DESIGN OF A GIRDER AND CANTILEVER
IN REINFORCED CONCRETE
FOR
THE BALCONY OF THE PASADENA AUDITORIUM
PASADENA. CALIFORNIA

DESIGNED BY

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A GENERAL DISCUSSION

In the proposed Pasadena Auditorium, there is a large balcony, the seating capacity of which is a little more than one thousand. In the plans as provided, the construction of the balcony is to be in the main of structural steel members, the whole fire-proofed with concrete, with the floors, seat risers, etc., of reinforced concrete.

The general dimensions and some tentative drawings were available, enough material, however, to make a comprehensive study of such construction. The length of the main girder is 96' between centers of the supporting columns, with a maximum allowable depth in the center of 16', this depth increasing toward the ends to about 23'. The minimum over-hang of the cantilever-ed portion is 33' 4"; the radius of curvature of the front of the balcony being 42' 9", the over-hang increases toward the sides of the auditorium.

Since the number of cantilevers was lacking in the drawings, from various structural portions shown, it was assumed that there were to be six cantilevers in all, two over the columns and four between these columns, making the distance between centers of cantilevers 19.2 feet.

The total load, including live, dead, and any assumed impact forces was given as 235 pounds per square foot of horizontal projection of the balcony. Since this desige is to be made in reinforced concrete, a small amount was added to take care of the additional weight caused by this type of construction, making the total load 250 pounds per square foot.

Having determined the minimum over-all dimensions, etc., we are ready for the design of the individual parts. It being beyond the scope of this report to investigate the floor system of the balcony proper, this portion has been omitted, and the load caused by small concentrations where the risers join the cantilevers has been assumed as uniformly distributed over the length of the cantilever, which gives a trifle larger moment than if these were considered as concentrations at about 2' 10" apart.

For convenience in the solution of the various problems arrising in the design of the balcony girder and the cantilevers, the formulae most often used are grouped below.

1- Rectangular reinforced concrete beams:

Moment: $p = a_s/bd$.

$j = 1 - k/3$.

$M_c = f_c kjbd^2/2$, or $f_c = 2M/kjbd^2$.

$M_s = f_s pjbd^2$, or $f_s = M/a_s jd$.

$K = M/bd^2$.

Shear: $v = V/bjd$. $v_o = V/bd$.

Bond: $u = V/\ ojd$.

Stirrups: $n = \dfrac{bL\ (v - 40)^2}{4va_s f_s}$.

$x_1 = \dfrac{L\ (v - 40)}{2v}$.

Bent rods: $1' = f_s/4u$.

$x_2 =$ or $\dfrac{L}{2}(1 - \sqrt{m_2/m}\ )$.

$b = \left[2.5(n - 1) + 4\right] d_1$.

In these formulae

f_c = unit compressive stress in the concrete.

f_s = unit tensile stress in the steel.

a_s = area of cross-section of steel.

b = breadth of beam.

d = distance from compression surface to axis of the reinforcement.

M_c = resisting moment determined by the concrete

M_s = resisting moment determined by the steel.

M = resisting moment in general.

p = steel ratio.

k = ratio of depth of neutral axis to depth of steel.

j = ratio of lever arm of resisting couple to depth of steel.

V = total shear.

v = unit shear.

v_o = average shear.

u = unit bond.

o = circumference of one bar.

s = horizontal spacing of stirrups.

w = uniform load in pounds per foot.

L = length of beam.

x_1 = distance to last stirrup.

x_2 = distance from support where m_2 bars may be bent up.

m = actual number of bars.

m_2 = number of bars to be bent up.

2- T-Beams of reinforced concrete:

$$k = \frac{2pnd^2 + t^2}{2pnd^2 + 2td} .$$

$$z = \frac{3kdt - 2t^2}{6kd - 3t} .$$

$$jd = d - z .$$

$$M_c = f_c(1 - t/2kd)bt.jd .$$

$$M_s = f_s a_s jd \quad \text{or} \quad f_s = M/a_s jd .$$

$$f_c = f_s k/(n - nk) .$$

$$\text{Deflection} = D = \frac{c_1 \; WL^3 \; n}{E_s \; bd^3 \; B}$$

$$B = 1/3 \left[k - \left(\frac{b - b'}{b}\right)\left(\frac{kd - t}{d}\right)^3 + \frac{b'}{b}(1 - k)^3 + 3pn(1 - k)^2 \right]$$

$$k = \frac{np + \frac{1}{2}\left[\frac{b'}{b} - \frac{b'}{b}\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^2\right]}{np + \frac{b'}{b} - \frac{b'}{b}\left(\frac{t}{d}\right) - \frac{t}{d}} .$$

3- Beams with steel in top and bottom:

$$M_c = f_c bd^2 \left[\frac{k}{2}\left(\frac{3 - k}{3}\right) + \frac{np'}{k}\left(\frac{kd - d'}{d}\right)\left(\frac{d - d'}{d}\right) \right] .$$

$$M_s = f_s bd^2 \left[p\left(\frac{d - d'}{d}\right) - \frac{k^2}{2n(1 - k)} \left(\frac{kd - 3d'}{3d}\right) \right] .$$

$$k = \sqrt{2n\left(\frac{pd - p'd'}{d}\right) + n^2(p + p')^2} - n(p + p') .$$

4- Columns with longitudinal reinforcement:

$$f_c = \frac{P}{A}\left(\frac{t + 6x}{t}\right) .$$

$$P = f_c A \left[(n - 1)p + 1 \right] .$$

Where certain of the formulae were used repeatedly, curves were plotted for various values of the functions involved and the solutions were thus considerably simplified.

Formulae and other useful data were obtained from "Reinforced Concrete Construction" (Vol. 1) by G. A. Hool.

Loadings, allowable maximum stresses, etc.,

Loadings: total uniform live and dead load combined
250 pounds per square foot.

Concrete: mix 1:2:3 f_c = 800 pounds per sq.in.

Steel: 18,000 pounds per square inch.

The first step in the design will be to determine the loadings which affect the main girder directly. Since the girder is 96 feet long between centers of supports, there will be six cantilevers of sufficient size to support their respective portions of the total load. The spacing between centers of these cantilevers will be 19.2 feet (= 96/5) there being five spaces and six cantilevers. Their lengths are given in Figure 1.

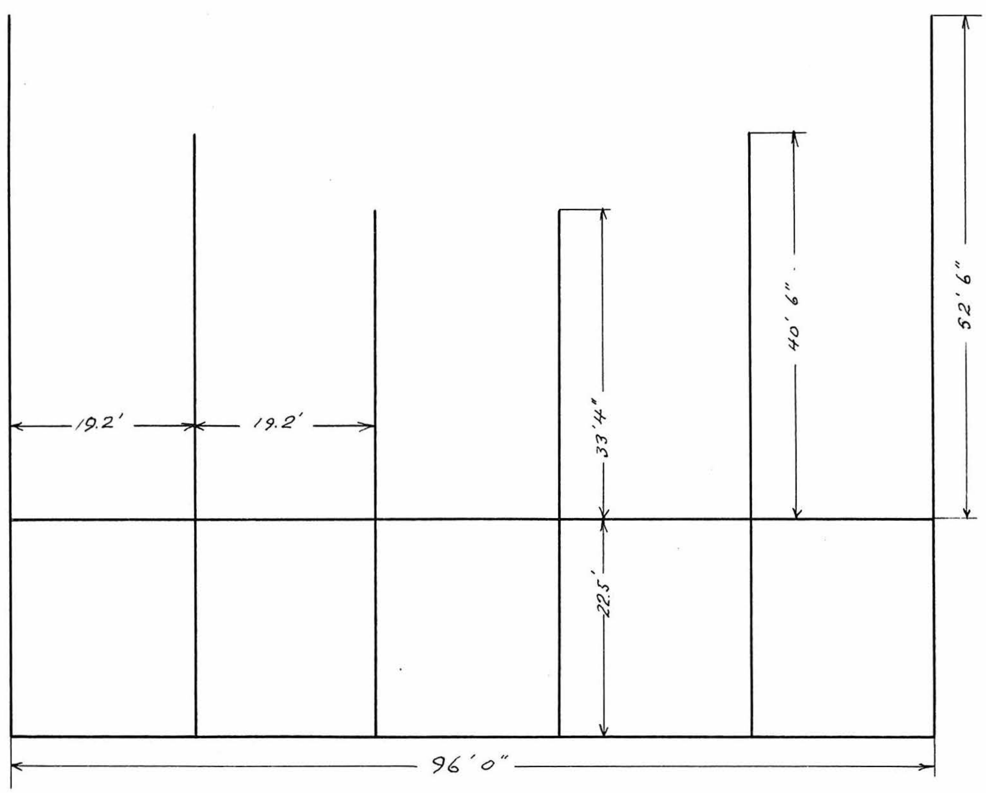


Fig. 1

Design of the Balcony Girder.

