.281		2.720	2.842	.122	
	Av			1.452	.766			1.229	.657			.1215	- 6.62
4	1	4.940	3.797	1.143		5.227	3.558	1.669		5.050	4.940	.110	
	2	5.577	4.419	1.158		5.824	4.150	1.674		5.050	4.939	.111	
	Av			1.150	.606			1.672	.895			.1105	+ 6.02
5	1	4.905	4.143	.762		5.707	4.062	1.645		5.002	4.750	.252	
	2	5.609	4.867	.742		6.053	4.413	1.640		5.368	5.113	.255	
	Av			.752	.396			1.642	.878			.2535	+13.80
6	1	4.588	4.185	.403		4.958	3.836	1.122		4.527	4.273	.254	
	2	4.398	4.017	.381		4.768	3.645	1.123		4.335	4.090	.245	
	Av			.392	.206			1.122	.602			.2495	+13.60
7	1	4.196	4.053	.143		4.164	3.640	.524		4.367	4.238	.129	
	2	4.379	4.238	.141		4.552	4.052	.500		4.363	4.238	.125	
	Av			.142	.075			.512	.274			.127	+6.90
8	1	4.310	4.290	.020	.011	4.348	4.270	.078	.042	4.321	4.278	.043	+2.34
	2												
	Av												

