A METHOD OF DESIGNING MEDIUM AND HIGH FREQUENCY ALTERNATORS WITH A SPECIAL REFERENCE TO THE INDUCTOR TYPE

THESIS

by

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CONTENTS

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SYNOPSIS. The introduction to the present paper contains a recapitulation of various modern applications of medium and high frequency alternators together with a brief description of their usual types. In Section I the peculiar features of medium and high frequency alternators are considered together with some experimental data, and fundamental equations and expressions are derived in terms of the ratio of the minimum flux density in rotor slots to the maximum flux density in the air-gap, i.e. in terms of $\alpha = \frac{B_{min}}{B_{max}}$. Various wave forms of the field flux and E M F are considered and numerical coefficients dependent upon them computed. A method for the predetermination of the principal dimensions of an inductor alternator is given. In Section II are presented alternative equations, formulas and numerical coefficients in terms of the ratio of the maximum flux linking one coil of the armature winding to the minimum flux, i.e. in terms of $\gamma = \frac{\phi_{max}}{\phi_{min}}$ which is regarded as one of the most fundamental characteristics of the inductor alternator. A method for the experimental determination of γ on existing machines is given. Section III is

devoted to the predetermination of γ ; for that purpose a general expression and particular formulas are obtained and discussed, their bases being partly theoretical and partly empirical. A method for the predetermination of the maximum voltage obtainable at no load is suggested and developed. Section IV contains the verification of the theory and methods presented in the foregoing sections and the discussion of the results. The following criteria are used for the verification: 1. Saturation curve; 2. Maximum voltage obtainable at no load; 3. Optimum pole width; 4. Particular limiting cases. The structural and experimental data of ten inductor alternators of different types, frequencies, capacities, makes and applications are used for the verification. There is a good agreement between the computed and experimental results.

Introduction

Modern applications of medium and high frequency alternators and their usual types.

Besidesthe well known application of medium and high frequency alternators in the realm of the radio wave transmission, which dates from long ago, the following more recent uses are to be pointed out:

1. A new system of generating high frequency currents by means of alternators of an intermediate frequency (say from 5000 to 10000 cycles) has recently been devised. It contains a special type of a frequency multiplier in the form of a transformer whose magnetic circuit is highly saturated by the primary alternating current itself without any use of direct current. The above alternators have successfully been used for the transmission of the shorter radio waves(in the band 280 - 900 - 2100m.)[1] and quite recently for broadcasting purposes. There is a good reason to expect that the system is capable of successful operation for the generation and transmission of short waves in the band below 100 m.

2. During the recent years medium and high frequency alternators have been used in metallurgy in this country and Europe - for the purpose of electric heating and melting. High frequency ironless induction furnaces which originated from that application show a steady progress as to the scope and variety of their uses.[2]

3. The recent application of medium frequency alternators for the power control on transmission lines has proved to be successful and very promising.[3]

4. High and medium (audio) freqnency alternators have been used with success for energizing submarine signalling systems since long ago.

5. There are modern mining prospecting methods in which medium (audio) frequency alternators are used to energize the region to be explored.

6. In the chemical industry the applications of medium and high frequency alternators are constantly increasing, the production of ozone being one of the oldest ones.[4]

7. Medium frequency alternators have successfully been used for supplying the current to high speed induction motors which drive directly, without gear, high speed machine tools for fashioning and shaping wood, centrifugal machines, pumps and etc.

8. Medium frequency current generated by alternators has found a new application in machines for testing materials.

9. High voltage direct current can be obtained by combined application of medium frequency alternators and rectifiers which system contains apparatus of small size.

10. High and medium frequency alternators are used in laboratory work as well as for industrial measurements.

Thus the manifold uses of medium and high frequency alternators justify the attention given in the present article to their design and computation from the electrical point of view.

The main features of inductor alternators are shown in Figs. 1, 1a, 1b, 2, 2a, 2b, 3, 4, 5, where the ~~current~~ usual types with different arrangements of teeth and slots are represented.

Figs. 1, 1a, 1b, 2, 2a, 2b show homopolar alternators with a cylindrical rotor; Fig. 3 represents a homopolar disk rotor type which is known as Alexanderson's high frequency alternator. In Figs. 4 and 5 are represented the special forms of peripheral toothed zones as devised by Latour. A similar arrangement though not identical has been previously invented by Alexanderson in connection with his alternator.

Fig. 6 gives an idea of Bethenod's heteropolar inductor alternator. An improved alternator of this type has lately been described by M. C. Spencer[5], whereas the modified type of a homopolar inductor alternator has quite recently been presented in a paper by Dr. K. Schmidt.[6]

In particular, Figs. 1 and 2a represent the alternator type which has been designed by the author and built by the Siemens and Halske Company of Petrograd. This is a slight modification of the usual homopolar inductor alternator(see Figs. 2 and 2b). Fig. 1a is a drawing on a larger scale of Fig. 1 in a paper by K. Schmidt.[7] Of course there are other types of medium and high frequency alternators, but those referred to appear to be most widely used. Furthermore, in the course of the discussion other papers will be mentioned which contain description of other systems.

In all the Figs. referred to, A is used to indicate field winding coils (d.c. coils), B armature a.c. winding, R rotor body, S stator body, Z rotor pole or tooth, S_1 laminated armature iron core, N' and S' designate north and south polarity respectively. The dotted closed lines in Fig. 1 represent the magnetic circuit in usual homopolar inductor generators.

When revolving, the rotor peripheral toothed zone makes the magnetic flux linked with the armature a.c. winding coils oscillate, inducing hereby an electromotive force.

Section I <u>General Principles and Methods of Designing and Fundamental Formulas</u>.

<u>First Method</u>.

The armature winding together with the magnetic circuit in both medium and high frequency alternators, besides meeting voltage and current requirements must also possess a definite amount of inductance. This is necessitated by the fact that in almost all applications these machines are working on the resonance principle. On the one hand, the alternator inductance must not exceed a given value lest the tuning of the circuit ~~should~~ become impossible, with the capacity introduced into the circuit by some special arrangement, as for instance by the antenna. On the other hand, it is desirable to limit the use of tuning coils or at least their size by concentrating all or most of the necessary inductance in the alternator itself. Therefore a designing engineer must consider with care the question of providing for a definite amount of inductance in such an alternator under design and must not have to chance its value.

This requirement introduces new equations into the problem of designing this type of alternators so that this method becomes different from the usual one applied to ordinary commercial generators.

Furthermore, the inductance of such an alternator changes with the relative position of a stator and rotor, as the latter revolves. The coefficient of self induction has, for instance, different values for the relative positions I and II represented in Fig. 7. Thus the question arises as to what value of inductance should be taken into account in designing an alternator of that type. Both the electromotive force and the current have their maximum(amplitude or crest value) at resonance at the position II (Figs. 7 and 10). It is the inductance at this instant that must be considered in determining the condition of resonance. This statement has already been published by the author in 1925[8]; it will now be considered more or less in detail.

At resonance the maximum electromagnetic energy (at the instant of the maximum current) is equal to the maximum electrostatic energy (at the instant of the maximum voltage across the equivalent circuit condenser) so that we have

$$\frac{L_r I^2}{2} = \frac{C_r V^2}{2} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

where L_r is the coefficient of self induction of the circuit at the relative position II,

I the amplitude value of the current,

C_r the equivalent capacity of the circuit, which is practically equal to the capacity of a condenser inserted in the resonant circuit and

V is the crest or amplitude value of the voltage across it. Substituting $I=\omega C_r V$, where $\omega=2\pi f$, f being the resonance frequency we get from (1) the well known relation $\omega = \frac{1}{\sqrt{C_r L_r}}$, which shows that the fundamental frequency is determined by the value of L_r inherent to the relative position II of the stator and rotor. In medium and high frequency inductor alternators the alternating reactive field penetrating into the poles (rotor teeth) is small in comparison with the effective direct current excitation field. This is due to the fact that in these machines the ratio of the effective d.c. field ampere turns to the reactive a.c. armature ampere turns corresponding to the said armature field is considerably greater than in the alternators of the usual commercial type so that the distortion of the excitation field flux will be much less than in the latter; the position of the vector of the electromotive force at load will almost coincide with that of the corresponding vector at no load, and the reactive armature flux will practically act like the leakage armature flux, say across stator slots, constituting thus one part of the entire resonant inductance, the other part being identified with the leakage armature flux.

Under the effective field ampere turns are understood only those ampere turns which correspond to $\frac{B_{max} - B_{min}}{2}$, where B_{max} is the maximum magnetic induction in the air gap, and B_{min} is the minimum magnetic induction in the rotor slot. This point will be considered in detail later in the section III.

The armature reactive flux is schematically represented in Fig. 10 by the paths like e d c, q r m n r q, q' r' q' etc., while the leakage flux is exemplified by the lines of forces such as x y z, h g f.

In determining the contribution of the reactive flux to the resonant inductance it is necessary to take into account both the rotation of the rotor and the mutual inductance between the two armature windings B and B on either half of the stator (Figs. 1 and 1a). The usual normal arrangement of those two windings is such that the coils of one of them are in one alignment, along a parallel to the axis of the rotor, with the corresponding coils of another winding, so that the two corresponding coils have one and the same radial median plane, the poles(rotor teeth) of one half of the rotor being also in one alignment with the corresponding poles of another.

Applying Kirchhoff's laws to the magnetic circuit, it is found that the effect of mutual inductance in each winding is equivalent to that of half a reactive flux represented in Fig. 10. Since the self inductance effect itself in each winding is contributed also by half a reactive flux because of the rotation of the rotor, the resultant inductance corresponds to the full amount of the reactive flux <u>at rest</u> plus the leakage flux. This conclusion is different from the view according to which only half a reactive flux must be taken into account in determining the value of the resonant inductance and which, for instance, has been expressed in a paper by Professor A. Blondel.[9]

The writer's conclusions are in agreement with the experimental results obtained by him on several inductor alternators of medium frequency. For instance, the inductive reactance $L_r\omega$ of one such alternator with the usual coil arrangement, determined by the resonance method gave the value of 10.18 ohms at a normal load current and at the frequency $f = 1000$. Direct measurement by means of alternating current of the same frequency flowing through the armature winding of the same machine at rest and supplied by another alternator gave for $L_r\omega$ the value from 10 to 10.4 ohms at the rotor position II, and from 7.35 to 8.85 ohms at the position I.[10] In both cases the upper values were determined at the lower values of the field current, and the experiment was so conducted as to reproduce load conditions of the tested alternator.

In interpreting these results, it must be borne in mind that the common armature flux interlinked with both the windings B and B' (Fig. 1 and 1a), as indicated, for example, in Fig. 1 by a closed dotted line X Y, practically vanishes when the experiment is made on an inductor alternator at rest, since that flux lies within the solid metal(iron or steel) parts along a considerable length and is therefore almost entirely damped at the frequency used. Consequently there remains only the flux represented in Fig. 10 whose magnetic paths lie in planes perpendicular to the rotor axis, and this result has been predicted on the basis of the outlined theory, namely: that the resonance inductance is determined by the entire flux at rest at the position II of Fig. 10 in so far as the usual normal inductor alternator is concerned.

In passing, it might be pointed out that the previous consideration of the mutual inductance combined with Blondel's theory of two reactances practically leads to the same result, since the computation made by the author shows that in medium and high frequency inductor alternators the values of two reactances are much closer to each other than in ordinary commercial alternators, and the subsequent use of Bethenod's diagram[11] completes the determination of the resonance inductance, with the result previously stated.

Another essential feature encountered in the design of inductor alternators is the fact that the polarity of the flux ϕ linked with any coil of the armature winding does not change its sign, as the flux value oscillates between ϕ max and ϕ min, the maximum and minimum values respectively. The following is the first of two methods by which the stated peculiar features of design are taken into consideration.

The first method

Notation

B_{max} denotes the maximum magnetic induction in the air gap corresponding to the median plane of a pole (rotor tooth) when it faces a stator tooth.

B_{min} denotes the minimum magnetic induction in the median plane of an interpolar space (rotor slot)

B_{max} and B_{min} have one and the same sign and are directed radially in machines with a cylindrical rotor, on the one hand, and unidirectionally and parallel to the axis of the shaft in machines with a disk rotor (see Figs. 1 and 3 respectively), on the other hand.

u denotes the linear peripheral velocity in cms per second of a rotor.

m denotes the number of parallel connected circuits or branches of the armature a.c. winding.

z denotes the number of conductors connected in series in one slot of the stator or the number of turns in series in one coil, two slots being used per coil.

S denotes the number of coils of the armature winding connected in series (i.e. in one branch)

L_a denotes the coefficient of self induction of the armature winding.

f denotes the frequency of E M F

T denotes its period so that $T = \frac{1}{f}$

L denotes the coefficient of self induction of one of the parallel branches of the armature winding so that $L = mL_a$

l denotes the gross iron length in cms of the armature core in the direction perpendicular to lateral faces of a pole i.e. measured parallel to the shaft axis in alternators with a cylindrical rotor, and radially in disk type machines (see Figs. 1 and 3).

ζ denotes the value of the flux linked with one coil of the armature winding per cm length of the iron core in a stator and per ampere turn in a stator slot. Sometimes ζ is termed as leakage magnetic conductance per cm of axial length of a stator iron core, the whole length being l (see Figs. 1 and 3).

E_{omax} denotes the maximum or crest value of a complex wave of E M F of an alternator.

E_o or E_{eff} denotes the R M S or effective value of a complex E M F wave.

$$\beta_o = \frac{E_o}{E_{omax}}$$

E_{1max} denotes the maximum or crest value of the fundamental sine wave of E M F.

E_1 denotes the R M S of same. $E_1 = \frac{E_{1max}}{\sqrt{2}}$

$$\beta_1 = \frac{E_{1max}}{E_{omax}}$$

$$\beta = \frac{E_1}{E_{omax}} = \frac{E_{1max}}{\sqrt{2}\, E_{omax}} = \frac{\beta_1}{\sqrt{2}}$$

E_{max} denotes the maximum or crest value of a sine E M F which is to be applied to the working circuit.

