A

THESIS

ENTITLED

THE THEORY, DESIGN, AND CONSTRUCTION

OF

SENSITIVE VACUUM THERMOPILES

Presented

by

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Foreword

The uses of thermopiles for measuring radiant energy in expermental physics, astrophysics, and even biology make it important to be able to use them at their maximum efficiency. The maximum efficiency can be obtained by applying a more exact theory to to their design and a careful regard for details in their construction.

It is the purpose of this thesis to enable one to calculate previously the results one can expect, to design the instrument for the highest efficiency, and finally to construct the thermopile.

PART I.

General Design and Theory

of

Sensitive Vacuum Thermopiles.

GENERAL DESIGN AND THEORY OF SENSITIVE VACUUM THERMOPILES.

-ABSTRACT-

A more complete theory of vacuum thermopile design for N junctions for four types of construction. Attention is paid to the practical construction and application of the thermopiles. The sensitivity and quickness of reaching thermal equilibrium are both calculated by considering radiation from the receivers and conduction through the wires; the other factors are proved theoretically and experimentally to be negligable for practical design. The design of a thermopile for sensitivity is made by considering the whole galvanometer circuit. If we assume the approximate correctness of the Wiedemann-Franz Law, simple optimum conditions are obtained. The theory of design is general; however, it is especially adapted to high thermoelectric power bismuth alloy wires at room temperature and for a critically damped galvanometer circuit. The design of a thermopile and choice of a galvanometer for maximum sensitivity depend on the sensitivites, resistances and characteristics of the galvanometers available and the size of the area into which the radiation to be measured can be concentrated. If it is impossible to concentrate the light into as small a receiver as can be made, the number of junctions to be used in series can be calculated. Curves are plotted to facilitate the

best choice of a galvanometer and design of a thermopile for use in many experiments. The formulae were verified experimentally at room temperature. Greater sensitivity was gained by having the thermopile at the temperature of liquid oxygen, as predicted by the theory.

cold junction is formed by the wires joining the conducting leades at the points A and B. The other cold junctions are cemented to an insulator. In Fig.2, the junctions are insulated electrically from the receiver.

If we denote by T the absolute temperature of the cold junctions and the surroundings, n the number of hot junctions, A the area of each receiver or the total receiving area divided by n, p_T the thermoelectric power of the wires at the temperature T, ΔT the temperature of the hot junctions above the cold junctions, I the intensity of the radiant energy in $Cal \cdot cm^{-2} \cdot sec^{-1}$, R_G the resistance of the galvanometer, R_t the resistance of the thermopile, R_e the external resistance in the circuit, σ the constant of radiation (1.37×10^{-12} $Cal \cdot cm^{-2} \cdot sec^{-1} \cdot degrees^{-4}$), λ_1 the thermal conductivity wire #1, λ_2 the thermal conductivity of wire #2, k_1 the electrical conductivity of wire #1, k_2 the electrical conductivity of wire #2, r_ν the reflecting power of the front surface of the receiver for the measured light, e_1 the emissivity of the front surface of the receiver for temperature T, e_2 the emissivity of the back surface of the receiver for temperature T ($\int_0^\infty I_\nu e_{T\nu} d\nu = I e_1$), and x_1 and x_2 the cross sectional areas of the wires divided by the lengths between the hot and cold junctions, the following equation holds for the equilibrium state:

INTRODUCTION

The sensitivity of thermopiles can often be increased several fold by selecting the proper galvanometer and thermopile.

For maximum efficiency, it is obvious that the light to be measured should be concentrated into as small an area as possible and also that the thermopile should recieve all of the light; however, the best choice of a galvanometer, number of junctions in the thermopile, and the resistance of the thermopile are only to be answered by careful analysis.

Practically there is at ones disposal only a limited number of galvanometers; therefore, it is best to design a thermopile for each galvanometer and select the most sensitive combination.

CONSTRUCTION 1.

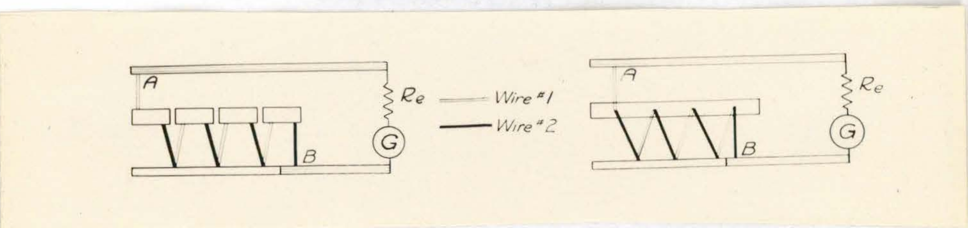


Fig. 1. Fig. 2.

Figs. 1 and 2 show two possible ways of constructing a thermopile of n junctions with no compensating receivers. The first construction is more difficult but it is quicker acting and it is more sensitive. In either construction, one

$Cal\cdot sec^{-1}$ gained = $Cal\cdot sec^{-1}$ lost

$$nIA(1-r_v) = n\{4\sigma T\ A(e_1 + e_2) + \lambda_1 x_1 + \lambda_2 x_2\}\Delta T$$

The current, J,[2,3] in microamps through the galvanometer is

$$J = \frac{nP_T\Delta T}{R} = \frac{nIP_T A(1-r_v)}{4\sigma T^3 A(e_1+e_2)+\lambda_1 X_1+\lambda_2 X_2} \times \frac{1}{R_G + R_e + \frac{n}{k_1 X_1} + \frac{n}{k_2 X_2}} \qquad (1)$$

The maximum J as a function of the two variables x and x is found by solving simultaneously $\frac{\partial J}{\partial X_1} = 0 = \frac{\partial J}{\partial X_2}$ according to La Grange.

$$\frac{\partial J}{\partial X_1} = 0 = 4\sigma T^3 A(e_1+e_2) - \frac{(R_e+R_G)\lambda_1 k_1 X_1^2}{n} - \frac{k_1\lambda_1 X_1^2}{k_2 X_2} + \lambda_2 X_2 \qquad (2)$$

$$\frac{\partial J}{\partial X_2} = 0 = 4\sigma T^3 A(e_1+e_2) - \frac{(R_e+R_G)\lambda_2 k_2 X_2^2}{n} - \frac{k_2\lambda_2 X_2^2}{k_1 X_1} + \lambda_1 X_1 \qquad (2')$$

These equations are satisfied if $k_1x_1 = k_2x_2$ and $\lambda_1 x_1 = \lambda_2 x_2$ and these conditions are compatible with the Wiedemann-Franz law.[1]

Therefore when the Wiedemann-Franz law holds, the loss of heat should be the same through each wire; however, for conditions when this law does not hold, the correct relation should be inserted into equations (2) and (2').

