THESIS.

Design for a 180' Concrete
Arch Bridge at Devil's Gate .

by

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The present structure over the Arroyo Seco at Devil's Gate is a steel deck truss of the Pratt type. The maximum allowable load on this bridge is a five ton truck. As this bridge is on the main line drive from Pasadena to La Canada and Glendale it would appear to be inadequate for the service which it might be expected to undergo in the very near future. Furthermore, there has been some talk of beautifying the Arroyo and making a public park of it. Any such proceedure would certainly necessitate the removal of the present structure at Devil's Gate. It is through the influence then of these two needs that I was induced to design a concrete arch bridge to take the place of the present one and I hope that it will not only satisfy the need of a stronger bridge but will fit into any improvement plan of the future. In order to satisfy the esthetic sense I decided to design the bridge with open spandrels and three centered arch ring. This type of structure will be appropriate for any plan of beautification beacuse of the already existing open spandrel arch on Colorado Street.

The method used in the design of the arch was to assume a given ring and then investigate it with the required loading. To better estimate the likely dimensions of the ring, it is advisable to look up the records of existing arches which are fulfilling conditions somewhat similar to the ones at hand. By such records I found that for the span required, i.e. 180', the following dimensions were somewhere in the vicinity of the correct ones; these dimensions are, rise 45 feet, ring thickness at crown 4 feet, at springing line 12 feet. Steel reinforcement equals $1\frac{1}{4}$% of area at crown, .625% is placed at top and bottom 2 inches from surface.

Having assumed these dimensions the next procedure is to investigate them. This investigation was carried out in a manner as outlined in "Principles of Reinforced Concrete Construction" by Turneaure and Maurer, and in "Concrete, Plain and Reinforced" by Taylor and Thomson. The method and results follow.

The arch ring is divided into 20 divisions of such a length that $\frac{ds}{I}$ is a constant, where ds = length of division measured along the arch axis, I = moment of inertia of any section = $I_{concrete} + 15\ I_{steel}$ (see pg.92 Turneaure and Maurer). The method of making this division is; let $i = \frac{1}{I}$, i_a = mean value of i found by taking "n" divisions of equal length, s = half length of arch ring measured along the axis, and n = desired number of divisions in ½ the arch ring. Then $\frac{ds}{I} = \frac{si_a}{n}$. The value of $\frac{ds}{I}$ being known, the length ds can be determined by the cut and try method, until Σ ds = s. Following are the required data and computations of $\frac{ds}{I}$.

Table I. DIVISION OF ARCH RING.

Assume 1¼% steel (1¼% of area at crown) placed as shown 2" from surface.

Preliminary Divisions.

No. of Division.	Depth in Feet	I_c $bh^3/12$.	15 I_s	I= I_c+ $15I_s$	i= 1/I.
1	4.00	5.33	2.52	7.85	.1275
2	4.10	5.75	2.66	8.41	.1185
3	4.30	6.63	2.95	9.58	.1045
4	4.60	8.13	3.41	11.54	.0865
5	5.00	10.41	4.08	14.49	.0691
6	5.60	14.62	5.21	19.83	.0504
7	6.50	22.85	7.15	30.00	.0333
8	7.70	38.01	13.30	51.31	.0195
9	9.10	62.82	14.40	77.22	.0129
10	10.90	107.80	20.90	128.70	.0077

Table I (cont).

$\Sigma\, i = .6299.$

Final Divisions.

No. of Division.	i	ds	No. of Division.	i	ds
1	.1290	5.31	6	.0970	7.06
2	.1260	5.44	7	.0825	8.07
3	.1215	5.63	8	.0699	9.80
4	.1150	5.95	9	.0486	14.10
5	.1070	6.40	10	.1066	41.20

$$\frac{ds}{I} = \frac{si_a}{n} = \frac{108.8}{10} \times \frac{.6299}{10} = .685$$

$s = \frac{1}{2}$ length of arch ring measured along N.Axis = 108.8

i_a = average i

n = number of divisions = 10.

Note : $I = I_c + 15\, I_s = bh^3/12 + 15 \times .0125\, b'h' \times a^2.$

where, bh = area of concrete section at point in question.

b'h' = " " " at crown.

.0125 = % steel.

a = distance from N.Axis to center of steel

$15 \times .0125 \times b'h' = .75$

After the divisions ds/I are laid off on the arch, the next step is to calculate H_0, V_0, and M_0 (Thrust, shear and moment at crown), for load of unity placed successively at the load points H, B, C, D, E, and F, at base of spandrel pier. Table II gives the necessary data for the calculations.

In Table II the points 1, 2, 3, etc. are the center points of the divisions ds. "X" is the horizontal distance from the crown to the point in question. "Y" is the vertical distance from the neutral axis at the crown to the point in question. "X^2" and "Y^2" need no explanation. The arch ring is assumed to be cut by a vertical plane passing through the crown perpendicular to the neutral axis. Each half of the ring is thus acting as a cantilever with V_0, H_0, and M_0 substituted in place of the half arch removed. Now M_R and M_L are the moments at the given points 1, 2, 3, etc. on these cantilevers due to all loads between the point in question and the crown. Thus for a load of unity placed at B, the moment M_R at point 2 is derived as follows. Let the distance from the crown to the load = X , then M_R at point 2 = $(X - X_1)1$ = 16.4 - 15 = 1.4.

Table II. Calculations for H_o, γ, M_o for Load of Unity placed successively at A, B, etc.