E denotes the R M S value of same

$$E = \frac{E_{max}}{\sqrt{2}}$$

p_r denotes the pole or rotor tooth pitch.

p_s denotes the stator slot pitch (or stator tooth pitch), usually $p_s = p_r/2$

a_r denotesthe peripheral width of a pole (rotor tooth)

a'_r denotes the width of a pole at its root.

b_r denotes the peripheral width of a rotor slot.

a_s denotes the peripheral width of a stator tooth.

b_s denotes the peripheral width of a stator slot (the opening at the air-gap)

a'_s denotes the least width of a stator tooth.

b denotes the width of a stator slot which corresponds to a'_s (See Figs. 7, 10 and 11)

$$\alpha = \frac{B_{min}}{B_{max}}$$

ϕ denotes the flux linked with one coil of the armature winding.

ϕ_{max} denotes its maximum value.

ϕ_{min} denotes its minimum value.

$$1/\gamma = \lambda = \frac{\phi_{min}}{\phi_{max}}$$

γ denotes the coefficient of flux oscillation $\dfrac{\phi_{max}}{\phi_{min}}$

δ denotes the single air-gap length, radial (see Fig. 1) or axial (see Fig. 3)

B_r denotes the maximum flux density in poles (rotor teeth).

B_s denotes the maximum flux density in stator teeth.

ψ denotes the armature core iron space factor, i.e. the ratio of the net iron length to the gross iron core length 1.

In Figs. 7, 10 and 11 are shown the coils of a.c. or armature winding of a stator, the cross signs indicating the left side with direct conductors and the points denoting the right side with return conductors of a coil. The field flux diagram of B in Fig. 7 shows that, while the magnetic field is unidirectional along the rotor periphery, one side of each of the coils is subject to the action of a pole flux which is greater and the other side is under the influence of a slot flux.

It follows therefrom that

$E_{omax} = (B_{max} - B_{min})$ z 1 S u 10^{-8} volts..........(2)

$E_o = \beta_o (B_{max} - B_{min})$ z 1 S u 10^{-8} volts..........(3)

Two limiting forms of a complex E M F wave, between which all complex wave forms in an alternator considered can be ranged are the rectangle, on the one hand, and the sine curve, on the other, and the corresponding values of are 1 and $\frac{1}{\sqrt{2}}$ respectively, so that actually

$$\frac{1}{\sqrt{2}} < \beta_o < 1$$

These two limiting forms of the complex E M F wave are dependent upon the two limiting forms of the magnetic field which are represented in the Fig. 8 by the broken line a'b'x'y'e'u'v'h', on the one hand, and the sine curve b'k'l'e'o'q'h', on the other, while a real form of the magnetic field is given with sufficient accuracy by the trapezoidal diagram b'c'd'e'f'g'h' to which a trapezoidal complex E M F wave corresponds.

The reader will be referred to Fig. 9 in which different trapezoidal complex E M F waves are represented (the right hand half of the figure).

Furthermore, as an alternator will have to feed a resonant circuit which is to be tuned to the fundamental frequency, it follows that the fundamental E M F sine wave should be taken into account and regarded as a working factor of the system. Therefore, it is necessary to introduce the values E_{1max} and E_1 into equations and final results.

For the two limiting cases we have

$\frac{E_{1max}}{E_{omax}} = \beta_1 = \frac{4}{\pi} = 1.27$ (Rectangular complex E M F wave)

and $\beta_1 = 1$ (Sine complex E M F wave) so that

actually $1 < \beta_1 < \frac{4}{\pi}$

Finally we get the following limiting values for

$$\frac{E_1}{E_{omax}} = \beta = \frac{\beta_1}{\sqrt{2}}$$

$\beta = \frac{1}{\sqrt{2}} = 0.708$ (sine complex E M F wave)

$\beta = \frac{4}{\pi\sqrt{2}} = 0.905$ (rectangular complex E M F wave)

so that actually

Sine wave		Rectangular wave	
0.708	$< \beta <$	0.905	....(3 bis)

In Fig. 9 different complex E M F waves of trapezoidal shape which suits the real wave form the best are represented, and respective values of β determined by the method just exemplified are given in Table. I. The trapezoidal E M F waves are to be represented each by Fourier series; then the computation of β can easily be carried out for each case.

As an average suitable for most cases <u>the value from 0.8 to 9.0, say 0.85, can be taken for</u> β .

If the pole pitch p_r is very small involving also a very small width of the rotor pole itself or a pole has a shape like that represented in the Fig. 1a, the complex E M F wave will not differ much from a sine curve; then it is safer to take for β the value from 0.708 to 0.80, say 0.75.

Now we have $E_1 = \beta E_{omax}$, and using (3) we get

$$E_1 = \beta(B_{max} - B_{min}) z\, 1 S u\, 10^{-8} \text{volts} \qquad (4)$$

and as $E_1 = E$

$$E = \beta(B_{max} - B_{min}) z\, 1 S u\, 10^{-8} \text{volts} \qquad (5)$$

since E is the R M S value of the sine E M F which an alternator has to supply to a resonant circuit.

Furthermore, as it has already been proved, such an alternator should possess a given or required value of inductive reactance $L_a\omega$ which requirement leads to the following expressions

$$L\omega = mL_a\omega$$

$$mL_a\omega = L\omega = \zeta\omega z^2\, 1\, S\, 10^{-8} \qquad (6) \quad \text{where } \omega = 2\pi f$$

Combining equations (5) and (6) and solving them, we get the following formulas for determining the characteristic data of the armature a.c. winding and therefore the size of an inductor generator

$$z = \beta \frac{\mathcal{L}\omega}{\zeta\omega} \frac{u(B_{max} - B_{min})}{E} = \beta \frac{\mathcal{L}\omega}{\zeta\omega} \frac{B_{max}(1-\alpha)u}{E} \quad \ldots (7)$$

$$S\ell = \frac{\mathcal{L}\omega}{\zeta\omega} \frac{1}{z^2} 10^8 \quad \ldots (8)$$

$$S\ell = \frac{\zeta\omega}{\mathcal{L}\omega} \frac{E^2}{\beta^2} \frac{10^8}{u^2 B_{max}^2 (1-\alpha)^2} \quad \ldots (9)$$

$$\left.\begin{matrix}(7)\\(8)\\(9)\end{matrix}\right\} \mathrm{I}$$

where B_{max} can be computed in a usual manner, since it corresponds to the median plane of a pole with δ_e as an equivalent air-gap length, or, if B_{max} given, field ampere-turns can be calculated as well. $\alpha = \frac{B_{min}}{B_{max}}$ which ratio can be either computed or determined by an experimental or graphical method, as it will be shown later in Sections II and III.

Investigating several medium frequency inductor alternators, it has been found by the author that ζ had almost the same value, namely: from 13 to 15, averagely 14, for closed slots of a stator(Figs. 1a, 2, 2b, 3) and about 10 for open slots(Figs.1, 2a)

Anyhow ζ can be computed or determined experimentally by means of a model of the magnetic circuit and armature winding.

The value of $\alpha = \frac{B_{min}}{B_{max}}$ depends upon the form of the toothed peripheral zones of the rotor and stator, in general, and upon the ratio $\frac{\delta}{p_z}$, in particular.

The same reasonings which have just been developed can also be applied to heteropolar a.c. generators of non-inductor type, if they are to satisfy certain conditions concerning inductance.

This will lead to the following formulas analogous to the foregoing ones:

$$\left.\begin{aligned} z &= 2\beta \frac{L\omega}{\zeta\omega} \frac{u B'_{max}}{E} \ldots\ldots (10) \\ lS &= \frac{L\omega}{\zeta\omega} \frac{1}{z^2} 10^8 \ldots\ldots (11) \\ lS &= \frac{\zeta\omega}{4L\omega} \frac{E^2}{\beta^2 B'^2_{max}} \frac{1}{u^2} 10^8 \ldots (12) \end{aligned}\right\} \text{II}$$

A heteropolar alternator is equivalent to an inductor generator in so far as, ceteris paribus, the maximum magnetic induction B'_{max}, regardless of its polarity in the former, is equal to $\frac{B_{max} - B_{min}}{2}$ in the latter. In fact, if in the set I of formulas (7), (8) and (9) $(B_{max} - B_{min})$ is replaced by $2B'_{max}$, the group II is obtained, and vice versa, by substituting $\frac{B_{max} - B_{min}}{2}$ for B'_{max}, we can transform the second set of formulas II into the first one I. This mutual transformation may be considered as verification of the presented formulas.

Now it would be in order to give some suggestions about how to use the foregoing equations.

First of all, it is necessary to carry out some preliminary calculations. The pole pitch p_r is to be determined from the admitted peripheral velocity u of a rotor and frequency f: $p_r = \frac{u}{f}$, and then the width a_r of a pole will be chosen as a fraction of p_r; for the most part $a_r \simeq 0.5\ p_r$ approximately.

In modern high frequency machines of the type considered u is equal to from 100 to 150 m per second, whereas in some exceptional cases it reaches 200 m per second and even more. In medium frequency generators u is considerably lower.

Choosing for the air-gap δ a certain value which in modern high and medium frequency alternators is only a fraction of 1 mm, it will be possible to determine experimentally or by computation the ratio $\alpha = \frac{B_{min}}{B_{max}}$.

Furthermore, taking as a basis the admitted peripheral velocity u and the number of revolutions per second, the diameter D of a rotor is directly determined; thereupon knowing pole pitch p_r and choosing a certain number m of parallel branches or circuits for the armature winding, we are able to determine the number of coils S in series contained in each branch.

The value of ζ is approximately constant for a given type of an inductor alternator to be designed; hence its value obtained by experimental investigations of other machines of the same type may be taken as a tentative value and substituted for ζ in the formulas of either of two groups I and II. In this way we obtain from equations (7) or (10) respectively, as the case may be, a tentative value for the number z of conductors in a stator slot, and from the other equations of the same set also the length l of the armature core. As z can be only an integer it should be rounded out accordingly. ζ must be determined for the position II (Figs. 7 and 10) as it has previously been shown in this paper.

The next step to be taken is the determination of the area of the cross section of conductors of the armature winding. As the value of the current supplied by an alternator is given, and the number m of the parallel branches of the armature winding has already been given, cross section of conductor can be easily determined. In middle frequency alternator the current density can be admitted much higher than in generator of commercial type. The writer can point out to a case when he admitted 12 amp. per 1 mm^2, and no over-heating occurred (see also a paper by Karl Schmidt[12]). Thus it will be possible to determine the shape and the cross section area of a stator slot for a selected z and a determined pole pitch. Of course, in outlining a stator slot allowance for insulation must be made.

Following the slot selection the peripheral forms of both a stator and rotor can be selected, and consequently it becomes possible to make a second and more accurate determination of the value of ζ . In its turn, an accurate value of ζ brings about a more precise determination of z and l by means of the sets of equations referred to (I or II).

It is seen that all the characteristic data of an alternator discussed: D, l, z, p_r, a_r, S, m, ζ, α the form and size of the slots, can be determined by means of this method, in general, and by the sets of equations I, II, in particular, so that an alternator designed should meet the requirements and conditions imposed.

It should be noted that contents of this chapter are not limited to the inductor type of alternator; the set II of equations makes the method applicable also to heteropolar non-inductor generators. The writer has applied this method with ~~quite~~ satisfactory results.

Section II

General Methods and

Fundamental Formulas
(continued)

Second Method

The foregoing formulas and equations contain the ratio $\alpha = \frac{B_{min}}{B_{max}}$. They will now be transformed in such a way as to introduce ⊛ $\gamma = \frac{\phi_{max}}{\phi_{min}}$ where ϕ_{max} denotes the value of the flux passing through a pole and corresponding to one half of the pole pitch i.e. to $\frac{p_r}{2}$, while ϕ_{min} is the value of the flux across a rotor slot corresponding to another half of the pole pitch, although the pole width a_r and that of a slot b_r may differ somewhat from $\frac{p_r}{2}$. Thus ϕ_{max} represent the total amount of the flux passing through the air-gap across that rectangular tangent strip area $1 \times \frac{p_r}{2}$ which is situated along the outer cylindrical surface of the pole symmetrically with respect to its median line (see Figs. 7 and 8).

The ratio $\lambda = \frac{\phi_{min}}{\phi_{max}}$ is an important factor in the operation of an inductor alternator; its reciprocal $\gamma = \frac{\phi_{max}}{\phi_{min}}$ will henceforth be called the coefficient of the flux oscillation.

We shall at first consider two limiting cases, i.e. a rectangular field flux distribution and a sine flux oscillation or distribution.

