THE SCATTERING OF ELECTRONS BY GASES.

THESIS

BY

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IN

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OUTLINE

SUMMARY OF RESULTS

It is found

(1) That high angle electron scattering in gases can be measured by using low pressures and high current densities.

(2) That the angular distribution function of electrons scattered by mercury is not a monotonic function of the angle, but has a minimum.

(3) That the angular position of this minimum depends upon the electron energy and moves to smaller angles for higher energies.

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THE SCATTERING OF ELECTRONS BY GASES.

I. THE PROBLEM

In the studies of electron scattering that were first made it was the experimenter's idea to measure the average area presented by an atom to an external moving electron. This was done most accurately by a method due to Ramsauer, and later refined and specialized by others. R. B. Brode in his Doctor's Thesis at this Institute (1925) gives a very full account of the method, with bibliography. The scattering was found to follow this equation:

$$I = I_0 e^{-\alpha p x}.$$

I is the current density in a beam of small divergence, I_0 the current density where $x=0$, "x" the distance in the direction of the beam to the point where I is measured, p the pressure in mm. of mercury reduced to $0°C$, and α the coefficient of absorbtion. "α" can be thought of as the number of effective square cm. of cross section presented by 1 c.c. of gas at 1mm. pressure and $0°C$, or it can be thought of as the probability of an effective encounter with an atom while the electron moves one cm. in gas of the same concentration. The latter would be the same as the reciprocal of the mean free path.

It was found that α was independent of p and independent of I_0 within the range of current densities used, but was dependent on the speed of the electrons in a most interesting way. For high speed electrons α was generally low, and changed with the speed in a slow monotonic manner, increasing for lower velocities. However, for electrons of less than ten $\sqrt{\text{volts}}$ velocity, in the "Ramsauer region", some of the curves of α with speed show maxima and minima, and in some cases the minima are actually the lowest part of the entire curve. (10)

Several independent investigators have covered and are still working on a large number of gases. In a recent paper by Brueche (9) a correlation is made that indicates that similar absorption coefficient curves belong to atoms that have similar external configurations.

While the measurements of the absorption coefficients and the corresponding technique have been refined to a great degree, a very closely allied problem has remained comparatively in its infancy. This problem is the study of the angular distribution of the scattered electrons with regard to their original direction. It is with this problem and the results obtained in answering it that this thesis will deal.

II. PRELIMINARIES

The first work on this problem was that of Dymond (5) in which scattering by helium was studied. He reported observation of "preferred angles" at which sharp peaks of scattered electrons were found. Later Harnwell (6) at this Institute tried to check Dymond's results, but could not do so. Dymond himself almost simultaneously reported that his preliminary results were due to lack of shielding in the glass apparatus. Harnwell surmounted this difficulty by sputtering the inside of the tube with magnesium. With the tube properly shielded Harnwell's measuring apparatus could detect nothing but the main beam of electrons, either in vacuo or in gas. The peaks as reported by Dymond had disappeared. Harnwell left the Institute before he had time to carry the investigations further. However, he has since obtained and published the scattering curves he was after. (12) The author inherited the apparatus and with the cooperation of Mr. Warren Arnquist began a series of investigations in an attempt to improve the technique. The principle improvements sought were:

1. Currents of larger density.
2. Lower speed electrons.
3. Better shielding.
4. Improved methods of measurement.
5. More monochromatic electrons.
6. A scattering region as nearly as possible free of either electric or magnetic fields.

Conditions three, four and six were difficult to meet in the design as then used. Shielding by magnesium coats was not permanent; insulation conditions were not ideal for use with an electrometer. The presence of a hot filament connected with an accelerating potential introduced stray electrostatic fields and space currents to the shielding in the scattering space. Higher current densities at lower speeds were, however, made possible by introducing a high accelerating field and then retarding the electrons obtained. However, the necessarily compact arrangement of the filaments and slits proved impractical owing to the sputtering of the filament to the mica used for insulation.

The parts of the old apparatus that were in vacuo are shown diagrammatically in Figure 1.

The axes of collimation of S_1 and S_2 were arranged to intersect at "O", which was the axis of rotation of the filament assembly. It is seen that the electrons reaching C_2 must have been deflected by the amount of the angle θ , due to an encounter with an atom near O . Rotation of the "electron gun" served to change θ as desired. This rotation introduced a difficulty when the beam was made intense; each time θ was changed the intense electron beam impinged upon a new area of the tube wall. The outgassing that resulted destroyed the pressure equilibrium that had existed. For this reason and also to attain the above improvements it was decided to construct an apparatus of entirely new design.

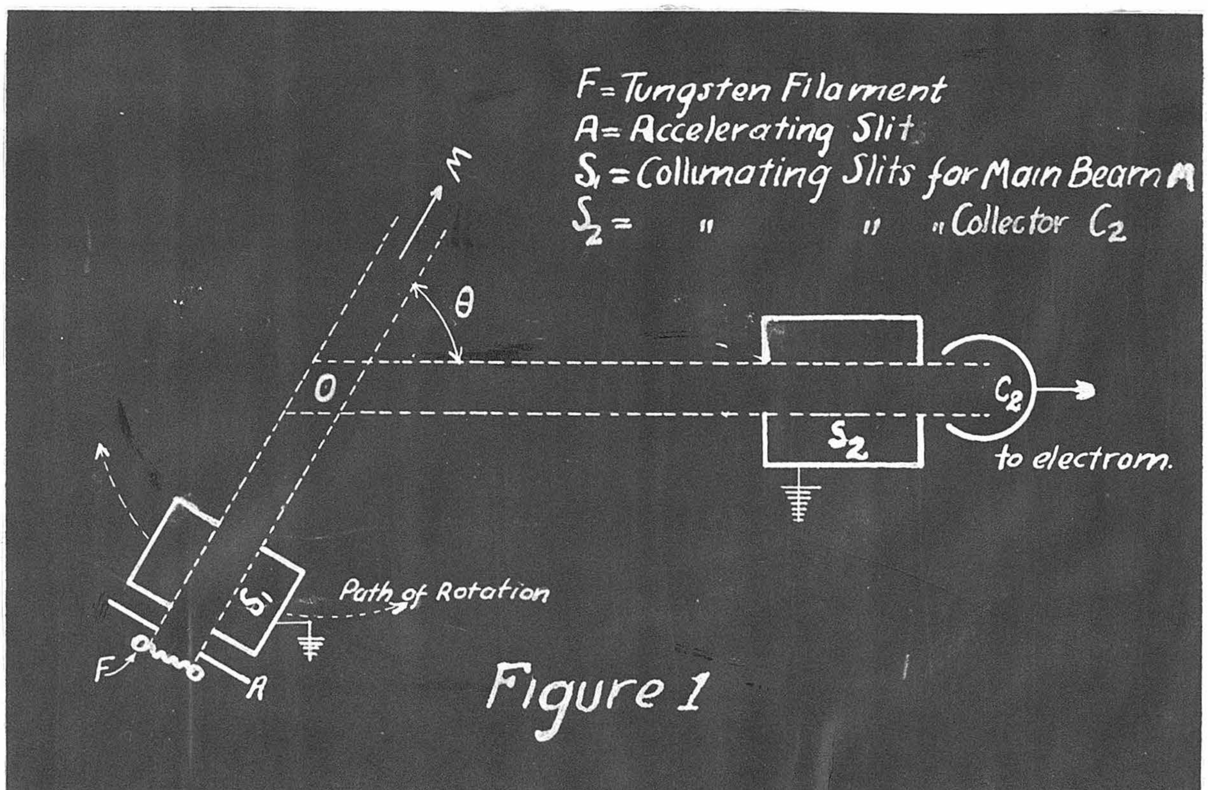


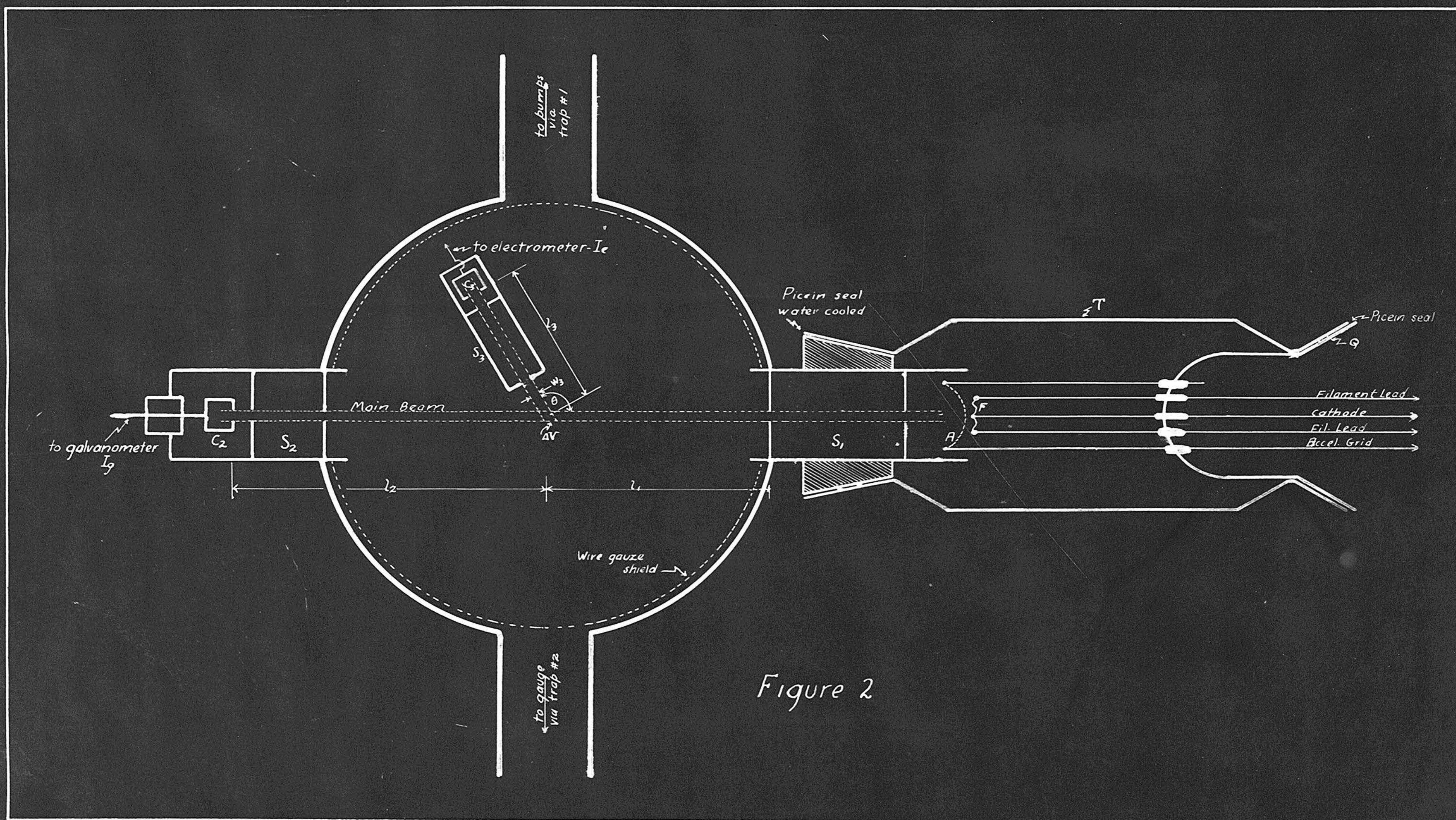
Figure 1

III. THE DESIGN.

Because of the difficulty caused by rotating the main beam it was decided that the source and the main beam, M, should be fixed and the collector, C_3, be rotated instead. This also had the advantage of making possible a less compact filament assembly and thereby avoiding insulation troubles. Because of the high intensity of the main beam to be used a separate, fixed collector, C_2 , was arranged to be connected with a galvanometer for its measurement. (See Figure 2.) This arrangement made it possible to test experimentally the shape of the main beam and also to test the scattering space for its freedom from magnetic fields, as will be explained later, (Appendix I). In the diagram of the apparatus, Figure 2, the width of the slits is purposely exaggerated to make it possible to differentiate between the sides of the beam. The actual slit width was 1.0 mm. for S_1 and less for the others in proportion to their respective spacings.