Load at A.

Point.	X	Y	X^2	Y^2	M_L
1	5.3	.15	28.15	.225	0
2	16.4	.90	268.00	.81	0
3	27.0	2.50	729.00	6.25	0
4	37.6	4.90	1415.0	24.00	0
5	48.0	8.00	2304.0	64.00	0
6	58.00	12.20	3364.0	149.0	0
7	67.6	17.60	4575.0	310.0	0
8	76.4	23.90	5840.0	572.0	0
9	84.4	31.10	7120.0	968.0	0
10	91.8	39.20	8450.0	1535.0	0
	Σ	140.45	34093.2	3629.3	00

Load at A.

Point.	M_R	$(M_L + M_R)Y$	$(M_R - M_L)X$
1	5.3	.795	28.15
2	16.4	14.76	268.0
3	27.0	67.50	729.0
4	37.6	184.1	1415.0
5	48.0	384.0	2304.
6	58.0	707.5	3364.
7	67.6	1190.0	4575.
8	76.4	1825.0	5840.
9	84.4	2625.0	7120.
10	91.8	3610.0	8450.
Σ	-512.5	-10608.7	-30493.2

Table II (cont). Load at B.

Point.	M_R	M_RY	M_RX
1			
2	1.4	1.26	22.95
3	12.0	30.0	324.0
4	22.6	111.0	849.00
5	33.0	264.0	1585.0
6	43.0	525.0	2490.0
7	52.6	926.0	3560.0
8	61.4	1465.0	4690.
9	69.4	2160.0	5850.
10	76.8	3010.0	7060.
Σ	-372.2	-8492.3	-26431.

Load at C.

Point	M_R	M_RY	M_RX
4	7.6	37.3	286.1
5	18.0	144.0	865.0
6	28.0	341.0	1625.0
7	37.6	661.0	2542.0
8	46.4	1110.0	3545.0
9	54.4	1695.	4585.0
10	61.8	2425.	5675.0
Σ	-253.8	-6413.	-19123.0

Table II (cont). Load at D

Point.	M_R	M_RY	M_RX
5	3.	24.0	144.0
6	13.	158.5	754.0
7	22.1	389.0	1510.0
8	31.4	751.0	2395.0
9	39.4	1225.0	3522.0
10	46.8	1835.0	4300.0
Σ	-155.7	-4382.5	-12425.

Load at E.

Point	M_R	M_RY	M_RX
7	7.6	134.0	514.0
8	16.4	392.0	1254.0
9	24.4	758.0	2060.0
10	31.8	1248.0	2920.0
Σ	-80.2	-2532.0	-6748.0

Load at F

Point	M_R	M_RY	M_RX
8	1.4	33.5	107.
9	9.4	292.4	793.
10	16.3	639.9	1500.0
Σ	-27.1	-965.8	-2400.

Load at G

Point	M_R	M_RY	M_RX
10	1.8	70.6	165.4
Σ	-1.8	-70.6	-165.4

Of course as the load advances from the crown to the springing line the computations become fewer and fewer. This is easily understood when it is remembered that the half arch is assumed to act as a cantilever and the moment on a cantilever is zero between a given load and the free end of the cantilever.

From Table II then, we have the necessary quantities to find H_o, V_o, and M_o due to a unit load at A, B, C, D, etc. These values are found by substituting in the following equations.

$$H_o = \frac{n \sum MY - \sum M \sum Y}{2\left[(\sum Y)^2 - n\sum Y^2\right]}$$

$$V_o = \frac{\sum (M_R - M_L)X}{2 \sum X^2}$$

$$M_o = -\frac{\sum M + 2 H_o \sum Y}{2 n}$$

In these equations, the general equations of the elastic arch, $\sum Y$, $\sum Y^2$ and $\sum X^2$ are for $\frac{1}{2}$ of the arch only. $\sum M$ is for the entire arch and is equal to $\sum M_R + \sum M_L$. The summation $\sum (M_R - M_L)X$ is a summation of the products $(M_R - M_L)X$ in which M_R and M_L are the bending moments at corresponding points in the right and left halves which have equal abscissae X; and the summation $\sum MY$ is for the entire arch and equals $\sum (M_R + M_L)Y$. The computations are given below.

Calculations of H_o, V_o, M_o, for load of unity placed at A, B, C, D, E, F, and G successively.

Load at A.

$$H_o = \frac{10\,(-10609) - (-512.5)(140.5)}{-33186} = +\ 1.025$$

$$V_o = \frac{-\ 34093}{2 \times 34093} = -\ .5$$

$$M_o = -\ \frac{-513 - 2 \times 1.025 \times 141}{20} = +\ 11.2$$

$$e = \frac{M_o}{H_o} = \frac{11.2}{1.025} = +\ 10.9$$

Load at B.

$$H_o = \frac{10\,(-8492) - (-372)(141)}{-33186} = +\ .98$$

$$V_o = \frac{-\ 26431}{68186.4} = -\ .388$$

$$M_o = -\ \frac{-372 - 2 \times .98 \times 141}{20} = +\ 5.77$$

$$e = \frac{M_o}{H_o} = \frac{5.77}{.98} = +\ 5.89$$

Load at C.

