An Experimental Test of Einstein's Time Transformation Equation

Thesis by

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Summary

The experiments which have been performed to test the fundamental equations of the special theory of relativity can be explained by the relativity theory or equally well by a theory having a Lorentz contraction and a universal time. The object of the work to be described is to test the Einstein time transformation equation. This is just as essential a part of the special theory of relativity as is the space transformation equation or Lorentz contraction.

The apparatus used is a Michelson interferometer with a large path difference. Light from an electrodeless discharge in mercury is made to interfere and the resulting fringes photographed. It is shown that changes in the velocity of the apparatus because of the earth's rotation combined with its total velocity should give no effect on the phase-difference of the recombining light rays on the basis of the theory of relativity and, on the theory with a fixed ether and a Lorentz contraction, should cause a shift in phase-difference of $\frac{2lv\Delta v}{\lambda c^2}$ waves where $2l$ is the total difference in the path lengths, v is the total velocity of the earth in its equatorial plane, Δv is the change in velocity, λ is the wave-length of the light used, and c is the velocity of light.

The results are in complete agreement with the relativity theory and would conflict with the other theory if the general velocity were greater than about 100 kilometers per second.

Introduction*

The special theory of relativity requires certain transformation equations for distances and time intervals for systems moving with respect to each other. Using the customary notation they are:

$$\Delta x' = \Delta x\sqrt{1-\frac{v^2}{c^2}}$$

$$\Delta y' = \Delta y$$

$$\Delta z' = \Delta z$$

$$\Delta t' = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$$

All theories agree that measurements in the Y and Z directions are not affected by the motion, hence the second and third equations are not in question. The first equation has been shown to be true by the Michelson-Morley experiment.[(1)] The last equation has not been adequately investigated. It is the object of the present work to subject it to an experimental test.

In order to know what to expect if the relativity time transformation is not valid, we shall consider a theory having a fixed ether and a Lorentz contraction. It assumes a universal time for all observers, t'= t. We shall show that it predicts a positive result for the present experiment while

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* This experiment was suggested by R. J. Kennedy (Physical Review, Volume 20, 1922). The work described here was done in collaboration with him. He is entirely responsible for the design and construction of most of the apparatus and for the measurement of the plates.

the theory of relativity requires a negative one, hence our results should decide between the two and thus test the relativity time transformation equation.

The essential part of the apparatus used in this work is an interferometer of the Michelson type having a large difference in the lengths of its paths. Changes in the phase-difference of the recombining light rays are determined by measurements of the changes in the diameters of the interference fringes.

Theory

The special theory of relativity requires that the motion of the interferometer through space produces no observable effect on it. The diameters of the fringes and, therefore, the difference in phase of the recombining light must remain unchanged. This follows from the postulate that it is impossible to measure absolute motion through space.

The theory with a fixed ether and a Lorentz contraction does not lead to such a simple result. We must compute the effect to be expected in this case. Consider a beam of light traveling down and back over some path l in the moving system. If the velocity of the moving system with respect to the ether is v, the path in the ether will be like that shown in Fig. 1. For an observer moving with the apparatus, the length of the path is: $l = \sqrt{x^2 + y^2}$ where x is the projection of l on the direction of motion and y is that perpendicular to the direction of motion.

For an observer stationary with respect to the ether, the distances are given by:

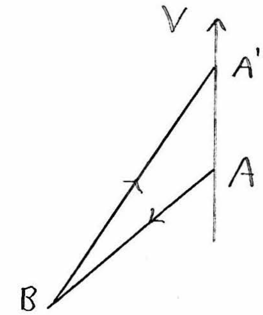

Fig 1.

$$X^0 = X\sqrt{1-\frac{v^2}{c^2}}$$

$$y^0 = Y$$

Now from the geometry of the figure, if t_1 is the time for the light to go from A to B and t_2 that required for it to go from B to A', we have:

$$c^2t_1^2 = y^2 + \left(x\sqrt{1-\frac{v^2}{c^2}} - vt_1\right)^2$$

$$c^2t_2^2 = y^2 + \left(x\sqrt{1-\frac{v^2}{c^2}} + vt_2\right)^2$$

Solving for t_1 and t_2

$$t_1 = \frac{-xv \pm c\sqrt{x^2+y^2}}{c^2\sqrt{1-\frac{v^2}{c^2}}}$$

$$t_2 = \frac{+xv \pm c\sqrt{x^2+y^2}}{c^2\sqrt{1-\frac{v^2}{c^2}}}$$

$$t_1+t_2 = \frac{2\sqrt{x^2+y^2}}{c\sqrt{1-\frac{v^2}{c^2}}} = \frac{2l}{c\sqrt{1-\frac{v^2}{c^2}}}$$

Hence, we see that the time required for the light to travel down and back over any length l is independent of the direction of l and is equal to $\frac{2l}{c\sqrt{1-\frac{v^2}{c^2}}}$

Consider an interferometer with path-lengths l_1 and l_2 and difference in path-length l. (These are the distances between the mirrors. The light traverses these distances twice, once in each direction). The difference in time required for the light to traverse the two paths is:

$$t = \frac{2l}{c\sqrt{1-\frac{v^2}{c^2}}}$$

The number of wave-lengths difference between the two paths will be:

$$n = \frac{t}{\tau} = \frac{2l}{\tau c\sqrt{1-\frac{v^2}{c^2}}} = \frac{2l}{\lambda\sqrt{1-\frac{v^2}{c^2}}}$$

where τ is the period of the light and λ is its wave-length. Expanding in powers of $\frac{v}{c}$:

$$n = \frac{2l}{\lambda}\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \text{etc.}\right)$$

Now suppose that v changes to $v+\Delta v$. Then n changes to $n+\Delta n$ where:

$$n+\Delta n = \frac{2l}{\lambda}\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{1}{2}\frac{2v\Delta v}{c^2} + \text{etc.}\right)$$

Hence:

$$\Delta n = \frac{2l v \Delta v}{\lambda c^2}$$

This change in the number of waves is equivalent to a shift in the interference fringes of exactly the same amount. Hence, when the velocity changes by an amount Δv, the fringes will be shifted Δn fringe-widths.