Let us look at a few of the more salient points of the design.

First. If the results are to be at all interpretable it must be certain that the scattered electrons being measured have been through only one encounter. This puts an upper limit on the pressure of the gas. From the experimental value of the absorbtion coefficient, α , the mean free path, λ , can be deduced. As long as the mean free path is greater or equal to the dimensions of the apparatus, the probability that a scattered electron which reaches C_3 has had more than one en-



to pumps via trap #1
to electrometer-I_e
C_3
S_3
l_3
w_3
θ
ΔV
Main Beam
Picein seal water cooled
T
Picein seal
Q
Filament Lead
cathode
Fil. Lead
Accel. Grid
F
A
S_1
S_2
C_2
to galvanometer I_g
l_2
l_1
Wire gauze shield
to gauge via trap #2

Figure 2

counter is practically zero. Under our conditions the mean free path is given by $\lambda = \frac{1}{35 p_{mm}}$ cm so that when $\lambda = 15$ cms, $p = 0.002$ mm as the upper limit.

Second. For given conditions of electron supply from slits S_1 the beam intensity, I_0, at "ΔV" in the center goes down exponentially with the pressure, p; at the same time the current scattered from ΔV is proportional to p, so that there is an optimum value of p. A simple calculation shows that this optimum "p" is much higher than the upper limit referred to above. Fortunately, however, the scattered current goes to zero much more slowly on the low pressure than on the higher pressure side of its maximum.

Third. In order to obtain an intense beam, especially at low velocities, it is necessary to use wide slits. This in turn means proportionately longer distances between slits and correspondingly lower pressures.

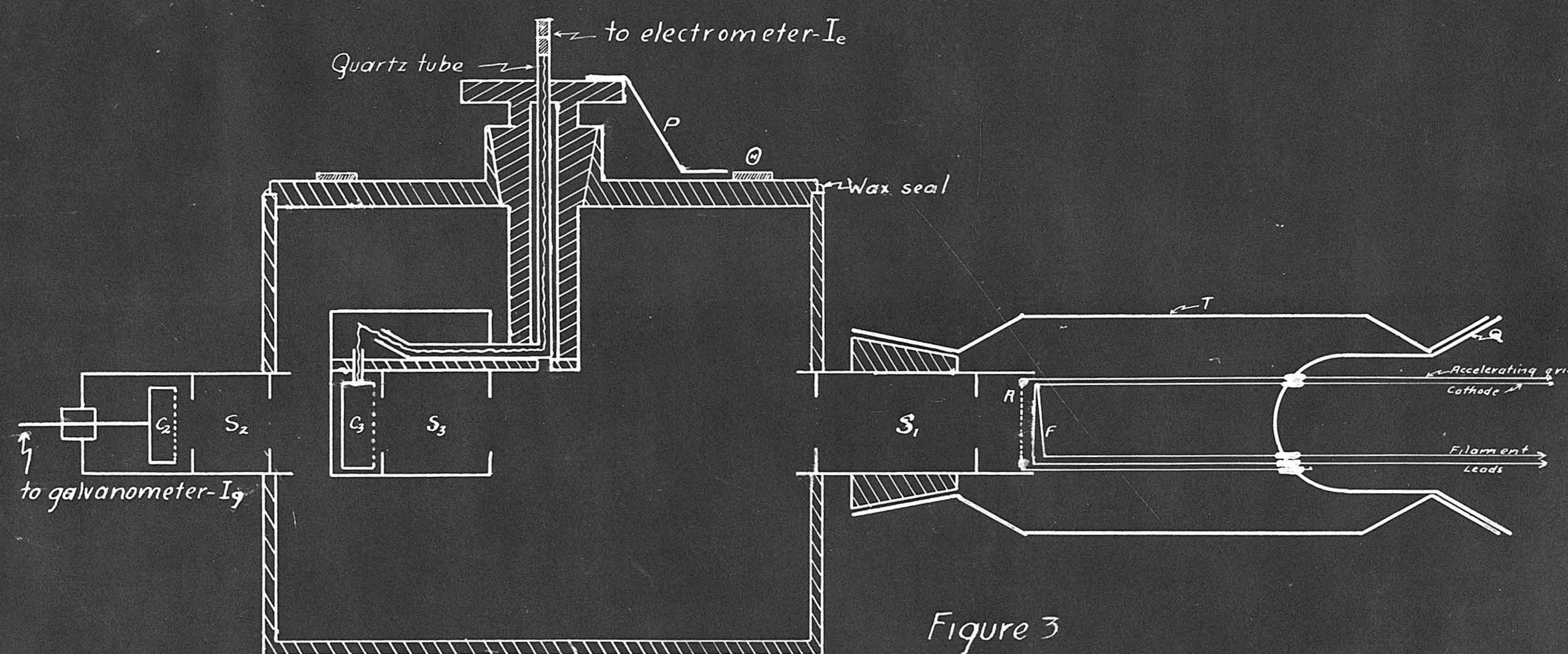
Fourth. As the source of the electrons and the accelerating grids are to be in the gas they must be so spaced that at the pressures used a discharge will not tend to start. Ionization is bound to occur, but self-sustained discharges or "arcs" are avoided by placing the accelerating grid very close to the filament. This grid, "A", is a fine mashed, nickel gauze semi-cylinder bent convex toward the filament and with its axis parallel to the filament. The filament used is an eight mil tungsten wire bent as a narrow hair pin to reduce its magnetic field. It is supported vertically by a nickel wire looped through it at the top. This wire is, incidentally, brought out through a seal to make a direct connection with the filament and to avoid the RI drop of the filament leads in the accelerating potential. The filament and grid are both spot welded to leads through the glass stopper, "G". The seals are so extended that the leakage path through sputtered tungsten is a maximum. Where bare the leads are close enough to make a spontaneous gas discharge improbable. The filament assembly fits into a ground joint at one end of the tube, T, the other end of which is fitted to the apparatus by another ground joint. These joints, which make the filament assembly very accessible for adjustment and repair, are sealed with picein.

With a view to obtaining as monochromatic a source as possible coated cathodes, indirectly heated, were tried. They were very successful in high vacuum, or when used at low emissions in a gas. However, when high emissions were tried, the positive ion bombardment shortly proved fatal to the coatings. On this account the heavy tungsten emitter was chosen. This yielded the desired current density at the cost of a wider range of velocities.

<u>Fifth.</u> To insure that the gas under examination be in a space as nearly field free as possible the entire apparatus is constructed of brass. The scattering chamber is circular in cross section as shown in Figure 2. The exploring collector is mounted in a ground metal joint in the cover. The cover with the rotating assembly is removable for adjustment. The electron beam enters the chamber after passing through the

two slits, S_1 , both of which are grounded. Diametrically opposite these slits and optically aligned with them are another similar pair of slits, S_2, that admit a portion of the electron beam to the collector, C_2. To make the absorbtion of the chamber walls for electrons as efficient as possible a fine mesh copper screen is placed immediately in front of them as indicated by the dots. The slits, S_1, both grounded, effectively prevent stray electrostatic fields from the grid from entering the chamber. The grounded slits, S_2, likewise shield the chamber from the field of the collector, C_2, that arises from the retarding potential.

The earth's magnetic field is balanced by two large pairs of Helmholtz coils. One pair is for the vertical and the other pair is for the horizontal component. The current needed to balance the horizontal component was determined before the apparatus was installed, with a magnetic pendulum. To allow for possible, minor, future variations in this component the apparatus was so mounted that the axis of the electron beam is horizontally north and south. Experiment showed that comparatively large variations in this component made no noticeable change in the beam. The current needed to balance the vertical component was found by experiment; it was adjusted so as to allow the maximum current to reach C_2. This arrangement also permits an analysis of the sharpness of the beam to be made. This is done by moving the beam back and forth across the slits, S_2, by varying the vertical magnetic component. An analysis of this is given in Appendix I.



to electrometer-Ie
Quartz tube
P
Wax seal
T
Accelerating grid
Cathode
Filament
Leads
R
F
S1
S2
S3
C2
C3
to galvanometer-Ig

Figure 3

Sixth. The rotating collector, being somewhat compact, uses narrower slits. A vertical section of the apparatus is given in Figure 3. This figure is taken in a plane through the slits S_1 and S_2, with $\theta = o$ for the collector. The ground joint holding the collector is in the center of the circular cover. This cover has a groove in its edge turned to fit snugly into the top of the chamber. A vacuum tight seal is made by filling only the outer edge of the crack with picein. This is done after pumping down to a few cm. pressure.

To insulate the lead to C_3, it is put through a quartz tube which in turn is completely surrounded by metal for shielding. Thus no electrons can reach the lead except via the slits S_3 and the collector C_3. Conversely the interior of the main chamber is free from the field of C_3 due to the retarding potential on C_3.

The angular position, θ, of the collector is read directly by the index, P, attached to the rotator, on the raised engraved scale Θ. Θ is a circle divided into 180 equal divisions.

Seventh. The electric circuits used to control the beam and to make the measurements are shown in Figure 4.

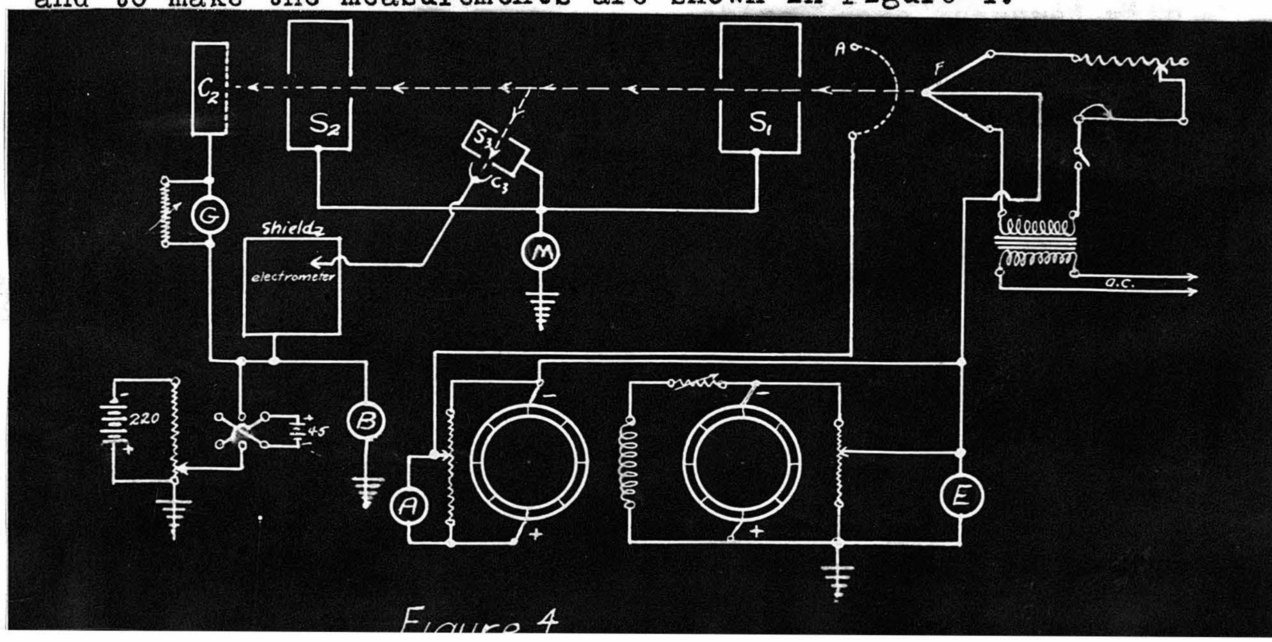


Figure 4

The filament is lit by A.C. through an insulating transformer and is controlled by coarse and fine adjustments as indicated. A D.C. generator with two separate windings and two commutators furnish independent accelerating and retarding voltages. The accelerating voltage hereafter called "A" is read by meter A, and the voltage determining the electron energy at the slits hereafter called "E" is read on meter E. Thus the energy of the electrons reaching and passing slits S_1 is dependent only on E, while the condition of the space charge near the filament can be independently controlled by A. It is found that about 60% of the emission current stops on the accelerating grid, and that the remainder reaches the slits S_1 and passes through the milliammeter, M, to ground. It is found that the current through the slits is constant for given values of A and E if the reading of M is constant, and provided A is high enough to draw the saturation current from the filament. M is kept constant by manual control of the filament current. An automatic control was tried for a while, but it was found that the limits of variation were wider than with the manual control.