The equations[2,3] $R_t^2 = \frac{n(R_G+R_e)\frac{\lambda}{k}}{\sigma T^3 A(e_1+e_2)}$ and $J_{max} = \frac{IP_T A(1-r_v)}{\frac{4}{R_t^2}\frac{\lambda}{k}(R_G+R_t+R_e)^2}$ (3')

are obtained by substituting $k_1x_1 = k_2x_2$ and $\lambda_1 x_1 = \lambda_2 x_2$ into equation (2) and substituting the result into equation (1). However, these equations are not true optimum relations for in their derivation the need for the galvanometer to be critically damped was ignored.

 Since the sensitivity of a galvanometer can not be

1) A. Sommerfeld, Zeit. F. Phys., Vol 27,27, (1928)
2) Dreish in Handbuch der Phys., Vol 19,830, (1928) derived a similar formula.
3) F.S.Brackett & E.D.McAlister, Rev. of Sci. Insts., p.181, March 1930.

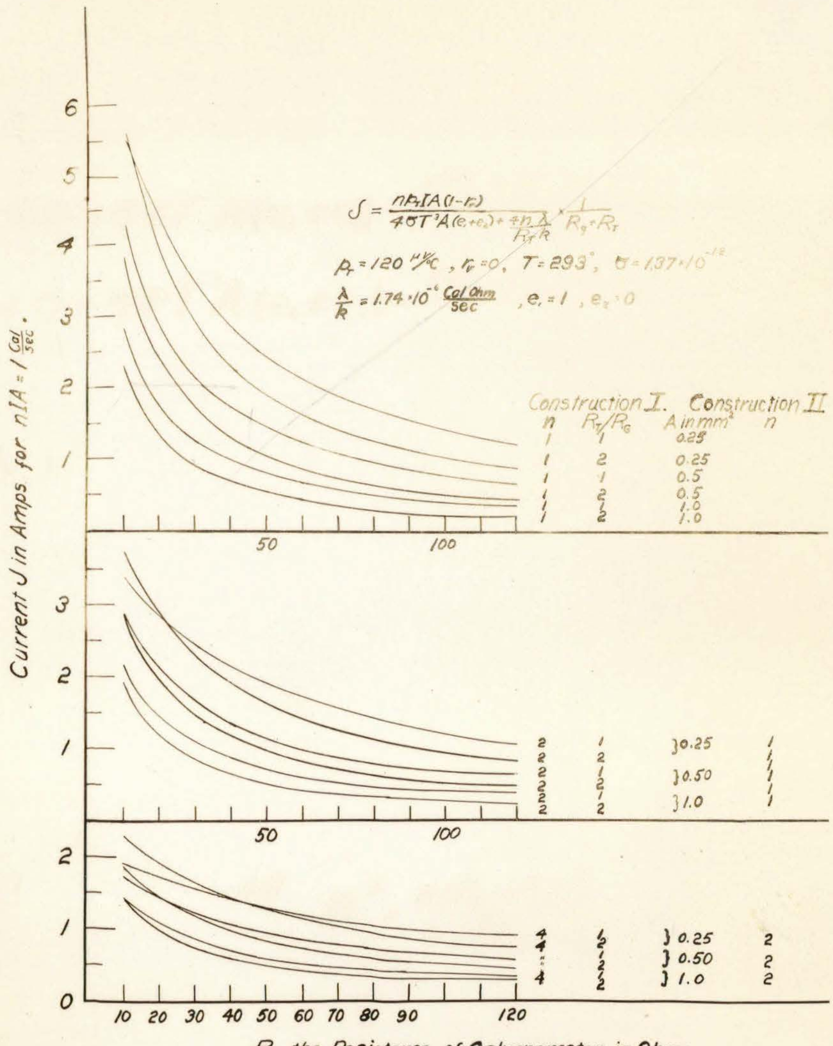

Current J in Amps for nIA = 1 Cal/sec.
R_g the Resistance of Galvanometer in Ohms.
J = nP_T IA(1-r_T) / (4σT³A(e1+e2) + 4nλ/(R_T k)) × 1/(R_g+R_T)
P_T = 120 μV/°C, r_T = 0, T = 293°, σ = 1.37·10^-12
λ/k = 1.74·10^-6 Cal Ohm/sec, e1 = 1, e2 = 0
Construction I. Construction II
n R_T/R_G A in mm² n
1 1 0.25
1 2 0.25
1 1 0.5
1 2 0.5
1 1 1.0
1 2 1.0
2 1 } 0.25 1
2 2 1
2 1 } 0.50 1
2 2 1
2 1 } 1.0 1
2 2 1
4 1 } 0.25 2
4 2
4 1 } 0.50 2
4 2
4 1 } 1.0 2
4 2
6 5 4 3 2 1
3 2 1
2 1 0
50 100
10 20 30 40 50 60 70 80 90 120

Fig. 3.

Fig. 3.

expressed as a simple function of its resistance, it was considered better not to attempt to correct the formulas (3') in that way.

If we put $2n/k_1 x_1 = 2n/k_2 x_2 = R_t$, and assume the validity of the Wiedemann-Franz law, the true optimum relation for J becomes

$$J = \frac{n p_r I A (1-\lambda_r)}{4\sigma T^3 A(e_1+e_2) + \frac{4n}{R_t}\frac{\lambda}{k}} \cdot \frac{1}{R_G + R_t + R_e} \qquad (3)$$

where $R_G + R_t$ is the critical damping resistance of the galvanometer.