In the experiment, a change in velocity is furnished by the earth's orbital motion and by its rotation. It does not seem practical to use the change caused by the orbital velocity as this would require the apparatus to be kept in operation over a long period of time. The rotation, however, takes place in a day and can be used. The velocity, v, now consists of the orbital velocity combined with any general motion of the solar system through the ether. The effect to be expected from the orbital motion alone is small and has the disadvantage of always occurring at the same solar time. It would be very difficult to observe. The effect from a general motion of the solar system might be expected to be larger and has the added advantage that it would follow sidereal time and could, therefore, be separated from spurious effects which would naturally follow solar time. It is impossible to tell the exact velocity of the solar system through the ether, but we may reasonably expect it to be at least as large as the velocity with respect to the nebulae and other distant objects, several hundred kilometers per second.(2)

If we take Δv to be zero when the velocity of the observer because of the earth's rotation has no component in the direction of the general motion, and if we consider Δv to be the change in the general velocity because of the earth's rotation, we get:

$$\Delta v = \omega r \cos\varphi \sin(\omega t + \varepsilon)$$

where ω is the angular velocity of the earth, r is its radius, φ is the latitude of the observer, t the time, and ε is a phase constant.

Substituting this into our equation for Δn and putting in the values of the known constants we get:

$$\Delta n = \frac{2 l v \omega r \cos\varphi \sin(\omega t + \varepsilon)}{\lambda c^2}$$

$l = 15.0 \text{ cm}$

$\omega = 7.28 \times 10^{-5} \text{ sec}^{-1}$

$r = 6.38 \times 10^{8} \text{ cm}$

$\varphi = 34^\circ 8'$

$\lambda = 5.46 \times 10^{-5} \text{ cm}$

$c = 3.00 \times 10^{10} \frac{\text{cm}}{\text{sec}}$

$$\Delta n = 2.35 \times 10^{-11} V \sin(\omega t + \varepsilon)$$

The displacement of the fringes will follow a simple sin curve with a period of one day, an amplitude depending on the velocity through the ether, and a phase constant depending on the direction of the motion and the zero of our time scale. If v, the velocity of the earth in the plane of its equator, were 425 kilometers per second, then the amplitude of the sin curve would be 1/1000 of a fringe-width. It will be shown later that this could easily be detected.

Description of Apparatus

The arrangement of the apparatus is shown in Fig. 2. The interferometer and the optical system are mounted on a concrete bench. This is placed in a small room which can be made dark for changing plates, and which may be thermostated.

Let us follow the path of the light through the apparatus. It leaves the source, which will be described in detail later, passes through the aperture A_1, and the lens L_1, which makes it plane-parallel. Then the direct vision prism D and the lens L_2 focus the green line of wave-length 5461 Å on a second aperture A_2. (A water cell was in front of this aperture in the run of January 1930, but not in the other two). The light is again rendered plane parallel by the lens L_3. It is polarized with its electric vector parallel to the bench

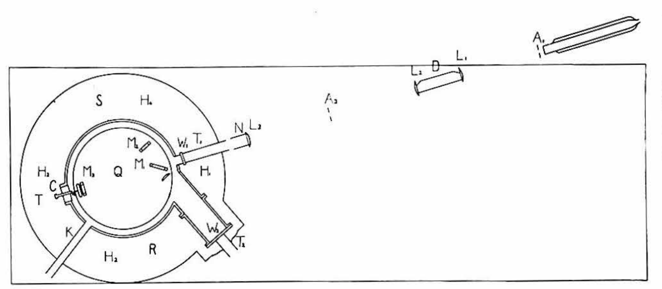

Fig. 2.

by a Nicol prism N, and enters the interferometer through the tube T_1 and the window W_1. Here it strikes the half-platinized surface of the mirror M_1 at the angle of complete polarization. Half of the light is reflected to M_2, and the remainder goes through the platinum film to M_3. After the light has recombined, it passes out through the window W_2 and tube T_2 to the photographic plate which is pressed lightly against the end of the tube. (Since the light is polarized as stated above, there will be no trouble from light reflected at the glass side of the half-platinized mirror).

The mirrors are mounted in invar supports which are ground into the heavy quartz disk Q. This rests on a felt pad which lies on the bottom of the vacuum chamber. The mirrors M_1 and M_2 are fixed, but M_3 can be adjusted by three differential screws. These may be turned from outside the vacuum chamber by three rods passing through ground joints in the cylinder C . (Only one of the three rods is shown in the diagram). The adjusting rods only touch the screws when adjustments are being made.

The vacuum line K connects the chamber to a mercury diffusion pump. A discharge tube connected to the line serves as a pressure gauge.

The interferometer was placed in a steel tub which was filled with water in order to keep the apparatus at constant temperature. A thermo-regulator was placed at R, a small electrically-driven stirrer at S, a Beckman thermometer at T, and heaters at H_1, H_2, H_3, and H_4. The thermo-regulator consisted of a pirex tube filled with toluene. A small amount of mercury was used to make contact with a piece of tungsten wire. The regulator was used to change the grid bias on a vacuum-tube in whose plate circuit was the relay which controlled the heating current. In this way practically no current was broken at the regulator contact, so the mercury did not become contaminated. The heaters were four 25 watt light bulbs connected in series-parallel. They could be operated either from the 115 volt a. c. supply or from storage cells.

