A Determination of e/m

from the

Zeeman Effect.

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Abstract

Values of e/m have been obtained from the Zeeman separations of the Cd line λ 6439 and the Zn line λ 6362, the g-factors of which have been carefully examined. Magnetic fields up to 7300 gauss are produced in a solenoid giving less than .1% variation in field strength over a length of 6 cm at the center. The current-field ratio K of the solenoid is determined, under operating conditions, in terms of the calculated ratios of measured single layer solenoids. The field strength during exposure is given by the product of K and the solenoid current. Evaporation of Zn and Cd in the short (6 cm) positive column of a helium discharge gives the desired lines, the Zeeman patterns of which are photographed with a Fabry-Perot interferometer. Measurement of the separations at orders of interference in the neighborhood of 100,000 gives the following values:

Cd λ 6439 : $e/m = 1.7578 \times 10^7$ e m units $\pm$ .06%

Zn λ 6362 : $e/m = 1.7576 \pm .05\%$

These results are stated as

$$e/m = (1.7577 \pm .0017) \times 10^7 \text{ abs e m units.}$$

A Determination of e/m from the Zeeman Effect.

I. Introduction

The precise evaluation of the specific charge of the electron has been frequently attempted during the last twenty years. Our feeling that e/m is a fundamental constant leads to the expectation that it should occur in the descriptions of various observed phenomena with a single definite value. The three types of phenomena which lend themselves best to precise determinations of e/m are 1) the acceleration of free electrons in electric and magnetic fields, 2) the Zeeman effect, and 3) the difference between the Rydberg constants of H and He^+. The disagreement between the values so obtained, more particularly between those arising from the same phenomenon, has been serious enough to justify further work.

In a critical survey of the measurements of e/m which had been made before 1926, Gerlach [1)] found that the more accurate determinations by each of the three methods were in good agreement, with a mean value of $e/m = 1.766 \times 10^7$ e.m. units per gram (hereafter the units and the factor 10^7 will be omitted). In his discussion, however, Gerlach followed Back [2)] in omitting from consideration the result $e/m = 1.761 \pm .001$ obtained by Babcock [3)] from the Zeeman effect of 116 spectral lines, for the reason that the Runge denominators for some of the lines were large and uncertain. Babcock made a later calculation[4)] based on 48 lines for which the denominators were well established, and

1) The references are listed at the end.

obtained $e/m = 1.7606 \pm .0012$. Meanwhile Houston [5] had obtained $e/m = 1.7606 \pm .0010$ from a determination of the Rydberg constants of H and He^+, and Wolf [6] had completed careful measurements on the deflection of electrons in a magnetic field, which gave $e/m = 1.7679 \pm .0018$.

The close agreement between the spectroscopically determined values of Babcock and Houston, and the fact that the 0.5% discrepancy between these and Wolf's value amounted to four times the probable error of the last, led Birge [7] to suggest that there were two values of e/m: the spectroscopic value, to be used with atomic electrons, and the deflection value appropriate to free electrons.

Subsequent to Birge's suggestion there have been published two more precision measurements of e/m for free electrons, and also several discussions of the possibility of correcting the deflection value by employing quantum mechanical expressions for the path of an electron beam in a magnetic field. The first of the new experimental work was that of Kirchner [8] who in 1929 announced preliminary measurements giving $e/m = 1.770$. His final value [9] was $1.7602 \pm .0025$. His method was a modification of that of Wiechert [10]. The velocity attained by electrons falling through a potential difference of 2500 volts was determined by passing them through an electrostatic velocity filter driven by a high frequency oscillator. Using the same method with accelerating potentials as high as 20,000 volts, Perry and Chaffee [11] obtained $e/m = 1.761 \pm .001$. The agreement between the two determinations is close. Perry and Chaffee, however, were under the objectionable necessity of employing magnetic fields to focus their beams. Kirchner considers their accuracy overestimated; his own value contains a purely estimated correction of .06% for the influence of contact potential within his apparatus. Before full reliance can be placed in these two recent values for free electrons, not only must Wolf's higher value be explained away, but also the values close to 1.766, obtained by Bucherer, Wolz, Neuman, Alberti, Bestelmeyer, and Busch 12). Perry and Chaffee attempt to explain the difference between their value and Wolf's as the result of retardation by the residual gas in Wolf's apparatus. Applying their extrapolation of the Thomson-Whiddington formula to the pressure of Hg vapor at the temperature (-20° C.) of Wolf's traps indicates less than .02% error from this cause. It is probably much less, as Wolf reports that he obtained a "dark" vacuum.

The attempt of Page 13) to explain the discrepancy between the deflection and spectroscopic values of e/m on the basis of a wave-mechanical treatment of magnetically deflected electrons has resulted in some controversy. The most conclusive work on the subject is that of Uhlenbeck and Young 14) and that of Huff 15). The former treat the 'streaming' of electrons in a semi-infinite magnetic field, and find that an error of only .0001% is introduced by classical computation of deflection experiments. Huff examined the same problem, making use of the Dirac electron to include possible spin effects, and obtains the same result. The question of whether or not there is a real difference between the spectroscopic and deflection values is still an open one. No adequate theoretical explanation has appeared, and the experimental evidence is divided.

The work described in this paper was planned by Dr. Houston to approach the problem from another direction, namely, to test the agreement between the spectroscopic values by making a new determination from the Zeeman effect which differed in experimental detail as widely as possible from Babcock's prodedure. The value which has been obtained is

$$e/m = (1.7577 \pm .0017) \times 10^7 \text{ abs e m u.}$$

General Requirements of a Determination from the Zeeman Effect.

It was the plan of this work to examine the Zeeman effect of only a few lines whose behavior in a magnetic field could be reasonably well established. It was also necessary that the lines have a simple Zeeman pattern and be both sharp and free from hyperfine structure.

We may consider two terms having the g-factors g and g'. In a magnetic field, H, a transition between the sublevels characterized by m_i and m_j' will differ from the field-free line by

$$1.) \qquad \Delta\nu_{ij} = a \frac{e H}{4\pi m_0 c}$$

where $$a = m_i g - m_j' g'$$

If there is no departure from Russell-Saunders coupling the g-factors may be accurately calculated from the Lande formula. With two-electron systems in which a departure from ls coupling is evidenced by the close approach of the singlet term to the triplet term, the theory of Houston[16)] may be applied to calculate the g-factors from the observed separation of the fine-structure levels. The disturbance of the g-factors from levels in other configurations has not been treated.

In the Zeeman effect of singlet lines several combinations of the m-levels may give the same $\Delta\nu$ when g has the value of 1 given by the Lande formula. The g-factors calculated by Houston's method for slightly disturbed terms, however, differ somewhat from 1 and the lines due to the several combinations do not coincide. To the center of gravity of the observed Zeeman component is related a value of a averaged over the contributing transitions according to their intensities, I_{ij}. The appropriate value of a to be applied to the σ components of singlet lines is thus

$$\bar{a} = \sum_{ij} I_{ij} \, (m_i g - m_j' g') / \sum_{ij} I_{ij} \qquad 2)$$

summed over the combinations giving approximately the same $\Delta\nu$.

The equation for determining e/m becomes

$$e/m = \frac{4\pi c}{a\,H} \Delta\nu \qquad 3)$$

where e/m is given in absolute electromagnetic units when c is the velocity of light in vacuo, H the magnetic field in abs-gauss, and is expressed in cm^{-1} reduced to vacuum. The essential measurements are those of the magnetic field H and the Zeeman displacement $\Delta\nu$. In the present determination the magnetic field was produced in an air-core solenoid. Two lines were employed, the $'P$ - $'D$ lines Cd λ 6439 and Zn λ 6362. These satisfy the requirements of sharpness and simplicity, and show only small deviations from unity in the calculated values of $\bar{a}$. For the cadmium line $g_D = 1.00049$, $g_P = 1.00216$, and $\bar{a} = .999655$. For the Zinc line $g_D = 1.00003$, $g_P = 1.0002$, and $\bar{a} = .999945$. The longitudinal Zeeman effect was photographed with a Fabry-Perot interferometer crossed with a prism spectrograph.