The current J is greater when e_1 and e_2 are small and when R_e is zero. Usually, the only external resistance besides the thermopile in the circuit is in the galvanometer leads and that is negligible. The necessity for the galvanometer to be critically damped imposes a relation between R_G and R_t depending on the galvanometer.

The maximum sensitivity is obtained by concentrating the light to be measured onto as small an area as it is possible. After this has been calculated, the best design of the thermopile, whether to make it a single or multiple receiver type and of what resistance, depends on the galvanometers available. To facilitate the best choice of the galvanometer and design of the thermopile, curves are plotted in Fig. 3 of equation (3) for different conditions. The choice of the constants is sufficiently correct for practical design of thermopiles to be used at room temperature. The current sensitivity, resistance, and external critical damping resistance are usually given by the manufacturer. To calculate the current sensitivity from the voltage sensitivity, it is necessary to know if the voltage sensitivity was taken across the galvanometer terminals or in a critically damped circuit.

The usefulness of a thermocouple may depend on its quickness of reaching thermal equilibrium and in all cases the time so required should be less than the period of the galvanometer. The quickness[4] is sometimes defined as t_e, the time for the hot receiver to reach 1 percent of ΔT after the measured radiation is stopped.

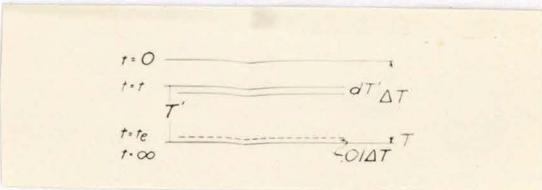


Fig. 4.

In Fig.4, is represented the temperature of the hot receiver above the surroundings at any time t.

Cal·sec^{-1} lost by from the receiver by radiation and conduction by wires = Cal·sec^{-1} lost from mass of receiver and wires.

Therefore

$$t_e = \frac{4.61(m_r c_r + m_1 c_1 + m_2 c_2)}{4\sigma T^3 A(e_1 + e_2) + \frac{4n}{R_t}\frac{\lambda}{k}} \qquad (4)$$

where m_1 and m_2 are the masses of the wires #1 and #2 between a hot and cold junction, c_1 and c_2 are the specific heats of the two wires and m_r is of the receiver and c_r its specific heat. Rigerously $m_r c_r = m_{r'} c_{r'} + m_p c_p$

4) Moll and Burger, Phil Mag. & J. of S. Vol 50, 621 (1925).

where r' refers to the metal receiver and p to the blackening paint. The factor 4.61 is due to t_c being defined as the time to reach 1 percent of equilibrium.

It is to be noticed that the quickness is increased by making the wires very short and of the proper cross sectional area to give the wires the necessary resistance. A thermocouple at the temperature of liquid air is slower than at room temperature.

CONSTRUCTION II.

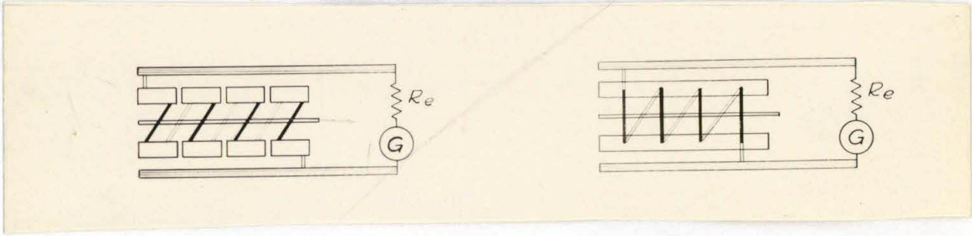


Fig. 5. Fig. 6.

Figures 5 and 6 show two ways of constructing a thermopile of n hot junctions and n compensating junctions. The wires are cemented to the middle insulating support to make the instrument more rugged and to permit it to be repaired. It will be proved that the middle support actually increaces the sensitivity of the thermopile. The junctions in Fig.6 are cemented to the receivers and are insulated electrically from it. The construction in Fig.5 is more difficult than in Fig.6 but it is more sensitive and it is quicker acting.

The analysis for optimum sensitivity is almost identical to that for Construction I. since the compensating junctions essentially represent an external resistance in the galvanometer circuit of Construction I.

The relations for optimum sensitivity is

$$J = \frac{n p_T I A(1-\lambda_r)}{4\sigma T^3 A(e_1+e_2)+\frac{8n\lambda}{R_t k}} \cdot \frac{1}{R_G+R_t+R_e} \qquad (5).$$

The curves in Fig. 3 are helpful in making the best selection of a galvanometer and thermopile of this construction. If the energy is concentrated into half the number of receivers for this conctruction, the sensitivity will be the same as for Construction I.

The quickness is the same as for Construction I,

$$t_c = \frac{4.61(m_\lambda c_\lambda + m_1 c_1 + m_2 c_2)}{4\sigma T^3 A(e_1+e_2)+\frac{8n\lambda}{R_r k}} \qquad (4').$$

In some cases, it is possible to use a thermopile of this construction and allow the light to fall first on one receiver and then on the other[5]. This arrangement gives twice the deflection of a galvanometer. The quickness is unchanged.

CONSTRUCTION III.

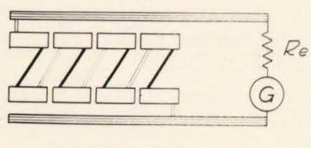


Fig. 7.

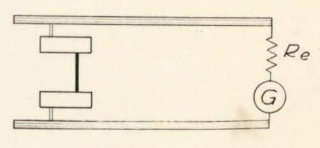


Fig. 8.

Figures 7 and 8 represent a type of construction essentially the same as Construction II without the middle suppot. This type of construction is more often used than Construction II but it is less sensitive, more difficult to build, and usually cannot be repaired.

5) Badger and Cartwright, Phys. Rev., Vol 33, 696 (1929).

A disadvantage of this construction is that all but one of the cold junctions is gaining heat by conduction from two warm receivers and only losing heat by radiation. The best design of this construction is a thermocouple of one hot and one cold receiver and this will be proved to have the same sensitivity as a thermocouple made by Construction II.

Figure 8 shows the best design for a compensating thermocouple of this construction. The previous notation will be used with ΔT_A and ΔT_B to denote the temperature of the receivers A and B above the surroundings.