The scattered electrons reaching C_3 are measured with an electrometer inside the electrometer shield. For this a modified Dolezalek electrometer was used. A description of its construction and use is given in an article by the author as Appendix II. In order to supply the collectors C_2 and C_3 with the necessary retarding potential "B" the galvanometer return is connected to the electrometer shield, and the whole is raised to the desired potential with respect to ground. The retarding potential or bias is read on voltmeter B.

At first the potentiometer supplying B was fed from the low potential D.C. commutator, but since the small fluctuations in voltage induced charges on collector C_3 of the order of magnitude of those being measured, the electrometer readings were quite unsteady. For this reason batteries were substituted as a supply for B. The auxiliary 45 volt battery shown, in Figure 4, connected to a reversing switch is used either to read a bias higher than the range of the potentiometer, or to give a small positive bias instead of a negative bias.

Let us now consider the current that we can hope to get on C_3:

Let $f(\theta)\,d\theta$ be the probability under the given conditions that an electron be scattered in a range, $d\theta$, at an angle θ with its original direction. It is really this $f(\theta)$ that we are after.

Let I_s stand for the total current of electrons scattered from the volume ΔV common to the paths of collimation of slits S_1 and S_3. (See Figure 2)

Because of the slits, S_3, only a fraction, K_3, of the electrons scattered between θ and $\theta + d\theta$ are received on C_3 and the number is still further reduced by scattering along the path, of length l_3, from the scattering volume to C_3. Thus the current received, I_e, is

$$I_e = K_3 I_s f(\theta) e^{-\alpha p l_3} d\theta.$$

Let now I_0 be the intensity of the main beam, M, just as it enters the scattering volume ΔV on its way to S_2. Then of this main beam a certain fraction, K_2, is lost on slits S_2, and it is also further reduced by scattering along its path, of length l_2 , from the center of the chamber to C_2. Hence the galvanometer current I_g is

$$I_g = K_2 I_0 e^{-\alpha p l_2} .$$

As the intensity of the main beam is given at a distance x (positive to the left of ΔV) by

$$I = I_0 e^{-\alpha p x}$$

We have the amount of current scattered in a distance dx given by

$$dI = -I_0 \alpha p\, e^{-\alpha p x}\, dx .$$

so if we set $dx = \frac{W_3}{\sin\theta}$ and $x = 0$

where $W_3 =$ width of slits, (S_3) then

$$dI = -I_0\, \alpha p\, \frac{W_3}{\sin\theta} .$$

but since

$$I_s = (-dI)_{x=0} .$$

we have

$$I_s = \alpha p I_0 \frac{W_3}{\sin\theta} .$$

Thus we have for the current to C_3 :

$$I_e = \alpha p I_0 \frac{W_3}{\sin\theta} K_3 f(\theta)\, e^{-\alpha p l_3} d\theta .$$

and as before

$$I_g = K_2 I_0\, e^{-\alpha p l_2} .$$

dividing to eliminate I_0,

$$\frac{I_e}{I_g} = \frac{\alpha p\, d\theta\, K_3 W_3\, e^{\alpha p(l_2 - l_3)}}{K_2} \cdot \frac{f(\theta)}{\sin\theta} .$$

whence

$$f(\theta) = \frac{I_e}{I_g} \cdot \frac{\sin\theta}{p} \cdot \left\{ \frac{K_2\, e^{-\alpha p(l_2 - l_3)}}{\alpha\, d\theta\, K_3 W_3} \right\} .$$

Consider now the expression in brackets. The exponent $e^{-\alpha p(l_2-l_3)}$ is practically unity when the pressure is low enough to satisfy the condition for single scattering. ($p \leq 10^{-3}$ mm.)

The value of α changes very slowly with the electron speed in the range of speeds used, and is a constant for any one speed. (See Brode[(3)]). ($\alpha = 35$ to 50 cm.^{2}/ cm.^{3}). The value of K_2 and K_3 (especially K_2), however, were quite critically dependent on the pressure, on electron velocity and on current density as we shall see later in examining the results. The effective width, W_3 , of the slits is constant. The value of $d\theta$, the limits of the angular divergence of the scattered electrons received, is slightly dependent on the angle θ , but principally is determined by the slit width and slit length. A detailed analysis of this dependence is given in Appendix III. Hence, if E , p , M , and A are constant the quantity in the brackets is constant $\left(\frac{1}{K}\right)$ so that we have

$$\frac{I_e}{I_q} \cdot \frac{\sin\theta}{p} = K f(\theta).$$

The final adjustment of the apparatus gave $d\theta$ such that 90% of the current arriving at C_3 was within $\pm$ 1.5° of θ .

Objection was once raised against the use of long slits because it is conceivable that an electron reaching C_3 might have been scattered through an angle considerably greater than θ . But as the analysis shows, the objection is not supportable, especially at angles greater than $\theta = 30°$, so that the value of $d\theta$ is essentially that determined by slit width only. However, this brings up another and rather more serious objection; the scattering volume ΔV, proportional to $\frac{W_3}{\sin\theta}$ increases quite rapidly for $\theta <$ 30°, so that

$$I_s \neq \alpha p I_0 \Delta x$$

(page 12)

This latter expression assumes that ΔV is so small that the current density, I_0 , is essentially constant throughout it, an assumption that is invalid when $\theta < 30^{\circ}$.

IV. PROCEDURE IN MAKING MEASUREMENTS

First the resistance, R, in the electrometer (see Appendix II) was measured by allowing it to discharge a condenser. When first made, R was not constant, but became so in a few months. Measurements taken a year later agree quite well with the original constant value, $1.68 \cdot 10^{11}$ ohms. This resistance is quite small compared to the insulation resistance of the shielded lead to C_3. With a bias of 200 volts the leakage resistance of the quartz tube was found to be of the order of 10^{15} ohms. The deflection, at this bias, produced on the electrometer was of the order 3 to 10% of the smallest currents which were read, so that for lower biases correction for leakage was unnecessary.

Great difficulty was found at first in getting successive readings of I_e, scattered current, and of I_q, main beam, to agree. This was because the great dependence of K, in

$$K f(\theta) = \frac{I_e}{I_q} \frac{\sin\theta}{p}$$

on p was not fully appreciated. (See page 13). When p was under control and the whole apparatus in equilibrium, successive readings would check regularly within 5% and usually to about 3%.

As mercury vapor was the most readily available gas, and since the scattering up to 30° had been measured by Arnot (2) it was decided to investigate the higher angle scattering in mercury. The arrangement is as follows in Figure 5.

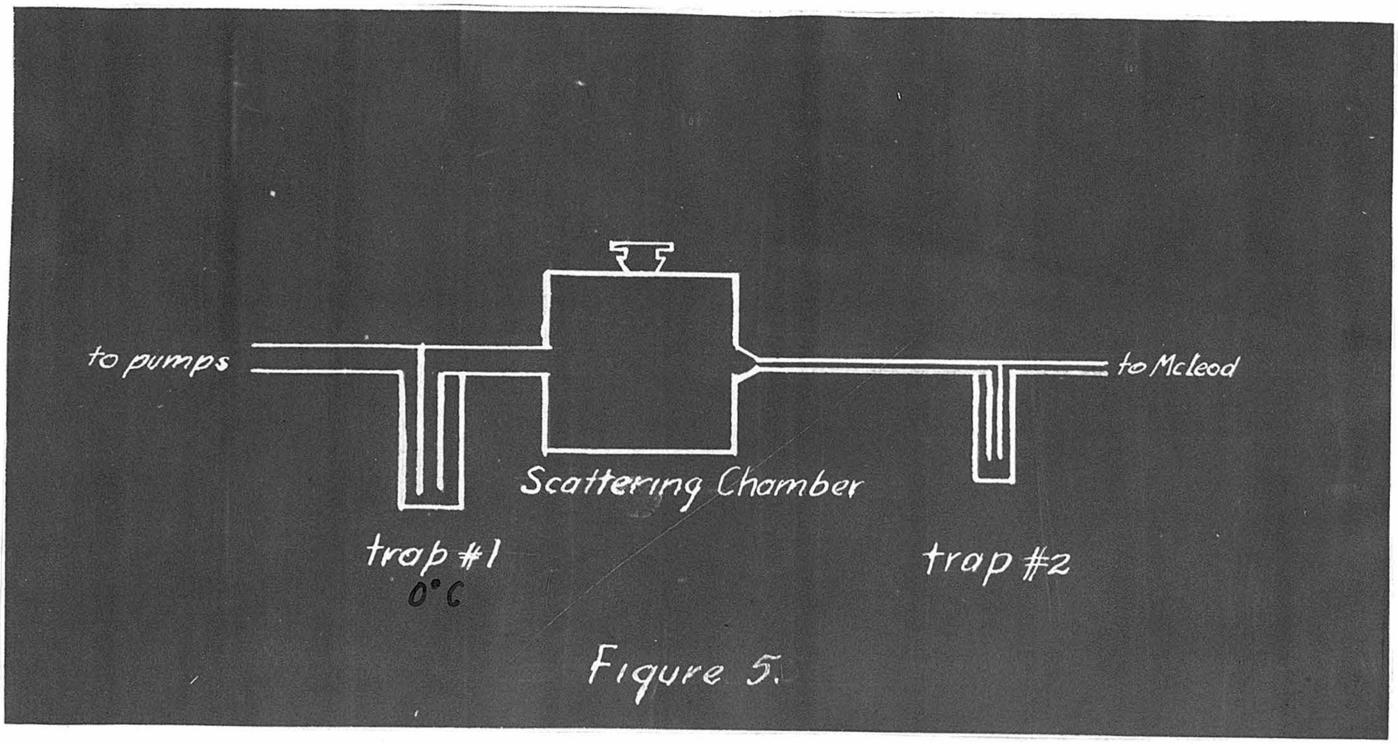


Figure 5.

The diffusion pumps are connected through trap number 1 with a short, large diameter tube to the scattering chamber. On the other side a comparatively small tube about a meter long passes through trap number 2 to a McLeod gauge. A mixture of ice and water placed on trap number 1 is constantly and thoroughly agitated by a stream of air bubbles. A thermometer with its bulb in the bottom of the ice bath is used to check the uniformity of the temperature. Liquid air is put on trap number 2. Under these conditions mercury vapor is constantly distilling from the pump to trap number 1 and coming to pressure equilibrium there with metallic mercury at 0°C. Because of the large tube the vapor at this equilibrium pressure can readily diffuse into the scattering chamber, which is at room temperature; thus mercury cannot condense there. Because of the constriction of the long, narrow tube to trap number 2 the temperature of this trap has comparatively little to do with the pressure in the main apparatus. The pressure of mercury vapor at 0°C. ($1.8 \cdot 10^{-4}$ mm.) is exactly in the range for which the apparatus is designed (10^{-3} to $5. 10^{-5}$ mm.) The essential condition for single encounter scattering is amply satisfied. (Electron m.f.p. 140 cm.)

The room temperature is quite constant, between 19° and 21° C., so that everything else being the same, readings from day to day can be checked. However, the density of vapor in the chamber is extremely sensitive to the temperature of trap number 1. If the air agitator becomes a little less vigorous so that the bottom of the bath changes from 0° to $0.^{\circ}1$ C., the main beam reading would drop as much as 20%. Because of this sensitivity to temperature control, and the time needed to reach equilibrium (two hours), measurements were confined to this pressure.

The procedure in making a run is this: The apparatus is first brought to pressure and temperature equilibrium. The absence of non-condensible gases is checked by the McLeod, which reads zero. While this is going on, the filament is lit and the slits bombarded with a heavy current. Next the values of A, E, M, and the current "i" in the vertical component coils are experimentally set to give maximum current to the collector C_2. By adjusting these values, each to produce the maximum effect, small changes in any of them make little difference in the beam save in the case of "i" as will be shown later.