The light source caused some difficulty. The requirements are: very homogeneous light of extremely constant frequency and reasonably great and constant intensity. During the first part of the work, an ordinary water-cooled mercury arc was used. This gave good interference, but the fringes were not steady. It appeared that the wave-length of the light was varying, perhaps because of Doppler shifts caused by the motion of the emitting particles. The type of source finally adopted was an electrodeless discharge in mercury. The contrast of the fringes is not quite so good here, but they are much steadier.

The form of the tube employed is shown in Fig. 3. The main tube M is surrounded by a jacket J, filled with carbon tetrachloride. This is connected to the atmosphere by the vertical tube T which is cooled by a water jacket J_2, thus preventing the liquid from escaping. When the lamp is in operation, the carbon tetrachloride is boiling gently, keeping the temperature of the tube at 76° C. The back end of the main tube is connected to the mercury reservoir R, and to the pumps through the capillary C. The oil bath B which surrounds the mercury reservoir is kept at a constant temperature of about 60° C by a small mercury thermo-regulator and an electric heater. This keeps the vapor pressure of the mercury in the lamp constant. The front end of the lamp is heated with a Bunsen burner to prevent the mercury from condensing there and to drive off a coating which otherwise forms. The inductance coil of the high-frequency oscillator is wound around the main tube outside of the jacket. This type of source is quite intense and gives good interference; however the frequency and intensity do not seem to be quite constant. This will be discussed in the section on systematic errors.

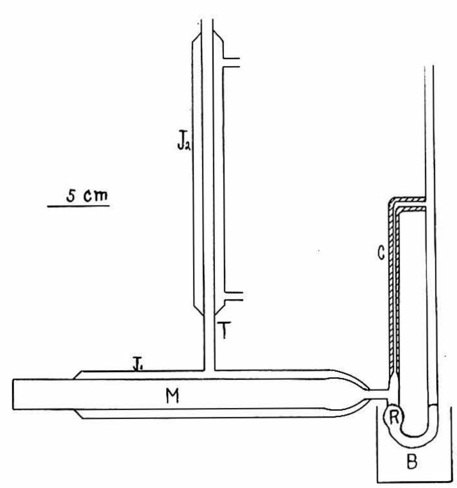

Fig. 3.

The circuit of the oscillator which furnished the power to drive the electrodeless discharge is shown in Fig. 4. This arrangement gave a wave-length of 20 meters.

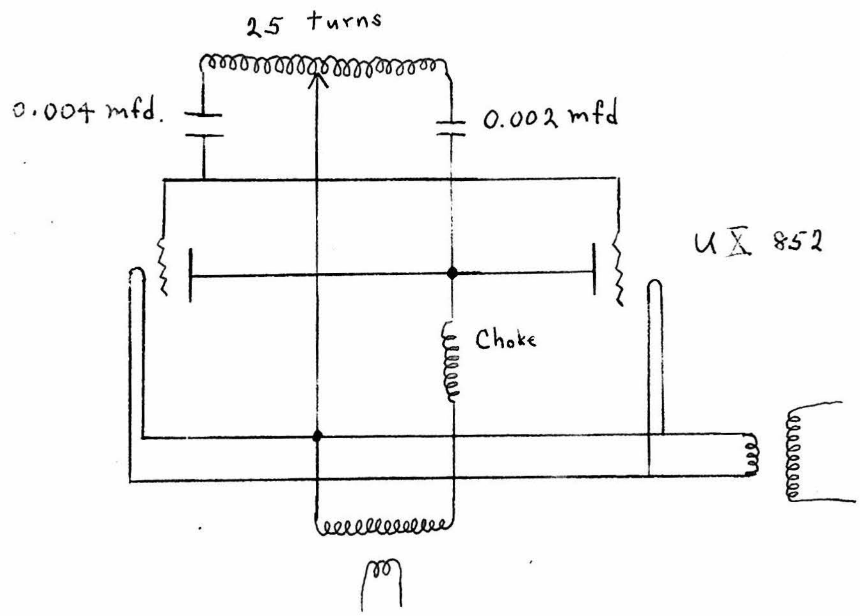

Fig. 4.

The filament transformer was supplied from the 115 volt a. c. line. A (UV-886) ballast tube was in series with it to smooth out voltage fluctuations. The plate transformer was connected to the 230 volt line. Here, two ballast tubes in parallel were used in series with the transformer. The plate transformer ratio was 10 to 1.

The small room in which the interferometer was housed was thermostated at all times. The temperature and barometric pressure here were recorded. During the run in October 1929, the outside room was kept at roughly constant temperature, and during the run in January 1930 it was quite carefully thermostated.

The photographs of the interference fringes were at first measured on a recording microphotometer. This gave satisfactory results but was very tedious. In order to read the plates more quickly, Dr. Kennedy devised a comparator which is shown schematically in Fig. 5. This was used in all of the work described here.

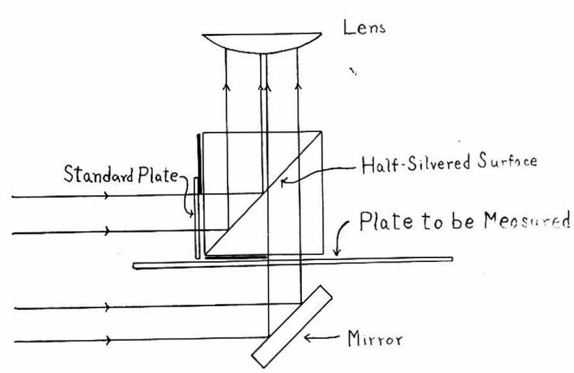

Fig. 5.

Two 45° prisms, one of which has been half-silvered on the diagonal face, are cemented together and mounted so that they move perpendicular to the paper in the diagram. A standard plate is clamped to them while the plate to be measured is stationary. The observer sees a set of fringes, one half of which is from the standard plate and one half from the other. By means of a micrometer, he moves the prisms until one fringe of the two systems coincides. Then by setting on the same fringe on the opposite side of the center, he can measure the difference in the diameters of a standard fringe and of the one to be measured. In practice, settings are taken on nine fringes of a photograph and the results averaged by an automatic device.

