Evaporation as a Function of Insolation,

Thesis by Burt Richardson, In Partial Fulfillment of the
Requirements for the Degree of Master of Science, California
Institute of Technology, Pasadena, California, 1930.

In the Western States water is scarce and valuable, and rates of evaporation are high. Heretofore, evaporation losses have been ascertained by direct measurements. The writer considers it possible to approach the problem by a study of the physical cause of evaporation. The principal cause is insolation which may be defined, briefly as ''exposure to the rays of the sun.''

This paper is concerned with the effect of insolation and with the measurement of the quantity of radiant energy imparted by the sun and sky to the water and the bottom of a reservoir. In a sense, this effect may be considered a measure of insolation. For example, it may be said that, in Pasadena,California, on a sunny day, the insolation is roughly 600 calories per sq. cm. per day.

Introduction

The quantitative effect of insolation is variable and depends on the sun's elevation,the length of the day, cloudiness, smoke, etc. It may be determined by three independent methods:

(1) By means of a Weather Bureau, recording, thermo-electric pyrheliometer.
(2) By computing the quantity as a geometric problem from a knowledge of the sun's radiation, altitude, etc.,together with the latitude of the reservoir or lake.
(3) By tracing the radiant heat energy which strikes the water surface of a heat-insulated tank.

Having determined the value of insolation for a particular site it is possible to show what part of it goes into evaporation and to place definite limits on the quantity of evaporation. By using either Method (1) or Method (2) in combination with Method (3) the evaporation from a lake or proposed reservoir may be determined.

Notation

The notation adopted for use in this paper is as follows:

A = correction coefficient = $\frac{\text{solar radiation}}{\text{total radiation}}$.
a = the Stefan-Boltzmann radiation constant.
B_r = the radiation from water.
b = a constant which permits the use of the air temperature above the body of water instead of the sky temperature.
C = a factor representing the conduction of heat through the walls of the tank.
C' = the percentage of cloudiness that occurred during a certain month, as given in Monthly Weather Review reports.
δ = the sun's declination for a certain day of the year.
E = quantity of water evaporated.
F = factor denoting the relative clearness of the sky above the particular body of water in question.
H = original vapor pressure of the air passing over the surface of the water.
i = angle of incidence of energy striking water surface.
I = the quantity of radiant heat, in calories per minute, that reaches a certain area on the surface of the earth.
I_{ex}= insolation incident upon the exterior of the earth's atmosphere.
K = the solar constant.
k = the ratio of the incident to the reflected intensity of radiation.
L = the latent heat of evaporation of water.
n = index of refraction of water.
P = atmospheric pressure.
P_w = original vapor pressure of the air in contact with the water surface.
Q = the observed radiant energy that passes through local clouds.
Q_s = the observed radiant energy that passes through a clear sky.
R = the ratio, convection to evaporation.
r = the angle of refraction.
S = sensible heat measured by the warming or cooling of the water.
T_a = original temperature,centigrade, of the air passing over the surface of the water.
T_w = original temperature, centigrade, of the air in contact with the water surface.
t = time in minutes.
ω = $\frac{2\pi}{24 \times 60}$ radians per min. = angular velocity of the earth.
l = latitude of the station or lake.
ρ = the transmission coefficient of the earth's atmosphere.
ρ' = the ''mean'' transmission coefficient of the atmosphere at a particular place.
α = the zenith angle of the sun.

(1) Insolation Recorded by Thermo-Electric Pyrheliometers

The Weather Bureau of the U.S.Department of Agriculture maintains six stations at which recording thermo-electric pyrheliometers (1) are used and their records of insolation are published in the Monthly Weather Review. Due to the difference in latitude, cloudiness,etc., the value of insolation is, of course, not the same for all stations. For example, the average number of calories reaching each square centimeter of the earth's surface per day during the month of July,1928, at certain places is indicated in Table 1.

Table 1. Values of Insolation at Seven Stations in the United States.

Location.	Insolation in calories per square centimeters per day.
New York City,N.Y.	378.5
Washington D.C.	540.8
Chicago,Ill.	412.8
Madison,Wis.	509.2
Lincoln,Nebr.	531.2
Twin Falls,Idaho	737.2
Scripps Institution of Oceanography,La Jolla,Calif.	544.7

The insolation varies from day to day at any one station, as illustrated by data observed at the Scripps Institution of Oceanography at La Jolla, Calif. While this is not a U.S.Weather Bureau Station, it uses the U.S.Weather Bureau Pyrheliometer,No.17 which was taken by the writer from the California Institute of Technology in June 1928. Since July 1928 the hourly values of insolation have been observed and computed by the writer and these values appear monthly in the Monthly Weather Review. Table 2, which follows, gives the observations for each day of July,1928.

Table 2. Insolation as Recorded by a U.S.Weather Bureau Pyrheliometer No.17, at the Scripps Institution of Oceanography,La Jolla, Calif.