II. Apparatus

Solenoid. The solenoid constructed for this experiment was designed to fulfill the requirements that:

(a) The variation of the field strength over a light source 6 cm long and placed longitudinally at the center of the coil should not exceed 0.1%.

(b) A method of cooling should be provided to permit continuos operation at full power.

(c) Subject to the demands of (a) and (b) the maximum possible field strength should be obtained from the available D.C. power supply. The chief features of the method of design are contained in the Appendix ~~I~~ (p. i).

Figure (1) shows a cross section of the solenoid as it was constructed by the Institute shop. The winding is continuous and consists of 2449 turns of No. 4,B and S. (5.2 mm) square d.c.c. copper wire in 18 layers. The coil proper is 80 cm long, with an outer diameter of 39.7 cm and an inner diameter of 7.6 cm. The coil was wound on the heavy brass inner tube T, between the cast-brass spiders S, and was insulated from the tube by a layer of 3/16" micarta strips laid longitudinally, and from the spiders by micarta strips attached to the spider arms. Between each layer of the coil was placed a layer of black fiber spacers, each 6.5 mm x 3.2 mm x 80 cm, parallel to the axis of the tube and spaced so as to leave passages through the coil. A similar layer of spacers insulated the wire from the brass tube, 40 cm in diameter, into which the completed coil was forced. The ends

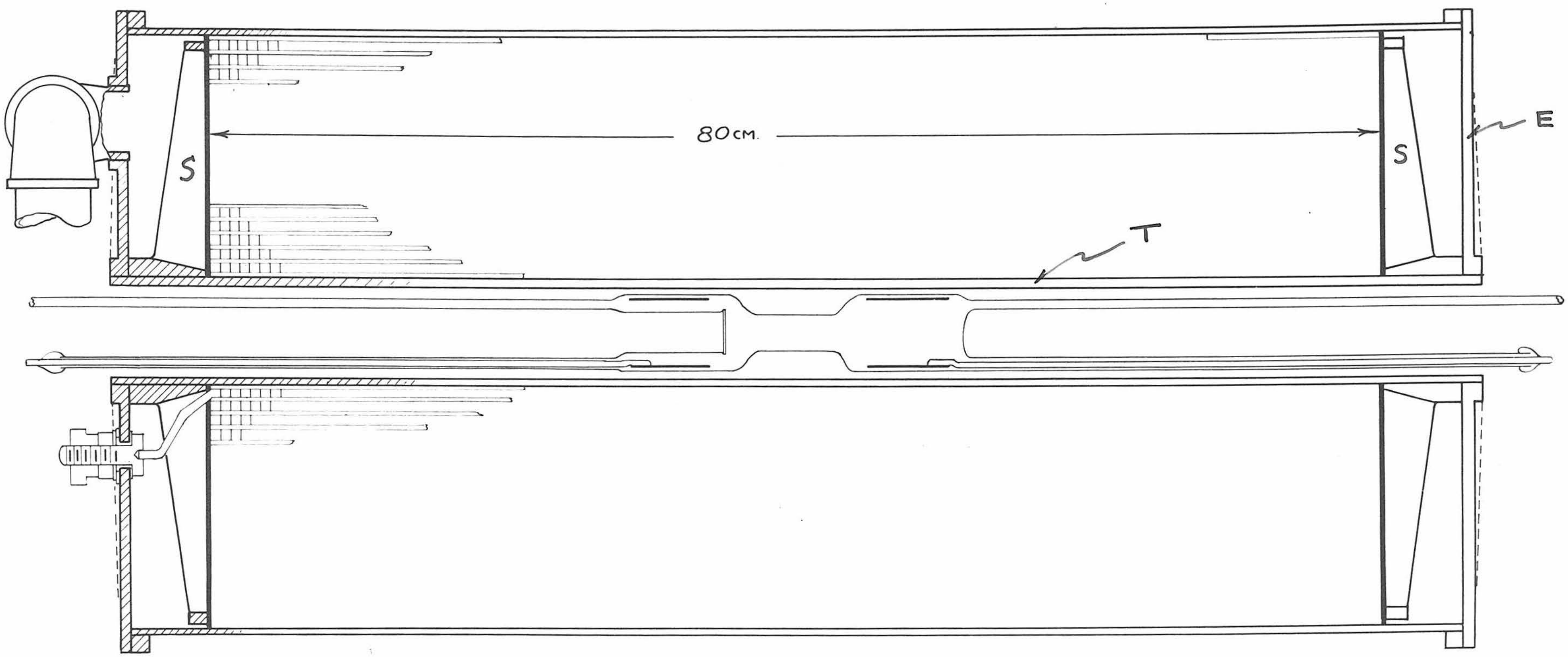


FIG. 1.

of the shell were closed by the cast-brass plates E, which were screwed to the inner tube and to a flange on the outer tube. The entire assemblage, weighing 1200 lbs., was supported by a wooden platform in the construction of which only brass screws were used. In order to obviate the accidental presence of iron in the spiders and the end plates they were cast from ~~a~~ freshly alloyed copper and zinc.

The solenoid was cooled by pumping kerosene through the passages left between the fibre spacers. A centrifugal pump maintained an estimated flow of 200 liters per minute through a circuit consisting of the solenoid and eight automobile radiators in which the kerosene was cooled. The radiators, assembled in a unit and connected in series-parallel with welded or soldered joints, were immersed in a tank of running water. To protect the oil against contamination with water in the event of leaks, the radiators were placed on the discharge side of the pump~~s~~ so that the pressure of the kerosene was always higher than that of the water. To test the insulation afforded by the kerosene, we frequently measured the resistance between the winding and the brass shell, and always found it to be between 10^5 and 10^6 ohms. The leakage resistance across the coil was necessarily of the same magnitude. As the resistance of the coil was 1.3 ohms between terminals, the difference between the measured current and the effective current was negligible. With the method in which the solenoid was used, moreover, the accuracy of the measurements would have been uninfluenced by a considerable leak, provided that the leak was constant and that Ohm's law was obeyed.

Two compound generators connected in series supplied the solenoid with a full-load current of over 200 amperes at 270 volts.

The field circuit of one of the generators was controlled by means of a coarse and a fine rheostat placed near the solenoid. A field of 7300 gauss, requiring 54 kilowatts could be maintained continuously without exceeding a temperature of 50° C. in the circulating kerosene. During an exposure the exciting current was measured by means of a .001 ohm Leeds and Northrup shunt and a Brooks type deflection potentiometer, and was held constant by controlling the field current of the generators. During the winding of the solenoid the diameter of each layer was measured with calipers; the number of turns in each layer was read from a revolution counter. From this data the field-current ratio could be computed either as (a) the sum of the constants of 18 single layers having the measured diameters, or (b) from the virtual outside dimensions of the coil, using equation (1), Appendix I. The sum of the 18 separate computations in (a) gave

$$K = 36.68 \text{ gauss per ampere}$$

while (b) gave

$$K = 36.67 \text{ gauss per ampere.}$$

These values are interesting only insofar as they are over 0.5% lower than the measured value discussed in Sec. III. The difference is in the proper direction to be explained as the result of the compression of the inner layers by the tension of those wound over them.

The field variation of the completed coil is given in Table I. The values in the second column were computed from Eqs. (1), (2) and (3) in the Appendix. The observed values in the third column were obtained from ballistic measurements made with a flip coil 3 cm long.