The energy distribution of the electrons is next investigated by changing the retarding bias, β , and reading the galvanometer. Because of the type of electron emitter used, as explained in the discussion of the filament (page 7) the electron voltage distribution half width is about 2 to 3 volts. Consequently, it was decided practical to investigate only elastically scattered electrons. The bias, β, is chosen so as to be just less than the energy of the full speed electrons. Under these conditions it is found that the collectors, C_2 and C_3 , being negative to ground, collect all the positive ions that are diffused into their neighborhood as well as the full speed electrons. The resulting positive current is, in order of magnitude, usually about 10% of the amount of the negative current of the scattered electrons. (For very high angles it is sometimes 100% or greater). However, the positive current is easily accounted for by increasing β to cut out all electrons and by taking the difference in observed currents, which is the electron current that was cut out. This is checked by the fact that although the positive ion component might change slowly by a factor of two or three, the current difference, or the electron current stays constant within our limits of observation.

At each angle two sets of readings are made of I_c and of the maximum I_g. If during a run these readings check to 5%, and readings made at the beginning and end of the run at the same angles agree, the run is accepted.

Check runs made on different days under the same conditions give for $Kf(\theta)$ surprisingly good agreements in actual values, for all electrons above 100 volts energy. For slower electrons the ordinates of curves made on different days differ by a constant factor, but the curve shapes are exactly alike. The significance of this is probably discovered in the analysis of the spread of the main beam to be discussed later. (Page 18)

V. OBSERVATIONS AND RESULTS

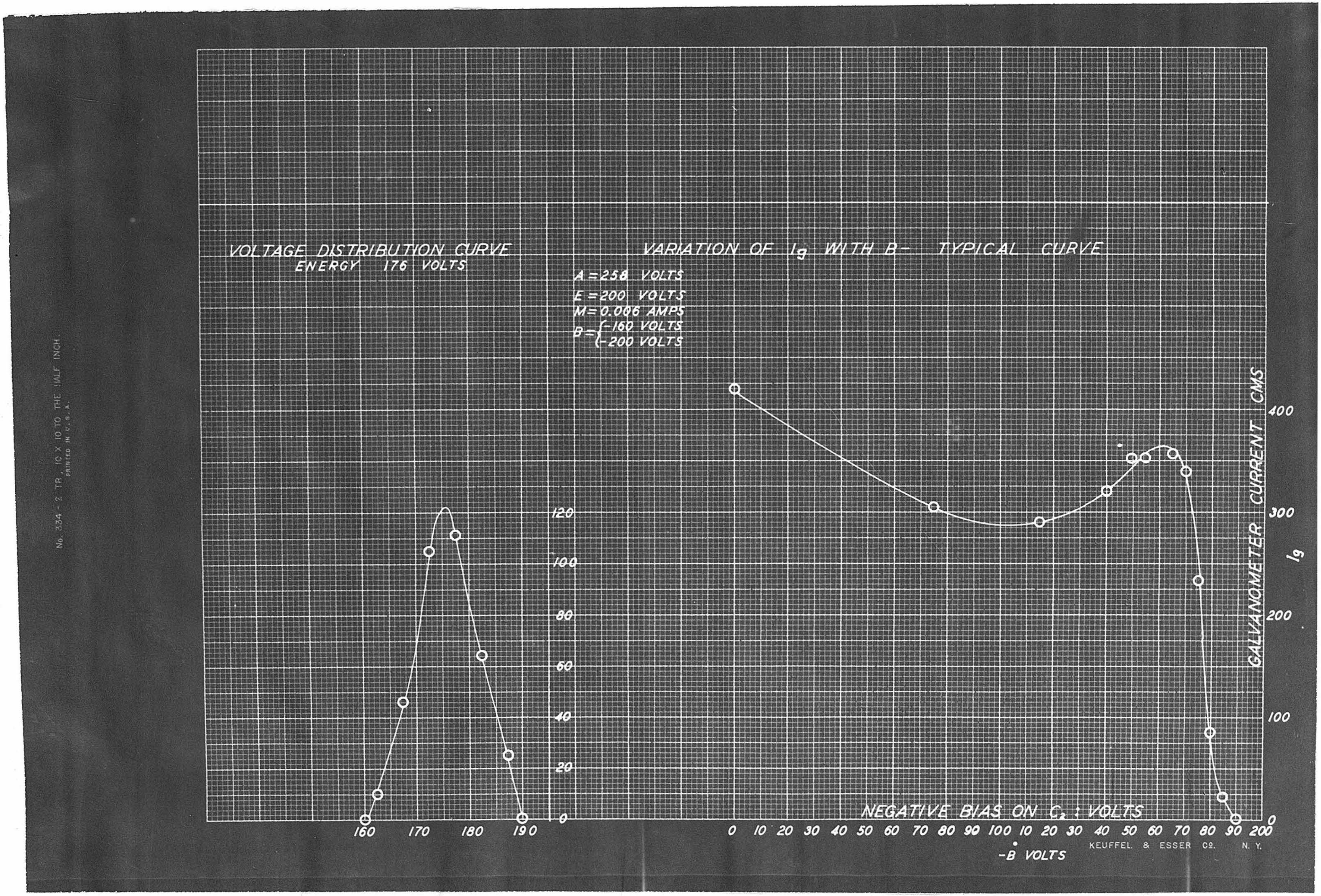
1. Focusing effect. All runs for the reasons above explained (page 16) were made at a vapor pressure of $1.8 \cdot 10^{-4}$ mm. We observed at some certain velocities the "focusing effect" due to the positive ion sheath. This effect was most marked in the case of 128 volt electrons as shown in curve A. Here the broad flat peak was observed at hard vacuum conditions and the sharp peak at equilibrium under the running conditions at 128 volts energy. Most of the area of the low peak is within an angle of 1.2° of the peak, which corresponds quite well with the geometric calculation in Appendix III. However, the same percentage of area in the beam focused by ions is within 0.5° of the peak. (The peak current, I_g, here is a little over 4 ~~micro~~amperes.) It is now quite plain why it was necessary always to read only the maximum I_g, determined by varying "i" to control the vertical magnetic component. A small change in the magnetic field at 128 volts would change the galvanometer current by a factor of two or three. (This amounts to a change in K_2, see page 13 .) At other energies the focusing effect was not nearly as strong.

2. Energy of Electrons. As explained in the discussion of the procedure the actual electron energy was measured at

each voltage by reading the galvanometer current I_g against the retarding bias, β. The graphs of these data show an interesting phenomenon. They display a maximum just before the bias cuts off the current. A typical curve is given here. (Energy 176 volts.) Before the mercury vapor pressure was under control, we could not reproduce these curves in the region between zero and maximum bias. They did, however, check under equilibrium conditions. In making the runs the bias that just preceded the steep part of the curve was used to collect only elastically deflected electrons. At this energy, -160 volts was used to collect only full speed electrons, and -200 volts was used to read only the positive ions.

An explanation of this type of curve, in view of its sensitivity to pressure, is in the formation of positive ions near C_2 by the very intense electron beam. Due to the negative bias these ions would be collected and thus reduce the negative current indicated by the galvanometer. However, when the bias became so high as nearly to stop the electrons, the electron velocities near C_2 were much lower. Consequently, fewer ions were formed near C_2, and the indicated current would rise. For low biases the secondary, photo, and inelastic electrons were collected, causing another rise.

The derivatives of these curves give the electron energy distributions. In every case the peak of the distribution was from 10 to 20 volts lower than the voltage. (See page 22). This was consistently the case and repeated itself under the same conditions either in gas or in vacuum. Considering the very high current densities used this effect might


VOLTAGE DISTRIBUTION CURVE
ENERGY 176 VOLTS
VARIATION OF Ig WITH B- TYPICAL CURVE
A = 258 VOLTS
E = 200 VOLTS
M = 0.006 AMPS
B = { -160 VOLTS, -200 VOLTS
0 20 40 60 80 100 120
160 170 180 190
GALVANOMETER CURRENT CMS
Ig
0 100 200 300 400
NEGATIVE BIAS ON C2 : VOLTS
0 10 20 30 40 50 60 70 80 90 100 10 20 30 40 50 60 70 80 90 200
-B VOLTS
KEUFFEL & ESSER CO. N.Y.
No. 334 - 2 TR, 10 X 10 TO THE HALF INCH.
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be ascribed to space charge. It has been observed by others, and some experiments with this type of electron gun are referred to by Whiddington in a recent note to Nature, about November, 1929.

3. Scattering Curves. Runs were made at given settings but different times to study the reproduceability of the runs; also to make sure that the changes in the runs could be accounted for. The sensitivity of the results to small changes in any of the numerous variables was observed as above outlined. At energies above 100 volts the values of $Kf\theta$ were quite easily checked any time. (See curve E). Near and below 100 volts $Kf(\theta)$ would change by a constant factor from time to time, but the minimum at a given voltage was always found at a given angle within the limits of the resolving power. This meant, of course, that for electrons of lower energies the "K" in $Kf(\theta)$ was not reproduceable. One explanation of this might be in the fact that the energy dissipated in bombardment of the slits was so much lower at lower voltages that conditions there of occluded gases were unconstant.

Because of the methods used, i.e., simultaneous readings of I_e and I_q (see page 17) and because of the great change in the scattering volume at low angles (see page 13) the scattered electrons were taken only from the range $30^\circ \leq \theta \leq 150^\circ$.

Over the whole range of velocities studied one very interesting characteristic of the curves becomes apparent; it is the distinct minimum whose locus is a function of the energy. The curves appear as though they would be quite monotonic except for the fact that at one angle for each velocity the scattered electrons are almost missing. Because of the somewhat wide energy distribution the minima at low energies are less sharp than at higher energies, but are nevertheless quite distinct and reproducible. That these minima have not been reported before is probably due to the fact that other observers have found that at higher angles, (greater than about 30° , which is where these measurements start) the intensity of the scattered electrons received was of the order of magnitude of the accidental variations of the measuring apparatus and of the positive ion current. This would in such cases effectively mask the minima.

In one of our curves there appeared to be another very weak minimum near 100° (176 volts). But this is extremely doubtful because we were there again at the bottom of the range of measurements and, besides, the cage containing C_3 was getting near the very intense main beam.

Curves B to F give graphically the data and interpretations. The legends explain the conditions at which the data were taken. The curves are arranged in the order of increasing voltages. The effect of the changes in K from run to run can be seen as above mentioned (page 20) to decrease with increasing energy. In the case of curve E at 154 colts the actual values for for the positive angles do not exactly check those at negative angles, as they do in the cases of curves F, 176 volts, and B, 128 volts. This is probably because the negative angles were read first before the apparatus was exactly at equilibrium. The positive angles were taken in the afternoon some hours later. The check points were taken again in the morning several days later and agree quite exactly with the original curve.

Curve G is a plot of the loci of the minima as a function of the energy. The extended scale of this curve exaggerates the displacements due to uncertainties. However the displacement of the smooth curve, as drawn, from the points as derived from the data is adjusted in accordance with the uncertainties of the derived points. In order to be liberal this allowance was made $\pm$ 2 volts by $\pm 1.5^{\circ}$, and the curve was drawn closest to the points at the higher voltages.

VI. CONCLUSION

It is found:

1. That high angle electrons scattering in gases can be measured by using low pressures and high current densities.

2. That the angular distribution function of electrons scattered by mercury is not a monotonic function of the angle, but has a minimum. and a maximum.

3. That the angular position of this minimum depends upon the electron energy, moving to smaller angles for higher energies.

During the preliminary work and during the making of the final runs the following phenomena were observed and are considered worthy of future experimental examination:

1. The focusing effect at high current densities and low pressures.

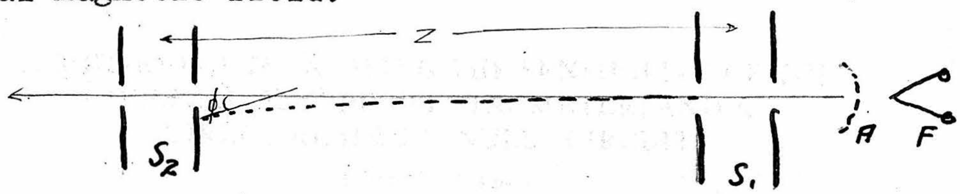
2. The difference in the energy of the electrons in the beam as compared with the accelerating field E.