For the equilibrium state,

$\text{Cal}\cdot\text{sec}^{-1}$ gained by A = $\text{Cal}\cdot\text{sec}^{-1}$ lost by A.

$$IA(1-\lambda_v) = 4\sigma T^3 A(e_1+e_2)\Delta T_A + \lambda_1 X_1 \Delta T_B + \lambda_2 X_2(\Delta T_A - \Delta T_B).$$

$\text{Cal}\cdot\text{sec}^{-1}$ gained by B = $\text{Cal}\cdot\text{sec}^{-1}$ lost by B,

$$\lambda_2 X_2(\Delta T_A - \Delta T_B) = 4\sigma T^3 A(e_1+e_2)\Delta T_B + \lambda_1 X_1 \Delta T_B .$$

J_{max} is found by solving $\frac{\partial J}{\partial X_1} = 0 = \frac{\partial J}{\partial X_2}$ simultaneously.

$$\frac{\partial J}{\partial X_1} = 0 = \frac{\partial}{\partial X_1}\frac{P_T(\Delta T_A - \Delta T_B)}{R} = -\left(R_G + R_e + \frac{2}{k_1 X_1} + \frac{1}{k_2 X_2}\right)\lambda_1 + (h + \lambda_1 X_1 + 2\lambda_2 X_2)\frac{2}{k_1 X_1^2}$$

$$\frac{\partial J}{\partial X_2} = 0 = \frac{\partial}{\partial X_2}\frac{P_T(\Delta T_A - \Delta T_B)}{R} = -\left(R_G + R_e + \frac{2}{k_1 X_1} + \frac{1}{k_2 X_2}\right)2\lambda_2 + (h + \lambda_1 X_1 + 2\lambda_2 X_2)\frac{1}{k_2 X_2^2}$$

A solution compatible with the Wiedemann-Franz law is

$$\lambda_1 X_1 = 2\lambda_2 X_2 \quad \text{and} \quad k_1 X_1 = 2k_2 X_2 .$$

This gives exactly the same sensitivity as for Construction II. The increase in temperature of the hot junction is just off-set by the heating of the cold junction by conduction.

CONSTRUCTION IV

Figure 9 represents a thermopile of n junctions in series and m parallel circuits.

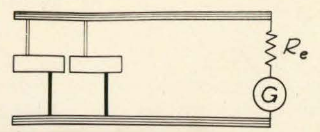


Fig. 9.

For equilibrium,

$Cal\cdot sec^{-1}$ gained = $Cal\cdot sec^{-1}$ lost

$$nmIA(1-r_r) = nm\{4\sigma T^3 A(e_1+e_2)+\lambda_1 X_1+\lambda_2 X_2\}\Delta T$$

$$J = \frac{n p_T \Delta T}{R} = \frac{n I p_T A(1-r_r)}{4\sigma T^3 A(e_1+e_2)+\lambda_1 X_1+\lambda_2 X_2}\cdot\frac{1}{R_G+R_t+R_e}$$

For optimum design, $\lambda_1 x_1 = \lambda_2 x_2$, $k_1 x_1 = k_2 x_2$, and $2n/mkx = R_t$; therefore,

$$J = \frac{nmIp_T A(1-r_r)}{m\sigma T^3 A(e_1+e_2)+\frac{4n}{R_t}\frac{\lambda}{k}}\cdot\frac{1}{R_G+R_t+R_e}$$

This construction, for a given amount of energy falling on a given receiving area, gives exactly the same ideal sensitivity as calculated for Construction I for n junctions. Construction I is much simpler for only 1/m as many receivers are required each having m times the area. This result can be seen intuitively for it has been assumed that there is no radiation from the wires and the receiver is all at the same temperature.

DISCUSSION.

Many thermopiles were constructed with various sizes of gold leaf receivers and wires (diameter 2µ -20µ) and complete agreement with the formulas was obtained by using the following reasonable constants:

p_T = 120 µv/°C, r_r = 0.05, e_1 = 0.85, e_2 = 0.05, $\lambda/k = 1.74\cdot 10^{-6}$.

The derivation of the formulas was made with the following assumptions:

1. The loss of heat from the receiver is due to radiation from the receiver and conduction through the wires. The cooling by conduction and convection of the air is made negligible by the high vacuum. The radiation from the wires can usually be made negligible by using fine short wires so the radiating area is small compared with the area of the receiver. The wires also have a low emissivity for the infra-red corresponding to their temperature.

2. The receiver is all at the same temperature. This can be realized in actual construction very nearly by making the wires very small (diameter 2μ - 8μ) and the receivers of gold leaf which has a specific heat conductivity about 100 times greater than the wires. This assumption was verified experimentally by using different shapes for the receivers and fastening the wires at different positions. No difference was observed whether the same amount of energy was distributed over the entire receiver or concentrated onto a small part of it.

3. The Peltier heat is negligible. The Peltier heat is completely balanced out except for Construction I. The effect can be calculated by a differential method by calculating the cooling of the hot junctions by the current as found from Eq.(1). This correction is usually less than 0.01 percent.

4. The Joule heat is negligible. The Joule heat would increase the sensitivity for Construction I and would be completely balanced out for Constructions II and III; however, the Joule heat J•R is very small.

5. The Thomson heat is negligible. The Thompson heat is ineffective for it occurs in the high resistance wires and in all four types of construction, the heating of one wire is balanced by the cooling of the other to a first aproximation.

As an example in using the curves in Figure 3, let us suppose that the light to be measured can only be concentrated into an area of 1.0 mm^2 and there is at one's disposal a very sensitive galvanometer of 50 ohms resistance ant it requires 50 ohms for the external critical damping resistance. If the stray radiation is sufficiently small to permit Construction I, for 1 x 10^{-6} Cal/sec striking the receiver, 0.8 μA will go through the galvanometer for a receiver of 1mm^2, 1.15 μA for two receivers each 0.5 mm^2, and 1.32 μA for four receivers each 0.25 mm^2. If the galvanometer had a resistance of 20 ohms and required an external critical damping resistance of 20 ohms, then 1.72 μA would go through the galvanometer for one receiver of 1mm^2, 2.15 μA for two receivers each 0.5 mm^2, and 1.70 μA for four receivers each 0.25mm^2.