Referring to Fig. 8 where different forms of field distribution are represented we get in the first case (rectangular distribution)

⊛ $\lambda = \frac{\phi_{min}}{\phi_{max}}$ or its reciprocal

$$\phi_{max} = \frac{B_{max}+B_{min}}{2}\frac{lp_r}{2} + \frac{B_{max}-B_{min}}{2}\frac{lp_r}{2}$$

$$\phi_{min} = \frac{B_{max}+B_{min}}{2}\frac{lp_r}{2} - \frac{B_{max}-B_{min}}{2}\frac{lp_r}{2}$$

$$\phi_{max} - \phi_{min} = (B_{max} - B_{min})\frac{lp_r}{2}$$

$$\phi_{max}\left(1 - \frac{\phi_{min}}{\phi_{max}}\right) = B_{max}\left(1 - \frac{B_{min}}{B_{max}}\right)\frac{lp_r}{2}$$

$\phi_{max}(1 - \lambda) = B_{max}(1 - \alpha)\frac{l\ p_r}{2}$, and as

$B_{max}\frac{l\ p_r}{2} = \phi_{max}$, we get finally for a rectangular field

$$\left.\begin{aligned} 1 - \lambda &= 1 - \alpha \\ \lambda &= \alpha \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots(13)$$

In the second case (sine wave flux oscillation or distribution) we have from Fig. 8

$$\left.\begin{aligned} \phi_{max} &= \frac{B_{max}+B_{min}}{2}\frac{lp_r}{2} + 2/\pi\,\frac{B_{max}-B_{min}}{2}\frac{lp_r}{2} \\ \phi_{min} &= \frac{B_{max}+B_{min}}{2}\frac{lp_r}{2} - 2/\pi\,\frac{B_{max}-B_{min}}{2}\frac{lp_r}{2} \\ \phi_{max} - \phi_{min} &= 2/\pi\, B_{max}\frac{lp_r}{2}\left(1 - \frac{B_{min}}{B_{max}}\right) \\ \phi_{max}(1-\lambda) &= 2/\pi\, B_{max}\frac{lp_r}{2}\left(1 - \frac{B_{min}}{B_{max}}\right) \\ 1-\lambda &= 2/\pi\,\frac{B_{max}}{\phi_{max}}\frac{lp_r}{2}(1-\alpha) \end{aligned}\right\} \quad \ldots(14)$$

Substituting for ϕ_{max} its expression from (14), we get

$$1 - \lambda = \frac{2}{\pi} \frac{B_{max} \frac{1}{2} p_r (1 - \alpha)}{\frac{B_{max} + B_{min}}{2} \frac{1}{2} p_r + \frac{2}{\pi} \frac{B_{max} - B_{min}}{2} \frac{1}{2} p_r}$$

and after obvious reductions we have

$$1 - \lambda = \frac{4}{\pi} \frac{(1 - \alpha)}{1 + \alpha + \frac{2}{\pi}(1 - \alpha)} = \frac{2(1 - \alpha)}{\frac{\pi}{2}\left[2 - (1 - \alpha)\right] + 1 - \alpha} \quad \ldots\ldots\ldots\ldots (15)$$

from which equation we obtain

$$1 - \alpha = \frac{\pi}{2} \frac{1 - \lambda}{1 + (1 - \lambda)\frac{1}{2}\left(\frac{\pi}{2} - 1\right)} = k(1 - \lambda) \quad \ldots\ldots (16)$$

where $$k = \frac{\pi}{2} \frac{1}{1 + 0.285(1 - \lambda)} \quad \ldots\ldots\ldots\ldots (16\ \text{bis})$$

and $\lambda = 1/\gamma$

In applying the expressions (16) and (16 bis) we shall discriminate between medium and high frequency alternators: while in the former the coefficient of the flux oscillation $\gamma = 1/\lambda$ can be considered as having an average value equal to about 3.5-4 and $\lambda = 0.25 - 0.285$, in the latter $\gamma = 1.75 - 2.25$ and $\lambda = 0.45 - 0.57$. The reason for the discrimination lies in different values of $\frac{p_r}{\delta}$ in medium and high frequency alternators: in the former $\frac{p_r}{\delta}$ is the greater.

Consequently, if we take $\gamma = 4$ and $\gamma = 1.75$ respectively, we get

$$k = \frac{\pi}{2} \; \frac{1}{1 + 0.75 \times 0.285} = 1.29 \text{ for medium frequency}$$

$$k = \frac{\pi}{2} \; \frac{1}{1 + 0.43 \times 0.285} = 1.40 \text{ for high frequency}$$

and $k = 1.345$ for intermediate cases between medium and high frequency alternators.

It follows that we have generally

$$1 - \alpha = k\,(1 - \lambda) \quad \ldots\ldots\ldots\ldots\ldots (17)$$

where $k = 1$ for a rectangular distribution of the field flux i.e. for a rectangular wave of the E M F, and k 1.29 - 1.40 in the case of the sine E M F (sine field flux oscillation or distribution).

Since the actual form of the field flux distribution will be of an intermediate shape, k will assume values between those limiting figures. The intermediate forms of the field flux as well as of the E M F can be represented by trapezoids. This can be done with sufficiently good approximation. In Fig. 9 four trapezoids I, II, III, IV show a gradual transition from a rectangular shape to the sine wave of both the field flux and the E M F, the trapezoid IV being very close to the sine curve. Now the following computation will be applied to the wave shapes intermediate between rectangular and sine form.

With the notation of Fig. 9 we have

$$\Phi_{max} - \Phi_{min} = \frac{B_{max} + B_{min}}{2}\, l a_p + \frac{B_{max} - B_{min}}{2}\, \frac{l(a_p + a_p')}{2} -$$

$$- \frac{B_{max} + B_{min}}{2}\, l a_p + \frac{B_{max} - B_{min}}{2}\, \frac{l(a_p + a_p')}{2}$$

$$\Phi_{max} - \Phi_{min} = (B_{max} - B_{min})\, \frac{l(a_p + a_p')}{2} \quad \ldots\ldots (17\,bis)$$

$$1-\lambda = \frac{B_{max}-B_{min}}{\Phi_{max}} \; \frac{l(a_p+a_p')}{2}$$

$$1-\lambda = \frac{1}{2}\;\frac{B_{max}(1-\alpha)\,l(a_p+a_p')}{(B_{max}+B_{min})\frac{la_p'}{2}+\frac{(B_{max}-B_{min})}{2}\frac{l(a_p+a_p')}{2}}$$

$$1-\lambda = \frac{2(1-\alpha)}{(1+\alpha)\frac{2a_p}{a_p+a_p'}+(1-\alpha)} = \frac{2(1-\alpha)}{\left[2-(1-\alpha)\right]\frac{2a_p}{a_p+a_p'}+1-\alpha} \quad \cdots\cdots(18)$$

whence we get

$$1-\alpha = \frac{2a_p}{a_p+a_p'}\;\frac{1-\lambda}{1+\frac{1}{2}(1-\lambda)\left(\frac{2a_p}{a_p+a_p'}-1\right)} \quad \cdots\cdots(19)$$

or

$$1-\alpha = k(1-\lambda) = k(1-1/\gamma) \quad \cdots\cdots\cdots\cdots(20)$$

where $$k = \frac{2a_p}{a_p+a_p'}\;\frac{1}{1+0.5(1-1/\gamma)\left(\frac{2a_p}{a_p+a_p'}-1\right)} \quad \cdots\cdots(21)$$

It is seen that the relationship between $1-\alpha$ and $1-\lambda = 1-1/\gamma$ is of the same simple form for intermediate wave shapes as for the limiting forms of the field flux and E M F, the coefficient k being determined as equal to 1 or from the equations (21) or (16bis) for the rectangular, trapezoidal and sinusoidal wave form respectively.

Substituting in the set I of equations (7), (8) and (9)

$k(1-\lambda)$ for $1-\alpha$, and $1/\gamma$ for λ, and putting

$$k\beta = \sigma \qquad \text{...............(22)}$$

we get the third set of equations

$$\left.\begin{aligned} z &= \sigma\left(1-1/\gamma\right)\frac{\mathcal{L}\omega}{\varsigma\omega}\,\frac{u\,B_{max}}{E} \;\ldots\ldots (23) \\[2ex] S\ell &= \frac{\mathcal{L}\omega}{\varsigma\omega}\,\frac{1}{z^2}\,10^8 \;\ldots\ldots\ldots (24) \\[2ex] S\ell &= \frac{1}{\sigma^2\left(1-1/\gamma\right)^2}\,\frac{\varsigma\omega}{\mathcal{L}\omega}\,\frac{E^2}{u^2 B_{max}^2}\,10^8 \;\ldots\ldots (25) \end{aligned}\right\} \mathrm{III}$$

This set of equations makes possible the computation of the principal dimensions of alternator in terms of γ rather than α, which is an advantage because it is easier to predetermine γ than α. The method of computing γ by means of formulas presented by the author will be given in the next section.

The verification of the whole method by checking computed results against experimental data obtained from observations made on various medium and high frequency alternators will be presented later in a special section.

The numerical values of β, k and σ are grouped together in Table I for different values of γ and various wave forms corresponding to the trapezoids of Fig. 9 with the rectangular and sine waves of the field flux and E M F as the limiting cases.

TABLE I NUMERICAL VALUES OF AND

Wave form of Field flux and E M F	γ = 1.25			γ = 1.75			γ = 2.00			γ = 2.25		
	β	k	σ	β	k	σ	β	k	σ	β	k	σ
Rectangle	0.905	1	0.905	0.905	1	0.905	0.905	1	0.905	0.905	1	0.905
Trapezoid I	0.895	1.08	0.967	0.895	1.07	0.957	0.895	1.065	0.953	0.895	1.063	0.951
Trapezoid II	0.865	1.175	1.015	0.865	1.15	0.995	0.865	1.142	0.988	0.865	1.138	0.985
Trapezoid III	0.812	1.29	1.047	0.812	1.24	1.007	0.812	1.23	1	0.812	1.22	0.992
Trapezoid IV	0.745	1.425	1.06	0.745	1.35	1.005	0.745	1.333	0.993	0.745	1.317	0.982
Sine wave	0.707	1.485	1.05	0.707	1.40	0.992	0.707	1.375	0.973	0.707	1.35	0.955

TABLE I (continued)

Wave form of Field flux and E M F	γ = 2.50			γ = 2.75			γ = 3.00			γ = 3.25		
	β	k	σ	β	k	σ	β	k	σ	β	k	σ
Rectangle	0.905	1	0.905	0.905	1	0.905	0.905	1	0.905	0.905	1	0.905
Trapezoid I	0.895	1.061	0.95	0.895	1.06	0.948	0.895	1.058	0.947	0.895	1.056	0.945
Trapezoid II	0.865	1.13	0.978	0.865	1.128	0.975	0.865	1.125	0.973	0.865	1.122	0.97
Trapezoid III	0.812	1.21	0.984	0.812	1.205	0.98	0.812	1.2	0.975	0.812	1.193	0.972
Trapezoid IV	0.745	1.303	0.972	0.745	1.294	0.965	0.745	1.285	0.957	0.745	1.28	0.954
Sine wave	0.707	1.34	0.947	0.707	1.33	0.941	0.707	1.32	0.934	0.707	1.31	0.93

TABLE I (continued)

Wave form of Field flux and E M F	$\gamma = 3.50$			$\gamma = 4.00$			$\gamma = 5.00$		
	β	k	σ	β	k	σ	β	k	σ
Rectangle	0.905	1	0.905	0.905	1	0.905	0.905	1	0.905
Trapezoid I	0.895	1.055	0.945	0.895	1.05	0.94	0.895	1.05	0.94
Trapezoid II	0.865	1.12	0.968	0.865	1.115	0.965	0.865	1.11	0.96
Trapezoid III	0.812	1.19	0.967	0.812	1.183	0.962	0.812	1.175	0.957
Trapezoid IV	0.745	1.272	0.948	0.745	1.261	0.94	0.745	1.25	0.932
Sine wave	0.707	1.30	0.92	0.707	1.29	0.913			

In general, β is to be computed from Fourier expansion in trigonometric series corresponding to a wave form considered (see Fig. 9) and k to be determined by means of equation (16 bis) or (21) according to the case, with reference to Fig. 9, σ being equal to $k\beta$.

Examining the values of σ given in Table I for $1.25 \leqq \gamma \leqq 5$ and for different wave forms corresponding to Figs. 8 and 9 it is seen that for every practically possible case

$$0.905 \leqq \sigma \leqq 1.05 \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

so that, if σ is given an average value 0.95, the error will not exceed 5.5 per cent in any practical case.

But in general, it is quite possible to assume for σ a value that would be more suitable for a given individual case than the above average value, taking into account the value of γ and a probable form of the field flux.

For instance, in a medium frequency alternator it is quite reasonable to assume the wave form to be close to the rectangular distribution of the field, since the ratio $\frac{p_r}{\delta}$ is considerably larger than in high frequency alternators and, taken in itself, may be very great, reaching sometimes the value of about one hundred. In this case the toothed peripheries of both the stator and rotor are supposed to be normal (see Figs. 7 and 10) with $a_r = \frac{p_r}{2}$ and $a_s \approx a_r$ approximately.

In another case represented in Fig. 1a where a special form of rotor poles is selected it is advisable to consider the flux wave as approaching the sine flux oscillation curve and accordingly to choose an appropriate value of σ from Table I or compute it from the foregoing formulas. In most cases it will be possible to determine σ within 2 per cent of error.

As in medium and high frequency alternators only one stator slot is used <u>per effective pole</u>, in general, and only one conductor per slot is to be found in high frequency alternators, in particular, the E M F wave as a function of time may be considered identical with the corresponding field flux wave regarded as a function of peripheral arc length.

Therefore, the graph of Fig. 9 can be used to represent both the waves each with its own appropriate scale.

In the sets of formulas I and III E is equal to the R M S value of the fundamental E M F wave or, in other words, denotes an assigned R M S value of a sine E M F which is to be supplied to a resonant circuit. Very often it is, however, desirable and even necessary to have relations between E_o, i.e. that R M S value of a complex E M F wave which is directly measured by a hot wire voltmeter, on the one hand, and electrical and mechanical data of an alternator, on the other.

For this purpose the following formula is given

$$E_o = \frac{\sqrt{2}}{\rho} E \quad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

where for a sine E M F complex wave $\rho = \sqrt{2}$ and for a rectangular complex wave $\rho = 4/\pi = 1.27$, since in this case $E_o = E_{omax}$, $E = \frac{E_{max}}{\sqrt{2}}$ on the one hand, and $E_{max} = \frac{4}{\pi} E_o$, on the other; consequently $E_o = \frac{\sqrt{2}E}{4/\pi}$

For the wave forms intermediate between a rectangular one and a sine curve we have assumed the trapezoids in the Fig. 9. Therefore E_o i.e. the R M S of E M F of the complex wave is determined by the relation

$$E_o = \sqrt{\frac{E_{omax}^2 a_p' + 2\int_0^{\frac{a_p - a_p'}{2}} \frac{E_{omax}^2 x^2 dx}{\left(\frac{a_p - a_p'}{2}\right)^2}}{a_p}} \quad \text{(see Fig. 9)}$$

On integration we get

$$\left.\begin{aligned} E_o &= E_{omax}\sqrt{\frac{1}{3} + \frac{2}{3}\frac{a_p'}{a_p}} \\ E_o &= \beta_o E_{omax} \\ \text{where} \quad \beta_o &= \sqrt{\frac{1}{3} + \frac{2}{3}\frac{a_p'}{a_p}} \end{aligned}\right\} \quad \ldots\ldots(28)$$

Furthermore, we have, taking into account the second of the equations (28)

$$E_{max} = E_{1max} = \beta_1 E_{omax}$$

$$\sqrt{2}\,E = \sqrt{2}\,E_1 = \beta_1 E_{omax} = \beta_1/\beta_o\, E_o \quad \ldots\ldots\ldots\ldots(29)$$

whence it follows

$$E_o = \frac{\sqrt{2}\,E}{\beta_1/\beta_o} = \frac{\sqrt{2}\,E_1}{\beta_1/\beta_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(30)$$

or putting

$$\beta_1/\beta_o = \rho \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(31)$$

we fall back into the formula (27)

Now taking, as intermediate wave forms, the trapezoids represented in Fig. 9, it is possible to compute β_o from the third equation of the group (28) and β_1 from the expansion of the trapezoids in Fourier series, and thus to determine ρ from (31) for each of the trapezoids I, II, III, IV of the figure 9.