Further development of the apparatus may make possible a study of the behavior of the distribution function, at high angles, of the inelastically deflected electrons.

In conclusion I should like to thank Professor R. A. Millikan for his interest, advice and encouragement in this work; Dr. C. C. Lauritsen for his assistance in untangling some of the difficulties; Mr. W. Arnquist for the great amount of time and energy that he has contributed toward the attack and solution of this problem; Mr. Julius Pearson, and Mr. William Clancy for their effort and skill contributed to the construction of the apparatus.

APPENDIX I

Analysis of the width of the main beam by variation of the vertical magnetic field:



When the magnetic field, H, perpendicular to the paper is zero, the beam will go straight through as shown by the solid line. When it is not zero, the beam on reaching S_2 will be slightly deflected by the total angle ϕ along the arc of a circle of radius "R" as shown by the dotted path. It is proposed to calculate ϕ in terms of the magnetic field and electron speed. For an electron of V volts energy moving perpendicular to a field of H gauss the radius of curvature is

$$R = 3.36 \frac{\sqrt{V}}{H} \text{ cms.}$$

For small angles $\phi = \frac{Z}{R}$ where Z is the distance from the center of the slits S_1 to the center of S_2.

$$\therefore \quad \phi = \frac{ZH}{3.36\sqrt{V}} \text{ radians.}$$

The field strength F of the vertical component coils is given from the dimensions by $F = 1.100\, i$ gauss, when "i" is amperes. Thus if i_0 is the field current giving the maximum current through slits S_2, the resulting value of H is $H = 1.100\,(i - i_0)$

Z is 15.3 cms.

$$\therefore \quad \phi = \frac{16.83}{3.36} \frac{(i - i_0)}{\sqrt{V}} = 5.07 \frac{i - i_0}{\sqrt{V}} \text{ radians.}$$

$$= 287 \frac{i - i_0}{\sqrt{V}} \text{ degrees.}$$

APPENDIX II

A METHOD OF INCREASING THE SENSIBILITY OF THE DOLEZALEK TYPE ELECTROMETER; AND A DIRECT READING "NULL" CIRCUIT

By John M. Pearson

In connection with some studies being made on the scattering of electrons by gases, we have found ourselves in need of a means of measuring currents over a range of several orders of magnitudes. The method developed below has been found so satisfactory and convenient, that it is thought, perhaps, to be of general interest.

The scheme makes use of a Dolezalek electrometer, of comparatively rugged construction, which has been slightly modified to increase its sensibility. This instrument is used as an element in a null circuit that is direct reading, and that can be adjusted electrically to cover a wide range of orders of magnitude of current.

I. SYNOPSIS OF THE THEORY

Let $C_1(\theta)$ = Capacity of needle and quadrants "1."
Let $C_2(\theta)$ = Capacity of needle and quadrants "2."
(See Fig. 1.)
Let V_1 = Potential of quadrant "1" above the instrument case.
Let V_2 = Potential of quadrant "2" above the instrument case.
Let V_3 = Potential of needle above the instrument case.
Let T_0 = Moment of torsion of fiber.
Let θ = Angular displacement of needle.

Then we can write the total potential energy of the suspended system as

$$E=(V_3-V_1)^2\frac{C_1}{2}+(V_3-V_2)^2\frac{C_2}{2}+\frac{1}{2}T_0\theta^2.$$

In equilibrium, $-dE/d\theta=0$. So if V_3 = constant.

$$-\frac{dE}{d\theta}=(V_3-V_1)C_1\frac{dV_1}{d\theta}+(V_3-V_2)C_2\frac{dV_2}{d\theta}$$

$$-\frac{(V_3-V_1)^2}{2}\frac{dC_1}{d\theta}-\frac{(V_3-V_2)^2}{2}\frac{dC_2}{d\theta}-T_0\theta=0.$$

Then we have:

(1) The mechanical zero is at $\theta=0$.

(2) At the electrical zero,

$$V_1=V_2=\frac{dV_1}{d\theta}=\frac{dV_2}{d\theta}\equiv 0. \quad V_3\neq 0$$

so

$$\frac{V_3^2}{2}\left\{\frac{dC_1}{d\theta}+\frac{dC_2}{d\theta}\right\}+T_0\theta=0$$

Now, $dC_1/d\theta>0$ and $dC_2/d\theta<0$ by the convention of the direction of increase of θ. Hence if the electrical and mechanical zeros coincide, $|(dC_1/d\theta)_{\theta=0}|=|(dC_2/d\theta)_{\theta=0}|$ since $\theta=0$.

(3) Voltage sensitivity can be defined by

$$S_v=\left(\frac{\partial\theta}{\partial V_1}\right)_{V_2,V_3\ \textit{constant.}}$$

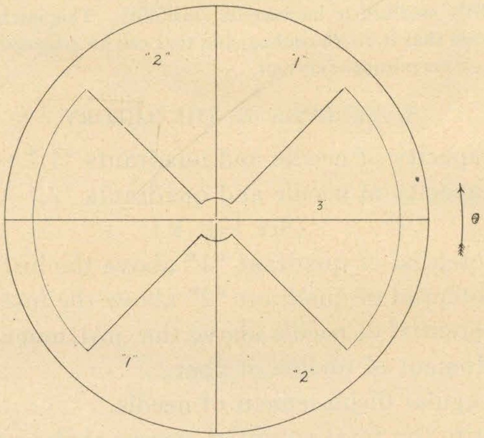


FIG. 1.

Where in equilibrium,

$$\left.\begin{array}{ll} V_1\neq 0 & \dfrac{dV_1}{d\theta}\equiv 0 \\ V_2\neq 0 & \\ V_3\neq 0 & \dfrac{dV_2}{d\theta}\equiv 0 \end{array}\right\}\ \therefore\ -\frac{(V_3-V_1)^2}{2}\,\frac{dC_1}{d\theta}-\frac{(V_3-V_2)^2}{2}\,\frac{dC_2}{d\theta}-T_0\theta=F=0.$$

So

$$S_v=\left(\frac{\partial\theta}{\partial V_1}\right)_{\textit{equilibrium}}=-\frac{\dfrac{\partial F}{\partial V_1}}{\dfrac{\partial F}{\partial\theta}}=+\frac{(V_3-V_1)\dfrac{dC_1}{d\theta}}{\dfrac{(V_3-V_1)^2}{2}\,\dfrac{d^2C_1}{d\theta^2}+\dfrac{(V_3-V_2)^2}{2}\,\dfrac{d^2C_2}{d\theta^2}+T_0}.$$

The denominator can be made as small as desirable, since $T_0>0$, $(V_3-V_1)^2>0$ and $(V_3-V_2)^2>0$, if the second derivatives are negative.

When these derivatives are negative we have what Compton[1] calls "negative electrostatic control"; and when they are positive, "positive electrostatic control." The first case amounts to a reduction, the second to an increase in the effective moment of torsion.

(4) A null method is suggested by the following inspection: Consider the needle always in equilibrium at an angle θ, kept there by properly adjusting V_2 as V_1 changes. As before,

$$-\frac{(V_3-V_1)^2}{2}\frac{dC_1}{d\theta}-\frac{(V_3-V_2)^2}{2}\frac{dC_2}{d\theta}-T_0\theta=0.$$

$$\therefore V_3-V_2=+\sqrt{-\frac{(V_3-V_1)^2\dfrac{dC_1}{d\theta}+2T_0\theta}{\dfrac{dC_2}{d\theta}}} \qquad (0<V_3>V_2).$$

Then

$$\frac{\partial V_2}{\partial V_1}=\left\{\frac{\partial(V_3-V_2)}{\partial(V_3-V_1)}\right\}_{\substack{V_3=const.\\ \theta=const.}}=\sqrt{-\frac{1}{\dfrac{dC_2}{d\theta}}}\;\frac{(V_3-V_1)\dfrac{dC_1}{d\theta}}{\sqrt{(V_3-V_1)^2\dfrac{dC_1}{d\theta}+2T_0\theta}}.$$

$$=\sqrt{-\frac{1}{\dfrac{dC_2}{d\theta}}}\;\frac{\dfrac{dC_1}{d\theta}}{\sqrt{\dfrac{dC_1}{d\theta}+\dfrac{2T_0\theta}{(V_3-V_1)^2}}}.$$

Thus if we take the mechanical zero, $\theta=0$, as a null point we have:

$$\frac{\partial V_2}{\partial V_1}=\sqrt{-\frac{\left(\dfrac{dC_1}{d\theta}\right)_{\theta=0}}{\left(\dfrac{dC_2}{d\theta}\right)_{\theta=0}}},$$

which is a constant independent of needle or quadrant voltages. Experimentally, of course, how small a "dV_1" can be detected at $\theta=0$ depends upon S_v which in turn depends on V_3, etc. If the electrical and mechanical zeros coincide, by (2)

[1] A. H. Compton and K. T. Compton, Physical Review, *14*, p. 90; 1919.

$$\left(\frac{dC_1}{d\theta}\right)_{\theta=0} = -\left(\frac{dC_2}{d\theta}\right)_{\theta=0} \quad \therefore \frac{\partial V_2}{\partial V_1} = 1 \quad \text{and} \quad V_2 = V_1 + V_0$$

but since the electrical zero is $\theta = 0$, $V_0 = 0$.

II. EXPERIMENTAL ARRANGEMENT

(5) The arrangement to make $d^2C_1/d\theta^2 < 0$ and $d^2C_2/d\theta^2 < 0$ is as follows: The needle is first bent on its axis as in Fig. 2, so that when the quadrants are level the right hand end "A" is about one fifth the way from the bottom to the top, and the left hand end "B" the same distance down from the top to the bottom of the quadrants. Next the instrument is tilted about the axis MN, down in back as seen in Fig. 2.

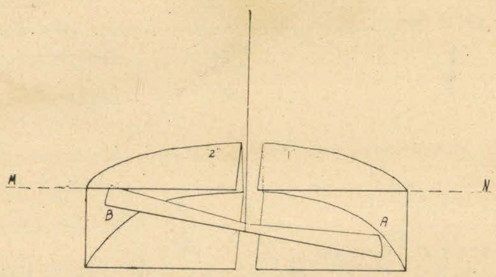


FIG. 2. *Front halves of quadrants and needle removed.*

This brings about the desired condition, as can be seen from Fig. 3. The arrows indicate the positive direction of θ. It is seen that for the "A" end of the needle, positive rotation not only moves the needle out of quadrant "2" and into quadrant "1," but also increases the spacing of the needle from the bottom of quadrant "2." Therefore, since the capacity contributed by the top side of the needle is much smaller than that contributed by the bottom side, $d^2C_2/d\theta^2 < 0$. And in moving into quadrant "1" the advancing edge of the "A" end of the needle gets farther from the bottom so $d^2C_1/d\theta^2 < 0$. Exactly the same kind of argument holds for the "B" end of the needle and the tops of the quadrants.

The instrument used had two levelling screws in a line parallel to MN, and one centrally in back as seen in Fig. 2. By adjustment of the first two screws, and the height of the needle, the electrical and mechanical zeros could be brought approximately together. Further adjustments, as for zero drift, are made with the third screw. A very convenient means of doing this would be that described by Watson[2] in his recent paper on "Current Measurement with the Compton Electrometer."

[2] E. E. Watson, Proceedings of The Cambridge Philosophical Society, *25*, Part I, p 74; 1929.

By raising the needle voltage V_3, the instrument can easily be made to approach instability. We used a Dolezalek instrument with the suspension which gives normally 800 mm/volt. By making the above changes we could operate regularly at 8000 mm/volt. We could attain double this, but only temporarily since the instrument was on a stand that would not remain accurately enough level with slight temperature changes.