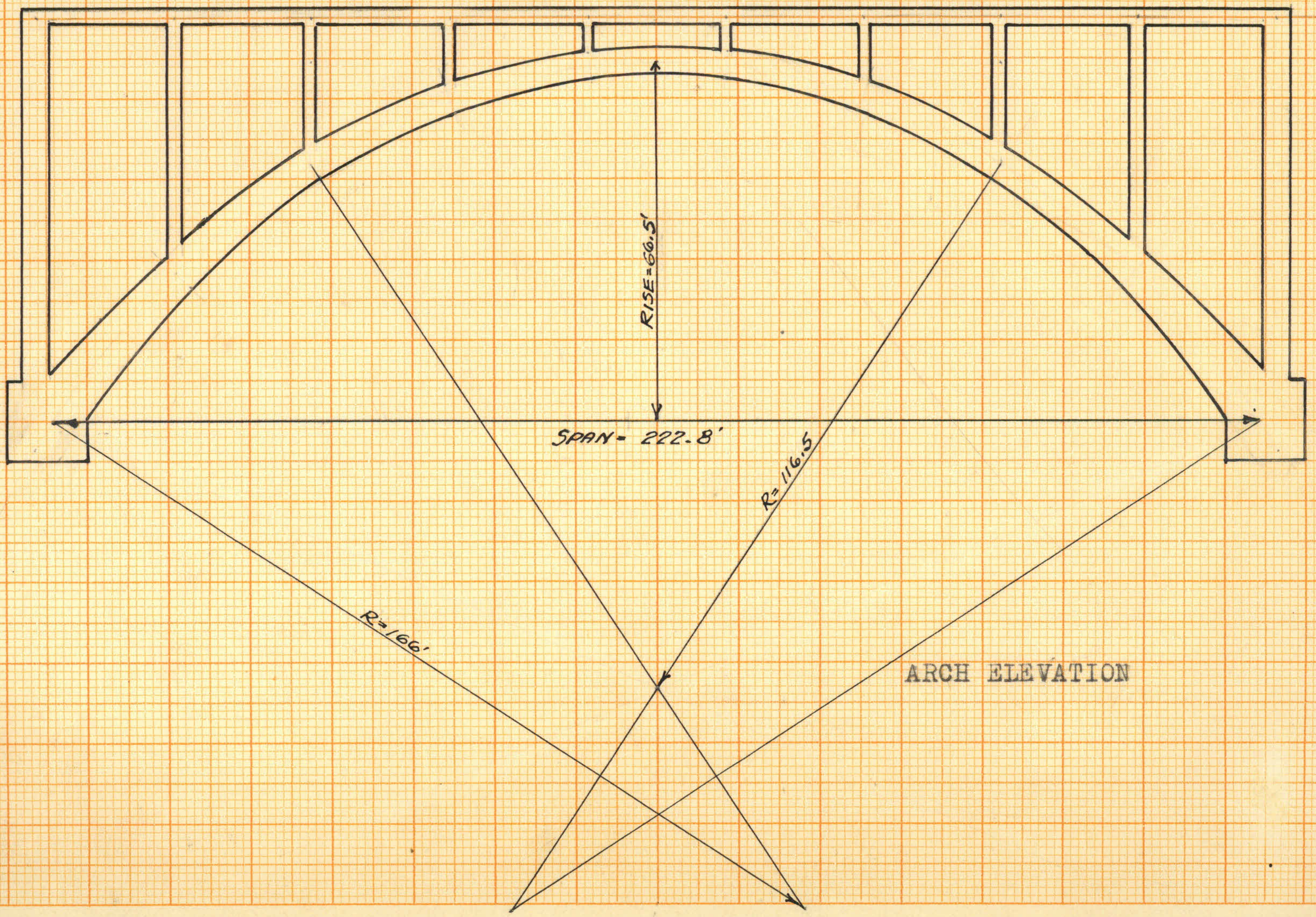


PLATE 1

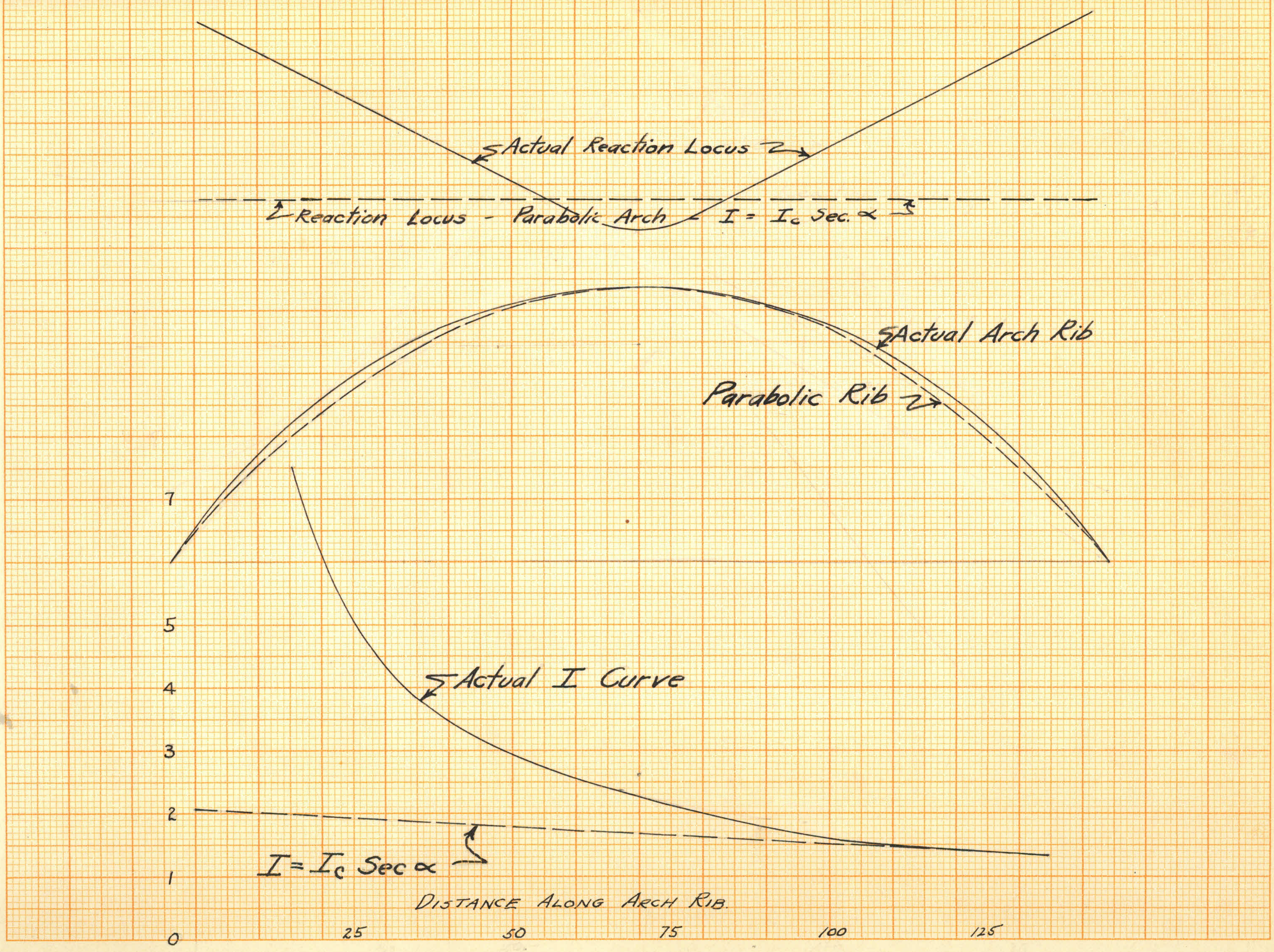
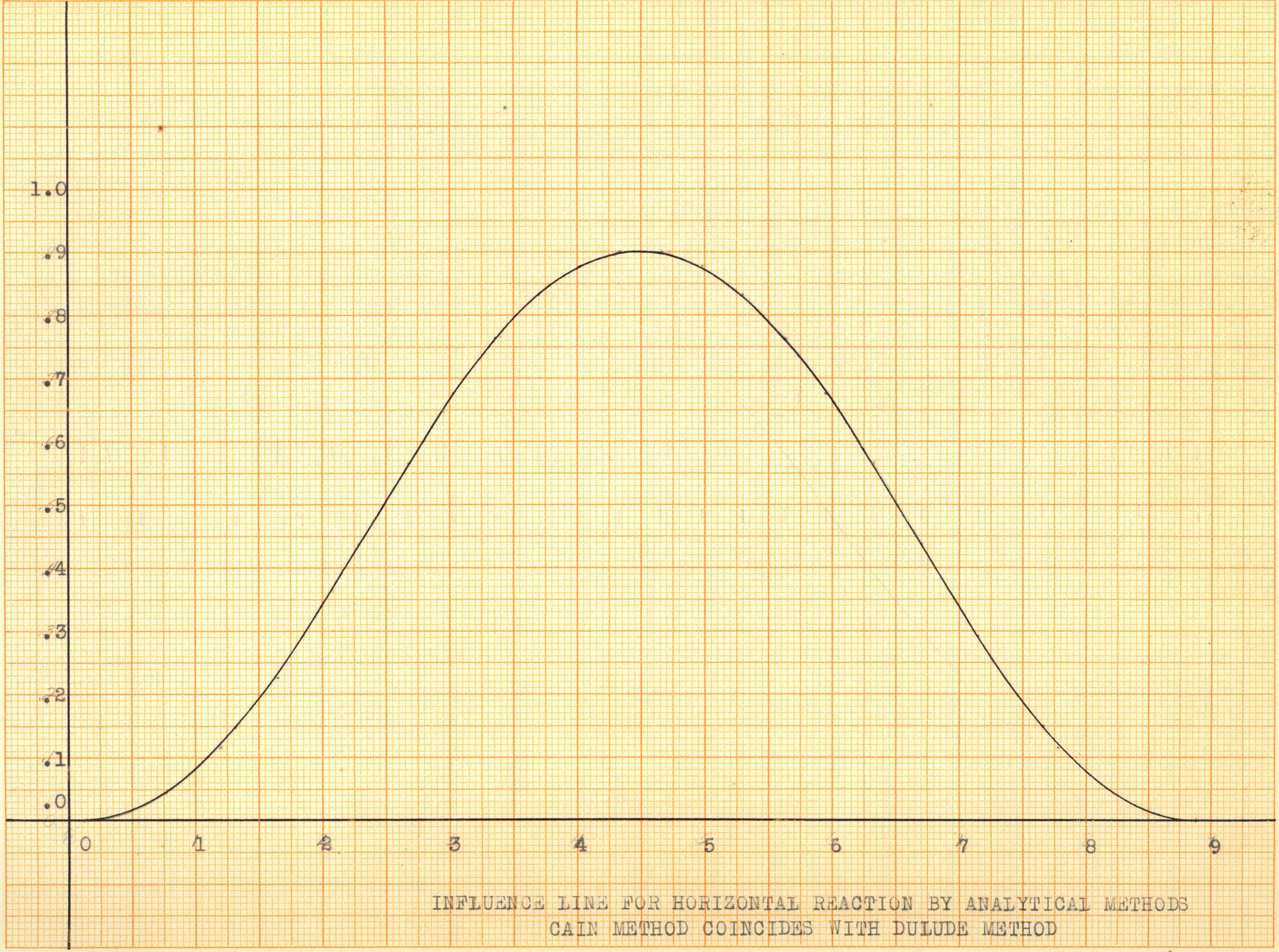
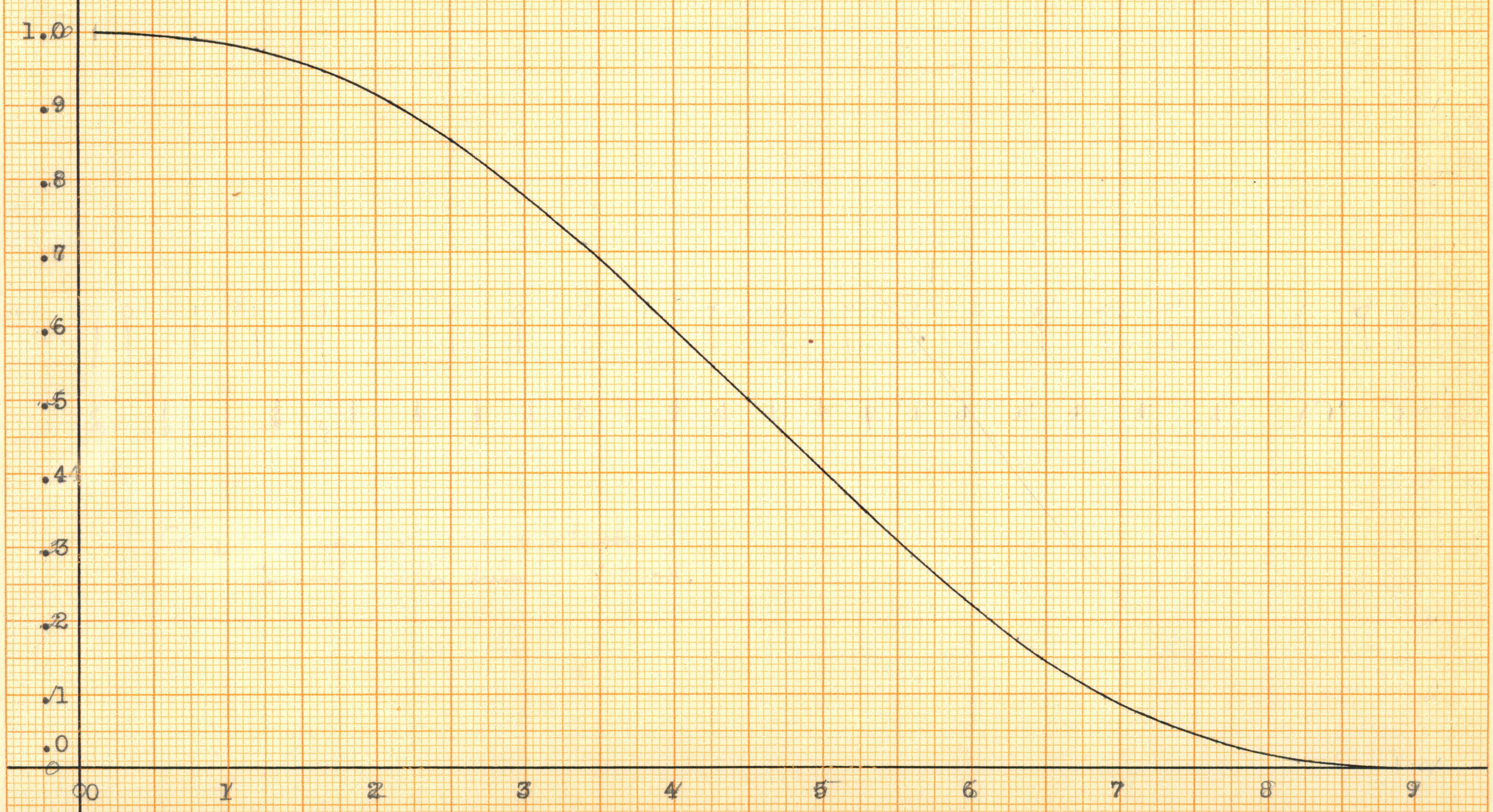