$$H_o = \frac{10\,(-6413) - (-253.8)(141)}{-33186} = +\ .854$$

$$M_o = -\ \frac{-253.8 - 2 \times .854 \times 141}{20} = +\ 0.65$$

$$V_o = \frac{-19123}{681864} = -\ .28$$

$$e = \frac{M_o}{H_o} = \frac{.65}{.854} = +\ .762$$

Calculations of H_o, V_o, M_o for load of unity placed at A, B, C, D, E, and F successively. (cont)

Load at D.

$$H_o = \frac{10\ (-4382.5) - (-155.7)(141)}{-33186} = +\ .688$$

$$V_o = \frac{-\ 12425}{68186.4} = -\ .1825$$

$$M_o = -\ \frac{-155.7 - 2 \times .688 \times 141}{20} = -\ 1.95$$

$$e = \frac{M_o}{H_o} = \frac{-\ 1.95}{.688} = -\ 2.87$$

Load at E.

$$H_o = \frac{10\ (-2532) - (-80.2)(141)}{-\ 33186} = +\ .421$$

$$V_o = \frac{-\ 6748}{68186.4} = -\ .0988$$

$$M_o = -\ \frac{-80.2 - 2 \times .421 \times 141}{20} = -\ 1.92$$

$$e = \frac{M_o}{H_o} = \frac{-\ 1.92}{.421} = -\ 4.56$$

Load at F.

$$H_o = \frac{10\ (-965.8) - (-27.1)(141)}{-33186} = +\ .1765$$

$$V_o = \frac{-2400}{68186.4} = -\ .0352$$

$$M_o = -\ \frac{-\ 27.1 - 2 \times .1765 \times 141}{20} = -\ 1.135$$

Load at F (cont).

$$e = \frac{M_o}{H_o} = \frac{-1.135}{.1765} = -6.43$$

Load at G.

$$H_o = \frac{10\ (-70.6) - (-1.8)(141)}{-33186} = +.0136$$

$$V_o = \frac{-165.4}{68186.4} = -.00243$$

$$M_o = -\frac{-1.8 - 2 \times .0204 \times 141}{20} = -.1975$$

$$e = \frac{M_o}{H_o} = \frac{-.1975}{.0136} = -14.5$$

These equations and calculations to follow pg. 9.

$$H_o = \frac{n \sum MY - \sum M \sum Y}{2\ \left[(\sum Y)^2 - M \sum Y^2\right]}$$

$$V_o = \frac{\sum (M_R - M_L) X}{2 \sum X^2}$$

See pg. 338 Turneaure and Maurer.

$$M_o = -\frac{\sum M + 2\ H_o \sum Y}{2\ n}$$

$$2\ \left[(\sum Y)^2 - M \sum Y^2\right] = 2(\overline{140.45}^2 - 10 \times 3629.3) = -33186.$$

$$2 \sum X^2 = 2 \times 34093.2 = 68186.4$$

$$2\ n = 2 \times 10 = 20.$$

Influence lines of Fibre Stress at Sections A ------ G Inclusive.

GRAPHICAL SOLUTION.

An influence line of fibre stress shows the character and amount of stress at any section due to a load of unity passing over the bridge.

In order to solve graphically for the fibre stress at any section, the force polygon of plate II was layed with the load line MN equal to unity. Then in order to determine the directions of the thrust lines, the shear (V_o), caused by the load at any point, is laid off on MN from N and the corresponding horizontal thrust (H_o) is layed off as the pole distance. The rays drawn from M and N to the end of H_o give the directions of the thrust lines. The distance below or above the neutral axis, at the crown, of this thrust line is given by M_o/H_o. If this quantity is positive, the thrust line passes above the neutral axis, if negative below. With these polygons drawn, the bending moment at any section, due to a unit load at any section is given by the vertical distance from the neutral axis at that section to the proper thrust line, multiplied by the corresponding pole distance H_o. The quantity thus found is the true bending moment at the given section. The fibre stress at that section can be computed by the equation $f_c = Mu/I = N/A$, where f_c = fibre stress, M equals the true moment at the section, u = the distance from the neutral axis to the fibre in question. (For our purpose u = distance to extreme fibre.) I = the moment of inertia of the section, N = the thrust normal to the section and A = area of the section.

The above method is correct but it is too cumbersome. The following method is much shorter. M = Ne, where M = moment, N = thrust and e = eccentricity. If r = radius of gyration, $Ar^2 = I$, then the expression $f_c = Mu/I \pm N/A$ can be written,

$$f_c = Neu/I \pm \frac{N\frac{r^2}{u} \times u}{I}$$

$$= \frac{N\left(e \pm \frac{r^2}{u}\right)u}{I} \qquad (1)$$

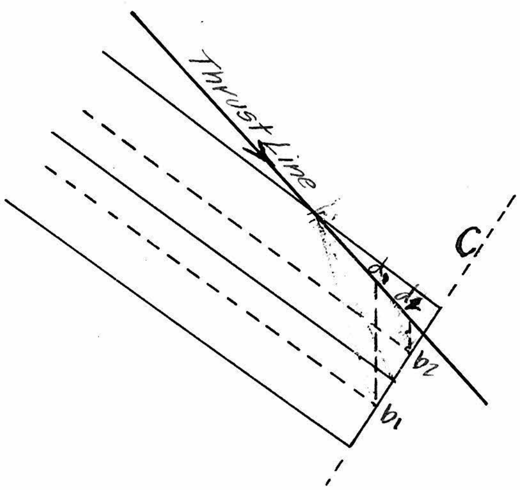
But $u = d/2$

$r^2 = I/A = d^3/12d = d^2/12$ with unit width.

