THESIS

Design and Construction
of an
Harmonic Analyzer.

by

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Pasadena, California.

1916.

INTRODUCTORY.

With the advent of the long distance high voltage power transmission line and the use of polyphase circuits the need for an accurate analysis of wave forms has become imperative. There is today no simple inexpensive device on the market for mechanically accomplishing an analysis of waves plotted to rectangular coordinates. An Harmonic Analyzer is manufactured by the Westinghouse Electric and Manufacturing Company but is adapted only to the analysis of waves plotted to polar coordinates, and is moreover too expensive for the average laboratory. Michelson has patented a machine which will both analize and synthesize waves in rectangular coordinates but it is excessively complicated and the price is prohibitive. Moreover while excellent in theory the operation of the machine depends on the addition of the forces exerted by helical springs so that its practical accuracy is dubious. In view of these facts the following design of an Harmonic Analyzer adapted to analyze rectangular coordinate curves just as they come out of the ordinary oscillograph without replotting cannot be considered out of place.

In this paper under the title of "Theory", a discussion of the determination of the Coefficients of Fournier's series will be given togehter with a method of representing $\frac{1}{\pi}\int_0^{2\pi} f(\theta) \begin{pmatrix} \sin n\theta \\ \cos n\theta \end{pmatrix} d\theta$

graphically which as far as I know is quite new. This graphical method which I have named the "Cylindrical Graph", was a "by-product" in the design of the machine. The problem was attacked from the point of view of a machine which would evaluate the integral just mentioned. In explaining the theory of operation of the Harmonic Analyzer it greatly simplifies matters however to reverse this order of events by first developing the cylindrical graph and then showing how the analyzer takes advantage of it.

Under the heading "Design" a complete description of the analyzer and its mode of operation supplemented with detail and assembly drawings will appear.

The section entitled "Operations" is intended to give concise directions for starting and using the Analyzer. Some space is also devoted to a few simple tests of curves which may be made by inspection and will save much time in a complete analysis.

At the date of this writing the Analyzer is still under construction. If time will permit a fourth section will be added to this paper when the Analyzer is completed giving the results of a test of the machine.

A number of patterns were made of the parts of the

Analyzer and these will be left in the Electrical department of Throop College in case it should become necessary or desirable to make duplicate parts.

In the theoretical development of this machine constant reference was made to Byerly's text entitled, "Fourhier's Series and Spherical Harmonics".

In the construction of the machine I owe much to the patience and skill of several of my fellow students. Mr. Prosser, Mr. Kramer, Mr. Burns, Mr. Heywood and Mr. Wilcox. Without their help the machine would never have been completed. I wish also to express my thanks to Dr. Seares of the Mt. Wilson Solar Observatory who made it possible for me to have the planimeter divided on the dividing engine at the Observatory shops.

Above all I wish to thank Professor Ford of Throop College for the kindly interest he has taken in both the work of design and construction. His many suggestions and modifications have made the Harmonic Analyzer a success when otherwise it would have been a failure.

THEORY.

The discovery of the fact that any single valued finite function of a variable can be expressed as a sum of sine and cosine functions of different frequencies of that variable was made by FourXier in 1822. It may be expressed thus :

(1) $f(\theta) = A_1 \sin\theta + A_2 \sin 2\theta + A_3 \sin 3\theta + * * *$
$+ B_o + B_1 \cos\theta + B_2 \cos 2\theta + B_3 \cos 3\theta + * * *.$

It is evident that the coefficients A_1, A_2, * *, B_o B_1, B_2, * *, here represent the amplitudes of the harmonic components of $f(\theta)$. The presence of the constant B_o is harmonized with the remainder of the expression by regarding it as the coefficient of the term $B_o \cos 0\theta$. Obviously $A_o \sin 0\theta$ is omitted.

The expression (1) is perfectly general for any finite single valued function of θ. Then function need not even be continuous so long as it possesses only a finite number of discontiniuties.

But the curious and altogether fortunate thing about this expression is that for any given function $f(X)$ the coefficients A_1, A_2, * * B_o, B_1, B_2, * *, are each and all uniquely determined. The immense utility of the FourXier series is entirely due to this fact.

To understand this a little more clearly see plate

III. This shows some simple sine curves of different periods and the resultant curve obtained by adding corresponding ordinates. The component of lowest frequency is called the fundamental, that of three times the frequency the third harmonic, etc. Now the principle we wish to make clear is that given any curve whatsoever such as f (θ) in the figure III there is one and only one amplitude of fundamental which will go to make up this curve--one and only one amplitude of second, third or nth harmonic. Or to put it negatively, in reproducing the curve we cannot have less that the proper amplitude of one harmonic and make up for this deficiency by having more than the proper amplitude of another harmonic. The form of the function alone determines immediately the amplitude of all the harmonic components.

It is desirable then to find the law by which the amplitudes of the various harmonics depend on the form of f (θ). We have the expression given :

(1) $f(\theta) = A_1 \sin\theta + A_2 \sin 2\theta + A_3 \sin 3\theta + * * + A_n \sin n\theta$
$+ B_0 + B_1 \cos\theta + B_2 \cos 2\theta + B_3 \cos 3\theta + * * + B_n \cos n\theta.$

If we multiply both members by $\sin\theta\, d\theta$ we obtain $f(\theta) \sin\theta\, d\theta = A_1 \sin^2\theta\, d\theta$ + terms of the form $A_n \sin\theta \sin n\theta\, d\theta + B_0 \sin\theta\, d\theta + B_1 \cos\theta \sin\theta\, d\theta$ + terms of the form $B_n \cos n\theta \sin\theta\, d\theta$.

If we now integrate both sides between the limits 0 and 2π we obtain

$$\text{(II)} \quad \int_0^{2\pi} f(\theta) \sin\theta \, d\theta = A_1 \int_0^{2\pi} \sin^2\theta \, d\theta .$$

All the other terms of the right hand member reduce to zero when integrated between these limits as the following proof will show.

To prove that :

Terms of the form $A_n \int_0^{2\pi} \sin\theta \sin n\theta \, d\theta = 0$

Integrating by parts :

$$\int \sin\theta \sin n\theta \, d\theta = \frac{n\cos n\theta \sin\theta - \sin n\theta \cos\theta}{1 - n^2}$$

Between 0 and 2π this is evidently 0 except in the case when $n = 1$. The fraction then becomes indeterminate $(\frac{0}{0})$ and not can be evaluated. Thus the case of $n = 1$ gives $A_1 \int_0^{2\pi} \sin^2\theta \, d\theta$ and is the only term in the sine series which remains. Let us see if any cosine terms remain.

To prove that :

Terms of the form $B_n \int_0^{2\pi} \cos n\theta \sin\theta \, d\theta = 0$

Integrating by parts :

$$\int \cos n\theta \sin\theta \, d\theta = \frac{n\sin n\theta \sin\theta - \cos n\theta \cos\theta}{n^2 - 1}$$

This evidently vanishes between the limits 0 and 2π except when $n = 1$. The fraction then becomes indeterminate and it is necessary to investigate it further. The case of $n = 1$ gives the integral, $B_n \int_0^{2\pi} \cos\theta \sin\theta \, d\theta$

But $\cos\theta \sin\theta = \frac{1}{2}\sin 2\theta$, $\quad \frac{1}{2}\sin 2\theta \, d\theta = -\frac{1}{4}\cos 2\theta$

This evidently vanishes between the limits $\theta = 0$ and $\theta = 2\pi$.

Thus it is seen that equation (II) is valid, all terms having vanished from the right hand member except the one shown.

$$\int \sin^2\theta \, d\theta = \frac{\theta}{2} - \frac{\sin 2\theta}{4}$$

Between the limits $\theta = 0$ and $\theta = 2\pi$ this evidently becomes π and Equation II becomes,

$$\int_0^{2\pi} f(\theta) \sin\theta \, d\theta = A_1 \times \pi$$

Solving for A_1:

$$\text{(III)} \quad A_1 = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \sin\theta \, d\theta .$$

Here then is the desired relation between the coefficient A_1 and $f(\theta)$. Note that it is entirely independent of the other coefficients in the series and depends for its magnitude completely on the form of $f(\theta)$.

This is only one coefficient however. All the others including those of the cosine terms must be found. To do this multiply both members of Equation I by $\sin n\theta \, d\theta$ when n is the number of the sine term whose coefficient is desired. Integrating both members between the limits $\theta = 0$ and $\theta = 2\pi$ as before all the terms will vanish in the right hand member except the nth term. The result is $\int_0^{2\pi} f(\theta) \sin n\theta \, d\theta = A_n \int_0^{2\pi} \sin^2 n\theta \, d\theta = A_n \pi$

(IV) $A_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \sin n\theta \, d\theta$

Here then is a general formula for any coefficient of a sine term in the series.

By multiplying both members of (I) by $\cos n\theta \, d\theta$ and proceeding similarly we get :

(V) $B_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \cos n\theta \, d\theta$

This formula is not exactly valid when n is zero as will be evident when the method of its derivation is recalled, for the multiplying of both members of (I) by $\cos 0\theta \, d\theta$ is identical to multiplying by $d\theta$. The entire right hand member is then either of the form $\int_0^{2\pi} \sin n\theta \, d\theta = 0$ or of the form $\int_0^{2\pi} \cos n\theta \, d\theta$, which is evidently zero except for the term $n = 0$, where it becomes,

$B_0\int_0^{2\pi} d\theta = 2\pi B_0$, thus $\int_0^{2\pi} f(\theta) \, d\theta = + 2\pi B_0$.

(VI) $B_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta) \, d\theta$.

The Cylindrical Graph.

We now have sufficient formulae to obtain all the coefficients :

(IV) $A_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \sin n\theta \, d\theta$.

(V) $B_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \cos n\theta \, d\theta$, with the exception

of (VI) $B_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta) \, d\theta$

It is well never to lose sight of the geometric interpretation of the problem as it affords a concrete conception for the mind to grasp and retain.

How can we represent grahpically $\frac{1}{\pi}\int_0^{2\pi} f(\theta) \begin{pmatrix} \sin n\theta \\ \cos n\theta \end{pmatrix} d\theta$?

Let this be regarded as the integral of the product of a differential quantity ($\sin n\theta \, d\theta$) by some finite quantity $f(\theta)$. Suppose we let $\sin n\theta \, d\theta = d\lambda$. Then the thing to be found is $\int f(\theta) \, d\lambda$.

Clearly this will appear as an area if we plot values of λ as abscissa and $f(\theta)$ as ordinates. But what is λ?

$$\lambda = \int \sin n\theta \, d\theta$$

$$\lambda = -\frac{1}{n} \cos n\theta + c.$$

Consequently plot values of $(-\frac{1}{n} \cos n\theta + c)$ as abscissa and values of the $f(\theta)$ as ordinates. Now if this is done between the limits $\theta = 0$ and $\theta = 2\pi$, a curve will be obtained whose area should be the desired integral, and if we divide the area of this curve by π we will get the value of A_n. Here then is a simple method of obtaining graphically the coefficients in the Fourier series for any function $f(\theta)$. By ruling paper with vertical lines so spaced that their distances measured from an f (θ) axis will be values of cos nθ for equal simple increments of nθ the work of plotting can be reduced, for it remains then only to assign to each vertical line its proper height according to the value of f (θ) at that point and to connect the ends of these ordinates with a continuous smooth curve.

If it is desired to find the coefficient of a term in the cosine series similar reasoning will show that it is merely necessary to plot values of $\frac{1}{n}$ sin nθ against the given f (θ) and the area divided by π will be the desired coefficient.

