THESIS

REINFORCED CONCRETE CHIMNEY DESIGN

L. M. MILLER

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Formulas for Reinforced Concrete Chimney Design.

The analysis which follows is based upon the several fundamental assumptions adopted in reinforced concrete beam design with the additional assumption that, since the concrete is usually thin as compared to the diameter of the chimney, no appreciable error is involved in assuming all material as concentrated on the mean diameter of the shell.

The principles involved in the demonstration of the thickness of steel and concrete are taken from the analysis of Messrs. C. Percy Taylor, Charles Glenday, and Oscar Faber as given by them in the March 13, 1908 issue of "Engineering" (London).

Notation

W = weight in pounds of the chimney above the section under consideration.

M = moment in inch pounds of the wind pressure about that section.

P = total compression in concrete.

T = total tension in steel.

$n = \frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to concrete.

f_c = maximum compression in concrete in pounds, per square inch (measured at the mean circumference).

f_s = maximum tension in the steel in pounds per square inch.

D = mean diameter of sheel in inches

r = mean radius of shell in inches.

t = total thickness of shell in inches.

t_c = thickness in inches of concrete only.

t_s = thickness in inches of an imaginary steel shell of mean radius r, and having a cross-sectional area equivalent to the total area of reinforcing bars.

A_s = total cross-sectional area, in square inches, of reinforcing bars in the section under consideration.

Notation (continued)

k = ratio of distance of neutral axis, from mean circumference on compression side, to diameter D.

j, z, C_p, and C_T = constants for any given value of k.

jD = distance between center of compression and center of tension.

zD = distance from center of compression to center of force due to weight.

Analysis.

Referring to Fig. A, if f_c is the maximum intensity of stress in the concrete at the mean circumference on the compression side, then the intensity of compression in the steel at that point is nf_c. Since f_s is the maximum intensity of stress in the steel at the mean circumference on the tension side, then the variation of

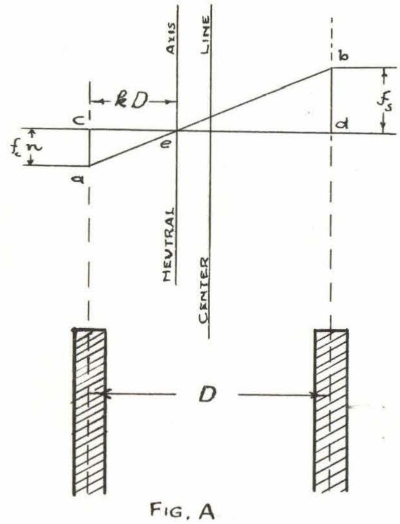


FIG. A

the stress in the steel, across the section "cd", is represented by the straight line "ab" which cuts the line "cd" at "e", thus locating the neutral axis or the line of zero stress. Having assumed a constant value for the modulus of elasticity of the concrete in compression, it therefore follows that, at any point of a given section, the stress in either the concrete or steel is directly proportional to the distance of that point from the neutral axis.

Calling kD the distance of the neutral axis from the mean circumference on the compression side as shown in Fig. A, we have by similar triangles

$$\frac{kD}{D} = \frac{nf_c}{f_s + nf_c}$$

whence

$$k = \frac{1}{1 + \frac{f_s}{nf_c}} \qquad (1)$$

By this formula the position of the neutral axis may be determined for any combinations of f_c,

f_s, and n.

If now, as shown in Fig. B, α represents half the angle subtended at the center by the portion in compression, we have

$$\cos\alpha = (1-2k)$$

from which, for any given value of z, $\cos\alpha$ becomes known as well as α and $\sin\alpha$. Thus having located the neutral axis for any given combinations of