Table I

x cm	% calc.	$\frac{H_x - H_o}{H_o}$	% obsd.
0	0		0
1	.0072		-
2	.0288	+	.047
3	.0635	*	.105
4		+	.115
5	.16		

* Taken in front of the center
\+ Taken behind the center

The computed values are much more reliable than the observed values. The latter are differences between single readings, and should, moreover, be somewhat large because of the size of the flip coil used. A curve plotted from the calculated values was used to graphically average the field-strength over the light-source.

Source of radiation.

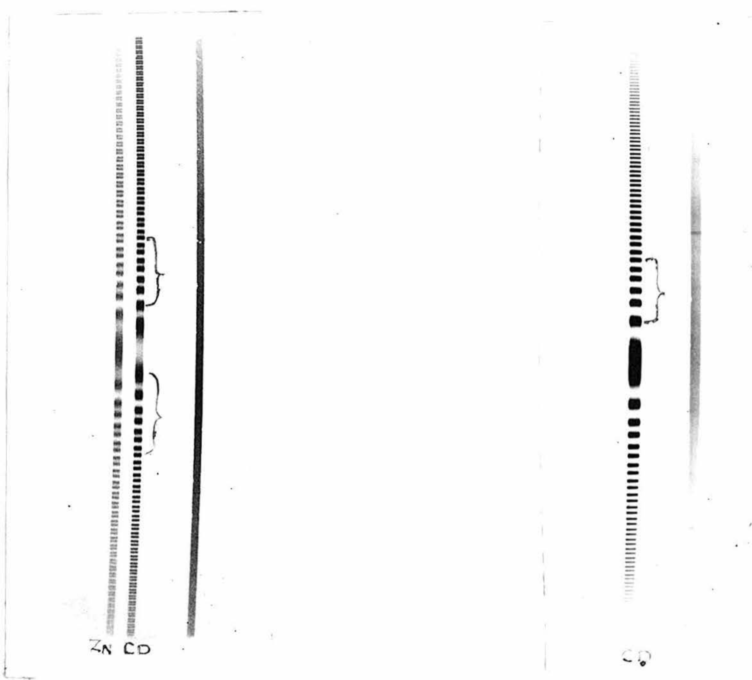
The light was produced by evaporating zinc and cadmium in the positive column of a D.C. discharge through helium. The tube, shown in Fig. 1, was constructed entirely of quartz. The illumination could be confined to a constriction, 6 cm long and elliptical in cross section (1 cm x 3 cm), which was placed at the center of the solenoid. The electrodes were short sections of heavy copper tubing 2 cm in diameter. A re-entrant window extended through the anode to within 2 cm of the constriction, which was viewed end-on. The proportions of the tube were such that neither of the electrodes was visible from the lens used to focus the light on the slit of the spectrograph. Thus by adjusting the pressure within the tube so that the cathode glow was confined to within a few mm of the cathode no light entered the spectrograph save that originating in the region of maximum field.

The helium filling the tube was continuously circulated by means of a Kurth type two-stage diffusion pump, and purified by passing through two charcoal traps kept in liquid air. The heat of the discharge was sufficient to vaporize zinc and cadmium shavings placed in the constriction and on the cathode; wrappings of copper foil and asbestos maintained a uniform temperature within the tube. At currents above 600 m.a. the spectra of Cd and Zn were produced with great intensity, entirely suppressing the He spectrum. All exposures, however, were made at somewhat lower currents, as the Zn and Cd red lines were found to be considerably sharper when their intensity was approximately one-third that of He λ 5875. The current for the discharge was furnished by four 500 v., D.C. generators in series and was regulated by means of a series resistance.

3. Optical Apparatus

The Fabry-Perot interferometer is particularly suited to accurate measurement of the longitudinal Zeeman separation of singlet lines. As only the two displaced σ components are present, the dispersion of the instrument may be so adjusted, by altering the distance between the mirrors, that the fringes due to one component lie midway between adjacent orders of the other. This avoids the shifting effect encountered with closely but unevenly spaced photographic images. The spacing of the fringes can also be adjusted by changing the field-strength, but where the measurements are to be made as precise as possible it is advantageous to obtain the proper spacing at maximum field.

The interferometer used in this investigation was of a modified Hilger type with plates 2.5 cm in diameter. The adjustment of the plates to parallelism by means of differential screws was both sensitive and stable. The interferometer was placed between the collimator and prism of a spectrograph. The sputtered gold surfaces of the interferometer plates were sufficiently dense to show twenty visible reflections of the filament of a 10 watt lamp. The resolving power may be roughly calculated as half of the number of reflected beams multiplied by the order of interference. Most of the exposures were made with the plates separated 3.5 cm, corresponding to an order of over 100,000 and a resolving power greater than 10^6.



a. FIG 2 b

At this order of interference, the normal Zeeman separation with the maximum field of 7300 gauss caused the components to overlap, with a separation of 4.5 orders. In Figure 2a the Zn and Cd red lines are shown at this separation. Two Zeeman components belonging to the same order are indicated with a bracket. Because of the uneven spacing of the components, this exposure was not measured. Fig. 2 shows the Zeeman pattern of the cadmium red line with the components separated approximately 3.5 orders, at a field of 6860 gauss. The spectroscopic apparatus was mounted on a heavy concrete slab supported by twelve tennis balls which were prevented from rolling. This arrangement proved stable, and effectively protected the interferometer from the unavoidable vibrations from the cooling pump. The supports of the collimator, interferometer and camera were independently fastened to the concrete base, permitting each part of the optical system to be separately aligned. To protect the interferometer and prism from temperature variations a tight wooden box was provided, from which the slit and the camera projected through felt gaskets. The box, which rested on a felt pad covering the concrete base, could be removed without disturbing the apparatus. The temperature was observed with a Beckmann thermometer and regulated by electrical heating to within .05° C. The spectroscopic equipment was placed 2 m from the solenoid, a distance sufficient to prevent disturbances either in the apparatus or in the field within the solenoid.

Although a 1 cm diaphragm was used between the interferometer mirrors, exposures ranging from 30 to 90 seconds, on Ilford Extra Rapid Panchromatic Plates, were sufficient.

III Measurements

Magnetic Calibration

The intensity of the magnetic field during an exposure was determined as

$$4) \qquad H = K\,I = K\,R\,P$$

where K is the magnetic constant in gauss per ampere, and I is the exciting current measured in international amperes obtained from a potentiometer reading of the voltage P across the shunt of resistance R. As the ratio of the int. ampere to the abs. ampere is .99995 there is no appreciable error in regarding H as being given in absolute gauss [17]. The value of K was obtained by comparing the field produced by a measured current in the main solenoid with the field in a single-layer standard solenoid, the magnetic constant of which could be calculated from the turn-density and the dimensions. The comparison has been made by two methods.

The first of these methods is a null method affording a direct comparison between the two constants. It is limited, however, to measurements of K where I is less than 1 ampere. It is undesirable to place too much reliance on the constancy of K over a wide range of currents, because of the possible influences of ferromagnetic surroundings, heating and slight distortions of the coil due to internal electromagnetic forces. Therefore the chief use of determinations by the null method has been to test both the constancy of K and the accuracy of the second method, with which determinations are made at both high and low currents.

1.) Standard Solenoids.