$\therefore \frac{r^2}{u} = \frac{d^2}{12} \times \frac{2}{d} = \frac{d}{6}$ = a length.

$\therefore N\left(e \pm \frac{r^2}{u}\right)$ is a moment which may be written as M'. This new moment is equal to the thrust N multiplied by the eccentricity e $\pm$ the additional distance $\frac{r^2}{u} = \frac{d}{6}$. Then if we take the center of moments at a distance $e \pm \frac{d}{6}$ below or above the gravity axis, or at the lower or upper edge of the middle third according as upper or lower fibre stress is desired we can obtain the fibre stress direct by multiplying this moment M' by u/I for the section. As will be seen from the above, the fibre stress varies directly with the moment M'. Therefore an influence line drawn for such a moment will serve as an influence line for fibre stress.

It should be noted that in the actual calculation of the moment M' it is much simpler to measure the vertical distances $b_1\ d_1$ and $b_2\ d_2$, from the edge of the middle third or kern, to the proper thrust line. These distances multiplied by the proper pole distance gives M' for upper and lower fibre stress respectively.



For values of M' with loads on the left half of arch consider the load on the right half and the corresponding section on the left half.

Thus values of M' can be computed for any section for loads at any point. If the thrust line passes above the upper edge of the kern, the values of M' are plotted as positive ordinates showing compression in the upper fibre and tension in the lower. If the thrust line passes through the kern H_o x (b_1d_1) is positive showing compression in upper fibre, H_o x (b_2d_2) is negative showing compression in the lower fibre. If the thrust line passes below the kern both H_o x (b_1d_1) and H_o x (b_1d_1) are negative showing tension in upper fibre and compression in lower fibre.

Table III gives the values of H_o x (b_1d_1) and H_o x (b_1d_1) at sections A_1B_1------G for a unit load moving across the arch.

Plate III shows these values of M' plotted to scale. A word of explanation may be needed for Plate III. For any section the value of the ordinates at any point give values of M' due to a load of unity at that point on the arch. The actual stress = M'u/I.

Table III - SOLUTION BY USE OF INFLUENCE LINES.

Unit Load at A. $H_o = 1.025$

			M	
Section.	b_1d_1	b_2d_2	$H_ob_1d_1$	$H_ob_2d_2$
A	11.47	9.133	11.76	9.35
B	5.10	3.70	5.23	3.79
C	.30	1.30	0.308	1.33
D	2.90	4.7	2.97	4.82
E	4.00	6.10	4.10	6.25
F	2.00	4.80	2.05	4.92
G	10.00	6.10	10.25	6.25
B'	5.00	3.60	5.13	3.69
C'	0.20	1.40	0.205	1.44
D'	3.00	4.60	3.08	4.72
E'	4.00	6.00	4.10	6.15
F'	1.90	4.60	1.95	4.72
G'	10.0	6.10	10.25	6.25

Unit load at B. $H_o = .98$

			M	
Section.	b_1d_1	b_2d_2	$H_ob_1d_1$	$H_ob_2d_2$
A	6.6	5.0	6.47	4.90
B	13.0	11.50	12.75	11.25

Table III (cont)

Unit load at B (cont)

Section	$b_1 d_1$	$b_2 d_2$	$\frac{M}{H_o b_1 d_1}$	$\frac{M}{H_o b_2 d_2}$
C	6.5	4.70	6.37	4.61
D	1.0	0.80	0.98	0.78
E	2.0	4.20	1.96	4.12
F	2.0	4.80	1.96	4.71
G	6.8	2.70	6.66	2.65
B'	1.5	0.0	1.47	0.0
C'	2.0	3.5	1.96	3.43
D'	3.9	5.4	3.82	5.29
E'	3.4	5.4	3.33	5.29
F'	0.0	2.5	0.0	2.45
G'	14.0	10.5	13.7	10.3

Unit load at C. $H_o = 0.854$

Section	$b_1 d_1$	$b_2 d_2$	$\frac{M}{H_o b_1 d_1}$	$\frac{M}{H_o b_2 d_2}$
A	1.6	0.0	1.36	0.0
B	6.8	5.4	5.81	4.61
C	13.9	12.6	11.85	10.75
D	6.0	4.1	5.13	3.50
E	0.0	2.4	0.0	2.05
F	3.2	6.2	2.73	5.29
G	1.1	3.8	.94	3.25
B'	2.6	4.0	2.22	3.42
C'	5.0	6.6	4.27	5.64
D'	6.0	7.6	5.13	6.49
E'	4.6	6.6	3.93	5.64
F'	0.2	2.6	0.17	2.22
G'	15.3	12.0	13.05	10.25

Table III (cont)

Unit load at D.		$H_o = 0.688$		
Section	b_1d_1	b_2d_2	$H_ob_1d_1$	$H_ob_2d_2$
A	2.2	3.7	1.51	2.55
B	2.3	0.9	1.58	0.62
C	8.4	7.0	5.78	4.81
D	16.2	14.8	11.15	10.18
E	6.6	3.8	4.54	2.61
F	1.3	5.0	0.895	3.44
G	4.4	10.6	3.03	7.3
B'	5.4	7.0	3.72	4.81
C'	7.0	8.6	4.81	5.91
D'	7.0	8.6	4.81	5.91
E'	4.8	6.6	3.30	4.54
F'	0.4	2.0	0.28	1.38
G'	17.2	14.0	11.85	9.64
Unit load at E.		$H_o = 0.421$		
A	3.7	5.3	1.55	2.23
B	0.0	1.4	0.0	0.59
D	12.5	11.0	5.26	4.64
C	5.7	4.2	2.40	1.77
E	21.5	20.0	9.05	8.43
F	4.4	0.6	1.85	0125
G	19.6	28.8	8.25	12.10
B'	6.3	7.8	2.65	3.28
C'	7.3	8.8	3.07	3.70