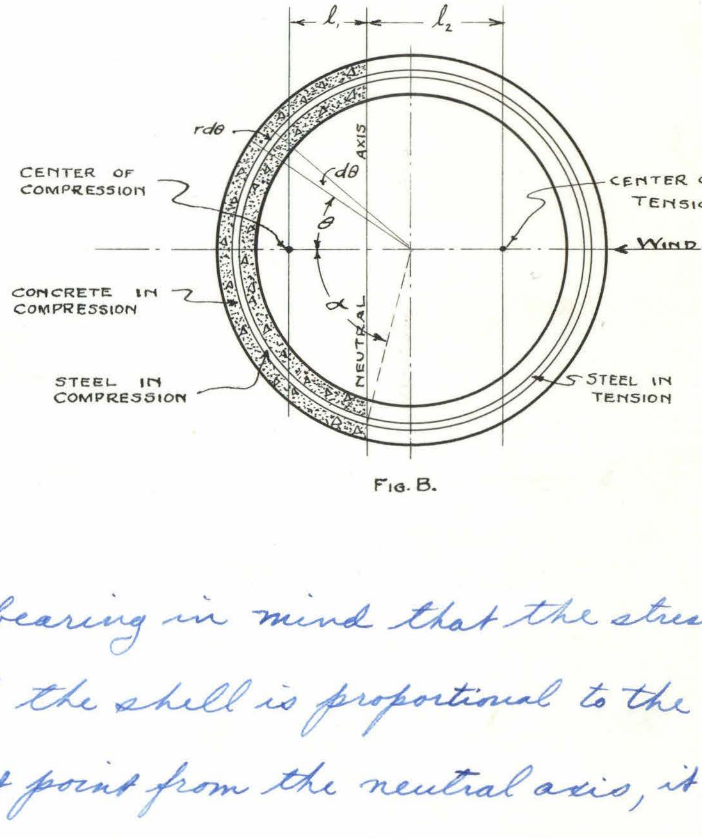


FIG. B.

f_c, f_s and n and bearing in mind that the stress at any point of the shell is proportional to the distance of that point from the neutral axis, it is now possible to determine the total force on the compression side, the total force on the tension side, and also the location of the center of compression and the center of tension.

Considering a small radial element subtending an angle $d\theta$, as shown in Fig. B, we have in this element, since the length of an arc is its radius times the angle,

$$\text{area of concrete} = t_c\, r\, d\theta$$

$$\text{area of steel} = t_s\, r\, d\theta$$

The distance of the element from the neutral axis is $r(\cos\theta - \cos\alpha)$, while the distance from the neutral axis to the point of extreme stress f_c is $r(1-\cos\alpha)$. Therefore the intensity of stress on this elemental area is

$$f_c \frac{r(\cos\theta - \cos\alpha)}{r(1 - \cos\alpha)} \text{ in the concrete}$$

and

$$f_c n \frac{r(\cos\theta - \cos\alpha)}{r(1 - \cos\alpha)} \text{ in the steel.}$$

Assuming these intensities at the mean circumference to represent the average for the entire element, we have the total force on the elemental area (concrete and steel)

$$dP = (t_c + n t_s)\, r\, d\theta \frac{f_c\, r(\cos\theta - \cos\alpha)}{r(1 - \cos\alpha}$$

The total force P on the compression side of the section is therefore

$$P = (t_c + n t_s)\, 2 \int_0^{\alpha} \frac{f_c r(\cos\theta - \cos\alpha)}{(1-\cos\alpha)}\, d\theta$$

Integrating this expression, gives

$$P = f_c r(t_c + n t_s) \frac{2}{(1-\cos\alpha)} (\sin\alpha - \alpha\cos\alpha)$$

Since any given position of the neutral axis determines α, as shown above, this equation may take the form

$$P = C_p f_c r(t_c + n t_s) \qquad (2)$$

in which C_p is a constant for a given position of the neutral axis.

Having determined the magnitude of P, its location, with respect to the neutral axis, may best be found by taking its moment about that axis and dividing by P, thus giving the distance from the neutral axis to the center of compression l, as shown in Fig. B.

As before, the compressive force

on an elemental area is

$$dP = (t_c + nt_s)\, r\, d\theta \,\frac{f_c r(\cos\theta - \cos\alpha)}{r(1-\cos\alpha)}$$

The distance of this force from the neutral axis being $r(\cos\theta - \cos\alpha)$, we have as its moment about that axis

$$dM_c = (t_c + nt_s)\, r\, d\theta \,\frac{f_c r^2(\cos\theta - \cos\alpha)^2}{r(1-\cos\alpha)}$$

while the moment of the total compressive force P is

$$M_c = (t_c + nt_s)\, 2\int_0^\alpha r\, \frac{f_c r(\cos\theta - \cos\alpha)^2}{(1-\cos\alpha)}\, d\theta$$

$$= (t_c + nt_s)\,\frac{2 f_c r^2}{(1-\cos\alpha)}\left[\int_0^\alpha \cos^2\theta\, d\theta - 2\cos\alpha\int_0^\alpha \cos\theta\, d\theta + \cos^2\alpha\int_0^\alpha d\theta\right]$$

Integrating, we have

$$M_c = (t_c + nt_s) f_c r^2\, \frac{2}{(1-\cos\alpha)}\left[(\alpha\cos^2\alpha - \tfrac{3}{2}\sin\alpha\cos\alpha + \tfrac{1}{2}\alpha)\right]$$

Dividing M_c by P we have

$$l_1 = \frac{M_c}{P} = \frac{(\alpha\cos^2\alpha - \tfrac{3}{2}\sin\alpha\cos\alpha + \tfrac{1}{2}\alpha)}{(\sin\alpha - \alpha\cos\alpha)}\, r \qquad (3)$$

Following a similar method of procedure it is possible to determine the total tension and the location of the center of tension.