The forces acting directly on the girder are those brot to it thru the four inner cantilevers: a, b, c, and d. These forces will now be determined. A typical cantilever is shown in Figure 2, and the loadings are indicated on the figure.

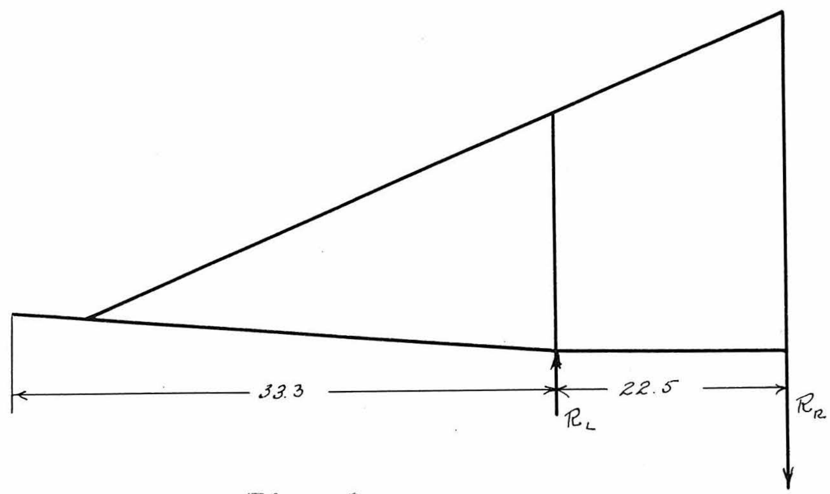


Fig. 2

Distance between cantilevers = 19.20 feet.

Live and dead load per sq. ft. = 250 pounds.

Load per foot of each cantilever = 19.2 x 250 = 4800#

1- Cantilevers b and c:

$L = 55.8$ ft.

$L_1 = 22.5$ ft. $L_2 = 33.3$ ft.

Taking moments about R_r, and solving for R_1, we find

$$55.8 \times \frac{55.8}{2} \times 4800 - 22.5R_1 = 0$$

$R_1 = 324{,}000$ pounds

$R_r = 56{,}000$ pounds.

2- Cantilevers a and d:

$L = 63.0$ feet.

$L_1 = 22.5$ " $L_2 = 40.5$ ft.

Taking moments about R_r, we find

$$63.0 \times \frac{63.0}{2} \times 4800 = 22.5\ R_r.$$

R_r = 423,000 pounds and R_1 = 121,000 pounds.

3- Cantilevers e and e':

L = 75.0 ft. L_1 = 22.5 ft. L_2 = 52.5 ft.

$$22.5\ R_1 = 75.0 \times \frac{75.0}{2} \times 4800$$

R_1 = 600,000. R_r = 240,000 pounds.

Having determined the reactions on the girder caused by the cantilevers, it is then necessary to find the maximum bending moment caused by these loads. The positions of the loads is given in Fig. 3.

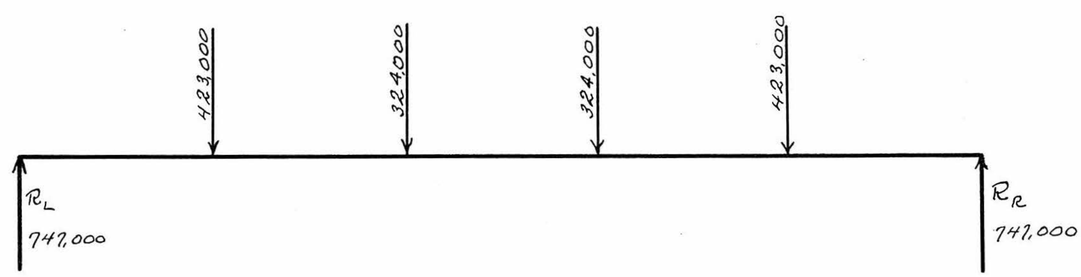


Fig. 3

$$M_c = (747 \times 46.5 - 423 \times 28.8 - 324 \times 9.6)12 \times 1000$$
$$= (34700 - 12200 - 3100)12 \times 1000 = 233,000,000 \text{ inch pounds..}$$

It is now necessary to assume a cross-section for the girder which will approximately carry the required moment. Fig. 4 shows one that will be investigated. The section has been chosen because there will be considerable saving in dead weight due to the T-beam since that part of the concrete below the neutral axis does not increase the effectiveness of the beam in any way. The flange will be the widest in the center of the girder and gradually be reduced in width toward the ends, also to reduce weight.

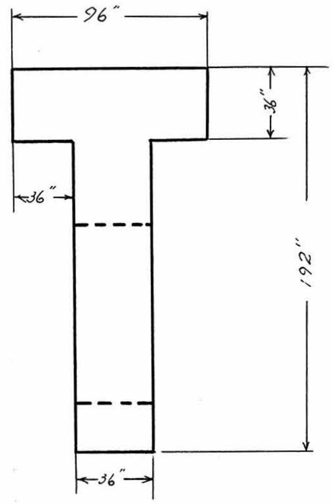


Fig. 4

The Moment due to the weight of the girder will now be found.

Stem: $M_c = 16 \times 3 \times 150 \times \frac{46.5^2}{2} \times 12 = 93{,}200{,}000$ inch pounds.

Flange: $M_c = 3 \times 3 \times 150 \times \frac{46.5^2}{2} \times 12 -$

$3 \times 3 \times 150 \times \frac{46.5^2}{2 \times 3} \times 12 = 11{,}800{,}000$ inch pounds.

Openings: $M_c = 7 \times 3 \times 150 \times 7.5 \times 12\ (46.5 - 4)$

$= -11{,}600{,}000$ inch pounds.

Total Bending Moment = 326,400,000 inch pounds.

$d = 15' = 180''$. $b = 8' = 96''$. $b' = 36''$.

$t = 36''$. $t/d = 36/180 = 0.20$ $j = 0.9$ (nearly)

$a_s = M/f_s jd = 326{,}400{,}000/(\ 18000 \times .9 \times 180) = 111.9$ sq. in.

$p = a_s/bd = 111.9/96 \times 180 = .00654$

$k = \frac{.00654 \times 15 + .5 \times .2 \times .2}{.00654 \times 15 + .2} = \frac{.098 + .02}{.098 + .2} = 0.396$ . . . . k.

$z = \frac{3 \times .396 \times 180 - 2 \times 36}{2 \times .396 \times 180 - 36} = 15.95$ . . . . z.

$j = \left(\frac{d - z}{d}\right) = \frac{180 - 15.95}{180} = 0.912$ . . . . j.

f

$$f_c = \frac{326400000}{\left(1 - \dfrac{36}{2x.396x180}\right)96x36 \text{ x } .912x180} = 775 \ \#/in^2 \quad \text{(OK)}$$

Using the new value of (j) as found in the above computations a_s = 110.4 sq.in. p = .0064 . These new values will not materially affect the preceeding computations. Therefore, they will be used where-ever necessary.

a_s = 110.4 sq.in. We shall try 1 1/8 in. square deformed bars. The number will be $110.4/(1\ 1/8)^2$ = 87.

b' = 36.

$$36 = \left[(n - 1)\frac{9}{8}x2.5 + 4\right]$$

n = 13 (nearly). The bars will be arranged in 7 rows, six having 13 bars each and one having 11 bars (total = 87) as shown in Fig. 5.

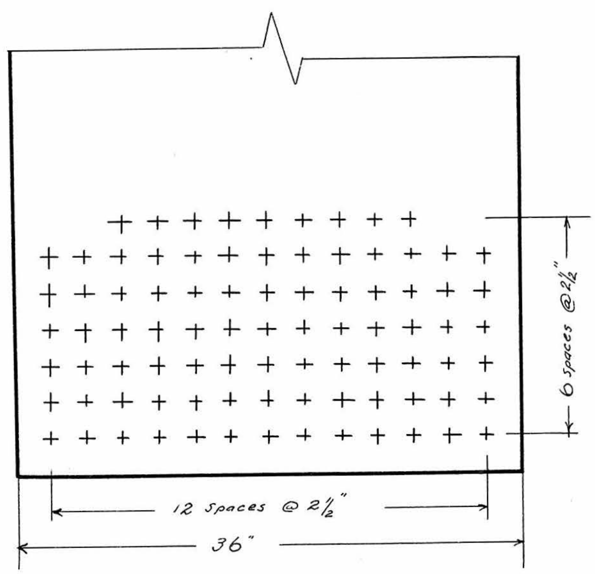


Fig. 5

Showing Spacing of Tension Steel in the Girder.