Two standard solenoids were employed in the calibrations. One consisted of a layer of No. 12 B and S bare copper wire wound on a bakelite tube which had been threaded 10 turns to the inch. The bakelite was of linen stock in order to avoid the reputed ferromagnetism of paper stock bakelite. The other solenoid was wound with No. 20 enamelled wire on a brass tube which had been threaded 28 turns to the inch and upon which a layer of insulating varnish had been baked. Both were of a size which permitted them to be placed within the inner tube of the large solenoid. The number of turns per cm in the winding of each was measured in terms of two scales, one a Starrett steel meter, the other a glass scale ~~made by~~ taken from cathetometer BL8166. Both scales were calibrated against a Gaertner Type M1001 meter at Pomona College, by a method of simultaneous readings made with a microscope provided with a micrometer eyepiece. At 20.5° the glass scale showed an excess length of 0.032%; the steel scale a .008% excess. The errors were uniform to within the accuracy of the calibration, and have been applied to all measurements.

The uniformity of the windings on the standard solenoids was carefully examined. The brass solenoid could be adequately checked by scale measurements. Owing to the difficulty of reading small distances between the larger turns of the bakelite solenoid its uniformity was examined with the use of a microscope mounted rigidly on the carriage of a Pratt and Whitney precision lathe. A total of 125 readings were made on every tenth turn along 4 sides of the coil. The distances were determined in terms of the screw of the lathe by reading a revolution counter at each setting. The winding was found to be uniform to within the estimated reading error of .002 cm. In addition these measurements provided a third independent evaluation of the turn-density of the bakelite solenoid.

Table II gives the data from measurements of the two solenoids, together with the values of their constants computed from the relation

$$6) \qquad K_s = .4 \pi n \cos \alpha \cos \varphi$$

Table II

Bakelite Solenoid

Date	Scale	No. readings	No. intervals	Wt	N (turns/cm)	Prob. error
8/10/30	Starrett	20	10	2	3.9324	.001
12/2/30	P. & W. Lathe	32	16	1	3.9346	.002
5/14/31	Glass	32	70	3	3.9319_2	.0004

3.9325 ± .02%
(= 9.9886 turns/inch)

Mean diameter = 6.0 cm

Length of winding = 89.7 cm

End corr:
$\cos\alpha$ = .9978 ± .0045%

Pitch angle
$\cos\varphi$ = .99991

K_1 = 4.9301 gauss/amp ± .02%

Brass Solenoid

Date	Scale	Readings	Intervals	Wt	N (turns/cm)	Prob. error
4/5/31	Starrett	8	4	1	11.0236	.001
5/14/31	Glass	12	35	2	11.0234_6	.0004

11.0235 ± .005%
(= 27.9999 turns/inch)

Mean diameter = 5.86 cm

Length of winding = 90.0 cm

$\cos\alpha$ = .9979 ± .001%

$\cos\varphi$ = 1.0000

K_2 = 13.823 gauss/amp ± .006%

where n is the number of turns per cm., α is the angle subtended at the center by the radius at the end, and φ is the pitch angle of the winding.

2.) <u>The null method.</u>

The arrangement of apparatus for determining the ratio between the constants of the large solenoid and the standard is shown in Fig. 4. The standard solenoid was placed within the inner tube of the big solenoid and connected so that the two fields were in opposition. A large flip coil wound with 10,000 ohms of No. 40 B and S wire was placed at the common center of the two solenoids and connected to a Leeds and Northrup wall-type ballistic galvanometer. The ratio of the currents in the two solenoids was varied until a balance was indicated by zero galvanometer deflection when the flip coil was operated. The ratio of the constant of the solenoid to that of the standard is then

$$5) \qquad A = \frac{K}{K_s} = \frac{I_s}{I}$$

where the subscript s denotes the standard solenoid. The currents I and I_s were measured by means of two Brooks Type deflection potentiometers, and standard resistances.

In practice the currents were read at a series of valves giving small galvanometer deflections. A plot of the deflections against the current ratios gave the balance point as the intersection of two curves. By reversing the currents in both circuits, the influence of the earth's field was eliminated. The potentiometers were interchecked, and showed a maximum departure of 0.02%. The same standard cell was used with both potentiometers.

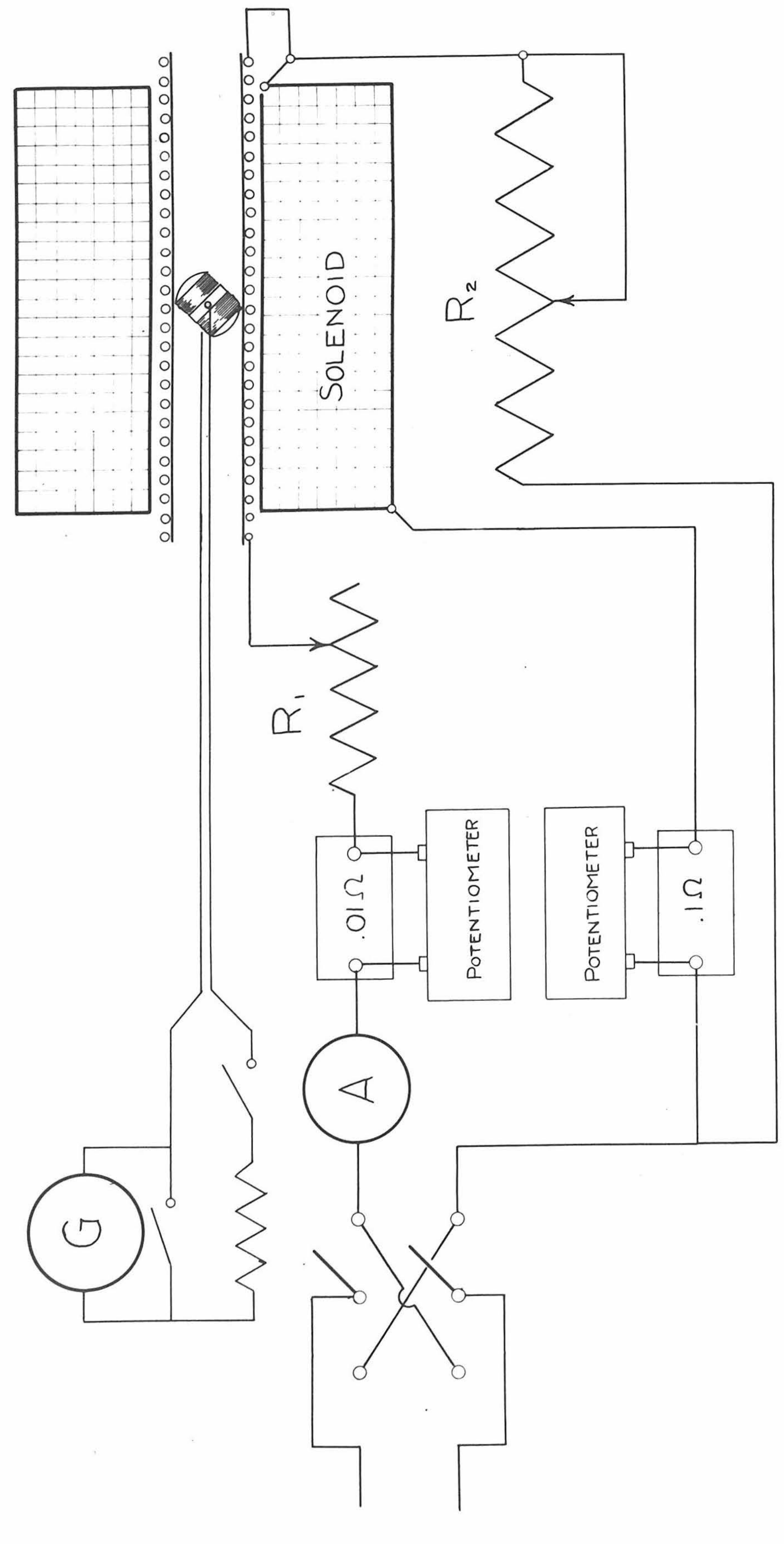


Fig. 4.