Plate (I) shows an analysis of the curve f (θ) = 1.5 sin θ by this method. The unit for both abscissa and ordinates is one inch. With this unit the ordinary planimeter will give the correct area directly. These curves will be recognized as the familiar composition of two simple harmonic motions at right angles. In general, however, the curves are not restricted to this type as the form of the curve depends entirely on the form of f (θ). Since f (θ) = 1.5 sin θ, if our method is correct we should obtain zero for all the cosine coefficients and also for all the sine coefficients except A_1 the coefficient in A_1 sin θ . Obviously A_1 should turn out to be 1.5. If the curve for A_1 in Plate (I) be measured with a planimeter, it will be found to

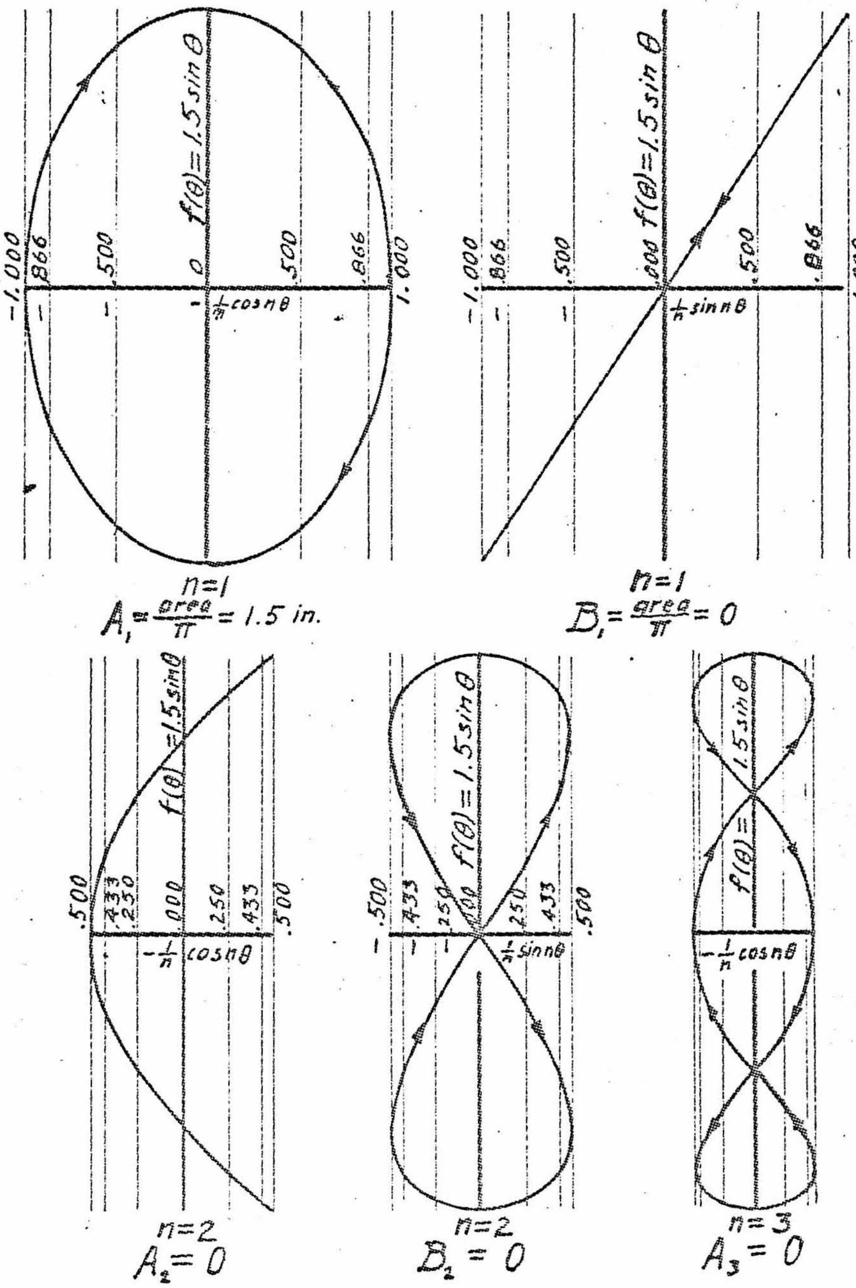

f(θ)=1.5 sin θ
-1.000 .866 .500 0 .500 .866 1.000
-1/n cos nθ
n=1
A₁ = area/π = 1.5 in.
f(θ)=1.5 sin θ
-1.000 .866 .500 .000 .500 .866 1.000
1/n sin nθ
n=1
B₁ = area/π = 0
f(θ)=1.5 sin θ
.500 .433 .250 .000 .250 .433 .500
-1/n cos nθ
n=2
A₂ = 0
f(θ)=1.5 sin θ
-.500 .433 .250 .000 .250 .433 .500
1/n sin nθ
n=2
B₂ = 0
f(θ)=1.5 sin θ
-1/n cos nθ
n=3
A₃ = 0

Plate (I)

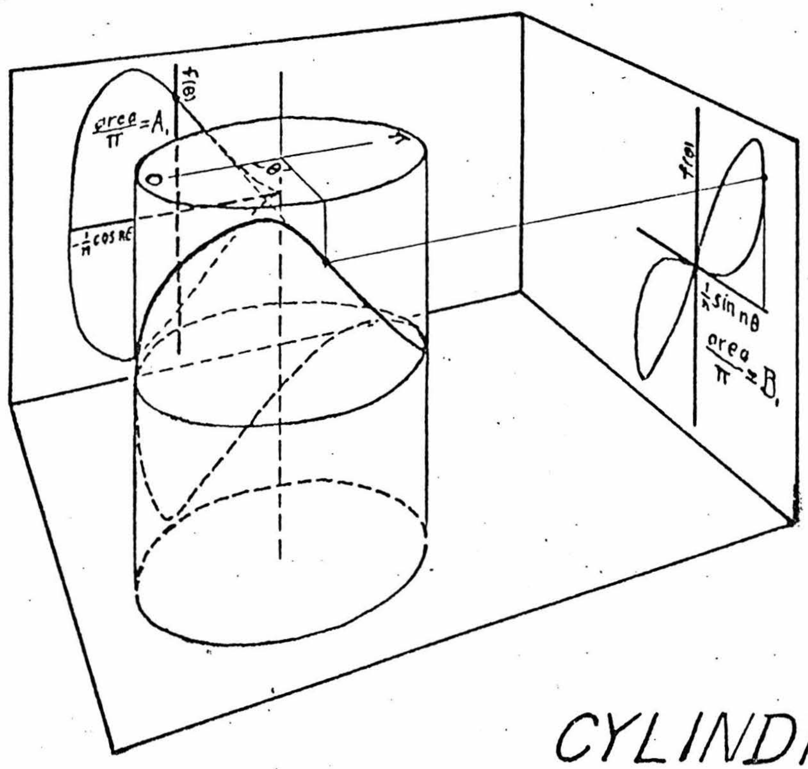
have an area of $1.5 \times 1 \times \pi$. This is obviously the case since the curve is an ellipse and its area is π times the product of the semi-major by the semi-minor axes. Dividing this area by π we obtain 1.5 for the coefficient as we should expect. Of course the same reasoning applies to any amplitude r of $f(\theta) = r \sin \theta$ just as well as for 1.5. Notice moreover that in tracing this curve from $\theta = 0$ to $\theta = 2\pi$ (that is, from $(-\frac{1}{n} \cos \theta) = -1$ to $(-\frac{1}{n} \cos \theta) = -1$), the point moves clockwise. If $f(\theta)$ had been $-1.5 \sin \theta$ the point would have traced around the curve in the counter-clockwise direction, as a trial will quickly show. The area would then have been negative and we should and would obtain -1.5 for the coefficient. As for the coefficients of all the other terms of the series it will be seen that as the point moves from $\theta = 0$ to $\theta = 2\pi$ it traces either around no area at all or else around equal positive and negative areas so that the result is zero. This again is what we should expect.

To visualize the whole matter definitely suppose that the given $f(\theta)$ which is to be analized into harmonic components is plotted against θ on a sheet of transparent paper, and that this sheet is then rolled into a right circular cylinder so that the ordinates become elements of the cylindrical surface and the point $\theta = 0$ coincides with the point $\theta = 2\pi$. The projections of this curve upon a plane parallel to the axis of the cylinder or its appearance viewed at infinite distance would then be obviously the same as the curve for A_1 or B_1 in Plate (I). Plate (II) is an attempt to show this in perspective. If the sheet were rolled twice before bringing $\theta = 0$ and $\theta = 2\pi$ into coincidence the curve would appear like those in Plate (I) which represent A_2 or B_2. Here then at last we have a concrete idea of

$$\left.\begin{matrix} A_n \\ \\ B_n \end{matrix}\right) = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \begin{pmatrix} \sin\theta \\ \\ \cos\theta \end{pmatrix} d\theta .$$

We can now look at curves to be analyzed with "eyes that see", for it is merely a matter of visualizing the curve rolled up and remembering that the apparent area which is seen is a measure of the amplitude of the curve's nth harmonic if the curve has been rolled n times. (Note the cylinder should be viewed so that $\theta = 0$ appears on the extreme left for the sine series and directly in the center for the cosine series).

We have so far purposely avoided the constant term in the cosine series because it does not harmonize well with the rest. Let us make both our imagination and our transparent sheet of paper very elastic. Then if we stretch the curve $f(\theta)$ on the sheet once around an immense cylinder of infinite diameter bringing $\theta = 0$ and $\theta = 2\pi$ togehter at the center as before, the apparent curve will certainly be as $\underset{a \doteq 0}{}$ $(\frac{1}{a}\cos a\theta)$, $(f(\theta))$, and the area divided by π should be the constant term. This would be quite convenient if it were not that



Analysis of

$f(\theta) = \sin\theta + .5\sin 2\theta.$

Sin and cos terms for

$n = 1$

CYLINDRICAL GRAPH.

while the point in tracing from $\theta = 0$ to $\theta = 2\pi$ moves around an infinite distance it invariably encloses what would be a zero area if the distance were anything but infinite. For, whatever the curve be, if it is stretched out to infinite length any picture we can show of it will be a straight line parallel to the stretched axis. Consequently any attempt to measure the area will invariably result in an indeterminate of the form $\infty - \infty$. Moreover we cannot view the cylinder from any position which will help us to evaluate this difference. The writer realizes the inadequacy of this explanation. The only way to comprehend the matter is for the reader himself to experiment with some sample curves (known to contain a constant term) on this infinite cylinder. The problem is fascinating but useless.

Fortunately the constant term is obtained very simply if one gives up the idea of its being the coefficient of cos (0)θ. Thus as has already been shown :

$$B_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)\, d\theta$$

or in other words B_0 is the average cordinate. This can easily be obtained from the curve directly with a planimeter.

In order to further make clear the analysis of curves into harmonic functions we will now take a curve apart and put it together again both algebraically and graphically. We will start with the curve $f(\theta) = \theta$, a simple straight line passing thru zero, and analyze it into harmonic components by the formulae IV, V, and VI. We will then take a finite number of these components and adding then together obtain an approximation to f $(\theta) = \theta$. Then, taking this approximation we will, by the cylindrical graph, resolve it into its original components again.

If f $(\theta) = \theta$.

$$A_1 = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \sin\theta \, d\theta = \frac{1}{\pi}\int_0^{2\pi} \theta \sin\theta \, d\theta.$$

$$A_1 = \frac{1}{\pi}(\sin\theta - \theta\cos\theta)_0^{2\pi} = -2.$$

And in general :

$$A_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \sin n\theta \, d\theta = \frac{1}{\pi}\int_0^{2\pi} \theta \sin n\theta \, d\theta$$

$$A_n = \frac{1}{\pi}\left(\frac{\sin n\theta - n\theta\cos n\theta}{n^2}\right)_0^{2\pi} = -\frac{2}{n}.$$

Let us see if f $(\theta) = \theta$ contains cosine terms :

$$B_1 = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \cos\theta \, d\theta = \frac{1}{\pi}\int_0^{2\pi} \theta \cos\theta \, d\theta.$$

$$B_1 = \frac{1}{\pi}(\cos\theta - \theta\sin\theta)_0^{2\pi} = 0.$$

And in general :

$$B_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \cos n\theta \, d\theta = \frac{1}{\pi}\int_0^{2\pi} \theta \cos n\theta \, d\theta.$$

$$B_n = \frac{1}{\pi}\left(\frac{\cos n\theta - n\theta\sin n\theta}{n^2}\right)_0^{2\pi} = 0$$

except when n = 0 in which case

$$B_0 = \frac{1}{2\pi}\int_0^{2\pi} \theta \, d\theta = \pi .$$

Evidently there are no cosine terms except B_0 which is equal to π . The following then is the expression for $f(\theta) = \theta$.

$$f(\theta) = \theta = \pi - 2 \sin \theta - \sin 2\theta - \frac{2}{3} \sin 3\theta - * * * - \frac{2}{n} \sin n\theta - * * * * .$$

On Plate (III) will be found the graphs of $f_0(\theta) = \pi$, $f_1(\theta) = -2 \sin \theta$, $f_2(\theta) = -\sin 2\theta$, $f_3(\theta) = -\frac{2}{3} \sin 3\theta$ and $f_4(\theta) = -\frac{1}{2} \sin 4\theta$.

These have been graphically added together on the curve sheet with the result shown at A. It will be seen that the resultant already with only the first four terms of the infinite series approaches the ideal curve B quite nearly. θ is plotted in radians with a scale of one inch equals one radian and $f(\theta)$ is plotted to units of one-half inch each.