Table III (cont)

Unit load at E (cont) $H_o = 0.421$

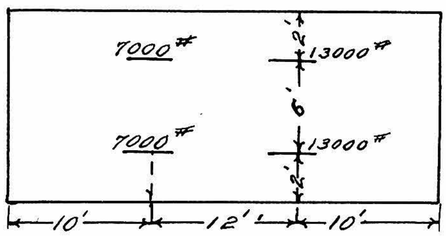
Section	b_1d_1	b_2d_2	$\frac{M}{H_o b_1 d_1}$	$\frac{M}{H_o b_2 d_2}$
D'	6.6	8.2	2.78	3.45
E'	3.8	5.5	1.60	2.31
F'	2.1	0.1	0.885	0.092
G'	19.8	16.8	8.33	7.06

Unit load at F. $H_o = 0.176$

A	5.7	7.2	1.001	1.27
B	2.0	3.4	0.353	0.60
C	3.3	1.8	0.582	0.318
D	10.2	8.8	1.86	1.55
E	19.1	17.6	3.37	3.11
F	30.5	28.9	5.38	5.1
G	51.4	71.6	9.06	12.7
B'	8.0	9.6	1.41	1.69
C'	8.9	10.3	1.57	1.82
D'	8.0	9.6	1.41	1.69
E'	5.1	6.8	0.90	1.20
F'	0.9	1.4	0.247	0.15
G'	18.6	15.8	3.28	2.79

FLOOR DESIGN.

The floor system is designed fro a loading, equivalent to a 20 ton truck. This loading gives two concentrated loads of 13000# each over the rear wheels of the truck as shown in the diagram. Since the maximum floor space is 5 feet the floor is designed to carry a concentrated load of 13000# at the center of the span.



The moment coefficients used are for continuous beams. (See "Hool's Concrete Construction", volume II, pages 374-376.) An allowance for impact is given in the coefficents.

The beams are designed as simple T beams, span 18', and loading equivalent of 20 ton truck.

The girders are designed as continuous rectangular beams. Intermediate spans are 15', end spans are 25'. The complete calculations are given below.

Note: For moment coefficients see Hool, pages 374-376; floor is designed for impact due to live load.

Intermediate spans.

Span = 5'- 0".

Live load = 13000#, concentrated at center of span, due to twenty ton truck.

Dead load = 150# ft^2 (Floor assumed 12" thick)

M = .191 x 13000 x 5 x 12

$M = 149\ 000$ inch pounds due to live load, including impact.

$M = .085 \times 150 \times 5 \times 5 \times 12$

$= 3\ 830$ inch pounds due to dead load.

Total $M = 149\ 000 + 3\ 830 = 153\ 000$ inch pounds.

$M_c = f_c/2 \times bkjd^2$

$= 325 \times 12 \times .38 \times .875\ d^2 = 153\ 000.$

$$d = \frac{153\ 000}{325 \times 12 \times .38 \times .875}$$

$d = 10.85$ inches net depth. (12" gross depth).

<u>Steel Area</u>

$M_s = Af_c jd = 153\ 000$ inch pounds.

$$A = \frac{153\ 000}{16\ 000 \times .875 \times 11} = .995 \text{ square inches of steel.}$$

Use $\frac{1}{2}$" square twisted rods throughout floor spaced 3".

<u>End Spans</u>

Span = 5'.

Loading same as for intermediate spans.

Live $M = .211 \times 13\ 000 \times 5 \times 12$

$= 164\ 500$ inch pounds (includes impact)

Dead $M = .116 \times 150 \times 5 \times 5 \times 12$

$= 5\ 220$ inch pounds

Total $M = 164\ 000 + 5\ 220 = 170\ 000$ inch pounds

$M_s = f_c \times bkjd2 = 170\ 000$

$$d^2 = \frac{170\ 000}{325 \times 12 \times .38 \times .875}$$

$d = 11.41$ inches net depth of floor

<u>Steel Area</u>

$$A = \frac{170\ 000}{16\ 000 \times .875 \times 11} = 1.27 \text{ square inches.}$$

BEAM DESIGN.

Span 18'.

Designed as simple T beams for loading equivalent to a twenty ton truck;

Moment coefficients increased in ratio of $\frac{191}{130}$, for impact (see Hool, page 376).

Dead load of floor = 750 lbs. per ft. of beam

" " " beam = 250 " " " " "

Live load Moment = $13\ 000 \times 6 \times 12 \times \frac{191}{130}$

= 1 370 000 inch pounds.

Dead load Moment = $.13 \times 1\ 000 \times 18 \times 18 \times 12$

= 505 000 inch pounds.

Total Moment = 1 875 000 inch pounds.

$$M_c = \frac{f_c}{2} bt(d - \tfrac{1}{2}t)$$

$$d = \frac{1\ 875\ 000}{325 \times 30 \times 12} + 6.$$

= 22 inches, minimum allowable depth.