Table III gives the results of several null runs. Values obtained in August, 1930, with two other standard solenoids, also by the null method, are included because of the wide divergence between the brass standard and the bakelite standard. This serious difference appeared also in calibrations by the mutual inductance method. It constitutes the greatest uncertainty in the experiment. It is a difference far greater than the limits of error in measuring the brass and bakelite solenoids. The errors given in the table are the uncertainties in linear measurement and ~~is~~ are in a direction to be explained by ferromagnetism of the brass tube, or electrical leakage between the turns of the brass solenoid. As the turns are wound with an air gap between them, and frequent tests showed no leakage to the tube, the latter alternative is not probable. The two solenoids introduced in Table III were wound on 2 inch pyrex tubing, one with No. 12 B and S D.C.C. wire, the other with No. 18 B and S wire. They were measured carefully on a large comparator which had been calibrated against the Starrett scale.

Two points are to be noted in Table III. The first is the fact that the ratio between the main solenoid constant and the bakelite solenoid constant has changed by .037% over a period of six months, only two times the mean deviation of the 5 August 1930 values. The second point is the probability that the brass solenoid has some defect. The mean of the four values of K is 36.866. The value of K from the bakelite standard differs from the mean by .032%, while the difference for the brass standard is .23%, or seven times as great. This will be taken as a measure of the relative weights to be assigned to the two solenoids in the second calibration, to be described.

Table III NULL METHOD CALIBRATIONS

Standard Solenoid		Bakelite	Brass	Glass (No 12 wire)	Glass (No 18 wire)
K_s $(\frac{gauss}{amp})$		4.930 + .02%	13.823 + .006%	5.354 + 0.1%	9.827_8 + 0.05%
$A = \frac{I_s}{I}$	Aug. 1930	7.476 7.477 7.474 7.478 7.478 7.4766 (mean dev = .017%)		6.879	3.747_3
	March 1931	7.473_8	2.6730 2.6732 2.6731		
Constant of large sol. $K = AK_s$ (gauss/amp)		36.854	36.952	36.830	36.82_8

Calibrations with Mutual Inductance. In order to permit determinations of the solenoid constants to be made under the conditions which prevailed during exposures, the arrangement shown in Fig. 3 was adopted. The current in the primary of the mutual inductance, M, is varied until its reversal gives a deflection of the galvanometer, G, equal to that produced when the flip coil at F is operated in the field of the standard solenoid alone. The galvanometer shunt, R_4, is such that full scale deflections are secured. Since the total resistance in the galvanometer circuit is the same for both deflections we have, in effect, a calibration of the mutual inductance and the flip coil in terms of the standard solenoid constant and the ratio of the currents. Using the nomenclature

K_s = constant of std. solenoid, (gauss/amp)

I'_s = current in std. solenoid, (amperes)

$H_s = K_s I_s$ (gauss)

F = magnetic area of flip coil

M = mutual inductance

I'_m = current in inductance

we have

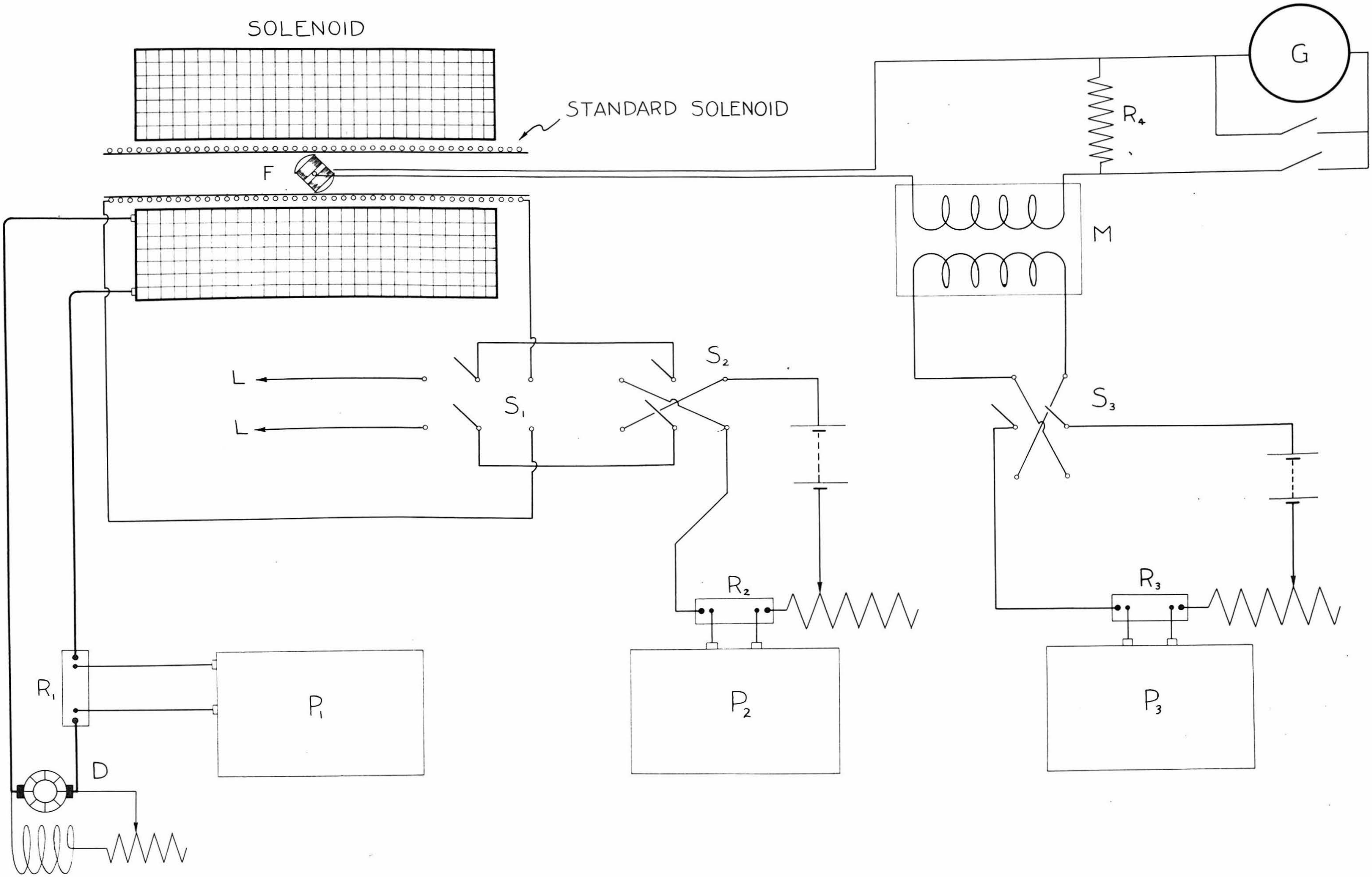
$$2 F H_s = 2 M I'_m$$

or

$$F K_s I'_s = M I'_m$$

$$M/F = K_s I'_s / I'_m \qquad (7)$$

Having calibrated the mutual inductance we can now use it to determine the constant K of the large solenoid at full current. The galvanometer shunt R_4 is increased until a suitable deflection is obtained when the flip coil is operated in the field of the main solenoid. The current in the primary of the mutual inductance is now raised until a