In accordance with our assumption that the concrete is to take no tensile stress it is evident that in considering the forces on the tension side of the section we are concerned merely with the steel. On the tension side a small element therefore has an area $= t_s r \, d\theta$.

The intensity of stress on this element, being proportional to its distance from the neutral axis, is

$$f_s \frac{r(\cos\theta + \cos\alpha)}{r(1 + \cos\alpha}$$

while the total tension on the small element is

$$dT = t_s r \, d\theta \, f_s \frac{(\cos\theta + \cos\alpha)}{(1 + \cos\alpha)}$$

The total force T, on the tension side of the section is therefore

$$T = 2\int_0^{(\pi-\alpha)} t_s r f_s \frac{(\cos\theta + \cos\alpha)}{(1 + \cos\alpha)} \, d\theta$$

Integrating, we have

$$T = f_s r t_s \frac{2}{(1+\cos\alpha)} [\sin\alpha + (\pi-\alpha)\cos\alpha]$$

Since, as before, any given position of the neutral axis determines α, this equation may take the form

$$T = C_T f_s r t_s \tag{4}$$

in which C_T is a constant for a given position of the neutral axis. By a method similar to that used in considering the force on the compression side we may write the moment, about the neutral axis, of the force on a small element on the tension side as

$$d M_T = t_s r \, d\theta f_s \frac{r(\cos\theta + \cos\alpha)^2}{(1+\cos\alpha)}$$

while the moment of the total tensile force T about this axis is

$$M_T = 2\int_0^{(\pi-\alpha)} t_s r f_s \frac{r(\cos\theta + \cos\alpha)^2}{(1+\cos\alpha)} \, d\theta$$

Integrating, we have

$$M_T = t_s r^2 f_s \frac{2}{(1+\cos\alpha)}\left[(\pi-\alpha)\cos^2\alpha + \tfrac{3}{2}\sin\alpha\cos\alpha + \tfrac{1}{2}(\pi-\alpha)\right]$$

Dividing M_T by T we have as the distance of the center of tension from the neutral axis

$$l_2 = \frac{\left[(\pi-\alpha)\cos^2\alpha + \tfrac{3}{2}\sin\alpha\cos\alpha + \tfrac{1}{2}(\pi-\alpha)\right]}{\left[\sin\alpha + (\pi-\alpha)\cos\alpha\right]}\, r \qquad (5)$$

From formulas (3) and (5) it is evident that the distance between the total force in compression and the total force in tension (i.e., $l_1 + l_2$) may, for any given position of the neutral axis, be expressed as a constant times the diameter D. Thus $l_1 + l_2 = jD$ as shown in Fig. C, zD may represent the distance of the center of compression from the center of the chimney, z also being a constant for any given position of the neutral axis.

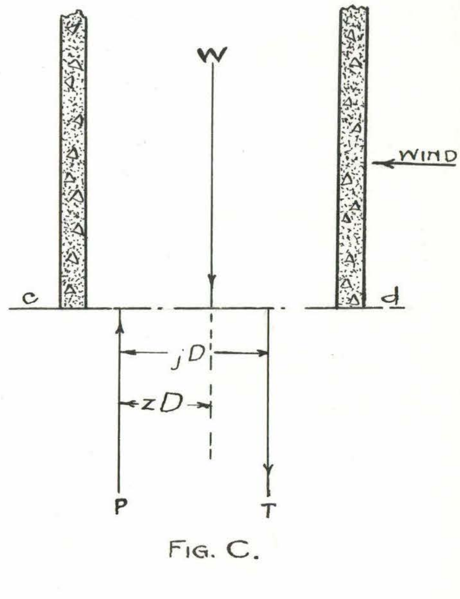


FIG. C.

In a chimney the tensile and compressive stresses which we have been considering are produced by a combination of wind pressure and the weight of the chimney. Thus, on any horizontal section cd, as shown in Fig. C, the forces external to that section are: the horizontal pressure of the wind, causing a moment M about the section, and a central vertical load W representing the weight of that portion of the chimney above the section under consideration. These forces are resisted, and held in equilibrium, by the forces P and T which represent the compressive and tensile stresses in the concrete and steel.