Use 28 inches for "d" otherwise % steel runs too high.

$$M_s = A_s f_s\ (d - \tfrac{1}{2}t)$$

$$A_s = \frac{1\ 875\ 000}{16\ 000 \times 22} = 5.32 \text{ square inches of steel.}$$

$$\%\ \text{steel} = \frac{5.32}{900} = .59\%$$

Shear Considerations

Allowable unit shear on concrete = 100# (with stirrups)

Area carrying shear = width of stem x $(d - \frac{1}{2}t)$

Effective area for beam in question = 12 x 22 = 264 in.^{2}

100 x 264 = 26 400 lbs. shear carried by concrete.

Total shear on beam = 22 000 lbs.

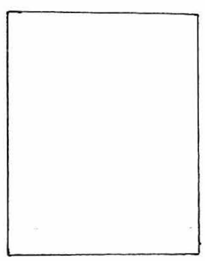
Use stirrups of $\frac{1}{4}$" steel wire, spaced 15 inches.

Intermediate Girder Design

Span 15'.

Note: Girders designed as continuous rectangular beams.

Loading = 28 000 lbs. @ 1/3 points (Impact factor applied = $\frac{191}{130}$.



Moment coefficients from Hool's Concrete Construction Volume II, page 376.

Assume 16 x 20 Girder.

Load per foot of Girder due to its own weight equals 330 pounds.

Moment = .122 x 28 000 x 15 x 12

= 615 000 inch lbs. due to beam loads at $\frac{1}{3}$ pts.

Dead Load Moment = .086 x 330 x 15 x 15 x 12

= 76,700 in.lbs. due to wt. of gir.

Total Moment = 692 000 inch pounds

$M_c = \frac{f_c}{2} \times bkjd^2 = 692\ 000$

$$d^2 = \frac{692\ 000}{325 \times 16 \times .38 \times .875} = 400.$$

d = 20 inches, net depth

Use 22 " gross " below slab.

$M_s = A_s f_s jd$

$$A_s = \frac{692\ 000}{16\ 000 \times .875 \times 20} = 2.47 \text{ square inches of steel.}$$

$$\%\ \text{Steel} = \frac{2.47}{16 \times 20} = .775\%$$

Shear.

Maximum shear = 20 500 pounds.

Allowable unit on concrete if stirrups are used = 100#

Allowable Total Shear = 100 x 16 x 20 = 32 000#

Use $\frac{1}{4}$" wire stirrups spaced at 10 inches.

Girders (End Span)

Span 25 feet. Assume 20 x 26 Girder.

Load from beams at $\frac{1}{5}$ points = 28 000# (Includes impact)

Moment over support = 4.47 x 28 000 x 12 =

1 500 000 in. lbs. (From beams) (See Hool's Con. Const. p359)

Dead load Moment = .107 x 540 x 15 x 15 x 12
(Wt.Girder)
= 156 000 inch pounds.

Total Moment = 1 656 000 inch pounds.

$M_c = \frac{1}{2} f_c bkjd^2$

$$d^2 = \frac{1\ 656\ 000}{325 \times 20 \times .38 \times .875} = 765.$$

d = 27.7 inches

Use 28 inches net depth = 20 x 30 beam (Overall)

Shear.

Maximum shear = 62 000 pounds

100 x 20 x 26 = 52 000 pounds carried by concrete

Use $\frac{1}{4}$" wire stirrups spaced at 14 inches.

Stress in stirrups = $\frac{10000}{28}$ x 14 = 5 000 lbs.

At 120 pounds per square inch concrete will carry total shear.

Bridge Loading

From a consideration of the influence lines on plate III we see that any loading which will give a maximum fibre stress at G will probably be a safe loading for which to investigate the arch. Consequently live loads were placed so as to give maximum fibre stress at section G, i.e. at F', E', D', C', B', A and B. Since spandrel coulm are used to take the floor load to the ring it was thought unnecessary to use a concentrated floor loading when investigating the arch,hence a uniform loading, which would give a column load equal to or greater than the assumed concentrated load was used. The loading as finally decided upon was 150 lbs. per square foot extending from the left end of the arch up to point B. The total load per column per foot width of arch ring is shown on plate IV.

Now having the loads at the various points F' E' --- F given, we can find the fibre stress at any section caused by any load by simply multiplying the proper ordinate as scaled from the influence line on Plate III, by the corresponding load. The total stress is found by summing algebraically the products of the ordinates and the corresponding loads and multiplying the quantity thus found by $\frac{u}{I}$ for the section in question. For example take section C. The algebraic sum of the rpoducts of ordinates times loads equals 111,940 for upper fibre and 108,050 for lower fibre. The stress equals

$$\frac{111\ 940 \times .199}{144} = 165 \text{ lbs. per in}^2 \text{ compression in upper fibre}$$

or

$$\frac{108\ 050 \times .199}{144} = 149 \text{ lbs. per in}^2 \text{ compression in lower fibre}$$

Table IV gives the complete data and calculations for fibre stress at sections A------G inclusive.

From these calculations we see that the maximum compression stress is at section A with a value of 422 pounds per square inch.

The maximum tensile stress is at G, with a value of 21 pounds per square inch.

Both of these values are entirely within safe limits, especially since we have .625% of steel in top and bottom of arch.

Table IV. Note: Positive signs show tension.

Negative " " compression.

FIBRE STRESS - Section A.