The use of multiple junctions is an advantage when the the resistance of the thermopile and galvanometer is great.

The best number of junctions to use can be calculated by putting nA = constant (the smallest area into which the light can be concentrated) and differenting Eq. (3) or (5) with respect to n.

The best number of junctions for Construction I is

$$n^2 = \frac{R_t \sigma T^3 (e_1 + e_2) A}{\frac{\lambda}{k}}$$

and for Construction II,

$$n^2 = \frac{R_t \sigma T^3 (e_1 + e_2) A}{2 \frac{\lambda}{k}}$$

The derivations for the optimum conditions have been made by assuming the correctness of the Wiedemann-Franz law; if a better relation is known for the wires to be used, the same general analysis can be applied.

CONCLUSION

1. If stray radiation is sufficiently small, Construction I is more sensitive than Construction II.

2. Construction II is preferable in all cases to Construction III.

3. A construction with the junctions in series-parallel gives no increase in sensitivity.

4. In actual construction, it is possible to obtain the ideal sensitivity for a vacuum thermopile.

5. Only when the Wiedemann-Franz law holds does the optimum condition exist that the loss of heat from a receiver should be the same through each of the two wires.

6. The radiant energy to be measured should be

concentrated onto as small a receiver as possible.

7. The design of a thermopile for maximum sensitivity depends on the galvanometers available.

8. A receiver should only be blackened on the front side and preferably with a material only black to the wave lengths of the light to be measured.

9. A gain in sensitivity was obtained by operating a thermocouple at liquid oxygen temperature. The thermo-electric power of some metals also increases for lower temperatures such as Bi + 11% Sb.

10. The optimum number of junctions to be used can be calculated by a simple formula.

11. The use of multiple junctions is important if the resistance of the galvanometer is great or if the radiant energy to be measured can not be concentrated onto a small receiver.

PART II.

Construction of Sensitive Vacuum Thermocouples.

CONSTRUCTION OF SENSITIVE VACUUM THERMOCOUPLES

- ABSTRACT -

The thermocouple herein described had a sensitivity of about 1μv for 10^{-8} Cal. sec^{-1} falling on the receiver, and reached practical thermal equilibrium in 0.1 sec. The sensitivity in vacuum was usually 20 to 40 times the sensitivity in air. Very short fine wires of high thermoelectric power, bismuth alloys, were soldered to a gold leaf receiver which was blackened on one side by electroplating it with platinum black. The lightness of the gold leaf receivers made the instrument extremely rugged.

Greater sensitivity was obtained for visible radiation by using a receiver of silver leaf tarnished on one side with H_2S; however, the sensitivity decreased for wave-lengths longer than 1μ.

CONSTRUCTION OF SENSITIVE VACUUM THERMOCOUPLES

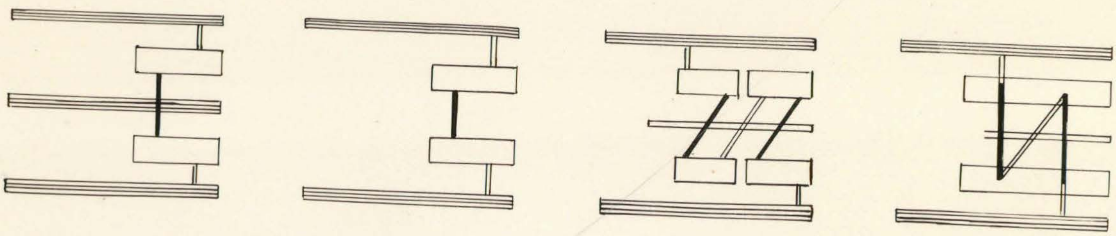


Fig. 1 Fig. 2 Fig. 3 Fig. 4

It was proved that a compensating junction thermocouple as shown in Fig. 1 has exactly the same ideal sensitivity as the one in Fig. 2. The first construction is simpler and makes a much more rugged instrument that can be easily repaired. The thermocouple in Fig. 1 can be used as a single receiver without a compensating junction, for then the copper leads form the cold junction. This gives greater sensitivity and it can be used if stray radiation is sufficiently small.

Fig. 3 shows a multiple junction thermocouple of very rugged construction that can be easily repaired. The wires are cemented to the middle insulating support. Fig. 4 shows a simpler construction often used. The junctions are cemented to the receivers and are insulated from them. This type is a little less sensitive and does not reach thermal equilibrium as quickly.

The wires can be bought from a manufacturer. If not, it is easy to make good monocrystals by the Taylor process. Wires of Bi + 3% Sb and Bi + 10% Sn have a thermoelectric power of about 120 μV/°C and about the same specific resistance. It is desirable to use monocrystals, otherwise the wires are too brittle and break. The wires are first soldered to the copper supports and then made the proper length (usually about 0.5 mm) by touching them with a small soldering iron. The soldering iron can be made by electrically heating a fine sewing needle. It should be tinned with a solder made with 52.5% Bi, 32% Pb and 15.5% Sn (melting point 96°C). The diameter of the wires is usually from 2μ to 15μ depending on the resistance required. For optimum sensitivity, the resistance of the thermocouple wires should be the same, so that the receiver will lose an equal amount of heat through each wire. With the small soldering iron and a 40 power microscope, a small cone of solder can be put on the end of each wire. It is important to use as little solder as possible. The soldering flux I use consists of $ZnCl_2$, alcohol and a little glycerine. The proportion seems unimportant.

The receivers are made of the thinnest gold leaf (approximately 0.1μ thick) and are either blackened by electroplating one side with platinum black or by painting them. Gold leaf can be electroplated with platinum black by soldering it to a platinum wire and dipping it into a plating solution consisting of 0.5 g platinum chloride, 5 mg lead acetate, and 15 cc of water. a 6 volt battery is connected to the gold leaf and to a platinum electrode. The gold leaf floats on the solution and is therefore only plated on one side. A sufficiently black surface can usually be obtained in a few seconds, for it is best to keep the mass of the receivers very small. The gold leaf can be lifted from the solution on a platinum slip. It is then allowed to float on water to clean it, removed as before, dried over a flame, and taken from the platinum slip with a razor. The gold leaf can also be blackened by stretching it across an iron washer and painting it with two parts camphor soot and one part platinum black in dilute lacquer. The part over the hole in the washer is cut out and divided into small receivers of the desired size with a razor. It is convenient to cut the gold leaf receivers on the emulsion side of an unused photographic plate. The painted receivers are slightly less sensitive and not as light as those electroplated. For work in the extreme infra-red, it was found best to paint the receivers with white lead.