The system of forces as shown in Fig. C. must be in equilibrium. Hence taking moments about the force P, we may write

$$T j D = M - W z D$$

But

$$T = C_T f_s r t_s$$

Therefore

$$C_T f_s r t_s j D = M - WzD$$

Whence

$$r t_s = \frac{M - WzD}{C_T f_s j D}$$

The total area of steel $A_s = 2\pi r t_s$

Therefore

$$A_s = \frac{2\pi (M - WzD)}{C_T f_s j D} \qquad (6)$$

From Table I, page 20, it may be seen that the constant j changes but slightly for a considerable variation in the position of the neutral axis.

Taking $\frac{2\pi}{j} = 8$ for all cases, equation (6) may be

$$A_s = \frac{8(M - WzD)}{C_T f_s D} \qquad (7)$$

While this formula is not exact, the error involved is inappreciable for almost any case so that formula (7) may always be used instead of formula (6).

Applying now the condition that the summation of all vertical forces must be zero, we have

$$P - T = W$$

Substituting values of P and T as previously found, the equation becomes

$$C_P f_c r(t_c + n t_s) - C_T f_s r t_s = W$$

Transposing and solving for t_c we obtain

$$t_c = \frac{W + (C_T f_s - C_P f_c n) r t_s}{C_P f_c r}$$

The total thickness of the shell is

$$t = t_c + t_s$$

whence

$$t = \frac{W + (C_T f_s - C_P f_c n) r t_s}{C_P f_c r} + t_s$$

For convenience in use, after having determined A_s by the formula given above, by substituting $r = \frac{D}{2}$ and $t_s = \frac{A_s}{\pi D}$, this formula for "t" may best be written

$$t = \frac{2W + (C_T f_s - C_P f_c n)\frac{A_s}{\pi}}{C_P f_c D} + \frac{A_s}{\pi D} \qquad (8)$$

In Table II, page 20, is given values of "k", the location of the neutral axis, for various combinations of f_c, f_s and n; while Table I, page 20, gives the corresponding values of the constants C_P, C_T, z and j for various positions of the neutral axis.

Shear or Diagonal Tension.

Having determined the necessary thickness of shell and vertical reinforcement, the size and spacing of the circular steel hoops must be considered. The external forces produce shear and diagonal tension which may be analyzed similarly to like stresses in rectangular beams, and the reinforcement necessary to resist the diagonal tension, which is a function of the vertical tension, may be determined. Usually this reinforcement is not so great as that which it is advisable to insert for the proper distribution of temperature stresses, but nevertheless it should be determined to be sure that it is sufficient in quantity.

The concrete should never be relied upon to carry any tension or vertical shear because the expansion from the heat may cause vertical cracks in the concrete. These need not be considered dangerous if sufficient horizontal reinforcement is provided. Considering the stresses due to vertical shear, it may be easily shown that at any horizontal section of a chimney the vertical shear per inch of height is the total horizontal shear on that section divided by the distance between centers of tension and compression, jD. With this as a basis there may be developed a formula for practical use in determining the necessary area and spacing of horizontal steel hoops at any given section. Thus let

h_t = height, in feet, of chimney above section under consideration.

F = effective wind pressure against chimney in pounds per square foot.

f_s = allowable tensile stress in pounds per square inch in steel hoops.

D = mean diameter of shell in inches.

P_o = ratio of area of steel hoop to area of concrete.

At any horizontal section of a chimney the total shear on that section is equal to

$$\frac{D}{12} h_e F$$

while the maximum shear per inch of height is

$$\frac{D\, h_e F}{12\, jD}$$

Having seen that for all positions of the neutral axis j remains practically the same, and giving j an average value of, say, 0.783, the expression for the maximum vertical shear per inch of height becomes

$$0.106\, h_e F$$

while the shear or diagonal tension in one foot of height is $12 \times 0.106\, h_e F$.

The area of steel in one foot of height of chimney will be $12\, t\, p_o$ and the stress the hoops in this height are capable of sustaining on their two sections is

$$2 \times 12\, t\, p_o\, f_s$$

Equating these we have

$$12 \times .106 h_e F = 2 \times 12 \, t \, p_o f_s$$

whence

$$p_o = \frac{h_e F}{18.8 f_s t}$$

This ratio of steel is for shear or diagonal tension only. To provide for temperature stresses or rather to distribute the strains so as to prevent the localization of cracks, an additional amount of horizontal steel is needed. This may be provided for arbitrarily by assuming 0.25% steel or rather .0025 for temperature stress in addition to the steel for shear. Expressing this as a formula for ratio of steel gives

$$P_o = \frac{h_e F}{18.8 f_s} + .0025 \qquad (9)$$

Small rods spaced 6 to 10 inches apart except in the upper part of the stack where the spacing may be greater are advised.