SOLENOID
STANDARD SOLENOID
F
G
R_4
M
S_1
S_2
S_3
L
L
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R_3
P_1
P_2
P_3
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reversal with the switch S_3 gives the same deflection. The currents I and I_m in the solenoid and mutual inductance, respectively, giving the same deflection, are read on the potentiometers P_1 and P_3. From considerations similar to those leading to Equ. (7), we obtain

$$8) \qquad K = \frac{M}{F} \cdot \frac{I_m}{I}$$

with Equ. 7) this gives

$$9) \qquad K = K_s \frac{I'_s I_m}{I'_m I}$$

Certain assumptions made in arriving at Equ. 9) must be fulfilled if its application is to lead to accurate results:

a) The galvanometer constant must not change during the calibration of the inductance, nor during the calibration of the solenoid from the inductance. The Leeds and Northrup Type P ballistic galvanometer was placed 70 feet from the solenoid, the field of which had some effect upon the galvanometer constant at shorter distances. Furthermore, very slight dependence was placed upon the proportionality of galvanometer deflection to current impulse. A series of deflections, covering a small range, was made using the flip coil. Intermingled with these readings was a second series of deflections, in the same small range, obtained by reversing the current in the primary of the mutual inductance. Each deflection in both series was then plotted against the current at which it was read. Graphical interpolation to the same galvanometer deflection in both series gave the desired current ratio (Im/I or $I'm/I's$).

b.) The method also requires that the mutual inductance have the same value when being compared with the large solenoid as it has during the comparison with the standard solenoid. As the two types of comparison were intermingled, any permanent change would have shown itself as a change in the ratio I'm/I's given by the sucessive calibration against a given standard solenoid. No such change appeared. (v. Table IV, page 23) Errors due to ferromagnetic surroundings were minimized by suspending the inductance midway between the ceiling and floor of an adjoining room. The construction of the inductance was such as to guard against changes of its value when the necessary changes in primary current were made. A low resistance primary of No. 12 D.C.C. wire was wound on a micarta tube 8 in. in diameter and 15 in. long. The secondary, consisting of 12 lbs. of No. 32 S.C.C. enameled wire wound in three separate coils, was rigidly supported within the primary.

c.) The required constancy of the flip coil is indicated by the coherence of the readings.

d.) The galvanometer shunt resistance, R_4, needed to remain constant only during each half of the calibration. Separate coils of chromel wire were prepared for each required value of resistance.

Using this method determinations of the main solenoid constant were made at solenoid currents of 200 amperes, 150 amperes, and 1 ampere. Although the comparisons between the mutual inductance and the bakelite solenoid, and those between the inductance and the large solenoid were made alternately, the results have been collected separately in tables IV and V. A series of comparisons with the brass solenoid was also made. These are averaged into the final result with the weight of 1/7 which was estimated from the null method. The three values of K given in Table V for the three different currents show no definite trend. The error in the value obtained at 200 amperes is the propagated mean deviation.

A Leeds and Northrup Type K potentiometer and two deflection potentiometers were used in these calibrations. The currents in all three potentiometers were checked against the same standard cell. Since the value of K as given in Equ. (9) depends only upon relative current values, the e.m.f. of the standard cell is immaterial. Upon being interchecked the potentiometers showed a maximum disagreement of .03%, and average deviations of the order of .001%. The shunts were calibrated by the Southern California Edison Company's testing laboratory against resistances having Bureau of Standards certificates. The total correction in e/m for the shunt errors entering both calibrations and current measurements during exposures is less than .004%

Spectroscopic Measurements

Since the solenoid field during an exposure is $H = KI_e$, where K is the solenoid constant and I_e the current, Equ. (3) becomes

$$10) \qquad e/m = \frac{4\pi c}{a\,K}\,\frac{\Delta\nu}{I_e}$$

The difference $2\Delta\nu$ between the wave numbers of the two Zeeman components is found from the difference in order of their fringe systems at the center of the interferometer pattern.

The relation for the fractional order in terms of the diameters of the fringes is

$$11) \qquad p = \frac{D_i^2}{D_i^2 - D_{i-1}^2} - i$$

TABLE IV

CALIBRATIONS OF THE MUTUAL INDUCTANCE

Bakelite Standard

I_s/I_m :	39.813
	846
	852
	854
	833
	827
	834
	842
	834
	851
	829
	840

Mean = 39.838

Mean deviation = ±.025%

$K_s = 4.930$ (± .02%)
(From Table II)

∴ $K_s I_s/I_m = 196.40 \pm .032\%$

Weight : 7

Brass Standard

I_s/I_m :	14.225
	234
	230
	223
	234
	227

Mean = 14.229

Mean dev. = ± .027%

$K_s = 13.823$ (± .006%)

∴ $K_s I_s/I_m = 196.68$

Weight : 1 (p. 17)

Difference : 0.14%

Weighted means :

$K_s I_s/I_m = 196.44$ (± .04%)

Table V

CALIBRATION OF LARGE SOLENOID

	200 amp.	150 amp	1 amp
I/I_m :	5.3282		
	365		
	331		
	305		
	289		
	262		5.3266
	282	5.3278	315
	300	322	289
	283	311	297
	299	311	285
Mean :	5.3298	5.3305	5.3290
Mean dev. :	0.034%	0.026%	0.022%
$K = \frac{K_s I'_s}{I'_m} \cdot \frac{I_m}{I}$:	36,857 (± .05%)	36.852	36.862

where D_i is the linear diameter of the i-th ring from the center of the pattern. Two approximations are made in deriving Equ. (11): tan θ has been substituted for θ, and $1 - \frac{\theta^2}{2}$ for cos θ. The errors introduced are partially self-cancelling and for our purpose quite negligible since θ, the angular diameter of the largest fringe measured, was less than .04 radian~~s~~. The error is even further reduced by the fact that we employ the difference between the p's of the two Zeeman components.

The diameters, D_i, of roughly twenty fringes of each component were measured on a comparator. The value of p for each component was then calculated as follows: From the table of values of D_i^2 a mean value of $D_i^2 - D_{i-1}^2$ was obtained. Dividing this into each D_i^2 gave, (all of which should = p) by Equ. (11), a table of i + p. The mean of the fractional parts, p, is the desired fractional order at the center. From the difference in order, p - p' between the fringes corresponding to $+\Delta\nu$ and $-\Delta\nu$ in the Zeeman pattern, the separation in cm^{-1} is given by the expression

$$11) \qquad 2\Delta\nu = \frac{p - p'}{2d}$$

in which the maximum error introduced by approximation is the reciprocal of the order of interference, or .001%. $\Delta\nu$ must be reduced to vacuum in order to give the correct result in Equ. (10). This was necessary because ~~accomplished automatically by determining~~ the interferometer plate distance d was measured in terms of the wavelengths of neon secondary standard lines, the ~~values~~ wavelengths of which are given ~~for vacuum conditions~~ in I.Å. units. The method of Lord Rayleigh[18] was followed in evaluating d, giving an accuracy of one or two parts in 1000,000. The order of interference was changed after every three or four exposures.

During the exposures the solenoid current was measured by the same shunt and potentiometer that were used in the 200 ampere calibration runs. By constant regulation of the generator field the current was maintained with average fluctuations of 0.1%. A week before the exposures were started, the standard cell was checked with a cell newly arrived from the Bureau of Standards. This and other comparisons which have been made through the kindness of Dr. Smythe indicate a constancy to within one part in ~~1.0100~~ 10186. over a period of one and a half years.

Thirty one separate reductions of nineteen interferometer patterns gave the values of $2\Delta\nu/I_e$ listed in Table VI. More than half of the patterns were independently measured and reduced by two persons, whose results showed an average absolute discrepancy of .03% and a maximum discrepancy of .13%. The mean for each pattern is weighted according to the number of times it was reduced. The values of e/m given have been calculated by Eqn. (10) using $\bar{a} = .999655$ for the Cd line and $\bar{a} = .999945$ for the Zn line. The solenoid constant, $K = 36.857$, is taken from the 200 amp. calibrations. (Table V) The following corrections have also been introduced:

+ 0.005 %, int. to abs. gauss

+ 0.004 %, total correction for shunt resistances

+ 0.020 %, for average field over the light source (Table I).

− 0.027 % " reduction of $\Delta\nu$ to vacuum.