The shear diagram for the girder is shown in Fig. 6, the light lines being the shear diagram for the weight of the girder only, while the heavy line is the total shaer diagram.

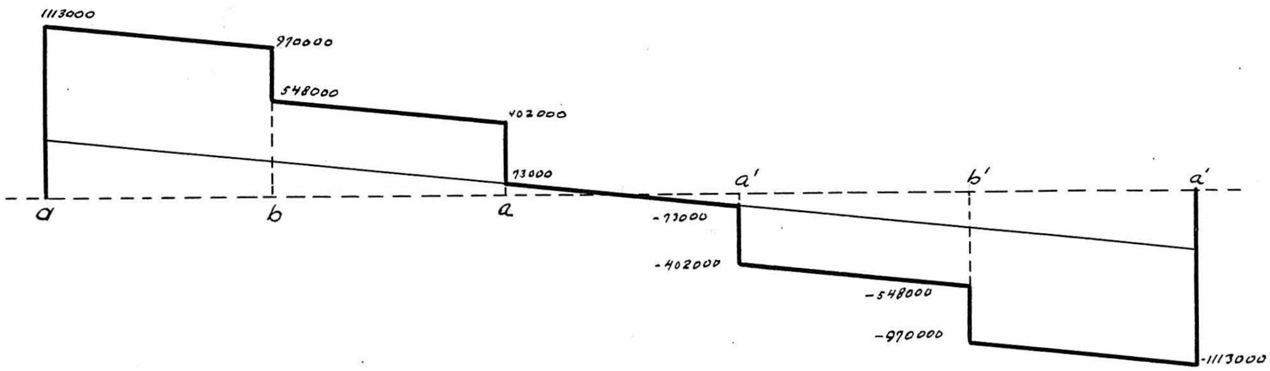


Fig. 6. Shear Diagram.

Shear at the supports:

Cantilever <u>a</u>	324,000 pounds.
Cantilever <u>b</u>	423,000 "
Girder	366,000 "
	1,113,000 pounds.

The allowable unit shear for beams with vertical stirrups and bent rods is 150 pounds per square inch (1:2:3 concrete).

$v_{ab} = V/bjd = 1113000/36(.912x15.3 -4)12 = 144.5\ \#/in^2.$

If stirrups with six verticals are used, $a_s = 1.5$ sq.in.

Then $s = 3a_sf_sjd/2V$

$$= \frac{3x1.5x18000x216}{2x1113000} = 7.84 \text{ ins. (c-c)}$$

If stirrups with 8 vertical legs are used, s = 10.4 ins. Therefore these latter will be used, the spacing being 10.0 ins. c-c, betweer the cantilevers <u>e</u> and <u>a</u>'.

$v_{bc} = 92.5\ \#/in^2$

Stirrups with 6 vertical legs will be used.

a_s = 1.5 sq. ins.

$$s = \frac{1.5 \times 18000\ jd}{92.5bjd - 50bjd} = 17.8 \text{ ins.}$$

s will be taken as 17.5 ins. c-c.

N_{ab} = 23 stirrups, at 10" c-c.

N_{bc} = 14 stirrups, at 16.5" c-c.

The next point to consider is the unit bond in the horizontal bars, or rather how many bars are required to develop the proper tensile stress.

The allowable unit bond stress for deformed bars hooked at the ends, u = 180 lbs/in^2.

$$o = V/ujd = 1113000/(180x216) = 28.6 \text{ inches.}$$

Twenty-six (26) bars will be used, tho the perimeters is excessive. u = 44.1 lbs./in.

The points at which bars will be bent up will next be determined.

Since the girder is carrying four concentrated loads, the usual simple formula for determining the points at which the longitudinal bars may be bent up does not apply, since that formula is applicable only for a uniform loading. It is then necessary to plot the moment diagram at a convenient scale, the abscissas of which represent the length of the girder, and the ordinates to the curve represent the bending moment at any point. Then, it is simply the problem of finding the points at which the moment is reduces an amount egual to that moment which a certain number of bars would resist. Thus: one 1 1/8" square bar in this girder has a resisting moment of 3,750,000 inch pounds. Then for each

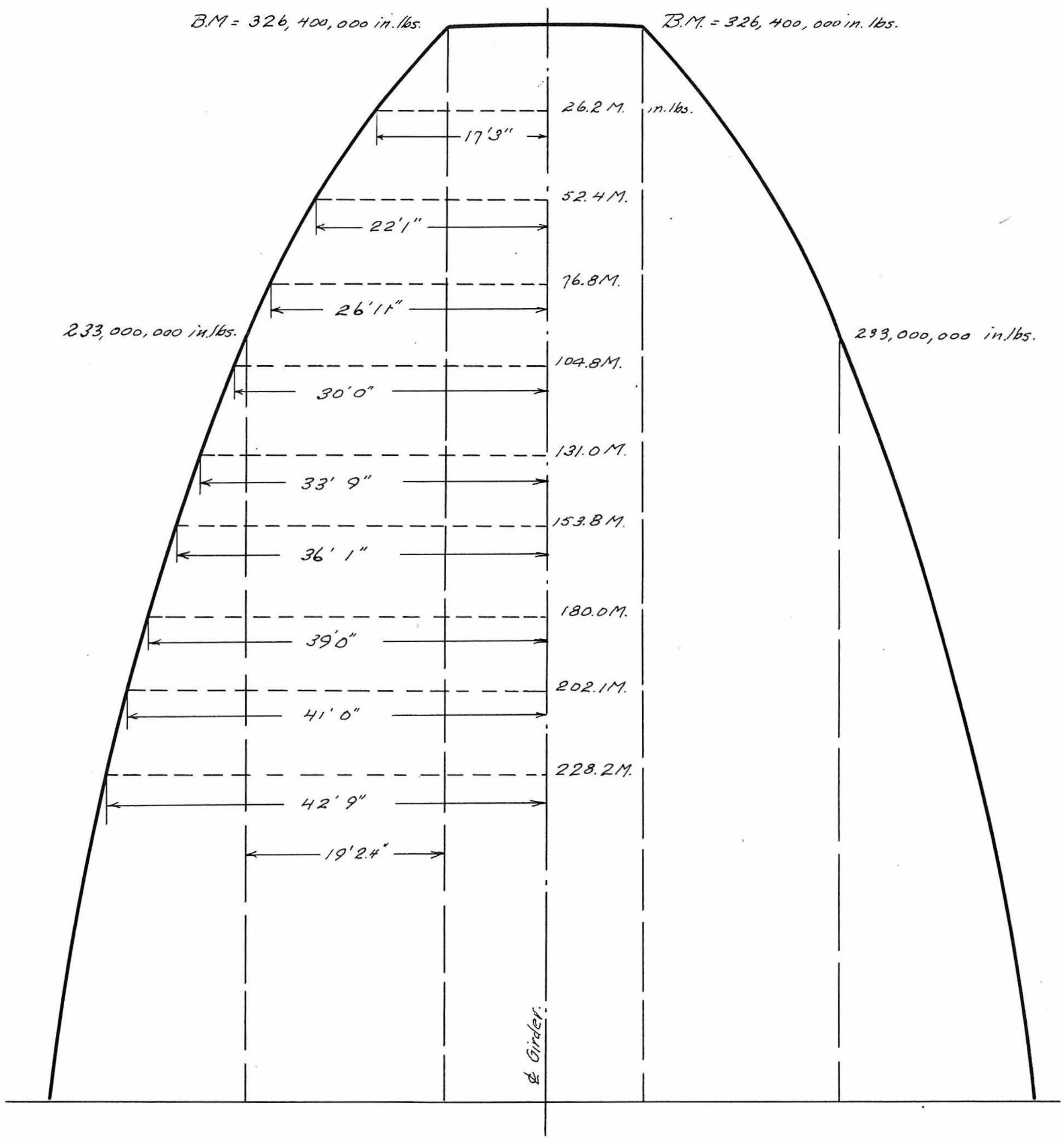


Fig. 6-a.