TABLE I

k	C_P	C_T	z	j
0.050	0.600	3.008	0.490	0.760
0.100	0.852	2.887	0.480	0.766
0.150	1.049	2.772	0.469	0.771
0.200	1.218	2.661	0.459	0.776
0.250	1.370	2.551	0.448	0.779
0.300	1.510	2.442	0.438	0.781
0.350	1.640	2.333	0.427	0.783
0.400	1.765	2.224	0.416	0.784
0.450	1.884	2.113	0.404	0.785
0.500	2.000	2.000	0.393	0.786
0.550	2.113	1.884	0.381	0.785
0.600	2.224	1.765	0.369	0.784

TABLE II

Maximum tensile stress, f_s	k Ratio of depth of neutral axis to depth of steel below most compressed surface.				
	$n = 15$				
	Maximum compressive stress in concrete, f_c				
	300	400	500	600	700
8,000	.360	.428	.484	.530	.568
9,000	.334	.400	.454	.500	.538
10,000	.310	.375	.428	.474	.512
11,000	.290	.353	.405	.450	.488
12,000	.272	.334	.384	.428	.466
13,000	.257	.316	.366	.409	.447
14,000	.243	.300	.349	.391	.428
15,000	.231	.286	.334	.375	.417
16,000	.220	.272	.319	.360	.396
17,000	.210	.261	.306	.346	.387
18,000	.200	.250	.294	.334	.368
19,000	.192	.240	.283	.322	.356
20,000	.184	.231	.272	.310	.344

Design.

Problem: Design a reinforced concrete chimney for a boiler capacity of 2000 H.P.

From Table 45, Steam Power Plant Engineering, Gebhart: At 5 lbs. fuel per boiler horsepower, and for 2000 H.P. Height of stack = 225 ft. Diameter = 90 inches, Area = 44.18 square ft. Effective Area = 40.19 square ft.

Wind force equivalent to a 150 miles per hour gale = 50 lbs. per square ft. of area. Effective diameter on a cylinder exposed to wind force = .6 true diameter. Thus a force due to wind of $.6 \times 50 = 30\#/ft^2$ of projected chimney area is used.

This value for wind force is the most common one used by designers of chimneys. The Weber Chimney Co. uses this value.

Distance down from top of chimney	Inside diameter of chimney
45 ft	102 inches
65 "	107 "
85 "	113 "
105 "	118 "
125 "	123 "
145 "	129 "
165 "	134 "
185 "	139 "
195 "	142 "
205 "	145 "
215 "	147 "
225 "	150 "

Using $f_c = 500\#/in^2$ and $f_s = 14{,}000\#/in^2$

$$t = \frac{2W + 6500\,A_s}{820\,D} + \frac{A_s}{\pi D}$$

$$A_s = \frac{8(M - Wz\,D'')}{32{,}700\,D''}$$

$$P_0 = \frac{h_e F}{18.8 f_s t} + .0025 = \frac{225 \times 30}{18.8 \times 14{,}000 \times 15} + .0025 = .0042$$

Area of bottom spiral steel $= 225 \times 12 \times \frac{445}{2} \times .0042 =$
$= 108$ sq. in.

∴ Use ½" φ Spial 8 inch pitch.

Height of chimney above section, $H = 45$ ft $= h$

Assumed shell thickness, $t = 4$ inches $= .33$ ft

Mean shell diameter, $D' = 8.33$ ft $= 100$ inches

Mean outside shell diameter, $\delta'' = 104$ inches

Moment due to wind force, $M = 15\delta'' h^2 = 3{,}160{,}000$ in lbs.

Weight of shell above section, $W = 472\, D'H't' = 58{,}800$ #

$$A_s = \frac{8(3{,}160{,}000 - .427 \times 58{,}800 \times 100)}{32{,}700 \times 100} = 1.59 \text{ sq. in}$$

Use $10\text{-}\frac{3}{4}''\phi = 5.6$ sq in

$$t = \frac{2 \times 58{,}800 + 6500 \times 1.59}{820 \times 100} + \frac{1.59}{\pi \times 100} = 1.56 \text{ in.}$$

$\therefore$ 4 inch thickness is Ok.

Height of chimney above section, $H = 65$ ft

Assumed shell thickness, $t = 4$ inches $= .33$ ft.

Mean shell diam, $D = 8.54$ ft $= 102.5$ inches

Mean outside shell diam, $\delta'' = 104$ inches

Moment due to wind force $= 6{,}770{,}000$ inch lbs.