The wires that have been tinned are then moistened with a little flux and the receiver is stuck to them. The wires are then securely soldered to the gold leaf by touching the blackened surface with the soldering iron or by bringing a hot wire near it. However, the last method involves a certain amount of risk and because of this, often ruins the entire junction.

It is necessary to have the junction of dissimilar metals heated as high as possible for maximum sensitivity. This was done by bending the wires out of the plane of the copper conductors, so the receiver was soldered only to the tips of the wires. The receiver was considered to have been divided into two equal parts and the tips of the wires were soldered to the center of each part. The heat then could most easily flow to the junction and heat it. The electrical resistance offered by the gold leaf in the circuit was negligible.

If the wires are too large, they can be rolled flat and cut with a razor into small wedges. The tip of the wedge is then soldered to a receiver. This construction was often used for it was structurally very strong.

Two factors which determine the sensitivity of the thermocouple are the size of the receiver and the resistance

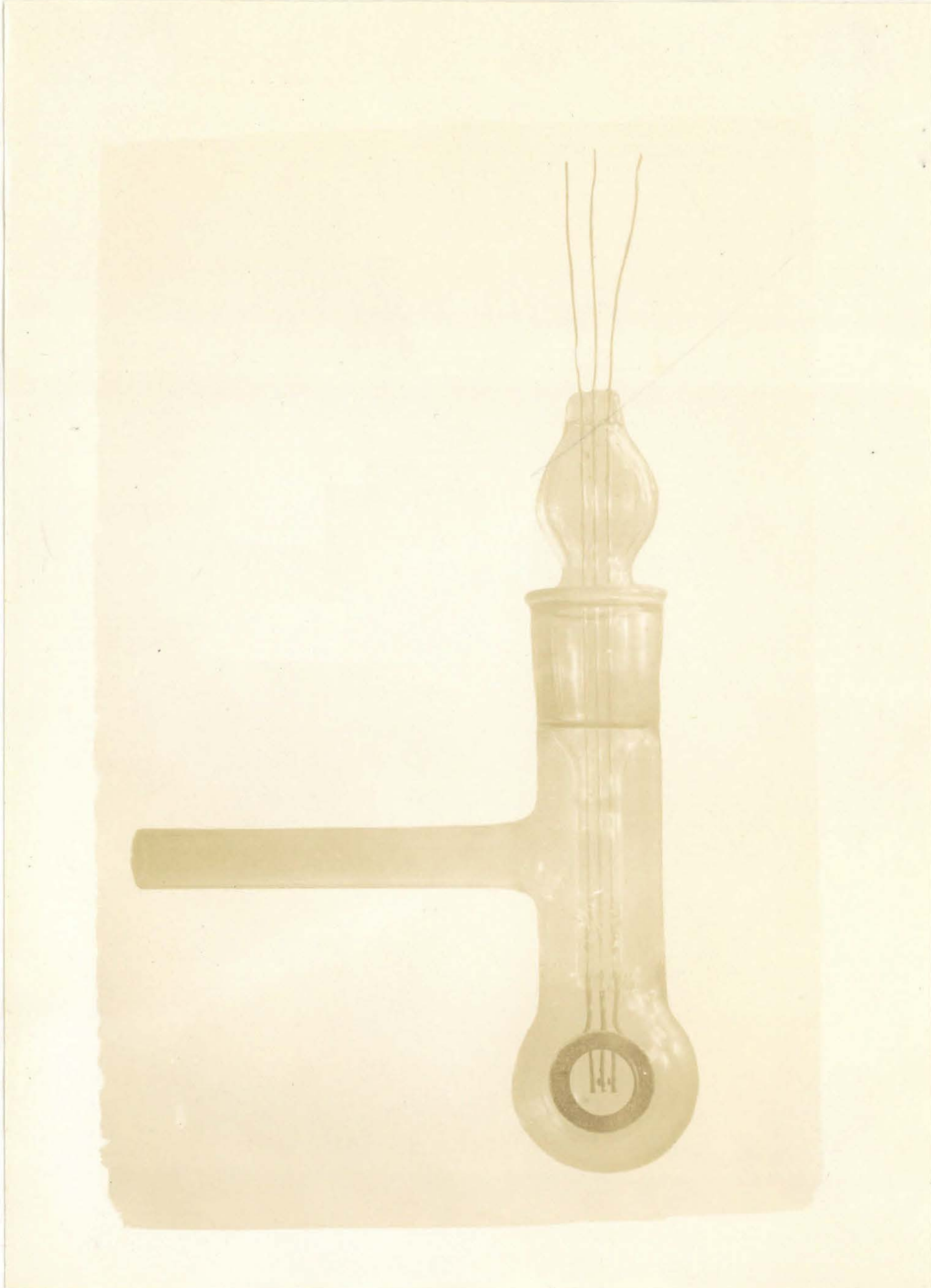


Fig. 5.

of the wires. For a receiver 0.5 mm^2 and a resistance of 20 ohms, the thermocouple gave better than 1 microvolt for 1×10^{-8} $Cal \cdot Sec.^{-1}$ falling on the receiver at room temperature. For a receiver of 0.25 mm^2 and a resistance of 100 ohms, 3×10^{-9} $Cal \cdot Sec.^{-1}$ gave 1 microvolt.

With the cooperation of Dr. Edison Pettit, the dependence of sensitivity on pressure was determined. Different thermocouples had different characteristics, but it was usually necessary to have the pressure under 10^{-4} mm of mercury to gain maximum sensitivity. We found the practical quickness to be 0.1 sec.