The following Table II is obtained

Table II Numerical values of ρ.

Wave form	ρ
Rectangle	1.27
Trapezoid I	1.33
Trapezoid II	1.375
Trapezoid III	1.4
Trapezoid IV	1.405
Sine wave	1.41

From equation (5) it follows

$$E = E_1 = \beta B_{max}(1 - \alpha)\, zlSu\ 10^{-8}$$

and on account of (20) and (22) it is obtained

$$E = E_1 = \sigma(1 - 1/\gamma)B_{max} zlSu\ 10^{-8} \ldots\ldots\ldots\ldots (32)$$

Substituting in (27) for E (or E_1) the foregoing expression it obtains

$$E_o = \frac{\sqrt{2}\,\sigma}{\rho}(1 - 1/\gamma)B_{max} zlSu\ 10^{-8} \ldots\ldots\ldots\ldots (33),$$

and putting

$$\eta = \frac{\sqrt{2}\,\sigma}{\rho} \ldots\ldots\ldots\ldots (34),$$

$$E_o = \eta(1 - 1/\gamma)B_{max} zlSu\ 10^{-8} \ldots\ldots\ldots\ldots (35)$$

In Table III are grouped the values of η corresponding to different wave forms of the field flux and E M F as well as to the numerical values of γ which actually occur in medium and high frequency alternators.

The formula (35) will be used later for the verification of the theory.

From the equations (23) and (25)

TABLE III NUMERICAL VALUES OF η

Wave form of E M F	$\gamma = 1.25$	$\gamma = 1.75$	$\gamma = 2.00$	$\gamma = 2.25$	$\gamma = 2.50$	$\gamma = 2.75$	$\gamma = 3.00$	$\gamma = 3.25$	$\gamma = 3.50$	$\gamma = 4.00$	$\gamma = 5.00$
Rectangle	$\eta = 1.00$	$\eta = 1.00$	$\eta = 1.00$	$\eta = 1.00$	$\eta = 1.00$	$\eta = 1.00$	$\eta = 1.00$	$\eta = 1.00$	$\eta = 1.00$	$\eta = 1.00$	$\eta = 1.00$
Trapezoid I	" 1.025	" 1.015	" 1.01	" 1.008	" 1.006	" 1.004	" 1.003	" 1.002	" 1.001	" 0.998	" 0.997
Trapezoid II	" 1.04	" 1.018	" 1.013	" 1.01	" 1.003	" 1.00	" 0.999	" 0.997	" 0.994	" 0.99	" 0.985
Trapezoid III	" 1.05	" 1.01	" 1.007	" 1.00	" 0.993	" 0.988	" 0.983	" 0.977	" 0.975	" 0.97	" 0.962
Trapezoid IV	" 1.06	" 1.005	" 0.993	" 0.982	" 0.972	" 0.965	" 0.957	" 0.944	" 0.948	" 0.94	" 0.932
Sine Wave	" 1.05	" 0.992	" 0.973	" 0.955	" 0.947	" 0.941	" 0.93	" 0.929	" 0.92	" 0.913	

For the fundamental component of the E M F wave

$$E_1 = E = \sigma(1 - 1/\gamma)\frac{L\omega}{\zeta\omega}\frac{uB_{max}}{z} \quad ......(36)$$

$$E_1 = E = \sigma(1 - 1/\gamma)\sqrt{\frac{L\omega}{\zeta\omega}}\sqrt{SluB_{max}}\ 10^{-8} \quad .....(37)$$

Furthermore, in view of the equations (27) and (34) we have

For the complex E M F wave

$$E_o = \eta(1 - 1/\gamma)\frac{L\omega}{\zeta\omega}\frac{uB_{max}}{z} \quad ....(38)$$

$$E_o = \eta(1 - 1/\gamma)\sqrt{\frac{L\omega}{\zeta\omega}}\sqrt{Sl}\ uB_{max}\ 10^{-8} \quad .....(39)$$

The equations (35), (38) and (39) establish certain relations between the electrical and mechanical features of an alternator of the inductor type and the observed E M F at no load.

Examination of Table III shows that for high frequency alternators, usually characterized by lower values of γ, the coefficient η has almost constant value for all practically possible wave forms of the flux, namely $\eta \approx 1$. This same result holds for normal medium frequency alternators, since it has been shown that the wave form of the field flux in them approaches the rectangular distribution ~~of the field~~. In general, it is easy in any case to select from Table III an appropriate value of η.

The expression (35) makes it possible to determine experimentally the coefficient of flux oscillation γ. Since η can be evaluated within a few per cent of error, it is quite permissable to use the equation (35) solving it with respect to γ.

The solution is found to be

$$\gamma = \frac{\eta B_{max} zlSu}{\eta B_{max} zlSu - 10^8 E_o} \quad ....(40)$$

All the quantities in the right hand side of the last equation are known for a given alternator: some of them are capable of being computed, for instance B_{max}; some constitute structural features of the machine, as z, l and S, and others are observed, as for example E_o.

It is noteworthy that a definite wave form of the field flux is connected with a definite upper limiting value of γ. This may be seen from the equation (20) and (19) aw well as from (16) and (16 bis).

In fact, the lowest possible value of α is zero, since in inductor alternators the field flux is unidirectional at any given point on the stator periphery, α being equal to $\frac{B_{min}}{B_{max}}$; therefore it follows from the above mentioned equations that the limiting value of $\gamma = \frac{\Phi_{max}}{\Phi_{min}}$ is to be determined from the equation

$$k\,(1 - 1/\gamma) = 1 \ldots\ldots\ldots\ldots\ldots (41)$$

Referring to the equations (16) and (16 bis) it is seen that

$$1 = \frac{\pi}{2}\,\frac{1 - 1/\gamma}{1 + 0.285\,(1 - 1/\gamma)} \ldots\ldots (42)$$

for the sine wave.

Solving (42) for γ it is found that

$$\gamma_{max} = 4.55 \text{ for the sine flux oscillation wave} \ldots (43)$$

By the same method it will be found that

$$\left.\begin{array}{lllll} \text{for the trapezoid} & & & \text{IV} & \gamma_{max} = 5 \\ \text{"} & \text{"} & \text{"} & \text{III} & \gamma_{max} = 7 \\ \text{"} & \text{"} & \text{"} & \text{II} & \gamma_{max} = 11 \\ \text{"} & \text{"} & \text{"} & \text{I} & \gamma_{max} = 23 \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (44)$$

(See Fig. 9)

But these upper limiting values of γ depending upon the wave form of the field flux may, only in exceptional cases, interfere with the design of an inductor alternator, since usually γ is less than the lowest of those permissible upper limits of γ_{max}.

In those exceptional cases care must be taken of an appropriate form of the field wave so that it should be compatible with the value of γ. As seen from (43) and (44), the closer the wave form of the field flux is to the rectangle, the higher is the upper limiting value of γ.

The foregoing theory has revealed the extraordinary importance of the coefficient of flux oscillation γ. It must be a keystone of any method of designing the inductor alternator.

In one other respect γ plays an important role, namely: in determing the ratio of the effective field d.c. ampere-turns to the reactive a.c. ampere-turns of an armature per pole. This ratio is exceedingly important in the operation of a.c. generators and synchronous motors, since it partly determines the relative reaction of the armature.

The determination of this ratio in inductor alternators is complicated by the fact that only a part of the field d.c. ampere-turns must be taken into account, which part may be termed as effective field ampere-turns. These correspond not to B_{max} but only to $\frac{B_{max} - B_{min}}{2}$. Furthermore, in view of (20) we have

$$\frac{B_{max} - B_{min}}{2} = \frac{B_{max}(1 - \alpha)}{2} = \frac{B_{max}}{2} k\,(1 - 1/\gamma) \ldots\ldots (45)$$

The effective field ampere-turns are defined and determined as such field ampere-turns that are capable of producing the field whose magnetic induction or flux density is equal to $\frac{B_{max}}{2} k(1-1/\gamma)$

Even under this restriction the ratio of the field ampere-turns in inductor medium and high frequency generators to the reactive a.c. ampere-turns per pole is much greater than in ordinary low frequency alternators. This is accounted for by the fact that in the former the a.c. ampere-turns per pole are unusually small in comparison with the field ampere-turns. That property of medium and high frequency inductor alternators has already been used in determining the resonant inductance; it is due to the fact that there is only one stator slot per effective pole and the cross-sectional area of a slot is small.

The suggestions at the end of Section I about how to use the fundamental equations apply also to those of Section II with the only modification that we have to deal with the coefficients λ or γ instead of α.

The results of Sections I and II are independent of the method by which γ is determined.

SECTION III Predetermination of the Coefficient of Flux Oscillation γ. Experimental Formulas for γ; Their Rational Basis.

The saturation curve of inductor alternators shows quite peculiar features which are different from those observed on ordinary a.c. generators of heteropolar type.

As the excitation field increases, the E M F at no load increases up to a certain limit beyond which a further increase in the field brings about a decrease in the E M F at no load.

The importance of that fact is seen at once. Furthermore, it has been shown by several research workers, for instance by Latour, Osnos, Bethenod, ~~and others~~ that the point at which an inductor alternator operates with maximum efficiency corresponds to the greatest E M F obtainable at no load. In making a design for an inductor alternator it is of advantage to assume that its normal operation conditions correspond to those in the neighborhood of the maximum E M F at no load; in other words, the normal excitation field must be close to that at which the E M F reaches its maximum at no load.

The maximum value of the E M F at no load is determined by the maximum of the product $(1 - 1/\gamma)B_{max}$ as it is seen from the expressions (35), (37) and (39). As B_{max} is increased, γ decreases because of the increase in the reluctance of the magnetic path lying within the pole and stator tooth.

The product $(1 - 1/\gamma)B_{max}$ is dependent upon two factors, one of which, namely $(1 - 1/\gamma)$ decreases as another is increased, with the result that finally B_{max} reaches the value beyond which the product $(1 - 1/\gamma)B_{max}$ begins to decrease with a further increase in B_{max}.

Thus the expressions (35), (37) and (39) show in the most evident manner the characteristic property of the saturation curve of the inductor alternator. If it is known how γ varies with B_{max}, it is possible to predetermine the saturation curve, in general, and both B_{max} at which the maximum E M F at no load occurs and the value of that maximum itself, in particular. ϕ_{min} which is the flux passing through the rotor slot (interpolar space) is in parallel with ϕ_{max}, ~~i.e.~~ the flux through the pole face linked with an a.c. coil; therefore, if the reluctance along paths within the pole and stator tooth increases as B_{max} assumes higher values, the ratio $\gamma = \frac{\phi_{max}}{\phi_{min}}$ is decreased. Now the problem arises as to how to determine γ as a function of B_{max}. This problem is superimposed upon another one; namely, how γ should depend upon the ratios: $\frac{\delta}{p_r}$, $\frac{a_r}{p_r}$, $\frac{a_s}{p_s}$, (See Notation and Figs. 10 and 11).

The whole problem of determining γ is very complicated and has not yet been solved in a satisfactory manner.

There exist no general analytical solutions of the problem. The well known analytical solutions given by F. W. Carter[13,14], relate to the particular cases; namely, (1) to the case when the reluctance of a pole may be considered as equal to zero and generally negligible, and (2) to the case of highly saturated poles or teeth, which case occurs in electrical machines of usual commercial type. The same remark applies also to a recent paper by Coe and Taylor[24]

But neither of those cases occurs in medium and high frequency inductor alternators; the reluctance of poles and rotor teeth does not play a negligible part in determining the coefficient of flux oscillation γ, nor do the magnetic induction densities in them, in the working part of the saturation curve, reach its rectilinear saturated part. This point will be considered in detail later. Furthermore, the mentioned particular solutions lose their accuracy as soon as one has to deal with toothed peripheries of both a rotor and stator, and that is precisely the case of inductor alternators.

The graphical methods of solution devised by Lehmann[15] are painstaking, require much time and labor and represent, in a geometrical form, the application of the method of successive approximations. That method becomes even more cumbersome and loses its accuracy when the reluctance of the teeth and poles is to be taken into account as is the case with inductor alternators.

Although Lehmann and Stevenson's method has already been used by Messrs Roth, Metzler ~~Stevenson~~ and Wieseman in application to inductor alternators, other methods more handy and convenient are wanting in determining the value of γ.

The method which has been adopted in the present paper by the author embodies the working out of an empirical formula for γ on a rational basis connected with a special and more definite view taken of the saturation curve of inductor alternators and of the causes that make it differentiate from the usual type of saturation curve.

The numerical results obtained on the basis of the outlined method are checked against the experimental data observed on a number of inductor alternators of various capacities, frequencies, tooth arrangements, as well as of different make and application.

The results of the verification are very satisfactory.