Accidental Errors

The uncertainty of a measured quantity caused by purely random errors can easily be computed from the theory of probability. It may be made small by taking a large number of observations. Hence, random errors are serious only in that they make the work tedious. We are not very much concerned with them because they have already been made small compared with the systematic ones. If the systematic errors can be eliminated, the random ones will limit the precision of the experiment.

Systematic Errors

The effect to be observed is a small change in the diameters of the fringes having a period of one day. Any other factor which produces such a daily shift will introduce a spurious result. It is errors of this sort which make the experiment particularly difficult. We shall first consider errors coming from the source, then from the interferometer, and finally from the measurement of the plates.

A change in the intensity of the source will naturally change the density of the photograph. This in turn changes the diameters of the fringes as will be shown later. Hence the intensity of the source must have no daily period. Now the intensity may depend on the lamp temperature, the pressure of the mercury vapor in the lamp, or the power supplied by the oscillator. The lamp temperature depends on the atmospheric pressure, as this determines the boiling point of the carbon tetrachloride surrounding the lamp. Therefore, one might expect an error caused by changes in the atmospheric pressure. There does not seem to be an appreciable one.

The pressure of the mercury vapor in the lamp depends on the temperature of the little oil bath surrounding it, hence changes in this might affect the intensity. Tests do not show an appreciable effect of this sort.

The final and important factor determining the intensity of the source is the power supplied by the oscillator. When the plate or filament voltages are changed, the density of the photograph does change. Since the line voltage has a daily period, this will introduce a serious error. The correction which must be made for this will be discussed later.

A change in the wave-length of the light will obviously cause a spurious effect. In fact, a change of 10^{-5} Å will cause a shift of about 1/1000 of a fringe-width. This might be caused by a Doppler effect if the emitting particles moved in the line of sight with a velocity of 50 cm. per second. Such an effect would probably depend on the line voltage if it were present.

There might be a Zeeman effect from changes in the intensities of the magnetic fields present. Careful tests showed that the fields present were not large enough to cause an appreciable error in the results.

There might be a reversal of the lines depending on the temperature of the front end of the tube. No such effect was found.

There might be Stark effects. In fact, the displacements do seem to depend on the oscillator voltage more than can be explained by the changes produced in the densities of the plates. The exact reason is not clear, but the fact is reasonably certain. This is serious as the oscillator voltage has a period of one day. The data must be corrected for it.

There are a number of things about the interferometer which might cause trouble. The optical system might become slightly moved by the expansion of its parts. This would change the angle at which the light enters the interferometer and therefore produce a change in the diameters of the fringes. A computation based on the known variation of the temperature shows the effect to be negligible.

A change in the distance between the mirrors of 3×10^{-8} cm. corresponds to a shift of about 1/1000 fringe width. This might be caused by a change in the bath temperature. Fortunately a rough determination of the coefficient of expansion of the apparatus shows this to be negligible.

The length of the optical path depends upon the index of refraction of the air in the interferometer. Since the index of refraction of a gas depends upon its pressure, this must be kept constant. If it varies by 5×10^{-3} mm. of mercury, the fringes should shift 1/1000 of a fringe-width. In practice the pressure is such that there is no discharge visible in the pressure gauge. This indicates that the pressure is always below 10^{-3} mm. of mercury. It is hard to see how there could be a daily change greater than a small fraction of this.

Some other sources of errors which have been investigated are: changes in the camera magnification by expansion of its tube, the effect of the photographic plates not being plane, and the warping of the interferometer by stresses set up in the vacuum line. The first two are clearly negligible. The third is made as small as possible by keeping the room temperature constant and by avoiding all jarring.

To sum up, there does not seem to be anything about the interferometer which might cause an appreciable daily variation in the diameters of the fringes.

The errors which are introduced in the measurements of the plates are mainly accidental. However, there is a systematic one depending on the density, which has caused considerable trouble. In the first place, the plates become heated and expanded by the passage of light through them. This has been shown by measuring a plate when it has first been put in the comparator and again after it has had time to warm up. Because of this effect, dark exposures will expand more than light ones and therefore give readings which are too large. A second factor may also come in. Since any one fringe is not symmetrical, an increase in density may throw the center of gravity toward the side where the intensity falls off more slowly. Because of this, one would expect dense plates to be too small. Plates taken to test this gave widely discordant results, perhaps because the importance of the first effect was not appreciated at the time.

To sum up the whole matter of systematic errors, we may say that the voltage on the oscillator transformers and the density of the plates are the things for which corrections must be applied. They are not independent as the density depends on the voltage, but one correction cannot be applied for them both as the density is also affected by the sensitivity of the plates, and this seems to vary quite badly.

The voltage correction was found by actually changing it by a large amount, and noting the resulting change in the diameters of the fringes. This was not done very carefully, as a thorough investigation would take considerable time and we hope to eliminate the difficulty. However, the approximate value of the correction is known. (See appendix).

The correction for density was made as follows. Since the plates are always denser at the ends and since there is no reason why the displacements should have a period of this sort except for the changes in density (unless there is some systematic creepage of the emulsion on the glass plate, which no one has observed)[3], we assume that any difference in the average displacement between the center and the ends of the plate is caused by the change in density. The densities of all of the exposures were measured for the runs of October 1929 and January 1930, so the relation between density and displacement could be found. (See appendix). The results for the two runs were in close agreement. They were averaged and used for all three runs.

Method of Taking the Data

The interference fringes were photographed on astronomical green-sensitive plates made by the research laboratory of the Eastman Kodak Company. The exposure time was one half of an hour, so that 48 photographs were taken each day, twelve on a plate. An automatic camera shifted the plates at the required times. If the external conditions remained absolutely constant, this would give all the required data, but of course many things which might affect the diameters of the fringes varied, so these were measured and recorded. (The data taken for the run of January 1930 are shown in the appendix).