Now let us start with the curve A in Plate III and by the cylindrical graph analyze it into the same components out of which we constructed it. The work is shown on Plate IV. For each value of θ we plot a point whose ordinate is the ordinate of curve A for that value of θ and whose abscissa is $-\frac{1}{n} \cos n\theta$. It is evident from an inspection of these figures that the tracing point has moved counter-clockwise around the major part of all of the

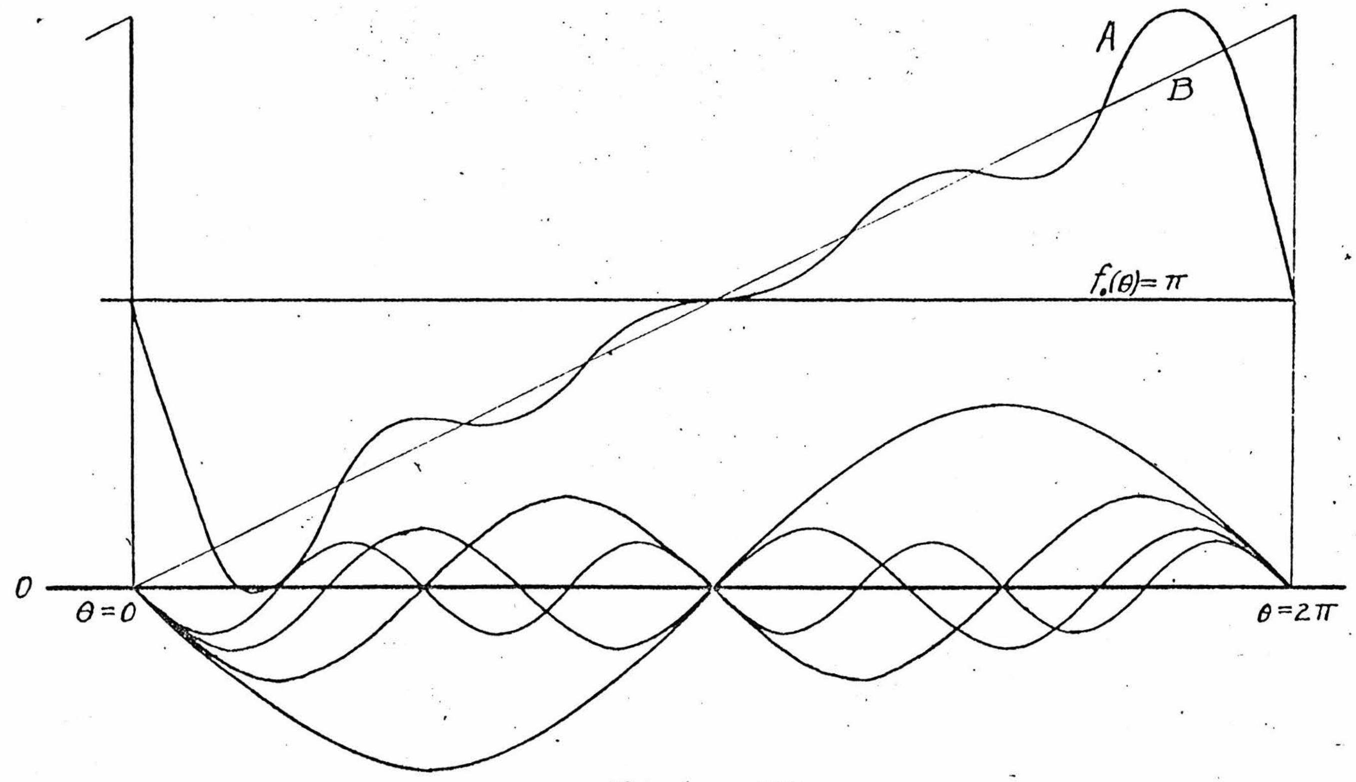


Plate III.

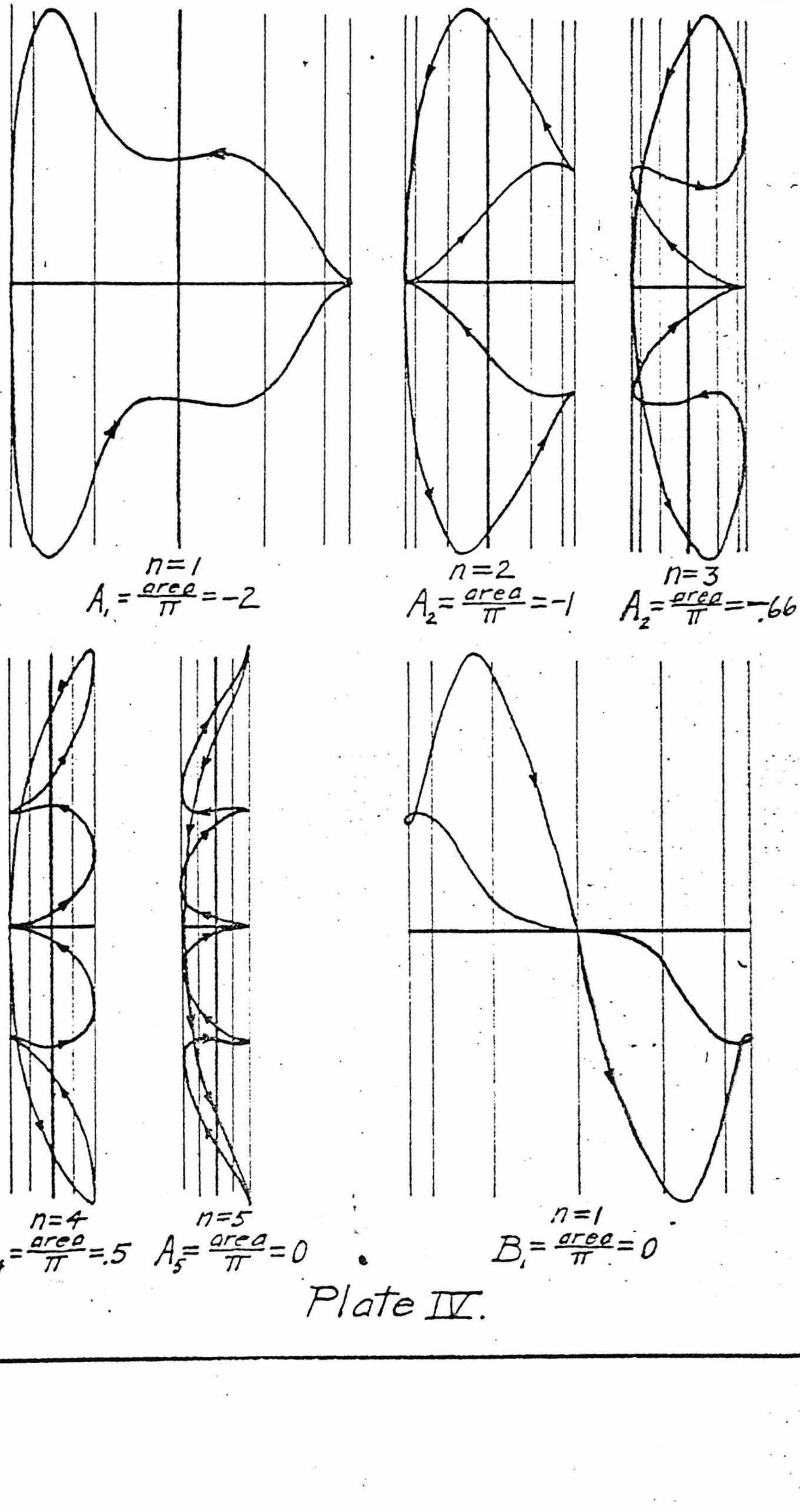

n=1
A₁ = area/π = -2
n=2
A₂ = area/π = -1
n=3
A₂ = area/π = -.66
n=4
A₄ = area/π = .5
n=5
A₅ = area/π = 0
n=1
B₁ = area/π = 0

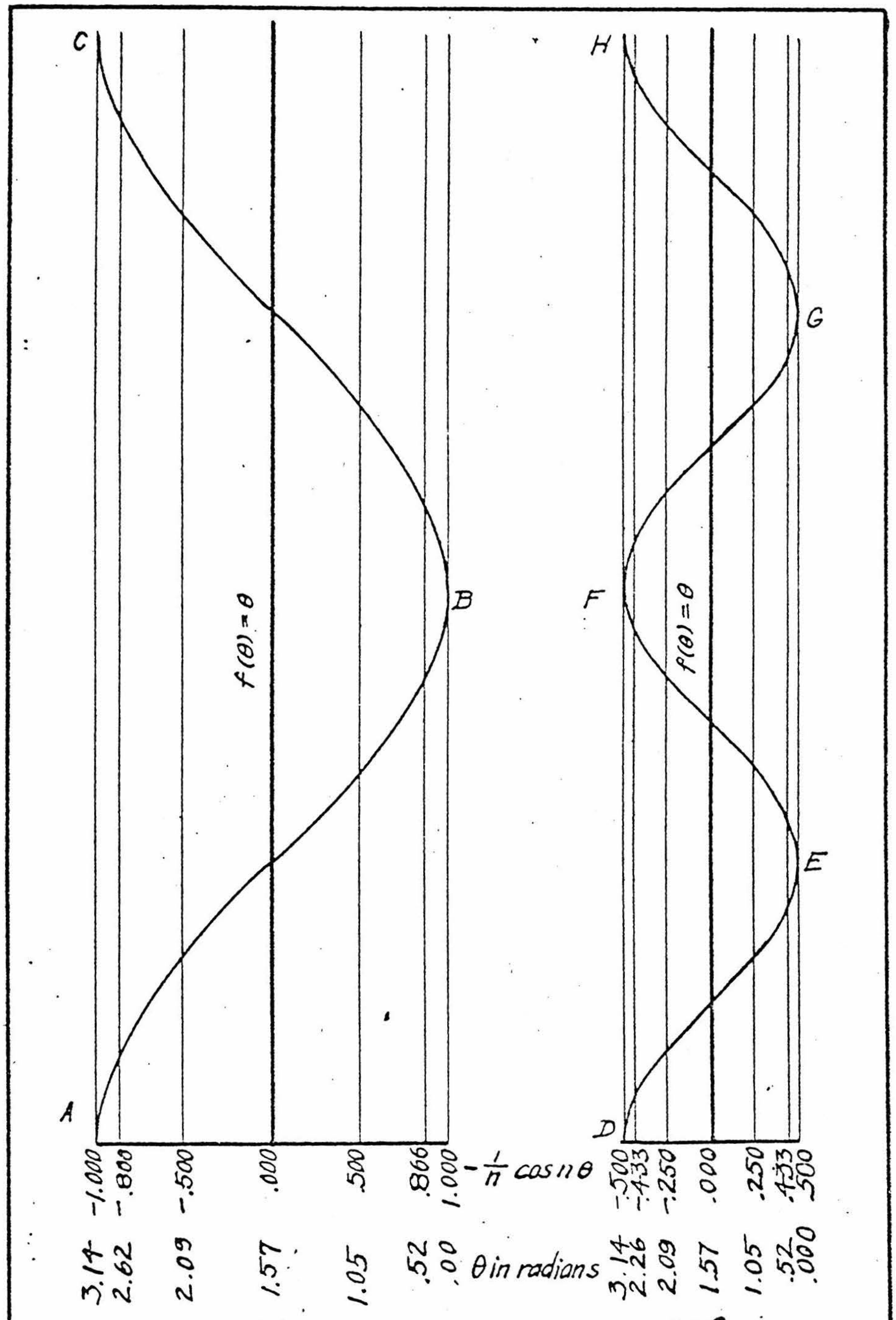
Plate IV.

areas up to the graph where n = 5. These areas then are negative and when divided by π are equal to the coefficients of the first four sine terms. The graph for n = 5 however registers a zero area with a planimeter for the closed areas traversed in a clock-wise direction exactly equal those traversed in a counter-clockwise direction. The graph for the fundamental cosine term has also been plotted and appears in the lower right hand corner. It is of course zero consisting as it does of equal positive and negative areas. Thus we have analyzed the curve back into the components from which it was built, the graphs showing conclusively that the curve is made up of four negative sine terms and no cosine terms. As for the constant term of the cosine series, its value is evidently π since that is the average ordinate of the curve as can be seen by a glance at Plate III from the symmetry of the figure. The scale in these figures is unity equals one inch, both for the $(-\frac{1}{n}\cos n\theta)$ axis and the $f(\theta)$ axis.