A graphical method of determining the points at which longitudinal reinforcing bars may be bent up.

reduction of this amount, one bar may be bent up. However, since there are thirteen (13) bars in each row with the exception of one which contains nine (9), it is more advantageous to bend them up six (6) or seven (7) at a time. Then for seven bars, the

R.M. = 26,200,000 inch pounds.

It is then only necessary to determine the number of bars to bent up at any one time, determine the R.M. of all the bars bent up to that section, plot this value as a straight line with an ordinate equal to the R.M., and scale the intercept between the curve of bending Moments. For further details, see Fig. 6-a.

Below is a table giving the number of bars to be bent, the total bent, the R.M., and the distance from the center at which they may be bent.

Number of bars to be bent.	Number of bars bent.	Resisting Moment.	Distance from center of girder.
7	7	26,200,000	17' 3"
7	14	52 400 000	22' 1"
7	21	78 600 000	26' 11"
7	28	104 800 000	30' 0"
7	35	131 000 000	33' 9"
6	41	153 800 000	36' 1"
7	48	180 000 000	39' 0"
6	54	202 100 000	41' 0"
7	61	228 200 000	42' 9"

The angle at which the longitudinal bars are bent is 45°, and the radius of the bend is not to be less than 12d = 13.5". The ends of all the bars are to be hooked, the radius of the hook being 5d = 5.625 inches.

Since it is practically impossible to obtain bars of sufficient length to avoid splices, there will have to be a certain amount of splices. This will be accomplished by using U-bolts of 1/2" steel rod. The reinforcing bars shall lap not less than four (4') feet, and clamped with six (6) U-bolts. The splices are to be made at such points that there will be as few at any section as possible. The individual units will be separated by either concrete blocks 1.5" thick or by 1/4"x 1 1/2" strap iron.

The columns to support this girder will be investigated.

The total load on each of the columns is 1113000 - 600000 = 1,713,000 pounds.

Since the girder is 36 inches wide, this will limit the overall dimensions of the columns to 32" x 32".

For a 1:2:3 mix concrete the allowable axial compression is 900 pounds per sq.in. when about 1% spiral reinforcement is used.

$A = 32 \times 32 = 1024$ sq.ins.

$P = Af_c(p(n-1) + 1)$.

$1713000 = 1024 \times 900(14p + 1)$

$p = 0.061$

$a_s = 0.061 \times 1024 = 62.5$ sq.ins. steel required.

Use 40-1.25" square bars (= 62.5 sq.ins.); the bars will be arranged in the form of a square with eleven (11) bars on a side at three (3") inches c-c of bars.

A spiral consisting of 1/2" high carbon steel square bars with a pitch of 3" gives p' = 0.0104 which is within the required limits:

$$p' = \frac{.25 \times 4 \times 32}{3 \times 32 \times 32} = 0.0104 = 1.04\%.$$

In order that the secondary stresses in these columns be reduced to a negligible quantity, there has been ample allowance made for increased stresses due to any deflection in the girder, by reducing the allowable concrete stress from 1000 pounds (for 1:2:3 concrete) to 900 pounds and by using a little larger area of steel than required.

For complete details of the balcony girder, see Plate I, at the end of this report.

Deflections due to the total live and dead load on the Balcony Girder will be determined, so the the required camber may be calculated.

$n = 15$; $p = .00654$; $b = 96"$; $b' = 36"$; $k = .396$;

$t = 36"$; $t/d = .20$; $b'/b = .375$; $L = 93'$;

$W = 2,226,000$ pounds.

Substituting the values in the equation given for the determination of <u>B</u> given on Page 6, we find

$$B = .333\left[\,\overline{.396}^3 - (1 - .375)(.396 - .2)^2 + 3 \times 15(1 - .396)^2 \times .00654\right] =$$

$$= .333(.0622 - .005 + .0825 + .1308)$$

$$= .09$$

$$D = \frac{c}{E_s}\frac{WL^3}{bdd}\frac{n}{B}\text{, where } c = \text{a constant} = 5/384$$

$$= \frac{5}{384} \cdot \frac{2226000 \times 93^3 \times 12^3}{30000000 \times 96 \times 16^3 \times 12^3} \cdot \frac{15}{.090}$$

$$= .328 \text{ inches.}$$

Therefore, inorder that the girder shall be in its proper (or theoretical) position when completed, there should be a camber of at least 3/8 inch. However, for appearance's sake the camber should be materially increased, in oredr to reduce any tendency of a sagging appearance due to optical illusion. Therefore, this camber is exaggerated, and will be made six (6') inches in the middle of the Girder. The curve of the bottom of the Girder will be parabolic, the other dimensions being given on Plate I, which is to be found in another part of this report.

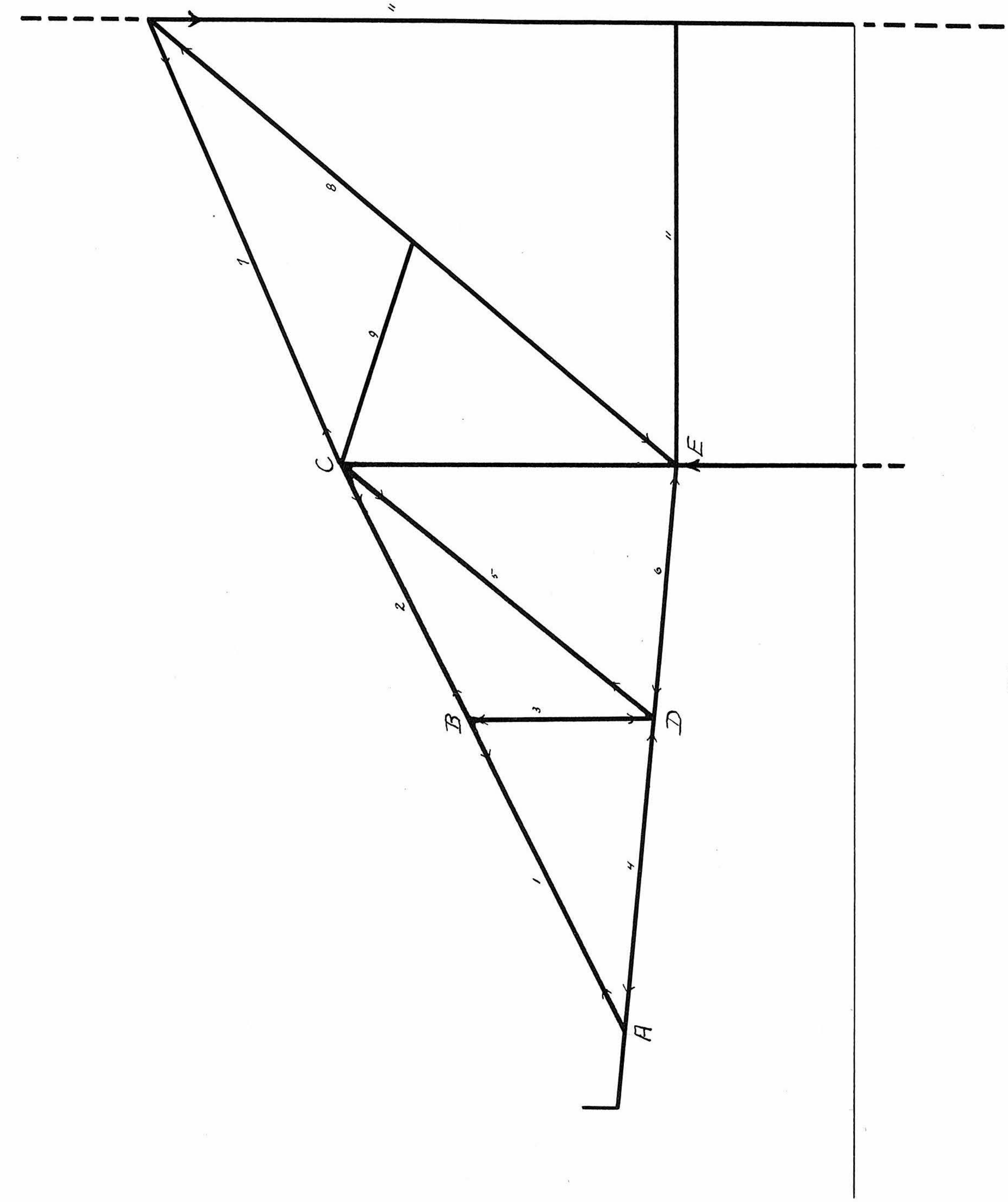


Fig. 7.