Date,July,1928	Calories per square centimeter per day.	Date,July,1928	Calories per square centimeter per day.
1	662.2	16	600.6
2	627.1	17	604.1
3	669.1	18	579.6
4	679.1	19	632.3
5	620.6	20	481.9
6	614.0	21	534.2
7	610.3	22	432.8
8	591.3	23	353.6
9	665.9	24	275.5
10	605.1	25	238.0
11	586.5	26	517.4
12	471.9	27	592.4
13	482.9	28	489.2
14	633.6	29	474.7
15	571.4	30	410.0
		31	579.0

The total insolation for the 31 days was 16,886.3 calories, which is the equivalent of 10.35 in. of evaporation, or an average of 544.7 calories per day.

Considering that it requires 585.4 calories to evaporate 1 cc of water at 20° C., it is easily seen how much a given water surface would drop each day due to evaporation, should all the insolation go into evaporation. Actually, the ratio of observed evaporation to observed insolation is between 40 and 60 per cent.

Table 3 gives all the records of insolation that have been observed at stations in the United States. It has been compiled from various sources. Items Nos.1 to 12, inclusive, are records of six stations of the U.S.Weather Bureau where insolation is observed;(2) Items Nos.13 to 27, inclusive, are records reported from sundry other sources;(3) and the observations listed for the last three stations are those observed by the writer, using the pan method.(4)

Table 3. The Mean Monthly Quantities of Observed Insolation. Also, the Mean Monthly Insolation Computed by Equation (12) Incident upon the Exterior of the Earth's Atmosphere.

Insolation, in Calories per Sq.Cm. per Day

Station	Nature of data	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
New York City	I	128	235	262	354	374	358	402	276	264	195	114	91
	I_{ex}	373	560	650	793	964	999	999	888	700	518	377	297
Washington D.C.	I	178	242	338	419	472	489	473	420	360	279	188	143
	I_{ex}	377	577	678	820	994	999	998	894	705	543	410	326
Chicago, Ill.	I	98	151	223	328	432	335	390	380	336	167	87	78
	Iex	317	547	643	785	961	1019	1036	890	683	510	363	286
Madison, Wis.	I	165	245	315	414	450	519	505	434	334	221	139	125
	I_{ex}	303	534	629	777	947	1003	1085	891	684	524	357	279
Lincoln, Nebr.	I	208	300	374	451	475	556	570	486	392	290	203	171
	I_{ex}	324	561	650	794	965	999	1020	889	690	515	379	298
Twin Falls, Ida.	I	162	261	250	345	380	614	717	708	567	390	180	150
	I_{ex}	308	550	647	787	954	1018	1024	887	688	504	354	280
Vancouver, B.C.	I	81	147	204	283	366	357	326	304	199	131	74	53
	I_{ex}	196	342	549	792	943	1006	942	791	571	347	196	153
Victoria, B.C.	I	94	157	259	339	414	405	388	335	255	153	90	71
	I_{ex}	249	396	592	815	957	1011	954	813	618	403	253	202
St.Johns N.B.	I	114	206	270	345	397	424	422	364	284	165	108	78
	Iex	249	396	592	815	957	1011	954	813	618	4L3	254	202
Boston Mass.	I	139	223	327	402	449	477	421	376	317	235	153	120
	I_{ex}	331	477	655	848	964	1014	965	851	673	475	332	282
Eureka, Calif.	I	148	246	313	435	486	524	442	382	318	233	151	129
	Iex	331	477	655	848	964	1014	965	851	673	475	332	282
Lower Calif., Mex.	I	306	364	415	482	476	482	449	437	420	341	301	281
	I_{ex}	494	615	761	898	973	997	973	898	774	618	496	453
Pasadena, Calif.	I	302	383	427	532	568	512	582	568	490	400	365	316
	I_{ex}	430	545	704	849	960	1011	986	902	779	616	474	418
La Jolla, Calif.	I	264	316	365	433	487	504	545	438	285	291	298	275
Fort Collins, Colo.	I									370			
San Francisco, Calif.	I								564				
San Diego, Calif.	I							662					

(2). Insolation Computed as a Geometric Problem

It is possible to determine the insolation value from a knowledge of the sun's radiation, its altitude, etc., and the latitude of the reservoir or lake. A formula may be set up for insolation, as:

$$I = K \int_{t_1}^{t_2} \cos \alpha \, \rho^{\sec \alpha} \, dt \qquad (1)$$

Differentiating with respect to time to obtain the rate of insolation,

$$\frac{dI}{dt} = K \cos \alpha \, \rho^{\sec \alpha} \qquad (2)$$

The solar constant, K, as determined by Abbot,(5) is 1.93 calories per square cm. per minute. To illustrate the use of Equation (2) consider the parallel of 40°N. latitude. Use Abbot's value for the solar constant, ($K = 1.93$); for simplicity assume a transmission coefficient of $\rho = 0.7$, and insert proper values for α, the zenith angle of the sun, for each minute of each day of each month, successively. The values of I thus obtained are given in the following table.