As an additional example the ideal curve $f(\theta) = \theta$ shown at B in Plate III has been analyzed for its first two harmonics by the cylindrical graph method. The work is shown in Plate V. The scale of this graph like the others is unity equals one inch. Since the graph is not a closed curve attention must be paid to the area desired. To go back to fundamentals it was shown that,

$$A_n = \frac{1}{\pi}\int_{\theta = 0}^{\theta = 2\pi} f(\theta)\, d\lambda.$$

where $\lambda = -\frac{1}{n}\cos n\theta$ = the abscissa of our graph. This integral is obviously the area under the curve as we pass from 0 to 2π, areas passed over in the positive direction being positive and vice-versa. The area desired is evidently then in this case by subtraction the area between the curve ABC and the ordinate AC. This graph is obviously a sine curve being the projection of an inclined line wrapped around a cylinder. Since this sine curve has an amplitude of 1 and a period of 2π, the area desired is evidently -2π: dividing this by π we get for the value of A_1, -2, which is what we obtained by the method of direct calculus. Similarly the area of the graph divided by π for n = 2 is -1; and a little consideration of the method of construction of the figure will show that for any graph where n = n the area divided by π will be $-\frac{2}{n}$, which is what it should be. The area traced over can be thought of as closed if we consider that the tracing point finally passes down the ordinate from C to the starting point at A. This corresponds to the vertical part of the curve shown in Plate III. If this device of closing in the curve on the vertical ordinate is always strictly observed with the planimeter the sign of the area will automatically take care of itself and we thus have a perfectly general method of graphical analysis capable of meeting all problems.



$n=1$ $\qquad$ $n=2$

$A_1 = \frac{area}{\pi} = -2$ $\quad$ Plate (V.) $\quad$ $A_2 = \frac{area}{\pi} = -1$

Having seen the curve on Plate III the reader will probably ask why, if it is intended to reproduce the ideal curve B, it does not continue on to infinity as $f(\theta) = \theta$ does. At 2π it drops off vertically to zero.

The form of the curve depends entirely on the amplitudes A_1, A_2, A_3 etc. If at any point on the curve these amplitudes have not the proper value the ideal curve will not be reproduced. Now the value of these amplitudes it will be remembered was obtained by solving an equation. (See equation II above) But this equation possesses the curious property that it is only an equality between $\theta = 0$ and $\theta = 2\pi$ for if some other limits had been used, the undesirable terms of the form $\int \sin m\theta \sin n\theta \, d\theta$ would not have vanished and Equation II would not have been true as it stands. Therefore when θ becomes greater than 2π the formula IV does not give the correct value for A_n and consequently the curve depending as it does on the A_n's cannot even approximate the ideal $f(\theta)$. By a proper choice or change of scale however 2π can be made to come anywhere on the $f(\theta)$ and thus we can represent as much of it by the Fourier's series as we like, provided we are satisfied with a finite amount. We will now pass on to the design of a machine which will automatically determine the integral :

$$\frac{1}{\pi}\int_0^{2\pi} f(\theta)\begin{pmatrix}\sin n\theta\\ \cos n\theta\end{pmatrix} d\theta.$$

given the $f(\theta)$.

DESIGN . .

In the study of alternating current wave forms the wave is now almost invariably obtained with an oscillograph. This instrument causes a fine point of light to move transversely on a very sensitive photographic film in unison with the variation of the instantanious value of the current wave while at the same time the film has a uniform longitudinal motion. The period in inches of the plot thus formed may have any size whatever within the practical limits of the oscillograph. For the sake of simplicity in the resulting Fourier series it is important that the units of the curve shall be so assumed as to make the period equal to 2 π. To avoid replotting the curve to a new scale it is desirable then to have the machine devised so it will multiply ~~ordinates~~ whatever length the curve may happen to have by the proper factor to make it 2 π.

In the oscillographic record of the curve the value of the scale in amperes per inch is unknown. A calibration once for all is impossible as the oscillograph requires continual adjustment. In order then to ascertain the scale to which the oscillograph has plotted the wave it is customary to use a voltmeter or ammeter at the time the graph of the wave is taken, thus obtaining the root mean square value of all the instantaneous values of current or voltage graph over a period 2 πn. It is desirable then that the instrument shall be capable of obtaining this root mean square ordinate in inches. The ampere or volt reading, effective value may then be divided by this R.M.S. ordinate thus giving the scale of the curve in amperes or volts per inch. Then having obtained the values of the amplitudes of the harmonics in inches their values in amperes can readily be obtained. A third requirement of the machine whihc is most imperative of all is that its expense be small--a total cost of $75 being the limit.

With these things in view the machine was designed as follows. Referring to the assembly drawing, A is a table sliding upon a pair of rails BB. Projecting under this table is a large pin C capable of turning without lost motion and having a hole in its enlarged head to take the rod D, a good sliding fit. This rod D also passes through the pin E and through a third and similar pin O projecting under the carriage F, which slides on the rods GG. Thus the carriage F and the table A have a uniform motion along their guides with respect to each other, the relative magnitude of which can be varied by fixing the carriage of the pin E anywhere along its guides HH.

The carriage F is fitted with a half nut which meshes with the screw J. The end K of the screw J is adapted to take change gears which transmit motion between the shafts J and L. On the shaft L is a worm M which drives the gear N thus turning the disc and quill Q on the stationary stud R. The disc Q has a hole bored exactly one inch from the center which contains a pin S with a diametral hole through its enlarged head. The aluminum table T slides upon the rods UU and carries a rod V which passes through the diametral hole in S. As Q is driven round uniformly by the worm M it is evident that the table T will execute an harmonic motion of amplitude one inch. At the same time the carriage F will translate uniformly causing a proportional motion of the table A. The columns WW are fitted with small V-grooved pulleys in which rests the long rod X. A planimeter Y rests with its wheel on the table T, its stationary point being fixed in the outrigger of that table. The tracing point P of the planimeter is attached to the long rod X and partakes of any motion which X may have. A pointer Z on this same rod comes within a short distance of the top of the table A.

If a curve to be analyzed is mounted on this table the pointer Z can be made to follow the f (θ) ordinate of this curve as the curve slides along under the pointer. This means that the tracing point of the planimeter P is exectuing the same motion. What figure would this tracing point P, if it were equipped with a pencil, discribe on the aluminum table T as that table maves under it? The ordinate of the curve is evidently f (θ) and its abscissa is either of the form $a \sin b\theta$ or $a \cos b\theta$ where ~~a~~ ~~and~~ b is ~~are~~ determined ~~respectively~~ by the ~~initial~~ position of the point E and the gear ratio connecting K and L. Consequently the figure traced by P upon the table T is the "cylindrical graph" of the given curve on table A. It is evident that the area of this graph is measured by the planimeter. If the planimeter is properly proportioned the reading on the wheel can be made to give the traced area divided by π and thus the reading on the wheel would give the amplitude of the desired harmonic directly were it not for a circumstance shortly to be explained. We must first give consideration to the units to which the cylindrical graph is traced.

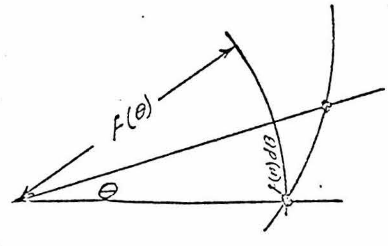
The wheel N has 72 teeth and since M is a single threaded worm 72 revolutions of L cause one revolution of N. The screw J is cut with twelve threads per inch. Thus to move the carriage F six inches, 72 revolutions of K are necessary. With certain rare exceptions the travel of the carriage F is always six inches for one complete integration from 0 to 2π. Now if the gear ratio between K and L is one to one, the crank disc Q will make one complete revolution during the normal six inch travel of the carriage, F. If the gear ratio is such that L makes 2 revolutions for each one made by K then obviously the crank disc will make two complete revolutions during the travel of the carriage F. The machine is provided with gears which by ringing various changes will give all the integral ratios from one to sixteen, inclusive. The initial position of the carriage F is marked upon one of the guide rods G. The shaft K has at the opposite end from the gears a crank provided with a hole through which a pin can be inserted into the bracket behind. While making the set-up preliminary to an analysis the position of the shaft K is always fixed by this means. If the carriage F is put approximately on the mark and the half nut dropped into place, the carriage will take the proper initial position with great accuracy. This position is determined by means of the adjustable thrust collars on shaft K so that the swinging rod D shall be exactly parallel to its guides HH at the start. This prevents the table A from changing its initial position when the position of the point E is changed.

Having selected the positions on the curve that are to be the "zero" and "2π" points, mount the curve on the table A with thumb tacks so that the zero line shall be parallel with the guides along which the table is to move. The small marks at the ends of table A facilitate this. With the carriage F in the initial position see that the zero point of the curve to be analyzed is directly under the tracing point Z. Unlock the half nut on the carriage F and move the latter to its extreme position six inches along the guide rods. The half nut will locate the carriage accurately when it dropp into the threads. Adjust the pivot E in such a position that the point 2π on the curve will come directly under the tracing point Z. As above stated this will not affect the adjustment already made for the initial position. Clamp the pivot E with its set screw. When the tracing point Z has covered the distance zero to 2π of the curve the carriage F will have moved its normal six inches. This will have produced exactly one complete revolution of the crank disc Q if the gear ratio between K and L is one, or n revolutions if that ratio is n. It is evident that the crank disc passes through equal angles as the table A travels over equal distances. If the position of the crank disc Q is initially as shown in the assembly drawing, the pin S as it moves around counter-clockwise will be displaced from the axis of the rodX, a distance <u>in inches</u> equal to the sine of $n\theta$, n being the gear ratio and θ the abscissa of the given curve on table A, because the radius in inches of the circle described by S is exactly one. The table T reciprocates with the pin S. If a point had been marked on an extension of this table directly under the pin P on the rod X while the table was in its initial position, then at any instant the distance of P from that point would be sin $n\theta$, positive distances being measured on the table T from this point in the direction marked by the small arrow in the drawing and negative distances in the opposite direction.

Thus it is evident that if the point P on the rod X be considered as tracing a curve on an extension of table T it will at all times be sin nθ inches from the above mentioned initial mark in the direction of the arrow and f (θ) inches from this same initial point in the direction in which the long rod X slides, the positive directions here also being indicated by an arrow. Thus it will trace a curve whose area in square inches will be n times as great as the area of the cylindrical graph explained under "Theory". It will be n times as great because the "cylindrical graph" was plotted between the coordinates $\frac{1}{n}$ sin nθ and f (θ) while here the curve is plotted between sin nθ and f (θ). Consequently if we wish to obtain in inches the amplitude of the nth cosine harmonic it will ne necessary to divide the area of this curve by π n. Now it is evident that the area of this graph is measured by the planimeter as the curve is traced, for the tracing point of the planimeter has followed the graph with respect to the table on which the planimeter lies. The diameter of the wheel of the planimeter and the length of its tracer arm are such that the area in square inches divided by π is the scale reading. Thus it is necessary only to divide the area obtained on the planimeter by n the number of the harmonic for which the curve is being analyzed, to obtain the desired amplitude of that harmonic in inches. If the initial position of the pin S had been 90° clockwise from that specified above a little consideration will make it clear that the abscissa of the graph thus traced would have been $-\cos n\theta$. The area thus obtained divided by n is then the amplitude of the sine term in the curve. Both of these initial points are plainly marked on the crank disc and are located with great accuracy when the initial set-up is made by fixing the position of the shaft L. This shaft, like K, has a crank on one end with a hole to permit sliding a pin through into the bracket behind.

Now that means have been provided for obtaining the sine and cosine terms, the next step is to develop a method of adapting the instrument to find root mean square values of curves.

In "Alternating Current Phenomena" by C.P.Steinmetz, the following method of obtaining graphically the R.M.S. value of any curve is developed. Suppose the given curve be plotted in polar coordinates with $f(\theta)$ as the radius vector and θ as the argument. The small figure on the right shows a differential triangle on this plot.



It is evident that the area of this triangle is

$$ds = \frac{f(\theta)^2 \, d\theta}{2}$$

and the total area of the curve between the limits 0 and 2π is

$$\int_0^{2\pi} ds = s\Big)_0^{2\pi} = \tfrac{1}{2}\int_0^{2\pi} f\,(\theta)^2 d\theta.$$

Now, the root mean square ordinate of any function f (θ) is

$$\text{R.M.S.} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} f\,(\theta)^2\, d\theta.}$$

Comparing the last two equations, it is evident that

$$\text{R.M.S.} = \sqrt{\frac{s}{\pi}}$$

Thus it is evident that if the given curve f (θ) be plotted in polar coordinates it is merely necessary to divide the area by π to obtain the mean square value and the root mean square value is of course the square root of this.

In order to obtain R.M.S. values with the Harmonic Analyzer it is merely necessary to lift the reciprocating table T off its guides UU taking with it the pin S which slides easily out of the hole in the crank disc Q. Before doing this the long rod X and the planimeter are removed.

The irregular shaped aluminum casting shown on the right in the assembly drawing is then fitted onto the top of the crank disc Q and secured with the thumb screws. The long rod X and the planimeter are then replaced with the planimeter wheel resting on the table of the aluminum casting. The pole of the planimeter swings on a pivot on the outrigger of this casting. The tracing point of the planimeter is pivoted as before in the rod X.