Thermocouples constructed in this way had a sensitivity in vacuum about 30 times that in air. The increase in sensitivity depended on the size of the receiver and the resistance of the wires.

For absorbing visible radiation, a more sensitive thermocouple was made by making the receivers of silver leaf tarnished on one side with H_2S gas. The sensitivity for such a thermocouple began to decrease for radiation of wave-lengths greater than 1μ due probably to the transparency of the tarnished surface to the infra-red.

Fig. 5 shows a thermocouple of the construction in Fig. 1. The copper leads supporting the junctions are made heavy enough so that they will not vibrate if the instrument is dropped. The extreme lightness of the receivers, even with wires 2µ in diameter, makes the instrument surprisingly rugged.

A thermocouple was wrapped in cotton and placed in a paper box and thrown several times twenty feet into the air and allowed to fall on the cement sidewalk. The thermocouple was not damaged.

Although it is much more difficult to construct a thermocouple of fine wires and gold leaf, the added sensitivity, speed, and ruggedness makes it worth while.

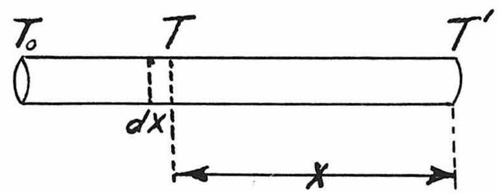
PART III.

Special Problems

SPECIAL PROBLEM I.

A more exact formula for the sensitivity of a thermocouple can be derived by taking into account the radiation from the wires.

Consider the wires to be of circular cross section of length L.



Let T_o denote the temperature of heavy conducting leads and surroundings, T' the temperature of hot junction, p the perimiter of wire, a the area of cross section of wire, ρ the density, c the specific heat, r_3 the reflectivity, T the temperature at distance x from the hot junction, λ the specific heat conductivity, f the emissivity of the surface of the wire (f is equal to $(1 - r_3)\ 4\sigma T_o^3$),and σ the Planch's constant of radiation, then:

$Cal \cdot Sec^{-1}$ across x = $-\lambda a/dx\ dT = -\lambda\ dT/dx\ a$,

$Cal \cdot Sec^{-1}$ across x + dx = $-\lambda(dT/dx + d^2T/dx^2\ dx)a$;

therefore, the gain in $Cal \cdot sec^{-1}$ in length dx $= \lambda\ d^2T/dx^2\ dxa$.

The loss of $Cal \cdot sec^{-1}$ in length dx by radiation is $pdxf(T-T_o)$.

The loss of $Cal \cdot sec^{-1}$ from volume adx = $c\rho\ dT/dt\ adx$.

The $Cal \cdot sec^{-1}$ gained = $Cal \cdot sec^{-1}$ lost by conduction and radiation.

$$\lambda\ d^2T/dx^2\ dxa = dT/dt\ c\rho a\ dx + pdxf(T-T_o)$$

Therefore,

$$\frac{dT}{dt} = \frac{\lambda}{c\rho}\,\frac{d^2T}{dx^2} - \frac{pf\,(T-T_0)}{c\rho a}$$

Let $k = \lambda/c\rho$ and $h = pf/c\rho a = \dfrac{p(1-r_3)\,4\sigma T_0^{\,3}}{c\rho a}$;

then, $dT/dt = k\, d^2T/dx^2 - h(T-T_0)$

For equilibrium state, $dT/dt = 0$, so $d^2T/dx^2 = h/k\,(T-T_0)$.

If we introduce a new variable defined by $u = T-T_0$, for the equilibrium state,

$$d^2u/dx^2 = h/k\ u.$$

The general solution is

$$\mu = A\,e^{\sqrt{\frac{h}{k}}\,x} + B\,e^{-\sqrt{\frac{h}{k}}\,x} .$$

If we substitute $\mu^2 = \dfrac{p(1-r_3)\,4\sigma T_0^{\,3}}{a\lambda} = \dfrac{p}{a\lambda} f$, the general solution becomes $u = T-T_0 = A\,e^{\mu x} + B\,e^{\mu x}$.

The boundary conditions are

$x = L$, $T = T_0$,

$x = 0$, $T = T'$,

and $x = o$, $-\lambda\, dT/dx\ a = \lambda C =$ Cal$\cdot$sec^{-1} lost by receiver (by wire).

If we substitute the boundary conditions into the general solution:

$$o = A\,e^{\mu L} + B\,e^{-\mu L} ,$$

$$-C/a = A\mu - B\mu .$$

The arbitrary constants are

$$A = \frac{-C}{a\mu(1 - e^{2\mu L})}$$

$$B = \frac{C}{a\mu(1 - e^{-2\mu L})} .$$

Therefore,

$$T' = T_0 + C/a\mu \left(\frac{1}{1+e^{-2\mu L}} + \frac{1}{1+e^{2\mu L}} \right)$$

where $C\lambda$ has been defined as the $Cal \cdot sec^{-1}$ lost by the receiver through the wire.

If R is the electrical resistance of the wire, k_1 its specific electrical conductivity, and r its resistance,

$R = L/ak_1 = L/k_1 \pi r$ and $p/a = 2\pi r/\pi r = 2/r$.

If we let $\beta = \sqrt{\frac{f}{\lambda}} \times \pi \sqrt{2}$

then $a\mu = \pi r^2 \sqrt{\frac{p}{a}} \cdot \sqrt{\frac{f}{\lambda}} = \pi r^2 \sqrt{\frac{2}{r}} \times \sqrt{\frac{f}{\lambda}} = r^{3/2}\beta$

and $L\mu = \sqrt{\frac{2}{r}} \cdot \sqrt{\frac{f}{\lambda}} \times R \cdot k_1 \cdot \pi r^2 = R k_1 r^{3/2} \beta$

Therefore $T' = T_0 + \frac{C}{r^{3/2}\beta} \left(\frac{1}{1+e^{-2 r^{3/2} \beta R k_1}} - \frac{1}{1+e^{2 r^{3/2} \beta R k_1}} \right)$.

This can be simplified by defining H so that

$$T' = T_0 + \frac{C}{r^{3/2}\beta H}$$

As was proved in the Design of Vacuum Thermopiles, the two wires from the receiver should have the same resistance and size. The voltage, E, produced by such a thermocouple can be derived (using the used for the Design of Vacuum Thermopiles).