Design of a Typical Cantilever for the Balcony Support.

Fig. 7 on the next page, gives the general dimensions and construction of a typical Balcony cantilever. For the purposes of this problem, only one cantilever will actually be designed, tho there will be found elsewhere in this report, a table of the stresses in each of the members of the other two cantilevers.

The loadings per square foot is the same for all cantilevers: 250 pounds per square foot. Since these supports are 19.2' c-c, the load per lineal foot of the cantilever is

$$w = 250 \times 19.2 = 4800 \text{ lbs/ft.}$$

Before any design is undertaken, the first step is to calculate the stresses in each of the members which make up the trussed cantilever.

JOINT B:

Figure 8 shows the forces acting at this point.

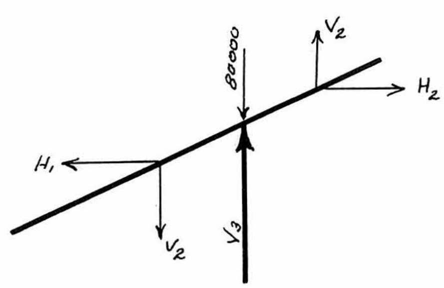


Fig. 8.

$$\sum H = 0 = H_1 - H_2.$$

$$\sum V = 0 = V_1 - V_2 - 80000 + V_3.$$

$$V_3 = 80000 \text{ pounds (c)}$$

JOINT D:

Pass a vertical section thru 2, 5, and 6, and take moments about the point A where 1 and 4 intersect. (see Fig.9.).

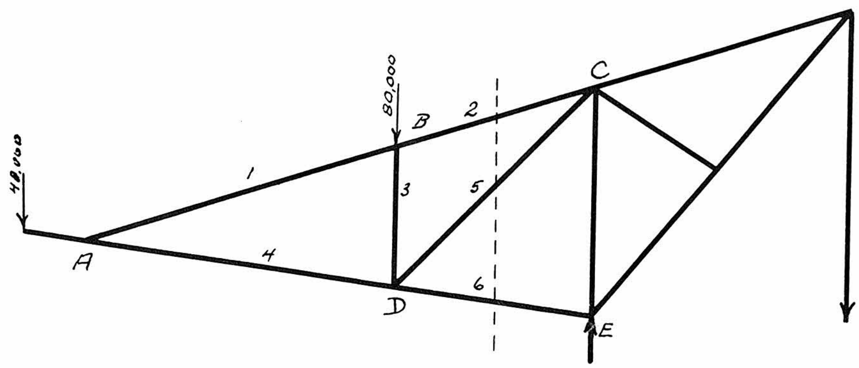


Fig. 9.

$H_5 = V_5$, because this member is constructed at an angle of 45° with the horizontal.

$$\sum M_A = -80000 \times 14.5 + 48000 \times 5.5 + 14.5V_5 + 1.75H_5 = 0$$

$$= -1160000 + 266000 + 16.25V_5 = 0$$

$$V_5 = H_5 = 55{,}000 \text{ lbs.}$$

$$S_5 = 55{,}000\sqrt{2} = 77{,}800 \text{ pounds (t).}$$

MEMBERS __1__ and __2__:

Taking moments about __D__, we have (see Fig. 10.)

$$\sum M_D = 0 = 48000 \times 20 - 8.25H_1.$$

$$H_1 = 116{,}400 = H_2.$$

$$V_1 = H_1 \times 6.5/14.5 = 116{,}400 \times 6.5/14.5$$

$$= 52{,}200 = V_2.$$

$$S_1 = S_2 = 127{,}500 \text{ pounds (t).}$$

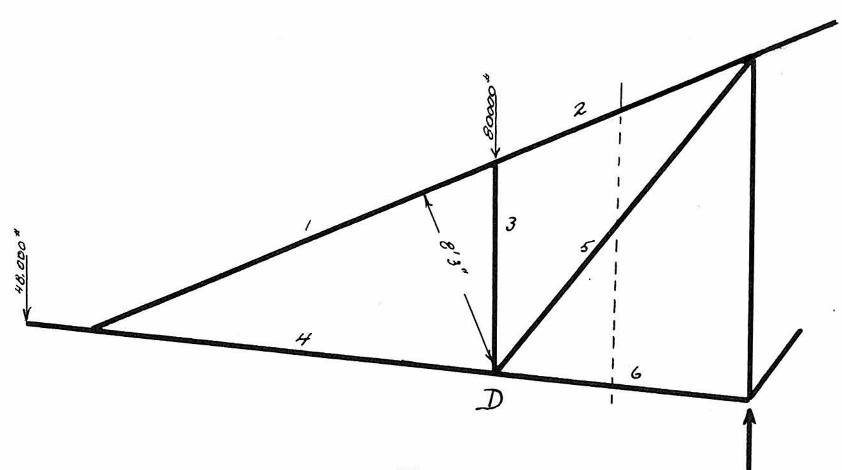


Fig. 10.

MEMBER 6: (See Fig. 11)

To find the stress in 6, pass a vertical section thru 2, 5, and 6, and take moments about C.

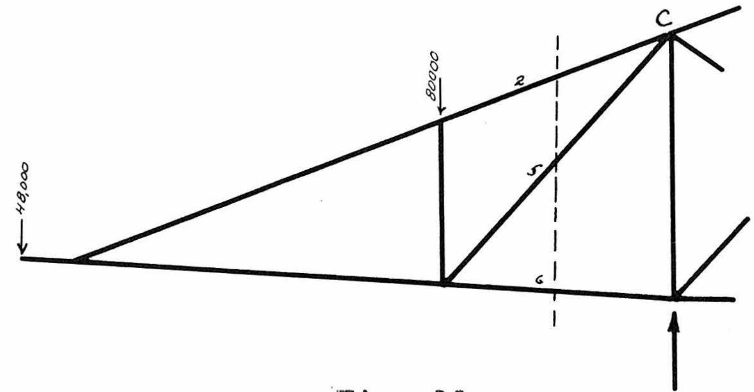


Fig. 11.

$\sum M_D = -48000 x 33.3 - 80000 x 13.3 - 15.1 H_6 = 0.$

$H_6 = 183,000$ pounds.

$V_6 = 183,000 \times 1.5/15.1 = 20,200$ pounds.

$S_6 = 184,200$ pounds (c).

MEMBER 4: (See Fig. 12)

$\sum H_A = 0 = 116,400 - H_4.$

$H_4 = 116,400$ pounds.

$\sum V_A = 0 = 48,000 - 52,000 + V_4.$

$V_4 = 4, 200$ pounds.

$S_4 = 117,000$ pounds, (c).

Member 1	Stress = 127,500	pounds	tension.
Member 2	Stress = 127,500	"	" .
Member 3	Stress = 80,000	"	compression.
Member 4	Stress = 117,000	"	" .
Member 5	Stress = 77,800	"	tension.
Member 6	Stress = 184,000	"	compression.