Weight of shell above section $= 87{,}000$ #

$$A_s = \frac{8(6{,}770{,}000 - .427 \times 87000 \times 102.5)}{32{,}700 \times 102.5} = 7.07 \text{ sq. in.}$$

Use $13 - \frac{3}{4}''\phi = 7.27$ sq. in.

$$t = \frac{2 \times 87{,}000 + 6500 \times 7.07}{820 \times 102.5} + \frac{7.07}{\pi \times 102.5} = 2.62 \text{ in}$$

$\therefore t = 4''$ is ok.

Height of chimney above section, H= 85 ft.

Assumed shell thickness, t = 4 inches = .33 ft.

Mean shell diam. D = 117 inches = 8.8 ft

Mean outside diam, δ" = 109.5 inches

Moment due to wind = 11,850,000 in. lbs.

Weight of shell above section = 116,000 #

$$A_s = \frac{8(11,850,000 - .427 \times 116,000 \times 117)}{32,700 \times 117} = 12.6 \text{ sq. in}$$

Use 23-3/4" ⌀ = 12.9 sq. in.

$$t = \frac{232,000 + 6500 \times 12.6}{820 \times 117} + \frac{12.6}{117\pi} = 3.30 \text{ in.}$$

∴ 4" assumed is OK.

Height of chimney above section, H= 105 ft.

Assumed shell thickness, t = 4 inches = .33 ft.

Mean shell diam. D = 122 inches = 10.16 ft

Mean outside diam, δ" = 112 inches

Moment due to wind = 18,530,000 in. lbs.

Weight of shell above section = 148,500 #

$$A_s = \frac{8(18,530 - .427 \times 148,500 \times 122)}{32,700 \times 122} = 21.6 \text{ sq. in.}$$

$$t = \frac{297,000 + 6500 \times 21.6}{820 \times 122} + \frac{21.6}{122\pi} = 4.43$$

∴ 4 inches is not sufficient

Recalculate @ t = 4.5 inches

Height of chimney above section, $H = 105$ ft

Assumed shell thickness, $t = 4.5$ inches $= .375$ ft

Mean shell diam. $D = 122.5$ inches $= 10.16$ ft

Mean outside diam. $\delta = 112$ inches

Moment due to wind $= 18{,}530{,}000$ in. lbs.

Weight of shell above section $= 152{,}000$#

$$A_s = \frac{8(18{,}530{,}000 - .427 \times 152{,}000 \times 122.5)}{32{,}700 \times 122.5} = 21.1 \text{ sq. in.}$$

Use 38-3/4" ϕ = 21.3 sq in.

$$t = \frac{304{,}000 + 6500 \times 21.1}{820 \times 122.5} + \frac{21.1}{122.5\pi} = 4.4 \text{ in.}$$

∴ 4.5 inches is ok.

Height of chimney above section, $H = 125$ ft.

Assumed shell thickness, $t = 5 = .417$ ft.

Mean shell diam. $D = 128$ inches 10.67 ft.

Mean outside diam. $\delta'' = 116.5$ inches

Moment due to wind $= 27{,}300{,}000$ in. lbs.

Weight of shell above section $= 193{,}800$#

$$A_s = \frac{8(27{,}300{,}000 - .427 \times 193{,}800 \times 128)}{32{,}700 \times 128} = 32 \text{ sq in.}$$

$$t_s = \frac{387{,}600 + 6500 \times 32}{820 \times 128} + \frac{32}{128\pi} = 5.68 \text{ in}$$

∴ 5 inches is not sufficient

Recalculate @ 6 inches

Height of chimney above section, $H = 125$ ft.

Assumed shell thickness, $t = 6$ inches $= .5$ ft

Mean shell diam. $D = 129$ inches

Mean outside diam. $\delta = 118.5$ inches

Moment due to wind $= 27,700,000$ in. lb.

Weight of shell above section $= 203,000\#$

$$A_s = \frac{8(27,700,000 - .427 \times 203,000 \times 129)}{32,700 \times 129} = 31.7 \text{ sq. in.}$$

Use 57 - $\frac{3}{4}''$ ⌀ $= 31.9$ sq. in.

$$t = \frac{406,000 + 6500 \times 31.7}{820 \times 129} + \frac{31.7}{129\pi} = 5.86 \text{ in.}$$

Height of chimney above section, $H = 145$ ft.

Assumed shell thickness, $t = 7$ inches $= .583$ ft

Mean shell diam. $D = 136$ inches

Mean outside diam. $\delta = 120.5$ inches

Moment due to wind $= 37,800,000$ inch lbs.