$Cal \cdot sec^{-1}$ gained by receiver = $Cal \cdot sec^{-1}$ lost by receiver

$$IA(1 - r_v) = \{4\sigma T^3 A(e_1 + e_2)\}\Delta T + 2C\lambda \, .$$

Also $\Delta T = T' - T_0 = \frac{C}{r^{3/2}\beta H}$

The thermoelectric power is, therefore,

$$E = P_T \Delta T = \frac{P_T IA(1 - r_v)}{4\sigma T^3 A(e_1 + e_2) + 2\lambda r^{3/2}\beta H} \, .$$

It so happens that for thermocouples of practical dimentions the factor H is nearly equal to unity.

<u>SPECIAL PROBLEM II.</u>

Thermocouples can also be used for amplifing the sensitivity of a galvanometer.

The light from the promary galvanometer is made to fall on a thermo-relay. The deflections of the secondary galvanometer, connected to the thermo-relay, are made greater than those of the primary galvanometer by any desired amount.

If the period of the secondary galvanometer is several seconds, the effect of higher frequency fluctuations of the primary galvanometer is integrated. The theoretical limit for the sensitivity of a galvanometer due to Brownian motion can thus be extended.

Two types of thermo-relays are used.

In one type, the mirror of the primary galvanometer moves the image of a geometric patern over another patern. The light transmitted by the second patern is collected and made to fall on a thermocouple. The amplification can be made perfectly linear by the proper design of the geometric paterns.

The other type of thermo-relay is shown in Fig.1.

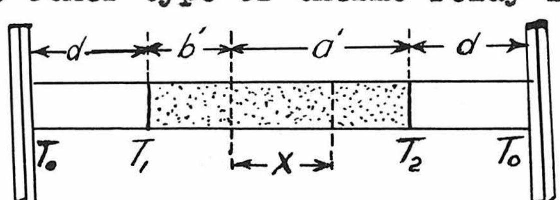


Fig.I.

The strips are extremely thin and of uniform cross sectional area a and width b. The thickness can be considered negligible compared to the width. The portion of the length b' and a' is blackened and it forms one of the thermoelectric elements.

Consider a quantity of heat q $Cal \cdot sec^{-1}$ falling on the receiver at x = 0.

The thermoelectric power produced is

$$E = p_T (T_2 - T_1) .$$

If we use the notation used previously in this thesis, for the section from x to x + dx,

$Cal \cdot sec^{-1}$ gained = $Cal \cdot sec^{-1}$ lost

$$\lambda \, d^2T/dx^2 \cdot dx \cdot a = dT/dt \; c\rho a \; dx + b dx \; f(T-T_0).$$

For the steady state of equilibrium, $dT/dt = 0$;

so $$\frac{d^2T}{dX^2} = \frac{fb}{\lambda a} (T-T_0) .$$

If we let $\gamma^2 = \frac{fb}{\lambda a}$, the general solution is

$$T - T_0 = A e^{\gamma x} + B e^{-\gamma x} \quad \text{for } x = -b' \text{ to } x = a'.$$

The boundary conditions are:

x = a' $\quad$ $T = T_2$,

x = -b' $\quad$ $T = T_1$,

and $q = q_1 + q_2 + q_3 + q_4$,

where $q_1 = \int_0^{a'} bf(T-T_0)\,dX$ is the heat lost by radiation between x = 0 and x = a'; $q_2 = \int_0^{b'} bf(T-T_0)\,dX$ is the heat lost by radiation between x = 0 and x = -b'; $q_3 = \frac{\lambda a}{d}(T_1 - T_0)$ is the heat lost by conduction by the lead between x = -b' and x = -b' - d; and $q_4 = \frac{\lambda a}{d}(T_2 - T_1)$ is the heat lost by the lead between x = a' and x = a' + d.

If we substitute the boundary conditions into the general solution,

$$A = \frac{(T_1 - T_0) - (T_2 - T_0)\ell^{\gamma(a'-b')}}{\ell^{\gamma b'} - \ell^{\gamma(2a'-b')}} ,$$

$$B = \frac{(T_1 - T_0) - (T_2 - T_0)\ell^{\gamma(b'-a')}}{\ell^{\gamma b'} - \ell^{\gamma(a'-2b')}} ,$$

and $q_1 = \int_0^{a'} bf(A\ell^{\gamma x} + B\ell^{-\gamma x})dx = bfA\int_0^{a'}\ell^{\gamma x}dx + bfB\int_0^{a'}\ell^{-\gamma x}dx$

Hence

$$q_1 = bfA\left(\frac{\ell^{\gamma a'} - 1}{\gamma}\right) - bfB\left(\frac{\ell^{-\gamma a'} - 1}{\gamma}\right) ,$$

$$q_2 = bfA\left(\frac{\ell^{\gamma b'} - 1}{\gamma}\right) - bfB\left(\frac{\ell^{-\gamma b'} - 1}{\gamma}\right) ,$$

$$q_3 = \frac{\lambda a}{d}(T_1 - T_0) ,$$

$$q_4 = \frac{\lambda a}{d}(T_2 - T_0) ,$$

and $q = q_1 + q_2 + q_3 + q_4$.

Also (by anology from Special Problem I),

$$T_{x=0} = T_1 + \frac{(q_2+q_3)}{a\mu\lambda}\left(\frac{1}{1+\ell^{-2\mu b'}} + \frac{1}{1+\ell^{2\mu b'}}\right)$$

and $$T_{x=0} = T_2 + \frac{(q_1+q_4)}{a\mu\lambda}\left(\frac{1}{1+\ell^{-2\mu a'}} + \frac{1}{1+\ell^{2\mu a'}}\right)$$

where $\mu^2 = fb/a\lambda$

For a specific problem, the numerical values can be substituted and the thermal electric power $E = p_T(T_2 - T_1)$ can be obtained.

The expressions are simplified if the radiation from the receivers is considered to be negligible compared with the loss of heat by conduction through the leads. For this case, $E = p_T(T_2 - T_1) \propto qa'$,

and therefore the amplification is linear.