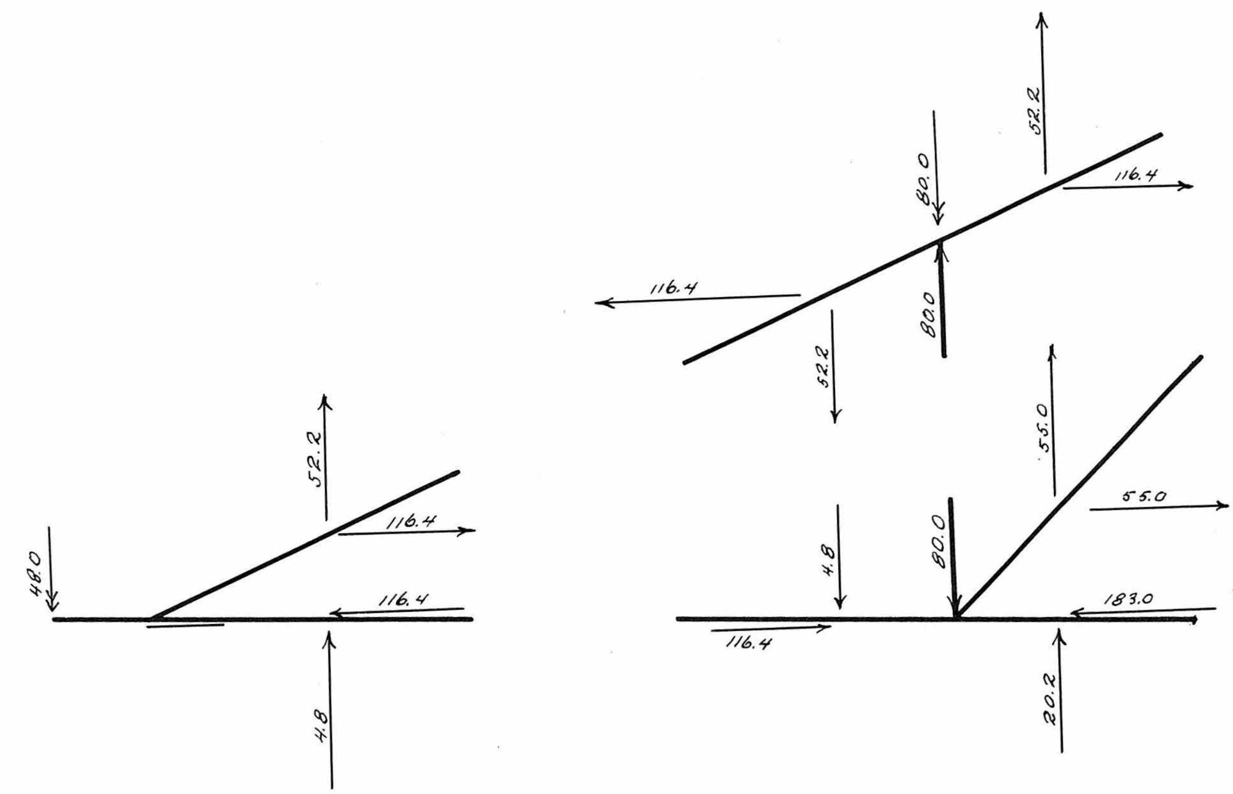


Fig. 12.

Stresses resolved into H and V components.

The design of the different parts of the cantilever will now be undertaken.

(1) Tension member, also acting as a beam.

a_s = 127,500/16000 = 8.0 sq.ins.

3/4" square bars will be used as much as possible.

N = 8.0/.5625 = 15 (nearly) 3/4" bars.

As a beam, L = 13.3' or L' = 10' out to out of supports.

M_c = 4800 x 100 x 12 / 10 = 576,000 inch pounds.

K = 107.4; b = 13" ; d = ?

d = $\sqrt{M/Kb}$ = 20.25 inches. D = 22.0 inches.

a_s = 2.03 sq. ins. Use 4- 3/4" square bars.

V = 25,200 pounds

v = 25200/13x.875x20.25 = 110#/in^2.

N_s = 4; 1' = 37"; s = 3", 11", 20", 31".

For the position of the steel, see Fig. 13.

To take care of the negative moment over the supports, additional steel will be required in the top of these (1 and 2) beams.

M = $WL^2/8$ = 720,000 inch pounds.

a_s = 2.5 sq. ins. Therefore to take care of this added tension in the top of these members, the uppermost row of 3/4" sq. bars will be replaced by 4- 1" squares and 1- 1.25" square bars which makes a_s = 2.47 sq.ins. which is with in the proper limits.

The 1.25 inch bar may be cut at a distance of four (4') feet on either side of column (3) and a short 7/8" bar be inserted at the other ends of these two members to take up the negative BM.

(2) Tension member. $S = 127,500$ pounds.

$a_s = 127,500/16000 = 8.0$ sq.ins. Use 15- 3/4" square Bars.

$M_c = 576,000$ in. lbs.

$b = 13"$; $d = 20.25"$; $D = 22.0"$

$a_s = 2.03$ sq.ins. Use 4- 3/4" square Bars.

$V = 21,600$ pounds.

$v = 95$ lbs./sq. in.

$N_s = 3$; $l' = 31"$; $s = 4"$, $13"$, $24"$.

For placing of steel, see Fig. 13.

(3) Compression member. $P = 80,000$ pounds.

$a_s = 10$- 3/4" sq. bars = 5.62 sq.ins. (assumed)

$$\frac{80000}{A} = \left[\frac{14 \times 5.62 + A}{A}\right] 550.$$

$A = (80000-43300)/550 = 66.8$ sq.ins.

$b = 13"$; $b' = 7.5"$.

Core dimensions: 9" x 7.5".

Out dimensions: 13" x 11.5".

The steel will be placed as shown in Fig. 14.

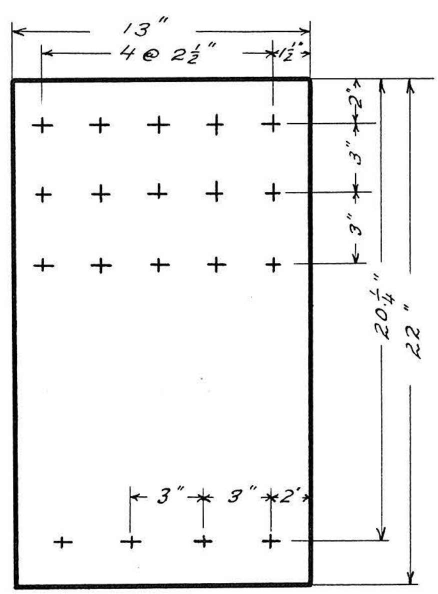


Fig. 13.

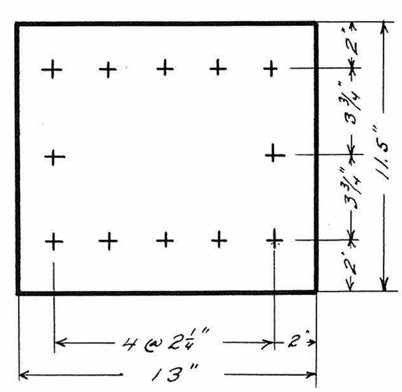


Fig. 14.

(4) Compression member. P = 117,000 pounds.

p = .05 (assumed)

$P/A = f_c[(n - 1)p + 1]$.

A = 125 sq. ins. d = 9"; D = 13"; b = 14"; B = 18".

w = 240 lbs./ft.

$M = wL^2/8$ = 34,500 in. lbs.

f = Mc/I = 144 pounds

a_s = 6.75 sq. ins. (assumed) 12- 3/4" square bars.

117,000 + 150 = 550(14x6.75 + A)

A = 118.0 sq. ins.

Core dimensions: 9" x 13.5".

Outside dimensions: 13" x 17.5".

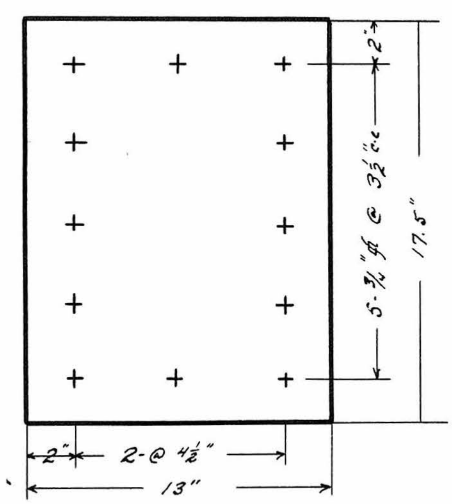
Steel spacing is shown in Fig. 14.a.

(5) Tension member. S = 77,800 pounds.

a_s = 77,800/16,000 = 4.86 sq. ins. Use 8- 3/4" square bars.)
2- 1/2" square bars.)

Outside dimensions (fire-proofing) 8" x 13".

Steel spacing is shown in Fig. 15.



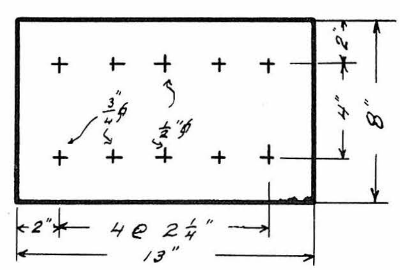


Fig. 15.

($\underline{6}$) Compression member. P = 184,000 pounds.

a_s = 10.15 sq. ins. (= 18- 3/4"sq. bars, assumed)

$A = (184{,}000 - 550 \times 14 \times 10.15)/550 = 193.0$ sq. ins.

$b = 9"$; $d = 21.5"$; $B = 13"$; $D = 25"$.

Should the portion under the floor of the balcony and supported by this member be used as a storage space, 100 lbs./sq.ft. will be assumed. This is live plus dead load.

$w = 100 \times 19.2 \times .5 = 960$ lbs./ft.

$M = 0.1 \times 960 \times 13 \times 13 \times 12 = 194{,}500$ inch lbs.