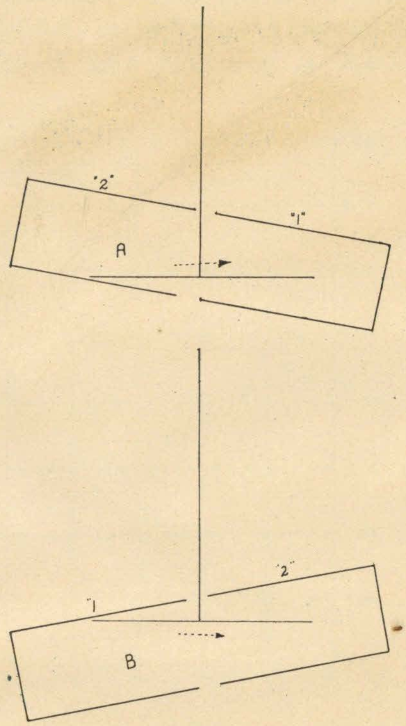


FIG. 3.

A curve of sensibility with needle voltage, obtained by letting $V_2=0$ and $V_1=0.001$ volt, is given in Fig. 4. With this adjustment the point of instability was 97 volts. It was found possible to adjust so that there was a range of several centimeters about the null point in which S_v was sensibly constant.

(6) The null method: Fig. 5 shows the diagram of the circuit used. By filtering the V_2 and V_3 leads with condensers and resistances as shown, it was found unnecessary to shield them. The condensers "C" are 2μf; and the resistances "r" are 1/2 megohm leaks. These

resistances, besides taking part in the filter, make it possible to ground any part of the instrument without measureably disturbing the external circuits.

The potentiometer P_1 is used to control the needle voltage.

The potentiometer P_2 supplies a known voltage to an Ayrton shunt "S," from which V_2 is derived. The voltmeter can conveniently have a range of 10 volts.

The result that $V_2 = V_1$ of (4), above, has been checked as closely as the meters could be read from 0.0001 to 10.0 volts. When V_1 and V_2 become appreciable as compared with V_3, S_v is automatically

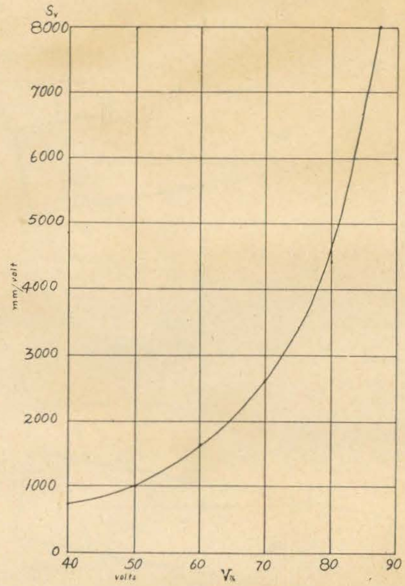


FIG. 4.

reduced provided the polarity of V_3 is chosen correctly. Otherwise V_3 can be reduced for high voltages to make the needle less agile.

The resistance R is a line of india ink on soft drawing paper, thoroughly boiled out and sealed in paraffin. After many trials we succeeded in making one that is constant at $2 \cdot 10^{11}$ ohms. Using this R the instrument and circuit can measure currents over a range from 10^{-15} to $0.5 \cdot 10^{-10}$ Ampères directly.

In manipulating the operator can use P_2 and the reversing switch to set the spot of light on the null point. With a little practice this can be done in a few seconds and a reading made. When measuring very small currents, because of the inertia of such a large needle, it is necessary to wait a few seconds to be sure the needle is balanced.

With so high a value of R, and if the circuit connected to quadrants "1" has a low enough capacity, the condition that $dV_1/d\theta = 0$ is not satisfied for rapid changes in θ. This operates to make the instrument

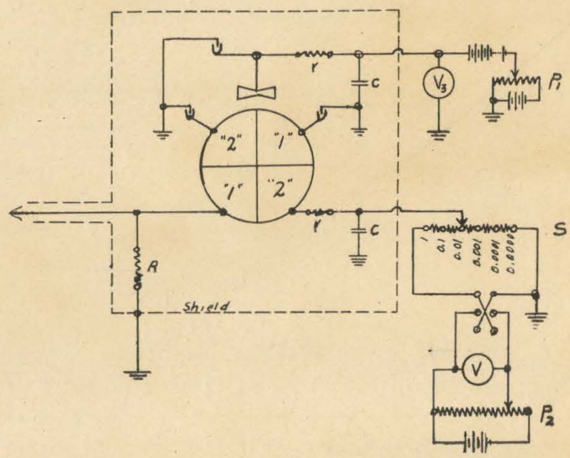


FIG. 5.

more stable to sudden changes in V_2, by virtue of the charge induced on quadrants "1" by the motion of the needle, without introducing an appreciable "lag."

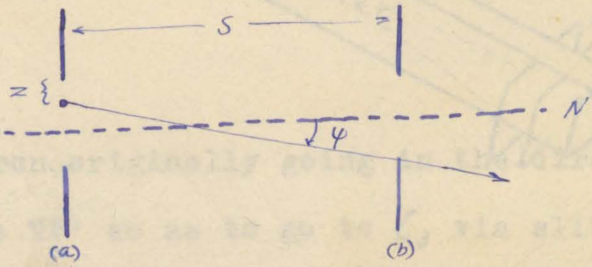
In conclusion I should like to thank Mr. W. Arnquist for his patient assistance in adjusting and using this circuit, and Dr. L. A. DuBridge for the technique used in making R.

NORMAN BRIDGE LABORATORY OF PHYSICS,
CALIFORNIA INSTITUTE OF TECHNOLOGY,
PASADENA, CAL., AUGUST 1, 1929.

APPENDIX III

Value of .

1. With respect to slit width:



Assume slit "a" to be uniformly illuminated with particles having all directions. Then the à priori probability that a particle moving at angle ψ with the axis MN passes through slit "b" is proportional to the distance z from one edge of illuminated slit "a", which is the width of the illuminated area from which such particles can come, and still get through slit "b".

Let W = width of slits
" S = distance between slits
" P_ψ = à priori probability for a particle at angle ψ.

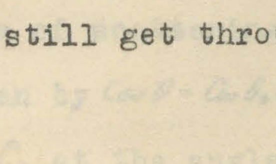
set $P_\psi = \frac{z}{W}$. Then, $\frac{W-z}{S} = \tan\psi$; $z = W - S\tan\psi$.

$\therefore P_\psi = 1 - \frac{S}{W}\tan\psi$.

$\therefore$ if $W << S$, $P_\psi = 0$ when $\tan\psi = \frac{W}{S}$ or $\psi = \frac{W}{S}$ radians.

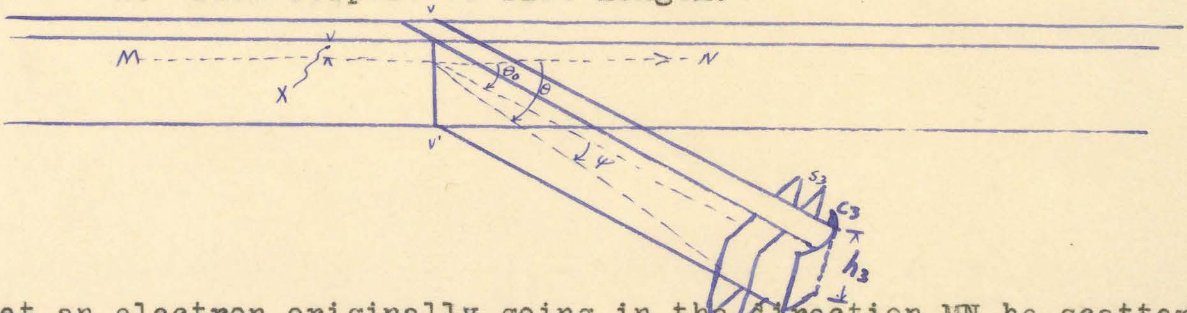
and $P_\psi = 1 - \frac{S}{W}\psi$.

The graph of this is:



where the shaded area is 90% of the total area. In the adjustment used, $\frac{W}{S} = \frac{.8}{25.4} = .031$ radians. Thus 90% of the beam was within 1.6° of the axis MN.

2. With respect to slit length.

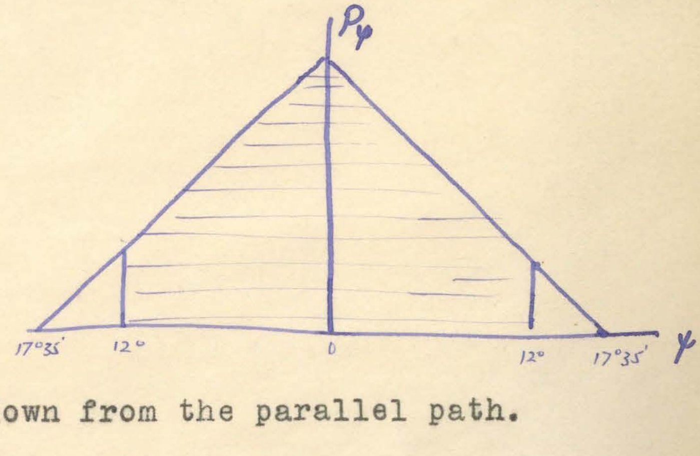


Let an electron originally going in the direction MN be scattered in the volume VV' so as to go to C_3 via slits S_3. As before, the distance from VV' to C_3 is l_3. Let h_3 be the length of slits S_3, and the heighth of the rectangular beam of collimation. Let the path MN be at a distance χ down from the top of this beam. Our scattered electron now, instead of moving parallel to the axis of collimation of slits S_3, might move down at an angle ψ. Thus the actual angle of scattering θ, instead of being θ_0 as read, will be given by $\cos\theta = \cos\theta_0 \cos\psi$. The number of electrons that can arrive at C_3 at the angle ψ is, by the same reasoning as in (1), proportional to the maximum value of χ for which an electron can get through S_3. If we let P_ψ be the à priori probability as before, $P_\psi = K\chi_{max} = K(h_3 - l_3 \tan\psi)$

Since $l_3 = 41^{mm}$; $h_3 = 13^{mm}$; $\psi_{max} = 17°35' = .307$ radians.

so that P_ψ plotted is:

and 90% of the area is shown shaded.



Thus it is seen that 90% of the area is within $\pm 12°$; or, 90% of the electrons reaching C_3 have deviated 12° or less up or down from the parallel path.

Let us now examine what effect this has on the real angle of scattering θ as compared with the measured angle θ_0 :

$$\cos\theta = \cos\theta_0 \cos\psi .$$

but $|\psi| \leq 12^\circ$. $\therefore 1 \leq \cos\psi \leq .928$. Moreover, for reasons explained in the text, $30^\circ \leq \theta_0$. Hence under the worst conditions, $(\theta = 30^\circ)$ $30^\circ \leq \theta \leq 31.9^\circ$, and the "$d\theta$" due to the length of the slits is 1.9° . As θ_0 nears 90°, this "$d\theta$" lessens to exactly zero, independent of ψ .