Method of Computation

Some days of a run are more self-consistent than others. These seem to deserve more weight. Hence, the data are weighted by days, the weights being determined by the deviations of the data of one day from its average. The deviations should strictly be taken from a prelimary sin curve, but this is laborious and unnecessary. The weights are inversely proportional to the squares of the deviations. The weighted measurements of exposures taken at the same times of day are averaged for all the days of a run. The problem then is to find the most probable sin curve through these data. We see by the equation developed in the theory that the displacement should be given by an equation of the form:

$$u = a \sin(\theta - \varepsilon) \tag{1}$$

where u is the displacement. In practice we do not measure the displacement from an average plate but from one chosen at random, hence a constant must be added. This gives for the complete equation:

$$u = a \sin(\theta - \varepsilon) + k \tag{2}$$

If u_i is any measured displacement, we must have by the principle of least squares:

$$\sum_0^{2\pi} (u - u_i)^2 = \sum_0^{2\pi} \left(a \sin(\theta_i - \varepsilon) + k - u_i\right)^2 \tag{3}$$

a minimum. This requires that:

$$\sum_0^{2\pi} 2\left[a \sin(\theta_i - \varepsilon) + k - u_i\right] \sin(\theta_i - \varepsilon) = 0 \tag{4}$$

and

$$\sum_0^{2\pi} 2\left[a \sin(\theta_i - \varepsilon) + k - u_i\right] a \cos(\theta_i - \varepsilon) = 0 \tag{5}$$

From (4)

$$a = \frac{\sum_0^{2\pi} (u_i - k) \sin(\theta_i - \varepsilon)}{\sum_0^{2\pi} \sin^2(\theta_i - \varepsilon)} \tag{6}$$

Now

$$\sum_{0}^{2\pi} \sin^2(\theta_i - \varepsilon) \approx \frac{n}{2\pi} \int_{0}^{2\pi} \sin^2(\theta_i - \varepsilon)\, d\theta_i = \frac{n}{2} \tag{7}$$

where n is the number of observations per day.

$$\therefore\ a = \frac{2}{n} \sum_{0}^{2\pi} (u_i - k) \sin(\theta_i - \varepsilon) \tag{8}$$

$$a = \frac{2}{n} \sum_{0} u_i \sin(\theta_i - \varepsilon) \tag{9}$$

Expanding (5)

$$a \sum_{0}^{2\pi} \sin(\theta_i - \varepsilon) \cos(\theta_i - \varepsilon) = \sum_{0}^{2\pi} u_i \cos(\theta_i - \varepsilon) - k \sum_{0}^{2\pi} \cos(\theta_i - \varepsilon)$$

Now

$$\sum_{0}^{2\pi} \cos(\theta_i - \varepsilon) = 0 \tag{10}$$

$$\sum_{0}^{2\pi} \sin(\theta_i - \varepsilon) \cos(\theta_i - \varepsilon) = 0 \tag{11}$$

$$\therefore\ \sum_{0}^{2\pi} u_i \cos(\theta_i - \varepsilon) = 0 \tag{12}$$

$$\tan \varepsilon = - \frac{\sum_{0}^{2\pi} u_i \cos \theta_i}{\sum_{0}^{2\pi} u_i \sin \theta_i} \tag{13}$$

There is a correction which must be applied to this result. It arises from the fact that the interferometer mirrors are gradually changing their positions thus causing a drift in the displacements. This introduces a spurious effect which must be taken out.

We seek the amplitude and phase of the first harmonic of

$$f(\theta) = b\,\theta$$

where b is the increase in the displacement per radian. By the usual Fourier analysis one gets:

$$a = 2b$$

$$\epsilon = \pi$$

where a is the spurious amplitude and ϵ is the phase constant.

Combining this with the previous result, we get the complete effect. The effect of the drift may be included in the first solution and the same result obtained, but the method followed here shows more clearly the exact effect of the drift of the mirrors.

Results

Three runs were taken in which the conditions were considered good enough to warrant measuring the plates. The first one, from April 19 to May 12, 1929, contained 19 good days out of the 24. In the second, from October 8 to 22, 1929, 14 days out of the 15 were good. The last one, from January 6 to 13, 1930, contained 7 good days and 1 that had to be discarded. The conditions of the experiment have been gradually improved so that the later runs are about as reliable as the early ones, in spite of the smaller amount of data taken.

The results are tabulated below.

April 1929

	a		ε
Displacement	3.58×10^{-4}	fringe-widths	$71.^{\circ}$
Correction for Slope	1.74		$180.^{\circ}$
Correction for Density	1.56		$87.^{\circ}$
Correction for Voltage	0.38		$90.^{\circ}$
Correction for Room Temp.	4.05		$-57.^{\circ}$

The final amplitude obtained from these is 2.7×10^{-4} fringe-widths. The displacement is a maximum at 10 hrs. 20 min. civil time.

The correction for room temperature comes from the fact that the outer room was not thermostated during this run. The ballast tubes used in the oscillator circuit are quite sensitive to temperature changes, hence the room temperature affects the voltage on the oscillator transformers, and thus the size of the fringes.

October 1929

	a		ε
Displacement	3.47×10^{-4}	fringe-widths	$-28.^{\circ}$
Correction for Slope	0.44		$180.^{\circ}$
Correction for Density	1.57		$121.^{\circ}$
Correction for Voltage	0.42		$-34.^{\circ}$

The final amplitude is 2.2×10^{-4} fringe-widths. The displacement is a maximum at 17 hrs. 20 min. civil time.

January 1930

	a	ϵ
Displacement	2.79×10^{-4} fringe-widths	$9.^\circ$
Correction for Slope	0.80	$180.^\circ$
Correction for Density	4.34	$108.^\circ$
Correction for Voltage	2.07	$145.^\circ$

The final amplitude is 4.9×10^{-4} fringe-widths. The displacement is a maximum at 13 hrs. 40 min. civil time.