Table 4. Computed Values of I at 40° North Latitude.

Month	Calories per sq.cm. per day (average)	Month	Calories per sq.cm. per day (average)
January	140.0	July	553.3
February	244.2	August	457.3
March	356.5	September	347.7
April	465.7	October	240.0
May	573.7	November	136.0
June	586.7	December	100.1

At this point something might be said about ρ, the transmission coefficient(6) of the earth's atmosphere used in Equations (1) and (2). The only available values of this coefficient are as shown in Table 5. These are weighted values computed from the mean solar intensity and coefficient for a particular wavelength.

Table 5. Observed Values of ρ and A.

Station	North latitude	ρ	Values of A for Zenith Angles α 0°	7.5°	25°	30°	55°	60°
Mt.Wilson, Calif.	34°-13'	0.841	0.87	0.86	0.86	0.86	0.80	0.80
Washington D.C.	38°-56'	0.689	0.87	0.85	0.84	0.84	0.81	0.80
Mt.Whitney Calif.		0.882						
Madison,Wis.	40°-51'		0.87	0.86	0.85	0.85	0.82	0.81
Lincoln,Nebr.	43°-05'		0.87	0.86	0.84	0.84	0.79	0.76

Values of the transmission coefficient for other places in the United States have not been computed so that research on this coefficient is highly desirable. The writer suggests that this coefficient may be computed directly by means of Equation (2). The left side of this equation which represents the rate of insolation is known from the pyrheliometer record by direct reading for a particular minute of the day, and the zenith angle(7) corresponding to this minute is also known, leaving the one unknown, ρ.

Total energy, including scattered sky radiation, is recorded by a pyrheliometer. Therefore, it is necessary to correct the pyrheliometer records by a coefficient, A, which represents the ratio of solar radiation to total radiation. Typical values of A for four stations(8) are given in Table 5.

The rate of insolation from the sun (not including sky radiation) equals:

$$\frac{dI'}{dt} = A\,\frac{dI}{dt} = \bar{K}\cos\alpha\;\rho^{\sec\alpha} \qquad (3)$$

and, after making the necessary transpositions:

$$\rho = \log^{-1}\left[\frac{\log\left(\dfrac{A\,\dfrac{dI}{dt}}{K\cos\alpha}\right)}{\sec\alpha}\right] \qquad (4)$$

To illustrate this method by which the transmission coefficient of the earth's atmosphere may be computed, the pyrheliometer records of fourteen days of 1926 for Pasadena, Calif., were used, and when the monthly values were plotted, a mean value of 0.751 for ρ was obtained.

The value of insolation is not constant along any parallel of latitude, but changes at points of different longitude, due to variation in the transmission coefficient, humidity, local clouds, smoke, dust, etc. Therefore, in order to use Equation (2) for points of different longitude, values of the transmission coefficient must be observed as was done by Abbot or computed by Equation (4).

(3) Insolation by Tracing Radiant Heat Energy

The third method (9) for measuring insolation is to trace the heat energy which comes to water contained in an open tank. In this case insolation (when corrected for reflection) will be equal to the summation of: (a)Evaporation; (b)convection losses, due to the circulation of air above the water; (c) sensible heat, measured by the warming of the water; (d) conduction, due to loss of heat through the walls of the tank; and (e) radiation, from water to colded air. This is simply a statement of the Law of Conservation of Energy.

The ratio,k, of the incident to the reflected intensity of radiation is given by Fresnel's equation:

$$k = \frac{1}{2}\frac{\sin^2(i-r)}{\sin^2(i+r)} - \frac{1}{2}\frac{\tan^2(i-r)}{\tan^2(i+r)} \qquad (5)$$

in which, i and r are the angles of incidence and of refraction, respectively, and are connected by the relation $n = \frac{\sin i}{\sin r}$ = index of refraction.

Let n, for water, be equal to 1.33; then r may be found for corresponding values of i and these, substituted in Equation (5) will give values of k for any angle of incidence. This is the correction factor for reflection.

Insolation is a positive quantity during the day, but at night it is equal to zero. Evaporation is always positive, while the other terms - convection, sensible heat, conduction, and radiation - may be either positive or negative. In symbols, then:

$$I = LE + LER + S + C + B_r \qquad (6)$$

in which, L is the latent heat of evaporation (585.4 at 20°C.); E is the number of cubic centimeters of water evaporated; and R is the ratio of convection to evaporation.