Actual Reaction Locus
Reaction Locus - Parabolic Arch - I = Ic Sec. α
Actual Arch Rib
Parabolic Rib
Actual I Curve
I = Ic Sec α
Distance Along Arch Rib.
0
1
2
3
4
5
7
25
50
75
100
125

PLATE 1A



INFLUENCE LINE FOR HORIZONTAL REACTION BY ANALYTICAL METHODS
CAIN METHOD COINCIDES WITH DULUDE METHOD

PLATE 2



INFLUENCE LINE FOR VERTICAL REACTION BY ANALYTICAL METHODS.

CAIN METHOD COINCIDES WITH DULUDE METHOD.

PLATE 3

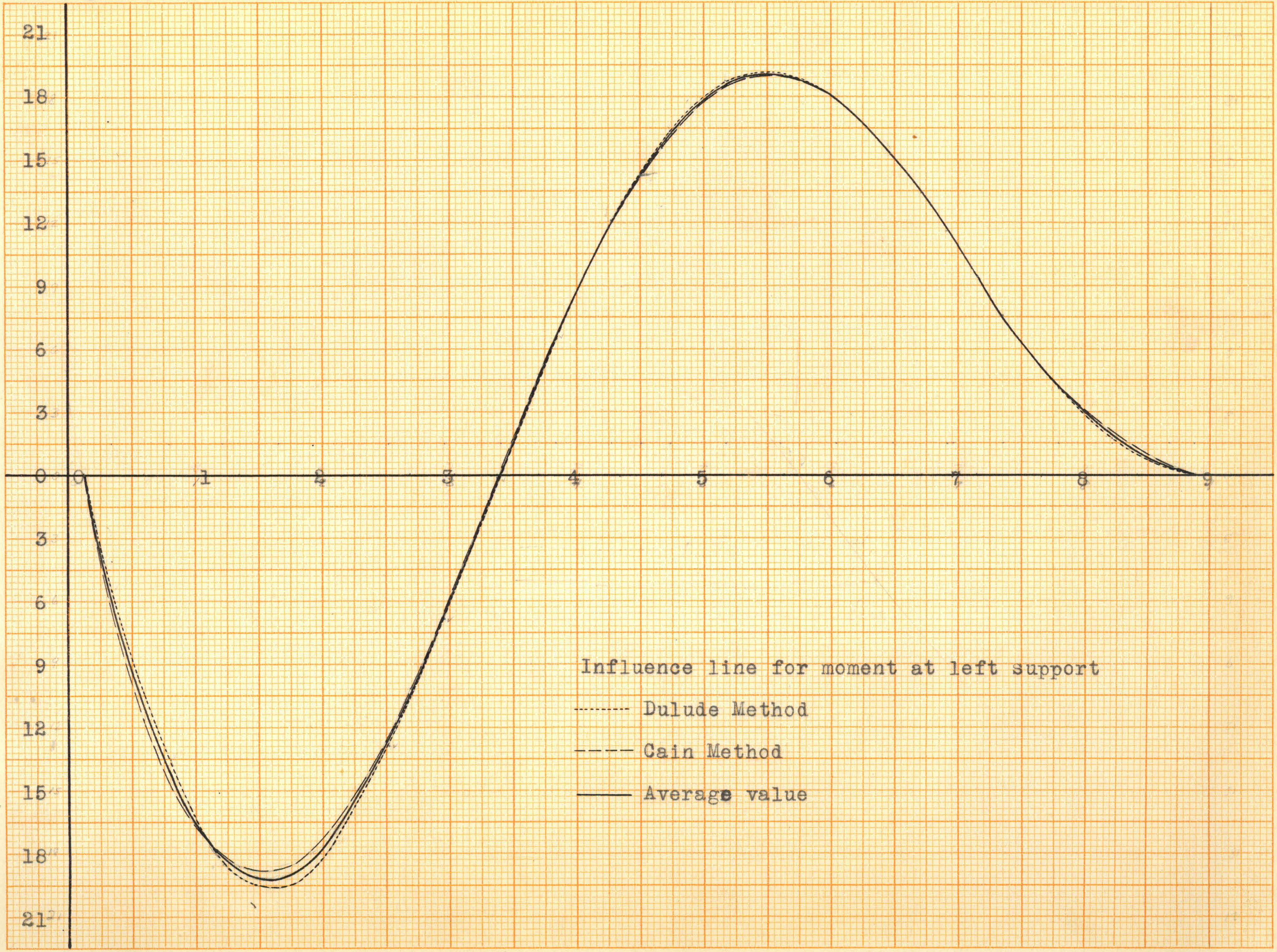

21
18
15
12
9
6
3
0
3
6
9
12
15
18
21
0
1
2
3
4
5
6
7
8
9
Influence line for moment at left support
Dulude Method
Cain Method
Average value