Now, if the gear ratio between shafts K and L is made one to one it is evident that the aluminum casting will turn once around while the curve on table A moves from 0 to 2π carrying with it the planimeter. If, meanwhile the tracing point Z has been guided along the given curve it should be evident that with respect to the aluminum casting the point P has traced the $f(\theta)$ plotted in polar coordinates and that the readings on the planimeter give the area of that plot divided by π. If a pencil had been secured at P so as to mark on a piece of paper fastened to the aluminum casting and rotating with it, this polar plot would actually have been drawn. Since the planimeter divides the area in inches by π automatically it is evident that the reading on the planimeter gives directly in square inches the mean square ordinate of the curve $f(\theta)$. To obtain the root mean square extract the square root of the planimeter reading.

In order to allow the operator to concentrate his whole attention on following the curve correctly with the tracing point the driving of the machine is accomplished with a motor geared to the shaft L. Two small brass bevel gears are used. The motor is adapted to run directly on alternating current at 110 volts so it is merely necessary to plug the cord connection of the motor to an ordinary electric light socket.

ADJUSTMENTS.

The harmonic analyzer has but few adjustments. In fact, the only adjustments aside from those incident to each set-up are the settings of the thrust collars on the shafts K and L. The entire accuracy of the analyzer depends on these collars however so that care should be taken not to disturb them. It is by means of the collars on L that the crank disc is finally adjusted accurately for its initial sine or cosine position. A good method of locating the initial point is to follow the process commonly used in locating the "dead centers" of a steam engine. With the table T in place on its guides and the thrust collars on L in any convenient setting, turn L by means of the crank until the table T is approximately 90° from its "dead center" and lock the crank. Scribe a light mark somewhere on the table T at right angles to its line of motion and set up a micrometer microscope above this mark so that it is exactly under the cross hair. Now unlock the crank and turn L until the table T having gone once forward and back has brought the mark again under the micrometer microscope. Count the turns. Unless the collars on L are already in adjustment the crank will not be in position to lock. Turn it around however until it <u>is</u> in position and lock. The crank should have made an <u>even</u> number of revolutions. If it has not, make an extra revolution before locking. The mark will now not appear under the cross hairs of the micrometer microscope. With the micrometer screw measure the distance of the separation and divide it by two. Set the cross hair in the mid-point of the discrepancy by turning off this half distance on the micrometer screw. Now adjust the thrust collars until the mark comes directly under the cross hair and lock them. Now if the crank is turned back the same number of revolutions as it was turned forward, the mark on the table T should return to its position exactly under the cross hair when the crank is locked. If it does not, this is either because the first discrepancy was too large or because the adjustment was made too near the dead center of the table. This second discrepancy should then be halved as before and when the table is again reciprocated the mark should come exactly under the cross hair when the crank is locked. The dead center for the table then is half the number of revolutions of the crank back from its present position. It is evident now why an even number of revolutions should be made. The number could not be halved otherwise. In order to find the position 90° away from the dead center merely rotate the crank 18 times.

The adjustment of the thrust collars on the shaft K controll the parallelism of the rod D with the guides EH when the carriage is in the initial position. This as explained above is necessary to facilitate setting the pivot E in the proper place for the scale of the given curve. The pivot E must be adjusted to the proper place along its guides so as to bring the point 2π on the curve under the tracing point when the carriage E occupies its final position. After making this adjustment the point $\theta = 0$ of the curve would no longer be under the tracing point in the initial position of the carriage F were it not for the initial parallelism of these rods. The following process is used to set the thrust collars so that this parallelism shall be obtained.

Set the carriage F in the initial position and lock the crank on rod K. Slide E as near the pivot under table A as possible. Mark a point on table A and set the cross hairs of a micrometer microscope exactly over this point. Now slide E as near F as possible. Unless the rod D was already parallel with its guides the table will move slightly carrying the mark away from the cross hair in the micrometer microscope. Now adjust the thrust collars until the mark is brought back under the cross hair. Lock the thrust collars. Now slide E back as near A as possible. This may move the mark away from the cross hair. If so, bring the cross hairs to the mark with the micrometer screw. Push the pivot E again as near to F as possible. If the mark again moves away from the cross hairs adjust ~~that~~ the thrust collars as before until it comes back. Continue this cycle of processes until the mark does not move appreciably under the cross hairs no matter what position E occupies. Not more than two or three repetitions will ever be necessary. Remember always to bring the mark and cross hairs together by adjusting the micrometer microscope when E is near A and by adjusting the thrust collars when E is near F. If this rule is not observed the discrepancy will tend to increase instead of diminish. If the thrust collars are firmly locked with their set screws there is no reason why the machine should not run indefinitely without needing adjustment.

The wheel of the planimeter has two hundred graduations, each hundred of which is numbered by tens from one to ten in the same directions around the wheel. The whole revolution counter is turned by a ten tooth pinion meshing a single thread worm and has twenty graduations upon it numbered from one to twenty. Thus each graduation represents a half turn of the planimeter wheel, that is it represents the passage of one hundred wheel divisions past the vernier. In taking a reading then it is merely necessary to take the first figure or two figures off the scale of the whole-revolution-counter and the remaining three figures off the planimeter wheel. In taking these readings it is better to start with the whole-revolution-counter somewhere near ten for the initial reading, because the final reading is just as likely to be less than the initial one as it is to be greater. This depends on whether the amplitude of the harmonic for which the curve is being analyzed is positive or negative. If the initial reading were taken near zero the counter might rotate backward through zero without being noticed by the operator and then what would appear to be a very large positive reading would in fact be a small negative reading.

The tracer arm of the planimeter can be varied slightly in length as can been seen in the detail drawing of the planimeter, by moving the small block containing the pivot hole to the desired position. This calibration is best made by causing the planimeter to measure the area of a circle of one inch radius. When it is properly adjusted the planimeter will read one for this area which we know in reality to be π . A simple method of causing the planimeter to measure such an area without removing it from the rod X is as follows: Make a set up for root mean square values. Place the pointer Z exactly over the zero line of the table A. Without moving the pointer Z run the analyzer through one complete analysis. If the reading on the planimeter does not change it indicates that the point P is exactly over the center of the disc Q when the pointer Z is on zero. If the reading changes it may be that the block B on the Rod X has been moved or that the pivots of the V-grooved pulleys carrying the rod X have been changed so that the rod X is no longer over the center of the disc Q. In either case this should be corrected before attempting to adjust the planimeter itself. When the planimeter finally remains stationary under the above named conditions set the pointer Z exactly one inch away from the zero line on table A and in this position make one complete R.M.S. analysis. The planimeter should register a difference of ~~four~~ one square inch. If it registers high the arm of the planimeter should be lengthened slightly. If it registers low it should be shortened slightly.

PRACTICAL SUGGESTIONS

For The Analysis Of Curves * * *

Any curve to be expressed completely (that is from $-\infty$ to $+\infty$) by a Fourier's series must have a continuum of periodically recurring values. By this is meant that if the proper length were chosen with a pair of dividers, the points of these dividers placed anywhere along the θ axis of the curve would locate ordinates identically equal. We will call this length a period. If a curve does not fulfill this condition it is useless to try to fit a Fourier's series to it throughout its entirety. Any finite amount of such a curve can be expressed as a Fourier's series however. The first step then in a Fourier analysis is to decide exactly what portion of the given curve we desire to reproduce. If the curve is periodic obviously the reproduction will be most effective if a portion of the curve one period in length be chosen. We will omit any consideration of non-periodic curves. It is not always as easy to find the period of a periodic curve as one might suppose. Sometimes what appears to be a period of the curve is merely the period of some very predominant harmonic. Careful examination will show that ordinates separated what one imagines to be a period are not exactly equal but are perhaps decreasing. Plate VI shows an excellent example of this. If a small portion only of this curve were seen, the distance AB would immediately have been assumed (and wrongly) as the period. In fact the period is as long as CA since it is only until this distance has been passed over that the values of the curve begin to repeat. This curve as illustrated has no fundamental nevertheless it has a fundamental period which must be made the limits of reproduction if a reproduction to infinity in both directions is desired. Having determined the period of the given curve it is next necessary to decide what particular point on the curve will be called 0, and what point 2π. No matter what we choose there is a Fourier's series which will represent it. The question is, "What point will give, if used, as zero, the simplest resulting Fourier's series?"

THERE WILL BE NO COSINE TERMS IN THE FOURIER EXPANSION OF A CURVE IF THAT CURVE IS SYMMETRICAL ABOUT THE POINT π AS A POLE.

Obviously it is desirable to choose the initial point of the curve to be analyzed so that if possible the above statement will be true. Inother words the zero of the

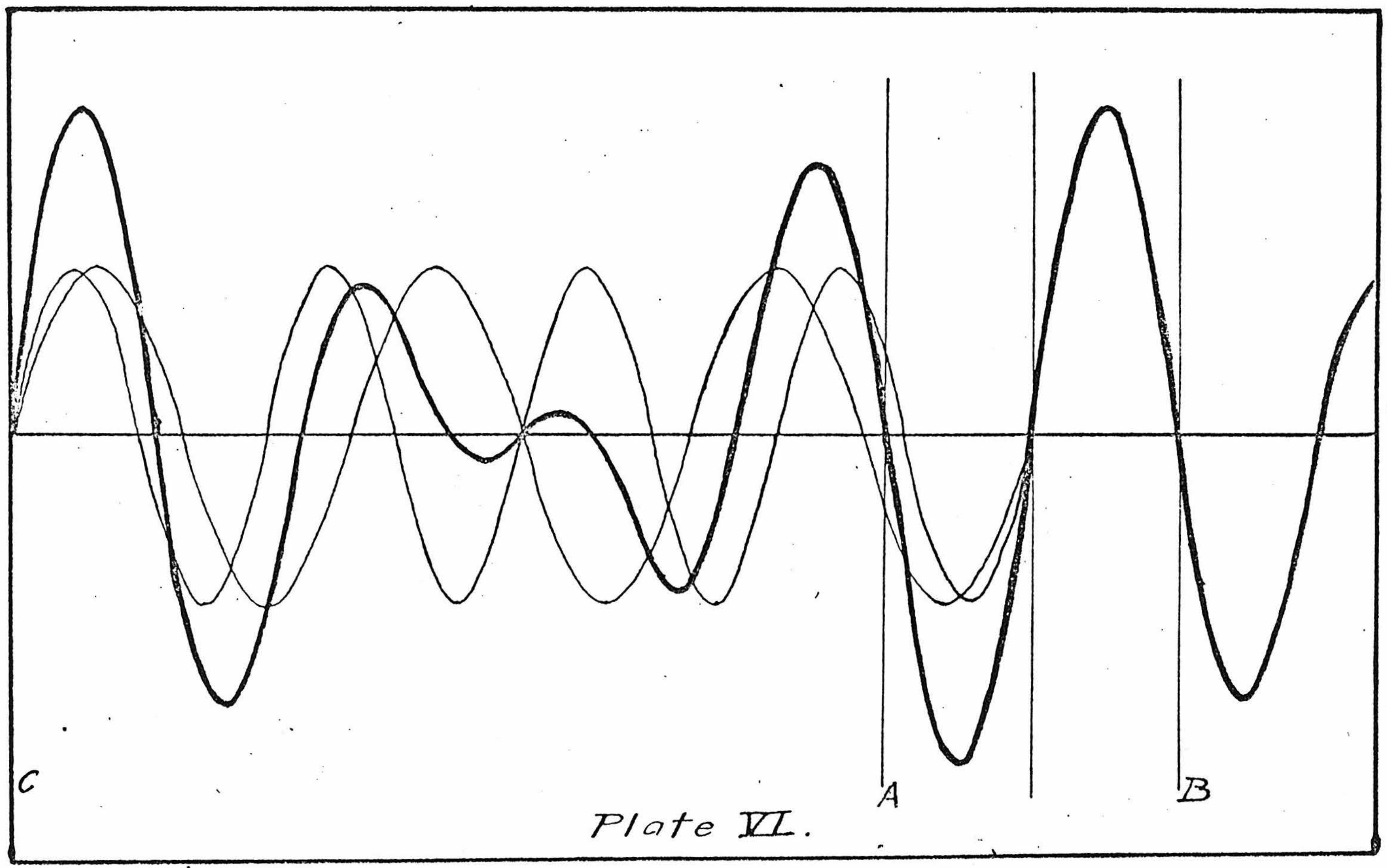


Plate VI.

curve should be chosen, if possible, so that every ordinate on the left of the center or 180° point will be equal and opposite in sign to its corresponding ordinate the same distance to the right of the center. Not all curves can be made to fulfill this requirement but nearly all alternator curves can be made to do so. This property immediately eliminates the necessity of analyzing for cosine terms thus cutting the work exactly in half. The following is a proof of the above proposition . If the curve is symmetrical about the point π as a pole this means that

$$f(\theta) \equiv -f(2\pi - \theta).$$

that is,

$$A_1 \sin\theta + A_2 \sin 2\theta + * * * + B_0 + B_1 \cos\theta + B_2 \cos 2\theta + *** \equiv -A_1 \sin(2\pi - \theta) - A_2 \sin 2(2\pi - \theta) + * * * - B_0 - B_1 \cos(2\pi - \theta) - B_2 \cos 2(2\pi - \theta) - ***$$

Simplifying the right hand member,

$$A_1 \sin\theta + A_2 \sin 2\theta + * * * + B_0 + B_1 \cos\theta + B_2 \cos 2\theta + * * * \equiv A_1 \sin\theta + A_2 \sin 2\theta - * * * - B_0 - B_1 \cos\theta - B_2 \cos 2\theta - * * *.$$

Since the two members are identically equal the terms of like frequency are separately equal to each other. But the only value which the coefficients B_0, B , B_2, etc. can have to make the separate cosine terms on either side equal is evidently zero therefore there are no cosine terms.