Bowen(10) has found that:

$$R = \frac{0.46\ (T_w - T_a)\ P}{(P_w - H)\,760} \qquad (7)$$

in which, T_a and H are the original temperature and vapor pressure of the air passing over the surface of the water; T_w and P_w are the corresponding quantities for the layer of air in contact with the water surface; P is the atmospheric pressure; S is the sensible heat measured by the warming or cooling of the water; and C represents the conduction of heat through the walls of the tank.

B_r is the radiation from water(4) and may be computed by the formula:

$$B_r = (0.906\ a\ T_w^4 - b\ a\ T_a^4) \qquad (8)$$

in which the decimal, 0.906, represents an important constant which takes care of the fact that water does not radiate as a perfectly black body; a is the Stefan - Boltzmann constant, amounting to

5.7×10^{-5} ergs per sq. cm. per sec. per degree to the fourth power; while b permits the use of air temperature in the formula in place of temperature of the sky which receives the radiant heat energy. The average value of the term , b,(which remains quite constant for a particular place) in the radiation Equation (8) has been observed at four stations to be as follows:

Station	Mean value of b.
Tank at Pasadena, California	0.815
Tank at Fort Collins,Colorado	0.760
Murray Lake, San Diego, California	0.794
Crystal Springs Lake, San Francisco,Calif.	0.757
Average (weighted)	0.790

When the temperature of the water and air is known, by using the mean value of 0.790 for b, radiation may be computed by Equation (8). However, when these temperatures were not known, it was possible, in thecase of California water surfaces, to estimate the value of radiation from the data collected at other places. The radiation from the water to the sky, in calories per square centimeter per day, has been found for five places for five places as shown in Table 6.

Table 6. Radiation from the Water to the Sky, in Calories per Square Centimeter per Day.

Location.	Radiation.
Pasadena,California	117.0
Fort Collins,Colorado	132.0
Murray Lake, San Diego,California	128.0
Crystal Lake,San Francisco,California	148.0
La Jolla, California	130.3
Average value (weighted)	131.4

From Equation (6) the exact amount of radiation may be determined for the night, since during the night is zero and all the other quantities of terms in the equation are measurable. Thus, the insolation may be determined by adding all the heat energy coming to the body of water. Table 7 indicates the order of magnitude of each term that enters into insolation when using the pan method. A similar tabulation for observations at Murray Lake, San Diego, California (July7 - July 27),gave corresponding average daily values, as follows:

Factors	Calories per square centimeter per day.
Evaporation	485.9
Sensible heat	0.8
Convection	34.4
Back radiation	128.2
Conduction	13.0
Observed insolation	662.3

Table 7. Items That Make Up Pan Insolation, As Observed At Crystal Springs Lake, San Francisco,Calif. August 10-30 1927.

Day	Evaporation	Sensible Heat	Convection	Radiation	Conduction	Pan Observed Insolation
1	389.1	0	103.2	147.1	19.3	658.7
2	232.5	-1.9	40.5	145.5	17.6	434.2
3	438.9	-28.0	51.1	141.3	12.5	615.8
4	403.0	13.0	6.7	144.5	9.6	576.8
5	351.8	83.7	26.0	148.0	14.4	623.9
6	494.2	9.4	101.4	171.4	30.6	707.0
7	355.5	- 9.3	94.5	165.0	27.6	633.3
8	326.4	- 3.6	87.7	162.5	26.2	599.2
9	395.3	33.5	66.6	158.8	20.2	674.4
10	412.2	-11.2	14.3	139.0	10.5	564.8
11	393.4	0	- 8.0	144.0	2.8	532.2
12	392.5	63.2	19.2	136.2	10.9	622.0
13	327.4	48.4	10.5	139.0	11.1	536.4
14	311.5	- 5.5	22.6	146.5	13.1	488.2
15	381.8	-13.0	15.3	145.8	13.1	543.0
16	370.2	- 9.4	29.8	141.5	11.2	543.3
17	315.0	7.5	65.4	150.2	15.1	553.2
18	243.8	-40.8	33.6	136.1	8.7	381.4
19	383.4	63.3	19.5	139.6	8.2	614.0
20	379.2	26.1	24.6	142.5	12.1	584.5
Total 20 days	7,197.1	225.4	824.5	2,944.5	294.8	11,486.3
Average per day	359.9	11.3	41.2	147.2	14.7	574.3

As a test of this method for measuring insolation, tanks of different areas and depths have been measured hourly over long periods of time with a resulting difference of less than 3 per cent. As one illustration, Table 8 will show a comparison of the results obtained from three tanks of different depths and areas and a Weather Bureau No.17 pyrheliometer over a period of five days, at the Scripps Institution of Oceanography, La Jolla, California.