In order to give an idea of the consistency of the data, the points from which the displacements were computed are shown in Fig. 6. They appear very rough, but when one considers the scale employed, this is not surprising. The probable error for the uncorrected displacement curve is about

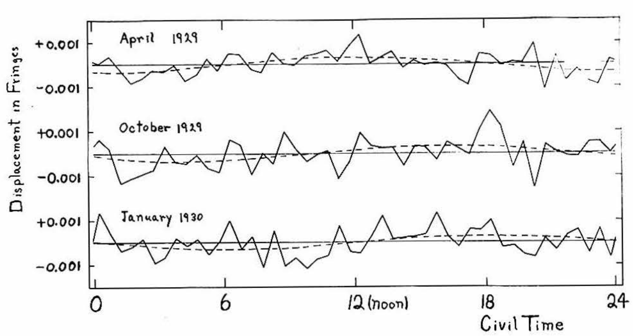

Fig. 6.

0.8×10^{-4} fringe-widths in amplitude and 2 hours in phase. This was computed for the April data before they were weighted, but it will be roughly right for all of the runs. As has been mentioned in discussing the errors, this is not a serious difficulty at present. The real trouble is in the corrections to be applied. Some of these are of the same size as the displacements, and they are not at all well-known. Because of this, we estimate that the uncertainty in the amplitude is about 2.5×10^{-4} fringe-widths. This corresponds to a velocity of about 100 kilometers per second. The data show that there is not an effect following sidereal time with an amplitude as large as this. Hence we conclude that, within the experimental error, the motion of the apparatus through space produces no observable effect. The limit set by our experimental error is still high, but if one considers that the velocity of the solar system might reasonably be expected to be several hundred kilometers per second, or more, it is evident that these results support the Einstein time transformation equation and form a real addition to the experimental evidence for the theory of relativity.

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Bibliography

1. D. C. Miller, Science, Volume 63, 1926.

   R. J. Kennedy, Proceedings of the National Academy of Sciences, Volume 12, 1926.

   K. K. Illingworth, Physical Review, Volume 30, 1927.

   A. A. Michelson, F. G. Pease, and F. Pearson, Journal of the Optical Society of America and Review of Scientific Instruments, Volume 18, 1929.

2. G. Stromberg, Astrophysical Journal, Volume LXI, 1925.

3. F. E. Ross, Monograph on the Theory of Photography from the Eastman Kodak Company, No. 5.

Appendix

Plate Transformer Voltage Correction

Exposure	Transformer Voltage		Displacements x 10^{-4} in. fringe-widths	
	Plate	Filament	First Reading	Second Reading
1	183.6	58.5	32	2
	182.7	58.5		
2	183.7	58.5	22	11
	183.7	58.5		
3	183.8	58.5	-23	-19
	183.5	58.7		
4	178.0	58.5	-09	-17
	177.6	58.7		
5	177.8	58.4	-07	-18
	177.5	58.5		
6	177.9	58.7	03	-34
	177.4	58.8		
	177.4	58.5		
7	182.5	58.7	39	32
	183.5	58.7		
8	183.7	58.6	24	-12
	183.5	58.6		
9	183.5	58.6	32	15
	183.5	58.6		

Averaging exposures 1,2,3,7,8, and 9 we get:

Plate Transformer Voltage	183.4
Filament Transformer Voltage	58.6
Displacement	13

Averaging exposures 4,5, and 6 we get:

Plate Transformer Voltage	177.6
Filament Transformer Voltage	58.6
Displacement	-14

An increase of 1 volt on the plate transformer causes an increase of 4.7×10^{-4} fringe-widths. This is the basis for the correction factor used.

Density Correction

From the data given in the following pages, we obtained the averages of the densities and the displacements for exposures in different positions on the plates. These are the results:

	Density in m.m. of wedge	Displacement in fringe-widths
End exposure	51.0	20.9×10^{-4}
Second exposure	45.4	17.6
Third exposure	40.9	16.8
Fourth exposure	37.9	16.0
Fifth exposure	36.8	15.1
Middle exposure	35.4	16.2

These are plotted against each other and a straight line drawn through the points. An increase of 1.0 m.m. of wedge corresponds to an increase in displacement of about 0.4×10^{-4} fringe-widths.

This and similar date from the October run forms the basis for the density correction.

Data of January 1930

Outer Room Temperature (In degrees Centigrade)

Time	Jan. 6	7	8	9	10	11	12	13	14
12:30	26.7	26.7	26.8	26.9	26.8	26.8	26.6	26.8	26.7
3:30	---	---	---	---	.8	---	.6	.4	.8
6:30	.7	.7	.9	.8	.9	.9	.8	.5	.7
9:30	---	.6	.7	.7	.8	---	.6	.8	.8
12:30(noon)	.7	.7	.7	.8	.8	.7	.8	.7	.7
3:30	---	.8	.7	.6	.7	.7	.8	.8	.7
6:30	.7	.7	.6	.7	.7	.6	.9	.8	.7
9:30	---	.8	---	.7	.8	---	.8	.7	---

Inner Room Temperature (In degrees Centigrade)

Time	Jan. 6	7	8	9	10	11	12	13	14
12:30	26.8	26.8	26.9	27.0	26.9	26.8	26.9	27.0	26.8
3:30	---	---	---	---	6.9	---	6.8	6.9	6.9
6:30	6.8	6.8	6.9	6.9	6.9	6.9	6.9	6.9	6.9
9:30	---	6.9	6.9	6.9	6.9	---	---	6.9	6.9
12:30(noon)	6.9	6.9	6.9	6.9	6.9	6.9	6.9	6.9	6.9
3:30	---	6.9	6.8	6.9	6.9	6.9	6.9	6.9	6.9
6:30	6.9	6.9	6.9	6.9	6.9	6.9	6.9	7.0	6.9
9:30	---	6.9	---	6.9	6.3	---	6.8	6.9	---

Interferometer Bath Temperature

The left hand figures give the temperature at which the bath heaters went on and the right hand ones the temperature at which they were shut off. All temperatures are in $\frac{1}{1000}$ of a degree Centigrade.