$f = Mc/I = 174$ lbs./in.sq.

$$f_c = \frac{P}{A}\left(\frac{t + 6x}{t}\right)$$

Considering this column acting as a T-beam for any stored load,

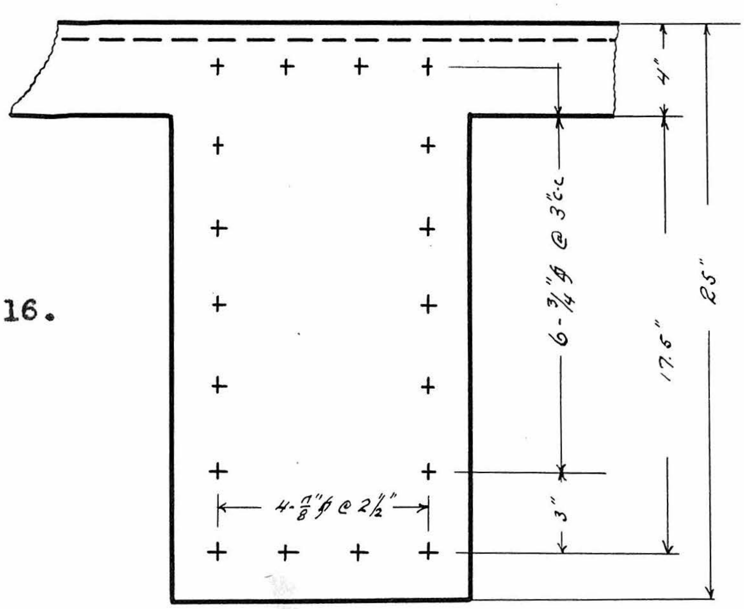
$t = 4"$; $t/d = .186$; $a_s = .63$ sq.ins; $p = 0.00147$

$b = 29"$; $b' = 13"$; $f_c = 350$; $K = 40$; $f_c = 350$ OK.

a_s (column) = 10.15 sq.ins. (= 18- 3/4" square bars)

a_s (T-beam) = 0.63 sq.ins. (Replace 4 lowest bars of column by 4- 7/8" square bars. The arrangement of the steel is shown in Fig. 16.

Fig. 16.



The calculation of the stresses in members 7, 8, 9, 10, and 11 will now be made.

Determine the loading which will give the maximum tensile stress in 7, and the maximum compressive stress in 8. These values are found to occur when the cantilever portion only is fully loaded.

Taking Moments about O, Fig. 17, having passed a vertical section thru 7, 8, 9, and 11,

(10) $$\sum M_O = 0 = 33.3 \times \frac{33.3}{2} \times 4800 - 21.0\, V_{10}$$

$$V_{10} = 127{,}500 \text{ pounds tension.}$$

$$V_{10} \text{ for fully loaded balcony} = \frac{55.8^2 \times 4800 \times 5.4}{2 \times 21.0}$$

$$= 68{,}500 \text{ pounds.}$$

(8) $$\sum M_x = 0 = 127{,}000 \times 21.0 - 9.5\, S_8.$$

$$S_8 = 281{,}000 \text{ pounds (c)}$$

(7) $$\sum M_o = 0 = 127{,}000 \times 21 - 15S_7.$$

$$S_7 = 177{,}500 \text{ pounds (t).}$$

$$\sum M_o = 0 = 33.3 \times \frac{33.3}{2} \times 4800 - 15S_7.$$

$$S_7 = 177{,}500 \text{ pounds (t) (CHECK)}$$

(9) $S_9 = 0$. Its purpose it to reduce the effective length of 8 , thereby reducing the value of L/r to less than 15.

(11) 11 acts purely as a beam to support the floor of the passageway (Foyer) and will be designed as such.

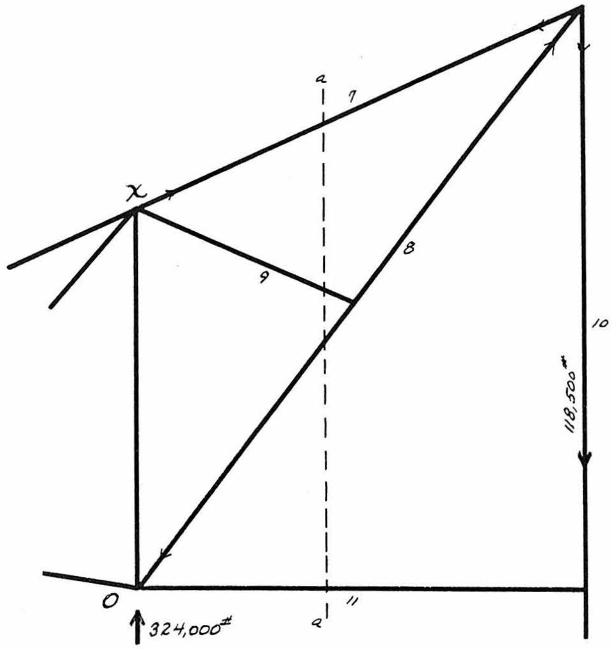


Fig. 17.

The design of the various members in this portion of the Cantilever system will now be made.

MEMBER <u>8</u>: Compression.

P = 281,000 pounds. A = 9 x 15 = 135 sq.ins.

281,000 = 900 x 135 (14p + 1)

p = .0935. a_s = 12.7 sq.ins. Use 17- 7/8" sq. bars.

p' = .01 3/8" round HC steel, spiral, pitch = 3".

MEMBER <u>9</u>: No stress; except secondary stresses in <u>8</u>.

10" square column with 4- 3/4" square bars.

MEMBER <u>7</u>: Tension and Beam.

a_s = 177,500/16,000 = 11.1 sq.ins. Use 20- 3/4" sq. bars.

$M_c = WL^2/8$ = 1,845,000 inch pounds.

b = 15"; d = 30"; K = 137.0 (too large)

p = .01 = p'. K' = .0089; L' = .250; k =.349.

$bd^2 = M/f_sK'$ or M/f_cL'.

bd^2 = 11300 or 11250
b

b = 15"; d = 27.5"; D = 29.5"

a_s = 4.12 sq.ins = a'_s.

$a_s = a'_s = 4.12$ sq.ins. Use 4- 7/8" & 1- 1/2" sq. bars. in the tension side.

The tension steel will be recalculated. Assume that there is an initial tension in the steel. There is consequently some tension in the concrete, i.e., when the beam is acting purely as a tension member. When acting as a beam, this stress is somewhat reversed. T = 177,500 lbs. C = 40,000 lbs. (=4.12x15x650) The net stress is then 137,500 lbs. tension. If it should be considered as carrying no load (live) the stress will be increased 22,500 lbs., or the total will be 160,000 pounds tension. Then a_s = 160,000/16,000 = 10 sq.ins. Therefore use 18- 3/4" sq. bars.

See Figures 18, 19, 20, and 21 for cross-sections and other dimensions for members 7, 8, 9, and 11.

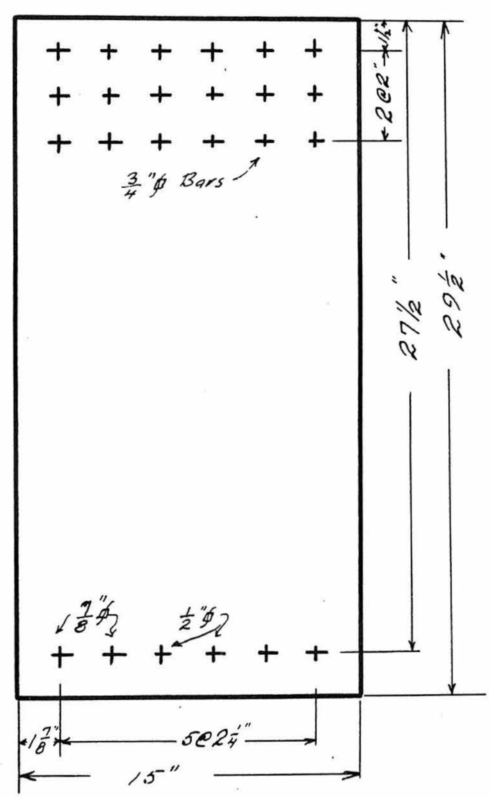


Fig. 17. #(7).