Table 8. Comparison of Insolation Quantities Observed by Different Methods

Method of observation	Insolation, in calories per square centimeter per day. July9	July10	July11	July12	July13	Aver.
Pyrheliometer	662.5	614.2	583.0	468.2	476.2	560.8
Tank No.1	653.6	617.7	594.0	463.4	496.3	565.0
Tank No.2	629.0	590.3	565.2	445.9	480.7	542.2
Tank No.3	638.3	626.7	578.9	474.5	443.4	552.3

Description of Method for Measuring Evaporation

By transposing Equation (6) which is in metric units, and converting to inches, it may be written:

$$E = \frac{I - S - C - B_r}{2.54\ L\ (1 + R)} \qquad (9)$$

This expression, which gives evaporation as a function of insolation, water temperature, wet and dry bulb temperatures, and vapor and atmospheric pressures, is a true equation;but the tedious process of evaluating each term hourly (as was done in the case of data mentioned herein) would defeat the purpose of this method for practical use. As a result of research, it is possible to assign limiting values to each term of Equation (9) and by considering certain variables constant, an equation may be obtained which is simple to handle. Thus, by considering sensible heat and conduction negligible, by using 585 for the value of latent heat of evaporation, and adopting the value 0.22 for Bowen's ratio,R, Equation (9) becomes, in inches:

$$E = \frac{I - B_r}{2.54 \times 585 \times 1.22} = \frac{I - B_r}{1814} \qquad (10)$$

Thus far, no mention has been made of the effect of wind on evaporation, because in the expression for insolation given in Equation (6) this effect would be measured, calorie for calorie, in changes in sensible heat and in convection. Sensible heat as well as the value of conduction for a short period or over a yearly cycle, as determined from a study of data collected in California, is negligible. Furthermore, no serious error will result from considering R as a constant equal to 0.22. This may be seen by referring to Table 9 which gives the value of R computed for dissimilar circumstances and over a period of time sufficiently long with hourly readings.

Table 9. Comparison of Values of R Computed Under Dissimilar Circumstances.

Location	Date	Number days	Ratio, R.
Pasadena, California	June 1926	10	0.193
Fort Collins, Colorado	Sept. 1926	10	0.221
Murray Lake, San Diego, Calif.	July 1927	20	0.165
Crystal Spr. Lake, San Francisco, Calif.	Aug. 1927	20	0.280
Scripps Institution, La Jolla, Calif.	July 1928	5	0.180
Pacific Ocean, Santa Barbara, Calif.	July 1928	12	0.280
U.S. Naval Air St. San Diego Bay	Aug. 1928	31	0.190
" " " "	Sept. 1928	30	0.208
" " " "	Oct. 1928	30	0.171
" " " "	Mar. 1929	31	0.170
" " " "	Apr. 1929	30	0.152

The maximum deviation in these values of R from the mean of 0.22 is 0.06, and since evaporation equals on the average 50% of insolation, the resulting error in insolation caused by taking R equal to a constant, 0.22, instead of the largest or smallest value of R computed, would be 3%.

It is interesting to note that by using the insolation quantity as computed by Method No. 2 and by using the value of radiation, (B_r), given for any one of four places mentioned in Table 6, the evaporation computed for the fifth place was within 5% of the observed evaporation in each case.

Before taking up the practical application of Equation (10) it is well to consider Table 10, which contains good evaporation data for stations in the United States. Unfortunately these records were from tanks not of the same size. It is well also to bear in mind that the amount of evaporation from a small tank next to a lake is not the same as that from the larger body of water, but simply represents an index. The data in Table 10 were compiled from records presented in various publications.(11) Equation (10) may be solved by substituting observed values of I from sources such as Table 3 and by using values of B_r determined by using Equation (8). Table 11 gives a comparison of the observed and computed evaporation near pyrheliometer stations.

Table 11. Comparison of Observed and Computed Evaporation.

Station	Insolation by pyrheliometer in inches per year divided by 1.22	Radiation in inches per year divided by 1.22	Evaporation in inches per year.	
			Computed	Observed
Boston, Massachusetts	60.0	17.3	42.7	39.11
New York City, N.Y.	50.4	17.3	33.1	31.76
Cincinnati,Ohio	66.0	17.3	48.7	49.99
Great Lakes	49.5	17.3	32.2	27.96
Birmingham,Ala.	66.1	15.0	51.1	51.30
Mitchell, Nebraska	73.7	17.0	56.7	65.67
Boise,Idaho	78.0	17.3	60.7	77.43
Klamath,Oregon	62.7	17.3	45.4	54.40
Lake Tahoe, California	62.7	17.3	45.4	42.20

Evaporation also has been computed for the stations of Table 10 with good agreement between the computed and observed values. The results of three other computations were as follows: Roger C.Wells(12) of the U.S.Geological Survey, determined the evaporation of Chesapeake Bay to be 120.9 cm. per year, while the computed amount,using his data, was 124.3 cm. per year. McEwen(13) by one method and Grunsky by another determined the evaporation of Lake Mendota (Madison,Wis,, a pyrheliometer station)for a 5-month period, to be 41.45 and 45.50 cm. respectively, while the computed amount was 39.66 cm. The evaporation of Swiss Alps

Table 10 - Observed Evaporation at Thirty-Two Stations in the United States, in Inches per Month.