Attention has been directed to the known peculiar property of the saturation curve of the inductor alternator, namely: to the existence of a maximum E M F at no load for a definite value of the field current so that a further increase in the current is connected with the decrease in the E M F. A further step has been made in the present paper by relating the maximum E M F obtainable at no load to the values of flux density in the poles and stator teeth corresponding to the beginning of the knee in the magnetization curve of the material of which the said poles and rotor teeth are built. In other words, since the bending down of the saturation curve of the inductor alternator is rather sharp at the beginning and then tends to assume an assymptotic form, it is clear that this bending down and consequently the maximum E M F which belongs to that part of the saturation curve can be connected only with the knee in the magnetization curve of the corresponding material. The examination of magnetization curves has shown that in general the beginning of the knee corresponds to flux densities from 8000 to 9000 for steel castings and to 10000-11500 for laminations, steel forgings and solid iron. From the moment that those flux densities have been reached in the poles or stator teeth, the E M F must be close to the maximum voltage obtainable at no load. This is the basis of the method for the determination of the maximum E M F and the value of B_{max} (in the air gap) corresponding to it. The outlined method has been applied by the author with very satisfactory results and will be developed further in detail. Since that part of the saturation curve which extends beyond the maximum E M F has but a little practical importance, if any, the problem is confined to the determination of γ only in the region of the magnetization curve not extending beyond the beginning of the knee in the poles and stator teeth.

The successive steps by which the formula for γ has been worked out and verified can be represented as follows:

I. The formula for γ was first established by the author on the basis of general considerations together with experimental and constructional data relating to the following inductor alternators:

No. 1. 1000 cycle inductor homopolar alternator investigated by the author at the laboratory of The Siemens-Halske Company of Petrograd.

No. 2. 500 cycle inductor homopolar alternator designed by the author and built by The Siemens-Halske Company of Petrograd.

No. 3. 10000 cycle inductor homopolar alternator described by B. G. Lamme[16].

II. The formula obtained has been verified by the experimental results observed on the following inductor alternators:

No. 4. 450 cycle inductor homopolar alternator described by Karl Metzler[17].

No. 5. 8000 cycle inductor homopolar alternator described by Karl Schmidt[18].

No. 6. 6500 cycle inductor homopolar alternator
No. 7. 30000 cycle " " "
} Both described[19] by Laffoon in the Electric Journal.

No. 8. 20000 cycle inductor homopolar alternator described by V. P. Vologdin[20].

No. 9. 500 cycle inductor homopolar alternator described by K. Schmidt[21].

No. 10. 500 cycle inductor homopolar alternator described by K. Schmidt[22].

This list of alternators will be referred to in the next section.

III. Besides the experimental information contained in the above papers and articles, some supplementary data relating to constructional features and observed magnitudes as the pole and tooth form, number of parallel circuits in the armature winding, maximum E M F observed at no load etc., in the listed alternators have been kindly communicated to the author, in reply to his letters, by Messrs. Bethenod of Paris, Calvert of Westinghouse Electric and Manufacturing Company, Schmidt of C. Lorenz A. G., Berlin, Alexanderson and Wieseman of General Electric Company, Vologdin of Leningrad.

IV. In some particular limiting cases it has proved possible to get from the empirical formula for γ the same results as those obtainable by means of Carter's analytical method as well as by Lehmann and Stevenson's graphical solution of the problem in the form given by R. Wieseman[23]. It has been possible to determine the optimum ratio a_r/p_r both from the empirical formula worked out for γ in the present paper and by the graphical method according to Wieseman, and in both cases the results have proved to be sufficiently close to one another. This is also regarded as a substantiation of the empirical formula for γ.

Furthermore, the optimum ratio $\frac{a_r}{p_r}$ and the optimum pole width (opt. a_r) determined from that formula, are in good agreement with the respective magnitudes in the alternators referred to.

Likewise some examples of the optimum ratio $\frac{a_r}{p_r}$ given by Karl Schmidt[22] are in satisfactory agreement with the results obtained on the basis of the said formula for γ .

The following is a set of expressions for γ given in general and particular form. They satisfy the requirements imposed upon them by the foregoing paragraphs I, II, III and IV.

The general formula can be presented as follows:

$$\gamma = 1 + \frac{p_r\left[1+y-y\frac{2a_r}{p_r}\right]\frac{2a_r}{p_r}\left[1+y-y\frac{a_s}{p_s}\right]\frac{a_s}{p_s}}{A\delta + C B_{max}^{q}} \quad \ldots\ldots\ldots(46)$$

(See Notation and Figs. 10 and 11);

a_r, p_r, a_s, p_s, <u>are measured in millimeters when introduced in (46)</u>; C and q may, for a certain type of generator, be considered as constant and taken, for a cylinder rotor type of the alternator which is most widely used (see Figs. 1, 1a, 2, 2a),

$$C = 10^{-7}$$
$$q = 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(47)$$

These values hold for the part of saturation curve, not exceeding the maximum voltage at no load. Furthermore, C must have appropriate dimensions to satisfy the requirement of dimensional homogeneity.

The coefficients A and y are slightly dependent upon the ratio $\frac{\delta}{p_r}$, but within certain domains for $\frac{\delta}{p_r}$ may be regarded as constant; B_{max} represents the maximum flux density at no load in the air gap along the central line of a pole at the instant when the said line coincides with the central line of a stator tooth.

With reference to the preceding paragraph I, of this section, the general expression (46) is found to assume the following particular forms.

Inductor alternator class a, high frequency, 10000 cycles and up:

$$\frac{\delta}{p_r} \geqq 0.0794$$

$$\gamma = 1 + \frac{p_r\left[4 - 3\frac{2a_r}{p_r}\right]\frac{2a_r}{p_r}\left[4 - 3\frac{a_s}{p_s}\right]\frac{a_s}{p_s}}{10\delta + B_{max}^2\,10^{-7}}$$

This expression is obtained from the general one (46) by putting $A = 10$ and $y = 3$.

Inductor alternator class b, medium frequency, 8000 cycles and down:

$$\frac{\delta}{p_r} \leqq 0.049$$

$$\gamma = 1 + \frac{p_r\left(2.5 - 1.5\frac{2a_r}{p_r}\right)\frac{2a_r}{p_r}\left(2.5 - 1.5\frac{a_s}{p_s}\right)\frac{a_s}{p_s}}{A\delta + B_{max}^2\,10^{-7}}$$

where $A = 12$ for $\frac{\delta}{p_r} \geqq 0.01$

$A = 11$ for $0.01 < \frac{\delta}{p_r} \leqq 0.025$

$A = 10$ for $\frac{\delta}{p_r} > 0.025$

The particular expression (49) is obtained from the general one (46) by putting $y = 1.5$

Both the expressions (48) and (49) have been worked out on the basis of certain general considerations and the experimental results obtained on the alternators Nos. 1, 2, and 3, and later they have been verified using the structural data and experimental tests of the alternators Nos. 4, 5, 6, 7, 8, 9, 10, (see paragraph II of this Section) with the supplementary information contained in the correspondence referred to in the paragraph III of this Section.

Furthermore, it is found that it is advisable, although not necessary, to subdivide the class b ($\frac{\delta}{p_r} \leqq 0.49$, medium frequency) into two sub-classes: b_1 for $0.01 < \frac{\delta}{p_r} \leqq 0.49$ and b_2 $\delta/p_r < 0.01$. The following expression for γ is recommended to be used for Inductor alternator sub-class b_2, $\frac{\delta}{p_r} \leqq 0.01$.

$$\gamma = 1 + \frac{p_r\left(2 - \frac{2a_r}{p_r}\right)\frac{2a_r}{p_r}\left(2 - \frac{a_s}{p_s}\right)\frac{a_s}{p_s}}{12\delta + B_{max}^2 10^{-7}} \quad \ldots\ldots (50)$$

which expression is obtained from the general one (46) by putting $y = 1$ and $A = 12$. The expression (49) holds for sub-class b_1.

For all the alternators whose data and tests have been used for working out or verifying the expressions for γ (48), (49) (see paragraph II) we have either $\frac{\delta}{p_r} \geqq 0.0794$ or $\frac{\delta}{p_r} \leqq 0.049$ with $y = 3$ and $y = 1.5$ respectively. Therefore it is only natural to assume for the intermediate class of alternators with $0.049 < \frac{\delta}{p_r} < 0.0794$ that $y = \frac{3 + 1.5}{2} = 2.25$, the average of the above values of y for the two classes of alternators. Thus the third class of alternators, c, arises whose characteristic feature is that $0.049 < \frac{\delta}{p_r} < 0.0794$ and $y = 2.25$. Therefore putting in (46) $y = 2.25$, it is obtained for $0.049 < \frac{\delta}{p_r} < 0.0794$

Inductor alternator, class c; $0.049 < \frac{\delta}{p_r} < 0.0794$

$$\gamma = 1 + \frac{p_r\left(3.25 - 2.25\frac{2a_r}{p_r}\right)\frac{2a_r}{p_r}\left(3.25 - 2.25\frac{a_s}{p_s}\right)\frac{a_s}{p_s}}{10\delta + B_{max}^2 10^{-7}} \quad \ldots (51)$$

It is seen that in the general expression for (46) y is a function of $\frac{\delta}{p_r}$ such that it increases as the ratio $\frac{\delta}{p_r}$ increases; but it is found that within wide limits y can be considered as a constant so that three values of y are sufficient to cover practically all cases, while the range of frequencies extends from 450 to 30000 cycles inclusive and that of the characteristic ratio $\frac{\delta}{p_r}$ varies from 0.01 to 0.113. Although it is quite possible to come out with three values of y for the classes of alternators designated a, b, c respectively in order to practically meet all requirements, it is advisable to assign a special value of y=1 for very small ratios $\frac{\delta}{p_r} \leqq 0.01$. but this is not necessary as the experimental verification of the formulas for γ will show .

The expressions for γ afford the possibility of determining the optimum ratio $\frac{a_r}{p_r}$ of the width of a pole to the pole pitch. Taking the expression for in its general form (46) and differentiating it with respect to the ratio $\frac{a_r}{p_r}$, it is found to be

$$\text{Optimum } \frac{a_r}{p_r} = \frac{y+1}{4y} \quad \text{.......(52)}$$

Therefore for the class a " $\frac{a_r}{p_r} = 0.333$

" " " " b " $\frac{a_r}{p_r} = 0.4165$ ....(53)

" " " " c " $\frac{a_r}{p_r} = 0.361$

For these values of $\frac{a_r}{p_r}$, γ reaches its maximum and therefore E M F will have its highest value.

The values obtained for optimum $\frac{a_r}{p_r}$ from the expressions for γ are in satisfactory agreement with the experimental results, on the one hand, and those obtained by Wieseman[23] by means of graphical methods, on the other; this is regarded as a strong evidence in favor of the expressions for γ presented in this article.

Furthermore, it is possible to predetermine from the same formulas the changes in E M F at no load with the variations of the ratio $\frac{a_r}{p_r}$. In one particular case it has been found possible to compare the results obtained by the methods presented in this paper with those arrived at by means of a graphical method used by Wieseman in the quoted article, and both the results have proved to be close to one another. This is also regarded as one of the substations of the formulas and expressions for γ .

Normal form of the inductor alternators

If $\frac{a_r}{p_r} = 0.5$ and $\frac{a_s}{p_s}$ is very close to 1(see Figs. 10 and 11), the inductor alternator is, in the present article, called and referred to as normal. In this particular case the general expression for γ (46) in which the ratios $\frac{2a_r}{p_r}$ and $\frac{a_s}{p_s}$ are to be put equal to 1 will assume the form as follows

$$\gamma = 1 + \frac{p_r}{A\delta + B_{max}^2 10^{-7}} \quad \cdots\cdots (54)$$

where as before $A = 12$ for $\frac{\delta}{p_r} \leqq 0.01$

$A = 11$ for $0.01 < \frac{\delta}{p_r} \leqq 0.025$

$A = 10$ for $\frac{\delta}{p_r} > 0.025$

If the case is taken in which B_{max} is very small so that it can be considered negligible, i.e. $B_{max} \simeq 0$, then the foregoing expression for γ (54) is reduced to the following one

$$\gamma = 1 + \frac{p_r}{A\delta} \quad \ldots\ldots (55)$$

with the same value of **A** as in (54) or in (49).

This is the case where γ can be determined using Carter's analytical method as presented in his article[13].

When $\frac{a_r}{p_r} = 0.5$, or, in other words, $a_r = b_r$ and $\frac{a_s}{p_s} = 1$ and the value of the flux density B is not taken into consideration, which amounts to the foregoing condition that in the expression for γ (54) B_{max} is taken equal to zero, then it follows from Carter's analytical theory of the distribution of the magnetic flux in the air-gap and slot that

$$\frac{\phi_{max}}{\phi_{min}} = \gamma = \frac{1}{1-\sigma} \quad \ldots\ldots (56)$$

where $$\sigma = \frac{1}{90}\tan^{-1}\frac{p_r}{4\delta} - 1.466\,\frac{2}{p_r}\log\left(1+\frac{1}{4}\,\frac{p_r^2}{4\delta^2}\right) \quad \ldots..(57)$$

In Notation of the present article(Figs. 10 and 11), the angle being expressed in degrees and the logarithm being a common one (Carter's paper, p.885). It is obvious that in the expression for γ (56) the meaning of the symbol σ is different from that in the Table I and in the set III of expressions (23), (24), (25).

Although the application of Carter's analytical method to inductor alternators is very restricted being confined to the considered case ($a_s = p_s$; $a_r = b_r = \frac{p_r}{2}$, B_{max} 0), and the case itself has but a little practical importance, its methodological significance lies in the possibility of checking the expression for γ in the limiting case of $B_{max} \approx 0$. The result of this verification is very satisfactory, since the values of γ computed from the author's formula for γ (55) are very close to those obtained on the basis of Carter analytical theory from the expressions (56) and (57), for different values of the characteristic ratio $\frac{\delta}{p_r}$. The numerical results will be given in the next section where the verification of the author's method will be presented. It might be stated that Carter's analytical methods have been devised for generators of commercial type, and whatever conclusions may be reached concerning their extension to inductor alternators, they will not lessen the importance of Carter's theory for commercial generators and motors.