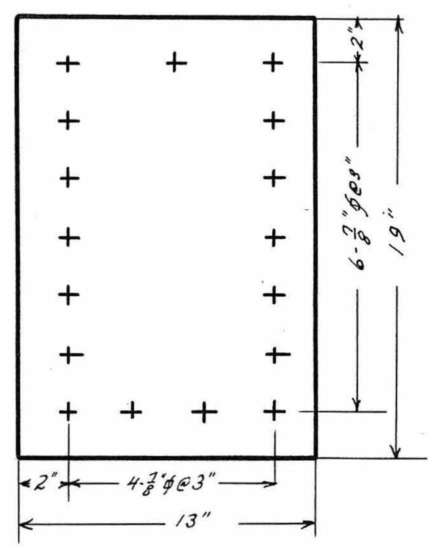


Fig. 18. #(8).

Negative moments over the supports is equal to $WL^2/20$, or 740,000 inch pounds.

The steel necessary to carry this is $740{,}000/f_s jd$ and is equal to 1.92 sq.ins. For this moment, use an additional 2- 7/8" and 2- 1/2" bars in the top of the beam at the supports and extending 4 ft. toward the center from the supports.

$$V = \frac{8 \times 4800}{15 \times .87 \times 27.5} = 106.0 \text{ pounds/sq.in.}$$

$N_s = 7$

$l' = 63"$

$s = 3", 7", 16", 28", 42", 57"$.

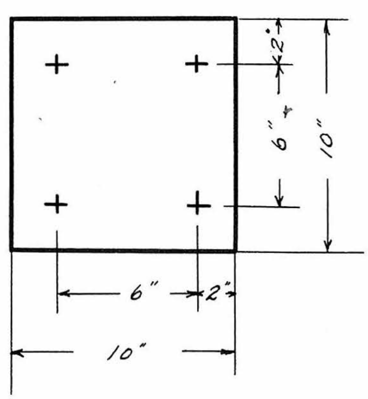


Fig. 19. #(9)

TIES: Except where other wise specified, i.e., where there is hooped construction, there shall be ties made of 3/8" square steel bars, bent into the proper form to thoroly tie the bars in the columns together, and shall have a distance between ties not greater then 9" where 3/4" square bars are used, and not greater than 10" where 7/8" square bars are used.

MEMBER (11): Tension.

F = 118,500 pounds (maximum).

a_s = 118,500/16,000 = 7.42 sq.ins.

Use 10- 7/8" sq. bars (a_s = 7.65 sq.ins.). Inorder to prevent overturning of the balcony, sufficient weight will have to be suspended from the point M , Fig. 17, to produce the required tension: 118,500 pounds. The weight of the wall and columns will be investigated. The thickness of the wall is 15", and w = 150 pounds per cubic foot. Then W (on the point M will be

W = 19.2 x 1.25 x 36 x 150 = 130,000 pounds. This is sufficient weight for the two center cantiliver counter-balances, but quite in-sufficient for the other four. However this calculation is made on the assumption that the weight of the wall above the balcony does not contribute any weight to this load, when in reality, the load is increased 30% if this is added, making a sufficient load for all similar tension members.

MEMBER (12): Beam. Floor load = 125 lbs./sq.ft. (total). See Fig. 20 for general dimensions and loadings.

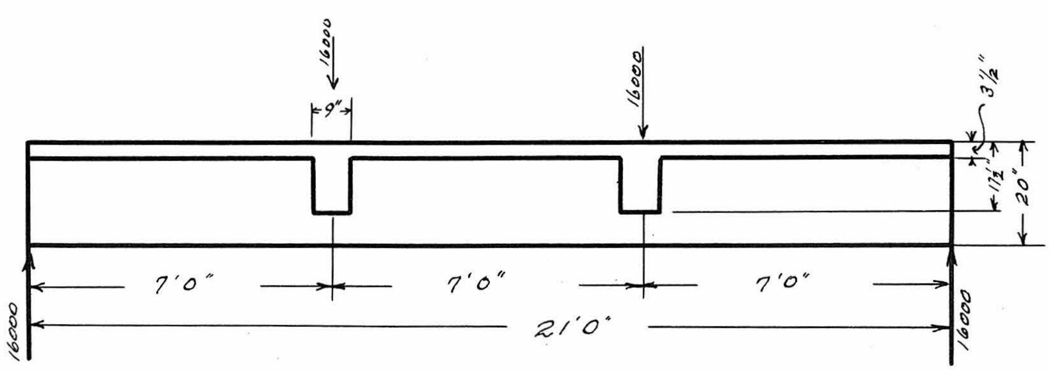


Fig. 20.

(a) Slab: $w = 125$ #/sq.ft. total. $L = 7'$

$M_c = wL^2/10 = 7350$ in.lbs.

$f_c = 750$; $f_s = 16000$; $K = 134$; $p = .0095$

$bd^2 = 7350/134 = 54.8$. $d = 2.13$ in. Take $d = 2.5"$

$f_c = 600$; $f_s = 16000$; $K = 98.5$; $j = .91$.

$a_s = .21$ sq.ins./ft. Use 3/8" sq. bars @ 8" c-c.

Bars may be bent up at 1/5 points.

(b) Beams: $w = 875$#/ft.

$M_c = wL^2/12 = 290,000$ inch pounds.

$bd^2 = M/K = 2140$. If $b = 9"$, $d =$ 15.5 (nearly)

$M/bd^2 = 134$ (check) Steel for negative M = positive M.

$a_s = 1.29$ sq.ins. Use 2- 3/4" sq's. & 1- 1/2" sq. bars.

$V = 7890$ #.

$v = V/bjd = 62$ lbs./sq.in.

$l' = 42"$.

$N_s = 4$.

$s = 5", 15", 25", 35"$.

(c) Girders: (10) Two concentrated loads at third points

$P = 16,000$ pounds

$M_c = PL/3 = 1,345,000$ inch pounds.

$b = 45"$; $b' = 13$; $d = 18"$; $K = 92.7$; $t = 3.5"$.

$f_c = 680$; $f_s = 16000$. $t/d = .195$; $j = .91$.

$a_s = 5.13$ sq.ins. Use 8- 3/4" & 2- 1/2" sq. bars.

Negative BM = 1345000 x .4 = 540,000 inch pounds.

$K = M/bd^2 = 128$; $f_c = 725$; $j = .88$

$a_s' = 2.14$ sq.ins. Use 4- 3/4" sq. bars.

$V = 16,000$. $v = V/bjd = 78$ lbs./in.sq.

$l' = 84"$.

$N_s = 9$.

$s = 4.5", 13.5", 22.5", 31.5"$, etc., to $85.5"$.

In all cases where stirrups are required for shear in the members of the cantilever trusses, except when otherwise specified are U-shaped made of 3/8" square bars, of the proper length.

Stresses occurring in the bottom of the girder caused by the intersection of the two compression members must be transferred to the top by means of some tension steel in the girder itself.

Fig. 7.

$\sum H_E = 0 = 324{,}000 - 20{,}000 - 232{,}000 = 72{,}000$ pounds.

$a_s = 4.5$ sq.ins.

Use 8- 3/4" square bars. (See Plate I).

For a detailed drawing of the cantilever construction, see Plate II at the end of this report.

(4a) Cantilever-ed beam.

$\sum M_A = 0 = 4 \times 4800 \times 2 \times 12 = 462{,}000$ inch pounds.

$b = 13"$; $K = 260$; $p = .0179$; $p' = .0236$.

$d^2 = M/Kb = 136.5$. $d = 11.66"$. Take $d = 11.75"$; $D = 13.5"$

$a_s = 2.72$ sq.ins. (tension). Use 5- 3/4" sq. bars.

$a'_s = 3.61$ " " . (compression) Use 6- 3/4" & 1- 1/2".

$V = 19{,}200$ pounds. $v = V/bjd = 140.5$ #/sq.in.

$l' =$ 48" (v = constant).

N_s = 15 stirrups.

s = 3.25" c-c.

Compression steel arranged in two rows as shown in Fig. 22.

c.g. of bars = 0.25" above centers of lower row.

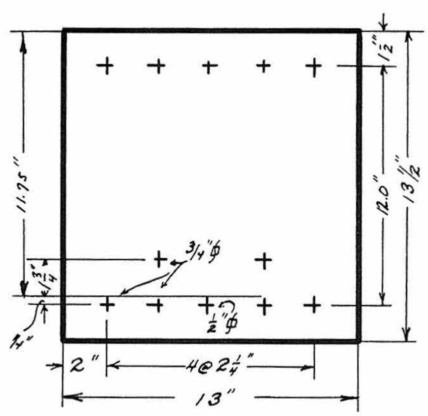


Fig. 22.

Section AA.