Time	Jan. 6	7	8	9	10	11	12	13	14
12:30	- -	- -	6 6	7 7	4 6	6 6	7 8	6 7	7 7
3:30	- -	- -	- -	- -	6 6	- -	7 7	7 -	- -
6:30	- -	- -	6 6	7 7	6 6	6 6	7 7	7 7	6 7
9:30	- -	6 -	- -	7 -	6 6	- -	- -	- -	- 7
12:30(noon)	- -	6 6	6 7	7 7	6 6	5 6	7 7	6 8	6 6
3:30	- -	- -	6 -	6 6	6 6	5 6	6 7	5 7	- 5
6:30	- -	6 6	6 8	6 6	4 6	6 7	6 7	6 8	5 7
9:30	- -	6 -	- -	6 -	6 -	- -	7 8	6 8	- -

Lamp Bath Temperature (In degrees Centigrade)

Time	Jan. 6	7	8	9	10	11	12	13	14
12:30	--	--	60.4	60.0	60.0	60.1	60.2	60.0	60.0
3:30	--	--	--	--	--	--	--	--	--
6:30	--	--	.5	.2	.3	.3	.0	.2	.0
9:30	--	.4	--	--	--	--	--	--	--
12:30(noon)	--	.4	--	.3	.4	.2	--	.3	.1
3:30	--	--	--	--	--	--	--	--	--
6:30	--	.4	.0	--	.5	.0	.2	.4	.3
9:30	--	--	--	--	--	--	--	--	--

Atmospheric Pressure (In inches of mercury)

Time	Jan. 6	7	8	9	10	11	12	13	14
12:30	29.--	--	.31	.40	.12	.16	.40	.30	.33
3:30	--	--	--	--	.12	--	.38	.21	.30
6:30	--	--	.37	.39	.13	.04	.40	.32	.28
9:30	--	.22	.43	.40	.16	--	--	.36	.31
12:30(noon)	--	.20	.40	.32	.11	.12	.36	.36	.28
3:30	--	.18	.39	.22	.10	.13	.26	.34	.28
6:30	--	.22	.41	.18	.13	.22	.24	.34	.28
9:30	--	.28	--	.12	.18	--	.26	.34	.28

In the next table, the figures in () are not actual readings but the means of two exposures before and after that one. The densities and voltages correspond to the exposures used.

Displacements (In 10^{-4} fringe-widths)

Time	Jan. 6	7	8	9	10	11	12	13
12:30(noon)	(18)	--	40	19	66	10	-12	-10
	44	--	21	44	48	13	7	28
	-14	--	29	46	21	5	21	25
	19	--	36	28	18	32	-11	47
	24	--	42	50	30	11	-22	16
	-5	--	21	53	24	29	6	22
	17	--	49	48	65	1	11	22
	32	--	30	15	37	30	-3	-19
	3	--	57	9	5	21	14	0
	43	--	(30)	32	21	24	5	-9
	26	--	(30)	53	20	25	1	(-13)
	17	--	(30)	36	53	41	6	(-14)
6:30	10	--	(30)	27	25	21	0	-31
	(10)	--	19	(33)	34	12	0	-14
	(10)	--	14	(33)	8	26	-4	-20
	-7	--	17	27	10	8	10	12
	21	--	25	43	16	32	-13	36
	11	--	-5	30	21	8	11	15
	-9	--	44	42	39	22	0	10
	23	--	43	27	43	12	-3	29
	21	--	17	39	22	12	-31	38
	9	--	39	46	44	6	-1	57
	18	--	36	(44)	19	-4	-29	-10
	41	--	19	(44)	47	6	23	24

Displacements (In 10^{-4} fringe-widths)

Time	Jan. 7	8	9	10	11	12	13	14
12:30 (midnight)	17	--	45	(44)	40	-6	3	-13
	-22	--	32	(44)	3	(-8)	24	33
	17	--	44	26	29	(-8)	-7	19
	8	--	52	65	30	-20	-10	18
	28	--	8	37	20	-13	-26	3
	26	--	37	44	12	1	-28	-34
	40	--	18	17	33	-13	16	13
	41	--	49	21	18	20	-30	5
	26	--	62	38	14	5	-9	-9
	52	--	16	0	37	-8	-20	12
	36	--	26	26	10	28	-13	27
	47	--	(24)	56	40	21	-19	45
6:30	(32)	--	(24)	32	(26)	-31	-4	9
	(32)	--	38	37	33	19	-8	-48
	16	--	17	39	22	-30	-10	-58
	28	--	8	61	45	-4	3	-10
	27	--	15	20	21	-11	-13	-41
	25	--	51	36	13	-11	-27	-10
	11	--	10	13	6	12	-8	-18
	24	--	17	51	(13)	-12	-18	-20
	16	--	58	10	(13)	-1	-8	-2
	46	--	50	36	(13)	24	3	(-10)
	28	--	(43)	39	(13)	-16	-12	(-10)
	38	--	(43)	33	22	-20	-22	-8

Density (In m.m. of wedge)