Having thus located the period points of the curve to be analyzed it is desirable to know whether there are even harmonics present. The ordinary alternator rarely perhaps never, gives even harmonics. In order to ascertain the absence of even harmonics slide the curve π untis to the left until it comes directly under the left half. If the two halves are then symmetriacl about the θ axis the curve contains no even harmonics. This is proven as follows. The above condition is equivalent to stating that $f(\theta) = -f(\theta + \pi)$.

or $A_1 \sin\theta + A_2 \sin 2\theta + * * * + B_o + B_1 \cos\theta + B_2 \cos 2\theta + * * * \equiv -A_1 \sin(\theta + \pi) - A_2 \sin 2(\theta + \pi) + * * * - B_o - B_1 \cos(\theta + \pi) - B_2 \cos 2(\theta + \pi) + * * *$/

Simplifying the right hand member,

$A_1 \sin\theta + A_2 \sin 2\theta + * * * + B_o + B_1 \cos\theta + B_2 \cos 2\theta + * * * = A_1 \sin\theta - A_2 \sin 2\theta + * * * - B_o + B_1 \cos\theta - B_2 \cos 2\theta + * * *$.

Since the expressions are identically equal terms of the same frequency are separately equal. But the only values of A_2, A_4, etc. B_o, B_2, B_4, etc., that can make the even numbered terms in the right hand member equal to the corresponding terms in the left hand member are zero. Therefore all the even terms including the constant term are zero.

These two tests can be easily made by inspection and are independent of each other. If they are both fulfilled the knowledge whihc they give saves three-quateres of the work which otherwise would be necessary in the analysis.

Testing the half curve for symmetry about an ordinate set up at the point $\frac{\pi}{2}$ is sometimes suggested. It is not however as general as the tests above given. The fulfillment of this test indicates the absence of even harmonics provided there are no cosine terms. If there are cosine terms it indicates absence of even sign harmonics and odd cosine harmonics. Occasion might arise when this test would be valuable. The curve might fulfill none of the tests except this particular one in which case one half the work of analysis would be avoided. Such could hardly be the case however with curves resulting from alternating current generators.

PROCEEDURE IN SETTING UP .

Having decided the location of zero, π and 2π on the curve to be analyzed, cut off a length of the oscillogram about $7\frac{1}{2}$ inches long to include the zero, π and 2π points. The zero point should be about three-quarters of an inch from one end. Lock the shaft K by pinning its crank to the cast iron bracket. Place the carriage F in its initial position by sliding to the marks on the guide rods and dropping the half nut into place on the screw. Now take the section cut from the oscillogram and secure it to the table A with thumb tacks is sucha position that the θ axis or "zero line" corresponds with the guide marks on the small brass inserts at either end of the table. Standing at the table A end of the analyzer facing the machine the positive direction on the θ axis of the curve will be to the right and the positive direction of the ordinates will be away from the observer. Next lift the half nut out of the thread and slide the carriage F to its extreme position according to the marks on the guide rods. Drop the half nut back in the threads. The oscillograph curve will have moved along its guides but the 2π point will probably not be under the pointer Z. Loosen the thumb screw U and slide the carriage E until the 2π point on the curve comes exactly under the pointer and fix the pivot in this position by tighteneing the thumb screw. Return the carriage F to its original position.

Now lock the shaft L by means of its crank, having first turned the disc to the position marked sine or cosine, depending on the term desired, by means of the crank. Next set up a gear ratio between the shafts K and L equal to the number of the harmonic desired. The following table gives the ratios obtainable with the gears furnished with the analyzer:

Ratio.	L	Idlers.			K
1	20		60		20
2	20		60		40
3	20		30		60
4	15		30		60
5	16	40		30	60
6	20	40		20	60
7	20	40		20	70
8	15	60		30	60
9	20	60		20	60
10	15	60		16	40
12	15	60		20	60
14	15	60		20	70
15	16	60		15	60
16	15	60		15	60

Note on 40.

To obtain the eleventh and thirteenth harmonics with the gears supplied special setting of the carriage F is reqtired. Instead of sttting carriage F on final point as per instructions for adjusting E, it should be set at point marked 11 if the eleventh harmonic is de-sired or on 13 if the thirteenth harmonic is desired. In making the analysis it should be allowed to run no farther than the mark at which the setting was made. For 11 and 13 the same gear combination is used as for 12, that is 15, 60, 20, 60. This could be avoided if special gears were cut to give ratios of 11 and 13. Any harmonic can be obtained by useing special gears.

The cranks remain locked while the gears are being secured by means of the thumb nuts that hold them by friction. ?

See that the planimeter reads about nine or ten on the counter disc. Take an exact reading of the planimeter with the pointer Z exactly over the zero point of the curve. Now unlock the brass cranks and start the analyzer either by turning the crank on shaft L, or by starting the motor on that same shaft. As the table A moves keep the pointer Z exactly over the curve under test until the final 2π point is reached, then stop immediately. If the ordinate of the curve is not zero here move the pointer back to the θ axis along a vertical line. Now read the planimeter and take the difference between the first reading minus this final reading. If the difference is negative the result obtained shows that there is a negative amplitude of harmonic of the order just analyzed for. If positive, the amplitude is positive. The reading of the planimeter is the amplitude in inches multiplied by the number of the harmonic sought. In very accurate work the process may be repeated as often as desired, the carriage F being returned to its initial position without turning the shafts. The initial and final readings can then be taken and their difference divided by the number of the harmonic and the number of repetitions. Thus almost any degree of accuracy can be obtained. This device is hardly necessary except when the amplitude is small.

WHEN THE CURVE IS TRACED BUT ONCE THE DIFFERENCE OF READINGS ON THE PLANIMETER DIVIDED BY THE NUMBER OF THE HARMONIC SOUGHT GIVES THE AMPLITUDE OF THE HARMONIC IN INCHES.

Having determined the amplitude of as many harmonics as isthought desirable it is in order to determine the scale of the curve in amperes or volts per inch. The "Root Mean Square" value of the curve in amperes or volts is known from the reading of an ammeter or voltmeter taken at the time when the oscillogram was made. The next step is to determine the "Root Mean Square" value of the curve in <u>inches</u>.

DETERMINATION OF R.M.S. VALUES.

DETERMINATION OF R.M.S. VALUES

Remove the long rod X and with it the planimeter. The planimeter may be lifted from the pivot on the outrigger of table T and folded under the rod X where it can be secured from swinging. The rod X is then removed from the pulleys by swinging it up and over out of the way.

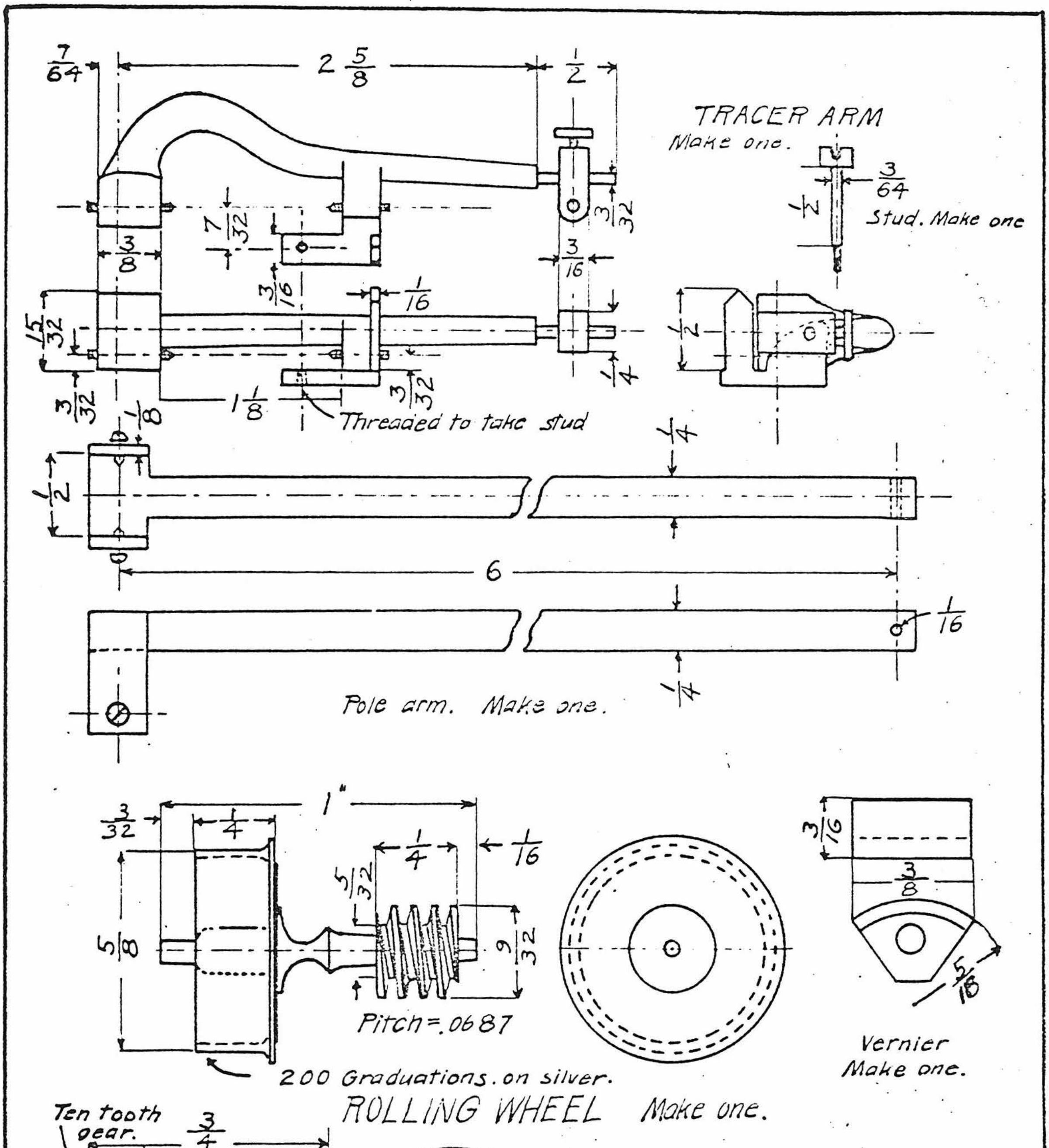
Remove the table T and with it the small steel pin S which will slide out of the hole in the bronze disc Q.