Weight of shell above section $= 261,300$ #

$$A_s = \frac{8(37,800,000 - .427 \times 261,300 \times 136)}{32,700 \times 136} = 40.8 \text{ sq. in.}$$

Use 73-3/4" ⌀ = 40.8 sq. in.

$$t = \frac{527,600 + 6500 \times 40.8}{820 \times 136} + \frac{40.8}{136\pi} = 7.16 \text{ in}$$

∴ 7 inches is O.K.

Height of chimney above section, $H = 165$ ft

Assumed shell thickness, $t = 8$ inches

Mean shell diam. $D = 142$ inches

Mean outside diam. $\delta = 124$ inches

Moment due to wind $= 50,600,000$ in. lbs.

Weight of shell above section $= 335,800$ #

$$A_s = \frac{8(50,600,000 - .427 \times 335,800 \times 142)}{32,700 \times 142} = 52.2 \text{ sq. in}$$

$$t = \frac{671,600 + 6500 \times 52.2}{820 \times 142} + \frac{52.2}{142\pi} = 9 \text{ inches}$$

∴ 8 inches is not sufficient.

Recalculate @ 9 inches.

Height of chimney above section, $H = 165$ ft.

Assumed shell thickness, $t = 9$ inches $= .75$ ft.

Mean shell diam. $D = 143$ inches

Mean outside diam. $\delta = 125$ inches

Moment due to wind $= 51,000,800$ in. lbs.

Weight of shell above section $= 345,000$ #

$$A_s = \frac{8(51,000,000 - .427 \times 345,000 \times 143)}{32,700 \times 143} = 51.5 \text{ sq. in.}$$

Use 92-3/4" ⌀ = 51.5 sq. in.

$$t = \frac{691,000 + 6500 \times 51.5}{820 \times 143} + \frac{51.5}{143\pi} = 9.07 \text{ in}$$

∴ 9 inches is O.K.

Height of chimney above section, $H = 185$ ft

Assumed shell thickness, $t = 12$ inches

Mean shell diam. $D = 151$ inches

Mean outside diam. $\delta = 130$ inches

Moment due to wind $= 66,600,000$ in. lbs.

Weight of shell above section $= 464,500$ #

$$A_s = \frac{8(66,600,000 - .427 \times 464,500 \times 151)}{32,700 \times 151} = 59.6 \text{ sq. in.}$$

Use 101-3/4" ⌀ = 59.6 sq. in.

$$t = \frac{929,000 + 6500 \times 59.6}{820 \times 151} + \frac{59.6}{151\pi} = 10.73 \text{ in.}$$

∴ 12 inches is o.k.

Height of chimney above section, $H = 195$ ft.

Assumed shell thickness, $t = 13$ inches $= 1.08$ ft.

Mean shell diam. $D = 155$ inches

Mean outside diam. $\delta = 131$ inches

Moment due to wind $= 74,600,000$ in. lbs.

Weight of shell above section $= 596,000$ #

$$A_s = \frac{8(74,600,000 - .427 \times 596,000 \times 155)}{32,700 \times 155} = 56.7 \text{ sq. in.}$$

Use 101 - 3/4" φ = 56.6 sq. in.

$$t = \frac{1,192,000 + 6500 \times 56.7}{820 \times 155} + \frac{56.7}{155\pi} = 12.72 \text{ in}$$

∴ 13 inches is O.K.

Height of chimney above section, $H = 205$ ft.

Assumed shell thickness, $t = 15$ inches

Mean shell diam. $D = 160$ inches

Mean outside diam. $\delta = 132$ inches

Moment due to wind $= 83,100,000$ in. lbs.

Weight of shell above section $= 674,400$ #

$$A_s = \frac{8(83,100,000 - .427 \times 674,400 \times 160)}{32,700 \times 160} = 56.7 \text{ sq. in.}$$

Use 101 - 3/4" φ = 56.6 sq. in.

$$t = \frac{1,348,800 + 6500 \times 56.7}{820 \times 160} + \frac{56.7}{160\pi} = 13.5 \text{ in.}$$

∴ 15 inches is O.K.

Height of chimney above section, $H = 215$ ft.

Assumed shell thickness, $t = 15$ inches

Mean shell diam. $D = 162$ inches

Mean outside diam. $\delta = 133$ inches

Moment due to wind $= 92,100,000$ in lbs.

Weight of shell above section $= 753,900$ #

$$A_s = \frac{8(92,100,000 - .427 \times 753,900 \times 162)}{32700 \times 162} = 60.3 \text{ sq in.}$$

Use $106 - \frac{3}{4}''\phi = 59.3$ sq in.

$$t = \frac{1,507,800 + 6500 \times 60.3}{820 \times 162} + \frac{60.3}{162\pi} = 14.4 \text{ in.}$$

$\therefore$ 15 inches is O.K.