Since the probable errors found for $\Delta\nu/I$ cover the uncertainties in both $\Delta\nu$ and I, the probable relative error in e/m is given by the square root of the sum of the squares of the relative errors in K and

TABLE VI

SPECTROSCOPIC MEASUREMENTS

Cd λ 6439

Plate		Wt.
1	.0034395	1
2	34352	2
3	34421	2
4	34391	2
5	34425	2
6	34364	2
7	34347	1
8	34411	1
12	34405	1
13	34408	2
14	34345	2
15	34366	2
17	34350	1

Mean : .00343815

Mean dev.: .08%

Prob. error : .02%

e/m = ~~1.7583~~ 1.7578 abs. e.m.u.

Prob. error : .06%

Zn λ 6362

Plate		Wt.
9	.0034387	3
11	34387	1
12	34375	2
14	34383	2
16	34379	1

Mean : .00343827

Mean dev. : .03%

Prob. error : .01%

e/m = ~~1.7581~~ 1.7576 abs. e.m.u.

Prob. error .05%

and $\Delta V/I$. The uncertainties given in Table VI are obtained in this way, and are seen to depend chiefly upon the probable error of .05% in K. Because the mean deviation between the null-method values using four standard solenoids was .11%, and because of the consequent uncertainty in assigning weights to the two standard solenoids used in the final calibration, the result of these measurements may be stated as

$$e/m = (\overset{1.7577}{\cancel{1.750_2}} \pm 0.1\%) \times 10^7 \text{ e.m.u. per gm.}$$

This value is 0.16% lower than the value 1.7606 obtained by Babcock and Houston, and is ~~.07%~~ 0.1% lower than the value 1.7595 indicated by Millikan 19) to fit with e, h, and N on the basis of the Lewis and Adams expression for the fine-structure constant.

The writer wishes to express his gratitude for the advice and energetic collaboration of Professor Houston, who suggested this work.

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Appendix I

Design of the Solenoid

The type of solenoid selected was that of a cylindrical continuous coil having a rectangular cross section. Cooling was to be accomplished by pumping kerosene directly through the coil, the layers being separated in such a way by long fibre spacers that a large number of rectangular channels extended from one end of the coil to the other. An outline will be given of the method used in selecting the dimensions of the coil and the sizes of wire and spacers to fulfil the required conditions of (a) uniformity of field, (b) maximum ratio of field strength to power consumption, and (c) cooling.

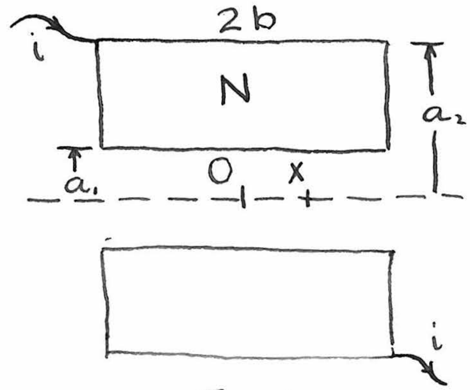

FIG. 1.

Figure I shows the section of a coil of N coaxial turns which are densely and uniformly distributed throughout a space limited by an outer cylinder of diameter $2a_2$ cm, an inner cylinder of diameter $2a_1$ cm, and two planes 2b cm apart and perpendicular to the common axis of the cylinders. In what follows it will be assumed that the internal diameter $2a_1$ is predetermined by the size of the apparatus to be accomodated in the solenoid. If a current of i amperes flows through each of the turns, the field strength at the geometric center, O, is

$$1) \qquad H_0 = \frac{2\pi Ni}{10(a_2 - a_1)} \ln \frac{a_2 + \sqrt{a_2^2 + b^2}}{a_1 + \sqrt{a_1^2 + b^2}} \text{ gauss}$$

At any axial point distant x cm from 0, the field is parallel to the axis and has the strength

$$2)\quad H_x = \frac{\pi N i}{10 b(a_2 - a_1)}\left[(b - x)\ \ln \frac{a_2 + \sqrt{a_2^2 + (b-x)^2}}{a_1 + \sqrt{a_1^2 + (b-x)^2}} + (b + x)\ \ln \frac{a_2 + \sqrt{a_2^2 + (b+x)^2}}{a_1 + \sqrt{a_1^2 + (b+x)^2}}\right]$$

H_x is an even function in x with a maximum at x = 0, for which value Equ. (2) reduces to Equ. (1). The maximum relative variation in the field over an interval on the axis extending x cm on either side of the center is then $H_x - H_o$, and condition (a) may be stated as

$$3)\qquad \frac{H_x - H_o}{H_o} \leqq V_x\ ,$$

where V_x is the permitted relative.variation over the interval of 2 x cm. We will not consider the radial variation of the field, as it is neglible compared with the longitudinal variation.

The resistance of the coil is

$$4)\qquad R = \frac{2\pi N \bar{r} \rho}{S}$$

where ρ is the specific resistance, $\bar{r}$ the mean radius of the coil, and S the cross sectional area of the wire. From the geometry of the coil this may be written as

$$5)\quad R = \frac{2\pi N \cdot \frac{a_2 + a_1}{2} \rho}{\frac{2b(a_2 - a_1)\ \lambda}{N}} = \frac{\pi N^2 \rho (a_2 + a_1)}{2b\ \lambda\ (a_2 - a_1)}$$

where λ is the space-filling factor, or fraction of the total coil-volume which is occupied by wire.

Combining Equs. (5) and (1) to eliminate N, we have

$$6)\quad \frac{H_o^2}{i^2R} = \frac{8\pi b \;\lambda}{100(a_2^2 - a_1^2)\rho} \left(\ln \frac{a_2 + \sqrt{a_2^2 + b^2}}{a_1 + \sqrt{a_1^2 + b^2}}\right)^2$$

as an expression for the efficiency. With the substitution of $\alpha = a_2/a_1$, $\beta = b/a_1$, 6) becomes

$$7)\quad H_o = \sqrt{\frac{w\lambda}{a_1\rho}}\; G$$

where $w = i^2R$, the power expended in the coil.

$$8)\quad G = 2\left[\frac{2\pi\;\beta}{\alpha^2 - 1}\right]^{1/2} \ln \frac{\alpha + \sqrt{\alpha^2 + \beta^2}}{1 + \sqrt{1 + \beta^2}}$$

is the shape-factor of efficiency, which contains the entire dependence of H_o upon the shape of the coil. The size dependence of H_o is contained in the factor $1/a_1$ of Equ. (6).

G has a maximum value of .179 at $\alpha \cong 2$, $\beta \approx 3$. A chart of the values of G for a wide range of α and β has been published by Cockcroft [1]. The values of a_2 and b which, with the adopted value of a_1, give a maximum G, subject to condition that they satisfy Equ. (3), are the desired dimensions of the coil. They are best found by trial, because of the complexity of the expressions for G and H_x.