There are other limiting cases in which the results obtained from the general expression for γ (46) coincide with those obtained theoretically with indisputable precision.

It is clear that for great values of either δ or B_{max}, γ must assume the value equal to 1, since for great values of the air-gap δ there must be $\phi_{max} = \phi_{min}$, and to great values of B_{max} in the air-gap δ between a pole and armature core correspond high flux densities in the poles and in the teeth of a stator, which causes very high reluctance in the corresponding parts of the magnetic circuit of an inductor alternator, thus obliterating differences between ϕ_{max} and ϕ_{min} and making γ equal to 1.

The same conclusions are drawn from the general expression for γ (46), putting either δ or B_{max} or both equal to very great values.

Furthermore, in justifying the introduction of the factor $\left[(y+1)-y\,\frac{2a_r}{p_r}\right]\frac{2a_r}{p_r}$ into the right-hand side of the expression for (46) it is seen that for $\frac{a_r}{p_r} = 0.5$ the right-hand side assumes the form of the expression for γ (54) as it must be for the normal type of the inductor alternator according to the foregoing considerations. Moreover γ is reduced to 1 for $\frac{2a_r}{p_r} = 0$, i.e. for $a_r = 0$, as it must also happen, since this would mean that the rotor periphery is smooth and δ is great, the shrinking of the poles making the rotor periphery smooth and the air gap δ great. The reader is referred to Figs. 10 and 11.

Besides satisfying the limiting conditions, the above factor accounts for the maximum value of γ being confined to the region of the ratio $\frac{a_r}{p_r}$ such that $0 < \frac{a_r}{p_r} \leqq 0.5$. (See (52) and (53)).

That this is actually the case is clear from the considerations as follows: Suppose that starting from $a_r = 0.5p_r$ the width of a pole is decreased; then, although both ϕ_{max} and ϕ_{min} are decreased in their absolute values, the rate of the decrease is different, being the greater for ϕ_{min} on account of the greater increase in the slot reluctance in the neighborhood of $a_r = 0.5p_r$, consequently γ increases to a certain limit. Since, for $a_r = 0$, γ equals 1 and is less than for $a_r = 0.5p_r$, there exists a maximum for γ in the region $0 < \frac{a_r}{p_r} \geqq 0.5p_r$.

It is only natural to assume a simple form for the factor accounting for the above changes in γ with the variations of the ratio $\frac{a_r}{p_r}$ and further to adapt it to experimental results arising from the tests of alternators of the type under consideration. It has been found by the author to be possible to solve this problem by means of the factor

$$\left[y + 1 - y\frac{2a_r}{p_r}\right]\frac{2a_r}{p_r} \quad \ldots\ldots\ldots\ldots(58)$$

in the general expression for γ (46)

The same or analogous considerations apply to the second factor in the expression for γ, namely to

$$\left[y + 1 - y\frac{a_s}{p_s}\right]\frac{a_s}{p_s} \quad \ldots\ldots\ldots\ldots(59)$$

where the ratio $\frac{a_s}{p_s}$ plays the same role as the ratio $\frac{2a_r}{p_r}$ in the first factor (58). When a_s ~~shrinks to nothing~~ tends to zero so that in the limit $a_s = 0$, ~~it amounts to saying that~~ there is no iron core within the coils of the armature winding, and consequently the air-gap δ assumes a great value inconsistent with the variations of the magnetic flux, with the final result that $\gamma = 1$. The same conclusion is obtained from the general expression for γ (46), putting $a_s = 0$, on the basis of the factor (59).

On the other hand, if $a_s = p_s$ the case will be that of the normal type of the inductor alternator, and at the same time, in agreement with this result the factor (59) is reduced to 1.

It is found unnecessary to make the general expression for γ (46) fit the cases where $\frac{a_r}{p_r} > 0.5$, since in practice such cases could hardly occur for the simple reason that the value of γ steadily decreases to 1, as $\frac{a_r}{p_r}$ is increased from 0.5 to 1. In this case ϕ_{min} will increase, while ϕ_{max} will remain practically invariable, with the result that γ - which is equal to $\frac{\phi_{max}}{\phi_{min}}$ - will decrease toward 1. Since this would be an obvious disadvantage, the arrangement with $\frac{a_r}{p_r} > 0.5$ is not commendable. In discussing these questions the reader is referred to Figs. 10 and 11.

Now it remains to derive some expressions for the determination of the flux densities B_r and B_s in poles and stator teeth respectively. It has already been pointed out that the problem of determining the maximum E M F at no load has been connected with finding those flux densities in the poles and stator teeth which correspond to the beginning of the knee in the magnetization curve of the material of which the corresponding part of an alternator has been made.

Now consider, with reference to Figs. 8 and 9 and equation(14), the expression $\phi_{max} + \phi_{min} = \frac{B_{max} + B_{min}}{2} p_r l_r$ ..(60)
Since the radial length of a pole is, in general, such that only an insignificant part of the slot flux ϕ_{min} reaches the bottom of a rotor slot, practically the entire flux ϕ_{min} enters a pole

at its lateral sides, and the expression (60) represents a total flux within a pole in the neighborhood of its root, ϕ_{max} being a flux at the top of a pole. Then, denoting the width of a pole near its root by a'_r, and putting the ratio $\frac{2a'_r}{p_r} = t$, it is found to be

$$\phi_{max} + \phi_{min} = \frac{B_{max} + B_{min}}{2} p_r l = B_{max} \frac{\left(1 + \frac{B_{min}}{B_{max}}\right)}{2} \frac{2a'_r l}{t} \quad \ldots\ldots(61)$$

$$B_r = \frac{\phi_{max} + \phi_{min}}{a'_r l} = \frac{B_{max}}{t}\left(1 + \frac{B_{min}}{B_{max}}\right)$$

Since $1 - \alpha = k\left(1 - \frac{1}{\gamma}\right)$ and $\alpha = \frac{B_{min}}{B_{max}}$ on account of (20) and Notation, it follows

$$B_r = \frac{B_{max}}{t}\left[2 - k\left(1 - \frac{1}{\gamma}\right)\right] \quad \ldots\ldots\ldots\ldots\ldots(62)$$

where the numerical value of k is to be taken from Table I or computed by means of the expression (21). The expression (62) holds for solid poles; in the case of laminated rotor teeth it is necessary to introduce in the formula (62) the iron space factor ψ ; then for laminated poles

$$B_r = \frac{B_{max}}{\psi t}\left[2 - k\left(1 - \frac{1}{\gamma}\right)\right] \quad \ldots\ldots\ldots\ldots\ldots(63)$$

Expressions (62) and (63) give the possibility of determining B_r under various conditions connected with the maximum flux density B_{max} in the air-gap.

For finding the expression for B_s the following derivation is available with respect to the trapezoid wave form of the field. From Fig. 9 it is found to be on account of equation (17 bis)

$$\phi_{max} + \phi_{min} = \left(B_{max} - B_{min}\right) \frac{l(a_p + a'_p)}{2}$$

$$\phi_{max}\left(1 - \frac{1}{\gamma}\right) = B_{max}\left(1 - \alpha\right) \frac{l(a_p + a'_p)}{2}$$

$$\phi_{max} = B_{max} \frac{1 - \alpha}{1 - \frac{1}{\gamma}} \frac{l(a_p + a'_p)}{2}$$

Now denoting with a'_s the minimum width of a stator tooth ~~at the level of a stator slot~~ so that $a'_s = p_s - b$, where b is the maximum width of a stator slot and p_s the stator slot pitch (See Figs. 10 and 11), the expression for B_s becomes

$$B_s = \frac{\phi_{max}}{\psi a'_s l} = \frac{B_{max}}{\psi} \frac{1 - \alpha}{1 - \frac{1}{\gamma}} \frac{a_p + a'_p}{2a_s} \quad \text{............(64)}$$

Furthermore, using the equation (19) where $1 - \lambda = 1 - \frac{1}{\gamma}$ and remembering that $a_p = a_s$ in Fig. 9, it is seen that for a trapezoidal wave $B_s = \frac{k' p_s}{\psi a'_s} B_{max}$ ....(65)

where $$k' = \frac{1}{1 + \left(1 - \frac{1}{\gamma}\right)\left(\frac{a_p}{a_p + a'_p} - 0.5\right)} \quad \text{....(66)}$$

It is seen that the coefficient k' depends upon the wave shape of the trapezoidal field flux, namely upon the ratio $\frac{a_p}{a_p + a'_p}$ (Fig. 9)

For the rectangular wave form of the field $k' = 1$, since $a_p = a'_p$, and the expression (65) becomes simply (rectangular wave) $B_s = \frac{B_{max}}{\psi} \frac{p_s}{a'_s}$ ....(67)

For sine flux oscillation on account of (14 bis) there is $\phi_{max} - \phi_{min} = \frac{2}{\pi}\left(B_{max} - B_{min}\right) \frac{p_r l}{2}$ (See Fig. 8)

$$\phi_{max}\left(1 - \frac{1}{\gamma}\right) = \frac{2}{\pi} B_{max}\left(1 - \alpha\right) \frac{p_r l}{2} \quad \text{where } \frac{p_r}{2} = p_s$$

$$\phi_{max} = \frac{2}{\pi} \frac{1}{\left(1 - \frac{1}{\gamma}\right)} B_{max} p_s l$$

and using the equation (17) where $\lambda = \frac{1}{\gamma}$ it obtains

$$\left.\begin{aligned} B_s &= \frac{B_{max}}{\psi}\,\frac{2}{\pi}\,k\,\frac{p_s}{a'_s} \\ \text{(Sine wave field flux oscillation)}\quad B_s &= \frac{k'}{\psi}\,\frac{p_s}{a'_s}\,B_{max} \end{aligned}\right\} \qquad \text{.................(68)}$$

where $k' = \frac{2}{\pi}k$, and the numerical values for k are to be taken from Table I or computed from the formula (16 bis).

It is clear that the expressions (65), (66), (67), (68) can be applied to the determination of the flux density at the upper part of a pole as distinguished from the flux density at its root, with the only difference that the value of the width of a pole at its top or upper part must be substituted for a'_s, namely: a_r should be used instead of a'_s.

The predetermination of the saturation curve by means of the equations (35), or (38), or (39) on the one hand, and the expressions for γ (46), or (48), or (49), or (50), or (51), on the other, must be accompanied by the computation of the flux densities B_r and B_s in both rotor and stator teeth, for which purpose the method presented in this section provides a set of expressions and formulas.

When B_r or B_s assumes the critical value corresponding to the beginning of the knee in the magnetization curve of the material which a corresponding tooth is made of, it means that E_o is close to its maximum value obtainable at no load. Usually the critical values of B_r and B_s occur almost simultaneously hand in hand; but it is not always so due to some particular arrangement of a toothed periphery on either the rotor or stator or on both. For instance, in the case of a tooth arrangement shown in Fig. 1a, it is clear that the magnetization phenomena occurring in the poles will be of much more importance than those in the stator teeth, since the latter become decidedly narrowed only in one cross-sectional plane, and their radial length is considerably less than that of the poles, so that with the circular slot or semi-circular slot arrangement on one of the two elements of an inductor alternator the critical effect of the flux density is to occur first in the teeth of the other one. In this case it would be quite justifiable to confine consideration to the alternator element (rotor or stator) other than that in which slots are circular or semi-circular.

Sometimes the edges of a pole at the periphery, i.e. at the air-gap, are rounded as shown in Fig. 12; in this case the width of a pole a_r at the top is considered in the cross-sectional plane where the narrowing of a pole due to the rounding of the edges begins, the latter being taken care of by assuming a special wave form of the field flux (sine-like or close to it).

Section IV

Verification of the theory and method presented in the foregoing sections.

The following criteria are used for the verification of the theory and method presented in this article.

1. E_o saturation curve.
2. Maximum voltage obtainable at no load.
3. Optimum ratio $\frac{a_r}{p_r}$ and optimum pole width (opt. a_r).
4. Particular limiting cases including that of the normal inductor alternator ($a_r = \frac{p_r}{2}$, $a_s \approx p_s$) for small values of B_{max}. ($B_{max} \approx 0$).

I. E_o Saturation curve

Table IV

Alternators numbered as in Section III	Voltage Computed	Voltage Observed	Field Current i or Field ampere-turns ni
No. 1; f = 1000	81.3	82	i = 0.17 ampere
	116.5	114.5	i = 0.25 ampere
	185.5	180	i = 0.44 ampere
No. 2; f = 500	174	170	i = 0.36 ampere
	207.5	207	i = 0.48 "
No. 3; f = 10000	160.5	167	i = 0.64 "
	401	400	i = 1.8 "
No. 4; f = 450	83.3	88	ni = 1191
	162	165.5	ni = 2477
	191	190	ni = 3326
	198	195	ni = 4000

It is seen that the computed results are in good agreement with the experimental ones.

2. Maximum voltage obtainable at no load and corresponding critical values of B_s and B_r, the flux densities in stator and rotor teeth respectively.

Table V

Alternators numbered as in Sec.III	Computed			Observed
	Maximum voltage	B_s formula (65)	B_r formula (62)	Maximum Voltage
No. 4; f = 450	191 198	10070 10530	8380 8980	195
No. 5; f = 8000	398.5	Circular stator slots	10250 Rotor considered as steel forging	400
	375		8950 Rotor considered as steel casting	
No. 6; f = 6500	275	10000	6300	250
No. 7; rating f = 30000; operated at f = 13200	77	11000	7300	83.5
No. 8; f = 20000	192.5	7570	10000 Rotor considered as steel forging	About 190
	182	6700	8700 Rotor considered steel casting	
No. 9; f = 500	2900	11350	Semicircular rotor slots	3075

The available data relating to the alternators Nos. 5 and 8 do not specify the material of which their rotors are made. Therefore two possibilities are considered, and the results in both cases are substantially in good agreement with experiment.

In accordance with the method adopted the critical values of B_s and B_r determining those of voltage at no load in the neighborhood of its maximum lie within the region 10000 - 11350 for laminations and steel forgings, with a suitable reduction for steel castings; and besides that, in special cases of circular or semicircular slots in one of the alternator elements, rotor or stator, only flux densities in another are considered.