Load at Section	Ordin. to Influ. Line. Up. Fib.	Low.Fib	Total Load	Moment = M' Upper fibre −	+	Lower fibre −	+
F'	+1.20	-1.50	44 000		52800	66000	
E'	+1.65	-2.40	26 000		42800	62400	
D'	+1.65	-2.65	20 000		33000	53000	
C'	-1.35	0.00	19 000	25750		0000	
B'	-6.40	+4.90	18 000	115000			88300
A	-11.75	+9.35	18 000	210000			168500
B	-6.75	+5.10	16 000	108000			81600
C	-1.35	0.00	16 000	21600			0000
D	+1.65	-2.65	17 000		28100	45100	
E	+1.65	-2.40	23 000		38000	55200	
F	+1.20	-1.50	39 000		46750	58500	
			Σ	480350	241450	340200	338400

Upper fibre stress = $\left(\frac{480350 - 241450}{144}\right) .255 =$

422 pounds per square inch compression.

Lower fibre stress = $\left(\frac{340200 - 338400}{144}\right) .255 =$

3.19 pounds per square inch compression.

FIBRE STRESS - Section B.

Load at Section	Ordin.to Influ. Line Up.Fib.	Low.Fib.	Total Load.	Moment = M' Upper Fibre −	+	Lower Fibre. −	+
F'	+1.40	-1.65	44 000		61600	72600	
E'	+2.60	-3.20	26 000		67600	83200	
D'	+3.75	-4.75	20 000		75000	95000	

Table IV (Cont) FIBRE STRESS - Section B. (Cont).

Load at Section	Ordin. to Influ. Line Up.Fib.	Low.Fib.	Total Load.	Moment = M' Upper fibre −	Upper fibre +	Lower fibre −	Lower fibre +
C'	+2.10	-3.35	19 000		39900	63750	
B'	-1.50	0.00	18 000	27000		0000	
A	-5.30	+3.75	18 000	95500			67500
B	-12.80	+11.00	16 000	205000			176000
C	-6.00	+4.80	16 000	96000			76750
D	-1.70	+0.70	17 000	28900			11900
E	00.00	-0.59	23 000	0000		13600	
F	+ .353	- .60	39 000		13800	23400	
			Σ	452400	257900	531550	332150

Upper Fibre stress = $\left(\frac{452400 - 257900}{144}\right) .2415 =$
326 pounds per square inch compression.

Lower Fibre stress = $\left(\frac{351550 - 332150}{144}\right) .2415 =$
32.6 pounds per square inch compression.

FIBRE STRESS - Section C.

Load at Section	Ordin. to Influ. Line Up.fib.	Low.fib.	Total Load.	Moment = M' Upper fibre −	Upper fibre +	Lower fibre. −	Lower fibre. +
F'	+1.57	-1.82	44 000		69100	80100	
E'	+3.07	-3.70	26 000		79900	96200	
D'	+4.81	-5.91	20 000		96000	118000	
C'	+4.27	-5.64	19 000		81200	107400	
B'	+1.96	-3.43	18 000		35300	61700	
A'	- .308	-1.40	18 000	5540		25200	
B	-6.37	+4.61	16 000	102000			73600
C	-11.85	+10.75	16 000	190000			172000

Table IV (Cont) FIBRE STRESS - Section C (Cont).

Load at Section	Ordin.to Influ. Line Up.fib.	Low.fib	Total Load.	Moment = M' Upper fibre −	Upper fibre +	Lower fibre −	Lower fibre +
D	-5.78	+4.81	17 000	98000			81800
E	-2.40	+1.77	23 000	55200			40750
F	- .582	+ .318	39 000	22700			12400
			Σ	473440	361500	488600	380550

Upper Fibre stress $= \left(\frac{473440 - 361500}{144}\right) .199 =$
165 pounds per square inch compression.

Lower fibre stress $= \left(\frac{488600 - 380550}{144}\right) .199 =$
149 pounds per square inch compression.

FIBRE STRESS - Section D.

Load at Section	Ordin.to Influ. Line Up.fib.	Low.fib.	Total Load.	Moment = M' Upper fibre −	Upper fibre +	Lower fibre −	Lower fibre +
F'	+1.41	-1.69	44 000		62000	74400	
E'	+2.78	-3.45	26 000		72400	89750	
D'	+4.81	-5.91	20 000		96100	118500	
C'	+5.13	-6.49	19 000		97800	123500	
B'	+3.82	-5.29	18 000		68800	95100	
A	+2.98	-4.72	18 000		53800	85000	
B	- .98	- .78	16 000	15750		12500	
C	-5.13	+3.50	16 000	82250			56100
D	-11.15	+10.18	17 000	189500			173000
E	-5.26	+4.64	23 000	121000			106500
F	-1.80	+1.55	39 000	70400			60500
			Σ	478900	450900	598750	396100

Table IV (Cont)   FIBRE STRESS - Section D (Cont).

Upper Fibre stress = $\left(\frac{478900 - 450090}{144}\right) .1724 =$

34.5 pounds per square inch compression,

Lower Fibre stress = $\left(\frac{598750 - 396100}{144}\right) .1724 =$

242 pounds per square inch compression.

FIBRE STRESS - Section E.