Station	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
Boston,Mass	0.90	1.20	1.80	3.10	4.61	5.86	6.78	5.49	4.09	2.95	1.63	1.20	39.61
Rochester,N.Y.	0.86	0.86	1.67	2.39	3.45	4.38	4.81	4.54	3.54	2.61	1.46	1.21	31.76
Cincinnati	1.00	1.50	2.50	4.12	5.07	6.21	7.20	7.26	5.63	3.00	1.50	1.00	45.99
Birmingham	1.50	1.50	2.25	4.45	5.91	7.28	7.36	7.34	6.00	4.00	2.25	1.50	51.34
Great Lakes	0.66	0.84	1.03	2.12	2.76	3.67	4.38	3.98	3.21	2.56	1.67	1.08	27.96
Mitchell Neb	1.75	1.75	3.00	4.50	6.25	8.05	10.95	9.39	7.44	5.59	4.00	3.00	65.67
Snake R.Ida.	2.25	2.50	4.00	7.00	11.21	12.31	15.00	13.50	11.00	8.50	5.75	3.50	96.52
Boise Ida.	2.00	2.75	4.25	6.00	7.90	9.57	10.59	12.26	9.15	5.42	5.52	2.00	77.43
Fallon Nev.	1.75	1.75	2.25	3.25	5.25	7.86	9.86	8.70	5.13	3.35	2.50	2.00	53.65
Hermiston Ore	1.25	1.25	3.00	7.28	7.89	9.54	12.04	11.07	7.35	3.88	2.00	1.50	68.05
Klamath Ore.	0.50	1.25	3.57	6.64	7.15	6.99	8.01	9.21	6.13	2.50	1.00	0.50	53.45
N.Yacima Wn.	1.75	2.50	6.25	7.91	8.36	8.90	10.74	9.41	5.51	3.15	2.00	1.50	67.88
Lake Tahoe	1.75	1.75	1.75	2.00	3.00	4.25	6.17	7.10	6.22	3.60	2.62	2.00	42.21
Salton Sea	3.41	5.09	5.95	8.75	10.50	13.00	14.03	12.19	12.08	9.24	5.96	5.25	105.45
Indio Calif.	3.18	5.08	7.50	12.05	15.84	16.11	16.34	13.78	12.37	8.91	5.17	3.00	119.33
Mecca Calif.	2.92	5.00	8.07	10.87	12.72	14.23	15.21	13.22	10.29	8.17	4.13	2.98	107.81
Brawley Calif.	3.05	5.00	8.00	10.74	13.79	13.68	14.14	11.26	10.15	6.99	4.09	2.66	103.55
Mammoth Calif.	4.24	5.67	8.99	12.02	15.52	16.75	18.00	13.73	12.16	9.49	5.26	3.70	125.53
Phoenix Ariz	4.25	4.40	5.25	7.00	9.50	12.00	12.75	12.50	11.00	8.31	6.56	4.22	97.74
Lee's F.Ariz	1.74	3.52	5.87	7.16	11.70	13.70	13.70	11.33	8.85	6.29	3.98	1.91	89.75
Roosevelt D.Ar	2.29	3.10	5.33	7.32	10.37	12.67	12.34	10.56	8.60	5.78	3.52	2.13	84.02
MesaExp.Ariz	2.78	3.68	5.74	7.78	10.27	11.17	10.55	8.29	6.39	4.73	3.36	2.62	77.36
Wilcox Ariz	3.59	4.73	7.29	10.10	11.24	12.20	10.64	9.03	8.10	6.70	4.67	3.41	91.70
Yuma Ariz	3.09	3.89	5.72	7.23	8.24	8.91	10.24	9.96	7.85	5.39	3.40	2.62	76.54
Carlsbad N.M.	4.50	4.50	5.51	7.45	10.12	11.05	12.88	12.00	9.50	7.00	5.75	4.50	94.76
N.Mex Agri C	2.87	4.50	7.41	9.37	11.10	11.91	11.15	9.79	8.09	5.93	3.63	2.52	88.27
ElButte N.M.	2.78	4.49	7.56	10.44	13.43	14.48	12.22	10.83	9.07	7.76	4.32	3.05	100.43
L.Avalon N.M.	2.34	3.26	5.49	7.49	7.87	8.70	10.13	9.60	8.57	6.51	3.17	2.42	75.55
Santa Fe N.M.	1.53	2.13	3.96	6.15	8.51	10.14	8.93	8.06	6.55	4.81	2.55	1.93	65.25
Spur Tex.	2.79	3.54	4.82	5.48	6.66	8.68	9.33	7.88	5.59	4.48	3.42	2.34	65.01
Hill RanchTex	2.47	3.43	5.29	6.01	6.81	7.95	8.97	8.72	6.36	4.96	3.18	2.44	66.59
Beeville Tex	2.69	3.26	4.26	4.79	6.21	7.35	8.14	8.14	5.89	4.72	3.06	2.17	60.68

(14)

Lakes amounted to 198.3 cm., while the computed amounted to 193.7 cm.