No. 334 - 2 TR, 10 X 10 TO THE HALF INCH.
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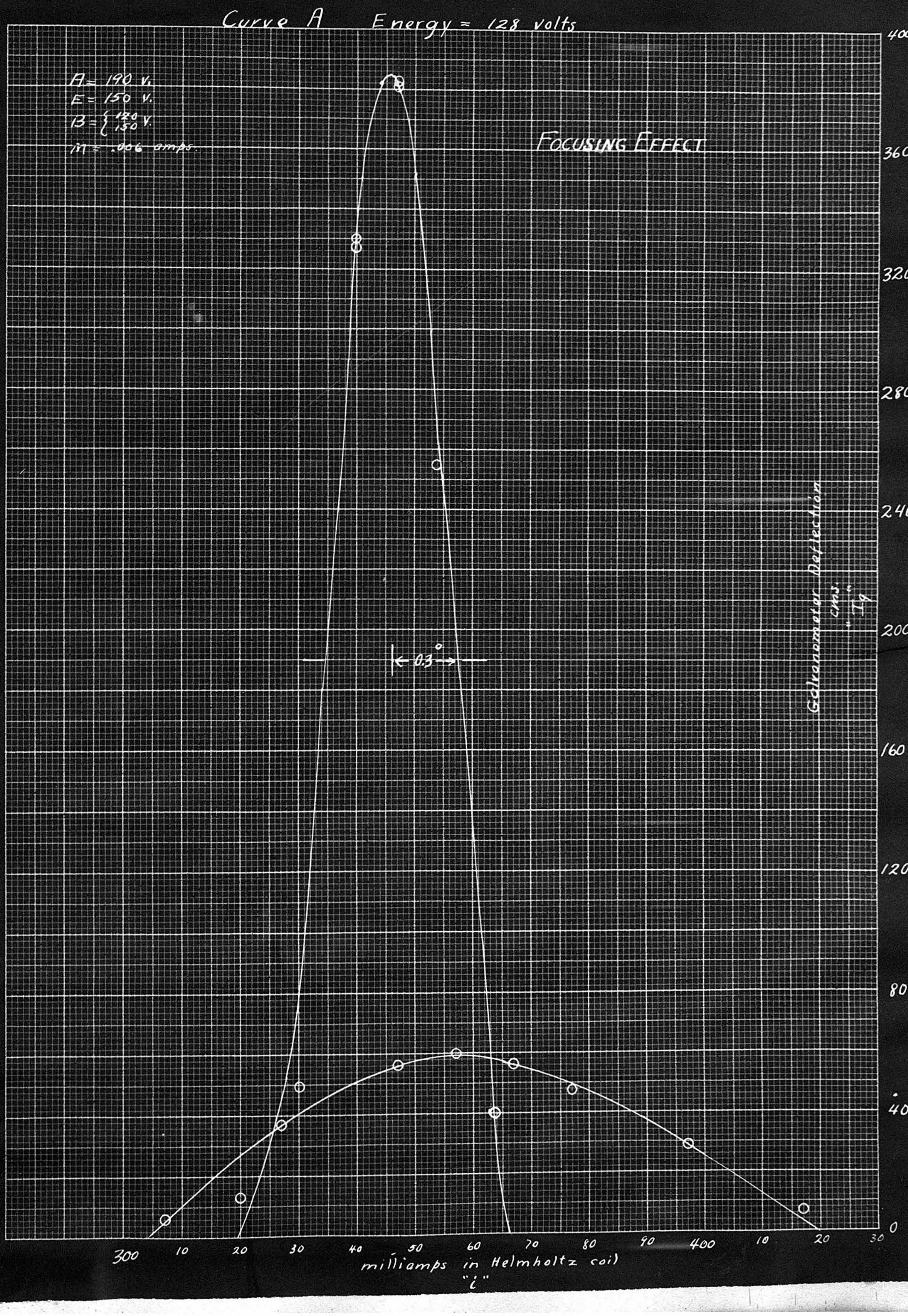
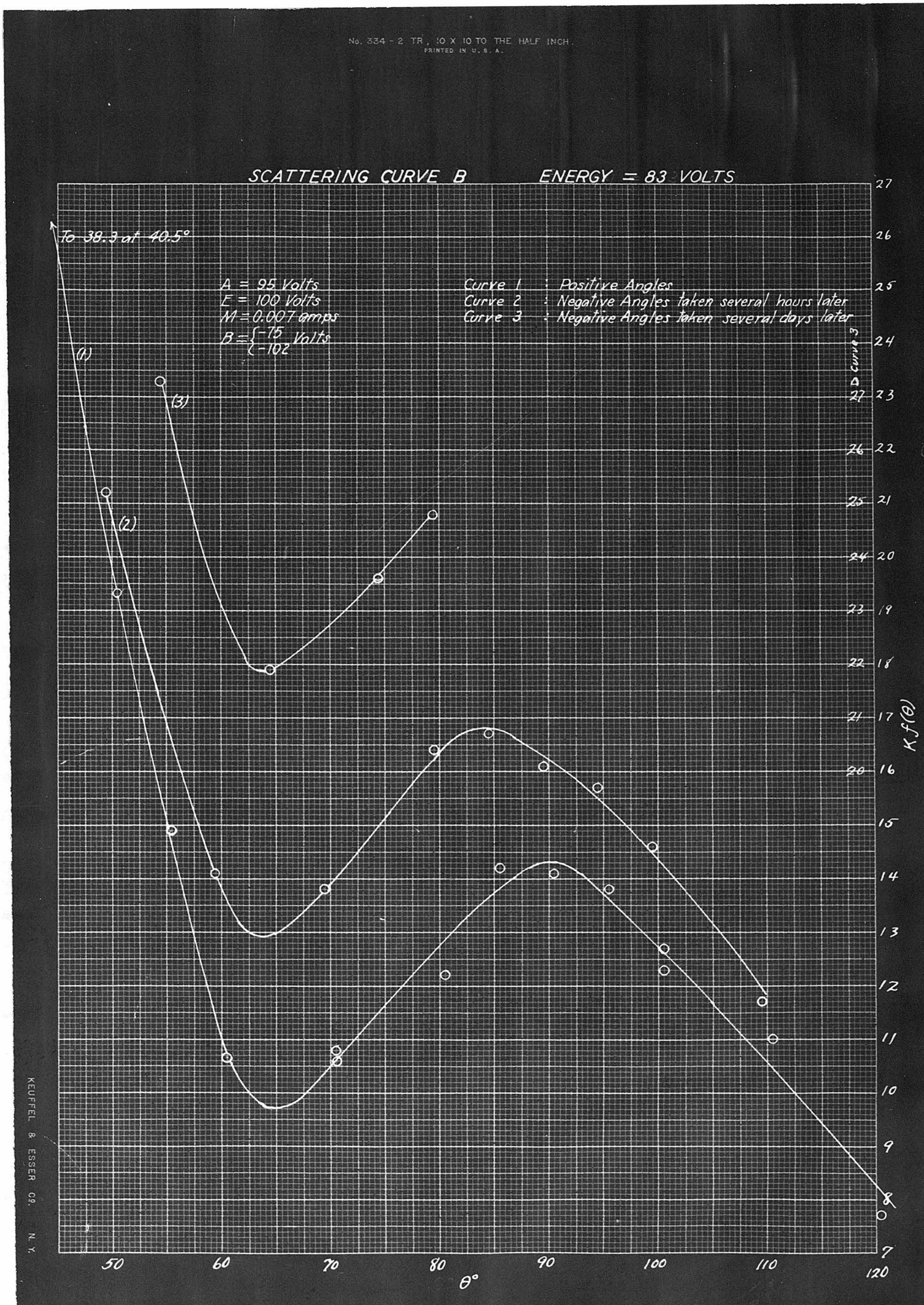


KEUFFEL & ESSER CO. N. Y.