PLATE 4

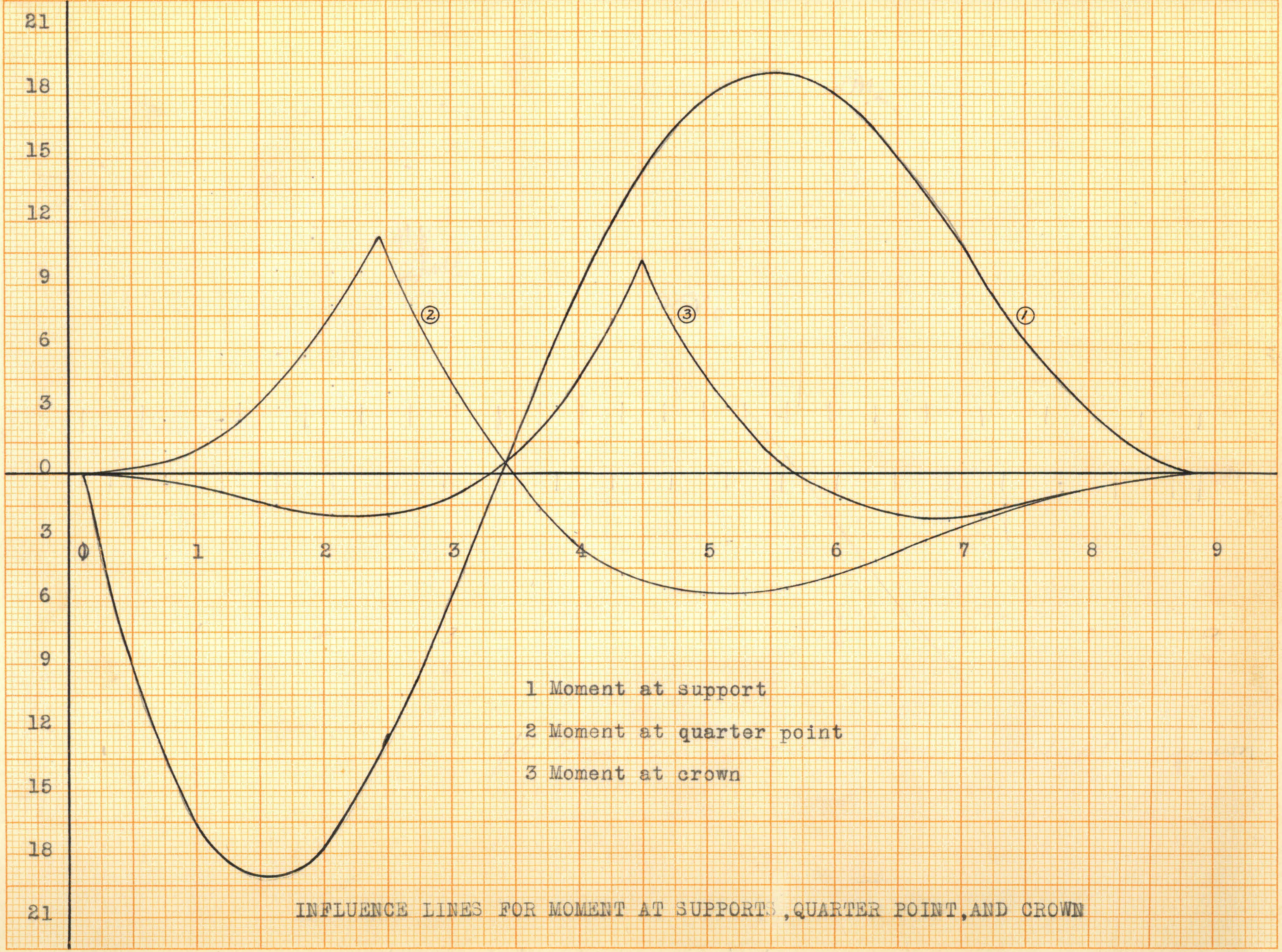
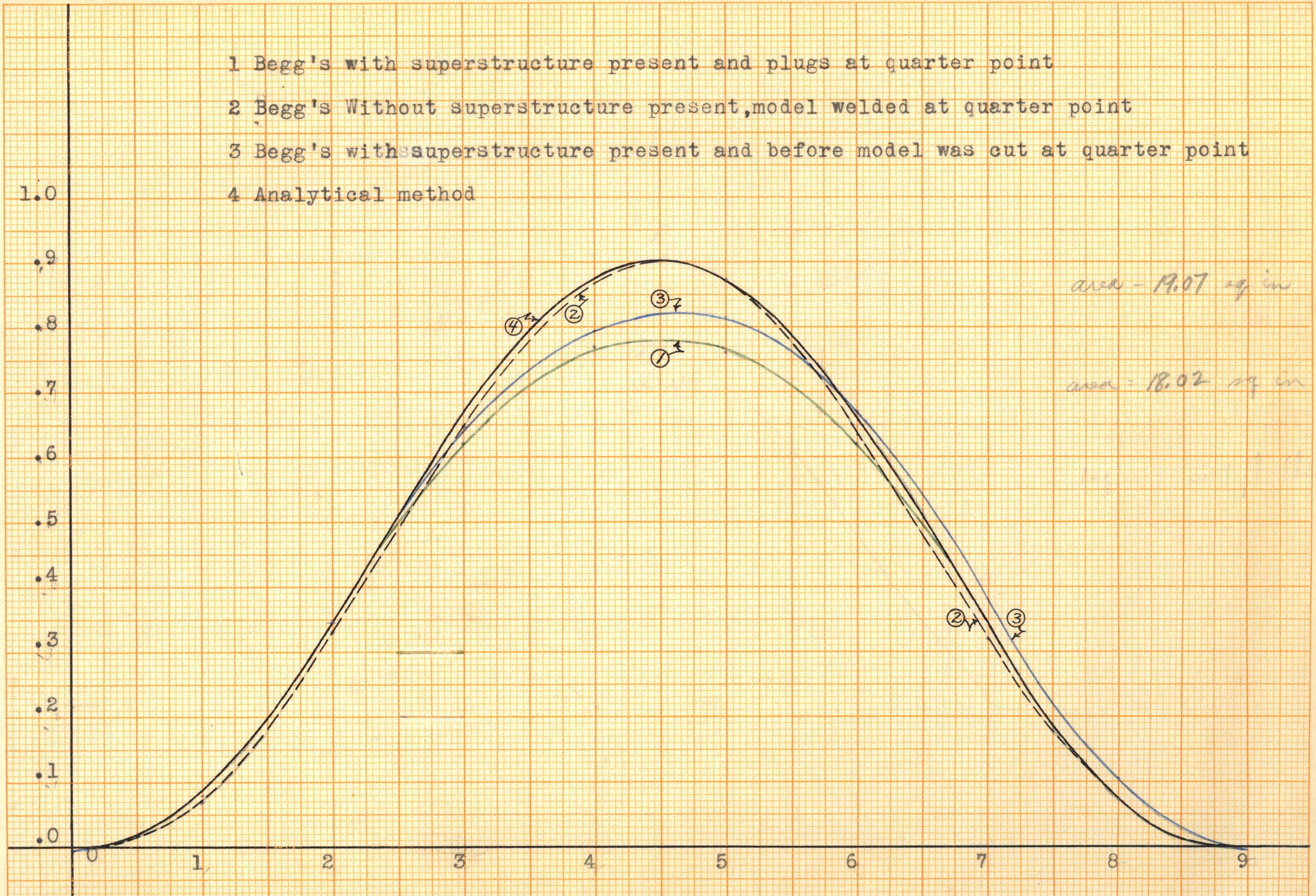
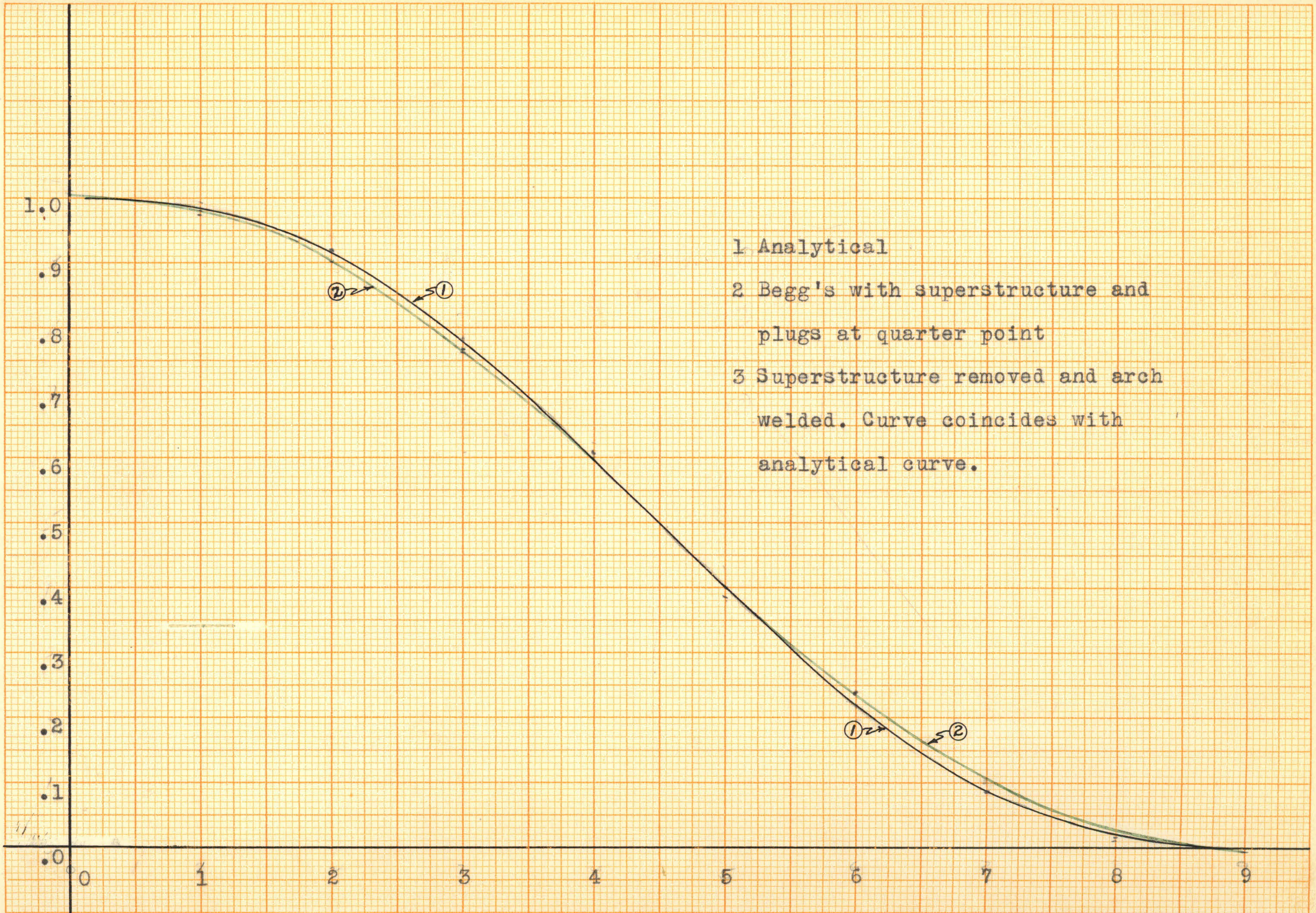