Time	Jan. 6	7	8	9	10	11	12	13
12:30 (noon)	64	--	120	112	35	74	11	16
	69	--	114	80	29	62	14	24
	68	--	105	52	24	52	9	21
	55	--	103	44	35	54	10	27
	60	--	103	44	28	46	9	24
	54	--	100	50	36	50	8	32
	60	--	91	45	34	45	8	30
	50	--	92	52	47	47	11	36
	55	--	87	47	37	44	11	38
	46	--	55	56	55	44	21	46
	66	--	55	54	55	34	26	38
	43	--	55	79	70	35	35	38
6:30	101	--	55	157	103	39	31	33
	78	--	104	95	73	24	33	35
	65	--	72	95	34	23	28	30
	39	--	62	96	32	30	28	37
	36	--	50	50	22	31	21	32
	32	--	52	31	28	32	22	35
	35	--	45	27	18	29	18	31
	23	--	45	18	18	36	13	36
	30	--	40	23	12	31	10	24
	22	--	38	21	16	35	16	26
	34	--	33	47	14	35	14	21
	27	--	36	47	12	40	19	22

Density (In m.m. of wedge)

Time	Jan. 7	8	9	10	11	12	13	14
12:30 (midnight)	23	--	44	47	12	43	21	51
	36	--	53	47	19	35	25	49
	31	--	52	103	19	35	24	37
	31	--	52	40	25	30	26	36
	34	--	43	27	27	25	23	25
	28	--	45	23	30	31	24	27
	29	--	42	13	26	30	19	17
	23	--	43	20	30	38	23	21
	28	--	36	15	34	31	15	16
	22	--	48	24	47	31	18	28
	36	--	66	27	70	43	12	19
	59	--	64	44	117	56	14	27
6:30	64	--	64	22	56	11	66	53
	64	--	83	30	16	17	62	52
	82	--	58	32	22	12	49	44
	76	--	46	40	25	22	48	43
	71	--	36	39	24	18	42	47
	69	--	42	41	31	27	44	48
	78	--	37	40	26	23	36	43
	67	--	39	45	44	30	42	45
	78	--	32	46	44	34	36	39
	73	--	46	54	44	37	36	40
	74	--	67	51	44	41	36	40
	84	--	67	67	43	51	21	41

Plate Transformer Voltage (From recording voltmeter)

Time	Jan. 6	7	8	9	10	11	12	13
12:30 (noon)	163.2	--	2.2	3.4	2.7	2.6	1.8	2.4
	3.2	--	2.1	3.8	2.8	3.0	2.7	2.4
	3.0	--	2.2	3.2	2.8	2.8	2.6	2.4
	3.9	--	3.0	2.5	2.6	3.0	2.8	1.3
	3.8	--	2.7	2.7	2.8	3.0	2.6	1.4
	3.9	--	2.2	3.0	2.8	3.0	2.7	1.3
	3.9	--	2.4	3.0	2.3	2.9	2.1	1.4
	3.8	--	2.3	3.1	2.7	3.0	2.3	1.4
	3.6	--	2.7	3.7	2.8	3.1	2.5	1.5
	3.8	--	2.8	3.2	3.8	3.0	2.7	1.4
	3.7	--	2.8	3.2	3.5	2.7	1.8	1.4
	3.7	--	2.8	2.9	3.0	2.9	1.5	1.4
6:30	3.7	--	2.8	3.2	3.0	2.8	1.4	1.5
	3.6	--	3.1	3.1	3.0	2.8	1.3	1.4
	3.6	--	3.0	3.1	3.0	2.4	1.4	1.4
	3.5	--	3.2	3.1	3.2	2.2	1.5	1.3
	3.5	--	3.2	3.2	3.1	2.4	1.4	1.4
	3.5	--	3.3	3.5	3.2	2.5	1.4	1.5
	3.6	--	3.5	2.9	3.1	2.5	1.5	1.3
	3.5	--	2.8	2.6	3.1	2.7	1.8	1.3
	3.0	--	3.1	2.1	2.9	2.3	1.8	1.2
	2.5	--	2.5	1.8	2.7	2.1	1.8	1.1
	2.5	--	2.4	2.1	2.4	2.5	1.3	1.3
	2.4	--	2.8	2.1	2.7	2.8	1.3	1.2

Plate Transformer Voltage (From recording voltmeter)

Time	Jan. 7	8	9	10	11	12	13	14
12:30 (midnight)	2.4	--	2.5	2.1	2.7	2.4	1.4	1.3
	2.7	--	2.2	2.1	2.7	2.3	1.6	1.2
	2.0	--	2.5	2.0	2.7	2.3	1.8	1.2
	1.7	--	2.5	2.5	2.8	2.0	2.0	1.2
	1.7	--	2.5	2.5	2.7	2.1	1.9	1.4
	1.6	--	2.6	2.5	1.8	2.1	1.2	1.4
	1.6	--	2.4	2.5	1.5	2.0	1.8	1.5
	1.6	--	2.3	2.5	1.2	2.1	1.3	1.5
	1.8	--	2.3	2.5	1.2	1.9	1.7	1.3
	1.7	--	2.1	2.4	2.1	1.7	2.2	1.5
	1.7	--	2.2	2.3	2.3	1.7	2.5	1.4
	1.8	--	2.4	2.0	2.5	1.8	2.4	1.6
6:30	2.2	--	2.4	1.8	2.5	1.7	2.5	1.4
	2.2	--	2.8	2.5	2.5	1.6	2.6	1.1
	2.7	--	2.3	2.7	2.8	1.5	2.5	1.0
	2.5	--	2.6	2.9	2.0	1.5	1.9	1.0
	2.6	--	2.2	2.9	2.6	1.6	1.9	0.8
	2.5	--	2.4	2.1	2.7	1.8	1.9	1.0
	2.5	--	2.6	2.5	2.6	1.7	2.1	0.8
	2.5	--	3.3	2.6	2.7	1.7	2.7	0.8
	2.0	--	2.7	2.7	2.7	1.9	2.6	0.9
	2.4	--	2.8	2.8	2.7	1.8	2.6	0.9
	3.0	--	3.2	2.6	2.7	1.8	2.5	0.9
	2.8	--	3.2	2.6	3.0	1.6	2.5	0.9