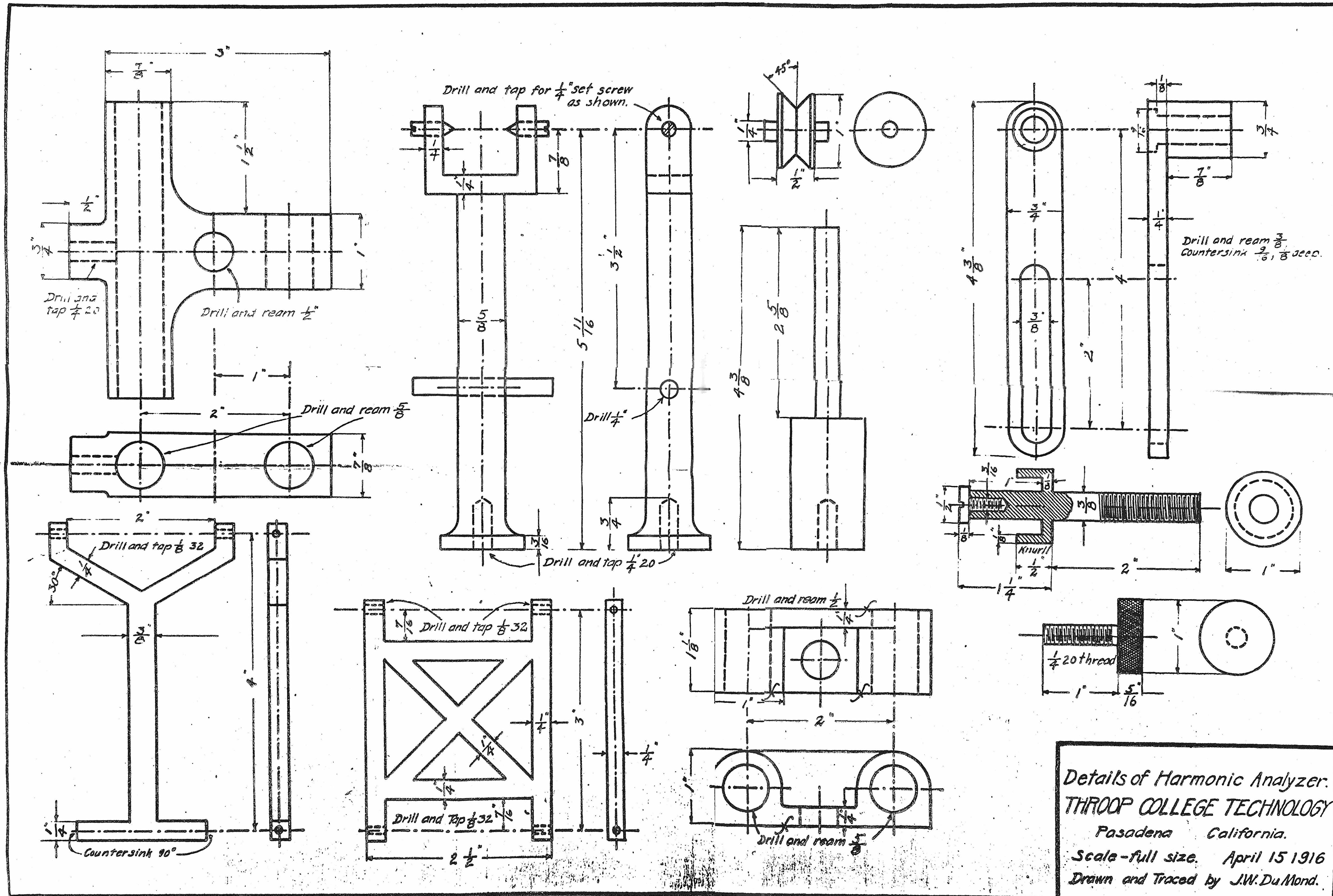
Secure the irregular aluminum casting upon the top of the disc Q. The exact horizontal angle at which it stands makes no difference.

Replace the rod X on its pulleys. Unfold the planimeter and attach its pole end to the pin in the outrigger of the irregular aluminum casting allowing the wheel to rest on the projecting table of same. Now start the analyzer with a one to one ratio on the gears as though an analysis for fundamental amplitude were being made. Follow the curve with the pointer Z as before taking initial and final readings on the planimeter. The difference of these readings is the mean square ordinate of the curve in square inches if the latter has been traced over but once. The root mean square ordinate is evidently the square root of this difference. The root mean square ordinate in amperes (ammeter reading) divided by the root mean square ordinate in inches gives the amperes per inch scale of the curve.

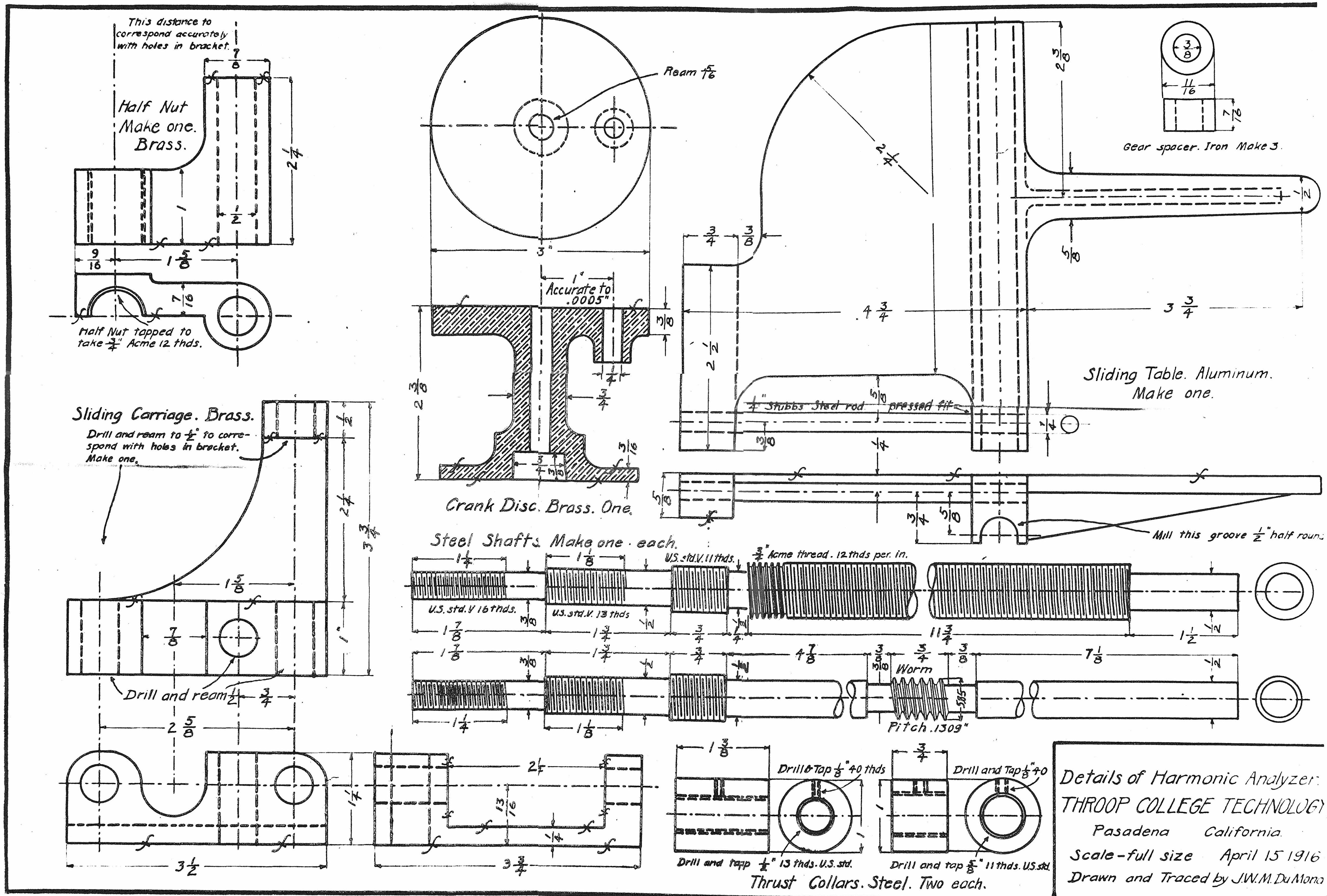
Then all of the harmonic amplitudes which have been found in inches can be multiplied by this factor to get their value in ~~inches~~ Amperes. As nearly all formulae applying to alternating sine waves of current are constructed for effective or root mean square values, it will be then convenient to change the values of the amplitudes (or maximum values) just obtained to effective values by dividing them by 1.414 or $\sqrt{2}$.

It is rarely of any importance to know the relative phase of the harmonics of different frequency in power work. It is their size in which one is more often interested. If both sine and cosine terms of the same order or frequency have been found by analysis their root mean square or effective values may be combined by taking the square root of the sum of the squares of the two separate effective values. This result is then the total effective value of the particular harmonic in question and may be used in impedance formulae, etc.

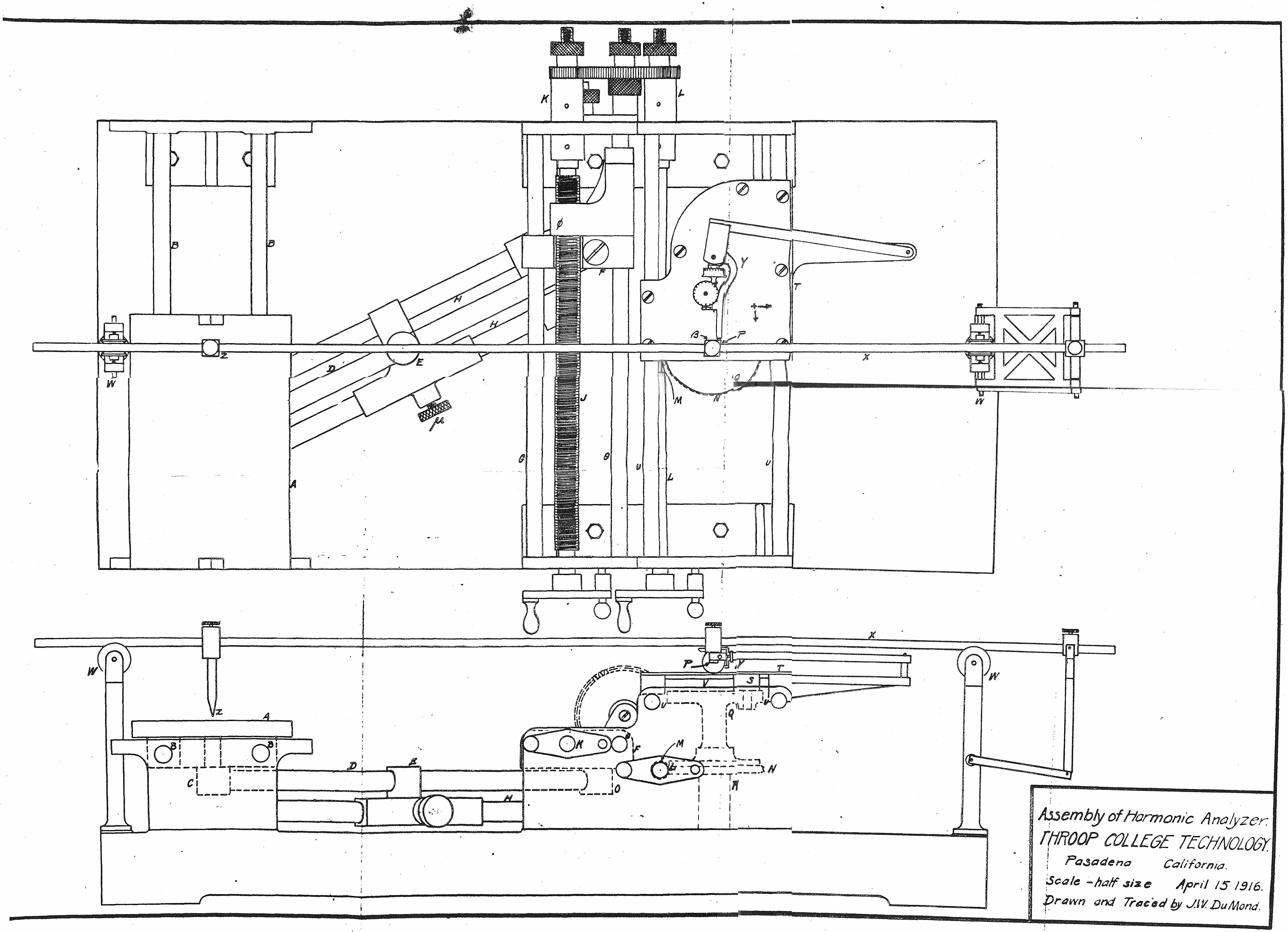

TRACER ARM
Make one.
Stud. Make one
Threaded to take stud
Pole arm. Make one.
Pitch = .0687
200 Graduations on silver.
ROLLING WHEEL Make one.
Vernier
Make one.
Ten tooth gear.
3/64 Hole
RECORDING DISC Make one.
DETAILS OF PLANIMETER
THROOP COLLEGE OF TECHNOLOGY.
Pasadena California.
Scale { 2" = 1" / 1" = 1"
April 25 1916
Drawn and traced by J.W. DuMond.