Height of chimney above section $= 225$ ft.

Assumed shell thickness, $t = 15.5$ inches

Mean shell diam. $D = 165.5$ inches

Mean outside diam. $\delta = 135$ inches

Moment due to wind $= 105,000,000$ in. lbs.

Weight of shell above section $= 837,700$ #

$$A_s = \frac{8(105,000,000 - .427 \times 837,700 \times 165.5)}{32,700 \times 165.5} = 59.2 \text{ sq. in.}$$

Use $106 - \frac{3}{4}''\phi = 59.4$ sq. in.

$$t = \frac{1,674,400 + 6500 \times 59.2}{820 \times 165.5} + \frac{59.2}{165.5\pi} = 15.3 \text{ in.}$$

$\therefore$ 15.5 inches is O.K.

Design of Lining.

Height = 100 ft

Top mean diam. = 111 inches

Bottom mean diam. = 138 inches

Thickness, t = 4"

$$\text{Weight} = \frac{(111+138)\times 100\times\pi\times 4\times 150}{2\times 12\times 12} = \frac{249\times 100\pi\times 25}{12} = 162,800\#$$

$$\text{Concrete stress at bottom section} = \frac{162,800}{\pi\times 138\times 4} = 94.\#/\text{sq in}$$

Theoretically no steel required.

Use 1% vertical steel.

$A_s = .01\times\pi\times 138\times 4 = 17.6$ sq. in. Use 32-3/4"φ 13"-cc.

Use 3/8"φ Spiral looping. 8" pitch.

Design of Footing.

Total Wind Force = 76,000#

Weight of outer shell = 837,700#

" " inner " = 162,800

Total weight of shells = 1,000,500#

Depth of footing to develop punching shear =

$$d = \frac{1,000,500}{\pi\times 150\times 120} = 17.7 \text{ inches.}$$

Bearing Capacity:

Maximum allowable = 8000 #/sq ft.

Average " = 4000 #/sq ft.

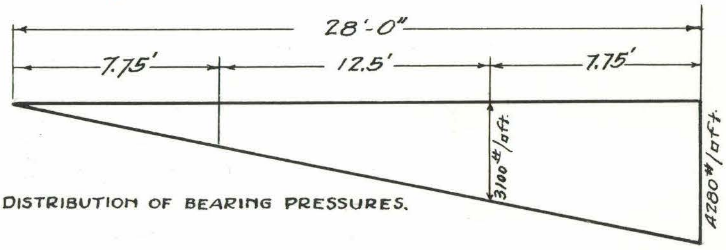
Assume a 28'x28' footing.

Total weight of shells and footing required =

$$W = \frac{76,000 \times 103 \times 6}{28} = 1,680,000^{\#}$$

Net weight of footing = (1,680,000 − 1,000,000) = 680,000#

$$\text{Depth of footing} = \frac{680,000}{28 \times 28 \times 150} = 5'-9''$$



DISTRIBUTION OF BEARING PRESSURES.

$$\text{Average Bearing Pressure} = \frac{1,680,000}{28 \times 28} = 2140 \ \#/\text{sq ft.}$$

Max. " " = twice ave. = 4280 #/sq ft.

Moment on Cantilever of footing =

$$M = 12\left(\frac{7.75^2}{2} \times 3100 + \frac{7.75^2 \times 180}{2 \times 3}\right) = 1,140,000 \text{ in.lbs}$$

$$K = \frac{1,140,000}{12(5.5 \times 12)^2} = 21.8$$

$p = .0017$

$A_s = .0017 \times 66 \times 12 = 1.35$ sq. in per lineal ft.

Use 1" ∅ 9¼" c.-c.

$$\text{Bond} = \frac{V}{\Sigma o j d} = \frac{7.75\left(\frac{3100+4280}{2}\right)}{1.3 \times 4 \times .875 \times 66} = 95 \text{ \#/sq. in } \underline{ok}.$$

Moment on Diagonal: Length $= 12.3'$

$$\therefore A_s' = \frac{12.3}{7.75} \times 1.35 = 2.14 \text{ sq in/ft.}$$ (total steel required)

A_s contributes .707 of rod section in each direction $= 1.414\,A_s$. Therefore the net diagonal steel so needed $= (2.14 - 1.91) = .23\,\square''/\text{ft}$.

Use $\frac{1}{2}''\,\phi$ $12''$ c-c.

$$\text{Shear} = \frac{7.75 \times 3690}{12 \times .875 \times 66} = 41 \text{ \#/sq in.}$$

$\therefore$ no stirrups needed.