When the value of insolation is not known from pyrheliometer records it is possible to compute the insolation, I_{ex}, incident upon the exterior of the earth's atmosphere for any latitude, and, later, to convert this into actual Insolation, I, by means of the two coefficients, ρ' and F, the mean transmission coefficient at a particular place and the clearness factor, respectively. The relation may be expressed by the integral:

$$I_{ex} = \int_0^t 2K \cos \alpha \, dt \qquad (11)$$

But $\cos \alpha = \sin l \sin \delta + \cos l \cos \delta \cos \omega t$ so Equation (11) upon integration becomes:

$$I_{ex} = (\, 2K\, t \sin l \sin \delta \,) + \frac{2K}{\omega}(\cos l \cos \delta \sin \omega t\,) \qquad (12)$$

in which, l = the latitude of the station; δ = the sun's declination for a certain day of the year; and $\omega = \frac{2\pi}{24x\ 60}$ = earth's angular velocity. The time, t, is the number of minutes of half-day, and K, the solar constant, has a value of 1.93 calories per sq. cm. per min. The values of declination, δ, vary from +23° on June 21 to -23° on December 21 of each year. The results of applying these values in Equation (12) to obtain quantitatively the insolation striking the exterior of the earth's atmosphere are given in Table 3 in ''Computed Insolation'! Table 12 gives additional values of exterior insolation for eleven different latitudes extending from Canada to Mexico, together with the equivalent, in inches of evaporation. These values of evaporation would represent the physical maximum because they assume no clouds and a transmission coefficient of 1.00

Clearness Factors

(15)

By using Kimball's formula we have a means of estimating the

Table 12.- Evaporation as Determined by Computing the Insolation on the Exterior of the Earth's Atmosphere.

Values of l, in degrees No.Lat.	Values of I_{ex}, in Calories per Square Centimeter per Day.											
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
52	189	305	512	743	916	993	953	818	527	387	226	161
49	229	347	548	765	929	1003	965	837	562	425	270	198
48	241	359	557	770	932	1000	964	838	579	441	283	211
45	286	404	596	796	945	1008	976	861	611	478	323	255
42	322	440	623	809	943	1004	974	870	638	510	361	291
41	336	450	634	815	951	1002	976	871	647	521	376	291
37	392	505	678	844	963	1032	993	896	696	572	433	367
33	449	556	717	864	963	1003	985	908	731	619	489	424
30	487	585	735	871	960	992	977	911	799	646	519	461
29	491	600	746	875	956	990	974	913	807	651	533	476
20	618	702	714	904	947	958	851	925	856	746	642	595

Values of l, in degrees No.Lat.	Values of Evaporation,E, in Inches of Evaporation from a Water Surface Corresponding to the Above Values of I_{ex}.											
52	3.9	5.7	11.6	14.9	18.9	19.9	19.7	16.9	10.5	8.0	4.5	3.4
49	4.7	6.5	11.3	15.3	19.2	20.1	19.9	17.3	11.2	8.8	5.4	4.1
48	5.0	6.7	11.5	15.4	19.3	20.1	19.9	17.3	11.6	9.1	5.7	4.4
45	5.9	7.5	12.3	15.9	19.5	20.2	20.2	17.8	12.2	9.9	6.5	5.3
42	6.7	8.2	12.9	16.2	19.5	20.1	20.2	18.0	12.8	10.5	7.2	6.0
41	6.9	8.4	13.1	16.3	19.6	20.1	20.2	18.0	12.9	10.8	7.5	6.3
37	8.1	9.4	14.0	16.9	19.9	20.6	20.5	18.5	13.9	11.8	8.7	7.6
33	9.3	10.4	14.8	17.3	19.9	20.1	20.4	18.8	14.6	12.8	9.8	8.8
30	10.1	10.9	15.2	17.4	19.8	19.8	20.2	18.9	16.0	13.3	10.4	9.5
29	10.3	11.2	15.4	17.5	19.8	19.8	20.1	18.9	16.1	13.4	10.7	9.8
20	12.8	13.1	14.8	18.1	19.6	19.2	19.6	19.1	17.1	15.4	12.8	12.3

Table 13. - The Ratio of Observed Insolation and the Computed Insolation Striking a Horizontal Surface on the Exterior of the Earth's Atmosphere for All Stations Where Insolation Records Have Been Kept.