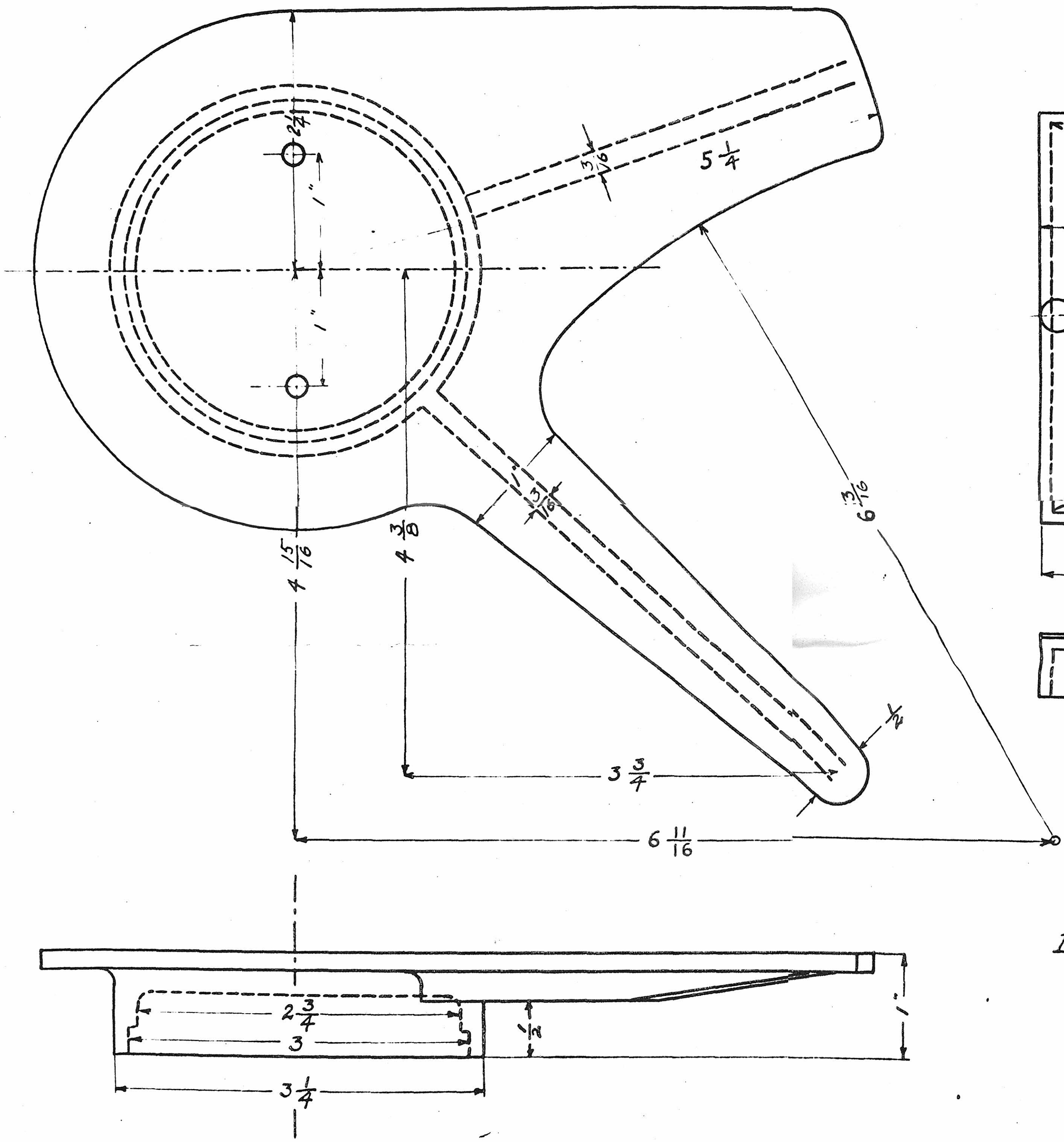
Details of Harmonic Analyzer.
THROOP COLLEGE TECHNOLOGY
Pasadena California.
Scale-full size. April 15 1916
Drawn and Traced by J.W. Du Mond.



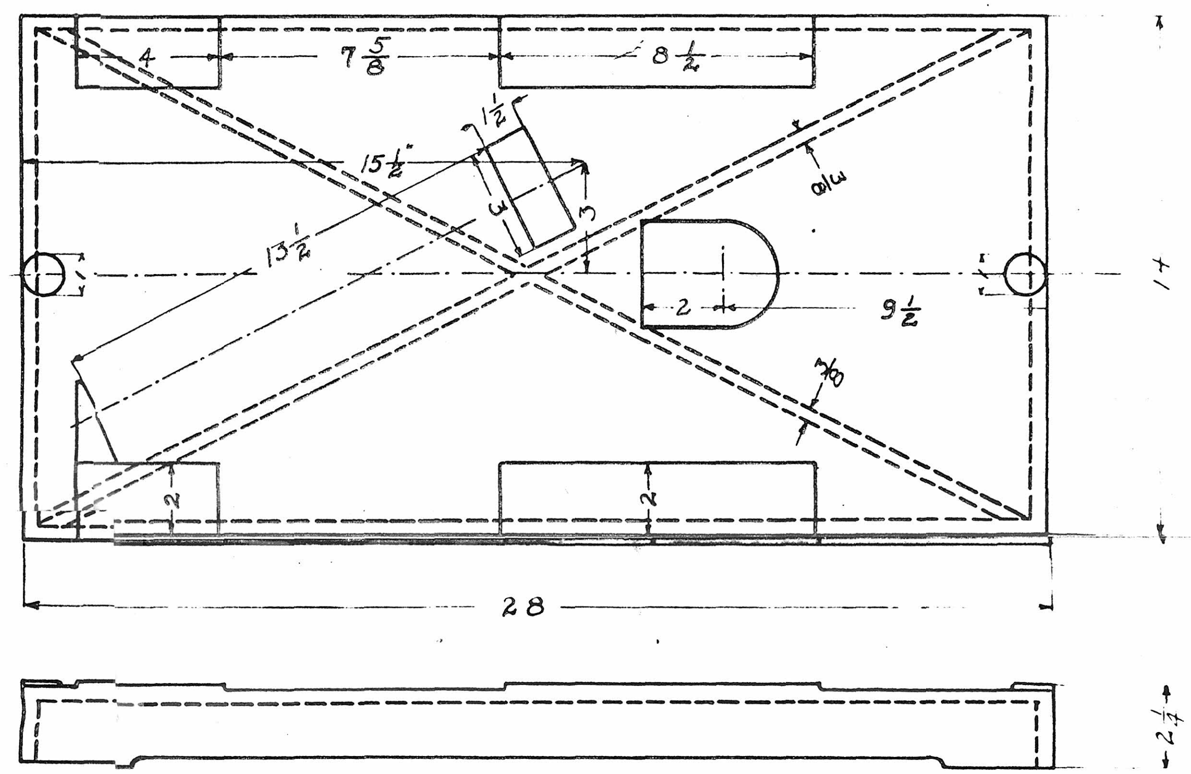
This distance to correspond accurately with holes in bracket.
Half Nut
Make one.
Brass.
Half Nut tapped to take 3/4" Acme 12 thds.
Ream 5/16
Accurate to .0005"
Crank Disc. Brass. One.
Gear spacer. Iron Make 3.
Sliding Table. Aluminum.
Make one.
1/4" Stubbs Steel rod pressed fit
Mill this groove 1/2" half round
Sliding Carriage. Brass.
Drill and ream to 1/2" to correspond with holes in bracket.
Make one.
Drill and ream 1/2
Steel Shafts. Make one each.
U.S. std.V 16 thds.
U.S. std.V. 13 thds
U.S. std.V. 11 thds.
3/4" Acme thread. 12 thds per. in.
Worm
Pitch .1309"
Drill & Tap 1/8" 40 thds
Drill and Tap 1/8" 40
Drill and tapp 1/2" 13 thds. U.S. std.
Drill and tap 5/8" 11 thds. U.S. std.
Thrust Collars. Steel. Two each.
Details of Harmonic Analyzer.
THROOP COLLEGE TECHNOLOGY
Pasadena California.
Scale-full size April 15 1916
Drawn and Traced by J.W.M. Du Mond



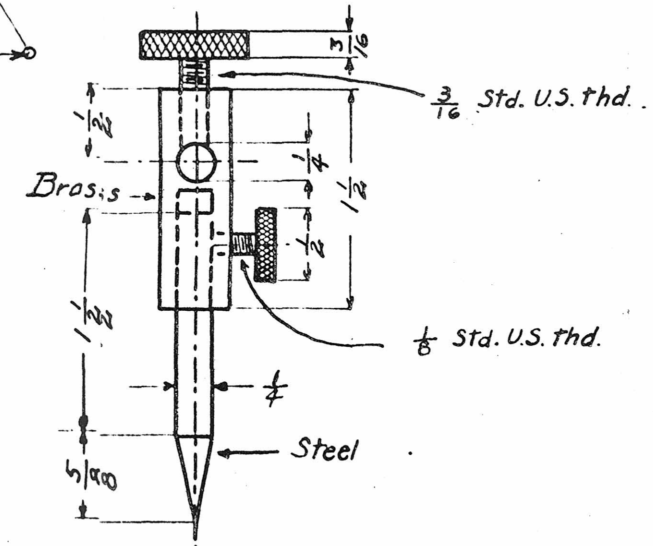
Assembly of Harmonic Analyzer.
THROOP COLLEGE TECHNOLOGY.
Pasadena California.
Scale - half size April 15 1916.
Drawn and Traced by J.W. Du Mond.



R.M.S. Casting, Aluminum.

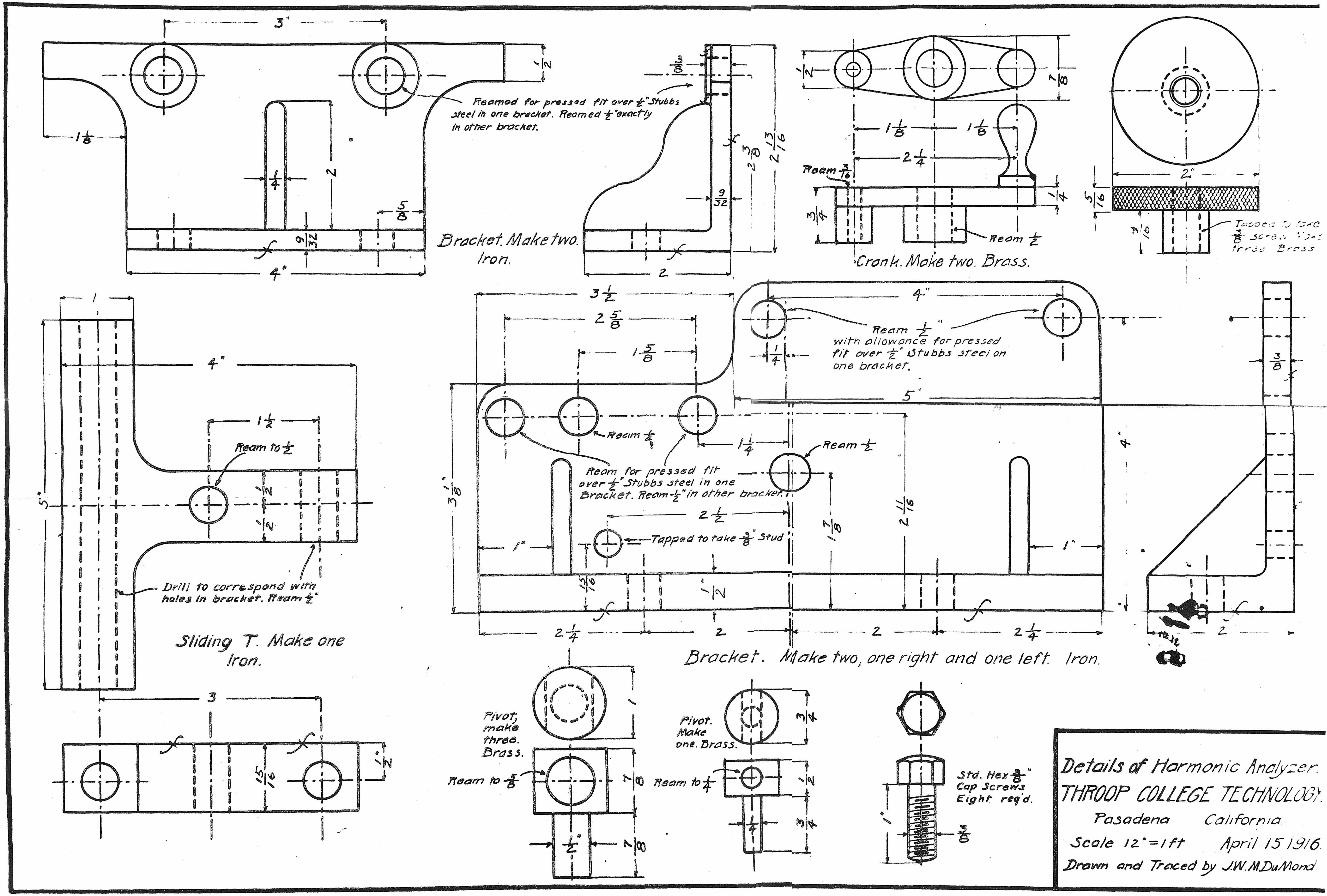


Base, Cast Iron 1/4 Size.



Pointer.

Details of Harmonic Analyzer
THROOP COLLEGE TECHNOLOGY
Pasadena California
Scale - full size May 23 1916
Drawn and Traced by J.W. Du Mond.



Details of Harmonic Analyzer.
THROOP COLLEGE TECHNOLOGY.
Pasadena California.
Scale 12" = 1 ft April 15 1916.
Drawn and Traced by J.W.M. DuMond.