In making the foregoing computations, average magnetic properties of materials have been considered. It is clear that the knowledge of individual magnetization curves will serve better the purpose of the method presented in this article.

The reader is, for further discussion, referred to Appendix which contains detailed computations.

3. <u>Optimum ratio of the width of a pole to the polepitch</u> $\frac{a_r}{p_r}$

a. According to Schmidt's experiment on an inductor alternator No. 10 with $\delta = 1$ mm, $p_r = 26$ mm and characteristic ratio $\frac{\delta}{p_r} = \frac{1}{26} = 0.0385$, the optimum width of a pole $a_r = 10$ mm. According to the presented method (opt. a_r) $= 10.8$ mm, in the same alternator.

b. According to Schmidt, for $\delta = 2$ mm and $p_r = 26$ mm the optimum width of a pole is expected to be $a_r = 8.5$ mm According to the author's method (opt. a_r) $= 8.87$ mm It is seen that the optimum width of a pole in both cases is in very satisfactory agreement with the results obtained on the basis of the present theory.

c. It is found to be

in alternator No. 3 (opt. a_r) = 3.33 mm computed; a_r = 3.175 mm actual
" " No. 5 (opt. a_r) = 5.05 mm " ; a_r = 4.7 mm "
" " No. 6 (opt. a_r) = 10.78 mm " ; a_r = 10.15 mm "
" " No. 7 (opt. a_r) = 2.37 mm " ; a_r = 2.8 mm "
" " No. 9 (opt. a_r) = 32.6 mm " ; a_r = 30 mm "

This coincidence shows that the alternators have been designed to realize the optimum pole width and is regarded as an evidence in favor of the present method.

d. Optimum ratio of the width of a pole to the pole pitch $\frac{a_r}{p_r}$ according to the present method, on the one hand, and Wieseman results obtained by graphical method, on the other, are put together in Table VI.

Table VI

	Author's results	Wieseman's results
for $\frac{\delta}{p_r} \geqq 0.0794$	(opt. $\frac{a_r}{p_r}$) = <u>0.333</u>	(opt. $\frac{a_r}{p_r}$) = 0.34 - 0.355 average = <u>0.3475</u>
for $\frac{\delta}{p_r} \leqq 0.049$	(opt. $\frac{a_r}{p_r}$) = <u>0.4165</u>	(opt. $\frac{a_r}{p_r}$) = 0.38 - 0.49 average = <u>0.435</u>
for $0.049 < \frac{\delta}{p_r} < 0.0794$	(opt. $\frac{a_r}{p_r}$) = <u>0.361</u>	(opt. $\frac{a_r}{p_r}$) = 0.355 - 0.38 average = <u>0.3675</u>
for $\frac{\delta}{p_r} \leqq 0.01$	(opt. $\frac{a_r}{p_r}$) = <u>0.5</u>	(opt. $\frac{a_r}{p_r}$) = 0.49 - 0.5 average = <u>0.495</u>

Furthermore, from Wieseman's findings (by graphical method) it results that for $\frac{\delta}{p_r} = 0.05$

$$\frac{E_2}{E_1} = 1.083, \quad \frac{E_3}{E_1} = 0.905$$

while by the author's method, for the same value of the ratio $\frac{\delta}{p_r} = 0.05$

$$\frac{E_2}{E_1} = 1.09 \ , \quad \frac{E_3}{E_1} = 0.965$$

where E_1 is E M F corresponding to $\frac{a_r}{p_r} = 0.5$, E_2 to $\frac{a_r}{p_r} = 0.4$ and E_3 to $\frac{a_r}{p_r} = 0.20$

This shows ~~quite~~ a satisfactory agreement between the results of the two methods.

4. Limiting cases.

Normal type of inductor alternator($a_r = \frac{p_r}{2}$, $a_s \approx p_s$) and $B_{max} \approx 0$

Table VII

Numerical values of γ for different values of $\frac{\delta}{p_r}$

$\frac{\delta}{p_r}$	from Computed by the formula (56) obtained on the basis of Carter's theory.	from Computed by the formula (55)obtained by the author's method
0.125	1.785	1.8
0.083	2.22	2.2
0.0625	2.595	2.6
0.05	2.98	3.0
0.04165	3.33	3.4
0.03125	4.035	4.2
0.025	4.65	4.635
0.01	9.32	9.32

This table shows that in the limiting case considered the author's formula gives practically the same results as Carter's analytical method, but the latter, as it has already been pointed out, is not applicable to inductor alternators in actual cases. The other limiting cases have been considered in Section III and found to be in favor of the formula (46)presented by the author.

Appendix

In order to illustrate in detail the application of the method presented in this paper the following examples are given.

Inductor alternator No. 4 (Mediumfrequency)

The structural and experimental data relating to this inductor alternator are all the more interesting as they make it possible to use several criteria for the verification of the method presented in this paper.

Predetermination of the saturation durve.

Structural and experimental data are as follows: $f = 450$; $\delta = 2.25$ mm; $p_r = 62.8$ mm; $a_r = b_r = 31.4$ mm; $a_s = 27.4$ mm; $p_s = 31.4$ mm; $l = 7$ cm; $u = 2825$ cm p s; $S = 2 \times 20$; $z = 5$; $a_s' = 23$ mm; $\frac{\delta}{p_r} = 0.0359$; $\frac{a_r}{p_r} = 0.5$; $\frac{a_s}{p_s} = 0.873$. This is the "normal" type in view of the values of $\frac{a_r}{p_r}$ and $\frac{a_s}{p_s}$. As a field flux wave form the trapezoid I is to be assumed since the value of the ratio $\frac{a_s}{p_s}$ in the alternator considered is somewhat in excess of the value of $\frac{a_p'}{a_p}$ for the trapezoid I (Fig 9) ($\frac{a_s}{p_s} = 0.873$, $\frac{a_p'}{a_p} = 0.833$). Therefore in view of the values of γ found later it follows from Tables I and III that $k = 1.055$ and $\eta = 1$.

In view of the characteristic ratio $\frac{\delta}{p_r}$ which is equal to 0.0359 the expression for γ (49), with $A = 10$, must be applied to the alternator considered.

Now using the expressions (65) and (62) for B_s and B_r, it is also possible, while predetermining the saturation curve, to find out the above flux densities and compare them with those determined by K. Metzler who applied a graphical method.

The values of B_{max}, the maximum flux densities in the air gap, have been computed by the same writer from the saturation curve and data of the magnetic circuit and will be used in the following computation to predetermine the saturation curve, as well as B_s and B_r according to the theory presented in this article.

Point 1. $B_{max} = 2860$; $\gamma = 1 + \dfrac{62.8 \times (2.5 - 1.5 \times 0.873) \times 0.873}{22.5 + \dfrac{2860^2}{10^7}} = 3.8$

$1 - \frac{1}{\gamma} = 0.737$; $\eta = 1$; consequently from the expression(35)

$E_o = 0.737 \times 2860 \times 2825 \times 5 \times 2 \times 20 \times 7 \times 10^{-8} = 83.3$ volts against 88 observed, according to the table published in p.70 and the saturation curve represented in p.58 of the article by K. Metzler. The use of the expression (56) based on Carter's theory gives $E_o = 93.5$ volts.

Point 2. $B_{max} = 5720$; $\gamma = 3.535$

$1 - \frac{1}{\gamma} = 0.717$

$E_o = 0.717 \times 5720 \times 2825 \times 5 \times 2 \times 20 \times 7 \times 10^{-8} = 162$ volts against 165.5 observed. The use of the expression (56) gives $E_o = 186.5$ volts. (On the basis of Carter's theory).

Point 3. $B_{max} = 6850$; $\gamma = 3.4$; $1 - \frac{1}{\gamma} = 0.706$

$E_o = 0.706 \times 6850 \times 2825 \times 5 \times 2 \times 20 \times 7 \times 10^{-8} = 191$ volts against 190 observed. The use of the expression (56) gives $E_o = 222.5$ volts. (On the basis of Carter's theory).

Point 4. $B_{max} = 7150$; $\gamma = 3.37$; $1 - \frac{1}{\gamma} = 0.703$

$E_o = 198$ volts against 195 observed.

The use of the expression (56) gives $E_o = 332$ volts.

It is seen that the saturation curve determined by the method of the present paper is very close to that actually observed. This is not, however, the case when the expression (56) based on Carter's analytical theory is used.

The next step will show that the flux densities B_s computed from the expressions (65) and (66) are very close to those obtained by K. Metzler by means of a graphical method.

Predetermination of the flux densities B_s and B_r in the stator and rotor teeth respectively for the foregoing points of the saturation curve.

Since in the case considered the field flux wave form, as has already been shown, is close to the trapezoid I, it is easy to see that $\frac{2a_p}{a_p + a_p'} = \frac{24}{22}$ (Fig. 9) and $k = 1.055$ for the foregoing values of γ (from 3.37 to 3.8); then the computation gives the results as follows:

Point 1. $B_r = 2860\ (2 - 0.737 \times 1.055) = 3550$;

$\psi = 0.9$ for medium frequency alternators

$$B_s = \frac{31.4}{23}\ \frac{1}{0.9}\ \frac{2860}{1 + 0.5 \times 0.737 \times (\frac{24}{22} - 1)} = 4200$$

against 4300 as found by Metzler.

Point 2. $B_r = 7120$;

$B_s = 8400$ against 8600 as determined by Metzler.

Point 3. $B_r = 8380$;

$B_s = 10070$ against 10300 according to Metzler.

Point 4. $B_r = 8980$;

$B_s = 10530$.

There is a close agreement between the results obtained by either method.

Predetermination of the maximum voltage obtained at no load.

According to the view expressed in Section III the maximum voltage at no load corresponds to flux densities of 8000 - 9000 in steel castings and 10000 - 11500 in laminations, solid iron teeth and steel forgings. The examination of the values of B_s and B_r shows that for $B_{max} = 6850 - 7150$ (points 3 and 4) E M F which equals 191-198 volts must be close to the maximum. The maximum voltage observed at no load is equal to 195 volts (pp. 58 and 62 of Metzler's paper) which shows a good agreement between two results.

It would be in place to point out how simple and short all those computations are when compared with the painstaking and lengthy graphical solution exemplified in Metzler's paper, while the accuracy of final results is practically the same. If the trapezoid II is assumed as the field flux wave the results remain substantially the same.

Inductor Alternator No. 3 (High frequency)

Its characteristic features are as follows: $\delta = 0.794$mm; $p_r = 10$ mm; $a_r = 3.175$ mm; $p_s = 5$ mm; $a_s = 4.206$ mm; $u = 10000$ cm ps. The edges of the poles at the air-gap are rounded; $\frac{\delta}{p_r} = 0.0794$; $\frac{2a_r}{p_r} = 0.635$; $S = 400$; $l = 2.7$ cm; $z = 1$; $\frac{a_s}{p_s} = 0.8412$; $\frac{a_r}{p_r} = 0.3175$.

In view of the rounded edges of the poles as well as of the values of the ratios $\frac{a_s}{p_s}$ and $\frac{a_r}{p_r}$ the field flux oscillation is obviously represented by a sine wave.

For $i = 1.8$ amp. (field current) $E_o = 400$ volts from the experimental saturation curve, and $B_{max} = 6900$ as determined from the magnetic circuit and the number of ampere-turns.

In view of the value of the ratio $\frac{\delta}{pr} = 0.0794$ the expression for γ (48) is to be applied with $A = 10$, hence it follows:

$$\gamma = 1 + \frac{10 \times (4 - 3 \times 0.635) \times 0.635 \times (4 - 3 \times 0.8412) \times 0.8412}{7.94 + \frac{6900^2}{10^7}} = 2.3$$

$$1 - \frac{1}{\gamma} = 0.565$$

Accordingly for sine wave and $\gamma = 2.3$

$\eta = 0.955$ from Table III

From the formula (35) E_o is found to be

$E_o = 0.955 \times 0.565 \times 10000 \times 6900 \times 2 \times 200 \times 2.7 \times 10^{-8} = 401$ volts against 400 observed.

It is found from the structural data and experimental saturation curve to be, for $i = 0.6375$ amp., $B_{max} = 2400$ and $E_o = 167$ volts. The computation gives $\gamma = 2.94$, $1 - \frac{1}{\gamma} = 0.66$.

Since $\eta = 0.935$ from Table III.

$E_o = 0.935 \times 0.66 \times 10000 \times 2400 \times 400 \times 2.7 \times 10^{-8} = 160$ volts against 167 volts observed.

Section along CDEFMNOPQL.