Load at Section	Ordin. to Influ. Line Up. fib.	Low. fib.	Total Load.	Moment = M' Upper fibre -	Upper fibre +	Lower fibre -	Lower fibre +
F'	+ .90	-1.20	44 000		39700	52800	
E'	+1.60	-2.31	26 000		41600	59900	
D'	+3.30	-4.50	20 000		66000	90000	
C'	+3.93	-5.64	19 000		74800	107500	
B'	+3.33	-5.29	18 000		60000	95100	
A	+4.10	-6.15	18 000		73900	111000	
B	+1.96	-4.12	16 000		31400	66000	
C	0.00	-2.05	16 000		00000	32800	
D	-4.54	+2.61	17 000	77100			44400
E	-9.05	+8.43	23 000	208000			194000
F	-3.37	+3.11	39 000	131500			121500
			Σ	416600	387400	615100	359900

Upper fibre stress = $\left(\frac{416600 - 387400}{144}\right) .1250 =$

24.5 pounds per square inch compression.

Lower fibre stress = $\left(\frac{615100 - 359900}{144}\right) .1250 =$

221 pounds per square inch compression.

Table IV (Cont) FIBRE STRESS - Section F.

Load at Section	Ordin. to Influ. Line Up.fib.	Low.fib.	Total Load.	Moment = M' Upper fibre -	-	Lower fibre -	-
F'	- .247	-0.15	44 000	10900		6600	
E'	- .885	-0.042	26 000	23000		1100	
D'	- .28	-1.38	20 000	5600		27700	
C'	+ .17	-2.22	190000		3240	42200	
B'	0.00	-2.45	18 000		0000	44000	
A	+2.00	-4.90	18 000		36000	88000	
B	+1.96	-4.71	16 000		31400	75500	
C	+2.73	-5.29	16 000		43600	84500	
D	+ .895	-3.74	17 000		15300	63700	
E	-1.85	-0.25	23 000	42600		5750	
F	-5.38	+5.1	39 000	210000			199000
			Σ	292100	129540	439050	199000

Upper fibre stress = $\left(\frac{292100 - 129540}{144}\right)$ .0975 =
110 pounds per square inch compression.

Lower fibre stress = $\left(\frac{439050 - 199000}{144}\right)$ .0974 =
163 pounds per square inch compression.

FIBRE STRESS - Section G.

Load at Section	Ordin. to Influ. Line Up.fib.	Low.fib	Total Load.	Moment = M' Upper fibre -	+	Loer fibre. -	+
F'	-3.28	+2.79	44 000	144000			123000
E'	-8.33	+7.09	26 000	217000			184000
D'	-11.85	+9.64	20 000	237000			193000
C'	-13.05	+10.25	19 000	249000			194500
B'	-13.70	+10.30	18 000	247000			185000

Table IV (Cont) FIBRE STRESS - Section G (Cont).

Load at Section	Ordin. to Influ. Line Up.fib.	Low.fib.	Total Load.	Moment = M' Upper fibre -	+	Lower fibre. -	-
A	-10.25	+6.25	18000	184000			112800
B	-6.66	+2.65	16000	106800			42400
C	-0.94	-3.25	16000	15100		52000	
D	+3.03	-7.30	17000		51600	124000	
E	+8.25	-12.10	23000		190000	278000	
F	+9.06	-12.70	39000		354000	495000	
			Σ	1421900	595600	949000	1033700

Upper fibre stress = $\left(\frac{1421900 - 595600}{144}\right) .0355 =$
- 204 pounds per square inch compression.

Lower fibre stress = $\left(\frac{1069700 - 889000}{144}\right) .0355 =$
21 pounds per square inch tension.

PROPERTIES of SECTIONS A-----G.

Section	Depth	u = d/2	I = $I_c - 15I_s$	u/I
A	4.0'	2.0'	7.85	.2550
B	4.2	2.1	8.70	.2415
C	4.6	2.3	11.54	.1990
D	5.0	2.5	14.50	.1724
E	6.0	3.0	24.00	.1250
F	7.4	3.7	38.00	.0974
G	12.0	6.0	169.00	.0355

FINAL THRUST LINE CALCULATIONS.

The final calculations of H_0, V_0, and M_0 due to assumed loads are given in Table V. The quantities all refer to plate IV. M_R and M_L were obtained graphically as explained on page 348 of Turneaure and Maurer. After the values of H_0, V_0, and M_0 were found, the true equilibrium polygon or thrust line was drawn in as follows.

The loads at F', E', -------F are layed off vertically to scale as the load line MN. From the middle of load A, V_0 is layed off towards M (V_0 is positive). From the upper end of V_0, H_0 is layed off as a pole distance. The rays are drawn in as usual. The strings of the equilibrium polygon are drawn in parallel to the proper rays of the force polygon. The eccentricity e of the equilibrium polygon at the crown is given by $\frac{M_0}{H_0}$.

It is seen that the thrust line falls within the kern at all but two points, i.e. D and G. According to the analytic solution "G" is the only section at which the thrust falls outside the kern. This seems to show an error but upon inspection it will be seen that at D the thrust is very close to the lower edge of the kern. This indicates that at this section the upper fibre is in tension, whereas actually there is 34.5 pounds compression in the upper fibre. This discrepancy simply lies in the slight error in the graphival solution. On the whole, however, the thrust line checks the analytic solution very well, i.e. it lies close to the upper edge of the kern where the compression is large in the upper fibre, and close to the lower edge of the kern where the compression is large in the lower fibre.

This completes the analysis of the arch stresses.

Plate V shows the completed structure in elevation, and section.