Station	Years observed	Values of the Ratio $\frac{I}{I_{ex}}$											
		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
New York City N.Y.	1924-29	.34	.42	.40	.45	.39	.36	.40	.31	.38	.38	.30	.31
Washington D.C.	24-29	.37	.42	.50	.51	.48	.48	.46	.47	.51	.51	.46	.44
Chicago Ill.	24-29	.31	.28	.35	.42	.45	.33	.38	.43	.49	.33	.24	.27
Madison Wis.	24-29	.55	.46	.50	.53	.48	.52	.47	.49	.49	.42	.39	.45
Lincoln Nebr.	24-29	.64	.53	.58	.57	.49	.56	.56	.55	.57	.56	.54	.57
Twin Falls, Ida.	27-28	.53	.48	.39	.44	.40	.60	.70	.80	.82	.77	.51	.54
Vancouver B.C.	1924	.41	.43	.37	.36	.39	.35	.35	.38	.35	.38	.38	.35
Victoria B.C.	''	.38	.40	.44	.42	.43	.40	.41	.41	.41	.38	.36	.35
St. Johns N.B.	''	.46	.52	.46	.42	.42	.42	.44	.45	.46	.41	.43	.39
Boston (Boat)	''	.42	.47	.50	.47	.47	.47	.44	.44	.47	.50	.46	.43
Eureka Calif.	''	.45	.52	.48	.51	.51	.52	.46	.45	.47	.49	.46	.46
Florida (Boat)	''	.43	.40	.48	.47	.47	.44	.44	.44	.42	.42	.49	.43
Lower Calif. Mex.	''	.62	.59	.55	.54	.49	.48	.46	.49	.54	.55	.61	.62
Santiago, Cuba.	''	.54	.58	.54	.53	.48	.52	.49	.49	.51	.52	.52	.52
Pasadena, Calif.	1926-27	.70	.70	.61	.63	.59	.51	.59	.63	.63	.65	.76	.75
Fort Collins, Colo.	1926									.57			
San Francisco, Calif.	1927								.65				
San Diego, Calif.	1927							.69					
La Jolla	1928-29							.56					

percentage of clear days:

$$F = \frac{Q}{Q_s'} = 0.22 = 0.78 \; (\; 1.00 - C' \;) \qquad (13)$$

in which, F is the clearness factor; Q is the observed radiant energy that passes through local clouds; Q_s, the observed radiant energy that passes through a clear sky; and C', the percentage of cloudiness that occurred during a certain month as given in Monthly Weather Review reports.

The value of F is useful inconverting the exterior insolation, I_{ex}, into surface insolation, I, and then into evaporation. The following equation is useful and is sufficiently accurate to give good results:

$$I = I_{ex} \rho' F \qquad (14)$$

Equation (14) demonstrates the combined effect of relative cloudiness and the atmospheric transmission coefficient and the product of the two terms, $\rho'F$, give the interesting ratio of the energy we receive to that which strikes the exterior or the atmosphere. Table 13 includes values of this ratio.

Conclusions

It is possible to determine the value of insolation by three distinct methods, and the results obtained by these different methods check experimentally within 5%. When it is not possible to get a ratio of evaporation to insolation from pyrheliometer records, it is necessary to compute the insolation on the exterior of the earth's atmosphere and to correct this quantity by a transmission coefficient for the earth's atmosphere and by a clearness factor (see Equation (14)). The evaporation formula (Equation (9)) given in terms of insolation and radiation checks experimentally with observed evaporation in California, and when applied to bodies of water outside California it gives satisfactory results.

List of References Cited:

(1) Kimball and Hobbs, Journal, Optical Society of America, Vol.7 Sept.1923.

(2) Monthly Weather Review, Vol.57, January 1929.

(3) Monthly Weather Review, October 1928, p.430.

(4) Richardson and Montgomery, Nat.Res, Council Bulletin 68, Apr.1929
Monthly Weather Review, July 1929, p.300

(5) and (6) Abbot, Annals, Astrophy.Observ.Smithsonian Inst.Vol.4, 1922

(7) Bowditch, ''American Practical Navigator''.

(8) Monthly Weather Review, April 1927, p.156.

(9) Cummings and Richardson, Physical Review, October 1927 p.527.

(10) Bowen, Physical Review, June 1926, p.779.

(11) Freeman, ''Regulation of the Great Lakes'', October 1926.
Monthly Weather Review, July 1927, p.320; Eng.News 1910.

(12) Wells, Journal Wash. Academy Sci. October 19, 1928.
Wells, U.S.Geol. Survey, Dept.Int.Prof.Paper 156C. Mar.14, 1929.

(13) McEwen, Tech.Series, Scripps Inst. U. of Cal. Vol.2, No.6 September 1929.

(14) Monthly Weather Review, August 1925, p.355.

(15) Monthly Weather Review, April 1927, p.156.

Note: This thesis published Am.Soc.Civil Eng.Proceedings May 1930.