The process may be illustrated graphically. Inspection of Equs. (1) and (2) shows that if we put $x = \xi a_1$, Equ. (3) is of the form (omitting the inequality)

$$V_\xi = f(\xi, \alpha, \beta)$$

where $\alpha = a_2/a_1$, $\beta = b/a_1$ as in Equ. (8) for G. As we have assigned

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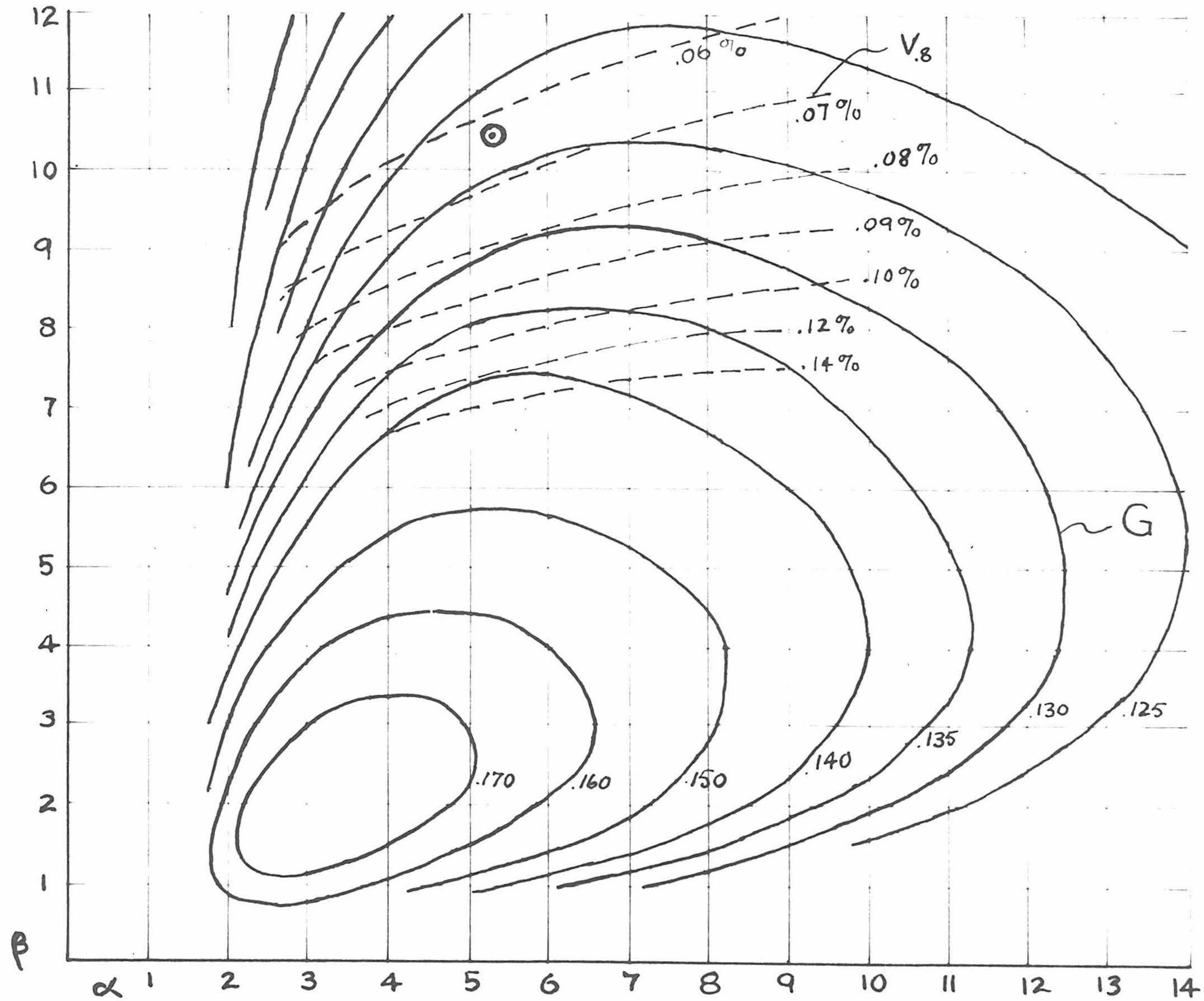

12
11
10
9
8
7
6
5
4
3
2
1
β
α
1
2
3
4
5
6
7
8
9
10
11
12
13
14
$V_{.8}$
.06%
.07%
.08%
.09%
.10%
.12%
.14%
G
.170
.160
.150
.140
.135
.130
.125

FIG 2a.

fixed values of x and a_1, ξ is a constant and V_ξ may be treated as a function of α and β alone. In Fig. 2 the contours G = const. and $V_{.8}$ = const. have been roughly plotted against α and β . The values of G are those of Cockcroft (l.c.); V is the variation for x = 3 cm, a_1 = 3.8 cm (ξ = .8). On each V contour there is a point for which G is a maximum. Its coordinates α and β give the required dimensions: a_1; $a_2 = \alpha a_1$; $b = \beta a_1$. Since neither G nor V changes rapidly on the curve $V_{.8}$ = .001, only a tentative selection of α, and β need be made, subject to alteration to fit the stock sizes of wire and tubing.

The dimensions of the coil now being roughly determined, it remains to decide on the number of turns, N, the space filling factor λ , and the quantities related to N and λ , namely the cross sections of the wire and the spacers. We will assume the wire to be square in section having a side m cm. The spacers also will be assumed square, of side d cm. We now obtain relationships which may be solved for the four quantities N, λ , m, and d.

Since the available power W and the voltage V are given, we have, by Equ. (4),

$$9) \qquad W = \frac{V^2}{R} = \frac{2V^2 b \lambda (a_2 - a_1)}{\pi N^2 \rho (a_2 + a_1)}$$

in which only N and λ are unknown. From the geometry of the coil, the cross section of the wire may be expressed as

$$10) \qquad m^2 = 2b \lambda (a_2 - a_1)/N$$

The remaining conditions depend upon the cooling requirements. The heat resulting from the dissipation of the power W is to be carried away by a total flow F cc/sec of a liquid having density ρ' and specific heat σ. If the permitted rise in the temperature of the wire is ΔT, the required flow is

11) $$F = W/J\sigma\rho'(\Delta T - \tau)$$

where J is the heat equivalent and τ is the highly uncertain temperature difference which exists between the wire and the cooling medium. If we now assume the spacers, which are d cm square, to be spaced d cm apart in each inter-layer, then each of the ducts is d cm square, and the number of ducts is approximately

12) $$n = \frac{1/2\ (\text{Total vol} - \text{vol wire})}{\text{vol of 1 duct}} = \frac{\pi a_2^2(1-\lambda)}{2d^2}$$

The flow through each duct is then

13) $$f = \frac{F}{n} = \frac{2Fd^2}{\pi a_2^2(1-\lambda)}$$

The arrangement we have made of the spacers also results in the approximate relation

14) $$\lambda = \frac{md}{d(m+d)}$$

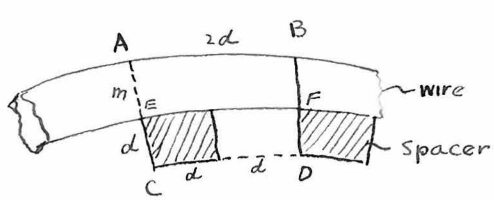

FIG. 3.

This is made clear by Fig. 3 in which the approximate rectangle ABCD is the section of a parallelopiped containing one spacer, one duct and the wire immediately above them. The entire volume of the coil may be divided into n parallelopipeds identical with the one shown in section as ABCD, which is thus representative of the entire volume. The ratio of wire volume to total volume is then that of the areas $\overline{ABEF}/\overline{ABCD}$ which leads at once to Equ. (14).

Equ. (13) for the flow may now be extended. To a first approximation the flow in a square pipe 1) d cm on a side is

15) $$f = \frac{1}{12}\frac{d^4}{\mu}\frac{dp}{dz} \quad cc/sec$$

where μ is the absolute viscosity (poise) and $\frac{dp}{dz}$ the pressure gradient. In the present instance the ducts are 2b cm long, and we may estimate the pressure p against which the circulating pump will deliver F cc/sec. Then $\frac{dp}{dz} = \frac{p}{2b}$ and Equ. 15) becomes

16) $$f = \frac{d^4 p}{24 \mu b}$$

Combining Equs. (16) and (13) results in

17) $$d^2 = \frac{24\mu Fb}{\pi p a_2^2 (1 - \lambda)}$$

We now have the four equations (9), (10), (14), and (17) which may be solved for the four quantities N, λ, m, and d. All of the other quantities are regarded as known.

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