21
18
15
12
9
6
3
0
3
6
9
12
15
18
21
0
1
2
3
4
5
6
7
8
9
1
2
3
1 Moment at support
2 Moment at quarter point
3 Moment at crown
INFLUENCE LINES FOR MOMENT AT SUPPORTS, QUARTER POINT, AND CROWN

PLATE 5



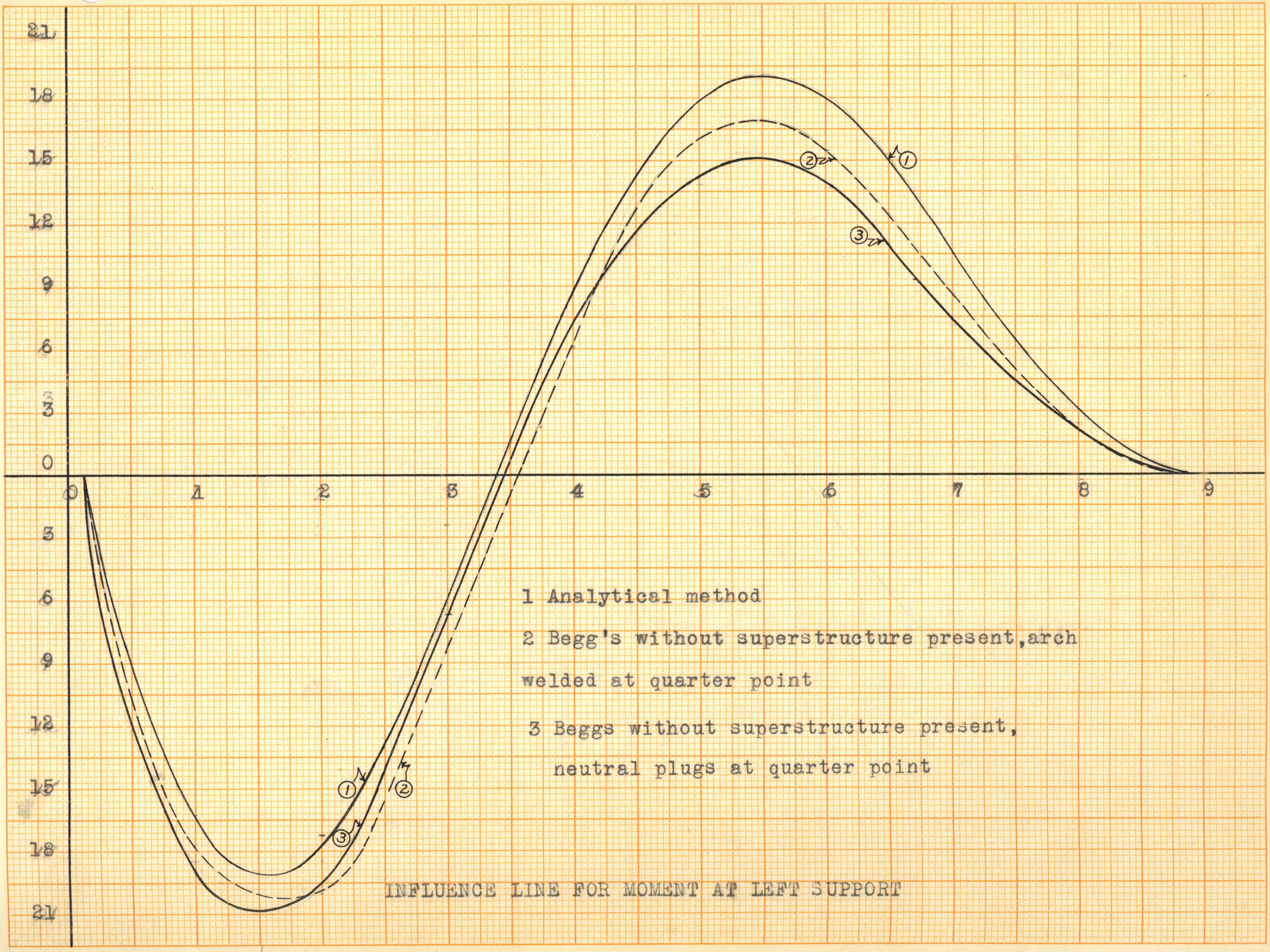
INFLUENCE LINE FOR HORIZONTAL REACTION

PLATE 6



Influence Line for Vertical Reaction at Left Support.