No. 334 - 2 TR, 10 X 10 TO THE HALF INCH.
PRINTED IN U.S.A.
SCATTERING CURVE B
ENERGY = 83 VOLTS
To 38.3 at 40.5°
A = 95 Volts
E = 100 Volts
M = 0.007 amps
B = {-75, -102 Volts
Curve 1 : Positive Angles
Curve 2 : Negative Angles taken several hours later
Curve 3 : Negative Angles taken several days later
(1)
(2)
(3)
Curve 3
K f(θ)
θ°
50
60
70
80
90
100
110
120
7
8
9
10
11
12
13
14
15
16
17
18
19
20
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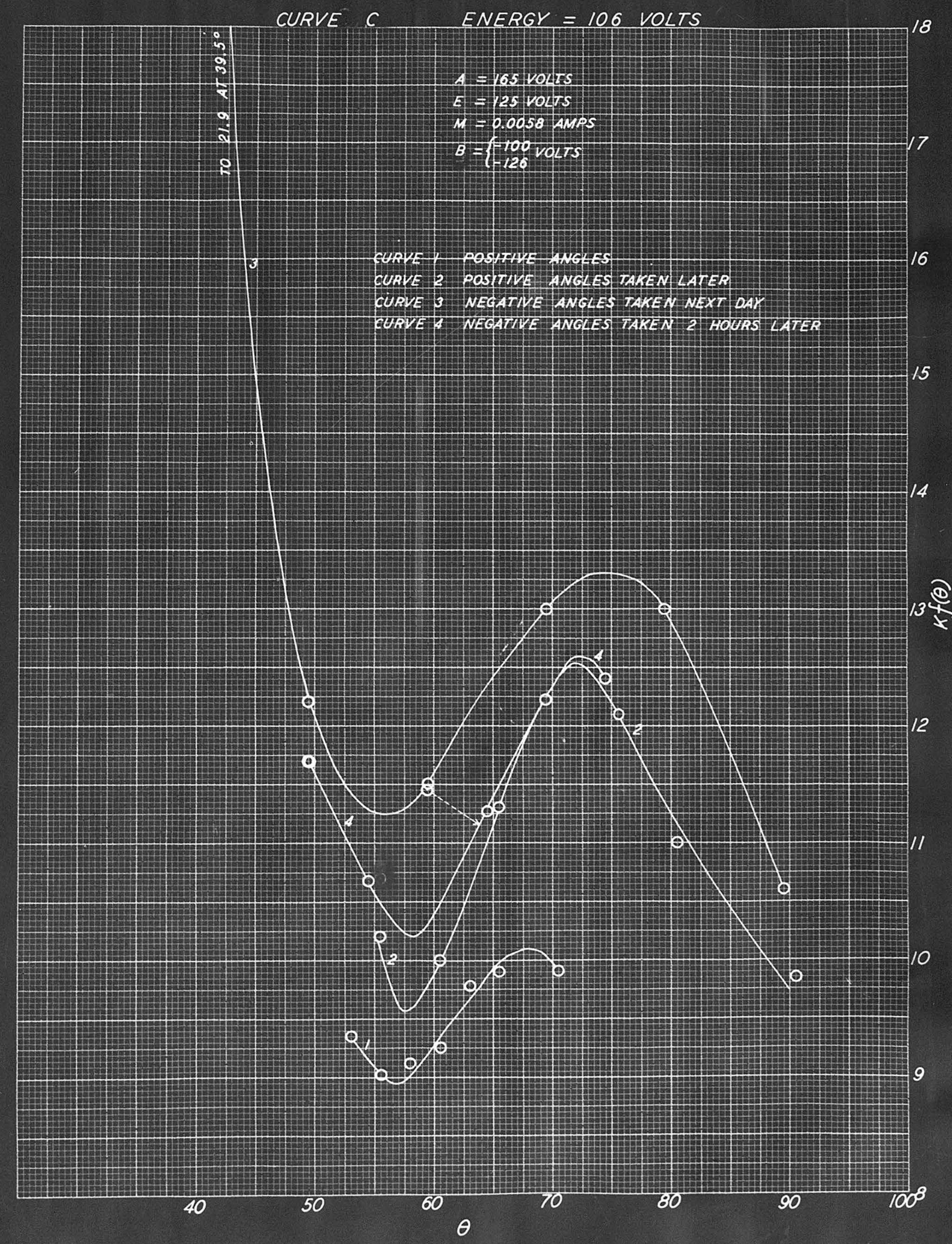
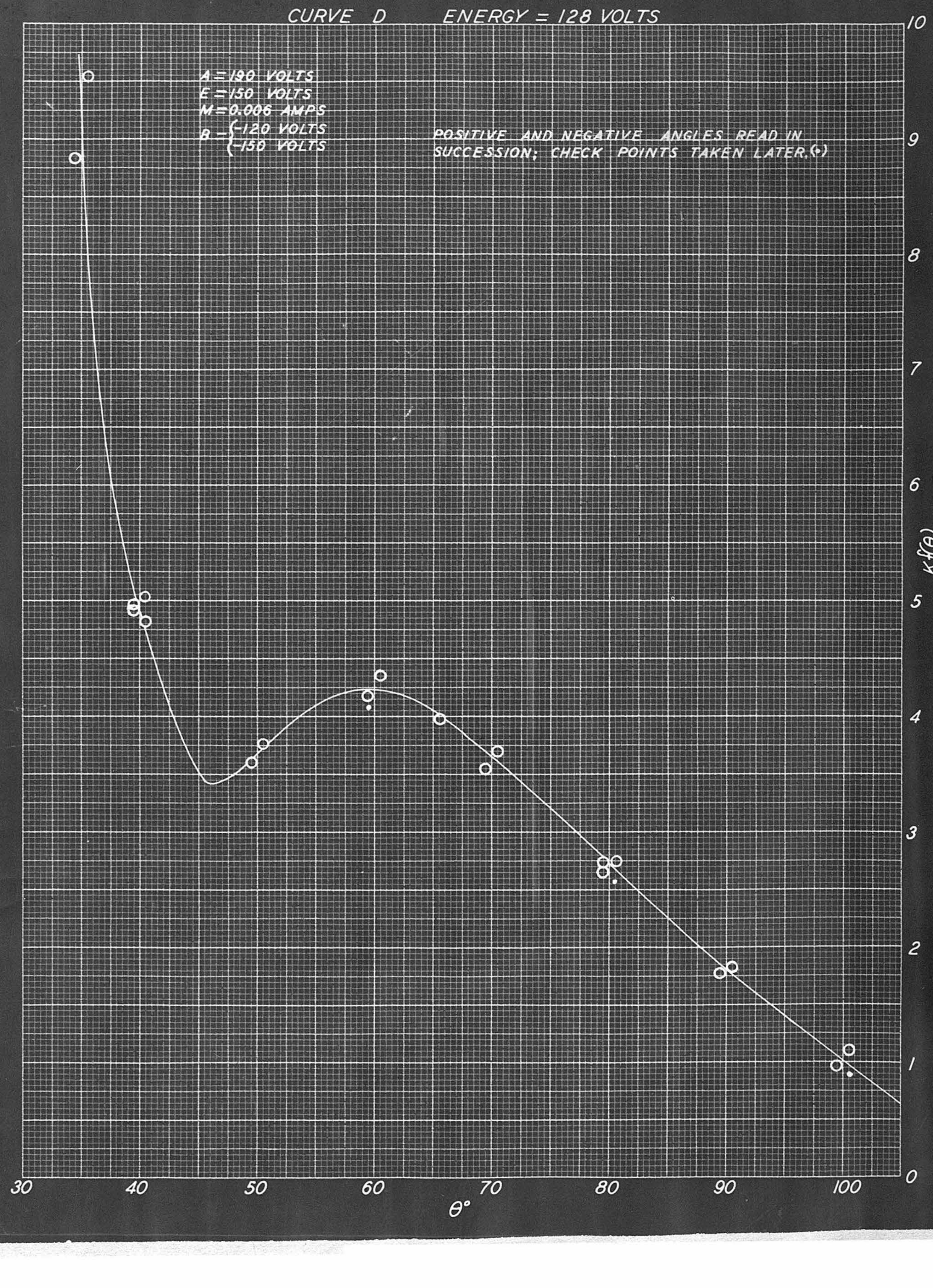
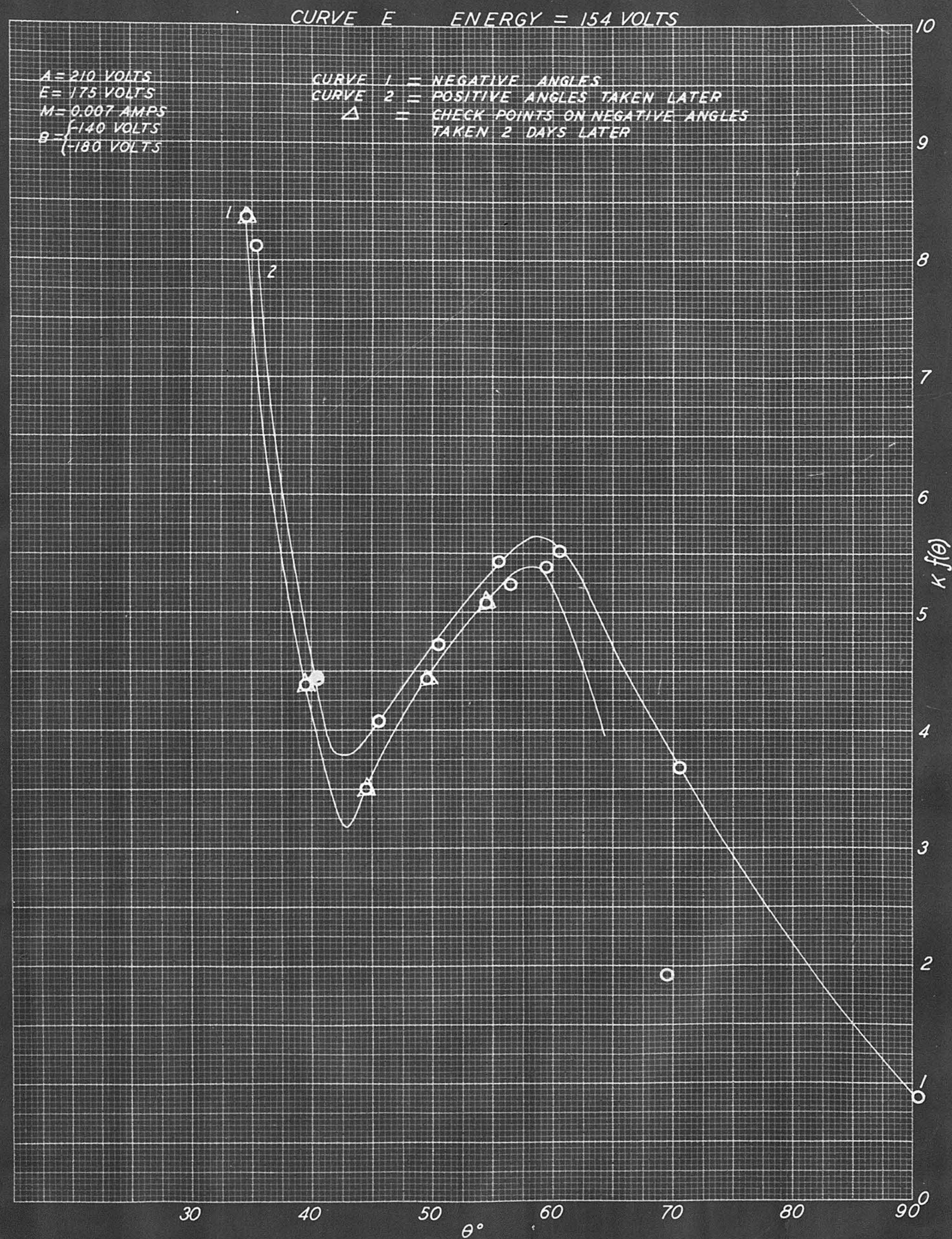
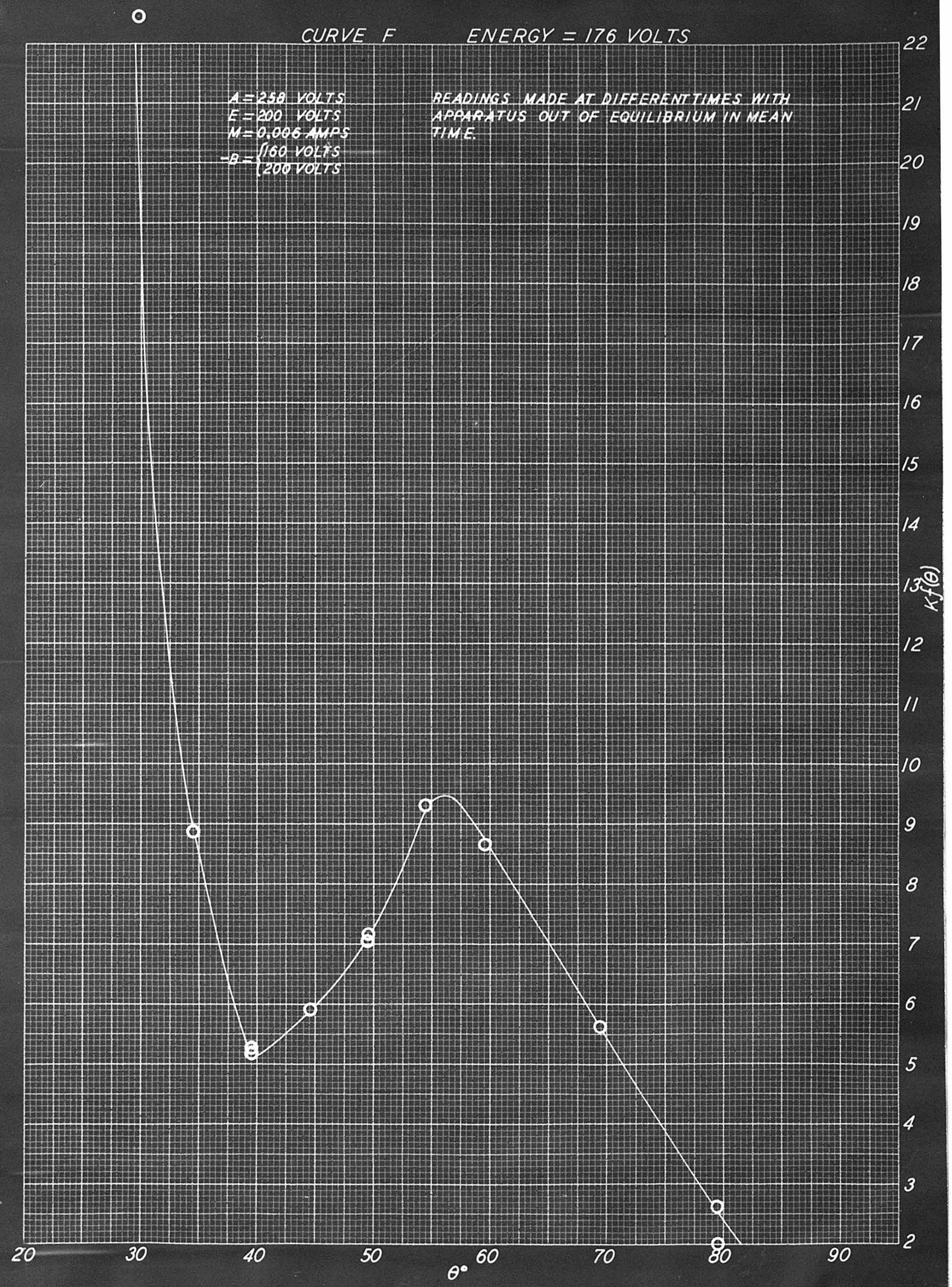
KEUFFEL & ESSER CO. N.Y.


CURVE C ENERGY = 106 VOLTS
A = 165 VOLTS
E = 125 VOLTS
M = 0.0058 AMPS
B = {-100, -126} VOLTS
TO 21.9 AT 39.5°
CURVE 1 POSITIVE ANGLES
CURVE 2 POSITIVE ANGLES TAKEN LATER
CURVE 3 NEGATIVE ANGLES TAKEN NEXT DAY
CURVE 4 NEGATIVE ANGLES TAKEN 2 HOURS LATER
1
2
3
4
40
50
60
70
80
90
100
θ
8
9
10
11
12
13
14
15
16
17
18
Kf(θ)


CURVE D ENERGY = 128 VOLTS
A = 190 VOLTS
E = 150 VOLTS
M = 0.006 AMPS
B — {-120 VOLTS / -150 VOLTS}
POSITIVE AND NEGATIVE ANGLES READ IN SUCCESSION; CHECK POINTS TAKEN LATER. (•)
K f(θ)
θ°
30
40
50
60
70
80
90
100
0
1
2
3
4
5
6
7
8
9
10


CURVE E ENERGY = 154 VOLTS
A = 210 VOLTS
E = 175 VOLTS
M = 0.007 AMPS
B = -140 VOLTS
-180 VOLTS
CURVE 1 = NEGATIVE ANGLES
CURVE 2 = POSITIVE ANGLES TAKEN LATER
△ = CHECK POINTS ON NEGATIVE ANGLES TAKEN 2 DAYS LATER
1
2
30
40
50
60
70
80
90
θ°
0
1
2
3
4
5
6
7
8
9
10
κ f(θ)


CURVE F ENERGY = 176 VOLTS
A = 258 VOLTS
E = 200 VOLTS
M = 0.006 AMPS
-B = {160 VOLTS / 200 VOLTS
READINGS MADE AT DIFFERENT TIMES WITH APPARATUS OUT OF EQUILIBRIUM IN MEAN TIME.
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
K f(θ)
20
30
40
50
60
70
80
90
θ°

CURVE G LOCI OF MINIMA