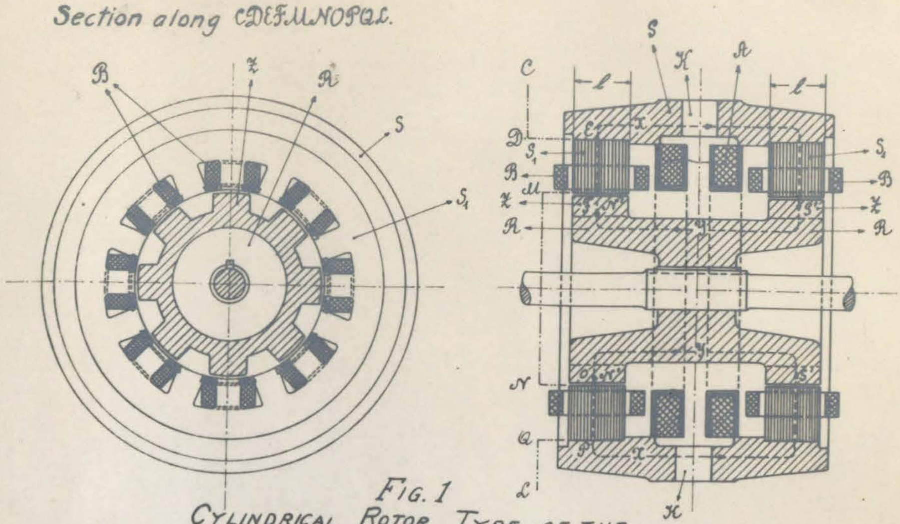

Fig. 1
CYLINDRICAL ROTOR TYPE OF THE
INDUCTOR ALTERNATOR
N.M. OBOUKHOFF

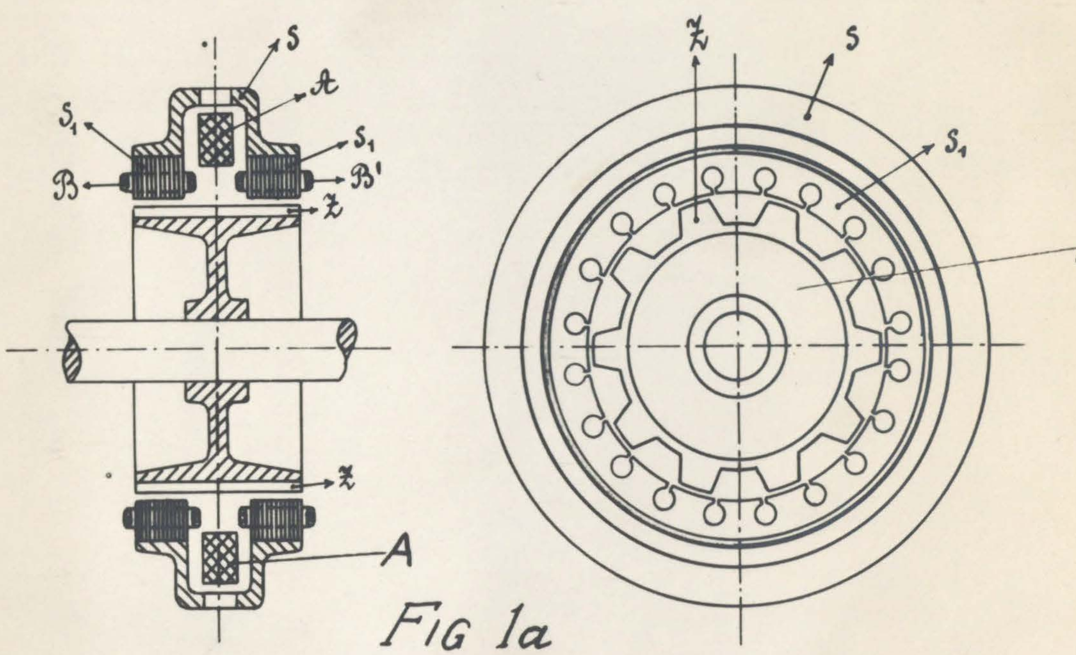

FIG 1a
INDUCTOR ALTERNATOR
N.M. OBOUKHOFF

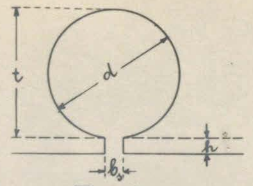

FIG. 1B
CIRCULAR STATOR SLOT
N.M.OBOUKHOFF

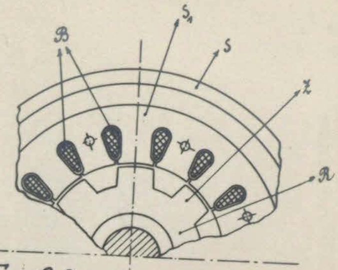

FIG.2 STATOR SLOT FORM
N.M. OBOUKHOFF

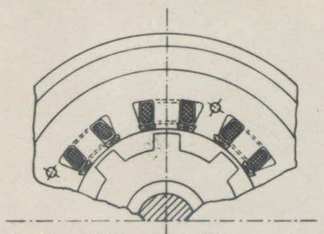

FIG.2a SPECIAL FORM OF
INDUCTOR ALTERNATOR
N.M OBOUKHOFF

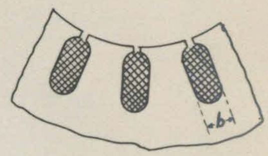

FIG. 2B
STATOR SLOT FORM
N.M.OBOUKHOFF

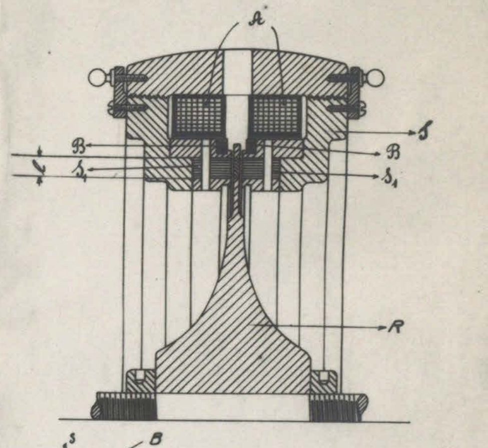

FIG 3
DISK ROTOR TYPE
OF THE
INDUCTOR ALTERNATOR
N.M. OBOUKHOFF

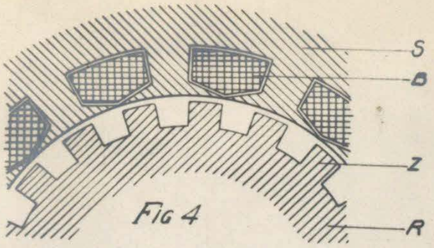

Fig 4

Latour's Inductor Alternator
N.M.Oboukhoff

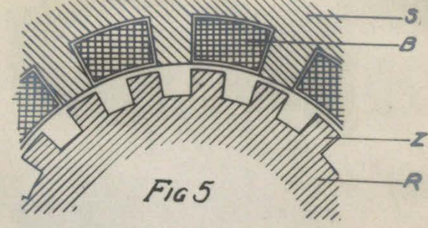

Fig 5

Latour's Inductor Alternator
N.M.Oboukhoff

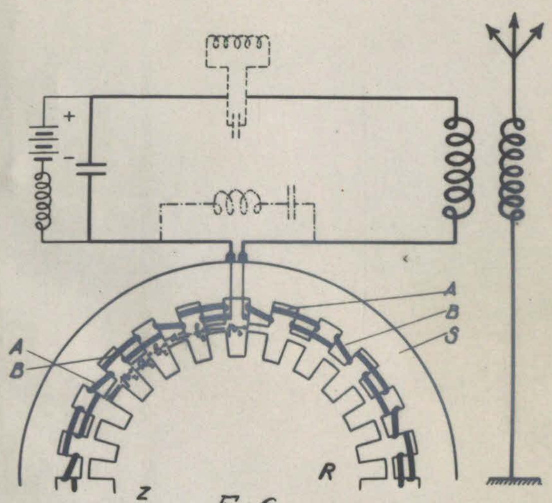

Fig 6

Bethenod's Inductor Alternator
N.M.Oboukhoff

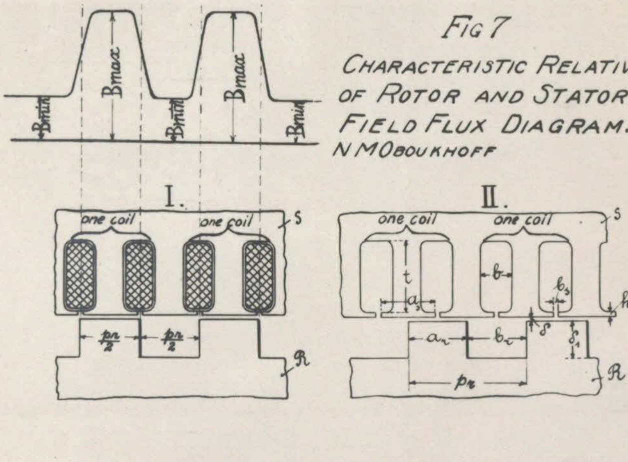

Fig 7

Characteristic Relative Positions of Rotor and Stator and Field Flux Diagram.
N M Oboukhoff

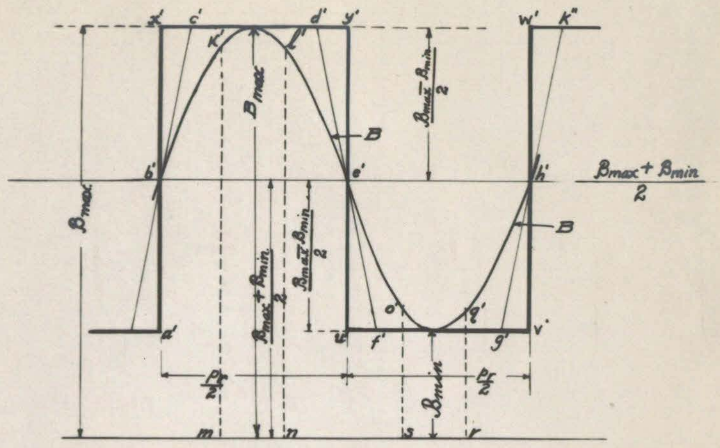

Fig 8

FIELD FLUX DISTRIBUTION FOR THREE CHARACTERISTIC CASES — RECTANGLE, TRAPEZOID, AND SINELIKE DISTRIBUTION.

N. M. OBOUKHOFF

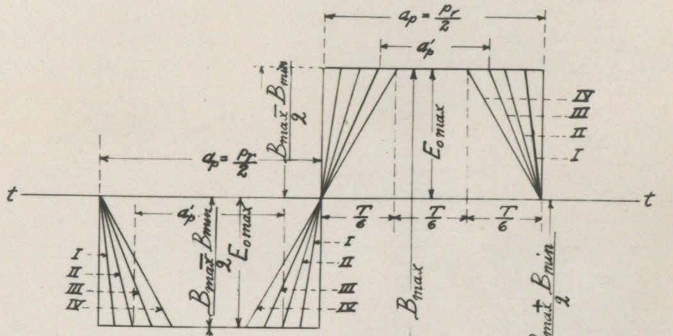

Fig. 9

TYPICAL WAVE FORMS OF FIELD FLUX AND EMF AT NO LOAD. COMBINED DIAGRAM.

N. M. OBOUKHOFF

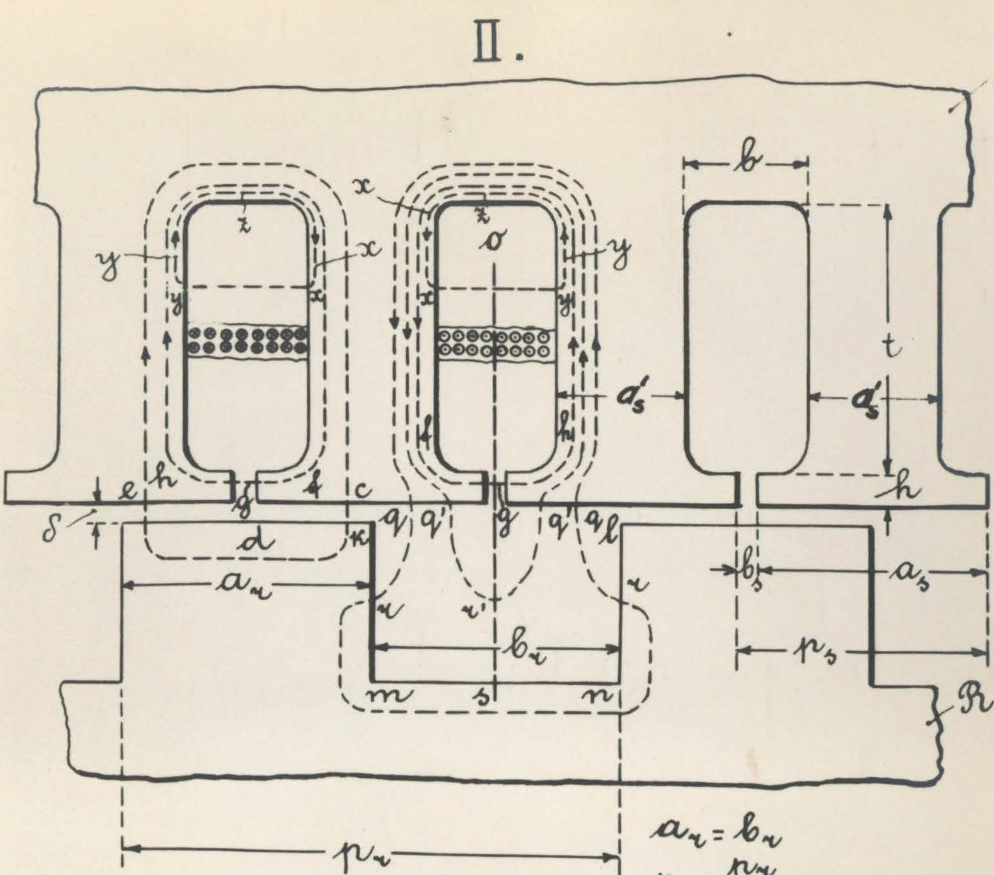

FIG 10

FIELD FLUX DISTRIBUTION AT THE CHARACTERISTIC POSITION II

N. M. OBOUKHOFF

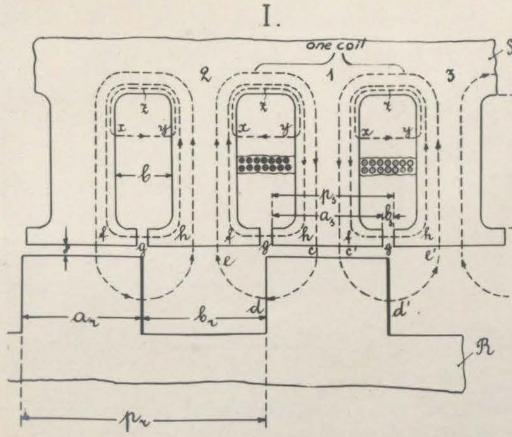

FIG. 11

FIELD FLUX DISTRIBUTION AT THE CHARACTERISTIC POSITION I

N. M. OBOUKHOFF

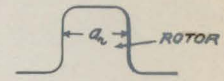

FIG 12

SPECIAL POLE FORM (ROTOR TOOTH FORM)

N. M. OBOUKHOFF

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