PLATE 7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18
15
12
9
6
3
0
3
6
9
12
15
18
21
0
1
2
3
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5
6
7
8
9
1
2
3
1
2
3
1 Analytical method
2 Begg's without superstructure present, arch welded at quarter point
3 Beggs without superstructure present, neutral plugs at quarter point
INFLUENCE LINE FOR MOMENT AT LEFT SUPPORT

PLATE 8

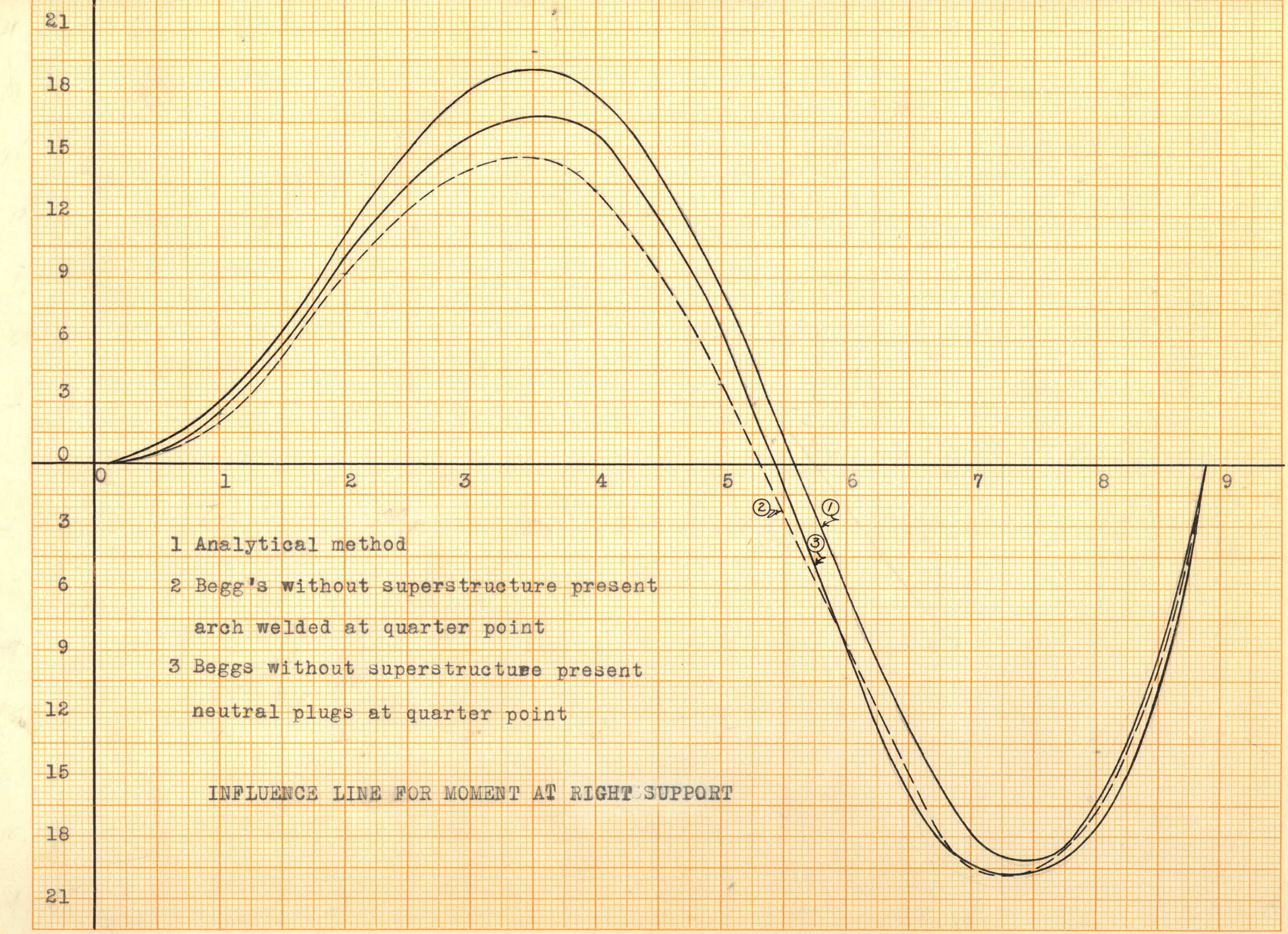

21
18
15
12
9
6
3
0
3
6
9
12
15
18
21
0
1
2
3
4
5
6
7
8
9
1 Analytical method
2 Begg's without superstructure present
arch welded at quarter point
3 Beggs without superstructure present
neutral plugs at quarter point
INFLUENCE LINE FOR MOMENT AT RIGHT SUPPORT

PLATE 9

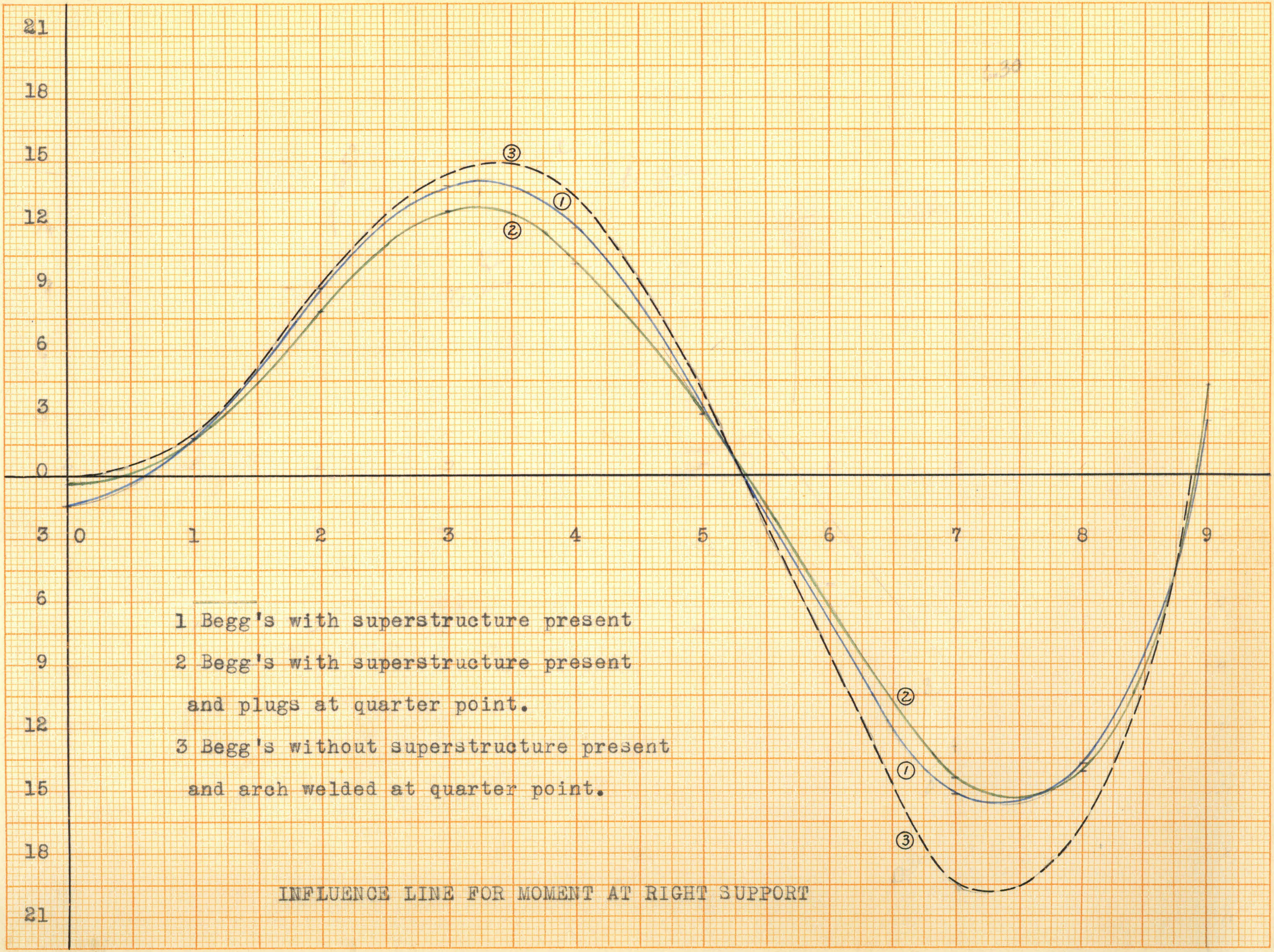
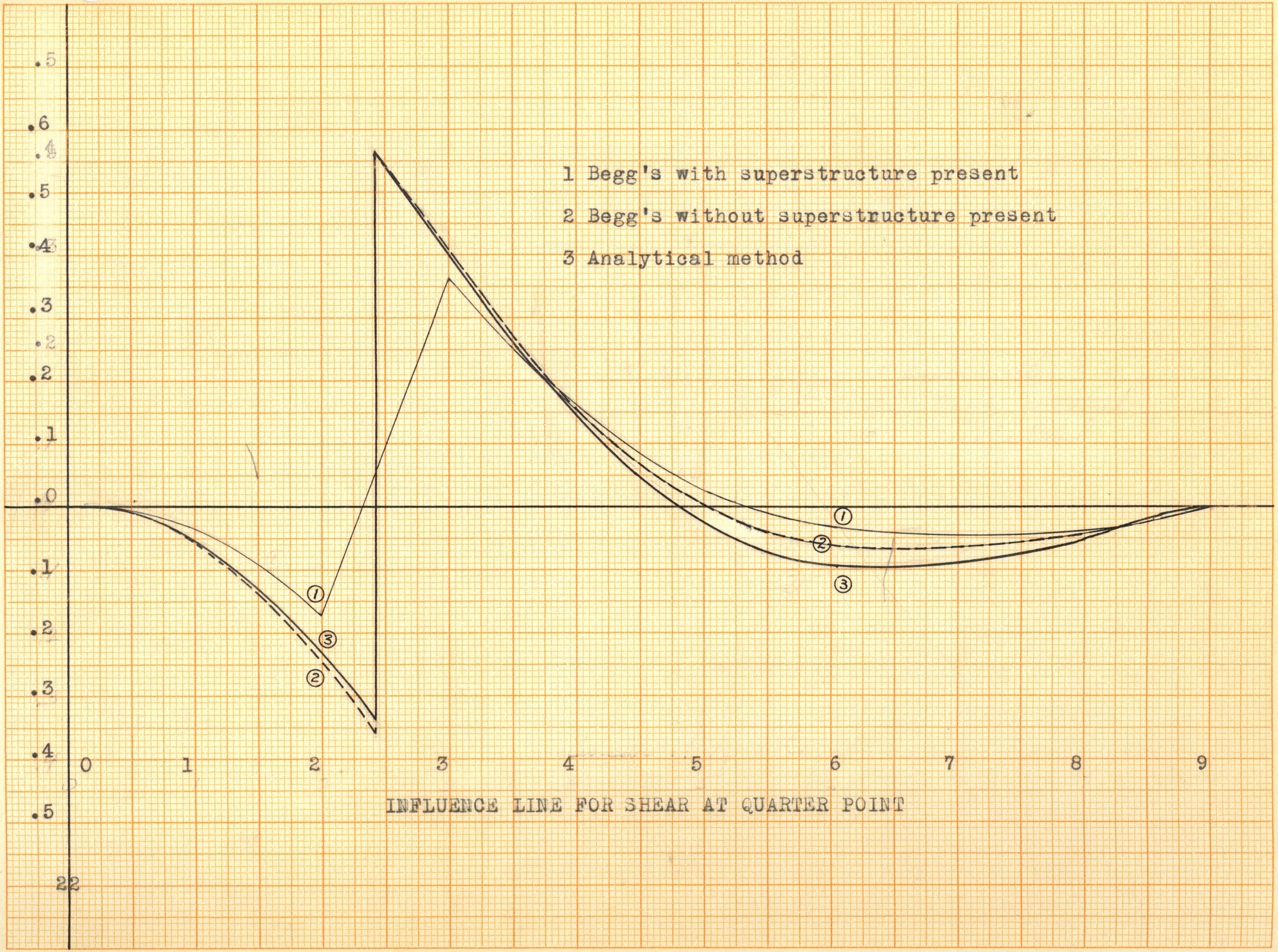

21
18
15
12
9
6
3
0
3
6
9
12
15
18
21
0
1
2
3
4
5
6
7
8
9
1 Begg's with superstructure present
2 Begg's with superstructure present
and plugs at quarter point.
3 Begg's without superstructure present
and arch welded at quarter point.
INFLUENCE LINE FOR MOMENT AT RIGHT SUPPORT

PLATE 10



INFLUENCE LINE FOR SHEAR AT QUARTER POINT

PLATE 11

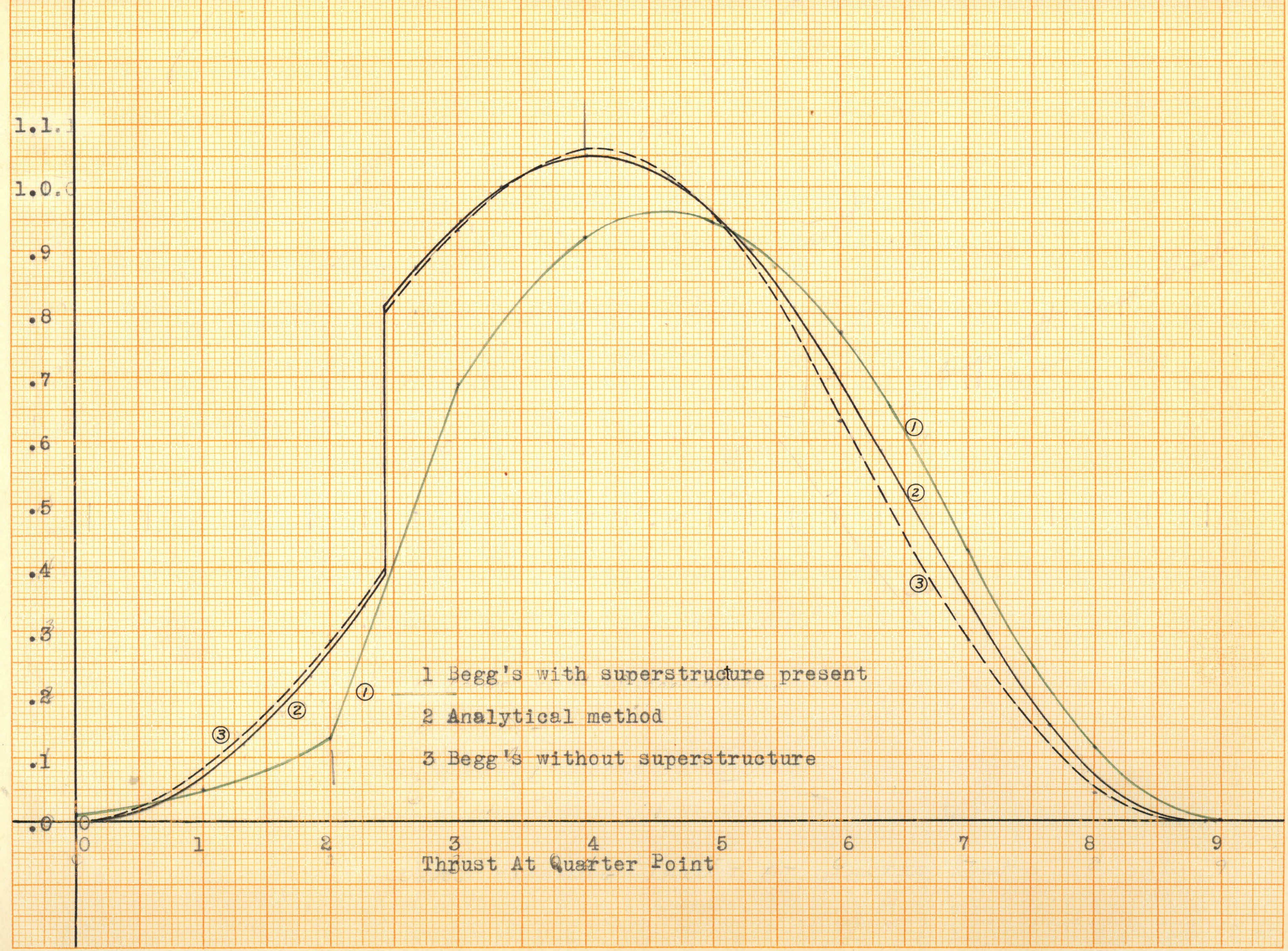

1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
0
1
2
3
4
5
6
7
8
9
Thrust At Quarter Point
1 Begg's with superstructure present
2 Analytical method
3 Begg's without superstructure

PLATE 12

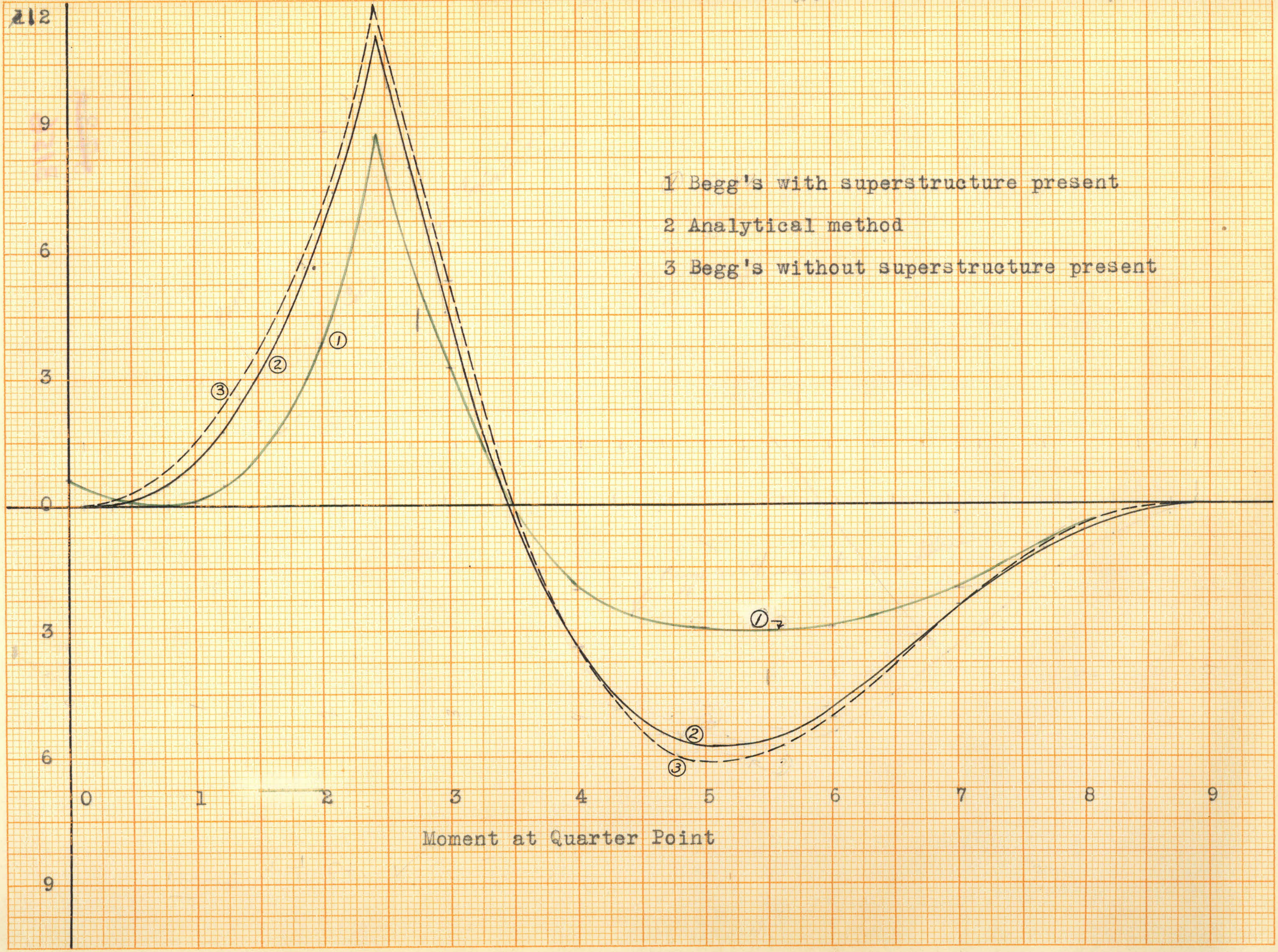

12
9
6
3
0
3
6
9
0
1
2
3
4
5
6
7
8
9
1 Begg's with superstructure present
2 Analytical method
3 Begg's without superstructure present
Moment at Quarter Point

PLATE 13